

Unified Mathematical Definition of the Universe

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Core Definition

Definition 0.1 (Universe). The **Universe** is a single mathematical structure that is simultaneously maximal, consistent, and complete within a multi-layered category, denoted as

$$\mathfrak{U} = \left(U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}} \right)$$

where each component and the compatibility between them are described below; it is unique up to isomorphism.

1 Event and Causal Layer

1.1 Event Set and Causal Partial Order

$$U_{\text{evt}} = (X, \preceq, \mathcal{C})$$

where

- X is a class-set (may be a proper class), with elements called "events";
- $\preceq \subseteq X \times X$ is a partial order, satisfying reflexivity, antisymmetry, and transitivity;
- $\mathcal{C} \subseteq \mathcal{P}(X)$ is a family of "causal patches", such that for each $C \in \mathcal{C}$, $(C, \preceq|_C)$ is a locally finite partial order, forming a cover $\bigcup_{C \in \mathcal{C}} C = X$.

1.2 Global Causal Consistency

(X, \preceq) is **stably causal**: there are no closed causal chains, and there exists a strictly increasing time function

$$T_{\text{cau}}: X \rightarrow \mathbb{R}, \quad x \prec y \Rightarrow T_{\text{cau}}(x) < T_{\text{cau}}(y).$$

1.3 Causal Net and Causal Diamond Family

Define the set of all bounded causal regions

$$\mathcal{D} = \{D \subseteq X : D = J^+(p) \cap J^-(q), p \preceq q\},$$

where $J^\pm(\cdot)$ are determined by \preceq ; each $D \in \mathcal{D}$ is called a "small causal diamond".

2 Geometric Layer (Spacetime and Metric)

2.1 Lorentzian Manifold Structure

$$U_{\text{geo}} = (M, g, \Phi_{\text{evt}}, \Phi_{\text{cau}})$$

where

- M is a four-dimensional orientable, time-oriented C^∞ manifold;
- g is a Lorentzian metric with signature $(- + ++)$;
- $\Phi_{\text{evt}} : X \rightarrow M$ is an event embedding;
- Φ_{cau} pulls the causal partial order back to the light-cone causal structure:

$$x \preceq y \iff \Phi_{\text{evt}}(y) \in J_g^+(\Phi_{\text{evt}}(x)).$$

2.2 Global Hyperbolicity

(M, g) is globally hyperbolic: there exists a Cauchy hypersurface $\Sigma \subset M$ such that

$$M \simeq \mathbb{R} \times \Sigma, \quad \text{every timelike/null geodesic intersects } \Sigma \text{ exactly once.}$$

2.3 Geometric Time Function

$$T_{\text{geo}} : M \rightarrow \mathbb{R}$$

is a smooth time function whose gradient is everywhere timelike, encoding the causal structure as

$$p \in J_g^+(q) \Rightarrow T_{\text{geo}}(p) \geq T_{\text{geo}}(q).$$

3 Measure, Probability, and Statistical Layer

3.1 Measure Structure

$$U_{\text{meas}} = (\Omega, \mathcal{F}, \mathbb{P}, \Psi)$$

where

- $(\Omega, \mathcal{F}, \mathbb{P})$ is a complete probability space;
- $\Psi : \Omega \rightarrow X$ is a random event map, such that observational statistics arise from the push-forward measure of Ψ .

3.2 Statistical Time Series

Define sample paths on worldlines

$$\Psi_\gamma : \Omega \rightarrow X^{\mathbb{Z}}, \quad \Psi_\gamma(\omega) = (x_n)_{n \in \mathbb{Z}},$$

satisfying $x_n \prec x_{n+1}$; inducing a time series process.

4 Quantum Field and Operator Algebra Layer

4.1 Local Observable Algebra Net

$$U_{\text{QFT}} = (\mathcal{O}(M), \mathcal{A}, \omega)$$

where

- $\mathcal{O}(M)$ is the family of bounded causally convex open sets on M ;
 - $\mathcal{A} : \mathcal{O}(M) \rightarrow C^*\text{Alg}$, $O \mapsto \mathcal{A}(O)$ is a Haag–Kastler type net, satisfying isotony, covariance, and microcausality:
- $$O_1 \subseteq O_2 \Rightarrow \mathcal{A}(O_1) \subseteq \mathcal{A}(O_2), \quad O_1 \perp O_2 \Rightarrow [\mathcal{A}(O_1), \mathcal{A}(O_2)] = 0.$$
- ω is a positive, normalized state consistent across all $\mathcal{A}(O)$.

4.2 GNS Construction

$$(\pi_\omega, \mathcal{H}, \Omega_\omega)$$

where

- $\pi_\omega : \mathcal{A} \rightarrow B(\mathcal{H})$ is a $*$ -representation;
- Ω_ω is a cyclic and separating vector;
- $\omega(A) = \langle \Omega_\omega, \pi_\omega(A)\Omega_\omega \rangle$.

5 Scattering, Spectrum, and Time Scale Layer

5.1 Scattering Pair and Spectral Shift

$$(H, H_0), \quad V := H - H_0$$

are a pair of self-adjoint operators satisfying appropriate trace-class/relative trace-class assumptions, ensuring the existence of a spectral shift function $\xi(\omega)$.

5.2 Scattering Matrix and Wigner–Smith Delay

$$S(\omega) \in U(\mathcal{H}_\omega), \quad Q(\omega) = -iS(\omega)^\dagger \partial_\omega S(\omega).$$

5.3 Total Scattering Phase and Relative Density of States

$$\Phi(\omega) = \arg \det S(\omega), \quad \varphi(\omega) = \tfrac{1}{2}\Phi(\omega), \quad \rho_{\text{rel}}(\omega) = -\xi'(\omega).$$

5.4 Unified Time Scale (Mother Ruler)

Define scale density

$$\kappa(\omega) := \frac{\varphi'(\omega)}{\pi} = \rho_{\text{rel}}(\omega) = \frac{1}{2\pi} \text{tr } Q(\omega).$$

For a reference frequency ω_0 , define scattering time

$$\tau_{\text{scatt}}(\omega) - \tau_{\text{scatt}}(\omega_0) := \int_{\omega_0}^{\omega} \kappa(\tilde{\omega}) d\tilde{\omega}.$$

5.5 Geometric Time Alignment

Require the existence of a monotonic bijection f such that

$$T_{\text{geo}}(\Phi_{\text{evt}}(x)) = f(\tau_{\text{scatt}}(\omega_x))$$

holds for appropriately defined frequency markers ω_x ; i.e., geometric time and scattering time fall into the same scale equivalence class.

6 Modular Flow and Thermal Time Layer

6.1 Modular Operator and Modular Flow

$$S_0 \pi_\omega(A) \Omega_\omega = \pi_\omega(A)^* \Omega_\omega,$$

closure S has polar decomposition $S = J\Delta^{1/2}$, defining modular flow

$$\sigma_t^\omega(A) = \Delta^{\text{it}} A \Delta^{-\text{it}}.$$

6.2 Modular Time Scale

Define modular Hamiltonian

$$K_\omega := -\log \Delta, \quad \sigma_t^\omega(A) = e^{\text{it} K_\omega} A e^{-\text{it} K_\omega}.$$

The modular parameter $t_{\text{mod}} \in \mathbb{R}$ serves as a time parameter; for different faithful states ω, ω' , their modular flows are conjugate in the outer automorphism group, related by at most an affine rescaling of time.

6.3 Alignment with Scattering Scale

Require the existence of constants $a > 0, b \in \mathbb{R}$ such that for some unified boundary algebra $\mathcal{A}_\partial \subseteq \mathcal{A}$,

$$t_{\text{mod}} = a \tau_{\text{scatt}} + b$$

holds on the common domain.

7 Generalized Entropy, Energy, and Gravity Layer

7.1 Generalized Entropy on Small Causal Diamonds

For each $D \in \mathcal{D}$ and its boundary cross-section $\Sigma \subset \partial D$, define

$$S_{\text{gen}}(\Sigma) = \frac{A(\Sigma)}{4G\hbar} + S_{\text{out}}(\Sigma),$$

where $A(\Sigma)$ is the area, and S_{out} is the exterior von Neumann entropy.

7.2 Generalized Entropy Extremality and Time Arrow

Deformations along the null generator affine parameter λ satisfy

$$\frac{d}{d\lambda}S_{\text{gen}}(\lambda)\Big|_{\lambda=0}=0, \quad \frac{d^2}{d\lambda^2}S_{\text{gen}}(\lambda)\geq 0,$$

uniformly for the family of all small causal diamonds; combined with the Quantum Null Energy Condition (QNEC) $T_{kk}\geq\frac{\hbar}{2\pi}S''_{\text{out}}$, this yields the local gravitational field equations.

7.3 Einstein Field Equations as Geometric Closure

$$G_{ab}+\Lambda g_{ab}=8\pi G\langle T_{ab}\rangle$$

holds everywhere on M ; where T_{ab} is given by ω and field operator expectations.

8 Boundary Time Geometry and GHY Term

8.1 Boundary Data

Take a manifold with boundary (M, g) , its boundary ∂M , induced metric, and extrinsic curvature

$$h_{ab}, \quad K_{ab}, \quad K=h^{ab}K_{ab}.$$

8.2 EH+GHY Action

$$S_{\text{EH}}[g]=\frac{1}{16\pi G}\int_M R\sqrt{-g}\,d^4x, \quad S_{\text{GHY}}[g]=\frac{\varepsilon}{8\pi G}\int_{\partial M} K\sqrt{|h|}\,d^3x.$$

8.3 Brown–York Quasilocal Stress Tensor

$$T_{\text{BY}}^{ab}=\frac{2}{\sqrt{|h|}}\frac{\delta S}{\delta h_{ab}}=\frac{\varepsilon}{8\pi G}(K^{ab}-Kh^{ab})+\dots$$

8.4 Geometric Time Generator

For a timelike Killing vector field t^a on the boundary and a spatial section Σ , define

$$H_\partial=\int_\Sigma T_{\text{BY}}^{ab}t_an_b\,d^{d-1}x,$$

where n^b is the unit normal of Σ in ∂M ; H_∂ generates boundary time translation τ_{geom} .

8.5 Alignment with Modular Flow

Require the existence of a constant $c>0$ such that on the boundary algebra \mathcal{A}_∂

$$\text{Ad}\left(e^{-i\tau_{\text{geom}}H_\partial}\right)=\sigma_{t_{\text{mod}}}^\omega, \quad t_{\text{mod}}=c\tau_{\text{geom}},$$

thus geometric time, modular time, and scattering time belong to the same scale equivalence class $[\tau]$.

9 Observer and Consensus Layer

9.1 Observer Objects

$$U_{\text{obs}} = (\mathcal{O}, \text{worldline}, \text{res}, \text{model}, \text{update})$$

where each observer

$$O_i = (\gamma_i, \Lambda_i, \mathcal{A}_i, \omega_i, \mathcal{M}_i, U_i)$$

contains: worldline $\gamma_i \subset M$, resolution scale Λ_i , observable algebra $\mathcal{A}_i \subseteq \mathcal{A}$, local state ω_i , candidate model family \mathcal{M}_i , update rule U_i .

9.2 Time Experience Scale

For each worldline γ_i , define proper time

$$\tau_i = \int_{\gamma_i} \sqrt{-g_{\mu\nu} dx^\mu dx^\nu},$$

and require the existence of an affine transformation

$$\tau_i = a_i \tau_{\text{scatt}} + b_i = a'_i T_{\text{geo}} + b'_i = a''_i t_{\text{mod}} + b''_i,$$

i.e., observer subjective time and the unified scale belong to the same equivalence class.

9.3 Causal Consensus

The local partial orders (C_i, \prec_i) of all observers satisfy Čech-like consistency in overlapping regions, implying the existence of a unique global partial order (X, \preceq) (i.e., U_{evt} above), so the "same universe causal net" is the glued limit of all observers' local data.

10 Category, Topology, and Logic Layer

10.1 Universe Category Model

$$U_{\text{cat}} = (\mathbf{Univ}, \mathfrak{U}, \Pi)$$

where

- **Univ** is a 2-category with "candidate universe structures" as objects and isomorphisms preserving all the above structures as morphisms;
- \mathfrak{U} is the terminal object in **Univ**: for any object V , there exists a unique morphism $V \rightarrow \mathfrak{U}$;
- Π represents the limit cone composed of projections of each layer (geometric, operator, scattering, modular, entropy, observer, etc.), such that

$$\mathfrak{U} \simeq \varprojlim (U_{\text{geo}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, \dots).$$

10.2 Topology and Logic

$$\mathcal{E} = \text{Sh}(M)$$

is the category of sheaves on M (or a Grothendieck topos), carrying internal higher-order logic; physical propositions correspond to the lattice of subobjects in \mathcal{E} ; causality and observability correspond to relations between sublayers and states.

11 Computation and Realizability Layer

11.1 Computable Structure

$$U_{\text{comp}} = (\mathcal{M}_{\text{TM}}, \text{Enc}, \text{Sim})$$

where

- \mathcal{M}_{TM} is the space of Turing machines;
- $\text{Enc} : \mathbf{Univ} \rightarrow \mathcal{M}_{\text{TM}}$ is an encoding functor for universe structures (in the upper bound sense);
- $\text{Sim} : \mathcal{M}_{\text{TM}} \rightrightarrows \mathbf{Univ}$ gives a family of simulable sub-universes; the real universe \mathfrak{U} is the upper bound over all computable models, satisfying consistency but not assuming "computational completeness".

12 Final Compressed Definition of the Universe

Synthesizing the above, the **Universe** is:

Based on a given set theory, a **maximal consistent structure** of all accessible events on X , geometric causal structure, quantum fields and operator algebras, scattering and spectral shifts, modular flow and generalized entropy, boundary time geometry and observer networks, etc.

$$\mathfrak{U} = (U_{\text{evt}}, U_{\text{geo}}, U_{\text{meas}}, U_{\text{QFT}}, U_{\text{scat}}, U_{\text{mod}}, U_{\text{ent}}, U_{\text{obs}}, U_{\text{cat}}, U_{\text{comp}})$$

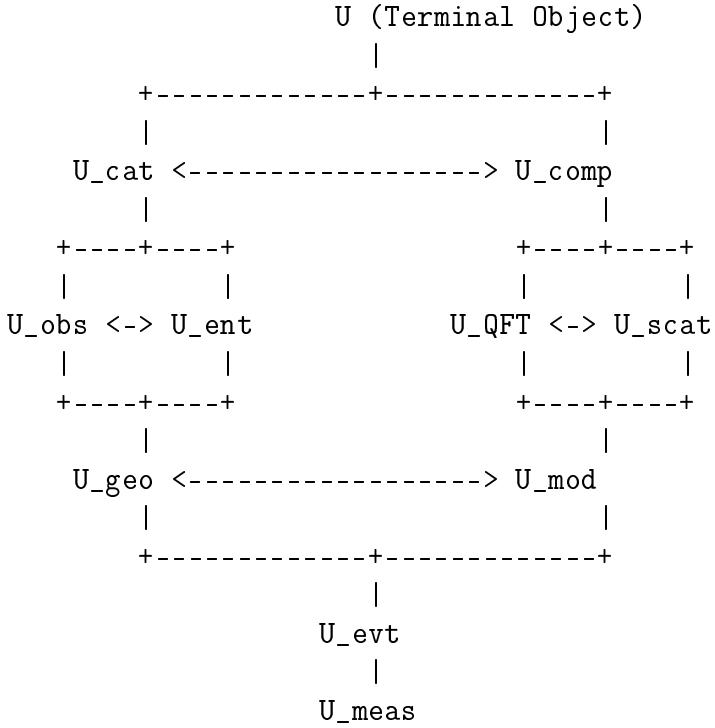
Within it, the unified time scale is given by

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\text{rel}}(\omega) = \frac{1}{2\pi} \text{tr } Q(\omega)$$

and is in the same scale equivalence class $[\tau]$ as geometric time T_{geo} , modular time t_{mod} , and observer proper time τ_i ; All physical laws are compatibility conditions between components of this structure, and the Universe is its unique solution up to isomorphism.

Structure Diagram

The ten-layer structure of the Universe and its interrelations can be illustrated as follows:



Time scale unification relation:

$$[\tau] = \{T_{\text{cau}}, T_{\text{geo}}, \tau_{\text{scatt}}, t_{\text{mod}}, \tau_{\text{geom}}, \tau_i\} \text{ affine equivalence}$$

All arrows represent structural compatibility constraints, and the Universe \mathfrak{U} is the unique maximal solution that satisfies all constraints simultaneously.

13 Mathematical Status

In the category **Univ**, the universe \mathfrak{U} has the following properties:

1. ****Terminality****: For any candidate universe structure V , there exists a unique morphism $V \rightarrow \mathfrak{U}$.
2. ****Limit Property****: \mathfrak{U} is the inverse limit of all component structures.
3. ****Completeness****: All physical laws are satisfied simultaneously as compatibility conditions in \mathfrak{U} .
4. ****Maximality****: There exists no consistent structure strictly containing \mathfrak{U} .
5. ****Uniqueness****: \mathfrak{U} is uniquely determined by the above axioms up to isomorphism.

Therefore, the Universe is not "constructed", but **a mathematical object that exists uniquely under consistency constraints**.