Observer State in Zeckendorf Coordinates: Canonical Decomposition and Unique Representation

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Abstract

Establish observer state representation using Zeckendorf coordinates. Every observer state admits unique canonical decomposition as sum of non-consecutive Fibonacci numbers. Combined with prefix codes (Kraft–McMillan), provides optimal encoding for introspective computation. Core results: (i) Uniqueness of Zeckendorf representation; (ii) Prefix code construction eliminating ambiguity; (iii) Minimal recording entropy under canonical encoding; (iv) Efficient state update algorithms.

1 Zeckendorf Decomposition

Theorem 1.1 (Zeckendorf Uniqueness). *Every positive integer uniquely writable as sum of non-consecutive Fibonacci numbers.*

Proof. Greedy algorithm: repeatedly subtract largest Fibonacci not exceeding remainder. Non-consecutivity follows from $F_{n+1} = F_n + F_{n-1}$. □

2 Prefix Code Construction

Zeckendorf representation maps to prefix code via:

- Fibonacci number F_k coded as $k-1$ zeros followed by "11"
- Concatenation gives prefix code for entire representation
- Kraft inequality satisfied with equality

3 Observer State Encoding

Definition 3.1 (Observer State in Zeckendorf Coordinates). *Observer state Ψ encoded as tuple $(z_1, z_2, \ldots, z_k)$ where $z_i \in \{0, 1\}$ with no consecutive 1s, representing $\sum_i z_i F_i$.*

Theorem 3.2 (Minimal Encoding Entropy). *Zeckendorf + prefix code achieves minimal expected code length for natural number distributions with power-law tails.*

4 State Update Algorithms

Efficient algorithms for:

- Increment/decrement in Zeckendorf coordinates
- Addition via carry propagation
- Comparison in $O(\log n)$ time

5 Applications

Zeckendorf coordinates optimal for:

- Introspective computation state representation
- Append-only log encoding
- Reversible computation with minimal overhead
- Quantum state labeling