Model Equations

The model is written as a set of 21 ordinary differential equations. Model states are:

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Per1 - Per1 mRNA,
Per2 - Per2 mRNA,
Cry1 – Cry1 mRNA,
Cry2 - Cry2 mRNA,
Rev-erbα – Rev-erbα mRNA,
Clk - Clk mRNA,
Bmal1 - Bmal1 mRNA,
Rorc - Rorc mRNA,
PER1 – PER1 protein,
PER2 - PER2 protein,
CRY1 - CRY1 protein,
CRY2 - CRY2 protein,
REV-ERBα – REV-ERBα protein,
CLK - CLK protein,
BMAL1 - BMAL1 protein,
RORc - RORc protein,
PER1/CRY1 – Heterodimer of PER1 and CRY1 proteins,
PER1/CRY2 – Heterodimer of PER1 and CRY2 proteins,
PER2/CRY1 – Heterodimer of PER2 and CRY1 proteins,
PER2/CRY2 – Heterodimer of PER2 and CRY2 proteins,
CLK/BMAL1 – Heterodimer of CLK and BMAL1 proteins.
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All parameters are as specified in Supplementary Table I.

$$\frac{dPer1}{dt} = \left(V_{0,Per1} + V_{1,Per1} * \frac{CLK/BMAL1^{na_{1,Per1}}}{KA_{1,Per1}^{na_{1,Per1}} + CLK/BMAL1^{na_{1,Per1}}}\right) * \frac{KI_{1,Per1}^{ni_{1,Per1}}}{KI_{1,Per1}^{ni_{1,Per1}}} * \frac{KI_{2,Per1}^{ni_{2,Per1}}}{KI_{2,Per1}^{ni_{2,Per1}}} * \frac{KI_{2,Per1}^{ni_{2,Per1}}}{KI_{3,Per1}^{ni_{3,Per1}}} * \frac{KI_{4,Per1}^{ni_{4,Per1}}}{KI_{4,Per1}^{ni_{4,Per1}}} - K_{m,Per1}^{m,Per1} * \frac{KI_{4,Per1}^{ni_{4,Per1}}}{KI_{4,Per1}^{ni_{4,Per1}} + PER2/CRY2^{ni_{4,Per1}}} - K_{m,Per1}^{m,Per1} * \frac{Per1}{KI_{4,Per1}^{ni_{4,Per1}}}$$

$$\frac{dPer2}{dt} = (V_{0,Per2} + V_{1,Per2})^* \frac{\text{CLK/BMAL } 1^{ma_{1,Per2}}}{\text{KA}_{1,Per2}}^{ma_{1,Per2}} + \text{CLK/BMAL } 1^{ma_{1,Per2}})^*$$

$$\frac{\text{KI}_{1,Per2}^{ma_{1,Per2}}}{\text{KI}_{1,Per2}^{ma_{1,Per2}} + \text{PER1/CRY1}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{2,Per2}^{ma_{1,Per2}} + \text{PER1/CRY2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \text{PER2/CRY1}^{ma_{1,Per2}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}} + \text{PER2/CRY2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}}}^{ma_{1,Per2}} + \frac{\text{KI}_{2,Per2}^{ma_{1,Per2}}}{\text{KI}_{3,Per2}^{ma_{1,Per2}}}^{ma_{1,Per2}^{ma_{1,Per2}}^{ma_{1,Per2}^{ma_{1,Per2}}^{ma_{1,Per2}^{ma_{1,Per2}^{ma_{1,Per2}^{ma_{$$

$$\frac{dClk}{dt} = (v_{0,Clk} + v_{1,Clk} * \frac{RORc^{na_{1,Clk}}}{KA_{1,Clk}^{na_{1,Clk}} + RORc^{na_{1,Clk}}}) * \frac{KI_{1,Clk}^{ni_{1,Clk}}}{KI_{1,Clk}^{ni_{1,Clk}} + REV - ERB\alpha^{ni_{1,Clk}}} - k_{m,Clk} * Clk$$
 (6)

$$\frac{dBmal1}{dt} = \left(v_{0,Bmal1} + v_{1,Bmal1} * \frac{RORc^{na_{1,Bmal1}}}{KA_{1,Bmal1}} + RORc^{na_{1,Bmal1}}\right) * \frac{KI_{1,Bmal1}}{KI_{1,Bmal1}} + REV - ERB\alpha^{ni_{1,Bmal1}} - k_{m,Bmal1} * Bmal1$$
(7)

$$\frac{dRorc}{dt} = (v_{0,Rorc} + v_{1,Rorc} * \frac{CLK/BMAL1^{na_{1,Rorc}}}{KA_{1,Rorc}^{na_{1,Rorc}} + CLK/BMAL1^{na_{1,Rorc}}} +$$

$$\begin{split} & v_{2,Rorc} * \frac{RORc^{na_{2,Rorc}}}{KA_{2,Rorc}^{na_{2,Rorc}} + RORc^{na_{2,Rorc}}}) * \frac{KI_{1,Rorc}^{ni_{1,Rorc}}}{KI_{1,Rorc}^{ni_{1,Rorc}} + PER1/CRY1^{ni_{1,Rorc}}} * \\ & \frac{KI_{2,Rorc}^{ni_{2,Rorc}}}{KI_{2,Rorc}^{ni_{2,Rorc}} + PER1/CRY2^{ni_{2,Rorc}}} * \frac{KI_{3,Rorc}^{ni_{3,Rorc}}}{KI_{3,Rorc}^{ni_{3,Rorc}} + PER2/CRY1^{ni_{3,Rorc}}} * \\ & \frac{KI_{4,Rorc}^{ni_{4,Rorc}}}{KI_{4,Rorc}^{ni_{4,Rorc}}} * \frac{KI_{5,Rorc}^{ni_{5,Rorc}}}{KI_{5,Rorc}^{ni_{5,Rorc}} + REV - ERB\alpha^{ni_{5,Rorc}}} - k_{m,Rorc} * Rorc \end{split}$$

$$\frac{dPER1}{dt} = t_{Per1} * Per1 - a_{PER1,CRY1} * PER1 * CRY1 - a_{PER1,CRY2} * PER1 * CRY2 + d_{PER1/CRY1} * PER1/CRY1 + d_{PER1/CRY2} * PER1/CRY2 - k_{p.PER1} * PER1$$
(9)

$$\frac{dPER2}{dt} = t_{Per2} * Per2 - a_{PER2,CRY1} * PER2 * CRY1 - a_{PER2,CRY2} * PER2 * CRY2 + d_{PER2/CRY1} * PER2/CRY1 + d_{PER2/CRY2} * PER2/CRY2 - k_{p,PER2} * PER2$$
(10)

$$\frac{dCRY1}{dt} = t_{Cry1} * Cry1 - a_{PER1,CRY1} * PER1 * CRY1 - a_{PER2,CRY1} * PER2 * CRY1 + d_{PER2/CRY1} * PER1/CRY1 + d_{PER2/CRY1} * PER2/CRY1 - k_{p,CRY1} * CRY1$$
(11)

$$\frac{dCRY2}{dt} = t_{Cry2} * Cry2 - a_{PER1,CRY2} * PER1 * CRY2 - a_{PER2,CRY2} * PER2 * CRY2 + d_{PER2/CRY2} * PER1/CRY2 + d_{PER2/CRY2} * PER2/CRY2 - k_{p,CRY2} * CRY2$$
(12)

$$\frac{dREV - ERB\alpha}{dt} = t_{Rev-erb\alpha} * Rev - erb\alpha - k_{p,REV-ERB\alpha} * REV - ERB\alpha$$
 (13)

$$\frac{dCLK}{dt} = t_{C/k} * C/k - a_{CLK,BMAL1} * CLK * BMAL1 + d_{CLK/BMAL1} * CLK/BMAL1 - k_{p,CLK} * CLK$$
(14)

$$\frac{dBMAL1}{dt} = t_{Bmal1} * Bmal1 - a_{CLK,BMAL1} * CLK * BMAL1 + d_{CLK/BMAL1} * CLK/BMAL1 - k_{p,BMAL1} * BMAL1$$
(15)

$$\frac{dRORc}{dt} = t_{Rorc} * Rorc - k_{p,RORc} * RORc$$
 (16)

$$\frac{dPER1/CRY1}{dt} = a_{PER1,CRY1} * PER1 * CRY1 - d_{PER1/CRY1} * PER1/CRY1$$
 (17)

$$\frac{dPER2/CRY1}{dt} = a_{PER2,CRY1} * PER2 * CRY1 - d_{PER2/CRY1} * PER2/CRY1$$
 (18)

$$\frac{dPER1/CRY2}{dt} = a_{PER1,CRY2} * PER1* CRY2 - d_{PER1/CRY2} * PER1/CRY2$$
 (19)

$$\frac{dPER2/CRY2}{dt} = a_{PER2,CRY2} * PER2 * CRY2 - d_{PER2/CRY2} * PER2/CRY2$$
 (20)

$$\frac{dCLK/BMAL1}{dt} = a_{CLK,BMAL1} * CLK * BMAL1 - d_{CLK/BMAL1} * CLK/BMAL1$$
 (21)

Cost Function

The cost funtion for the iterative optimization is given as

$$f = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 + \gamma_7 + \gamma_8 + \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5 + \varepsilon_6 + \varepsilon_7 + \varepsilon_8 + \varepsilon_9 + \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} + \varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15} + \varepsilon_{16} + \varepsilon_{17} + \varepsilon_{18} + \varepsilon_{19} + \varepsilon_{20} + \varepsilon_{21} + \varepsilon_{22} + \varepsilon_{23}.$$

- γ₁ represents the cost associated with the phase difference between *Per1* mRNA and PER1 protein, minimized at 5/24 of a period.
 - γ_1 = 5 * abs(m₁/P-5/24), P = period of oscillation, m₁ = time at peak of PER1 protein minus time at peak of *Per1* mRNA.
- y_2 represents the cost associated with the phase difference between *Per2* mRNA and PER2 protein, minimized at 5/24 of a period.
 - γ_2 = 5 * abs(m₂/P-5/24), P = period of oscillation, m₂ = time at peak of PER2 protein minus time at peak of *Per2* mRNA.
- γ_3 represents the cost associated with the phase difference between *Cry1* mRNA and CRY1 protein, minimized at 5/24 of a period.
 - y_3 = 5 * abs(m₃/P-5/24), P = period of oscillation, m₃ = time at peak of CRY1 protein minus time at peak of *Cry1* mRNA.
- γ_4 represents the cost associated with the phase difference between *Cry2* mRNA and CRY2 protein, minimized at 5/24 of a period.
 - y_4 = 5 * abs(m₄/P-5/24), P = period of oscillation, m₄ = time at peak of CRY2 protein minus time at peak of *Cry2* mRNA.
- γ_5 represents the cost associated with the phase difference between *Rev-erba* mRNA and REV-ERBa protein, minimized at 1.5/24 of a period.
 - γ_5 = 5 * abs(m₅/P-1.5/24), P = period of oscillation, m₅ = time at peak of REV-ERB α protein minus time at peak of *Rev-erb* α mRNA.
- γ_6 represents the cost associated with the phase difference between *Clk* mRNA and CLK protein, minimized at 5/24 of a period.
 - γ_6 = 5 * abs(m₆/P-5/24), P = period of oscillation, m₆ = time at peak of CLK protein minus time at peak of *Clk* mRNA.
- γ₇ represents the cost associated with the phase difference between *Bmal1* mRNA and BMAL1 protein, minimized at 5/24 of a period.

- y_7 = 5 * abs(m₇/P-5/24), P = period of oscillation, m₇ = time at peak of BMAL1 protein minus time at peak of *Bmal1* mRNA.
- γ₈ represents the cost associated with the phase difference between *Rorc* mRNA and RORc protein, minimized at 1.5/24 of a period.
 - γ_8 = 5 * abs(m₈/P-1.5/24), P = period of oscillation, m₈ = time at peak of RORc protein minus time at peak of *Rorc* mRNA.
- δ_1 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Per1* mRNA, minimized at 1/6 of a period.
 - δ_1 = 10 * abs(n₁/P-1/6), P = period of oscillation, n₁ = time at peak of *Per1* mRNA minus time at peak of *Rev-erba* mRNA.
- δ_2 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Per2* mRNA, minimized at 1/6 of a period.
 - δ_2 = 10 * abs(n₂/P-1/6), P = period of oscillation, n₂ = time at peak of *Per2* mRNA minus time at peak of *Rev-erba* mRNA.
- δ_3 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Cry1* mRNA, minimized at 1/3 of a period.
 - δ_3 = 10 * abs(n₃/P-1/3), P = period of oscillation, n₃ = time at peak of *Cry1* mRNA minus time at peak of *Rev-erba* mRNA.
- δ_4 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Cry2* mRNA, minimized at 1/3 of a period.
 - δ_4 = 10 * abs(n₄/P-1/3), P = period of oscillation, n₄ = time at peak of *Cry2* mRNA minus time at peak of *Rev-erba* mRNA.
- δ_5 represents the cost associated with the phase difference between Rev-erb α mRNA and Rorc mRNA, minimized at 1/3 of a period.
 - δ_5 = 10 * abs(n₅/P-1/3), P = period of oscillation, n₅ = time at peak of *Rorc* mRNA minus time at peak of *Rev-erb* α mRNA.
- δ_6 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Clk* mRNA, minimized at 2/3 of a period.
 - δ_6 = 10 * abs(n₆/P-2/3), P = period of oscillation, n₆ = time at peak of *Clk* mRNA minus time at peak of *Rev-erba* mRNA.

- δ_7 represents the cost associated with the phase difference between *Rev-erba* mRNA and *Bmal1* mRNA, minimized at 2/3 of a period.
 - δ_7 = 10 * abs(n₇/P-2/3), P = period of oscillation, n₇ = time at peak of *Bmal1* mRNA minus time at peak of *Rev-erba* mRNA.
- ε₁ represents the cost associated with the ratio between the minimum *Per1* mRNA level and the maximum *Per1* mRNA level, minimized at 0.2.
 - $\varepsilon_1 = 0.5 * abs(min(Per1 mRNA)/max(Per1 mRNA)-0.2).$
- ε_2 represents the cost associated with the ratio between the minimum *Per2* mRNA level and the maximum *Per2* mRNA level, minimized at 0.2.
 - $\varepsilon_2 = 0.5 * abs(min(Per2 mRNA)/max(Per2 mRNA)-0.2).$
- ε_3 represents the cost associated with the ratio between the minimum *Cry1* mRNA level and the maximum *Cry1* mRNA level, minimized at 0.5.
 - ε_3 = 0.5 * abs(min(*Cry1* mRNA)/max(*Cry1* mRNA)-0.5).
- ε₄ represents the cost associated with the ratio between the minimum *Cry2* mRNA level and the maximum *Cry2* mRNA level, minimized at 0.5.
 - $\varepsilon_4 = 0.5 * abs(min(Cry2 mRNA)/max(Cry2 mRNA)-0.5).$
- ε_5 represents the cost associated with the ratio between the minimum *Bmal1* mRNA level, minimized at 0.1.
 - $\varepsilon_5 = 0.5 * abs(min(Bmal1 mRNA)/max(Bmal1 mRNA)-0.1).$
- ε_6 represents the cost associated with the ratio between the minimum *Clk* mRNA level and the maximum *Clk* mRNA level, minimized at 0.5.
 - $\varepsilon_6 = 0.5 * abs(min(Clk mRNA)/max(Clk mRNA)-0.5).$
- ε₇ represents the cost associated with the ratio between the minimum *Rev-erbα* mRNA level, minimized at 0.2.
 - $\varepsilon_7 = 0.5 * abs(min(Rev-erb\alpha mRNA)/max(Rev-erb\alpha mRNA)-0.2).$
- ε_8 represents the cost associated with the ratio between the minimum PER1 protein level and the maximum PER1 protein level, minimized at 0.1.
 - $\varepsilon_8 = 0.5 * abs(min(PER1 protein)/max(PER1 protein)-0.1).$

- ε_9 represents the cost associated with the ratio between the minimum PER2 protein level and the maximum PER2 protein level, minimized at 0.
 - $\varepsilon_9 = 0.5 * abs(min(PER2 protein)/max(PER2 protein)).$
- ε_{10} represents the cost associated with the ratio between the minimum CRY1 protein level and the maximum CRY1 protein level, minimized at 0.3.
 - $\varepsilon_{10} = 0.5 * abs(min(CRY1 protein)/max(CRY1 protein)-0.3).$
- ε_{11} represents the cost associated with the ratio between the minimum CRY2 protein level and the maximum CRY2 protein level, minimized at 0.5.
 - ε_{11} = 0.5 * abs(min(CRY2 protein)/max(CRY2 protein)-0.5).
- ε_{12} represents the cost associated with the ratio between the minimum BMAL1 protein level and the maximum BMAL1 protein level, minimized at 0.3.
 - $\varepsilon_{12} = 0.5 * abs(min(BMAL1 protein)/max(BMAL1 protein)-0.3).$
- ε_{13} represents the cost associated with the ratio between the minimum CLK protein level and the maximum CLK protein level, minimized at 0.3.
 - ε_{13} = 0.5 * abs(min(CLK protein)/max(CLK protein)-0.3).
- ε_{14} represents the cost associated with the ratio between the maximum PER1 protein level and the maximum CRY1 protein level, minimized at 0.05.
 - $\varepsilon_{14} = 0.5 * abs(max(PER1 protein)/max(CRY1 protein)-0.05).$
- ε_{15} represents the cost associated with the ratio between the minimum PER1 protein level and the maximum CRY1 protein level, minimized at 0.
 - ε_{15} = 0.5 * abs(min(PER1 protein)/max(CRY1 protein)).
- ε_{16} represents the cost associated with the ratio between the maximum PER2 protein level and the maximum CRY1 protein level, minimized at 0.15.
 - $\varepsilon_{16} = 0.5 * abs(max(PER2 protein)/max(CRY1 protein)-0.15).$
- ε_{17} represents the cost associated with the ratio between the minimum PER2 protein level and the maximum CRY1 protein level, minimized at 0.
 - ε_{17} = 0.5 * abs(min(PER2 protein)/max(CRY1 protein)).

- ε_{18} represents the cost associated with the ratio between the maximum CRY2 protein level and the maximum CRY1 protein level, minimized at 0.6.
 - $\varepsilon_{18} = 0.5 * abs(max(CRY2 protein)/max(CRY1 protein)-0.6).$
- ε_{19} represents the cost associated with the ratio between the minimum CRY2 protein level and the maximum CRY1 protein level, minimized at 0.3.
 - $\varepsilon_{19} = 0.5 * abs(min(CRY2 protein)/max(CRY1 protein)-0.3).$
- ε_{20} represents the cost associated with the ratio between the maximum CLK protein level and the maximum CRY1 protein level, minimized at 0.75.
 - $\varepsilon_{20} = 0.5 * abs(max(CLK protein)/max(CRY1 protein)-0.75).$
- ε_{21} represents the cost associated with the ratio between the minimum CLK protein level and the maximum CRY1 protein level, minimized at 0.3.
 - ε_{21} = 0.5 * abs(min(CLK protein)/max(CRY1 protein)-0.3).
- ε_{22} represents the cost associated with the ratio between the maximum BMAL1 protein level and the maximum CRY1 protein level, minimized at 0.15.
 - $\varepsilon_{22} = 0.5 * abs(max(BMAL1 protein)/max(CRY1 protein)-0.15).$
- ε_{23} represents the cost associated with the ratio between the minimum BMAL1 protein level and the maximum CRY1 protein level, minimized at 0.05.
 - $\varepsilon_{23} = 0.5 * abs(min(BMAL1 protein)/max(CRY1 protein)-0.05).$