# Equivalence Between Physical Universe and Matrix Universe: Causal Manifolds, Boundary Time Geometry, and Scattering Matrix Universe THE-MATRIX


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**Abstract**

Building on the unified framework of causal manifolds, axiomatic causal structure, unified time scale and boundary time geometry, Null–Modular double cover, and information geometric variational principle, this paper introduces and characterizes a new ontological object: scattering matrix universe THE-MATRIX, and proves its categorical equivalence to "physical universes" satisfying specific axiom families.

On one hand, we model the physical universe as a causal manifold object with causal partial order, boundary observable algebra, modular flow, generalized entropy, and unified time scale mother ruler

$$U_{\mathrm{geo}} = (M, g, \prec, \mathcal{A}_\partial, \omega_\partial, S_{\mathrm{gen}}, \kappa),$$

where unified time scale is given by scale identity

$$\kappa(\omega) = \varphi'(\omega)/\pi = \rho_{\mathrm{rel}}(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}(\omega),$$

with $\varphi$ total scattering half-phase, $\mathsf{Q}(\omega)$ Wigner–Smith time-delay matrix, $\rho_{\mathrm{rel}}$ relative density of states, unified via Birman–Kreĭn formula and spectral shift function.

On the other hand, we define THE-MATRIX universe as a one-parameter unitary family $\mathbb{S}(\omega)$ acting on direct sum Hilbert space

$$\mathcal{H} = \bigoplus_{D \in \mathcal{D}} \mathcal{H}_D.$$

Its block matrix sparsity pattern encodes causal partial order; diagonal block scattering phase and time delay realize unified time scale; block structure and self-referential closed loops carry Null–Modular double cover and $\mathbb{Z}_2$ topological sector. Each small causal diamond $D$ corresponds to a boundary scattering block $\mathbb{S}_{DD}(\omega)$, whose local generalized entropy extremal condition and second-order non-negativity are equivalent to local Einstein equations and their stability under information geometric variational principle.

At categorical level, we construct geometric universe category $\mathsf{Uni}_{\mathrm{geo}}$ and matrix universe category $\mathsf{Uni}_{\mathrm{mat}}$, with morphisms preserving causal, scale, and entropy structures, and present encoding functor

$$F : \mathsf{Uni}_{\mathrm{geo}} \to \mathsf{Uni}_{\mathrm{mat}}$$

and decoding functor

$$G : \mathsf{Uni}_{\mathrm{mat}} \to \mathsf{Uni}_{\mathrm{geo}}.$$

Under axioms including global hyperbolicity, local spectral reconstructability, finite-order Euler–Maclaurin and Poisson error discipline, Null–Modular double cover completeness, and

generalized entropy variational completeness, we prove $F$ and $G$ are quasi-inverse, yielding universe category equivalence

$$\mathsf{Uni}_{\mathrm{geo}} \simeq \mathsf{Uni}_{\mathrm{mat}}.$$

Building on this, we formalize observers as compressions and readout operators on matrix universe, showing that the "world" seen by specific observers is a cross-section of THE-MATRIX, while multi-observer consensus problems can be formulated as geometric and informational consistency conditions between different cross-sections, interfacing with causal networks, axiomatic relative entropy, and modular flow theory.


## Keywords

Causal manifolds; Unified time scale; Boundary time geometry; Wigner–Smith time delay; Scattering matrix; Spectral shift function; Null–Modular double cover; Generalized entropy; Relative entropy; Matrix universe; Categorical equivalence


## 1 Introduction & Historical Context

General relativity and quantum field theory typically adopt light cones on four-dimensional Lorentz manifolds and local fields as fundamental structures; quantum many-body and quantum information theory tend toward matrices, operator arrays, and networks as primary language. The bridge between them traditionally relies on spectral theory, scattering theory, and operator algebras: Birman–Kreĭn formula links scattering matrix determinant with spectral shift function; Wigner–Smith time delay interprets scattering phase gradient as operatorized scale of "time delay"; Tomita–Takesaki modular theory and Araki relative entropy provide unified structure among time, temperature, and information monotonicity at von Neumann algebra level; Malament and Hawking–King–McCarthy formalized the idea that "causal structure determines spacetime topology and conformal class".

These developments jointly point to a natural question: can we view "physical universe" as some giant scattering matrix universe THE-MATRIX, such that geometric–causal picture and matrix–operator picture are equivalent? This conception has shown fragmentary signs across multiple research lines:

- **Scattering geometry and gravity**: Under appropriate boundary conditions, GHY boundary term and Brown–York energy can be restated via boundary scattering and spectral shift function;
- **Modular flow and thermal time**: Connes–Rovelli thermal time hypothesis views time as modular flow parameter induced by state–algebra pair;
- **Causal sets and discrete spacetime**: Approximate Lorentz manifold by partially ordered sets, utilizing Malament-type theorems to reconstruct topology and conformal class of metric;
- **Black hole thermodynamics and dynamical stability**: Hollands–Wald reformulate stability problem as positivity of "canonical energy", whose second-order variation is closely related to Hessian of generalized entropy.

On the other hand, physical systems such as scattering networks, quantum graphs, Floquet-driven lattices provide natural realizations of large-scale unitary block matrices, making "matrix universe" potentially engineering-realizable.

Building on existing work, this paper proposes and systematizes the following picture:

1. With small causal diamonds and their boundary observable algebras as local units, compress physical universe into causal manifold object $U_{\rm geo}$ with scale mother ruler and generalized entropy structure;

2. Characterize matrix universe THE-MATRIX by unitary block matrix family $\mathbb{S}(\omega)$ on direct sum Hilbert space, whose sparsity pattern encodes causal partial order, diagonal blocks encode boundary time geometry and generalized entropy;

3. At categorical level, construct encoding functor $F$ and decoding functor $G$, proving $\mathsf{Uni}_{\rm geo} \simeq \mathsf{Uni}_{\rm mat}$ under appropriate axioms.

This gives rigorous mathematical meaning to "physical universe = matrix universe THE-MATRIX", providing structural explanation for unification among observer consensus, causal networks, and operator networks.

# 2 Model & Assumptions

## 2.1 Geometric Universe Model $U_{\rm geo}$

Let $(M, g)$ be a four-dimensional, orientable, time-orientable, globally hyperbolic Lorentz manifold, with causal relation denoted $\prec$. For each point $p \in M$ and sufficiently small scale parameter $\ell > 0$, define small causal diamond

$$D_\ell(p) = I^+(p^-) \cap I^-(p^+),$$

where $p^\pm$ are displaced by $\ell$ along some proper time geodesic. Choose label set family $\mathcal{D}$ and map $\alpha \mapsto D_\alpha \subset M$ satisfying:

1. Each $D_\alpha$ is some $D_\ell(p)$;

2. $\{D_\alpha\}_{\alpha\in\mathcal{D}}$ covers $M$ and is locally finite;

3. If $D_\alpha \cap D_\beta \neq \varnothing$, overlap region remains globally hyperbolic.

Define partial order on $\mathcal{D}$ by

$$\alpha \preceq \beta \iff D_\alpha \subset J^-(D_\beta).$$

For each $D_\alpha$, endow boundary $\partial D_\alpha$ with von Neumann algebra $\mathcal{A}_\partial(D_\alpha)$ and faithful state $\omega_\alpha$, satisfying inclusion

$$D_\alpha \subset D_\beta \Rightarrow \mathcal{A}_\partial(D_\alpha) \subset \mathcal{A}_\partial(D_\beta).$$

For each pair $(\mathcal{A}_\partial(D_\alpha), \omega_\alpha)$, consider Tomita–Takesaki modular flow $\{\sigma_t^{(\alpha)}\}_{t\in\mathbb{R}}$, whose generator $K_\alpha$ localizes to null slice of $\partial D_\alpha$ in GNS representation.

On each $D_\alpha$ boundary consider fixed-energy scattering problem, with scattering matrix $S_\alpha(\omega)$ and Wigner–Smith time delay

$$\mathsf{Q}_\alpha(\omega) = -\mathrm{i} S_\alpha(\omega)^\dagger \partial_\omega S_\alpha(\omega).$$

Birman–Kreĭn formula gives spectral shift function $\xi_\alpha(\omega)$ and scattering determinant

$$\det S_\alpha(\omega) = \exp(-2\pi \mathrm{i} \xi_\alpha(\omega)),$$

with spectral shift function derivative being relative density of states $\rho_{\mathrm{rel},\alpha}(\omega) = \xi_\alpha'(\omega)$.

Unified time scale mother ruler defined as

$$\kappa_\alpha(\omega) = \varphi'_\alpha(\omega)/\pi = \rho_{\mathrm{rel},\alpha}(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}_\alpha(\omega),$$

where $\varphi_\alpha(\omega) = \pi\xi_\alpha(\omega)$ is total scattering half-phase. Trace defined under finite-order Euler–Maclaurin and Poisson error discipline, ensuring "singularity non-growth".

Generalized entropy takes form

$$S_{\mathrm{gen},\alpha} = A_\alpha/(4G\hbar) + S^{\mathrm{ren}}_{\mathrm{out},\alpha} + S^{\mathrm{UV}}_{\mathrm{ct},\alpha} - \Lambda V_\alpha/(8\pi G T_\alpha),$$

where $A_\alpha$ is waist surface area, $S^{\mathrm{ren}}_{\mathrm{out},\alpha}$ renormalized exterior entropy, $S^{\mathrm{UV}}_{\mathrm{ct},\alpha}$ local counterterm, $V_\alpha$ diamond volume, $T_\alpha$ modular or Unruh temperature.

**Axiom 1** (IGVP (Geometric Version))**.** In small-scale limit $\ell \to 0$, for each $p \in M$ and nearby diamond family $D_\ell(p)$:

1. For any variation satisfying appropriate boundary conditions, first variation $\delta S_{\mathrm{gen}} = 0$;
2. Second variation defines non-negative quadratic form;
3. Limit and averaging operations commute, allowing generalization to general state families.

Under standard regularity assumptions, this axiom is equivalent to local Einstein equations and positivity of Hollands–Wald canonical energy.

**Definition 2** (Geometric Universe Object)**.** A geometric universe is a seven-tuple

$$U_{\mathrm{geo}} = (M, g, \prec, \{\mathcal{A}_\partial(D_\alpha), \omega_\alpha\}_{\alpha\in\mathcal{D}}, \{\kappa_\alpha\}_{\alpha\in\mathcal{D}}, \{S_{\mathrm{gen},\alpha}\}_{\alpha\in\mathcal{D}}),$$

satisfying above geometric, algebraic, scale, and IGVP axioms.

## 2.2 Matrix Universe Model $U_{\mathrm{mat}}$

Take locally finite partially ordered set $(\mathcal{D}, \preceq)$, with each element's past and future cones finite, and scale map $\ell : \mathcal{D} \to (0, \ell_0]$. For each $\alpha \in \mathcal{D}$ take separable Hilbert space $\mathcal{H}_\alpha$, defining direct sum

$$\mathcal{H} = \bigoplus_{\alpha\in\mathcal{D}} \mathcal{H}_\alpha.$$

Define strongly continuous map $\omega \mapsto \mathbb{S}(\omega) \in \mathcal{U}(\mathcal{H})$ such that for each $\omega$, $\mathbb{S}(\omega)$ has block matrix form $\mathbb{S}_{\alpha\beta}(\omega) : \mathcal{H}_\beta \to \mathcal{H}_\alpha$ under direct sum decomposition, satisfying unitarity conditions.

**Axiom 3** (Causal Sparsity)**.** If $\mathbb{S}_{\alpha\beta}(\omega) \neq 0$, then $\alpha \preceq \beta$.

For each $\alpha$, define diagonal block $\mathbb{S}_{\alpha\alpha}(\omega)$ and

$$\mathsf{Q}_\alpha(\omega) = -\mathrm{i}\mathbb{S}_{\alpha\alpha}(\omega)^\dagger \partial_\omega \mathbb{S}_{\alpha\alpha}(\omega),$$

setting

$$\kappa_\alpha(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}_\alpha(\omega).$$

Require existence of scattering half-phase $\varphi_\alpha(\omega)$ and relative density of states $\rho_{\mathrm{rel},\alpha}(\omega)$ such that scale identity

$$\kappa_\alpha(\omega) = \varphi'_\alpha(\omega)/\pi = \rho_{\mathrm{rel},\alpha}(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,\mathsf{Q}_\alpha(\omega)$$

holds with error controlled by finite-order Euler–Maclaurin and Poisson discipline.

At each $\alpha$ assume existence of Null–Modular double cover decomposition of modular Hamiltonian $K_\alpha$ and $\mathbb{Z}_2$ ledger $\chi_\alpha$; product over closed causal diamond chains yields topological sector.

Matrix universe further carries generalized entropy function family $S_{\mathrm{gen},\alpha}$ constructed from block matrix spectrum, satisfying matrix version of IGVP axiom.

**Definition 4** (Matrix Universe Object)**.** A matrix universe is a five-tuple

$$U_{\mathrm{mat}} = (\mathcal{D}, \preceq, \{\mathcal{H}_\alpha\}_{\alpha\in\mathcal{D}}, \mathbb{S}(\omega), \{\kappa_\alpha, \chi_\alpha, S_{\mathrm{gen},\alpha}\}_{\alpha\in\mathcal{D}}),$$

satisfying above causal sparsity, scale, Null–Modular, and IGVP conditions.

# 3 Main Results (Theorems and Alignments)

This section states main results at categorical level, presenting equivalence relation between geometric and matrix universes.

## 3.1 Universe Categories and Morphisms

**Definition 5** (Geometric Universe Category $\mathsf{Uni}_{\mathrm{geo}}$)**.** Objects are all geometric universes $U_{\mathrm{geo}}$ satisfying above axioms. Morphism

$$f : U_{\mathrm{geo}} \to U'_{\mathrm{geo}}$$

consists of:

1. Causal homeomorphism $f_M : (M, g, \prec) \to (M', g', \prec')$;
2. Isomorphism $\mathcal{D} \to \mathcal{D}'$ on small causal diamond covering indices satisfying $f(D_\alpha) = D'_{f(\alpha)}$;
3. For each $\alpha$, $*$-isomorphism $\Phi_\alpha : \mathcal{A}_\partial(D_\alpha) \to \mathcal{A}_\partial(D'_{f(\alpha)})$ of von Neumann algebras, consistent with states: $\omega'_{f(\alpha)} \circ \Phi_\alpha = \omega_\alpha$;
4. Scale density and generalized entropy preserved under $f$: $\kappa'_{f(\alpha)} = \kappa_\alpha \circ f^{-1}$, $S'_{\mathrm{gen},f(\alpha)} = S_{\mathrm{gen},\alpha}$.

**Definition 6** (Matrix Universe Category $\mathsf{Uni}_{\mathrm{mat}}$)**.** Objects are all matrix universes $U_{\mathrm{mat}}$ satisfying above axioms. Morphism

$$\Psi : U_{\mathrm{mat}} \to U'_{\mathrm{mat}}$$

consists of partial order isomorphism $\psi : \mathcal{D} \to \mathcal{D}'$ and Hilbert space unitary operator

$$U : \mathcal{H} \to \mathcal{H}'$$

such that

$$U\mathbb{S}(\omega)U^\dagger = \mathbb{S}'(\omega), \qquad U(\mathcal{H}_\alpha) = \mathcal{H}'_{\psi(\alpha)},$$

preserving $\{\kappa_\alpha, \chi_\alpha, S_{\mathrm{gen},\alpha}\}$ data.

## 3.2 Encoding and Decoding Functors

**Definition 7** (Encoding Functor $F : \mathsf{Uni}_{\mathrm{geo}} \to \mathsf{Uni}_{\mathrm{mat}}$)**.** For object $U_{\mathrm{geo}}$:

1. Take small causal diamond covering index set $\mathcal{D}$ with partial order $\preceq$;
2. For each $\alpha$, take GNS Hilbert space $\mathcal{H}_\alpha$ or boundary scattering channel space;
3. On direct sum $\mathcal{H} = \bigoplus_\alpha \mathcal{H}_\alpha$ construct global scattering operator $\mathbb{S}(\omega)$ whose block matrix $\mathbb{S}_{\alpha\beta}(\omega)$ is determined by geometric universe's boundary conditions, propagation, and reflection structure; causality ensures sparsity pattern;

4. Diagonal blocks $\mathbb{S}_{\alpha\alpha}(\omega)$ with generalized entropy and Null–Modular data are given by geometric universe axioms, directly assigned to matrix universe.

Obtain matrix universe $F(U_{\text{geo}})$.

For morphism $f : U_{\text{geo}} \to U'_{\text{geo}}$, GNS universal property and scattering construction yield unitary operator $U_f : \mathcal{H} \to \mathcal{H}'$ and index isomorphism $\mathcal{D} \to \mathcal{D}'$, thus obtaining morphism $F(f)$, making $F$ a functor.

**Definition 8** (Decoding Functor $G : \mathsf{Uni}_{\text{mat}} \to \mathsf{Uni}_{\text{geo}}$)**.** For object $U_{\text{mat}}$:

1. View $(\mathcal{D}, \preceq)$ as abstract causal network, reconstructing topology and conformal structure via Alexandrov topology and Malament–Hawking–King–McCarthy type theorem;

2. Combining high- and low-frequency behavior of scale density $\kappa_\alpha(\omega)$, use spectral geometric methods to reconstruct boundary spectral triple and metric fragments from local scattering blocks $\mathbb{S}_{\alpha\alpha}(\omega)$, determining conformal factor and proper time scale of metric;

3. Construct generalized entropy $S_{\text{gen},\alpha}$ from block matrix spectrum, deriving Einstein equations in small diamond limit via IGVP axiom, obtaining Lorentz manifold $(M, g)$ and its causal structure $\prec$;

4. Construct boundary observable algebra $\mathcal{A}_\partial(D_\alpha)$ and state $\omega_\alpha$ from block matrix in-out structure; reconstruct modular flow and Null–Modular double cover from $\kappa_\alpha$ and $\chi_\alpha$.

Obtain geometric universe $G(U_{\text{mat}})$.

For morphism $\Psi : U_{\text{mat}} \to U'_{\text{mat}}$, partial order isomorphism and Hilbert space unitary operator induce causal homeomorphism and boundary algebra isomorphism, yielding $G(\Psi)$, making $G$ a functor.

### 3.3 Main Equivalence Theorem

To state main result, introduce the following mutual reconstructability axiom.

**Axiom 9** (Geometric–Matrix Mutual Reconstructability)**.**

1. For any $U_{\text{geo}} \in \mathsf{Uni}_{\text{geo}}$, encoding $F(U_{\text{geo}})$ satisfies matrix universe axioms, preserving all topological, scale, and generalized entropy information;

2. For any $U_{\text{mat}} \in \mathsf{Uni}_{\text{mat}}$, decoding $G(U_{\text{mat}})$ satisfies geometric universe axioms, with reconstructed causal manifold and boundary time geometry unique up to isomorphism;

3. All spectral–geometric reconstruction uses only finite-order Euler–Maclaurin and Poisson expansion, satisfying "singularity non-growth" principle;

4. Scale function $\ell(\alpha)$ of index set $\mathcal{D}$ is sufficiently dense so small diamond limits and Radon-type closures are well-defined;

5. $\mathbb{Z}_2$ ledger $\chi_\alpha$ and Null–Modular data completely record topological sectors, allowing complete reconstruction of matrix universe topological structure at geometric level.

**Theorem 10** (Categorical Equivalence of Geometric and Matrix Universes)**.** *Under above mutual reconstructability and regularity axioms, encoding functor $F : \mathsf{Uni}_{\text{geo}} \to \mathsf{Uni}_{\text{mat}}$ and decoding functor $G : \mathsf{Uni}_{\text{mat}} \to \mathsf{Uni}_{\text{geo}}$ are quasi-inverse, yielding categorical equivalence*

$$\mathsf{Uni}_{\text{geo}} \simeq \mathsf{Uni}_{\text{mat}}.$$

## 4 Proofs

This section provides proof outline of main equivalence theorem, with more technical arguments in appendices.

### 4.1 Fullness and Faithfulness of $F$

**Proposition 11** (Fullness). *If two geometric universes $U_{\text{geo}}, U'_{\text{geo}}$ satisfy*

$$F(U_{\text{geo}}) \cong F(U'_{\text{geo}}),$$

*then $U_{\text{geo}} \cong U'_{\text{geo}}$.*

*Proof outline*:

1. **Causal network isomorphism**: Matrix universe nonzero block sparsity pattern determines abstract causal network $(\mathcal{D}, \preceq)$. Matrix universe isomorphism implies partially ordered set isomorphism, hence isomorphism of small causal diamond covering indices and their partial orders of two geometric universes;

2. **Local geometric reconstruction**: For each $\alpha$, block matrix diagonal element $\mathbb{S}_{\alpha\alpha}(\omega)$ scattering spectrum coincides; combined with Birman–Kreĭn formula and spectral geometry theory, uniquely reconstructs boundary spectral triple and conformal class of local metric fragment $g|_{D_\alpha}$;

3. **Scale and volume information**: High-frequency behavior of scale density $\kappa_\alpha(\omega)$ gives coefficients of boundary Dirac spectrum counting function, determining waist surface area and small volume quantities; combining causal structure and volume information, Malament–Hawking–King–McCarthy type theorem reconstructs conformal factor of metric;

4. **IGVP layer constraint**: Equivalence of generalized entropy and its variation ensures consistency of Einstein equations and matter stress–energy tensor, excluding residual degrees of freedom;

5. **Gluing uniqueness**: Scattering matrix and entropy data consistency in overlap regions ensures unique gluing of metric and algebra, yielding global causal homeomorphism and boundary algebra isomorphism.

Therefore $U_{\text{geo}}$ and $U'_{\text{geo}}$ are isomorphic in $\mathsf{Uni}_{\text{geo}}$.

**Proposition 12** (Faithfulness). *If two morphisms between geometric universes $f, g : U_{\text{geo}} \to U'_{\text{geo}}$ satisfy $F(f) = F(g)$, then $f = g$.*

*Proof outline*:

1. $F(f) = F(g)$ implies their unitary realizations $U_f$ and $U_g$ on $\mathcal{H}$ coincide, with consistent action on index set;

2. GNS representation universal property ensures von Neumann algebra and state isomorphisms completely determined by corresponding unitary;

3. Thus geometric and algebraic level morphisms must coincide, i.e., $f = g$.

Hence $F$ is fully faithful.

### 4.2 $G \circ F \simeq \mathrm{id}_{\mathsf{Uni}_{\mathrm{geo}}}$

For any $U_{\mathrm{geo}}$, first encode to obtain $F(U_{\mathrm{geo}})$, then decode to obtain $G(F(U_{\mathrm{geo}}))$. By construction:

1. Abstract causal network $(\mathcal{D}, \preceq)$ isomorphic to original small causal diamond covering;
2. Local scattering blocks $\mathbb{S}_{\alpha\alpha}(\omega)$ and scale density $\kappa_\alpha(\omega)$ directly given by geometric universe;
3. Decoding process, per mutual reconstructability axiom, necessarily returns to original $(M, g, \prec)$ and boundary time geometry, up to causal homeomorphism and algebra–state unitary isomorphism.

Collect these isomorphisms into natural transformation $\eta : G \circ F \Rightarrow \mathrm{id}_{\mathsf{Uni}_{\mathrm{geo}}}$.

### 4.3 $F \circ G \simeq \mathrm{id}_{\mathsf{Uni}_{\mathrm{mat}}}$

For any $U_{\mathrm{mat}}$, first decode to obtain $G(U_{\mathrm{mat}})$, then encode to obtain $F(G(U_{\mathrm{mat}}))$. Mutual reconstructability axiom ensures:

1. **Causal network and topology**: Small causal diamond covering of reconstructed $(M, g, \prec)$ isomorphic to original index set $\mathcal{D}$;
2. **Local scattering blocks and scale**: $\mathbb{S}_{\alpha\alpha}(\omega)$ and $\kappa_\alpha(\omega)$ reconstructed from geometric universe coincide with original matrix universe;
3. **Off-diagonal blocks** uniquely determined by propagation paths and causal structure; after encoding, global $\mathbb{S}(\omega)$ is unitarily equivalent to original matrix universe.

Thus there exists natural transformation $\epsilon : F \circ G \Rightarrow \mathrm{id}_{\mathsf{Uni}_{\mathrm{mat}}}$.

### 4.4 Naturality of Equivalence

For any morphism $f : U_{\mathrm{geo}} \to U'_{\mathrm{geo}}$, encoding-decoding and natural isomorphisms satisfy

$$\eta_{U'_{\mathrm{geo}}} \circ G(F(f)) = f \circ \eta_{U_{\mathrm{geo}}}.$$

Similarly for any matrix universe morphism $\Psi$,

$$\epsilon_{U'_{\mathrm{mat}}} \circ F(G(\Psi)) = \Psi \circ \epsilon_{U_{\mathrm{mat}}}.$$

This completes proof of categorical equivalence.

## 5 Model Applications

This section shows how this equivalence framework rewrites observers, consensus, and Null–Modular double cover structures.

## 5.1 Observers as Matrix Compression and Readout

In geometric universe, an observer can be abstracted as multi-component object

$$O_i = (C_i, \prec_i, \Lambda_i, \mathcal{A}_i, \omega_i, \mathcal{M}_i, U_i, u_i, \{\mathcal{C}_{ij}\}_j),$$

where $C_i \subset M$ is accessible causal domain, $\Lambda_i$ resolution scale, $\mathcal{A}_i$ observable algebra, $\omega_i$ state, $\mathcal{M}_i$ model family, $U_i$ update operator, $u_i$ utility function, $\mathcal{C}_{ij}$ communication channels.

In matrix universe, this corresponds to:

1. Index subset $\mathcal{D}_i \subset \mathcal{D}$ representing observer-accessible small causal diamonds;
2. Hilbert subspace $\mathcal{H}_i = \bigoplus_{\alpha \in \mathcal{D}_i} \mathcal{H}_\alpha$;
3. Projection operator $P_i : \mathcal{H} \to \mathcal{H}_i$;
4. Submatrix family $\mathbb{S}^{(i)}(\omega) = P_i \mathbb{S}(\omega) P_i^\dagger$;
5. State family and update operators on $\mathcal{B}(\mathcal{H}_i)$ describing observer's belief and learning process.

Observer's "world cross-section" can be understood as weighted section

$$\{(\alpha, \omega_{i,\alpha})\}_{\alpha \in \mathcal{D}_i},$$

whose evolution is determined by submatrix $\mathbb{S}^{(i)}(\omega)$ and communication operators with other observers.

## 5.2 Consensus and Conflict

Multi-observer consensus can be decomposed into three consistencies:

1. **Causal consistency**: On overlap region $\mathcal{D}_i \cap \mathcal{D}_j$, sparsity pattern and partial order must be compatible:
$$\mathbb{S}^{(i)}_{\alpha\beta}(\omega) \neq 0 \iff \mathbb{S}^{(j)}_{\alpha\beta}(\omega) \neq 0;$$
2. **Scale consistency**: On common frequency window and common diamonds, scale density and logarithmic derivative coincide:
$$\kappa^{(i)}_\alpha(\omega) = \kappa^{(j)}_\alpha(\omega),$$
corresponding to unified time scale equivalence class;
3. **State and model consistency**: States on common observable algebra converge to same fixed point through iterative communication and Umegaki relative entropy monotonicity; model family intersection contracts to unique true model under data accumulation.

Matrix universe provides operatorized expression for these consistency conditions: all observer cross-sections $\mathbb{S}^{(i)}$ are compressions of same THE-MATRIX; consensus existence equivalent to existence of global matrix universe $U_{\text{mat}}$ and projection family $\{P_i\}$ such that all cross-sections and compression conditions are compatible.

### 5.3 Null–Modular Double Cover and Self-Referential Scattering Networks

Closed-loop structure in matrix universe naturally supports self-referential scattering networks and $\mathbb{Z}_2$ topological sectors. Two-layer decomposition $K_\alpha = K_\alpha^+ + K_\alpha^-$ of modular Hamiltonian on null-measure boundary and relative determinant

$$\chi_\alpha = \operatorname{sgn} \det_2 \exp(\mathrm{i}\pi K_\alpha)$$

give sector parity on block matrix closed loops, forming Null–Modular double cover.

When $\mathbb{S}(\omega)$ comes from Floquet-driven scattering network, discrete translational symmetry in frequency–index space manifests as "time crystal" structure in matrix universe, with $\mathbb{Z}_2$ sector transitions closely related to time-delay parity jumps.

## 6 Engineering Proposals

Although this paper's main thrust is ontological and mathematical structure, matrix universe THE-MATRIX still has potential experimental and engineering realization paths. Combining existing scattering and spectral measurement techniques, we can propose following layered engineering scheme:

1. **Microwave networks and quantum graph platforms**

   Construct microwave networks composed of coaxial cables, waveguides, and couplers on two-dimensional surfaces, whose scattering matrices can be precisely measured with vector network analyzers in wide frequency bands. By carefully designing node and edge connections, block matrix sparsity pattern approximates some causal network $(\mathcal{D}, \preceq)$, measuring Wigner–Smith time delay $\mathsf{Q}(\omega)$ and trace $\operatorname{tr} \mathsf{Q}(\omega)$ to verify scale identity and matrix universe axioms.

2. **THE-MATRIX fragments on integrated photonics platforms**

   Realize multi-port interference networks and delay lines on silicon photonic chips, implementing high-dimensional unitary block matrices;

   Utilize frequency combs and interference measurements for high-precision readout of phase and time delay, realizing finite fragments of matrix universe in controllable mesoscopic systems.

3. **Scattering networks in cold atoms and atomic waveguides**

   Construct multiple $\delta$ potential wells and coupling regions in one-dimensional or quasi-one-dimensional atomic waveguides, forming tunable scattering center arrays;

   Through Brillouin zone engineering and modulation, realize Floquet–matrix universe fragments, examining time crystal periodic structures and $\mathbb{Z}_2$ sector transitions.

These engineering proposals do not require directly "realizing universe", but verify compatibility of scale identity, causal sparsity patterns, and Null–Modular double cover through finite-dimensional matrix universe fragments, providing testable signals for ontological framework.

## 7 Discussion (Risks, Boundaries, Past Work)

Applicability and risks of this equivalence framework mainly manifest in following aspects.

1. **Dependence on global hyperbolicity**

   Geometric universe model assumes $(M, g)$ globally hyperbolic with good small causal diamond covering. This is natural in many physical solutions, but may not hold in universes with naked singularities or topological complexity; matrix universe reconstructability needs reexamination then.

2. **Technical assumptions of spectral–geometric reconstruction**

   Reconstructing metric from scattering matrix and scale density relies on spectral geometric techniques such as high-frequency asymptotics of Dirac spectrum and trace formula. Existing literature mostly establishes these on compact manifolds or with good boundary conditions; generalization to general non-compact universes with timelike infinity requires additional analysis.

3. **Regularity of relative and generalized entropy**

   IGVP axiom requires generalized entropy differentiable in small diamond limit with non-negative second variation, related to relative entropy monotonicity and QNEC/QFC type inequalities. Existing results mostly proved in free field or controllable interaction background fields; fully general case remains open problem.

4. **Relations with existing unification schemes**

   - **With AdS/CFT**: Matrix universe in some sense generalizes boundary CFT S-matrix picture, not requiring AdS asymptotic structure, but should reduce to standard holographic narrative in AdS case;
   - **With causal sets**: When ignoring all block matrix spectral data in matrix universe, retaining only sparsity pattern, reduces to causal set model;
   - **With quantum graphs and network theory**: Matrix universe can be viewed as extreme generalization of quantum graph theory at "universe scale".

5. **Boundaries of ontological interpretation**

   This framework views "universe" and "matrix universe" as categorically equivalent two descriptions, not asserting which is more "real", but emphasizing equal power at observable structure and variational principle level. This attitude is compatible with standard physical practice, but philosophical interpretation requires further discussion.

# 8 Conclusion

Building on unified framework of causal manifolds, boundary time geometry, Null–Modular double cover, and information geometric variational principle, this paper introduces scattering matrix universe THE-MATRIX, constructs geometric universe category $\mathsf{Uni}_{\mathrm{geo}}$ and matrix universe category $\mathsf{Uni}_{\mathrm{mat}}$, presents encoding functor $F$ and decoding functor $G$, and proves their categorical equivalence under explicit mutual reconstructability and regularity axioms.

This result gives rigorous mathematical meaning to "physical universe = matrix universe THE-MATRIX": any geometric–causal universe satisfying axioms can be equivalently represented as giant scattering matrix universe with causal sparsity pattern, unified scale mother ruler, and generalized entropy structure, and vice versa. Observers and consensus processes are naturally characterized as consistency problems among compressions, readouts, and cross-sections of THE-MATRIX.

Future work can advance in three directions: construct finite fragments of matrix universe through engineering platforms and verify fingerprints of scale and topological sectors; perfect general proofs of spectral–geometric reconstruction and IGVP at rigorous mathematical level; explore ontological meaning and inference forms of "universe as matrix" at philosophical and information-theoretic level.

## Acknowledgements & Code Availability

This work builds on extensive literature in scattering theory, spectral geometry, operator algebras, and general relativity, with default acknowledgment to developments in related fields. All calculations and arguments in this paper are primarily analytical reasoning, not dependent on public code implementations. No dedicated numerical or symbolic computation code has been released yet; future numerical verification can be implemented based on general scattering matrix and spectral analysis libraries to simulate and verify matrix universe fragments.

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Additional literature related to IGVP, QNEC/QFC, AdS/CFT can be supplemented in specific applications.

## Appendix A: Matrix Reformulation of Scale Identity and Boundary Time Geometry

This appendix provides derivation framework of scale identity in matrix universe, explaining its correspondence with boundary time geometry.

### A.1 Birman–Kreĭn Formula and Spectral Shift Function

Consider diagonal block scattering operator $\mathbb{S}_{\alpha\alpha}(\omega)$, assume it comes from pair of self-adjoint operators $(H_\alpha, H_{0,\alpha})$, with $H_{0,\alpha}$ free Hamiltonian, $H_\alpha$ potential Hamiltonian, and difference $V_\alpha = H_\alpha - H_{0,\alpha}$ trace class. By Birman–Kreĭn theory, introduce spectral shift function $\xi_\alpha(\lambda)$ satisfying trace formula for appropriate $f$:

$$\operatorname{tr}(f(H_\alpha) - f(H_{0,\alpha})) = \int \xi_\alpha(\lambda) f'(\lambda)\, d\lambda.$$

Simultaneously there exists scattering matrix $S_\alpha(\lambda)$ such that

$$\det S_\alpha(\lambda) = \exp(-2\pi \mathrm{i} \xi_\alpha(\lambda)).$$

Define total scattering half-phase

$$\varphi_\alpha(\lambda) = \pi \xi_\alpha(\lambda),$$

then relative density of states

$$\rho_{\mathrm{rel},\alpha}(\lambda) = \xi'_\alpha(\lambda) = \varphi'_\alpha(\lambda)/\pi.$$

### A.2 Wigner–Smith Time Delay and Trace Formula

For each $\alpha$, define

$$\mathsf{Q}_\alpha(\omega) = -\mathrm{i}\mathbb{S}_{\alpha\alpha}(\omega)^\dagger \partial_\omega \mathbb{S}_{\alpha\alpha}(\omega).$$

In finite-dimensional case, using unitarity $\mathbb{S}_{\alpha\alpha}^\dagger \mathbb{S}_{\alpha\alpha} = \mathbb{I}$, obtain

$$\operatorname{tr} \mathsf{Q}_\alpha(\omega) = -\mathrm{i}\partial_\omega \log\det \mathbb{S}_{\alpha\alpha}(\omega) = 2\varphi'_\alpha(\omega).$$

In infinite-dimensional case, introduce spatial truncation projection $\chi_R$, define

$$\operatorname{tr} \mathsf{Q}_\alpha(\omega) = \lim_{R\to\infty} \operatorname{tr}(\chi_R \mathsf{Q}_\alpha(\omega) \chi_R).$$

Under finite-order Euler–Maclaurin and Poisson expansion, can prove this limit exists and

$$(2\pi)^{-1} \operatorname{tr} \mathsf{Q}_\alpha(\omega) = \varphi'_\alpha(\omega)/\pi = \rho_{\mathrm{rel},\alpha}(\omega).$$

Thus scale identity

$$\kappa_\alpha(\omega) = \varphi'_\alpha(\omega)/\pi = \rho_{\mathrm{rel},\alpha}(\omega) = (2\pi)^{-1} \operatorname{tr} \mathsf{Q}_\alpha(\omega)$$

holds.

### A.3 Interpretation in Boundary Time Geometry

In boundary time geometry framework, scale density $\kappa_\alpha(\omega)$ has dual meaning:

1. As scattering phase derivative, it measures phase shift change rate per unit frequency bandwidth;
2. As relative density of states, it gives local DOS relative to background;
3. As time-delay trace, it measures total delay time of all channels in periodic unit.

Incorporating $\kappa_\alpha(\omega)$ into spectral distribution of boundary spectral triple, one can define time scale in Dirac spectrum, linking scattering–scale with modular flow–thermal time geometry.

## Appendix B: From Causal Network and Scale Density to Lorentz Manifold

This appendix provides argument outline for reconstructing Lorentz manifold $(M, g)$ from abstract causal network $(\mathcal{D}, \preceq)$ and scale density $\kappa_\alpha$.

### B.1 Causal Structure and Topology

Under local finiteness and scale density assumptions, define Alexandrov basic open set on $\mathcal{D}$:

$$U_{\alpha\beta} = \{\gamma \in \mathcal{D} : \alpha \prec \gamma \prec \beta\}.$$

Under appropriate conditions, topology generated by these open sets can be completed to Hausdorff, second-countable space. Using Malament theorem: "continuous class of timelike curves determines spacetime topology", when satisfying past/future distinguishing condition, causal structure uniquely determines topological structure.

By mapping each $\alpha$ to center point of corresponding small causal diamond in limit manifold $M$, requiring causal relation compatible with Alexandrov topology, obtain manifold structure.

### B.2 Conformal Class and Conformal Factor

Malament–Hawking–King–McCarthy framework shows that under appropriate energy conditions and regularity assumptions, causal structure and volume measure determine conformal class of metric. High-frequency asymptotic behavior of scale density $\kappa_\alpha(\omega)$ relates to local DOS via scattering–spectral theory, extracting local volume element and waist surface area, determining volume measure.

Therefore, causal structure and volume information derived from $\kappa_\alpha$ jointly determine conformal class of metric.

Determining conformal factor relies on time scale: relationship between scale density and proper time fixed via boundary–bulk correspondence and Hamilton–Jacobi type structure, giving complete Lorentz metric $g$.

### B.3 IGVP and Einstein Equations

On reconstructed Lorentz manifold, for each small causal diamond $D_\alpha$, construct generalized entropy $S_{\mathrm{gen},\alpha}$ from matrix universe spectrum and Null–Modular data. In small-scale limit $\ell(\alpha) \to 0$, impose on all allowed variations:

1. First-order extremal condition $\delta S_{\mathrm{gen}} = 0$;
2. Second variation non-negative;

Through "entropy variation  Einstein equations" idea established in Jacobson–Hollands–Wald works, can prove these conditions equivalent to $G_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$, ensuring local stability.

Therefore, matrix universe data suffices to reconstruct causal manifold $(M, g, \prec)$ satisfying Einstein equations.

# Appendix C: First- and Second-Order Variations of IGVP Matrix Version

This appendix shows how to perform first- and second-order variations of generalized entropy in matrix universe, connecting to canonical energy and relative entropy Hessian.

## C.1 Matrix Expression of Generalized Entropy

For given index $\alpha$, generalized entropy written as

$$S_{\text{gen},\alpha} = A_\alpha/(4G\hbar) + S^{\text{ren}}_{\text{out},\alpha} + S^{\text{UV}}_{\text{ct},\alpha} - \Lambda V_\alpha/(8\pi G T_\alpha).$$

Expression of each term in matrix context:

1. Area term $A_\alpha$: From Weyl asymptotics of boundary Dirac spectrum,

   $$A_\alpha \sim c_d \int^{\Lambda_{\text{UV}}} \lambda^{d-2}\, dN_\alpha(\lambda),$$

   $N_\alpha(\lambda)$ spectral counting function;

2. Exterior entropy $S^{\text{ren}}_{\text{out},\alpha}$: Given by von Neumann entropy of reduced density matrix spectrum determined by diagonal and off-diagonal blocks;

3. Counterterm $S^{\text{UV}}_{\text{ct},\alpha}$: Written as coefficients on finite-order local operator basis, reflecting renormalization conditions;

4. Volume term $V_\alpha/T_\alpha$: Extracted from low-energy behavior of DOS and scale density and modular flow parameter.

## C.2 First Variation and Field Equations

Consider parameter family $U_{\text{mat}}(\lambda)$, with block matrices and scale density varying with $\lambda$. First variation of generalized entropy:

$$\frac{d}{d\lambda} S_{\text{gen},\alpha} = \frac{1}{4G\hbar}\frac{dA_\alpha}{d\lambda} + \frac{dS^{\text{ren}}_{\text{out},\alpha}}{d\lambda} + \frac{dS^{\text{UV}}_{\text{ct},\alpha}}{d\lambda} - \frac{\Lambda}{8\pi G}\frac{1}{T_\alpha}\frac{dV_\alpha}{d\lambda} + \cdots,$$

where omitted terms include temperature and scale rescaling corrections.

By relating block matrix variation to metric variation $\delta g_{ab}$ and matter field variation $\delta\phi$, above expression can be rewritten as

$$\frac{d}{d\lambda} S_{\text{gen},\alpha} = \frac{1}{8\pi G}\int_{\text{waist}} (G_{ab} + \Lambda g_{ab} - 8\pi G T_{ab})\, \delta g^{ab}\, d\Sigma + \text{boundary terms}.$$

Requiring first variation vanishes for all $\delta g^{ab}$ satisfying appropriate boundary conditions yields Einstein equations.

### C.3 Second Variation and Canonical Energy

Second variation

$$\frac{d^2}{d\lambda^2} S_{\mathrm{gen},\alpha}\big|_{\lambda=0}$$

in matrix context can be expressed as quadratic form on metric and matter field perturbations

$$\mathcal{Q}_\alpha[\delta g, \delta\phi].$$

Using convexity and monotonicity of Araki relative entropy and Hollands–Wald canonical energy construction, can relate $\mathcal{Q}_\alpha$ to second-order variation of canonical energy $\mathcal{E}$:

$$\mathcal{Q}_\alpha[\delta g, \delta\phi] \geq 0 \iff \mathcal{E}[\delta g, \delta\phi] \geq 0.$$

This provides consistency between IGVP axiom second-order layer and dynamical stability, ensuring small perturbations on matrix universe do not trigger negative canonical energy modes, providing information geometric criterion for THE-MATRIX stability.