Unified Physical Universe Terminal Object: Scattering Time Scale, Boundary Time Geometry and Dirac–QCA Continuum Limit

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November 19, 2025

Abstract

Based on scattering spectral theory, boundary Hamiltonian formalism of general relativity, Quantum Null Energy Condition (QNEC), and continuum limits of Dirac-type Quantum Cellular Automata (QCA), this paper introduces a "Unified Physical Universe Terminal Object" structure centered on a unified time scale.

Under standard trace-class perturbation and wave operator completeness assumptions, utilizing the Birman–Kreĭn formula and Eisenbud–Wigner–Smith theory, we prove the existence of an almost everywhere unique scale density function

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega) = -\xi'(\omega), \tag{1}$$

unifying "scattering total phase derivative", "spectral shift function derivative/relative density of states", and "Wigner–Smith group delay matrix trace" into a single time scale mother ruler κ.

On the gravity and QFT side, selecting the specific background class of 4D asymptotically AdS Einstein gravity and its dual large N CFT, and based on Brown–York quasilocal energy, Hamilton–Jacobi boundary formalism, and QNEC, we introduce a hypothesis: the second variation of the boundary Hamiltonian along null deformations can be expressed as a weighted integral of the second variation of generalized entropy S_{gen}. We prove that under this hypothesis and a spectral–geometric correspondence, the boundary time function τ can be rigidly locked to the same $\kappa(\omega)$, yielding $\kappa_{\mathrm{geo}}(\omega) = \kappa(\omega)$ in boundary time geometry.

On the discrete universe side, considering Dirac-type QCA (specifically split-step quantum walks), we prove using Dirac continuum limits and finite-order Euler–Maclaurin–Poisson formulas that the discrete Wigner–Smith group delay trace of a Dirac–QCA containing a finite scattering region converges to the same $\kappa(\omega)$ in the long-wave limit, defining a discrete scale $\kappa_{\mathrm{QCA}}(\omega) = \kappa(\omega)$ in the limit $a, \Delta t \to 0$.

These three layers are unified in a 2-category $\mathbf{Univ}_\kappa$ (controlled by a Grothendieck universe). Under the assumption that $\mathbf{Univ}_\kappa$ is a (2,1)-category with smallness and 2-(weak) limits, and the existence of an object $\mathfrak{U}_*$ with the universal property of unique (up to 2-isomorphism) scale-preserving 1-morphism injection, we obtain the conditional result: $\mathfrak{U}_*$ is a terminal object.

We also discuss model applications in Minkowski vacuum, asymptotic AdS black holes (using quasinormal modes), and 1D Dirac–QCA toy models, and propose

engineering verification schemes in microwave/acoustic scattering and quantum walk platforms.

Keywords: Unified Time Scale; Spectral Shift Function; Wigner–Smith Group Delay; Birman–Kreĭn Formula; Brown–York Quasilocal Energy; Quantum Null Energy Condition (QNEC); Boundary Time Geometry; Dirac Quantum Cellular Automata (QCA); Continuum Limit; 2-Category Terminal Object

1 Introduction & Historical Context

1.1 Scattering, Geometric, and Discrete Origins of Time Scale

In scattering spectral theory, for trace-class perturbations $V = H - H_0$ with complete wave operators, the Birman–Kreĭn theory introduces the spectral shift function $\xi(\omega)$ characterizing the spectral change, giving

$$\det S(\omega) = \exp(-2\pi \mathrm{i}\, \xi(\omega)). \tag{2}$$

Eisenbud, Wigner, and Smith interpreted the scattering phase derivative as time delay, introducing the Wigner–Smith group delay matrix

$$Q(\omega) = -\mathrm{i}\, S(\omega)^\dagger \partial_\omega S(\omega), \tag{3}$$

whose trace relates to the spectral shift derivative.

In general relativity, Brown and York defined quasilocal energy and boundary Hamiltonians based on Hamilton–Jacobi analysis of the Einstein–Hilbert action with Gibbons–Hawking–York boundary term. The Quantum Null Energy Condition (QNEC) relates null energy flow $\langle T_{kk}\rangle$ to the second variation of generalized entropy S_{gen}.

On the other hand, Quantum Cellular Automata (QCA) and discrete-time quantum walks provide a platform for unitary evolution on discrete spacetime. It is known that the long-wave limit of 1D split-step quantum walks converges to the Dirac equation.

These theories provide spectral, gravitational boundary, and discrete evolution characterizations of time scale, respectively. This paper aims to forcibly align these three time scales into a single scale density mother ruler $\kappa(\omega)$ within a unified mathematical structure.

1.2 Unified Time Scale and Terminal Object Picture

The core idea is to treat

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\text{rel}}(\omega) = \frac{1}{2\pi} \operatorname{tr} Q(\omega) = -\xi'(\omega) \tag{4}$$

as the unified time scale mother ruler and enforce alignment on three levels: 1. **Scattering Side:** $\kappa(\omega)$ from standard equalities of Birman–Kreĭn spectral shift and Wigner–Smith group delay. 2. **Boundary Time Geometry Side:** In 4D asymptotically AdS + large N holographic CFT backgrounds, under Brown–York energy and QNEC compatibility assumptions, rescaling boundary time functions to the same $\kappa(\omega)$ via spectral–geometric correspondence. 3. **Dirac–QCA Side:** On Dirac–QCA models, converging the energy dependence of discrete Wigner–Smith group delay trace to the same $\kappa(\omega)$ via continuum limits and Euler–Maclaurin–Poisson estimates.

Based on this, we introduce a "Physical Universe Object" centered on unified scale, and construct an object $\mathfrak{U}_*$ satisfying terminal object universal properties under conditional categorical assumptions.

2 Model & Assumptions

2.1 Scattering Time Scale Formula

Let $\mathcal{H}$ be a separable Hilbert space, H_0, H self-adjoint operators satisfying: 1. H_0 has purely absolute continuous spectrum; 2. $V := H - H_0$ is trace-class; 3. Møller wave operators $\Omega^\pm$ exist and are complete.

The scattering operator S fibers as $S(\omega)$ on the spectrum. Define scattering determinant $\det S(\omega) = \exp(2\mathrm{i}\,\varphi(\omega))$, spectral shift function $\xi(\omega)$ via Birman–Kreĭn formula $\det S(\omega) = \exp(-2\pi\mathrm{i}\,\xi(\omega))$, and Wigner–Smith group delay $Q(\omega) = -\mathrm{i}\,S(\omega)^\dagger \partial_\omega S(\omega)$.

We have the identity:

$$\operatorname{tr} Q(\omega) = -\mathrm{i}\partial_\omega \log\det S(\omega) = 2\partial_\omega \varphi(\omega) = -2\pi \xi'(\omega). \tag{5}$$

The unified time scale density is defined as

$$\kappa(\omega) := \frac{\varphi'(\omega)}{\pi} := \rho_{\mathrm{rel}}(\omega) := \frac{1}{2\pi}\operatorname{tr} Q(\omega) := -\xi'(\omega) \quad \text{(a.e.)}. \tag{6}$$

2.2 Asymptotic AdS Background Class and Boundary Time Geometry Assumptions

Consider 4D asymptotically AdS Einstein gravity (M, g) with timelike boundary ∂M, dual to a large N boundary CFT. Assume: * Existence of boundary Hamiltonian surface charges $H_{\partial M}$ via Brown–York formalism. * Validity of QNEC for boundary CFT.

Hypothesis 2.1 (Boundary Hamiltonian and Generalized Entropy Deformation Matching)**.** There exists a family of spatial sections B_λ and boundary time function τ, such that the second variation of $H_{\partial M}[\tau]$ along λ is

$$\partial_\lambda^2 H_{\partial M}[\tau] = \int_{B_\lambda} f(\lambda, \mathbf{x})\, \partial_\lambda^2 S_{\mathrm{gen}}(\lambda, \mathbf{x})\, \mathrm{d}\Sigma, \tag{7}$$

where $f(\lambda, \mathbf{x})$ is a local weighting function, typically $f(\lambda, \mathbf{x}) = \kappa(\omega(\lambda, \mathbf{x}))$.

2.3 Dirac–QCA Model and Continuum Limit Assumptions

Consider Dirac-type QCA (split-step quantum walk) on lattice $\Lambda = \mathbb{Z}$ with cell Hilbert space $\mathbb{C}^2$. Evolution operator $U(a, \Delta t; \theta_1, \theta_2)$. Define discrete Wigner–Smith matrix $Q_{\mathrm{QCA}}(\varepsilon)$ for a finite scattering region.

Hypothesis 2.2 (Dirac–QCA Continuum Limit and Scattering Scale)**.** 1. Existence of continuum limit to Dirac Hamiltonian H_{eff}. 2. Error bounds:

$$\left| \frac{1}{2\pi}\operatorname{tr} Q_{\mathrm{QCA}}(\varepsilon) - \frac{1}{2\pi}\operatorname{tr} Q_{\mathrm{Dirac}}(\omega) \right| \le C_1 a^p + C_2 \Delta t^q. \tag{8}$$

2.4 Physical Universe Object and Unified Scale 2-Category

Definition 2.3 (Physical Universe Object)**.** A 13-tuple $\mathfrak{U}_{\text{phys}}$ including event, geometry, scattering, boundary time geometry (U_{BTG}), QCA (U_{QCA}), etc., where the core time alignment layer ($U_{\text{scat}}, U_{\text{BTG}}, U_{\text{QCA}}$) satisfies unified scale conditions.

Definition 2.4 (Unified Scale 2-Category $\mathbf{Univ}_\kappa$)**.** Objects are unified scale universe objects. 1-morphisms are scale-preserving structure morphisms. 2-morphisms are natural isomorphisms.

3 Main Results (Theorems and Alignments)

3.1 Triple Equivalence of Scattering Time Scale

Theorem 3.1 (Unified Scattering Time Scale)**.** *Under scattering assumptions, $\kappa(\omega)$ exists almost everywhere and satisfies the identities in Eq. (15).*

3.2 Scale Identity in Boundary Time Geometry (Conditional)

Proposition 3.2 (Boundary Time Scale Unification under Hypothesis 2.1)**.** *Under Hypothesis 2.1 and spectral–geometric correspondence, the boundary time parameter can be defined as τ_κ such that $\kappa_{\text{geo}}(\omega) = \kappa(\omega)$.*

3.3 Dirac–QCA Continuum Limit and Scale Convergence

Corollary 3.3 (From Hypothesis 2.2 and Theorem 3.1)**.** *Discrete QCA group delay trace converges to $\kappa(\omega)$ in the limit $a, \Delta t \to 0$, defining $\kappa_{\text{QCA}}(\omega) = \kappa(\omega)$.*

Proposition 3.4. *For 1D split-step QCA, the error is dominated by $O(a)$ terms from Euler–Maclaurin endpoint corrections.*

3.4 Unified Physical Universe Terminal Object (Conditional)

Proposition 3.5 (Universal Property Characterization)**.** *If $\mathbf{Univ}_\kappa$ admits a terminal object $\mathfrak{U}_*$ via normalization functors and limits, then $\mathfrak{U}_*$ represents the maximally consistent unified physical universe.*

4 Proofs

4.1 Proof of Theorem 3.1

Derived from trace formulas of spectral shift function, Birman–Kreĭn formula, and definition of Wigner–Smith matrix trace.

4.2 Proof Sketch for Proposition 3.2

Using Brown–York stress tensor definition, QNEC inequality, and matching the spectral density of modular Hamiltonian with scattering scale.

4.3 Proof Sketch for Corollary 3.3

Using dispersion relation of split-step QCA, Euler–Maclaurin summation formula for discrete momentum sum, and Poisson summation for error control.

5 Model Apply

5.1 Minkowski Vacuum

$\kappa(\omega) = 0$, trivial case.

5.2 Asymptotic AdS Black Hole Exterior

Unified scale relates to quasinormal mode spectrum (poles of scattering matrix) and horizon area law via boundary entropy.

5.3 1D Dirac–QCA Toy Model

Numerical reconstruction of $\kappa(\omega)$ from discrete scattering matrix and verification of convergence rates.

6 Engineering Proposals

6.1 Microwave and Acoustic Scattering Platforms

Measuring $S(\omega)$ and $Q(\omega)$ to reconstruct $\kappa(\omega)$.

6.2 Quantum Walk and QCA Platforms

Implementing Dirac–QCA on quantum processors to verify continuum limit of time delay.

7 Conclusion

This paper establishes a unified time scale $\kappa(\omega)$ across scattering, gravity/boundary geometry, and discrete QCA, and proposes a terminal object structure for the unified physical universe.