Abstract

Construct unified framework of time and observer with boundary as fundamental stage. Assume in gauge field state without observer readings, boundary data of spacetime and fields are determined, but no preferred time parameter exists; time only appears as "geodesic parameter" when observer performs readings and their attention selects boundary section family.

First, on scattering theory end, based on scale identity

$$\varphi'(\omega)/\pi = \rho_{\mathrm{rel}}(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,Q(\omega),$$

define observable time scale as unified reading of boundary scattering phase, relative spectral density, Wigner–Smith time delay trace.

Second, on geometry and gravity end, based on Einstein–Hilbert–Gibbons–Hawking–York action variation, view boundary as "generator converting normal flux into readable quantities," characterize "generalized entropy geodesics" via extremality of generalized entropy S_{gen} and quantum focusing inequality.

Introduce "attention sections" and "section universe": observer's attention generates time axis calibrated by scale identity and selected by generalized entropy geodesics through selecting boundary section family and accessible algebra; different observers' time axes correspond to different section families on same boundary geometry, explaining phenomena like double-slit interference where "observation changes pattern" geometrically expressible as "entering different sections."

Finally provide series of testable predictions and engineering proposals, including group delay–generalized entropy alignment experiments based on electromagnetic scattering networks, black hole information readings based on island formula analog, double-slit experiment paths controlling classical–quantum transition by adjusting observation resolution.

Keywords: Boundary Time Geometry; Attention Sections; Generalized Entropy Geodesics; Wigner–Smith Time Delay; Gibbons–Hawking–York Boundary Term; Quantum Extremal Surface; Page–Wootters Relational Time; Double-Slit Interference

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1 Introduction and Historical Context

1.1 Observer-less Time and Boundary Priority

In standard formulations of general relativity and quantum field theory, time often viewed as pre-given external parameter: in general relativity as parametrized geodesic on manifold, in quantum theory as parameter generating unitary evolution. However, considering gravitational action with boundary, pure Einstein–Hilbert bulk action insufficient for well-defined variation; must add Gibbons–Hawking–York boundary term

$$\mathcal{S}_{\mathrm{GHY}} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} \epsilon\sqrt{h}\,K$$

to eliminate normal derivative contributions, making variation well-defined when boundary geometry fixed. This fact shows: in presence of boundary, boundary not passive "endpoint" but generative condition for bulk geometry and dynamics.

In quantum gravity and holography research, generalized entropy

$$S_{\text{gen}}[\sigma] = \frac{\text{Area}(\sigma)}{4G\hbar} + S_{\text{out}}[\sigma]$$

becomes fundamental quantity describing boundary sections, where σ is codimension-two section, S_{out} is quantum field entropy on its exterior. Quantum Extremal Surface (QES) given by stationary condition of S_{gen}, core for constructing gravitational system entropy and island formula.

In parallel, Quantum Focusing Conjecture (QFC) proposes: along any null geodesic bundle orthogonal to σ, "quantum expansion" of generalized entropy monotonically non-increasing, unifying Bousso light-sheet bound with classical focusing theorem, implying quantum null energy condition.

These structures jointly point to idea: in "gauge field state" without observer readings, boundary geometry and generalized entropy flow are determined, but no preferred time parameter exists; everything just "block universe" geometry and entropy functions. For time to become "experiential quantity," requires additional input: observer and their readings.

1.2 Scattering Scale, Phase, Time Delay

In scattering theory, Wigner–Smith time delay matrix

$$Q(\omega) = -i\, S(\omega)^{\dagger} \partial_{\omega} S(\omega)$$

describes group delay of incident state at frequency ω relative to free propagation. Its trace satisfies scale identity with total scattering phase $\Phi(\omega) = \arg\det S(\omega)$ derivative and spectral shift function:

$$\varphi'(\omega)/\pi = \rho_{\text{rel}}(\omega) = (2\pi)^{-1} \operatorname{tr} Q(\omega),$$

where $\varphi(\omega) = \frac{1}{2}\Phi(\omega)$, ρ_{rel} is Krein spectral shift density.

This type of scale identity shows: for given scattering system, observable time scale essentially determined by boundary scattering matrix $S(\omega)$, not by bulk preset "absolute time" variable. Time reading is derived quantity from boundary spectral data.

1.3 Relational Time and Internal Reference Frame

In quantum time problem, Page–Wootters mechanism proposes: in overall static state $|\Psi\rangle$ satisfying constraint $\hat{H}_{\text{tot}}|\Psi\rangle = 0$, if interpreting certain subsystem as "clock," conditional state relative to clock pointer results can satisfy effective evolution equation, thus "time" appears as parameter of system–clock correlation. This mechanism represents "internal reference frame" perspective: time not pre-given but generated by observational correlation and conditioning.

These developments jointly form this paper's historical background: boundary term ensures variational consistency, generalized entropy and QES/QFC provide dynamical constraints on "boundary sections," scattering scale identity gives unified proxy for boundary time readings, Page–Wootters relational time schemes show time derivable from conditional probabilities and attention selection.

1.4 Core Questions and Approach

This paper attempts to answer three related questions:

1. In gauge field state without observer readings, how does boundary "exist without time"? 2. When observer performs readings, how does attention select section family on boundary, thus forced to move along geodesic, how does this geodesic become observer's "time axis"? 3. In this sense, how are pattern changes in double-slit interference, coexistence of "all possible sections" in universe unified as "section universe" geometry, with observer merely selecting path with attention?

For this, introduce unified framework of "Boundary Time Geometry," under rigorous model assumptions give provable theorems, characterizing time axis as "attention geodesic" calibrated by scale identity and selected by generalized entropy extremality and focusing conditions.

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2 Model and Assumptions

2.1 Underlying Geometry and Boundary Data

Let $(\mathcal{M}, g_{\mu\nu})$ be Lorentzian manifold with boundary $\partial\mathcal{M}$, outward unit normal vector n^μ, induced boundary metric h_{ab}. Gravitational action taken as

$$\mathcal{S}_{\rm grav} = \frac{1}{16\pi G}\int_{\mathcal{M}} \sqrt{-g}\,(R-2\Lambda)\,d^4x + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} \epsilon\sqrt{h}\,K\,d^3y,$$

where K is extrinsic curvature trace, $\epsilon = \pm 1$ distinguishes spacelike/timelike boundaries. Matter field bulk action denoted $\mathcal{S}_{\rm matter}$, total action $\mathcal{S} = \mathcal{S}_{\rm grav} + \mathcal{S}_{\rm matter}$.

Assume bulk field dynamics given by renormalizable or effective field theory, making well-defined path integral or partition function $Z[\partial\mathcal{M}]$ exist under given boundary conditions. In semiclassical limit, dominant contribution for given boundary data $(\partial\mathcal{M}, h_{ab}, K_{ab})$ comes from stationary geometries satisfying Einstein equations.

2.2 Boundary Observable Algebra and States

Let $\mathcal{A}_\partial$ be observable algebra attached to boundary $\partial\mathcal{M}$, taken as well-defined local operator algebra's C^* or von Neumann algebra; states represented by positive normalized linear functionals $\omega : \mathcal{A}_\partial \to \mathbb{C}$. Given partition $\partial\mathcal{M} = R \cup \bar{R}$, can define local subalgebra $\mathcal{A}_R \subset \mathcal{A}_\partial$ and its restricted state ω_R.

In case with gravitational dual, assume for each well-defined boundary region R exists codimension-two section σ_R (like HRT surface or QES) as corresponding "entropy boundary," whose generalized entropy $S_{\rm gen}[\sigma_R]$ equals von Neumann entropy of ω_R plus geometric area term.

2.3 Generalized Entropy Geodesics and Quantum Focusing

Given smooth codimension-two section family $\{\sigma(\lambda)\}$ deformed along null geodesic bundle N parametrized by λ. Define generalized entropy

$$S_{\rm gen}(\lambda) = \frac{\mathrm{Area}[\sigma(\lambda)]}{4G\hbar} + S_{\rm out}[\sigma(\lambda)].$$

Quantum Focusing Conjecture proposes generalized entropy's "quantum expansion"

$$\Theta(\lambda) = \frac{4G\hbar}{\mathrm{Area}[\sigma(\lambda)]} \frac{dS_{\mathrm{gen}}}{d\lambda}$$

monotonically non-increasing along N: $d\Theta/d\lambda \le 0$, implying quantum null energy condition and Bousso light-sheet bound in appropriate limits.

Call curve families satisfying following conditions "generalized entropic geodesics": • Each section $\sigma(\lambda)$ makes S_{gen} stationary under small transverse variations: $\delta S_{\mathrm{gen}}[\sigma(\lambda)] = 0$; • Quantum expansion monotonically non-increasing along family direction: $d\Theta/d\lambda \le 0$.

In semiclassical limit, above conditions degenerate to area extremality and classical focusing theorem, aligning with usual extremal surface and geodesic concepts.

2.4 Scattering Scale Mother Ruler and Time Reading

Consider scattering process on boundary with S-matrix spectral decomposition $S(\omega)$. Define scale mother ruler

$$\kappa(\omega) = \frac{\varphi'(\omega)}{\pi} = \rho_{\mathrm{rel}}(\omega) = \frac{1}{2\pi} \operatorname{tr} Q(\omega),$$

where $Q(\omega) = -iS(\omega)^\dagger \partial_\omega S(\omega)$. Scale identity provable under appropriate trace-class perturbation conditions via relation between Birman–Krein spectral shift function and Wigner–Smith time delay matrix.

In engineering context, can implement time scale calibration by measuring group delay spectrum $(2\pi)^{-1} \operatorname{tr} Q(\omega)$ or scattering phase derivative $\varphi'(\omega)/\pi$. Here view $\kappa(\omega)$ as "boundary time scale mother ruler"—scale structure already existing in observer-less background but not yet selected as specific time axis.

2.5 Observer, Attention Sections, Time Axis

Observer $\mathcal{O}$ characterized by three elements:

1. Timelike curve (candidate worldline) $\gamma : \tau \mapsto x^\mu(\tau)$, parameter τ not necessarily identified with proper time;
2. "Attention section" family $\{\Sigma_\tau\}$, each Σ_τ codimension-one section embedded in $\partial\mathcal{M}$, transverse to $\gamma(\tau)$;
3. Accessible algebra $\mathcal{A}^\Lambda_{\Sigma_\tau} \subset \mathcal{A}_\partial$ and corresponding conditional state $\omega^\Lambda_{\Sigma_\tau}$ attached to Σ_τ, where Λ represents observation resolution or coarse-graining scale.

"Attention" formalized as family of completely positive trace-preserving maps $\mathcal{E}_{\tau,\Lambda} : \mathcal{A}_\partial \to \mathcal{A}^\Lambda_{\Sigma_\tau}$ plus Bayes-type update from global state ω to conditional state $\omega^\Lambda_{\Sigma_\tau}$.

Observer's time axis selected by joint condition of τ and γ, making τ simultaneously satisfy: • Maintain linearity relative to scale mother ruler $\kappa(\omega)$ reading: $\tau \sim \int \kappa(\omega)\, w(\omega)\, d\omega$, where $w(\omega)$ weight function related to experimental apparatus; • Corresponding section family $\{\sigma(\tau)\}$ (determined by Σ_τ and bulk geometry) satisfies generalized entropy geodesic conditions.

In this sense, time axis is "generalized entropy geodesic selected by attention and calibrated by scattering scale mother ruler parameter."

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3 Main Results (Theorems and Alignments)

This section gives three main results. Proofs expanded in Section 4 and appendices.

Theorem 3.1 (Observer-less Time and Boundary Block Structure). *Let $(\mathcal{M}, g_{\mu\nu})$ satisfy Einstein equations, given gravitational–matter boundary data on $\partial\mathcal{M}$ making generalized entropy $S_{\text{gen}}(\lambda)$ well-defined for any codimension-two section family $\{\sigma(\lambda)\}$. If not selecting any attention section family $\{\Sigma_\tau\}$ and accessible algebra family $\{\mathcal{A}^{\Lambda}_{\Sigma_\tau}\}$, then:*

1. Can define global scale mother ruler $\kappa(\omega)$, but no selected single time parameter τ exists, only "scale field" without preferred direction;

2. Any description of "evolution" restatable as automorphism on $(\partial\mathcal{M}, \mathcal{A}_\partial, \omega)$, equivalent to coordinate rescaling on block universe.

Thus in observer-less sense, boundary has determined geometric–entropy–scale structure but no preferred time axis.

Theorem 3.2 (Attention Geodesic Theorem). *Under assumptions 2.1–2.5, assume exists timelike curve candidate family γ and attention map $\mathcal{E}_{\tau,\Lambda}$ satisfying:*

1. For each τ, codimension-two section $\sigma(\tau)$ determined by $\gamma(\tau)$ and boundary data makes $S_{\text{gen}}[\sigma(\tau)]$ stationary for small transverse variations;

2. Along null geodesic bundle determined by $\sigma(\tau)$, generalized entropy's quantum expansion strictly monotonically non-increasing;

3. Time reading τ given by scale mother ruler, i.e., exists non-negative integrable weight function $w(\omega)$ making

$$\frac{d\tau}{d\lambda} = \int \kappa(\omega)\, w_\lambda(\omega)\, d\omega,$$

where w_λ only depends on $\mathcal{E}_{\lambda,\Lambda}$ and boundary scattering structure.

Then exists one-to-one correspondence (modulo monotonic reparametrization) between: • "Attention time axis" families $\{\gamma, \tau, \mathcal{E}_{\tau,\Lambda}\}$ satisfying above 1–3 conditions; • Timelike geodesic families extremizing action functional $\mathcal{J}[\gamma] = \int L_{\text{eff}}(x, \dot{x})\, d\tau$, where L_{eff} is effective Lagrangian including generalized entropy and scale terms.

In other words, attention time axes satisfying scale identity and generalized entropy geodesic conditions equivalent to geodesics on certain effective geometry.

Theorem 3.3 (Section Universe and Observation Branches). *Under Theorem 3.2 setting, consider family of different observation resolutions $\{\Lambda_i\}$ and corresponding attention section families $\{\Sigma^{(i)}_\tau\}$, defining attention time axis families $\{\gamma^{(i)}, \tau^{(i)}\}$. If for each pair (i,j) exists completely positive map family between sections*

$$\Phi_{ij,\tau} : \mathcal{A}^{\Lambda_i}_{\Sigma^{(i)}_\tau} \to \mathcal{A}^{\Lambda_j}_{\Sigma^{(j)}_{\tilde{\tau}}}$$

satisfying relative entropy monotonicity

$$S(\omega^{(i)}_{\Sigma^{(i)}_\tau} \| \omega^{(i)}_{\Sigma^{(i)}_\tau} \circ \Phi_{ij,\tau}) \geq 0,$$

then can:

1. Embed all attention time axes into "section universe" space $\mathfrak{S}$, whose points are equivalence classes $[\Sigma, \mathcal{A}^{\Lambda}_{\Sigma}, \omega^{\Lambda}_{\Sigma}]$;

2. Understand each observer's experiential history as path on $\mathfrak{S}$; differences between observers correspond to selecting different geodesic families on $\mathfrak{S}$.

When applying this structure to double-slit interference experiment, cases with/without path detector correspond to different attention paths in section universe: appearance/disappearance of interference fringes is geometric expression of "section selection" not "single wavefunction collapse."

Proposition 3.4 (Macroscopic Gravity–Microscopic Time Delay Bridge)**.** *In weak field, slowly varying gravitational background, Shapiro delay produced by macroscopic gravitational potential Φ_{grav} along observer worldline writable at first-order approximation as effective contribution of boundary scattering phase, thus unifying macroscopic gravitational time delay with microscopic Wigner–Smith time delay on scale mother ruler. Can interpret "gravitational potential" as geometric effect of scale mother ruler integration, not existence of additional force.*

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4 Proofs

This section provides proof skeletons of Theorems 3.1–3.3 and Proposition 3.4; detailed calculations postponed to appendices.

4.1 Theorem 3.1

Key to Theorem 3.1 is separating "existing structure" from "selected parameter."

1. From given boundary data and bulk field equations, can define path integral $Z[\partial\mathcal{M}]$ or corresponding boundary state $|\Psi_\partial\rangle$ containing all allowed bulk histories.

2. For any codimension-two section family $\{\sigma(\lambda)\}$, can calculate generalized entropy $S_{\text{gen}}(\lambda)$ via semiclassical methods, obtaining scalar function family.

3. Scale mother ruler $\kappa(\omega)$ completely determined by boundary scattering matrix $S(\omega)$, which determined by $(\mathcal{M}, g_{\mu\nu}, \text{boundary conditions})$, independent of observer existence.

However, combining $\kappa(\omega)$ and $S_{\text{gen}}(\lambda)$ into "one-dimensional time parameter" τ requires selecting: • Specific curve family $\{\gamma\}$; • Specific section family $\{\Sigma_\tau\}$ and accessible algebra family; • Specific weight function family $w_\lambda(\omega)$.

Without these selections, $\kappa(\omega)$ and $S_{\text{gen}}(\lambda)$ just fields and functions on block universe, no preferred parametrization. Any "evolution" viewable as automorphism of boundary state ω, i.e., coordinate rescaling, similar to overall static state $|\Psi\rangle$ situation in Page–Wootters formalism. Thus Theorem 3.1 directly holds.

[Proofs of Theorems 3.2, 3.3, and Proposition 3.4 condensed due to length...]

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5 Model Applications

This section demonstrates framework applications in three typical scenarios: black hole information, cosmological redshift, double-slit interference.

5.1 Black Hole Information and Island Formula

In black hole evaporation phenomenon, late-time Hawking radiation von Neumann entropy grows with time until Page time vicinity reaching peak, then declines; overall curve given by “island formula”

$$S(R)=\min_{\mathrm{QES}}\left\{\frac{\mathrm{Area}(\partial I)}{4G\hbar}+S_{\mathrm{bulk}}(I\cup R)\right\},$$

where R is radiation region, I is island region.

In this framework: • Distant observer’s attention section family $\{\Sigma_\tau^{\mathrm{out}}\}$ mainly supported near infinity boundary, corresponding QES penetrates black hole interior; • Free-fall observer’s section family $\{\Sigma_\tau^{\mathrm{in}}\}$ local to near-horizon region, corresponding QES different from external sections.

Two observer types’ attention time axes correspond to different paths in section universe $\mathfrak{S}$. For external observer, island region information “encoded in radiation”; for internal observer, same information exists locally inside black hole. Information preserved but distributed across different section families, relieving expressive tension of “where is information.”

5.2 Cosmological Redshift and Time Scale Rescaling

In standard FRW cosmology, comoving observer’s redshift–time relation controlled by scale factor $a(t)$, frequency satisfies $\omega\propto a^{-1}$. In this framework, scattering matrix on cosmological horizon or large-scale boundary evolves with time, its phase derivative gives scale mother ruler $\kappa(\omega)$. Cosmological redshift restatable as:

• Background geometry determines scale mother ruler rescaling with frequency; • Observers at different cosmic epochs select different weight functions $w_\lambda(\omega)$ via respective attention section families and accessible frequency bands; • Time axis comparison becomes scale integral comparison between different observers, not flow of some absolute time.

This restatement provides unified boundary–scattering–scale interpretation for “cosmic time,” facilitating analysis of relations among gravity–dark energy–information flow.

5.3 Section Selection in Double-Slit Interference

Consider standard double-slit experiment. Without introducing path detector, incident particle state $|\psi_{\mathrm{in}}\rangle$ evolves through two slits to detection screen boundary state $|\psi_{\mathrm{out}}\rangle$, intensity distribution controlled by interference terms. Adding path detector achievable via entanglement with environment, leading to effective state on screen approximately mixed state without interference terms.

In this paper’s framework:

• Without detection, attention section family $\{\Sigma_\tau^{\mathrm{free}}\}$ corresponds to accessible algebra maintaining two-slit coherence, its generalized entropy structure allowing cross-slit coherence;

• With detection, attention map $\mathcal{E}^{\mathrm{path}}_{\tau,\Lambda}$ compresses accessible algebra to path-distinguishable subalgebra, its relative entropy structure and environmental information flow determining new section family $\{\Sigma_\tau^{\mathrm{decoh}}\}$.

Two section families correspond to different paths in section universe $\mathfrak{S}$. Experimenter by choosing whether to enable detector actually selects their attention time axis in section universe, not directly "changing universe state." Universe structurally accommodates all section paths; observation merely selects one.

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6 Engineering Proposals

6.1 Scale–Entropy Alignment Experiment Based on Electromagnetic Scattering Networks

Construct multi-port electromagnetic scattering network, use vector network analyzer to measure frequency-dependent S-matrix $S(\omega)$, obtain Wigner–Smith matrix $Q(\omega)$ and its trace via numerical differentiation. Simultaneously, partition network interior into "accessible region" and "environment region," control generalized entropy S_{out} distribution by varying coupling strength and lossy element configuration.

Measurement scheme includes: 1. Estimate von Neumann or Rényi entropy approximation of output state via port power spectrum; 2. Verify correlation between group delay spectrum changes and entropy changes, test scale identity stability in controlled environment; 3. Implement double-slit-like scenarios via tunable switches and path-identifiable detection ports, compare relations among group delay–entropy–pattern under different attention sections in same network.

6.2 Page–Wootters Scheme Based on Cold Atoms and Quantum Optics

Implement Page–Wootters relational time experiments on cold atom or trapped-ion platforms, use part of degrees of freedom as "clock," another part as "system," prepare overall approximately static constrained state. Reconstruct system's effective evolution via conditional measurement of clock results.

Combining this framework: • Approximate experimental apparatus optical boundary as $\partial\mathcal{M}$; • Reconstruct state changes on boundary observable algebra via quantum tomography, verify consistency of time parametrizations corresponding to different attention section families; • Measure conditional state entropy changes, test approximate conditions for generalized entropy geodesics.

6.3 Gravitational Lensing and FRB Time Delay

Use multi-image time delays and frequency dispersion characteristics of Fast Radio Bursts (FRBs) or FRB-like pulse signals to analyze relations between macroscopic gravitational potential and microscopic frequency scale. In this framework, observer's attention section families at different image surfaces correspond to different boundary scattering paths, their group delay differences on scale mother ruler directly giving macroscopic gravitational effects. Combining statistics of numerous events, potentially give astronomical-level tests of Proposition 3.4 bridge relation.

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7 Discussion (Risks, Boundaries, Past Work)

7.1 Risks and Applicability Domain

Main risks of this framework:

• Dependence on QES and island formula makes strict applicability limited to states with semiclassical gravitational duals; for general QFT backgrounds, construction of generalized entropy geodesics and section universe may be insufficient.

• Assumption of QFC currently still conjectural status; though partial proofs and strong support examples exist, lacks completely general mathematical proof.

• Scale identity strictness in scattering theory depends on specific trace-class perturbation conditions and spectral assumptions; requires careful extrapolation for complex open systems.

Thus this framework should be understood as unified picture under triple conditions of "semiclassical gravity + well-behaved scattering + controllable observation"; extrapolation to strong quantum gravity, non-local interactions, highly non-equilibrium systems requires additional arguments.

7.2 Relation to Existing Work on Observer–Time–Measurement

This paper closely connected to Page–Wootters, internal reference frames and relational time, quantum measurement and decohering histories directions. This paper's features:

• View "time" not just as relational parameter but as "attention time axis jointly selected by boundary scale mother ruler and generalized entropy geodesics";

• Via scale identity unify scattering phase, group delay, spectral shift function as proxies for time reading, placing macroscopic gravity and microscopic scattering in same boundary framework;

• Via section universe $\mathfrak{S}$ construction, provide formalization for "many-worlds" "branching" based on boundary and information geometry, without introducing additional ontology.

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8 Conclusion

This paper in unified framework of boundary–scattering–generalized entropy–attention gives provable and testable formulations for three types of questions: "how does boundary exist without time in absence of observer" "how does observer attention generate time axis" "how does double-slit interference reflect section selection."

Core elements include:

• Scale mother ruler $\kappa(\omega) = \varphi'(\omega)/\pi = \rho_{\rm rel}(\omega) = (2\pi)^{-1}\,\mathrm{tr}\,Q(\omega)$ as unique proxy for time reading;

• Generalized entropy geodesics and quantum focusing as geometric criterion for "attention time axis";

• Section universe $\mathfrak{S}$ as structural space accommodating all attention paths; observer's experience merely one geodesic on it.

In this picture, universe structurally simultaneously contains "all possible sections"; observer's attention selects among these sections a path consistent with generalized entropy–

scale, this path perceived as "time axis." Time thus no longer independent reality but product of stable alignment between boundary scale and attention sections.

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References

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A Scale Identity and Wigner–Smith Matrix Rigorous Derivation Outline

[Birman–Krein formula, trace relations...]

B Generalized Entropy Geodesics and Quantum Focusing

[QFC forms, Euler–Lagrange equations...]

C Section Universe and Relative Entropy Monotonicity

[Section objects and morphisms, quotient space construction...]

D Page–Wootters Formalism Attention Time Axis

[Clock–system decomposition, conditional states...]