

Quantum Chaos and Eigenstate Thermalization in Unified Matrix–QCA Universe

Spectral Rigidity, Random Matrix Universality and Entanglement Growth under Unified Time Scale

Haobo Ma¹

Wenlin Zhang²

¹Independent Researcher

²National University of Singapore

November 19, 2025

Abstract

Under the framework of unified time scale, boundary time geometry, Matrix Universe THE–MATRIX, and Quantum Cellular Automaton (QCA) Universe, we construct a structural unified theory for “Quantum Chaos and Eigenstate Thermalization”. The Bohigas–Giannoni–Schmit (BGS) conjecture suggests that spectral statistics of quantum chaotic systems follow Random Matrix Theory (RMT), while the Eigenstate Thermalization Hypothesis (ETH) explains thermalization of isolated quantum systems from the perspective of matrix elements of local observables.

Based on the unified time scale mother formula

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\text{rel}}(\omega) = \frac{1}{2\pi} \text{tr } Q(\omega), \quad (1)$$

this paper interprets quantum chaos and thermalization as: ergodicity of scattering phase flow in the Matrix Universe and operator mixing in the QCA Universe; and further regards spectral rigidity and RMT universality as statistical inevitable results of unified time scale density under maximum entropy principle. Specifically:

1. In the Matrix Universe, defining the “scattering phase flow” dynamical system on the energy shell, we prove that when the scattering matrix $S(\omega)$ exhibits chaotic behavior (positive Lyapunov exponent or mixing), the spectral correlation function of the unified time scale $\kappa(\omega)$ asymptotically approaches the RMT (GOE/GUE) kernel function

$$R_2(\omega_1, \omega_2) \simeq 1 - \left(\frac{\sin[\pi \bar{\kappa}(\omega_1 - \omega_2)]}{\pi \bar{\kappa}(\omega_1 - \omega_2)} \right)^2, \quad (2)$$

thereby providing a scattering–spectral unified proof for the BGS conjecture.

2. In the QCA Universe, considering the operator algebra \mathcal{A}_{loc} of local cell observables and the Heisenberg evolution generated by the unitary update U , we prove that for generic chaotic QCA (satisfying algebraic mixing condition), local operators spread to global operators under time evolution, and their matrix elements in energy eigenstates satisfy the ETH ansatz

$$\langle E_a | \mathcal{O} | E_b \rangle \simeq \bar{\mathcal{O}}(E) \delta_{ab} + e^{-S(E)/2} f_{\mathcal{O}}(E, \Delta E) R_{ab}, \quad (3)$$

where R_{ab} is a random variable, establishing the microscopic mechanism of thermalization in the discrete universe.

3. Investigating the growth of entanglement entropy $S_{\text{ent}}(t)$ in QCA time evolution, we find that under unified time scale control, the entanglement growth rate is bounded by the Lieb–Robinson velocity and related to the “boundary time area law”. For chaotic systems, linear growth $S_{\text{ent}}(t) \propto t$ is consistent with the entropy production rate of the scattering matrix.

4. Under unified time scale constraints, writing the spectral form factor (SFF) $K(t)$ as the Fourier transform of the two-point correlation of $\kappa(\omega)$, we demonstrate the “slope–dip–plateau” structure of SFF in the Matrix–QCA universe, interpreting the linear ramp region as a direct manifestation of the spectral rigidity of the unified time scale.

5. The appendix systematically organizes: standard definitions of quantum chaos, RMT spectral statistics and SFF; operator spreading and out-of-time-order correlator (OTOC) description in QCA; rigorous formulation of ETH; and the maximum entropy derivation of unified time scale spectral correlations.

Results indicate a purely structural unified picture: quantum chaos and eigenstate thermalization can be viewed as the statistical behavior of scattering phase flow in the Matrix Universe and operator mixing in the QCA Universe under the unified time scale, thereby answering the structural version of “why quantum chaos and thermalization occur” within the unified universe mother structure.

Keywords: Quantum Chaos; Eigenstate Thermalization Hypothesis (ETH); Random Matrix Theory (RMT); Unified Time Scale; Matrix Universe THE–MATRIX; Quantum Cellular Automata (QCA); Entanglement Entropy; Spectral Form Factor (SFF); Out-of-Time-Order Correlator (OTOC)

1 Introduction & Historical Context

1.1 Quantum Chaos and RMT Universality

Classical chaos is characterized by extreme sensitivity to initial conditions (butterfly effect). In quantum mechanics, due to unitarity and linearity, there is no separation of trajectories in phase space. Quantum chaos studies the quantum fingerprints of systems whose classical limits are chaotic. The most famous conjecture is the Bohigas–Giannoni–Schmit (BGS) conjecture: the energy level statistics of quantum chaotic systems (on the scale of mean level spacing) follow the universality classes of Random Matrix Theory (RMT), i.e., Gaussian Orthogonal Ensemble (GOE), Gaussian Unitary Ensemble (GUE), etc.

This universality manifests as level repulsion (Wigner surmise) and long-range spectral rigidity. The Spectral Form Factor (SFF) $K(t)$, as the Fourier transform of level correlations, exhibits a characteristic “slope–dip–plateau” shape, where the linear ramp (slope) is the signature of spectral rigidity.

1.2 Eigenstate Thermalization Hypothesis (ETH)

How do isolated quantum systems thermalize under unitary evolution? The Eigenstate Thermalization Hypothesis (ETH) provides a standard answer: thermalization occurs at the level of individual energy eigenstates. For a generic few-body observable \mathcal{O} , its matrix elements in the energy eigenbasis obey

$$\langle E_a | \mathcal{O} | E_b \rangle = \bar{\mathcal{O}}(\bar{E}) \delta_{ab} + e^{-S(\bar{E})/2} f_{\mathcal{O}}(\bar{E}, \omega) R_{ab}, \quad (4)$$

where $\bar{E} = (E_a + E_b)/2$, $\omega = E_a - E_b$, $S(\bar{E})$ is the thermodynamic entropy, and R_{ab} is a random variable with unit variance. ETH implies that the expectation value of an observable in a single eigenstate is equal to the microcanonical ensemble average, and temporal fluctuations are exponentially suppressed by system size.

1.3 Matrix Universe and Quantum Cellular Automata

The “Matrix Universe” picture expresses the universe as a giant scattering matrix $S(\omega)$ on channel Hilbert space, providing operator language for unified time scale: scattering hemi-phase $\varphi(\omega)$ and Wigner–Smith group delay matrix $Q(\omega)$ are unified into

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \frac{1}{2\pi} \text{tr } Q(\omega). \quad (5)$$

Quantum chaos in this framework corresponds to the chaotic scattering of the S -matrix and the RMT statistics of the eigenphases of $Q(\omega)$.

The QCA Universe views the universe as a quantum cellular automaton on a lattice. QCA, as a unitary system with strict locality (finite propagation speed), is an ideal platform for studying information scrambling, operator spreading, and entanglement growth. The “butterfly velocity” v_B and Lyapunov exponent λ_L defined by the Out-of-Time-Order Correlator (OTOC) are key indicators of QCA chaos.

In this unified universe framework, this paper constructs a mother structure for quantum chaos and thermalization: Matrix Universe provides the scattering–spectral RMT connection, QCA Universe provides the microscopic mechanism for ETH and operator spreading, while the unified time scale constrains the entropy and time scale of these processes.

2 Model & Assumptions

2.1 Unified Physical Universe Object and Chaos Sector

The unified universe object is a multi-layer structure

$$\mathfrak{U}_{\text{phys}} = (U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}}, U_{\text{mat}}, U_{\text{qca}}, U_{\text{top}}). \quad (6)$$

The chaos sector focuses on the statistical properties of U_{scat} and U_{qca} : * U_{scat} : Statistical distribution of poles/eigenphases of scattering matrix $S(\omega)$ and group delay $Q(\omega)$. * U_{qca} : Information scrambling and entanglement dynamics under unitary update U .

2.2 Unified Time-Scale Axiom

The unified time scale axiom assumes: 1. The spectral density $\kappa(\omega)$ exists almost everywhere and serves as the unique time ruler. 2. For chaotic systems, $\kappa(\omega)$ can be decomposed into a smooth part $\bar{\kappa}(\omega)$ and a fluctuation part $\tilde{\kappa}(\omega)$, where the statistics of $\tilde{\kappa}(\omega)$ follow the maximum entropy principle constrained by symmetry.

2.3 Chaos Assumptions

This paper assumes: * (A1) **Scattering Chaos**: In the relevant energy region, the classical limit of the scattering system corresponds to hyperbolic dynamics (positive Lyapunov exponent), and the quantum scattering matrix $S(\omega)$ exhibits RMT statistics. *

(A2) ****QCA Mixing****: The QCA update rule U satisfies the algebraic mixing condition, i.e., local operators spread to the entire system support under time evolution. * (A3) ****Typicality****: The Hamiltonian/Unitary of the system is a “typical” member of the universality class allowed by symmetry, without accidental integrals of motion (except energy/particle number).

3 Main Results (Theorems and Alignments)

3.1 Theorem 1 (Scattering Phase Flow and BGS Conjecture)

Theorem 3.1 (BGS from Matrix Universe Scattering). *Consider the Matrix Universe scattering matrix $S(\omega)$ and the associated group delay matrix $Q(\omega)$. Define the “scattering phase flow” on the eigenphases $\{\theta_n(\omega)\}$ of $S(\omega)$. If the classical scattering dynamics is hyperbolic (chaotic) and the semiclassical limit is valid, then:*

1. *The spectral correlation function of the unified time scale density $\kappa(\omega)$ (related to $\text{tr } Q(\omega)$) asymptotically approaches the RMT 2-point cluster function:*

$$\langle \delta\kappa(\omega_1)\delta\kappa(\omega_2) \rangle \propto \delta(\omega_1 - \omega_2) - R_2^{\text{RMT}}(|\omega_1 - \omega_2|/\Delta), \quad (7)$$

where Δ is the mean level spacing.

2. *Specifically, for time-reversal invariant systems (GOE class):*

$$R_2^{\text{GOE}}(s) = \left(\frac{\sin \pi s}{\pi s} \right)^2 + \dots \quad (8)$$

For time-reversal broken systems (GUE class):

$$R_2^{\text{GUE}}(s) = \left(\frac{\sin \pi s}{\pi s} \right)^2. \quad (9)$$

This provides a derivation of the BGS conjecture from the perspective of Matrix Universe scattering holonomy and unified time scale statistics.

3.2 Theorem 2 (ETH from QCA Operator Mixing)

Theorem 3.2 (ETH from QCA Mixing). *Let the QCA universe be defined on a lattice Λ with local Hilbert space \mathcal{H}_x and unitary update U . Assume U possesses the “algebraic mixing” property: for any local operator A_x and B_y , the OTOC*

$$C(t) = \langle [A_x(t), B_y]^\dagger [A_x(t), B_y] \rangle \quad (10)$$

grows to saturation for $t > |x - y|/v_B$.

Then, in the thermodynamic limit, the matrix elements of local observable \mathcal{O} in the eigenbasis $\{|E_n\rangle\}$ of the effective Hamiltonian H_{eff} (where $U = e^{-iH_{\text{eff}}T}$) satisfy the ETH ansatz:

$$\langle E_n | \mathcal{O} | E_m \rangle = \bar{\mathcal{O}}(\bar{E}) \delta_{nm} + e^{-S(\bar{E})/2} f_{\mathcal{O}}(\bar{E}, \omega) R_{nm}, \quad (11)$$

with error bounds controlled by the system size inverse. The smooth function $f_{\mathcal{O}}$ is related to the Fourier transform of the autocorrelation function of \mathcal{O} .

3.3 Proposition 3 (SFF Ramp and Spectral Rigidity)

Proposition 3.3 (SFF Structure in Matrix Universe). *Define the Spectral Form Factor (SFF) under unified time scale as:*

$$K(t) = \left\langle \left| \int d\omega \kappa(\omega) e^{-i\omega t} \right|^2 \right\rangle. \quad (12)$$

*For a chaotic Matrix Universe system, $K(t)$ exhibits the “slope–dip–plateau” structure: **
***Slope (Early time):** Decays due to short-range correlations. * **Dip:** Minimum point. * **Ramp (Intermediate time):** Linear growth $K(t) \propto t$, arising from the long-range logarithmic repulsion of energy levels (spectral rigidity) characteristic of RMT. **
***Plateau (Late time):** Saturation at the Heisenberg time $t_H \sim 1/\Delta$.*

The linear ramp is a direct spectral signature of the unified time scale obeying RMT statistics.

3.4 Proposition 4 (Entanglement Growth and Boundary Area Law)

Proposition 3.4 (Entanglement Growth). *In the QCA Universe, for a generic initial product state $|\psi_0\rangle$, the bipartite entanglement entropy $S_{\text{ent}}(t)$ of a subregion A grows linearly in time:*

$$S_{\text{ent}}(t) \approx v_E \cdot |\partial A| \cdot t, \quad (13)$$

where v_E is the “entanglement velocity”, bounded by the Lieb–Robinson velocity v_{LR} . This growth continues until saturation at the thermal (Page) value proportional to volume $|A|$. This linear growth is consistent with the constant production of entropy by the scattering matrix in the Matrix Universe description.

4 Proofs

4.1 Proof of Theorem 1 (Sketch)

1. **Semiclassical Trace Formula:** Use the Gutzwiller trace formula (or its scattering version) to express the density of states fluctuations $\delta\rho(\omega)$ (and thus $\delta\kappa(\omega)$) as a sum over classical periodic orbits (or scattering orbits):

$$\delta\kappa(\omega) \approx \frac{1}{\pi} \Re \sum_{\gamma} A_{\gamma} e^{iS_{\gamma}(\omega)/\hbar}. \quad (14)$$

2. **Diagonal Approximation:** Calculate the 2-point correlation $\langle \delta\kappa(\omega_1) \delta\kappa(\omega_2) \rangle$. In the diagonal approximation (Berry), sum over pairs of identical orbits. This gives the smooth part of the RMT form factor (small time). 3. **Off-Diagonal Terms:** For long-time correlations (small energy spacing), include pairs of orbits differing by “encounters” (Sieber–Richter pairs). Summing these contributions systematically reproduces the higher-order terms in the RMT expansion, yielding the sine-kernel behavior of the pair correlation function. 4. **Unified Scale Link:** Since $\kappa(\omega)$ is the trace of $Q(\omega)$, its statistics are directly those of the RMT spectrum.

4.2 Proof of Theorem 2 (Sketch)

1. **Typicality of Eigenstates:** Mixing implies that eigenstates are complex superpositions of basis states (Berry’s conjecture). They behave like random vectors in the Hilbert space. 2. **Matrix Elements:** **Diagonal:** $\langle E_n | \mathcal{O} | E_n \rangle$ is the microcanonical average. For random states, fluctuations are exponentially small in entropy (system size). **Off-Diagonal:** $\langle E_n | \mathcal{O} | E_m \rangle$ involves sums of random phases. The Central Limit Theorem implies these are Gaussian distributed with variance proportional to $1/\dim(\mathcal{H}) \sim e^{-S}$. 3. **Dynamic Link:** The function $f_{\mathcal{O}}(\omega)$ is shown to be the Fourier transform of the correlation function $\langle \mathcal{O}(t) \mathcal{O}(0) \rangle$, linking ETH to dynamic relaxation.

5 Model Apply: Black Hole Information

This framework can be applied to the black hole information paradox. **Matrix Universe:** The Black Hole S-matrix is modeled as a chaotic unitary matrix. **QCA:** The horizon dynamics are a chaotic QCA (scrambling). **Unified Time:** The SFF ramp provides evidence for the discrete spectrum and unitarity of the black hole evolution, resolving the information loss problem (at the level of spectral statistics).

6 Conclusion

The unified Matrix–QCA Universe naturally accommodates Quantum Chaos and ETH. Chaos is the generic behavior of the scattering phase flow and QCA update. RMT universality and Eigenstate Thermalization are not ad-hoc assumptions but emergent statistical properties of the unified time scale and operator algebra in a complex, interacting universe. This unifies the microscopic mechanism (QCA mixing) with the macroscopic statistical description (RMT/ETH).

A RMT and SFF Definitions

(Standard definitions of GOE, GUE, GSE, and Spectral Form Factor)

B OTOC and Butterfly Velocity

(Definition of Out-of-Time-Order Correlator and its relation to chaos)

C Rigorous ETH Statement

(Mathematical formulation of ETH bounds)