TDSÑ:

Linealidad $ax_1[n]+bx_2[n] \to ay_1[n]+by_2[n]$

Invarianza $x[n] \to y[n]$, $y[n-n_0]$
$x[n-n_0] \to y[n-n_0]$

Causalidad $h[n]=0 \; n<0$ POLOS $\geq$ CEROS

Estabilidad $x[n]$ acotada, $y[n]$ acotada. $\sum_{n=-\infty}^{\infty}|h[n]|<\infty$
Ej: $u[n]$

SECUENCIAS ENERGÍA:

Absolutam. sumables $|X(e^{j\omega})|<\infty$
limitadas o infinitas con crecimiento $\leq 1/n^2$

Cuadráticamente sumables
$\sum_{-\infty}^{\infty}|x[n]|^2<\infty$

POISSON:
$\tilde{x}[n]=\sum_{-\infty}^{\infty}x[n-kN] \xrightarrow{F} \frac{2\pi}{N}\sum_{-\infty}^{\infty}X(e^{j\omega})\delta(\omega-\frac{2\pi k}{N})$

PARES DE TRANSFORMADAS:

$\delta[n] \longleftrightarrow 1$

$\delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$

$a^n u[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}}$

$u[n]-u[n-M] \longleftrightarrow \frac{\operatorname{sen}(\frac{\omega M}{2})}{\operatorname{sen}(\frac{\omega}{2})}\cdot e^{-j\omega\frac{(M-1)}{2}}$

PARES TRANSF. PERIODICAS:

$\frac{\operatorname{sen}(\omega_c n)}{\pi n} \longleftrightarrow X(e^{j\omega})=\begin{cases}1 & |\omega|\leq|\omega_c| \\ 0 & |\omega_c|\leq|\omega|<\pi\end{cases}$

$e^{j0n}=\cos 0n \longleftrightarrow 2\pi\delta(\omega) \quad |\omega|<\pi$

$e^{j\omega_0 n} \longleftrightarrow 2\pi\delta(\omega-\omega_0) \quad |\omega|<\pi$

$\cos\omega_0 n \longleftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \quad |\omega|<\pi$

$u[n] \longleftrightarrow \frac{1}{1-e^{-j\omega}}+\pi\delta(\omega) \quad |\omega|<\pi$

$\operatorname{sign}[n] \longleftrightarrow \frac{1}{1-e^{-j\omega}}$

PROPIEDADES:

$x[n]$ real $\longleftrightarrow X^*(e^{-j\omega})=X(e^{j\omega})$
$\operatorname{Re}\{X(e^{j\omega})\}$ par
$\operatorname{Im}\{X(e^{j\omega})\}$ impar
$|X(e^{j\omega})|$ par
$\measuredangle X(e^{j\omega})$ impar

$x[n]$ real $\to x[n]=x_{par}[n]+x_{impar}[n]$ $\quad x_{par}[n]=\frac{x[n]+x[-n]}{2}$

$x_{par}[n] \longleftrightarrow X(e^{j\omega})=\operatorname{Re}\{X(e^{j\omega})\}$

$x_{impar}[n] \longleftrightarrow X(e^{j\omega})=j\operatorname{Im}\{X(e^{j\omega})\}$ $\quad x_{impar}[n]=\frac{x[n]-x[-n]}{2}$

$x[n]=x_{Re}[n]+jx_{Im}[n]$

$X(e^{j\omega})=X^*(e^{-j\omega}) \quad X(e^{j\omega})=-X^*(e^{-j\omega})$

NYQUIST PASO BANDA:

$-\Omega_0 \; -\frac{2}{3}\Omega_0 \quad \frac{2}{3}\Omega_0 \; \Omega_0$

$\begin{Bmatrix} -\frac{2}{3}\Omega_0+k\Omega_s\leq\frac{2}{3}\Omega_0 \\ -\Omega_0+(k+1)\Omega_s\geq\Omega_0\end{Bmatrix}$

$\frac{2\Omega_0}{k+1}\leq\Omega_s\leq\frac{4/3\,\Omega_0}{k}$

$k=0,1,2\ldots \to$ Intervalos Ω_s

$x[n]*y[n] \longleftrightarrow X(e^{j\omega})\cdot Y(e^{j\omega})$

$x[n]\cdot y[n] \longleftrightarrow \frac{1}{2\pi}[X(e^{j\omega})*Y(e^{j\omega})]$

$e^{j\omega_0 n}x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$

$x[-n] \longleftrightarrow X(e^{-j\omega})$
$\quad \xrightarrow{x[n]\text{ real}} X^*(e^{j\omega})$

$x[n-n_0] \longleftrightarrow X(e^{j\omega})e^{-j\omega n_0}$

PARSEVAL:
$\sum_{n=-\infty}^{\infty}|x[n]|^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\overbrace{X(e^{j\omega})Y^*(e^{j\omega})}^{|X(e^{j\omega})|^2}d\omega$

TRANSFORMADA Z:

$X(z)=\sum_{-\infty}^{\infty}x[n]z^{-n}$ + ROC; $\quad x[n]=\frac{1}{2\pi j}\oint X(z)z^{n-1}\cdot dz$

ROC: $x[n]$ duración finita: $\forall z \begin{cases}-\{0\} & \text{causal} \\ -\{\infty\} & \text{anticausal}\end{cases}$

$x[n]$ de derechas: hacia afuera del polo más alejado origen (excepto 0 con $n_0<0$)

$x[n]$ de izquierdas: hacia adentro del polo más cercano al origen (excepto 0 con $n_0<0$)

$x[n]$ bilateral: anillo limitado por 2 polos.

PARES DE TRANSFORMADAS Z:

$\delta[n] \longleftrightarrow 1 \quad$ ROC: $\forall z$

$\delta[n-n_0] \longleftrightarrow z^{-n_0} \quad$ ROC: $\forall z \begin{cases}-\{0\} & \text{si } n_0>0 \\ -\{\infty\} & \text{si } n_0<0\end{cases}$

$u[n] \longleftrightarrow \frac{1}{1-z^{-1}} \quad$ ROC: $|z|>1$

$-u[-n-1] \longleftrightarrow \frac{1}{1-z^{-1}} \quad$ ROC: $|z|<1$

$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \quad$ ROC $|z|>a$

$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \quad |z|<a$

$n\cdot a^n u[n] \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2} \quad |z|>a$

$\cos\omega_0 n\cdot u[n] \longleftrightarrow \frac{1-\cos\omega_0\cdot z^{-1}}{1-2\cos\omega_0 z^{-1}+z^{-2}} \quad$ ROC: $|z|>1$

PROPIEDADES TRANSFORMADA Z:

$x[n]$ real: TZ simetría eje real.

$ax_1[n]+bx_2[n] \longleftrightarrow aX_1(z)+bX_2(z)$
ROC: $R_{x_1}\cap R_{x_2}$

$x[n-n_0] \longleftrightarrow z^{-n_0}\cdot X(z)$

$z_0^n x[n] \longleftrightarrow X(z/z_0)$ ROC: $|z_0|R_x$

$x[-n] \longleftrightarrow X(1/z)$ $\quad$ Roc: $1/R_x$
$\quad \xrightarrow{\text{si real}} X^*(z)$

$x_1[n]*x_2[n] \longleftrightarrow X_1(z)X_2(z)$ ROC: $R_{x_1}\cap R_{x_2}$

$x[n]$ causal $\begin{cases} x[0]=\lim_{z\to\infty}X(z) \\ x[\infty]=\lim_{z\to 1}(z-1)X(z)\end{cases}$

PROCESOS ESTACIONARIOS:

Estacionario: indep. del tiempo

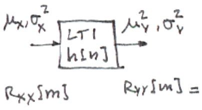
$E[x]=\int_{-\infty}^{\infty}x\cdot f(x)dx=\mu_x$

$\sigma^2=E[(x-\mu)^2]=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

$R_{xx}[m]=E[x_1,x_2]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_1x_2 f(x_1,x_2)dx_1dx_2=E\{x[n]x[n+m]\}$ $\quad S_{xx}(e^{j\omega})=TF\{R_{xx}[m]\}$

$C_{xx}[m]=E[(x_1-\mu_1)(x_2-\mu_2)]$

$P=E\{x[n]x[n]\}=R_{xx}[0]=\mu^2+\sigma^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}S_{xx}(e^{j\omega})d\omega$



$\mu_y=E[y[n]]=\mu_x H(e^{j0})$

$R_{yy}[m]=R_{xx}[m]*C_{hh}[m]$

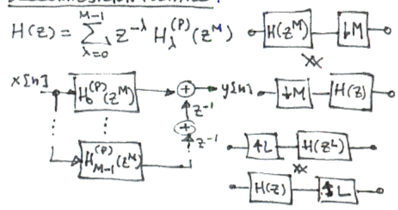
$R_{yx}[m]=\cancel{R_{xx}}$ $\quad C_{hh}[m]=h[m]*h^*[-m]$

$S_{yy}(e^{j\omega})=|H(e^{j\omega})|^2\cdot S_{xx}(e^{j\omega})$

$S_{xy}(e^{j\omega})=H(e^{j\omega})S_{xx}(e^{j\omega})$

DESCOMPOSICIÓN POLIFASE:

$H(z)=\sum_{\lambda=0}^{M-1}z^{-\lambda}H_\lambda^{(P)}(z^M)$



MUESTREO:

$x_s(t)=\sum_{-\infty}^{\infty}x_c(nT)\delta(t-nT) \quad \Omega_s\geq 2\Omega_{max}$

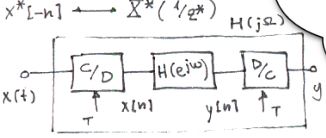
$X_s(j\Omega)=\frac{1}{T}\sum_{-\infty}^{\infty}X_c(j(\Omega-\frac{2\pi k}{T})) \xrightarrow{NYQ} \frac{1}{T}X_c(j\Omega) \quad |\Omega|\leq\frac{\pi}{T}$

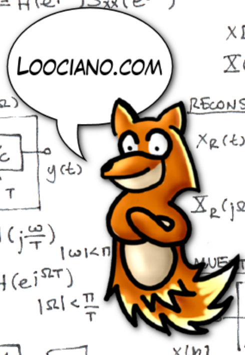
$x[n]=x_c(nT)$

$X(e^{j\omega})=X_s(e^{j\Omega})\Big|_{\omega=\Omega T}=\frac{1}{T}\sum_{-\infty}^{\infty}X_c(j(\frac{\omega}{T}-\frac{2\pi k}{T})) \xrightarrow{NYQ} \frac{1}{T}X_c(j\frac{\omega}{T}) \quad |\omega|<\pi$

$nx[n] \longleftrightarrow -z\frac{dX(z)}{dz}$

$x^*[-n] \longleftrightarrow X^*(1/z^*)$





$H(e^{j\omega})=H(j\Omega)\Big|_{\Omega=\frac{\omega}{T}}=H(j\frac{\omega}{T}) \quad |\omega|<\pi$

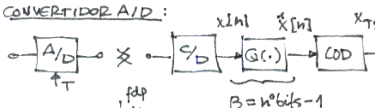
$H(j\Omega)=H(e^{j\omega})\Big|_{\omega=\Omega T}=H(e^{j\Omega T}) \quad |\Omega|<\frac{\pi}{T}$

RECONSTRUCCIÓN:

$x_R(t)=\sum_{-\infty}^{\infty}x[n]\frac{\operatorname{sen}(\frac{\pi}{T}(t-nT))}{\pi/T\,(t-nT)}$

$X_R(j\Omega)=\frac{H_R(j\Omega)}{T}\sum_{-\infty}^{\infty}X_c(j(\Omega-\frac{2\pi k}{T})) \xrightarrow{NYQ} \frac{H_R(j\Omega)}{T}\cdot X_c(j\Omega) \quad |\Omega|<\frac{\pi}{T}$

CONVERTIDOR A/D:



$B=$ nºbits-1

$\Delta=\frac{2x_m}{2^{B+1}}=\frac{x_m}{2^B}$

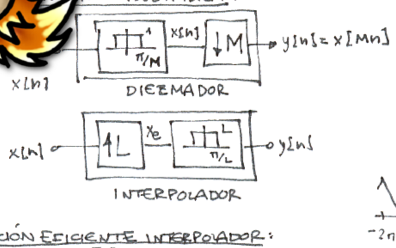
fdp $1/\Delta$ $\quad -\Delta/2 \quad \Delta/2$

$\sigma_e^2=\frac{\Delta^2}{12}$

$x[n]=\hat{x}[n]+e[n]$
$-\Delta/2\leq e[n]\leq\Delta/2$

SNR(dB) $=6{,}02$ ... $-1{,}7$ ...
Si nºbits >5:
SNR(dB) $\approx 6\cdot$nºbi...

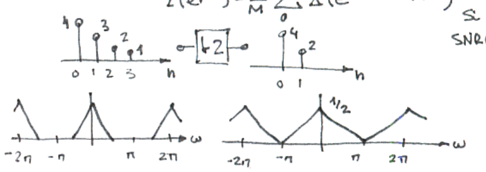
MUESTREO MULTITASA:



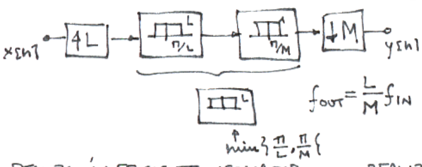
COMPRESOR:

$x[n] \to \boxed{\downarrow M} \to y[n]=x[Mn]$

$Y(e^{j\omega})=\frac{1}{M}\sum_{0}^{M-1}X(e^{j(\frac{\omega}{M}-\frac{2\pi k}{M})})$



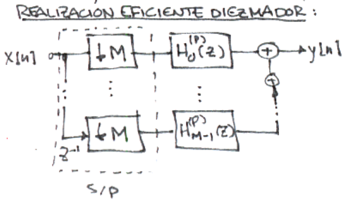
CAMBIO FREC. MUESTREO:



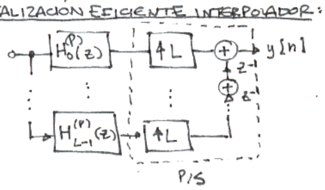
$f_{OUT}=\frac{L}{M}f_{IN}$

$\min\{\frac{\pi}{L},\frac{\pi}{M}\}$

REALIZACIÓN EFICIENTE DIEZMADOR:

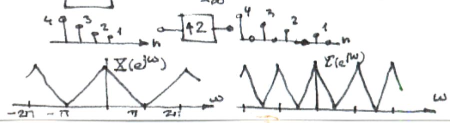


REALIZACIÓN EFICIENTE INTERPOLADOR:

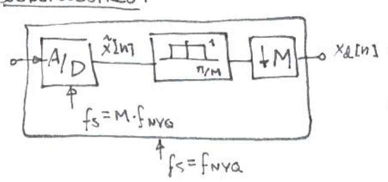


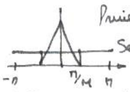
EXPANSOR:

$x[n] \to \boxed{\uparrow L} \to x_e[n]=\sum_{-\infty}^{\infty}x[k]\delta[n-kL] \begin{cases} x[n/L] & n=0,\pm L,\pm 2L \\ 0 & \text{resto}\end{cases}$



SOBREMUESTREO:





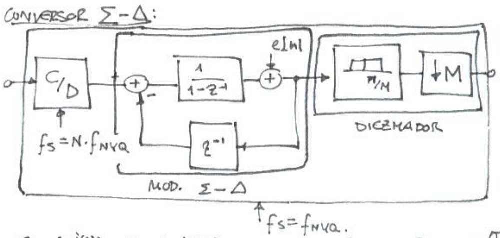
$P_{ruido} = \frac{\sigma_e^2}{M}$

$S_{ee}(e^{j\omega}) = \sigma_e^2$

$SNR\,(dB) = SNR|_{cuant\ directa} + 3{,}01r$

$M = 2^r$

CONVERSOR $\Sigma-\Delta$:



$H(e^{j\omega})| = 1$ $e[n]=0$

$H(e^{j\omega})| = 1 - e^{-j\omega}$ $x[n]=0$

$|H(e^{j\omega})|^2 = 2(1-\cos\omega)$

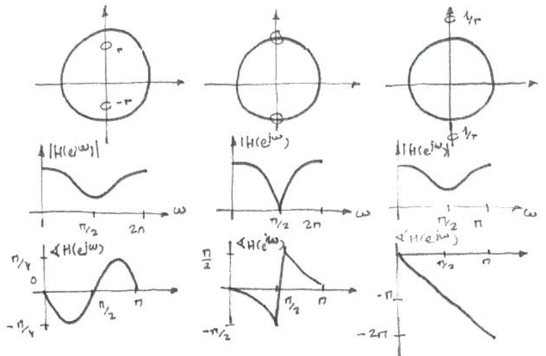
$S_{yy}(e^{j\omega}) = S_{yy}(e^{j\omega})|_{x[n]} + S_{yy}(e^{j\omega})|_{e[n]}$

$P_{ruido} = \frac{\sigma_e^2 \pi^2}{3M^3}$

$SNR(dB) = SNR|_{cuant\ directa} - 10\log_{10}\frac{\pi^2}{3} + 9{,}03r$ $M=2^r$

$S_{yy}(e^{j\omega}) = |H(e^{j\omega})|^2|_{x[n]} S_{xx}(e^{j\omega}) = S_{xx}(e^{j\omega})$

$S_{yy}(e^{j\omega})|_{e[n]} = |H(e^{j\omega})|^2 S_{ee}(e^{j\omega}) = \sigma_e^2 2(1-\cos\omega)$



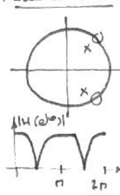
FIR-IIR:

	FIR	IIR
Ec. dif.	$y[n] = \sum_0^M b[k]x[n-k]$	$\sum_0^N a[k]y[n-k] = \sum_0^M b[k]x[n-k]$
Resp. impulso	$h[n] = \sum_0^M b[k]\delta[n-k]$	infinita
F. Transf.	$H(z) = \sum_0^M b[k]z^{-k}$	$H(z) = \frac{B(z)}{A(z)}$
Diagrama polo-cero	sólo ceros (polos en origen)	Polos y ceros fuera origen
Estabilidad	Estable	Pueden ser inestables
Diseño fase lineal	Sí	No
Efic. computacional	Baja (100-150)	Alta (4)

Retardo de grupo: $-\frac{d}{d\omega}\measuredangle\{H(e^{j\omega})\}$

Retardo de fase: $-\frac{1}{\omega}\measuredangle H(e^{j\omega})$

Filtro ranura:

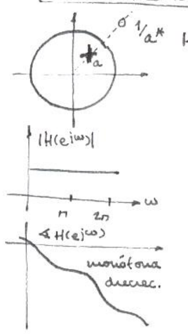


$H(z) = \frac{1 - 2\cos\omega_0 z^{-1} + z^{-2}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$

FASE MÍNIMA:

- Mín. fase
- Mín retardo grupo
- Concentrar energía primeros términos
- Sist. inverso causal estable.

SISTEMAS PASO TODO (IIR)

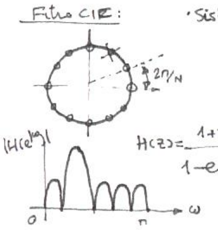


$H(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}}$

SISTEMA PASOTODO REAL.

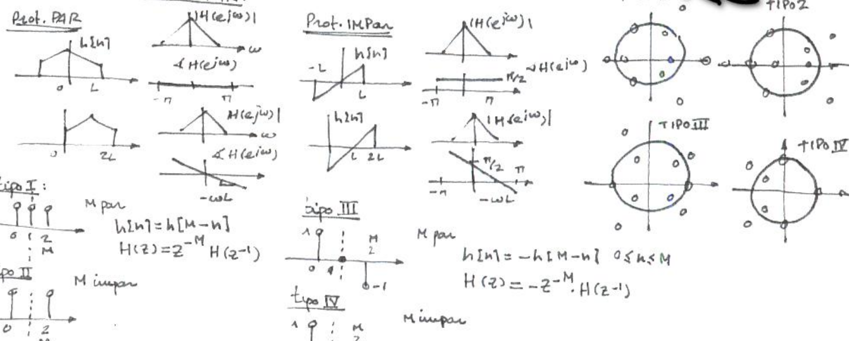
$H_{AP} = \prod_{k=1}^{N} \frac{z^{-1} - a_k}{1 - a_k z^{-1}} \cdot \prod_{k=1}^{M} \frac{r^2 - 2r\cos\omega_k z^{-1} + z^{-2}}{1 - 2r\cos\omega_k z^{-1} + r^2 z^{-2}} = \frac{z^{-M} A(z^{-1})}{A(z)}$

Filtro CIR:



$H(z) = \frac{1 + z^{-N}}{1 - e^{j\frac{2\pi k}{N}} z^{-1}}$

SISTEMAS FASE LINEAL: FIR.



tipo I: M par

$h[n] = h[M-n]$

$H(z) = z^{-M} H(z^{-1})$

tipo II: M impar

tipo III: M par

$h[n] = -h[M-n] \quad 0 \le n \le M$

$H(z) = -z^{-M} H(z^{-1})$

tipo IV: M impar

FASE MÍNIMA: ceros dentro circunferencia unidad

FIR: Sólo ceros (polo en origen)

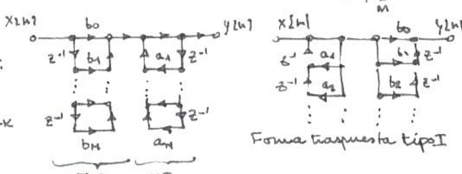
IIR: ceros y polos fuera del origen

PASO TODO: ceros y polos compensados (fuera y dentro c.u.)

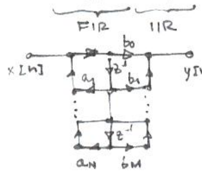
FLUJOGRAMAS:

Forma directa tipo I:

$H(z) = \frac{\sum_0^M b[k] z^{-k}}{1 - \sum_1^N a[k] z^{-k}}$

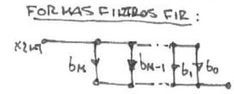


Forma directa tipo II o canónica.

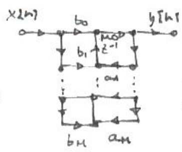


```
M1=M2=M3...=0
for i=1:datalength
  A = x[i]+a[1]*M1+...a[N]M_N
  y[i] = b[0]·A+b[1]·M_1+...+b_M M_N
  M_N = M_{N-1}
  M3 = M2
  M2 = M1
  M1 = A
end
```

FORMAS FILTROS FIR:



$y[n] = \sum_{k=0}^{M} b[k]\,x[n-k]$



Forma traspuesta tipo II

```
M0=M1=M2...=0
for i=1:datalength
  y[i] = b[0]x[i]+M0
  M0 = M1+b1 x[i]+a1 y[i]
  M1 = M2+b2 x[i]+a2 y[i]
  :
end
```

DFS:

$\tilde{x}[n] = \tilde{x}[n+N]$

$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi k}{N}n} = \sum_0^{N-1} \tilde{x}[n] W_N^{kn}$

$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi k}{N}n} = \sum_{k=0}^{N-1} \tilde{X}[k] W_N^{-kn}$

PROPIEDADES:

$a\tilde{x}_1[n] + b\tilde{x}_2[n] \longleftrightarrow a\tilde{X}_1[k] + b\tilde{X}_2[k]$

$\tilde{x}[n] = x[((n))_N] \longleftrightarrow \tilde{X}[k]$

$\tilde{x}[n-n_0] = x[((n-n_0))_N] \longleftrightarrow e^{-j\frac{2\pi k}{N}n_0}\tilde{X}[k] = W_N^{kn_0}\tilde{X}[k]$

$\tilde{X}[k] = \tilde{X}(e^{j\omega})|_{\omega = \frac{2\pi k}{N}}$

POISSON:

$\tilde{x}[n] = x[((n))_N] = \sum x[n-kN] \longleftrightarrow \frac{2\pi}{N}\sum X(e^{j\omega})\delta(\omega - \frac{2\pi k}{N})$

DFT: $X[k]$

PROPIEDADES:

$x[n]$ real: $X[k] = X^*[((-k))_N]$

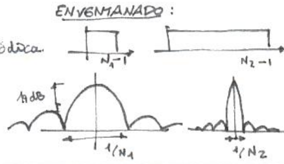
$Re\{X[k]\}$ par periódica $|X[k]|$ par periódica

$Im\{X[k]\}$ impar periódica $\measuredangle X[k]$ impar periódica

Equivalencia convoluciones

orden conv. circular $\ge L+P-1$

ENVENTANADO:



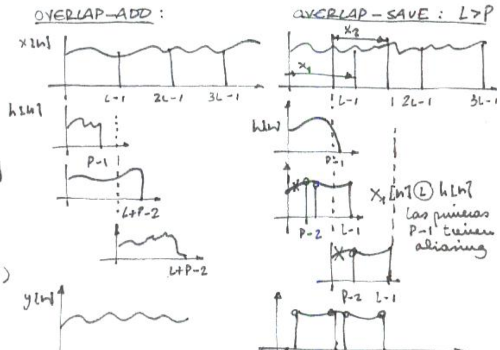
TIPOS:

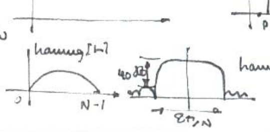
$rect[n] = u[n] - u[n-M]$

$Rect(e^{j\omega}) = \frac{sen(\frac{\omega N}{2})}{sen(\frac{\omega}{2})} e^{-j\frac{(N-1)}{2}\omega}$

OVERLAP-ADD:

OVERLAP-SAVE: L>P





$hamm[n] = 0{,}53836 - 0{,}46164\cos(\frac{2\pi n}{N})$