Chuletario de Comunicaciones Digitales

¿Te han servido estos apuntes?

Para sugerencias, agradecimientos o para comunicar erratas, escríbeme a:

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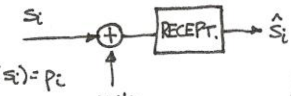
CMDG: LUCIANO RUBIO ROMERO HOJA 1/2

$s_i(t) + n(t) \to \boxed{h(t)} \xrightarrow{t_0} X(t)$ Típico: $X(t) = \underbrace{\langle s_i, h(t_0-t)\rangle}_{f} + \langle n, \underbrace{h(t_0-t)}_{f}\rangle \equiv N(\mu=\mu_i, \sigma=\|f\|^2 \frac{N_0}{2})$, $\mu_i = \langle s_i, f\rangle$

1 RECEPTORES ÓPTIMOS



$P(s_i) = p_i$, $n(t)$ RBG $N_0/2$

$$f(x/A) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-(x_i-\mu_{iA})^2}{2\sigma^2}}$$

$$f(\vec{x}/A) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{\frac{-\|\vec{x}-\mu\|^2}{2\sigma^2}}$$

Estimación Bayesiana: Caso discreto

$I \to$ Incógnita — Prob. a priori $P(I=s_i)$

$O \to$ Observación — Prob. cond. de la observ. $P(O=r_j \mid I=s_i)$

$$P(I=s_i/O=r_j) = \frac{P(O=r_j/I=s_i)}{P(O=r_j)} \cdot P(I=s_i)$$

$$P_A = P(I=g(O)) = \sum_{j=1}^{N} \underbrace{P(I=g(r_j)/O=r_j)\cdot P(O=r_j)}_{\text{Maximizamos este término para aumentar } P_A}$$

$$g(r_j) = \arg\max_{s_i} P[O=r_j/I=s_i]\cdot P[I=s_i]$$

Estimación Bayesiana: caso continuo.

$I \to$ Incógnita discreta — Prob. a priori $P(I=s_i)$

$X \to$ Incógnita continua $\to f(\vec{x}/s_i)$, $\vec{x} = (x_1, \ldots, x_L)$

$$P(I=s_i \mid X=\vec{x}) = \frac{f(\vec{x}/s_i)P(I=s_i)}{f(\vec{x})}$$

$$g(\vec{x}) = \arg\max_{s_i} f(\vec{x}/s_i)P[s_i]$$

Teorema de Filtrado:

$x(t) \to \boxed{h(t)} \to y(t)$

$S_x(f)$ — $S_y(f) = S_x(f)\cdot|H(f)|^2$

$\eta_x \uparrow TF$ — $\eta_Y = \left[\int_{-\infty}^{\infty} h(t)dt\right]\eta_x$

$R_x(\tau)$ — $R_Y(\tau) = R_x(\tau) * h(\tau) * h^*(-\tau)$

$R_x(\tau) = E[x(t)+x(t+\tau)]$

$$\mathcal{E}(s) = \int_{-\infty}^{\infty}|s(t)|^2 dt$$

$\|s(t)\| = \sqrt{\mathcal{E}(s)}$

$$\langle r,s\rangle = \int_{-\infty}^{\infty} r(t)s^*(t)dt \overset{\text{PARSEVAL}}{=} \int_{-\infty}^{\infty} R(f)S^*(f)df = \langle R,S\rangle$$

GRAM-SCHMIDT: $\{s_1, \ldots, s_L\} \to \{\varphi_1, \ldots, \varphi_L\}$ SIST. ORTONORMAL

$$\varphi_1 = \frac{s_1}{\|s_1\|}$$

$$\varphi'_{r+1} = s_i - P(s_i)_{\{\varphi_1 \ldots \varphi_r\}} \Rightarrow \varphi_{r+1} = \frac{\varphi'_{r+1}}{\|\varphi'_{r+1}\|}$$

$$P(s_i) = \langle s_i, \varphi_1\rangle\varphi_1 + \langle s_i, \varphi_2\rangle\varphi_2 + \ldots$$

RUIDO:

$h(t)$, $\{\varphi_i\}_{i=1..L}$ — $n_i = \langle n(t), \varphi_i(t)\rangle \equiv N(0, \sqrt{\frac{N_0}{2}})$

$\mu_i = E[n_i] = 0$

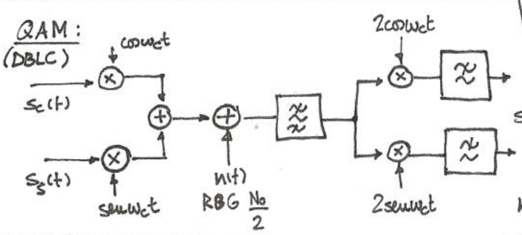
$$\sigma_{ij}^2 = E[(n_i-\mu_i)(n_j-\mu_j)] = \begin{cases} \frac{N_0}{2} & i=j \\ 0 & i\neq j \end{cases}$$

RECEPTOR ÓPTIMO:

$$f(\vec{r}/s_i) = \frac{1}{(2\pi)^{L/2}\sigma^L} e^{-\frac{\|\vec{r}-\vec{s}_i\|^2}{2\sigma^2}}$$

$$\hat{s}_i = \arg\max_{s_i}[f(\vec{r}/s_i)P_i]$$

QAM: (DBLC)



$s_1(t) + n_{comp}(t)$

$n_{comp}(t) = n_c(t) + jn_s(t)$

$\vec{r} = \vec{s} + \vec{n} = N(\vec{\mu} = \vec{s}_i, \Sigma = \sigma^2 I_d)$

$$\hat{s}_i = \arg\max_{s_i}\left[P_i \exp\left(\frac{-\|\vec{r}-\vec{s}_i\|^2}{2\sigma^2}\right)\right] = \arg\max_{s_i}\left[\ln P_i - \frac{1}{2\sigma^2}\left(-2\,\underbrace{Re\langle\vec{r},\vec{s}_i\rangle}_{\langle r, s_i\rangle} + \mathcal{E}[s_i]\right)\right]$$

$\vec{n} = (n_1, \ldots, n_L) = (n_{1c}, n_{1s}, \ldots, n_{Lc}, n_{Ls})$

$E[n_{ic}, n_{jc}] = N_0\delta_{ij}$

$E[n_{is}, n_{js}] = N_0\delta_{ij}$

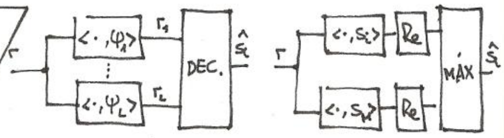
$E[n_{is}, n_{jc}] = 0$

- Señales equiprobables

(1) $\hat{s}_i = \arg\min_{s_i} \|\vec{r}-\vec{s}_i\|$

- Señales equiprobables con $\mathcal{E}[s_i] = cte$

(2) $\hat{s}_i = \arg\max Re[\langle\vec{r}, \vec{s}_i\rangle]$



(1) (2)

PRODUCTOS ESCALARES:

$\langle r(t), \varphi(t)\rangle$

$r(t) \to \boxed{h(t)} \xrightarrow{t_0}$, $\varphi^*(t_0 - t)$

$$f(r_1, r_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{\sigma_2^2(r_1-\mu_1)^2 + \sigma_1^2(r_2-\mu_2)^2 - 2\rho\sigma_1\sigma_2(r_1-\mu_1)(r_2-\mu_2)}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right]$$

2 INTERFERENCIA ENTRE SÍMBOLOS

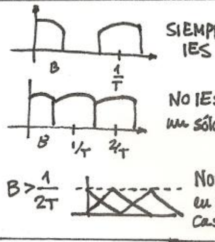
$\{s_i(t)\}$

$s_i(t) = A_i\varphi(t)$

$v(t) = \sum s(t-nT) = \sum A_{I(n)}\varphi(t-nT)$

$v(t)+n(t) \to \boxed{\varphi^*(t_0-t)} \xrightarrow{t_0}$ $A_{I(0)}\underbrace{\langle\varphi(t),\varphi(t)\rangle}_{1} + \underbrace{\sum_{n\neq 0} A_{I(n)}\langle\varphi(t-nT), \varphi(t)\rangle}_{IES} + \langle n, \varphi\rangle$

ANCHO DE BANDA MÍNIMO PARA NO IES:



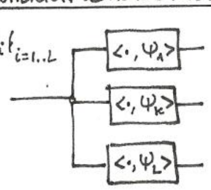
$B < \frac{1}{2T}$ SIEMPRE IES

$B = \frac{1}{2T}$ NO IES en un sólo caso

$B > \frac{1}{2T}$ NO IES en varios casos

CONDICIÓN DE NO IES MULTIDIMENSIONAL:

$\{\varphi_i\}_{i=1..L}$



Condición: (tiempo)

$\langle\varphi_i(t-nT), \varphi_k(t)\rangle = 0$

Para $i = k$:

$\langle\varphi_k(t-nT), \varphi_k(t)\rangle = \delta(n)$

$x_k(t) = \varphi_k(t) * \varphi_k^*(-t)$

Condición (frecuencia)

$$\sum_{-\infty}^{\infty} X_k\left(f-\frac{l}{T}\right) = cte \qquad \left|\Psi_k\left(f-\frac{l}{T}\right)\right|^2$$

- Condición de no IES: (tiempo)

$\langle\varphi(t), \varphi(t-nT)\rangle = 0$ $\forall n \in \mathbb{Z}, n \neq 0$

$x(nT) = 0$ $\forall n \in \mathbb{Z}, n \neq 0$

$x(nT) = \langle\varphi(t), \varphi(t-nT)\rangle$

- Condición de no IES (frecuencia)

$$x(nT) = x(t)\sum\delta(t-nT) = x(0)\delta(t) = \delta(t) \overset{TF}{\longleftrightarrow} X(f) * \frac{1}{T}\sum\delta\left(f-\frac{k}{T}\right) = 1$$

$$\Downarrow$$

$$\sum X\left(f-\frac{k}{T}\right) = cte$$

ANCHO DE BANDA MÍN. SIN IES: $B = \frac{1}{2T}\cdot L$

- Con DBLC:

$B_{min} = \frac{L}{T}$

TEOREMILLA:

Si en un dominio, la señal tiene la n-ésima derivada continua, en el otro dominio la señal decrece como $\frac{1}{x^{n+2}}$

Para una determinada velocidad de símbolo V_S que cumpla no IES, cumplen también sus submúltiplos enteros

$$W_{min}\ \text{sin IES} = \frac{R\cdot \overset{\text{dimensión}}{L}}{2\log_2 \underset{\text{nº señales}}{M}}$$

③ EVALUACIÓN DE MOD. DIGITALES

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$$\boxed{\gamma_{b\,\text{aprox}} = \gamma_b(P_E) \approx \frac{2}{\beta \log_2 M}\left[Q^{-1}(P_E)\right]^2} \qquad \beta = \frac{d^2_{min}}{\mathcal{E}_{av}}$$

$\mathcal{E}_0$: energía media recibida

- sin modulación $\mathcal{E}_0 = \mathcal{E}_{av} \Rightarrow \frac{\mathcal{E}_{av}}{\sigma^2} = \frac{2\mathcal{E}_0}{N_0}$
- con modulación $\mathcal{E}_0 = \frac{1}{2}\mathcal{E}_{av} \Rightarrow \frac{\mathcal{E}_{av}}{\sigma^2} = \frac{2\mathcal{E}_0}{N_0}$

$\gamma_0 = \frac{\mathcal{E}_0}{N_0}$ $\qquad \frac{\mathcal{E}_{av}}{\sigma^2} = 2\gamma_b \log M$

$\mathcal{E}_b = \frac{\mathcal{E}_0}{\log_2 M}$: energía por bit

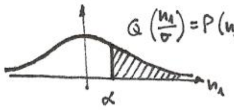
$\gamma_b : \frac{\mathcal{E}_b}{N_0}$: energía por bit normaliz.

$$P_E = P_b \cdot \log_2 M$$

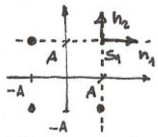
Cálculo aproximado:

$$P_{A_i} = P[\vec{r} \in R_i | s_i] = P[\vec{s}_i + \vec{n} \in R_i] = P[\vec{n} \in R_i - \vec{s}_i]$$

FUNCIÓN Q:



$Q\left(\frac{n_1}{\sigma}\right) = P(n_1 \geq \alpha)$



$$P_{A_1} = P[n_1 \geq -A \text{ y } n_2 \geq -A] \underset{\substack{\uparrow \\ n_1, n_2 \\ \text{indep.}}}{=} P[n_1 \geq -A] \cdot P[n_2 \geq -A] = \left(1 - Q\left(\frac{A}{\sigma}\right)\right)^2$$

Condiciones para aplicar γ_b aprox.

$d_{min,i} = d_{min}\ \forall i$

La dist. entre todas las señales es la dist. mínima

- Cálculo exacto:

$$P_{A_i} = P[\vec{r} \in R_i / s_i] = P[n_1 \in I_1 - s_{i1}, \dots, n_L \in I_L - s_{iL}] = P[n_1 \in I_1 - s_{i1}] \cdots P[n_L \in I_L - s_{iL}]$$

$$P_{A_1} = \left[1 - Q\left(\frac{A/\sqrt{2}}{\sigma}\right)\right]^2$$

$$Q\left(\frac{d_{min}/2}{\sigma}\right) \leq P_{E_i} \leq \nu_i\, Q\left(\frac{d_{min}/2}{\sigma}\right) \Rightarrow \boxed{Q\left(\frac{d_{min}/2}{\sigma}\right) \leq P_E \leq \nu_{max}\, Q\left(\frac{d_{min}/2}{\sigma}\right)}$$

$\nu_{max} \equiv$ "vecinos"

$P_E = \frac{1}{M}\sum_{i=1}^{M} P_{E_i}$

$\nu_{max} = \max_i \nu_i$

sabiendo:
$Q\left(\frac{d_{min}/2}{\sigma}\right) \leq P_E \Rightarrow \frac{d_{min}/2}{\sigma} \geq Q^{-1}(P_E)$

$$\boxed{\frac{2}{\beta \log_2 M}\left[Q^{-1}(P_E)\right]^2 \leq \gamma_b \leq \frac{2}{\beta \log_2 M}\left[Q^{-1}\left(P_E/\nu_{max}\right)\right]^2} \rightarrow \gamma_{b\,min} \leq \gamma_b \leq \gamma_{b\,max}$$

Error relativo: $\frac{|\gamma_{b\,aprox} - \gamma_b|}{\gamma_b} \leq \frac{\gamma_{max} - \gamma_{b\,min}}{\gamma_{b\,min}}$

PROPIEDADES P_E:

- Giro y traslación de constelación no afectan porque mantienen la distancia entre señales.
- Escalado idéntico señal y ruido:

$$\left.\begin{array}{l} s_i \rightarrow \alpha s_i \\ \sigma \rightarrow |\alpha| \sigma \\ N_0 \rightarrow |\alpha|^2 N_0 \end{array}\right\} P_{A_i} \text{ no varían}$$

$\Downarrow$

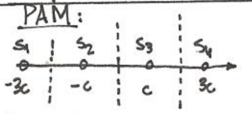
$\mathcal{E}_{av} \rightarrow |\alpha|^2 \mathcal{E}_{av}$

- Acotación:

$d_{ij} = \|s_i - s_j\|$

$d_{min,i} = \min_{j \neq i} d_{ij}$

PAM:



(4-PAM)

$d_{min} = 2c$

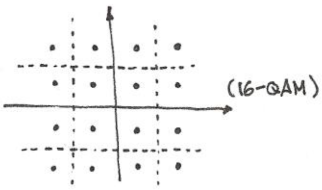
$$\left.\begin{array}{l} P_{E_{ext}} = P[n_1 \geq c] = Q\left(\frac{c}{\sigma_n}\right) \\ P_{E_{int}} = 2Q\left(\frac{c}{\sigma_n}\right) \end{array}\right\} P_E = \frac{1}{M}\left(2P_{E_{ext}} + (M-2)P_{E_{int}}\right) = \frac{2M-2}{M} Q\left(\frac{c}{\sigma_n}\right)$$

$$\mathcal{E}_{av} = \frac{1}{M}\sum_i \mathcal{E}[s_i] = \frac{c^2}{3}(M^2-1)$$

$$P_E = \frac{2M-2}{M} Q\left(\sqrt{\frac{3\mathcal{E}_{av}}{\sigma_n^2 (M^2-1)}}\right) \Downarrow \quad P_E = \frac{2M-2}{M} Q\left(\sqrt{\frac{6\gamma_b \log_2 M}{M^2-1}}\right)$$

$$\beta = \frac{d_{min}}{\mathcal{E}_{av}} = \frac{12}{M^2-1} \Downarrow \quad \beta \log_2 M = \frac{12 \log_2 M}{M^2-1}$$

QAM:



(16-QAM)

$$P_{A, M^2\text{-QAM}} = \left(P_{A\text{-}M\text{-PAM}}\right)^2 \qquad P_A = P[A_i + n_c \in R_i] \cdot P[B_j + n_s \in R_j] = \left(P_{A, \sqrt{M}\text{-PAM}}\right)^2 = \left[1 - \frac{2\sqrt{M}-2}{\sqrt{M}} Q\left(\frac{c}{\sigma_n}\right)\right]^2$$

$$P_{E, M^2\text{-QAM}} = 1 - \left(1 - \frac{2M-2}{M} Q\left(\frac{c}{\sigma_n}\right)\right)^2$$

$$\mathcal{E}_{av\,M^2\text{-QAM}} = 2\,\mathcal{E}_{av\,M\text{-PAM}} = 2\frac{c^2}{3}(M^2-1)$$

$\Downarrow$

$$P_{A\,M\text{-QAM}} = \left[1 - \frac{2\sqrt{M}-2}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma_b \log_2 M}{M-1}}\right)\right]^2$$

$$\beta_{M\text{-QAM}} = \frac{6}{M-1}$$

Ganancia asintótica:

$$g_{\substack{M^2\text{-QAM} \\ \text{sobre} \\ M\text{-PAM}}} = \frac{\beta_{M\text{-PAM}} \cdot \log_2 M}{\beta_{M^2\text{-QAM}} \log_2(M^2)} = \frac{\frac{12}{M^2-1}\log_2 M}{\frac{6}{M^2-1} 2\log_2 M} = 1 \quad ; \quad \frac{\gamma_{b\,M^2\text{-PAM}}(P_E)}{\gamma_{b\,M\text{-PAM}}(P_E)} \xrightarrow[P_E \to 0]{} 1$$

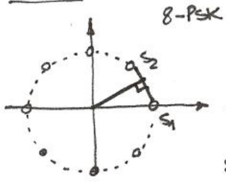
Ancho de Banda:

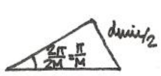
$$W_{min\,M\text{-PAM}} = \frac{1}{2T} \qquad R = \frac{\log_2 M}{T} \qquad \eta_{M\text{-PAM}} = \frac{R}{W} = 2\log_2 M$$

$$W_{min\,M^2\text{-QAM}} = \frac{1}{2T} \cdot L \cdot K = \frac{1}{T} \qquad \eta_{M^2\text{-QAM}} = \frac{R}{W} = 2\log_2 M$$

PSK:

8-PSK





A

$$\operatorname{sen}\frac{\pi}{M} = \frac{d_{min}/2}{A}$$

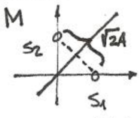
$$d_{min} = 2A \operatorname{sen}\frac{\pi}{M}$$

$$\beta \log_2 M = 4 \operatorname{sen}^2\frac{\pi}{M} \log_2 M \approx 4\pi^2/M^2 \cdot \log_2 M$$

$$\beta = \frac{d_{min}^2}{\mathcal{E}_{av}} = 4 \operatorname{sen}^2\frac{\pi}{M}$$

MOD. ORTOG.

$L = M$



$d_{min} = \sqrt{2}A$

$$\frac{2}{\beta \log_2 M}\left[Q^{-1}(P_E)\right]^2 \leq P_E \leq \frac{2}{\beta \log_2 M}\left[Q^{-1}\left(\frac{P_E}{M-1}\right)\right]^2$$

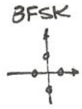
$$\eta = \frac{R}{W} = \frac{\log_2 M / T}{\frac{1}{2T} \cdot L} = \frac{2\log_2 M}{M}$$

FSK:

η : efic. espect.

$\rho = \gamma^{-1}$: eficac. potencial

BFSK



$$C = W \log_2\left(1 + \frac{S}{N}\right)$$

Mod. Ideal:

$$R = W \log_2\left(1 + \frac{\mathcal{E}_b \cdot R}{N_0 \cdot W}\right)$$

$$\eta = \log_2(1 + \eta/\rho)$$

Ejemplo de interés: Hallar regiones de decisión:

Sabemos que $f(r_1, r_2/s_j) \equiv N(\mu_{ij}, \sigma_{ij})$ $\quad r_1, r_2$ indep.

$$f(r_1, r_2/s_j) = \left(\frac{1}{\sqrt{2\pi}\sigma_{1j}} \exp -\frac{(r_1-\mu_{1j})^2}{2\sigma_{1j}^2}\right)\left(\frac{1}{\sqrt{2\pi}\,\sigma_{2j}} \exp \frac{-(r_2-\mu_{2j})^2}{2\sigma_{2j}^2}\right) = \frac{1}{2\pi\sigma_{1j}\sigma_{2j}} \exp\left[-\left(\frac{(r_1-\mu_{1j})^2}{2\sigma_{1j}^2} + \frac{(r_2-\mu_{2j})^2}{2\sigma_{2j}^2}\right)\right]$$

$$\text{decisión} = \arg\max P_i\, f(r_1, r_2/s) = \frac{P_i}{2\pi\sigma_{1j}\sigma_{2j}} \exp\left[-\left(\frac{(r_1-\mu_{1j})^2}{2\sigma_{1j}^2} + \frac{(r_2-\mu_{2j})^2}{2\sigma_{2j}^2}\right)\right]$$

La frontera entre símbolos j y k:

$$P_j \cdot f(r_1, r_2/s_j) = P_k\, f(r_1, r_2/s_k) \Rightarrow \ln\left(\frac{\sigma_{1k}\sigma_{2k}P_j}{\sigma_{1j}\sigma_{2j}P_k}\right) - \left[\frac{(r_1-\mu_{1j})^2}{2\sigma_{1j}^2} + \frac{(r_2-\mu_{2j})^2}{2\sigma_{2j}^2}\right] = -\left[\frac{(r_1-\mu_{1k})^2}{2\sigma_{1k}^2} + \frac{(r_2-\mu_{2k})^2}{2\sigma_{2k}^2}\right]$$