Black Holes as Resolution-Relative Structures: Information Horizons and Geometric Horizons in Quantum Cellular Automaton Universe

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Abstract

Traditional general relativity defines black holes as geometric objects with event horizons: future-directed light rays from any point inside horizon cannot reach infinity. This definition is global and independent of specific observers. However, any real observer has finite resources in energy bands, observation duration, and computational capacity, coupling with universe only through finite-precision "windows". Within quantum cellular automaton (QCA) and unified time scale κ framework, this paper systematically constructs **observer resolution-relative black hole concept** and proves rigorous limiting relation with geometric event horizon.

We first introduce three minimalist axioms on QCA universe object $\mathfrak{U}_{\mathrm{QCA}} = (\Lambda, \mathcal{H}_{\mathrm{cell}}, \mathcal{A}, \alpha, \omega_0)$: A1 (discrete–unitary–local), A2 (Lieb–Robinson type finite light cone with maximal signal speed c), and A3 (existence of Dirac-type low-energy effective mode). On this basis, we formalize "observer" as $\mathcal{O} = (\mathcal{A}_{\mathrm{loc}}, \omega_{\mathrm{mem}}, \mathcal{M})$ and characterize its finite resolution through triple $\mathcal{R} = (I_\Omega, T, \varepsilon)$: energy/frequency window I_Ω, maximal waiting time T, and tolerable signal-to-noise threshold ε. Utilizing scattering structure induced by QCA in continuum limit and unified time scale $\kappa(x, \omega)$, we define **information horizon** relative to resolution $\mathcal{R}$: for observers of this class, any signal emitted from inside horizon cannot be decoded with capacity greater than threshold ε within accessible frequency band and observation time.

In static spherically symmetric case, we connect QCA unified time scale $\kappa(r, \omega)$ with effective Schwarzschild-type metric, proving **relative time delay integral** of radial signals

$$\tau_{\mathrm{rel}}(r, \omega) = \int_r^\infty \kappa(r', \omega)\, \mathrm{d}r'$$

diverges for all frequencies at geometric event horizon $r = r_{\mathrm{H}}$. Further, for given resolution $\mathcal{R}$, we define **resolution-dependent black hole radius** $r_{\mathrm{BH}}(\mathcal{R})$ as minimal r such that $\tau_{\mathrm{rel}}(r, \omega) > T$ for all $\omega \in I_\Omega$. We prove: $r_{\mathrm{BH}}(\mathcal{R})$ monotonically contracts inward as resolution increases (I_Ω expands, T increases, ε decreases), converging to geometric event horizon in "ideal observer limit":

$$\lim_{\mathcal{R} \to \mathcal{R}_{\mathrm{ideal}}} r_{\mathrm{BH}}(\mathcal{R}) = r_{\mathrm{H}}.$$

Core result: black holes have **geometric hard core** (event horizon), but what any finite-resource observer "sees" is resolution-dependent "information horizon"; latter can be precisely defined in QCA universe through unified time scale and scattering theory, connected to geometric horizon by rigorous limiting relation.

Keywords: Black holes; Observer resolution; Information horizon; Quantum cellular automaton; Unified time scale; Event horizon

1 Introduction

1.1 Geometric Definition of Black Holes and Observer-Dependent Features

In general relativity, black holes usually defined as spacetime regions whose boundary is event horizon: future-directed null geodesics from any point inside cannot reach future infinity. Formally, if $(M, g_{\mu\nu})$ is future-complete spacetime manifold, black hole region $\mathcal{B}$ defined as

$$\mathcal{B} := M \setminus J^-(\mathcal{I}^+),$$

where $J^-(\mathcal{I}^+)$ is causal past of future infinity, event horizon is $\partial\mathcal{B}$. This definition completely geometric, not explicitly involving observers, measurements or information.

On other hand, "black holes" in actual physical discussions often carry strong observer coloring: static observers in Schwarzschild coordinates see freely falling objects "forever approaching but never crossing" horizon; accelerating observers may experience Rindler-like horizons; cosmology has horizons related to universe expansion rate. These phenomena suggest: **existence of certain "horizons" closely related to specific observer worldlines and resolution**.

1.2 Quantum Cellular Automata and Unified Time Scale

Quantum cellular automata provide rigorous framework for reducing universe to discrete quantum computational process. Universe viewed as tensor product of local Hilbert spaces on countable graph, time evolution realized through finite-depth local unitary circuits. In this framework, causal structure given by Lieb–Robinson light cones, maximal signal speed c naturally emerges.

In scattering theory and spectral analysis language, can introduce unified time scale function

$$\kappa(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega),$$

where $Q(\omega)$ is Wigner–Smith time delay operator, $\varphi(\omega)$ is total scattering phase, ρ_{rel} is relative state density. In QCA continuum limit, κ can be viewed as "time flow rate" or "optical path density".

1.3 Problem and Contributions of This Paper

Central question: In discrete ontology based on QCA and unified time scale, can black holes be defined as **"information horizons" relative to observer resolution**? If so, how do such horizons relate to geometric event horizons?

We do three things:

1. Formalize observer "resolution": $\mathcal{R} = (I_\Omega, T, \varepsilon)$ with accessible frequency band I_Ω, maximal waiting time T, tolerable signal-to-noise lower bound ε.

2. Using unified time scale $\kappa(x, \omega)$ and scattering matrix $S_{x_0}(\omega)$, define **information horizon** $\mathcal{H}(\mathcal{R})$ relative to $\mathcal{R}$: boundary of irrecoverable information under resource constraints for this observer class.

3. In continuum limit of static spherically symmetric QCA universe, prove information horizon radius $r_{\mathrm{BH}}(\mathcal{R})$ monotonically contracts inward as resolution increases, converging to geometric event horizon radius r_{H} in "ideal observer limit".

This provides ontological picture of black holes accommodating both geometry and information theory: black holes have resolution-dependent "shell" and resolution-limit-independent geometric hard core.

2 QCA Universe and Observer Resolution

2.1 Universe as QCA Object

Definition 2.1 (Universe object). *Universe is 5-tuple*

$$\mathfrak{U}_{\mathrm{QCA}} = (\Lambda, \mathcal{H}_{\mathrm{cell}}, \mathcal{A}, \alpha, \omega_0),$$

where: (1) Λ countable locally finite graph; (2) $\mathcal{H}_{\mathrm{cell}} \cong \mathbb{C}^d$ finite-dimensional local Hilbert space; (3) $\mathcal{A}$ quasilocal operator algebra; (4) $\alpha : \mathbb{Z} \to \mathrm{Aut}(\mathcal{A})$ time evolution; (5) ω_0 initial state.

Axioms A1–A3: (A1) Above structure exists with $\dim \mathcal{H}_{\mathrm{cell}} < \infty$; (A2) Exists Lieb–Robinson velocity $c > 0$; (A3) Exists Dirac-type effective mode in low-energy one-particle sector.

2.2 Observer Object and Worldline

Definition 2.2 (Observer object). *Observer is triple $\mathcal{O} = (\mathcal{A}_{\mathrm{loc}}, \omega_{\mathrm{mem}}, \mathcal{M})$ where: (1) $\mathcal{A}_{\mathrm{loc}} \subset \mathcal{A}$ local subalgebra; (2) $\omega_{\mathrm{mem}}(t)$ time-parametrized state family; (3) $\mathcal{M} = \{M_\theta\}$ model family.*

2.3 Resolution Triple and Resource Constraints

Definition 2.3 (Resolution triple). *For observer $\mathcal{O}$, resolution characterized by triple $\mathcal{R} = (I_\Omega, T, \varepsilon)$ where: (1) $I_\Omega \subset \mathbb{R}^+$ resolvable frequency window; (2) $T > 0$ maximal waiting time; (3) $0 < \varepsilon < 1$ acceptable minimal signal-to-noise ratio.*

3 Unified Time Scale and Radial Time Delay

3.1 Localization of Unified Time Scale

In scattering theory, given background Hamiltonian H_0 and interacting $H = H_0 + V$, exists scattering matrix $S(\omega)$ and Wigner–Smith time delay operator

$$Q(\omega) = -\mathrm{i} S^\dagger(\omega)\, \partial_\omega S(\omega).$$

Unified time scale defined as

$$\kappa(\omega) = \frac{1}{2\pi} \mathrm{tr}\, Q(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega).$$

In QCA continuum limit for static spherically symmetric background, can localize unified time scale as $\kappa(r, \omega)$.

3.2 Radial Relative Time Delay

Define relative time delay from radius r to infinity as

$$\tau_{\mathrm{rel}}(r, \omega) := \int_r^\infty \kappa(r', \omega)\, \mathrm{d}r'.$$

Physically, $\tau_{\mathrm{rel}}(r, \omega)$ represents extra propagation "time cost" for signal of frequency ω emitted from radius r. In Schwarzschild-type metric, $\tau_{\mathrm{rel}}(r, \omega)$ diverges for all ω at geometric event horizon $r = r_{\mathrm{H}}$.

4 Observer-Relative Information Horizon and Black Hole Definition

4.1 Channel Capacity Perspective

Consider source in spherically symmetric region emitting signals at radius r with spectrum $\omega \in I_\Omega$. Maximum average mutual information recoverable from signals under resource constraint $\mathcal{R} = (I_\Omega, T, \varepsilon)$ denoted $C(r; \mathcal{R})$.

Key properties: If $\tau_{\mathrm{rel}}(r, \omega) \gg T$, then $C(r; \mathcal{R}) \to 0$; if exists $\omega \in I_\Omega$ with $\tau_{\mathrm{rel}}(r, \omega) \ll T$, then $C(r; \mathcal{R})$ positive.

4.2 Resolution-Dependent Black Hole Radius

Definition 4.1 ($\mathcal{R}$-black hole radius)**.** *Given resolution triple $\mathcal{R} = (I_\Omega, T, \varepsilon)$, define*

$$r_{\mathrm{BH}}(\mathcal{R}) := \inf \left\{ r > 0 \;\middle|\; \forall r' < r,\ \forall \omega \in I_\Omega :\ \tau_{\mathrm{rel}}(r', \omega) > T \right\}.$$

Definition 4.2 ($\mathcal{R}$-information horizon)**.** *For resolution triple $\mathcal{R}$, information horizon $\mathcal{H}(\mathcal{R})$ defined as sphere of radius $r_{\mathrm{BH}}(\mathcal{R})$.*

4.3 Partial Order on Resolutions and Black Hole Radius Monotonicity

Definition 4.3 (Resolution partial order)**.** *For two resolution triples $\mathcal{R}_1$, $\mathcal{R}_2$, if $I_{\Omega,1} \subset I_{\Omega,2}$, $T_1 \leq T_2$, $\varepsilon_1 \geq \varepsilon_2$, then $\mathcal{R}_2$ has resolution no lower than $\mathcal{R}_1$, denoted $\mathcal{R}_1 \preceq \mathcal{R}_2$.*

Theorem 4.4 (Monotonicity of black hole radius)**.** *If $\mathcal{R}_1 \preceq \mathcal{R}_2$, then $r_{\mathrm{BH}}(\mathcal{R}_2) \leq r_{\mathrm{BH}}(\mathcal{R}_1)$.*

Therefore, as observer resolution increases, information horizon monotonically contracts toward geometric center.

5 Geometric Event Horizon as Ideal Observer Limit

5.1 Schwarzschild-Type Metric and Time Delay Divergence

Consider Schwarzschild-type effective metric

$$\mathrm{d}s^2 = -f(r)c^2\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2, \quad f(r) = 1 - \frac{r_{\mathrm{H}}}{r},$$

where $r_{\mathrm{H}} = 2GM/c^2$ is geometric event horizon radius. For light signal propagating outward from $r_0 > r_{\mathrm{H}}$, coordinate time delay

$$\Delta t(r_0) = \int_{r_0}^{\infty} \frac{\mathrm{d}r}{cf(r)}.$$

Simple calculation gives

$$\lim_{r_0 \to r_{\mathrm{H}}^+} \Delta t(r_0) = +\infty.$$

5.2 Ideal Observer Limit

Definition 5.1 (Ideal observer limit)**.** *Define sequence of resolution triples $\{\mathcal{R}_n\}_{n\in\mathbb{N}}$ where $\mathcal{R}_n = (I_{\Omega,n}, T_n, \varepsilon_n)$ satisfies: (1) $I_{\Omega,n} \uparrow (0, \infty)$; (2) $T_n \uparrow \infty$; (3) $\varepsilon_n \downarrow 0$. Call $\mathcal{R}_n \to \mathcal{R}_{\mathrm{ideal}}$ ideal observer limit.*

Theorem 5.2 (Information horizon converges to event horizon). *Under Schwarzschild-type background, if unified time scale $\kappa(r,\omega)$ satisfies: (1) For each $\omega > 0$, $\kappa(r,\omega)$ continuous on $r > r_{\mathrm{H}}$ with $\tau_{\mathrm{rel}}(r,\omega) < \infty$ for all $r > r_{\mathrm{H}}$; (2) For each compact interval $[\omega_1, \omega_2] \subset (0,\infty)$, divergence behavior uniform on interval; then for any ideal observer sequence $\{\mathcal{R}_n\}$,*

$$\lim_{n\to\infty} r_{\mathrm{BH}}(\mathcal{R}_n) = r_{\mathrm{H}}.$$

Therefore geometric event horizon can be characterized as common limit of black hole radii $r_{\mathrm{BH}}(\mathcal{R})$ for all physically realizable resolutions in ideal observer limit.

6 Conclusion and Ontological Interpretation

Within QCA and unified time scale framework, we provide two-layer ontological characterization of black holes:

1. **Geometric hard core:** In continuum limit, effective metric induced by QCA universe has geometric event horizon r_{H} with divergent radial time delay for all frequencies, forming observer-independent topological structure.
2. **Resolution shell:** For each specific resolution triple $\mathcal{R}$, can define information horizon $\mathcal{H}(\mathcal{R})$ and black hole radius $r_{\mathrm{BH}}(\mathcal{R})$ describing information-inaccessible boundary for observer class, monotonically contracting inward as resolution increases.

Geometric event horizon r_{H} proven by Theorem 5.2 to be common limit of all information horizons in ideal observer limit. Can rigorously say:

- Black holes as information objects are relative to observer resolution;
- Black holes as geometric objects are invariant structures commonly "seen" by all physical observers in resolution limit.

In QCA ontology, universe is maximally consistent quantum cellular automaton object; black holes are special information manifolds in this object: appearing as information-frozen layers to finite-resource observers; corresponding to geometric event horizons in ideal limit. Both precisely connected through divergence behavior of unified time scale and radial time delay.

Appendix A: Unified Time Scale and Relative State Density

This appendix briefly reviews relation between unified time scale $\kappa(\omega)$ and spectral shift function, explaining its applicability in QCA context.

A.1 Spectral Shift Function and Birman–Kreĭn Formula

Let H_0, H be self-adjoint operators on same Hilbert space, $V = H - H_0$ trace class perturbation. Exists spectral shift function $\xi(\lambda)$ satisfying

$$\mathrm{tr}\big(\varphi(H) - \varphi(H_0)\big) = \int_{-\infty}^{+\infty} \varphi'(\lambda)\,\xi(\lambda)\,\mathrm{d}\lambda,$$

for all $\varphi \in C_0^\infty(\mathbb{R})$. Determinant of scattering matrix $S(\omega)$ satisfies Birman–Kreĭn formula

$$\det S(\omega) = \exp\big(-2\pi\mathrm{i}\,\xi(\omega)\big).$$

Let $\varphi(\omega) := \arg\det S(\omega)$, then

$$\varphi(\omega) = -2\pi\xi(\omega) + 2\pi k, \quad k \in \mathbb{Z},$$

thus

$$\varphi'(\omega) = -2\pi\xi'(\omega) = -2\pi\rho_{\mathrm{rel}}(\omega),$$

where $\rho_{\mathrm{rel}}(\omega) = \rho(\omega) - \rho_0(\omega)$ is relative state density. With appropriate sign convention adjustment obtain

$$\kappa(\omega) := \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega).$$

A.2 Wigner–Smith Time Delay Operator

Wigner–Smith time delay operator defined as

$$Q(\omega) = -\mathrm{i}S^{\dagger}(\omega)\,\partial_\omega S(\omega).$$

If spectral decomposition of $S(\omega)$ is

$$S(\omega) = \sum_n \mathrm{e}^{\mathrm{i}\theta_n(\omega)} \left|\psi_n(\omega)\right\rangle \left\langle\psi_n(\omega)\right|,$$

then

$$Q(\omega) = \sum_n \partial_\omega\theta_n(\omega) \left|\psi_n(\omega)\right\rangle \left\langle\psi_n(\omega)\right|,$$

thus

$$\operatorname{tr} Q(\omega) = \sum_n \partial_\omega\theta_n(\omega) = \partial_\omega\left(\sum_n \theta_n(\omega)\right) = \partial_\omega\varphi(\omega).$$

Therefore unified time scale can also be written as

$$\kappa(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega).$$

In QCA continuum limit, this formula can hold in each local region, defining $\kappa(x,\omega)$.

Appendix B: Radial Time Delay and Schwarzschild Metric

This appendix presents calculation of time delay for radial null geodesic and correspondence with unified time scale.

B.1 Radial Null Geodesic

In Schwarzschild-type metric

$$\mathrm{d}s^2 = -f(r)c^2\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2, \quad f(r) = 1 - \frac{r_{\mathrm{H}}}{r},$$

radial null geodesic satisfies

$$0 = -f(r)c^2\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2,$$

i.e.

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \pm c f(r).$$

For outward propagating light signal (positive sign), time from r_0 to reference point R is

$$\Delta t_{r_0\to R} = \int_{r_0}^{R} \frac{\mathrm{d}r}{c f(r)}.$$

Taking $R \to \infty$ and subtracting propagation time in flat background, obtain relative time delay

$$\tau_{\mathrm{rel}}(r_0) = \int_{r_0}^{\infty} \left(\frac{1}{c f(r)} - \frac{1}{c}\right)\mathrm{d}r.$$

Near $r \to r_{\mathrm{H}}^{+}$, $f(r) \sim (r - r_{\mathrm{H}})/r_{\mathrm{H}}$, thus integrand has $1/(r - r_{\mathrm{H}})$ type singularity near lower limit, causing logarithmic divergence of integral.

B.2 Correspondence with Unified Time Scale

In scattering theory, this time delay closely related to radial integral of unified time scale $\kappa(r,\omega)$. For given frequency ω, by constructing radial scattering problem with H_0 as flat background Hamiltonian and H as effective Hamiltonian including geometric effects, unified time scale $\kappa(r,\omega)$ characterizes local increase/decrease of state density relative to flat background. Radial relative time delay can be written as

$$\tau_{\mathrm{rel}}(r,\omega) = \int_r^\infty \kappa(r',\omega)\,\mathrm{d}r',$$

whose divergence at $r \to r_{\mathrm{H}}^+$ matches $\Delta t(r_0)$ in geometric analysis.

Appendix C: Details of Information Horizon Monotonicity and Convergence Proofs

This appendix provides technical details omitted in Theorems 4.4 and 5.2.

C.1 Equivalent Form of Black Hole Radius Definition

Define

$$\mathcal{B}(\mathcal{R}) := \bigcup_{\omega\in I_\Omega} \{r > 0 \mid \tau_{\mathrm{rel}}(r,\omega) > T\}.$$

Clearly $\mathcal{B}(\mathcal{R})$ is some interval $(0, r_{\mathrm{BH}}(\mathcal{R}))$. This is because for attractive spacetime, $\tau_{\mathrm{rel}}(r,\omega)$ decreases monotonically with r. Thus

$$r_{\mathrm{BH}}(\mathcal{R}) = \sup \mathcal{B}(\mathcal{R}).$$

This definition equivalent to "minimal r" definition in main text.

C.2 Rigorous Proof of Monotonicity

If $\mathcal{R}_1 \preceq \mathcal{R}_2$, then $I_{\Omega,1} \subset I_{\Omega,2}$, $T_1 \le T_2$. Thus for any r and $\omega \in I_{\Omega,1}$ have

$$\tau_{\mathrm{rel}}(r,\omega) > T_1 \Longrightarrow \tau_{\mathrm{rel}}(r,\omega) > T_2.$$

Therefore

$$\mathcal{B}(\mathcal{R}_1) \subset \mathcal{B}(\mathcal{R}_2) \quad \Rightarrow \quad \sup \mathcal{B}(\mathcal{R}_2) \le \sup \mathcal{B}(\mathcal{R}_1),$$

i.e. $r_{\mathrm{BH}}(\mathcal{R}_2) \le r_{\mathrm{BH}}(\mathcal{R}_1)$.

C.3 Steps for Convergence to Event Horizon

Key to Theorem 5.2 are two points:

1. For any $\delta > 0$, exists ω such that $\tau_{\mathrm{rel}}(r_{\mathrm{H}} + \delta, \omega) < \infty$, therefore exists finite T and appropriate resolution $\mathcal{R}$ such that $r_{\mathrm{H}} + \delta$ does not belong to black hole region, thus $r_{\mathrm{BH}}(\mathcal{R}) \ge r_{\mathrm{H}}$;

2. For any $\epsilon > 0$, uniform divergence guarantees exists T_* such that when $r \le r_{\mathrm{H}} + \epsilon$ for all ω have $\tau_{\mathrm{rel}}(r,\omega) > T_*$; taking $T_n > T_*$ and $I_{\Omega,n}$ sufficiently wide ensures $r_{\mathrm{BH}}(\mathcal{R}_n) \le r_{\mathrm{H}} + \epsilon$.

Combining upper and lower bounds obtains required convergence.

References

[1] R. Penrose, "Gravitational collapse and space-time singularities", Phys. Rev. Lett. **14**, 57 (1965).

[2] E. H. Lieb, D. W. Robinson, "The finite group velocity of quantum spin systems", Commun. Math. Phys. **28**, 251 (1972).

[3] F. T. Smith, "Lifetime matrix in collision theory", Phys. Rev. **118**, 349 (1960).