Scale-Gauged Cosmological Observation Theory

— Rigorous Equivalence of "Expansion ≡ Resolution Enhancement", Axiomatic Readout, Relativistic Reformulation, Information Boundary, and Turing Semantics

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Abstract

Abstract: We establish a cosmological observation theory centered on the duality between the scale gauge (denoted $a(t)$) and the internal observational metric (c-lock, denoted $R(t)$). Setting $a(t) = R(t)^{-1}$ and $\kappa(t) := \dot{a}/a = -\dot{R}/R$, cosmological redshift and time dilation unify as Mellin dilation on the energy axis: $1 + z = a(t_0)/a(t_e) = R(t_e)/R(t_0)$, $\nu_0 = \nu_e/(1 + z)$, $\Delta t_0 = (1 + z)\Delta t_e$. All readouts are aligned on the "mother scale":

$$\boxed{\rho(E) = -\xi'(E) = \frac{1}{2\pi}\,\mathrm{tr}\,\mathsf{Q}(E) = \frac{1}{2\pi i}\,\partial_E \log\det S(E) = \frac{\varphi'(E)}{2\pi}\,, \quad \mathsf{Q} := -i\,S^\dagger S'},$$

where ρ is the relative density of states, Q is the Wigner–Smith group delay matrix, and φ is the total scattering phase. The mother scale is equivalently characterized by the Birman–Kreĭn formula and the group delay definition, providing cross-device comparable calibration unity (fixed units; dimension E^{-1}). Readout errors obey "Nyquist–Poisson–Euler–Maclaurin (NPE) finite-order closure": Nyquist cutoff eliminates aliasing, Poisson summation bridges discrete–continuous, and finite-order Euler–Maclaurin (EM) encapsulates endpoint errors with Bernoulli layers and tail bounds. Under Landau density threshold and Wexler–Raz dual frame conditions, window shrinking $r \downarrow$ maintains **non-increasing singularity**; whereas the monotonicity and scaling of Fisher information with respect to r **depend on noise model and normalization choice**, and no universal monotonicity or r^{-2} lower bound exists in general. The linear stabilizer preserving "light cone + mother scale" uniquely corresponds to the Lorentz group; in the FRW background, the unified frequency shift law $1 + z = ((k_\mu u^\mu)_e)/((k_\mu u^\mu)_o)$ naturally yields Etherington's distance duality $D_L = (1 + z)^2 D_A$ and Tolman's surface brightness $(1 + z)^{-4}$. Via reversible causal automaton (RCA/QCA) semantics, we provide a unified formulation of "light", "redshift", "shared master frequency", and "c-limited allocation of attention/action resources". This paper distinguishes "gauge" from "true dynamics" and proposes falsifiability criteria and an engineering "resolution allocation matrix" approach.

1 Notation, Axioms, and Conventions

1.1 Observational Objects and Scattering Geometry

1. Background Hilbert space $\mathcal{H}$; energy parameter $E \in \mathbb{R}$.
2. Scattering matrix $S(E)$ and Wigner–Smith matrix $\mathsf{Q}(E) = -i\,S^\dagger(E)\,\partial_E S(E)$.
3. Total scattering phase $\varphi(E) = \arg\det S(E)$.
4. **Trinity mother scale**:

$$\rho(E) := -\xi'(E) = \frac{1}{2\pi}\,\mathrm{tr}\,\mathsf{Q}(E) = \frac{1}{2\pi i}\partial_E \log\det S(E) = \frac{\varphi'(E)}{2\pi}\,,$$

where the equivalence follows from the Birman–Kreĭn formula $\det S(E) = e^{-2\pi i\,\xi(E)}$ (with $\xi(E)$ the spectral shift function) and the group delay definition; thus $\rho(E) = -\xi'(E)$ holds. This relation holds in generalized scattering and geometric settings [1].

1.2 Scale Gauge / c-lock

The external scale factor $a(t)$ and the internal observational metric $R(t)$ satisfy

$$a(t) = R(t)^{-1}, \qquad \kappa(t) := \frac{\dot{a}}{a} = -\frac{\dot{R}}{R}.$$

Let the internal master frequency be $f_{\text{clk}}(t) := \frac{c}{\ell_{\text{clk}}(t)} = \frac{c}{R(t)\,\ell_*}$, where ℓ_* is a fixed mother scale length constant and $R(t)$ is the dimensionless internal metric satisfying $a(t) = R(t)^{-1}$. Any "resolution enhancement" operation refers to $r \downarrow$ or sampling density $\text{dens}(\Lambda) \uparrow$.

1.3 NPE Finite-Order Closure (Non-Asymptotic)

- **Nyquist:** For band-limited targets, aliasing is zero when sampling rate exceeds twice the bandwidth; for non-band-limited cases, aliasing terms are explicitly accounted for [2].
- **Poisson:** Discrete–continuous bridging via Poisson summation, allowing lattice sums to switch to frequency-domain comb spectra [3].
- **Euler–Maclaurin (finite-order):** Endpoint layers and tail bounds given by Bernoulli polynomials, with truncation order p fixed and error bounds auditable [4].

Error notation definition: Denote by $\varepsilon_{\text{alias}}(r) \geq 0$ the L^1-upper bound of **aliasing terms** introduced via Poisson summation; when the Nyquist condition is satisfied, $\varepsilon_{\text{alias}}(r) = 0$. Denote by $\varepsilon_{\text{EM}}(\delta; r; p) \geq 0$ the "endpoint layer + tail" budget after truncating the Euler–Maclaurin formula at fixed order p (δ is the sampling step/equivalent grid spacing). There exists a constant $C_{2p}(r)$ such that

$$\varepsilon_{\text{EM}}(\delta; r; p) \leq C_{2p}(r)\,\delta^{2p}.$$

In general, **no universal power-law relation** with r is claimed; if further regularity such as $w \in W^{2p,1}$, $h * \rho \in L^1_{\text{loc}}$ is given, one may derive $C_{2p}(r) \lesssim r^{-2p}$, whence $\varepsilon_{\text{EM}}(\delta; r; p) = O(\delta^{2p} r^{-2p})$ as a **model-dependent** conclusion. This order bound requires assuming $h * \rho$ is piecewise C^{2p} on the thickening $K^{\uparrow}$ (where $K^{\uparrow} := K + \text{supp}(w_r * h)$) of the working compact domain K with bounded derivatives; **or** under weaker BV assumptions, all jump contributions are incorporated into the Bernoulli endpoint layers before estimation (yielding only a BV-version bound, **not equivalent to piecewise** C^{2p}). Beyond this regularity regime, this paper does not claim such order.

1.4 Frame and Density Threshold

We adopt the Gabor/Weyl–Heisenberg framework: for window w and lattice Λ, we require Landau necessary density and Wexler–Raz duality to ensure stable invertible reconstruction [5].

2 Expansion $\equiv$ Resolution Enhancement: Mellin Dilation and Unified Frequency Shift

Definition 2.1 (Scale Gauge)**.** *A choice (a, R) satisfying $a(t) = R(t)^{-1}$ is called a **scale gauge**. Under this gauge, external "expansion" and internal "resolution enhancement" are rigorously equivalent to Mellin dilation on the energy axis.*

Theorem 2.2 (Redshift–Dilation Equivalence)**.** *For the same photon observed at emission t_e and observation t_0, we have*

$$1+z=\frac{a(t_0)}{a(t_e)}=\frac{R(t_e)}{R(t_0)},\qquad \nu_0=\frac{\nu_e}{1+z},\qquad \Delta t_0=(1+z)\Delta t_e.$$

Proof sketch. In the FRW metric, the frequency is $\omega=-k_\mu u^\mu$, hence $1+z=((k_\mu u^\mu)_e)/((k_\mu u^\mu)_o)$; parallel transport of k^μ along null geodesics yields $\omega\propto a^{-1}$. Substituting $a=R^{-1}$ completes the proof [6]. □

Proposition 2.3 (Etherington Duality and Tolman Decay)**.** *If photon number is conserved, geometry is described by metric gravity, and light follows unique null geodesics, then*

$$D_L=(1+z)^2D_A,\qquad I_{\rm obs}=\frac{I_{\rm em}}{(1+z)^4}.$$

This conclusion is independent of the choice of scale gauge and is a geometric–counting invariant [7].

3 Relativistic Windowed Reformulation: Stabilizer of Light Cone + Mother Scale

Theorem 3.1 (Lorentz Group = Stabilizer)**.** *The group of* ***linear*** *transformations preserving the Minkowski light cone structure and the mother scale is isomorphic to $SO^+(1,3)$.*

Argument. After fixing the origin (removing translational freedom), the group of linear automorphisms preserving causal order is generated by $\mathbb{R}_+\times SO^+(1,3)$; requiring mother scale invariance (excluding global dilation) leaves only $SO^+(1,3)$. This is consistent with Alexandrov–Zeeman-type theorems [8]. □

Proposition 3.2 (GR Local Covariantization and Unified Frequency Shift)**.** *By locally flattening "light cone + mother scale" at each point of the manifold, the unified frequency shift law*

$$1+z=\frac{(k_\mu u^\mu)_e}{(k_\mu u^\mu)_o},$$

is compatible with the stationary-phase condition of geodesic equations and consistent with standard SR/GR kinematics [9].

4 Essence of Resolution Enhancement: Information Geometry and Singularity Conservation

Let the **normalized window** $w_r(x):=\frac{1}{r}w(x/r)$, where $w\ge 0$, $w\in W^{1,1}(\mathbb{R})$, $\int_{\mathbb{R}}w=1$ (optionally: $\int x\,w(x)\,dx=0$), convolution kernel h, and observable

$$g_r(E)=(w_r*h*\rho)(E).$$

Proposition 4.1 (Scale Bound and Convergence of Gradient Response)**.** *Let $w\ge 0$, $w\in W^{1,1}(\mathbb{R})$, $\int w=1$, and $h*\rho\in L^1_{\rm loc}$* ***(or** *BV**)**, with $g_r=w_r*h*\rho$. For compact domain K,*

$$|\partial_E g_r|_{L^1(K)}\le\frac{|w'|_{L^1}}{r}\,|h*\rho|_{L^1(K^\uparrow)}.$$

Convergence by cases:

(i) If $h * \rho \in W^{1,1}(K^{\uparrow})$ and $w \geq 0$, $\int w = 1$, then

$$\lim_{r \downarrow 0} |\partial_E g_r - (h * \rho)'|_{L^1(K)} = 0, \qquad |\partial_E g_r|_{L^1(K)} \leq |(h * \rho)'|_{L^1(K^{\uparrow})}.$$

(ii) If $h * \rho \in BV(K^{\uparrow})$ (not necessarily in $W^{1,1}$), then

$$g_r \xrightarrow[r \downarrow 0]{} h * \rho \quad \text{in } L^1(K), \qquad |\partial_E g_r|_{L^1(K)} \leq \mathrm{TV}(h * \rho; K^{\uparrow}),$$

and $\partial_E g_r \overset{*}{\rightharpoonup} D(h * \rho)$ in the weak sense in measure space. We do not claim convergence of $|\partial_E g_r - (h * \rho)'|_{L^1}$ in this case.*

NPE estimator version *(for discrete implementation $\widehat{g}_r$):*

$$|\partial_E \widehat{g}_r|_{L^1(K)} \leq \frac{|w'|_{L^1}}{r} |h * \rho|_{L^1(K^{\uparrow})} + \varepsilon_{\mathrm{alias}}(r) + \varepsilon_{\mathrm{EM}}(\delta; r; p).$$

If the Nyquist condition is satisfied, $\varepsilon_{\mathrm{alias}}(r) = 0$, leaving only the EM endpoint–tail budget.

The above shows: reducing r improves edge **approximation**, but does not produce a universal $1/r$ **lower bound** growth. Here $K^{\uparrow} := K + \mathrm{supp}(w_r * h)$ denotes the thickening of the compact domain K by the effective support of the convolution kernel [4].

Proposition 4.2 (Model Dependence of Fisher Information). *Under the premise that Nyquist and NPE error budgets ($\varepsilon_{\mathrm{alias}}$, $\varepsilon_{\mathrm{EM}}$) are auditable, the monotonicity and scaling of $\mathcal{I}_r(\theta)$ with respect to r **depend on noise model and normalization choice**; in general, **no universal** r^{-2} **lower bound or monotonicity conclusion exists**. Once noise statistics (e.g., AWGN/Poisson) and window normalization (e.g., $\int w = 1$ or $|w_r|_2$ fixed) are specified, one may derive the corresponding r-scaling and comparison results [2].*

Theorem 4.3 (Non-Increasing Singularity). *Legitimate window switching ($w \mapsto w_r$ with fixed-order EM budget) corresponds to smoothing that does not **introduce new** singularities of $h * \rho$; thus under alias control and auditable EM error, resolution enhancement does not "manufacture spurious peaks". The **location and order** of singularities may be affected by smoothing; this paper makes no invariance claims [3].*

5 Stable Reconstruction and Frame Threshold

Theorem 5.1 (Landau Necessary Density). *Stable sampling of band-limited Paley–Wiener-type spaces requires lower Beurling density not less than the bandwidth volume constant; if insufficient, reconstruction condition number explodes [5].*

Theorem 5.2 (Wexler–Raz Duality and Tight Frames). *For Gabor systems, the Wexler–Raz biorthogonality relation characterizes the orthogonality condition of dual windows; there exists a parameter regime where tight frames hold, making reconstruction robust. Multi-window fusion reduces estimation variance from σ^2 to approximately σ^2/K under statistical independence approximation [10].*

Remark 5.3 (Balian–Low Barrier). *Orthonormal bases at critical density cannot simultaneously achieve good time-frequency localization (Balian–Low), suggesting the need for redundant frames rather than critical bases [11].*

6 Gauge vs. True Dynamics: Falsifiability Fingerprints

Define cosmological **state fingerprints**: deceleration $q := -\ddot{a}a/\dot{a}^2$, jerk $j := \frac{d^3a/dt^3}{aH^3}$. Define

$$\eta(z) := \frac{D_L}{(1+z)^2 D_A}.$$

Criterion: If $\eta(z) \equiv 1$, and under NPE budget closure and mother scale invariance there are **no new singularities/spurious peaks**, then it belongs to **gauge layer consistency**; if $\eta(z) \neq 1$ or **new singularities/spurious peaks** appear, it points to **true dynamics/new physics** (such as optical depth, non-metric effects, or photon non-conservation) [12].

7 RCA/QCA Semantics: Light Cone, Redshift, and c-Limited Allocation

Definition 7.1 (Causal Cone and "Light")**.** *Local reversible update lattice dynamics satisfying Lieb–Robinson bounds induce an effective "light cone"; the minimal notation flow saturating this bound is called "light" [13].*

Proposition 7.2 (Discrete Formulation of Redshift)**.** *Timing with master frequency $f_{\rm clk}(t) = \frac{c}{R(t)\,\ell_*}$, the discrete period of the same symbol stream observed satisfies*

$$P_0 = (1+z)P_e, \qquad \nu_0 = \nu_e/(1+z),$$

i.e., cosmological redshift's discrete time dilation, consistent with the continuous formulation. (Direct discretization of the unified frequency shift law from §2.2.)

Proposition 7.3 (c-Limited Allocation of Attention/Action)**.** *Let resource density–flux pair (ρ, J) satisfy conservation $\partial_t \rho + \nabla \cdot J = s$ and constraint $|J| \leq c\,\rho$; then influence can propagate within the causal cone only at speeds not exceeding c; this "scheduling light speed" is consistent with the Lieb–Robinson velocity [14].*

8 Information Boundary and Velocity Limit

Proposition 8.1 (Processing Rate Upper Bound: Quantum Speed Limit)**.** *The Mandelstam–Tamm and Margolus–Levitin bounds give the shortest evolution time and maximum state change rate; thus under given energy/power budget, any "resolution enhancement–processing rate" is limited by them, not relaxed by scale gauge choice [15].*

9 Operational Protocol for Observation–Reconstruction–Duality Consistency

Protocol A (Mother Scale Triple Closure): For the same object, compute simultaneously $\varphi'(E)/(2\pi)$, $(2\pi)^{-1}\operatorname{tr}\mathsf{Q}(E)$, $\rho(E)$, requiring curve and directional pole consistency to verify calibration unity and Birman–Kreĭn–Wigner–Smith mutual verification [1].

Protocol B (NPE Budget): For each data pipeline, report "alias $= 0/\neq 0$, EM order p, tail bound"; under Nyquist satisfaction and specified noise/normalization, $r \downarrow$ **reduces bias** but **variance typically increases** (bandwidth optimization needed); **no new singularities, no spurious peaks** guaranteed by §4.3 "non-increasing singularity" and alias/EM budget [2].

Protocol C (Geometric Duality Check): Construct $\eta(z) = D_L/[(1+z)^2 D_A]$ and perform Tolman exponent regression (expecting $n = 4$) as "gauge vs. dynamics" consistency evidence [7].

Protocol D (Resolution Allocation Matrix): In time/frequency/angle/scale–phase coordinates, take

$$M^\star = \arg \max_{M\succeq 0,\, \operatorname{tr} M=\chi} \big\langle M,\, \nabla_{\mathbf{r}}\mathcal{I}\, \nabla_{\mathbf{r}}\mathcal{I}^\top \big\rangle,$$

where $\chi > 0$ is a fixed resource budget constant (independent of $\kappa(t) = \dot{a}/a$), $\mathbf{r} = (t, \omega, \vartheta, s)$ collects time/frequency/angle/scale–phase coordinates (tailorable by task). Report Fisher information gain and condition number improvement.

10 Minimal Sufficiency of the Theory

1. **Scale gauge $a = R^{-1}$:** Unifies "external expansion" and "internal resolution enhancement" as the same Mellin dilation, without changing intrinsic singularities.
2. **Mother scale calibration:** Unifies readout via $\rho = -\xi' = \frac{1}{2\pi}\operatorname{tr}\mathsf{Q} = \frac{1}{2\pi i}\partial_E \log\det S = \frac{\varphi'}{2\pi}$, cross-device comparable [1].
3. **NPE finite-order closure:** Closes error budget with Poisson–EM finite-order discipline; Nyquist eliminates aliasing [3].
4. **Frame threshold:** Landau necessary density and Wexler–Raz duality ensure stable invertible reconstruction [16].
5. **Relativistic consistency:** Stabilizer of light cone + mother scale yields Lorentz group; in FRW, unified frequency shift law, Etherington, and Tolman naturally hold [8].

11 Appendix: Correspondence of Standard Results with This Paper's Structure

- **Wigner–Smith group delay and "density–phase derivative" triple equivalence:** Group delay matrix definition and experimental measurability, and the Birman–Kreĭn relation between $\det S$ and spectral shift function, support mother scale calibration [17].
- **Covariant formulation of redshift:** $\omega = -k_\mu u^\mu$ and $1 + z = ((k_\mu u^\mu)_e)/((k_\mu u^\mu)_o)$; standard composition and duality of cosmological distance measures [9].
- **Tolman $(1 + z)^{-4}$ and duality test:** Observational calibration and methodological guidance [18].
- **Alexandrov–Zeeman theorem:** Causal structure determines (up to global dilation) Lorentz–Poincaré group; removing global dilation yields Lorentz group [8].
- **Landau density, Wexler–Raz, Balian–Low:** Three-point balance of stable sampling–duality–impossibility of simultaneous localization [5].
- **Lieb–Robinson and QCA:** Effective "light speed" on lattice and causal cone of reversible updates [13].
- **Quantum speed limit:** Resolution enhancement and processing rate uniformly constrained by MT/ML-type bounds [15].

Proof Appendix (Selected)

A. Birman–Kreĭn–Group Delay–Phase Derivative Trinity

Let $S(E)$ be a unitary scattering matrix. By Smith's definition $\mathsf{Q}(E) = -i\, S^\dagger S'$,

$$\operatorname{tr}\mathsf{Q} = -i\ \operatorname{tr}(S^\dagger S') = -i\,\partial_E \log\det S\,,$$

using $\partial_E \log\det S = \operatorname{tr}(S^{-1}S') = \operatorname{tr}(S^\dagger S')$ (since S is unitary). By $\det S(E) = e^{-2\pi i \xi(E)}$ and $\operatorname{tr}\mathsf{Q} = -i\,\partial_E \log\det S$, we get $\frac{1}{2\pi}\operatorname{tr}\mathsf{Q} = -\xi'(E)$. Also $\varphi(E) = \arg\det S(E) = -2\pi\xi(E)$, hence $\varphi'(E) = -2\pi\xi'(E)$. Thus

$$\rho(E) = -\xi'(E) = \frac{1}{2\pi}\operatorname{tr}\mathsf{Q}(E) = \frac{1}{2\pi i}\partial_E \log\det S(E) = \frac{\varphi'(E)}{2\pi}\,.$$

[1]

B. Geometric Origin of Etherington and Tolman

Under unique null geodesics, photon number conservation, and metric gravity, transformation of angular area element and intrinsic luminosity yields $D_L = (1+z)^2 D_A$; combining the $(1+z)^{-1} \times (1+z)^{-1}$ factor of photon energy/arrival rate per unit time–unit area flux with the $(1+z)^{-2}$ scaling of visual angle area, we obtain Tolman surface brightness decay $(1+z)^{-4}$ [7].

C. Alexandrov–Zeeman Stabilizer to Lorentz Group

After fixing the origin, linear maps preserving causal order are generated by $\mathbb{R}_+ \times SO^+(1,3)$; invoking mother scale invariance removes global dilation, leaving $SO^+(1,3)$ [8].

D. Landau–Wexler–Raz–Balian–Low Frame Triangle

Landau lower bound gives necessary sampling point density; Wexler–Raz characterizes dual windows making reconstruction operator identity; Balian–Low declares orthonormal bases at critical density cannot be simultaneously well-localized, hence engineering uses redundant tight frames [16].

Tooling Definitions and Symbol Index

- $a(t)$: scale factor; $R(t) = a(t)^{-1}$: internal metric; $\kappa = \dot a/a$.
- $S(E)$, $\mathsf{Q}(E)$, $\varphi(E)$, $\rho(E)$: trinity mother scale objects [1].
- NPE: Nyquist (aliasing account/cutoff)–Poisson (summation bridge)–Euler–Maclaurin (finite-order Bernoulli layers and tail) [2].
- Frame density and duality: Landau necessary density, Wexler–Raz duality, Balian–Low restriction [5].
- Unified frequency shift: $1 + z = ((k_\mu u^\mu)_e)/((k_\mu u^\mu)_o)$ [9].

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