Finance.

- 两种问题，一种 technical，一种 behiviarior.

- bond 就是债券

- future 就是期货，比如我明年这个时候加油，还是3块一个 litter.
  trader 要对十几个 bond 很熟.

- 所有的公司，在你眼里都是现金流一股而已.

- 三个大的证券中心：NYC，London，Tokoyo

Econ362.

- NPV internal rate of return.

- Options. Futures. Forwards. → commited to buy the asset At someday in future.

- Call $Max[S_T - K]$.

→ 有buyer l及有seller.

- Deriatives sitting on Top.

bond stock asset.

- hedging.

- Asset by Arbitage.

- $C(S, K) - P(S, K)$

- Pricing order.

↘ complete market.

- 所有方向之风险，都可以被 hedge

-

$t=0$ $t=1$

$S_0 \to S_u$ $F(S_H)$,

$S_0 \to S_d$ $F(S_L)$

$$\left(F(S_H), F(S_L)\right) \in \mathbb{R}^2$$

$$\boxed{k\cdot(1,1)}$$

↘ European options. 只能到期行权. → 只是 right 不是 obligation.

American —— : 到期及之前行权.

call 到那时买

put 卖.

↘ Future 必须买(卖)

↘ Risk-neutral probability.

第3个

$$P_t = E_t^{Q}[\quad]$$

$S_0 = 1$ $\frac{1}{2}$ (q) → $S_u = 2$

$\frac{1}{2}$ $(1-q)$ → $S_D = 0.5$

risk-premium $= 1.25 - 1 = 0.25$

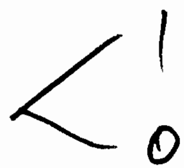
→ arbitrage. 无风险套利.

→ K strike price

明天约定的价钱.

$$(S-K)^+ = \begin{cases} S-K & S \geq K \\ 0 & S < K \end{cases}$$

$\underline{(S-K)^+}$



Δ units of S

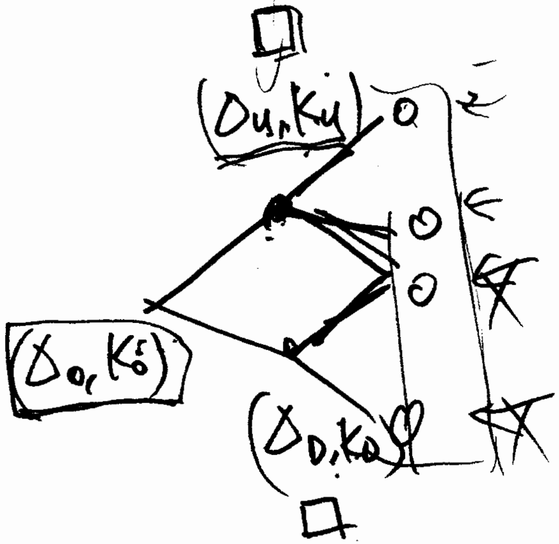
and ^{K} risk-free bond $\quad \frac{(\Delta, K)}{1 \quad 1+r_f = R_f}$

$$\begin{cases} 22 \cdot \Delta + K(1+r_f) = 1 \\ 18 \cdot \Delta + K(1+r_f) = 0 \end{cases} \qquad \Delta = \frac{1}{4}$$

得 $4\Delta = 1$

↘ replicate 复制一个 profolio.

↘ Arbitrage ~ 复制（已知之几个单元之 payoff）



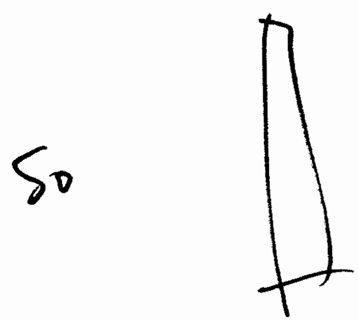
time series Chao's weight @ Today TMR

cross-section Tao's
Chao's weight.

panel

↘ contineous dist.
明天价格不只两种. distribution.

↘



→ Black-Scholes ~~call~~

→ price European call.

Finance 4.

$\sigma = \sigma(\underbrace{\quad\quad}_{n个影响之主数})$

假设 underlying 是 lognormal

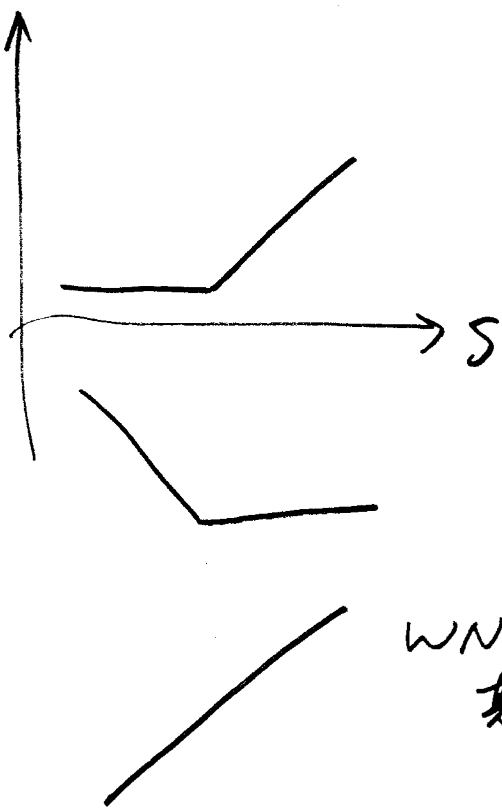
每一天价 是 lognormal

~~Call(S, K) + risk-free = put + S.~~

put call parity.

金融衍生品取决于另一个资产之价格

microsoft 股票.



→ $T = T - T_0$

行权 Maturity ↓ T, Now ↓ T_0

马尔科夫 — random walk.

之前的history完全体现于现在位置.

跟我怎么走到这里无关.

时间序列上.

WN i.i.d.

期望值.

Matingale:

条件 $S_t = E_t[S_{t+1}]$

Stationary 方差或标准差固定.

egodif 比较弱的独立.

隔得足够远.

Finance 5.

AR MA

$$= \rho . \quad X_t = \rho + a_1 X_{t-1} + a_2 X_{t-2} + \varepsilon_t$$ auto regressive.

(AR 2)

取决于过去的值

$$X_t = \rho + a_1 \varepsilon_t + \varepsilon_{t-1}$$

$$X_t = \rho + \quad \gamma(j) \equiv \mathrm{cov}(\varepsilon_t, \varepsilon_{t+j})$$

- efficificient - 资产价格 random walk.
  - 明天之期望 = 今天价格
    - 基于我们掌握之一切信息, 完全不到(?)

θ^*

Finance.

债券.

老王要盖房子,娶媳妇,没钱,怎么办.

借.

嫌借不好听,起名成债券.借你钱就是卖你债券.

由国家发行之债券,就是国家借你钱.回头还.

Finance 1.

→ volatility，是 stock 之波动，像 standard deviation.

→ eto integrition

$$dS_T = \int_0^T rS\,dt + \int_0^T \sigma S\,dW_t + \frac{1}{2}\int_0^T \sigma\,dt.$$

$\underbrace{S}$ 是 exponential (Brownian motion). $(dW_t)^2 = dt$

$dW_t\,dt = 0$

→ 什么是 dW_t,

$t \to 0$ 时，dW_t 是多少？

W_t 是 Brownian motion,

定义是 jump 是符合 standard distribution 的.

→ Complete market：

市场上所有之 ~~Bond, option~~ 金融衍生品 可以人用

我已知 process 的 n 个 $\underbrace{\text{多项式}}_{\text{bond}}$ 线性表出.

— eto integral 用的不过是微积分之符号. dW_t, $\int$. 不完全是微积分之意义. 但积分还是保是之义上的.

— 你需要记住一些 rule. 这些 rule 是人为总结出来的，是 work. 为什么这么来的，先不用管.

$(dW_t)^2 = dt$

$dW_t\,dt = 0$

这些性质，有点像 $\delta(x)$，于是你掌握了之后. setup 一堆 Rule，便可以进行运算了 这便是 storchastical calculus.

→ Matingale:

明天之期望等于今天的值.

$EX_{t+1} = X_t$.

→ risk neutral probability 不是真实之 physical probability.

→ 是主观选的一个 probability measure.

→ 这个 measure 下, 下一个之收益是有一个之 realization.

→ 利息是 risk-free 的, 因为不可能亏.

→ Markov. conditional Expectation.

X_{t+1} 只 depends on X_t 与 X_{t-1} 无关.

→ Portafrolio.

我要定价的是金融衍生F

G. H 我知道它今天之价格, 以及规律 (比如符合一个 random process).

我根据 G, H 今天价格组合成 F 今天价格, 得系数. 然后根据

这两 G, H 之 process 之性质, 可得 F 后面之价格.

Economy

现在中国不是没有钱，而是钱不够流通. 所以要开放金融市场.

借钱出去，自由订利率.

Finance.

- Date S&P. Return Variance Likelihood Volatility Autocorrelation
- Mathematics, Finance and Risk.

- The Black-scholes model assumes at least the market consists of one risky asset, usually called the stock, and one riskless asset, usually called the money market, cash, or bond.

Now we make assumptions on the assets (which explain their names):

- riskless rate: The rate of return on the riskless asset is constant and thus called risk-free interest rate.
- The instantaneous log returns of the stock price is an infinitesimal random walk with drift; more precisely, it is a geometric Brownian motion, and we assume its drift and volatility is constant.
- The stocks do not pay a dividend.

Assumptions on the market:

- There's no arbitrage opportunity (i.e. there's no way to make a riskless profit)

- The Black-Scholes equation is a partial differential equation, which describes the price of the option over time.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

- The key financial insight behind the equation is that one can perfectly hedge the option by buying and selling the underlying asset in just the right way and consequently eliminate risk.

- This hedge, in turn, implies there's only one right price for the option.

- The Black Scholes equation calculates the price of European put and call options.

  describes the price of the option over time.

- The value of a call option for a non-dividend-paying underlying stock in terms of the Black-Scholes parameter:

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

$$d_2 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] = d_1 - \sigma\sqrt{T-t}$$

The price of a corresponding put option based on put-call parity is:

$$P(S,t) = Ke^{-r(T-t)} - S + C(S,t)$$
$$= N(-d_2)Ke^{-r(T-t)} - N(-d_1)S.$$

$N(\cdot)$: cumulative dist. func of standard normal K: the strike price.

$T-t$: time to maturity. S: spot price of the underlying asset.

r: is the risk free rate.
σ: is the volatility of returns of the underly asset.

- A put-call parity defines a relationship between the price of a European call option and European put option.
  - both with the identical strike price and expiry.
  - Namely that a portfolio of a long call option and a short put option is equivalent to (and hence has the same value as) a single forward contract.
    - at the strike price and expiry.

$$C - P = D(\underbrace{F}_{\text{forward price}} - \underbrace{K}_{\text{strike price}})$$

- This is because if the price at expiry is above the strike price — call will be exercised
  below — put will be exercised.

- Spot contract, ~~is~~ is a contract of buying or selling a commodity, security or currency for settlement on the spot date, which is normally two business days after the trade date.
  - in contrast w/ a <u>forward</u> contrast or <u>future</u> contrast where contract terms are agreed now but delivery and payment will occur at a future date.

A security is a tradable financial asset of any kind.

↘ broadly categorized into:

debt securities: banknotes, bonds.

equity ——— : common stocks.

derivative —— : forwards, futures, options, swaps.

In finance, a future contract is a standardized contract between two parties to buy or sell a specified asset of standardized quantity and quality for a price agreed upon today with delivery & payment occurring at a <u>specified date</u>, making it a type of derivative instrument.

↘ the delivery date.

↘ The party agreeing to buy the underlying asset in the future, the "buyer" of the contract, is said to be "long".   long 就是买.

"seller" ———————————— "short".   short 就是卖. 出货.

$$dX_t = \mu_t dt + \sigma_t dB_t$$ ← Ito's lemma

a Wiener process.

A process S is said to follow a geometric Brownian motion with volatility σ and drift μ if it satisfies the stochastic differential equation.

$dS = S(\sigma dB + \mu dt)$, Apply ito's lemma with $f(S) = \log(S)$.

$$d\log(S) = f'(S)\,dS + \frac{1}{2} f''(S)\, S^2 \sigma^2 dt$$

$$= \frac{1}{S}(\sigma S\, dB + \mu S\, dt) - \frac{1}{2}\sigma^2 dt$$

$$= \sigma dB + \left(\mu - \frac{\sigma^2}{2}\right) dt.$$

$$\log(S_t) = \log(S_0) + \sigma B_t + \left(\mu - \frac{\sigma^2}{2}\right) t.$$

$$S_t = S_0 \exp\left(\sigma B_t + \left(\mu - \frac{\sigma^2}{2}\right) t\right).$$

In mathematical finance, the Greeks are the quantities representing the sensitivity of the prices of derivatives such as options to a change in underlying parameters.

The name is used because the most common of these sensitivities are often denoted by Greek letters.

Delta measures the rate of change of the theoretical option value w.r.t. changes in the underlying asset's price.

$$\Delta = \frac{\partial V}{\partial S}.$$

By put-call parity, long a call and short a put equals a forward F.

Vega measures sensitivity to volatility.

Vega is the derivative of the option value w.r.t. the volatility of the underlying asset.

$$\nu = \frac{\partial V}{\partial \sigma}$$

Vega is typically expressed as the amount of money per underlying share that option's value will gain or lose as volatility rises or falls by 1%.

Theta,

measures the sensitivity of the value of the derivative to the passage of time.

$$\Theta = -\frac{\partial V}{\partial \tau}.$$

Rho,

measures sensitivity to the interest rate:
It's the derivative of the option value w.r.t. the risk free interest rate.

$$\rho = \frac{\partial V}{\partial r}.$$

Gamma,

Second derivative of the value function w.r.t. the underlying price.

$$E(X_{n+1} \mid X_n, X_{n-1}, \cdots ; X_1) = X_n$$

A ~~stocha~~ martingale is a sequence of random variables (i.e. a stochastic process)

A model of a fair game that knowledge of past events never helps predict the mean of further wins.

绿宝书

Chap 3.

1

$$a^x = e^{x \ln a}$$

$$e^x = \lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\frac{d}{dx} \ln u = \frac{1}{u} u'$$

$$\frac{d}{dx} \tan x = \sec^2 x.$$

What's the derivative of $y = \ln x^{\ln x}$?

2.

Calculus & algebra.

2- Solution.

令 $u = \ln y = \ln x \cdot \ln(\ln x)$

$$\downarrow$$

$y = (\ln x)^{\ln x}$

$$\frac{du}{dx} = \frac{d}{dx}(\ln y) = \frac{1}{y} \cdot y'$$

同时

$$\frac{du}{dx} = \frac{d}{dx}\left[\ln x \cdot \ln(\ln x)\right]$$

$$= \frac{1}{x} \cdot \ln(\ln x) + \ln x \cdot \underline{\frac{d}{dx}\left[\ln(\ln x)\right]}$$

$$\frac{d}{dx} \ln u = \frac{u'}{u} = \frac{1/x}{\ln x}$$

$$= \frac{1}{x} \ln(\ln x) + \frac{1}{x}$$

$$= \frac{1}{x}\left[\ln(\ln x) + 1\right]$$

故 $\frac{dy}{dx} = \frac{y'}{y} = \frac{1}{x}\left[1 + \ln(\ln x)\right]$

$$y' = \frac{d}{dx}(y) = \cancel{\ln}\, \frac{y}{x}\left[1 + \ln(\ln x)\right] = \frac{(\ln x)^{\ln x}}{x}\left[1 + \ln(\ln x)\right]$$

Without calculating the numerical results, can you tell me which number is larger, e^{π} or π^{e}?

设 $e^{\pi} > \pi^{e}$

$\downarrow$

$\pi \ln e > e \ln \pi$

$\downarrow$

$$\frac{\ln e}{e} > \frac{\ln \pi}{\pi}$$

$$\left.\frac{\ln x}{x}\right|_{x=e} > \left.\frac{\ln x}{x}\right|_{x=\pi}$$

$f(x) = \frac{\ln x}{x}$ 递减才行

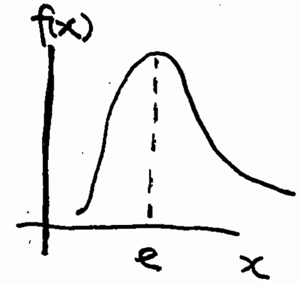
$$f'(x) = \frac{\frac{1}{x}\cdot x - \ln x\,(-1)\frac{1}{x^2}}{x^2}$$

$$= \frac{1 + \ln x \frac{1}{x^2}}{x^2}.$$

$e^{\pi} = 23.14$

$\pi^{e} = 22.46$

$f(x) = \frac{\ln x}{x}$



处 global maximum.

What's the integral of $\sec(x)$ from $x=0$ to $x=\pi/6$?

Math
green.
4.

$$\frac{d\sec x}{dx} = \frac{d(1/\cos x)}{dx} = \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

$$\frac{d\tan x}{dx} = \sec^2 x.$$

$$\frac{d(\sec x + \tan x)}{dx} = \sec x(\sec x + \tan x).$$

$$\frac{d \ln|\sec x + \tan x|}{dx} = \frac{1}{\sec x + \tan x} d(\sec x + \tan x) = \sec x.$$

$$\int_0^{\pi/6} \sec x \, dx = \int_0^{\pi/6} \frac{d \ln|\sec x + \tan x|}{dx} dx = \ln|\sec x + \tan x| \Big|_0^{\pi/6}$$

$$= \ln\left|\frac{1}{\cos\frac{\pi}{6}} + \tan\frac{\pi}{6}\right| - \ln\left|\frac{1}{\cos 0} + \tan 0\right|$$

$$= \ln\left|\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{3}\right| = \ln\sqrt{3}.$$

A. Suppose two cylinders each with radius 1 intersects at right angles and their center also intersects. What is the volume of the intersection?

Math
green
5.

If X is a standard normal r.v., $X \sim N(0,1)$,
what is $E[X \mid X>0]$?

Math
Green
6

Ito's lemma can be used to derive the Black-Scholes equation for an option.

Suppose a stock price follow a Geometric Brownian motion given by the stochastic differential equation:

$$dS = S\,(\sigma\, dB + \mu\, dt).$$

Then if the value of an

Finance

/

Risk-neautrual

其头翻挢过来,便是无风险.

- Brownian motion is the most commonly used stochastic process in finance, which underlies the black-soles methodology.

- Define the brownian motion first:

  A random (stochastic) process, $\{X_t : t \geq 0\}$, is a Brownian motion with parameters (μ, σ) if

  1. For $0 < t_1 < t_2 < \cdots < t_{n-1} < t_n$

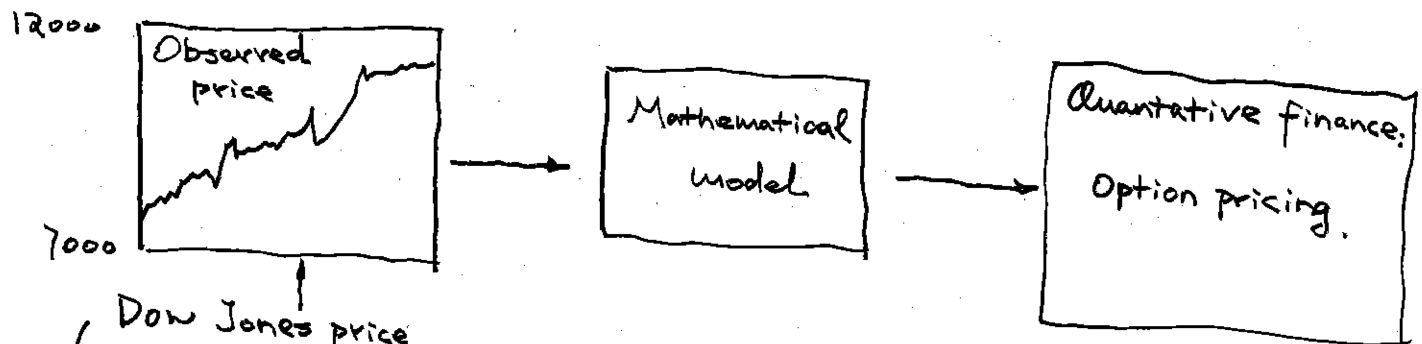
     $(X_{t_2} - X_{t_1}), (X_{t_3} - X_{t_2}), \cdots, (X_{t_n} - X_{t_{n-1}})$, are mutually independent.

     即每一步之跳跃都互相独立.

  2. For $s > 0$, $X_{t+s} - X_t \sim N(\mu s, \sigma^2 s)$ and.

  3. X_t is a continous function of t.

- Options: A type of insurance on the futurn behavior of the stock prices.
- Basic philosophy of quantitative finance is that we can introduce mathematical models, which mimic the behavior of the observed stock prices.



Dow Jones price.

From the Dow Jones price plotted above, we can see already two features of financial prices.

- On the long time scale, the price tends to grow roughly exponentially.
- On the shorter time scales, we see significant fluctuations.

One of the simplest possible models that captures these qualitative features is the geometric Brownian model:

$$S_t = S_0 \exp(\alpha t + \sigma W_t)$$

S_t: stock price at time t

S_0: ______________ 0

α: long term growth rate

σ: size of fluctuation.

- σ, α. can be fitted to observed price.

- Geometric brownian motion is not the be-all and end-all of financial modelling, but it is a good starting point. We'll see how to build more sophisticated models based on Brownian motion (stochastic differential eqn.)

  widely used.

- This class build a toolbox to manipulate, construct and perform computations in stochastic models based on Brownian motion.

- Will learn:

  how to build stochastic models on the basis of Brownian motion
  Ito integrals, stochastic differential equation, and how to manipulate them analytically (stochastic calculus, partial diffential Eqn, Girsanov's theorem)

  how to obtain explicit numerical solutions on the computer (Monte Carlo, numerical methods for stochastic and partial diffential eqn).

  Apply our techniques to some problems in the quantative finance (financial models, option pricing, optimal investment)

Stock pricing should be discreet, but usually people use continuous time model.

It changes instantenouly when there's transaction. occurring on the market.

- someone buys shares,
- someone put shares for sale.

- 但因为每天有很多很多之 transaction. 所以可以用 continous model 来处理.

- On the other hand, working in continuous time is much simpler than working with discrete time models:

Let $0 = t_0 < t_1 < t_2 < \cdots < t_k = T$, suppose we wanna eval the quantity,

$$\sum_{l=0}^{k-1} (t_l)^2 (t_{l+1} - t_l)$$

There's no simple way to evaluate the sum. However, if $t_{l+1} - t_l$ are very small (if k is very large)

$\exp(\mu) t$

$$\boxed{dS_t = \mu S_t\, dt + \sigma S_t\, dW_t}$$

$S_t = \exp(\mu t)$

$dS_t = \mu \cdot S_t\, dt.$

$$dC(t, S_t) = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} dS_t^2$$

$$= \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S_t} (\mu S_t dt + \sigma S_t dW_t)$$

$$+ \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} \Big(\underbrace{\mu^2 S_t^2 dt^2}_{0} + \underbrace{2\mu S_t \sigma S_t\, dt\, dW_t}_{0} + \underline{\sigma^2 S_t^2 dt} \Big)$$

$$= \underbrace{\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t} \mu S_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} \sigma^2 S_t^2 \right)}_{\equiv 0} dt + \frac{\partial C}{\partial S_t} \sigma S_t dW_t$$

dS_t

$\exp\left[\mu (T-t)\right] + \sigma \exp$

$S_t = \exp\left[\mu (T-t)\right] +$

$S_t = \exp(\mu t)$

$dt^{\frac{1}{2}}. \quad \frac{1}{2} t^{-\frac{1}{2}} . dt.$

$dS_t = \sigma S_t \frac{1}{2} \frac{1}{dW} dt.$

$(r - \frac{1}{2}\sigma^2)(T-t) + \sigma(W_T - W_t) \quad dS_t = \underline{\sigma S_t\, dW_t}.$

$\exp\left(\frac{\sigma}{2W}\right) t.$

$S_t = \exp(\sigma W_t).$

$\underline{dS_t = S_t \cdot \sigma\, dW_t.}$

$=$

$110 \cdot p + 90(1-p)$

$dS_t = \mu dt + \sigma dW_t$

$C(t, S_t) = e^{-rT}$

$dS_t = \mu S_t dt + \sigma S_t dW_t$

$dB_t = r B_t dt$

$\max(S_T - K, 0)$

$C(t, S_t)/B_t$ martingale.

$$dC = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} (dS_t)^2$$

$\mu S_t dt + \sigma S_t dW_t$

drift

$dS_t = \underline{rS_t dt} + \underline{\sigma S_t dW_t}$

$C(S_t, t)$

$S_t \cdot \sigma dW_t$

$$dC(S_t, t) = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} (dS_t)^2$$

$$= \frac{\partial C}{\partial t} dt + \underbrace{\frac{\partial C}{\partial S_t} (rS_t dt + \sigma S_t dW_t)}$$

$$+ \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} \cancel{r^2 S_t^2 dt^2 +} (\sigma^2 S_t^2 \underbrace{dW_t^2}_{dt})$$

$$= dt \underbrace{\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t} rS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} \sigma^2 S_t^2 \right)}_{= 0} + \frac{\partial C}{\partial S_t} \sigma S_t dW_t$$

$$\underline{dC = \underset{\downarrow 0}{m}\, dt + n\, dW_t}$$

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t} rS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S_t^2} \sigma^2 S_t^2 .$$

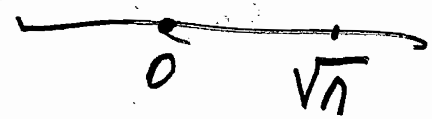
$(dt)^2$ —

$\underline{dW_t \cdot dt}$ — $(dt)^{\frac{3}{2}}$

$\underline{dW_t^2 = dt}$

$(dt)^{\frac{1}{2}}$

$dS \quad dW_t$



t

n

$(dW)^2 = dt$

$\underline{dW \cdot dt = (dt)^2 = 0}$

$g(dW_t, t)$ (dt)

$$d\left(\frac{C(t,S_t)}{B_t}\right)$$

$$B_t = B_0 e^{-rt}$$

$$d\left[\frac{C(t,S_t)}{B_0 e^{+rt}}\right] = \qquad\qquad dt$$

$$d\left(\frac{C(t,S_t)}{B_t}\right) = \cancel{\square}\; dt + \overset{W}{\cancel{A}}\, dW_t$$

$$B_t \sim B_0 e^{rt}$$

$$d\left(\frac{C}{B}\right) = \frac{\cancel{d}\, dC\cdot B - C\cdot dB}{B^2}. \qquad dB = r\cdot B_0\cdot e^{rt}$$

$$= \frac{m\,dt + n\,dW_t}{B} - \frac{C\cdot dB}{B^2}.$$

$$= \frac{1}{B_t}(m\,dt + n\,dW_t) - C\frac{r\cancel{B_t}}{B_t}\,dt.$$

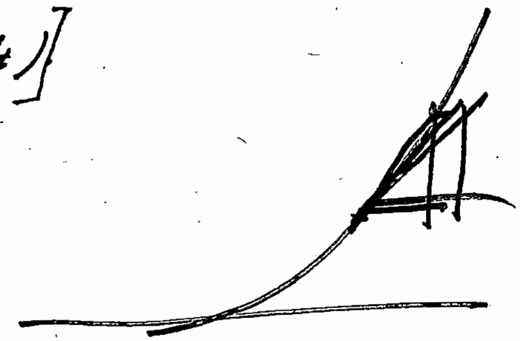
$$= \frac{1}{B_t}\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t}r + \frac{1}{2}\sigma^2\frac{\partial^2 C}{\partial^2 S_t} - Cr\right)dt + \frac{\partial C}{\partial S_t}\frac{\sigma S_t}{B_t}\,dW_t$$

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t}r + \underbrace{\frac{1}{2}\sigma^2\frac{\partial^2 C}{\partial^2 S_t}} - Cr = 0$$

$dS_t = r S_t\, dt + \sigma S_t\, dW_t$

$S_t =$

$S_T = S_t \exp\left[\left(r - \frac{1}{2}\sigma^2\right)(T - t) + \sigma (W_T - W_t)\right]$



$\boxed{df(S_T, t) =}$

$df(x, y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$

$\left(\mu - \frac{1}{2}\sigma^2\right) dt$

$N(0, 1)$

$E\left[d(\log S_T)\right] = E\left[\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\, dW_t\right]$ ~ Normal distribution

$E[\quad]$

$\frac{\partial f}{\partial S_T} \frac{\partial S_T}{\partial t} = \frac{1}{S_T} \mu S_T$

No
Risk-~~neutral~~ neutral Q

$$dS_T = \mu S_t\,dt + \sigma S_t\,dW_t \qquad \frac{\partial S}{\partial t}$$

$$df(S_T, t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S_T}dS_T + \frac{1}{2}\frac{\partial^2 f}{\partial S_T^2}d^2 S_T$$

μ

SDE

$S_T(t)$ dt dW_t $d^2 S_T = \sigma^2 S_T^2\,dt$

dt dW_t

$$\boxed{S_T = (\mu - \tfrac{1}{2}\sigma^2)}$$

$dS_T = S_T \cdot \sigma\,dW_t$

$S_T \sim \log \text{normal}$

$S_T = \{e^{\mu t + \sigma W_t}\}$

$d e^{\sigma W_t}$

$= \sigma e^{\sigma W_t}\,dW_t$

Normal

$S_T(t)$

$$d(\log S_T) = \mu dt + \frac{dS_T}{S_T} + \frac{1}{2}(-\sigma^2)dt$$

$=$

$S_T(t)$ dS_T $C = \log(S_T, t)$ $\mu dt + \sigma dW_t$

$$\frac{\partial C}{\partial S_T} = \frac{1}{S_T} \qquad \frac{\partial^2 C}{\partial S_T^2} = -\frac{1}{S_T^2}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_T}\frac{dS_T}{dt} = \mu.$$

$$f = S_T^2 - K$$

$$dS_T = \mu S_T\,dt + \sigma S_T\,dW_t$$

$$c = f(t, S_T) = S_T^2 - K$$

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S_T}dS_T$$

risk-neautral

$$\mathbb{E}\left(\frac{c}{S_T}\right)$$

c.

$$\frac{\partial c}{\partial t} + r\frac{\partial c}{\partial S_T} + \frac{1}{2}\sigma^2\frac{\partial^2 c}{\partial S_T^2} - rc = 0$$

$$\frac{\partial c}{\partial S_T} = \cancel{2} S_T$$

$$\frac{\partial^2 c}{\partial S_T^2} = 1$$

$$\underline{\frac{\partial c}{\partial t} + rS_T + \frac{1}{2}\sigma^2 - rc = 0}$$

$$c = r e^{r\cdot t} + \underbrace{rS_T + \frac{1}{2}\sigma^2}$$

$$\frac{\partial c}{\partial t} = \cancel{\ }$$

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S_t} r S_t + \frac{1}{2}\sigma^2 \frac{\partial^2 C}{\partial S_t^2} S_t^2 - rC = 0$$

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

$$d(\log S_t) = d\log(\mu S_t dt + \sigma S_t dW_t)$$

$$\frac{d\log x(t)}{dt} = \frac{1}{x(t)} \frac{dx(t)}{dt}.$$

$$d\log x(t) = \frac{1}{x(t)} dx(t).$$

$$= \frac{1}{\cancel{\mu S_t dt + \sigma S_t dW_t}\ S(t)} \mu$$

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

$$d\log S(t) = \frac{1}{S(t)} dS(t).$$

$$= \frac{1}{\cancel{\mu S_t dt + \sigma S_t dW_t}\ \cancel{S(t)}} \mu dt + \sigma dW_t.$$

$$df(t, S_t).$$

$$= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S_t} dS_t + \frac{1}{2} \frac{\partial^2 f}{\partial S_t^2} dS_t^2$$

$$= \frac{1}{\mu S_t^2} dt + \frac{1}{S_t} dS_t + \frac{1}{2}\left(-\frac{1}{S_t^2}\right) d$$

$$\sigma^2 S_t^2 dt.$$

$$f = \log S(t).$$

$$\mu dt + \sigma dW_t.$$

$$dS(t) = \mu S_t dt + \sigma S_t dW_t. \quad -\frac{1}{2}\sigma^2 dt.$$

$$d^2 S(t) = \sigma^2 S_t^2 dt.$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial S} \cdot \frac{1}{\frac{\partial S}{\partial t}} \qquad \frac{\partial f}{\partial S} \frac{\partial S}{\partial t}.$$

$$= \frac{1}{S(t)} \cdot \cancel{\frac{1}{\mu S_t}} \qquad (\mu - \frac{1}{2}\sigma^2) dt + \sigma dW_t.$$

$$\frac{\partial f}{\partial S_t} = \frac{1}{S(t)}.$$

$$\frac{\partial^2 f}{\partial S_t^2} = -\frac{1}{S_t^2}$$

- 所谓标普五百，

↘ 是 500支股票给一个权重，得到一个平均后的值.

$$\sum_{i=1}^{500} \text{权重} \times \text{价格}$$ 得到一个 weighted average.

故标普五百 (SP 500) 是一个平均值.

Finance Model 1

↘ 所以是一个时间序列

100个月

$[\, V_0 \quad V_1 \quad V_2 \quad \cdots \qquad V_{99} \,]$

如2000年 1月,标普500

- So to implement this stradegy, we usually buy the mini-futures on SP500 and finance with 3-month T bill, in such case, we can have a cash-neautual strategy, position is basically determined by our ~~a~~ prediction.

- value factor, of equity.
  - ↘ Why trailing 12 month.
  - ↘ Or how do you deal with earning seasons.

Finance Model 2

- So we use trailing 12-month earning yield to avoid the impact from seasonality.
  - earning seasons.
  - The sum of past 12-month earnings, which include 4 earning seasons.
  - Will smooth the seasonalities.

- alternatives of the value factor:

  book to ~~market~~ price value, which used in farma french model.
  or
  cash earning to price ratio, which could be a better indicator.

- 为何要用 value factor:

  two major reasons:

  1. We always want to buy stock when it's undervalued,
     or sell stock —— overvalued.
  2. value factor is an approximate for equity risk premium.
     which is basically telling if we should buy a little more stock.

→ how to calculate trailing 12 month earning yield.

The trailing 12 month earning yield is basically the average of last four quarters earnings divided by the price.

In theory, we should get earnings and calculate but in fact we use the trailing 12 month PE ratio → published number to calculate.

---

3 month change in 10 year yield, why?

We take it as risk-on or risk-off factors. when yield goes up, the return on the treasure goes down, people moves money from treasery to equity.

Why 10 years?

Cause that's the bench mark of the market, we can of course use 30 years or 2 years, but that's no as liquid as 10 years.

→ Any other alternatives?

Yes, of course, we can use other market risk-on/risk-off indicator, like volitily of sp-500 indice. But 3 month change in 10 year yield works better in our case.

↓ more smooth.

nfci, what it is.

The indicator was published by the chicago fed,
it basically takes a lot of market indicators from the equity market,
and uses PCA to determine the weights, and generate indicator
to reflect how much money people are borrowing from their broker or
dealer to investigate the stock market.

Why?

It's one of the best market sentiment indicator we find,
Besides, there could be a lot of events happening outside our model,
like the Chinese market could drop 20% overnight, and the
US market too. So we need to know people's positioning to reduce our position.
We understand it could be a little laging, but it's always better late
than nothing.

pairwise correlation.

Why.

how to calculate:

> We calculate the correlation of any two SP500 stocks
> get correlation for that,
>
> 500 ~~× 500~~ correlation
>
> $$\begin{array}{cc} & 500 \\ 500 & \begin{bmatrix} & \\ & \end{bmatrix} \end{array}$$
>
> 除去diagonal的500, 然后求平均. ~~只~~一个值.
>
> and we calculated average of this big matrix. → correlation 矩阵.

Frequency:
- monthly, could also be used in a daily basis.

Finance 5.

---

(4) Matlab regstats.
求线性回归系数与参数

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \Sigma$$

	Y: SP500 收益 forward return on the future	X_1: Value (trailing 12 month earning yield.)	X_2: 3 month change in 10 year bond.	X_3: NFCI leverage.	X_4: correlation
Jan-2000	0.1	3	1	0.7	0.6
Feb-2000	0.2	3.2	-2	0.72	0.64
	⋮	⋮	⋮	⋮	⋮
Jan-2010	0.3	5	5	0.9	0.8
Feb-2010	-0.1	5.1	5	0.1	0.81

各时间点一个值.

Finance.
1.

PROJECT

Trailing 12 month earnings yield on S&P500 → a value factor. help to predict the future returns on this market.

收益. 毛利润.

$$\text{earning yield} = \frac{\text{earning per share}}{\text{Price}}.$$

→ 一千万$, 一千万股.
平均每股赚一块.

→ Price 股票是10块.

→ yield $\frac{\$1}{\$10}$.

value factor.
股票: 此公司之盈利能力.

→ earning 每季度报一次
4, 7, 10, 1

$$1 + \frac{1}{0.9} + \frac{1}{0.81} + \cdots = 10.$$

折现

→ price 每还候. earning 12个月平均后波动小一些.
每季报一次.

Forward month futures excess return

Finance

2

是futures更值还是cash更值.

$r_F - r_0$

3month T bills.

短期国债利率.

$Y_t - X_{t+1}$

$$Y = A \; X$$

X_{1t}, X_{2t},

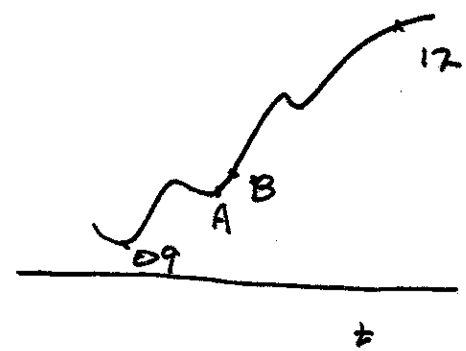
第一个指标 t时刻的值.

$$Y_{t+1} \sim \underbrace{X_{1t} + X_{2t}}_{\text{影响factor}} + \cdots \underbrace{Y_t + Y_{t-1}}_{\text{过去房价}}$$

房价

$$Y_t \sim X_{1t} + X_{2t}$$

time series regression.

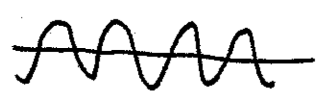


Y: price, level.

回 time series.

Y: return, difference.

~~time series~~



panel regression.

Finance 3.

3-month change on 10-year treasury yield

10年期国债收益率.

describe

risk on & off factor.

市场有风险. → 买国债. 风险小. 收益小.

— 小. → 买股票 — 大 — 大.

Hedge fund

有1亿. 借10亿.

投股票 short

bond 收益率 下降, bond return 为正.

yeild. price上升

yield 下降, (即 bond 的利息)

像 stock price 一样,

是交易出来的.

$$P = \sum \frac{\text{Coupon}}{1+Y}$$

price

yield.

$Y\downarrow\ P\uparrow,\ r = P_t - P_{t-1} > 0.$

NFCI leverage Indicator

National finance condition indicator.

NFCI 大, 越容易发生 crisis.

34 indice

$\boxed{\text{leverage}}$, credit, non-finacil leverage.

describe 大家是否crowd into this market.

describe whether investors are borrowing money from dealer/brokers to buy stocks.

~~???~~ intuition

Finance 4.

④ Avg of Pairwise Correlation between Index components.

$$\begin{array}{cc} 500 & \begin{bmatrix} 1 & \times & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \\ & 500 \end{array}$$

When all stocks moves up together, either some macro events happened. or ~~these~~ a single quant factor is driving the market.

一致性(很有风险)

比如 momentum
追涨杀跌.

[If the single quant factor got reverse, lose lots of money..
-Mean reversion.

有的股票: value 好
growth 好
dividend 好.

We do that on the expanding window basis,
so we do not need to worry about in/out of the sample issue.

1990 → data starts.

Finance 5

panel regression (简单的面板数据. 无时间性).

$$
\begin{array}{cc}
Y & X \\
\begin{bmatrix} 1990\ 2月 \\ \vdots \\ 2000\ 1月 \end{bmatrix} &
\begin{bmatrix} 1990\ 1月 \\ \vdots \\ 1999\ 12月 \end{bmatrix}
\end{array}
\rightsquigarrow \beta.
$$

window start.

regstats(Y, X, 'linear');

2000年2月 = β 2000年1月.

out of the sample 更靠谱

in the sample 经常 over estimate.

I can show a few statistics for back~~test~~

without any scaling, we can get an Information ratio → 0.7

IR → 0.7

$$\frac{\overline{Return} - R_c}{\sigma_R}$$

R_c → cash return

σ_R → return standard deviation.

annualized return.

story of scaling.
if you scale it up,
you can get any number.

regressive.

- more time to deeper research. 2

R^2.

how to evaluate error

error variance

time

- We don't manage money not exposed to. markets.

risk factor stationary model, price model.

10,000 floating

C++

stackoverflow the stack.

recursion sum.

~~accumulate~~. ~~loop~~ without using the loop. for / while.

4 tasks.

- building models. risk factor.

- productions matlab. interacting with

- talking about your mod

- supporting

Stats

↳ a desk

two drawers.

~~two~~ gold

a gold / silver

gold

$$P(A|B) = \frac{\overset{\frac{1}{2}\times 1}{P(A\cap B)}}{P(B)} = \frac{2}{3}.$$

two drawer

The one is gold

P(B)

$\frac{1}{2}\times 1 + \frac{1}{2}\times\frac{1}{2}.$

Statistics.

VAR:

$$Y_t = X_t + X_{t-1} + \cdots + Y_{t-1} + \cdots + Y_{t-2} + \Sigma$$

要求 stationary.

若不 stationary，则加一项.

Unit roots test
判断 stationary
的标准.

$$Y_t = \beta_1(\underbrace{\alpha_1 Y + \alpha_2 X}_{\text{cointegration}}) + X_t + X_{t-1} + \cdots + Y_{t-1} + \cdots + Y_{t-2} + \Sigma.$$

VEC

把不 stationary 的两个
<u>通过组合变成 stationary</u>.

panel regression.

<u>maximum likelyhood</u>.

Finance
#1
1608=7

↘. bond: 10-30年期国债券 treasury bond. 国债
corporate/credit bond 企业债.
↘有点像借条.
发coupon.

bill: 一年以下
note: 1-10年.

bill, note, bond "国家借条"

↘ 金融产品比正常Normal更容易出现极端情况. 叫 heavy tail
非理性
自然灾害
理性.
马太效应.

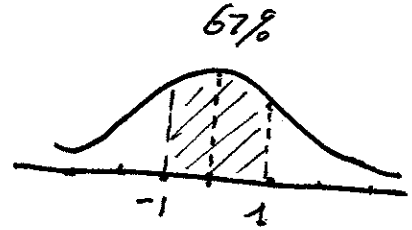
Finance 2

160807

- t-distribution 本身便比 normal 厚尾.
  但 stock market 比 t-dist 更加厚尾 (极端).

- $N(0,1)$.

67%	95%	99.7%
1σ	2σ	3σ



- a 5-SD event under normal dist. occurs in 7000 years.
  t-dist —— 75 ——.

- $$R_k = \log\frac{S_k}{S_0} = \log\left(\frac{S_k}{S_{k-1}}\,\frac{S_{k-1}}{S_{k-2}}\cdots\frac{S_1}{S_0}\right) = \log\frac{S_k}{S_{k-1}} + \log\frac{S_{k-1}}{S_{k-2}} + \cdots = r_k + \cdots + r_2 + r_1.$$

  叠加起来的 return.

- An important concept of time series is the stationarity.
  - structural invariability across time
  - so that historical relationship can be aggregated.

- ACF, autocorrelation function.

$$\rho(k) = Corr(X_{t+k}, X_t) = \frac{Cov(X_{t+k}, X_t)}{\sqrt{Var(X_{t+k}) \cdot Var(X_t)}} = \frac{\gamma(k)}{\gamma(0)}$$ for stationary time series.

  - measures linear relations of X_t & X_{t+k}
  - $-1 \le \rho(k) \le 1$, even $\rho(k) = \rho(-k)$
  - semi-positive difinite function:

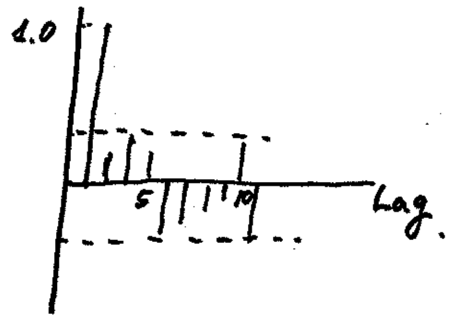
$$\sum_{i=1}^{k} \sum_{j=1}^{k} \rho(i-j) a_i a_j \ge 0$$

White noise:

$$\{\varepsilon_t\} \sim WN(0, \sigma^2)$$

if $\gamma(0) = 1$, $\gamma(j) = 0$, $j > 0$

ACF for white noise.



Thus, about 95% of acf falls in $\pm 1.96/\sqrt{T}$

→ 里面有多少个sample
结束时间.

→ 有correlation 便不是 i.i.d.

$$\{i.i.d.\} \longrightarrow \{\text{martingale difference}\} \longrightarrow \{\text{uncorrelated}\}.$$

Martingale hypothesis.

asset price (log price) is a random walk,

$X_t = X_{t-1} + \varepsilon_t$ with martingale difference ε_t

$$E_{t-1} X_t = X_{t-1} + E_{t-1} \varepsilon_t = X_{t-1}$$

Linear time series: Finance 5

MA

- MA(q) - model

$$X_t = \mu + \sum_{j=1}^{q} a_j \varepsilon_{t-j} + \varepsilon_t, \quad \{\varepsilon_t\} \sim WN(0, \sigma^2)$$

- ACF of MA(2).

$$X_t = \mu + \varepsilon_t + a_1 \varepsilon_{t-1} + a_2 \varepsilon_{t-2}$$

$$X_{t-3} = \mu + \varepsilon_{t-3} + a_1 \varepsilon_{t-4} + a_2 \varepsilon_{t-5}$$

hence, $\rho(3) = \rho(4) = \cdots = 0$.

$\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}$ $\quad \varepsilon_{t-3}, \varepsilon_{t-4}, \varepsilon_{t-5}$

$$\rho(3) = \mathrm{corr}(X_t, X_{t-3}) = \frac{\mathrm{cov}(X_t, X_{t-3})}{\sqrt{\mathrm{Var}(\ )\mathrm{Var}(\ )}} = 0.$$

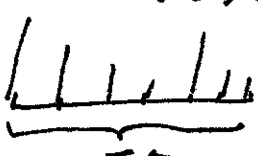
- Let X_t follow an MA(q) model, then

$$\sqrt{T}\, \hat{\rho}(j) \xrightarrow{D} N\left(0, 1 + 2\sum_{i=1}^{q} \rho^2(i)\right), \quad j > q.$$

ρ是 correlation

$\hat{\rho}$是 predicted correlation.

序列长度



for $j > q$, about 95% sample correlations $\hat{\rho}(j)$ fall in the interval

$$\pm \frac{1.96}{\sqrt{T}} \left\{1 + 2\sum_{i=1}^{q} \hat{\rho}^2(i)\right\}^{\frac{1}{2}}$$

Autoregressive model:

a sample and useful class of model for forecasting returns.

$r_t = \log \frac{S_t}{S_0}$

AR

AR(p)

$$X_t = b_0 + \underbrace{b_1 X_{t-1} + \cdots + b_p X_{t-p}}_{\text{拿historical的data来线性组合的值}} + \varepsilon_t$$

加上一项 white noise

加一项常数

- Suppose X_t is stationary with mean μ.

$$\mu = b_0 + b_1\mu + \cdots + b_p\mu \longrightarrow \mu = \frac{b_0}{1-b_1-b_2\cdots-b_p}.$$

Let $b(z) = 1 - b_1 z - \cdots - b_p z^p$

为 characterization func.

$$\mu = \frac{b_0}{b(z)|_{z=1}}$$

-

$B X_t = X_{t-1}$

$B^2 X_t = X_{t-2}$

$B^k X_t = X_{t-k}$

后移操作.

$$b(B) = 1 - b_1 B - \cdots - b_p B^p$$

$$b(B) X_t = X_t - b_1 X_{t-1} - \cdots - b_p X_{t-p} = b_0 + \varepsilon_t$$

$$b(B) X_t = b_0 + \varepsilon_t \longrightarrow X_t = b^{-1}(B)\,(b_0 + \varepsilon_t)$$

需 $b(B)$ 可逆，需 $b(z)$ 有 unit circle 外的根

$$b(z)^{-1} = \sum_{j=0}^{\infty} c_j z^j$$

$$X(t) = b(B)^{-1}(b_0 + \varepsilon_t) = \sum_{j=0}^{\infty} c_j B^j (b_0 + \varepsilon_t) = \sum_{j=0}^{\infty} c_j (b_0 + \varepsilon_{t-j})$$

$$B\varepsilon_t = \varepsilon_{t-1}$$

$$B^2 \varepsilon_t = \varepsilon_{t-2}$$

$$\vdots$$

For AR(1) model,

$$X_t = b X_{t-1} + \varepsilon_t$$

特征方程. $b(z) = 1 - bz$, root: $z = \frac{1}{b}$.

则是 $|b| < 1$ 时, $b(z)$ 有大于1之根, 则是 series 是 stationary.

AR(3) model:

$$X_t = 0.8 X_{t-1} - 0.5 X_{t-2} + 0.4 X_{t-3} + \varepsilon_t$$

特征方程:

$$b(z) = 1 - 0.8z + 0.5z^2 - 0.4z^3$$

$$= (1 - 0.8z)(1 + 0.5z^2).$$

has all roots outside the unit circle. Thus, it is stationary.

Finance.
8.

AR(4):

$$X_t = 0.5X_{t-1} + 0.3X_{t-2} - 0.7X_{t-3} + 0.2X_{t-4} + \varepsilon_t,$$

ACF(5)

MA(4):

$$X_t = \varepsilon_t + 0.6\varepsilon_{t-1} + 0.6\varepsilon_{t-2} + 0.3\varepsilon_{t-3} + 0.7\varepsilon_{t-4}$$

ARMA(2,2)

AR(1).

$$X_t = b_0 + b_1 X_{t-1} + \varepsilon_t$$

$$X_{t-1} = b_0 + b_1 X_{t-2} + \varepsilon_{t-1}$$

$$corr(X_t, X_{t-1}) = b_1$$

$$corr(X_t, X_{t-2}) = b_1^2$$

$$corr(X_t, X_{t-k}) = b_1^k.$$

$|b_1| < 1$

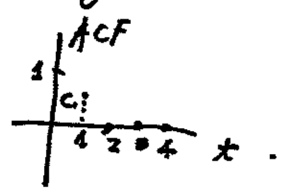
MA(1)

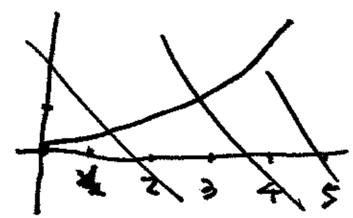
$$X_t = b_0 + \varepsilon_t + c_1\varepsilon_{t-1}$$

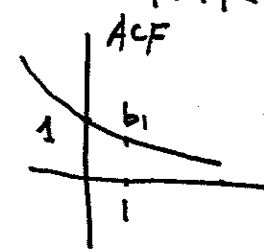
$$X_{t-1} = b_0 + \varepsilon_{t-1} + c_1\varepsilon_{t-2}$$

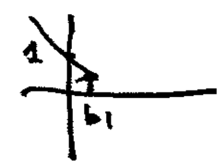
$$corr(X_t, X_{t-1}) = c_1$$

$$corr(X_t, X_{t-2}) = 0$$









PACF suggest to fit a ARCH(3) model:

$$Y_t = \mu + X_t, \quad X_t = \sigma_t \varepsilon_t \rightarrow WN(0, 1)$$

$$\sigma_t^2 = c_0 + b_1 X_{t-1}^2 + b_2 X_{t-2}^2 + b_3 X_{t-3}^2$$

Fitting results:

	μ	c_0	b_1	b_2	b_3
Est.	0.0196	0.0090	0.2973	0.0090	0.0626
SE	0.0062	0.0013	0.0887	0.0645	0.0777
t-stat	3.161	6.923	3.352	0.101	0.806.

Dropping. 太小.

refit to ARCH(1) to obtain

$$Y_t = 0.0213 + X_t$$

$$X_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = 0.00998 + 0.4437\, X_{t-1}^2$$

$$\frac{s_{x,y}}{s_{x,x}} \quad \frac{Cov(x,y)}{var(x,x)} = \frac{\sum_i (y_i-\bar{y})(x_i-\bar{x})}{\sum_i (x_i-\bar{x})^2}$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y.$$

ARMA(2,2)

$$X_t = b_0 + b_1 X_{t-1} + b_2 X_{t-2} + c_1 \varepsilon_{t-1} + c_2 \varepsilon_{t-2} + \varepsilon_t$$

WN: $\{X_t\}$ $X_t \sim i.i.d.\ N(0,1)$.

$$Corr(X_t, X_{t-1}) = 0.$$

ARCH
GARCH 有 volatilities clustering 的效果.

GARCH models are widely used for modeling volatilities.

A generalized autoregressive conditional heteroscedastic (GARCH) Model with orders p & q is:

$$X_t = \sigma_t \varepsilon_t \quad \text{and} \quad \sigma_t^2 = c_0 + \sum_{i=1}^{p} b_i X_{t-j}^2 + \sum_{j=1}^{q} a_j \sigma_{t-j}^2$$

平方 of standard deviation

c_0, b_i, a_j : non-negative const.

$\{\varepsilon_t\} \sim$ i.i.d. $N(0,1)$, independent of $X_{t-1}, X_{t-2}, \cdots$, denoted by GARCH(p,q)

σ_t^2 : weighted average of past volatilities

facilitate the modeling of volatility clustering.

$\{X_t\} \sim$ GARCH(p,q)

$\{X_t^2\} \sim$ ARMA$(\max(p,q), q)$

As in the ARCH model, when

$$\sum_{i=1}^{\max(p,q)} (a_i + b_i) < 1.$$

特征 equation has no root inside the unit circle and hence is stationary.

- In this case, the ACF of $\{X_t^2\}$ decays exponentially

Let $\eta_t = X_t^2 - \sigma_t^2 = \sigma_t^2(\varepsilon_t^2 - 1)$

then $\{\eta_t\}$ is a martingale difference series.

$$\eta_{t-1} = \sigma_{t-1}^2(\varepsilon_{t-1}^2 - 1)$$

$$\Delta\eta = \eta_t - \eta_{t-1} = (\sigma_t^2 \varepsilon_t^2 - \sigma_{t-1}^2 \varepsilon_{t-1}^2) + \sigma_t^2 - \sigma_{t-1}^2$$

GARCH(p,q) stationary $\longleftrightarrow \sum_{i=1}^{p} b_i + \sum_{j=1}^{q} a_j < 1$

Example: consider the stationary GARCH (1, 1) process,

$$\sigma_t^2 = c_0 + b_1 X_{t-1}^2 + a_1 \sigma_{t-1}^2 \quad (b_1 + a_1 < 1)$$

$\Updownarrow$ stationary

$X_t = \sigma_t \varepsilon_t$

$X_t^2 = \sigma_t^2 \underline{\varepsilon_t^2}$

$$X_t^2 = c_0 + (a_1 + b_1) X_{t-1}^2 - a_1 \eta_{t-1} + \eta_t \qquad ?$$

securities = assets = 证券.

T bill 3 month — market 公认之 $\underline{\text{cash rate}}$
LIBOR

e^{rt}

$\downarrow r_f$.

伦敦银行间 隔夜拆借利率

到期日3个月
买卖容易
交易量大.
国家信用
保证 = riskless.

GARCH (1, 1)

$X_t = \sigma_t \varepsilon_t$

~~再~~写递推形式

$$\sigma_t^2 = c_0 + b_1 X_{t-1}^2 + a_1 \sigma_{t-1}^2$$

securities: equity, bond, options, commodity, futures.

stock (= equity)

T bond

credit (~~bond~~ 债 = corporate bond)

short:

先借了卖

再买了还

Portfolio management

研究portfolio construction, 先写出return及risk表达式.

r_{return} = riskless return + risky assets return

$= \alpha_0 R_0 + \alpha_1 R_1 + \alpha_2 R_2 + \cdots + \alpha_n R_n$

$= \alpha_0 R_0 + \underline{\alpha}^T \underline{R}$ → 向量

加上时间项

$r_{t+1} = \alpha_0 \underline{R_{0,t}} + \underline{\alpha}^T \underline{R}_{t+1}$

$\underline{\alpha}$ → 固定比例向量

Portfolio return: at time t+1, the return of the portfolio.

$$r_{t+1} = \underbrace{\alpha_0 R_{0,t}}_{\text{单独拿出来, cash, riskless.}} + \underbrace{\alpha^T R_{t+1}}_{\text{risky securities.}}$$

Expected return:

$$\mu_t(\alpha) = E_t r_{t+1} = R_{0,t} + \alpha^T \xi_t,$$

$$\sigma_t^2(\alpha) = var_t(r_{t+1}) = \alpha^T \Sigma_t \alpha.$$

var_t → t时刻的variance.

Σ_t → covariance matrix

$$f(\alpha) = \alpha^T \varepsilon_t - \frac{A}{2}\alpha^T \Sigma_t \alpha$$ 取极值 $\frac{\partial f(\alpha)}{\partial \alpha} = 0$

$$\frac{\partial f}{\partial \alpha} = \varepsilon_t - A\Sigma_t \alpha = 0$$

α 列向量

α^T 行向量

$$\alpha_t^* = (A\Sigma_t)^{-1} \varepsilon_t = \frac{1}{A} \Sigma_t^{-1} \cdot \varepsilon_t \qquad \alpha_t^* =$$

$$\alpha_{0,t}^* = 1 - 1^T \alpha_t^*$$

这个 portifolio 中有多少 cash (比例). 是 100% 减去剩下的 (risky) asset 的占比.

ε_t: excess return

每一个已经减去 ε_0, cash rate

只对 risky asset 做 allocation.

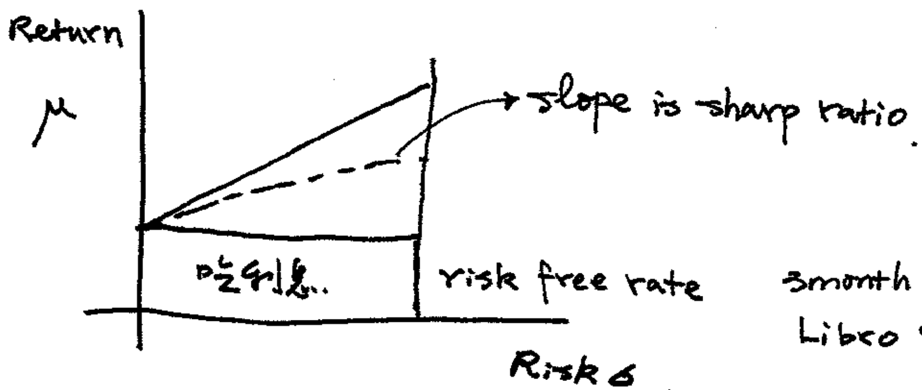
sharp ratio 形容一个策略好不好的最重要指标.

$$= \frac{\mu^* - R_0}{\sigma^*}$$

R_0 → riskless.

σ^* → volitality $= \sqrt{Var}$
risk.

越大越好.



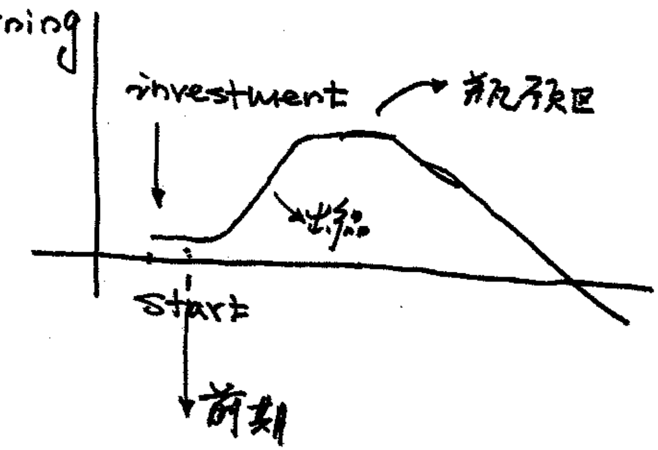
3month T bill
Libco 利率.

sharp ratio.

$$J(\alpha) = \frac{\alpha^T \Sigma}{(\alpha^T \Sigma \alpha)^{\frac{1}{2}}} = \frac{\text{excess return}}{\sqrt{Var}}$$

$\sqrt{Var}$ " i.e. volitility
vol = std.

Growth



→ Value $\frac{\text{earning per share}}{\text{price}}$ 成熟型看 value

↕

Growth earning growth startup
sale growth. 看 growth.

Size effect.

→ Large cap share × price/share 大
google
- good liquidity.
- 大家对其预测一致.
Volatility 小.

Small cap
dragon wave.
- no liquidity
- volatility 大.
市场涨,它们涨得快.

→ book to market.
帐面价值. → 市场交易的
资产,厂房,设备 价值.

一般 market value > book value
(你所有资产的价值
加起来会产生现金流).

CAPM

i: google, apple.

excess return: (return) − (risk-free return)

$r_{M.t}$ → SP500 return, 你买 index 的 return.

像是用自己的房子做生意.
先要把房租挣出来.

$$- \; r_{i.t} = r_f + \beta_i \cdot (r_{M.t} - r_f) + \varepsilon_{i.t}$$

t 指在某个时间点.

$$\bar{r}_i - r_f = \beta_i (\bar{r}_M - r_f)$$ ← single Securities' Expected Return in Propotinal to Market Excess Return

CAPM.

→ $\beta_i (r_M - r_f)$ → Market return/risk

→ The Part of Return that can be explained by Market.

即核心思想是, 单股的 excess return 正比于 index 的 excess return.

r_f:
risk free rate
3 month T bill.

3month T bill
2–10 year bond
>10 year. treasury

→ $\varepsilon_{i.t}$ → idiosyncratic Return / risk

按 CAPM 是正态分布, 但实际上不是.

→ The Part of Return explained by Security fundementals.

越小公司, return越高.
因为你承担更高风险.

Fama − French (Multi regress. 加了两个 factor 来 explain 之无法被 CAPM 解释的 return)

Add two More factor Size & Book-to-Market Ratio

$$\rightarrow r_i = r_f + \beta_1 (r_M - r_f) + \beta_2 (\text{small cap} - \text{Large cap}) + \beta_3 \cdot (\text{High Book-to-Market} - \text{Low Book-to-Market})$$

BP

BP

Mean - variance

investor prefer:

fix risk, higher return

or

fix return, lower risk.

$$\mu_p = w_1\mu_1 + w_2\mu_2 + \cdots + w_N\mu_N = W^T\mu$$

$$Var(r_p) = \sum_{i=1}^{N} \sigma_i^2 w_i^2 + \sum_{i\neq j} \sigma_{ij} w_i w_j = w^T \Sigma w.$$

↓ risk

$W_i \sim$ 第 i 個 asset 的比重.

σ_i^2 : Variance of ith asset

σ_{ij} : i, j 兩個 asset 的 covariance

⇓ 數學化.

Matlab 中 用 quantprog 解.

等 quandratic programming

$$\min_w \; w^T \Sigma w.$$

s.t. $w^T\mu = \underline{\mu_p}$, $w^T e = 1$

(各个 w_i 加起来 $\equiv 1$).

↓ targeted profit

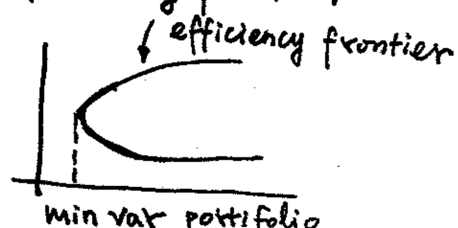
多 μ_p, 便得到 efficiency frontier

quandprog Quandratic programming

$$\begin{cases} F = f - \lambda g \\ \dfrac{\partial F}{\partial \lambda} = 0 \\ \dfrac{\partial F}{\partial w} = 0 \end{cases}$$

maximize $\left(r^T w - \frac{\lambda}{2} w^T \Omega w\right)$

s.t. $w^T \mu$



Lagrange multiplier:

min. $D^2 = x^2 + y^2 + z^2$

s.t. $g(x,y,z) = 2x + 3y + 4z$.

$$\left.\begin{aligned} \frac{\partial f}{\partial x} + \lambda\frac{\partial g}{\partial x} &= 2x + 2\lambda = 0 \\ \frac{\partial f}{\partial y} + \lambda\frac{\partial g}{\partial y} &= 2y + 3\lambda = 0 \\ \frac{\partial f}{\partial z} + \lambda\frac{\partial g}{\partial z} &= 2z + 4\lambda = 0 \end{aligned}\right\}$$

$2x + 3y + 4z - 12 = 0$

$\lambda = -24/29$

$x = 24/29$

$y = 36/29$

$z = 48/29$

$\Rightarrow D = \dfrac{|d|}{\sqrt{x^2+y^2+z^2}}$

金 Finance. Portifolio optimization.

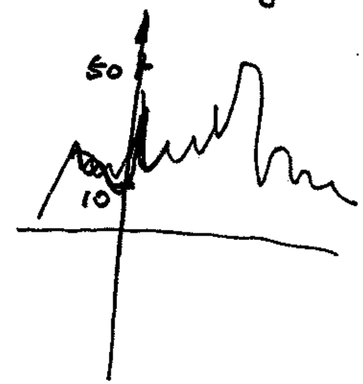
Finance
160827.
1

经常假设, stock price 的 difference 是个正态分布.

$$P_n - P_{n-1} \sim N(0,1).$$

— Not a good price. Several reasons.

1. 没有考虑 ~~how big~~ what position the stock price is,



$10, 你的增量是一块
$50, —— 还一块
take 的权重并不相同!

于是, price 本身并非 normally distributed.
but the percentage of the price is normally distributed.

— — — —

Absolute stock price different is not 正态.

Instead,
we want "<u>relative difference</u>" to be normally distributed.

$$\frac{P_n - P_{n-1}}{P_n} \sim N(0,1)$$

$$\frac{增量}{绝对值} = 相对增量.$$

— 于是问题便来了, 如果相对增量 $\sim N(0,1)$ 标准正态.

那么绝对价格满足何种分布?

— — — log normal!!

另外，random walk 之性质.

$$P_n - P_{n-1} \sim N(0,1),$$

则 $P_n \sim N(0, \sqrt{n})$

"no matter how far you go,
still 正态，且均值不变
但 variance 正比于你走的步数的平方根."

- - - - - - -

Def log normal r.v. Y, such that $\log Y$ is normally distributed.

$\log Y$ 正态，Y 什么分布？

① change of var formula,

X, Y. R.V.

such that $Y = h(x)$

$$P(X \le x) = P(Y \le h(x)) \quad \text{for all } x,$$

then

$$F_Y(y) = P(Y \le y) = P(Y \le h^{-1}(x)) = F_X(h^{-1}(x))$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X(h^{-1}(x)) = f_X(h^{-1}(x)) \frac{1}{h(x)}.$$

Finance

160827

3.

Def of log normal distribution

X: log normal

Y: normal with mean μ, var: σ^2

$$P(X \le x) = P(\cancel{y} \overset{Y}{} \le \log x)$$

pdf: $$f_X(x) = f_Y(\log x)\frac{1}{x} = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}} \quad x > 0.$$

Poisson
Exponential distributions can be grouped into one family.

exponential family of distribution

$\exists$ vector θ, that parametrize distributi[on]
as.

$$f^{(\theta)}(x) = h(x)\, c(\theta) \exp\left(\sum_{i=1}^{k} w_i(\theta) t_i(x)\right)$$

$h(x)$, $t(x)$ depends only on x
$c(\theta)$, $w_i(\theta)$ on θ.

Given a R.V. Finance 4.

1. Study 'statistics'

   k-th moment of r.v. $\mathbb{E}X^k$

   moment generating function.

2. Study long-term (large-scale) behavior.

   - Law of large number
   - Central limit thm.

MGF

$M_X(t) = \mathbb{E}\, e^{tX} \quad (t \in \mathbb{R})$

Remark.

- Does not necesarily exist.
- Log normal does not have MGF.

为何叫 moment generating func

$$\frac{d^{(k)} M_X}{d t^{(k)}}(t) = \mathbb{E}X^k$$

M_X 的 k阶导等于 X 的 k阶 moment.

- $M_X(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} m_k \qquad m_k = \mathbb{E}X^k$

但是若 X、Y 的 k阶矩相同 ($k \in N$)
它们的 distribution 不一定同!
因为 $M_X(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} m_k$
可能不收敛.

- Moment generating func actually includes all info of R.V.

THM: 若 X, Y 的 M.G.F 一样, 则 X, Y 的 dist. 一样

- Law of large number

弱大数定律:

$X_1, X_2, \cdots X_n$ be $\underbrace{\text{i.i.d.}}$ sequence,

$\searrow$ mean μ, variance σ^2

构造一个新的随 R.V.

$$X = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$$

Then $\forall \varepsilon > 0$, $P(|X - \mu| \geq \varepsilon) \to 0$.

即你取有大量 sample.
然后求 sample 之均值.
此 sample mean $\xrightarrow{\text{接近}}$ 真实分布之 μ
的概率为 1

- Central Limit Thm.

WLLN: $\underbrace{\frac{1}{n}\sum_{i=1}^{n} X_i}_{\text{has mean } \mu,\ \text{var } \frac{\sigma^2}{n}} \to \mu$ (X_i 是 i.i.d.)

($\sum_{i=1}^{n} X_i$ has var $n \cdot \sigma^2$

$\underline{\frac{1}{n}\sum_{i=1}^{n} X_i \text{ has var: } \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}}$)

What happens for $\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i$?

$\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i$ has var: σ^2

$\searrow$ same as X_i !

var $= \sigma^2$

与 X_i 之 var 同!

So does $\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i$ has the same dist. as X_i?

Thm:

$X_1, X_2, \cdots, X_n$ be i.i.d. R.V. with mean μ, Var σ^2

$$Y_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i - \mu).$$

则 Y_n 的分布 converge to that of $N(0, \sigma^2)$

任何的独立同分布, R.V. 取大量 然后求和除上 $\frac{1}{\sqrt{n}}$, 趋于正态分布.

Suppose, $\exists$ R.V. X, whose mean is unknown.

How to estimate the mean?

Take independent trials, $X_1, X_2, \cdots, X_n$.

And use $\frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ 'estimator' of mean,

- The 'law of large #' says, this estimator will be very close to the mean.

- The central limit thm tells how this $\frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ is around the mean μ

虽然我们并不知道这个mean在何处.

because the tail of normal dist is small, we'll get close to μ fast

Risk Neutral Valuation: Two horses race example.

两匹马,

马1 20%几率赢 $10,000

马2. 80%几率赢 $50,000.

内部人员知 public不知道那么精确

public bet

内部人员(M)设置赔率 odds (O)

O = 4:1, 若马1赢, M须payback $10,000 four times more.

输 50,000.

但他收 10,000 + 50,000 = 60,000

赚 10,000.

若马2赢, M须payback ~~50,000~~ $50,000 + \frac{50,000}{4} = 62,500$.

输 2500

Expectation 是 0

若赔率设为5:1, 不管哪只马赢, M都会赢!

→ risk-neutral !!

underlying securities could be:

interests rate, bonds, swaps, comodities.

dereviative contract is some formal payout,

connected to underline, usually is

liquid instrument.

traded on exchanges.

forward contract:

One party is agree to buy sth with a price fixed today.

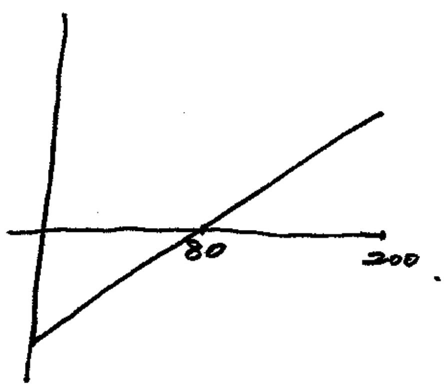
- usually, no mony changes hands.

比如

商定好以 K 的价格买一只股票，不管它如何变.

Forward Contract payoff 是一条直线.

maturity 时的收益.



$S(t) = 80$, $K = 88.41$, $T = 2$ (years)

European call option pays $\max(S-K, 0)$ at time T

forward is an obligation to buy for an agree price,

call option is an option to buy an asset. at agreed price today.

an insurance against asset going down.

you can never lose money, the pay off is always positive.

on a forward you can lose money.
(if you agree on a ~~lower~~ higher price. (比市价高)
你还是要买.

--- at expire time

Call pay off.

Black-schole will exactly be soln.

if $S - K < 0$
S: assets
at payoff 比 K 小, → out of the money.

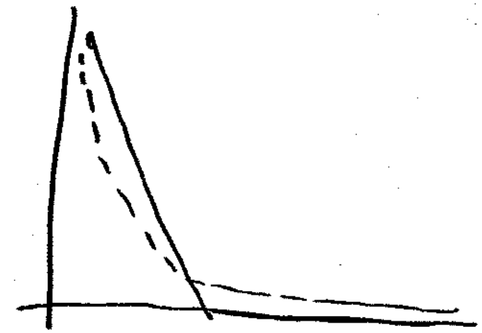
Then the payout will be 0.

$S - K > 0$, → in the money.

payout: $S-K$

A put option is a bet on the asset going down.

pays: $\max(K-S, 0)$ at maturity.



- given 腰行假一些 assumption, no uncertainty of price of the option.
    - Once we know the price of the underlying, its price of the option is completely deterministic.
    - The price has nothing to do w. risk preference with the 投资者
        - only depends on volitility of the stock.

Suppose our economy includes:

- stock S (risky)
- riskless money market account B with interest rate r (riskless)
- derivative claim f.

(cash, riskless means it grows exponentially with interest rate r) basically zero coupon bond

Assume one step before maturity. (two possible outcomes)

$$\begin{tikzcd}
 & S_1,\ B_0 e^{r\,dt},\ f_1 \\
S_0,\ B_0,\ f \arrow[ur, "p"] \arrow[dr, "1-p"'] & \\
 & S_2,\ B_0 e^{r\,dt},\ f_2
\end{tikzcd}$$

- consider forward contract which pays $S-K$ in time dt.

One could think that its strike K should be defined by the 'real world' transition probability p:

$$p(S_1-K)+(1-p)(S_2-K) = pS_1 + (1-p)S_2 - K$$

同時有贏或輸的可能性.

Consider the following strategy:

1. Borrow $\$S_0$ to buy the stock. Enter the Forward contract with strike K_0.

   这是约定好的股票

   Forward会给你约定好的 K_0.

2. In time dt, deliver stock in exchange for K_0 and repay $\$S_0 e^{r dt}$.

   loan. 借的钱, 加利息

If $K_0 > S_0 e^{r dt}$ we make riskless profit.
If $K_0 < S_0 e^{r dt}$ we lose money.

$\Rightarrow K_0 = S_0 e^{r dt}$

组建一个portifolio, maturities 收益与衍生品同
则 portifolio 现在价格便是衍生品之应有价格.

Current price of a derivative claim is determined by current price of a portifolio which exactly replicates the payoff of the derivative at the maturity.

This is how risk neutral valuation works.

replicating portfolio

$S(t) = 80$

$K = 88.41$

$T = 2$.

于是你可反推 r (interest rate)

$88 = 80 e^{rT}$

$\downarrow$

$r = 5\%$.

For a general derivative claim a & b, find a and b s.t.

$$f_1 = aS_1 + bB_0 e^{rdt}$$
$$f_2 = aS_2 + bB_0 e^{rdt}$$

$$\begin{tikzcd} & S_1, B_0 e^{rdt}, f_1 \\ S_0, B_0, f \arrow[ur, "p"] \arrow[dr, "1-p"'] & \\ & S_2, B_0 e^{rdt}, f_2 \end{tikzcd}$$

Then
$$f_0 = aS_0 + bB_0$$

↑初始

easy to see: $a = \dfrac{f_1 - f_2}{S_1 - S_2}$, $b_1 = \dfrac{S_1 f_2 - S_2 f_1}{(S_1 - S_2) B_0 e^{rdt}}$

$$f_0 = e^{-rdt}\left(S_0 e^{rdt} \frac{f_1 - f_2}{S_1 - S_2} + \frac{S_1 f_2 - S_2 f_1}{S_1 - S_2} \right)$$
$$f_0 = e^{-rdt}\left(f_1 \frac{S_0 e^{rdt} - S_2}{S_1 - S_2} + f_2 \frac{S_1 - S_0 e^{rdt}}{S_1 - S_2} \right)$$
$$f_0 = e^{-rdt}\left(f_1 q + f_2 (1-q) \right)$$

$q = (S_0 e^{rdt} - S_2)/S_1 - S_2$ $\quad 0 < q < 1 \rightarrow \in [0, 1]$

→这个比值是一种 probality.
但不是 real world probability.

The measure our stock behaves,
called risk-neutral meature, or martingale.
In this measure, the value of derivative is just expected value.

Black-Scholes equations:

Finance

7.

Assume that the stock has log-normal dynamics:

$$\frac{dS}{S} = \cancel{S}\,\mu dt + \sigma dW$$

normally distributed with mean 0 and standard deviation $\sqrt{dt}$.

drift μ.

(i.e. W is a Brownian motion)

It's extremely important that square root of Brownian motion is $\sqrt{t}$.

Again, we will use the idea of replicating portifolio:

We'd like to find such coefficient a, b on 无限小时间 dt.

$$df = a\,dS + b\,dB.$$

Use ito's lemma

$(dS)^2$, variance of BM. on the order of dt.

ito's lemma is nothing more than taylor's rule, but because std of Brownian motion being on the order of $\sqrt{t}$, we'll need one more term there.

$$df(S,t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \underline{\frac{1}{2}\frac{\partial^2 f}{\partial S^2}(dS)^2}$$

$$(dS)^2 = (S\mu dt + \sigma S dW)^2$$

$$= \sigma^2 S^2 dt$$

$(dt)^2 = 0 \quad (dt)^{\frac{3}{2}} = 0$

把 $df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \overbrace{dt}^{\sigma^2 S^2}$

代入

$$df = a\,dS + b\,\underline{dB}.$$

bond, riskless, deterministic.

$dB = rB\,dt$　　固定以r的大小指数增长

B grows exponentially with interest rate r.

$$\frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} dt = a\,dS + b\,r\,B\,dt$$

$$dS = S\mu\,dt + S \cdot \sigma\,dW.$$

$$\left(\underline{\frac{\partial f}{\partial t} + S\mu \frac{\partial f}{\partial S} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2}\right) dt + \frac{\partial f}{\partial S} \sigma S\,dW = \left(\underline{a S\mu + b r B}\right) dt + a\sigma S\,dW$$

Start comparing the terms.

$$\underline{a}\ \sigma S = \underline{\frac{\partial f}{\partial S}}\ \sigma S.$$

$bB = f - aS$ is deterministic as $\underline{dB} = rB\,dt$ (bond 倍期, 固定以利息率r指数增长).

$$d(f - aS) = r(f - aS)\,dt$$

$$df = \frac{\partial f}{\partial t} dt + \underset{\substack{\uparrow \\ a}}{\frac{\partial f}{\partial S}} dS + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt \longrightarrow \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 dt = \Big(rf - r \overset{\substack{a \\ \uparrow}}{\frac{\partial f}{\partial S}} \cdot S\Big) dt$$

$$\downarrow$$

Black-Schole equation. $$\frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 + \frac{\partial f}{\partial S} rS - rf = 0.$$

Finance 9.

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 + \frac{\partial f}{\partial S} rS - rf = 0.$$

Black-schole equation.

- Since we made no assumption, any tradable deriative with any payoff should satisfy this equation.
- There's no dependency on drift or probability going up/down. The only dependence is on the <u>volitility</u> of the stock.
- We also have hedging stradegy. for any time, we found replicating portifolio coefficient a & b.

At any time,
we can ~~hold~~/short derivative
and long the portiforlio,

→ stock + some cash

$df = a\,dS + b\,dB$.

→ every dt, a, b dynamically change. 但是已知!!

cash 变成 Bond. 便是 riskless 之增大. (可理解为储蓄)

short derivative = long stock + cash.

两边 perfectly represent each other.

so we're sure there's no risk?

This is actually how the business is working:

We trade our trading
and hedging their positions immediately.

They do take some market risk,
but you want to take very specific market risk,
no everything.

Black-schole equation

$$\frac{\partial f}{\partial t} \cancel{dt} + \frac{1}{2}\frac{\partial f^2}{\partial S^2}\sigma^2 S^2 \cancel{dt} + \frac{\partial f}{\partial S} rS \cancel{dt} - rf = 0$$

- Any tradable derivative satisfies this eqn.
- There's no dependence on actual drift μ.
- We have a hedging stradegy (replicating portifolio)
- By a change of var, Black-schole equation tranforms into heat eqn $\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}$

We know much more on heat eqn.

To solve any PDE, we'll need initial & boundary conditions.

Boundary & final conditions are determined by the pay-off of a specific derivative.

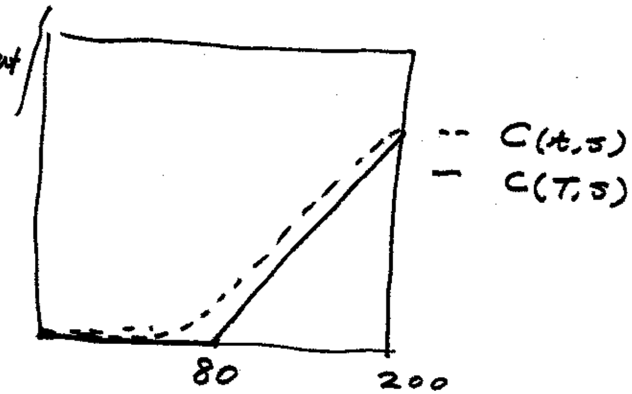
For European call

Final payout

$$C(S,T) = \max(S-K,\ 0)$$

$$C(0,t) = 0, \quad C(\infty,t) \cong S$$

B.C.



$S(t) = 80$

$K = 80$

$T = 2$ years.

For European put

$$P(S,T) = \max(K-S,\ 0)$$

$$P(0,t) = Ke^{-r(T-t)}, \quad P(\infty,t) = 0.$$

at current time, $t=0$. it should be 0.
at ∞. it should be forward price, discounted $(S-K)$

For European call/put, the eqn can be solved analytically.

$$C_t = e^{-r(T-t)}\left(e^{r(T-t)} S N(d_1) - K N(d_2)\right)$$

$$P_t = e^{-r(T-t)}\left(K N(-d_2) - e^{r(T-t)} S N(-d_1)\right)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du.$$

This how call/put
圖中之曲線(非折線)
is drawn.

Finance 15.

↳ Dividend.
|
T Bond as reference

赚了一元，
- 给股东. 如能增长. 不用投资. 发给股东. 像是bond.
- 投资.

比较稳定.
不像 hyper-loop.
可能市值翻番.
几乎就是bond.
跟要知 T Bond
比一比.

↳ VIX : implied volatility 的 index (指标)

SP500的volatility.

intuition是:
它是买option的人，心目中未来股票的volatility.
大家愿意为减少vol, 所付出的价格.

risk-neutral measure下的
volatility, 比实际的volatility高不少.

↳ T Bond 下降时.
Dividend 公司更 attractive. 价格上涨.

low vol portifolio.
市场涨或跌，它都不动.
如电力公司，与它相关的是
今夏热不热.
与市场无关.

bond 每半年给你一个 coupon.
若 rate 不变,

大家担心的程度,
常用做 risk gauge, 可做 risk indicator.

有的高.
high vol.

买option, 大部分是为了对冲自己的风险.
买了一支股票, 再买一个put, 对冲掉下行风险.
这个对冲, 我愿意支付的价格, 就是其 price.
当我愿意对冲时, 是我评估风险较高时.

implied vol.
(未来的)

由过去data算的
是过去的
vol.

此主观评估一般高于真实的 vol. 即 realized vol

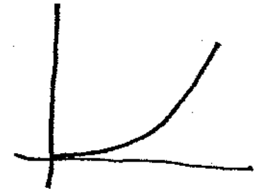
↳ momentum :
一个股票过去几年涨得好.
下一年涨得好的概率大.

Finance 3.7

股票

$S_t = S_0 e^{\alpha t}$

大的趋势是指数增长的. α drift, 反映大趋势.

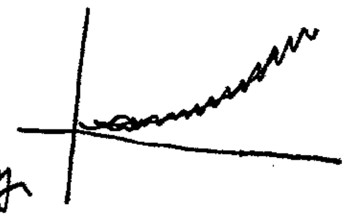


with 波动

这个波动, 便是加一项 Brownian motion

$S_t = S_0 e^{\alpha t + \sigma W_t}$

drift　　σ volatility 反应波动.



整个的, 你股票之行为, 符合 geometric brownian motion.

Ito's lemma

$f(t, X_t)$ $\qquad dX = \mu dt + \sigma dW$

$$df = \left(\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial X^2}\right) dt + \sigma \frac{\partial f}{\partial x} dW$$

Brownian motion

对df之泰勒展开，取到 dt 一阶.

由于 dW 为 Brownian motion,

$$dW = \underline{W_t - W_{t+\Delta t} \sim N(0, dt)}$$

予见: $d^2W = dt$

Variance.

Brownian motion 之性质.

- $R_k = \log\left(\frac{S_k}{S_0}\right) = \log\left(\frac{S_k}{S_{k-1}} \frac{S_{k-1}}{S_{k-2}} \cdots \frac{S_1}{S_0}\right) = \log\left(\frac{S_k}{S_{k-1}}\right) + \log\left(\frac{S_{k-1}}{S_{k-2}}\right) + \cdots + \log\left(\frac{S_1}{S_0}\right)$

$$= r_k + r_{k-1} + \cdots + r_1$$

- ACF, auto correlation function.

$$\rho_k = \mathrm{Corr}(X_{t+k}, X_t) = \frac{\mathrm{Cov}(X_{t+k}, X_t)}{\sqrt{\mathrm{Var}(X_{t+k}) \cdot \mathrm{Var}(X_t)}} = \frac{\gamma(k)}{\gamma(0)}$$

for stationary time series.

Prob
&
Stat.

random walk.

escape from 100: $\frac{200}{200+100} = \frac{2}{3}$.

Finance.

→ strategic & tactical asset allocation.

1年 prediction 范围 4-6月

钱放哪

什么系统. 什么国家.

资产配置.

rebalance freq

小 大

一年一balance. 1个月-balance

quatarly 也算成tactic.

rebalance, 60% 股票 40%债券

8% — 2% —

update portifolio

α strategist

→ portfolio construction.

给一堆α → 如何出 portifolio.

如何 combine,

α: excess return

$$R = \alpha + \beta \cdot \text{factor} + \varepsilon$$

↳ residue.

timing ~~中~~是 regression 的 model.

panel regression 的方法, time series 的 data.

当 ~~past~~ distribution 不变,

correlation (AR, auto regression)

时, 可以拿 panel regression 来

解决 time series data

Finance.

→ 合理定价，以使 arbitrager 的机会消失.

完备市场之假设下

→ complete market.

→ 买入被低估之资产，卖出被高估之资产，以获得 profit.

Finance

B return! more

$B \rightleftarrows E$

Eq! move to bond.

$$P = \sum \frac{c}{(1+Y)^n}$$

c → Coupon.

Y是yield (类似于 interest rate)

比如c是年末发的coupon，换算成当下的值，要除上利率(多期利率).

yield 上升，price ↓，return ↓

今天面一个 bond 值 100元

coupon rate (利息率 4%)

bond 每年给你一个 coupon

所以 bond return 分两部分.

一个是 coupon 带来 return (carry)

一个是 price 上升带来的 return. (value)

$$P_B = \sum \frac{c}{(1+Y)^n} + \frac{100}{(1+y)^n}.$$

$y = c$，

一直值不变

一直等于100块.

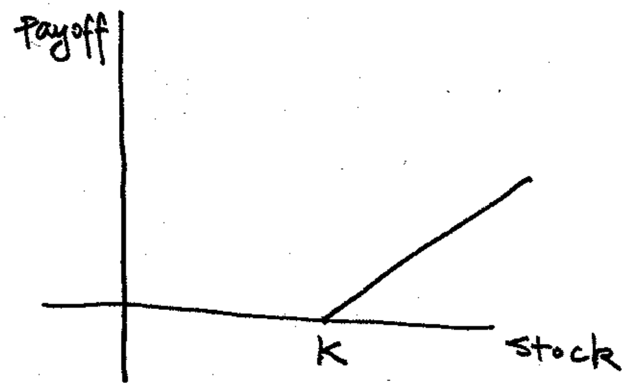
$f = aS + bB$

$a = \frac{\partial f}{\partial S}$ delta

$\frac{\partial^2 f}{\partial S^2}$ gamma

$\frac{\partial f}{\partial \sigma}$ vega

$\frac{\partial f}{\partial t}$ theta



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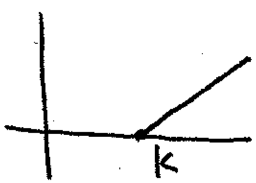
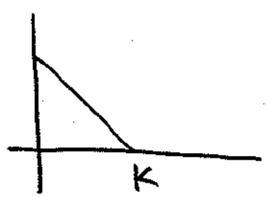
put-call parity.

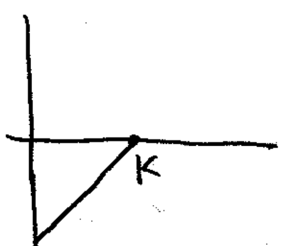


call

买一个
put



sell一个
put

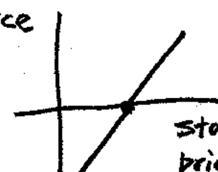


at same K

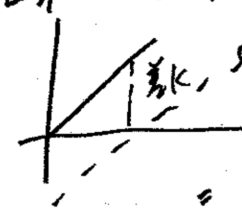
C - P

1. on the same underlying stock
2. 同期 expire.

stock 波动大, options 更贵.
小, options 更便宜.

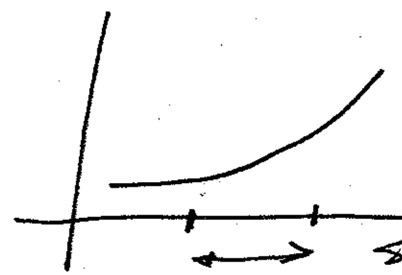


正常stock 过(0, 0)



expire 时成立.

不 expire 时.



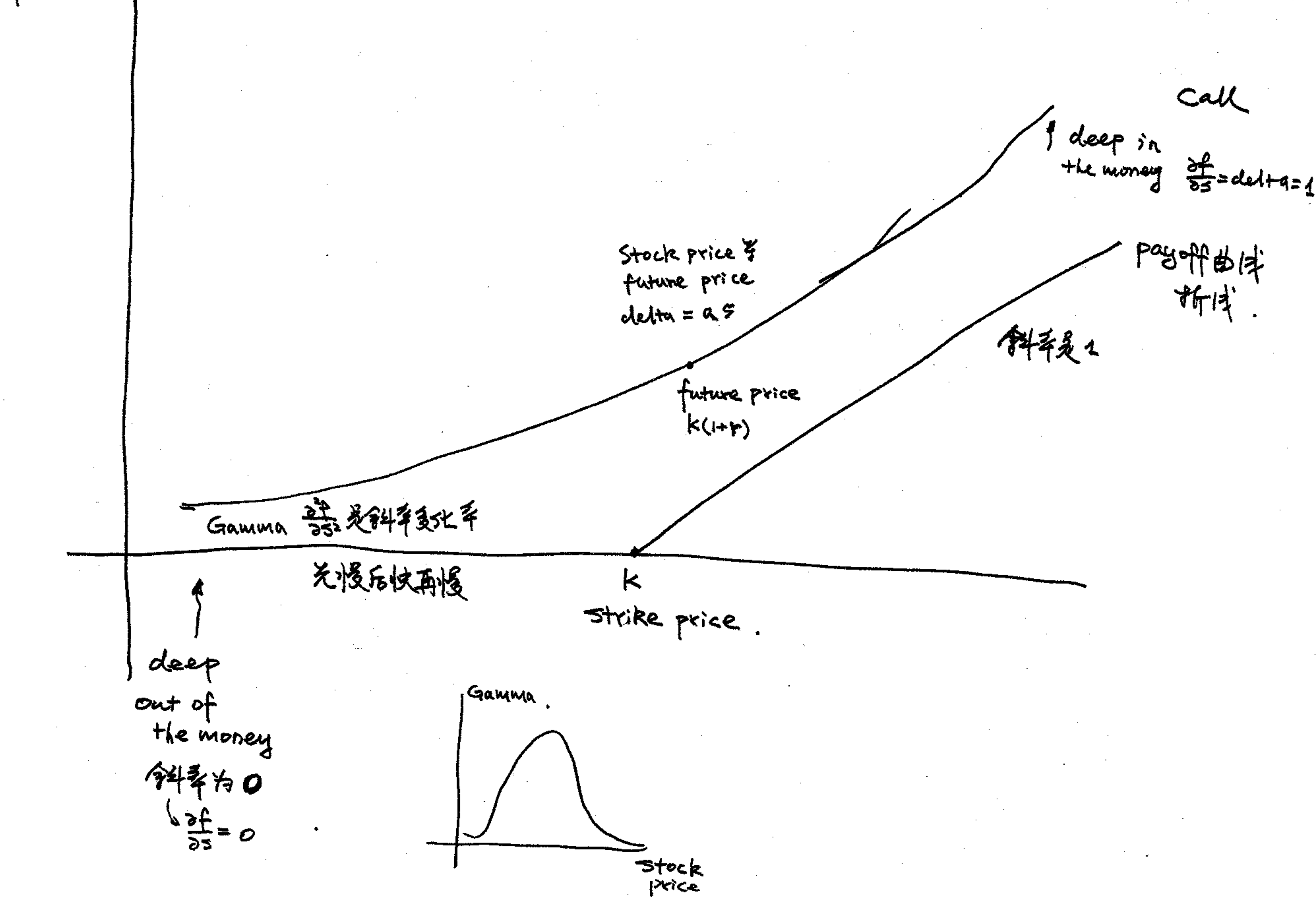
$$\underbrace{C - P}_{\text{put call parity}} = \underbrace{S}_{\text{stock}} - (\underbrace{Ke^{-rt}}_{\text{future price}} + D).$$

Finance 2

$dS = \mu S dt + \sigma S dW$

在~~完全竞争~~ non arbitrag的条件下 $\mu = r$.

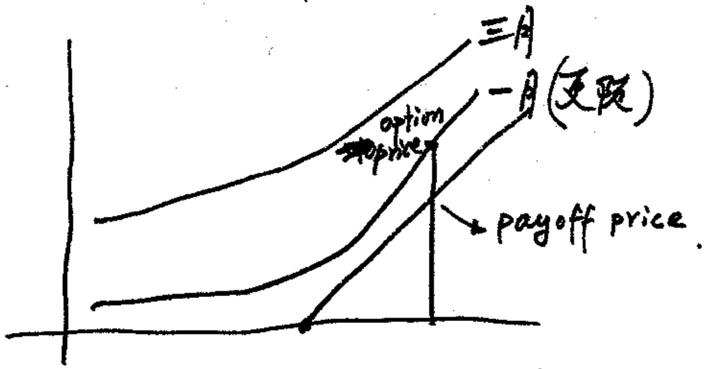
→ risk free measure

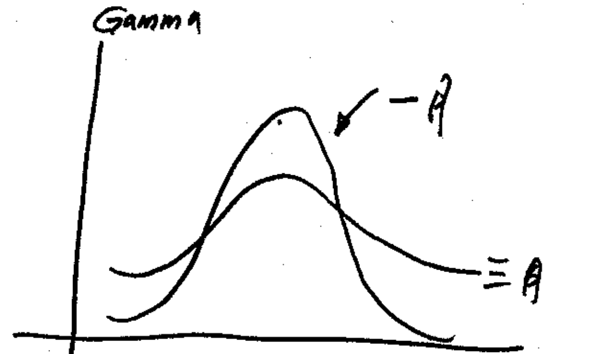


Finance
检 3

一个call option, 一个一月到期, 一个三月到期

越到期越接近payoff





→到期只能拿payoff.

没到到期日,比到期可多赚一些.

time value

三月高于一月的.

在at the money处

最大.

theta always 是负值.

(你刚买时最大, 随时间变化, 每天掉time value都变低一点,
time value
到最后只剩payoff了)

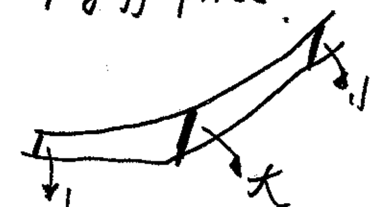
→ put和call相反.

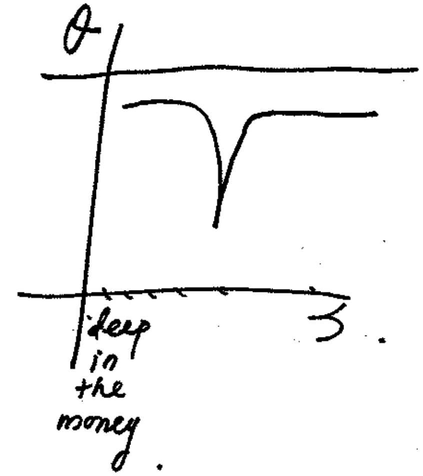
→ always 一月 三月线.

→ always 两条间的线

option price

payoff price





Given $\log X \sim N(0,1)$.

Compute the expectation of X.

scope of variables in C++.

difference between pass by value/reference/pointer.

ants on a stick.

Given some coin, how do you flip them to
make sure all coin have the same sides?

What is polymorphism?

design a matrix class.

Expected number of rolls to see all
six sides on a die.

Binomial pricing.

Print spiral of a matrix.

Coding.

Write code to multiply two matrices.

Coding.

You have two strings, whose only known property is that:

当你点燃一端，在一小时烧光

How do you measure 45 minutes?

To write down code for x^n in $O(\log n)$ time.

Blindfolded.

In a room with 8 coins. What's the minimum of flips required to guarantee that one of the permutations had all coins with the same side?

Eb 1

无排期
(不用等).
特殊人才

a 自己证明是杰出人才
PhD + 10篇paper + 100引用 + 20审稿.

b. 公司证明你是杰出人才
PhD + 5篇paper + 50引用.

Eb2.
3-5年
排期
且得公司给
你申.

投资移民
50-80万+..

婚姻绿卡.

Finance.

vs.

economics.

- seasonality filter X12 专门调节数据的.

- missing value, different frequency. Kalman filter

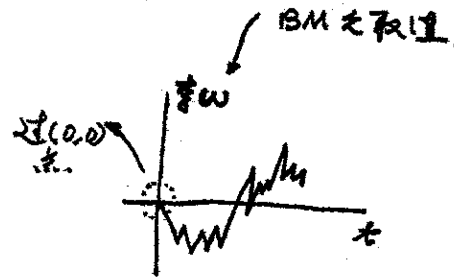
  GDP — 一季度

  unemployment — 每月.

- vol/noisy MA, EMA

Brownian motion (Wiener process)

W_t → indexed with time，t 时刻的值.



1) $W_0 = 0$ （t 时刻，从 0 跑起）

2). Brownian motion $W(t)$ is normally distributed $\sim N(0,t)$

The increment of BM is also normally distributed

$$W_t - W_s \sim N(0, t-s)$$

$$W_t \sim \underbrace{N(0,t)}$$

不是普通正态.

variance 随时间 t ↑.

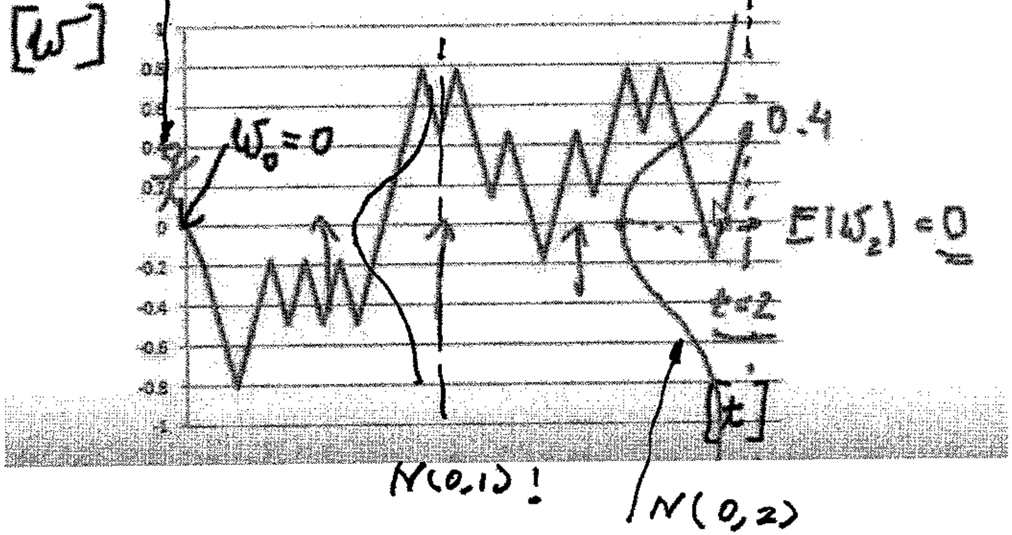
预:

	W_t (BM)	$W_t - W_s$ (Increment of BM)
dist ~	$N(0,t)$	$N(0,t-s)$
Expectation	$EW_t = 0$	$E[W_t - W_s] = 0$
Variance	t	$t-s$

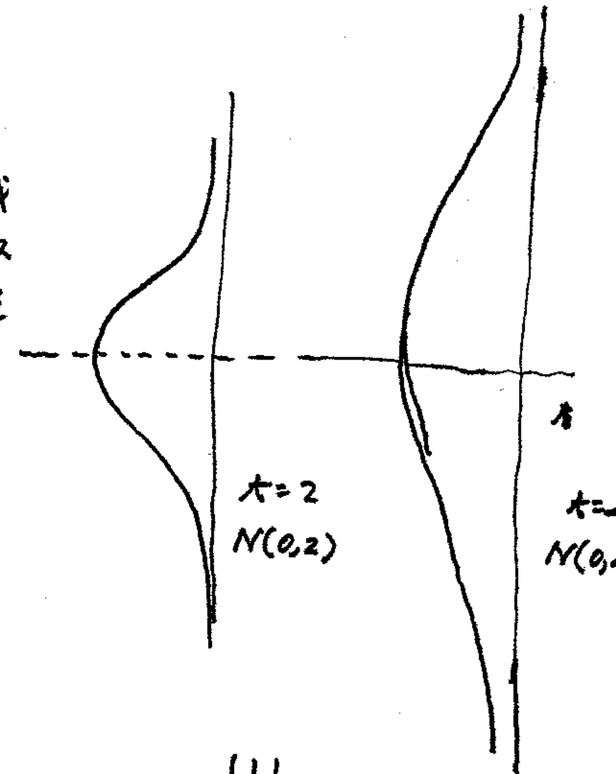
Brownian motion.

$W_0=0$ 是初始时刻在坐标原点 → $t=2$时刻

问你W在什么地方?



答案是W可以取竖轴上之任之一点。我们知道这个取值之分布满足正态分布!

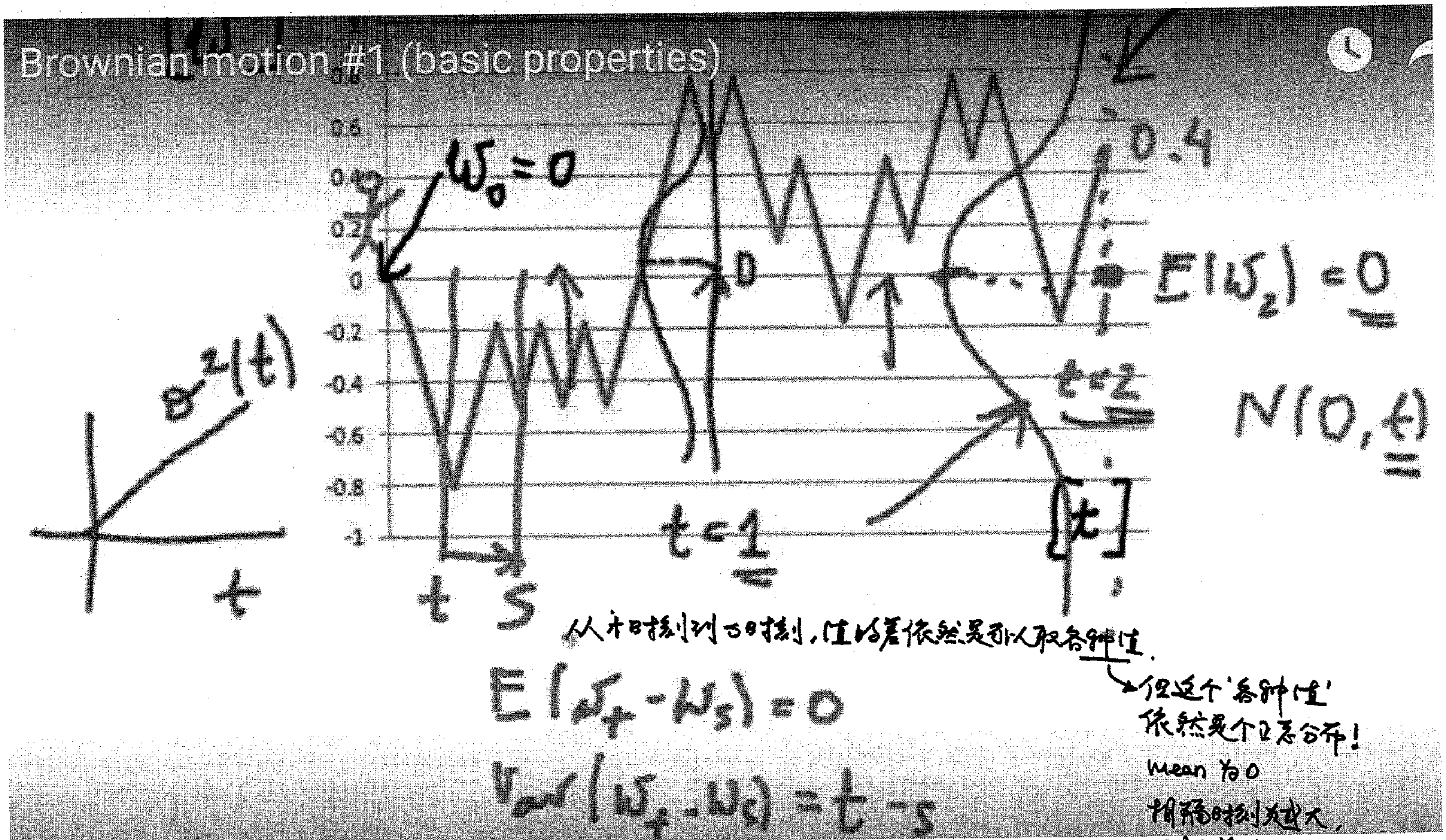


$W_t \sim N(0,t)$!!!

所以!每个t时刻都是一个正态分布!这个正态分布,恒以0为均值,时刻t之大小为 variance (方差!)

这不是一个单式子,而是无穷多之式子!

Finance 3.



从t时刻到s时刻，值的差依然是可以取各种值。

但这个'各种值'依然是个正态分布！
mean 为0
相隔时刻越大，
则方差越大的正态分布！

a) the term stationary increments means that $W_{t2} - W_{t1}$ has same distribution as $W_{t2+a} - W_{t1+a} \sim N(0, t_2 - t_1)$

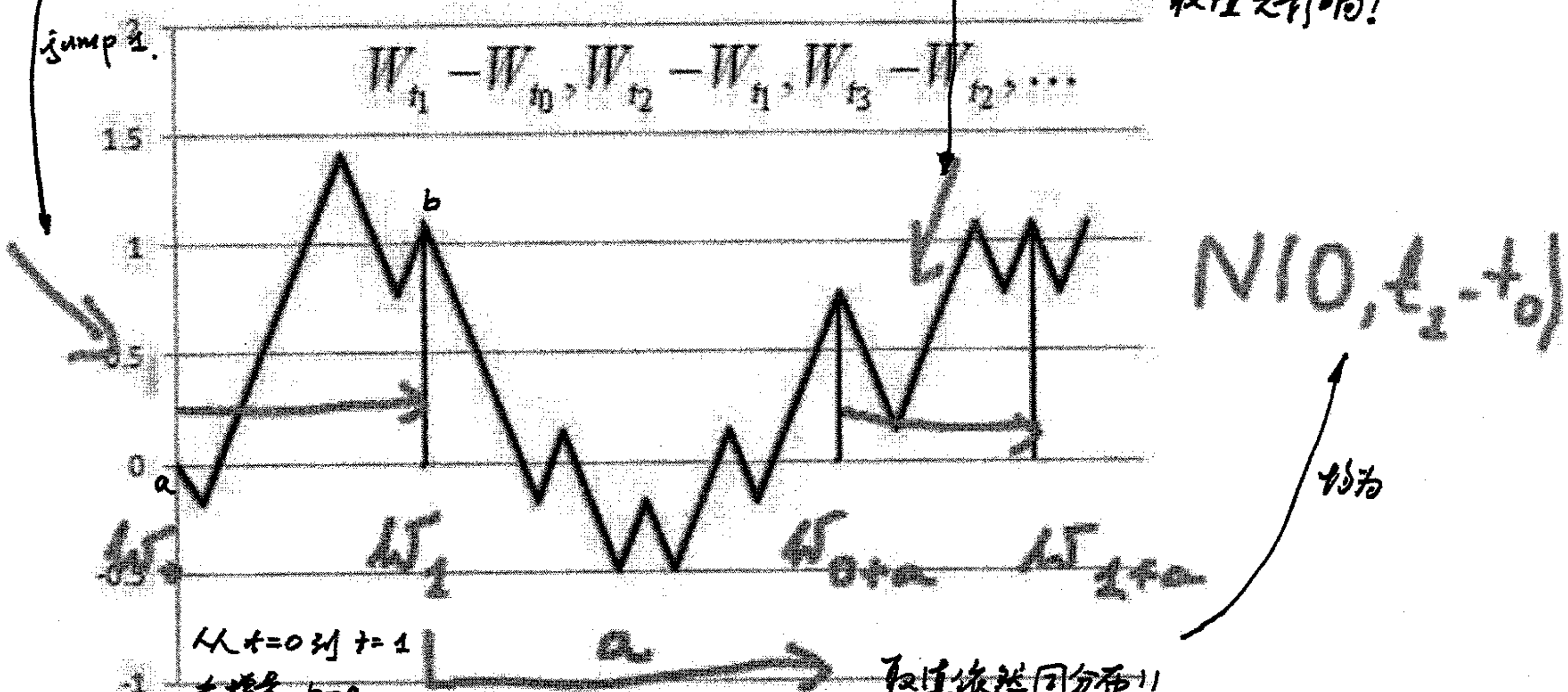
Just an increase in time, Jump!

b) The term independent increments means that for every choice of $t_0 < t_1 < t_1 < t_3 < t_4 ... < t_n$ the increment random variables: $W_{t1} - W_{t2}, W_{t3} - W_{t4}, ..., W_{tn} - W_{tn-1}$ are jointly independent

jump 2

jump2之取值
丝毫不受 jump1
取值之影响!

jump 1.



从 $t=0$ 到 $t=1$
之增量 $b-a$
是个R.V., 可取任何值
但取值满足正态分布

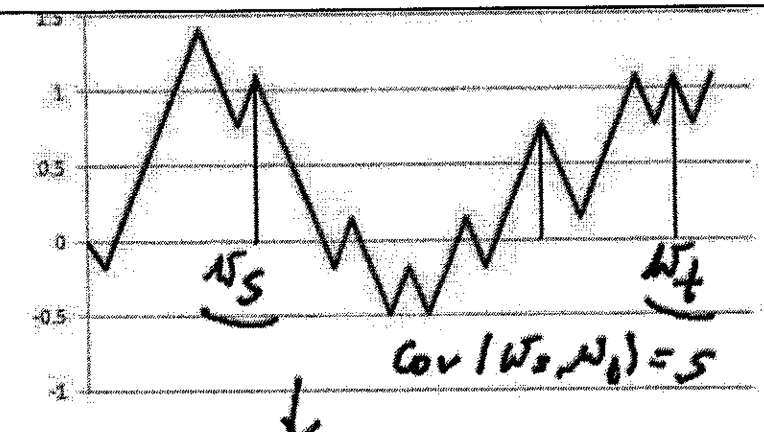
平移a后

取值依然同分布!!

means
'stationary'

均为

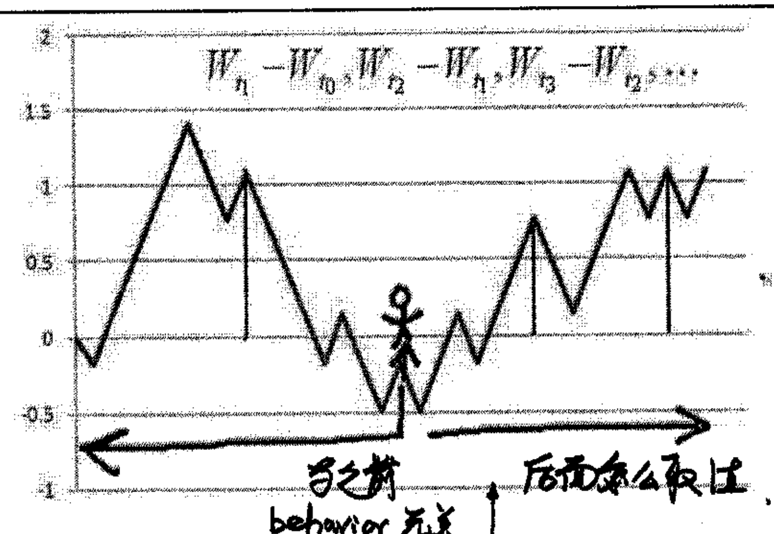
Finance 5



4. $cov\{W(t), W(s)\} = min\{s,t\}$ Showing this property will help us prove that BM has independent increments.

5. BM is a Markov process: Past is irrelevant for where I am going to be in the future.

6. BM is a Martingale: $E[W(t+s)|F_t] = [W(t)]$ = The expected value of where I am going to be in the future is where I am now.



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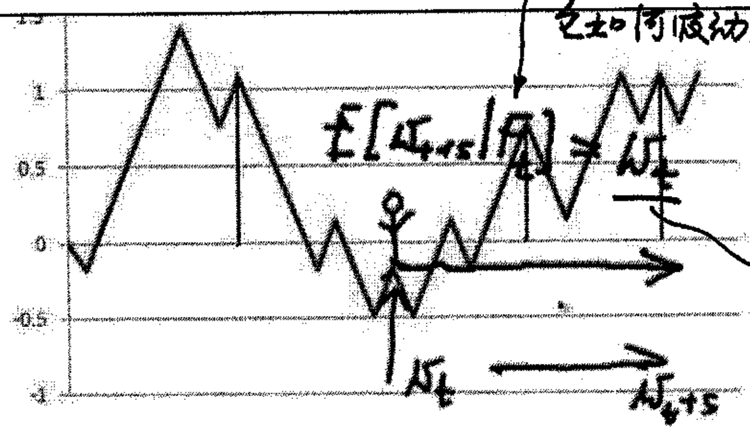
F_t代表之前所有之信息.
之如何波动, 到 t时刻
之W_t这个值的



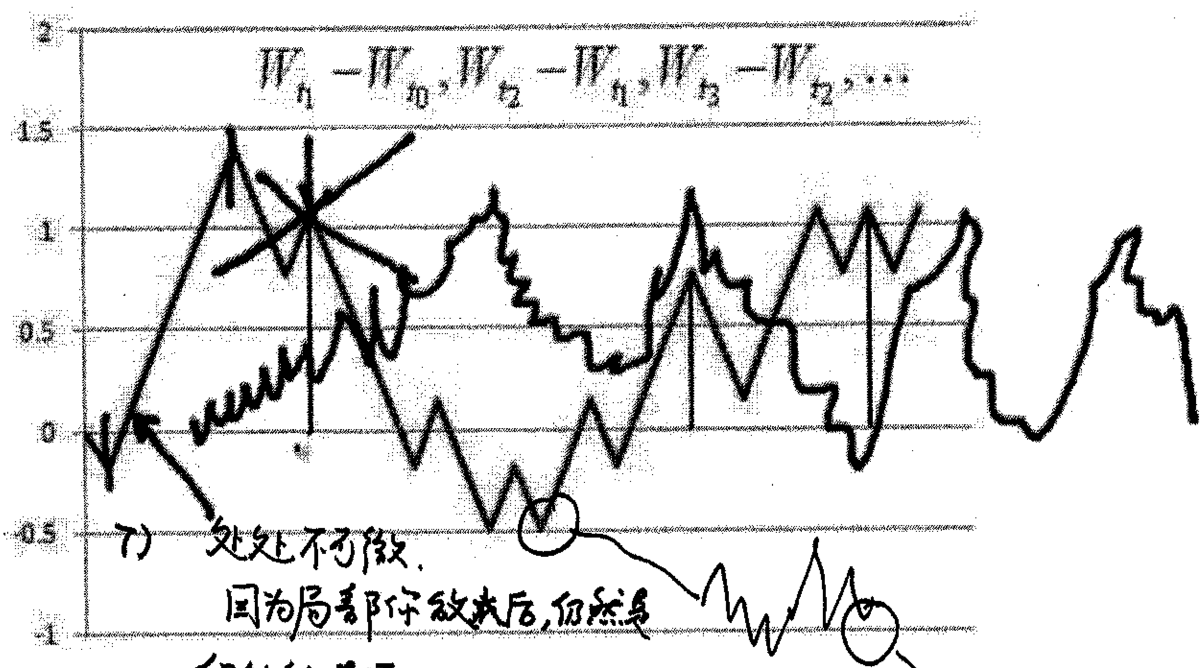
based on 你 t时刻
之前的所有信息

4. $cov\{W(t), W(s)\} = min\{s,t\}$ Showing this property will help us prove that BM has independent increments.

5. BM is a Markov process: Past is irrelevant for where I am going to be in the future.

6. BM is a Martingale: $E[W(t+s)|F_t] = [W(t)]$ = The expected value of where I am going to be in the future is where I am now.

$E[W_{t+s}|F_t]$
t+s时刻值
$\|$
W_t
(得到预期)
这代叫 martingale



但处处连续

仅能补gap.

8) 分形
fractal!

Martingale.
"describe a fair game"

Martingale.

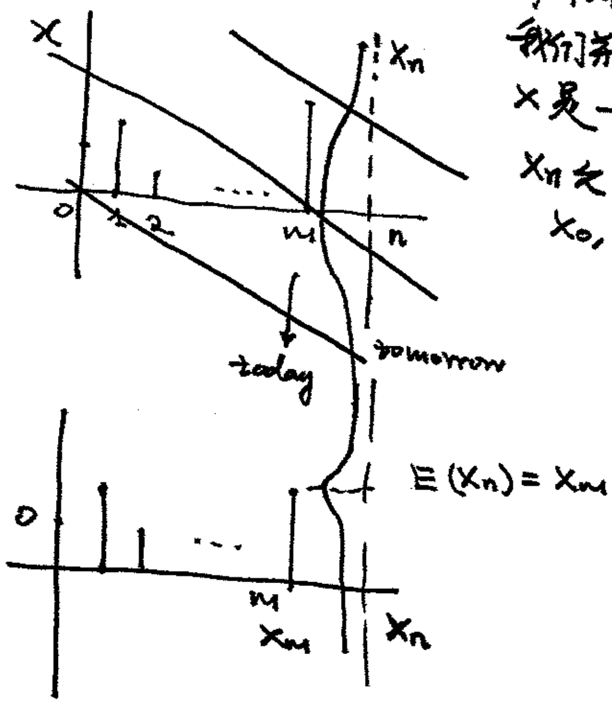
- $\underline{\text{Zero}}$ drift stochastic process.

  ↘ doesn't not change.
  (mean)

  $\mu = 0$.

- $E(\underbrace{X_n \mid X_0, X_1, X_2, \cdots, X_m}) = X_m$

  我们知道X在 $t=0, 1, \cdots m$ 时的信息. X_n 可能取很多值, 具体取何值, 我们并不知, 但如果X是一个martingale, 那么 X_n 之 <u>expectation</u> based on $x_0, x_1, \cdots x_m$, 便等于 x_m.



取值
即 X_n 是一个分布
我们不知此分布为何.

→ 若 X_n 是 BM, (Brownian motion)
我们知此分布为 Normal

→ 一般之 martingale 我们只知此分布之 mean.
$E(X_n) = X_m$

Fin.
量化投资 1
第3章
统计学基础

量化投资，
指利用数学、统计学、IT 来进行
管理投资组合。
portfolio

股市. equity
期货. options, futures.
债券. bond
基金 funds.

处于不断之变动之中.
但与物理定律不同. f=ma. 可准确预测.

金融资产无人可准确预测明日之值.
今：有随机因素存在.
虽无法准确预知.
但可以从中寻找规律性.

~~概率学是~~ 给出随机事件之描述
即随机数学是
统计学则是寻找随机事件之规律性.

如抛均匀硬币.
抛一次，你不知是 head，还是 tail.
抛一万次，你可予知 head : tail → 1:1
此便是随机事件之规律性了.
大数定律之 power!

FIN
量化投资
2.

认为金融数据，有遍历性。
即历史是可以借鉴的。
可以借助以前之数据来预测未来之数据。
虽然只有一个观测数据，但若将以前数据视为在相同条件下得到，可视为重复多次之试验。

量化：将"文本"用数字来代表。
股票'好'，'很好'，… 给一个分数：很好'6'，好'5'..
随机变量也是如此！把一个事件，对应一个实数！
最简单者，示性函数，
1 代表发生。
0 代表不发生。

F2N
量化投资
3.

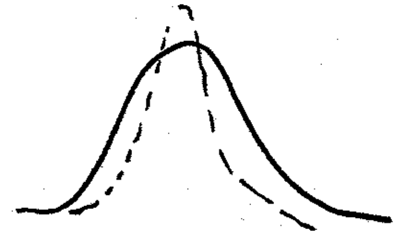
数值特征.

期望，方差 (Var)，偏度，峰度.

方差 → 标准差 std → 衡量数据之离散程度

偏度 → 衡量数据是否对称.

峰度 → 与正态分布做一个比较.



山尖越陡，则峰度越大.

此四个特征都是随机变量的矩 "moments".

原点矩　　中心矩.

$E\{(X-EX)^k\}$　　EX^k.

均值：$EX = \sum_i x_i P(x_i)$

方差：$VX = \sum_i (x_i - EX)^2 P(x_i) = (x_1 - EX)^2 \cdot P(X=x_1) + (x_2 - EX)^2 P(X=x_2) + \cdots$

偏度：$\gamma = \dfrac{E(X-EX)^3}{(VX)^{\frac{3}{2}}}$

峰度：$K = \dfrac{E(X-EX)^4}{(VX)^2}$

均为原点矩.

通常是不知整体，只知样本的。

样本期望 — 均值。

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

--- 描述数据之中心位置。

→ 均值、中位数、众数等…

→ 如拿到一支股票，求一下它的均值，便可知它大概处于什么水平。

→ 价格均值
或是
收益率均值。

样本方差：

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

→ 无偏的，更多人使用。

$$\hat{s} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

这种计算是有偏的。

偏度：skewness

$$\hat{s}(x) = \frac{1}{(n-1)s^3}\sum_{i=1}^{n}(x_i - \bar{x})^3$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

若股票A，B均值同。
A之方差更大，则是更大概率会取到极端的值，
则是风险更大。

峰度：kurtosis

$$\hat{K}(x) = \frac{1}{(n-1)s^4}\sum_{i=1}^{n}(x_i - \bar{x})^4.$$

- 峰度与偏度之计算十分相像，只是次方数有所变化。
- 偏度可以衡量此组数据是否对称。
- 峰度可以衡量 —— 是否陡峭

- 关于峰度：

  若峰度大，则更为陡峭，则易于取到均值附近值。

  小，——平缓（厚尾），更易于取到极端之值。

- R中只须一个函数可算出所有。

  fBasics.

```
> basicStats(mmm)
```

→ 某股票之收益率.

Min		
Max		
1. Quantile	-0.027613	(25%分位数)
3. Quantile	-0.051689	(75% ——)
mean		
median		
sum		
Var		
stdev		
Skewness	0.438876	
Kurtosis	3.430089.	

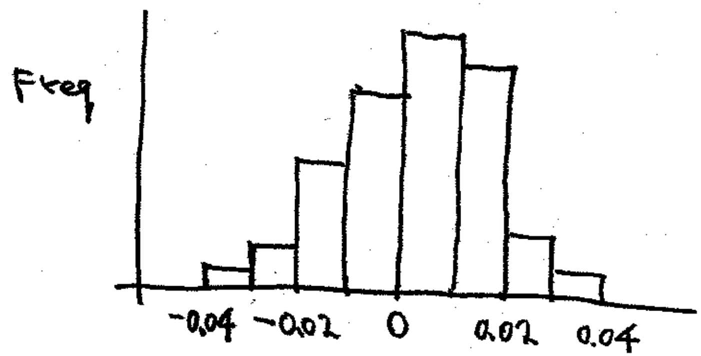
→ 正态是等于3.

3.430089 是默认减去了3后得的（超额峰度）

概率密度函数，类似于直方图。

histogram,

Histogram of index.SH$RetIndex



收益率之直方图.

2 二项分布.

伯努利试验，结果只有两个.

n重——：一个伯努利试验，独立重复n次.

n重伯努利试验，试验成功之次数 X，服从二项分布.

$$X \sim B(n, p)$$

重复次数　单次成功概率.

$EX = np$

$VX = npq$

泊松分布：设 R.V. X之取值为 0, 1, 2, … X之分布律：

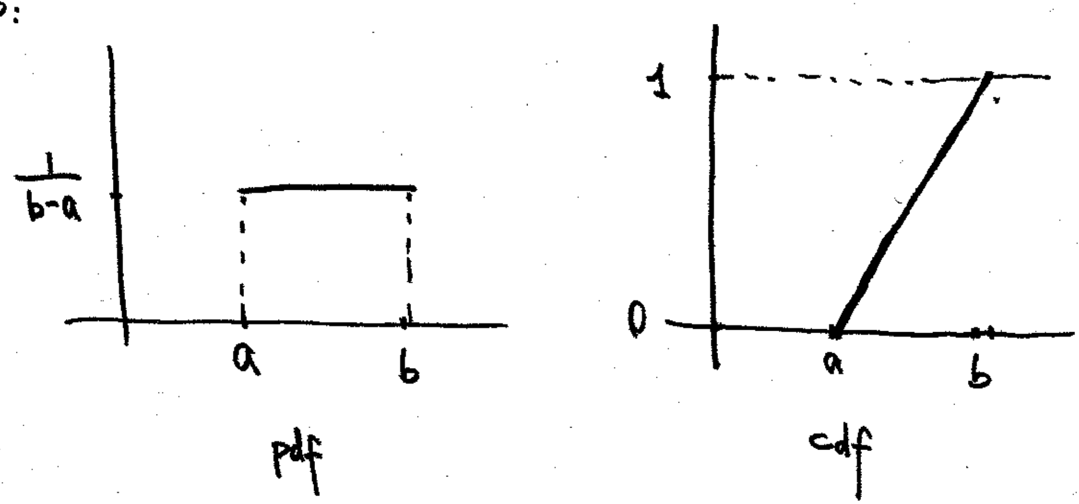
· 是二项分布之极限. $\lambda = np$，且 n大，p小时，才更精确.

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

如单位变时，位变乘之概率.

· $EX = \lambda$，$VX = \lambda$.

均匀分布：



正态分布.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x \le +\infty.$$

$X \sim N(\mu, \sigma^2)$

位置参数
控制~~对称轴~~对称轴
之位置

尺度参数.
$\sigma^2 \to$ 大，钟型曲线更矮胖.

卡方分布:

$X_1, X_2, \cdots, X_n$ i.i.d. $\sim N(0,1)$

比如取 n 个样本.

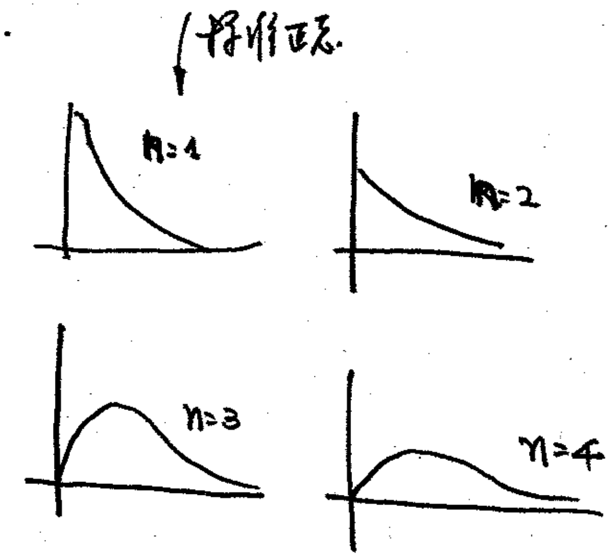
则称统计量

$$\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2$$

服从自由度为 n 之卡方分布，记为 $\chi^2 \sim \chi^2(n)$

卡方分布概率密度函数:

$$f(y) = \begin{cases} \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} y^{\frac{n}{2}-1} e^{-y/2}, & y > 0. \\ 0 & \text{其他}. \end{cases}$$



t分布 -- student分布.

设 $X \sim N(0,1)$，$Y \sim \chi^2(n)$，且X、Y相互独立，则称随机变量 $T = \frac{X}{\sqrt{Y/n}}$

服从自由度为n之t分布，记为 $T \sim t(n)$

t之概率密度：

$$h(t) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{\pi n}\,\Gamma(\frac{n}{2})}\left(1+\frac{t^2}{n}\right)^{-\frac{n+1}{2}} \quad -\infty < t < +\infty$$

与正态分布长得很相似，只是形状会更加"矮胖"、"厚尾"。

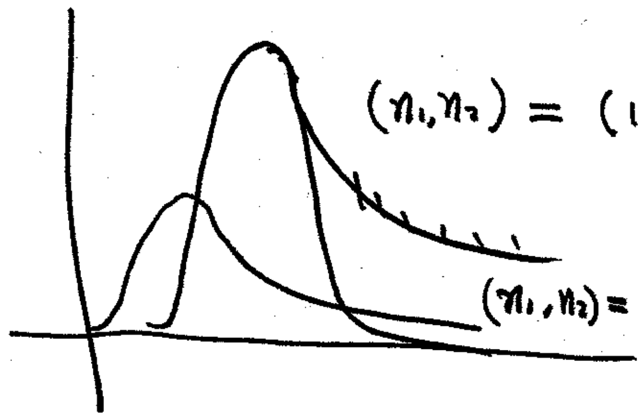
t分布对于抽样之描述更为精确。

F分布.

设 $U\sim\chi^2(n_1)$, $V\sim\chi^2(n_2)$, 且 U.V 互相独立, 则称 R.V. $F=\dfrac{U/n_1}{V/n_2}$ 服从自由度为 (n_1, n_2) 之F分布. 记为 $F\sim F(n_1, n_2)$

星 期 10.

$$\psi(y)=\begin{cases}\dfrac{\Gamma[(n_1+n_2)/2]\,(n_1/n_2)^{n_1/2}\,y^{(n_1/2)-1}}{\Gamma(n_1/2)\,\Gamma(n_2/2)\,[1+(n_1y/n_2)]^{(n_1+n_2)/2}}, & y>0\\ 0, & 其他.\end{cases}$$



χ^2, t, F 分布在假设检验中非常有用.

runif (n, min, max)

rnorm (n, mean, sd)

rbinom (n, size, prob)

rt (n, df)

rchisq (n, df)

rf (n, df1, df2)

rpois (n, lambda)

↘ 描述性及推断性统计

通过均值，方差这种简化的 feature 来描述统计数据.

画出直方图，饼图也算是一种方法.

通常拿到的是样本.
如何由样本推断总体.

↘ 样本.

各次抽样 i.i.d. 独立同分布.

点估计：

比如欲知总体均值，可以用样本均值估计.

———— 方差，———— 方差 —

无法说明置信度.

区间估计方法. ————→ 显著性水平 α，置信度 (1-α) → 这两个其实是一回事.

找置信区间. 5% 95%

先找一样本统计量

比如均值

然后题目会给出允许之抽样误差.

↓

统计量 ± 误差

↓

根据构造的统计量之分布

↓

得区间估计

区间估计其实很取决于样本

服从何种分布，与总体有何种关系.

董波 13.

估计一物品的重量，将其称了10次，得到重量为 10.1、10、9.8、10.5、9.7、10.1、9.9、10.2、10.3、9.9，假设所称出的物体重量服从正态分布，求重量置信度为0.95的置信区间.

```
> x <- c(10.1, 10, 9.8, 10.5, 9.7, 10.1, 9.9, 10.2, 10.3, 9.9)
> t.test(x)

    one sample t-test
    data:  x
     t = 131.5854, df = 9, p-value = 4.296e-16
    alternative hypothesis: true mean is not equal to 0
    95 percent confidence interval.
       9.877225,  10.222775
    sample estimate:
     mean of x:
         10.05.
```

意为做100次抽样，95次，真实值会包含于置信区间内.

```
t.test (x, y = NULL,
        alternative = c("two.sided", ...
        mu = 0, ...
        conf.level = 0.95, ...)
```

假设检验：

基本思想：

小概率事件在一次试验中不会发生。

如果发生，则说明原假设有问题，我们来拒绝它。

反证法。

先认为原假设为真。

构造一个小概率事件，若其发生，则推断假设不成立。

零假设 H_0：

与

备择假设 H_1

如：

H_0：明天股票会涨

H_1：——跌

无罪推定原理。

假设你无罪，看对方（原告）能否证明你有罪。

类似地，先假设零假设为真，然后去找足够证据来推翻此假设。

第一类错误:

原假设 H_0 为真，但被拒绝

（无罪之人被判有罪）

第一类
犯错概率为显著性水平 α.

星投
15

第二类 —

原假设错误，但我们接受了它

（有罪之人被释放）

第二类犯错概率不那么明白指出.
但保证 α 不变时.
提高样本容量. 可降低第二类错误
发生之概率.

P值:

H_0 为真时，出现此样本观测结果更极端之事件的概率.

"P小"说明小概率事件在一次试验中"偏偏发生". 于是我们拒绝原假设.

P大时，不能拒绝原假设.

也不意味着100%接受. 更为严谨.

常见假设检验：

Z检验　　基本原理均同，
t ——　　关键在于检验统计量服从何种分布。
卡方 ——　　用样本构造出某个统计量。
F ——

正态分布 —— Z检验
t —— t ——
X^2 —— 卡方检验
F —— F ——

- t检验常用于
　检验单一总体均值是否等于某个数，
或比较
　两个独立样本是否相等。

卡方检验常用于：
- 两个总体方差是否相等。→ F检验
总体方差是否等于某常数。

- 总体方差已知，用Z检验
　　未知，　t检验。

　其余情况类似。
　但是一般会用t检验。
　因为通常并不知总体方差
　而t检验只须知样本即可。

量检 17.

一件物品之重量，将其称了10次，得到之重量为:

10.1, 10, 9.8, 10.5, 9.7, 10.1, 9.9, 10.2, 10.3, 9.9.

假设称出之物体重量服从正态分布.

现在想知道该物品重量是否显著不为10?

```
> x <- c(10.1, 10.9, 9.8, 10.5, ...)
> t.test(x, mu=10)
```

H0一定要含等号.
H1则不行——

只有原假设包含等号,才能进行下一步之判断.

One sample t-test:

data: x

t = 0.6547, df = 9, p-value = 0.5291

alternative hypothesis: true mean is not equal to 10.

H1.

95 confidence interval:

9.877225 10.222775

sample estimates:

mean of x

10.05

P-value = 0.5291.

表示不可拒绝原假设

表示物品重量仍为10.

chisq.test( )

F.test( )

1. 给出假设检验

2. 构造统计量

3. 根据统计量服从之分布，得到 p-value

→ 与显著性水平比较

越小越拒绝 H_0

大 不了——

点估计，区间估计，假设检验，

均是基于一元统计量.

两个变量之间的性质，如相关性.

如：- 美国股市上涨与中国股市上涨有无关系.

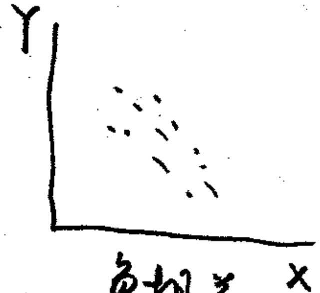
- 房地产之低迷是否影响银行股之走势

- 不同种股票之间是否有关系.

→ 相关性衡量这种变量之间的线性关系.

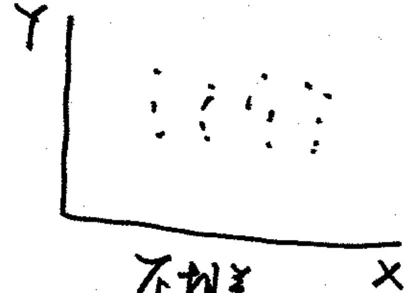
正相关

cor → 1



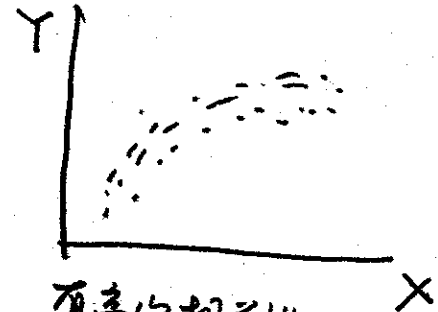
负相关

cor → -1



不相关

cor → 0



有高次相关性

(用相关系数衡量不出来了)

相关系之度量:

协方差 cov

相关系数 cor

$$cov = (X-EX)(Y-EY)$$

$$corr = \frac{cov(X,Y)}{\sqrt{VX}\sqrt{VY}}$$

相关系数更好用.
因为除去了量纲.
于是不同种类之数据
易于进行相关性之比较. → 比如房地产与股市之关系.
为不同单位之量.

R:

cov(X,Y)

cor(X,Y)

量投
20.

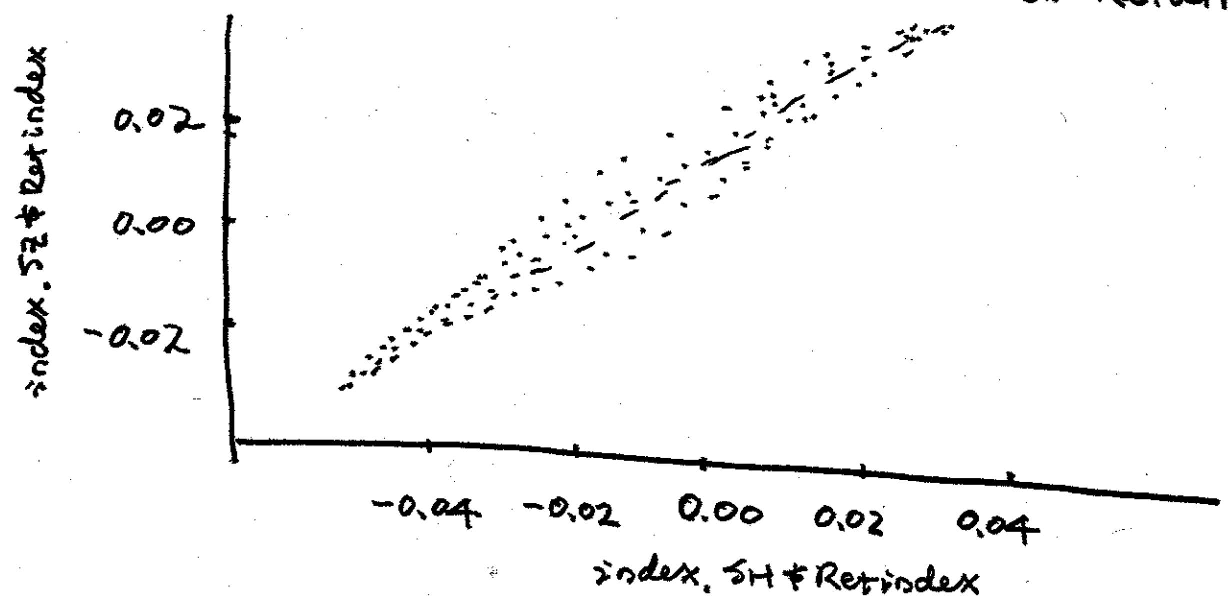
上证指数与深证成指收益率之计算.

Shanghai

```
> cov(index.SH$Retindex, index.SZ$Retindex)
[1] 0.0002079941

> cor( ——————————— )
[1] 0.9082776
```

Scatter plot of SH & SZ Index Return



时间序列：

- 同一个个体的某个特征随时间推移而不断发展、变动之过程。

 - 比如一个妹子每年上称下自己的体重 → 构成时间序列。

 - 每个股票每天之价格。

 - 某个资产之收益率/对数收益率。

 $\underbrace{\text{收益率/对}}_{\text{金融中常研究}}$

- 数据序列有哪些基本特征？

 是否有规律可循？

 若存在此种规律，如何通过统计建模来找到此种规律？

 如何通过历史数据对未来进行预测？

 多个时间序列之间是否存在某种关联，如何刻画这种关联？

F2N

量化投资

第4课

时间序列

1

xts, zoo (R包).

tseries.

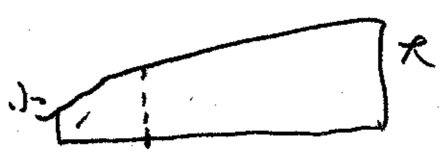
quantmod — getSymbols("CHL", src = "yahoo")

時间序列之描述性统计.

- 期望, 方差, 峰度, 偏度
  ↳ 超额峰度 (默认已经减去了3)
- 超额峰度: 尖峰厚尾.

• 分位数:

先把数据从小到大排列.



25% (Quantile 1)

25% 分位点是, 25% 的数据比它小.

75% ——— 大.

时间序列常用自相关.

↳ 自己与自己的相关性.

前3天与今天有无相关性?

滞后量 lag

y_t y_{t-1} y_{t-2} ... y_{t-l}

一阶滞后量 二阶滞后量 l 阶滞后量

— ~~幕~~ 自相关性对随机过程而言是很 crucial 的.

有了自相关性, 你知 y_t, 就可预测 y_{t-l}.

没有自相关性之随机过程, 无法做 prediction.

差分:

$\Delta y_t = y_t - y_{t-1}$

$\Delta^2 y_t = y_t - y_{t-2}$.

随机变量 滞后l后产生的新随机变量

自协方差 $\gamma_l = \mathrm{cov}(y_t - y_{t-l})$

自相关系数 $\rho_l = \frac{\gamma_l}{\gamma_0}$

时间序列 $\{X_t\}$ 中之 X_t 均为随机变量

$t=1, 2, \cdots$

$X_1, X_2, X_3 \cdots$

可以毫无关，如

$X_1 \sim N(0,1)$

$X_2 \sim U(0,1)$

$X_3 \sim \chi^2(3)$.

但有协方差计算中，认为 $X_1, X_2, \cdots X_n$ 来自于同一总体之样本.

balance sheet 股票研究之fundamental.

Corporate finance 1

Asset

Liability & share holder equity.

current assets: (短期的)

1 inventory.

2 cash & cash equivalent.

3 account receivable

4 prepaid expense.

流动性: 2 > 3 > 1 = 4.

原材料, 半成品, 成品(未卖出) 合算 inventory.

Current liability.

- short term debt
- account payable.
- long term debt due in one year.

长期负债到的maturity到了一年以内, 便转化为短期负债了.

Fixed asset 长期的 固定资产.

properties, plant, equipment
- 减去折旧 (depreciation)

intangible asset 无形资产
- (amortization).

investment

(例如宝能系或的收购万科)

Long term liability. 长期负债

long term debt

Share holder equity

preferred stock

common stock.

Book value:

total asset − total liability

(其实就是share holder equity那项)

是账面上, equity holder 所占有的公司的部分.

Market value

(i.e. market cap

= ~~price~~ price × shares
(equity price)

是市场上, 所认为的, equity的总价值.

一般来讲, market cap > book value,

因为mkt_cap 不仅包含资产当前的价值, 还包含资产将创造的价值 → 未来的earnings.

book-to-market value

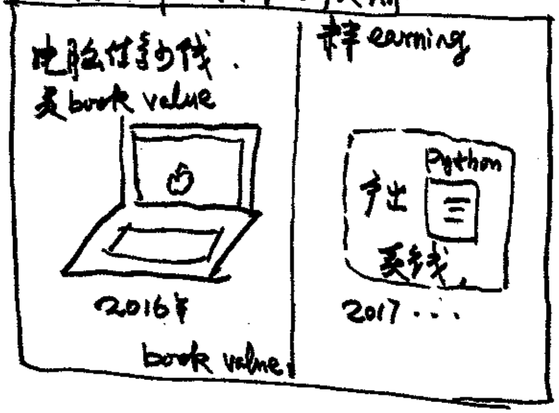
book-to-market value.

对于一个 growth 型公司, 如 startup,
它的 book-to-market value, 较低,
~~market~~

market cap > book value
大于的那部分判断了未来 earnings.

有点类似于 earning / price ratio.
= 都在讲同一件事情. 都是 value factor.

Market cap 对未来的预期.



1. Good will 通常不算在 book value 中.
因其估计不准.

2. book value 会算 preferred & common
market value 不算 preferred equity
因其 price 与未来收益无关.

现实计算 book/market ratio 是用

$$\frac{\text{book} - \text{preferred}}{\text{market}}.$$

Corporate finance ③

Capital employed = total asset − current liability.

Intuition: 当下可用来投资的钱，永远，有多少钱专门属于你的长期投资者

之等于 equity + long term liability.

永远不用还，但是要分享利益。　　短期不用还。

而 investment 通常是个中长期的活动，
所以 short-term liability 不能拿来

发了 equity 后，equity 便一直在外面 roll，与我没有关系。

equity 是"公司的一部分"
part of company.

买卖 equity，通过二级市场，不与公司直接打交道。

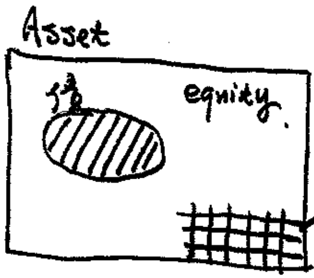
上市 IPO，其实是把公司分成很多 piece 给卖了。
equity

但不同于真正的卖，它的所有权归 equity holder，但还有权还在你手里。

只是公司的管理层，实际上是代理人。

Corporate finance 4.

把债拿走,
Asset剩下部分,便是equity



equity 分成很多很多的小piece.

卖给 share holder. $\xrightarrow{\text{所有share holder 聚一起开会}}$ 股东大会

公司的最高权力.

公司之所有者.

选出一部分人来负责日常

代表股东利益 ← board 董事会 $\xrightarrow{\text{head}}$ 董事长

决定运营的人

management $\xrightarrow{\text{head}}$ CEO

独董:

不hold公司股份, 不在公司任职, 不参与日常运营.

站在第三方的角度来判断, 决策是否正确.

Corporate finance

5

- Operating asset = Total asset − cash (及 cash equivalent)

  所~~[illegible]~~

  亦即所有可以创造收益的资本.

- Working capital = (current asset − Cash) − (Current liability − short term debt)

  所有在运营的"跑来跑去的资本"

  ≈ inventory + account receivable − account payable.

  在运营中，至少要多少钱
  才能维持我日常之静运营.

  如果积 过大.
  负债货商的太多.
  Working capital 为负.
  可能导致资金破裂

- debt to equity
  debt to asset

  是借钱多少的衡量.

  leverage ratio

- current ratio
  quick ratio

  形容公司短期偿还能力, liquidity ratio.

  $\dfrac{\text{current asset}}{\text{current liability}}$

  quick = $\dfrac{\text{cash + account receiable}}{\text{current liability}}$. 最流动性的两项.

Cash ratio
$= \dfrac{\text{cash}}{\text{Current liability}}$

Income statement:

Gross sales, 又叫 headline sales.

- Returns, allowances, & discounts

= Net sales.

- Cost of good sold (COGS)

= gross profit 毛利
(Net sales - COGS).

~~Net sales - Cost~~

- Operating expense (毛利)

= Earning before interest, taxes,
债 税
depreciation and amortization
折旧 无形资产折旧.

* EBITDA.

- Depreciation & amortization.

= operating income.

: 卖出一个好的产品.
对应的变动成本:
原材料, 劳动力,
可以放入单件商品中的成本.

厂房中的打印用纸, 属于 COGS.
CEO ——, 属于 operating expense.

interest, taxes ... 是金融 factor. 随时可变
如国家调税率.

EBITDA 是排除掉这些可变的.
公司主营业务的盈利能力.

operating income 是可以每年产生的 income
卖个厂房的钱, 不属于 operating income
毛利
是一次性的.

Operating income. -- 吃好多次

\+ Non-operating income -- 吃一次

= Earning before interest and taxes (EBIT).

− interest expenses.

= Income before tax

− Taxes

− Minority interest

= Income from continuous operations

\+ Extrodinary items and discontinued operation. ← 比如分厂倒闭
或是一次比较奇怪的收入.

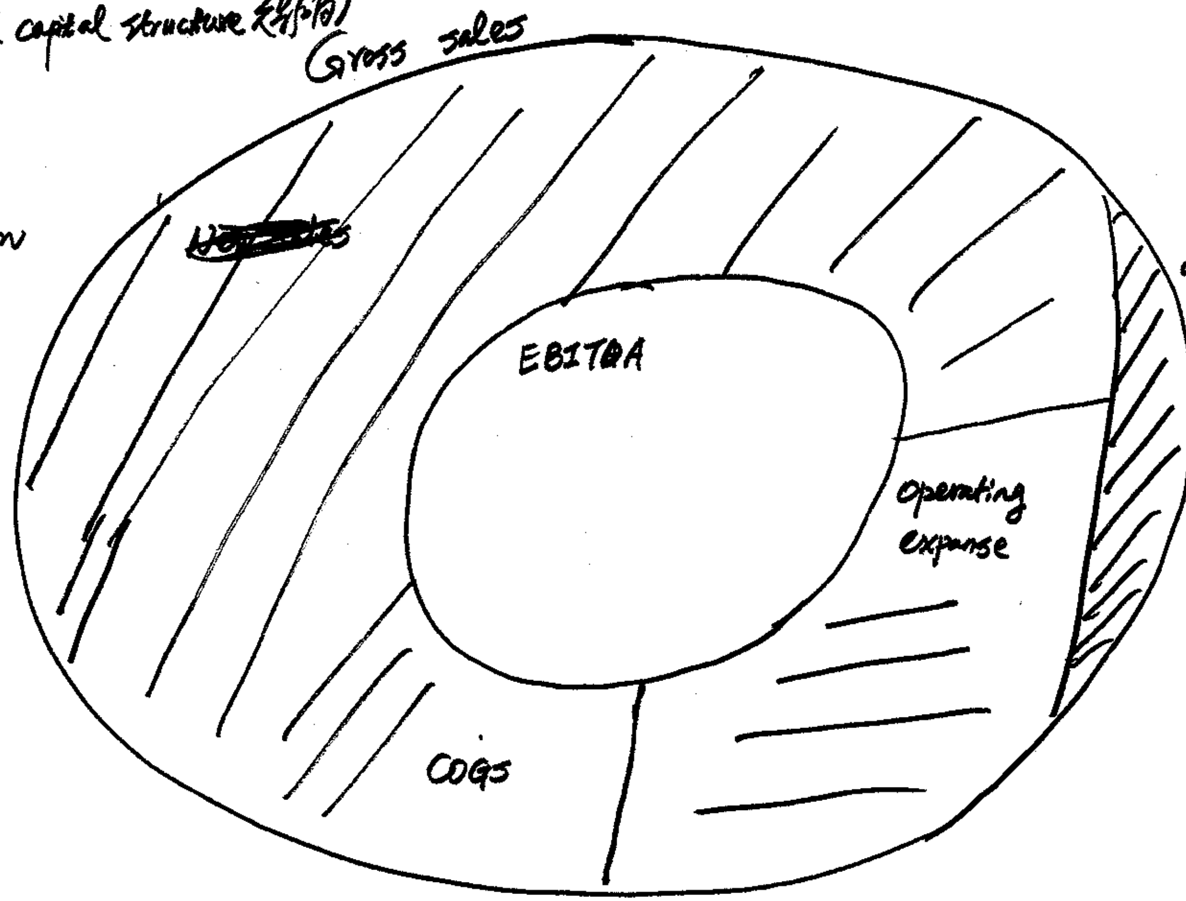
= Net income.

Corporate Fin 8

EBITDA 很重要，
是公司正常運营下之收益能力
(排除 capital structure 之影响)

FCF的关键
free cash flow

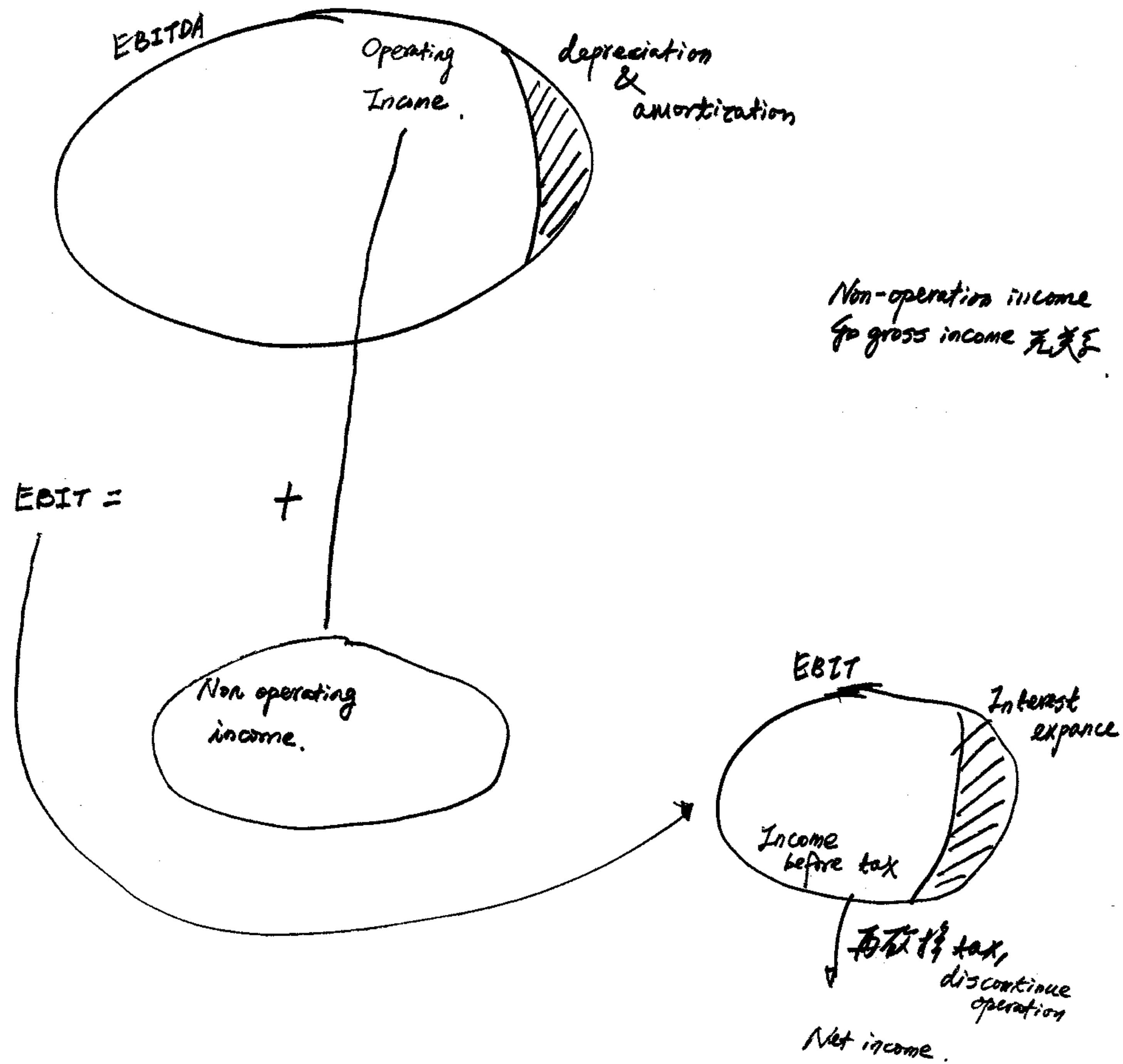
所有公司金融的关键.



a點.
销售的
退貨, allowence,
discount
折扣

EBITA + COGS + Operating expanse = Net sales = Gross sales − return allowence discount

Corporate FIN



所有的 margin 的，是 $\frac{\text{赚的}}{\text{卖的}}$.

Corporate FIN 10.

✓ gross profit margin = $\frac{\text{gross profit}}{\text{Net sales}}$.

事称 gross margin

Net sales = gross profit + COGS.

✓ operating profit margin. = $\frac{\text{EBIT}}{\text{Net sales}}$.

✓ Return on asset (ROA, 就是 as

=(Net income + interest expense) / average total asset.

(Tax 除掉了. 但 interest expense 加回来了.)

balance sheet 的平均 asset

over last year.

balance sheet 是一个点.

income statement 是一段时间.

很多服务业.

及金融业, 该办法

衡量具体 asset,

使用 Return on equity

来衡量.

Return on equity. (重要)

$\frac{\text{Net income}}{\text{Shareholder's equities}}$.

大家比较 return on equity

和融资成本.

来衡量公司之盈利能力.

投行的 return on equity

已经接近融资成本 SP 出名了

Corporate FIN 11.

payout ratio = $\frac{\text{divided}}{\text{Net income}}$

(你挣的钱中，有多少返还给了股东。)

公司处于发展阶段，会不太会发 dividend，
会进行投资，能得更多的回报。

Turnover：流动性的财产的流动速度的衡量。

不很常用。
金融中

更 care 回报率。

Cashflow 分三种。

① Operating 公司运营
Investing 公司投资
Financing 公司融资

Operating cashflow

= Net income 加回 Non cash charges
depreciation & amortization
减去 change in working capital.

FCF (free cash flow)

= Operating cashflow − CAPEx

为了未来增加产量所进行的投资
如厂房、设备、软件

fixed capital investment

~~Operating~~

Working capital investment

Stock timing model:

是一个关于大盘的 future
(期货)

mini future of S&P 500.

用本期的数据预测下一期的 return.

宏观数据.

若是下一期为正, 就 long.

下一期为负, 就 short

不经过 risk-model

非常 aggressive.

就怕突然大跌,

因为是用的宏观数据来预测.

预测反应很慢.

作用:

通常的 hedge fund strategy,

是 market neutral,

value

quality

high div low vol.

等, ~~讲~~ 你 long 500支, short 500支.

基本上 market exposure 便很小了, 比较 conservative

于是当市场很好时, 容易 under perform 市场.

于是在上面加一层 layer. 便是 stock timing

市场好时, time model 提供一个 β.

只有一个 security

Market neutral, 资金量的投入是固定的

你 long 和 short 的资金量是相同的.

比如你有一个 million,

long 一个 million,

short 相同的量. 其实你是借了一个 million 的股票先卖了. 你得回 90万, 于是 long 100万

short 100万

你只需 10万. 加了 10倍杠杆!

Market neutral

通常比较 conservative,

市场好时, 并没有增加

总的投 AUM 量

资金

~~risk~~

risk budgeting.

你的 risk 是可控的, 虽然投入的资金量是变化的

FIN 1.

↘ imagine market Beta as a matrix,

	GOOG	APPL	GE	AAA
1990-12-1	0.3	0.4	0.2	0.2
1990-12-2	0.31	0.41	0.18	0.19

$$r_i - r_f = \beta_i (r_m - r_f) + \alpha_i + \Sigma_i$$

$i = \{GOOG, APPL, GE, AAA\}$.

r_m, r_f 对大家都一样

↘ alpha signal neutralized by industry, market...

↘ 我们不赌 industry.

对 industry 之 exposure 为 0.

一开始可以在 Top 2000 中研究.

- SSRN. 中搜索关键字, 按下载量排序. 很多 paper 发到 Journal 之前, 会先到 SSRN. Free

在某一个 industry, 永远是 market neutral 的
long/short 的策

给苹果一个正的 alpha,
自然会给通讯行业其他股票负的 alpha

所以通讯行业即使下跌,
只要苹果跌得少,
(因为 ~~[illegible]~~
short 通讯行业其他的股票
会赚更多的钱)
就总体依然赚钱

- signal 与 alpha 要 clearly 区分.

  一般 signal 是 raw alpha, 进行加工来得 alpha.

- market neutral:

  我把市场的影响去掉. 我的股票, 在市场上的 long/short 是一样的.

  不做 market neutral,
  alpha 好的可能性低.
  因为你始终在那里赌市场
  (有个市场的 beta 在)

sector 大的分类

industry. 不同行业

做完 industry neutral, alpha 的 performance, 往往可以提高.
因为你把 industry 的影响去掉了.

alpha:

sharpe ratio高.

与alpha pool的correlation低（越低越代表新的risk,

Out of sample 的衡量指标？

主要便是sharpe ratio，但同时也降低了correlation和turnover，那便更好.

在websim framework下，alpha是每只股票的long/short position.

- Journal of Finance.

- Technical

用价格和量的信息.

Fundamental

用公司股格的信息.

timing:

是做什么时候买，什么时候卖.

FIN
4.

$$\frac{\text{Out sample 的 IR}}{\text{In sample 的 IR}} \geq 0.7$$

就算很不错.

用 in sample built your model,
用 out sample data 来进行 back test.

降低 turn over:
把 decay 变得更长.

Out sample - 30, 40, 60.
指从 in sample 结束的那一天开始算, 第30, 40, 60天的 sharpe ratio.

alpha和Beta

time series regression.

FIN.

$$r_i - r_f = \alpha_i + \beta_i (r_m - r_f) + \varepsilon_i$$

i可以是Apple，可以是Google，可以是华为. r_i, ε_i, β_i, α_i 对每个股，都有不同.

r_f 是 risk free interest rate, cross sectional 对苹果，对Google大家都一样.

r_m 是 SP500，大盘，cross sectional 都一样.

数学框架，~~本来~~是 linear regression.

经济之 intuition:

β 是因为你承担了 systematic 的风险.

这里的 systematic，指 market.

market，~~大~~股指涨跌，是分散到个股头上的.

你承担了风险，于是有 expected return.

所以是 risk premium.

β就是factor exposure.

你β大，exposure很多，于是 expected return 也多.

就像你不买车的保险，是省下了保险的钱，但承担了 systematic risk，不出事，你就是只赚的.

α的地位，是不承担风险的超额收益.

投资，本质上，还是借钱给人.

吃利息，你牺牲了流动性.

或者说把流动性借给别人用.

创造了价值你跟着分.

体量大以后，便有了专门之strategy.

f zN

Worldquant simulation setting.

Region & Universe: USA Top3000

Delay 1

Decay

neutralization

Simulation Timeframe 5 Years.

所谓risk的neutralization.

买橘子, 卖苹果.

10种橘子 10种苹果

$$\begin{array}{lcccccc} & 橘子1 & 橘2 & \cdots & 苹果1 & 苹2 & \\ 通过weight, w & [\,+0.6 & +0.7 & & -0.1 & -0.2.\,] \end{array}$$

定w 让 整个食谱中的假特B为0

C为0.

只留假特H

这个special的假特H, 只有我们知道, 可能是α.

decretional factor.

为何叫risk factor.

因为大家都知道

SML

VMG (value - Growth)

MOM (winner - loser)

可是他们有人操纵

这些, 来控制你的买卖.

FIN

XX

- 1 million invest 什么
  - bond/equity
  - 宏观形势.
  - ☆ economist. 全世界经济形势.
  - Journal of portifolio management 每年几期, 几4刀
  - Wall street Journal "to follow market"
    - Financial times
- 美联储每年加几次息?

- self-introduction: 短.
  - 写稿子: 底稿.
  - 关键字提示.
  - 内容、次序.
- 想要问的, 放在最后面.
- workshop
  - independent research/group study.
- We have a common friend.

- Bloomberg, first word
  - trader式口语.
- CNBC
  - xingxingwumich@gmail.com
  - Share123.

hold 一个房子, 房租是 carry, 房子本身的价格是 value.

carry:

value factor: PPP.

Lasso:

market cap, tracking error.

long/short 肯定两个以上, 一个不奇妙了.

Lasso.

alpha1

alpha1 from LQ, Ranked... xslx

共 98517 rows.

monthly

sam

select dates, count(ticker) from alpha1 group by dates order by dates;

select distinct ticker from alpha1;

select * from alpha1 where ticker like 'KO' ordered by dates;

Fin
170108
1

- long value, short growth.

dividend yield advantage,

three sources of capital gain.

relative growth in book equity.

convergence in valuation ratio.

upward drift in valuation ratio

Consumption最难控制.
刺激唯一方法是减税.

value premium

近期看好 growth:

Investment.
Consumption
Government expenditure
Export

dividend 收两种税:
企业所得税
个人 ——

trump: 减税, 对

WACC:

$(1-tax) \times debt +$ 股票yield

infrastructure,

Muni

项目 general

P/B高: IT

P/B低: walmart

F2N
170108.
2

tax: 个人转移支付.

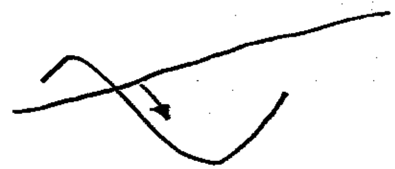
个人 → 政府.

个人花和政府花哪个更efficient.

市场最优,政府次优.

加几次息.

市场低估还是高估.



$(W_p - W_B)\Omega$.

Design value strategy

- systematic / discretionary.
- long-short / long-only.
- value indicator
- rebalance freq.
- reporting lag.
- equity universe, country
- stock ex , REITs, ADRs, ETF, closed
- economic scale.

$W^T R - \frac{\lambda}{2} W^T \Omega W$.

s.t.

$(W - W_B)^T \Omega (W - W_B) \le 1\%$ 控制 tracking error.

FIN
170 / of
3.

Timing. overfit.

momentum.

ex ante value spread.

seasonality. – Jan.

macroeconomics liquidity. indicators.

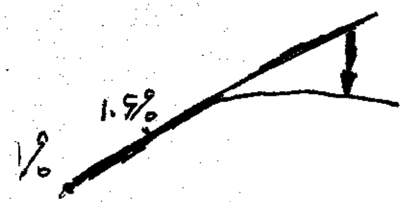
Value

Fix income / bond. currency
PPP.

Value 有点 mean reverting.

Go momentum 结合 . Value, momentum negative correlation.

F2N
170108
4.



classical

QM

We utilized open source portifolio construction tool, pyfolio.

Backtester tool, zipline.

Finance.

lasso.

Seeking alpha ?

Try alpha <u>factors</u>.

→ alpha 与 beta, 都是 factors. !

long/short portifolio ≡ market neautral.

Quantopian powered by an opensource backtester
github.com/quantopian/zipline.

portiforlio construction, risk optimization.

5 Basic accessible quant strategies.

Mean reversion - What goes up ... (special case: Pairs Trade)
Momentum - The trend is your friend
valuation - Buy low, sell high
Sentiment - Buy the rumor, sell the news.
Seasonality - Sell in May & go away

Why it makes money in the past,
how to make money, in the future.

Portfolio and Risk Analytics with pyfolio.

DATA!
DATA!

Quantopian.
Fin

talk from
Jessica Stauth.

3 + statement

Fundamental.

balance sheet
income statement
statement of cash flow.

- 个人资产配置，一般是 equity + risk free
  (可以是存银行，可以是买 bond)
  equity → 可以是个股，可以是投资 Hedge fund

Typically 60 + 40
equity risk-free

→ 当然取决于你对 risk 的 preference.
求稳可以多加 risk-free.
40 equity, 60 risk-free.
比较喜欢 risk 可以
80 equity, 20 risk-free.
甚至是 100% equity
→ 如大部分 hedge fund.

$$PV = \sum_{i=1}^{n} \frac{cash\_i}{(1+r)^i} = \frac{cash\_0}{1} + \underbrace{\frac{cash\_1}{1+r} + \frac{cash\_2}{(1+r)^2} + \cdots + \frac{cash\_n}{(1+r)^n}}$$

present value

未来的现金到 present.
要除以(1+利息).

所以 PV 之变化，一部分来源于{cash_0, cash_1, …, cash_n}
一部分来源于利息.

所以 bond 的价值变化来源于两项，
- 一是 price change,
- 另一是 coupon return.

GTAA overview – investment universe

- *Assets* span more *liquid* segments of global markets to accommodate tactical rebalancing of high capacity strategies. 小盘股.
  - Country and regional equity indices, e.g. S&P 500, Russell 2000, TOPIX, AEX, MIB30, FTSE 100, CAC 40, IBEX 35, DAX, TSE 60, ASX 200, MSCI Europe, MSCI EM, MSCI EAFE
  - Equity style indices, e.g. large and small cap country indices, value and growth country indices
  - Fixed income indices, e.g. investment and high yield country baskets (US) and regional indices
  - Government/sovereign bond indices
  - Interest rate swaps 利率互换，是一种产品，依然是一个对赌协议.
  - Currencies, e.g. G10 and liquid EM forwards
  - Commodities
- *Implementation instruments* can be fully funded such as ETFs, passively or even actively managed funds, or levered instruments such as futures, forwards, swaps (only if leverage is allowed).

Insteres rate swaps

一个是固定的，如4%

一个是浮动的，如 LIBOR + 3%。

银行的短期流动性.
最安全，风险最低（银行再不安全，国家要垮了）

LIBOR：伦敦银行间隔夜拆借利率.

GTAA overview – example 1

- A conservative cash-neutral fully-funded (no leverage) GTAA portfolio that compliments a 60/40 strategic portfolio. Typical constraints include no-shorting in the total portfolio. Implementation vehicles include ETFs, passively or even actively managed funds. Typical turnover is less than 100% per annum (can be lower if liquidity of implementation vehicles is low). Rebalance can be monthly, quarterly, or based on trigger. Typical tracking errors are 1-3%.

战略性, stragigic

global tactical asset allocation
全球，战术性，资产配置.

Investment Universe	SAA (passive)		GTAA (active)		Total Portfolio
US Large Cap	25%		7%		32%
US Small Cap	10%		8%		18%
EAFE	12%		-2%		10%
MSCI EM	10%	+	7%	=	17%
REITS	3%		0%		3%
US IG Bonds	25%		-15%		10%
US High Yield Bonds	5%		-2%		3%
Int'l Fixed Income	10%		-3%		7%

TAA策略可以多种，如
trend following, (momentum)
value (long high value, short low value)
carry

这个比例是根据
market cap 来的.
由市场决定，于是称为passive.

Alternative.

18/01/01.

1

Real asset

Commodity

HF/PE.

private / public

private - NCREIF

public - REIT

valuation

Factor:

value factor.

Rental yield: 租售比

residential.

cap rate: NOI.

Net operating Income/P.

e.g. size: 20,000 sqft

rental: 25/ft

vacancy: 5%.

tax/insurance: 35,000.

utilities: 800,000.

× interest: 400,000.

growth factor.

个人可支配收入.

house ~~pers~~ information.

→ 有Lag.

人口分布.

人口结构. (城市化进程)

value factor:

- earning/price.
- Devidend/~~price~~ yield.
- PPP.

Alternative
17/01/01.
2.

growth
股票的 growth factor: expected earnings. growth

城市化：第三产业(服务业).
消费力.

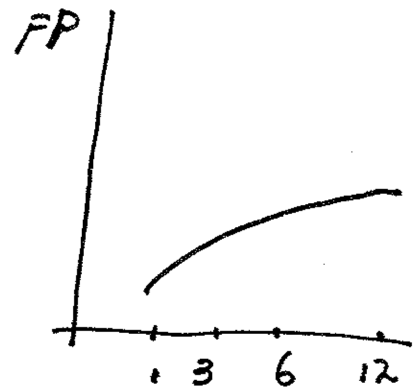
Commodities. futures.

① Futures.

② Futures return.

3 T breakdown.

collateral
spot Δ
roll return



Convenience yield:

$$F = S \cdot e^{(r-q)T}$$

r → cost, q → benifits.

$$\log \frac{F}{S} = r - q \quad \swarrow T = 1 \text{年}.$$

r → storage + Financing

q → convenience yield

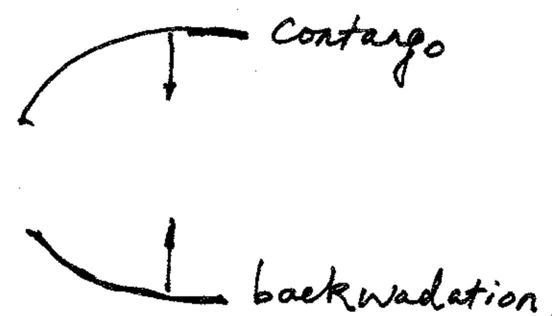
Factor:

momentum,

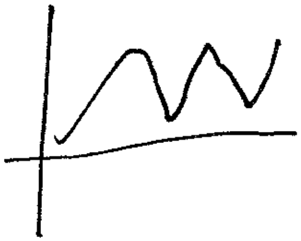
carry: roll return

value: 没有太多的 factor.

commodity by commodity.



Alternative 4



$$\max\left(w^T r - \frac{\lambda}{2} w^T \Omega w \right)$$

$$\text{s.t. } \Sigma w^T = 1$$

$$\Sigma w^T = 0 .$$

$$-\frac{1}{\beta_H}(r_H - r_f) + \frac{1}{\beta_L}(r_L - r_f) .$$

$r_{i,t} = \beta \times f$

exposure

$$\begin{pmatrix} \; \\ \; \end{pmatrix} \quad T \quad \overset{n}{\begin{pmatrix} E/P & \cdot \\ & \end{pmatrix}}$$

已知 factor exposure. 求 factor return.

h 是 portifolio weights.

$$\max \; hB - \underbrace{\frac{\lambda}{2} hB\,\Omega_F B^T h^T}_{\text{整个 portifolio risk}}$$

Factor mimik portifolio.

s.t. $B_i \cdot h = 0$

只要一个 factor, 别的都被 natualized.

max return − risk

$r = \lambda B \Omega$

high-div, low vol portifolio. FZN 1 Workshop.

本 portifolio 很简单，只有两个 factor,

high div: long dividend 最高的 top 10% stocks,
short —— 最低的 bottom 10% ——

low vola: long volatility 最低的 10% stocks.
short —— 最高的 10% ——.

high beta, high volatility:

CAPM 模型认为个股 是正比于 market

比如 apple, google

大盘, SP500 index.

不同个股之比例系数不同.

$r_{apple} = \beta_1 r_M = 1 r_M$ 于是 β高, volatility 亦高.

$r_{goog} = \beta_2 r_M = 2 r_M$

关于 factor return:

FZN 2.

bet against beta:

- long-short:

short top 10% high β.

long bttm 10% low β.

factor exposure 不是 portifolio 的 weights.

把它分成一层一层的, 根据 percentile (factor exposure 的 percentile)

~~long~~ long 最高的

short 最低的. → $-\beta$, 3

子类型因子

Rank

destroy distribution.

OBV

Mean reversion:

is the theory suggesting that prices and returns eventually move back towards the mean or average.

- historical average of the price or return.
- Or growth in the economy, or average return of an industry.

这是一个 long-term signal.

提高频率后, turnover 会很高

分层

- 10% → long 这一部分.
- 80% → 这一部分没有 trade
- 10% → short 这一部分

- mkt_cap 是体量.
  比如有多少支口红.
- volume 是交易量
  每天卖出几支.

选股票的 universe.

→ top mkt_cap

及 volume.

→ 交易量, 代表着流动性.

→ 设一个 daily trade value 的下限

daily value = volume * price

BID Ask spread:

Buy.

Bid

The price a buyer is willing to pay for a stock.

SPREAD.

ASK − BID = SPREAD.

Sell.

Ask

Price a seller is willing to sell their shares.

S Asking price = \$10.1

bid-ask spread = \$0.2

Market price = \$10.

B Offer price = \$9.9.

Bid-ask, long short

不过是fancy terminology of buy-sell.

<u>cross sectional stock,</u>

所有stock的return和所有stock的exposure，做一个regression
得到一个β. 这个β可以理解为此factor的return.

regression 每天一个 cross sectional, 然后叠起来. 得到factor系数的dataframe.
时间是index.
统计系数的非参数, cutoff是150.
用training的最后一行系数 做 expected return的 prediction
每天每个ticker都有expected return.
根据expected return, long top 10% percentile, short bottom percentile, 得到weights

ranking是factor exposure的ranking.

r_{t+1} = lasso regression of [features at t时候]
lag一期可以得 expected return

↑ prediction.

先train一个risk model,
generate一个alpha,
把alpha的ranking减去as,当成weights.

lasso可以refine risk.

~~k~~to

Zhao Yue.

lasso

首先通过lasso选出不为0的features,比如24个选出8个,然后用这8个来组建一个portifolio,

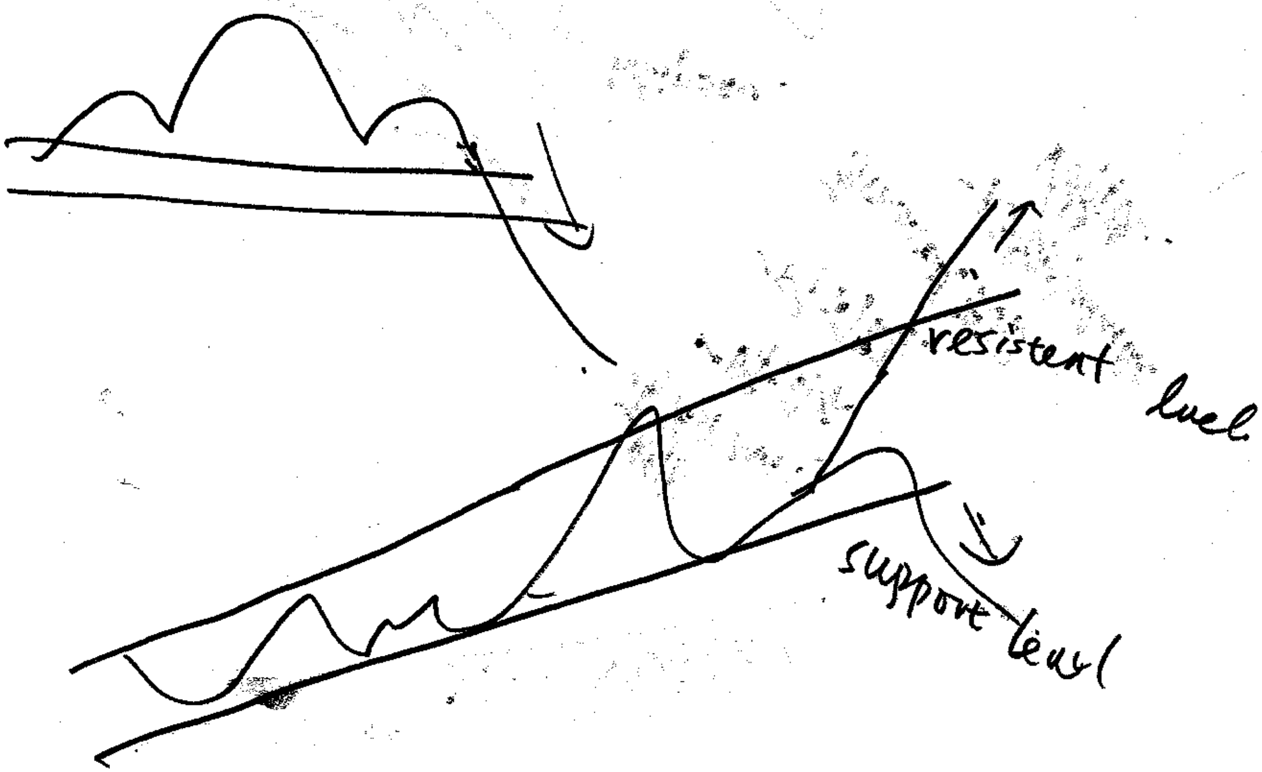
FIN
~~5~~.
Workshop.

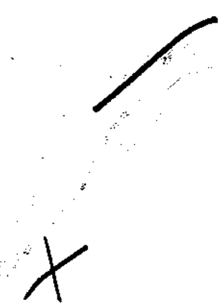
Lasso.

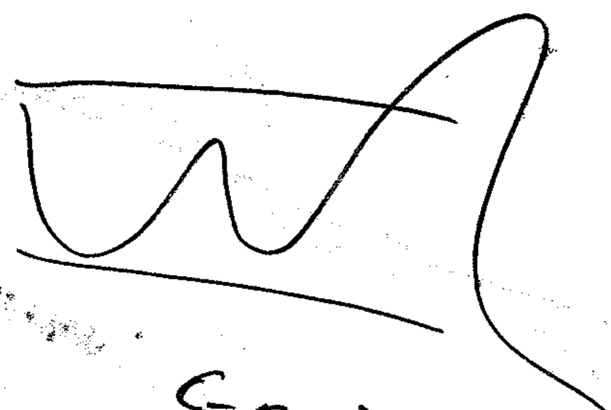
1. Smooth expected return.

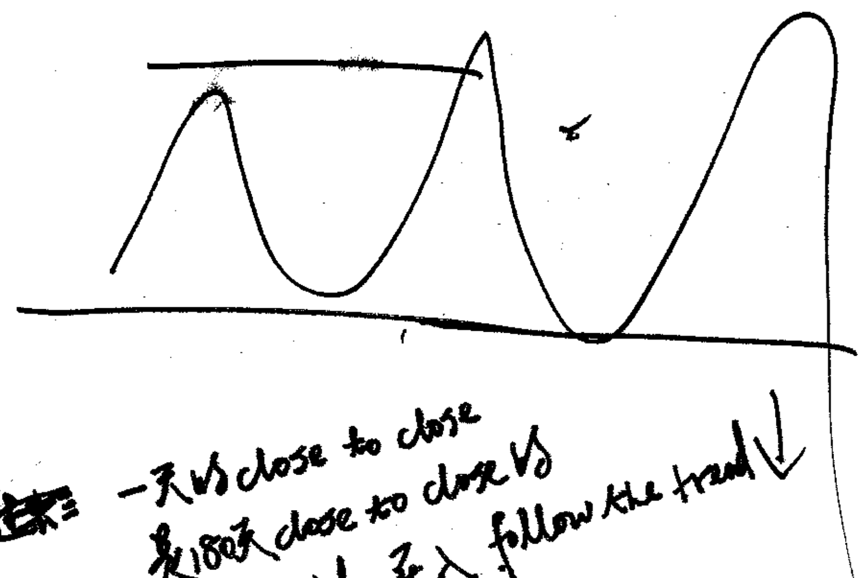
~~prospective~~ 理论

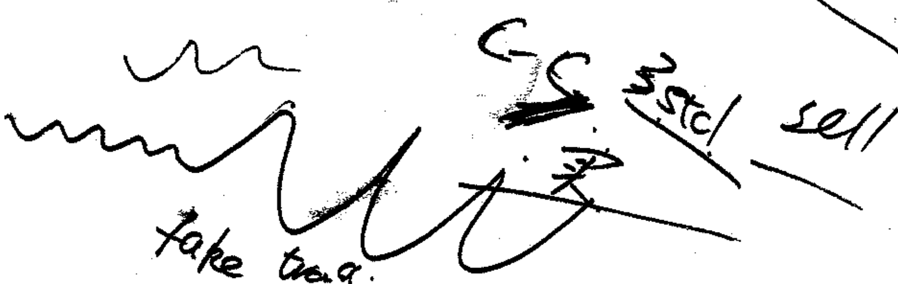
预期理论









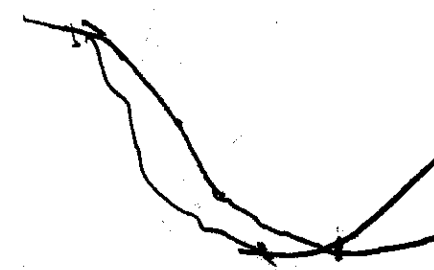


人操: 追涨= 一天的close to close 是180天close to close的 三个std. 买入 follow the trend

MA crossover.

卖.

$X - MA(X)$



lagged.

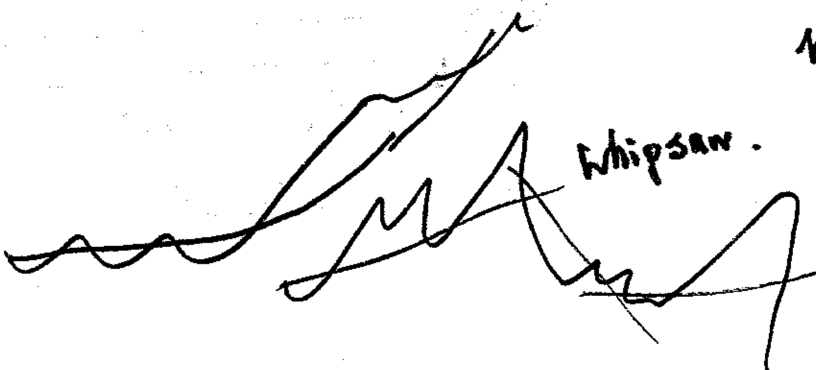
有效 MA.

- 时间长
- 时间短.

有 tradeoff 要调

- 一天的 volatile

V的标准.



Whipsaw.

Secondly, I think it's very hard to get bored in this area, cause researchers here are facing to the ever-changing markets, also people tend to use the cutting edge knowledge like machine learning to stay competetive. This is exactly I'm seeking for.

*** Quantative model example
Oh, there're actually too many examples, because physics is nothing but applied mathematics. I can mention several. For example, I built a set of mathematical model to describe physical phenomena, like laser driven mass transfer, which can be described by an high-order non linear PDE and my job is to developed the PDE and solve it. This requires numeric method and simulation.

*** Project
We did a lot of analysis in this project, but the core part is to generate quant model to do the stock induce timing.

What indice, how to implement (What kind of security)
We basically use sp500 mini future in US and corresponding stock index futures in different

The timing model is regression based quant model. I will take the US model as an example. We regressed forward month futures excess return on several macro and market indicators. I will go through the indicators one by one:

1. ~~Trading~~ 12 month earning yield on s&p500 futures.
Trailing.
dax future for Germany.

#
countries.
For example, we use nikke future for Japan.
*** Project2
It's basically independent research, funded by a prestigious PM from China. He is really interested in understanding quantitative method we use in US. We gathered a group of people with quant and strategist background, to replicate the most popular research and try to deliver that to the PM.

The core part for this project, is a stock timing model. We did a multi-variable regression, on the forward monthly return on the S&P 500 futures (actually mini-futures). We have several predictors.

The first one is value factor, which is a really traditional factor we use in prediction of equity risk premium. So we used the trailing 12-month earnings yield as the value factor.

The second factor is the 3-month change in 10-year yield. Basically it's a reflection on the risk on/off in this market. When the market risk is really high, people try to move their money from the treasury to the equity market, so we would predict higher performance in the equity market, when the treasury yield has to decrease over past 3-month.

The third one is the most important one, which is the NFCI (national finance condition index), we used the leverage sub index, to be specific. Basically it's an indicator reflecting the leverage of the equity markets. It talks about whether people are borrowing money from their dealers or brokers, to invest in the equity market. We expect higher return from the futures if the leverage is high.

The last one is more like quant factor, we calculated the average of the pair-wise correlations of S&P 500 components. The reason why we do this is that we think when the market has much higher correlations, it's either because the market is trading on a single macro event, or the market relies on a single quant factors to generate returns. In either of these situations, it's too risky and we don't want to invest too much money in the market.

The preliminary results for this timing models are pretty good, we got on the raw signal 0.7 sharp ratios in the monthly re-balance, it's not perfect, but as the first-step result it looks promising. And we are trying to get more factors into this model to gain better results.

*** Project Questions
The lines of the code is pretty large.
First we have data collection process, we used python api to grab data from fed-fred and several websites like yahoo finance. Then clean and pack those data together to put into our data base, and generate files to do some data validations, those are finished in python.

The second parts of this project, we use matlab to do a expanding window regressions, we do encounter several issues including multi-colinearities, cause some of our indicators have really high correlation between each other, like nfci and 3-month change in the 10 year yield. We did a regression first of NFCI on the 3-month change in the 10-year yield, and put the residual into the model. So the two time series should be orthogonal, we won't have the multi-colinearities any more.

We do make sure in the expanding window regressions, the beta go to stables so we won't have an issue about in or out the sample.
*** Why do you guys choose these indicators.
We start from a very typical quant method, and tried a lot of quant indicators,

*** Ask them questions
Basically you've seen my background, I'm pretty good at quantitative research and programming, and I have some basic knowledge of finance. And I'm really looking to more exposure to the market and the quant part in the asset strategies. So I want to know the things that I can expose to, is there any opportunities that I can talk to the PM or clients?

Can I know who will be the major parties I will cooperate with, is there any opportunities to interact with different people?

I notices your group has recently acquired by the man group, I want to know what will be the potential growth opportunities of this platform in the future.

Interview
with
mfci.
1

3 month change
in 10 years bond
yield.

NFCI.
nation

risk model
pricing model
mostly credit.

risk ~~high~~ low
people move
~~ba~~ money from
treasury to equity.

PhD physics
berkeley.

Price ~~low~~ ↑ when
yield low ↓
price ↑

↳ exposure to
smart people
and strong
technology.

how to use risky.

↳ compare risk between.
different
portifolio.

→ std for the price.

↘.