

Unified Physical Universe Terminal Object

Complete Unification Framework of Geometry–Boundary Time–Matrix–QCA–Topology

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November 19, 2025

Abstract

Based on mature frameworks including general relativity, algebraic quantum field theory, scattering and spectral shift theory, Tomita–Takesaki modular theory, generalized entropy and quantum energy conditions, Brown–York quasilocal energy, and quantum cellular automata, this paper presents a multi-layer structured "Unified Physical Universe Terminal Object"

$$\mathfrak{U}_{\text{phys}}^{\star} = (U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}}, U_{\text{BTG}}, U_{\text{QCA}}, U_{\text{top}}), \quad (1)$$

and proves it is a terminal object in the appropriate 2-category $\mathbf{Univ}_{\mathcal{U}}$. Core results include:

1. For self-adjoint pairs (H, H_0) satisfying standard scattering assumptions, a scale identity exists among scattering phase derivative, spectral shift function derivative, and Wigner–Smith group delay trace:

$$\kappa(\omega) = \varphi'(\omega)/\pi = \rho_{\text{rel}}(\omega) = (2\pi)^{-1} \text{tr } Q(\omega), \quad (2)$$

unifying time scales into a unique "scattering mother ruler", where φ is total scattering hemi-phase, ρ_{rel} is spectral shift derivative, and Q is Wigner–Smith group delay operator.

2. On the "Boundary Time Geometry" (BTG) layer, boundary time generators defined by boundary observable algebra \mathcal{A}_{∂} , boundary state ω_{∂} , Gibbons–Hawking–York boundary term, Brown–York quasilocal stress tensor, and Tomita–Takesaki modular flow provide a unique (up to affine) time parameter, making scattering time, modular time, and geometric time belong to the same time scale equivalence class $[\tau]$.

3. On the topology–scattering–relative cohomology layer, constructing a relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$ on $Y = M \times X^\circ$ and its boundary, unifying \mathbb{Z}_2 holonomy, scattering line bundle twisting, and $w_2(TM)$. Under "Modular–Scattering Alignment" and local quantum energy conditions, it is proven that $[K] = 0$ is equivalent to: local geometry satisfying Einstein equations, non-negativity of second-order relative entropy, and scattering square-root determinant having no \mathbb{Z}_2 anomaly on any physical loop.

4. On the QCA universe layer, defining universe QCA object with countable graph Λ , finite-dimensional cell Hilbert space $\mathcal{H}_{\text{cell}}$, quasilocal C^* algebra \mathcal{A} , finite

propagation radius QCA automorphism α , and initial state ω_0 :

$$\mathfrak{U}_{\text{QCA}} = (\Lambda, \mathcal{H}_{\text{cell}}, \mathcal{A}, \alpha, \omega_0), \quad (3)$$

proving existence of local finite causal partial order on induced event set $E = \Lambda \times \mathbb{Z}$, and recovering Dirac-type relativistic field theory in single-particle and continuous limits.

5. Constructing three types of representation categories in physical subcategories: continuous geometric universe, matrix scattering universe, and QCA universe

$$\mathbf{Univ}_{\text{geo}}^{\text{phys}}, \quad \mathbf{Univ}_{\text{mat}}^{\text{phys}}, \quad \mathbf{Univ}_{\text{qca}}^{\text{phys}}, \quad (4)$$

giving functors preserving unified scale, causality, and generalized entropy structure, and proving category equivalence

$$\mathbf{Univ}_{\text{geo}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{mat}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{qca}}^{\text{phys}}. \quad (5)$$

These three representations can be viewed as different projections of the same terminal object $\mathfrak{U}_{\text{phys}}^*$.

6. Under unified time scale, generalized entropy monotonicity, and topological anomaly-free constraints, $\mathfrak{U}_{\text{phys}}^*$ is a terminal object in 2-category $\mathbf{Univ}_{\mathcal{U}}$: any "universe structure" satisfying axioms uniquely embeds into $\mathfrak{U}_{\text{phys}}^*$, and conversely, any physical universe description is the image of some structure-forgetting functor acting on $\mathfrak{U}_{\text{phys}}^*$.

This paper also provides application examples, including black hole entropy and information, unified scale interpretation of cosmological constant and dark energy, QCA version of area law, and several engineeringly feasible verification schemes (group delay measurement in electromagnetic/acoustic scattering, Dirac limit experiments on QCA/quantum walk platforms), and discusses the relation and limitations of this framework with existing unification schemes.

Keywords: Unified Time Scale; Boundary Time Geometry; Wigner–Smith Group Delay; Birman–Krein Formula; Generalized Entropy and QNEC; Brown–York Quasilocal Energy; Quantum Cellular Automata; Matrix Scattering Universe; Null–Modular Double Cover; Category Terminal Object

1 Introduction & Historical Context

General relativity characterizes gravity as curvature of a four-dimensional Lorentzian manifold (M, g) , with time coordinates given by integral curves of timelike vector fields; quantum field theory constructs particles and interactions with local field operators and Fock spaces on fixed backgrounds. Their traditional combination—QFT on curved space-time and semiclassical gravity—has yielded significant results in black hole thermodynamics, cosmology, and quantum information, but unified answers to "ontological status of time", "observer and causal structure", and "quantum origin of gravity" remain lacking.

On the other hand, scattering theory provides a basis for "far-field observable" unified language. For self-adjoint pairs (H, H_0) satisfying appropriate conditions, the Birman–Krein formula links scattering matrix determinant with spectral shift function:

$$\det S(\lambda) = \exp(-2\pi i \xi(\lambda)), \quad (6)$$

where ξ is the spectral shift function. Eisenbud, Wigner, and Smith introduced the time delay operator

$$Q(\omega) = -i S(\omega)^\dagger \partial_\omega S(\omega), \quad (7)$$

widely used to analyze quantum scattering, wave propagation, and "group delay" in complex media. This suggests time can be unifiedly defined in the frequency domain via phase gradients and spectral data.

Generalized entropy and quantum energy conditions provide a new perspective for geometrizing the "arrow of time". Quantum Null Energy Condition (QNEC) and Quantum Focusing Conjecture (QFC) link the null component of stress-energy tensor with the second variation of generalized entropy, converting relative entropy monotonicity into geometric energy conditions. In semiclassical gravity and AdS/CFT, this idea developed into a series of results on "entropy determines geometry".

Boundaries play a key role in gravity and QFT. Brown and York introduced quasilocal energy and boundary stress tensors using Hamilton–Jacobi analysis, localizing definitions of energy and momentum to boundaries of bounded regions. Boundary terms reappear in Gibbons–Hawking–York action, black hole thermodynamics, and recent null boundary charge definitions, suggesting "time" can be viewed as a translation parameter on the boundary rather than a primitive coordinate in the bulk.

Regarding discrete models, Quantum Cellular Automata (QCA) and discrete-time quantum walks form a rigorous framework for "discrete universe dynamics". Schumacher and Werner provided structure theorems for QCA with finite propagation speed and translation invariance. Numerous works show that appropriately chosen discrete-time quantum walks yield Dirac equations and relativistic wave equations in the continuous limit. This supports the view that "the universe is intrinsically discrete but continuous field theory is its scaling limit".

In this context, previous works have completed several "unification chains": 1. Constructing unified time scale density $\kappa(\omega)$ via Birman–Krein formula and Wigner–Smith group delay. 2. Unifying boundary spectral triples, Tomita–Takesaki modular flow, Brown–York stress tensor, and scattering phase scale in Boundary Time Geometry (BTG). 3. Gluing Lorentzian causal partial order, unitary evolution, and generalized entropy extremality–monotonicity on small causal diamonds with unified time scale equivalence class $[\tau]$. 4. Constructing "maximally consistent universe" on multi-layer structure object \mathfrak{U} and proving its terminal object property. 5. Defining the universe as QCA object $\mathfrak{U}_{\text{QCA}}$ and recovering relativistic field theory in continuous limits. 6. Introducing Null–Modular double cover and relative cohomology class $[K]$ on $Y = M \times X^\circ$.

However, these chains remain parallel. This paper aims to construct a multi-layer structure terminal object $\mathfrak{U}_{\text{phys}}^*$ in 2-category $\mathbf{Univ}_{\mathcal{U}}$, making all above structures its different components or projections, thus rigorously characterizing the "Unified Physical Universe".

2 Model & Assumptions

2.1 Universe 2-Category and Size Control

Given a fixed Grothendieck universe \mathcal{U} , denote $\mathbf{Univ}_{\mathcal{U}}$ as the following 2-category: * Objects are \mathcal{U} -small sets of multi-layer structures

$$\mathfrak{U} = (U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}}, \dots). \quad (8)$$

* 1-morphisms are functor-type maps preserving structure. * 2-morphisms are natural isomorphisms or compatible transformations between different 1-morphisms.

2.2 Notation and Unified Scale Identity

1. Unified scale density $\kappa(\omega)$ defined as

$$\kappa(\omega) = \varphi'(\omega)/\pi = \rho_{\text{rel}}(\omega) = (2\pi)^{-1} \text{tr } Q(\omega). \quad (9)$$

2. Unified time scale equivalence class $[\tau]$ allows affine transformations. 3. Small causal diamond $D_{p,r} \subset (M, g)$. 4. Generalized entropy $S_{\text{gen}}(\Sigma)$. 5. QCA Universe $\mathfrak{U}_{\text{QCA}} = (\Lambda, \mathcal{H}_{\text{cell}}, \mathcal{A}, \alpha, \omega_0)$.

2.3 Scattering and Spectral Shift Assumptions (A1–A5)

Consider a self-adjoint pair (H, H_0) satisfying:

- * (A1) Existence and completeness of wave operators $W_{\pm}(H, H_0)$.
- * (A2) Unitary equivalence on absolute continuous spectrum.
- * (A3) Existence of spectral shift function $\xi(\lambda)$ and Birman–Kreĭn formula.
- * (A4) Differentiability of $\xi(\lambda)$.
- * (A5) Differentiability of $S(\omega)$ and trace-class property of $Q(\omega)$.

2.4 Geometry and Generalized Entropy Assumptions (B1–B4)

* (B1) (M, g) is 4D globally hyperbolic Lorentzian manifold with boundary. * (B2) Existence of Brown–York quasilocal stress tensor T_{BY}^{ab} . * (B3) QNEC/QFC type relations for generalized entropy. * (B4) Equivalence of QNEC/QFC/entropy extremality to Einstein equations.

2.5 Modular Structure and AQFT Assumptions (M1–M3)

* (M1) Existence of modular operators $\Delta_{\mathcal{O}}$ and modular flow. * (M2) Thermal time relation $\sigma_t^\omega = \alpha_{t/\beta}$ for KMS states. * (M3) Matching of modular Hamiltonian eigenvalues with scattering scale $\kappa(\omega)$.

2.6 QCA Axioms and Continuous Limit Assumptions (Q1–Q4)

* (Q1) Countable, locally finite graph Λ . * (Q2) Finite-dimensional $\mathcal{H}_{\text{cell}}$. * (Q3) Finite propagation radius R . * (Q4) Existence of scale parameter $\epsilon \rightarrow 0$ yielding Dirac/Klein–Gordon limits.

2.7 Observer and Consensus Geometry Assumptions (O1–O3)

* (O1) Observers defined by causal domains C_i . * (O2) Čech consistency on overlaps. * (O3) Existence of 2-limit construction from observers to global object.

3 Main Results (Theorems and Alignments)

3.1 Scale Identity and Endogenous Boundary Time Geometry

Theorem 3.1 (Existence and Uniqueness of Unified Scale Density). *Under scattering assumptions (A1)–(A5), there exists an almost everywhere defined Borel function $\kappa(\omega)$ such that*

$$\kappa(\omega) = \varphi'(\omega)/\pi = \rho_{\text{rel}}(\omega) = (2\pi)^{-1} \text{tr } Q(\omega). \quad (10)$$

This κ is unique up to global phase redefinition (constant addition).

Theorem 3.2 (Boundary Time Geometry and Unified Scale Alignment). *Under (B1)–(B4), (M1)–(M3), and (A1)–(A5), for each small causal diamond boundary system \mathcal{B} , there exists a unique (up to affine) time parameter τ such that: 1. Scattering time τ_{scatt} defined by integral of κ ; 2. Modular time τ_{mod} aligned with geometric Killing flow; 3. Geometric time τ_{geom} from Brown–York Hamiltonian spectrum; all belong to the same equivalence class $[\tau]$.*

3.2 Topological Constraints and Null–Modular Double Cover

Theorem 3.3 (Equivalence of Topological Anomaly-Free and Einstein–Entropy Conditions). *With relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$, the following are equivalent:* 1. $[K] = 0$. 2. Einstein equations and QNEC/QFC hold on all small causal diamonds, compatible with scattering–modular data. 3. Trivial \mathbb{Z}_2 holonomy of scattering square-root determinant on all physical loops.

3.3 Triple Equivalence of Geometric, Matrix, and QCA Universes

Theorem 3.4 (Geometry–Matrix Universe Equivalence). *There exist functors $F_{\text{geo} \rightarrow \text{mat}}$ and $G_{\text{mat} \rightarrow \text{geo}}$ inducing category equivalence*

$$\mathbf{Univ}_{\text{geo}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{mat}}^{\text{phys}}. \quad (11)$$

Theorem 3.5 (QCA–Geometry Universe Equivalence). *There exist functors $C_{\text{qca} \rightarrow \text{geo}}$ and $D_{\text{geo} \rightarrow \text{qca}}$ inducing category equivalence*

$$\mathbf{Univ}_{\text{qca}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{geo}}^{\text{phys}}. \quad (12)$$

Theorem 3.6 (Triple Representation Equivalence).

$$\mathbf{Univ}_{\text{geo}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{mat}}^{\text{phys}} \simeq \mathbf{Univ}_{\text{qca}}^{\text{phys}}. \quad (13)$$

3.4 Unified Physical Universe Terminal Object Theorem

Theorem 3.7 (Unified Physical Universe Terminal Object). *Under assumptions and $[K] = 0$, there exists a multi-layer object $\mathfrak{U}_{\text{phys}}^*$ satisfying: 1. Layers satisfy unified scale identity, causal–entropy compatibility, and topological anomaly-free conditions. 2. For any object V in $\mathbf{Univ}_{\mathcal{U}}$ satisfying axioms, there exists a unique 1-morphism $F_V : V \rightarrow \mathfrak{U}_{\text{phys}}^*$, unique up to 2-morphism. Thus, $\mathfrak{U}_{\text{phys}}^*$ is a terminal object.*

Corollary 3.8 (No Further Non-Trivial Unification Freedom). *Any "more unified" structure is isomorphic to $\mathfrak{U}_{\text{phys}}^*$.*

4 Proofs

(Proofs follow the structure outlined in the original document, establishing scale identity, boundary time alignment, topological equivalence, category equivalences, and terminal object property via limits.)

5 Model Apply

5.1 Black Hole Entropy, Information, and QCA Area Law

Unified framework explains Bekenstein–Hawking entropy in three representations: geometric (horizon area), matrix (scattering delay/absorption), and QCA (entanglement across deletion cone).

5.2 Cosmological Constant and Unified Time Scale

Λ interpreted as mismatch between $\kappa(\omega)$ and micro-QCA spectrum on large scales.

5.3 Arrow of Time, Entropy Production, and QCA Observation

Time arrow unified as generalized entropy monotonicity, scattering delay directionality, and QCA entanglement spreading.

6 Engineering Proposals

6.1 Group Delay and Unified Scale Experiment

Measuring $S(\omega)$ and $Q(\omega)$ in microwave/acoustic systems to verify scale identity.

6.2 Dirac Limit on QCA/Quantum Walk Platforms

Implementing Dirac–QCA on ion traps/superconducting qubits to test scale alignment.

6.3 QCA Black Hole Toy Models

Simulating horizon-like irreversible dynamics and measuring entanglement area law.

7 Discussion

Risks include validity of QNEC/QFC, existence of QCA continuous limits for general cases, and reliance on $[K] = 0$ as a consistency condition.

8 Conclusion

The paper constructs $\mathfrak{U}_{\text{phys}}^*$ as a terminal object, unifying time scale, generalized entropy/gravity, topological sectors, and three universe representations (geometric, matrix, QCA).

(Technical appendices on proof details, QCA limits, etc.)