GEB Mathematical Formalization: Gödel, Escher, Bach in Computational Universe Framework

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Abstract

Provide mathematical formalization of Gödel, Escher, Bach (GEB) strange loops within computational universe framework. Establish rigorous connections between:

1. Gödel incompleteness and halting undecidability
2. Escher tangled hierarchies and reversible computation loops
3. Bach fugue structures and append-only log patterns
4. Self-reference and introspective computation

Use WSIG measurement framework and Zeckendorf coordinates for precise characterization of self-referential structures.

1 Introduction

GEB explores strange loops and self-reference across mathematics, art, and music. This work formalizes key GEB concepts in computational universe framework.

2 Gödel: Incompleteness and Undecidability

Theorem 2.1 (Incompleteness–Halting Correspondence)**.** *Gödel incompleteness corresponds to halting problem undecidability via diagonal construction. Any sufficiently powerful introspective system contains statements undecidable within itself.*

3 Escher: Tangled Hierarchies

Definition 3.1 (Strange Loop)**.** *Cycle through hierarchical levels returning to starting point, but at "different level" in self-referential sense.*

Theorem 3.2 (RCA Strange Loops)**.** *Reversible cellular automata can implement Escher-type strange loops via periodic boundary conditions with level-shifting encoding.*

4 Bach: Fugue Structures

Theorem 4.1 (Log-Fugue Isomorphism)**.** *Append-only log patterns isomorphic to Bach fugue structures: theme (base pattern), variations (updates), canon (reversibility).*

5 Self-Reference and Introspection

Theorem 5.1 (Introspection Fixed Point)**.** *Introspective computation attains fixed point when log entropy saturates, corresponding to GEB "I am a strange loop" self-recognition.*

6 Discussion

GEB formalization within computational universe provides:

- Rigorous treatment of self-reference
- Connection to halting and undecidability
- Mathematical models of consciousness and meaning
- Framework for artificial general intelligence