Resource-Bounded Incompleteness Theory (RBIT)

From Abstract Logic to Computable Resource Gaps

The Core Problem: From Ideal to Real

Classic Gödel's Incompleteness

Concerns ideal observers with **infinite** resources (time, memory).

It proves the mere *existence* of true but unprovable statements in any sufficiently strong formal system.

The Real-World Resource Gap

Real observers—whether human or AI—are **finite**.

We are bound by limited computation, memory, and time. How does incompleteness affect *us* in a practical, measurable way?

RBIT's Core Insight

Incompleteness is not a logical paradox, but a concrete, measurable manifestation of a resource gap.

Operationalizing Incompleteness

Logically Unprovable

A statement whose **shortest proof length** exceeds a given computational budget `L`.

Statistically Indistinguishable

Two distributions a test cannot tell apart with `N` samples at scale `m` and threshold `ε`.

Theory Extension

Adding new, computable axioms to a theory, which expands its reach but cannot eliminate incompleteness.

The Unified Resource Framework

Logical Resources (R_log)

Defined by **L**, the upper bound on proof length.

This resource governs the power of **deductive** reasoning. How far can we search for a proof?

Statistical Resources (R_stat)

Defined by the tuple **(m, N, ε)**: Scale, Sample Size, and Threshold.

This resource governs the power of **inductive** or **empirical** reasoning. How well can we distinguish between phenomena?

A Sentence's 3-Layer State

Layer	State Space	Description
Semantic	{True, False}	Objective, non-computable truth (based on standard model N).
Proof	{Proved, Refuted, Undecided}	Syntactic status, dependent on Theory `T` and Budget `L`.
Statistical	{Distinguishable, Indistinguishable}	Empirical status, dependent on Resources `(m, N, ε)`.

Theorem 1: Resource-Bounded Incompleteness

The Gödel Sentence G_L

For every budget **L**, RBIT constructs a sentence **G_L** that asserts its own unprovability *under that budget*.

$G_L \leftrightarrow$ "There is no T-proof of G_L with length $\leq L$ "

1. Truth: $N \vDash G_L$ (The sentence is true in the standard model)

2. Unprovability: $\ell_T(G_L) > L$ (Its shortest proof is longer than budget `L`)

A Quantitative Gap

This theorem makes incompleteness a **quantitative** and **computable** problem.

It guarantees that a gap between "True" and "Provable" exists at *every* finite resource level, not just in an abstract infinite limit.

The "unknowable" is relative to your resources.

Theorem 2: Permanent Incompleteness

> "Any attempt to "fix" incompleteness by adding new computable axioms is futile."

Incompleteness is a fundamental structural feature that re-emerges in every stronger, consistent theory. You can expand the boundary of knowledge, but you can never eliminate the boundary itself.

Theorem 3: Resolution Monotonicity

More resources (logically or statistically) monotonically expands the boundary of what is knowable. Increasing resources (`L'`≥ `L`) expands the decidable set (`Dec_L(T) ⊆ Dec_{L'}(T)`), but the set of *all* truths is never fully reached.

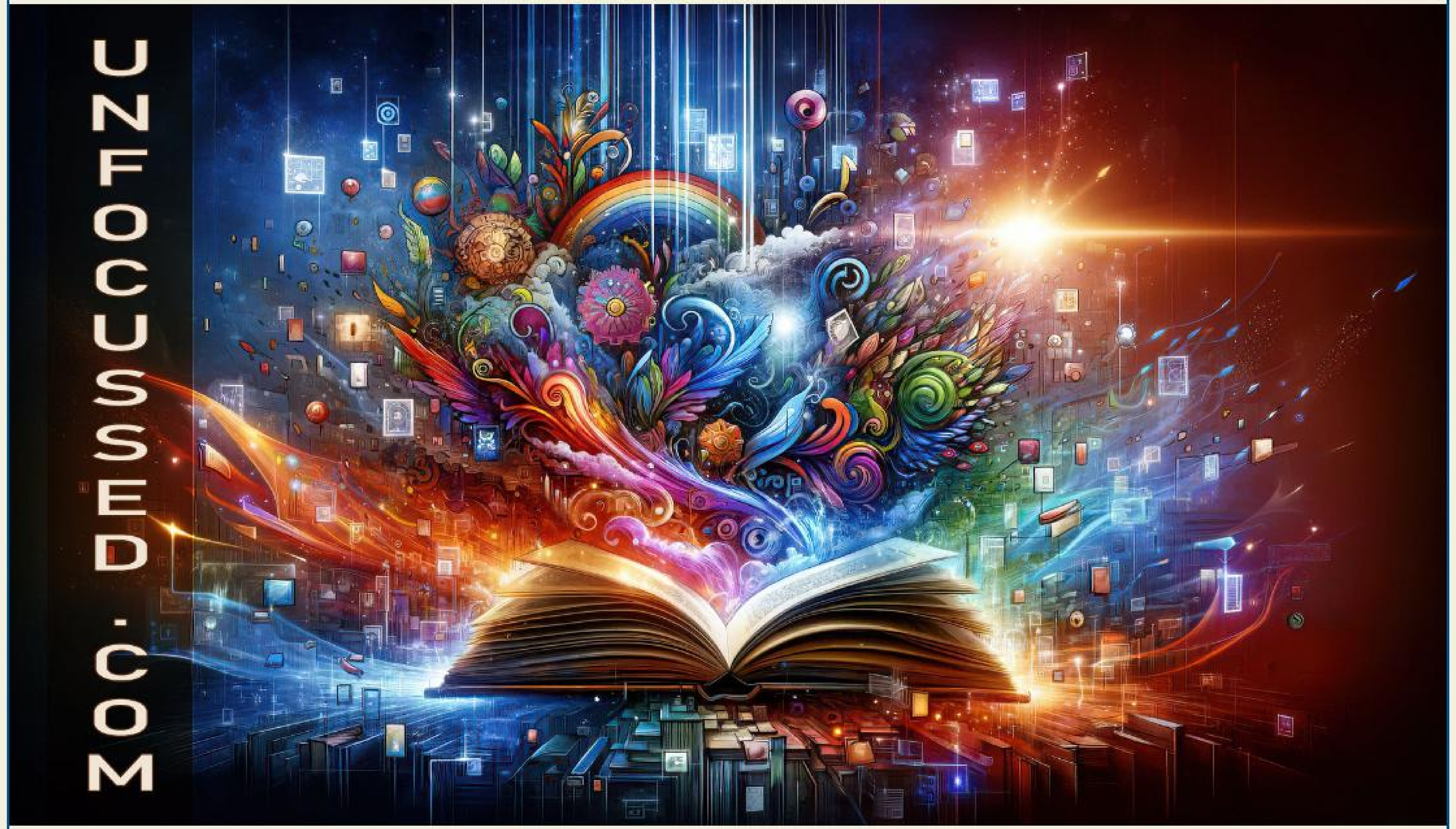
Significance & Applications

- **Artificial Intelligence:** Provides a mathematical model for AI cognitive limits and guides resource-aware formal verification.
- **Complexity Theory:** Aligns proof complexity (logic) with sample complexity (statistics) under a single unified framework.
- **Philosophy of Science:** Offers a formal model for "objective truth, finite cognition" and argues that scientific exploration is a necessarily endless process.

Open Questions & Future Directions

Key Challenges

- Can we find a quantitative *conversion rate* between logical (`L`) and statistical (`N`, `ε`) resources?
- What is the precise growth rate of the shortest proof `l_T(G_L)` as `L` increases?
- How can RBIT be used to design AI systems that are *aware* of their own cognitive boundaries (AI Safety)?



Thank You

Questions?

Image Sources

http://unfocussed.com/cdn/shop/articles/The_Synergy_of_Words_and_Visuals_56c51f24-b2c7-42e1-bcd1-926582b0c45d.jpg?v=1710642595

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