Fields/Electromagnetic induction

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1 Induced EMFs

Tool: Induced EMFs
Category: Fields/Electromagnetic induction

A current produces a magnetic field and conversely a magnetic field can produce a current. Actually, the magnetic field *induces* an EMF which, in turn, can produce a current.

This EMF can be generated in 2 ways,

- By relative movement between a magnet and a conductor.
- Changing the magnetic field.

2 Lenz's law

Tool: Lenz's law
Category: Fields/Electromagnetic induction

The direction of the induced EMF is such that it tends to oppose the flux change, and it does oppose it if current flows.

Lenz's law is a direct consequence of the principle of conservation of energy. Let's say relative motion produces an induced EMF which causes a current to flow. The magnetic field generated by this current can *only* help or hinder the motion, it cannot have a neutral effect. Helping the motion would result in the creation of a perpetual motion machine which violates the conservation of energy.

Fleming's left hand rule can be modified to decide the direction of the induced current and therefore the direction of the induced EMF. Just reverse the direction of the 'current' finger.

3 Magnetic flux

Tool: Magnetic flux
Category: Fields/Electromagnetic induction

Since the flux density B is the number of flux lines per unit area, then it logically follows that the magnetic flux is

$$\begin{aligned} \phi &= \vec{B} \cdot \vec{A} \\ &= BA\cos\theta \qquad (1) \end{aligned}$$

ϕ is the magnetic flux.
B is the magnetic flux density.
A is the area.
θ is the angle between the direction of the magnetic flux and the vector perpendicular to the plane of the area.

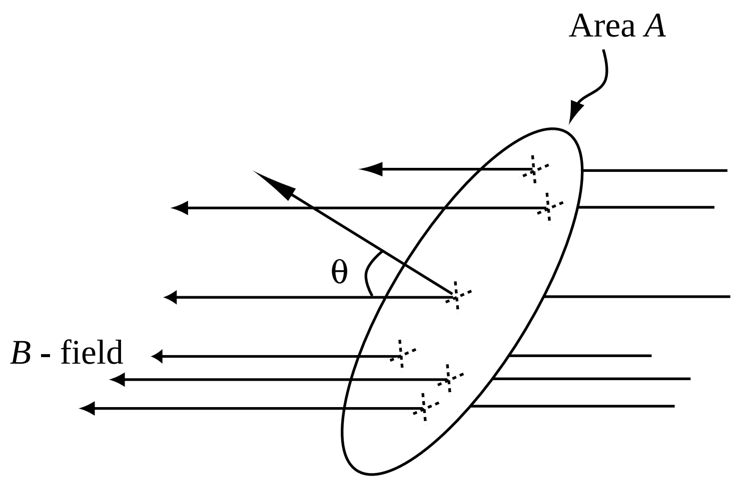

Figure 1: Magnetic flux through an area.

4 Motional EMF

Tool: Motional EMF
Category: Fields/Electromagnetic induction

If a long, straight conductor of length l is moved perpendicularly with respect to a magnetic field, the charge carriers within will experience a force according the the following relationship.

$$F_B = Bqv \sin \theta \tag{2}$$

F_B is the force on each charge carrier due to motion in a magnetic field.
B is the magnetic flux density.
q is the charge on each carrier.
v is the velocity with which the conductor is moved.
θ is the angle between the direction of motion and the direction of the mag-

netic field. In this case $\theta = 90°$

The force will be directed along the length of the conductor. As charge carriers flow to one end of the conductor, a corresponding electric field is set up.

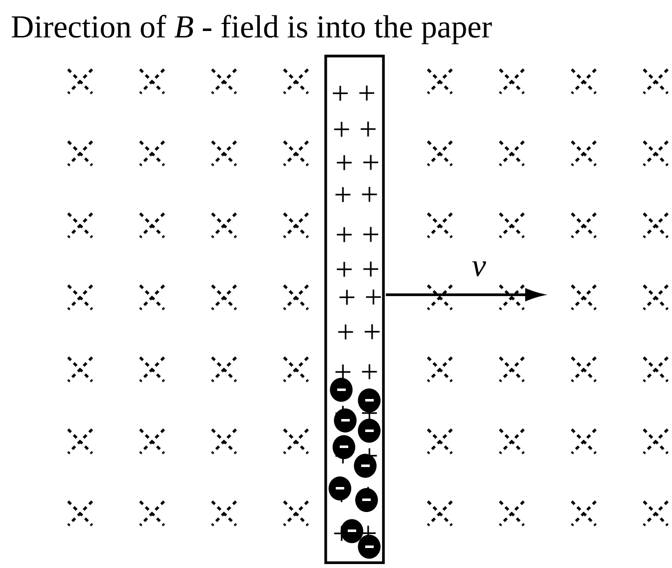

Figure 2: Separation of charges causing an electric field to be produced.

Therefore, the electric field force experienced by each charge carrier is

$$\begin{aligned} F_E &= Eq \\ &= \frac{V}{l}q \end{aligned} \tag{3}$$

F_E is the force on each charge carrier due to the electric field.
E is the electric field strength.
V is the potential difference across the conductor.
l is the length of the conductor.

At equilibrium, the force on each carrier due to the magnetic field is balanced by the force of the electric field.

$$F_E = F_B$$

$$\begin{aligned}\frac{V}{l}q &= Bqv \\ V &= Blv \end{aligned} \tag{4}$$

5 Faraday's law

Tool: Faraday's law
Category: Fields/Electromagnetic induction

The induced EMF around a closed loop is directly proportional to rate of change of magnetic flux through the loop.

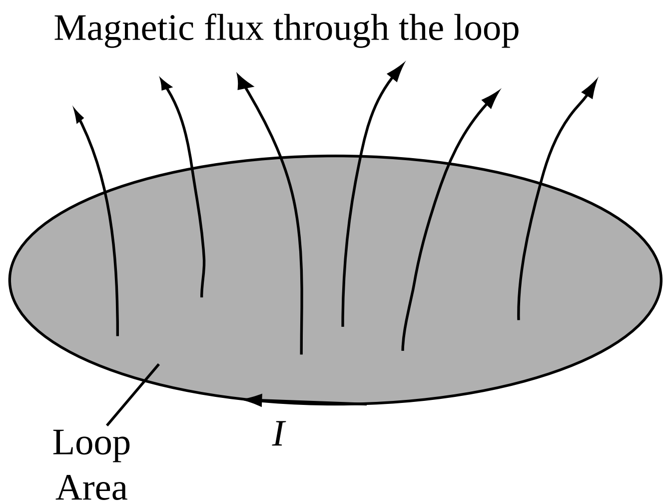

Figure 3: Time varying magnetic flux through a conducting loop which induces a current.

Mathematically speaking, Faraday's law can be expressed as

$$\varepsilon = -N\frac{d\phi}{dt} \tag{5}$$

ε is the induced EMF.
N is the number of conducting loops or coils.
ϕ is the magnetic flux through the loops.
t is the time.

Due to Lenz's law, a minus sign appears in the expression.

6 Consistency of motional EMF with Faraday's law

Tool: Consistency of motional EMF with Faraday's law
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Say we have a frictionless, conducting track upon which rests a long, thin conductor of length l. Supposing the conductor is pushed along the track with velocity v and through a magnetic field. Also, the direction of motion is perpendicular to the magnetic field's direction.

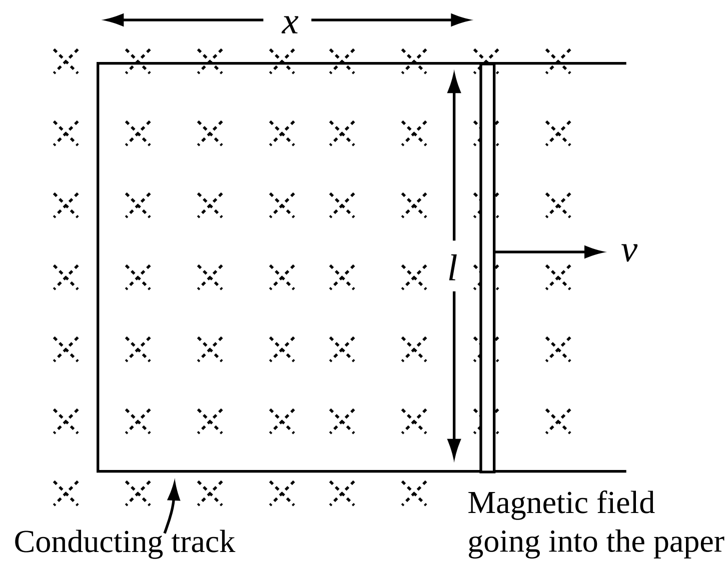

Figure 4: Conductor moving on a conducting track in a magnetic field.

To use Faraday's law, a loop has to be specified. In this case, it's obvious to the conducting track and the conductor form a viable loop. So by Faraday's law,

$$\begin{aligned} \varepsilon &= N\frac{d\phi}{dt} \\ &= N\frac{Blx}{dt} \\ &= Bl\frac{dx}{dt} \\ &= Blv \end{aligned} \tag{6}$$

ε is the induced EMF.
N is the number of loops or turns in the coil. $N = 1$ in this case.
ϕ is the magnetic flux through the loop.
B is the magnetic flux density.
l is the length of the conductor or the of the track.
x is the distance from the end of the track traveled by the conductor.
t is the time.
v is the velocity of the conductor.

However, it's already known that the motional EMF of a long conductor moving in a magnetic field is $V = Blv$. Therefore, motional EMF and Faraday's law are consistent.

7 Induced EMF for common devices

Tool: Induced EMF for common devices
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a. Disc spinning in a magnetic field – The induced EMF is generated between the center and the edge of the disc and the plane of the disc is perpendicular to the direction of the magnetic field.

$$\varepsilon = \frac{1}{2}Br^2\omega \tag{7}$$

ε is the induced EMF.
B is the magnetic flux density.
r is the radius of the disc.
ω is the angular velocity of the disc.

b. Rotating coil – The axis of rotation is perpendicular to the magnetic field.

$$\varepsilon = BAN\omega \sin \omega t \tag{8}$$

A is the area of the coil.
N is the number of turns in the coil.
t is the time.

8 Eddy currents

Tool: Eddy currents
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A 'proper' complete circuit is not necessary for currents caused by induced EMFs to flow. Microscopic currents can flow within conductors and these are known as *eddy currents*.

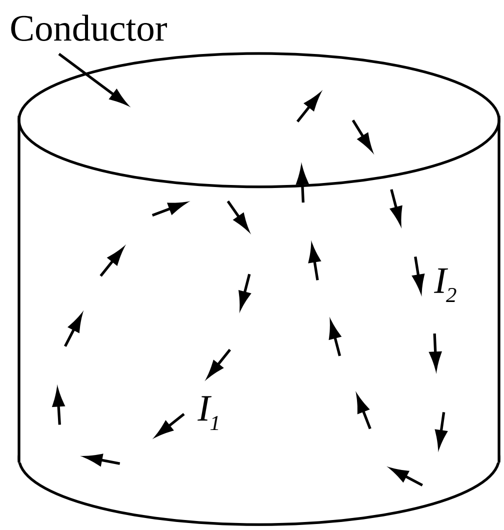

Figure 5: Eddy currents flowing in a conductor.

Such currents can cause magnetic and heating effects that, depending on the application, may or may not be useful. In things like transformers and dynamos, every precaution is taken to minimize eddy currents because their very presence means energy is being wasted. It is easier for eddy currents to flow in 'thick' conductors, therefore, one way to reduce them is to divide up conductors into *laminations* – thin sheets separated by insulating layers.

Some applications that make use of eddy currents are:

a. Electromagnetic damping – By Lenz's law, eddy currents flow in such a way as to oppose the motion that causes them, acting like a brake on a moving body.

b. Induction heating – Current flows cause heating effects and eddy currents are no different.

9 Electric motors and generators

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a. **Generators**

All generators are based on the premise of rotation a coil in a magnetic field (or rotating a magnet around a stationary coil). Naturally, this produces an EMF which is naturally alternating. Therefore, an AC generator is a relatively simple affair where the EMF is directly tapped from the rotating axle via 2 carbon brushes that press against 2 copper slip-rings.

In a DC generator, a split-ring commutator is used instead of slip rings so that the EMF is rectified (the polarity is always in the same direction).

b. **Motors**

Actually, a DC motor is just a DC generator run in reverse (supplied with electrical power). On the other hand, most AC motors are *induction* motors. In an induction motor, AC current is used to produce a changing or moving magnetic field which, in turn, causes nearby conductors to move.

10 Inductance or self-inductance

Tool: Inductance or self-inductance
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Current running in a coil creates a magnetic field. If the current changes overtime, the magnetic flux also changes, resulting in an EMF being induced in the coil. By Lenz's law, the induced EMF will oppose the change that caused it. This phenomenon is known as self-inductance or simply inductance and such a coil is known as an *inductor*.

For an air-core solenoid, the magnetic flux density through the center is

$$B = \frac{\mu_0 N I}{l} \tag{9}$$

B is the magnetic flux density.

μ_0 is the permeability.
N is the number of turns in the solenoid.
I is the current carried by the solenoid.
l is the length of the solenoid.

Applying Faraday's law to the solenoid, the induced EMF is

$$\begin{aligned} \varepsilon &= -N\frac{d\phi}{dt} \\ &= -N\frac{BA}{dt} \\ &= -N\frac{d}{dt}\left[\frac{\mu_0 NI}{l}A\right] \\ &= -\frac{\mu_0 N^2 A}{l}\frac{dI}{dt} \\ &= -L\frac{dI}{dt} \end{aligned} \tag{10}$$

ε is the induced EMF.
ϕ is the magnetic flux through the solenoid.
A is the cross-sectional area of the solenoid.
t is the time.
L is the inductance.

In this case, $L = (\mu_0 N^2 A)/l$. Of course, for other coil geometries and materials of construction, L is a different value.

11 Mutual inductance

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Mutual inductance is when the changing current in a coil, say coil **A**, induces an EMF in a nearby coil (coil **B**). Transformers make use of this effect to step-up and step-down the voltages of AC current. Coil **B**'s induced EMF is given by

$$\varepsilon_B = -M\frac{dI_A}{dt} \tag{11}$$

ε_B is the EMF induced in coil **B**.
M is the mutual inductance.
I_A is the current flowing in coil **A**.

12 Energy stored in an inductor

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Integrating the electrical power with respect to time gives the energy stored in an inductor.

$$\begin{aligned} U &= \int_0^I P dt \\ &= \int_0^I \varepsilon I dt \\ &= \int_0^I LI \frac{dI}{dt} dt \\ &= \int_0^I LI dI \\ &= \frac{1}{2} LI^2 \end{aligned} \tag{12}$$

U is the stored energy in the inductor.
P is the electrical power.
t is the time.
ε is the induced EMF. Since it's the energy *stored* we're after, the minus sign can be ignored.
I is the current flowing.
L is the inductance.

13 Transformers

Tool: Transformers
Category: Fields/Electromagnetic induction

A transformer changes the voltage of an *alternating* current. It uses 2 coils of wire, one called the *primary* and the other, the *secondary*, wrapped around a core of ferro-magnetic material (soft-iron). The primary coil is connected to the AC source, creating a time varying current which induces an EMF in the secondary coil via mutual inductance. If the transformer is ideal, then the voltage across the secondary coil is related to the source voltage by the following equation.

$$V_2 = \frac{N_2}{N_1} V_1 \tag{13}$$

V_2 is the voltage across the secondary coil.
N_2 is the number of turns in the secondary coil.
V_1 is the source voltage or voltage across the primary coil.
N_1 is the number of turns in the primary coil.

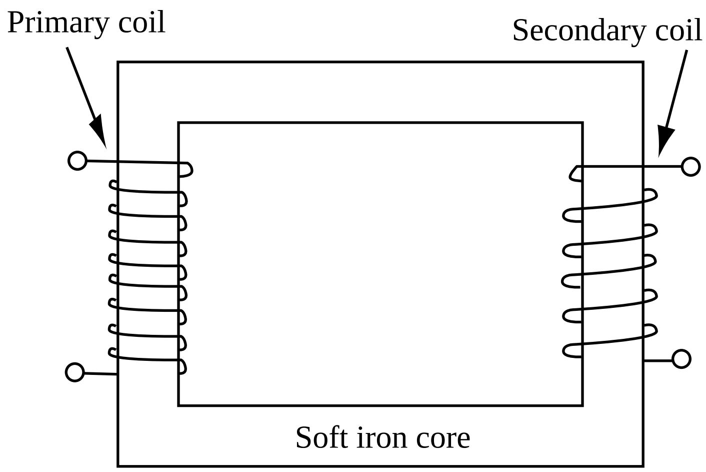

Figure 6: A typical transformer configuration.

An ideal transformer is conservative and in reality, transformers are highly efficient devices. For calculation purposes, transformers are taken to be ideal unless specified otherwise.