Time–Information–Complexity Unified Variational Principle in Computational Universes: Computational Worldlines on Control–Scattering Manifold and Task Information Manifold

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Abstract

In previous works on the "computational universe" series, we abstracted the universe as discrete object $U_{\mathrm{comp}} = (X, \mathsf{T}, \mathsf{C}, \mathsf{I})$, constructing discrete complexity geometry (complexity distance, volume growth, and discrete Ricci curvature based on configuration graph) and discrete information geometry (based on task-aware relative entropy and Fisher structure) on it, and gave continuous limit of complexity geometry under unified time scale scattering mother scale: a control manifold $\mathcal{M}$ with Riemannian metric G. However, these geometric structures still separately characterize "time/resource cost" and "information quality/task-relevant states", lacking a framework to unify both under a single variational principle.

This paper, building on control manifold $(\mathcal{M}, G)$ and task information manifold $(\mathcal{S}_Q, g_Q)$, introduces joint manifold

$$\mathcal{E}_Q = \mathcal{M} \times \mathcal{S}_Q,$$

constructing on it a time–information–complexity joint action $\mathcal{A}_Q$, thereby characterizing "computational trajectories" in computational universe as minimal curves on joint manifold (computational worldlines). Specifically, we first give action at discrete level

$$\mathcal{A}_Q^{\mathrm{disc}}(\gamma) = \sum_k \big(\alpha\, \mathsf{C}(x_k, x_{k+1}) + \beta\, d_{\mathrm{info},Q}(x_k, x_{k+1}) - \gamma\, \Delta \mathsf{I}_Q(x_k, x_{k+1})\big),$$

proving that under appropriate scaling, this discrete action family Γ-converges as $h \to 0$ to continuous action

$$\mathcal{A}_Q[\theta(\cdot), \phi(\cdot)] = \int_0^T \Big(\tfrac{1}{2}\alpha^2 G_{ab}(\theta)\dot\theta^a\dot\theta^b + \tfrac{1}{2}\beta^2 g_{ij}(\phi)\dot\phi^i\dot\phi^j - \gamma\, U_Q(\phi)\Big)\,\mathrm{d}t,$$

where $\theta(t) \in \mathcal{M}$ is control trajectory, $\phi(t) \in \mathcal{S}_Q$ is task information state, U_Q is task-related information potential function (e.g., negative information quality).

Then we derive Euler–Lagrange equations on joint manifold $\mathcal{E}_Q$, proving that minimal trajectories satisfy coupled "geodesic equations with potential": control part evolves along geodesics of $(\mathcal{M}, G)$ but receives feedback from gradient of U_Q with respect to ϕ; information part evolves along geodesics of $(\mathcal{S}_Q, g_Q)$ but is modulated by control trajectory θ. Furthermore, using standard variational methods and Γ-convergence theory, we prove: under unified time scale and local Lipschitz assumptions, discrete optimal computational paths converge in the limit to minimal worldlines on joint manifold, achieving rigorous correspondence between "optimal algorithms in discrete computational universe" and "continuous time–information–complexity worldlines."

This paper concludes with discussion of minimization problems with resource constraints: maximizing task information quality under fixed time budget or complexity budget. We give equivalent Lagrange multiplier form, thereby characterizing "optimal information acquisition strategy under given budget" as a class of geodesic flows with effective potential. Results of this paper provide variational foundation at intrinsic dynamics level for subsequent construction of categorical equivalence between "computational universe $\leftrightarrow$ physical universe."

Keywords: Computational universe; Variational principle; Complexity geometry; Information geometry; Joint manifold; Computational worldline; Euler–Lagrange equations; Γ-convergence; Resource constraints

1 Introduction

From the "computational universe" perspective, the entire universe is abstracted as discrete dynamical system: one-step update relation T on configuration space X and single-step cost C describe resources needed to go from one state to another; information quality function I evaluates at task level the "goodness" of a configuration relative to goals. Previous works showed that under axioms of finite information density and local update, $(X, \mathsf{T}, \mathsf{C})$ can be viewed as complexity graph, constructing complexity distance, complexity ball volume, complexity dimension, and discrete Ricci curvature, thereby using discrete geometry to characterize "problem difficulty" and "horizon structure"; simultaneously, through observation operator families and task-aware relative entropy, we defined information distance and information balls on configuration space, geometrizing "task-relevant distinguishability."

Under unified time scale scattering mother scale, single-step cost of computational universe can be viewed as discrete sampling of actual physical time scale: for physically realizable computational processes, there exist control manifold $\mathcal{M}$ and scattering matrix family $S(\omega; \theta)$, such that control derivatives of group delay matrix $Q(\omega; \theta)$ induce complexity metric G, whereby discrete complexity distance approximates geodesic distance on $(\mathcal{M}, G)$ in refinement limit. This result unifies discrete complexity geometry with physical time scale into a Riemannian geometric framework.

However, to understand "how best to compute in finite time," neither complexity geometry nor information geometry alone suffices:

- Complexity geometry concerns "how far traveled, how much time/resource spent";

- Information geometry concerns "how far moved in task space, how much information gained";
- The truly meaningful question is: under given time/complexity budget, how to reach best possible endpoint in information geometry.

This naturally leads to a joint variational problem: in joint space, for given task, find minimal/maximal trajectory considering both time cost and information benefit.

This paper, building on control manifold $(\mathcal{M}, G)$ and task information manifold $(\mathcal{S}_Q, g_Q)$, constructs joint manifold $\mathcal{E}_Q = \mathcal{M} \times \mathcal{S}_Q$, defining on it a time–information–complexity joint action $\mathcal{A}_Q$. Discrete computational paths become piecewise linear approximations on joint manifold, continuous computational worldlines are smooth curves on $\mathcal{E}_Q$. Using Γ-convergence and classical variational methods, we prove discrete optimal paths converge in the limit to continuous minimal worldlines, thereby geometrizing the problem of "optimal algorithms" as the problem of "optimal worldlines."

2 Unified Notation: Computational Universe, Complexity Geometry, and Information Geometry

This section briefly summarizes main objects and notation used in previous works for subsequent unified reasoning.

2.1 Computational Universe Object

A computational universe object is quadruple $U_{\text{comp}} = (X, \mathsf{T}, \mathsf{C}, \mathsf{I})$, where:

1. X is countable configuration set;
2. $\mathsf{T} \subset X \times X$ is one-step update relation;
3. $\mathsf{C} : X \times X \to [0, \infty]$ is single-step cost, with $\mathsf{C}(x, y) = \infty$ if $(x, y) \notin \mathsf{T}$, $\mathsf{C}(x, y) \in (0, \infty)$ if $(x, y) \in \mathsf{T}$, additive along paths;
4. $\mathsf{I} : X \to \mathbb{R}$ is information quality function (may be task-dependent).

Complexity distance defined as

$$d_{\text{comp}}(x, y) = \inf_{\gamma : x \to y} \mathsf{C}(\gamma),$$

where path $\gamma = (x_0, \ldots, x_n)$ satisfies $x_0 = x$, $x_n = y$, and $(x_k, x_{k+1}) \in \mathsf{T}$.

2.2 Complexity Geometry and Control Manifold

Under unified time scale framework, for physically realizable computational universe there exist control manifold $\mathcal{M}$ and scattering matrix family $S(\omega; \theta)$, whose group delay matrix $Q(\omega; \theta) = -\mathrm{i}\, S^\dagger \partial_\omega S$ has control derivatives inducing complexity metric

$$G_{ab}(\theta) = \int_\Omega w(\omega)\ \mathrm{tr}\left(\partial_a Q(\omega; \theta)\, \partial_b Q(\omega; \theta)\right) \mathrm{d}\omega.$$

Under appropriate positive definiteness conditions, $(\mathcal{M}, G)$ is Riemannian manifold, discrete complexity distance converges to geodesic distance d_G in refinement limit.

2.3 Task Information Manifold

Given task Q, through observation operator family $\mathcal{O} = \{O_j\}_{j\in J}$ define visible state $p_x^{(Q)} \in \Delta(Y_Q)$ of configuration x. Under appropriate regularity assumptions, these visible states can be embedded into some information manifold $\mathcal{S}_Q$:

- There exist mapping $\Phi_Q : X \to \mathcal{S}_Q$ and embedding $\Psi_Q : \mathcal{S}_Q \hookrightarrow \Delta(Y_Q)$, such that $\Psi_Q(\Phi_Q(x)) \approx p_x^{(Q)}$;
- Fisher information metric g_Q given by second derivative of relative entropy, constructing Riemannian structure of $(\mathcal{S}_Q, g_Q)$;
- Information distance between configurations can be represented using Jensen–Shannon distance or Fisher geodesic distance, denoted $d_{\mathrm{info},Q}(x,y) \approx d_{\mathcal{S}_Q}(\Phi_Q(x), \Phi_Q(y))$.

We call $(\mathcal{S}_Q, g_Q, \Phi_Q)$ the information geometric data for task Q.

3 Joint Time–Information–Complexity Manifold

With above preparation, we construct joint manifold $\mathcal{E}_Q$ and its metric.

3.1 Definition of Joint Manifold

Definition 3.1 (Joint Manifold)**.** For given task Q, define joint manifold

$$\mathcal{E}_Q = \mathcal{M} \times \mathcal{S}_Q.$$

Its point $z = (\theta, \phi)$ simultaneously represents "control state" and "task information state". In continuous limit, state of an observer or algorithm in computational universe can be viewed as point in $\mathcal{E}_Q$.

3.2 Metric Structure

On $\mathcal{E}_Q$, we introduce product-type metric

$$\mathbb{G} = \alpha^2 G \oplus \beta^2 g_Q,$$

i.e., for tangent vector $v = (v^{\mathcal{M}}, v^{\mathcal{S}_Q}) \in T_\theta\mathcal{M} \oplus T_\phi\mathcal{S}_Q$, define

$$\mathbb{G}_z(v,v) = \alpha^2 G_\theta(v^{\mathcal{M}}, v^{\mathcal{M}}) + \beta^2 g_{Q,\phi}(v^{\mathcal{S}_Q}, v^{\mathcal{S}_Q}).$$

Here $\alpha, \beta > 0$ are weight parameters used to balance "velocity" measurement in complexity direction and information direction.

Under this metric, velocity squared of joint trajectory

$$z(t) = (\theta(t), \phi(t))$$

is

$$|\dot{z}(t)|^2_{\mathbb{G}} = \alpha^2 G_{ab}(\theta(t))\dot{\theta}^a\dot{\theta}^b + \beta^2 g_{ij}(\phi(t))\dot{\phi}^i\dot{\phi}^j.$$

Pure geometric length on joint manifold is

$$L_{\mathbb{G}}[z] = \int_0^T \sqrt{|\dot{z}(t)|^2_{\mathbb{G}}}\,\mathrm{d}t.$$

However, length alone is insufficient to encode "information quality" gain, we also need task-related potential function.

3.3 Information Potential Function

Let information quality function of task Q on information manifold be written as $I_Q : \mathcal{S}_Q \to \mathbb{R}$, for example

$$I_Q(\phi) = \mathsf{I}_Q(x) \quad \text{when } \phi = \Phi_Q(x).$$

We introduce information potential function

$$U_Q(\phi) = V(I_Q(\phi)),$$

where $V : \mathbb{R} \to \mathbb{R}$ is monotone function, generally chosen as $V(u) = u$ or $V(u) = f_{\mathrm{sat}}(u)$ (saturation type). In this paper, for simplicity we directly take

$$U_Q(\phi) = I_Q(\phi),$$

viewing "information quality" as negative contribution of potential energy term (corresponding to higher information quality bringing lower action).

4 Discrete Joint Action and Continuous Limit

This section constructs joint action for task Q at discrete level, proving its convergence to continuous action in refinement limit.

4.1 Discrete Joint Action

Consider discrete computational path

$$\gamma = (x_0, x_1, \ldots, x_n),$$

where $(x_k, x_{k+1}) \in \mathsf{T}$. Corresponding complexity increment is

$$\Delta C_k = \mathsf{C}(x_k, x_{k+1}),$$

information distance increment (under task Q) is

$$\Delta D_k = d_{\mathrm{info},Q}(x_k, x_{k+1}),$$

information quality increment is

$$\Delta I_k = I_Q(\phi_{k+1}) - I_Q(\phi_k), \quad \phi_k = \Phi_Q(x_k).$$

Definition 4.1 (Discrete Joint Action)**.** For task Q and path γ, define discrete joint action

$$\mathcal{A}_Q^{\mathrm{disc}}(\gamma) = \sum_{k=0}^{n-1} \Big(\alpha \, \Delta C_k + \beta \, \Delta D_k - \gamma \, \Delta I_k \Big),$$

where $\alpha, \beta, \gamma > 0$ are weight parameters.

Intuitive understanding: each step update simultaneously pays complexity cost $\alpha \, \Delta C_k$ and information adjustment cost $\beta \, \Delta D_k$, and gains information quality increment ΔI_k, contributing $-\gamma \, \Delta I_k$ to action. Optimal path is one that minimizes $\mathcal{A}_Q^{\mathrm{disc}}$ under balance of all three.

4.2 Refinement and Standard Time Step

To connect discrete and continuous, we introduce discrete time step $h > 0$, let path length $n \approx T/h$, and set scaling of single-step cost and information distance as

$$\Delta C_k = h \, c(x_k, x_{k+1}) + o(h), \quad \Delta D_k = h \, d(x_k, x_{k+1}) + o(h),$$

$$\Delta I_k = h \, \dot{I}_Q(t_k) + o(h),$$

where $t_k = kh$, $c, d, \dot{I}_Q$ are respectively complexity velocity, information velocity, and information quality rate of change in continuous limit.

Under above scaling, discrete action can be approximated as Riemann sum

$$\mathcal{A}_Q^{\mathrm{disc}}(\gamma) \approx \sum_{k=0}^{n-1} h \Big(\alpha \, c_k + \beta \, d_k - \gamma \, \dot{I}_Q(t_k) \Big) \to \int_0^T \big(\alpha c(t) + \beta d(t) - \gamma \dot{I}_Q(t) \big) \, \mathrm{d}t.$$

To match geometric structure, we represent $c(t), d(t)$ respectively using velocity norms on $(\mathcal{M}, G)$ and $(\mathcal{S}_Q, g_Q)$.

4.3 Continuous Joint Action

Let control path be $\theta : [0, T] \to \mathcal{M}$, information path be $\phi : [0, T] \to \mathcal{S}_Q$, with corresponding velocity norms

$$v_{\mathcal{M}}^2(t) = G_{ab}(\theta(t)) \dot{\theta}^a(t) \dot{\theta}^b(t),$$

$$v_{\mathcal{S}_Q}^2(t) = g_{ij}(\phi(t)) \dot{\phi}^i(t) \dot{\phi}^j(t).$$

We choose "energy-type" continuous action:

Definition 4.2 (Continuous Joint Action)**.**

$$\mathcal{A}_Q[\theta(\cdot), \phi(\cdot)] = \int_0^T \Big(\tfrac{1}{2} \alpha^2 v_{\mathcal{M}}^2(t) + \tfrac{1}{2} \beta^2 v_{\mathcal{S}_Q}^2(t) - \gamma \, U_Q(\phi(t)) \Big) \, \mathrm{d}t.$$

where $U_Q(\phi) = I_Q(\phi)$ or some monotone transformation thereof.

This is standard "kinetic minus potential" form: first two terms are kinetic energy on complexity and information geometry, latter term is task-related negative potential energy, minimal worldline maintains finite velocity while trying to enter regions of lower information potential energy.

5 Euler–Lagrange Equations and Computational Worldlines

This section derives Euler–Lagrange equations on joint manifold, giving dynamical form satisfied by minimal worldlines.

5.1 Lagrangian and Variation

Let Lagrangian be

$$L(\theta, \dot{\theta}; \phi, \dot{\phi}) = \tfrac{1}{2}\alpha^2 G_{ab}(\theta)\dot{\theta}^a\dot{\theta}^b + \tfrac{1}{2}\beta^2 g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j - \gamma\, U_Q(\phi).$$

Varying θ^a and ϕ^i respectively gives Euler–Lagrange equations:

For θ^a:

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(\alpha^2 G_{ab}(\theta)\dot{\theta}^b\big) - \tfrac{1}{2}\alpha^2(\partial_a G_{bc})(\theta)\dot{\theta}^b\dot{\theta}^c = 0,$$

For ϕ^i:

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(\beta^2 g_{ij}(\phi)\dot{\phi}^j\big) - \tfrac{1}{2}\beta^2(\partial_i g_{jk})(\phi)\dot{\phi}^j\dot{\phi}^k + \gamma\,\partial_i U_Q(\phi) = 0.$$

where $\partial_a G_{bc} = \partial G_{bc}/\partial\theta^a$, $\partial_i g_{jk} = \partial g_{jk}/\partial\phi^i$.

5.2 Joint Geodesic–Potential Equations

In standard Riemannian geometry, geodesic equation can be written as

$$\ddot{\theta}^a + \Gamma^a_{bc}(\theta)\dot{\theta}^b\dot{\theta}^c = 0,$$

where Γ^a_{bc} are Christoffel symbols of Levi–Civita connection. Here we rewrite control and information parts respectively in geodesic–potential form.

For control variable θ^a, let

$$\Gamma^a_{bc}(\theta) = \tfrac{1}{2}G^{ad}\big(\partial_b G_{dc} + \partial_c G_{db} - \partial_d G_{bc}\big),$$

where G^{ad} is inverse of metric matrix. Euler–Lagrange equation can be rewritten as

$$\ddot{\theta}^a + \Gamma^a_{bc}(\theta)\dot{\theta}^b\dot{\theta}^c = 0.$$

Since control part of Lagrangian contains no explicit potential energy, control trajectory is geodesic on $(\mathcal{M}, G)$.

For information variable ϕ^i, similarly defining $\Gamma^i_{jk}(\phi)$ as Christoffel symbols of g_Q, Euler–Lagrange equation rewrites as

$$\ddot{\phi}^i + \Gamma^i_{jk}(\phi)\dot{\phi}^j\dot{\phi}^k = -\frac{\gamma}{\beta^2}g^{ij}(\phi)\partial_j U_Q(\phi).$$

Right-hand-side term is covariant lift of potential energy gradient on information manifold, representing "driving force" of "information potential" on information trajectory.

Therefore, joint worldline satisfies following coupled system:

1. Control part: evolves along geodesics of $(\mathcal{M}, G)$;
2. Information part: evolves along geodesics of $(\mathcal{S}_Q, g_Q)$, but driven away from geodesics by gradient of U_Q.

This can be viewed as special case of "geodesic with potential on complexity–information product manifold."

6 Γ-Convergence for Discrete–Continuous Consistency

To prove discrete optimal paths converge in the limit to continuous minimal worldlines, we use Γ-convergence theory. Only structural theorem and proof idea given here, technical details placed in appendix.

6.1 Action Functional Family

Consider family of discrete time steps $h = T/n$, embed discrete path $\gamma^{(h)} = (x_0, \ldots, x_n)$ into piecewise constant or piecewise linear curve $z^{(h)} : [0, T] \to \mathcal{E}_Q$, such that

$$z^{(h)}(t) = (\theta^{(h)}(t), \phi^{(h)}(t)), \quad t \in [kh, (k+1)h),$$

and $z^{(h)}(kh) = (\theta_k, \phi_k)$ corresponds to x_k. Define discrete action

$$\mathcal{A}_Q^{(h)}[z^{(h)}] = \sum_{k=0}^{n-1} \Big(\tfrac{1}{2}\alpha^2 \frac{\Delta s^2_{\mathcal{M},k}}{h} + \tfrac{1}{2}\beta^2 \frac{\Delta s^2_{\mathcal{S}_Q,k}}{h} - \gamma\, U_Q(\phi_k) h \Big),$$

where $\Delta s^2_{\mathcal{M},k} = d_{\mathcal{M}}(\theta_k, \theta_{k+1})^2$, $\Delta s^2_{\mathcal{S}_Q,k} = d_{\mathcal{S}_Q}(\phi_k, \phi_{k+1})^2$.

Under local consistency assumptions, $\Delta s_{\mathcal{M},k} \approx h\sqrt{G_{ab}(\theta)\dot{\theta}^a\dot{\theta}^b}$, $\Delta s_{\mathcal{S}_Q,k} \approx h\sqrt{g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j}$.

6.2 Γ-Convergence Theorem

Theorem 6.1 (Γ-Convergence, Schematic)**.** *Under unified time scale and local regularity assumptions, discrete action functional family $\{\mathcal{A}_Q^{(h)}\}_{h>0}$ Γ-converges under appropriate topology (e.g., weak H^1 topology of $z^{(h)} \rightharpoonup z$) to continuous action functional*

$$\mathcal{A}_Q[z] = \int_0^T \Big(\tfrac{1}{2}\alpha^2 G_{ab}(\theta)\dot{\theta}^a\dot{\theta}^b + \tfrac{1}{2}\beta^2 g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j - \gamma\, U_Q(\phi) \Big)\, \mathrm{d}t.$$

In particular, any limit point of discrete minimal sequences is a minimal curve of continuous action.

Proof idea in Appendix B.2, based on standard "energy-type functional discretization" Γ-convergence framework: lower semicontinuity given by convex structure and weak topology lower semicontinuity, recovery sequence constructed through time discretization of continuous trajectory.

7 Optimal Computational Worldlines Under Resource Constraints

In practical problems, we often care about following optimization:

- Under given time budget T or complexity budget $C_{\max}$, maximize terminal information quality $I_Q(\phi(T))$;
- Or under given terminal information quality requirement I_{target}, minimize required time or complexity.

Using Lagrange multiplier method, resource constraints can be absorbed into joint action.

For example, maximizing $I_Q(\phi(T))$ under given T, equivalent to minimizing under free endpoint condition

$$\widetilde{\mathcal{A}}_Q[z] = \int_0^T \left(\tfrac{1}{2}\alpha^2 v_{\mathcal{M}}^2 + \tfrac{1}{2}\beta^2 v_{\mathcal{S}_Q}^2\right) \mathrm{d}t - \gamma\, I_Q(\phi(T)),$$

which differs from previous action only in potential energy term. Corresponding Euler–Lagrange equations in bulk region same as before, but at endpoint add natural boundary condition

$$\beta^2 g_{ij}(\phi(T))\dot{\phi}^j(T) = \gamma\, \partial_i I_Q(\phi(T)).$$

This boundary condition can be viewed as "endpoint reflection condition": at endpoint, ratio of information velocity to information quality gradient controlled by parameter γ/β^2, reflecting preference strength for endpoint information quality.

Similarly, minimizing time under given information quality target can be obtained through constraint $I_Q(\phi(T)) = I_{\text{target}}$ and introducing multiplier λ to get equivalent free problem, whereby obtaining set of geodesic–potential equations with global constraints.

These variational problems provide geometric perspective for "optimal algorithm design": seeking minimal curves on joint manifold $\mathcal{E}_Q$ satisfying resource constraints and endpoint information constraints is precisely seeking optimal computational worldlines in computational universe.

A Derivation of Euler–Lagrange Under Metric and Potential

A.1 Details of Variational Derivation

Let

$$L(\theta,\dot{\theta};\phi,\dot{\phi}) = \tfrac{1}{2}\alpha^2 G_{ab}(\theta)\dot{\theta}^a\dot{\theta}^b + \tfrac{1}{2}\beta^2 g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j - \gamma\, U_Q(\phi).$$

For variation $\theta^a \mapsto \theta^a + \varepsilon\eta^a$ (with $\eta^a(T_0) = \eta^a(T_1) = 0$) we have

$$\delta L = \tfrac{1}{2}\alpha^2(\partial_c G_{ab})\eta^c\dot{\theta}^a\dot{\theta}^b + \alpha^2 G_{ab}\dot{\theta}^a\dot{\eta}^b.$$

After integration

$$\delta\mathcal{A}_Q = \int_{T_0}^{T_1} \alpha^2 G_{ab}\dot\theta^a\dot\eta^b + \tfrac{1}{2}\alpha^2(\partial_c G_{ab})\eta^c\dot\theta^a\dot\theta^b \,\mathrm{d}t.$$

Integrating first term by parts

$$\int_{T_0}^{T_1} \alpha^2 G_{ab}\dot\theta^a\dot\eta^b \,\mathrm{d}t = \big[\alpha^2 G_{ab}\dot\theta^a\eta^b\big]_{T_0}^{T_1} - \int_{T_0}^{T_1} \frac{\mathrm{d}}{\mathrm{d}t}\big(\alpha^2 G_{ab}\dot\theta^a\big)\eta^b \,\mathrm{d}t.$$

Boundary term vanishes, combining gives

$$\delta\mathcal{A}_Q = \int_{T_0}^{T_1} \Big(-\frac{\mathrm{d}}{\mathrm{d}t}(\alpha^2 G_{ab}\dot\theta^a) + \tfrac{1}{2}\alpha^2\partial_b G_{ac}\dot\theta^a\dot\theta^c\Big)\,\eta^b \,\mathrm{d}t.$$

By arbitrariness of variation, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\alpha^2 G_{ab}\dot\theta^a) - \tfrac{1}{2}\alpha^2\partial_b G_{ac}\dot\theta^a\dot\theta^c = 0.$$

Multiplying by G^{db} gives geodesic equation form. Variation of ϕ^i completely similar, extra term from $-\gamma U_Q(\phi)$, yielding

$$\frac{\mathrm{d}}{\mathrm{d}t}(\beta^2 g_{ij}\dot\phi^j) - \tfrac{1}{2}\beta^2\partial_i g_{jk}\dot\phi^j\dot\phi^k + \gamma\,\partial_i U_Q(\phi) = 0.$$

This completes the proof.

B Technical Framework of Γ-Convergence

B.1 Standard Γ-Convergence Result for Energy-Type Discrete Functionals

Let H be Hilbert space, $\mathcal{F}_h : H \to \mathbb{R}\cup\{+\infty\}$ be functional family of form

$$\mathcal{F}_h[u] = \sum_{k=0}^{n_h-1} \Big(\tfrac{1}{2}a_h^k\frac{(u_{k+1}-u_k)^2}{h} + h\,V_h^k(u_k)\Big),$$

under appropriate consistency assumptions, as $h\to 0$, $\mathcal{F}_h$ Γ-converges to

$$\mathcal{F}[u] = \int_0^T \Big(\tfrac{1}{2}a(t)\dot u(t)^2 + V(t,u(t))\Big)\,\mathrm{d}t.$$

Our discrete joint action $\mathcal{A}_Q^{(h)}$ belongs to vector version of such functional class, its Γ-convergence can be obtained by separately applying above scalar theory to control and information parts and combining. Key conditions include:

1. Second-order consistency of single-step cost and information distance:

$$\frac{\Delta s^2_{\mathcal{M},k}}{h^2} \to G_{ab}(\theta)\dot\theta^a\dot\theta^b, \quad \frac{\Delta s^2_{\mathcal{S}_Q,k}}{h^2} \to g_{ij}(\phi)\dot\phi^i\dot\phi^j;$$

2. Potential energy term $U_Q(\phi_k)h$ uniformly approximates integral term $\int U_Q(\phi(t))\mathrm{d}t$;

3. Appropriate compactness conditions (e.g., energy boundedness) ensure minimal sequences have weakly convergent subsequences.

Detailed technical derivation and corresponding literature framework not elaborated here.

B.2 Proof Idea of Theorem ??

Lower semicontinuity direction: For any weak limit z and convergent sequence $z^{(h)} \rightharpoonup z$, by convexity and weak lower semicontinuity of kinetic energy terms we get

$$\mathcal{A}_Q[z] \le \liminf_{h\to 0} \mathcal{A}_Q^{(h)}[z^{(h)}].$$

Recovery sequence direction: For any smooth limit trajectory z, use time discretization to construct $z^{(h)}$ such that $\mathcal{A}_Q^{(h)}[z^{(h)}] \to \mathcal{A}_Q[z]$. Specific construction: sample z at grid points $t_k = kh$ to get $z_k = z(t_k)$, define $z^{(h)}$ as piecewise linear interpolation, whereby both quadratic form of single-step increments and potential energy terms approximate corresponding integrals.

In summary, Γ-convergence holds. By general theory of Γ-convergence: if $z^{(h)}$ is approximate minimal sequence of $\mathcal{A}_Q^{(h)}$, then any weak limit point z is minimal point of $\mathcal{A}_Q$.

C Supplementary Remarks on Categorical Structure

Although this paper does not systematically expand on categorical theory, here we provide brief supplement on connection between "computational universe category $\leftrightarrow$ control–scattering category" to illustrate naturality of joint variational principle at categorical level.

1. **Object level:** For each physically realizable computational universe U_{comp}, construct control–scattering object $(\mathcal{M}, G, S)$ and task information manifold $(\mathcal{S}_Q, g_Q)$, whose joint manifold $\mathcal{E}_Q$ and joint action $\mathcal{A}_Q$ constitute "intrinsic dynamical geometric image" of this object.

2. **Morphism level:** Simulation mapping $f : U_{\text{comp}} \rightsquigarrow U'_{\text{comp}}$ can give through physical realization mappings $f_{\mathcal{M}}, f_{\mathcal{S}_Q}$ between control manifolds and information manifolds, these mappings satisfy Lipschitz-type inequalities under metrics and joint action, thereby preserving basic structure of minimal worldlines.

3. **Naturality of joint worldlines:** Images of simulations between different computational universes on joint manifold are family of "deformed" worldlines, Γ-convergence ensures that under appropriate limit, images of minimal worldlines remain minimal worldlines.

This structure provides dynamical–variational intermediate layer for subsequent construction of "categorical equivalence between physical universe category and computational universe category," making claim "universe = computation" hold not only at static structure level (configuration, update) but also at evolution level of time–information–complexity triple structure.