Интерполирование сплайнами

2 декабря 2020 г.

[70]:
```
# to export to latex use "jupyter nbconvert --to latex file.ipynb"

from IPython.display import display, Latex, Markdown


def func_Runge(x: float): return round((1 + x ** 2) ** -1, 3)


def get_default_nodes(n: int):
    h = 2 / n
    nodes_x = [round(-1 + k * h, 3) for k in range(n + 1)]
    nodes_f = [func_Runge(x) for x in nodes_x]
    return nodes_x, nodes_f


def d2f(x): return (6 * pow(x, 2) - 2) / pow(pow(x, 2) + 1, 3)

def df(x): return -2 * x / pow(1 + x ** 2, 2)


def phi1(t): return ((1 - t) ** 2) * (1 + 2 * t)


def phi2(t): return (t ** 2) * (3 - 2 * t)


def phi3(t): return t * (1 - t) ** 2


def phi4(t): return (t ** 2) * (t - 1)


def tma(A: list, B: list, C: list, F: list) -> list:
    alpha = [-C[0] / B[0]]
```

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    beta = [F[0] / B[0]]

    for a, b, c, f in zip(A[1:], B[1:], C[1:], F[1:]):
        beta.append((f - a * beta[-1]) / (a * alpha[-1] + b))
        alpha.append(-c / (a * alpha[-1] + b))

    X = [(F[-1] - A[-1] * beta[-1]) / (B[-1] + A[-1] * alpha[-1])]
    alpha.reverse()
    beta.reverse()

    for a, b in zip(alpha[1:], beta[1:]):  # possible mistake
        X.append(a * X[-1] + b)

    X.reverse()
    return X


def spline1I(x: float, nodes_x: list, nodes_f: list) -> float:  # 5.a)
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] < x <= nodes_x[i + 1]:
            l = i
            break

    h = [nodes_x[i + 1] - nodes_x[i] for i in range(len(nodes_x) - 1)]
    mu = [h[i-1] * pow(h[i-1] + h[i], -1) for i in range(1, len(h))]
    lmbda = [1 - x for x in mu]

    c = [3 * (mu[i] * ((nodes_f[i + 1] - nodes_f[i]) / h[i]))
         + lmbda[i] * (nodes_f[i] - nodes_f[i - 1]) / h[i - 1]
         for i in range(1, len(h) - 1)]

    mu.insert(0, 1)
    lmbda.append(0)
    c.insert(0, 2 * df(nodes_x[0]))
    c.append(2 * df(nodes_x[-1]))
    m = tma(lmbda, [2 for i in range(len(c))], mu, c)
    t = (x - nodes_x[l]) / h[l]
    m.insert(0, (c[0] - mu[0] * m[0]) / 2)

    return phi1(t) * nodes_f[l] + phi2(t) * nodes_f[l + 1] \
           + m[l] * h[l] * phi3(t) + \
           m[l + 1] * h[l] * phi4(t)
```

```python
def spline1II(x: float, nodes_x: list, nodes_f: list) -> float:  # 5.б)
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] < x <= nodes_x[i + 1]:
            l = i
            break

    h = [nodes_x[i + 1] - nodes_x[i] for i in range(len(nodes_x) - 1)]
    mu = [h[i-1] * pow(h[i-1] + h[i], -1) for i in range(1, len(h))]
    lmbda = [1 - x for x in mu]

    c = [3 * (mu[i] * ((nodes_f[i + 1] - nodes_f[i]) / h[i]))
         + lmbda[i] * (nodes_f[i] - nodes_f[i - 1]) / h[i - 1]
         for i in range(1, len(h) - 1)]

    mu.insert(0, 1)
    lmbda.append(1)
    c.insert(0, 3 * (nodes_f[1] - nodes_f[0]) / h[0] - h[0]
             * d2f(nodes_x[0]) / 2)
    c.append(3 * (nodes_f[-1] - nodes_f[-2]) / h[-2] - h[-2]
             * d2f(nodes_x[-1]) / 2)
    m = tma(lmbda, [2 for _ in range(len(c))], mu, c)
    t = (x - nodes_x[l]) / h[l]
    m.insert(0, (c[0] - mu[0] * m[0]) / 2)

    return phi1(t) * nodes_f[l] + phi2(t) * nodes_f[l + 1] \
           + m[l] * h[l] * phi3(t) + \
           m[l + 1] * h[l] * phi4(t)


def spline2I(x: float, nodes_x: list, nodes_f: list) -> float:  # 6.a)
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] < x <= nodes_x[i + 1]:
            l = i
            break

    h = [nodes_x[i + 1] - nodes_x[i] for i in range(len(nodes_x) - 1)]
    mu = [h[i-1] * pow(h[i-1] + h[i], -1) for i in range(1, len(h))]
    lmbda = [1 - x for x in mu]

    d = [(6 / (h[i - 1] + h[i])) * ((nodes_f[i + 1] - nodes_f[i]) / h[i])
         -  (nodes_f[i] - nodes_f[i - 1])/ h[i - 1]
```

```python
         for i in range(1, len(h) - 1)]

    lmbda.insert(0, 1)
    mu.append(1)
    d.insert(0, 6 * ((nodes_f[1] - nodes_f[0]) / h[0] - df(nodes_x[0])) / h[0])
    d.append(6 * (df(nodes_x[-1]) - (nodes_f[-1] - nodes_f[-2]) / h[-2]) / h[-2])
    M = tma(lmbda, [2 for _ in range(len(d))], mu, d)
    t = (x - nodes_x[l]) / h[l]
    M.insert(0, (d[0] - lmbda[0] * M[0]) / 2)

    return (1 - t) * nodes_f[l] + t * nodes_f[l + 1]\
           - (pow(h[l], 2) * t * (1 - t))\
           * ((2 - t) * M[l] + (1 + t) * M[l + 1])


def spline2II(x: float, nodes_x: list, nodes_f: list) -> float:  # 6.6)
    l = 0
    for i in range(len(nodes_x) - 1):
        if nodes_x[i] < x <= nodes_x[i + 1]:
            l = i
            break

    h = [nodes_x[i + 1] - nodes_x[i] for i in range(len(nodes_x) - 1)]
    mu = [h[i-1] * pow(h[i-1] + h[i], -1) for i in range(1, len(h))]
    lmbda = [1 - x for x in mu]

    d = [(6 / (h[i - 1] + h[i])) * ((nodes_f[i + 1] - nodes_f[i]) / h[i])
         -  (nodes_f[i] - nodes_f[i - 1])/ h[i - 1]
         for i in range(1, len(h) - 1)]

    lmbda.insert(0, 0)
    mu.append(0)
    d.insert(0, 2 * d2f(nodes_x[0]))
    d.append(2 * d2f(nodes_x[-1]))
    M = tma(lmbda, [2 for _ in range(len(d))], mu, d)
    t = (x - nodes_x[l]) / h[l]
    M.insert(0, (d[0] - lmbda[0] * M[0]) / 2)

    return (1 - t) * nodes_f[l] + t * nodes_f[l + 1] \
           - (pow(h[l], 2) * t * (1 - t)) * ((2 - t) * M[l] + (1 + t) * M[l + 1])
```

Интерполирование сплайнами

Пусть $f(x) = \dfrac{1}{1+x^2}$, $x \in [-1;1]$. Вычислить значения функции для $x = -1 + \dfrac{h}{2}, -\dfrac{h}{2}, \dfrac{h}{2}, \ 1 - \dfrac{h}{2}, \left(h = \dfrac{2}{n}, n = 4,\ 10,\ 20,\ 40\right)$, применяя различные способы интерполирования: 5. Сплайн $S_{31}(f,x)$ с узлами $x_k = -1 + kh$, $k = 0,\ldots,n$, и параметрами $m_i = S'(f, 1x_i), i = 0,\ldots,n$, в котором). Граничные условия I типа - $S^{'}(f,1) = f^{'}(1)$, $S^{'}(f,-1) = f^{'}(-1)$;). Граничные условия II типа - $S^{''}(f,1) = f^{''}(1)$, $S^{''}(f,-1) = f^{''}(-1)$. 6. Сплайн $S_{31}(f,x)$ с узлами $x_k = -1 + kh$, $k = 0,\ldots,n$, и параметрами $M_i = S'(f,1x_i), i = 0,\ldots,n$, в котором). Граничные условия I типа - $S^{'}(f,1) = f^{'}(1)$, $S^{'}(f,-1) = f^{'}(-1)$;). Граничные условия II типа - $S^{''}(f,1) = f^{''}(1)$, $S^{''}(f,-1) = f^{''}(-1)$.

№5 а) Сплайн $S_{31}(f,x)$ с узлами $x_k = -1 + kh$, $k = 0,\ldots,n$, и параметрами $m_i = S'(f,1x_i), i = 0,\ldots,n$, в котором граничные условия I типа - $S^{'}(f,1) = f^{'}(1)$,$S^{'}(f,-1) = f^{'}(-1)$;

[71]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$S_{I}(f,%.3f)=%.4f$;"
                      %(x, spline1I(x, x_nodes, f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_I(f,-0.750) = 0.6542$;

$S_I(f,-0.250) = 0.8916$;

$S_I(f,0.250) = 0.9377$;

$S_I(f,0.750) = 0.6700$;

Случай, когда $n = 10$:

$S_I(f,-0.900) = 0.5568$;

$S_I(f,-0.100) = 0.9857$;

$S_I(f, 0.100) = 0.9874;$

$S_I(f, 0.900) = 0.5576;$

Случай, когда $n = 20$:

$S_I(f, -0.950) = 0.5265;$

$S_I(f, -0.050) = 0.9965;$

$S_I(f, 0.050) = 0.9967;$

$S_I(f, 0.950) = 0.5278;$

Случай, когда $n = 40$:

$S_I(f, -0.975) = 0.5130;$

$S_I(f, -0.025) = 0.9995;$

$S_I(f, 0.025) = 0.9993;$

$S_I(f, 0.975) = 0.5142;$

№5 б) Сплайн $S_{31}(f, x)$ с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$, и параметрами $m_i = S'(f, 1x_i), i = 0, \ldots, n$, в котором граничные условия II типа - $S''(f, 1) = f''(1)$,$S''(f, -1) = f''(-1)$;

[72]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$S_{II}(f,%.3f)=%.4f$;"
                      %(x, spline1II(x, x_nodes, f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_{II}(f, -0.750) = 0.6385;$

$S_{II}(f, -0.250) = 0.9229;$

$S_{II}(f, 0.250) = 0.9281;$

$S_{II}(f, 0.750) = 0.6773;$

Случай, когда $n = 10$:

$S_{II}(f, -0.900) = 0.5513;$

$S_{II}(f, -0.100) = 0.9855;$

$S_{II}(f, 0.100) = 0.9875;$

$S_{II}(f, 0.900) = 0.5607;$

Случай, когда $n = 20$:

$S_{II}(f, -0.950) = 0.5241;$

$S_{II}(f, -0.050) = 0.9965;$

$S_{II}(f, 0.050) = 0.9967;$

$S_{II}(f, 0.950) = 0.5287;$

Случай, когда $n = 40$:

$S_{II}(f, -0.975) = 0.5118;$

$S_{II}(f, -0.025) = 0.9995;$

$S_{II}(f, 0.025) = 0.9993;$

$S_{II}(f, 0.975) = 0.5145;$

№6 а) Сплайн $S_{31}(f, x)$ с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$, и параметрами $M_i = S'(f, 1x_i), i = 0, \ldots, n$, в котором граничные условия I типа - $S^{'}(f, 1) = f^{'}(1)$,$S^{'}(f, -1) = f^{'}(-1)$;

[73]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
```

```
        display(Latex("$S_{II}(f,%.3f)=%.4f$;"
                      %(x, spline2I(x, x_nodes, f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_{II}(f, -0.750) = 0.5807;$

$S_{II}(f, -0.250) = 0.7535;$

$S_{II}(f, 0.250) = 0.9498;$

$S_{II}(f, 0.750) = 0.7850;$

Случай, когда $n = 10$:

$S_{II}(f, -0.900) = 0.5451;$

$S_{II}(f, -0.100) = 0.9312;$

$S_{II}(f, 0.100) = 0.9847;$

$S_{II}(f, 0.900) = 0.6057;$

Случай, когда $n = 20$:

$S_{II}(f, -0.950) = 0.5261;$

$S_{II}(f, -0.050) = 0.9815;$

$S_{II}(f, 0.050) = 0.9955;$

$S_{II}(f, 0.950) = 0.5524;$

Случай, когда $n = 40$:

$S_{II}(f, -0.975) = 0.5128;$

$S_{II}(f, -0.025) = 0.9954;$

$S_{II}(f, 0.025) = 0.9991;$

$S_{II}(f, 0.975) = 0.5247;$

№6 б) Сплайн $S_{31}(f, x)$ с узлами $x_k = -1 + kh$, $k = 0, \ldots, n$, и параметрами $M_i = S'(f, 1x_i), i = 0, \ldots, n$, в котором граничные условия II типа - $S^{''}(f, 1) = f^{''}(1)$,$S^{''}(f, -1) = f^{''}(-1)$;

[74]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)

for X, n in zip(Arguments, N):
    x_nodes, f_nodes = get_default_nodes(n)
    answers = []
    display(Latex("Случай, когда $n=%i$:"%n), Markdown("<br>"))
    for x in X:
        display(Latex("$S_{II}(f,%.3f)=%.4f$;"
                      %(x, spline2II(x, x_nodes, f_nodes))))
    display(Markdown("<br>"))
```

Случай, когда $n = 4$:

$S_{II}(f, -0.750) = 0.5868$;

$S_{II}(f, -0.250) = 0.7615$;

$S_{II}(f, 0.250) = 0.9455$;

$S_{II}(f, 0.750) = 0.7942$;

Случай, когда $n = 10$:

$S_{II}(f, -0.900) = 0.5543$;

$S_{II}(f, -0.100) = 0.9311$;

$S_{II}(f, 0.100) = 0.9847$;

$S_{II}(f, 0.900) = 0.6082$;

Случай, когда $n = 20$:

$S_{II}(f, -0.950) = 0.5292$;

$S_{II}(f, -0.050) = 0.9815$;

$S_{II}(f, 0.050) = 0.9955$;

$S_{II}(f, 0.950) = 0.5526$;

Случай, когда $n = 40$:

$S_{II}(f, -0.975) = 0.5152$;

$S_{II}(f, -0.025) = 0.9954;$

$S_{II}(f, 0.025) = 0.9991;$

$S_{II}(f, 0.975) = 0.5251;$

Вычисленные значения функции f в требуемых точках:

[75]:
```
N = (4, 10, 20, 40)
Arguments = ((-1 + 1 / n, - 1 / n, 1 / n, 1 - 1 / n) for n in N)
for X in Arguments:
    for x in X:
        display(Latex("$f(%.3f)=%.4f$;"%(x, func_Runge(x))))
```

$f(-0.750) = 0.6400;$

$f(-0.250) = 0.9410;$

$f(0.250) = 0.9410;$

$f(0.750) = 0.6400;$

$f(-0.900) = 0.5520;$

$f(-0.100) = 0.9900;$

$f(0.100) = 0.9900;$

$f(0.900) = 0.5520;$

$f(-0.950) = 0.5260;$

$f(-0.050) = 0.9980;$

$f(0.050) = 0.9980;$

$f(0.950) = 0.5260;$

$f(-0.975) = 0.5130;$

$f(-0.025) = 0.9990;$

$f(0.025) = 0.9990;$

$f(0.975) = 0.5130;$