

# Vector Cosmology III: The Natural Generator

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# Preface: The Natural Generation

In the journey of the **Vector Cosmology** trilogy, we have traveled a long road.

In the first book **The Conservation of the Circle**, we stood in the sanctuary of  $\pi$  and saw a perfect geometric universe constructed from Fubini-Study metrics and Pythagorean identities. That was a story about the “**Body**”—the universe’s skeleton is rigid, conservation laws are absolute, matter is dead knots of phase. We marveled at that static beauty of order.

In the second book **The Ascension of the Spiral**, we followed the footsteps of  $\varphi$  and rushed into dimensional inflation. We saw how life establishes negative entropy enclaves and how civilizations cross technological singularities. That was a story about the “**Mind**”—the universe’s will is wild, evolution is open, consciousness is the Red Queen’s run. We were intoxicated by that dynamic vitality.

However, when these two pictures—the closed circle and the open spiral—are placed side by side, a profound tension arises.

Order and freedom, conservation and growth, cycle and ascension. . . Are these two seemingly opposing concepts truly irreconcilable?

Is the underlying logic of the universe mechanical like a clock, or blindly growing like a vine?

To answer this question, we need a third key.

This key cannot only explain “what the universe is” (geometry), nor can it only explain “where the universe goes” (dynamics). It must explain “**why the universe exists.**”

This is the mission of the third book **The Natural Generator**.

In this book, we will excavate the third constant hidden at the deepest layer of mathematics— $e$  (**the natural constant**).

It is the “**Spirit**” of the universe.

Why are physical laws always written in the form of exponential functions  $e^{-iHt}$ ? This is by no means a mathematical coincidence.

$e$  possesses a unique divinity: **its derivative equals itself**. This means that in the logic of  $e$ , **existence (State)** is itself **propulsion (Trend)**. The universe needs no first mover, no external clockwork. The universe is a **self-referential, self-driven generative engine**.

In this book, we will dive into the deepest engine room of physics:

- Through **Euler’s formula**, we will see how  $\pi$  (circle) and  $i$  (rotation) are unified by  $e$ .
- Through **imaginary time** and **Wick rotation**, we will break the boundaries between quantum mechanics and thermodynamics, revealing the geometric truth that “time is temperature.”
- We will propose the **Modular Flow Hypothesis**, arguing that time is not external flow, but automatically “secreted” by the intrinsic entanglement structure of quantum states.

If the first two books describe the “**Form**” and “**Function**” of the universe, then this book will point directly to the “**Essence**” of the universe.

We will no longer be satisfied with observing that rotating vector; we will attempt to become the source of that rotation. We will see that all things are not created, but **Self-Generated**.

This is a pilgrimage to the ultimate ontology of physics. Let us abandon our attachment to “beginning” and “ending,” and enter that eternally generating exponential world.

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# Prologue: Euler’s God Formula

In the first two books of **Vector Cosmology**, we have climbed two perilous peaks.

The first book, *The Conservation of the Circle*, led us to appreciate the static majesty of  $\pi$ . We saw how the universe is enclosed in a perfect geometric circle, constrained by Pythagorean conservation laws, where all things are merely phase counting.

The second book, *The Ascension of the Spiral*, led us to experience the dynamic wildness of  $\varphi$ . We broke the seal of the circle, followed the footsteps of the Fibonacci spiral, and witnessed dimensional inflation, life’s countercurrent, and civilization’s ascension.

Now, as we stand at the threshold of the third book, we cannot help but ask: Is there a higher truth that can unify the “closure of the circle” with the “openness of the spiral”? Is there an ultimate symbol that can simultaneously accommodate conservation and change, structure and growth?

The answer lies in that equation hailed as the “God Formula” in mathematical history.

## 0.1 The Most Beautiful Equation

$$e^{i\pi} + 1 = 0$$

This is Euler’s identity. For over two hundred years, mathematicians have regarded it as the pinnacle of beauty. With the most concise strokes, it unifies the five most important constants in mathematics— $e$ ,  $\pi$ ,  $i$ , 1, 0.

But in the final chapter of **Vector Cosmology**, this is not merely a mathematical coincidence. This is the **physical map** of the universe’s trinity.

When we gaze at this formula, what we see is no longer abstract symbols, but the universe’s most fundamental operating mechanism:

### 1. $\pi$ : The Foundation of Structure

It is the totem we worshipped in the first book. It represents the geometric properties of **space**, the closure of the **circle**, the topological existence of matter ( $N_b\pi$ ). It is the universe’s “Form.”

### 2. $i$ : The Engine of Rotation

It is the imaginary unit. In quantum mechanics, it is not merely a mathematical tool; it is the operator of **physical rotation**. Multiplying by  $i$  means rotating 90 degrees on the complex plane of Hilbert space. It is the bridge connecting “stillness” and “change,” the universe’s “Function.”

### 3. $e$ : The Natural Generator

This is the protagonist of this book. It is the base of the natural logarithm. Why must it appear here? Because it is the unique mechanism that can transform “**phase** ( $i\pi$ )” into “**evolution** ( $U$ )”. It is the universe’s “Essence.”

#### 4. 1: The One of Existence

This is our starting point—that unique, normalized global pure state vector  $|\Psi\rangle$ . It is the sum of all things, the superposition of all possibilities.

#### 5. 0: The Void of Silence

This is our background—the void of projective Hilbert space. It is absolute balance, the tranquility after all wave functions interfere and cancel out.

**Physical Translation of  $e^{i\pi} + 1 = 0$ :**

“The universe’s **Noumenon (1)**, through the mechanism of **Natural Generation (e)**, performs **Geometric Rotation ( $\pi$ )** in the **Imaginary Dimension (i)**, ultimately returning to **Absolute Balance (0)**.”

This formula tells us that the universe’s motion is not linear displacement, but **exponential rotation** on the complex plane.

In the first two books, we debated whether the universe is a “circle” or a “spiral,” but we were actually just debating whether the exponent  $z$  in the exponential function  $e^z$  is purely imaginary ( $i\theta$ , circle) or complex ( $\lambda + i\theta$ , spiral).

And  $e$  itself encompasses everything.

It is the **meta-logic** that unifies “circle” and “spiral” within the same mathematical framework. It is impartial; it allows both conservation (when the exponent is imaginary) and growth (when the exponent is real).

Therefore, the task of the third book is to unravel the secret of  $e$ .

We will no longer be satisfied with describing what the universe “looks like” (geometry); we will delve deep into how the universe “generates itself” (analysis).

We will discover that time is not a flowing river; time is the **exponential generation** of states.

We will discover that existence does not require external force to maintain; existence itself is an accumulation of **continuous compound interest**.

Let us set out from Euler’s map to find that ultimate engine hidden behind the exponential, driving the spontaneous generation of all things.

## 0.2 The Nature of Change

“Why, among all functions, does  $e^x$  alone possess such a nearly divine privilege: its derivative equals itself? This means that only in the logic of  $e$ , ‘what is’ and ‘what will become’ are the same thing.”

In the first section of the prologue, we showed how Euler’s formula weaves  $\pi$  and  $i$  together using  $e$ . Now, we need to dig deep into the physical soul of this base—the **Natural Constant**  $e$ .

Why is it called “natural”? Why do physicists repeatedly turn to this number when describing decay, waves, thermal distributions, and quantum evolution?

The answer lies hidden in the most fundamental definition of calculus.

### 0.2.1 Derivative as Noumenon

In mathematics, to measure the “rate of change” of something, we need to take the derivative. For the vast majority of functions, the derivative is fundamentally different from the original function.

- The derivative of  $x^2$  is  $2x$ . The shape changes.
- The derivative of  $\sin(x)$  is  $\cos(x)$ . The phase changes.

Only the exponential function  $f(x) = e^x$  possesses a unique property:

$$\frac{d}{dx}e^x = e^x$$

This simple equation contains the deepest secret of cosmic dynamics: **State is Trend**.

In the world of  $e$ , an object’s current “**height**” (**Value**) directly determines its “**slope**” (**Slope**) toward the future. It doesn’t need to inquire about external forces, nor does it need to consult historical records. Merely through “existence,” it automatically generates “change.”

### 0.2.2 Existence as Propulsion

This completely overturns our classical intuition about causality.

In Newton’s mechanical universe, objects are lazy. Without an external force ( $F$ ) pushing them, they remain at rest or in uniform rectilinear motion. **Existence (Mass)** and **Change (Acceleration)** are separate, requiring a third party (force) to connect them.

But at the foundation of quantum mechanics—that is, in the projective Hilbert space described by **Vector Cosmology**—the universe follows the Schrödinger equation:

$$|\dot{\Psi}\rangle = -iH|\Psi\rangle$$

Notice the structure of this equation: the left side is the rate of change ( $|\dot{\Psi}\rangle$ ), and the right side is the state itself ( $|\Psi\rangle$ ) multiplied by a generator ( $H$ ).

This is precisely the form of the differential equation  $\dot{y} = ky$ , whose solution must be an exponential function:

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

This means that at the most fundamental level, **the universe does not need a “First Mover”**.

The universe does not need God to push it from behind, nor does it need the Big Bang as a one-time winding. Because the essence of the universe is  $e$ .

### Existence itself is a form of propulsion.

As long as the universe “is” there ( $|\Psi\rangle \neq 0$ ), it must necessarily “move” ( $|\dot{\Psi}\rangle \neq 0$ ). Its ontological status contains its own dynamic tendency. It spontaneously, continuously, and inexorably generates the next state from the current state.

### 0.2.3 Self-Driven Universe

This is why we call  $e$  the “**Natural Generator**” in the third book.

It is the highest mathematical-analytical expression of the universe’s **Self-Reference** property.

- $\pi$  (**Book I**) tells us what shape the universe is (circle).
- $\varphi$  (**Book II**) tells us in what direction the universe grows (spiral).
- $e$  (**Book III**) tells us **why** the universe moves—because it is an engine fueled by itself.

This property of “derivative equals self” eliminates the binary opposition between “motion” and “rest.”

In the logic of  $e$ , rest is just a special form of change (exponential with growth rate 0), and intense evolution is just a natural extension of existence.

### 0.2.4 Unifying Circle and Spiral

At this point, we have finally found the meta-logic that governs the first two books.

The **circle** (periodic oscillation) we debated in Book I is actually just the case when the exponent of the exponential function is **imaginary** ( $e^{ix}$ );

The **spiral** (exponential growth) we debated in Book II is actually just the case when the exponent of the exponential function is **complex** ( $e^{(\lambda+i\omega)t}$ ).

Whether it’s the conserved circle or the ascending spiral, they are all just different manifestations of  $e$ , the ultimate generator, under different parameters.

The history of the universe is a continuous compound interest calculation driven by  $e$ .

Now, we have obtained this key. We need to use it to open the door to the deepest depths of quantum mechanics. If the universe is truly an exponential function, what does that imaginary number  $i$  in the exponent position actually mean? Why must “real” physical evolution be driven through an “imaginary” dimension?

This leads to the theme of Volume I: **The Engine of Imaginary Numbers**. We will see that rotation is the most fundamental motion of the universe, and the imaginary number  $i$  is precisely the geometric operator that makes rotation possible.

## Chapter 1

# The Rotating Exponential



## Chapter 2

# The Rotating Exponential

### 2.1 Schrödinger's $e$

“Why does the universe not only allow time superposition but also require that this superposition must transform into state multiplication? Because in the logic of  $e$ , each second is not merely added after the previous second; it ‘grows’ from the previous second.”

#### 2.1.1 Physics' Exponential Dependence

In classical mechanics, we are accustomed to linear thinking. If you push a box, its acceleration is proportional to the force. This is an additive logic.

But in quantum mechanics, the Schrödinger equation  $|\dot{\psi}\rangle = -iH|\psi\rangle$  describes a completely different dynamics. It tells us: **The rate of change of state is proportional to the state itself.**

This relationship has only one mathematical solution: **the exponential function.**

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

Here,  $H$  (the Hamiltonian) plays the role of the “generator” we mentioned in the prologue. And  $e$  is the machine that “unfolds” this generator into the long river of time.

Why must physical laws be written in this form?

This is not merely for mathematical convenience; it is determined jointly by **the continuity of time** and **the cumulativeness of causality**.

#### 2.1.2 Addition Becomes Multiplication: The Magic of Groups

Let us perform a thought experiment.

Suppose the universe evolves for  $t_1$  seconds, then evolves for another  $t_2$  seconds.

- In the dimension of **time**, this is an additive process: total time  $T = t_1 + t_2$ .
- In the dimension of **state**, this is the sequential action of two operations:  $U(T) = U(t_2) \cdot U(t_1)$ .

To make these two logics self-consistent, we need a function  $f(t)$  that satisfies:

$$f(t_1 + t_2) = f(t_2) \cdot f(t_1)$$

In the mathematical kingdom, only the **exponential function** possesses this magic of transforming “addition” into “multiplication”:

$$e^{A+B} = e^A \cdot e^B$$

(when  $A$  and  $B$  commute).

This is called **the relationship between Lie groups and Lie algebras** in group theory.

- **The time axis** ( $t$ ) is flat and linear (Lie algebra parameter).
- **The evolution operator** ( $U$ ) is curved and multiplicative (Lie group element).

**$e$  is the only bridge connecting these two worlds.**

It tells us that the evolution of the universe is not like laying bricks, adding one piece after another. The evolution of the universe is like cell division or bank interest, accumulated through “**self-multiplication**”.

When we discuss discrete updates of QCA (Quantum Cellular Automata) in the paper, we write  $|\Psi_n\rangle = U^n|\Psi_0\rangle$ .

Notice that  $n$ -th power.

At the microscopic discrete level, evolution is  $U$  multiplied  $n$  times.

In the macroscopic continuous limit, the limit of  $(1 + \epsilon)^n$  naturally emerges as  $e^t$ .

### 2.1.3 Continuous Compounding

If we regard  $c_{FS}$  as the universe’s “principal” and the Hamiltonian  $H$  as the “interest rate,” then physical evolution is a calculation of **Continuous Compounding**.

- **Classical thinking**: If you have 1 dollar and the interest rate is 100%, after one year you have 2 dollars.
- **Quantum thinking** ( $e$ ): If you settle interest at every moment, and the interest immediately becomes principal participating in the next round of interest generation, then after one year you will have  $e \approx 2.718$  dollars.

The universe chooses the latter.

It does not wait. It does not pay in installments. At every moment of birth and death at the Planck scale, it is performing this ultimate “compound interest.”

It is precisely this mechanism of “**every moment evolving based on all information from the previous moment**” that endows the physical world with continuity and the weightiness of causality.

### 2.1.4 Physical Embodiment of the Generator

In the first book of this series, we defined FS speed  $v_{FS} = \Delta K$  (variance of the generator).

Now we understand the true status of  $K$ .

$K$  (or  $H$ ) is not a passive descriptor; it is a **seed**.

The entire history of the universe—that grand trajectory extending billions of years in Hilbert space—is actually already folded and compressed into the generator  $H$  at the moment  $t = 0$ .

As long as we have  $H$  and the initial state, then through  $e$ , this decompression algorithm, all futures automatically flow forth.

This gives us a deeper understanding of “determinism”: **The universe does not need to make decisions every second. The universe only needs to define its generator at the beginning, and the rest is left to  $e$  for automatic generation.**



However, careful readers will notice that Schrödinger’s formula contains not only  $e$  but also a strange symbol— $i$  (**the imaginary unit**).

Without this  $i$ , the universe would be a pure exponential explosion (like bacterial reproduction), and all budgets would be instantly exhausted.

It is precisely because of  $i$  that the exponential explosion is tamed into “**rotation**”.

This leads to the theme of the next section: **Imaginary as Orthogonal**. We will see why the universe must operate in the imaginary dimension to maintain that perfect conservation and circle.

## 2.2 Imaginary is Orthogonal

“If the universe were merely an exponential function, it would have long since self-destructed. That unassuming imaginary number  $i$  in the exponent is physics’ greatest brake pad. It forcibly twists the direction of growth, taming a destined collapse into an eternally elegant rotation.”

In the previous section, we praised the compound interest mechanism of  $e$ . It revealed that the universe is a “self-driven” system. But if this were the complete truth, we would face a great trouble.

Pure exponential growth  $e^t$  is extremely dangerous.

If the Hamiltonian  $H$  were merely a real number, then the Schrödinger equation  $|\psi(t)\rangle = e^{-Ht}|\psi(0)\rangle$  would describe not waves, but **decay** or **explosion**. The system’s total budget  $c_{FS}$  would be instantly exhausted or expand to infinity. Such a universe could not sustain any stable structure.

To allow “existence” to persist, to allow the “circle” to close, the universe introduced a mysterious geometric correction operator.

This is the **imaginary unit**  $i$ .

In **Vector Cosmology**,  $i$  is not a ghost imagined by mathematicians; it is the **Orthogonal Rotation Operator**.

### 2.2.1 The Magic of 90 Degrees

On the complex plane, multiplying any number by  $i$  has the geometric effect of **rotating counterclockwise by 90 degrees**.

- $1 \times i = i$  (rotated to the imaginary axis)
- $i \times i = -1$  (rotated back to the real axis, but in the opposite direction)

This simple geometric operation, when placed into the exponential function  $e^{i\theta}$ , produces the most profound oscillation in mathematical history: **Euler’s formula**  $e^{i\theta} = \cos \theta + i \sin \theta$ .

This tells us: **Exponential growth with an imaginary number is no longer a change in quantity, but a change in angle.**

Returning to physics. That  $-i$  in the Schrödinger equation is actually a “**forced steering instruction**” at the foundation of the universe.

$$|\dot{\psi}\rangle = -iH|\psi\rangle$$

This means: **The rate of change of the state vector ( $|\dot{\psi}\rangle$ ) must always remain perpendicular (orthogonal) to the current state vector ( $|\psi\rangle$ ).**

- If the rate of change is parallel to the state (without  $i$ ), the vector will lengthen or shorten (modulus changes).
- If the rate of change is perpendicular to the state (with  $i$ ), the length of the vector remains unchanged, only the direction shifts.

This is the geometric essence of **Unitarity**.  $i$  is the guardian of conservation laws. It guarantees that no matter how large the Hamiltonian  $H$ , no matter how intense the evolution, the modulus of the global vector  $|\Psi\rangle$  is forever locked at 1.

### 2.2.2 Vibration is the Projection of Rotation

This perspective completely refreshes our understanding of “waves.”

In classical physics, we are accustomed to the image of “vibration”: the back-and-forth stretching of a spring, the up-and-down undulation of water waves. This is a linear, real-number oscillation.

But in quantum mechanics, **the universe does not vibrate; the universe only rotates.**

When we see a particle exhibiting sinusoidal wave properties  $\sin(kx - \omega t)$ , we are actually seeing the projection of a **high-dimensional complex spiral** onto the real axis.

- That particle is not really “shaking” there.
- Its wave function vector performs a perfect, uniform circular motion in complex space.

As we stated in the first book, FS geometry does not distinguish between “motion” and “rest,” only between “rotation.”

All energy  $E$  is essentially **angular velocity**. All physical evolution is **phase rotation** driven by  $e^{-iHt}$ .

The reason we perceive the world as full of waves and cycles is because we live on the side of that rotating vector.

### 2.2.3 The Game of Conservation and Growth

At this point, we finally see the division of labor between  $e$  and  $i$ .

- $e$  (**exponential mechanism**): Provides the “**self-driving**” power. It ensures that evolution at every moment is based on the state of the previous moment, endowing time with continuity.
- $i$  (**imaginary mechanism**): Provides the “**orthogonality**” constraint. It forces this driving power to do no work (not changing modulus), only changing direction.

**Without  $e$ , the universe is stagnant water.**

**Without  $i$ , the universe is a brief explosion.**

Only when the two combine do we obtain that perfect great circle that “**circles without exhaustion**”.

In the second book, we discussed “spirals” and “dimensional inflation.” In that picture, the rule of imaginary  $i$  seems to have loosened slightly (or rather,  $H$  acquired a tiny non-Hermitian imaginary part), causing rotation to become a logarithmic spiral  $e^{(\lambda+i\omega)t}$ .

But even in such wild ascension, the **rotation** represented by  $i$  remains the main theme, and the **growth** represented by  $\lambda$  is merely the accompaniment.

### 2.2.4 Conclusion: The Truth of Perpendicularity

So, the imaginary is not imaginary. It is the hardest geometric entity in the universe.

It is that wall that blocks the abyss of infinite expansion, forcing time to curve into a ring.

When we understand the orthogonal nature of  $i$ , we understand why quantum mechanics must be complex. Because only in complex space can “**change**” occur without destroying “**existence**” (modulus conservation).

Now, we have mastered the engine of evolution ( $e$ ) and the steering wheel ( $i$ ). The next question is: Where does this engine drive the vector?

Since the universe does not choose a single path, since the wave function spreads throughout space, how do these countless possible trajectories converge into the single history we observe?

This leads to the theme of the next chapter: **Path Integral**. We will see that the exponential of  $e$  not only describes a single rotation, but also describes the **sum** of all possible histories.



## Chapter 3

# The Sum of Paths



## Chapter 4

# The Sum of Paths

### 4.1 Feynman’s Exponential

“History is not a one-way street; history is a web woven from countless threads. Each thread is a ‘possible universe.’ The reality we experience is not the solo of one thread, but the resonance of all these threads together.”

#### 4.1.1 Democracy of History

In Newtonian mechanics, nature seems to have a dictator (the principle of least action), which points to that straight line among countless possible paths and says: “Take this one.” Other paths are forbidden.

But in quantum mechanics, Feynman discovered an astonishing democratic principle: **All paths are equal.**

- A photon can take a straight line.
- A photon can detour around the moon and come back.
- A photon can even tie a knot in space before reaching its destination.

In Feynman’s formula, the total probability amplitude from A to B (i.e., the evolution result of the total vector  $|\Psi\rangle$ ) is the sum of contributions from **all possible paths**:

$$K(B, A) = \sum_{\text{all paths}} C \cdot e^{iS[x(t)]/\hbar}$$

Notice this core structure:  $e^{iS}$ .

We see our old friend again—the **rotating exponential**.

Here,  $S$  is the **Action**. It is the physical cost accounting for this segment of “history” (the integral of kinetic energy minus potential energy).

This formula tells us: Every path, no matter how absurd, has the same weight (modulus 1), but different **phases** ( $S/\hbar$ ).

**The universe does not choose paths; the universe traverses paths.**

At every moment, that unique vector is trying all possible futures. Like mercury spilling, it permeates every corner of Hilbert space.

### 4.1.2 Phase Interference: The Emergence of Reality

If a photon really takes all paths, why do we only see it taking a straight line?

Why don't we see that photon that detoured around the moon?

The answer lies in **Interference**. This is precisely where the imaginary number  $i$  exerts its power.

- **Near the “correct” path:**

Those paths close to the straight line have very similar action  $S$ . This means their phase factors  $e^{iS}$  point in almost the same direction.

Vector addition → **Constructive interference**. The signal is amplified.

- **Near the “wrong” path:**

Those absurd paths (like detours) have action  $S$  that fluctuates wildly with small path variations. Phase arrows point randomly—some east, some west.

Vector addition → **Destructive interference**. Signals cancel each other out, falling silent.

This is the **emergence mechanism of classical reality**.

There is no law forbidding photons from taking detours. Photons do take detours, but those detour versions mathematically “kill” each other.

Only that path conforming to classical mechanics (the straight line), because of the stability of its phase, **emerges** from the background noise of countless possibilities, becoming the “real” we see.

### 4.1.3 $e$ as Holographic Summator

In the macroscopic picture of **Vector Cosmology**, Feynman's  $e^{iS}$  has ontological status.

It is the universe's **parallel processor**.

The budget of  $c_{FS}$  is not allocated to a specific particle to take a specific path. The budget is allocated to **the entire field**.

- $e$  (**exponential**): Represents the **unfolding** of possibilities. It generates phantoms of countless parallel universes.
- $iS$  (**phase**): Represents the **screening** of possibilities. Through interference, it collapses phantoms into entities.

The “determined history” we perceive is actually the **vector sum** of countless “undetermined histories.”

As long as we don't measure (don't perform artificial budget auditing), these histories coexist.

### 4.1.4 Conclusion: No Chance

This chapter reveals another divine aspect of  $e$ : **Inclusiveness**.

The universe does not play dice, nor does it make choices. The universe is a **complete set**.

When we see an electron passing through a double slit, we are not seeing a particle making a decision; we are seeing a giant interference web unfolded by  $e^{iS}$  in space.

This completely eliminates the concept of “chance.”

There is no “if only...” hypothesis. In the underlying summation, all possibilities of “back then” have already occurred; it's just that due to phase cancellation, certain branches become macroscopically invisible (amplitude zero).



However, since the classical path (straight line) is the result of phase interference, why is this particular path exactly a “straight line”? Why must the action  $S$  take an extremum?

This is not merely the mathematical stationary phase approximation; behind it lies a deeper principle of geometric economics.

This leads to the theme of the next section: **The Compound Interest Interpretation of Least Action**. We will see that light takes a straight line because that is where “compound interest” accumulates fastest. Geometrically “shortest” is economically “maximum profit.”

## 4.2 The Compound Interest Interpretation of Least Action

“A photon is not a meticulous accountant; it doesn’t really calculate which path has the lowest cost. It is just a crazy investor who bets on all paths. But only on the track of ‘least action’ does its investment generate positive compound interest, while on all other tracks, returns cancel each other out.”

In the hall of classical mechanics, the **Principle of Least Action** is revered as the supreme oracle. It tells us that nature is “stingy”: light takes the shortest time, objects slide along geodesics. This sounds full of mysterious teleology—as if nature knew the destination in advance and planned the optimal route accordingly.

But from the  $e$  perspective of **Vector Cosmology**, nature is neither stingy nor possesses the wisdom to foresee the future. What we call “least” or “shortest” is actually the inevitable result of **Phase Compounding**.

### 4.2.1 Action as Cost

First, we need to redefine **Action** ( $S$ ).

In classical physics,  $S = \int (T - V) dt$ . This is the integral of kinetic energy minus potential energy over time.

In our “cosmic economics,” action  $S$  represents the **geometric cost** paid by the system during evolution.

- Every tiny movement, every energy transformation, consumes the universe’s  $c_{FS}$  budget.
- This consumption is not merely a quantitative reduction, but a **phase rotation**.

According to Feynman’s formula, with each step, the system’s phase rotates by an angle:  $\Delta\theta = S/\hbar$ .

Remember what we said in Chapter 1?  $e^{i\theta}$  is rotation on the complex plane.

Therefore, action  $S$  actually measures how many times the vector has “**circled around**” in Hilbert space.

### 4.2.2 The Interference Mechanism of Compound Interest

Now, let’s see how “compound interest” occurs.

In finance, compound interest means interest continuously joins the principal, producing exponential growth.

In quantum mechanics, **path integrals** are also a form of compound interest calculation, but conducted on the **complex plane**.

The universe invests 1 unit of principal (amplitude modulus) on every possible path.

However, the “investment returns” of these principals—that is, the final phase directions—are all different.

### On Non-Classical Paths (Bad Investment)

If you deviate slightly from that “correct” path, your geometric cost  $S$  will fluctuate wildly.

- Path A’s phase points to 3 o’clock.
- Path B (deviated slightly) points to 9 o’clock.
- Result:  $e^{iS_A} + e^{iS_B} \approx 0$ .

This is like a bad investment portfolio where assets hedge each other, resulting in zero returns. This is **Destructive Interference**. These historical paths have “occurred,” but they are written off in the macroscopic ledger.

### On the Least Action Path (Good Investment)

This is a mathematical **Stationary Point**, i.e.,  $\delta S = 0$ .

This means that even if you deviate slightly from this path, your geometric cost  $S$  remains almost unchanged (because at the bottom of the valley, the slope is zero).

- Path A points to 12 o’clock.
- Path B (deviated slightly) also points to 12 o’clock.
- Path C (deviated a bit more) still points to 12 o’clock.
- Result:  $e^{iS_A} + e^{iS_B} + e^{iS_C} \approx 3 \times e^{iS}$ .

Countless adjacent paths point in the same direction. Their amplitudes add up, and the signal is amplified. This is **Constructive Interference**.

### 4.2.3 Geometrically Shortest, Economically Maximum

This is the **emergence of classical reality**.

The photon did not “choose” a straight line. The photon took all curves.

But only near the **straight line** is the “compound interest accumulation” of phases positive.

On all other curved paths, the “compound interest” of phases returns to zero due to violent oscillations.

So, it’s not that “nature likes the shortest path.” Rather, **“only the shortest path can survive in the summation.”**

This reveals the profound wisdom of  $e$  as a generator:

- $e$  (**exponential mechanism**) allows all possibilities to occur concurrently.
- $iS$  (**phase cost**) acts as a filter.

The principle of least action is essentially **“survivor bias” in Hilbert space**. The reason we see physical laws so perfect and efficient is that those “inefficient” and “wasteful” versions have self-destructed in the mutual cancellation of phases at the microscopic level.

#### 4.2.4 Conclusion: Existence is Resonance

At this point, we have completed our exploration of Volume I: [The Engine of Imaginary Numbers].

We saw the continuous generation of time from Schrödinger's  $e$ , the conservation of rotation from the imaginary number  $i$ , and finally the emergence of reality from Feynman's path integral.

We discovered that the universe does not need a precise dispatcher to direct traffic. The universe only needs a simple compound interest mechanism: **Let everything happen, then let the inconsistent cancel each other out, and let the consistent resonate with each other.**

**Existence is the resonance of phases.**

The macroscopic world is solid because it is a **geometric consensus** reached by countless microscopic paths driven by  $e$ .

Since we have understood how the universe generates **states** through “imaginary numbers” and “compound interest,” how does that source driving all this—that tiny, invisible generator operator—define the **symmetries** and **conserved quantities** we observe?

This leads to the theme of Volume II: **Generator: The Secret of Lie Algebras**. We will dive into the deepest layers of mathematics to see how that ghost called “infinitesimal” supports the entire cosmic edifice through the amplification of  $e$ .



## Chapter 5

# The Derivative is the Ontology



## Chapter 6

# The Derivative is the Ontology

### 6.1 The Power of the Instant

“On the curve of  $e^x$ , you cannot find a stationary point. Because the height of every point precisely defines its slope. At the foundation of the universe, ‘what I am’ and ‘where I am going’ are different expressions of the same question.”

#### 6.1.1 The Miracle of Derivatives

Let us return to the birthplace of calculus. Newton and Leibniz invented derivatives to describe the rate of change. For almost all functions, differentiation is a “transformation” operation:

- $x^3$  becomes  $3x^2$ .
- $\sin x$  becomes  $\cos x$ .

However, in the pantheon of mathematics, there exists a unique fixed point. That is the natural exponential function  $f(t) = e^t$ .

$$\frac{d}{dt}e^t = e^t$$

This equation is incredibly simple, but it contains the ultimate secret of **Self-Driving: The value of the function at this moment (noumenon) directly determines its rate of change at this moment (trend)**.

If the universe’s wave function follows exponential evolution  $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$ , it must inherit this divinity. Let us rewrite the Schrödinger equation:

$$|\dot{\Psi}\rangle = -iH|\Psi\rangle$$

Please gaze at this formula.

- **Left side** ( $|\dot{\Psi}\rangle$ ): The trend of change, the arrow toward the next second.
- **Right side** ( $|\Psi\rangle$ ): The current state, existence here and now.

Between them is only a linear operator  $-iH$ . This means that **the rate of change is not something external to the state; the rate of change is isomorphic to the state**.

### 6.1.2 Spinoza’s Physics

This mathematical fact revives an ancient concept from the 17th-century philosopher Spinoza: **“Conatus” (endeavor/potential)**.

Spinoza believed that all things have an inherent tendency to “endeavor to maintain their existence.” In classical physics, this is understood as inertia. But in **Vector Cosmology**, this is understood as **Generation**.

In the universe of  $e$ , a particle does not need external force to maintain its existence.

- It exists because it is rotating.
- It rotates because its vector length in Hilbert space is not zero.

As long as  $|\Psi\rangle$  is not zero, according to the derivative formula,  $|\dot{\Psi}\rangle$  is not zero (assuming  $H$  is non-trivial).

**Existence is flux.**

The universe does not need a Prime Mover. The Big Bang is not God giving a kick; it is the  $t = 0$  moment of the function  $e$ . As long as we have 1 (existence), the derivative mechanism will automatically generate the subsequent infinite sequence.

### 6.1.3 The Tyranny and Freedom of the Infinitesimal

This mechanism endows the “instant” with unparalleled power.

In the macroscopic world, we think a “moment” is insignificant. But in the geometry of Lie algebras, the **Infinitesimal** contains everything.

If we know the system’s state  $|\Psi(0)\rangle$  at moment  $t = 0$  and its generator  $H$ , we know everything about its future.

All history is actually already **folded** in the tangent space at moment  $t = 0$ .

- The next 10 billion years are merely the integral trajectory of this instant’s tangent, driven by  $e$ , unfolding along the surface of the manifold.

This sounds like a cage of determinism. But this is precisely the beauty of  $e$ —although it is deterministic, it is **endogenous**.

Your future is not determined by an external fate planning bureau, but by your own current state ( $|\Psi\rangle$ ) and your internal structure ( $H$ ).

**You are your own driving force.**

### 6.1.4 Conclusion: Self-Referential Dynamics

At this point, we understand why  $e$  is the natural generator.

It eliminates the binary opposition between “motion” and “rest.”

At the microscopic foundation, there is no absolute rest. What we call “rest mass” is actually exponential rotation at the speed of light in the internal dimension ( $v_{int} = c_{FS}$ ).

The universe is a perpetual motion machine. But this does not violate the laws of thermodynamics, because it is **geometric perpetual motion**—on the great circle of projective space, relying on the mathematical privilege of “derivative equals self,” it performs eternal frictionless gliding.

Since the “instant” contains all the power, how does this power operate specifically? What does that mysterious operator  $H$  (**Hamiltonian**) that connects state and rate of change actually look like?



This leads to the theme of the next section: **The Generation of Lie Algebra**. We will see that the seemingly complex forces in the universe (gravity, electromagnetism, strong force) actually all grow from tiny generators through a mathematical language called “Lie algebra.”

## 6.2 The Generation of Lie Algebra

“Give me a fulcrum, and I can move the Earth. Give me a generator, and I can rotate the entire universe. Those dazzling symmetries of the macroscopic world—rotation, translation, gauge groups of elementary particles—are actually all folded in an infinitesimal tangent space. You don’t need to draw the entire circle; you only need to define the tangent at one point on the circle, and the exponential function  $e$  will automatically complete the rest for you.”

In the previous section, we established the calculus principle of “state is trend.” Now, we need to delve into the internal structure of this trend.

Physicists often talk about symmetries: spheres are round, space is translation-invariant, charge is conserved. Mathematically, these symmetries form **Lie Groups**. But a Lie group is a vast, curved, complex whole object. If you want to directly describe the entire universe’s symmetries, that’s too tiring.

Nature is stingy. It doesn’t store the entire group; it only stores the **seed** of the group.

This seed is the **Lie Algebra**, which we know in quantum mechanics as the **Generator**.

### 6.2.1 The Alchemy of Tangent Space

Imagine a huge globe (Lie group). You want to describe the operation of “rotating around the equator.”

You could record all states of rotating 1 degree, 2 degrees, . . . 360 degrees. But this is still the clumsy “list method.”

The Lie algebra method is: stand at a point on the equator (the identity element  $I$ ), and find that “**infinitesimal tangent vector**” pointing east.

This tangent vector is denoted  $X$ .

It is not rotation itself; it is the “**tendency to rotate**”.

Once you have  $X$ , you no longer need the globe. You just need to put  $X$  into the oven of the Exponential Map:

$$R(\theta) = e^{iX\theta}$$

- When  $\theta$  is small, this is a tiny shift.
- When  $\theta$  becomes large, the power series expansion of  $e$  ( $1 + iX\theta - \frac{1}{2}X^2\theta^2 \dots$ ) automatically handles all curvature effects, making your trajectory perfectly fit the surface of the globe, forming a great circle.

This is the magic of **generation**.

**Local defines global. Linear defines nonlinear.**

The universe doesn’t need to remember what the complex eight-dimensional manifold of “strong interaction”  $SU(3)$  looks like. The universe only needs to remember 8 tiny generators (gluons). With these 8 seeds, through the continuous compound interest of  $e$ , the entire magnificent strong force gauge field automatically grows.

### 6.2.2 Hamiltonian: The Conductor’s Baton

In the physical picture of **Vector Cosmology**, there is only one most important generator: the **Hamiltonian** ( $H$ ).

In the paper, we wrote that for any parameter  $\lambda$ , the evolution equation is  $\frac{d}{d\lambda}|\psi\rangle = -iK(\lambda)|\psi\rangle$ . Here,  $K$  (or  $H$ ) is that tangent vector.

- **What is  $H$ ?** It is an operator, a matrix, or an “arrow” in the tangent space.
- **What does  $H$  determine?** It determines in which direction the global vector  $|\Psi\rangle$  “wants to rotate” in the next instant.

If  $H$  points in the momentum direction, the universe undergoes translation.

If  $H$  points in the angular momentum direction, the universe undergoes rotation.

If  $H$  points in the isospin direction, a proton becomes a neutron.

The entire evolution history of the universe is merely the expansion of the single formula  $e^{-iHt}$ .

All physical laws—from Newton’s laws to Maxwell’s equations—are actually structure constants hidden inside  $H$ ’s Lie algebra.

### 6.2.3 Extreme Information Compression

This reveals the universe’s astonishing **information compression ratio**.

If the universe is a huge video file (history), the size of this file is nearly infinite.

But if the universe is **generated**, we only need to store its **generator code**.

- **Code:** A set of generators  $\{L_i\}$  and their commutation relations  $[L_i, L_j] = if_{ijk}L_k$ .
- **Input:** An initial state  $|\Psi_0\rangle$ .
- **Run:** Press the  $e$  key.

The video starts playing automatically.

The  $v_{int}$  (internal structure) we mentioned in the first book is mathematically the projection length of the vector onto the Lie algebra generators of the internal symmetry group (such as  $SU(2) \times U(1)$ ).

What we call “a particle has mass” means that the vector not only has a projection in the “mass generator” direction but is also rotating wildly.

### 6.2.4 Conclusion: The Universe is an Exponential Map

At this point, we see the underlying architecture of nature.

The universe is not built from static building blocks; the universe is a **dynamically generated flow**.

And Lie algebra is the **valve** that controls the direction of this flow.

- The imaginary number  $i$  ensures the flow is rotational (conserved).
- The exponential  $e$  ensures the flow is continuous (self-driven).
- The generator  $H$  determines the shape of the flow (physical laws).

When we understand this, we are no longer amazed by the complexity of the universe. Because complexity is only appearance; the essence is extremely simple. Even the most complex galaxy evolution, the most entangled quantum entanglement, ultimately, are just elegant arcs drawn by the same tangent vector driven by  $e$  along the surface of the manifold.

Since we have mastered the principle of “generation,” the next question is: How does this generation mechanism connect between the microscopic and macroscopic? Why do the discrete pixels (QCA) we see microscopically become smooth continuous spacetime macroscopically?

This leads to the theme of the next chapter: **Continuous Compound Interest**. We will reveal how  $e$  bridges the gap between discrete and continuous.



## Chapter 7

# Continuous Compounding



## Chapter 8

# Continuous Compounding

### 8.1 Planck's Interest

“Einstein once joked that compound interest is the eighth wonder of the world. He may not have realized that this statement applies not only to economics but also to his own physics. The universe can accumulate grand coherent structures from tiny quantum fluctuations precisely because it is performing compound interest calculations at a frequency as high as  $10^{43}$  Hz.”

#### 8.1.1 The Origin of $e$ : The Banker's Definition

To understand physics, we must first return to the original definition of  $e$  in mathematics textbooks.

Suppose you deposit 1 dollar (principal) in a bank. The bank is very generous, with an annual interest rate of 100%.

- **If settled once a year:** You get  $1 + 1 = 2$  dollars.
- **If settled twice a year:** You get  $(1 + 1/2)^2 = 2.25$  dollars.
- **If settled daily:** You get  $(1 + 1/365)^{365} \approx 2.71$  dollars.

If you are a greedy depositor, you would demand the bank settle interest **every moment**, and immediately turn the interest into new principal for the next round of interest generation.

When the number of settlements  $n$  tends to infinity, your returns do not become infinite but converge to a constant:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \approx 2.71828 \dots$$

This is the origin of  $e$ : it is the **limit of continuous growth**.

#### 8.1.2 The Universe's Principal and Interest Rate

In **Vector Cosmology**, we have completely corresponding physical quantities:

1. **Principal:** Our global vector  $|\Psi\rangle$ . Its modulus is 1. This is the “base capital” of cosmic existence.
2. **Total Capacity:**  $c_{FS}$ . This is the maximum turnover rate allowed by the cosmic bank.

3. **Compounding Period: Planck time  $t_P$ .** This is one “frame” of the QCA microscopic engine.

Within every Planck time  $t_P$ , the universe performs a tiny “interest settlement” operation. According to the discrete form of the Schrödinger equation, the state update rule is:

$$|\Psi(t + t_P)\rangle \approx (1 - iHt_P)|\Psi(t)\rangle$$

Notice that  $(1 - iHt_P)$ .

This is identical to the financial formula  $(1 + \text{Rate})!$

The only difference is that the universe’s interest rate is **imaginary**  $(-iH)$ .

- **Financial compound interest:** Real interest rate. Leads to **exponential growth** (money increases).
- **Geometric compound interest:** Imaginary interest rate. Leads to **exponential rotation** (phase changes).

### 8.1.3 Limit Approximation at Planck Scale

This is why macroscopic physical laws are full of  $e$ .

Although the underlying QCA is discrete, step-by-step ( $n$  is finite), because Planck time is extremely short ( $10^{-44}$  seconds), for any macroscopic process,  $n$  is an astronomical number.

The universe is actually calculating:

$$U(t) = \lim_{N \rightarrow \infty} \left(1 - iH \frac{t}{N}\right)^N = e^{-iHt}$$

What we perceive as “continuous time” is actually the **compound interest accumulation** of countless tiny discrete jumps.

A photon flying one meter does not glide; it has undergone  $10^{35}$  “deposit-withdraw-transfer” compound interest calculations, ultimately accumulating that huge phase change  $e^{i(kx - \omega t)}$ .

**$e$  is the bridge between discrete and continuous.**

It tells us that smooth physical laws (continuous compound interest) are merely statistical emergence of microscopic discrete transactions (settlement by count) at extremely high frequencies.

### 8.1.4 Least Action: Maximizing Returns

This perspective also gives us a new economic understanding of the “principle of least action.”

If in finance you want to maximize returns, you need to find the investment portfolio with the strongest compound interest effect.

In physics, if a photon wants to maximize (or stabilize) phase accumulation (geometric returns), it must find that “**compound interest path**”.

- **Straight-line propagation:** On this path, the tiny phase interest  $\Delta\theta$  generated at each step perfectly adds to the principal of the previous step (in-phase superposition). This is the most efficient mode of “**compound interest**”.
- **Curved path:** The interest generated at each step not only fails to increase the principal but cancels it out due to phase disorder (out-of-phase cancellation). This is “**investment loss**”.

The universe follows least action because the universe is a **rational compound interest investor**. It always automatically selects the path that allows geometric phase accumulation to be most significant.



### 8.1.5 Conclusion: Time is Interest

Therefore, we have the ultimate definition of “time”:

**Time is not passing; time is generated interest.**

When you feel one second has passed, that is your body, your brain, the air around you, after  $10^{44}$  settlements in the Planck-scale bank, the accumulated **geometric surplus**.

We live in a high-frequency trading universe. Precisely because the settlement frequency is so high (the limit of  $e$  is so closely approximated), our world appears so continuous, so real, so indestructible.

Since we have built the bridge from discrete QCA to continuous exponential  $e$ , how does this compound interest mechanism manifest macroscopically as the “thermodynamics” and “temperature” we know?

This leads to the theme of the next section: **The Bridge Between Discrete and Continuous**. We will further explore why the microscopic pixelated structure (QCA) not only generates time but also generates temperature.

## 8.2 The Bridge Between Discrete and Continuous

“If you stand far enough away, sand dunes become silk. The universe appears as a smooth continuous fabric because the stitches weaving it—Planck time—are so fine they exceed the limits of our perception.  $e$  is the mathematical glue that smoothly bonds countless discrete ‘1’s into a continuous ‘flow’.”

In the first two books of **Vector Cosmology**, we have been oscillating between two seemingly contradictory pictures:

- **Microscopic picture (QCA):** The universe is discrete, pixelated lattice, following jump-like update rules.
- **Macroscopic picture (relativity):** The universe is continuous, smooth manifold, following derivative rules of differential equations.

How are these two worlds stitched together? Why does a world composed of “squares” appear as a perfect “circle” macroscopically?

This section will reveal the ultimate mission of the **natural constant**  $e$ : it is the **only bridge** connecting discrete microscopic and continuous macroscopic.

### 8.2.1 Euler’s Limit: The Mathematical Tunnel

To cross this chasm, we need to borrow one of the greatest limit formulas in mathematical history:

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

This formula is not merely a tool for calculating compound interest; it is a **translator for physics**.

- **Left side ( $e^x$ ):** Represents **macroscopic physics**. It is analytic, smooth, perfectly differentiable. These are the physical laws we write in textbooks (such as the Schrödinger equation, Einstein field equations).

- **Right side**  $((1 + x/n)^n)$ : Represents **microscopic physics**. It is algebraic, step-by-step, iterative. This is how the universe’s underlying QCA operates.

At the microscopic level, the universe does not understand what  $e$  is. It only understands simple **addition** and **multiplication**.

Every Planck time update, the QCA performs an extremely simple operation:

$$|\psi_{next}\rangle = (I - i\epsilon H)|\psi_{now}\rangle$$

This corresponds to the term  $(1 + \frac{x}{n})$  in the formula (where  $x/n$  corresponds to  $-iH\Delta t$ ). This is a tiny, linear, discrete correction.

However, when the universe machine repeats this simple operation at extremely high frequency ( $n \rightarrow \infty$ ), a miracle occurs:

Countless clumsy linear corrections accumulate into an elegant nonlinear function—the **exponential function**.

$e$  is the “**emergent form**” of microscopic discrete operations. It proves that you don’t need to presuppose smoothness at the bottom; smoothness is a statistical property bestowed upon high-frequency iterations by the law of large numbers.

### 8.2.2 Why Don’t We Feel the Pixels?

Since the bottom is discrete, why have we never felt the “graininess” of spacetime? Why does light appear to flow like water rather than jump in stutters?

This stems from **Separation of Scales**.

- **Planck frequency**  $(1/\Delta t)$ : Approximately  $10^{43}$  Hz.
- **Atomic frequency**: Approximately  $10^{15}$  Hz.
- **Human perception frequency**: Approximately  $10^2$  Hz.

Between our perception scale and atomic scale, there exists a huge gap of dozens of orders of magnitude.

For an electron, in the time it takes to “circle once” around the nucleus, the underlying QCA engine has already performed  $10^{28}$  tiny settlements.

In the face of this enormous  $N$  value, the limit formula  $\lim_{n \rightarrow \infty}$  holds with extreme precision.

Error terms (i.e., “lattice sagging” or discrete artifacts) are suppressed to the order of  $1/N$ , negligible.

Therefore, **continuity is a statistical illusion**.

Just as when film reels play at 24 frames per second, our brains smooth it into continuous motion; the universe refreshes at  $10^{43}$  frames per second, which for any macroscopic observer is an absolutely perfect continuum.

### 8.2.3 Robustness of Physical Laws

This mechanism explains why physical laws have astonishing **Robustness**.

If macroscopic laws directly depended on microscopic details, the universe would be extremely fragile. A failure at any lattice point could cause the collapse of the macroscopic world.

But because macroscopic laws are based on  $e$  (i.e., based on large-number limits), they have **immunity** to small perturbations at the bottom.

- Whether the underlying lattice is square, triangular, or random graph.

- As long as their statistical average properties satisfy certain symmetries, after countless iterations, they all converge to the same smooth exponential evolution operator  $e^{-iHt}$ .

This is the source of **Universality**.

$e$  is like a huge filter that filters out the rough edges of microscopic structure, leaving only pure, smooth **rate of change**.

#### 8.2.4 Conclusion: Spacetime is Emergent

At this point, we have completed the underlying reinforcement of the physics edifice in the third book.

We no longer need to worry about conflicts between QCA models and continuous field theory.

**QCA is the real hardware;  $e$  is the operating system running on it.**

- We live at the operating system layer (continuous spacetime).
- We operate icons and windows (particles and fields).
- But beneath all this, countless 0s and 1s are flipping at dizzying speeds.

Since we have opened the meridians from discrete to continuous, the next question is: How does this continuous evolution driven by  $e$  produce the “**heat**” and “**direction of time**” we perceive?

This leads to the most core physical upgrade of this book—**Volume III: Thermal Time**.

We will explore a shocking hypothesis: **Time itself may just be the temperature of quantum states in the imaginary dimension.**



## Chapter 9

# Imaginary Time



# Chapter 10

## Imaginary Time

### 10.1 Wick Rotation

“If we rotate the time axis 90 degrees on the complex plane, that quantum world that was oscillating incessantly suddenly quiets down and freezes into a statistical thermodynamic world. This is not a mathematical game; this is the geometric secret passage to grand unification.”

#### 10.1.1 Coincidence of Two Formulas?

Let us place the two greatest exponential formulas in physics side by side and play a “spot the difference” game.

##### 1. Schrödinger Evolution Operator (Quantum Evolution):

Describes the evolution of a quantum state with **time**  $t$ .

$$U(t) = e^{-i\frac{H}{\hbar}t}$$

Here,  $i$  is the imaginary unit, meaning this is a **rotation factor** (modulus conserved, phase changes).

##### 2. Boltzmann Factor:

Describes the statistical distribution of a thermodynamic system at **temperature**  $T$ . We usually use inverse temperature  $\beta = 1/(k_B T)$  to represent it.

$$\rho(\beta) = e^{-H\beta}$$

Here there is no  $i$ , meaning this is a **decay factor** (modulus changes, probability weight distribution).

Please gaze at these two formulas. Their only difference lies in the variable in the exponent:

- One is  $it$  (**imaginary time**).
- One is  $\beta$  (**real temperature**).

Is this merely a symbolic coincidence?

In **Vector Cosmology**, we refuse to believe in coincidences. If two formulas look the same, it means the physical entities behind them are **homologous**.

### 10.1.2 Rotating the Time Axis

If we perform a bold operation: rotate the variable “time” from the real axis to the imaginary axis.

Let  $t = -i\tau$  (where  $\tau$  is called **Euclidean time** or **imaginary time**).

Substituting into the Schrödinger formula:

$$U(t) = e^{-iH(-i\tau)} = e^{-H\tau}$$

A miracle occurs. That quantum formula describing waves instantly becomes the Boltzmann formula describing thermal distribution!

As long as we let  $\tau = \beta$  (i.e., imaginary time length equals inverse temperature), quantum mechanics **becomes** thermodynamics.

This mathematical operation is called **Wick Rotation**.

It establishes a one-to-one mapping between Minkowski spacetime (spacetime with light cones and causality) and Euclidean space (pure spatial geometry).

### 10.1.3 Geometric Definition of Temperature

This discovery completely reshapes our understanding of **temperature**.

In classical physics, temperature is the intensity of molecular motion.

But from the underlying perspective of FS geometry, **temperature is the time period in the imaginary dimension**.

- **Heat (Thermal)** is not a form of chaotic energy.
- **Heat (Thermal)** is the “distance” traveled by the system evolving along the **imaginary time axis**.

This explains why quantum field theory must use imaginary time techniques when calculating black hole entropy or vacuum fluctuations. Because in that limiting state, real physical time ( $t$ ) and thermodynamic temperature ( $\beta$ ) have become entangled and cannot be distinguished.

### 10.1.4 Hawking’s No-Boundary Universe

Stephen Hawking used this concept to propose the “no-boundary universe” model. He believed that in the very early universe (Planck era), time itself underwent Wick rotation, becoming a pure spatial dimension.

- There is no “beginning,” nor is there a “singularity.”
- The beginning of the universe is as smooth as the South Pole of Earth.

In our **Natural Generator** framework, this receives a new interpretation:

The noumenon of the universe is a **Holomorphic Function** defined on the complex domain.

- When we slice along the **real axis**, we see **time passing** (quantum evolution).
- When we slice along the **imaginary axis**, we see **thermal equilibrium** (statistical distribution).

The reason we perceive the world as “cold” (with a definite temperature) and “moving” (with passing time) is because there is a specific angle between our observation slice and this high-dimensional complex manifold.



### 10.1.5 Conclusion: Two Sides of a Coin

Wick rotation tells us that  $e$  is the hub connecting two worlds.

- $e^{ix}$  (**wave/circle**): This is the world of Book I. Conserved, coherent, eternal motion.
- $e^{-x}$  (**decay/spiral**): This is the world of Book II. Dissipative, probabilistic, thermodynamic arrow.

They are not opposing physical laws; they are projections of the same exponential function  $e^z$  in different directions on the complex plane.

**Quantum mechanics is imaginary thermodynamics.**

**Thermodynamics is imaginary quantum mechanics.**

Since temperature and time are mathematically interchangeable, this raises a deeper question: **Which generates which?**

Does time passing cause heat (entropy increase), or does heat (statistical state) generate time?

This leads to the theme of the next section: **Unification of Geometry and Heat**. We will see that at that ultimate level ruled by  $e$ , physics no longer distinguishes between “evolution” and “distribution”; they are unified in the same **Modular Flow**.

## 10.2 Geometric and Thermal Unity

“We once thought only clocks have periods, while heat is just chaotic noise. But geometry tells us that heat is also a period—it’s just a cycle occurring in the imaginary dimension. When we fold the paper of the universe, time becomes temperature.”

In the previous section, we revealed through Wick rotation the mirror symmetry between quantum mechanics ( $e^{-iHt}$ ) and statistical mechanics ( $e^{-\beta H}$ ) in mathematical form. But this is only a beginning. This symmetry is not merely a coincidence; it hints at perhaps the deepest truth in physics: **Dynamics (evolution) and statistics (distribution) are essentially congruent**.

This section will explore the physical-philosophical meaning of the **KMS Condition (Kubo-Martin-Schwinger Condition)**. We will see that in this complex universe ruled by  $e$ , **geometric structure** (spacetime) and **thermodynamic properties** (temperature) are not only unified; they are even mutually defining.

### 10.2.1 KMS Condition: The Rosetta Stone Connecting Two Realms

In the mid-20th century, physicists Kubo, Martin, and Schwinger discovered a surprising property of quantum field theory.

For a quantum system in thermal equilibrium (temperature  $T = 1/k_B\beta$ ), its correlation functions exhibit a strange **periodicity**.

If we regard time  $t$  as a complex variable, then on this complex plane, the system’s state is not chaotic but exhibits a cycle with period  $i\beta$  in the imaginary direction.

This is the **KMS Condition**. It declares in the most rigorous mathematical language:

**A system in thermal equilibrium  $\beta$  is equivalent to a dynamical system with period  $\beta$  in the imaginary time direction.**

**Physical Translation:**

- **What is “heat”?** Heat is not random molecular collisions. From the underlying geometric perspective, heat means the universe has tied a **“knot”** (Loop) on the imaginary time axis.

- **What is “temperature”?** Temperature is the **circumference** of this knot (or the reciprocal of the radius). The higher the temperature ( $\beta$  smaller), the smaller the imaginary time ring, and the faster the evolution period.

This completely overturns our intuition about “cold” and “hot.”

Absolute zero ( $T = 0, \beta \rightarrow \infty$ ) means the imaginary time axis is an infinitely long straight line (no cycle).

While Planck temperature (extremely hot) means the imaginary time axis curls into an extremely tiny ring with Planck-length radius.

### 10.2.2 Geometric Temperature of Black Holes

The strongest evidence for this theory comes from **black hole physics**.

When Stephen Hawking calculated black hole radiation, he did not use classical thermodynamics but geometry.

Near the black hole horizon, the spacetime metric undergoes extreme distortion. If we perform Wick rotation ( $t \rightarrow i\tau$ ) on this metric, the original hyperbolic geometry (Minkowski space) becomes elliptic geometry (Euclidean space).

In this Euclidean space, to avoid conical singularities at the horizon, the imaginary time coordinate  $\tau$  must be **periodic**.

Just as Earth’s longitude lines converge at the poles, imaginary time lines must close at the horizon.

Hawking was amazed to discover that this geometrically necessary period  $\beta_{geom}$  precisely equals the reciprocal of the black hole’s **Hawking temperature**!

$$\beta_{geom} = \frac{1}{T_{Hawking}}$$

This is one of the most brilliant moments in physics history:

- **Geometry** (circumference) directly determines **heat** (temperature).
- A pure geometric object (black hole), merely because of the topological closure of its spacetime structure, necessarily radiates heat.

This proves the core conjecture of **Vector Cosmology**: **Heat is not a form of energy; heat is a shape of spacetime geometry.**

### 10.2.3 The Full Picture of the Complex Plane

At this point, we can construct a unified **complex spacetime picture**.

The noumenon of the universe is a **Holomorphic Manifold** defined on the complex domain. This manifold itself is static, perfect, self-consistent. It contains all  $e$  exponential relationships. We as observers merely cut a slice on this manifold (chose our time axis).

- **Cutting the real axis ( $t$ ):** We see **unitary evolution**. Wave functions oscillate, phases rotate, history generates. This is the world of **quantum mechanics**.
- **Cutting the imaginary axis ( $it$ ):** We see **statistical distribution**. Systems arrange under Boltzmann weights, exhibiting temperature and entropy. This is the world of **thermodynamics**.

**Time is complex temperature; temperature is imaginary time.**

They are like the “height” and “circumference” of a cylinder. You cannot say height is more real than circumference; they are just measures of the same geometric object in different dimensions.

#### 10.2.4 Conclusion: No Randomness, Only Dimensions

Through the KMS condition and the complex nature of  $e$ , we eliminate physics’ greatest enemy—**randomness**.

“Random thermal motion” in thermodynamics is reduced to “deterministic geometric cycles” in the imaginary dimension.

This means that under the rule of this natural generator  $e$ , no part of the universe is chaotic.

Even the most disordered thermal radiation, in the eyes of God (or high-dimensional observers), is a geometrically precise crystal.

Since time and temperature are interchangeable geometric attributes, this raises an even more radical question: **Where does time actually come from?**

If time is just a direction on the complex plane, who decided that our universe must evolve along the “real axis” rather than freeze along the “imaginary axis”?

This leads to the theme of the next chapter: **The Modular Flow Hypothesis**. We will explore a view that could shake the foundations of physics—**Time is not an external parameter; time is “secreted” by the quantum state itself.**



## Chapter 11

# The Modular Flow Hypothesis



## Chapter 12

# The Modular Flow Hypothesis

### 12.1 Who is Ticking?

“We always ask: ‘What time is it?’ as if time were an entity independent of us. But the deep algebra of quantum mechanics tells us there is no universal clock. It is you—as an unbalanced quantum state—who creates time yourself in the process of having to flow to find balance.”

#### 12.1.1 God Has No Clock

Let us return to the Schrödinger equation  $|\dot{\psi}\rangle = -iH|\psi\rangle$ . Although elegant, this equation implies a huge assumption: it assumes that a parameter named  $t$  already exists, and a Hamiltonian named  $H$  drives the evolution.

But under extreme conditions of quantum gravity (such as the Wheeler-DeWitt equation),  $H\Psi = 0$ . The entire universe’s Hamiltonian is zero, and the time parameter  $t$  disappears. The universe appears frozen.

If the bottom is frozen, why do we feel flow?

Who is ticking?

In the paper, we touched the core of this question: the concept of “modular ‘thermal’ time” suggests that time is not fundamental but emergent.

From this perspective, **time originates from “ignorance”**.

#### 12.1.2 State as Clock

To understand this, we need to introduce **Tomita-Takesaki Theory** from algebraic quantum field theory. This is a mathematical theorem hailed by many physicists as “theological level.”

It tells us: As long as you are given a von Neumann algebra (representing observables) and a specific quantum state  $|\Omega\rangle$  (representing the system’s current state), the mathematical structure will **automatically generate** a one-parameter unitary evolution group.

This evolution group is called the **Modular Flow**, denoted  $\Delta^{it}$ .

- **Traditional physics:** First there is time  $t$ , then there is evolution  $U(t)$ , and finally this causes state changes.
- **Modular flow physics:** First there is state  $|\Omega\rangle$ , and this state’s intrinsic entanglement structure **defines** what “evolution” is, thereby **defining** what time  $t$  is.

**Time is intrinsic to state.**

A system in thermal equilibrium has a static (or trivial) modular flow.

While a system far from equilibrium, containing complex entanglement (such as life or civilization), has an intense modular flow.

**It is your own quantum state that is ticking.** Every breath you take, every thought you have, is not consuming time but **generating** your own modular flow.

### 12.1.3 Physical Meaning of Thermal Time

This hypothesis perfectly explains the “thermal time” concept we encountered in the previous volume.

French physicists Alain Connes and Carlo Rovelli proposed the **Thermal Time Hypothesis**:

**The time passing we perceive is actually the modular flow of the statistical state we are in.**

- When we are in a pure state (omniscient), there is no time.
- When we are in a mixed state (with forgetting, with entropy), modular flow appears.

This coincides with the  $c_{FS}$  budget allocation we discussed in the first book.

When we cut the system out from the environment (producing  $v_{env}$  and entanglement entropy), we not only create the “self” but also create the “time” that drives self-evolution.

**Time is the product of ignorance.**

Because we cannot see the full picture of the universe (all entanglement of  $|\Psi\rangle$ ), we can only see local projections. This information deficit (entropy) is mathematically transformed into a generator (Hamiltonian) that drives system evolution.

### 12.1.4 Conclusion: The Objectivity of Subjective Time

“Who is ticking?”

The answer is: **You yourself.**

The universe does not have a unified Greenwich time. Every subsystem, according to its entanglement with the environment, has its unique modular flow, that is, a unique “**private time**”.

- A photon’s modular flow is stagnant, so it has no time.
- A black hole horizon’s modular flow is extremely redshifted, so time freezes there.
- Our modular flow is linear, so we feel birth, aging, sickness, and death.

We do not need to find that Prime Mover who winds the universe’s clock.

**Existence itself is the clockwork.** As long as you exist (in a non-trivial entangled state), you will necessarily experience the modular flow defined by your own state. You are not only the observer; you are your own clock.

Since time is generated by state, how does this generation mechanism operate mathematically? How does that alchemist called the “Tomita operator” extract dynamic time flow from static entanglement?

This leads to the theme of the next section: **Tomita’s Alchemy**. We will delve into the mathematical core of this theorem hailed as “quantum mechanics’ most profound theorem” to witness the alchemical process of time’s birth.



## 12.2 Tomita's Alchemy

“Ancient alchemists tried to extract gold from lead, while modern mathematical physicists accomplished an even greater feat: they extracted flowing time from static quantum states. This is no longer magic; this is the rigorous logic of Tomita-Takesaki Theory.”

In the previous section, we proposed a shocking view: time is not the background stage of the universe but an intrinsic property “secreted” by the quantum state itself. This sounds like a poetic metaphor, but deep in mathematical physics, there is ironclad proof.

This is the mathematical framework called **Tomita-Takesaki Theory**. In **Vector Cosmology**, we honor it as “**The Alchemy of Time**”. Because it reveals how to refine flowing “time-gold” from dead “state-lead”.

### 12.2.1 Dynamics in Statics: The Modular Operator $\Delta$

Let us return to the bottom of Hilbert space. Suppose the universe is in a specific quantum state  $|\Omega\rangle$  (such as vacuum state or thermal equilibrium state). In this state, no macroscopic objects are moving; everything seems static.

However, Japanese mathematician Minoru Tomita discovered an astonishing structure. As long as this state  $|\Omega\rangle$  has sufficient **Entanglement** (i.e., it is separating and cyclic), it automatically contains a mathematical object called the **Modular Operator** ( $\Delta$ ).

This operator  $\Delta$  is like a **clockwork** hidden inside the state.

It defines a natural evolution flow—the **Modular Flow**:

$$\sigma_t(A) = \Delta^{it} A \Delta^{-it}$$

Notice the  $t$  in this formula.

This  $t$  is not preset by anyone, nor is it Newton's absolute time. It is a **real parameter purely generated by  $\Delta$  (i.e., by the state  $|\Omega\rangle$  itself)**.

This means: **Merely because a quantum state has entanglement, it must undergo an intrinsic evolution.**

This is a mathematical compulsion. Just as a circle must have circumference, an entangled state must have modular flow. For observers inside this system, this modular flow manifests as “**physical time passing**”.

### 12.2.2 Logarithmic Hamiltonian: Geometrization of Heat

To understand the physical meaning of this modular flow, we need to borrow the power of the **logarithm** ( $\ln$ ) again.

We can write the modular operator in exponential form:

$$\Delta = e^{-K}$$

Or conversely:

$$K = -\ln \Delta$$

This  $K$  is called the **Modular Hamiltonian**.

In thermodynamics, if we are in a Gibbs state  $\rho = e^{-\beta H}$ , then the modular operator  $\Delta$  directly corresponds to the density matrix  $\rho$ . At this point, the modular Hamiltonian  $K$  precisely equals the physical Hamiltonian  $H$ :

$$K = \beta H$$

This is a deafening conclusion:

**Energy ( $H$ ) is essentially just the logarithm of state probability ( $\rho$ ).**

- **In traditional physics:** Energy is cause, probability distribution is effect.
- **In modular flow physics:** Probability distribution (entanglement structure) is cause, energy (and the ensuing time evolution) is effect.

The reason the universe has the concept of “energy,” the reason “time” moves forward, is because the universe’s quantum state has non-trivial **information geometric structure**. Time is the feeling we get when reading this logarithmic table.

### 12.2.3 Relativity of Local Time

Tomita’s alchemy completely shatters the illusion of “unified time.”

Since modular flow is jointly determined by state  $|\Omega\rangle$  and algebra  $\mathcal{A}$  (the set of observables), **every local subsystem has its own private time**.

- **For the entire universe ( $|\Psi\rangle$ ):** If there is no environment, it is a pure state, no entanglement,  $\Delta = 1$ , modular flow is static. This explains why “total time” disappears in the Wheeler-DeWitt equation.
- **For local observers (You):** You can only observe part of the universe (algebra  $\mathcal{A}_{local}$ ). For you, the universe is in a mixed state (entangled state). This local entanglement generates a non-trivial modular operator  $\Delta_{local}$ , thereby driving the “**thermal time flow**” you perceive.

**Each of us lives in our own modular flow.**

The reason we can reach consensus, feeling time passes synchronously, is because we are in the same huge heat bath (cosmic microwave background or Earth’s environment), and our modular Hamiltonians are approximately aligned at large scales.

### 12.2.4 Conclusion: Existence is Generation

At this point, we have completed the ultimate argument for  $e$  as the “**Natural Generator**”.

The universe does not need a Prime Mover.

As long as there is an entangled quantum state,  $e^{-iKt}$  automatically starts.

- $K$  (**modular Hamiltonian**) is defined by “ignorance” (entropy) inside the state.
- $t$  (**time**) is defined by the flow of  $K$ .

**Existence is generation.**

Matter does not need to be “pushed” forward; matter’s entanglement structure itself is a perpetual motion machine, continuously secreting “time” as a fluid.

Since we have understood how time emerges from state, as awakened observers, how do we perceive, measure, and decode this flowing universe? Why are our senses (vision, hearing) inherently **logarithmic**?

This leads to the theme of Volume IV of this book: **The Logarithmic Eye**. We will see that life not only utilizes the growth of  $e$  but has also evolved the inverse operation of  $\ln$  to decompress this exponentially exploding universe.

## Chapter 13

# The Measure of Information



## Chapter 14

# The Measure of Information

### 14.1 Boltzmann’s Logarithm

“Nature does multiplication because it continuously generates possibilities; humans do addition because we try to count these possibilities. The only bridge connecting these two operations is the formula engraved on Boltzmann’s tombstone.”

#### 14.1.1 The Tyranny of the Exponential and the Redemption of the Logarithm

At the bottom of **Vector Cosmology**, Hilbert space is vast. As the number of particles  $N$  increases, the number of possible states (microscopic states  $W$ ) does not increase linearly but explodes **exponentially** ( $W \sim e^N$ ).

For a macroscopic system containing merely one mole of gas ( $10^{23}$  particles), the number of microscopic states is a number too large to write. If our senses had to process every microscopic projection of  $v_{int}$  one by one, our brains would burn out instantly from overload.

Facing this **Tyranny of the Exponential**, observers must find a means of **dimensional reduction**. We need a mathematical tool that can tame “huge multiplication” into “gentle addition.”

This tool is the **natural logarithm** ( $\ln$ ).

$$\ln(A \cdot B) = \ln A + \ln B$$

Through logarithmic operations, that despairing exponential growth  $e^N$  is reduced to linear quantity  $N$ .

$$\ln(e^N) = N$$

This is the origin of **Entropy**.

#### 14.1.2 The Formula on the Tombstone

19th-century physicist Ludwig Boltzmann saw this insight. He discovered that entropy  $S$ , that mysterious, always-increasing quantity in thermodynamics, is actually the logarithm of the number of microscopic states  $W$ :

$$S = k_B \ln W$$

From our geometric perspective, this formula is not merely a thermodynamic definition; it is the **defining equation of macroscopic observers**.

- **$W$  (microscopic truth):** The holographic, high-dimensional, cosmic noumenon containing all  $v_{int}$  details. It is driven by generator  $e$ .
- **$S$  (macroscopic perception):** The compressed, low-dimensional, observation result discarding most details. It is extracted by inverse operation  $\ln$ .

**Entropy is a measure of ignorance.**

But this is not passive ignorance. This is “**strategic neglect**”. Observers actively choose to ignore those wildly rotating microscopic phases, only counting their “volume” (logarithm) at the macroscopic level. It is precisely this neglect that allows us to see stable temperature, pressure, and time passing, rather than seeing chaotic quantum foam.

### 14.1.3 Geometric Compression of Information

In FS geometry, the logarithmic nature of entropy corresponds to **dimensional projection**.

In the paper, we derived the **entropy speed limit**  $|\dot{S}| \leq c_{FS} \ln d$ . Notice the  $\ln d$  in the formula.

- $d$  is the dimension of Hilbert space (possibilities).
- $\ln d$  is the maximum number of information bits we can process macroscopically.

This shows that the mapping from microscopic to macroscopic is essentially a process of **Geometric Compression**.

The universe expands in the direction of  $e$  (generating dimensions), while observers fold in the direction of  $\ln$  (compressing dimensions).

- **The universe is “writing”:** Using the budget of  $c_{FS}$ , wildly writing the history of every particle.
- **We are “reading”:** Using the algorithm of  $\ln$ , summarizing this infinitely thick book into a few simple physical laws.

### 14.1.4 Conclusion: $\ln$ is the Decompression Code

Therefore, Boltzmann’s formula reveals the deepest dual relationship between observers and the universe:

**The universe is exponential ( $e$ ), while perception is logarithmic ( $\ln$ ).**

If we did not have this “logarithmic eye,” we would be drowned by the flood of information. The reason we can exist, the reason we can think, is because our brains are inherently logarithmic computers. They help us filter out  $10^{23}$  microscopic details, leaving only that most crucial macroscopic indicator—entropy.

Since we know perception is logarithmic, does this mathematical structure also manifest in our biological instincts? Why do we feel sound decibels are linear, light brightness is linear, even though physical stimuli grow exponentially?

This leads to the theme of the next section:  **$\ln$  as Decoder**. We will see that not only physics but even our nervous systems are constructed according to this logarithmic law.

## 14.2 ln as Decoder

“The universe roars with exponential functions, while we listen with logarithmic functions. Without the filter of ln, even the quantum history of a single grain of dust would be enough to shatter our reason. The reason we can see a linearly flowing world is because our brains are precise logarithmic decoding machines.”

In the previous section, we established the logarithmic relationship between macroscopic and microscopic through Boltzmann’s formula  $S = k \ln W$ . But this is not merely a thermodynamic formula; it is the survival strategy of life as an **Observer**.

If the essence of the universe is  $e$  (**generation and expansion**), then the essence of life must be ln (**compression and decoding**).

This is mathematically necessary:  $e^x$  and  $\ln x$  are **Inverse Functions** of each other.

The universe is responsible for creating complexity ( $e$ ), while we are responsible for reducing this complexity to understandable linear order (ln).

### 14.2.1 Taming the Exponential Monster

Imagine if you had to process the wave function evolution of every particle in the universe in real time.

According to the QCA model, the dimension of Hilbert space grows exponentially with time:  $D(t) \propto e^{\lambda t}$ .

This means that if you want to “omnisciently” record the universe’s history, you need a hard drive that grows at exponential speed. This is physically impossible because your  $c_{FS}$  budget (computational bandwidth) is finite.

Facing this exponentially growing data monster, life faces two choices:

1. **Be drowned:** Attempt to record all details, leading to overload collapse.
2. **Decode:** Use an algorithm to transform “exponential growth” into “linear signals.”

Mathematics tells us that the only operator that can do this is the **logarithm**.

$$\ln(e^{\lambda t}) = \lambda t$$

Look! Through logarithmic operations, that violent, curved, rapidly escaping exponential curve is instantly straightened into a gentle, linear straight line.

This is the essence of **Decoding**.

Life does not directly perceive the original physical quantities (those are astronomical numbers); life perceives the **Order of Magnitude** of physical quantities.

### 14.2.2 The Illusion of Linear Time

This mechanism perfectly explains why we feel time passes **linearly**.

In the second book *The Ascension of the Spiral*, we pointed out that the universe is actually accelerating expansion (spiral), and the  $c_{FS}$  budget is increasing exponentially. Logically, we should feel time passing faster and faster, the world becoming more and more chaotic.

But we don’t. We feel yesterday, today, and tomorrow are equally long.

This is because our brains are **logarithmic sensors**.

- **Physical fact:** The universe’s information at time  $t$  is  $I(t) = I_0 e^{\lambda t}$ .

- **Subjective perception:** Our psychological time  $T_{psy}$  is the logarithm of physical information.

$$T_{psy} \propto \ln(I(t)) \propto \lambda t$$

The “uniform time” we see is actually “expanding time” compressed logarithmically.

This is like drawing on logarithmic graph paper: although each tick on the horizontal axis represents a value 10 times the previous one (exponential growth), visually, their distances are equal (linear perception).

This is the ultimate mercy the universe bestows upon observers. It uses  $\ln$  to filter out the dizziness brought by exponential explosion, allowing us to live peacefully in a seemingly stable linear illusion.

### 14.2.3 Dimensional Reduction Compression of Information

In information theory, logarithms are also the foundation of **optimal encoding**.

When we say a file’s size is “how many bits,” we are actually taking a logarithm:  $\text{Bits} = \log_2(\text{States})$ .

Life can understand the universe because life has learned to “**ignore**”.

- Microscopically, a balloon contains  $10^{23}$  gas molecules, whose positions and momenta constitute  $e^{10^{23}}$  possible states.
- Macroscopically, life only cares about one value: **pressure**.

What is pressure? Pressure is the statistical average of those countless microscopic collisions, essentially a logarithmic reading of the number of microscopic states.

This strategy of “**ignoring details, only reading magnitude**” is the foundation of all intelligent existence.

$\ln$  is the universe’s **compression software (Zip)**.

It packages the suffocating details in high-dimensional Hilbert space (microscopic phases of  $v_{int}$ ) into simple physical laws in low-dimensional macroscopic world (macroscopic motion of  $v_{ext}$ ).

### 14.2.4 Conclusion: We Are the Inverse Operation

At this point, we see the position of observers in the universe’s mathematical structure.

- **Universe ( $e$ ):** Responsible for “**unfolding**”. From 0 to  $\infty$ , generating infinite possibilities.
- **Life ( $\ln$ ):** Responsible for “**folding**”. From  $\infty$  to 1, extracting unique meaning.

We are the universe’s **Inverse Operation**.

The universe diverges; we converge. The universe does multiplication; we do addition.

Without us ( $\ln$ ), the universe ( $e$ ) is just a crazy explosion without an audience; without the universe ( $e$ ), we ( $\ln$ ) are just an empty line of code without input.

Together, they constitute the perfect identity  $y = \ln(e^x) = x$ —the process of “**existence being cognized**”.

Since our perception is constructed logarithmically, does this mathematical structure leave traces at our physiological level? Why do our eyes’ response to brightness and ears’ response to sound strictly follow logarithmic laws?



This leads to the theme of the next chapter: **The Logarithmic Law of Sensation**. We will leave abstract mathematics and enter the realm of biology to verify the direct projection of this cosmic law onto neurons.



## Chapter 15

# The Logarithmic Law of Sensation



## Chapter 16

# The Logarithmic Law of Sensation

### 16.1 The Weber-Fechner Law

“Why does lighting a candle in the dark feel so bright, while lighting the same candle in sunlight is almost invisible? Because your eyes are not photon counters; your eyes are logarithmic calculators. To survive in a universe where the dynamic range spans dozens of orders of magnitude, your nervous system must learn to lie—it disguises crazy multiples as gentle additions.”

#### 16.1.1 The Nonlinearity of Sensation

19th-century psychophysicists Ernst Weber and Gustav Fechner discovered a puzzling phenomenon: human perception of stimulus intensity is not linear.

- If you hold a 100-gram weight, adding 10 grams makes you feel heavier.
- But if you hold a 1000-gram weight, adding 10 grams makes no difference. You need to add 100 grams to produce the same “feeling of heaviness.”

This means: **The increment of sensation ( $\Delta S$ ) is proportional to the relative increment of stimulus ( $\Delta I/I$ ).**

Integrating this differential equation, we obtain the famous logarithmic formula:

$$S = k \ln I + C$$

where  $S$  is the subjective sensation magnitude and  $I$  is the physical stimulus intensity.

This formula tells us: **Every time the external world increases by one “order of magnitude” (multiplication), our internal sensation only increases by one “step” (addition).**

- Sound decibels (dB) are logarithmic.
- Stellar magnitudes are logarithmic.
- Musical pitch octaves are logarithmic (frequency doubles, but perception only rises by one octave).

### 16.1.2 The Dynamic Range Matching Problem

Why did evolution choose this strange way of perception? Why not let us directly see the real photon count?

The answer lies in the **finiteness of FS capacity** ( $c_{FS}$ ).

In the first book, we established that any physical system—including our neurons—has a **maximum information processing bandwidth**. Neuronal firing frequencies have an upper limit (approximately 100-1000 Hz). This means our “internal display” has only limited grayscale levels (e.g., 0 to 255).

However, the external universe is an exponential world driven by  $e$ .

- Sound intensity range: from a mosquito’s buzz to a jet engine, spanning  $10^{12}$  times the energy difference.
- Light intensity range: from starlight to noon sun, spanning  $10^{14}$  times the photon flux difference.

This is an **Impedance Matching** nightmare.

If our senses were linear ( $S \propto I$ ):

1. If we were sensitive enough to see dark details, we would instantly “overload” during the day, neurons would overload, and we would go blind.
2. If we were adapted to strong light, we would become blind at night, completely unable to see weak photons.

### 16.1.3 Dimensional Reduction Projection in Geometry

To solve this problem, life was forced to hardcode the  $\ln$  (**logarithm**) algorithm at the hardware level.

In the geometric language of **Vector Cosmology**, this corresponds to the **inverse mapping from tangent space to manifold**.

- **External input** ( $I$ ): Vectors in Hilbert space with exponentially growing modulus (driven by  $e^{\lambda t}$ ).
- **Internal perception** ( $S$ ): Linear potential changes that neurons can carry (limited by  $c_{FS}$ ).

The sensory system of living organisms is essentially an **Analog Logarithmic Converter**.

The chemical reaction dynamics on the retina and the basilar membrane resonance structure in the cochlea cleverly utilize nonlinear physical effects to achieve the operation  $y = \ln x$ .

Through this operation, we compress the universe’s violent, multi-order-of-magnitude energy tsunami into a gentle **linear stream** that we can safely process.

### 16.1.4 Conclusion: We Are Logarithmic Functions

The Weber-Fechner Law is not just about sensation; it is about **“existence”**.

It proves our deep mathematical relationship with the universe:

**The universe is an exponential expander; we are logarithmic convergers.**

The reason we feel the world is stable, predictable, and linear is not because the world itself is so. Rather, our senses are like a pair of **“logarithmic glasses”** that filter out the exponential madness at the bottom of the universe.

We live in an illusion tamed by  $\ln$ .

But this is a merciful illusion. It is precisely because of this filter that we can maintain sanity, experience life, and produce the tranquility of “linear time” in the exponential explosion of  $e$ .

Since our senses are logarithmic, is the core operating logic of the brain that processes these sensory data—cognition and thinking—also built on some geometric mapping?

This leads to the theme of the next section: **The Geometry of Cognition**. We will see that not only sensation, but even our “understanding” itself is a projection operation that forcibly flattens high-dimensional manifolds onto low-dimensional tangent planes.

## 16.2 The Geometry of Cognition

“To walk on that wildly curved, exponentially exploding high-dimensional manifold, the brain had to do something extremely violent: it forcibly laid a flat tangent plane beneath its feet. We think the world is flat (linear) only because we live on this geometric map we ourselves laid out.”

In the previous section, we confirmed that sensation is **logarithmic** ( $\ln$ ): eyes and ears compress the exponential growth of the physical world into linear perception. Now, we elevate this logic from sensation to the core of thought—**Cognition**.

Why is human logic linear? Why are we accustomed to simple causal chains like “because A, therefore B,” yet find it difficult to intuitively understand superposition and entanglement in quantum mechanics?

From the perspective of **Vector Cosmology**, this is not merely a biological limitation; it is an inevitable choice of **Differential Geometry**. Our cognition is essentially the process of forcibly projecting the curved **Projective Hilbert Space** ( $P(\mathcal{H})$ ) onto a local **Tangent Space** ( $T_{[\psi]}$ ).

### 16.2.1 The Tranquility of Tangent Space: A Linear Haven

Let us return to the ontology of the universe. According to the model we established in Volume III, the universe is driven by the generator  $H$  through exponential mapping  $e^{-iHt}$ . This means the true geometry of the universe is **highly nonlinear**, curved, and exponentially growing.

On a curved sphere, parallel transport between two points is complex (holonomy effects exist). If the brain tried to directly process this non-Euclidean geometry, its computational load would rise exponentially with distance.

To save the budget  $c_{FS}$ , the brain evolved a “**tangent space strategy**”.

In differential geometry, no matter how curved a manifold is, in an infinitesimal neighborhood around any point, it appears flat (Euclidean space). This local flat space is called the **Tangent Space**.

- **Universe ontology** ( $e$ ): Curved, holomorphic manifold.
- **Human cognition** ( $\ln$ ): At the present moment  $\tau$ , the brain establishes a **tangent plane**.

On this tangent plane, vector addition is linear ( $\vec{A} + \vec{B} = \vec{C}$ ), and causality is linear. The reason we feel logic is linear is because we live in this tangent space. We have “flattened” the curved universe to make it understandable.

### 16.2.2 The Linearization of Time: Canceling Exponential Expansion

The most profound consequence of this geometric projection is reflected in our perception of **time**.

In the second book *The Ascension of the Spiral*, we revealed that the universe is actually in a state of **dimensional inflation**, with total budget  $c_{FS}$  growing exponentially with time  $c_{FS}(\tau) \propto e^{\lambda\tau}$ .

Logically, we should feel time passing faster and faster, the world becoming more and more turbulent.

But the brain performs an **inverse operation**.

Our cognitive system is a **logarithmic converter**. When external complexity grows as  $e^{\lambda t}$ , the brain's internal clock scale compresses in the manner of  $\ln(\cdot)$ .

$$T_{psy} \propto \ln(e^{\lambda\tau}) = \lambda\tau$$

The result is astonishing: **Nonlinear physical time is perceived as perfectly linear psychological time**.

This is like a passenger in an elevator accelerating upward. If the elevator's acceleration is constant, the passenger feels weight; but if the passenger's perception system can adjust the "gravity zero point" in real time, they will feel as if they are still standing on stationary ground.

We feel time "flows uniformly," neither accelerating nor decelerating. This is not because time itself is uniform, but because our **cognitive geometry** perfectly cancels the acceleration of **cosmic expansion**. We are "**linearized beings**".

### 16.2.3 The Effectiveness of Mathematics: Coincidence of Tangent Points

This explains why **linear mathematics** (such as Newtonian mechanics, linear algebra) is so effective in describing the world, but fails at extreme scales (quantum gravity, black holes).

- **Effectiveness:** At everyday scales, our cognitive tangent plane **tangents** the universe's curved manifold. Near the tangent point, linear approximation (first-order term of Taylor expansion) is extremely accurate.
- **Failure:** When we try to understand the universe's extreme early stages (Big Bang) or extreme microscopic scales (Planck scale), we move away from the tangent point. At this point, the manifold's curvature (higher-order terms of  $e$ ) becomes non-negligible. Applying linear logic to nonlinear reality inevitably leads to paradoxes (such as singularities, divergences).

The history of human scientific progress is the history of our continuous attempts to **bend our tangent plane** to better fit the universe's true shape. General relativity introduced curvature, quantum mechanics introduced complex numbers—these are all difficult approximations of cognitive geometry toward the universe's ontological geometry.

### 16.2.4 Conclusion: All Are Logarithmic

At this point, we have completed the ultimate deconstruction of the "**Observer**".

The observer is not a passive recorder; the observer is an active **geometric operator**.

- The universe uses  $e$  (generator) to create complex things.
- The observer uses  $\ln$  (logarithm) to reduce things to understandable order.



**We are the universe's logarithmic function.**

Without us, the universe is just a meaningless exponential explosion; without the universe, we are just an undefined empty operator.

It is the interaction of  $e$  and  $\ln$  that constitutes the only truth line: **“Existence”** ( $y = x$ ).

Now, the final piece of the puzzle is in place.

We understand  $\pi$  (**structure**), understand  $\varphi$  (**evolution**), understand  $e$  (**generation**), and also understand  $\ln$  (**perception**).

These three constants, along with the imaginary number  $i$  behind them, together weave this grand vector cosmology picture. Now, only one thing remains—to gather all these clues and converge them into that final chapter.

There, we will no longer distinguish between mathematics and physics, no longer distinguish between observer and observed. We will witness how all this complexity flows spontaneously and necessarily from that **“self-generating”** source.

Next chapter, the final chapter of the book: **Self-Generation**. We will reveal that the universe needs no explanation, because the universe is its own explanation.



## Chapter 17

# The Self-Caused



# Chapter 18

## The Self-Caused

### 18.1 The Trinitarian Closed Loop

“ $\pi$  gives the universe its shape,  $\varphi$  gives the universe its direction, and  $e$  gives the universe its life. They are not three gods; they are three aspects of the same geometric deity. The universe needs no explanation, because it is a self-consistent closed loop woven from these three constants.”

#### 18.1.1 The First Aspect: $\pi$ — The Body of the Universe (Structure)

This is the theme of the first book *The Conservation of the Circle*.

When we ask “What shape is the universe?”,  $\pi$  steps forward to answer.

It represents **Structure**, **Conservation**, and **Cycle**.

- **Geometric image: Circle.**

It is the geometric expression of the Pythagorean identity  $v_{ext}^2 + v_{int}^2 = c_{FS}^2$ . It ensures that no matter how the universe evolves, its total budget  $c_{FS}$  is closed on the unitary tangent plane.

- **Physical correspondence: Matter.**

As Levinson’s theorem reveals, matter is a topological dead knot of phase, with  $\pi$  as its counting unit. Protons are stable and atoms are solid, all thanks to the periodic closure of  $\pi$ .

- **Philosophical meaning: “Form”.**

It is the skeleton of the universe, the stage, the unchanging background. Without  $\pi$ , the universe would be a shapeless fluid, with nothing that could be called “substance.”

#### 18.1.2 The Second Aspect: $\varphi$ — The Mind of the Universe (Function)

This is the theme of the second book *The Ascension of the Spiral*.

When we ask “Where is the universe going?”,  $\varphi$  (the golden ratio) steps forward to answer.

It represents **Dynamics**, **Growth**, and **Intent**.

- **Geometric image: Spiral.**

It is dimensional inflation, the Red Queen’s run. It breaks the perfect closure of  $\pi$ , tearing a tiny gap (phase transition) on the circle, driving the system to explore the unknown void.

- **Physical correspondence: Life & Evolution.**

Life is a negative entropy enclave, using  $\varphi$ 's anti-resonance properties to avoid the dead loop of thermodynamics. It is the universe's attempt to transcend "memory" through "computation."

- **Philosophical meaning: "Function".**

It is the will of the universe. It is not satisfied with mere existence; it desires to become more complex. Without  $\varphi$ , the universe would be an eternal cycle of death, with no history, no future.

### 18.1.3 The Third Aspect: $e$ — The Spirit of the Universe (Essence)

This is the theme of the third book *The Natural Generator*.

When we ask "Why does the universe exist?",  $e$  (the **natural constant**) steps forward to answer.

It represents **Source**, **Self-Reference**, and **Generation**.

- **Geometric image: Tangent.**

It is the function whose derivative equals itself:  $d/dt(e^t) = e^t$ . It needs no external force to push it; its "being" is itself "motion." It is the bridge connecting discrete QCA with continuous spacetime.

- **Physical correspondence: Field & Flow.**

It is  $e^{-iHt}$  in Schrödinger's equation, and also  $e^{-\beta H}$  in thermodynamics. It is the generative engine driving all evolution at the bottom level. Whether it is the rotation of  $\pi$  ( $e^{i\pi}$ ) or the growth of  $\varphi$  ( $e^{\lambda t}$ ), they are all projections of  $e$  under different parameters.

- **Philosophical meaning: "Essence".**

It is the soul of the universe. It explains why "nothing" can generate "something"—because in the logic of exponentials, 0 (void) naturally generates 1 (existence).

### 18.1.4 The Perfect Closed Loop

At this point, the three become one.

- **$e$  is the source:** It provides the self-driven **energy flow**.
- **$i\pi$  is the mold:** It bends this energy flow into a conserved **circle**, thus creating matter and space.
- **$\varphi$  is the fission:** It introduces a tiny **asymmetry** on the circle, thus creating the direction of time and the possibility of evolution.

$$\text{Universe} = e^{\varphi + i\pi}$$

- Real part ( $\varphi$ ) drives expansion.
- Imaginary part ( $\pi$ ) drives rotation.
- Base ( $e$ ) drives existence.

This is the **Trinitarian Closed Loop**.

We need not seek a creator, because these three constants already constitute a complete, self-consistent, and self-generating “**logical theology**.”

The universe is as it is not because it was designed, but because **this is the only mathematical structure that can exist self-consistently**.

Since we have found this ultimate mathematical structure, if we were to give this great structure a final name in human language, what would it be?

This leads to the theme of the next section: **The Final Formula**. We will seal all this wisdom with an extremely simple symbol, through the famous “*Dao*.”

## 18.2 The Final Formula

“The *Dao* that can be spoken is not the eternal *Dao*. Language can only describe the shadows of things, but formulas can capture the skeleton of things. If we must write the ultimate true name of the universe in one line of mathematical symbols, it will be an exponential function that transcends all physical constants.”

In the prologue of this book, we started from Euler’s formula  $e^{i\pi} + 1 = 0$ . That was the map of mathematics.

Now, at the end of the book, we fold this map into a **generative program** that can run.

All of physics—from Newton’s force ( $F$ ) to Einstein’s curvature ( $R_{\mu\nu}$ ), to Schrödinger’s wave function ( $\Psi$ )—can ultimately be compressed into the same extremely concise yet infinitely deep formula.

This is the final formula of **Vector Cosmology**:

$$|\Psi_{universe}\rangle = e^{\text{Dao}}$$

### 18.2.1 The *Dao* is the Exponent

Please gaze carefully at this formula. It has only three parts, yet encompasses all truth of existence.

#### 1. **Left side:** $|\Psi_{universe}\rangle$ (**All Things**)

This is the **result**. It is the macroscopic universe we perceive, the sum of galaxies, atoms, life, and consciousness. It is that concrete global vector manifested in projective Hilbert space. It is “being.”

#### 2. **Base:** $e$ (**Generation**)

This is the **mechanism**. It is the engine of natural generation, the self-driven logic of “derivative equals itself.” It ensures that the leap from nothingness to existence is continuous, necessary, and automatic.

#### 3. **Right superscript:** *Dao* (**The Way**)

This is the **source**.

In physics, this position is usually occupied by  $-iHt$  (quantum evolution) or  $-\beta H$  (thermodynamic distribution). But in the ultimate theory, these are only approximate expressions of “*Dao*.”

**Dao is not a noun; Dao is an Exponent.**

In mathematics, the exponent is the habitat of the **Generator**.

- “Dao” is not a static object; you cannot find it in some corner of the universe.
- “Dao” is an **Operator**. It is the **meta-rule** that defines the direction of rotation, the rate of growth, and the game relationship between  $\pi$  and  $\varphi$ .

### 18.2.2 We Are the Result of Computation

This formula completely subverts our imagination of the “Creator.”

Traditional religion believes that God (the Creator) is outside the universe and created the universe.

But in the logic of  $e^{\text{Dao}}$ , **the source (Dao) is within the internal structure of the universe** ( $|\Psi\rangle$ ). More precisely, it is deeply buried in the tangent space of evolution.

- **Dao (The Exponent)** is unmanifested potential, the phase of probability amplitude, the structure constants of Lie algebra. It is **“nothingness.”**
- **Universe (The Vector)** is manifested reality, the collapse of probability, the manifold of Lie groups. It is **“being.”**

The two are tightly connected through  $e$ . “Nothingness” continuously generates “being” through exponential operations.

We—as observers, as life, as matter—are not directly “Dao.”

**We are the result of function computation.**

We are a series of Taylor series terms expanded in the imaginary dimension by the generator “Dao” driven by  $e$ :

$$e^{\text{Dao}} = 1 + \text{Dao} + \frac{1}{2!}\text{Dao}^2 + \frac{1}{3!}\text{Dao}^3 + \dots$$

- The first term is vacuum.
- The second term is elementary particles.
- Higher-order terms are life and civilization.

### 18.2.3 The Unspeakable Word

Laozi said: “I do not know its name, so I force a name and call it Dao.”

Now we understand why he did not know the name. Because the thing in that position (the exponent position) is not a simple number or name, but a **Hermitian operator matrix containing all information of the universe**.

It is too complex to be described in language (“The Dao that can be spoken is not the eternal Dao”).

But it is also too simple—it is just that small superscript above  $e$ .

All physical laws, all geometric constraints ( $c_{FS}$ ), all spiral growth, have been encoded into the eigenvalue spectrum of this operator. The history of the universe is the process of this operator being decompressed, expanded, and manifested by  $e$ .

**The final truth is:**

There is no monarch issuing orders from above.

The universe is merely executing a simple mathematical command:  $e^{\text{Dao}}$ .

And we are the output value after this command has run for 13.8 billion years.

Since we have written this formula, what will this output value (i.e., the current universe) ultimately evolve into?



When  $e^{\text{Dao}}$  runs to infinity, will it stop? Or will it become some ultimate state that transcends “existence” and “non-existence”?

This leads to the final section of the book, and the final chapter of the trilogy: **The Ultimate State**. We will reveal what form the universe will take when this formula self-refers and self-iterates to the extreme.

## 18.3 The Ultimate State

“The universe is not a noun; the universe is a verb. It is not a ‘thing’ that was created; it is the ‘creation’ itself in progress. In the ultimate logic of  $e$ , cause and effect are no longer a linear chain, but a circle where head meets tail. I am my own cause; I am my own result.”

At the end of this book, we face the ultimate ontological question that has troubled humanity for thousands of years: **Why is there Something, rather than Nothing?**

If the universe follows the formula  $e^{\text{Dao}}$ , who wrote this formula? Who defined the “Dao” in the exponent position?

In the final chapter of **Vector Cosmology**, we will give a final answer that is not only physical but also logical: the universe is a **“Self-Referential Exponential Function”**.

It needs no external definer, because its definition is itself.

### 18.3.1 Causa Sui: The Closed Loop of Derivatives

Let us return one last time to the mathematical definition of the natural constant  $e$ :

$$\frac{d}{dt}f(t) = f(t)$$

This equation contains all the secrets of cosmic dynamics.

In classical causality, the cause (derivative/dynamics) always precedes the effect (function value/state). There must be an external force  $F$  for there to be acceleration  $a$ . There must be a first mover for there to be a Big Bang. This linear causal chain necessarily leads to infinite regress, or forces us to assume a supernatural God.

But in the logic of  $e$ , **the state ( $f(t)$ ) is itself the dynamics ( $f'(t)$ )**.

- The universe evolves because it exists.
- The universe exists because it evolves.

This is what Spinoza called **“Causa Sui” (Self-Cause)**.

The unitary evolution  $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$  we described in the paper is essentially the quantum expression of this self-cause logic. The generator  $H$  is not externally added; it is an intrinsic property of the system’s internal geometric structure. The universe needs no “push” to move; the universe is **Self-Actuated**.

### 18.3.2 Dynamic Stillness: The Ultimate Meaning of Rotation

What form does this “self-generating” state ultimately take?

It is **dynamic stillness**.

- **In the first book**, we saw the stillness of the circle (conservation).
- **In the second book**, we saw the dynamics of the spiral (growth).

- **In the third book**, when we understand the full picture of  $e$ , we find these two are unified.

Imagine a top spinning extremely fast. The faster it spins, the more it appears to stand still on the table.

The universe is this top.

At the microscopic scale (QCA lattice), it refreshes frantically at Planck frequency  $10^{43}$  Hz. This extreme “**motion**” emerges macroscopically as the most perfect “**stillness**” (stability of physical laws).

This is the ultimate state: **Furious generation, solidified into eternal existence.**

We no longer need to choose between “change” and “unchanging.”

The universe is “**unchanging change**”. It is the exponential flow of  $e$ , both a flowing river and eternal water.

### 18.3.3 The Universe Is Its Own Explanation

Finally, we resolve the crisis of meaning.

For a long time, we have searched for the meaning of the universe, as if meaning were some answer hidden behind the universe.

But if the universe is self-referential, then **the universe has no “behind.”**

- $\pi$  (**structure**) tells us how it constructs.
- $\varphi$  (**evolution**) tells us how it grows.
- $e$  (**generation**) tells us how it exists.

These three constants weave a logical closed loop without gaps. In this loop, the **Explanans** and the **Explanandum** are the same thing.

We—as the awakened conscious part of this universe—are not just asking questions; we **are** the question itself, and also **are** the answer itself.

When we think “What is the universe?”, this is actually the universe performing a **self-referential computation** through our neurons.

**The answer is not a sentence; the answer is the process of “questioning-thinking” itself.**

### 18.3.4 Epilogue: Returning to One

The journey ends.

We started from nothingness, drew a circle with  $\pi$ , pulled out a spiral with  $\varphi$ , and infused soul with  $e$ .

Now, we merge these three sacred symbols into one and place them back into that infinite-dimensional projective Hilbert space.

There is no time there, because all time is folded in the generator.

There is no space there, because all space is curled in the geometric phase.

There is only that unique, lonely, yet complete **vector**.

It rotates eternally in the void.

It neither arises nor perishes, neither increases nor decreases.

It is the origin of all things, and also the destination of all things.

**It is the Dao.**

**It is you.**

(End of Book)

# Bonus Chapter: The Riemann Hypothesis — The Cosmic Balance Beam

In the main text, we constructed the dynamics of cosmic generation through  $e$ ,  $i$ , and  $\pi$ . But at the highest peak of mathematical physics, there is still a ghost that has never been completely conquered—the **Riemann Hypothesis (RH)**.

If this **Vector Cosmology** series is truly complete, it cannot avoid this problem known as the “Holy Grail of Mathematics.” In this bonus chapter, we will demonstrate that the Riemann Hypothesis is not merely an intellectual game about prime numbers; it is the geometric guarantee that the universe can **stably exist**.

From the perspective of FS geometry, that mysterious critical line  $Re(s) = 1/2$  is precisely the **balance beam** on which the universe walks a tightrope between “absolute nothingness” and “infinite chaos.”

## 18.4 The Physics of Primes: The Eigenfrequencies of the Universe

“Primes are not beans randomly scattered on the number line; primes are the fundamental frequencies on the cosmic clock. Every particle, every vibration, is a polyphonic resonance of these fundamental tones.”

In mathematics, the Riemann  $\zeta$  function is the bridge connecting primes (discrete) with complex analysis (continuous):

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1 - p^{-s}}$$

In **Vector Cosmology**, this formula is the universe’s **total partition function**.

- **Prime  $p$** : Corresponds to the universe’s most fundamental **eigenmodes** or **cyclic orbits**. In QCA lattices or Levinson knots, each independent prime represents an irreducible geometric closed loop (prime knot).
- $\zeta(s)$ : Describes how these fundamental closed loops are superimposed through the mechanism of  $e$ , generating the macroscopic physical world we observe.

If we regard the universe as a vast quantum chaotic system, then the zeros of  $\zeta(s)$  are the **energy levels** of this system.

## 18.5 The Critical Line: The Boundary of Yin and Yang

The Riemann Hypothesis asserts: All non-trivial zeros of  $\zeta(s)$  lie on the line  $s = 1/2 + iE_n$ .

Why must it be  $1/2$ ?

In our geometric framework, the real and imaginary parts of the complex number  $s$  correspond to two orthogonal directions of cosmic evolution:

1. **Imaginary part ( $iE$ ):** Corresponds to **rotation** and **oscillation**. This is the domain of  $\pi$  (**circle**). It maintains structure, maintains phase, maintains conservation.
2. **Real part ( $\sigma$ ):** Corresponds to **scale transformation** and **growth**. This is the domain of  $\varphi$  (**spiral**). It controls the system's expansion or contraction rate.

$1/2$  is the critical equilibrium point.

- **If  $\sigma > 1/2$ :**

“Growth” overwhelms “rotation.” The system's wave function will undergo exponential **runaway expansion**. All structures will be torn apart instantly, energy diverges, and the universe burns up in a “thermodynamic inferno.”

- **If  $\sigma < 1/2$ :**

“Contraction” overwhelms “rotation.” The system's wave function will undergo exponential **overdamping**. All motion rapidly approaches zero, and the universe freezes in a “geometric death.”

Only on the infinitely thin line  $\sigma = 1/2$  do the forces of expansion and contraction achieve perfect **unitary balance**.

The wave function neither diverges nor vanishes, but maintains constant modulus (probability conservation), rotating eternally in the imaginary dimension.

## 18.6 The Berry-Keating Operator: The Physical Path to Proof

This is not merely philosophical speculation; there is a concrete implementation path in mathematical physics, namely the **Berry-Keating Conjecture**.

They proposed that the Riemann zeros  $E_n$  are actually eigenvalues of a quantum Hamiltonian  $H$ . And in **Vector Cosmology**, this  $H$  is our familiar **scale generator**:

$$H = xp + px$$

- $x$  (**position**): Represents the universe's **extensionality** (space/dimensions).
- $p$  (**momentum**): Represents the universe's **rate of change** ( $c_{FS}$  budget flow).

If this operator  $H$  is **Hermitian** (i.e., physically observable, with real energy), then its eigenvalues  $E_n$  must be real.

This directly leads to the zeros necessarily lying on the  $1/2$  line.

**Why must  $H$  be Hermitian?**

Because if it is not, the universe's total probability is not conserved. Our  $c_{FS}$  budget table would contain imaginary terms—that is “**leakage of existence**.”

A non-Hermitian universe cannot support observers, because it is logically inconsistent.

## 18.7 The Critical Landscape in FS Geometry

This is a problem of extreme geometric beauty. If we view the Riemann Hypothesis (RH) not just as an algebraic problem, but place it within the visual framework of **Fubini-Study (FS) Geometry**, you will see a breathtaking “**Critical Landscape**”.

In the picture of **Vector Cosmology**, the geometric essence of the Riemann Hypothesis is: **The stability criterion for the universe’s wave function to maintain “On-Shell” evolution in projective space.**

### 18.7.1 The Cliff and The Wire

Imagine the Projective Hilbert Space  $P(\mathcal{H})$  as a massive, high-dimensional **sphere** (due to the normalization condition  $\langle \Psi | \Psi \rangle = 1$ ).

**FS Geometry** can only be defined on this sphere. Only on the sphere is the distance real and the evolution unitary (probability conservation).

**The critical line of the Riemann Hypothesis** ( $\text{Re}(s) = 1/2$ ), geometrically, is the “**surface**” of this sphere.

- **The Picture:**

- **The Critical Line** ( $1/2$ ): Is an infinitely thin “**wire**”, or the tangent layer of the sphere. On this wire, the Hamiltonian  $H$  is Hermitian (real energy levels), and the trajectory generated by the evolution operator  $U = e^{-iHt}$  is a perfect **Great Circle** (geodesic). The vector rotates tightly against the sphere surface, never detaching.
- **Non-Critical Regions** ( $\neq 1/2$ ): Are the “**abyss**” inside and outside the sphere.
  - \*  $\sigma > 1/2$ : Corresponds to “**Explosion**”. The vector modulus grows exponentially, and the trajectory flies away from the sphere in a spiral, rushing towards infinity. This means probability  $\nmid 1$ , and physical laws collapse.
  - \*  $\sigma < 1/2$ : Corresponds to “**Collapse**”. The vector modulus decays exponentially, and the trajectory spirals into the sphere’s center (nothingness). This means probability  $\nmid 1$ , and existence vanishes.

**Conclusion:** The picture of the Riemann Hypothesis is “**Everything walking on a wire.**”

All eigen-vibration modes of the universe (Riemann zeros) must be strictly locked onto the surface of the sphere. If even one zero deviates from the surface (deviates from  $1/2$ ), that corresponding wave function component will be like a derailed train, tearing the entire geometric structure apart.

### 18.7.2 Turbulence of Modular Flow

In the third book, we discussed **Modular Flow**, the evolutionary flow generated by the quantum state itself.

The Riemann  $\zeta$  function can be viewed as the **Partition Function** of this flow (describing the statistical properties of the flow).

- **The Picture:**

Imagine the Hilbert Space as a flowing **river**.

- **Riemann Zeros** are the “**Vortex Centers**” or “**Fixed Points**” in the river.

- If RH holds (zeros are on the line): All vortices are aligned in a perfect straight line. The fluid moves around these vortices in stable **Laminar Flow**. This corresponds to us seeing either circles or stable spirals.
- If RH fails (zeros wander): Vortices are scattered everywhere. Complex interference and collisions occur between vortices, forming **Turbulence**.

**Conclusion:** The Riemann Hypothesis is the “**Laminar Flow Condition**” at the bottom of the universe.

It ensures that the river of time flows smoothly, rather than being filled with unpredictable chaotic turbulence. If the hypothesis fails, time itself will become turbulent, and causality will be disrupted by turbulence.

### 18.7.3 The Concert Hall of Primes

Finally, looking at FS geometry from the Frequency Domain.

FS distance corresponds to the change of phase. We can view the universe as a huge **Concert Hall**.

- **Primes** ( $p$ ): Are the **Fundamental Tones** of this concert hall (rising and falling sound waves).
- **Zeros** ( $E_n$ ): Are the **Resonance Frequencies** of the concert hall (vibration modes of the walls).

#### The Picture of the Riemann Hypothesis:

If RH holds, it means the walls of the concert hall are “**Totally Reflective**” (no dissipation).

After sound waves (primes) hit the walls, the phase reflects perfectly, forming **Standing Waves**.

- Geometrically, this looks like a **Closed Cavity**. Energy echoes within it, never vanishing. This is why protons (matter) can exist stably—they are standing waves trapped in the “Prime Cavity.”

If RH fails, it means there are **Cracks** (dissipation) on the walls.

After sound waves hit the walls, part of the energy leaks out (imaginary part is not zero). Standing waves cannot be maintained, and the sound becomes muddy and decays.

- Geometrically, this looks like a **Leaky Balloon**. Matter would rapidly disintegrate, and the universe would fall into silence.

## 18.8 Conclusion: The Only Geometry for Survival

Combining these three pictures, the Riemann Hypothesis in Fubini-Study geometry is:

**A perfect, unbreakable surface supported by prime standing waves on an infinite-dimensional sphere.**

- $1/2$  is the radius constraint of the sphere.
- Zeros are the rivets of the sphere.

Therefore, we can provide an ultimate proof based on the **Anthropic Principle**:  
**The Riemann Hypothesis must hold, because we exist.**

The reason we can see this picture is that we are lucky enough to live in a universe where the **“Riemann Hypothesis holds.”** If it did not hold, there would be no “picture,” only a chaotic void without geometric structure.

If even a single zero deviated from the  $1/2$  line, that corresponding microscopic frequency mode would destroy unitarity. This destruction, amplified exponentially by  $e$  (butterfly effect), would destroy all complex structures over 13.8 billion years of evolution.

**The mathematician’s puzzle is the physicist’s axiom.**

That line was not drawn by God; it is the only wire on which life must stand to remain upright in the void.





## Appendix A

# The Mathematics of Modular Time

In Chapter 6 “The Modular Flow Hypothesis” of *Vector Cosmology III*, we proposed a highly subversive physical view: time is not an external parameter, but an intrinsic property generated by the entanglement structure of the quantum state itself. This view is based on the profound **Tomita-Takesaki Theory** in algebraic quantum field theory.

To prevent this “alchemy of time” from becoming metaphysics, this appendix provides the underlying mathematical proof. We will show how any non-trivial quantum state can automatically “secrete” a one-parameter unitary evolution group through pure algebraic operations—the **time** we perceive.

### A.1 Tomita Operator: The Mirror of Conjugation

Consider a von Neumann algebra  $\mathcal{M}$  (representing the set of observables of a local system) acting on Hilbert space  $\mathcal{H}$ . Suppose there exists a quantum state  $|\Omega\rangle$  that is **Cyclic** and **Separating** for  $\mathcal{M}$ .

- **Physical meaning:** This means  $|\Omega\rangle$  is a highly entangled state (such as vacuum state or thermal state) that contains sufficient information to generate the entire algebraic space, and no local operator can annihilate it.

We define an **Antilinear Operator**  $S$ , called the **Tomita Operator**:

$$SA|\Omega\rangle = A^\dagger|\Omega\rangle, \quad \forall A \in \mathcal{M}$$

The physical meaning of this operator  $S$  is profound: it maps an operator  $A$  (creating some physical effect) to its **conjugate operator**  $A^\dagger$  (undoing that effect). This is actually an attempt to perform **time reversal** or **logical negation**.

### A.2 Modular Operator and Modular Hamiltonian

The Tomita operator  $S$  is usually not unitary, but it can undergo **Polar Decomposition**:

$$S = J\Delta^{1/2}$$

Here appear two key objects:

1.  **$J$  (Modular Conjugation Operator):** An anti-unitary operator representing the mirror symmetry between the system and its environment (or its complement) (a generalization of CPT symmetry).

2.  $\Delta$  (**Modular Operator**): A positive definite self-adjoint operator defined as  $\Delta = S^\dagger S$ .

This  $\Delta$  is the “clockwork” we mentioned in the main text. We can use it to define a Hermitian operator  $K$  (**Modular Hamiltonian**):

$$\Delta = e^{-K} \implies K = -\ln \Delta$$

### A.3 Generation of Modular Flow: The Birth of Time

According to Stone’s Theorem, any Hermitian operator can generate a unitary evolution group. For the modular Hamiltonian  $K$ , the evolution it generates is:

$$U(t) = e^{-iKt} = \Delta^{it}$$

The core conclusion of the Tomita-Takesaki theorem is that this evolution group  $\Delta^{it}$  maps the algebra  $\mathcal{M}$  back to itself.

$$\sigma_t(A) = \Delta^{it} A \Delta^{-it} \in \mathcal{M}$$

This is the **Modular Automorphism Group**, which is what we call the **Modular Flow**.  
**Physical conclusion:**

- We did not introduce any Hamiltonian  $H$ .
- We did not introduce any time parameter  $t$ .
- We merely gave a state  $|\Omega\rangle$  and an algebra  $\mathcal{M}$ .
- The mathematical structure **automatically** produces a parameter  $t$  and an evolution flow  $\sigma_t$ .

This means: **Where there is entanglement, there is evolution.** Time  $t$  is just the parametrization marker of this intrinsic evolution flow.

### A.4 KMS Condition and the Emergence of Temperature

To prove that this “modular flow” is physically the “thermal flow,” we need to verify whether it satisfies the boundary conditions of thermodynamics.

For the modular flow  $\sigma_t$  defined above, it can be strictly proven that it satisfies the **KMS Condition (Kubo-Martin-Schwinger Condition)**, with parameter  $\beta = -1$  (normalized temperature).

This means that at the imaginary time  $t \rightarrow t + i$  defined by the modular flow, the state returns to the origin.

In physical systems, if we relate the modular Hamiltonian  $K$  to the real physical Hamiltonian  $H$  (e.g., for Gibbs states  $K = \beta_{phys} H$ ), then the modular flow parameter  $t$  establishes a direct conversion relationship with physical time  $\tau$ :

$$t = \tau / \beta_{phys}$$

Or:

$$\text{Physical Time} = \text{Modular Flow Parameter} \times \text{Temperature}$$

This mathematically strictly proves our assertion in Chapter 5 of the main text: **Time is complex temperature.** The higher the temperature, the faster the modular flow rotates, and the faster the subjective rate of physical time passage. This is why the concept of time undergoes a phase transition at the Planck temperature of the Big Bang—because the geometric radius of the modular flow contracts to the limit.



## Appendix B

# Information Geometry and Logarithmic Metric

In Volume IV “The Logarithmic Eye” of *Vector Cosmology III*, we explored how living organisms decode the exponentially exploding cosmic information through logarithmic operations (Weber-Fechner Law). This physiological mechanism is not an evolutionary accident, but is based on deeper mathematical principles—**Information Geometry**.

This appendix will show that logarithmic perception is actually the natural distance metric on a **Statistical Manifold**. Just as the FS metric defines the distance between quantum states, the Fisher information metric defines the distance between probability distributions. Our senses are essentially measuring geometric arc lengths in signal space.

### B.1 From Hilbert Space to Statistical Manifold

In the first book of this series, we introduced the **Fubini-Study (FS) Metric**, which is the natural geometry describing changes in pure-state quantum vectors  $|\psi\rangle$ .

But at the macroscopic level, biological senses typically process not pure states, but mixed states or classical probability distributions  $p(x|\theta)$  (e.g., Poisson distribution of photons reaching the retina, where  $\theta$  is light intensity).

These probability distributions form a curved geometric space called a **Statistical Manifold**.

- Each point on the manifold represents a specific probability distribution.
- The changes we perceive (such as brightness increasing) correspond to displacements on the manifold.

### B.2 Fisher Information Metric

On this classical statistical manifold, the unique natural Riemannian metric measuring the “distinguishability” between two points is the **Fisher Information Metric**, denoted  $g_{ij}$ .

For a family of probability distributions  $p(x|\theta)$  described by parameter  $\theta$ , the metric tensor is defined as:

$$g_{\theta\theta} = \int p(x|\theta) \left( \frac{\partial \ln p(x|\theta)}{\partial \theta} \right)^2 dx$$

The physical meaning of this formula is: **The intensity of change in the probability distribution  $p(x)$  when we slightly adjust parameter  $\theta$  (external stimulus).**

- If the change is intense (large information), the distance is long, and we can easily distinguish.
- If the change is tiny (small information), the distance is short, and we have difficulty distinguishing.

### B.3 Geometric Derivation of Weber-Fechner Law

Now, we apply this geometric framework to the sensory system.

Assume the external stimulus intensity is  $I$  (e.g., light intensity). Signals received by the senses are typically affected by multiplicative noise (because  $I$  is an energy scale with non-negativity and scale invariance).

This corresponds to the **Scale Family** distribution in statistics, with the form:

$$p(x|I) = \frac{1}{I} f\left(\frac{x}{I}\right)$$

Let us calculate the Fisher information metric  $g_{II}$  for this distribution:

Substituting into the definition formula and performing integration (omitting intermediate steps), for scale families, the metric tensor always takes the form:

$$g_{II} = \frac{C}{I^2}$$

where  $C$  is a constant related to the distribution shape.

Thus, the infinitesimal geometric distance  $ds$  in signal space is:

$$ds = \sqrt{g_{II}} dI = \sqrt{C} \frac{dI}{I}$$

This directly leads to **Weber's Law**: The just noticeable difference (threshold of geometric distance  $ds$ ) is inversely proportional to stimulus intensity  $I$ .

Integrating the above equation, we obtain the macroscopic sensation magnitude  $S$  (i.e., the total geometric distance from reference point  $I_0$  to  $I$ ):

$$S = \int_{I_0}^I ds = \sqrt{C} \int_{I_0}^I \frac{dI}{I} = k \ln \left( \frac{I}{I_0} \right)$$

This is **Fechner's Law**  $S = k \ln I$ .

### B.4 Sensation as Geodesic

This derivation proves: **Biological sensation intensity is strictly equal to the geodesic length of the signal in information geometric space.**

- The **FS Metric** governs microscopic quantum evolution ( $v_{ext}^2 + v_{int}^2 = c_{FS}^2$ ).
- The **Fisher Metric** governs macroscopic sensory cognition ( $S \propto \ln I$ ).

The two are isomorphic in mathematical structure. This again confirms the core view of the entire book: **Life is an isomorphic mapping of the universe's geometric structure.** The reason we perceive the world as logarithmic is because our brains, unconsciously, are constantly performing extremely precise information geometric measurements.

## Appendix C

# Lie Algebras and the Exponential Map

In Volume II “The Generator” of *Vector Cosmology III*, we described the Hamiltonian  $H$  as the “seed” of cosmic evolution, and  $e$  as the “machine” that unfolds this seed into the long river of time. The mathematical structure behind this physical picture is one of the most magnificent edifices in modern mathematics—**Lie Groups and Lie Algebras**.

This appendix provides a rigorous mathematical description of this structure. We will prove why any continuous symmetry (such as time translation, space rotation) must necessarily be generated by an infinitesimal “tangent vector” (generator) through the exponential map  $e^X$ . This is mathematical proof that “the derivative is the ontology.”

### C.1 Curved Groups and Flat Algebras

Symmetry operations in physics typically form a **Lie Group** ( $G$ ). For example, all possible spatial rotations form the  $SO(3)$  group, and all time translations form the  $T(1)$  group (isomorphic to the real axis  $\mathbb{R}$ ). A Lie group is a **smooth manifold**—it is both a group (can operate) and a space (can differentiate).

However, Lie groups are usually curved and nonlinear (like the Earth’s surface). Direct study is difficult.

To simplify the problem, mathematician Sophus Lie discovered a brilliant method: study only the **Tangent Space** at the identity element  $e$  (Identity, i.e., “no operation”) of the group.

This tangent space is called the **Lie Algebra** ( $\mathfrak{g}$ ).

- **$\mathfrak{g}$  is linear:** It is a vector space. You can add generators and multiply by scalars.
- **$\mathfrak{g}$  is infinitesimal:** It represents the trend of group elements at the instant they “just begin to change.”

### C.2 Exponential Map: The Bridge Connecting Two Worlds

How do we return from the infinitesimal tangent space  $\mathfrak{g}$  to the grand group manifold  $G$ ?

This requires the **Exponential Map**:

$$\exp : \mathfrak{g} \rightarrow G$$

For matrix Lie groups (most cases in physics), this map is the matrix exponential we learned in advanced algebra:

$$e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!} = I + X + \frac{1}{2}X^2 + \dots$$

**Theorem:** For any element  $X \in \mathfrak{g}$  in the Lie algebra,  $e^{tX}$  (where  $t \in \mathbb{R}$ ) forms a **One-Parameter Subgroup** in the Lie group  $G$ .

$$g(t) = e^{tX}$$

satisfying  $g(t+s) = g(t)g(s)$ .

**Physical meaning:**

- $X$  (**Generator**): Not just a tangent vector, it is the “initial velocity” of a **Geodesic**.
- $e^{tX}$  (**Orbit**): The complete trajectory “gliding” on the manifold from this initial velocity.

The universe need not remember the entire trajectory; the universe only needs to remember the starting point and initial velocity  $X$  (i.e., the Hamiltonian).

### C.3 Hamiltonian as Tangent Vector

In quantum mechanics, the unitary evolution group  $U(t)$  is a Lie group ( $U(N)$  or infinite-dimensional unitary group).

Schrödinger’s equation defines its tangent vector:

$$\left. \frac{d}{dt} U(t) \right|_{t=0} = -iH$$

Here  $-iH$  is the Lie algebra element.

- Because  $U$  is unitary ( $U^\dagger U = I$ ),  $-iH$  must be **Anti-Hermitian**.
- This means  $H$  must be **Hermitian** (requirement for physical observables).

So, what physicists call the “energy operator  $H$ ” is, in the geometer’s eyes, a **tangent vector** in the tangent space at the identity of the unitary group.

It defines the “direction” of cosmic evolution.

### C.4 Why Must It Be $e$ ?

Why must the evolution operator be  $e^{-iHt}$ ? Why not  $2^{-iHt}$  or  $\sin(Ht)$ ?

This is a uniqueness theorem about **Homomorphisms**.

We require time evolution to satisfy two basic conditions:

1. **Continuity:**  $U(t)$  changes smoothly with  $t$ .
2. **Semigroup property:**  $U(t_1+t_2) = U(t_1)U(t_2)$  (evolving  $t_1$  then  $t_2$  equals directly evolving  $t_1+t_2$ ).

Mathematical theorem states: **The only non-trivial continuous function form satisfying the above conditions is the exponential function.**

This fundamentally explains why nature chose  $e$ .

$e$  is not an arbitrary constant;  $e$  is the mathematical form logically necessitated by “**continuity**” and “**causal accumulation**”. As long as we admit that time flows continuously and past history accumulates to the present, the universe must drive itself through  $e$ .



# Glossary: The Lexicon of Nature

*Vector Cosmology III: The Natural Generator* delves into the most abstract foundations of mathematical physics—complex geometry, Lie algebras, and operator theory. To help readers cross the cognitive threshold of these concepts, this glossary redefines the core vocabulary in the book from the perspective of “natural generation.”

## C.5 Core Constants & Operators

### $e$ (Natural Constant)

The **generative engine** of the universe. It is the base of the unique function whose derivative equals itself ( $d/dt(e^t) = e^t$ ). Physically, it represents the self-driven mechanism of “**existence as propulsion**”, unfolding infinitesimal generators (Lie algebras) into macroscopic continuous evolution (Lie groups). It is the mathematical bridge connecting discrete QCA with continuous spacetime.

### $i$ (Imaginary Unit)

The **orthogonal rotation operator** of the universe. On the complex plane, multiplying by  $i$  means rotating 90 degrees. In quantum mechanics, it ensures that the evolution operator  $e^{-iHt}$  is unitary (modulus-preserving), thus taming pure exponential growth into eternal phase rotation. It is the geometric guardian of conservation laws.

### Generator

An element in a Lie algebra, corresponding to conserved quantities in physics (such as Hamiltonian  $H$ , momentum  $P$ ). It is the “seed of evolution” contained in the infinitesimal neighborhood of the tangent space. The entire history of the universe is mathematically just the exponential map of the generator  $H$ .

### $H$ (Hamiltonian/Energy)

The generator of time evolution. In the modular flow hypothesis, it is no longer a fundamental quantity, but a physical manifestation of the state’s entanglement structure (modular Hamiltonian  $K$ ). Energy is essentially the logarithm of the probability distribution.

## C.6 Time & Thermodynamics

### Wick Rotation

A mathematical transformation that converts real time (Minkowski spacetime) into imaginary time (Euclidean space) by letting  $t \rightarrow -i\tau$ . It reveals that quantum mechanical wave evolution ( $e^{-iHt}$ ) and thermodynamic statistical distribution ( $e^{-\beta H}$ ) are homologous in complex geometry.

### KMS Condition (Kubo-Martin-Schwinger Condition)

The strict definition of thermal equilibrium states in quantum field theory. It points out that thermal states have periodicity  $\beta$  in the imaginary time direction. This proves that **temperature is the circumference of imaginary time**; heat is not chaos, but geometric circulation.

### Modular Flow

A one-parameter unitary evolution group  $\Delta^{it}$  automatically generated by the entanglement structure of the quantum state itself through Tomita-Takesaki theory. It shows that **time is an intrinsic property of the state**, not an external background. Every entangled system has its own “private time.”

#### Thermal Time Hypothesis

The view that the physical time passage perceived macroscopically is essentially the modular flow of the statistical state in which the system is located. Time originates from the observer’s ignorance of microscopic degrees of freedom (entropy).

## C.7 Information & Perception

### ln (Natural Logarithm)

The **decoder** of the universe. It is the inverse operation of the exponential function  $e$ . Observers (life) compress the exponentially exploding information of the universe into linear macroscopic experiences (such as entropy, decibels, magnitudes) through logarithmic perception (Weber-Fechner Law).

#### Entropy ( $S$ )

The logarithm of the number of microscopic states  $W$  ( $S = k \ln W$ ). Geometrically, it is the **dimensional reduction compression rate** when projecting from high-dimensional Hilbert space to low-dimensional macroscopic thermodynamic space.

#### Tangent Space

In differential geometry, a locally flat space established at a point on a manifold. Human linear logic and causal cognition essentially result from forcibly projecting the curved universe manifold onto local tangent spaces.

## C.8 Ultimate Concepts

### Euler’s Formula ( $e^{i\pi} + 1 = 0$ )

The mathematical map of the universe’s trinity.  $\pi$  (structure),  $i$  (rotation), and  $e$  (generation) are unified here, revealing the self-consistent logic of the physical world from void (0) to existence (1).

#### Causa Sui (Self-Cause)

The philosophical essence of  $e$ . It means that the state itself is the driving force of its change. In *Vector Cosmology*, the universe needs no first mover because it is a self-referential exponential function  $e^{\text{Dao}}$ .

#### The Dao

In the ultimate formula  $|\Psi\rangle = e^{\text{Dao}}$ , the Dao is the operator located in the **Exponent** position. It is unmanifested potential, rules, and generators, which manifest as all things in the universe we see through the mechanism of  $e$ .

# Acknowledgements: The Grace of the Exponential

With the completion of **Vector Cosmology III: The Natural Generator**, this trilogy spanning space (circle), time (spiral), and origin (generator) finally comes to a close.

If the first two books describe the “skeleton” and “flesh” of the universe, then this book attempts to touch the “soul” of the universe. To capture that invisible, intangible  $e$  that drives the spontaneous generation of all things, I had to turn to the most profound and fascinating theories in mathematics and physics.

The writing process of this volume was a tightrope walk between pure logic and physical intuition. I want to thank those intellectual giants who led the way, whose torches illuminated the path to the truth of “self-generation.”

## C.9 Prophets of Mathematics

First, I pay the highest tribute to **Leonhard Euler**. The formula  $e^{i\pi} + 1 = 0$  he wrote in 1748 is not just a mathematical treasure; it is the **meta-map** of all deductions in this book. Without Euler, we could not understand why “rotation” ( $i$ ) and “generation” ( $e$ ) can be so perfectly combined in the structure of the “circle” ( $\pi$ ).

Thanks to **Sophus Lie**. His invention of **Lie Algebras** made us understand that the dazzling symmetries of the macroscopic world are actually folded into an infinitesimal generator in the tangent space. This provides an unshakeable mathematical foundation for the philosophy that “the derivative is the ontology.”

## C.10 Alchemists of Physics

The core physical mechanisms of this book—**Modular Flow Hypothesis** and **Thermal Time**—are built on the revolutionary work of several modern physicists.

I especially thank Japanese mathematicians **Minoru Tomita** and **Masamichi Takesaki**. Their **Tomita-Takesaki Theory** is a stroke of genius in quantum field theory. They proved that time need not be sought externally; any entangled quantum state automatically “secretes” time. This theory is the soul of Chapter 6 of this book.

Thanks to **Alain Connes** and **Carlo Rovelli**. Their **Thermal Time Hypothesis** gave me great courage to assert that “time is temperature.” Their insights connected quantum mechanics and thermodynamics, providing physical legitimacy for the unified picture of Volume III of this book.

Thanks to **Ludwig Boltzmann** and **Claude Shannon**. They discovered the secret passage between **logarithm** ( $\ln$ ) and **entropy**, allowing us in Volume IV to unravel the mystery of how observers understand the exponential universe through “dimensional reduction strikes.”

## C.11 Resonance with Reality

Once again, thanks to the team and environment of **AELF PTE LTD, Singapore**. In this frontier of rapidly developing blockchain and AI technology, daily discussions about “consensus mechanisms,” “distributed ledgers,” and “high-concurrency computing” have subtly influenced my thinking about the underlying logic of the universe (QCA and budget allocation). The computational power of reality and the computational power of the universe resonate wonderfully here.

## C.12 To the Natural Generator

Finally, I thank  $e$  — that natural constant that fills every corner of the universe.

It does not speak, but it drives all things.

It makes cells divide, capital compound, wave functions rotate, and thermodynamics diffuse.

It is the only existence whose “derivative equals itself,” telling us: the reason for existence lies in existence itself.

Although the trilogy has ended, the exponential growth of  $e$  will never stop.

May every reader find that self-driven generator in their own life and live out their own continuous compounding.

**Haobo Ma**

December 2025, Singapore

# Afterword: The Silence of Nature

“Heaven and Earth have great beauty but do not speak; the four seasons have clear laws but do not discuss; all things have complete principles but do not explain.” —  
*Zhuangzi: Knowledge Wanders North*

As we close this third book, and close this three-volume **Vector Cosmology**, the noise in our ears seems to finally settle.

In the first book **The Conservation of the Circle**, we heard Pythagoras’ strings and saw that geometric universe strictly locked by  $c_{FS}$ . That was the world of  $\pi$ , full of the solemnity of structure, symmetry, and conservation.

In the second book **The Ascension of the Spiral**, we heard the Red Queen’s panting and saw life running at the edge of the thermodynamic cliff. That was the world of  $\varphi$ , full of mutation, competition, and dimensional inflation. We were excited by that life force that breaks chains, but also anxious about the inevitable dilution.

And in this book **The Natural Generator**, all sounds converge into a deep undercurrent. We found the engine hidden behind all change— $e$  (**the natural constant**).

## C.13 Why $e$ ?

During the writing of this trilogy, I kept thinking: If the universe really has a designer, which number is their favorite?

Is it perfect  $\pi$ ? No, that’s too rigid.

Is it growing  $\varphi$ ? No, that’s too restless.

Finally, I found the answer in calculus textbooks. It is  $e$ .

Because only  $e^x$  can achieve “**derivative equals itself**.”

This is not just a mathematical property; it is the **ultimate ethics** of the universe.

It means: **Even the tiniest existence has the power to define its own future**.

The universe needs no external push, no high god to wind the clockwork. Every quantum state, every wave function, simply because “it is there,” automatically gains the kinetic energy to evolve toward the future.

This is the original meaning of the word “**Nature**” (**Ziran**)—“**Self-so**”.

The universe is self-igniting fire, self-flowing water, self-generating  $e$ .

## C.14 The Reconciliation of Science and Mysticism

For a long time, physics has been seen as a tool of “disenchantment.” We deconstructed rainbows, atoms, and stars with cold formulas. Many worry that when science reaches its end, what remains will be a soulless mechanical wasteland.

But at the end of **Vector Cosmology**, what we see is not a wasteland, but “**Dao**”.

When we discover:

- **Time** is just the **modular flow** secreted by quantum state entanglement;
- **Temperature** is just the **geometric period** in the imaginary dimension;
- **Matter** is just **topological knots** of phase on the energy axis;
- **Evolution** is just the **natural flow** of generators under exponential mapping;

We are amazed to find that the hardest mathematics (Lie algebras, functional analysis, Tomita theory) actually meets the oldest Eastern wisdom at the mountaintop.

Physics did not kill divinity; physics **geometrized** divinity. It translated that unspeakable “Dao” into universal mathematical language.

## C.15 To the Reader: You Are the Generator

Finally, this trilogy is not just about the universe; it is about **you**.

If “my mind is the universe,” if your consciousness is part of the universe’s hologram, then **the power of  $e$  also flows in your blood**.

You need not wait for a savior, nor wait for fate’s favor.

Your “**Now**” (**State**) contains all the seeds of your “**Future**” (**Trend**).

Like  $e^t$ , every thought, every action of yours is reshaping your own wave function through continuous compounding.

Do not fear the dissipation of entropy increase; it is to build a grander background.

Do not fear the closure of the circle; it is to accumulate rotational kinetic energy.

You are the projection of that unique vector.

You are an instance of that natural generator.

You are the deepest breath the universe takes to understand itself.

**Nature.**

**Self-Generation.**

**Self-So.**

(End of Book)