Quantum–Classical Unification Theory Within the WSIG–EBOC–RCA Framework

Haobo Ma[1]
Wenlin Zhang[2]
[1]Independent Researcher
[2]National University of Singapore

(Windowed Scattering & Information Geometry · Eternal-Block Observer-Computing · Reversible Cellular A

November 19, 2025
Version: 1.15 (2025-11-02, Asia/Dubai)

Abstract

Using the trinity of phase–relative state density–group delay from Windowed Scattering–Information Geometry as the unique master scale for the energy axis, this paper establishes isomorphic semantics between static block geometry and reversible cellular automata, achieving quantum–classical unification. For a scattering pair (H, H_0), the multi-port scattering matrix $S(E)$ on the absolutely continuous spectrum of H_0 acts on the open channel subspace at energy E with $S(E) \in U(N(E))$. The Wigner–Smith delay matrix is defined as $\mathsf{Q}(E) := -i\, S(E)^\dagger \partial_E S(E)$, the ac density of relative state density as $\rho_{\mathrm{rel}}(E) := \xi'_{\mathrm{ac}}(E)$, and the half-determinant phase as $\varphi(E) := \pi\, \xi_{\mathrm{ac}}(E)$ (also written as $\frac{1}{2}\,\mathrm{Arg}_{\mathrm{ac}} \det S(E)$). At almost every Lebesgue point of the absolutely continuous spectrum of H_0, we have

$$\boxed{\varphi'(E) = \tfrac{1}{2}\ \mathrm{tr}\,\mathsf{Q}(E) = \pi\, \rho_{\mathrm{rel}}(E)}.$$

The semiclassical limit is closed to classical Hamiltonian flow, Poisson brackets, and Liouville dynamics through Egorov's theorem, Moyal deformation, and Wigner measure propagation. Readouts employ the Nyquist–Poisson–Euler–Maclaurin (NPE) error ledger to provide non-asymptotic upper bounds, with a variational optimality framework for single/multi-window–multi-kernel configurations (details in §6). The constants $(c, \hbar, e, G, k_{\mathrm{B}})$ achieve metrological correspondence within this system: c is calibrated by front support and group delay; $\hbar$ is the Weyl–Heisenberg central charge scale; e is anchored by the magnetic flux quantum; G is realized through curvature–energy flow correspondence; k_{B} is fixed by SI constantization. These structures possess realizable isomorphic semantics under EBOC's causal block universe and RCA's discrete light cone.

Keywords: Quantum-classical unification; WSIG; EBOC; RCA; Wigner-Smith delay; Semiclassical limit; Egorov theorem; Birman-Kreĭn formula
MSC 2020: 81Q20; 81S30; 47A40; 37N20; 35Q40

Contents

1 Axioms and Objects

A1 (Causal–Front): The world is a causal block (M, g). The null cone $ds^2 = 0$ determines front velocity c. In linear time-invariant (LTI) systems, if the impulse response $h(t)$ is supported on $t \geq 0$ and $h \in L^1(\mathbb{R})$ (or more generally is a tempered distribution with frequency response $H(\omega)$ in the Hardy class), then $H(\omega)$ is analytic in the upper half-plane and satisfies Kramers–Kronig; conversely, if H is bounded analytic in the upper half-plane with appropriate growth/decay, the corresponding h is causal. For time-varying systems, causality is stated via support of the retarded Green's function.

A2 (Master Scale–WSIG): Define

$$\mathsf{Q}(E) := -i\, S(E)^\dagger \partial_E S(E), \qquad \tau_{\mathrm{WS}}(E) := \hbar\, \mathrm{tr}\, \mathsf{Q}(E).$$

Further define the **per-port average group delay** for multi-port cases as

$$\overline{\tau}(E) := \frac{\hbar}{N(E)}\, \mathrm{tr}\, \mathsf{Q}(E), \qquad N(E) := \text{number of open channels (at that } E\text{)}.$$

Unless otherwise specified, trace and average are taken over the current open channel subspace; far from channel thresholds, $N(E)$ is constant.

Let the **scattering pair** (H, H_0) satisfy definable wave operators, with multi-port scattering matrix $S(E)$ on the absolutely continuous spectrum of H_0; for each energy E, $S(E)$ acts on the open channel subspace with $S(E) \in U(N(E))$.

BK Condition (ensuring applicability of spectral shift and determinant phase): Assume (H, H_0) forms a trace-class perturbation pair in the Birman–Kreĭn sense, e.g.,

$$(H - i)^{-1} - (H_0 - i)^{-1} \in \mathfrak{S}_1,$$

then the Kreĭn spectral shift function ξ exists, and at almost every Lebesgue point $E \in \sigma_{\mathrm{ac}}(H_0)$,

$$\det S(E) = \exp\big(2\pi i\, \xi(E)\big), \qquad -i\, \partial_E \log \det S(E) = 2\pi\, \xi'_{\mathrm{ac}}(E).$$

Denote the **ac density of relative state density** as

$$\rho_{\mathrm{rel}}(E) := \xi'_{\mathrm{ac}}(E).$$

To avoid multivaluedness ambiguity in Arg, define

$$\varphi(E) := \pi\, \xi_{\mathrm{ac}}(E) \quad \big(\text{also written as } \tfrac{1}{2}\, \mathrm{Arg}_{\mathrm{ac}} \det S(E)\big).$$

Then at almost every Lebesgue point of the absolutely continuous spectrum of H_0,

$$\mathrm{tr}\, \mathsf{Q}(E) = 2\pi\, \rho_{\mathrm{rel}}(E), \qquad \varphi'(E) = \tfrac{1}{2}\, \mathrm{tr}\, \mathsf{Q}(E) = \pi\, \rho_{\mathrm{rel}}(E).$$

Regularity and domain: The following equations involving $\partial_E S(E)$ and $\mathsf{Q}(E)$ are understood on $E \in \sigma_{\mathrm{ac}}(H_0)$ where $S(E)$ is locally absolutely continuous (or has weak derivative) with respect to E. This paper defaults to working energy windows far from channel thresholds and branch points; when discussing across thresholds, replace with traces/derivatives on the $N(E)$-dependent open channel subspace.

A3 (Bridge Constants): $\hbar$ is the central parameter of Weyl–Heisenberg; c is realized via front support and group delay metrological correspondence; e is anchored by the magnetic flux quantum; G is realized through curvature–energy flow correspondence; k_{B} is fixed by SI constantization.

A4 (Readout–Error): Any readout is a "window × kernel" weighted average, subject to NPE three-term error closure: aliasing/Poisson, finite-order Euler–Maclaurin remainder, bandwidth tail term.

A5 (Realizability–RCA, Reversibility Criterion): The influence domain of a radius-r reversible cellular automaton (RCA) is a discrete light cone. The necessary and sufficient criterion for reversibility is: the global map is bijective and its inverse is also a cellular automaton.

2 Trinity Master Scale and Main Theorem

2.1 Definition and Notation

Let the scattering pair (H, H_0) give $S(E)$ on the absolutely continuous spectrum of H_0 (acting on the open channel subspace at E), with $S(E) \in U(N(E))$. Take

$$\mathsf{Q}(E) := -i\, S^\dagger \partial_E S, \qquad \tau_{\mathrm{WS}}(E) := \hbar \operatorname{tr} \mathsf{Q}(E), \qquad \rho_{\mathrm{rel}}(E) := \xi'_{\mathrm{ac}}(E),$$

$$\det S(E) = \exp\big(2\pi i\, \xi(E)\big), \qquad -i\, \partial_E \log\det S(E) = 2\pi\, \xi'_{\mathrm{ac}}(E) \text{ (a.e. on } \sigma_{\mathrm{ac}}(H_0)\text{, under BK condition).}$$

2.2 Main Theorem (Trinity)

Theorem 2.1 (Trinity Identity)**.**

$$\boxed{\varphi'(E) = \tfrac{1}{2} \operatorname{tr} \mathsf{Q}(E) = \pi\, \rho_{\mathrm{rel}}(E)}$$

holds at almost every Lebesgue point of the absolutely continuous spectrum of H_0.

Proof. From $\det S(E) = \exp\big(2\pi i\, \xi(E)\big)$ (BK condition), at almost every Lebesgue point $E \in \sigma_{\mathrm{ac}}(H_0)$,

$$-i\, \partial_E \log\det S(E) = 2\pi\, \xi'_{\mathrm{ac}}(E).$$

Unitarity gives $\partial_E \log\det S(E) = \operatorname{tr}\big(S^\dagger \partial_E S\big)$. Thus on $E \in \sigma_{\mathrm{ac}}(H_0)$,

$$\operatorname{tr} \mathsf{Q}(E) = -i\, \operatorname{tr}\big(S^\dagger \partial_E S\big) = 2\pi\, \xi'_{\mathrm{ac}}(E) = 2\pi\, \rho_{\mathrm{rel}}(E),$$

and from the BK chain,

$$\varphi'(E) = \pi\, \xi'_{\mathrm{ac}}(E) = \tfrac{1}{2} \operatorname{tr} \mathsf{Q}(E)$$

at almost every Lebesgue point of the absolutely continuous spectrum of H_0. □

Single-channel verification: If $S(E) = e^{2i\delta(E)}$, then

$$\operatorname{tr} \mathsf{Q}(E) = 2\, \delta'(E), \qquad \rho_{\mathrm{rel}}(E) = \frac{\delta'(E)}{\pi}, \qquad \varphi'(E) = \delta'(E),$$

consistent with the Friedel relation.

3 Semiclassical Bridge: Egorov–Moyal–Wigner Measure ($\mathrm{Op}_\hbar$ Notation)

Egorov (leading order): For Weyl quantization $H = \mathrm{Op}_\hbar(p)$ and classical Hamiltonian flow Φ_t,

$$U_t^\dagger \,\mathrm{Op}_\hbar(a)\, U_t = \mathrm{Op}_\hbar\big(a \circ \Phi_t\big) + \mathcal{O}(\hbar),$$

which can extend to Ehrenfest time under appropriate regularity (with $\log(1/\hbar)$ corrections in chaotic cases).

Moyal deformation: Weyl calculus maps $\frac{i}{\hbar}[\hat{A}, \hat{B}]$ to the Moyal bracket, and

$$\{A, B\}_M = \{A, B\} + \mathcal{O}(\hbar^2).$$

Wigner measure propagation: The Wigner measure of a sequence of states propagates along the classical flow, converging to Liouville/Vlasov-type transport equations.

4 EBOC: Front Support, KK Causality, and Metrological Closure

Front support of retarded Green's function: For the three-dimensional wave equation, the retarded Green's function is

$$G_{\mathrm{ret}}(t, \mathbf{r}) = \frac{\delta\big(t - |\mathbf{r}|/c\big)}{4\pi\,|\mathbf{r}|},$$

with support exactly on the front $t = |\mathbf{r}|/c$. When **§1.A1 conditions are satisfied** ($h \in L^1$ or more generally tempered distribution with frequency response H in Hardy class with appropriate growth/decay), strict causality is **mutually equivalent** to upper half-plane analyticity and Kramers–Kronig dispersion relations; if these conditions fail, only directional implications hold or additional regularization is needed.

Decomposition and validity conditions for metrological closure: Suppose external links are uniform, and the free propagation phase for each open channel $n = 1, \ldots, N(E)$ can be written as diagonal factor

$$D_k(E) := \mathrm{diag}\big(e^{\,ik_n(E)L}\big), \qquad k_n(E) = \frac{E}{\hbar\, v_{p,n}(E)}.$$

Taking decomposition $S(E) = D_k(E)\,U(E)$, from $\dfrac{dk_n}{dE} = \dfrac{1}{\hbar\, v_{g,n}(E)}$ we obtain

$$\mathrm{tr}\,\mathsf{Q}(E) = \frac{L}{\hbar} \sum_{n=1}^{N(E)} \frac{1}{v_{g,n}(E)} \;-\; i\;\mathrm{tr}\big(U^\dagger \partial_E U\big).$$

If all channels satisfy $v_{g,n}(E) \approx v_g(E)$ in the given energy window (or $-i\;\mathrm{tr}(U^\dagger \partial_E U)$ has been eliminated with reference link), then

$$\overline{\tau}(E) = \frac{\hbar}{N(E)}\;\mathrm{tr}\,\mathsf{Q}(E) \;\approx\; \frac{L}{v_g(E)} \quad \big(\text{in vacuum, } v_g(E) = c\big).$$

If external links exhibit dispersion or reflection, or the scattering region contains localized states decoupled from ports, separate the continuous part and correct the above decomposition.

5 RCA: Reversibility Criterion, Discrete Light Cone, and Floquet Spectrum

On a finite alphabet and $\mathbb{Z}^d$ shift, a continuous global map commuting with shift is a cellular automaton; it is a reversible cellular automaton if and only if the global map is bijective and its inverse is also a cellular automaton. For radius-r, lattice spacing a, time step Δt, the influence domain at t steps is $\pm rt$, with discrete "speed of light" $c_{\text{disc}} = ra/\Delta t$; the continuum limit aligns with the EBOC front. The quasienergy spectrum of one-step evolution under periodic driving is given by Floquet–Sambe formalism, with energy scale $E = \hbar\omega$ compatible with group delay readouts.

6 NPE Error Ledger and Window/Kernel Optimization

Poisson–Nyquist (aliasing term): When band-limiting satisfies the Nyquist–Shannon condition, the aliasing term vanishes; for general windows, the aliasing term can be quantitatively bounded via Poisson summation.

Euler–Maclaurin (unified remainder bound): If $f \in C^{2m} \cap W^{2m,1}(\mathbb{R})$ with $f^{(k)}(\pm\infty) = 0$ $(0 \le k \le 2m-1)$, the even-order truncation remainder satisfies

$$\big|R_{2m}\big| \le \frac{2\,\zeta(2m)}{(2\pi)^{2m}} \int_{\mathbb{R}} \big|f^{(2m)}(x)\big|\,dx, \qquad m \in \mathbb{N}.$$

If decay/integrability fails, truncate with window function and incorporate boundary terms into R_{2m}, modifying this bound accordingly.

NPE readout notation and total error: Denote convolution $[\kappa\star f](E) := \int_{\mathbb{R}} \kappa(E-E')\,f(E')\,dE'$. For any window–kernel pair (w_R, κ), the readout is written

$$X_W = \frac{1}{2\pi} \int_{\mathbb{R}} w_R(E)\,[\kappa \star f](E)\,dE,$$

with composite error estimated as

$$\mathsf{Err} = \mathcal{O}(\hbar) + \mathcal{O}(\varepsilon_{\text{alias}}) + \mathcal{O}(\varepsilon_{\text{EM}}) + \mathcal{O}(\varepsilon_{\text{tail}}),$$

where $\varepsilon_{\text{alias}} = 0$ (when band-limited), ε_{EM} is bounded by the above, and $\varepsilon_{\text{tail}}$ is controlled by out-of-band mass and $|\operatorname{tr}\mathsf{Q}|_\infty$.

Multi-window–multi-kernel optimization: Under Parseval tight frames or Gabor frameworks, fit $\operatorname{tr}\mathsf{Q}$ with target kernel κ and minimize functional penalizing Err; feasibility and stability are guaranteed by biorthogonality relations and density theorems.

7 Typical Models and Unified Inferences

Free propagation link (unified formulation, including multimode): If external links are uniform and $-i\,\operatorname{tr}\big(U^\dagger\partial_E U\big) = 0$ (or this term has been canceled via reference link), then

$$\operatorname{tr}\mathsf{Q}(E) = \frac{L}{\hbar}\sum_{n=1}^{N(E)} \frac{1}{v_{g,n}(E)}, \qquad \tau_{\mathrm{WS}}(E) = L\sum_{n=1}^{N(E)} \frac{1}{v_{g,n}(E)}, \qquad \overline{\tau}(E) = \frac{1}{N(E)}\sum_{n=1}^{N(E)} \frac{L}{v_{g,n}(E)}.$$

If all channels satisfy $v_{g,n}(E) \approx v_g(E)$, this reduces to $\tau_{\mathrm{WS}}(E) \approx \dfrac{N(E)\,L}{v_g(E)}$ and $\overline{\tau}(E) \approx \dfrac{L}{v_g(E)}$. If these conditions fail, retain the correction term $-i\ \mathrm{tr}\big(U^\dagger \partial_E U\big)$.

Discrete light cone of RCA: For radius-r, lattice spacing a, time step Δt, the RCA influence domain at t steps is $\pm rt$, with discrete "speed of light" $c_{\mathrm{disc}} = ra/\Delta t$. In the continuum limit, it locally matches the group velocity of classical dispersion only in a linearized energy/wavenumber neighborhood E_0:

$$c_{\mathrm{disc}} \xrightarrow[\text{continuum limit}]{E\approx E_0} v_g(E_0),$$

with vacuum linear dispersion giving the special case $c_{\mathrm{disc}} \to c$.

Potential scattering DOS–phase–delay: If $S(E) = \mathrm{diag}\left(e^{2i\delta_j(E)}\right)$, then

$$\rho_{\mathrm{rel}}(E) = \frac{1}{\pi}\sum_j \delta_j'(E) = \frac{1}{2\pi}\,\mathrm{tr}\,\mathsf{Q}(E),$$

with delay peaks characterizing resonance lifetimes.

Quantum–classical dynamics reduction:

$$\frac{d}{dt}\langle \hat{A}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{A}]\rangle = \langle\{H,A\}\rangle + \mathcal{O}(\hbar^2),$$

with Wigner measure providing macroscopic transport limit.

8 Falsifiable Exits and Interfaces

Given multi-port $S(E)$ or port data, the trinity chain provides consistent predictions for $\varphi'(E)$, $\rho_{\mathrm{rel}}(E)$, $\mathrm{tr}\,\mathsf{Q}(E)$; any systematic deviation indicates window–kernel or model assumption mismatch. When §7 conditions ($-i\ \mathrm{tr}(U^\dagger \partial_E U) = 0$ or baseline canceled) are satisfied, multi-window regression $\overline{\tau}(E) \to L/v_g(E)$ rate is controlled by §6 bounds, experimentally verifiable. RCA prototype c_{disc}-calibration in continuum limit and linearized energy neighborhood E_0 locally matches $v_g(E_0)$; quasienergy spectrum readouts should be consistent with continuous links in that neighborhood.

9 Appendix: Technical Lemmas and Proof Outlines

9.1 BK–Kreĭn–WS Chain

Let the scattering pair (H, H_0) give $S(E)$ on the absolutely continuous spectrum of H_0 (with $S(E) \in U(N(E))$). Then

$$\det S(E) = \exp\big(2\pi i\,\xi(E)\big) \;\Rightarrow\; -i\,\partial_E \log\det S(E) = 2\pi\,\xi_{\mathrm{ac}}'(E)\ \text{(a.e. on } \sigma_{\mathrm{ac}}(H_0)),$$

$$\partial_E \log\det S(E) = \mathrm{tr}\big(S^\dagger \partial_E S\big), \qquad \mathsf{Q}(E) = -i\,S^\dagger \partial_E S, \qquad \rho_{\mathrm{rel}}(E) = \xi_{\mathrm{ac}}'(E),$$

$$\varphi(E) := \pi\,\xi_{\mathrm{ac}}(E)\ \big(= \tfrac{1}{2}\,\mathrm{Arg}_{\mathrm{ac}}\det S(E)\big), \qquad \varphi'(E) = \tfrac{1}{2}\ \mathrm{tr}\,\mathsf{Q}(E) = \pi\,\rho_{\mathrm{rel}}(E)\ \text{(a.e. on } \sigma_{\mathrm{ac}}(H_0)).$$

9.2 Egorov–NPE Composite Error

If $f_t = f \circ \Phi_t + \mathcal{O}(\hbar)$, then for

$$X_W = \frac{1}{2\pi} \int_{\mathbb{R}} w_R(E) \, [\kappa \star f_t](E) \, dE$$

we have

$$\mathsf{Err} = \mathcal{O}(\hbar) + \mathcal{O}(\varepsilon_{\text{alias}}) + \mathcal{O}(\varepsilon_{\text{EM}}) + \mathcal{O}(\varepsilon_{\text{tail}}),$$

where ε_{EM} is given by the Euler–Maclaurin bound in §6.

10 Conclusion

This paper establishes a quantum–classical unification framework across WSIG–EBOC–RCA: the trinity of phase–relative state density–group delay as the unique master scale for the energy axis; Egorov–Moyal–Wigner measure propagation implementing semiclassical reduction; NPE error ledger providing non-asymptotic verifiable bounds; front support and metrological closure calibrating bridge constants $(c, \hbar, e, G, k_{\text{B}})$. This system possesses isomorphic semantics between static block geometry and discrete reversible dynamics, providing an operational measurement–calibration chain for the quantum–classical interface.

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