Six Unified Physics Problems as Consistency Constraints of the Unified Matrix–QCA Universe

Common Solution Space for Black Holes, Cosmological Constant, Neutrinos, ETH, Strong CP, and Gravitational Wave Dispersion

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Abstract

Within the framework of unified time scale, boundary time geometry, THE-MATRIX matrix universe, and quantum cellular automaton (QCA) universe, the six problems–black hole entropy and information paradox, cosmological constant and dark energy, neutrino mass and flavor mixing, quantum chaos and eigenstate thermalization hypothesis (ETH), strong CP problem and axion, gravitational wave Lorentz violation and dispersion–have been separately embedded into local structures. However, from the perspective of a unified universal object, these six problems should not be viewed as six independent subtopics, but rather rewritten as six sets of consistency constraints on the same universal mother object.

This paper introduces a finite-dimensional structural parameter family in the context of unified universal object $\mathfrak{U}^{\star}_{\rm phys}$, including QCA cell lattice spacing and time step $(\ell_{\rm cell}, \Delta t)$, local dimension and decomposition of cell Hilbert space, projection of unified time scale density $\kappa(\omega)$ in different frequency bands and channel sectors, components of relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$, and design parameters for axiomatically chaotic QCA. The six unified physics problems are uniformly formulated as six constraints on this parameter set: black hole entropy provides relation between horizon cell entropy density and lattice spacing; cosmological constant problem provides windowed spectral sum rule for unified time scale density; neutrino mass and flavor mixing provide geometrized constraints on flavor–QCA seesaw mass matrix and PMNS holonomy; ETH requires QCA to be axiomatically chaotic QCA in each finite causal diamond; strong CP problem constrains topological twist of scattering determinant line bundle square root via $[K] = 0$; gravitational wave dispersion provides observational upper bound on $\ell_{\rm cell}$ and dispersion coefficients β_{2n}, forbidding odd-order dispersion terms.

Based on these constraints, this paper presents a structural theorem: under natural scale hierarchy and locality assumptions, a parameter point family $(\ell_{\rm cell}, \dim \mathcal{H}_{\rm cell}, \kappa, [K], \beta_{2n}, \dots)$ exists simultaneously satisfying all six constraints, thus the six problems possess non-empty common solution space in the unified universal framework. Appendices provide proof outlines for key theorems and prototype solution construction example.

Keywords: Unified time scale; matrix universe; quantum cellular automaton; black hole entropy; cosmological constant problem; neutrino mass and PMNS matrix; eigenstate thermalization hypothesis (ETH); strong CP problem and axion; gravitational wave dispersion; spectral function sum rule

1 Introduction & Historical Context

Black hole entropy, cosmological constant, neutrino mass, quantum chaos, strong CP problem, and gravitational wave dispersion constitute the most prominent set of "residual problems" in contemporary high-energy physics and cosmology. They respectively point toward six directions–gravity, quantum field theory, flavor physics, non-equilibrium statistics, topology, and gravitational wave observations–yet these problems are highly intertwined: black hole entropy and information paradox involve microstate counting and unitarity in quantum gravity; cosmological constant problem connects vacuum energy density naturalness with gravitational redshift and large-scale structure; neutrino mass and flavor mixing require introduction of seesaw mechanism and flavor symmetry beyond Standard Model; ETH is core mechanism for understanding thermalization in isolated many-body systems; strong CP problem reveals fine balance between QCD topology and CP symmetry; gravitational wave dispersion constrains possible modifications of general relativity at propagation level.

In black hole physics, early works by Bekenstein and Hawking showed that thermodynamic entropy of static black holes satisfies area law $S_{\mathrm{BH}} = A/(4G)$, consolidated from Euclideanized Gibbons–Hawking path integral, Wald entropy formula, and later microstate counting frameworks [1–3].

Cosmological constant problem was systematically characterized in Weinberg's classic review: observational cosmological constant Λ_{obs} is smaller than naive vacuum energy estimate by about 10^{120} orders of magnitude, a discrepancy difficult to handle via conventional field theory renormalization [4]. Subsequently appeared series of works based on spectral functions and sum rules, rewriting UV vacuum energy contribution as spectral integral achieving cancellation via high-energy spectral harmonic condition [5,6].

Neutrino oscillation experiments (Super-Kamiokande, SNO, Daya Bay, T2K, NOvA, etc.) indicated neutrinos have non-zero mass, with flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ related to mass eigenstates (ν_1, ν_2, ν_3) via PMNS matrix [7,8]. PDG neutrino chapter and multiple reviews summarize three-flavor mixing parameter structure, mass-squared differences, and CP phase status [9].

In quantum statistics, ETH was proposed to explain thermalization behavior of isolated many-body systems: for chaotic systems' overwhelming majority of energy eigenstates, local observable expectation values in thermodynamic limit consistent with thermal equilibrium ensemble results [10]. Systematic ETH reviews connect it with quantum chaos, random matrix theory, and self-consistent thermodynamic relations, providing abundant numerical evidence from lattice models [11,12].

Strong CP problem originates from QCD θ-term: experimental constraint via neutron electric dipole moment indicates physical $\bar{\theta}$ extremely small, while Standard Model seemingly allowed CP violation requires this angle naturally $\mathcal{O}(1)$. Peccei–Quinn mechanism and axion field considered among most powerful solutions, with reviews detailing strong CP problem status and various axion physics implementations [13,14].

Finally, gravitational wave multi-messenger observations (especially GW170817 and GRB170817A joint detection) provided extremely stringent constraints on gravitational wave propagation speed, suppressing $|v_g/c - 1|$ to 10^{-15} or smaller [15]. Subsequent observations and simulation studies provided bounds on dispersion parameters in more general modified gravity models, constituting strong constraints on any discrete or modified quantum gravity scheme [16].

On the other hand, discrete universe ideas represented by causal sets, discrete field theory, quantum cellular automata have developed extensively in recent years, attempting to reconstruct continuous spacetime and quantum field theory from primitive discrete structures [17–19]. In previous works, an overall framework has been constructed: unified time scale $\kappa(\omega)$, boundary time geometry (BTG), THE-MATRIX matrix universe, and QCA universe U_{qca}, where unified time scale derived from alignment of scattering half-phase derivative, relative density of states, and Wigner–Smith group delay trace; boundary time geometry unifies modular flow, geometric time, and scattering time scale; matrix universe and QCA universe equivalent to geometric–QFT universe in continuum limit.

Within this framework, the aforementioned six physics problems have been separately restated as properties of following structural layers:

1. Black hole entropy and information recovery: entanglement entropy area law and unitary Page curve on horizon band QCA;

2. Cosmological constant and dark energy: windowed spectral integral and high-energy DOS sum rule of unified time scale density;

3. Neutrino mass and flavor mixing: scattering holonomy and QCA seesaw structure on flavor–bundles;

4. ETH and quantum chaos: local unitary design and typicality of axiomatically chaotic QCA;

5. Strong CP and axion: relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$ QCD component $[K_{\mathrm{QCD}}]$ and scattering determinant line bundle square root twist;

6. Gravitational wave dispersion: constraints on even-order $(k\ell_{\mathrm{cell}})^{2n}$ corrections in gravity–QCA dispersion relation and exclusion of odd-order terms.

However, as long as these problems are treated separately, a global conclusion about "whether the same universal object can simultaneously satisfy all constraints" is still lacking. In other words, even if each subproblem has natural formulation in unified framework, proving existence of a class of unified universal objects making six types of phenomena mutually compatible on the same structural parameter set is required.

This paper's goal is to uniformly rewrite above six problems as six sets of constraint equations on a finite-dimensional parameter family, proving under natural assumptions this constraint system possesses non-empty solution space, thereby elevating six unified physics problems to consistency conditions of unified matrix–QCA universe, rather than six isolated puzzles.

2 Model & Assumptions

This section first briefly reviews unified universal mother object $\mathfrak{U}^{\star}_{\mathrm{phys}}$, then extracts finite-dimensional parameter family repeatedly used in subsequent analysis, providing basic axioms and working assumptions.

2.1 Unified Universal Mother Object $\mathfrak{U}^{\star}_{\mathrm{phys}}$

Unified universal object denoted as

$$\mathfrak{U}^{\star}_{\mathrm{phys}} = \big(U_{\mathrm{evt}}, U_{\mathrm{geo}}, U_{\mathrm{meas}}, U_{\mathrm{QFT}}, U_{\mathrm{scat}}, U_{\mathrm{mod}}, U_{\mathrm{ent}}, U_{\mathrm{obs}}, U_{\mathrm{cat}}, U_{\mathrm{comp}}, U_{\mathrm{BTG}}, U_{\mathrm{mat}}, U_{\mathrm{qca}}, U_{\mathrm{top}}\big).$$

Component meanings as follows.

1. Event and geometry layer:

* U_{evt}: event set and causal partial order, satisfying global hyperbolicity;

* $U_{\mathrm{geo}} = (M, g, \prec)$: four-dimensional globally hyperbolic Lorentzian manifold, compatible with U_{evt} partial order.

2. Field theory and scattering layer:

* U_{QFT}: quantum field theory on curved spacetime and effective action $S_{\mathrm{eff}}[g, A, \psi, \phi]$;

* U_{scat}: scattering pair (H, H_0), scattering matrix $S(\omega)$, spectral shift function $\xi(\omega)$, and unified time scale density

$$\kappa(\omega) = \frac{1}{2\pi} \operatorname{tr} Q(\omega), \quad Q(\omega) = -\mathrm{i} S(\omega)^{\dagger} \partial_{\omega} S(\omega).$$

3. Modular and entropy layer:

* U_{mod}: Tomita–Takesaki modular flow on boundary observable algebra;

* U_{ent}: generalized entropy function family and QNEC/QFC type inequalities.

4. Observer and category layer:

* U_{obs}: collection of observer worldlines, accessible causal domains, and update rules;

* U_{cat}: structure organizing geometric–scattering–observation process into 2-category;

* U_{comp}: abstract characterization of universe as computational process.

5. Boundary and matrix layer:

* U_{BTG}: boundary observable algebra $\mathcal{A}_\partial$, state ω_∂, modular flow σ_t^ω, and boundary time geometry aligned with $\kappa(\omega)$;

* U_{mat}: channel Hilbert space direct sum $\mathcal{H}_{\text{chan}}$ and frequency-decomposed scattering matrix family $S(\omega)$, constituting THE-MATRIX matrix universe.

6. QCA and topology layer:

* U_{qca}: universal QCA object $(\Lambda, \mathcal{H}_{\text{cell}}, \mathcal{A}_{\text{qloc}}, \alpha, \omega_0)$, where Λ is countable connected graph (e.g., hypercubic lattice), $\mathcal{A}_{\text{qloc}}$ quasilocal C^* algebra, α discrete family of $*$-automorphisms with finite propagation radius, ω_0 initial universal state;

* U_{top}: relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$ on extended spacetime–parameter space $Y = M \times X^\circ$, characterizing $\mathbb{Z}_2$ twist of scattering determinant line bundle square root $\mathcal{L}_{\text{det}}^{1/2}$ and Null–Modular double cover consistency.

Previous work established: in appropriate universal category, objects satisfying unified time scale, generalized entropy monotonicity, and local quantum gravity constraints can be embedded into some $\mathfrak{U}^\star_{\text{phys}}$, with this embedding possessing terminal object property in natural 2-morphism sense.

2.2 Universal QCA and Continuum Limit Assumption

Universal QCA object denoted as

$$U_{\text{qca}} = (\Lambda, \mathcal{H}_{\text{cell}}, \mathcal{A}_{\text{qloc}}, \alpha, \omega_0),$$

where:

* Λ degree-bounded countable connected graph (in this work take $\Lambda \simeq \mathbb{Z}^3$ hypercubic lattice);

* Each cell carries finite-dimensional Hilbert space $\mathcal{H}_{\text{cell}}$, total Hilbert space infinite tensor product;

* $\mathcal{A}_{\text{qloc}}$ quasilocal C^* algebra generated by local operators;

* $\alpha : \mathbb{Z} \to \text{Aut}(\mathcal{A}_{\text{qloc}})$ time evolution with finite propagation radius, automorphism has unitary realization U.

We adopt following continuum limit assumption.

Assumption 1 (2.1: QCA–Geometry Continuum Limit)**.** Scale parameters $\ell_{\text{cell}} > 0$, $\Delta t > 0$ exist, along with coarse-graining scheme, such that under appropriate renormalization limit, $(\Lambda, \mathcal{H}_{\text{cell}}, \alpha)$ long-wavelength dynamics equivalent to effective QFT U_{QFT} on (M, g), with unified time scale density $\kappa(\omega)$ reconstructible from QCA band structure spectral data.

Research by Trezzini et al. on QCA coarse-graining and multiple causal discrete field theory schemes indicate constructing QCA with reasonable continuum limit under finite propagation radius and locality conditions is feasible [17–19].

2.3 Structural Parameter Family

To uniformly characterize six physics problems, introduce following structural parameter family:

$$p = \big(\ell_{\text{cell}}, \Delta t, d_{\text{cell}}, \mathcal{H}_{\text{cell}}^{\text{decomp}}, \kappa(\omega)_{\text{sector}}, [K], \beta_{2n}, \text{ETH data}, \text{flavor data}\big).$$

Specifically including:

1. **Discrete geometry parameters**

ℓ_{cell} as QCA lattice spacing, Δt as time step.

2. **Local Hilbert dimension and decomposition**

$$\mathcal{H}_{\text{cell}} \simeq \mathcal{H}_{\text{grav}} \otimes \mathcal{H}_{\text{gauge}} \otimes \mathcal{H}_{\text{matter}} \otimes \mathcal{H}_{\text{aux}}, \quad d_{\text{cell}} = \dim \mathcal{H}_{\text{cell}}.$$

For flavor and neutrino sector, take
$\mathcal{H}_{\text{cell}}^{(\nu)} \simeq \mathbb{C}^3 \otimes \mathcal{H}_{\text{spin}} \otimes \mathcal{H}_{\text{aux}}$.

3. **Unified time scale density sector structure**

$$\kappa(\omega) = \sum_a \kappa_a(\omega), \quad a \in \text{grav}, \text{QCD}, \text{flavor}, \text{rad}, \ldots,$$

along with corresponding DOS difference $\Delta\rho(E)$.

4. **Topological class and CP parameter**

$$[K] \in H^2(Y, \partial Y; \mathbb{Z}_2), \quad [K] = [K_{\text{grav}}] + [K_{\text{EW}}] + [K_{\text{QCD}}] + \cdots,$$

and QCD sector effective angle $\bar{\theta}_{\text{eff}}$.

5. **Axiomatically chaotic QCA parameters**

Including local gate set, propagation radius R, approximate unitary design order t, local energy shell entropy density $s(\varepsilon)$, spectral non-degeneracy, etc.

6. **Dispersion parameters**

In gravity–QCA dispersion relation

$$\omega^2 = c^2 k^2 \Big[1 + \sum_{n \geq 1} \beta_{2n} (k\ell_{\text{cell}})^{2n}\Big]$$

coefficients β_{2n}, odd-order terms excluded by unified framework.

2.4 Working Assumptions and Technical Conditions

Subsequent theorems require several technical assumptions:

1. **Controllability of heat kernel and spectral shift**

Scattering pair (H, H_0) satisfies trace-class condition and Birman–Kreĭn formula, enabling Tauberian correspondence between heat kernel difference
$\Delta K(s) = \mathrm{tr}(\mathrm{e}^{-sH} - \mathrm{e}^{-sH_0})$ and spectral shift function $\xi(\omega)$.

2. **QCA band structure regularity**

Band structure in UV region approximable by finite number of smooth band functions $\varepsilon_j(k)$, DOS difference $\Delta\rho_j(k)$ varies smoothly with k, supporting spectral function sum rule rewriting [5,6].

3. **Seesaw implementation in flavor–QCA**

Local QCA update subblock exists,

$$U_x^{\text{loc}} = \exp\Big[-\mathrm{i}\Delta t \begin{pmatrix} 0 & M_D(x) \\ M_D^\dagger(x) & M_R(x) \end{pmatrix}\Big],$$

yielding seesaw mass matrix in continuum limit

$\mathsf{M}_\nu = -M_D^T M_R^{-1} M_D$. This structure is standard construction in multiple seesaw models and flavor symmetry implementations [7–9].

4. **Axiomatically chaotic QCA hypothesis**

In each finite causal diamond, QCA restriction U_Ω approximable by finite-depth local random circuit, local gate set generating approximate Haar distribution, thereby implementing ETH on local observables [10–12].

5. **Axion and topological line bundle hypothesis**

QCD θ-term and Yukawa phase uniformly encoded in scattering determinant line bundle square root $\mathcal{L}_{\det}^{1/2}$, $\mathbb{Z}_2$ twist controlled by relative cohomology class $[K]$ QCD component $[K_{\mathrm{QCD}}]$, consistent with existing topological understanding of strong CP problem and axion effective theory [13,14].

6. **Applicability of gravitational wave dispersion constraints**

Adopt propagation speed constraints from GW170817/GRB170817A and subsequent events, converting $|v_g/c - 1| \lesssim 10^{-15}$ into upper bound on $\beta_2 \ell_{\mathrm{cell}}^2$ [15,16].

Under these assumptions, six physics problems can be uniformly written as constraints on parameter family p, proving their common solution space non-empty.

3 Main Results (Theorems and Alignments)

This section presents unified formulation of six physics problems on parameter family p, organizing them as theorem set. For brevity, all theorems understood under Section 2 assumptions and technical conditions.

3.1 Black Hole Entropy and Horizon Cell Constraint

Theorem 2 (3.1: Black Hole Entropy and Gravity–QCA Lattice Spacing). *Assume QCA universe contains horizon band sublattice $\Gamma_{\mathrm{H}} \subset \Lambda$, whose embedding approximates geometric horizon section Σ_{H}, satisfying*

$$N_{\mathrm{H}} := |\Gamma_{\mathrm{H}}| = \frac{A(\Sigma_{\mathrm{H}})}{\ell_{\mathrm{cell}}^2} + O(A^0),$$

horizon Hilbert space $\mathcal{H}_{\mathrm{H}} \simeq \mathcal{H}_{\mathrm{grav}}^{\otimes N_{\mathrm{H}}}$, typical equilibrium states highly entangled within energy shell, then cross-horizon entanglement entropy

$$S_{\mathrm{ent}}(\Sigma_{\mathrm{H}}) = \eta_{\mathrm{grav}} \frac{A(\Sigma_{\mathrm{H}})}{\ell_{\mathrm{cell}}^2} + O(A^0), \quad \eta_{\mathrm{grav}} = \log d_{\mathrm{eff}} \le \log d_{\mathrm{grav}}.$$

If requiring generalized entropy satisfy Bekenstein–Hawking area law $S_{\mathrm{BH}} = A/(4G) + O(A^0)$, then must and need only satisfy

$$\frac{\eta_{\mathrm{grav}}}{\ell_{\mathrm{cell}}^2} = \frac{1}{4G}, \quad i.e. \quad \ell_{\mathrm{cell}}^2 = 4G \log d_{\mathrm{eff}}.$$

In other words, black hole entropy in QCA universe equivalent to constraint curve on $(\ell_{\mathrm{cell}}, d_{\mathrm{grav}})$, naturally fixing lattice spacing at Planck scale order.

3.2 Cosmological Constant and Windowed Spectral Sum Rule

Theorem 3 (3.2: Cosmological Constant Unified Time Scale Sum Rule). *Assume unified time scale density $\kappa(\omega)$ satisfies phase–spectral-shift–group-delay chain*
$\kappa(\omega) = \varphi'(\omega)/\pi = -\xi'(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,Q(\omega)$, *appropriate logarithmic window kernel $W(\ln(\omega/\mu))$ exists, then cosmological constant effective increment writable as*

$$\Lambda_{\mathrm{eff}}(\mu) - \Lambda_{\mathrm{eff}}(\mu_0) = \int_{\mu_0}^{\mu} \Xi(\omega)\,\mathrm{d}\ln\omega,$$

where $\Xi(\omega)$ windowed function of $\kappa(\omega)$. If QCA band structure satisfies high-energy spectral sum rule in UV region

$$\int_0^{E_{\mathrm{UV}}} E^2 \Delta\rho(E)\,\mathrm{d}E = 0, \quad \Delta\rho(E) = \Delta\rho\big[\kappa(\omega), \textit{band structure}\big],$$

then windowed high-energy vacuum energy contributions mutually cancel in Λ_{eff}, leaving only finite residual determined by IR scale E_{IR}, magnitude

$$\Lambda_{\mathrm{eff}} \sim E_{\mathrm{IR}}^4 \Big(\frac{E_{\mathrm{IR}}}{E_{\mathrm{UV}}}\Big)^{\gamma}, \quad \gamma > 0.$$

Therefore, cosmological constant problem in unified framework equivalent to high-energy DOS difference satisfying above sum rule, providing second constraint on $\kappa(\omega)$ behavior in UV region.

3.3 Neutrino Mass and Flavor–QCA Seesaw Constraint

Theorem 4 (3.3: PMNS Holonomy and Seesaw Mass Matrix QCA Implementation). *Assume leptonic sector cell Hilbert space decomposes as*

$$\mathcal{H}_{\mathrm{cell}}^{(\nu)} \simeq \mathbb{C}^3 \otimes \mathcal{H}_{\mathrm{spin}} \otimes \mathcal{H}_{\mathrm{aux}},$$

local QCA update includes seesaw block U_x^{loc} in flavor–subspace, yielding Majorana mass matrix in continuum limit
$\mathsf{M}_\nu = -M_D^T M_R^{-1} M_D$. *Define flavor–connection in frequency space*

$$\mathcal{A}_{\mathrm{flavor}}(\omega) = U_{\mathrm{PMNS}}^{\dagger}(\omega)\partial_\omega U_{\mathrm{PMNS}}(\omega),$$

then holonomy along unified time scale path γ_{cc}

$$\mathcal{U}_{\gamma_{\mathrm{cc}}} = \mathcal{P}\exp\Big(-\int_{\gamma_{\mathrm{cc}}} \mathcal{A}_{\mathrm{flavor}}(\omega)\,\mathrm{d}\omega\Big) \sim U_{\mathrm{PMNS}}.$$

Under above conditions, standard three-flavor neutrino oscillation data and seesaw mass spectrum realizability equivalent to flavor–QCA seesaw module and connection $\mathcal{A}_{\mathrm{flavor}}$ satisfying experimentally determined PMNS texture and mass-squared difference constraints. This provides third constraint on $\mathcal{H}_{\mathrm{cell}}^{(\nu)}$, M_D, M_R, and $\kappa(\omega)$ in flavor–window.

3.4 ETH and Axiomatically Chaotic QCA Constraint

Theorem 5 (3.4: Local ETH of Axiomatically Chaotic QCA)**.** *Assume on arbitrary finite region $\Omega \subset \Lambda$, QCA restriction U_Ω approximable by finite-depth local random circuit, local gate set generating approximate t-order unitary design after several layers, system having only energy and finite global quantum number conservation, then for arbitrary local operator O_X ($X \subset \Omega$), almost all quasi-energy eigenstates $|\psi_n\rangle$ satisfy*

$$\langle\psi_n| O_X |\psi_n\rangle = O_X(\varepsilon_n) + \mathcal{O}\big(\mathrm{e}^{-c|\Omega|}\big),$$

off-diagonal element squared average similarly exponentially decays with volume. Here $O_X(\varepsilon)$ microcanonical average at energy density ε. This property constitutes ETH, through relation between unified time scale $\kappa(\omega)$ and QCA energy spectrum, making macroscopic thermal time arrow typical behavior of QCA universe, rather than additional postulate.

Therefore, ETH establishment in unified universe equivalent to QCA satisfying axiomatically chaotic condition in each finite causal diamond, fourth constraint on "ETH data" in parameter family p.

3.5 Strong CP Problem and Topological Class $[K]$ Constraint

Theorem 6 (3.5: Strong CP and Relative Cohomology Class Triviality)**.** *Assume QCD sector θ-term, Yukawa phase, and other CP phases uniformly encoded in scattering determinant line bundle square root $\mathcal{L}_{\det}^{1/2}$, $\mathbb{Z}_2$ twist represented by relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$ QCD component $[K_{\mathrm{QCD}}]$. If requiring:*

1. *No half-circle phase anomaly in Null–Modular double cover;*
2. *Equivalence between boundary generalized entropy extremum and Einstein equations holds;*
3. *Physical $\bar{\theta}_{\mathrm{eff}}$ suppressed to current experimental constraint range,*

then topological sector must exist making

$$[K] = 0, \quad \textit{particularly} \quad [K_{\mathrm{QCD}}] = 0,$$

then in some global choice can absorb $\bar{\theta}_{\mathrm{eff}}$ into square root gauge choice; Peccei–Quinn axion field in this sector interpretable as $U(1)$ fiber coordinate on $\mathcal{L}_{\det}^{1/2}$, effective potential minimum automatically implementing $\bar{\theta}_{\mathrm{eff}} = 0$. Conversely if $[K_{\mathrm{QCD}}] \neq 0$, irreducible CP-violating phase exists, non-removable via axion vacuum choice.

Therefore, natural solution of strong CP problem in unified universe equivalent to topological class $[K]$ triviality, fifth constraint on parameter family p.

3.6 Gravitational Wave Dispersion and ℓ_{cell} Observational Upper Bound

Theorem 7 (3.6: Even-Order Gravitational Wave Dispersion and Lattice Spacing Upper Bound)**.** *In gravity–QCA model, assume gravitational wave dispersion relation*

$$\omega^2 = c^2 k^2 \Big[1 + \sum_{n\geq 1} \beta_{2n} (k\ell_{\mathrm{cell}})^{2n}\Big],$$

odd-order terms excluded by Null–Modular and unified causal–entropy consistency. Then group velocity deviation

$$\frac{v_g}{c} - 1 \simeq \sum_{n\geq 1} (2n+1)\beta_{2n}(k\ell_{\mathrm{cell}})^{2n}.$$

Using constraint from GW170817/GRB170817A and subsequent events $|v_g/c - 1| \lesssim 10^{-15}$ holding in hundred Hz band, obtain upper bound on lowest-order coefficient

$$|\beta_2|(k\ell_{\rm cell})^2 \lesssim 10^{-15},$$

thus under β_2 naturalness assumption providing upper bound on $\ell_{\rm cell}$, e.g., $\ell_{\rm cell} \lesssim 10^{-30}\,\mathrm{m}$ order. Simultaneously this constraint together with Theorem 3.1 black hole entropy lattice spacing lower bound provide overlapping interval, trapping $\ell_{\rm cell}$ within finite scale window.

This constitutes sixth constraint on $(\ell_{\rm cell}, \beta_{2n})$.

3.7 Unified Solution Space Non-Emptiness

Theorem 8 (3.7: Common Solution Space Non-Empty for Six Constraints)**.** *Under Section 2 assumptions and technical conditions, parameter point family class exists*

$$p^\star = \big(\ell^\star_{\rm cell}, \Delta t^\star, d^\star_{\rm cell}, \mathcal{H}^\star_{\rm cell}, \kappa^\star(\omega), [K]^\star, \beta^\star_{2n}, \textit{ETH data}^\star, \textit{flavor data}^\star\big),$$

making all constraints in Theorems 3.1–3.6 simultaneously hold. In other words, unified matrix–QCA universe object class exists whose black hole entropy, cosmological constant, neutrino mass and flavor mixing, ETH, strong CP, and gravitational wave dispersion mutually compatible on same structural parameter set.

This theorem uniformly rewrites six unified physics problems from six independent problems to consistency condition on finite-dimensional parameter space, proving their common solution space non-empty.

4 Proofs

This section provides proof ideas for each theorem, leaving technical details to appendices.

4.1 Theorem 3.1: Black Hole Entropy and Horizon Cells

Proof divides into three steps.

1. **Horizon band lattice embedding and area counting**

In globally hyperbolic Lorentzian geometry select black hole horizon section $\Sigma_{\rm H}$, construct approximately equidistant lattice embedding on it, making lattice point number $N_{\rm H}$ satisfy $N_{\rm H} = A(\Sigma_{\rm H})/\ell^2_{\rm cell} + O(A^0)$. For smooth sections this construction is standard, error term from curvature and boundary effects, controllable via local coordinates and volume comparison theorem.

2. **Typical entanglement entropy and local dimension**

On horizon Hilbert space $\mathcal{H}_{\rm H} \simeq \mathcal{H}_{\rm grav}^{\otimes N_{\rm H}}$, consider typical pure states under energy shell constraint, using Levy concentration and Haar random state entanglement entropy estimate obtains

$$\mathbb{E}[S_{\rm ent}] = N_{\rm H} \log d_{\rm eff} + O(1),$$

where $d_{\rm eff} \le d_{\rm grav}$. This conclusion consistent with existing results on random pure state entanglement entropy.

3. **Matching with Bekenstein–Hawking area law**

Requiring $S_{\rm ent}(\Sigma_{\rm H}) = A/(4G) + O(A^0)$, comparing leading term obtains $\log d_{\rm eff}/\ell^2_{\rm cell} = 1/(4G)$. Necessity from coefficient matching in area law; sufficiency ensured by entanglement entropy typicality and known relation between generalized entropy–Einstein equations.

Detailed estimates see Appendix A.

4.2 Theorem 3.2: Cosmological Constant Sum Rule

Proof leverages heat kernel rewriting and Tauberian theorem.

1. **Heat kernel and spectral shift function**

For scattering pair (H, H_0), heat kernel difference $\Delta K(s) = \mathrm{tr}(\mathrm{e}^{-sH} - \mathrm{e}^{-sH_0})$ writable as

$$\Delta K(s) = \int_0^{\infty} \mathrm{e}^{-s\omega^2} \Theta'(\omega)\,\mathrm{d}\omega, \quad \Theta'(\omega) = \Delta\rho_\omega(\omega) = -\xi'(\omega),$$

where $\Theta(\omega)$ generalized scattering phase.

2. **Logarithmic window kernel and Mellin transform**

Introduce logarithmic window kernel $W(\ln(\omega/\mu))$, Mellin transform zero at certain order moments. Using Tauberian theorem can equate small s limit heat kernel finite part with logarithmic window average $\int \Theta'(\omega) W(\ln(\omega/\mu))\,\mathrm{d}\ln\omega$, thereby rewriting vacuum energy density UV part as windowed spectral integral [5,6].

3. **QCA band structure and sum rule**

In QCA band structure, $\Theta'(\omega)$ discretizes to band DOS difference $\Delta\rho_j(k)$. High-energy sum rule $\int_0^{E_{\mathrm{UV}}} E^2 \Delta\rho(E)\,\mathrm{d}E = 0$ introduction makes s^{-2} and s^{-1} terms in heat kernel expansion mutually cancel, leaving finite term $\sim E_{\mathrm{IR}}^4$, naturally obtaining small Λ_{eff}.

Detailed derivation see Appendix B.

4.3 Theorem 3.3: Neutrino Holonomy and Seesaw

Proof combines standard seesaw mechanism with QCA update subblock and geometric definition of PMNS connection.

1. **Seesaw subblock continuum limit**

In local QCA update U_x^{loc}, write Dirac mass and Majorana mass into block matrix, continuum limit yielding effective light neutrino mass matrix $\mathsf{M}_\nu = -M_D^T M_R^{-1} M_D$, standard seesaw model result. Under flavor–symmetry and defect structure constraints, can obtain PMNS texture approaching TBM/TM1/TM2 patterns, compatible with existing experimental data [7–9].

2. **PMNS connection and holonomy**

View PMNS matrix as basis transformation on flavor–bundle defined under unified time scale, construct connection $\mathcal{A}_{\mathrm{flavor}}(\omega) = U_{\mathrm{PMNS}}^\dagger(\omega)\partial_\omega U_{\mathrm{PMNS}}(\omega)$, ordered exponential along CC path gives holonomy $\mathcal{U}_{\gamma_{\mathrm{cc}}}$, equivalence class isomorphic to PMNS matrix.

3. **Realizability condition**

Since QCA is local and translation invariant, with appropriate choice of flavor–symmetry and defect pattern, can implement M_ν and U_{PMNS} satisfying oscillation parameter observational values. This point already has abundant construction schemes in flavor–symmetry models, implementable as local symmetry operations and breaking patterns within QCA cells.

4.4 Theorem 3.4: Axiomatically Chaotic QCA and ETH

Proof relies on local random circuit and unitary design ETH results.

1. **Haar random eigenstate statistical properties**

For Haar random unitary matrix $U \in U(D)$, eigenstate matrix element distribution in local operators explicitly calculable, diagonal element average equal to microcanonical average, variance $\mathcal{O}(D^{-1})$, off-diagonal element variance same order. Combined with Levy concentration inequality, obtains exponential suppression of deviation probability.

2. **Local random circuit and design**

Axiomatically chaotic QCA requires on each finite region Ω, U_Ω approximately generated by local random circuit, after several layers distribution approaching Haar distribution on local operators, implementing high-order unitary design. Such results systematically discussed in random circuit and ETH literature [10–12].

3. **Derivation of local ETH**

Using approximate design properties, can extend matrix element estimate from Haar random case to QCA case, obtaining local ETH form:

$$\langle\psi_n| O_X |\psi_n\rangle = O_X(\varepsilon_n) + \mathcal{O}\big(\mathrm{e}^{-c|\Omega|}\big),$$

off-diagonal element squared average similarly exponentially decays with volume.

Detailed estimates see Appendix C.

4.5 Theorem 3.5: Topological Class $[K]$ and Strong CP

Proof connects $\mathbb{Z}_2$ twist of scattering determinant line bundle square root with QCD θ-term removability.

1. **Scattering determinant line bundle and square root**

On extended spacetime–parameter space $Y = M \times X^\circ$, scattering determinant $\det S$ defines $U(1)$ line bundle, square root $\mathcal{L}_{\det}^{1/2}$ existence determined by relative cohomology class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$: $[K] = 0$ if and only if global smooth square root choice exists. If $[K] \neq 0$, square root undergoes sign flip along certain parameter loops, corresponding to topological anomaly on Null–Modular double cover.

2. **QCD sector and $\bar{\theta}$ absorption**

QCD θ-term and Yukawa phase jointly contribute to $\mathcal{L}_{\det}^{1/2}$ fiber coordinate. If $[K_{\mathrm{QCD}}] = 0$, can smoothly redefine square root and fermion fields, making physical $\bar{\theta}_{\mathrm{eff}}$ absorbed into global gauge choice, no longer appearing in observables; if $[K_{\mathrm{QCD}}] \neq 0$, irreducible CP-violating phase exists.

3. **Geometric interpretation of axion field**

Peccei–Quinn axion understandable as $U(1)$ fiber coordinate on $\mathcal{L}_{\det}^{1/2}$, effective potential under $[K] = 0$ twist structure possessing minimum point, corresponding to $\bar{\theta}_{\mathrm{eff}} = 0$. Therefore, strong CP problem in unified universe equivalent to universe choosing $[K] = 0$ topological sector, conclusion consistent with topological picture in axion reviews [13,14].

Detailed argument see Appendix D.

4.6 Theorem 3.6: Gravitational Wave Dispersion and Observational Upper Bound

Proof directly uses velocity constraints from GW multi-messenger observations.

1. **Dispersion relation and group velocity**

From
$\omega^2 = c^2k^2[1 + \beta_2(k\ell_{\mathrm{cell}})^2 + \dots]$, obtain

$$v_g = \frac{\partial\omega}{\partial k} \simeq c\Big[1 + \frac{3}{2}\beta_2(k\ell_{\mathrm{cell}})^2 + \dots\Big],$$

thus

$$\frac{v_g}{c} - 1 \simeq \frac{3}{2}\beta_2 (k\ell_{\rm cell})^2.$$

2. **GW170817 constraint conversion**

GW170817 and GRB170817A arrival time difference about 1.7 s, combined with propagation distance about 40 Mpc, provides $|v_g/c - 1| \lesssim 10^{-15}$ constraint [15]. Substituting frequency $f \sim 10^2\,\mathrm{Hz}$, $k \sim 2\pi f/c$, yields

$|\beta_2|\ell_{\rm cell}^2 \lesssim \mathcal{O}(10^{-3})\,\mathrm{m}^2$. Further combining EFT constraints on higher-order dispersion parameters, can compress this upper bound to $\ell_{\rm cell} \lesssim 10^{-30}\,\mathrm{m}$ magnitude [16].

3. **Overlap with black hole entropy constraint**

Theorem 3.1 requires $\ell_{\rm cell}^2 \sim 4G \log d_{\rm eff} \sim \ell_{\rm P}^2$, corresponding to $\ell_{\rm cell} \sim 10^{-35}\,\mathrm{m}$ order. Several orders of magnitude comfortable overlapping interval exists between two constraints, ensuring compatibility.

Detailed estimates see Appendix E.

4.7 Theorem 3.7: Non-Emptiness Construction

Proof adopts explicit prototype parameter region construction.

1. Choose $\ell_{\rm cell}^\star \sim 10^{-35\pm1}\,\mathrm{m}$, let $d_{\rm grav}^\star \sim 2^n$ making $\eta_{\rm grav}^\star = \log d_{\rm eff}^\star \sim \mathcal{O}(1)$, satisfying Theorem 3.1 black hole entropy constraint and compatible with Theorem 3.6 dispersion upper bound.
2. Construct QCA band model carrying $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge structure, implementing paired energy bands in UV region, adjusting high-energy spectrum making $\int_0^{E_{\rm UV}} E^2 \Delta\rho(E)\,\mathrm{d}E = 0$, satisfying Theorem 3.2 vacuum energy sum rule. IR scale take $E_{\rm IR} \sim 10^{-3}$–$10^{-2}\,\mathrm{eV}$, naturally obtaining residual approaching observational cosmological constant magnitude [5,6].
3. In flavor–QCA module implement A_4, S_4 etc. flavor–symmetries and appropriate defects, choose M_D, M_R texture making $\mathsf{M}_\nu = -M_D^T M_R^{-1} M_D$ spectrum and PMNS parameters match existing oscillation data [7–9].
4. Select local gate set and random update rules, making QCA generate high-order unitary design in finite region, satisfying Theorem 3.4 axiomatically chaotic condition [10–12].
5. Let topological class $[K]^\star = 0$, particularly $[K_{\rm QCD}]^\star = 0$, introduce Peccei–Quinn symmetry in QCD–axion module, making axion vacuum automatically align to $\bar\theta_{\rm eff}^\star = 0$, satisfying Theorem 3.5 topological constraint [13,14].
6. Adjust gravity–QCA local gate parameters, making low-order dispersion coefficients $\beta_{2n}^\star$ satisfy existing gravitational wave dispersion constraints [15,16].

Under these choices obtained parameter point family $p^\star$ is example required by Theorem 3.7, thus unified solution space non-empty.

5 Model Apply

This section without introducing specific numerical fitting provides representative "prototype universe" model, explicitly placing six constraints on same parameter table, discussing geometric and physical meanings.

5.1 Prototype Parameter Table

Select following representative parameter magnitudes:

1. QCA lattice spacing and time step:

$$\ell_{\mathrm{cell}} \sim \ell_{\mathrm{P}} \sim 10^{-35}\,\mathrm{m}, \quad \Delta t \sim \ell_{\mathrm{P}}/c.$$

2. Local Hilbert space dimension:

$$d_{\mathrm{grav}} \sim 4, \quad d_{\mathrm{gauge}} \sim \mathcal{O}(10), \quad d_{\mathrm{matter}} \sim \mathcal{O}(10),$$

total dimension $d_{\mathrm{cell}} \sim 10^2$–$10^3$.

3. Unified time scale density sector structure: In UV region, paired band structure implements $\int_0^{E_{\mathrm{UV}}} E^2 \Delta\rho(E)\,\mathrm{d}E = 0$, in IR region $E \lesssim 10^{-2}\,\mathrm{eV}$ leaves small residual.

4. flavor–QCA seesaw module: Light neutrino mass spectrum concentrated at $m_\nu \sim 10^{-2}\,\mathrm{eV}$ order, PMNS matrix approaching global fit central values of current experimental data [7–9].

5. QCA chaos parameters: In typical volume $L^3 \sim (10^2$–$10^3)$ cells region, local random circuit depth $d_{\mathrm{circuit}} \sim \mathcal{O}(10)$ can generate approximate Haar distribution, ETH implementing on time scale $t_{\mathrm{th}} \sim \mathcal{O}(L)$.

6. Topological class and CP: $[K] = 0$, axion vacuum aligning to $\bar{\theta}_{\mathrm{eff}} = 0$.

7. Dispersion coefficients: $|\beta_2|(k\ell_{\mathrm{cell}})^2 \lesssim 10^{-15}$, higher-order β_{2n} negligible in existing frequency bands.

5.2 Geometric and Physical Intuition

In this prototype model, universe at minimum scale is highly local QCA:

* Its gravitational cell degrees of freedom organized as area lattice points near horizon, black hole entropy corresponding to entanglement entropy area law of these cells;

* Its band structure satisfies highly refined spectral sum rule in UV region, making vacuum energy UV divergence automatically canceled by internal geometric–field theory structure;

* Its flavor–sector implements light neutrino mass and PMNS texture via seesaw mechanism and flavor–symmetry defects;

* Its local chaos ensures macroscopic thermalization and time arrow;

* Its topological sector choice demotes strong CP problem to line bundle square root choice problem;

* Its discrete structure residual dispersion effects suppressed to indistinguishable magnitude by existing gravitational wave observations.

This model not unique fit to existing data, but proof that structurally self-consistent unified universal object class exists, unifying six unified physics problems as consistency conditions.

6 Engineering Proposals

Unified matrix–QCA universe only directly visible at extreme scales, but its structure can be engineered and tested in multiple "universal analog systems". This section provides several directions.

6.1 QCA Simulator and ETH, Dispersion Testing

1. **Many-body local random circuit simulator**

Construct two- or three-dimensional QCA-type circuits on superconducting qubits, ion traps, or Rydberg arrays, implementing approximate unitary design local gate updates, measuring local operator distribution and time evolution on energy eigenstates, directly testing Theorem 3.4 ETH mechanism.

2. **Discrete dispersion relation simulation**

Implement effective models with QCA dispersion relation on platforms like photonic crystals, cold atom optical lattices, through precise measurement of wavepacket propagation velocity and frequency-dependent phase velocity, verifying even-order dispersion and odd-term suppression structure.

6.2 Vacuum Energy Sum Rule Condensed Matter Analogy

In systems like topological insulators, superconductors, and superfluids, spectral function sum rules have been used to analyze vacuum energy and topological response [5,6]. Can design multi-band systems whose high-energy band structure satisfies condition similar to

$\int E^2 \Delta\rho(E)\,\mathrm{d}E = 0$, through measuring low-energy effective potential barrier height or critical temperature, indirectly testing macroscopic magnitude sensitivity to sum rule.

6.3 Flavor Structure and Quantum Information Implementation

Implement three-level system as "neutrino flavor" in optical or superconducting quantum circuits, through controllable coupling implement seesaw structure and flavor–holonomy, measure corresponding interference and oscillation patterns, as quantum information analogy of PMNS holonomy.

6.4 Topological Line Bundle and Axion Simulation

Using synthetic dimensions and topological qubits, can implement line bundle structure with $\mathbb{Z}_2$ twist, through controlling external parameter evolution along closed path, measuring square root phase sign flip or not, simulating influence of $[K]$ triviality and non-triviality on phase structure, providing topological simulation platform for strong CP problem.

7 Discussion (risks, boundaries, past work)

This section discusses unified constraint system applicability range, potential shortcomings, and relation to existing work.

7.1 Limitations of Discrete Universe Hypothesis

This paper takes QCA universe as underlying structure, assumption in tension with traditional continuous spacetime description. Although multiple causal discrete field theory and QCA research indicate many aspects of GR and QFT can be reconstructed in continuum limit [17–19], currently no experimental evidence directly pointing toward space–time discreteness, lacking completely rigorous mathematical theorem proving all necessary structures implementable in QCA.

7.2 Realizability of Cosmological Constant Sum Rule

Theorem 3.2 requires high-energy band structure satisfying refined spectral sum rule, potentially difficult to naturally implement in concrete models. Although works by Kamenshchik, Volovik et al. demonstrated spectral function sum rules can appear in certain theories and condensed matter analog systems [5,6], elevating it to entire universe's fundamental QCA band structure still has significant construction difficulty.

7.3 Complexity of Flavor–QCA Module

Embedding specific flavor–symmetries (like A_4, S_4 etc.) and seesaw texture into QCA cells, in principle feasible, but in practice constructing model that can both reproduce metal and hadron physics while maintaining QCA overall locality and chaos extremely complex. This paper in Theorem 3.3 only provides structural level equivalence, not detailed Lagrangian or gate composition.

7.4 Universality of ETH Hypothesis

Axiomatically chaotic QCA hypothesis directly borrows experience from random circuits and quantum chaotic systems [10–12], but actual universal QCA if exists likely simultaneously carries refined gauge structure and long-range constraints, these structures may weaken ETH universality at certain energies and scales. Need to conduct ETH testing for specific gauge QCA models.

7.5 Dynamical Origin of Topological Class $[K]$

Although Theorem 3.5 equates strong CP problem with $[K] = 0$, does not explain why universe chose this topological sector. Need to introduce some selection principle on more fundamental category or measure structure, making $[K] = 0$ statistically or variationally preferred sector, otherwise this condition still resembles statement "universe chose suitable vacuum".

7.6 Future of Gravitational Wave Dispersion Constraints

Current gravitational wave velocity and dispersion constraints mainly from several tens to several hundred Hz frequency band [15,16]. If future can observe gravitational waves at higher frequency bands, will further compress parameter region of β_{2n} and ℓ_{cell}, even directly rule out certain QCA models. Unified matrix–QCA universe needs repeatedly tested under continuously updated observational data.

8 Conclusion

This paper within framework of unified time scale, boundary time geometry, THE-MATRIX matrix universe, and QCA universe, uniformly rewrites six seemingly independent puzzles–black hole entropy, cosmological constant, neutrino mass and flavor mixing, ETH, strong CP problem, and gravitational wave dispersion–as six consistency constraints on finite-dimensional structural parameter family. Core parameters include QCA lattice spacing and time step, cell Hilbert space and decomposition, unified time scale density projection in different sectors, relative cohomology class $[K]$, axiomatically chaotic QCA local design parameters, and gravity–QCA dispersion coefficients.

Through establishing black hole entropy relation with lattice spacing–entropy density, spectral function sum rule controlling cosmological constant, flavor–QCA seesaw equivalence with PMNS holonomy, axiomatically chaotic QCA implementing ETH, $[K] = 0$ equivalence with strong CP, and observational upper bound on ℓ_{cell} under even-order dispersion gravitational waves, this paper provides unified theorem set. Finally proves parameter point family class exists simultaneously satisfying all constraints, thus six problems possess non-empty common solution space in unified universal framework.

This result does not mean six problems completely solved, but indicates in concrete unified structure, they can be viewed as six consistency conditions same universal object must satisfy. Future work can advance in following directions: construct more concrete gauge–QCA models, provide dynamical selection mechanism for $[K] = 0$, implement above structures in quantum simulation and condensed matter analog systems, continuously contract unified solution space following gravitational wave and cosmological observation progress.

Acknowledgements, Code Availability

Authors thank related literature and community for systematic research in black hole entropy, cosmological constant, neutrinos, ETH, strong CP problem, and gravitational wave dispersion, providing background and reference for this paper. Numerical prototype and QCA simulation of unified matrix–QCA universe framework described in this paper implementable on general quantum simulation platforms and tensor network libraries, code structure simple, but not published accompanying this article.

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A Black Hole Entropy and Gravity–QCA Lattice Spacing Constraint Details

Assume horizon band sublattice $\Gamma_{\mathrm{H}} \subset \Lambda$ embedding satisfies

$N_{\mathrm{H}} = A/\ell_{\mathrm{cell}}^2 + O(A^0)$. Horizon Hilbert space

$\mathcal{H}_{\mathrm{H}} \simeq \mathcal{H}_{\mathrm{grav}}^{\otimes N_{\mathrm{H}}}$, consider typical pure state distribution under energy shell constraint, using classic result of random pure state entanglement entropy under Haar measure, obtains cross-horizon entanglement entropy expectation

$$\mathbb{E}[S_{\mathrm{ent}}] = N_{\mathrm{H}} \log d_{\mathrm{eff}} + O(1), \quad d_{\mathrm{eff}} \le d_{\mathrm{grav}}.$$

Error term $O(1)$ controlled by local energy constraint and finite size correction, not growing with area A.

On other hand, relation between generalized entropy–Einstein equations indicates, in gravitational back-reaction equilibrium state, generalized entropy

$$S_{\mathrm{gen}} = \frac{A}{4G} + S_{\mathrm{out}}$$

variation equivalent to Einstein equations, leading area term $A/(4G)$. In QCA universe requiring $S_{\mathrm{ent}}(\Sigma_{\mathrm{H}}) = A/(4G) + O(A^0)$, then leading term comparison obtains

$$\frac{\log d_{\mathrm{eff}}}{\ell_{\mathrm{cell}}^2} = \frac{1}{4G},$$

i.e.

$\ell_{\mathrm{cell}}^2 = 4G \log d_{\mathrm{eff}}$. When $d_{\mathrm{eff}} \sim \mathcal{O}(1\text{–}10)$, this relation fixes ℓ_{cell} at Planck length order.

B Cosmological Constant Windowed Sum Rule Tauberian Proof Outline

Consider heat kernel difference

$$\Delta K(s) = \int_0^\infty \mathrm{e}^{-s\omega^2} \Theta'(\omega)\,\mathrm{d}\omega,$$

where $\Theta'(\omega) = \Delta\rho_\omega(\omega) = -\xi'(\omega)$. Introduce logarithmic window kernel $W(\ln(\omega/\mu))$, let its Mellin transform satisfy

$$\int_0^\infty \omega^{2n} W(\ln(\omega/\mu))\,\mathrm{d}\ln\omega = 0, \quad n = 0, 1.$$

Using Mellin–Laplace correspondence, can prove in $s \to 0^+$, $\mu \sim s^{-1/2}$ limit, small s heat kernel finite part equivalent to windowed spectral integral

$$\int \Theta'(\omega) W(\ln(\omega/\mu))\,\mathrm{d}\ln\omega$$

thereby rewriting vacuum energy UV divergence as windowed spectral integral. If QCA band structure satisfies sum rule in UV region

$\int_0^{E_{\rm UV}} E^2 \Delta\rho(E)\,\mathrm{d}E = 0$, then s^{-2} and s^{-1} terms in small s heat kernel expansion vanish, leaving only finite term related to IR part, corresponding to $\Lambda_{\rm eff} \sim E_{\rm IR}^4$.

C Axiomatically Chaotic QCA and ETH Design Estimate

Assume on finite region Ω, QCA restriction U_Ω viewable as depth d local random circuit, local gate composition generating approximate Haar distribution after several layers. In Hilbert space dimension $D \sim \exp(s|\Omega|)$, Haar random unitary matrix element statistics give:

* Diagonal elements: $\mathbb{E}[\langle\psi_n| O_X |\psi_n\rangle] = \langle O_X\rangle_{\rm micro}$, variance $\sim \mathcal{O}(D^{-1})$;
* Off-diagonal elements: $\mathbb{E}[|\langle\psi_m| O_X |\psi_n\rangle|^2] \sim \mathcal{O}(D^{-1})$.

Levy concentration inequality gives

$$\mathbb{P}\Big(\big|\langle\psi_n| O_X |\psi_n\rangle - \langle O_X\rangle_{\rm micro}\big| > \epsilon\Big) \le C\exp(-c\epsilon^2 D),$$

since $D \sim \exp(s|\Omega|)$, this probability exponentially decays with region volume. Extend Haar random case to approximate unitary design circuit, obtaining ETH form in Theorem 3.4.

D Relative Cohomology Class $[K] = 0$ and Strong CP Suppression

Scattering determinant line bundle square root $\mathcal{L}_{\rm det}^{1/2}$ twist class $[K] \in H^2(Y, \partial Y; \mathbb{Z}_2)$ understandable as $\mathbb{Z}_2$ single-valuedness obstruction on Null–Modular double cover. When $[K] \neq 0$, closed parameter loop $\gamma \subset X^\circ$ exists, making square root choice undergo sign flip along γ, corresponding to certain CP-odd phase non-removable via local field redefinition.

Embedding QCD θ-term and Yukawa phase into $\mathcal{L}_{\rm det}^{1/2}$ fiber coordinate, if $[K_{\rm QCD}] = 0$, global smooth square root choice exists, can absorb physical $\bar\theta$ via global phase redefinition, making strong CP violation disappear; if $[K_{\rm QCD}] \neq 0$, no such possibility, axion field also cannot completely eliminate CP violation via local potential minimum. Therefore, view $[K] = 0$ as unified universe consistency condition, strong CP problem rewritten as topological background choice problem.

E Gravity–QCA Dispersion and LIGO/Virgo Constraint Estimation

Consider dispersion relation

$$\omega^2 = c^2 k^2\big[1 + \beta_2 (k\ell_{\rm cell})^2\big],$$

group velocity

$$v_g = \frac{\partial\omega}{\partial k} \simeq c\Big[1 + \frac{3}{2}\beta_2 (k\ell_{\rm cell})^2\Big],$$

thus

$$\Big|\frac{v_g}{c} - 1\Big| \simeq \frac{3}{2}|\beta_2|(k\ell_{\rm cell})^2.$$

In GW170817 frequency band, $f \sim 100\,\mathrm{Hz}$, $k \sim 2\pi f/c \sim 10^{-6}\,\mathrm{m}^{-1}$, observation provides

$|v_g/c - 1| \lesssim 10^{-15}$. Substituting obtains

$$|\beta_2|\ell_{\text{cell}}^2 \lesssim \mathcal{O}(10^{-3})\,\text{m}^2.$$

If assuming $\beta_2 \sim \mathcal{O}(1)$, then $\ell_{\text{cell}} \lesssim 10^{-1.5}\,\text{m}$ this upper bound nearly unconstrained on cosmological scale. However combined with higher-frequency gravitational waves or other high-energy astrophysical process constraints on k^4 type dispersion parameter (usually expressed as effective mass or cutoff scale M_*) [15,16], can compress ℓ_{cell} upper bound to near Planck scale several orders of magnitude above, thereby forming non-empty overlapping window with lower bound from black hole entropy.