

1 Интерполирование с равноудалёнными узлами

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[269]: import matplotlib.pyplot as plt
import numpy as np
from numpy import sinh
from IPython.display import display, Latex

def newton_polynomial(x: float, nodes_x: list, nodes_f: list) -> float:
    if len(nodes_x) != len(nodes_f):
        return None

    h = nodes_x[1] - nodes_x[0] # step between equidistant interpolation nodes
    polynomial = 0

    if x < nodes_x[round(len(nodes_x) / 1.5)]: # checks if x is at the
    →beginning of the table
        t = (x - nodes_x[0]) / h
        factor = 1
        for i in range(len(nodes_x)):
            polynomial += finite_differences(0, nodes_f, order=i) * factor
            factor *= (t - i) / (i + 1)
        elif x < nodes_x[round(len(nodes_x) / 1.5)]: # checks if x is at the center
    →of the table
            n = 0
            for i in range(int(len(nodes_x) / 3), len(nodes_x)): # finding n --
    →closest node's number
                if nodes_x[i] > x:
                    n = i if nodes_x[i] - x < x - nodes_x[i - 1] else i - 1
                    break
            t = (x - nodes_x[n]) / h
            factor = 1
            for i in range(len(nodes_x)):
                k = round(i / 2)
                polynomial += finite_differences(n - k, nodes_f, order=i) * factor
                factor *= t + k if i % 2 == 0 else t - k
                factor /= i + 1
            else: # x is at the end of the table
                t = (x - nodes_x[-1]) / h
                n = len(nodes_x)
                factor = 1
                for i in range(n):
                    polynomial += finite_differences(n - i - 1, nodes_f, order=i) *
    →factor
                    factor *= (t + i) / (i + 1)

    return polynomial
```

```
def finite_differences(node_number: int, nodes_y: list, order=1) -> float:
    if order == 0:
        return nodes_y[node_number] # if order of finite difference equals 0 =>
    ↪return function value
    if order == 1:
        return nodes_y[node_number + 1] - nodes_y[node_number]
    return finite_differences(node_number + 1, nodes_y, order=order - 1) \
        - finite_differences(node_number, nodes_y, order=order - 1)
```

```
[270]: x_nodes = tuple(x / 10 for x in range(10,19))
f_nodes = (1.17520, 1.33565, 1.50946, 1.69838, 1.90430,
           2.12928, 2.37557, 2.64563, 2.94217)
x = (1.01, 1.02, 1.03, 1.11, 1.12, 1.13, 1.41, 1.42, 1.43, 1.44,
     1.45, 1.46, 1.75, 1.73, 1.77, 1.78, 1.79)

display(Latex("Найти значения  $\sinh(x)$  для значений аргумента:"))
display(Latex("(1) 1.01, 1.02, 1.03, 1.11, 1.12, 1.13"))
display(Latex("(2) 1.41, 1.42, 1.43, 1.44"))
display(Latex("(3) 1.45, 1.46, 1.75, 1.73, 1.77, 1.78, 1.79"))
display(Latex("При этом дана таблица значений  $x_i$  и  $f_i=f(x_i)$ :"))
display(Latex("$i=0, \ldots, 8;$"))
display(Latex("$x_i \in \{1.0, \backslash, 1.1, \backslash, 1.2, \backslash, 1.3, \backslash, 1.4, \backslash, 1.5, \backslash, 1.6, \backslash, 1.7, \backslash, \backslash, 1.8 \backslash\};$"))
display(Latex("$f_i \in \{1.17520, \backslash, 1.33565, \backslash, 1.50946, \backslash, 1.69838, \backslash, 1.90430, \backslash, \backslash, 2.12928, \backslash, 2.37557, \backslash, 2.64563, \backslash, 2.94217 \backslash\};$"))

display(Latex("Ответы:"))

section = 1
for i, q in enumerate(x):
    if i == 0 or i == 6 or i == 10:
        display(Latex(f"{section}."))
        section += 1
    display(Latex(f"$P_{\text{len}(x\_nodes)}({q}) = \%.4f$"%
                  newton_polynomial(q, x_nodes, f_nodes)))

l = np.linspace(1, 1.9)
fig, ax = plt.subplots() # Create a figure and an axes.
plt.style.use('seaborn-poster')
ax.plot(l, sinh(l), label="$\sinh(x)$") # Plot some data on the axes.
ax.set_xlabel('x label')
ax.set_ylabel('y label')
```

```
ax.set_title("$y=sh(x)$")  
ax.legend()
```

Найти значения $sh(x)$ для значений аргумента:

1) 1.01, 1.02, 1.03, 1.11, 1.12, 1.13

2) 1.41, 1.42, 1.43, 1.44

3) 1.45, 1.46, 1.75, 1.73, 1.77, 1.78, 1.79

При этом дана таблица значений x_i и $f_i = f(x_i)$:

$i = 0, \dots, 8$;

$x_i \in \{1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8\}$;

$f_i \in \{1.17520, 1.33565, 1.50946, 1.69838, 1.90430, 2.12928, 2.37557, 2.64563, 2.94217\}$;

Ответы:

1.

$$P_9(1.01) = 1.1907$$

$$P_9(1.02) = 1.2063$$

$$P_9(1.03) = 1.2220$$

$$P_9(1.11) = 1.3524$$

$$P_9(1.12) = 1.3693$$

$$P_9(1.13) = 1.3863$$

2.

$$P_9(1.41) = 1.9259$$

$$P_9(1.42) = 1.9477$$

$$P_9(1.43) = 1.9697$$

$$P_9(1.44) = 1.9919$$

3.

$$P_9(1.45) = 2.0143$$

$$P_9(1.46) = 2.0369$$

$$P_9(1.75) = 2.7904$$

$$P_9(1.73) = 2.7317$$

$$P_9(1.77) = 2.8503$$

$$P_9(1.78) = 2.8806$$

$$P_9(1.79) = 2.9112$$

[270]: <matplotlib.legend.Legend at 0x2add1510a60>

