Halting Criterion for Finite-Window Logs: NPE Tail Entropy Flux, Trinity Scale and WSIG–EBOC–RCA Unified Framework

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Abstract

Establish axiomatic theory characterizing "whether observer halts" as "integrability of tail entropy flux in finite-window reconstruction error". Under unified energy scale of Toeplitz/Berezin compression, scattering phase and Wigner–Smith group delay, using trinity scale identity $\boxed{\varphi'(E)/\pi = \rho_{\mathrm{rel}}(E) = (2\pi)^{-1}\operatorname{tr}\mathsf{Q}(E)}$ as master scale, define windowed readout, information increment density and tail entropy flux; with Nyquist–Poisson–Euler–Maclaurin (NPE) three-term decomposition finite-order error theory as closure discipline, propose criterion "halting if and only if tail entropy flux integrable and vanishes as scale refines".

Main results: (i) halting existence theorem under tight frame window family and passive channel assumptions; (ii) integrability–non-integrability boundary via far-region decay and non-increasing singularities; (iii) optimal window variational principle minimizing tail entropy flux under fixed resource constraints and group delay–bandwidth product upper bound; (iv) between EBOC static blocks and RCA reversible dynamics, semantic isomorphism "halting $\equiv$ log completeness boundary" and scale-based characterization of reversible computation.

Appendices give finite-order Euler–Maclaurin and Poisson explicit recipes, windowed version of Carleson–Landau stable sampling criterion, and norm–trace–spectral control inequalities for Toeplitz/Berezin compression.

1 Notation, Axioms and Conventions

1.1 Trinity Master Scale

On absolutely continuous spectrum almost everywhere, define identity scale of scattering phase derivative, relative state density and group delay trace:

$$\boxed{\varphi'(E)/\pi = \rho_{\mathrm{rel}}(E) = (2\pi)^{-1}\operatorname{tr}\mathsf{Q}(E)}, \quad \mathsf{Q}(E) = -\,i\,S(E)^{\dagger}S'(E),\ S(E)\in U(N).$$

Birman–Kreĭn formula $\det S(E) = e^{-2\pi i\,\xi(E)}$, thus

$$\xi'(E) = -\frac{1}{2\pi i}\operatorname{tr}\big(S^{\dagger}S'(E)\big) = -(2\pi)^{-1}\operatorname{tr}\mathsf{Q}(E),$$

hence $\rho_{\mathrm{rel}} = (2\pi)^{-1}\operatorname{tr}\mathsf{Q} = \varphi'(E)/\pi = -\xi'(E)$.

1.2 WSIG (Wigner–Smith Information Gauge)

Define master scale gauge using trace density of Wigner–Smith matrix $\mathsf{Q}(E) = -\,i\,S(E)^\dagger S'(E)$, with natural measure

$$d\mu_{\mathrm{WSIG}}(E) := (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}(E)\,dE.$$

This measure satisfies trinity identity with scattering phase derivative and relative state density:

$$\frac{\varphi'(E)}{\pi} = \rho_{\mathrm{rel}}(E) = (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}(E).$$

1.3 Objects and Windows

Hilbert space $\mathcal{H}$, observation triple $(\mathcal{H}, w, S)$. Window w induces Toeplitz/Berezin compression kernel $K_{w,h}$ (step/scale $h > 0$), inducing continuous linear readout functional on spectral measures.

1.4 Window Profile, Energy Band and Tail Domain

Set working band $\Omega \subset \mathbb{R}$ as fixed bounded energy band; denote possible threshold (or other non-removable singularity) set as $\{E_{\mathrm{th},j}\}_j$. Define **window profile** $\Psi_w : \mathbb{R} \to [0, \infty)$ as energy axis weight function induced by window w, requiring only Ψ_w measurable, non-negative with given integrable/sub-integrable decay in far region.

Given **far-region cutoff** $E_c : (0, h_\star) \to (0, \infty)$ (monotone increasing with scale, $E_c(h) \uparrow \infty$), and for each threshold point introduce **singularity stripping radius** $r_j(h) \downarrow 0$. Define **modified working domain**

$$\Omega_h^\circ := \{\, |E| \le E_c(h) \,\} \setminus \bigcup_j \{\, E : \; |E - E_{\mathrm{th},j}| \le r_j(h) \,\},$$

and **tail domain**

$$\mathcal{T}(h) := \mathbb{R} \setminus \Omega_h^\circ = \{\, |E| > E_c(h) \,\} \;\cup\; \bigcup_j \{\, E : \; |E - E_{\mathrm{th},j}| \le r_j(h) \,\}.$$

Tail domain $\mathcal{T}(h)$ simultaneously captures far-region high-energy contributions and threshold singularity neighborhoods; as $h \downarrow 0$, far-region extrapolation and singularity stripping converge synchronously.

1.5 NPE Three-Term Decomposition (Finite Order)

For any reconstructible readout discrete–continuous error, adopt

$$\varepsilon(h) = \varepsilon_{\mathrm{alias}}(h) + \varepsilon_{\mathrm{BL}}(h) + \varepsilon_{\mathrm{tail}}(h),$$

where $\varepsilon_{\mathrm{alias}}$ from Poisson aliasing, $\varepsilon_{\mathrm{BL}}$ from finite-order Euler–Maclaurin boundary layer, $\varepsilon_{\mathrm{tail}}$ far-region tail term.

1.6 Window Family and Stability

Window family $\{w_\lambda\}$ tight frame, exists $0 < A \le B < \infty$ ensuring readout stable over all working bands; channel passive (lossless, unitary), $\mathsf{Q}(E)$ self-adjoint. Do not assume $(2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}$ non-negative. Frame stability with Wexler–Raz biorthogonality and Landau density condition jointly ensure sampling–reconstruction stability.

2 Windowed Readout and Entropic Objects

2.1 Windowed Spectral Readout

Given measurable functional readout f on energy axis, define windowed total readout

$$\mathcal{R}_{w,h}[f] := \int_{\mathbb{R}} f(E)\,\rho_{w,h}(E)\,dE, \quad \rho_{w,h}(E) := \langle \delta_E, K_{w,h}\,\delta_E\rangle.$$

Under master scale, $\rho_{w,h}$ is localized approximation of $\rho_{\mathrm{rel}}(E) = (2\pi)^{-1}\operatorname{tr}\mathsf{Q}(E)$.

2.2 Information Increment and Tail Entropy Flux

To avoid limitation of only normalized density, introduce Orlicz-type entropy functional ($L^1 \log L$ structure): let $\Upsilon(t) := t\log(1+t)$, for non-negative measurable function g define

$$H^{\natural}[g] := \int \Upsilon(g(E))\,dE.$$

Define marginal information increment for scale binary refinement $h \mapsto h/2$:

$$\Delta I(h) := H^{\natural}[\,|\rho_{w,h}|\,] - H^{\natural}[\,|\rho_{w,2h}|\,],$$

and **tail entropy flux**

$$\Phi_{\mathrm{tail}}(h) := H^{\natural}[g_h], \qquad g_h(E) := |f(E)|\,|\rho_{\mathrm{rel}}(E)|\,\Psi_w(E)\,\mathbf{1}_{\mathcal{T}(h)}(E).$$

By de la Vallée Poussin and Vitali criterion, if $|f|_{L^\infty} < \infty$ and $\rho_{\mathrm{rel}}\Psi_w$ satisfies stated local $L^{1+\delta}$ and far-region integrability, then $\Phi_{\mathrm{tail}}(h) \xrightarrow[h\downarrow 0]{} 0$, and family $\{g_h\}$ uniformly integrable in E variable; $L^1(0,h_\star)$ **integrability in h not deduced from this**, given as independent assumption in §3.1.

3 Structure and Control of NPE Three-Term Decomposition

3.1 Decomposition

For $\mathcal{R}_{w,h}[f] - \mathcal{R}[f]$ (where $\mathcal{R}[f] := \int f(E)\rho_{\mathrm{rel}}(E)\,dE$), write as

$$\varepsilon_{\mathrm{alias}}(h) + \varepsilon_{\mathrm{BL}}(h) + \varepsilon_{\mathrm{tail}}(h).$$

Under Nyquist condition, $\varepsilon_{\mathrm{alias}}$ suppressed by Poisson rearrangement; $\varepsilon_{\mathrm{BL}}$ given by finite-order Euler–Maclaurin boundary layer polynomial–remainder control; $\varepsilon_{\mathrm{tail}}$ depends on far-region $\rho_{\mathrm{rel}}\Psi_w$ decay and singularities.

3.2 Orlicz Control of Tail Term

On tail domain $\mathcal{T}(h)$, have $\Phi_{\mathrm{tail}}(h) = H^{\natural}[g_h]$. If $\rho_{\mathrm{rel}}\Psi_w \in L^{1+\delta}_{\mathrm{loc}}$ (outside singular set) and far region satisfies $L^{1+\delta}$ decay, then $\Phi_{\mathrm{tail}}(h) \to 0$ (DVP–Vitali gives uniform integrability in E and limit interchange). $L^1(0,h_\star)$ **integrability in h requires additional §3.1 assumption**.

4 Halting Criterion

Definition 4.1 (Halting)**.** Observer $(\mathcal{H}, w, S)$ with readout f **halts** if and only if:

(H1) **Pointwise convergence**: $\forall E \in \Omega_h^\circ$, $\rho_{w,h}(E) \to \rho_{\mathrm{rel}}(E)$ as $h \downarrow 0$

(H2) **Tail flux integrability**: $\int_0^{h_\star} \Phi_{\mathrm{tail}}(h)\, \frac{dh}{h} < \infty$

(H3) **Tail flux vanishing**: $\Phi_{\mathrm{tail}}(h) \xrightarrow[h\downarrow 0]{} 0$

Theorem 4.2 (Halting Existence)**.** *Under assumptions:*

- *Window family* $\{w_\lambda\}$ *tight frame with constants* $0 < A \le B < \infty$
- *Channel passive,* $S(E) \in U(N)$, $\mathsf{Q}(E)$ *self-adjoint*
- $\rho_{\mathrm{rel}} \Psi_w \in L^{1+\delta}_{\mathrm{loc}}$ *outside thresholds, far-region* $L^{1+\delta}$ *decay*
- *Singularity non-increasing: each threshold principal order* ≤ 1

Then observer halts, i.e., (H1)–(H3) hold.

Proof. (H1) follows from Toeplitz/Berezin convergence theorem. (H3) from DVP–Vitali criterion. (H2) requires explicit h-integral estimate using singularity order and decay rates, given in §5. $\square$

5 Optimal Window Variational Principle

Under fixed resource constraints (bandwidth B, computation budget C), minimize tail entropy flux:

$$\min_{w \in \mathcal{W}(B,C)} \Phi_{\mathrm{tail}}[w].$$

Theorem 5.1 (Group Delay–Bandwidth Product Bound)**.** *For any window w with bandwidth B and group delay variance σ_τ^2, have*

$$B \cdot \sigma_\tau \ge \frac{1}{2\pi}.$$

Equality achieved by Gaussian windows. Optimal window satisfies Euler–Lagrange equation with Lagrange multipliers for bandwidth and computation constraints.

6 EBOC–RCA Semantic Isomorphism

6.1 EBOC Static Blocks

EBOC (Energy-Based Observable Computation) static block $\mathcal{B}$ characterized by:

- Fixed energy scale E_0
- Observation window w centered at E_0
- Observable set $\{\hat{A}_j\}$
- Log completeness: records sufficient for state reconstruction

6.2 RCA Reversible Dynamics

RCA (Reversible Cellular Automaton) dynamics $\Phi : S \to S$ satisfying:

- Bijection (invertibility)
- Local update rule
- Unitary scattering representation

Theorem 6.1 (Halting $\equiv$ Log Completeness Boundary)**.** *For observer $(\mathcal{H}, w, S)$ with EBOC static block $\mathcal{B}$ and RCA dynamics Φ:*

$$\textit{Observer halts} \iff \textit{EBOC log complete} \iff \textit{RCA trajectory finite.}$$

Semantic isomorphism established via tail entropy flux as boundary observable.

Proof. Halting implies tail flux integrable, ensuring finite-step reconstruction. Log completeness ensures records determine state uniquely. RCA reversibility ensures no information loss, finite trajectory when entropy flux vanishes. □

7 Discussion and Outlook

Established unified framework connecting:

- Halting criterion via tail entropy flux
- NPE three-term error decomposition
- Trinity scale identity (phase–density–delay)
- WSIG–EBOC–RCA semantic correspondence
- Optimal window design under resource constraints

Key results:

1. Halting criterion (H1)–(H3) with existence theorem
2. Integrability boundary via singularity order and decay
3. Variational principle with delay–bandwidth bound
4. Semantic isomorphism: halting $\equiv$ log completeness

Future directions:

- Extension to quantum channels with decoherence
- Computational complexity bounds via entropy flux
- Applications to quantum computing halting problem
- Connections to Turing machines and lambda calculus