Six Major Open Problems as Unified Constraint System:
Unified Time Scale, Universe Parameter Vector Θ and Joint Solution Space

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Abstract

Within the standard framework of general relativity, quantum field theory and precision cosmology, six problems remain in strong tension with a naive extrapolation of known principles: the microscopic origin of black hole entropy and the information problem, the naturalness of the cosmological constant and dark energy, the structure of neutrino masses and PMNS mixing, the range of validity of the eigenstate thermalization hypothesis (ETH), the strong CP problem in QCD, and possible dispersion or Lorentz violation in gravitational waves. These are usually treated as independent questions attached to different energy scales and sectors.

This work embeds all six into a single structural framework based on a unified time-scale in scattering theory, boundary time geometry and a parameterized quantum cellular automaton (QCA) / matrix universe description. A finite-dimensional parameter vector $\Theta \in \mathcal{P} \subset \mathbb{R}^N$ is introduced, from which a universe object $\mathfrak{U}(\Theta)$ is constructed. All low-energy effective constants and laws are treated as derived observables $\mathcal{O}(\Theta)$. The six "open problems" are rephrased as six scalar constraints on Θ, forming a single constraint map

$$\mathcal{C}(\Theta) = \big(\mathcal{C}_{\mathrm{BH}}, \mathcal{C}_{\Lambda}, \mathcal{C}_{\nu}, \mathcal{C}_{\mathrm{ETH}}, \mathcal{C}_{\mathrm{CP}}, \mathcal{C}_{\mathrm{GW}}\big)(\Theta) \in \mathbb{R}^6.$$

Technically, the construction relies on the unified time-scale identity in scattering theory,

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega),$$

which equates the derivative of the total scattering phase, the relative density of states and the trace of the Wigner–Smith delay operator under standard trace-class perturbation assumptions. Via a QCA/matrix-universe continuous limit, this frequency-domain time-scale controls small causal diamonds, black hole thermodynamics, the vacuum contribution to the effective cosmological constant and the propagation of long-wavelength gravitational waves. Jointly with an internal Dirac block for fermions and Yukawa textures, it also controls neutrino masses and mixing, ETH-like spectral statistics, and the effective QCD CP angle.

On the mathematical side, under natural differentiability and independence hypotheses, the zero set

$$\mathcal{S} = \{\Theta \in \mathcal{P} : \mathcal{C}(\Theta) = 0\}$$

is shown to be, locally, an embedded submanifold of dimension $N - 6$. When $N = 6$ and the Jacobian at a physical point has full rank, the solution set is locally discrete. In addition, strong-CP and topological-sector constraints force certain components of Θ to take values in a discrete set, so that the physically admissible parameter set is a finite or countable union of such lower-dimensional branches. This realizes, at the level of a well-defined map $\mathcal{C}(\Theta) = 0$, the idea that "our Universe" is one point (or a finite set of points) in a strongly constrained parameter space.

On the physical side, the unified constraint system couples sectors that are usually analyzed separately. Black hole entropy and gravitational-wave dispersion jointly constrain the high- and low-frequency behavior of $\kappa(\omega; \Theta)$; cosmological constant naturalness and ETH constrain the mid-frequency spectral density; neutrino mixing and strong CP link internal Dirac spectra, Yukawa phases and topological data. The framework thus yields qualitative cross-predictions between areas such as neutrino physics and cosmology, or black hole thermodynamics and gravitational-wave propagation, and defines a systematic target for model-building: construct explicit QCA/matrix-universe realizations for which the six-component constraint $\mathcal{C}(\Theta) = 0$ holds.

Keywords: Unified time scale; Scattering and spectral shift; Wigner-Smith group delay; Quantum cellular automaton; Matrix universe; Black hole entropy; Cosmological constant; Neutrino mass and PMNS matrix; Eigenstate thermalization hypothesis (ETH); Strong CP problem; Gravitational wave dispersion; Parameter space constraint

1 Introduction & Historical Context

1.1 Structural Tension in Six Major Open Problems

Within standard framework of general relativity and quantum field theory, observed universe is quantitatively very successfully described. However, at least six problems structurally expose "gaps" and naturalness difficulties in this framework:

1. **Black Hole Entropy and Information Problem**

   Works of Bekenstein and Hawking show black holes possess entropy proportional to horizon area, usually written as "one quarter of area divided by Planck area" law, satisfying four laws analogous to ordinary thermodynamics. However, microscopic degrees of freedom realization of this natural structure and its relationship with quantum information unitarity and axiomatized quantum gravity remain unified uncharacterized.

2. **Cosmological Constant and Dark Energy Problem**

   Observations show universe currently acceleratingly expanding, describable by extremely small but nonzero effective cosmological constant or dark energy density, while naive quantum field theory zero-point energy estimate exceeds by many orders of magnitude, forming so-called "cosmological constant fine-tuning problem". How

to explain this naturalness without relying on severe fine-tuning is long-standing open question.

3. **Neutrino Mass and PMNS Mixing Structure**

Neutrino oscillation experiments show neutrinos possess nonzero mass and significant flavor mixing, standard PMNS matrix simultaneously carries multiple mixing angles and possible large CP-violating phases, requiring mass generation mechanism beyond Standard Model. How to derive this mass hierarchy and mixing pattern from higher-level unified structure is important question in flavor physics and unified theory.

4. **Quantum Chaos and Eigenstate Thermalization Hypothesis (ETH)**

ETH provides eigenstate-level explanation for "why isolated quantum many-body systems exhibit thermalization behavior", connecting local observable expectation values of high-energy eigenstates with thermodynamic functions, suppressing off-diagonal elements to thermally exponentially small. However, ETH's applicability range, failure conditions and relationship with underlying microscopic dynamics (such as QCA or random matrix behavior) still lack unified description compatible with gravity and cosmology.

5. **Strong CP Problem**

QCD allows CP-violating term containing $\theta_{\mathrm{QCD}} F\tilde{F}$, whose natural value should be order-one constant. However, neutron electric dipole moment experiments constrain observable effective angle $\bar{\theta}$ to extremely small range, raising difficulty "why strong interaction almost does not break CP". Existing solutions include Peccei–Quinn mechanism and various non-standard model constructions, but fundamental explanation at quantum gravity and universe overall level remains unclear.

6. **Gravitational Wave Dispersion and Lorentz Violation**

Joint observation event GW170817/GRB 170817A shows gravitational wave speed highly consistent with light speed at celestial scales, strongly constraining dispersion corrections and propagation speed deviations in large class of modified gravity and Lorentz violation models. If underlying structure of gravity and matter is discrete, such as given by some QCA or lattice dynamics, why almost no dispersion traces left in observable frequency band constitutes nontrivial constraint.

In conventional research, these six difficulties attached to different energy scales, degrees of freedom and observation channels, viewed as "mutually independent" problem list. This work unifies them rewritten as six constraints on finite-dimensional "universe parameter vector" Θ, analyzes joint solution structure of these constraints in parameter space.

1.2 Unified Time Scale and Boundary Time Geometry

Starting point of unified time scale is scattering theory under bounded traceable perturbation. Let H_0 be free Hamiltonian, $H = H_0 + V$ traceable perturbation, define scattering matrix $S(\omega)$, spectral shift function $\xi(\omega)$, total scattering phase

$$\varphi(\omega) = \frac{1}{2\pi} \arg \det S(\omega),$$

and Wigner–Smith group delay operator

$$Q(\omega) = -\mathrm{i}\, S(\omega)^\dagger \partial_\omega S(\omega).$$

Under standard assumptions of Birman–Kreĭn formula and Lifshits–Kreĭn trace formula, spectral shift function derivative equals relative density of states. Combined with relationship between Wigner–Smith delay and state density, obtain unified identity

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi} \operatorname{tr} Q(\omega),$$

where $\kappa(\omega)$ interpreted as "time scale density" of each frequency mode, $\rho_{\mathrm{rel}}(\omega)$ relative state density.

In earlier work, $\kappa(\omega)$ can be connected to quantum null energy condition (QNEC) and its geometric version through relative entropy and energy conditions on small causal diamonds, thereby reconstructing several structures of general relativity on boundary time geometry, especially Raychaudhuri equation, focusing theorem and generalized entropy monotonicity. Thus, unified time scale connects scattering, spectrum and spacetime geometry on frequency domain scale.

1.3 QCA/Matrix Universe Characterization and Finite Information Hypothesis

On other hand, viewing universe as reversible QCA or "matrix universe" object is conception widely discussed at interface of quantum information and cosmology. Basic idea is to describe spacetime and matter using discrete cellular lattice sites, finite-dimensional local Hilbert space and finite propagation radius update rules, then reconstruct effective continuous spacetime and field theory through continuous limit and coarse graining.

In this work assume exists finite-dimensional parameter space

$$\mathcal{P} \subset \mathbb{R}^N, \quad \Theta = (\Theta^1, \dots, \Theta^N),$$

where Θ contains all independent universe-level free parameters: including local Hilbert dimension, coupling constants and phases of local update operators, internal symmetry groups and their breaking patterns, topological sector labels and boundary conditions. Physical motivation of finite-dimensionality is "finite distinguishable information" principle: if all physical constants and effective laws observable in some universe have bounded precision and finitely distinguishable orders, then its parameterized description should be compressible to finite-dimensional variables.

Central question of this paper thus stated as:

> Under unified time scale and QCA/matrix universe characterization, what geometric and topological joint action do constraints corresponding to six major open problems have on Θ? What structure does solution set $\mathcal{S}$ they define possess?

Following sections proceed in order "Model and Assumptions – Main Results – Proofs – Applications and Engineering Schemes – Discussion and Conclusion" to give systematic characterization.

2 Model & Assumptions

This section defines parameterized universe object $\mathfrak{U}(\Theta)$, derived physical quantities $\mathcal{O}(\Theta)$ and basic form of unified constraint map, explains main assumptions relied upon.

2.1 Unified Time Scale Master Formula

Starting from scattering theory, let H_0 and $H = H_0 + V$ be self-adjoint operators satisfying

1. $(H - H_0)(H_0 - \mathrm{i})^{-1}$ is trace-class operator;

2. Wave operators exist and complete, thus scattering operator S well-defined;

3. Dependence of energy-shell decomposition $S(\omega)$ sufficiently smooth, making $\partial_\omega S(\omega)$ exist and Wigner–Smith operator definable.

Birman–Kreĭn formula gives

$$\det S(\omega) = \exp\big(-2\pi \mathrm{i}\,\xi(\omega)\big),$$

where $\xi(\omega)$ is spectral shift function. Taking derivative with respect to ω yields

$$\frac{\varphi'(\omega)}{\pi} = \xi'(\omega), \quad \varphi(\omega) = \frac{1}{2\pi}\arg\det S(\omega).$$

On other hand, Lifshits–Kreĭn trace formula gives identity of spectral shift derivative and state density difference $\rho_{\mathrm{rel}}(\omega)$, thereby obtaining

$$\xi'(\omega) = \rho_{\mathrm{rel}}(\omega).$$

Finally, using relationship between Wigner–Smith delay trace and state density under trace-class perturbation, obtain

$$\rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega), \quad Q(\omega) = -\mathrm{i}S^\dagger(\omega)\partial_\omega S(\omega).$$

Combining gives unified time scale master formula

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi}\operatorname{tr} Q(\omega).$$

This formula will serve as core tool connecting frequency domain, scattering, spectral data and spacetime geometry throughout this paper.

2.2 Parameterized Universe Object

Assume universe describable as QCA or matrix universe object, parameterized by finite-dimensional vector

$$\Theta = (\Theta^1, \ldots, \Theta^N) \in \mathcal{P} \subset \mathbb{R}^N.$$

Components of Θ include:

- Local cellular Hilbert space dimension d_{loc};

- Local unitary gate coupling constants and phases;

- Internal fermion Dirac mass matrix parameters (Yukawa coupling texture);

- Gauge group representation labels and symmetry breaking scales;

- Topological sector labels (e.g., theta angles, winding numbers);

- Boundary condition parameters and initial state constraints.

From Θ construct universe object

$$\mathfrak{U}(\Theta) = (\Lambda, \{H_x\}, U(\Theta), \kappa(\omega; \Theta)),$$

where Λ lattice site set, H_x local Hilbert spaces, $U(\Theta)$ global update operator, $\kappa(\omega; \Theta)$ unified time scale density dependent on Θ.

Through continuous limit and effective field theory approach, $\mathfrak{U}(\Theta)$ determines all low-energy observables: effective gravitational constant $G_{\text{eff}}(\Theta)$, cosmological constant $\Lambda_{\text{eff}}(\Theta)$, Standard Model coupling constants, fermion mass matrices, etc. Write these derived quantities as

$$\mathcal{O}(\Theta) = \big(G_{\text{eff}}, \Lambda_{\text{eff}}, m_\nu, U_{\text{PMNS}}, \theta_{\text{QCD}}, \dots\big)(\Theta).$$

2.3 Six Constraint Components

We now formalize six major open problems as six scalar constraint functions on Θ.

1. **Black Hole Entropy Constraint** $\mathcal{C}_{\text{BH}}(\Theta)$

   For Schwarzschild black hole of mass M, Bekenstein–Hawking entropy

   $$S_{\text{BH}} = \frac{A}{4G},$$

   where A horizon area, G gravitational constant. Microscopic degrees of freedom counting requires entropy expressible as

   $$S_{\text{micro}}(\Theta) = \log \Omega(\Theta),$$

   where $\Omega(\Theta)$ number of accessible states. Constraint

   $$\mathcal{C}_{\text{BH}}(\Theta) = S_{\text{BH}} - S_{\text{micro}}(\Theta) = 0$$

   requires microscopic counting consistent with area law.

2. **Cosmological Constant Constraint** $\mathcal{C}_\Lambda(\Theta)$

   Observed effective cosmological constant extremely small,

   $$\Lambda_{\text{obs}} \sim 10^{-120} M_{\text{Pl}}^4,$$

   while naive vacuum energy estimate much larger. Define

$$\mathcal{C}_\Lambda(\Theta) = \Lambda_{\text{eff}}(\Theta) - \Lambda_{\text{obs}} = 0$$

as naturalness constraint, requiring effective cosmological constant to match observation without severe fine-tuning.

3. **Neutrino Mass and Mixing Constraint $\mathcal{C}_\nu(\Theta)$**

Neutrino oscillation data determines mass-squared differences and PMNS mixing matrix. From Θ derive neutrino mass matrix $M_\nu(\Theta)$ and mixing matrix $U_{\text{PMNS}}(\Theta)$, require

$$\mathcal{C}_\nu(\Theta) = \left|\Delta m^2(\Theta) - \Delta m^2_{\text{obs}}\right| + \left|U_{\text{PMNS}}(\Theta) - U_{\text{obs}}\right| = 0,$$

where norms appropriately defined to measure deviation from observed values.

4. **ETH Validity Constraint $\mathcal{C}_{\text{ETH}}(\Theta)$**

ETH requires high-energy eigenstate local observable matrix elements satisfy certain statistical properties. From $\mathfrak{U}(\Theta)$ spectrum and eigenstates, compute ETH validity measure $\mathcal{I}_{\text{ETH}}(\Theta)$ (e.g., off-diagonal suppression factor), require

$$\mathcal{C}_{\text{ETH}}(\Theta) = \mathcal{I}_{\text{ETH}}(\Theta) - 1 = 0,$$

where value 1 represents perfect ETH.

5. **Strong CP Constraint $\mathcal{C}_{\text{CP}}(\Theta)$**

QCD effective theta angle $\bar{\theta}(\Theta)$ obtained from Θ internal topological data and Yukawa phase structure, require

$$\mathcal{C}_{\text{CP}}(\Theta) = \bar{\theta}(\Theta) - \bar{\theta}_{\text{obs}} \approx \bar{\theta}(\Theta) = 0,$$

where $\bar{\theta}_{\text{obs}} \lesssim 10^{-10}$ from neutron electric dipole moment bound.

6. **Gravitational Wave Dispersion Constraint $\mathcal{C}_{\text{GW}}(\Theta)$**

Gravitational wave propagation at frequency ω influenced by unified time scale $\kappa(\omega; \Theta)$. Dispersion relation deviation from $\omega = ck$ writable as

$$\Delta c_{\text{GW}}(\omega; \Theta) = c_{\text{GW}}(\omega; \Theta) - c,$$

require

$$\mathcal{C}_{\text{GW}}(\Theta) = \left|\Delta c_{\text{GW}}(\omega_{\text{obs}}; \Theta)\right| = 0,$$

where ω_{obs} observed gravitational wave frequency.

Combining these six constraints gives constraint map

$$\mathcal{C}(\Theta) = \left(\mathcal{C}_{\text{BH}}, \mathcal{C}_\Lambda, \mathcal{C}_\nu, \mathcal{C}_{\text{ETH}}, \mathcal{C}_{\text{CP}}, \mathcal{C}_{\text{GW}}\right)(\Theta) \in \mathbb{R}^6.$$

Physical universe parameters Θ_{phys} must satisfy

$$\mathcal{C}(\Theta_{\text{phys}}) = 0.$$

3 Main Results

This section states main mathematical and physical conclusions about constraint map $\mathcal{C}(\Theta)$ and its zero set.

3.1 Geometric Structure of Solution Set

Theorem 3.1 (Solution Set as Submanifold). *Assume:*

1. *Parameter space $\mathcal{P} \subset \mathbb{R}^N$ open domain;*

2. *Constraint map $\mathcal{C} : \mathcal{P} \to \mathbb{R}^6$ continuously differentiable;*

3. *At some point $\Theta_0 \in \mathcal{S} = \{\Theta : \mathcal{C}(\Theta) = 0\}$, Jacobian matrix*

$$J_{\Theta_0} = \left.\frac{\partial \mathcal{C}}{\partial \Theta}\right|_{\Theta_0}$$

has rank 6.

Then locally near Θ_0, solution set $\mathcal{S}$ is smooth embedded submanifold of $\mathbb{R}^N$ with dimension $N - 6$.

In particular, when $N = 6$ and J_{Θ_0} full rank, solution set locally discrete: Θ_0 locally isolated point.

Proof. Direct application of implicit function theorem. See detailed proof in Appendix A. $\square$

3.2 Discrete Structure and Topological Constraints

Proposition 3.2 (Discrete Parameter Components). *Among components of Θ:*

1. *Local Hilbert dimension d_{loc} must be positive integer;*

2. *Topological sector labels (e.g., θ_{QCD} before Yukawa correction) periodic in 2π;*

3. *Certain symmetry group representations can only take discrete values.*

Therefore, physically admissible parameter set $\mathcal{P}_{\text{phys}}$ is union of countably many continuous branches

$$\mathcal{P}_{\text{phys}} = \bigcup_{k \in \mathbb{Z}} \mathcal{P}_k,$$

where each $\mathcal{P}_k$ continuous manifold with fixed discrete data labels.

Combining Theorem **??** and Proposition **??**, when $N = 6$ or slightly larger, physically admissible solution set $\mathcal{S}_{\text{phys}} = \mathcal{S} \cap \mathcal{P}_{\text{phys}}$ is finite or countable discrete point set.

3.3   Cross-Sector Coupling Structure

Proposition 3.3 (Cross-Predictions from Unified Constraints). *Unified constraint system $\mathcal{C}(\Theta) = 0$ couples traditionally separated sectors:*

1. *Black hole entropy constraint and gravitational wave dispersion jointly constrain high and low frequency behavior of $\kappa(\omega; \Theta)$;*

2. *Cosmological constant naturalness and ETH validity jointly constrain mid-frequency spectral density and vacuum structure;*

3. *Neutrino mixing and strong CP link internal Dirac spectra, Yukawa phases and topological data;*

4. *Any parameter change affecting one sector necessarily affects others through $\mathcal{C}(\Theta) = 0$ constraint surface.*

This yields qualitative cross-predictions, e.g.:

- *Neutrino physics parameter values may constrain cosmological constant effective value;*

- *Black hole thermodynamics may constrain gravitational wave propagation properties;*

- *ETH validity range may relate to strong CP solution mechanism.*

4   Proof Sketches and Technical Details

This section gives proof outlines of main results and explains key technical steps.

4.1   Proof of Theorem ??

Implicit function theorem standard form: if $F : \mathbb{R}^n \to \mathbb{R}^m$ continuously differentiable, $F(x_0) = 0$, and Jacobian $DF(x_0)$ rank m, then zero set near x_0 is $(n - m)$-dimensional smooth manifold.

In our case, $F = \mathcal{C}$, $n = N$, $m = 6$. Assumption ensures J_{Θ_0} rank 6, thus near Θ_0, $\mathcal{S}$ is $(N - 6)$-dimensional manifold.

When $N = 6$, manifold dimension 0, thus locally discrete. Detailed verification requires checking each constraint component $\mathcal{C}_i$ sufficiently independent, i.e., gradient vectors $\nabla_\Theta \mathcal{C}_i$ linearly independent. This can be verified by explicit computation or perturbation analysis around physical parameters.

4.2   Unified Time Scale and Six Constraints

Each constraint ultimately traces back to unified time scale $\kappa(\omega; \Theta)$ or internal structure parameters of Θ:

1. **Black Hole Entropy**: Microscopic counting through causal diamond chain on QCA lattice, each diamond entropy related to boundary degrees of freedom and $\kappa(\omega; \Theta)$ induced effective "boundary time". Total entropy satisfies

$$S_{\text{micro}}(\Theta) = \int_{\partial \Diamond} \kappa(\omega; \Theta)\, \rho_{\text{boundary}}(\omega)\, \mathrm{d}\omega,$$

   matching Bekenstein–Hawking formula requires specific relationship between κ and G_{eff}.

2. **Cosmological Constant**: Vacuum contribution from QCA zero-point energy depends on $\kappa(\omega; \Theta)$ frequency cutoff and integration range. Natural cancellation mechanism may emerge through special spectral structure of κ.

3. **Neutrino Mass**: From Θ internal fermion mass matrix and Yukawa texture determining neutrino sector effective Lagrangian. PMNS matrix eigenvalues and mixing angles computable from this effective Lagrangian.

4. **ETH**: QCA spectrum statistical properties controlled by $\kappa(\omega; \Theta)$ induced effective Hamiltonian spectrum. ETH validity related to level spacing distribution and eigenstate thermalization measure.

5. **Strong CP**: QCD theta angle obtained from Θ topological sector label and internal Yukawa phase structure. Natural smallness may come from special symmetry or topological cancellation mechanism.

6. **Gravitational Wave Dispersion**: Long-wavelength gravitational wave effective dispersion relation determined by $\kappa(\omega; \Theta)$ low-frequency behavior. Negligible dispersion requires κ approximately constant in observation band.

5 Applications and Model-Building Strategy

This section discusses how to use unified constraint framework to guide concrete QCA/matrix universe model construction.

5.1 Systematic Model-Building Target

Unified constraint framework defines clear model-building target:

Construct explicit QCA/matrix universe realization $\mathfrak{U}(\Theta)$ such that:

1. Parameter space dimension N not too large (ideally $N \lesssim 10$);
2. Six constraint map $\mathcal{C}(\Theta)$ explicitly computable;
3. Solution set $\mathcal{S} = \{\mathcal{C}(\Theta) = 0\}$ nonempty and contains physically reasonable points;
4. At these points, all low-energy observables $\mathcal{O}(\Theta)$ consistent with observations.

This transforms traditional "solve each problem separately" approach into "find joint solution in unified parameter space" task.

5.2 Testable Cross-Predictions

Unified constraint framework yields testable cross-sector predictions:

1. If future neutrino experiments update PMNS matrix parameters, through $\mathcal{C}_\nu(\Theta) = 0$ and $\mathcal{C}_\Lambda(\Theta) = 0$ coupling can give new constraints on cosmological constant fine structure;

2. If improved gravitational wave observations constrain dispersion, through $\mathcal{C}_{\mathrm{GW}}(\Theta) = 0$ coupling can constrain black hole microscopic entropy structure;

3. If strong CP problem solved through specific mechanism (e.g., Peccei–Quinn), through $\mathcal{C}_{\mathrm{CP}}(\Theta) = 0$ can reversely constrain neutrino mass matrix structure.

These predictions can serve as consistency checks and potential experimental falsification paths.

6 Discussion and Outlook

This work embeds six major open problems in fundamental physics into single finite-dimensional parameter space Θ unified constraint framework. Through unified time scale master formula and QCA/matrix universe characterization, originally seemingly independent six difficulties rewritten as six scalar constraint equations, forming well-defined constraint map $\mathcal{C} : \mathcal{P} \to \mathbb{R}^6$.

Mathematical analysis shows under reasonable assumptions, solution set $\mathcal{S} = \{\mathcal{C}(\Theta) = 0\}$ is low-dimensional (possibly even discrete) submanifold. Combined with discrete parameter component requirements, this realizes "universe as strongly constrained point in parameter space" concept.

Physical significance lies in: this framework couples traditionally separated sectors. Black hole thermodynamics and gravitational wave propagation, cosmological constant and quantum thermalization, neutrino physics and strong CP problem, all linked through unified parameter vector Θ. This opens possibility for cross-sector predictions and consistency checks.

Future work directions include:

1. Construct concrete QCA/matrix universe models satisfying all six constraints;

2. Develop numerical methods to search solution set $\mathcal{S}$;

3. Refine each constraint functional to incorporate latest observational data;

4. Explore extensions of framework to other open problems (e.g., dark matter, baryogenesis).

A Implicit Function Theorem and Solution Manifold

This appendix gives detailed proof of Theorem **??**.

A.1 Standard Implicit Function Theorem

Theorem A.1 (Implicit Function Theorem)*. Let $F : \mathbb{R}^n \to \mathbb{R}^m$ be continuously differentiable map, $F(x_0) = 0$. If Jacobian matrix $DF(x_0)$ has rank m, then exists neighborhood U of x_0 such that $F^{-1}(0) \cap U$ is $(n-m)$-dimensional smooth embedded submanifold of $\mathbb{R}^n$.*

A.2 Application to Constraint Map

In our case:

- $n = N$ (parameter space dimension)

- $m = 6$ (number of constraints)

- $F = \mathcal{C}$ (constraint map)

- $x_0 = \Theta_0$ (physical parameter point)

Jacobian matrix

$$
J_{\Theta_0} = \begin{pmatrix} \frac{\partial \mathcal{C}_{\mathrm{BH}}}{\partial \Theta^1} & \cdots & \frac{\partial \mathcal{C}_{\mathrm{BH}}}{\partial \Theta^N} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathcal{C}_{\mathrm{GW}}}{\partial \Theta^1} & \cdots & \frac{\partial \mathcal{C}_{\mathrm{GW}}}{\partial \Theta^N} \end{pmatrix}_{\Theta = \Theta_0}
$$

Assuming J_{Θ_0} rank 6 means six constraint equations locally independent, thus determining 6 degrees of freedom, leaving $N - 6$ free parameters.

When $N = 6$, all degrees of freedom constrained, solution set locally discrete.

B Coupling Between Unified Time Scale and Six Constraints

This appendix explains in detail how each constraint connects to unified time scale $\kappa(\omega; \Theta)$.

B.1 Black Hole Entropy and Boundary Time

From QCA causal diamond perspective, black hole horizon viewable as boundary of large causal diamond. Boundary degrees of freedom counting related to

$$
\int_{\partial \Diamond} \kappa(\omega; \Theta) \, f(\omega) \, \mathrm{d}\omega,
$$

where $f(\omega)$ mode density function. Bekenstein–Hawking area law

$$
S_{\mathrm{BH}} = \frac{A}{4G}
$$

requires specific form of κ ensuring proportionality between boundary integral and area.

B.2 Cosmological Constant and Vacuum Energy

QCA vacuum energy from zero-point contribution of all modes:

$$\rho_{\text{vac}}(\Theta) = \int_0^{\omega_{\text{cut}}} \omega \, \kappa(\omega; \Theta) \, \mathrm{d}\omega.$$

Natural smallness of cosmological constant requires $\kappa(\omega; \Theta)$ possesses special cancellation structure in low-frequency region, possibly through symmetry or topological constraint.

B.3 Neutrino Mass and Internal Structure

Neutrino mass matrix from Θ internal fermion sector parameters. While not directly involving $\kappa(\omega; \Theta)$, through unified framework couples to other constraints, especially strong CP constraint shares Yukawa phase structure.