1. POYNTING. · FRECUENCIA:

$$\frac{1}{2}\mathrm{Re}\left[\int_V -\vec{J}^*_{0ext}\cdot\vec{E}_c(\vec{r})\,dV\right] = \frac{1}{2}\mathrm{Re}\left[\int_S (\vec{E}_c(\vec{r})\times\vec{H}^*_c(\vec{r}))\,\underset{(saliente)}{d\vec{s}}\right] + \frac{\sigma}{2}\int_V \vec{E}_c(\vec{r})\cdot\vec{E}_c^*(\vec{r})\,dV + \frac{\omega_0\varepsilon''}{2}\int_V |\vec{E}_c(\vec{r})|^2 dV + \frac{\omega_0\mu''}{2}\int_V |\vec{H}_c(\vec{r})|^2 dV$$

FUENTES DE V · POTENCIA ESCAPA DE V POR S · POTENCIA DISIPADA (JOULE) · POTENCIA DISIPADA PÉRDIDAS DIELÉCT. · POTENCIA DISIPADA PÉRDIDAS MAGNÉTICAS

$$\frac{1}{2}\mathrm{Im}\left[\int -\vec{J}^*_{0ext}\cdot\vec{E}_c(\vec{r})\,dV\right] = \frac{1}{2}\mathrm{Im}\left[\int_S (\vec{E}(\vec{r})\times\vec{H}^*_c(\vec{r}))\cdot d\vec{s}\right] + \underbrace{2\omega_0\left[\int_V (\langle W_m\rangle - \langle W_e\rangle)\,dV\right]}_{\frac{\omega_0\mu'}{2}\int_V |\vec{H}_c|^2 dV - \frac{\omega_0\varepsilon'}{2}\int_V |\vec{E}_c|^2 dV}$$

FUENTES DE V · POTENCIA REACTIVA ESCAPA DE V POR S · ENERGÍA MAG. ALMACENADA EN V · ENERGÍA ELÉCT. ALMACENADA EN V


· TIEMPO:

$$\int_V (-\vec{J}_{0ext}\cdot\vec{E})\,dV = \int_S (\vec{E}\times\vec{H})\cdot d\vec{s} + \int_V \sigma(\vec{E})^2 dV + \int_V \left(\vec{E}\cdot\frac{d\vec{D}}{dt} + \vec{H}\frac{d\vec{B}}{dt}\right) dV \qquad \int_{x=0}^{x=\pi} \mathrm{sen}^3 x\,dx = \frac{4}{3}$$

FUENTES DE V · POTENCIA ESCAPA DE V POR S · POTENCIA DISIPADA (JOULE)

MEDIOS DE TRANSMISIÓN:

$\vec{E} = (-\nabla_t \Phi)e^{-\gamma_0 z}$

$\vec{H} = \frac{\hat{z}\times\vec{E}}{\eta} \Rightarrow \vec{E} = \eta(\vec{H}\times\hat{z})$

$V_0 = \Phi_{C_1} - \Phi_{C_2}$ ; $V_0 = \int_{C_2}^{C_1} \vec{E}\,d\vec{l}$

ONDA TENSIÓN:

$V(z) = V_0 e^{-\gamma_0 z}$

ONDA CORRIENTE

$I(z) = I_0 e^{-\gamma_0 z}$

$\vec{J}_s\big|_{S_{cond.}} = \hat{n}_{saliente\ cond.} \times \vec{H}_{//}\big|_{S_{cond.}}$

$I_0 = \int \vec{J}_s\,dl$

PARÁMETROS PRIMARIOS L.T.

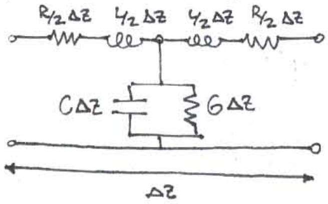
$Z_0 = \frac{V_0}{I_0}$ $\quad Z_0 = \eta\cdot \underset{geometría}{k_g}$ $\quad \gamma_0 = j\omega\sqrt{\mu\varepsilon}$ (contiene pérdidas dieléc. pero no las de conduct.)

CIRCUITO EQUIVALENTE:

condición:

$|\gamma_0\cdot\Delta z| \ll 1$

PÉRDIDAS DIELÉCT. Y CONDUCTOR:




$L = \mu k_g$ (H/m)

$C = \frac{\varepsilon'}{k_g}$ (F/m)

$G = \frac{\omega_0\varepsilon''}{k_g}$

$W_{PC} = \frac{R_s}{2}\int_{Cond.} (\vec{H}\cdot\vec{H}^*)\,dl$ POT. DISIP. CONDUCT

Cond. Leontovich espesor $\gg \delta$

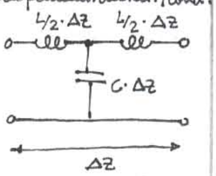
$R_S = \frac{1}{\sigma\delta}$ $\quad \delta = \frac{1}{\sqrt{\pi f_0 \mu\sigma}}$ $\quad \underset{(Np/m)}{\alpha_c} = \frac{W_{PC}}{2W_T}$ cte de aten. debida a pérdidas conductores

$\underset{(\Omega/m)}{R} = 2\alpha_c\sqrt{\frac{L}{C}}$ $\qquad \alpha_c = \frac{R}{2}\sqrt{\frac{C}{L}}$

$Z_C = \sqrt{\frac{R+j\omega_0 L}{G+j\omega_0 C}} = Z_0\left(1+\frac{\alpha_c}{\gamma_0}\right)$ $\qquad \gamma_c = \alpha_c + \gamma_0 \approx \alpha_c + \alpha_d + j\omega_0\sqrt{\mu\varepsilon'} = \sqrt{(R+j\omega_0 L)(G+j\omega_0 C)}$

CASO IDEAL:

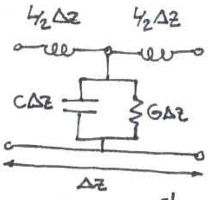
Sin pérdidas dielect. / cond.




$\underset{(H/m)}{L = \mu\cdot k_g}$ $\quad \underset{(F/m)}{C = \frac{\varepsilon}{k_g}}$

$Z_0 = \sqrt{\frac{L}{C}}$ $\quad \gamma_0 = j\omega\sqrt{LC}$

PÉRDIDAS DIELÉC.




$\underset{(H/m)}{L = \mu k_g}$ $\quad \underset{(F/m)}{C = \frac{\varepsilon'}{k_g}}$ $\quad \underset{(S/m)}{G = \frac{\omega_0\varepsilon''}{k_g}}$

$Z_0 = \eta k_g = \sqrt{\frac{j\omega_0 L}{G+j\omega_0 C}}$ $\quad \gamma_0 = j\omega\sqrt{\mu\varepsilon} \approx \underbrace{\frac{1}{2}\mathrm{tg}\delta\,\omega_0\sqrt{\mu\varepsilon'}}_{\alpha_d} + j\omega_0\sqrt{\mu\varepsilon'}$

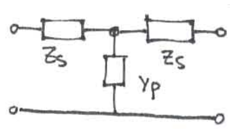
$\boxed{\mathrm{tg}\delta \ll 1 \text{ BAJAS PÉRDIDAS}}$

$\mathrm{tg}\delta = \frac{\varepsilon'' + \frac{\sigma_D}{\omega_0}}{\varepsilon'}$

Si $\sigma_D = 0$ $\Downarrow$ $\mathrm{tg}\delta = \frac{\varepsilon''}{\varepsilon'}$

$\underset{(Np/m)}{\alpha_d} = \frac{G}{2}\sqrt{\frac{L}{C}}$ cte de atenuación pérdidas dieléctr.

ESQUEMA GENERAL:

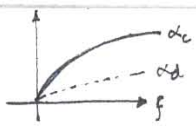



$Z_s = \frac{R}{2}\Delta z + j\omega_0\frac{L}{2}\Delta z = (R+j\omega_0 L)\frac{\Delta z}{2}$

$Y_p = G\Delta z + j\omega_0 C\Delta z = (G+j\omega_0 C)\Delta z$

$\underset{pérdidas\ dieléc-cond.}{Z_C} = \sqrt{\frac{2Z_s}{Y_p}} = \sqrt{\frac{R+j\omega_0 L}{G+j\omega_0 C}}$ $\qquad \underset{péd.\ dielect.}{Z_0} = \sqrt{\frac{2Z_s}{Y_p}} = \sqrt{\frac{j\omega_0 L}{G+j\omega_0 C}}$

$\alpha = \alpha_c + \alpha_d$

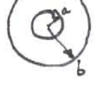



$\underset{(W)}{W_T} = W_T(z=0)\,e^{-2\alpha z}$

$\underset{(dBm)}{W_T(z=l)} = \underset{(dBm)}{W_T(z=0)} - \underset{(Np/m)}{\alpha}\cdot \underset{(dB/Np)}{8{,}68}\cdot \underset{(m)}{l}$ $\Rightarrow \underset{(W)}{W_{disip}} = \underset{(W)}{W_T(z=0)} - \underset{(W)}{W_T(z=l)}$

$W_T(z=0) = \frac{1}{2}\mathrm{Re}[V_0 I_0^*] = \frac{1}{2}\mathrm{Re}\left[\frac{1}{Z_0^*}\right]|V_0|^2$

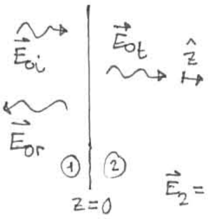
COAXIAL




$Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right)$ $\quad \mu_0 = 4\pi\cdot 10^{-7}$ (H/m) $\quad \varepsilon_0 = \frac{1}{36\pi\cdot 10^9}$

## INCIDENCIA NORMAL

• 2 medios




$$\boxed{\vec{E}_{ot} = \frac{2\eta_2}{\eta_2+\eta_1}\vec{E}_{oi} \qquad \vec{E}_{or} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}\vec{E}_{oi}}$$

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r = \vec{E}_{oi}e^{-\gamma_1 z} + \vec{E}_{or}e^{\gamma_1 z} \qquad \vec{E}_2 = \vec{E}_t = \vec{E}_{ot}e^{-\gamma_2 z}$$

$$\rho_1(z) = \frac{\vec{E}_{or1}}{\vec{E}_{oi1}}e^{2\gamma_1 z} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}e^{2\gamma_1 z} = \rho_1(z=0)e^{2\gamma_1 z}$$

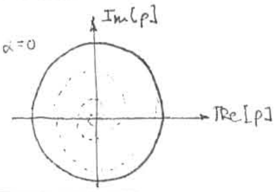
$$\vec{E}_1 = \vec{E}_{i1} + \vec{E}_{r1} = \vec{E}_{i1}\left[1+\rho_1(z)\right]$$

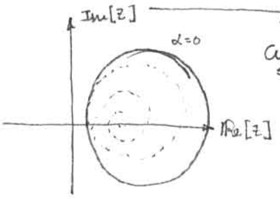
Cuidado con el signo de ρ

$$\vec{E}_2 = \vec{E}_{oi,2}e^{-\gamma_2 z} = \vec{E}_{oi,1}\left[1+\rho_1(0)\right]e^{-\gamma_2 z}$$

$$\boxed{\rho_1(z=0) = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}\eta_0 \qquad \boxed{\rho_n(z) = \frac{Z_n(z)-\eta_n}{Z_n(z)+\eta_n}}$$

Cuidado con el signo de ρ



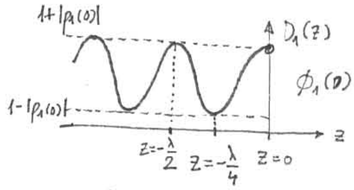



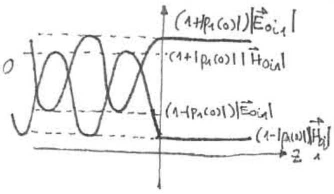

D.O.E:

• $|\vec{E}_1| = |\vec{E}_{oi,1}|\sqrt{1+|\rho_1(0)|^2+2|\rho_1(0)|\cos(2\beta_1 z+\phi_1(0))}$

• $D_1(z) = \dfrac{|\vec{E}_1|}{|\vec{E}_{oi1}|} = \sqrt{1+|\rho_1(0)|^2+2|\rho_1(0)|\cos(2\beta_1 z+\phi_1(0))}$

• $COE = \dfrac{D_{1\,max}}{D_{1\,min}} = \dfrac{1+|\rho_1(0)|}{1-|\rho_1(z)|} = \dfrac{|\vec{E}_1|_{max}}{|\vec{E}_1|_{min}}$

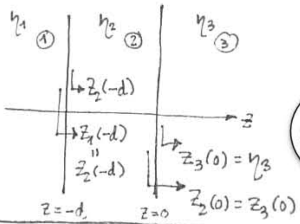


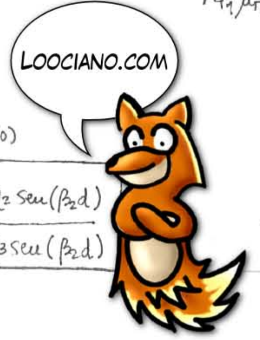

• Con pérdidas:

• $|\vec{E}_1| = |\vec{E}_{oi,1}|e^{-\alpha_1 z}\sqrt{1+|\rho_1(0)|^2e^{4\alpha_1 z}+2|\rho_1(0)|e^{2\alpha_1 z}\cos(2\beta_1 z+\phi_1(0))}$

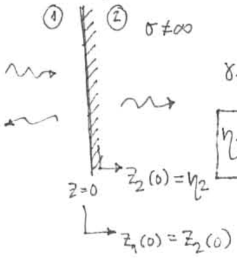
• $D_1(z) = \dfrac{|\vec{E}_1|}{|\vec{E}_{oi1}|} = \sqrt{1+|\rho_1(0)|^2e^{4\alpha_1 z}+2|\rho_1(0)|e^{2\alpha_1 z}\cos(2\beta_1 z+\phi_1(0))}$




$$\beta_1 = \frac{2\pi}{\lambda_1} \qquad \lambda_1 = \frac{\lambda_0}{\sqrt{\varepsilon_{r1}\mu_{r1}}}$$



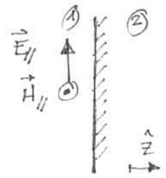

• Conductores




$$\gamma_2 = \frac{1+j}{\delta} \qquad \boxed{\eta_2 \approx \frac{1+j}{\sigma\delta}} \qquad \boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}}$$

$$Z_1(-d) = \eta_2\frac{Z_3 Ch(\gamma_2 d_2)+\eta_2 Sh(\gamma_2 d_2)}{\eta_2 Ch(\gamma_2 d_2)+Z_3 Sh(\gamma_2 d_2)}$$

$$\boxed{Z_1(-d) = \eta_2\cdot\frac{\eta_3\cos(\beta_2 d)+j\eta_2\,\mathrm{sen}(\beta_2 d)}{\eta_2\cos(\beta_2 d)+j\eta_3\,\mathrm{sen}(\beta_2 d)}}$$




En $z=0$:

$$\vec{E}_{//} \approx \frac{1+j}{\sigma\delta}(\vec{H}_{//}\times\hat{n})$$

En $z=0$: $\dfrac{\hat{z}\times\vec{E}_1(0)}{\vec{H}_1(0)} = \dfrac{\hat{z}\times\vec{E}_2}{\vec{H}_2} \simeq \dfrac{1+j}{\sigma\delta}$

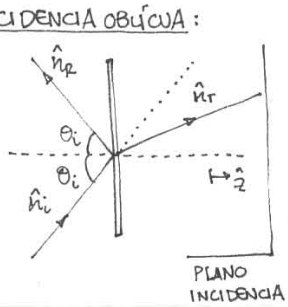
APROX. LEONTOVICH

$$\rho_1(0) = \frac{Z-\eta_1}{Z+\eta_1} = \frac{\frac{1+j}{\sigma\delta}-\eta_1}{\frac{1+j}{\sigma\delta}+\eta_1} \underset{\sigma\uparrow\uparrow}{\approx} -1+\frac{2(1+j)}{\eta_1\sigma\delta}$$

$$W_T = W_{i1}(1-|\rho_1|^2) = W_{disipada}$$

$$\frac{W_{dis}}{W_{i1}} = (1-|\rho_1|^2)$$

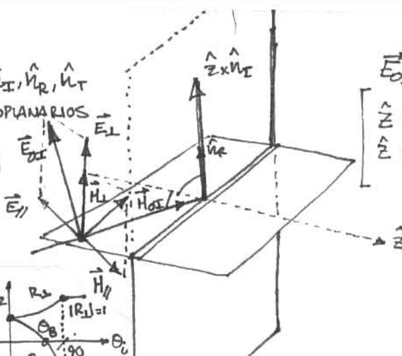
## • INCIDENCIA OBLÍCUA:




$$\left.\begin{aligned}&\hat{z}\times\hat{n}_R = \hat{z}\times\hat{n}_I\\&\gamma_2(\hat{z}\times\hat{n}_T) = \gamma_1(\hat{z}\times\hat{n}_I)\end{aligned}\right\}\ \hat{n}_I, \hat{n}_R, \hat{n}_T \text{ COPLANARIOS}$$

$$\underbrace{\sqrt{\varepsilon_{r2}}}_{n_2}(\hat{z}\times\hat{n}_T) = \underbrace{\sqrt{\varepsilon_{r1}}}_{n_1}(\hat{z}\times\hat{n}_i)$$

$$\sqrt{\varepsilon_{r2}}\,\mathrm{sen}\,\theta_T = \sqrt{\varepsilon_{r1}}\,\mathrm{sen}\,\theta_i$$




$$\vec{E}_{oI} = \vec{E}_\perp + \vec{E}_{//} \qquad \vec{E}_{o\perp} = \vec{E}_o\cdot\hat{n}_\perp\cdot\hat{n}_\perp$$

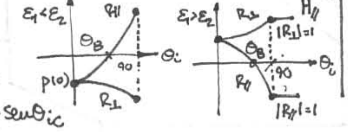
$$\left[\begin{aligned}&\hat{z}\times\vec{E}_{oT} - \hat{z}\times\vec{E}_{oI} - \hat{z}\times\vec{E}_{oR} = 0\\&\hat{z}\times\vec{H}_{oT} - \hat{z}\times\vec{H}_{oI} - \hat{z}\times\vec{H}_{oR} = 0\end{aligned}\right]$$

PROBLEMA:

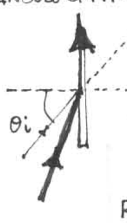
$$\vec{E}_{oI}\times\vec{H}_{oI} \doteq \hat{n}_I$$

$$\Downarrow$$

$$\underbrace{(\vec{E}_\perp\times\vec{H}_{//})}_{\text{PROBLEMA 1}} + \underbrace{(\vec{E}_{//}\times\vec{H}_\perp)}_{\text{PROBLEMA 2}} \doteq \hat{n}_I$$




· ÁNGULO CRÍTICO:




$$\mathrm{sen}\,\theta_T = 1 = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\,\mathrm{sen}\,\theta_{ic}$$

si $\varepsilon_1>\varepsilon_2$ $\quad \theta_{ic} = \mathrm{arcsen}\left(\sqrt{\dfrac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right)$

① $\vec{E}_{oR\perp} = \boxed{R_\perp}\vec{E}_{oi\perp}$

② $\vec{H}_{oR\perp} = \boxed{-R_{//}}\vec{H}_{oi\perp}$

$$R_\perp = \frac{\frac{\eta_2}{\hat{z}\cdot\hat{n}_T}-\frac{\eta_1}{\hat{z}\cdot\hat{n}_I}}{\frac{\eta_2}{\hat{z}\cdot\hat{n}_T}+\frac{\eta_1}{\hat{z}\cdot\hat{n}_I}} \qquad R_{//} = \frac{\eta_2(\hat{z}\cdot\hat{n}_T)-\eta_1(\hat{z}\cdot\hat{n}_i)}{\eta_2(\hat{z}\cdot\hat{n}_T)+\eta_1(\hat{z}\cdot\hat{n}_i)}$$

$$\theta_B = \mathrm{arctg}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

$\hookrightarrow \vec{E}_{oR} = \vec{E}_{oi\perp}$ (pol. lineal): si $\vec{E}_{oi} = \vec{E}_{//}$ TRANSMISIÓN TOTAL

Los campos paralelos a la discontinuidad son iguales en la separación de medios.

$$\vec{E}_{oi\perp} + \vec{E}_{oR\perp} = \vec{E}_{oT\perp}$$

$$\frac{\hat{z}\cdot\hat{n}_I}{\eta_1}\cdot\vec{E}_{oI\perp} = -\frac{\hat{z}\cdot\hat{n}_R}{\eta_1}\vec{E}_{oR} + \frac{\hat{z}\cdot\hat{n}_T}{\eta_2}\vec{E}_{oT\perp}$$