STLN:

TEMA 1:

Valor medio:

T.C: $\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\,dt = \langle x(t) \rangle$

T.D: $\lim_{N \to \infty} \frac{1}{2n+1} \sum_{x=-N}^{N} x[n] = \langle x(t) \rangle$

Valor de pico:

T.C. $x_p = \max \{ |x(t)| \}$

T.D. $x_p = \max \{ |x[n]| \}$

Energía:

$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$ $\qquad E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$

$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$ $\qquad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

Linealidad: Si $x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t) = a y_1(t) + b y_2(t)$

Invarianza temporal: Si $x(t)$ da $y(t)$, $x(t-t_0)$ da $y(t-t_0)$

Sistemas en T.C: derivadas:

$$\sum_{k=0}^{N} a_k \frac{d^{(k)} y(t)}{dt^k} = \sum_{k=0}^{N} \frac{d^{(k)} \cancel{x}(t)}{dt^k} + C.I \left\{ \begin{array}{l} y(t_0) = C_0 \\ y'(t_0) = C_1 \\ \vdots \\ y^{n-1}(t_0) = C_{n-1} \end{array} \right\}$$

- LINEAL:

  $C_0 = C_1 = \dots = C_{n-1} = 0$

- INVARIANTE

  $y(t_0 + t_1) = C_1$

  $y'(t_0 + t_1) = C_2$

- REPOSO INICIAL:

  Si $x(t) = 0 \quad \forall t < t_0 \longrightarrow y(t) = 0 \quad \forall t < t_0$

Sistemas en T.D: diferencias

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{N} b_k x[n-k] + C.I. \left\{ \begin{array}{l} y[n_0] = C_1 \\ y[n_0 - 1] = C_2 \\ \vdots \\ y[n_0 - N + 1] = C_{n-1} \end{array} \right.$$

- LINEAL:

  $C_1 = C_2 = \dots C_{n-1} = 0$

- INVARIANTE

  $y[n_0 + n_1] = C_1$

  $y[n_0 - 1 + n_1] = C_2$

  $\vdots$

- REPOSO INICIAL:

  $x[n] = 0 \quad \forall n < n_1 \longrightarrow y[n] = 0 \quad \forall n < n_1$

TEMA 2:

T.D.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

T.C.

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

Propiedades:

- Conmutativa: $x(t) * h(t) = h(t) * x(t)$
- Distributiva: $x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$
- Asociativa: $(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n]) = (x[n] * h_2[n]) * h_1[n]$
- Elem. neutro: $x(t) * \delta(t) = x(t)$

$$x(t-t_0) = x(t) * \delta(t-t_0)$$

- Memoria: si depende de instantes iniciales, tiene memoria: $y(t) = x(t)$ SIN MEMORIA; $y(t) = x(t) + x(t-3)$ CON MEMORIA
- Causalidad: es causal si sólo depende de la entrada en el instante o instantes anteriores.

CAUSAL: $h[n] = 0 \quad n<0$

ANTICAUSAL: $h[n] = 0 \quad n>0$

- Estabilidad

Para una entrada acotada, una salida acotada. $|x[n]| < A < \infty \longrightarrow |y[n]| < B < \infty$

Para un LTI:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$ Respuesta al impulso absolutamente sumable.

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

TEMA 3 :

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jk\omega_0 t}dt$$

$$a_0 = \frac{1}{T}\int_{\langle T\rangle} x(t)\,dt$$

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{-jk\omega_0 t}$$

TRANSFORMADA DE FOURIER

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

TRANSFORMADA INVERSA

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Condición de existencia:

$$\int_{-\infty}^{\infty}|x(t)|^2dt < \infty$$

$x(t)$ sea de energía finita

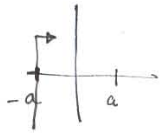
TRANSFORMADA DE LAPLACE :

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt$$

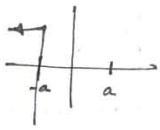
Condición de estabilidad:
Su región de convergencia contenga al eje $j\omega$.

$$x(t) = e^{-at}u(t)$$

$\sigma > -a$



$$x(t) = -e^{-at}u(-t)$$



TEMA 4 :

$$x[n] = \sum_{\langle N\rangle\ 0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$$

$$a_k = \frac{1}{N}\sum_{\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n}$$

TRANSFORMADA DE FOURIER :

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

(PERIÓDICA 2π)

TRANSFORMADA INVERSA

$$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega$$

CONVOLUCIÓN CIRCULAR :

$$y[n] = x_1[n]x_2[n] \longleftrightarrow Y(e^{j\omega}) = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X_1(e^{j\alpha})X_2(e^{j(\omega-\alpha)})d\alpha = \frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$$

TRANSFORMADA Z :

$$X(z) = \sum_{-\infty}^{\infty} x[n]z^{-n} \qquad |H(z)|^2 = H(z)\cdot H^*(z)$$

- Sistema estable :
  La Región Convergencia incluye a la circunferencia unidad
- Sistema Causal
  - Región de converg. contiene a ∞
- Sistema anticausal
  Región de converg. contiene a 0

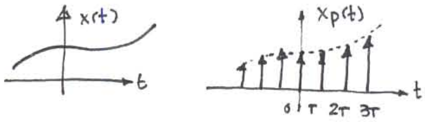
Sólo los sistemas estables tendrán T.F.

TEMA 5:

• MUESTREO IDEAL:

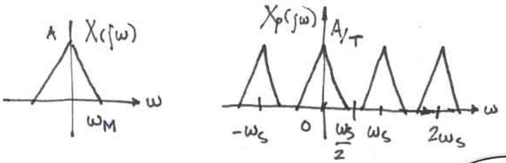
$$x(t) \longrightarrow \otimes \longrightarrow x_p(t) = \sum x(t)\delta(t-nT) = \sum x(nT)\delta(t-nT)$$

$$p(t) = \sum \delta(t-nT)$$



$$X(j\omega) \longrightarrow \otimes \longrightarrow X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * \frac{2\pi}{T}\sum \delta(\omega - k\omega_s) = \frac{1}{T}\sum X(j\omega) * \delta(\omega - k\omega_s) = \frac{1}{T}\sum_{-\infty}^{\infty} X(j(\omega - k\omega_s))$$

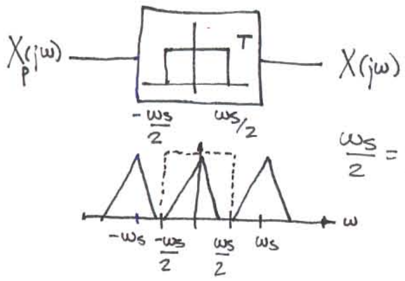
$$P(j\omega) = \frac{2\pi}{T}\sum \delta(\omega - k\omega_s)$$



CONDICIÓN DE NYQUIST

$$\boxed{\omega_s \geq 2\omega_M}$$

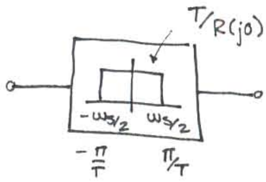
RECONSTRUCCIÓN DE LA SEÑAL:



$$\frac{\omega_s}{2} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

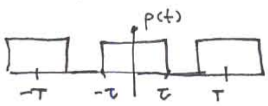
• MUESTREO NATURAL

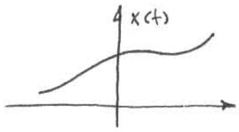
RECONSTRUCCIÓN:

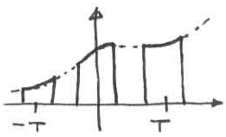


$$x(t) \longrightarrow \otimes \longrightarrow x_p(t) = x(t)\sum r(t-nT) = x(t)\cdot r(t) * \sum \delta(t-nT)$$

$$p(t) = \sum r(t-nT)$$

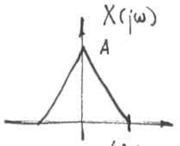


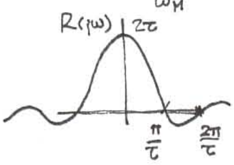


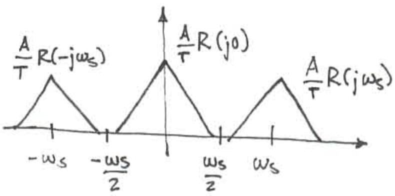


$$X(j\omega) \longrightarrow \otimes \longrightarrow X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{2\pi} X(j\omega) * R(j\omega)\frac{2\pi}{T}\sum \delta(\omega - k\omega_s) = \frac{1}{T}\sum R(jk\omega_s)\, X(j(\omega - k\omega_s))$$

$$P(j\omega) = R(j\omega)\cdot\frac{2\pi}{T}\sum \delta(\omega - k\omega_s)$$





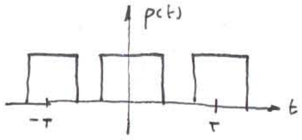


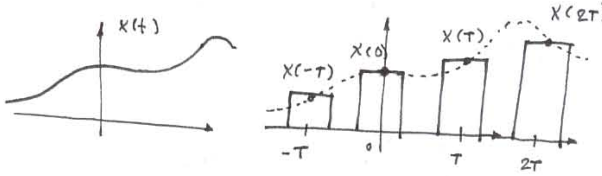
Cada muestra se multiplica por un valor cte de $R(j\omega)$

MUESTREO INSTANTÁNEO :

$$x(t) \rightarrow \otimes \rightarrow x_p(t) = \sum x(nT)\,r(t-nT) = r(t) * x(t) \sum \delta(t-nT)$$

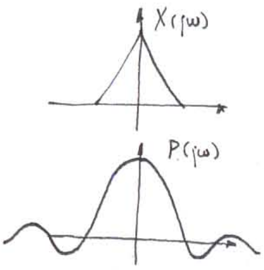
$$p(t) = \sum r(t-nT)$$

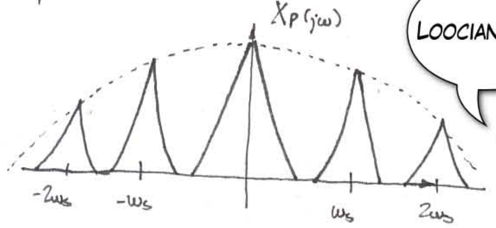




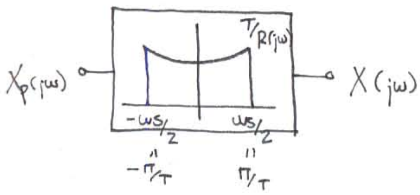
$$X(j\omega) \rightarrow \otimes \rightarrow X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * \frac{2\pi}{T} \sum \delta(\omega - k\omega_s) \cdot R(j\omega) = \frac{1}{T} R(j\omega) \sum X(j(\omega - k\omega_s))$$

$$P(j\omega) = \frac{2\pi}{T} \sum \delta(\omega - k\omega_s) \cdot R(j\omega)$$

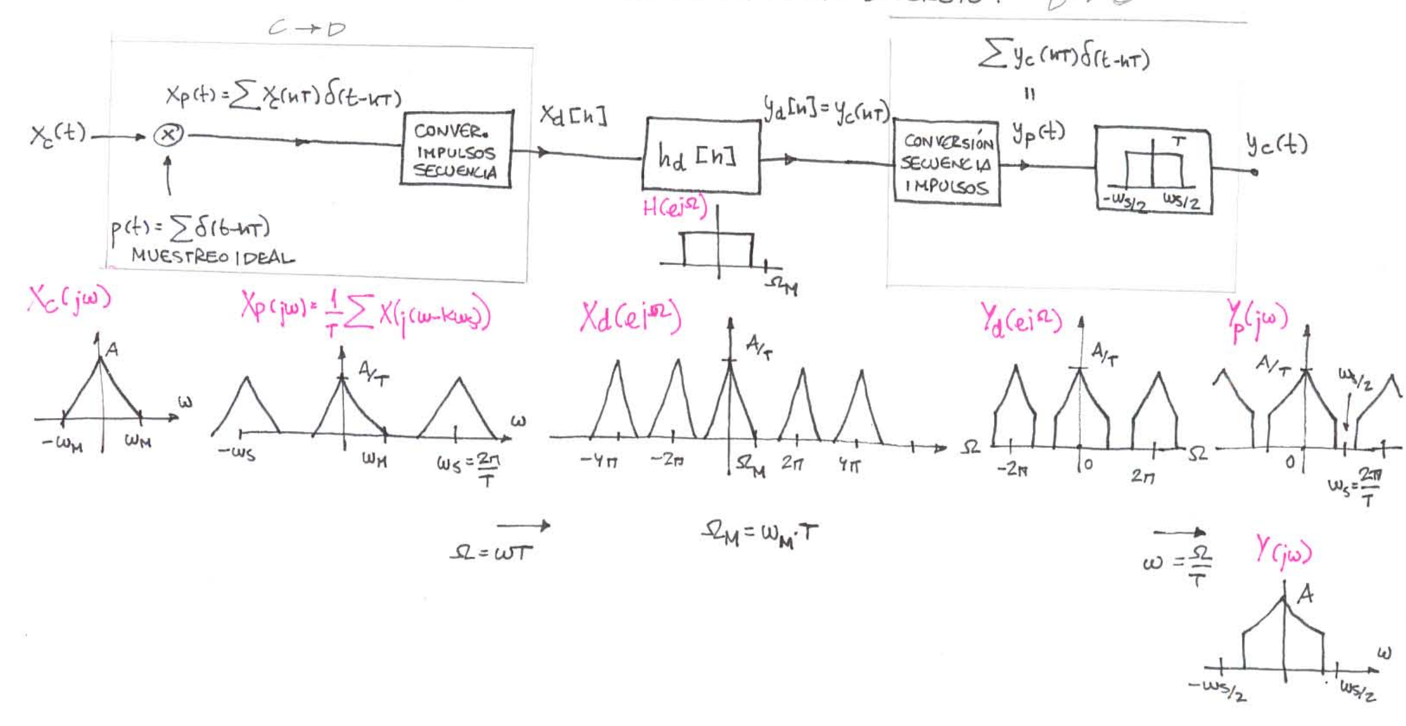




RECUPERACIÓN DE LA SEÑAL:



SIMULACIÓN EN TIEMPO CONTINUO DE SISTEMAS EN TIEMPO DISCRETO : D → C



TRUCO PARA PASAR EXP A SENO / COSENO :

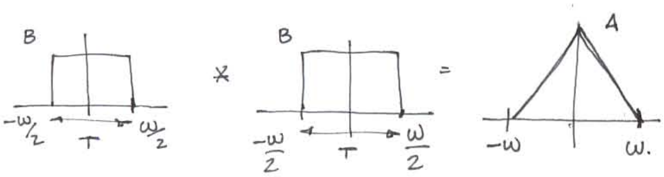
$$e^{j\omega a} + e^{-j\omega b}$$

se cogen las partes no comunes, se hace la media y se cambia signo:

$$\frac{a-b}{2} \rightarrow -\frac{a-b}{2} = \frac{b-a}{2}$$

$$\left(e^{j\omega a} + e^{-j\omega b}\right)\frac{e^{\frac{b-a}{2}}}{e^{\frac{b-a}{2}}} = \frac{2}{2}\left(e^{j\omega\left(\frac{a+b}{2}\right)} + e^{-j\omega\left(\frac{a+b}{2}\right)}\right)\frac{1}{e^{\frac{b-a}{2}}} = 2\cos\left(\omega\left(\frac{a+b}{2}\right)\right)e^{-j\omega\left(\frac{b-a}{2}\right)}$$

$$\left(e^{j\omega a} + e^{j\omega b}\right) = 2\cos\left(\omega\left(\frac{a-b}{2}\right)\right)e^{-j\omega\left(\frac{a+b}{2}\right)}$$



$$A = T \cdot B^2$$