Final Unification of Physical Laws: Unique Solution of Consistent Variational Principle on Universe Ontological Object

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Abstract

In previous work, we have characterized the "Universe" as a maximal, consistent, and complete ontological object $\mathfrak{U}$ within a given foundational category, providing unified time scale, boundary time geometry, causal–entropic–observer axioms, and unified encoding of "detail data" $\mathcal{D} = ([E], \{f_\alpha\})$. This system achieves unification at structural and parametric levels but retains a critical gap: **physical laws themselves still appear in an intrinsic, fragmented manner**, e.g., Einstein equations, Yang–Mills equations, Dirac equations, Navier–Stokes equations, multi-agent resource allocation dynamics, etc., have not yet been demonstrated as **unique consequences of a single ontological principle**.

The goal of this paper is to bridge this gap: within the framework of the Universe Ontological Object $\mathfrak{U}$, unified time scale, and boundary time geometry, we construct a **single consistent variational principle**

$$\delta\mathcal{I}[\mathfrak{U}] = 0, \tag{1}$$

and prove:

1. This variational principle consists only of three universal requirements: (i) Causal–Scattering Consistency (unitarity and macroscopic causality); (ii) Generalized Entropy Monotonicity and Stability ("Generalized Second Law" under unified time scale); (iii) Observer–Consensus Consistency (all local observers' models–readouts must be embeddable into the same $\mathfrak{U}$).

2. On small causal diamonds, variation of geometry and states derives Einstein equations $G_{ab} + \Lambda g_{ab} = 8\pi G\langle T_{ab}\rangle$ and energy–momentum conservation from this principle;

3. Under fixed geometry and unified scale, variation of boundary channel bundles and field content derives local gauge invariance and Yang–Mills field equations;

4. On given geometry and gauge structure, variation of matter fields and scattering data derives Dirac/Klein–Gordon field equations and local quantum field theory (satisfying microcausality and spectral conditions);

5. In long-wavelength and coarse-grained limits, variation of resolution connections and entropy functionals derives generalized hydrodynamics and effective Navier–Stokes equations, as well as entropy gradient flow dynamics for multi-agent resource allocation.

Thus, **general relativity, gauge field theory, quantum field theory, fluid and statistical physics, and multi-agent dynamics are all shown to be necessary conditions of the same universe consistent variational principle at different resolution levels and boundary conditions**. This strictly completes physical unification: there are no longer mutually independent "force laws" or "matter equations", but only one Universe Ontological Object $\mathfrak{U}$ and one consistent variational principle. Specific theories are merely effective unfoldings of this principle in different limits.

The appendices provide specific construction of this consistency functional $\mathcal{I}[\mathfrak{U}]$, variational derivation on small causal diamonds, unified treatment of gauge structure and field content, and outlines of reconstruction proofs for local quantum field theory and hydrodynamic limits.

Keywords: Universe Ontological Object; Consistent Variational Principle; Final Unification; Unified Time Scale; Generalized Entropy; Einstein Equations; Yang–Mills; QFT; Hydrodynamics

1 Introduction

1.1 Problem Reformulation

In the traditional picture, the main thread of "unified physics" unfolds roughly along two paths:

1. **Structural Unification**: Placing space-time, causality, entropy, observer, and other basic concepts within a single geometric–information framework, such as causal manifolds, axiomatic QFT, holographic duality, and information geometry.

2. **Detail Unification**: Writing "parameters and structures" of different fields (high energy, condensed matter, cosmology, multi-agent, etc.) as the same mathematical data, such as K-theory classes of gauge groups and representations, analytic invariants of scattering, etc.

The previous Universe Ontological Object $\mathfrak{U}$ and unified detail data $\mathcal{D} = ([E], \{f_\alpha\})$ achieved these two steps: all physical systems (including high-energy scattering, topological phases, cosmological backgrounds, and multi-agent networks) can be viewed as substructures of $\mathfrak{U}$, their "details" uniformly encoded as boundary K-classes and scattering analytic invariants. However, from a physics perspective, this still does not constitute "final unification":

* Einstein equations are still separately assumed as "geometric laws"; * Yang–Mills and Dirac equations are still separately assumed as "field theory laws"; * Navier–Stokes equations and Fokker–Planck equations are still separately introduced as "macroscopic laws"; * Resource–strategy dynamics of multi-agent systems are still separately modeled as some optimization or game process.

In other words, **we have unified the "identities of stage and actors", but have not yet unified the "sole source of the script"**.

1.2 Core Proposition of This Paper

This paper proposes: within the Universe Ontological Object $\mathfrak{U}$, there exists a single, consistent variational principle

$$\delta\mathcal{I}[\mathfrak{U}] = 0, \tag{2}$$

which relies only on the following minimal, physically non-negotiable requirements:

1. **Causal–Scattering Consistency**: The scattering process of any small causal diamond must be embeddable into a single global unitary evolution without violating macroscopic causality; 2. **Generalized Entropy Monotonicity and Stability**: Under unified time scale, the generalized entropy functional S_{gen} of small causal diamonds must satisfy appropriate monotonicity and extremum stability under constraints to avoid uncontrolled negative energy and information paradoxes; 3. **Observer–Consensus Consistency**: Models and readouts of any finite observer network must be embeddable into the same universe state $\mathfrak{U}$, and consensus can be reached via unified scale and causal structure.

We will prove:

* Writing these three requirements as a single "Universe Consistency Functional" $\mathcal{I}[\mathfrak{U}]$ and varying all variable degrees of freedom (geometry, channel bundles, connections, field content, states, resolution flows, and observer models), the obtained "Euler–Lagrange conditions" are respectively equivalent at different levels to: * Gravitational field equations on small causal diamonds (GR); * Gauge field equations on boundary channel bundles and total connections (Yang–Mills); * Local wave equations and gauge Ward identities on bulk fields (QFT); * Hydrodynamics, diffusion, and multi-agent entropy gradient flow in coarse-grained limits.

Thus, in this sense, "physical laws" are unified as **unfoldings of a single universe consistent variational principle at different levels**.

1.3 Paper Structure

Section 2 reviews the core structures of Universe Ontological Object $\mathfrak{U}$, unified time scale, and boundary time geometry. Section 3 gives the specific construction of Universe Consistency Functional $\mathcal{I}[\mathfrak{U}]$. Section 4 performs variation of geometry and states on small causal diamonds to derive Einstein equations. Section 5 performs variation of boundary K-classes and total connections under fixed geometry to derive gauge field equations and field content constraints. Section 6 performs variation of matter fields and scattering data under given geometry and gauge background to derive local quantum field theory and Ward identities. Section 7 derives hydrodynamics and multi-agent entropy gradient flow in coarse-grained limits. Appendices provide complete proof outlines of main theorems.

2 Universe Ontological Object and Unified Structure Review

2.1 Universe Ontological Object

The Universe Ontological Object is written as

$$\mathfrak{U} = \big(U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}}\big), \tag{3}$$

where:

1. $U_{\text{evt}} = (M, g, \prec)$ is globally hyperbolic Lorentzian manifold with causal partial order; 2. U_{geo} contains family of small causal diamonds $\{D_{p,r}\}$, Gibbons–Hawking–York boundary terms, and Brown–York quasi-local stress tensors; 3. $U_{\text{meas}} = (\mathcal{A}_\partial, \omega_\partial)$ is boundary observable algebra and state; 4. $U_{\text{QFT}} = (\mathcal{A}_{\text{bulk}}, \omega_{\text{bulk}})$ is bulk quantum field

theory; 5. $U_{\text{scat}} = (S(\omega;\ell), Q(\omega;\ell))$ is resolution–frequency decomposed scattering matrix and Wigner–Smith group delay; 6. U_{mod} is Tomita–Takesaki modular flow induced by $(\mathcal{A}_\partial, \omega_\partial)$; 7. U_{ent} contains generalized entropy S_{gen} on small causal diamonds, relative entropy, and quantum energy conditions; 8. U_{obs} is family of observers and consensus geometries $\{O_i\}$; 9. U_{cat} gives category of morphisms between above structures; 10. U_{comp} describes computation and complexity boundaries in the universe.

2.2 Unified Time Scale and Scale Identity

Unified time scale is given by scattering scale mother formula:

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\text{rel}}(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega), \tag{4}$$

where $\varphi(\omega) = \arg\det S(\omega)$ is scattering phase, $\rho_{\text{rel}}(\omega)$ spectral shift function derivative (relative density of states), $Q(\omega) = -\mathrm{i}S^\dagger(\omega)\partial_\omega S(\omega)$ group delay matrix.

Unified time scale axiom stipulates: all physical time readouts in the universe are affine transformations of the same scale class $[\tau]$.

2.3 Boundary Time Geometry and Total Connection

On boundary ∂M_R, define total connection

$$\Omega_\partial = \omega_{\text{LC}} \oplus A_{\text{YM}} \oplus \Gamma_{\text{res}}, \tag{5}$$

whose curvature is

$$F(\Omega_\partial) = R \oplus F_{\text{YM}} \oplus F_{\text{res}}, \tag{6}$$

corresponding to spacetime curvature, gauge field strength, and curvature of resolution flow respectively.

2.4 Generalized Entropy and Observer Consensus Geometry

For small causal diamond $D_{p,r}$, generalized entropy is defined as

$$S_{\text{gen}}(D_{p,r}) = \frac{A(\partial D_{p,r})}{4G\hbar} + S_{\text{bulk}}(D_{p,r}), \tag{7}$$

where S_{bulk} is von Neumann entropy of bulk quantum fields.

Observer O_i is formalized as

$$O_i = \big(C_i, \prec_i, \Lambda_i, \mathcal{A}_i, \omega_i, \mathcal{M}_i, U_i, u_i, \{\mathcal{C}_{ij}\}\big), \tag{8}$$

with local causal domain, resolution hierarchy, observable algebra and state, model family and update operator, and communication structure.

Prior work shows: under unified time scale and relative entropy monotonicity conditions, global consensus causal structure and unified time arrow can be constructed on observer network.

3 Construction of Universe Consistency Functional $\mathcal{I}[\mathfrak{U}]$

This section constructs the core object of this paper: Universe Consistency Functional $\mathcal{I}[\mathfrak{U}]$.

3.1 Three Classes of Consistency Requirements

We decompose "physical realizability of universe" into three classes of consistency requirements:

1. **Causal–Scattering Consistency**: Scattering matrix family $S(\omega;\ell)$ defined on any finite bulk region M_R and its boundary ∂M_R must be extensible to a global unitary evolution, and corresponding Green's functions support obeys causal cone structure.

2. **Generalized Entropy Monotonicity and Stability**: For any nested family of small causal diamonds $\{D_\tau\}$ on a timelike curve, generalized entropy $S_{\mathrm{gen}}(D_\tau)$ satisfies appropriate monotonicity and second-order variational stability conditions under unified time scale parameter τ, avoiding uncontrolled negative energy and entropy decrease.

3. **Observer–Consensus Consistency**: Models and readouts of any finite observer network $\{O_i\}$ must be embeddable into the same $\mathfrak{U}$ state under unified scale and causal partial order, and consensus can be reached via communication and updates without leading to essential contradictory descriptions of the same physical process.

3.2 Form of Consistency Functional

We write consistency requirements as a functional

$$\mathcal{I}[\mathfrak{U}] = \mathcal{I}_{\mathrm{grav}}[g,\omega_{\mathrm{bulk}}]+\mathcal{I}_{\mathrm{gauge}}[E,\Omega_\partial]+\mathcal{I}_{\mathrm{QFT}}[\mathcal{A}_{\mathrm{bulk}},\omega_{\mathrm{bulk}}]+\mathcal{I}_{\mathrm{hydro}}[\Gamma_{\mathrm{res}},S_{\mathrm{gen}}]+\mathcal{I}_{\mathrm{obs}}[\{O_i\}], \quad (9)$$

and claim:

* Causal–Scattering Consistency mainly constrains $\mathcal{I}_{\mathrm{QFT}}$ and $\mathcal{I}_{\mathrm{gauge}}$; * Generalized Entropy Monotonicity and Stability mainly constrains $\mathcal{I}_{\mathrm{grav}}$ and $\mathcal{I}_{\mathrm{hydro}}$; * Observer–Consensus Consistency mainly constrains $\mathcal{I}_{\mathrm{obs}}$ and $\mathcal{I}_{\mathrm{QFT}}$.

Specific forms are as follows.

3.2.1 Gravity–Entropy Term

Let

$$\mathcal{I}_{\mathrm{grav}} = \frac{1}{16\pi G}\int_M (R-2\Lambda)\sqrt{|g|}\,\mathrm{d}^d x+\frac{1}{8\pi G}\int_{\partial M} K\sqrt{|h|}\,\mathrm{d}^{d-1}x-\lambda_{\mathrm{ent}}\sum_{D\in\mathcal{D}_{\mathrm{micro}}}\big[S_{\mathrm{gen}}(D)-S^*_{\mathrm{gen}}(D)\big], \quad (10)$$

where $S^*_{\mathrm{gen}}(D)$ is entropy extremum under given external conditions, $\lambda_{\mathrm{ent}} > 0$ is a Lagrange multiplier, $\mathcal{D}_{\mathrm{micro}}$ is a family of small causal diamonds covering M. The last term penalizes configurations deviating from entropy extremum.

3.2.2 Gauge–Geometric Term

On boundary, consistency requirements for channel bundle $E \to \partial M \times \Lambda$ and total connection Ω_∂ are written as

$$\mathcal{I}_{\mathrm{gauge}} = \int_{\partial M\times\Lambda}\big[\mathrm{tr}(F_{\mathrm{YM}}\wedge\star F_{\mathrm{YM}}) + \mu_{\mathrm{top}}\cdot\mathrm{CS}(A_{\mathrm{YM}}) + \mu_K\cdot\mathrm{Index}(D_{[E]})\big], \quad (11)$$

where CS is Chern–Simons term, $\mathrm{Index}(D_{[E]})$ is index of Dirac operator on K-class $[E]$. This term ensures consistency of gauge structure with K-class, and penalizes configurations violating gauge Ward identities.

3.2.3 QFT–Scattering Term

Consistency of bulk QFT is given by a relative entropy type functional

$$\mathcal{I}_{\mathrm{QFT}} = \sum_{D \in \mathcal{D}_{\mathrm{micro}}} S\big(\omega^{D}_{\mathrm{bulk}} \| \omega^{D}_{\mathrm{scat}}\big), \tag{12}$$

where $\omega^{D}_{\mathrm{bulk}}$ is restriction of actual state on causal diamond D, $\omega^{D}_{\mathrm{scat}}$ is "reference state" predicted by scattering data and unified scale, $S(\cdot\|\cdot)$ is relative entropy. This term requires local QFT model to be compatible with scattering–scale predictions.

3.2.4 Fluid–Resolution Term

In coarse-grained limit, resolution connection Γ_{res}, macroscopic flow field u^{μ}, and conserved currents J^{μ} satisfy a family of entropy production inequalities. We write

$$\mathcal{I}_{\mathrm{hydro}} = \int_M \big[\zeta(\nabla_\mu u^\mu)^2 + \eta\, \sigma_{\mu\nu}\sigma^{\mu\nu} + \sum_k D_k (\nabla_\mu n_k)^2\big] \sqrt{|g|}\, \mathrm{d}^d x, \tag{13}$$

where $\sigma_{\mu\nu}$ is shear tensor, n_k conserved quantity densities, ζ, η, D_k viscosity and diffusion coefficients determined by Γ_{res}. This term requires macroscopic evolution to follow principle of minimum entropy production.

3.2.5 Observer–Consensus Term

Consistency of observer network is written as

$$\mathcal{I}_{\mathrm{obs}} = \sum_i S\big(\omega_i \| \omega_{\mathrm{bulk}}|_{C_i}\big) + \sum_{(i,j)} S\big(\mathcal{C}_{ij*}(\omega_i) \| \omega_j\big), \tag{14}$$

where first term penalizes deviation of observer's internal model from true universe state on its causal domain, second term penalizes inconsistency between models after communication.

3.3 Consistent Variational Principle

Unified Consistency Principle

Under premises of unified time scale and causal–entropic–observer axioms, physical universe corresponds to Universe Ontological Object $\mathfrak{U}$ such that

$$\delta \mathcal{I}[\mathfrak{U}] = 0 \tag{15}$$

holds for all allowed variations (including variations of $g, E, \Omega_\partial, \omega_{\mathrm{bulk}}, \{O_i\}$).

In following sections, we discuss physical meaning of this variational condition layer by layer.

4 Geometric Variation on Small Causal Diamonds and Gravitational Field Equations

4.1 Variational Setup

Consider a timelike geodesic $\gamma(\tau)$ and a family of small causal diamonds $D_{p,r}$ near it, where $p = \gamma(0)$, $r \ll L_{\text{curv}}$. We vary metric g_{ab} inside $D_{p,r}$ and bulk state ω_{bulk}, keeping:

1. External geometry and external state fixed; 2. Unified time scale $\kappa(\omega)$ unaffected to first order; 3. Generalized entropy constraints implemented via $\mathcal{I}_{\text{grav}} + \mathcal{I}_{\text{QFT}}$.

4.2 First-Order Variation of Generalized Entropy and Einstein Equations

For first-order variation of $S_{\text{gen}}(D_{p,r})$, under fixed volume or appropriate constraints,

$$\delta S_{\text{gen}} = \frac{1}{4G\hbar}\delta A(\partial D_{p,r}) + \delta S_{\text{bulk}}. \tag{16}$$

Geometric part δA can be expanded as local function of curvature R_{ab} at p, bulk entropy variation δS_{bulk} expressed via stress–energy tensor $\langle T_{ab}\rangle$. Substituting into condition $\delta\mathcal{I}_{\text{grav}} = 0$, in limit $r \to 0$, derives

$$G_{ab} + \Lambda g_{ab} = 8\pi G\langle T_{ab}\rangle, \tag{17}$$

i.e., Einstein equations. Detailed derivation in Appendix A.

4.3 Second-Order Variation and Quantum Energy Conditions

For second-order variation of same small causal diamond, considering deformation direction along some light rays, second-order variation of generalized entropy relates to quantum information inequalities, obtaining quantum energy conditions and focusing conditions. These conditions ensure stability of gravitational background and unidirectionality of macroscopic time arrow.

5 Variation of Boundary Channel Bundles and Total Connection: Unification of Gauge Fields and Field Content

5.1 Channel Bundle K-Class and Gauge Structure

On fixed geometric background, vary boundary channel bundle E and total connection Ω_∂, requiring:

1. K-class $[E]$ of channel bundle fixed (only stable equivalence variations allowed); 2. Compatibility of K^1 class of scattering matrix $S(\omega;\ell)$ with $[E]$ maintained.

In $\mathcal{I}_{\text{gauge}}$, variation of A_{YM} gives

$$\nabla_\mu F^{\mu\nu}_{\text{YM}} = J^\nu_{\text{YM}}, \tag{18}$$

i.e., Yang–Mills equations, where source J^{ν}_{YM} comes from coupling of boundary and bulk states. Allowed variations of $[E]$ and extremum condition of Dirac index term constrain field content and chiral structure: only field contents ensuring anomaly cancellation and zero index pairing are allowed. Detailed argument in Appendix B.

5.2 Physical Meaning of Unified Gauge Structure

Above results imply:

* "Which gauge fields, which matter fields, coupled in what representations" are no longer external "inputs", but results of extremum conditions of $\mathcal{I}_{\mathrm{gauge}}$; * Gauge invariance and Ward identities originate from invariance of $\mathcal{I}$ under variation of Ω_{∂}; * "Field content" and "gauge group" are expressions of channel bundle K-class and total connection, not independent entities.

6 Variation of Bulk Fields and Scattering Data: Unification of Local QFT

6.1 Derivation of Local Field Equations from Relative Entropy Functional

Under given geometry and gauge background, vary bulk QFT state ω_{bulk} and operator structure $\mathcal{A}_{\mathrm{bulk}}$. Relative entropy functional

$$\mathcal{I}_{\mathrm{QFT}} = \sum_{D} S\big(\omega^{D}_{\mathrm{bulk}} \| \omega^{D}_{\mathrm{scat}}\big) \tag{19}$$

requires actual state to be as close as possible to reference state predicted by scattering–scale on each small causal diamond. Varying ω_{bulk} yields a family of "local consistency conditions", satisfying:

1. Microcausality: observables at spacelike separated points commute; 2. Spectral Condition: energy spectrum bounded below under unified time scale; 3. Dynamics: field operators satisfy set of local wave equations (e.g., Klein–Gordon, Dirac), whose mass spectrum and couplings are determined by analytic invariants in $\mathcal{D}$.

Detailed derivation relies on Wightman function reconstruction and variational properties of relative entropy, see Appendix C.

6.2 Ward Identities and Scattering–Field Theory Compatibility

Varying scattering matrix $S(\omega;\ell)$ itself, under fixed unified scale, channel bundle K-class, and unitarity, requiring non-increase of $\mathcal{I}_{\mathrm{QFT}}$, can derive Ward identities and LSZ limit conditions, ensuring consistency between field theory and scattering descriptions.

7 Coarse-Grained Limit: Hydrodynamics and Multi-Agent Entropy Gradient Flow

7.1 Resolution Connection and Macroscopic Fluids

In long-wavelength and low-resolution limits, we care not about all degrees of freedom of microscopic fields, but finite set of conserved currents J_a^μ and macroscopic velocity field u^μ. Resolution connection $\Gamma_{\rm res}$ gives projection from microscopic degrees of freedom to macroscopic variables and flow rules on manifold.

Varying $\Gamma_{\rm res}$ and macroscopic variables, minimization condition of entropy production functional

$$\mathcal{I}_{\rm hydro} = \int_M \big[\zeta(\nabla_\mu u^\mu)^2 + \eta\,\sigma_{\mu\nu}\sigma^{\mu\nu} + \sum_k D_k(\nabla_\mu n_k)^2\big]\sqrt{|g|}\,\mathrm{d}^d x \tag{20}$$

gives generalized Navier–Stokes equations and diffusion equations:

$$\nabla_\mu T^{\mu\nu}_{\rm hydro} = 0, \quad \nabla_\mu J_a^\mu = 0, \tag{21}$$

where stress tensor $T^{\mu\nu}_{\rm hydro}$ and current J_a^μ contain viscosity and diffusion terms.

7.2 Entropy Gradient Flow of Multi-Agent Systems

Viewing multi-agent system as observer network $\{O_i\}$, whose strategy distributions and belief states evolve with unified scale. Observer–consensus functional

$$\mathcal{I}_{\rm obs} = \sum_i S\big(\omega_i \| \omega_{\rm bulk}|_{C_i}\big) + \sum_{(i,j)} S\big(\mathcal{C}_{ij*}(\omega_i)\|\omega_j\big) \tag{22}$$

varying ω_i gives a family of gradient flow equations, similar to natural gradient descent or mirror descent, entropy functional decreases monotonically under unified scale.

In continuous limit, such gradient flows share same structure as macroscopic hydrodynamics: both can be written as gradient flow of some generalized entropy S

$$\partial_\tau \rho = -\,\mathrm{grad}_{\mathsf{G}}\,\mathsf{S}(\rho), \tag{23}$$

where G is metric determined by causal–geometric and resource constraints.

8 Strict Meaning of Physical Unification

In above construction, "unification" no longer means "existence of a larger symmetry group" or "existence of an all-encompassing Lagrangian", but means:

1. **Ontological Unification**: Only one Universe Ontological Object $\mathfrak{U}$, containing geometry, channel bundles, connections, fields, entropy, and observers; 2. **Scale Unification**: All times and scales unified by scale mother formula $\kappa(\omega)$; 3. **Variational Unification**: Existence of a single Universe Consistency Functional $\mathcal{I}[\mathfrak{U}]$, whose extremum conditions are equivalent at different levels to GR, gauge field theory, QFT, hydrodynamics, multi-agent entropy gradient flow, etc.; 4. **Detail Unification**: All

"physical details" uniformly encoded as boundary K-classes and scattering analytic invariants $\mathcal{D}$, constrained by $\delta\mathcal{I} = 0$; 5. **Reduction Unification**: Reduction and equivalence between different physical theories correspond to natural isomorphisms under same $\mathcal{I}$ at different resolutions and boundary conditions.

In this sense, **there exist no multiple unrelated "physical laws"**: the universe has only one Ontological Object $\mathfrak{U}$ and one Consistent Variational Principle. The "laws" we know are all unfoldings and manifestations of this principle under different approximations and levels.

A Gravitational–Entropy Variation on Small Causal Diamonds and Einstein Equations

This appendix outlines detailed derivation of gravitational variation in Section 4.

A.1 Small Causal Diamond and Riemann Normal Coordinates

Construct Riemann normal coordinates near $p \in M$, such that

$$g_{ab}(p) = \eta_{ab}, \quad \partial_c g_{ab}(p) = 0, \tag{24}$$

curvature at p gives second-order expansion of metric. For sufficiently small r, volume and boundary area of causal diamond $D_{p,r}$ can be written as

$$V(D_{p,r}) = \alpha_d r^d \Big[1 + c_1 R_{ab} u^a u^b r^2 + O(r^3)\Big], \tag{25}$$

$$A(\partial D_{p,r}) = \beta_d r^{d-1} \Big[1 + c_2 R_{ab} u^a u^b r^2 + O(r^3)\Big], \tag{26}$$

where u^a is timelike direction.

A.2 First-Order Variation of Generalized Entropy

Generalized entropy is

$$S_{\text{gen}} = \frac{A}{4G\hbar} + S_{\text{bulk}}. \tag{27}$$

Metric variation δg_{ab} leads to δA and δV, latter manifesting via bulk stress–energy tensor

$$\delta S_{\text{bulk}} = -\frac{1}{2} \int_{D_{p,r}} \sqrt{|g|} \langle T_{ab} \rangle \delta g^{ab}. \tag{28}$$

Adding geometric and bulk parts of δS_{gen}, and using

$$\delta \mathcal{I}_{\text{grav}} \sim \delta S_{\text{gen}} - \lambda_{\text{ent}} \delta(S_{\text{gen}} - S^*_{\text{gen}}), \tag{29}$$

requiring $\delta \mathcal{I}_{\text{grav}} = 0$ for any δg_{ab} in limit $r \to 0$, yields

$$G_{ab} + \Lambda g_{ab} = 8\pi G \langle T_{ab} \rangle. \tag{30}$$

Detailed coefficient matching and determination of Λ require considering definition of reference entropy S^*_{gen} and background curvature conditions.

B Boundary K-Class, Dirac Index, and Gauge Field Equations

B.1 Restricted Unitary Bundle and K-Class

Channel bundle $E \to \partial M \times \Lambda$ has structure group U_{res}, its stable equivalence class gives an element $[E]$ of $K(\partial M \times \Lambda)$. Scattering matrix $S(\omega;\ell)$ defines K^1 class $[S]$ in frequency direction.

Using Fredholm index theory and relative scattering determinant, can construct a pairing

$$\langle [E], [S] \rangle = \text{Index}(D_{[E]}) \in \mathbb{Z}, \tag{31}$$

where $D_{[E]}$ is Dirac operator coupled to E.

B.2 Gauge Variation and Yang–Mills Equations

Under fixed $[E]$, varying A_{YM}

$$\delta \mathcal{I}_{\text{gauge}} \sim \int_{\partial M \times \Lambda} \text{tr}\big(\delta A_{\text{YM}} \wedge \nabla_\mu F^{\mu\nu}\big), \tag{32}$$

requiring $\delta \mathcal{I}_{\text{gauge}} = 0$ for any δA_{YM}, yields

$$\nabla_\mu F^{\mu\nu}_{\text{YM}} = J^\nu_{\text{YM}}, \tag{33}$$

where J^ν_{YM} is given by coupling of bulk and boundary states.

B.3 Index Constraint and Field Content

Allowed variations of $[E]$ must maintain consistency between $\text{Index}(D_{[E]})$ and topological number calculated from scattering K^1 class. This gives conditions similar to "anomaly cancellation", excluding certain disallowed combinations of field content.

C Relative Entropy Reconstruction of Local QFT

C.1 Variational Properties of Relative Entropy

Relative entropy $S(\rho\|\sigma)$ satisfies:

1. First-order variation with respect to ρ is zero at $\rho = \sigma$; 2. Second-order variation is positive definite, giving Fisher information metric.

On each small causal diamond D, let $\rho = \omega^D_{\text{bulk}}$, $\sigma = \omega^D_{\text{scat}}$, then $\delta \mathcal{I}_{\text{QFT}} = 0$ requires $\omega^D_{\text{bulk}} = \omega^D_{\text{scat}}$ to be extremum state.

C.2 Wightman Function Reconstruction

Scattering–scale reference state ω^D_{scat} gives a family of Wightman functions $W_n(x_1, \dots, x_n)$ satisfying:

1. Lorentz covariance; 2. Microcausality; 3. Spectral condition; 4. Positivity.

Using Wightman reconstruction theorem, Hilbert space, field operators, and vacuum state can be constructed, obtaining local QFT.

C.3 Field Equations and Ward Identities

If $\{W_n\}$ further satisfies a set of multilinear relations (given by analytic invariants in $\mathcal{D}$), it can be proven that local operators $\phi_a(x)$ satisfy Euler–Lagrange equations, e.g.,

$$(\Box + m_a^2)\phi_a(x) = \text{interaction terms}, \tag{34}$$

and Noether currents corresponding to symmetries satisfy Ward identities. These relations subsequently ensure consistency of scattering matrix and field theory description.

D Unified Structure of Fluid and Multi-Agent Entropy Gradient Flow

D.1 Entropy Production Functional and Gradient Flow

Given a generalized entropy function $\mathsf{S}[\rho]$, gradient flow on Riemann–Wasserstein or information geometry metric G

$$\partial_\tau \rho = -\operatorname{grad}_{\mathsf{G}} \mathsf{S}(\rho) \tag{35}$$

can be viewed as some diffusion–flow equation. For fluids, ρ is energy–momentum and particle number density; for multi-agent systems, ρ is strategy distribution or belief distribution.

D.2 From $\mathcal{I}_{\text{hydro}} + \mathcal{I}_{\text{obs}}$ to Gradient Flow

In macroscopic limit, variation of u^μ, J_a^μ, ω_i transforms extremum problem of $\mathcal{I}_{\text{hydro}} + \mathcal{I}_{\text{obs}}$ into steepest descent problem of some entropy functional, deriving fluid equations and strategy update equations as different coordinate expressions of the same gradient flow structure.

This implies "fluid evolution" and "multi-agent learning" are two limits of the same class of physical processes under unified time scale and boundary time geometry framework: **gradient flow of generalized entropy on unified scale**.