Representation and Inversion of "Time" in EBOC

— Characterizing "Sequence" and "Choice" in a Static Block Universe, and Deriving Consciousness Self-Linearization from Recursive Unfolding of Observation Windows

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Abstract

In a "timeless" static block universe (EBOC), all facts are given as once-for-all structure–measure objects; so-called "time" should be a secondary calibration endogenously invertible from that object, not a primitive coordinate. This paper provides a rigorous route under the unified semantics of EBOC: first, via the **window–consensus** paradigm, we define "sequence" as a **bidirectionally infinite path** (consensus chain) on the function graph driven by a **unified selector**, ensuring "unique successor" through preference aggregation and well-order disambiguation; then, via the identity **windowed trace = phase–density calibration** (phase derivative = relative density of states = Wigner–Smith group delay trace), we establish "time readout" as **window-weight density integral** and close it under **finite-order** Nyquist–Poisson–Euler–Maclaurin error discipline; finally, in KL/Bregman information geometry, we characterize the **recursive unfolding** of observation windows as an I-projection (minimal KL) sequence, thereby obtaining the **consciousness self-linearization** theorem and inversion parameters in **dual (expectation) coordinates**. The core conclusion is: the one-dimensionality of "narrative time" in EBOC can be endogenously inverted via the combined force of "structural selection + metric readout".

1 Notation & Axioms / Conventions

(Calibration Card I: Trinity) The calibration identity holding almost everywhere in the absolutely continuous spectrum

$$\boxed{\frac{\varphi'(E)}{\pi} = \rho_{\mathrm{rel}}(E) = \frac{1}{2\pi}\,\mathrm{tr}\,\mathsf{Q}(E)}, \qquad \mathsf{Q}(E) = -i\,S(E)^\dagger \frac{dS}{dE}(E).$$

where $S(E)$ is the scattering matrix, $\varphi'(E)$ is the total scattering phase derivative, ρ_{rel} is the relative density of states; the identity arises on one hand from the Birman–Kreĭn formula

$$\det S(E) = e^{2\pi i \xi(E)} \quad \Rightarrow \quad \xi'(E) = \rho_{\mathrm{rel}}(E) = \frac{1}{2\pi i}\frac{d}{dE}\log\det S(E).$$

Define total scattering phase $\varphi(E) := \dfrac{1}{2i}\log\det S(E)$ (continuous branch), then

$$\frac{\varphi'(E)}{\pi} = \frac{1}{2\pi i}\frac{d}{dE}\log\det S(E) = \xi'(E) = \rho_{\mathrm{rel}}(E),$$

fully consistent. On the other hand from Wigner–Smith time-delay matrix and Kreĭn–Friedel relation $\rho_{\mathrm{rel}}(E) = \frac{1}{2\pi}\,\mathrm{tr}\,\mathsf{Q}(E)$ [**?**].

(Calibration Card II: NPE Finite-Order Discipline) All windowed computations only allow **finite-order** Euler–Maclaurin (EM) and Poisson summation; error strictly decomposes as

$$\varepsilon = \varepsilon_{\text{alias}} + \varepsilon_{\text{EM}} + \varepsilon_{\text{tail}},$$

where under Nyquist sampling (band-limited signal, sampling rate $> 2B$) $\varepsilon_{\text{alias}} = 0$; EM remainder controlled by Bernoulli polynomials and higher-order derivatives of integrand; tail controlled by fast decay and band limitation. This discipline guarantees **non-increasing singularity** and "pole = primary scale" [**?**].

Windows and kernels. On energy axis $\mathbb{R}_E$, given even window $w_R \geq 0$ and front-end kernel $h \geq 0$ (band-limited, regular, and $\int_{\mathbb{R}} h(E)\, dE = 1$), convolution denoted $(h \star \rho)(E)$.

Working energy band. Denote

$$\mathcal{B} \;:=\; \operatorname{ess\,supp}\Big(\sum\nolimits_k w_{R_k}\Big) \;\subset\; \mathbb{R}_E,$$

the essential support of pointwise weight sum of the window family. All assertions in this paper about **coverage**, **bounded overlap (strong/weak)**, readout and inversion are stated on $\mathcal{B}$.

Integrability. Assume $\rho_{\text{rel}} \in L^1_{\text{loc}}(\mathcal{B})$; accordingly all $\int_{E_0}^{E(t)} \rho_{\text{rel}}$ appearing in this paper are well-defined on $\mathcal{B}$.

Window family coverage. Let window family $\{w_{R_k}\}$ satisfy $\displaystyle\sum_k w_{R_k}(E) > 0$ a.e. on $\mathcal{B}$.

Window family bounded overlap. Strong form: there exists $C < \infty$ such that $\displaystyle\sum_k w_{R_k}(E) \leq C$ a.e.; **Weak form:** there exist $M < \infty$ and $W_{\max} < \infty$ such that for any E, $\#\{k : w_{R_k}(E) > 0\} \leq M$ and $\sup_k \|w_{R_k}\|_\infty \leq W_{\max}$. Under either condition and $h \geq 0$, $\int h = 1$, we have $\displaystyle\sum_k w_{R_k}(E)\, \big(h \star \rho_{\text{rel}}\big)(E) \in L^1_{\text{loc}}$; **accordingly, F defined in §?? is a locally bounded variation (absolutely continuous) function on $\mathcal{B}$**; if further assuming $\displaystyle\int_{\mathcal{B}} \Big| \sum_k w_{R_k}(E)\, \big(h \star \rho_{\text{rel}}\big)(E) \Big|\, dE < \infty$ (e.g., finite window family, or $\sum_k w_{R_k} \in L^1(\mathcal{B})$ and $\rho_{\text{rel}} \in L^1(\mathcal{B})$), then F is **globally bounded variation** on $\mathcal{B}$.

Window family normalization (PUC) and approximate identity kernel. Let $\sum_k w_{R_k}(E) \equiv 1$ a.e. on $\mathcal{B}$, take nonnegative kernel family $\{h_\varepsilon\}_{\varepsilon>0}$ satisfying $\int h_\varepsilon = 1$ and for all $f \in L^1_{\text{loc}}(\mathcal{B})$ we have $h_\varepsilon \star f \to f$ in $L^1_{\text{loc}}(\mathcal{B})$ ($\varepsilon \to 0$). Under PUC and NPE finite-order discipline, the band-limited quantity $h_\varepsilon \star \rho_{\text{rel}}$ satisfies

$$F_\varepsilon(E) := \sum_k \int_{-\infty}^{E} w_{R_k}(E')\, \big(h_\varepsilon \star \rho_{\text{rel}}\big)(E')\, dE' = \int_{-\infty}^{E} \rho_{\text{rel}}(E')\, dE' + C_\varepsilon + \mathcal{O}(\varepsilon_{\text{EM}} + \varepsilon_{\text{tail}}),$$

where constant C_ε together with EM/tail terms give a uniform upper bound, and $C_\varepsilon \to C_0$ as $\varepsilon \to 0$.

Frames and band limitation. Multi-window Gabor/frame Parseval/Tight construction and Wexler–Raz biorthogonality provide stability and density criteria for windowed reconstruction and multi-channel cooperation; critical sampling constrained by Balian–Low phenomenon [**?**].

Information geometry. Adopt Legendre potential Λ and Bregman/KL construction: $\nabla\Lambda$ gives expectation coordinates, I-projection is minimal KL under linear moment constraints; KKT conditions characterize unique optimal point and give sensitivity [**?**].

2 Timeless Characterization of "Sequence" and "Choice"

2.1 Window Graph, Causal Compatibility, and Feasible Paths

Take window radius r and allowed fragment set $\mathcal{C}$. Construct De Bruijn-type **window graph** Γ: vertices are local fragments of length $2r$, edges are one-step slides; impose **causal compatibility** (advancing along edges does not violate underlying dependency preorder). Thus **feasible sequences** $X : \mathbb{Z} \to \mathcal{C}$ on the block correspond one-to-one with **bidirectional paths** on Γ [**?**].

2.2 Unified Selector and Function Graph Decomposition

For each vertex, aggregate multi-agent preferences as weighted extremum, and disambiguate with well-order, obtaining **unified selector** Sel and **deterministic successor**; this yields **function graph** Γ_{Sel} (each point out-degree = 1). Any finite out-degree-1 directed graph decomposes into **several directed cycles** plus their in-trees; periodic points form cycles, others are transient nodes. This paper defines **bidirectionally infinite consensus chain** as bidirectionally extended paths on cycles [**?**].

Proposition 2.1 (Function Graph Structure–Finite Fragment Case)**.** *Let allowed fragment set $\mathcal{C}$ be finite (equivalently: alphabet finite and window radius finite), then each connected component of Γ_{Sel} contains* ***exactly one*** *directed cycle, with other vertices flowing into that cycle via directed trees; the cycle admits bidirectional infinite paths, called consensus chains.* ***General infinite case:*** *each connected component contains* ***at most one*** *directed cycle.*

Proof. Function graphs are standard functional digraphs; their decomposition properties are as stated in the literature (cycles + in-trees) [**?**]. □

2.3 Linear Extension and Threshold Stability

For dependency preorder $\preceq$, by Szpilrajn's theorem any partial order extends to a total order; on the consensus chain image set take this **consistent linear extension** as the index coordinate $t \in \mathbb{Z}$. When weights and disambiguation have minimal gap, unique successor remains invariant under small perturbations (threshold stable) [**?**].

Definition 2.2 (Sequence and Choice)**.** *Choice: Given window state v,* $\mathrm{Sel}(v)$ *selects unique successor edge;*
Sequence: Bidirectional path $(v_t)_{t\in\mathbb{Z}}$ on Γ_{Sel} satisfying $v_t \to v_{t+1}$.

3 Representation of "Time": Phase–Density–Windowed Trace

3.1 Phase Derivative = Relative Density of States = Group Delay Trace

On the absolutely continuous spectrum, the Birman–Kreĭn formula connects spectral shift function ξ and $S(E)$:

$$\det S(E) = e^{2\pi i \xi(E)} \quad \Rightarrow \quad \xi'(E) = \rho_{\mathrm{rel}}(E) = \frac{1}{2\pi i}\frac{d}{dE}\log\det S(E).$$

On the other hand, Wigner–Smith defines $\mathsf{Q}(E) = -iS^\dagger S'$, Kreĭn–Friedel relation gives $\rho_{\mathrm{rel}}(E) = \frac{1}{2\pi}\operatorname{tr}\mathsf{Q}(E)$. Together yield the identity in Calibration Card I [**?**].

3.2 Windowed Readout and Non-Asymptotic Closure

Define **windowed trace readout**

$$\mathrm{Obs}(R;\rho_{\mathrm{rel}}) := \int_{\mathbb{R}} w_R(E)\,\big(h \star \rho_{\mathrm{rel}}\big)(E)\,dE.$$

Discrete implementation obeys **NPE three-way decomposition**: aliasing term (Poisson side), boundary Bernoulli layer (EM side), and tail (band-limited decay). When sampling satisfies Nyquist, $\varepsilon_{\mathrm{alias}} = 0$; EM remainder controlled by Bernoulli coefficients and higher-order derivative bounds; tail controlled by band limitation and window regularity, thus no new singularities introduced [?].

Definition 3.1 (EBOC Time Readout Functional)**.** *Given window family $\{w_{R_k}\}$ and kernel h, define*

$$\mathcal{T}[\rho_{\mathrm{rel}}] := \sum_k \int_{\mathbb{R}} w_{R_k}(E)\,\big(h \star \rho_{\mathrm{rel}}\big)(E)\,dE,$$

and under additional integrability assumption $\displaystyle\int_{\mathcal{B}} |h \star \rho_{\mathrm{rel}}|\,dE < \infty, \quad \sum_k w_{R_k} \in L^\infty(\mathcal{B}) \cap L^1(\mathcal{B})$ *(or finite window family), $\mathcal{T}[\rho_{\mathrm{rel}}]$ is finite and can be given a uniform upper bound via NPE finite-order error; under Nyquist $\varepsilon_{\mathrm{alias}} = 0$ [?].*

4 Inversion of "Time": Recovering Linear Order from Window Data

4.1 Phase Integral Index

Let consensus chain $C = \{v_t\}_{t\in\mathbb{Z}}$. In $\mathcal{B}$, $E(t)$ is taken from the monotone preimage of the readout functional: denote

$$F(E) := \sum_k \int_{-\infty}^{E} w_{R_k}(E')\,\big(h \star \rho_{\mathrm{rel}}\big)(E')\,dE'.$$

Under band limitation, Nyquist sampling, $w_{R_k} \geq 0,\ h \geq 0$ **and window family bounded overlap (strong or weak form)**, F is **locally bounded variation (absolutely continuous)** on $\mathcal{B}$; if further satisfying the global integrability condition given in the previous section, then F is **globally bounded variation** on $\mathcal{B}$. On the **absolutely continuous part** of $\mathcal{B}$, $\rho_{\mathrm{rel}} = \xi'$ a.e. holds, hence $\rho_{\mathrm{rel}} \geq 0$ **a.e.** $\Leftrightarrow$ ξ **is non-decreasing there**. Under this premise F is **monotone non-decreasing**; if further adding **window family coverage** and $\rho_{\mathrm{rel}} > 0$ (a.e.), then F is **strictly monotone**. Select step calibration $\Delta > 0$, define **effective index set**

$$\mathcal{T}_F := \{t \in \mathbb{Z} \mid t\Delta \in \mathrm{ran}\big(F|_{\mathcal{B}}\big)\}.$$

For $t \in \mathcal{T}_F$, take **right-continuous generalized inverse**

$$E(t) := F^{-1}(t\Delta), \qquad F^{-1}(y) := \inf\{E : F(E) \geq y\}.$$

To eliminate additive constant, take baseline index $t_\star \in \mathcal{T}_F$ and set $E_0 := E(t_\star)$, accordingly define

$$\tau(t) := \int_{E_0}^{E(t)} \rho_{\mathrm{rel}}(E)\,dE = \frac{1}{2\pi}\int_{E_0}^{E(t)} \mathrm{tr}\,\mathsf{Q}(E)\,dE,$$

by $\rho_{\mathrm{rel}} \in L^1_{\mathrm{loc}}(\mathcal{B})$ in Notation, the above integral is well-defined on $\mathcal{B}$. Thus $\tau(t_\star) = 0$; if $\rho_{\mathrm{rel}} \geq 0$ **(a.e.)** then τ is **monotone non-decreasing**, and when **window family coverage** and $\rho_{\mathrm{rel}} > 0$ (a.e.) hold, τ is **strictly increasing** [?].

Under the general condition of only satisfying "window family coverage + bounded overlap", F provides strictly monotone energy parameter $E(t)$ and order equivalence; phase coordinate τ still needs to be constructed through $\int \rho_{\rm rel}$. If further satisfying **PUC + approximate identity kernel**, then there exists constant C such that

$$\tau(t) = F(E(t)) - F(E_0) + \mathcal{O}(\varepsilon_{\rm EM} + \varepsilon_{\rm tail}),$$

thus "time readout" can be directly given by prefix windowed readout (up to constant) with error uniformly controlled by NPE discipline.

4.2 Inversion Theorem

Theorem 4.1 (Time Inversion)**.** *Under band-limited windows, Nyquist sampling, and finite-order EM conditions:*

(1) ***(General condition)*** *The linear order of any consensus chain C can be inverted from the prefix windowed readout F via the generalized inverse F^{-1} to a strictly monotone energy parameter $E(t)$, and accordingly obtain a bounded variation parameter equivalent to the chain index t; if $\rho_{\rm rel} \geq 0$* ***(a.e.)****, this parameter is* ***monotone non-decreasing****, and when* ***window family coverage*** *and $\rho_{\rm rel} > 0$ (a.e.) hold, it is* ***strictly increasing****.*

(2) ***(Additional PUC + approximate identity kernel)*** *Further we have $\tau(t) = F(E(t)) - F(E_0) + \mathcal{O}(\varepsilon_{\rm EM} + \varepsilon_{\rm tail})$, thus phase coordinates can be directly recovered from F (or its Nyquist sampling $\{F(E_j)\}$) within a uniform error bound.*

Proof sketch. (i) By Calibration Card I and Kreĭn–Friedel relation, reduce windowed trace to $\int \rho_{\rm rel}$; (ii) Nyquist closes aliasing to zero, EM remainder and tail controlled; (iii) By integrability assumption $\rho_{\rm rel} \in L^1_{\rm loc}(\mathcal{B})$ we know τ is well-defined on $\mathcal{B}$; when $\rho_{\rm rel} \geq 0$ **(a.e.)**, τ is monotone non-decreasing; under **window family coverage** and $\rho_{\rm rel} > 0$ (a.e.), τ is strictly monotone and invertible [**?**]. □

5 Recursive Unfolding of Observation Windows ⇒ Consciousness Self-Linearization

5.1 Submission = I-Projection (Minimal KL)

Let internal state be represented by natural parameter θ, potential function Λ of Legendre type, expectation coordinate $X = \nabla\Lambda(\theta)$. Each observation step updates target moment F_n to F_{n+1}, **submission/collapse** equivalent to

$$\theta_{n+1} = \arg\min_{\theta} \big\{ \mathrm{KL}(P_\theta \| P_{\theta_n}) \text{ s.t. } \mathbb{E}_\theta[T] = F_{n+1} \big\},$$

i.e., I-projection on linear constraints; unique solution exists and satisfies KKT. Bregman–Euclidean Pythagorean property gives optimal decomposition of projection [**?**].

5.2 Linear Response and Quasi-Linear Trajectory

When window change is "mild" and NPE order fixed, then

$$X_{n+1} - X_n = \nabla^2\Lambda(\theta_n)\,(\theta_{n+1} - \theta_n) + o\big(\|\theta_{n+1} - \theta_n\|\big),$$

KKT and strong convexity give first-order **linear response**; reparametrizing the iteration with τ from §**??**, can be viewed as approximate equal-step advance along some fixed vector v_* in expectation coordinates.

Theorem 5.1 (Consciousness Self-Linearization). *Let Λ be essentially smooth strictly convex potential; windows w_R and kernel h band-limited and satisfy Wexler–Raz/Parseval frame stability; sampling Nyquist. Then the submission states $\{X_n\}$ driven by recursive windows* ***admit a strictly increasing reparametrization map*** *$\sigma : \mathbb{Z} \to \mathbb{Z}$ and constant vector v_* in expectation coordinates, and there exists function $\varepsilon_{\rm micro}(R, \Delta) = o_{R\to\infty,\ \Delta\to 0}(1)$, such that for any baseline n and all integers m*

$$\|X_{\sigma(n+m)} - X_{\sigma(n)} - m\,v_*\| \ \le\ |m| \left[C\big(\varepsilon_{\rm EM} + \varepsilon_{\rm tail}\big) + \varepsilon_{\rm micro}(R, \Delta)\right].$$

Convention: *$\varepsilon_{\rm micro}(R, \Delta)$ depends only on window/kernel and NPE order, satisfying $\varepsilon_{\rm micro}(R, \Delta) \to 0$ (as $R \to \infty,\ \Delta \to 0$ with NPE order fixed), the bound reflects linear accumulation of error with step count $|m|$. Accordingly, consciousness exhibits* ***quasi-linear dominant trajectory*** *in its own dual coordinates.*

Proof sketch. Wexler–Raz and Parseval/Tight guarantee readout mapping and reconstruction stability; KKT and Bregman geometry's Pythagorean identity give first-order linearization of each I-projection step; NPE constraint ensures noise terms are entirely dominated by finite-order remainder [?]. □

6 Unified Trinity of "Sequence–Choice–Time"

- **From block to sequence:** Unified selector generates consensus chain (bidirectionally infinite path) on function graph, and via Szpilrajn assigns consistent linear extension;
- **From sequence to time:** Phase–density–group delay calibration identity makes "time readout" become integral of window-weighted density; Nyquist–EM guarantee non-asymptotic closure;
- **From time to consciousness:** I-projection on recursive windows enables dual coordinates to acquire quasi-linear principal axis, with τ as the endogenous parameter of that axis.

7 Thresholds, Singularities, and Implementation Notes

1. **Threshold/resonance:** Singularities (such as poles, branch points) of $\varphi'(E)$ (equivalently $\rho_{\rm rel}(E)$) correspond to continuous spectrum thresholds and resonances; zeros do not constitute general criteria. Windowing and finite-order EM do not increase singularity, maintaining "pole = primary scale" [?].
2. **Frames and density:** Multi-window Parseval/Tight and Wexler–Raz biorthogonality guarantee robust reconstruction; critical sampling constrained by Balian–Low, redundant sampling intervals recommended [?].
3. **Sampling and aliasing:** Band limitation and Nyquist sampling are sufficient conditions for closing aliasing; in engineering implementation, modulation–downsampling strategy can achieve in-band Nyquist [?].

8 Conclusion

From the EBOC static block perspective, "time" is not a primitive axis but generated in three steps: **(i) window–consensus** condenses choice into function graph's **consensus chain**; **(ii) phase–density** enables the chain to acquire invertible **time calibration** (windowed trace readout); **(iii) KL/Bregman** makes **submission** of recursive windows exhibit **self-linearization** in dual coordinates. This route is entirely anchored on verifiable criteria: function graph and linear extension, phase–density identity and NPE finite-order error discipline, Legendre–Bregman and KKT optimization structure, thereby reducing the one-dimensionality of "narrative time" to the result of **structural choice + metric readout**.

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