Интерполирование сплайнами

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[38]:
```
from math import cos, pi
from IPython.display import display, Latex, Markdown


def func_Runge(x: float):
    return round((1 + x ** 2) ** -1, 3)


def get_default_nodes(n: int):
    h = 2 / n
    nodes_x = [round(-1 + k * h, 3) for k in range(n + 1)]
    nodes_f = [func_Runge(x) for x in nodes_x]
    return nodes_x, nodes_f
```

Задание:

Пусть $f(x) = \frac{1}{1+x^2}$, $x \in [-1;1]$. Вычислить значения функции для $x = -1 + \frac{h}{2}, -\frac{h}{2}, \frac{h}{2}, 1 - \frac{h}{2}, \left(h = \frac{2}{n}, n = 4, 10, 20, 40\right)$, применяя различные способы интерполирования: 1. Интерполяционный полином Лагранжа (схема Эйткена) с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$. 2. Интерполяционный полином Лагранжа (схема Эйткена) с узлами $x_k = \cos\frac{2k+1}{2(n+1)}\pi$, $k = 0, \ldots, n$). 3. Сплайн $S_{11}(f, x)$ с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$. 4. Сплайн $S_{32}(f, x)$ с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$.

1.Интерполяционный полином Лагранжа

Схема Эйткена с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$.

[39]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)


def Aitken(x: float, nodes_x: list, nodes_f: list):
    coeffs = []
    n = len(nodes_f) - 1
    for i in range(1, n + 2):
```

```
        coeffs_n = []
        for j in range(n + 2 - i):
            if i == 1:
                coeffs.append(nodes_f)
                break
            pol = pow(nodes_x[j + i - 1] - nodes_x[j], -1) * (coeffs[i - 2][j] *␣
 ↪(nodes_x[j + i - 1] - x)
                                                              - coeffs[i - 2][j␣
 ↪+ 1] * (nodes_x[j] - x))
            coeffs_n.append(round(pol, 3))
            coeffs.append(coeffs_n)
    return coeffs[-1][0]


for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$L_{%i}(%.3f)=%.3f$;"%(n, x, Aitken(x, x_nodes,␣
 ↪f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$L_4(-0.750) = 0.656$;

$L_4(-0.250) = 0.931$;

$L_4(0.250) = 1.156$;

$L_4(0.750) = 1.331$;

Случай, когда $n = 10$:

$L_{10}(-0.900) = 0.555$;

$L_{10}(-0.100) = 1.019$;

$L_{10}(0.100) = 1.142$;

$L_{10}(0.900) = 1.666$;

Случай, когда $n = 20$:

$L_{20}(-0.950) = 0.526$;

$L_{20}(-0.050) = 1.018$;

$L_{20}(0.050) = 1.076$;

$L_{20}(0.950) = 1.622$;

Случай, когда $n = 40$:

$L_{40}(-0.975) = 0.513$;

$L_{40}(-0.025) = 1.007$;

$L_{40}(0.025) = 1.033$;

$L_{40}(0.975) = 1.527$;

2.Интерполяционный полином Лагранжа с узлами Чебышёва

Схема Эйткена с узлами $x_k = \cos\dfrac{2k+1}{2(n+1)}\pi,\ k = 0, \ldots, n.$

[40]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)


def get_chebyshev_nodes(n: int):
    nodes = []
    for k in range(n + 1):
        node = cos((2 * k + 1) * pi / (2 * n + 1))
        nodes.append(round(node, 4))
    return nodes, [func_Runge(x) for x in nodes]


for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_chebyshev_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$L_{%i}(%.3f)=%.3f$;"%(n, x, Aitken(x, x_nodes,␣
 ↪f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$L_4(-0.750) = 1.175$;

$L_4(-0.250) = 1.094$;

$L_4(0.250) = 0.921$;

$L_4(0.750) = 0.656$;

Случай, когда $n = 10$:

$L_{10}(-0.900) = 1.598$;

$L_{10}(-0.100) = 1.109$;

$L_{10}(0.100) = 0.993$;

$L_{10}(0.900) = 0.553$;

Случай, когда $n = 20$:

$L_{20}(-0.950) = 1.553$;

$L_{20}(-0.050) = 1.053$;

$L_{20}(0.050) = 0.999$;

$L_{20}(0.950) = 0.525$;

Случай, когда $n = 40$:

$L_{40}(-0.975) = 1.495$;

$L_{40}(-0.025) = 1.014$;

$L_{40}(0.025) = 0.989$;

$L_{40}(0.975) = 0.512$;

3.Сплайн $S_{11}(f, x)$

[41]:
```
def spline11(x: float, nodes_x: list, nodes_f: list):
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] <= x <= nodes_x[i + 1]:
            l = i
            break
    h = nodes_x[l + 1] - nodes_x[l]
    return nodes_f[i] + (x - nodes_x[i]) * (nodes_f[i + 1] - nodes_f[i]) / h


N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
```

```
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$S_{11}(f,%.3f)=%.3f$;"%(x, spline11(x, x_nodes,␣
↪f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_{11}(f, -0.750) = 0.650$;

$S_{11}(f, -0.250) = 0.900$;

$S_{11}(f, 0.250) = 0.900$;

$S_{11}(f, 0.750) = 0.650$;

Случай, когда $n = 10$:

$S_{11}(f, -0.900) = 0.555$;

$S_{11}(f, -0.100) = 0.981$;

$S_{11}(f, 0.100) = 0.981$;

$S_{11}(f, 0.900) = 0.555$;

Случай, когда $n = 20$:

$S_{11}(f, -0.950) = 0.526$;

$S_{11}(f, -0.050) = 0.995$;

$S_{11}(f, 0.050) = 0.995$;

$S_{11}(f, 0.950) = 0.526$;

Случай, когда $n = 40$:

$S_{11}(f, -0.975) = 0.513$;

$S_{11}(f, -0.025) = 0.999$;

$S_{11}(f, 0.025) = 0.999$;

$S_{11}(f, 0.975) = 0.513$;

4.Сплайн $S_{32}(f,x)$

[42]:
```
def df(x):
    return -2 * x / pow(1 + x ** 2, 2)

def spline32(x: float, nodes_x: list, nodes_f: list):
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] <= x <= nodes_x[i + 1]:
            l = i
            break
    h = nodes_x[l + 1] - nodes_x[l]
    t = (x - nodes_x[i]) / h
    A = -2 * (nodes_f[l + 1] - nodes_f[l]) / h + (df(nodes_x[l]) + df(nodes_x[l␣
 ↪+ 1]))
    B = -A + (nodes_f[l + 1] - nodes_f[l]) / h - df(nodes_x[l])
    return nodes_f[l] + (x - nodes_x[l]) * (df(nodes_x[l] + t * (B + t * A)))


N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$S_{32}(f,%.3f)=%.3f$;"%(x, spline32(x, x_nodes,␣
 ↪f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_{32}(f,-0.750) = 0.633$;

$S_{32}(f,-0.250) = 0.962$;

$S_{32}(f,0.250) = 1.107$;

$S_{32}(f,0.750) = 0.640$;

Случай, когда $n = 10$:

$S_{32}(f,-0.900) = 0.551$;

$S_{32}(f,-0.100) = 1.011$;

$S_{32}(f,0.100) = 1.019$;

$S_{32}(f,0.900) = 0.551$;

Случай, когда $n = 20$:

$S_{32}(f, -0.950) = 0.525$;

$S_{32}(f, -0.050) = 1.004$;

$S_{32}(f, 0.050) = 1.005$;

$S_{32}(f, 0.950) = 0.525$;

Случай, когда $n = 40$:

$S_{32}(f, -0.975) = 0.513$;

$S_{32}(f, -0.025) = 1.002$;

$S_{32}(f, 0.025) = 1.001$;

$S_{32}(f, 0.975) = 0.513$;