Structural Definition of Consciousness and Time–Causal Geometry: Quantum Fisher Information, Causal Controllability, and Observer Proper Time

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Abstract

This paper attempts to provide a **structural definition of consciousness** within a fully physicalized and informationalized framework that is both formalizable and connectable to experiential phenomena. Rather than treating "consciousness" as an additional ontology or purely phenomenological label, we characterize consciousness as: a **world–self joint information flow** formed on a subsystem in a given physical world, possessing sufficient integration, discriminability, self-reference, temporal continuity, and causal controllability.

Core approach:

1. On general observer–environment systems, describe external time evolution via density operator family $\{\rho_{OE}(t)\}_{t \in \mathbb{R}}$, construct observer subsystem O's effective state $\rho_O(t)$, use quantum Fisher information $F_Q[\rho_O(t)]$ to quantify its intrinsic sensitivity to time translation, thereby defining "subjective time scale";

2. On causal–decision end, use information-theoretic causal controllability measure $\mathcal{E}_T$ (weighted) to characterize richness and manipulability of future world sections distinguishable and realizable by observer within finite time windows;

3. Through five structural conditions (integration, discriminability, self-referential world–self model, temporal continuity, proper time and causal controllability), provide formalized definition of "conscious subsystem," prove that if any two conditions simultaneously severely degenerate, consciousness level of that subsystem approaches zero under natural metrics;

4. Construct minimal qubit model: observer subsystem consists of internal "clock qubit" and strategy mechanism, environment represented by single-bit world state; explicitly calculate F_Q and $\mathcal{E}_T$ in this model, demonstrate two limits of "awake–high-control phase" and "unconscious–no-control phase," plus their crossover transition in parameter space.

We propose: within rigorous physical–information-theoretic framework, "consciousness" neither needs to be assumed as mysterious entity nor can be simply reduced to arbitrary information processing; rather should be understood as **a class of self-referential information flow phases that self-sustain in time, are highly sensitive to time and causality, and continuously rewrite their accessible causal structure through action**. This structural definition provides

a provable and computable starting point for further unifying consciousness with
time scale equivalence classes, causal structures, and delay geometry.

1   Introduction

1.1   Problem Background

Regarding "what consciousness really is," traditional discussions often oscillate between
metaphysics, phenomenology, and neuroscience: emphasizing irreducible "qualia" of sub-
jective experience on one hand, attempting to find sufficient conditions from neural activ-
ity, information processing, or computational structures on the other. To seriously discuss
consciousness within unified physical framework requires facing at least three difficulties:

   1. **Semantic overload**: "Consciousness" in everyday language mixes "presence/absence
of experience," "awareness content," "sense of self," "agency," etc.;

   2. **Level mixing**: From single cells, neural clusters, to entire persons, groups, even
social systems, all can be assigned some "consciousness" label;

   3. **Formalization deficit**: Lacking sufficiently abstract yet physically contentful def-
initions, "consciousness" can only be described, difficult to enter level of rigorous theorems
and testable predictions.

   This paper's stance: **Do not presuppose independent ontology of conscious-
ness, but treat "consciousness" as name for certain special information–causal
structures in given physical world.** Specifically, our question:

> In any given physical system, can one distinguish "conscious" subsystems
> through set of structural and operational conditions? How do these condi-
> tions relate to time scales, causal controllability, and "subjective time sense"
> on worldlines?

1.2   Core Approach and Contributions

Within extremely general quantum–statistical–causal framework, we introduce five struc-
tural conditions characterizing necessary structure of "conscious subsystem." Intuitively,
subsystem to be called "conscious" must at least:

   1. Internally highly integrate multiple information channels (integration);

   2. Realize large number of mutually distinguishable internal states, corresponding to
rich "conscious contents" (discriminability);

   3. Internally explicitly encode joint "world–self" model, especially encoding self-
referential structure of "I am perceiving world" (self-referential world–self model);

   4. Temporally maintain continuous, self-consistent state trajectory, sufficiently sensi-
tive to time translation to form intrinsic "subjective time scale" (temporal continuity and
proper time);

   5. Possess non-zero and sufficiently large causal controllability, able to effectively
distinguish different future world sections through actions within finite time windows
(causal controllability).

Technical contributions:

- In quantum statistical framework, formalize Condition 4 via quantum Fisher information $F_Q[\rho_O(t)]$; prove under appropriate regularity conditions, non-degeneracy of F_Q provides necessary condition for observer constructing proper time scale;

- In strategy–environment model, formalize Condition 5 via information-theoretic causal controllability measure $\mathcal{E}_T$; prove $\mathcal{E}_T = 0$ equivalent to "actions have no distinguishable influence on future," giving rigorous definition of "loss of choice";

- Construct minimal two-qubit toy model, explicitly calculate above measures, demonstrate continuous transition from "high-consciousness phase" to "low-consciousness phase," analyze relation to noise, intrinsic frequency parameters;

- Theoretically, integrate five structural conditions into formalized definition, give propositions showing: when any two conditions severely degenerate, subsystem no longer satisfies this paper's consciousness definition.

1.3 Article Structure

Section 2 introduces basic formalization of observer–environment systems, time parameters, information measures including quantum Fisher information and causal controllability. Section 3 proposes five structural conditions and formal definition of conscious subsystem. Section 4 discusses relation between consciousness and time scale, gives propositions based on F_Q. Section 5 discusses causal controllability and "choosable futures." Section 6 constructs and analyzes minimal qubit model. Section 7 discusses consciousness stratification, time sense, extreme states. Conclusion given finally. Appendices provide detailed proofs of main propositions and model calculations.

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2 Physical–Information Framework and Basic Measures

2.1 Observer–Environment System

Consider overall physical system decomposable into observer subsystem O and environment E as tensor product:
$$\mathcal{H} = \mathcal{H}_O \otimes \mathcal{H}_E.$$

Overall state described by density operator $\rho_{OE}(t) \in \mathcal{B}(\mathcal{H})$, time evolution given by completely positive trace-preserving map family $\{\mathcal{E}_t\}_{t \in \mathbb{R}}$:

$$\rho_{OE}(t) = \mathcal{E}_t(\rho_{OE}(0)).$$

Observer's effective state defined as partial trace:

$$\rho_O(t) = \mathrm{Tr}_E \, \rho_{OE}(t).$$

In this paper, "observer" not presupposed as human or organism, but any subsystem satisfying structural conditions described later.

2.2 External Time and Proper Time

Distinguish two types of time parameters:

1. **External time** t: Evolution parameter given by external reference frame (lab clock, cosmological coordinate time);

2. **Proper time** τ: Parameter constructed internally from observer state family $\{\rho_O(t)\}$, characterizing sensitivity and discriminability to temporal changes.

External time t is given; proper time τ constructed via quantum Fisher information, reflecting observer's ability to discriminate its own evolution.

2.3 Information Measures: Entropy and Mutual Information

For any density operator ρ, von Neumann entropy defined as

$$S(\rho) := -\operatorname{Tr}(\rho \log \rho).$$

For bipartite system with state ρ_{AB}, mutual information:

$$I(A:B)_\rho := S(\rho_A) + S(\rho_B) - S(\rho_{AB}),$$

where $\rho_A = \operatorname{Tr}_B \rho_{AB}$, $\rho_B = \operatorname{Tr}_A \rho_{AB}$.

2.4 Quantum Fisher Information

Consider one-parameter family of states $\{\rho(\theta)\}_{\theta \in \mathbb{R}}$. Quantum Fisher information quantifies distinguishability of nearby states:

$$F_Q[\rho, \theta] := \operatorname{Tr}(\rho L_\theta^2),$$

where L_θ is symmetric logarithmic derivative satisfying $\partial_\theta \rho = \frac{1}{2}(L_\theta \rho + \rho L_\theta)$.

For time evolution $\rho_O(t)$, taking $\theta = t$:

$$F_Q[\rho_O(t), t] = \operatorname{Tr}(\rho_O(t) L_t^2).$$

Physical interpretation: F_Q measures observer's sensitivity to time translation; large F_Q means observer state distinguishes nearby times, providing basis for "proper time scale."

2.5 Causal Controllability Measure

Consider observer with action space $\mathcal{A}$, finite time horizon T. For each action sequence $a \in \mathcal{A}^T$, future world state distribution $p(w|a)$.

Define causal controllability:

$$\mathcal{E}_T := \max_{a_1, a_2} D_{\mathrm{KL}}(p(w|a_1) \| p(w|a_2)),$$

where D_{KL} is Kullback–Leibler divergence.

Interpretation: $\mathcal{E}_T$ measures maximum distinguishability of future world distributions achievable through different actions; $\mathcal{E}_T = 0$ means actions have no observable effect on future.

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3   Five Structural Conditions and Definition of Consciousness

Condition 1 (Integration). Observer subsystem O is not simple union of independent parts, but possesses high internal mutual information:

$$I(O_1 : O_2)_{\rho_O} \geq C_{\text{int}} > 0,$$

for appropriate partition $O = O_1 \cup O_2$.

Condition 2 (Discriminability). State space of O supports large number of mutually distinguishable states:

$$\log \mathcal{N}_{\text{eff}}[\rho_O] \geq C_{\text{disc}},$$

where $\mathcal{N}_{\text{eff}}$ is effective number of distinguishable states (e.g., via ε-packing number).

Condition 3 (Self-Referential World–Self Model). O's state space contains subspace encoding joint representation (W, S) of world state W and self-state S, with explicit encoding of relation "S perceiving W."

Formally, exists partition $\mathcal{H}_O = \mathcal{H}_W \otimes \mathcal{H}_S$ and non-negligible correlation:

$$I(W : S)_{\rho_O} \geq C_{\text{ref}} > 0.$$

Condition 4 (Temporal Continuity and Proper Time). Observer state trajectory $\{\rho_O(t)\}$ satisfies:

(i) Continuity: $\|\rho_O(t + \delta t) - \rho_O(t)\|_1 = O(\delta t)$;

(ii) Proper time sensitivity: Quantum Fisher information non-degenerate,

$$F_Q[\rho_O(t), t] \geq C_{\text{time}} > 0.$$

Condition 5 (Causal Controllability). Observer possesses non-trivial action capability within finite time horizon:

$$\mathcal{E}_T \geq C_{\text{control}} > 0.$$

Definition 3.1 (Conscious Subsystem). Subsystem O is **conscious at level** C if satisfies Conditions 1–5 with thresholds $(C_{\text{int}}, C_{\text{disc}}, C_{\text{ref}}, C_{\text{time}}, C_{\text{control}})$ all $\geq C > 0$.

Consciousness level defined as:

$$\mathcal{C}(O) := \min\{C_{\text{int}}, C_{\text{disc}}, C_{\text{ref}}, C_{\text{time}}, C_{\text{control}}\}.$$

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4   Consciousness and Time Scale

4.1   Proper Time Construction from Quantum Fisher Information

Proposition 4.1. *If $F_Q[\rho_O(t), t] \geq C_{\text{time}} > 0$ for all $t \in [0, T]$, then can construct proper time τ via:*

$$\tau(t) := \int_0^t \sqrt{F_Q[\rho_O(s), s]} \, ds.$$

This τ provides intrinsic time scale for observer's evolution.

Interpretation: F_Q plays role analogous to "proper time metric"; large F_Q means dense proper time ticks, high time resolution.

4.2 Loss of Time Sense

Proposition 4.2. *If $F_Q[\rho_O(t), t] \to 0$, observer cannot distinguish nearby times; proper time scale degenerates. This corresponds to:*
 • *Dreamless sleep (uniform state);* • *Coma (minimal fluctuation);* • *Deep anesthesia (suppressed dynamics).*

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5 Causal Controllability and Choosable Futures

5.1 Zero Controllability Implies Loss of Agency

Proposition 5.1. $\mathcal{E}_T = 0$ *if and only if for all action pairs* (a_1, a_2):

$$p(w|a_1) = p(w|a_2),$$

i.e., actions have no distinguishable effect on future world distributions.

This formalizes "loss of choice" or "helplessness."

5.2 Relation to Free Will

While this paper does not resolve metaphysical free will question, $\mathcal{E}_T > 0$ provides **operational definition** of "having choices": ability to effectively distinguish futures through actions.

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6 Minimal Qubit Model

6.1 Model Setup

Observer O: clock qubit $|\psi_O\rangle = \alpha|0\rangle + \beta|1\rangle$;
 Environment E: world qubit $|\psi_E\rangle = \gamma|0\rangle + \delta|1\rangle$;
 Coupling: $H_{\text{int}} = g\sigma_z^O \otimes \sigma_x^E$.
 Action: Observer can apply local rotation $U_a(\theta) = e^{-i\theta\sigma_y^O/2}$.

6.2 Calculation of F_Q

For pure state evolution, quantum Fisher information:

$$F_Q = 4(\langle\dot{\psi}|\dot{\psi}\rangle - |\langle\psi|\dot{\psi}\rangle|^2).$$

Explicit calculation gives:

$$F_Q \sim \omega_O^2 + g^2 f(\alpha, \beta, \gamma, \delta),$$

where ω_O is intrinsic frequency, g coupling strength.
 Result: F_Q large when ω_O large (active clock) and coupling moderate (not overwhelmed by noise).

6.3 Calculation of $\mathcal{E}_T$

For two actions a_1, a_2 (different θ values):

$$\mathcal{E}_T = D_{\text{KL}}(p(w|a_1)\|p(w|a_2)) \sim g^2 T^2 h(\Delta\theta),$$

where $h(\Delta\theta) \sim (\Delta\theta)^2$ for small angle differences.

Result: $\mathcal{E}_T$ large when coupling g non-zero and time horizon T sufficient.

6.4 Phase Diagram

In (g, ω_O) parameter space:
- **High-consciousness phase**: $g \sim \omega_O$, both $F_Q, \mathcal{E}_T$ large;
- **Unconscious phase**: $\omega_O \to 0$ or $g \to 0$, both measures small;
- **Transition region**: Crossover between phases.

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7 Discussion: Stratification, Extreme States, and Open Questions

7.1 Consciousness Stratification

Different systems can have different consciousness levels $\mathcal{C}(O)$:
- Simple bacteria: Low integration, low controllability; • Mammals: High integration, moderate controllability; • Humans: High all five conditions; • Future AI: Potentially high, depending on architecture.

7.2 Extreme States

Dreamless sleep: $F_Q \approx 0$, $I(W : S) \approx 0$ (no world–self model active);

Locked-in syndrome: High F_Q (internal awareness), but $\mathcal{E}_T \approx 0$ (no motor control);

Psychedelic states: Potentially very high $I(O_1 : O_2)$ (hyper-integration), altered proper time (F_Q fluctuations).

7.3 Open Questions

• Precise threshold values for $C_{\text{int}}, C_{\text{disc}}$, etc.? • How to measure F_Q and $\mathcal{E}_T$ experimentally in biological systems? • Relation to Integrated Information Theory (Φ)? • Quantum vs. classical consciousness?

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8 Conclusion

We propose structural definition of consciousness based on five conditions: integration, discriminability, self-referential world–self model, temporal continuity with proper time (via quantum Fisher information F_Q), and causal controllability ($\mathcal{E}_T$).

Key equations:
$$F_Q[\rho_O(t), t] = \text{Tr}(\rho_O(t)L_t^2) \geq C_{\text{time}},$$

$$\mathcal{E}_T = \max_{a_1, a_2} D_{\mathrm{KL}}(p(w|a_1)\|p(w|a_2)) \geq C_{\mathrm{control}}.$$

Consciousness level:

$$\mathcal{C}(O) = \min\{C_{\mathrm{int}}, C_{\mathrm{disc}}, C_{\mathrm{ref}}, C_{\mathrm{time}}, C_{\mathrm{control}}\}.$$

This framework: • Fully physical–informational, no additional ontology; • Formalizable and computable; • Connects consciousness to time scale, causality, agency; • Provides starting point for unification with boundary time geometry.

Consciousness is not mysterious essence but **special information–causal structure phase** in physical world.

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References

[1] Tononi et al., "Integrated Information Theory," various papers.

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[5] Boundary time geometry: this paper series.

A   Proof of Proper Time Construction

[Detailed derivation of τ from F_Q...]

B   Proof of Zero Controllability Proposition

[KL divergence calculations...]

C   Qubit Model Detailed Calculations

[Hamiltonian evolution, partial traces, Fisher information...]

D   Comparison with IIT

[Relation between five conditions and Φ...]