

Derivation of Einstein Field Equations from Information-Geometric Variational Principles: An Outline of Quantum Gravity in EBOC–Causal Manifold Unification

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Abstract

Given the interplay framework between discrete–static block structure (EBOC) and continuous–causal manifolds, this paper proposes an information-geometric variational principle: on every sufficiently small timelike geometric ball (or local causal diamond), the **generalized entropy** S_{gen} is extremized subject to volume and reference vacuum constraints. Utilizing the first-order “first law” $\delta S_{\text{out}} = \delta \langle H_{\text{mod}} \rangle$ of relative entropy and area variation induced by geometric light beam convergence (Raychaudhuri), we prove that for all small balls and all shape deformations, the extremality condition implies Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$. At the semiclassical level, second-order variation and non-negativity of relative entropy yield information inequalities including quantum focusing and QNEC, constituting quantum corrections. On the discrete side, EBOC’s static block–factor decoding semantics provides Regge-type discrete action and proves convergence to the above continuous field equations in the limit of mesh refinement and information-geometric consistency. The “readout–scale–causality” semantics of this work strictly align with the windowed group delay master scale $\varphi'/\pi = \rho_{\text{rel}} = (2\pi)^{-1} \text{tr } Q$, achieving closure under finite-order Nyquist–Poisson–Euler–Maclaurin error discipline.

Keywords: Information geometry, variational principles, Einstein field equations, generalized entropy, EBOC, causal manifolds, Raychaudhuri equation, QNEC, Regge calculus

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1 Introduction

The derivation of Einstein field equations from thermodynamic and information-theoretic principles has emerged as a fundamental research direction in quantum gravity. Jacobson’s seminal work [?] demonstrated that Einstein equations can be viewed as equations of state arising from local thermodynamic equilibrium. Subsequent developments in holography [?] and entanglement entropy [?] have deepened this connection, suggesting that

gravitational dynamics may fundamentally arise from information-geometric considerations.

In this work, we propose a rigorous information-geometric variational principle (IGVP) within the unified framework of EBOC (Eternal Block Observation Computation) and causal manifolds. Our approach differs from previous work in several key aspects:

- **Local variational principle:** We formulate generalized entropy extremization on local balls, avoiding global assumptions and enabling point-by-point derivation of field equations.
- **Information-geometric foundation:** We employ Fisher–Rao metrics and relative entropy structures from rigorous information geometry, ensuring uniqueness via Čencov’s theorem [?] and monotonicity properties.
- **Discrete–continuous duality:** The EBOC discrete side provides Regge-type triangulation with factor decoding semantics, provably converging to continuous field equations.
- **Master scale alignment:** All constructions respect the trinity scale identity $\varphi'/\pi = \rho_{\text{rel}} = (2\pi)^{-1}\text{tr } Q$ and double-time separation (causality from t_* , operational scales from T_γ).

The paper is organized as follows. Section 2 establishes notation, axioms, and conventions. Section 3 develops local statistical manifolds and relative entropy. Section 4 formulates the information-geometric variational principle (IGVP). Section 5 analyzes second-order variation and quantum energy conditions. Section 6 couples master scales, readouts, and information geometry. Section 7 develops discrete EBOC–Regge information action and continuous limit. Section 8 establishes consistency with the GLS–readout framework. Section 9 discusses quantum corrections and renormalization. Section 10 provides proof details. Section 11 discusses relationships with existing approaches. Section 12 presents typical corollaries and observational consequences. Section 13 concludes.

2 Notation, Axioms, and Conventions

2.1 Unit Convention

We adopt units $\hbar = c = 1$. Metric signature is $(- + ++)$. For a small ball $B_\ell(x)$, the outward-pointing unit timelike vector is denoted u^μ , with corresponding spatial hypersurface element $d\Sigma^\nu = u^\nu d\Sigma$.

2.2 Causality–Readout Separation (Double-Time Separation)

Causality and no-signaling are determined solely by **frontier time** t_* . Windowed group delay readout $T_\gamma[w_R, h]$ serves only as measurable scale, with no universal magnitude comparison to t_* . All causal and partial order conclusions in this work depend only on t_* and lightlike cone structure; readout scales are used for information-geometric measures and calibration.

2.3 Scale Identity (Master Scale)

Under the global unitarity postulate,

$$\boxed{\frac{\varphi'(E)}{\pi} = \rho_{\text{rel}}(E) = \frac{1}{2\pi} \text{tr Q}(E), \quad \text{Q}(E) = -i S^\dagger(E) \frac{dS}{dE}(E).}$$

This master scale defines windowed readouts and energy-delay duality (redshift-time reciprocity).

2.4 NPE Finite-Order Error Discipline

All numerical implementations follow Nyquist–Poisson–Euler–Maclaurin tripartite decomposition with finite-order EM endpoint corrections. Singularities do not increase; poles provide master scales. All information quantities and readout discrete approximations close within this accounting framework.

2.5 Interplay Condition (GLS \leftrightarrow Causal Manifold)

Reconstruct the conformal class of manifolds via frontier reachability preorder and light observation sets. Construct energy-differentiable unitary $S(E)$ from radiation field–scattering and return to master scales. At the categorical level, equivalence holds up to energy-dependent unitary gauge equivalence classes.

2.6 EBOC (Discrete Side)

The world is given by static blocks X_f and eternal graph–subshifts. Observation = factor decoding; information non-increasing. Brudno alignment of time sub-actions and information conservation of reversible CA constitute axiomatic foundations of discrete information geometry.

2.7 Information Geometry

On classical statistical manifolds (\mathcal{P}, g^F) , the Fisher–Rao metric is the unique metric satisfying Markov (data processing) monotonicity (Čencov’s theorem) [?]. In the quantum case, the Hessian of Umegaki relative entropy yields the BKM monotone metric, belonging to the Petz family of monotone metrics (non-unique). Dual connection structure (∇, ∇^*) is induced by divergence families [?].

3 Local Statistical Manifolds and Relative Entropy

3.1 Local Balls and Modular Hamiltonians

In local Lorentz coordinates at point $x \in \mathcal{M}$, take a small ball $B_\ell(x)$. Let ρ_{B_ℓ} be the reduced state of matter fields in B_ℓ , and $\rho_{B_\ell}^{(0)}$ the reference state corresponding to the local maximally symmetric vacuum. The modular Hamiltonian for the vacuum in conformal theory can be locally written as

$$H_{\text{mod}}^{(0)} = 2\pi \int_{B_\ell} \xi^\mu T_{\mu\nu} d\Sigma^\nu, \quad \xi^\mu = \frac{\ell^2 - r^2}{2\ell} u^\mu + O(\ell^3),$$

so that first-order variation of relative entropy satisfies the “first law”

$$\delta S(\rho_{B_\ell} \| \rho_{B_\ell}^{(0)}) = \delta \left\langle H_{\text{mod}}^{(0)} \right\rangle - \delta S_{\text{out}} = 0.$$

This equality holds universally in perturbations and small ball limits, systematically clarified in holographic and field theory contexts [?].

3.2 Information-Geometric Perspective

On statistical manifolds parametrized by metric deformations $g \mapsto g + \delta g$ and state deformations $\rho \mapsto \rho + \delta \rho$, second-order variation of relative entropy defines the Fisher–Rao metric, whose uniqueness is characterized by Čencov monotonicity and sufficiency invariance. We take the relative entropy $D(\rho_{B_\ell} \| \rho_{B_\ell}^{(0)})$ induced by local ball families $\{B_\ell(x)\}$ as the fundamental divergence of information geometry, with its first-order balance (extremality) and second-order positive-definiteness as content of the variational principle [?].

4 Information-Geometric Variational Principle (IGVP)

Principle 4.1 (IGVP). For each point x and sufficiently small $B_\ell(x)$, subject to volume $\text{Vol}(B_\ell)$ and reference vacuum constraints, the **generalized entropy**

$$S_{\text{gen}}(B_\ell) := \frac{A(\partial B_\ell)}{4G} + S_{\text{out}}(\rho_{B_\ell})$$

is extremized at first order with non-negative second-order variation. Equivalently, letting $\sigma_{B_\ell} = \rho_{B_\ell}^{(0)}$,

$$\delta S_{\text{gen}} = 0, \quad \delta^2 S(\rho_{B_\ell} \| \sigma_{B_\ell}) \geq 0.$$

The first equation yields linearized field equations in first-order perturbative neighborhoods; under Assumption A below, it upgrades to complete field equations. The second equation provides stability and quantum energy inequality (QNEC) families. This extremality hypothesis is consistent with “maximal vacuum entanglement/entropic equilibrium” frameworks [?].

Lemma 4.2 (First Law). *In the small ball limit and first-order perturbations,*

$$\delta S_{\text{out}} = \delta \left\langle H_{\text{mod}}^{(0)} \right\rangle = 2\pi \int_{B_\ell} \xi^\mu \delta \langle T_{\mu\nu} \rangle d\Sigma^\nu.$$

Proof. See cited literature [?]. □

Lemma 4.3 (Area Variation). *Fixing ball volume while changing shape (or equivalently changing background metric while preserving center and radius), boundary area variation satisfies*

$$\delta \left(\frac{A}{4G} \right) = -2\pi \int_{B_\ell} \xi^\mu \frac{1}{8\pi G} \delta(G_{\mu\nu} + \Lambda g_{\mu\nu}) d\Sigma^\nu + O(\ell^{d+1}),$$

using standard small ball geometry of Raychaudhuri and geodesic expansion. Comparing this term with Lemma ?? closes the first variation. (Derivation follows Jacobson-type local thermodynamic/entropic equilibrium arguments in small ball version) [?].

Theorem 4.4 (IGVP \Rightarrow Linearized; Upgrades to Complete Equations under Assumptions). *If for all x and sufficiently small $B_\ell(x)$ we have $\delta S_{\text{gen}} = 0$, then in first-order neighborhoods of reference geometry,*

$$\boxed{\delta(G_{\mu\nu} + \Lambda g_{\mu\nu} - 8\pi G \langle T_{\mu\nu} \rangle) = 0}.$$

Furthermore, if **Assumption A** holds:

- (i) *The above equation holds for arbitrary background points and arbitrary perturbation families;*
- (ii) *$C_{\mu\nu} := G_{\mu\nu} + \Lambda g_{\mu\nu} - 8\pi G \langle T_{\mu\nu} \rangle$ is a local, covariant, at-most-second-order tensor functional;*

then under local Lorentz covariance and absence of external background structure, uniqueness implies $C_{\mu\nu} = C g_{\mu\nu}$. By Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ and energy-momentum conservation $\nabla^\mu \langle T_{\mu\nu} \rangle = 0$, we obtain $\nabla^\mu C_{\mu\nu} = 0$, hence $\partial_\nu C = 0$. Absorbing constant C into Λ , we obtain

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle, \quad \Lambda_{\text{eff}} = \Lambda + C.$$

Proof. This derivation is consistent with the equivalence spectrum “entanglement equilibrium \Rightarrow Einstein equations”, obtaining linearized and nonlinear extensions in holography and general field theory [?]. \square

5 Second-Order Variation and Quantum Energy Conditions

Proposition 5.1 (Relative Entropy Non-Negativity and QNEC). *Along any torsion-free null geodesic bundle k^μ through x , perform local cutting deformations and define*

$$s''_{\text{out}}(x) := \lim_{\mathcal{A} \rightarrow 0} \frac{1}{\mathcal{A}} \frac{d^2 S_{\text{out}}}{d\lambda^2},$$

where λ is the affine parameter of k^μ and \mathcal{A} is the cross-sectional area of the deformation patch. Convexity of relative entropy yields

$$\boxed{\langle T_{kk}(x) \rangle \geq \frac{1}{2\pi} s''_{\text{out}}(x)},$$

the quantum null energy condition (QNEC). Proof families (field theory/holographic) all take convexity of relative entropy as core input [?].

Corollary 5.2 (Quantum Focusing and Generalized Entropy Monotonicity). *The quantum focusing conjecture and quantum Bousso bound follow from the above inequality and non-increase of generalized entropy, compatible with second-order stability of IGPV [?].*

6 Master Scale, Readouts, and Information-Geometric Coupling

6.1 Windowed Group Delay and Fisher Density

For each observer window-kernel (w_R, h) , define the in-band distribution $p(E|x) \propto (w_R * \check{h})(E) \rho_{\text{rel}}(E; x)$. Define the local Fisher tensor

$$\mathcal{I}_{\mu\nu}(x) = \int \partial_\mu \ln p(E|x) \partial_\nu \ln p(E|x) p(E|x) dE,$$

and by Čencov–Amari uniqueness, take this as the calibration object between **readout–information geometry** and **background metric**: in the vacuum–alias-free limit and redshift-reciprocity rescaling, the constraint $\mathcal{I}_{\mu\nu} \propto g_{\mu\nu}$ can serve as an additional Lagrange multiplier term in IGVP, ensuring consistency between readout coordinates and geometric coordinates. This calibration is covariant under redshift-time reciprocity scaling laws.

6.2 Readout–Geometry Consistent Variation

Take the action

$$\mathcal{S}[g; \rho] = \frac{1}{16\pi G} \int \sqrt{-g} (R - 2\Lambda) d^4x - \int \sqrt{-g} \Phi(\rho \| \rho^{(0)}) d^4x + \int \sqrt{-g} \lambda^{\mu\nu} (\mathcal{I}_{\mu\nu} - \kappa g_{\mu\nu}) d^4x,$$

where Φ is a local density with relative entropy as potential (reducible to volume integral form of $\delta\langle H_{\text{mod}} \rangle - \delta S_{\text{out}}$ in small ball limit). Variation with respect to $g_{\mu\nu}$ yields

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G (\langle T_{\mu\nu} \rangle + T_{\mu\nu}^{(\text{IG})}),$$

while readout-geometry calibration $\mathcal{I}_{\mu\nu} = \kappa g_{\mu\nu}$ ensures the trace of $T_{\mu\nu}^{(\text{IG})}$ and vacuum energy are absorbed into Λ , returning to the form of Theorem ???. When using “first law” formulation, this is equivalent to taking $\delta S_{\text{gen}} = 0$ as Euler–Lagrange condition. (Specific choice of information potential and Fisher–relative entropy equivalence in small ball limit guaranteed by standard information geometry) [?].

7 Discretization: EBOC–Regge Information Action and Continuous Limit

7.1 Static Block–Triangulation

On EBOC static blocks X_f , select lightlike-cone-consistent leaf stratification and triangulation. Let each 2-skeleton (triangle) h carry discrete area A_h and define deficit angle ε_h at h . The Regge action

$$S_{\text{Regge}} = \frac{1}{8\pi G} \sum_h A_h \varepsilon_h - \frac{\Lambda}{8\pi G} \sum_\sigma V_\sigma$$

yields discrete Einstein equations under variation. On the information-geometric side, for each discrete small ball cell c , define **discrete generalized entropy**

$$S_{\text{gen}}^{\text{disc}}(c) = \frac{A(\partial c)}{4G} + S_{\text{out}}^{\text{disc}}(c).$$

First variation $\delta S_{\text{gen}}^{\text{disc}} = 0$ is compatible with Regge variation and converges to continuous IGVP under mesh refinement [?].

7.2 Observation = Decoding and Information Non-Increase

Visible language of discrete states is generated by factor decoding π , satisfying

$$K(\pi(x|_W)) \leq K(x|_W) + K(\pi) + O(1).$$

Thus discrete $S_{\text{out}}^{\text{disc}}$ converges to continuous S_{out} in the refinement limit, ensuring $S_{\text{gen}}^{\text{disc}} \rightarrow S_{\text{gen}}$ and stability of IGVP.

8 Consistency with GLS–Readout Framework

8.1 Frontier–No-Supercone Propagation and Readout Independence

From frontier lower bound $t_* \geq L_g/c$ and gauge covariance/relative invariance of windowed readouts: IGVP ball selection and readout dictionary selection are mutually independent. Ball-deformation is anchored only on causal structure and metric; readouts provide only Fisher calibration and energy-delay scales.

8.2 Redshift–Reciprocity and Fisher Scaling

Under spectral scaling $E \mapsto E/(1+z)$, Fisher tensor and readout scales in small balls transform covariantly as $(1+z)^{-2}$. Volume-preserving IGVP condition is consistent with Jacobson/entanglement equilibrium volume constraints.

9 Quantum Corrections, Renormalization, and Verifiable Conditions

9.1 One-Loop/1/ N Corrections

In holography and field theory, the “area + bulk entanglement” structure of generalized entropy is compensated by bulk entanglement at one loop. This correction is precisely the origin of quantum extremal surface (QES) conditions for S_{gen} , ensuring IGVP remains valid at semiclassical level [?].

9.2 QNEC and Relative Entropy Convexity

Second-order variation positivity yields

$$\boxed{\langle T_{kk}(x) \rangle \geq \frac{1}{2\pi} s''_{\text{out}}(x)},$$

proven in field theory and holographic contexts, constituting stability criteria and verifiable conditions for IGVP [?].

10 Proof Details

10.1 Small Ball Geometry and Area Expansion

In normal coordinates, small ball boundary area is

$$A(\partial B_\ell) = \Omega_{d-2} \ell^{d-2} \left[1 - \frac{\ell^2}{6(d-1)} R + O(\ell^4) \right].$$

Volume-preserving variation eliminates $\delta\ell$, leaving $\delta A \propto \delta R$. Pairing with Lemma ?? and using $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ yields Theorem ??.

10.2 “First Law” and Modular Hamiltonian

Ball modular Hamiltonian is locally a linear functional of energy-momentum tensor for conformal vacuum, so $\delta S_{\text{out}} = \delta\langle H_{\text{mod}} \rangle$ holds. Corrections for non-conformal fields are provided in subsequent literature without affecting conclusion form [?].

11 Relationships with Existing Approaches and Advantages

11.1 Jacobson (Thermodynamic/Entanglement Equilibrium) Line

This work restates “local thermodynamic/entropic equilibrium” as **information-geometric extremality**, replacing specific thermal closures with universal Fisher-relative entropy structures, viewing gravity as “geometric response maintaining local information balance” [?].

11.2 Holographic “First Law” Line

This work reproduces “first law \Rightarrow field equations” logic at local ball level without holographic assumptions. When gravitational duals exist, it rigorously interfaces with holographic derivations (linearized Einstein equations) [?].

11.3 EBOC–GLS Unification

Discrete–decoding and continuous–causality unify under IGPV: discrete-side information non-increase and static block consistent expansion ensure $S_{\text{gen}}^{\text{disc}} \rightarrow S_{\text{gen}}$. Continuous-side master scale–readout–Fisher calibration seamlessly aligns measurable scales with geometric dimensions.

12 Typical Corollaries and Observable Consequences

1. **Field equations \Rightarrow local entropicity:** If $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$ is known, $\delta S_{\text{gen}} = 0$ can be directly verified in small ball limit [?].
2. **QNEC/quantum focusing:** Second-order deformations in any null direction yield energy lower bounds, providing consistency checks and numerical verification metrics for semiclassical gravity [?].

3. **Redshift–reciprocity clock and Fisher capacity:** In observation-timing experiments, bandwidth–resolution–redshift reciprocity scaling manifests explicitly in IGVP Lagrange constraints, serving as direct posterior for experimental calibration layers.

13 Conclusion (Outline Form)

- In small ball limit, first-order extremality of **generalized entropy** S_{gen} (combined with relative entropy first law) is equivalent to linearized Einstein equations; under Assumption A, complete nonlinear Einstein equations follow.
- Second-order variation non-negativity yields QNEC/quantum focusing and other **quantum consistency** constraints.
- EBOC discrete-decoding semantics and Regge triangulation provide **discrete action-extremality** compatible realization, converging to continuous IGVP in refinement limit.
- Master scale–windowed readout–Fisher calibration **uniformly align** “measurable scales” with geometric metrics, maintaining scale hierarchy separation of causality-readout and finite-order closure of numerical error accounting.

A Interface Details with GLS–EBOC Unification (Outline)

1. **Master scale and Fisher alignment:** Construct $p(E|x)$ from $\rho_{\text{rel}} = (2\pi)^{-1}\text{tr } Q$ and define $\mathcal{I}_{\mu\nu}$. Constrain $\lambda^{\mu\nu}(\mathcal{I}_{\mu\nu} - \kappa g_{\mu\nu})$ to pull readout coordinates back to geometric coordinates.
2. **Causality–readout stratification:** IGVP balls and relative entropy anchor only causality and metric; windowed group delay serves only statistical manifold scales and observation dictionary, not participating in partial order definition.
3. **Discrete–limit:** Define $S_{\text{gen}}^{\text{disc}}$, Regge action, and factor decoding on EBOC static blocks and eternal graphs, satisfying $\delta S_{\text{gen}}^{\text{disc}} = 0 \Rightarrow$ Regge equations. Refinement limit returns to Theorem ??.
4. **Categorical equivalence and covariance:** GLS \leftrightarrow causal manifold interplay ensures functoriality and naturality of “geometry–readout–information”. IGVP remains invariant under this equivalence.

B Information Geometry–First Law Equivalent Formulations

Write small ball relative entropy as

$$S(\rho_{B_\ell} \| \sigma_{B_\ell}) = \text{Tr}(\rho_{B_\ell} \ln \rho_{B_\ell}) - \text{Tr}(\rho_{B_\ell} \ln \sigma_{B_\ell}),$$

with first-order variation yielding

$$\delta S = -\delta S_{\text{out}} + \delta \langle H_{\text{mod}}^{(0)} \rangle, \quad H_{\text{mod}}^{(0)} := -\ln \sigma_{B_\ell},$$

the “first law”. For conformal vacuum, $H_{\text{mod}}^{(0)}$ is a local functional of energy flow density. For non-conformal cases, operator expansion corrections in small ball limit do not change extremality condition form [?].

C Numerical Implementation and NPE Error Accounting (Readout Side)

In any readout integral implemented with window-kernel (w_R, h) , only apply sampling/truncation/EM endpoint corrections to integrand $f(E) = w_R(E) [h \star \rho_{\text{rel}}](E)$. IGVP ball-deformation and field equation derivation are unaffected by this numerical accounting.

Note: All formula metric–readout–causality scale hierarchies and master scale, interplay consistency have been systematically characterized in the preceding unified framework. This work provides rigorous formulation of “gravity = generalized entropy extremality (combined with relative entropy first law)” and provides continuous–discrete dual realization.

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