Solution for Water Distribution Systems under Pressure-Deficient Conditions

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Abstract: In recent years, many researchers have tried to predict the behavior of the water distribution systems under pressure-deficient conditions. The root of the problem is that under these conditions the traditional demand-driven analysis will compute heads below the minimum required for outflow to occur physically at some or all of the nodes. The purpose of this paper is to present a novel algorithm for the solution of a water distribution network under pressure-deficient conditions, and is termed the pressure-deficient network algorithm (PDNA). The proposed model progressively introduces a set of artificial reservoirs into the network to initiate nodal flows, with the ultimate replacement of such reservoirs by full demand loads once it has become clear that the nodal flow can be satisfied. The foundation for the solution methodology is established using a series network for ease of discussion. The PDNA is presented in the form for coding into a computer program. For solving the flows in a looped network, the PDNA has to be used with a hydraulic network solver, as manual computation is too time consuming. Using the EPANET 2 hydraulic network solver, the PDNA is applied to both a single-source and a multiple-source network. The results show that the behavior of a water distribution system under pressure-deficient conditions is complex and nonintuitive.

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Introduction

The primary objective of a water distribution system is to provide water at a sufficient pressure and quantity to all its users. In traditional demand-driven analyses, the network solution is achieved by assigning the assumed demands for all nodes and computing the nodal pressure heads and link flows from the equations of mass balance and pipe friction headloss. For networks operating under normal/design conditions, the correct network solution for the specified demands is obtained, with the pressure at each demand node above the minimum required service level pressure. However, in the operational event of a (nonanticipated) pipe failure or fire-fighting flow requirement, a demand-driven analysis can yield nodal pressures that are lower than the required minimum or which even become negative. This condition of the network is termed pressure deficient. In the real network, the design demands would not be met. Such operational exigencies are the focus of this paper. Although this is a well-known problem that has been tackled by many researchers (e.g., Germanopoulos 1985; Jowitt and Xu 1993; and Gupta and Bhave 1996), it is still sometimes ignored.

Since the 1980s, researchers have proposed various methods to compute the actual flows of the network under such pressure-deficient conditions. Most of the proposed methods involve an assumption on the relationship between pressure and outflow at the demand nodes. These methods are generally termed head-driven analyses.

This paper presents a novel algorithm—termed the pressure-deficient network algorithm (PDNA)—to solve and provide additional insight into the behavior of water distribution networks operating under pressure-deficient conditions. The algorithm can be readily implemented into existing hydraulic network solvers. In this paper, the chosen hydraulic network solver is EPANET 2. Two network examples using the PDNA are presented. The first is a single-source network that is discussed at some length, in order to demonstrate the complex behavior of a water distribution network. The second example is a two-source network to show that the PDNA is equally applicable to multiple-source networks.

In the next section, the current methods for predicting the performance of a pressure-deficient network are briefly reviewed.

Review of Pressure-Deficient Solution Methods

The development of methods for computing the flows in a pressure-deficient network started in the early 1980s. Carey and Hendrickson (1984) assumed that the pipe capacities are limited by a maximum energy gradient, transforming the deficient-pressure network problem into a classical transhipment problem (Wagner et al. 1988a). Fujiwara and De Silva (1990) used this method to calculate the outflow Q_i at each demand node i and the results showed that the outflows tend to be overestimated.

Germanopoulos and Jowitt suggested the use of an empirical

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pressure-consumption relationship (Germanopoulos 1985) to predict the outflows at various nodal heads

$$Q_i = Q_i^{\mathrm{req}}(1 - a_i e^{-b_i H_i / H_i^*}) \tag{1}$$

where Q_i=actual outflow at demand node i; Q_i^{req}=desired flow at that node; H_i=available head; and H_i^*=nominal head required to satisfy demand Q_i^{req} at node i. The terms a_i and b_i are constants for node i. These constant terms could be calibrated for every demand node, or simply set semi-arbitrarily. The physical interpretation of the pressure-consumption relationship was not expressed in the paper. Essentially it was a pragmatic device to reflect the fact that local pressure deficiency in the network would result in localized failure to deliver the required demand. Jowitt and Xu (1993) used this relationship to predict the demand flows in a pressure-deficient network following network failures.

Reddy and Elango (1989) explored an alternative nodal demand-pressure relationship that was in essence like an emitter valve with $Q_i = Ke_i(H_i)^p$, where Ke_i is a constant and p is some exponent.

Chandapillai (1991) sought to relate the desirable head H_i^{des} node i and the minimum head H_i^{min} required to satisfy the demand through the expression

$$H_i^{\mathrm{des}} = H_i^{\mathrm{min}} + KC_i(Q_i^{\mathrm{req}})^n \tag{2}$$

where $KC_i(Q_i^{\mathrm{req}})^n$ head loss at consumer connection; KC_i=resistance coefficient appropriate to the consumer connection pipe; and n=exponent.

Wagner et al. (1988b) proposed the use of a parabolic curve to represent the pressure-outflow relationship at a demand node for the head between H_i^{min} and H_i^{des}. A very good review of this method and a summary of other pressure-deficient network predictors can be found in the paper by Gupta and Bhave (1996). This leads to an interesting discussion by Tanyimboh and Tabesh (1997) and a closure by Gupta and Bhave (1997). The resulting conclusion is that the behavior of a water distribution system under pressure-deficient conditions is complex and that further research was needed. The general form of the parabolic equation is given by

$$\text{If } H_i \leqslant H_i^{\mathrm{min}}, \quad Q_i = 0 \tag{3a}$$

$$\text{If } H_i^{\mathrm{min}} < H_i < H_i^{\mathrm{des}}, \quad Q_i = Q_i^{\mathrm{req}}\left(\frac{H_i - H_i^{\mathrm{min}}}{H_i^{\mathrm{des}} - H_i^{\mathrm{min}}}\right)^{1/n} \tag{3b}$$

$$\text{If } H_i \geqslant H_i^{\mathrm{des}}, \quad Q_i = Q_i^{\mathrm{req}} \tag{3c}$$

Gargano and Pianese (2000) and Ostfeld et al. (2002) used Eqs. (3a)–(3c) in their papers on reliability-based design of water distribution networks. With a traditional demand-driven solver, this requires some iteration, estimating network heads for the nominal demands, correcting the demands at these heads using Eqs. (3a)–(3c), and then re-estimating the heads and so on until sufficient convergence is obtained. This can lead to a high computational requirement and some researchers choose to use much simpler pressure outflow relationships. For example, Xu and Goulter (1999) and Khomsi et al. (1996) used a simple zero to one relationship for outflows in their computations of reliability, with demands satisfied when the nodal heads were greater than or equal to H_i^{des}, and otherwise zero.

Tanyimboh et al. (2001) used a modified version of the Eq. (3b) by relating the outflows to the head at the source, even though the actual relationship between the source head and outflow at any demand node is non-unique, and dependent on the topological layout of the network, the pipe parameters, and the demands at other nodes. This will be clearly illustrated in the next section, where the solution methodology to a pressure-deficient network is established.

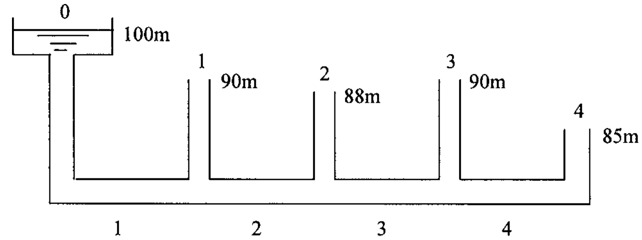

Fig. 1. Series network

The incorporation of pressure-dependent demands as suggested in the methods just described is essentially pragmatic, rather than fully calibrated. Although they produce a very useful element of pressure dependency, they do so in quite a crude way. What they don't do is model the fact that there is some pressure value at each node that might cause nodal outflows to cease altogether. Terms such as $Q_i = Q_i^{\mathrm{req}}(1 - a_i e^{-b_i H_i / H_i^*})$, or its emitter device equivalent, $Q_i = Ke_i(H_i)^p$, do not discriminate between pressure-sufficient and pressure-deficient nodes, but allow a smooth transition between the two conditions. This is probably not too serious on highly skeletized models, where "a demand node" might typically embody a number of properties, each with slightly different elevations at the consumer's tap. But it might be much more relevant at the finer scale in highlighting why some properties experience pressure difficulties while others in the same street might experience no such problems. In such cases, some unexpected behaviors might result, as will be shown subsequently.

Solution Methodology for Pressure-Deficient Networks

The series network (Fig. 1) from the paper by Gupta and Bhave (1996) is used to elucidate the solution methodology for pressure-deficient networks. The series network is supplied by a constant head source at Node 0. There are four demand Nodes 1, 2, 3, and 4 with demands of 2, 2, 3, and 1 m^3/min, respectively. The elevation at every node is shown in Fig. 1 and H_i^{min} for all nodes is the elevation itself. Effectively, this simulates the condition of a storage tank or simply an open standpipe at every demand node. Water will flow into the tanks or open pipes only when the head at the demand node is greater than the elevation of the node. A total of four distribution pipes 1, 2, 3, and 4 are in the series network, with diameters of 400, 350, 300, and 300 m, respectively. Each pipe has a length of 1000 m and a Hazen–Williams coefficient of 130. The corresponding resistance coefficients K_i for the four distribution pipes mains are 112.3 for K_1, 215.1 for K_2, and 455.3 for both K_3 and K_4. These data are as in Gupta and Bhave (1996). In addition, they assume a resistance coefficient in the consumer connection of KC_i=0.1, which is equivalent to a consumer connection pipe of diameter 400 mm, a length of just 0.89 m, and Hazen–William coefficient of 130. This represents a negligible head loss at the demand node.

When an additional fire fighting demand of 3 m^3/min occurs at Node 4, a demand-driven analysis indicates pressure-deficient conditions at Nodes 3 and 4. This is the condition modeled below.

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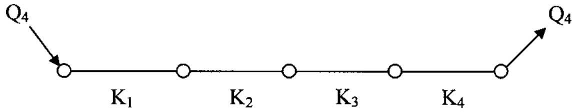
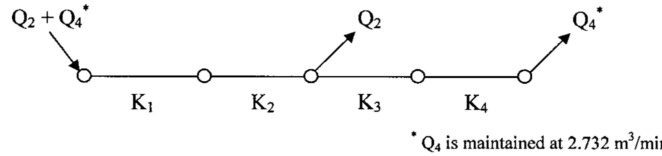


$$H_0 - H_4^{\min} = (K_1 + K_2 + K_3 + K_4) \times Q_4^{1.85} \quad (4)$$

Fig. 2. Outflow calculation for total source head of 85.00 m



$$H_0 - H_2^{\min} = (K_1 + K_2) \times (Q_2 + Q_4^{*})^{1.85} \quad (5)$$

Fig. 3. Outflow calculation for total source head of 89.08 m

In order to understand the foundation of the proposed PDNA solution, the behaviors of this simple series network was studied for a range of source heads, increased in small increments (e.g., 0.01 m) from 85.00 m. The Hazen–Williams equation is used to determine the head losses in the distribution mains. The head losses in the consumer pipes are effectively zero. As noted earlier, in Gupta and Bhave's analysis the resistance coefficient in the "consumer connections" is set at $KC_i=0.1$, but in the PDNA formulation no such assumption is required.

Two example calculations for the outflows at total source heads of 85.00 and 89.08 m are given in Figs. 2 and 3, respectively. A complete set of outflows and heads at every demand node for a range of total source heads (from 85.00 to 109.86) at Node 0 are shown in Table 1. Obviously, water will only start flowing through the system when the total head at the source is greater than 85.00 m (i.e., the elevation of the outlet at Node 4). At this head, there will be no flow at the other three nodes. Once the source head reaches 89.08, outflow is initiated additionally at Node 2, and in so doing, limits any further outflow at Node 4.

The most interesting behavior of the series network with the fire-fighting flow at demand Node 4 occurs when the total source head changes from 96.82 to 98.78 m. At a source head of 96.82 m, the demands at Nodes 1, 2, and 4 are fully met, but the head at Node 3 is only 88.04 m which is almost 2 m short of the required 90.00 m. In order for water to start flowing in Node 3, the total head has to be increased by a further 1.96 m. In the solution by Gupta and Bhave (1996), which uses the parabolic function as described by Eqs. (3*a*)–(3*c*), this behavior of the series network is not reflected. A plot of the head at source node H versus total outflow in the network Q is shown in Fig. 4, where the behavior is clearly marked on the curve with $\partial Q/\partial H=0$ between $H=96.82$ and 98.78 m before the total demand is met. An important point to note is that Eq. (3*c*) is assumed to hold, such that once the demand at a node is satisfied, any further increase in nodal pressure head does not result in an increase of outflow at that node. In practice, an increase in pressure head might result in a corresponding increase in outflow, unless the consumers throttle their taps to adjust to the assigned demands.

The most significant point to note at this stage is that the behavior of even this simple water distribution network is complex and non-intuitive. The assumption and use of a simplistic relationship between source head and outflow at a demand node does not anticipate this complex behavior, and the assumed rela-

Table 1. Outflows and Nodal Heads for Different Heads at Node 0

Head at Node 0 (m)	Outflow at node (m^3/min) / Head at node (m): 1	2	3	4	Total supply (m^3/min)
85.00	0.000	0.000	0.000	0.000[a]	0.000
	85.00	85.00	85.00	85.00	
89.08	0.000	0.000[a]	0.000	2.732	2.732
	88.71	88.00	86.50	85.00	
90.98	0.000	2.000[b]	0.000	2.732	4.732
	89.96	88.00	86.50	85.00	
91.03	0.000[a]	2.000	0.000	2.747	4.747
	90.00	88.03	86.52	85.00	
91.97	2.000[b]	2.000	0.000	2.747	6.747
	90.00	88.03	86.52	85.00	
96.82	2.000	2.000	0.000	4.000[b]	8.000
	94.11	91.08	88.04	85.00	
98.78	2.000	2.000	0.000[a]	4.000	8.000
	96.08	93.04	90.00	86.96	
100.00	2.000	2.000	0.3988	4.000	8.399
	97.04	93.62	90.00	86.96	
109.86	2.000	2.000	3.000[b]	4.000	11.000
	104.99	98.55	90.00	86.96	

水头没有升高
为什么供水量
变化了？

[a]Flow starts at node.

[b]Demand is met at node.

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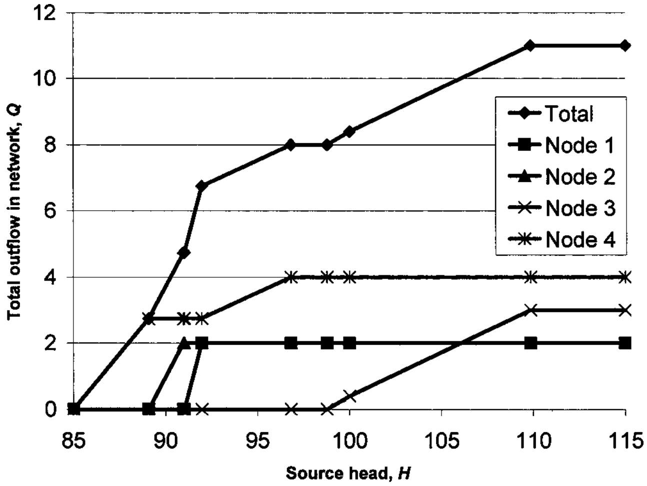

Fig. 4. Plot of head at source node versus total outflow in network

tionship is actually unnecessary. By carrying out the network computations using the PDNA approach outlined above, the behavior of the network becomes clear, and the actual relationship between source head and outflows at each demand node is generated as a byproduct of the analysis, rather than as an assumed input. The only requirement for the PDNA approach is knowledge of the minimum required delivery head. There are no other parameter assumptions.

From this simple example, it can be seen that the conceptual form of the PDNA solution methodology provides the outflows to a pressure-deficient network. The solution satisfies the basic continuity and energy equations. The next step is to outline the algorithm for the more general case.

Pressure-Deficient Network Algorithm

The PDNA now described is able to compute the outflows for both single-source and multiple-source water distribution systems. Compared to the other methods of head-driven analysis, which use parabolic or similar relationships between nodal/source heads and outflows, there is no requirement to assume initial heads at the demand nodes. For multiple-source and looped water distribution systems, the computation of outflows has to be dependent on hydraulic analysis programs, as hand calculation is too time consuming. The PDNA is presented in a form for coding into a computer program as follows (see also the flow diagram shown in Fig. 5). The essence of the algorithm is to progressively introduce a set of artificial reservoirs into the network to initiate nodal flows, with the ultimate replacement of such reservoirs with full demands once it has become clear that the nodal flow can be satisfied. The steps are as follows:

1. Perform the hydraulic analysis of the network with all demands set to zero (i.e., calculate the static heads in the network for zero demands).
2. Add artificial reservoirs with the same elevation as the demand node i, for all nodes that have $H_i - H_i^{\min} > 0$, together with a link joining each artificial reservoir to its demand node and an arbitrarily small resistance coefficient K_i.
3. Run the hydraulic analysis for the updated network and remove any artificial reservoirs that are supplying water to the water distribution network.
4. Repeat steps 2 to 3 until no demand node has $H_i - H_i^{\min} > 0$.

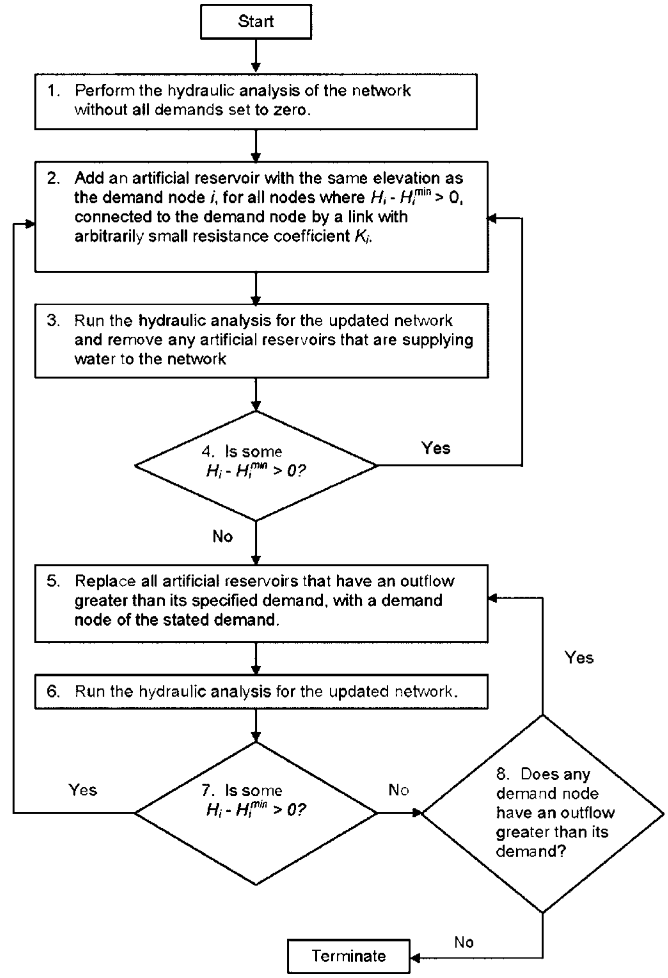

Fig. 5. Flow diagram for pressure-deficient network algorithm

5. Replace all artificial reservoirs that have an inflow greater than their specified demands, with a demand node of the stated demand.
6. Run the hydraulic analysis for the updated network.
7. Check the total head at every demand node. If there is any demand node with $H_i - H_i^{\min} > 0$ return to step 2, or else if there is any demand node with an outflow greater than its demand return to step 5, or else terminate the PDNA as the solution will now have been obtained.

The PDNA algorithm has been incorporated and tested using EPANET 2. On the range of examples tested, the PDNA converges smoothly and rapidly to a stable solution, although there might be conditions where this is not the case. It is also appropriate to note at this point that the hydraulic network solver EPANET 2 is able to model pressure dependent demands using emitter devices, which are essentially equivalent to the use of the empirical methods described earlier [e.g., the parabolic curves of Eqs. (3*a*)–(3*c*)], and which therefore require some assumption with respect of the empirical emitter parameters. The PDNA involves no such assumptions, and the key contribution of the proposed methodology is that it provides real insight into some of the complex behaviors that result from modeling pressure dependent nodes more realistically.

A further comment is appropriate with regard to network topography. Most network solvers (including such as EPANET 2) implicitly assume that the gradient of any pipe between any two adjacent nodes is uniform. If this is not the case, and the pipe

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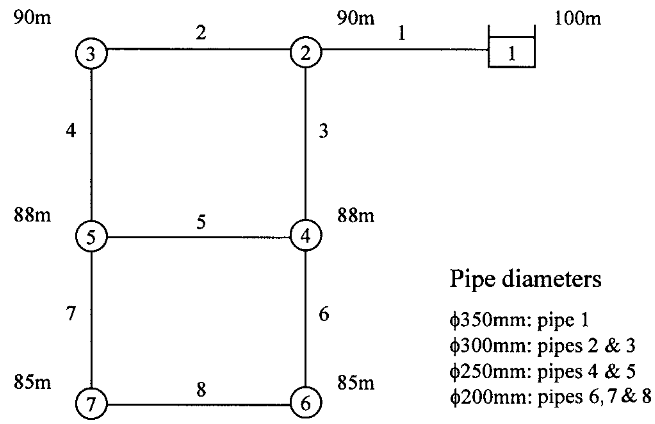

Fig. 6. Single-source network

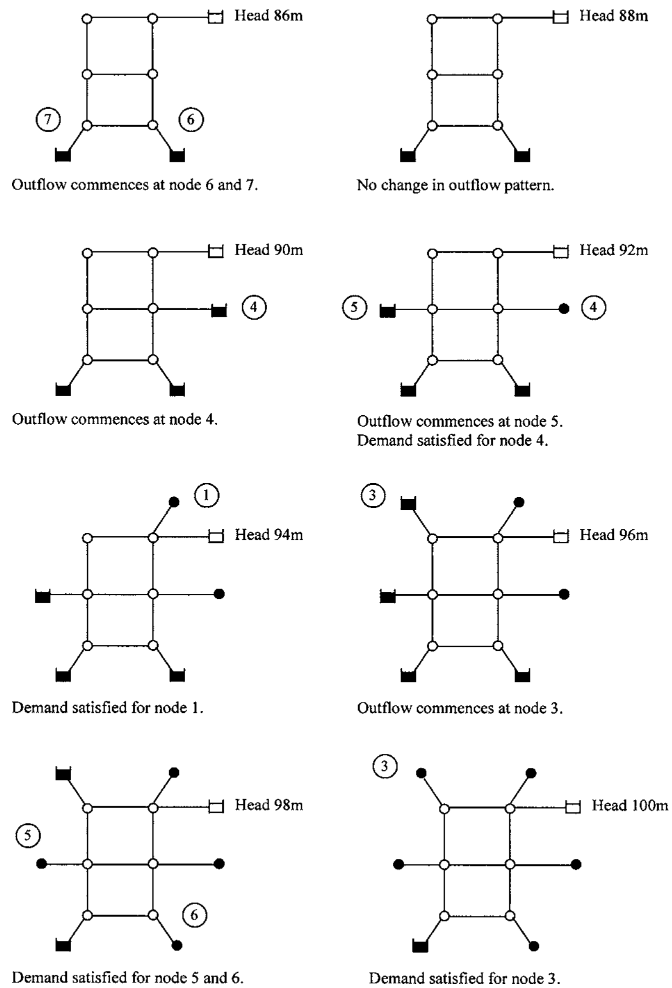

Fig. 7. Simplified diagrammatic illustration of data in Table 2

elevation somewhere along its length exceeds that of one or the other end nodes, then an appropriate dummy node should be inserted. The PDNA algorithm is no different in this regard.

Single-Source Networks

Normal Demands

The PDNA is applied to a single-source network with six demand nodes and eight pipes (Fig. 6). The nodal elevations and pipe diameters are shown in Fig. 6. The demand for all nodes under normal operating conditions is 25 L/s. All pipes are 1000 m long with a Hazen–Williams coefficient of 130. The head loss at the consumer connections (i.e., the customer connection pipes at the demand nodes) is assumed to be negligible and the minimum required head to initiate outflow at every demand node is equal to its elevation. A demand-driven analysis for the network showed that the nodal heads available are greater than their respective minimum required head under this normal operating condition.

Fire-Fighting Demand at Node 7

For an additional fire-fighting flow of 50 L/s at Node 7, a demand-driven analysis yields negative pressure at all demand nodes, i.e., $H_i - H_i^{\min} < 0$.

The PDNA was used to solve for the flows in this pressure-deficient condition by varying the total source head from 86.00 to 100.00 m, and inserting artificial reservoirs as required. For computational convenience, the pipe connections between these reservoirs and their corresponding nodes are assigned a diameter of 350 mm, length of 0.1 m, and Hazen–Williams coefficient of 130, which effectively constitute zero headloss. The results given by the PDNA are presented in Table 2.

A simplified diagrammatic illustration of this data is shown in Fig. 7, with the artificial reservoirs (■⎯) signifying those nodes where outflow has been initiated, and nodes and dummy links (—●) denoting nodes where demands are fully met for successive values of total head at the source.

Table 2. Outflows for Single-Source Network under Fire Fighting Flow at Node 7

Head at source (m)	Outflow at node (L/s)						Total supply (L/s)
	2	3	4	5	6	7	
86.00	0.00	0.00	0.00	0.00	10.29	10.15	20.45
88.00	0.00	0.00	0.00	0.00	18.62	18.38	37.00
90.00	0.00	0.00	9.87	0.00	22.77	22.72	55.36
92.00	0.00	0.00	25.00	9.04	22.98	22.86	79.88
94.00	25.00	0.00	25.00	14.61	23.27	22.91	110.79
96.00	25.00	7.04	25.00	24.15	24.36	22.91	128.46
98.00	25.00	20.99	25.00	25.00	25.00	24.38	145.38
100.00	25.00	25.00	25.00	25.00	25.00	29.93	154.93
117.56	25.00	25.00	25.00	25.00	25.00	75.00	200.00

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Table 3. Outflows for Single Source Network under Failure of Pipe Member 4

Head at source (m)	Outflow at node (L/s)						Total supply (L/s)
	2	3	4	5	6	7	
86.00	0.00	0.00	0.00	0.00	9.63	8.27	17.89
88.00	0.00	0.00	0.00	0.00	17.42	14.97	32.38
90.00	0.00	0.00	0.90	0.00	22.77	19.55	43.21
92.00	25.00	1.52	10.97	0.00	22.77	19.55	79.81
94.00	25.00	24.12	17.25	0.00	22.72	19.59	108.68
96.00	25.00	25.00	25.00	0.00	24.29	20.94	120.23
98.00	25.00	25.00	25.00	2.56	25.00	25.00	127.56
100.00	25.00	25.00	25.00	10.38	25.00	25.00	135.38
104.27	25.00	25.00	25.00	25.00	25.00	25.00	150.00

From Table 2, it can be seen that except for Nodes 6 and 7, the outflows from the remaining nodes start at different total source heads. Prior to the solution by the PDNA, it is very difficult to predict the order of outflow commencement for the nodes. For example, the outflow at demand Node 3 is the last to initiate, which is quite surprising even when the difference in elevations is accounted for. The fire-fighting flow at Node 7 is satisfied only when the total head at source is 117.56 m.

Pipe Breakage at Pipe 4

The next pressure-deficient condition to be considered is a pipe breakage at Pipe Member 4. The network is analyzed after assuming Pipe 4 has been isolated. Demand-driven analysis shows that Nodes 4, 5, 6, and 7 have negative pressures. Once again, the PDNA is used and the outflows for each demand node are shown in Table 3. The simplified diagrammatic illustration of the data in Table 3 is shown in Fig. 8.

Inspection of Table 3 and Fig. 8 shows that the order and commencement of nodal outflows are again not particularly intuitive. The total source head versus outflows at demand Node 4 is plotted for various pressure-deficient conditions in the looped network (Fig. 9). For the case of pipe failure in Pipe Member 3, the outflow at Node 4 only begins at a total source head of 108.96 m, which is significantly higher than the other pressure-deficient conditions. Referring back to Fig. 6, it can be seen that Pipe Member 3 is a crucial pipe linking demand Node 4 to the source, which explains the required higher total source head.

Another interesting observation is when Pipe Member 7 fails, which reveals that the outflow at Node 4 starts at a lower total source head compared to the case of no pipe failure. This can be explained by the fact that both the outflows to Nodes 6 and 7 have to pass through Node 4, and the higher friction head loss involved will cause a faster rise in head at demand Node 4. From the variation of outflows at demand Node 4 in Fig. 9, it is clearly shown that the nodal outflows are not only dependent on the total source head, but also on the network layout and the prevailing network conditions. Hence the approach suggested by Tanyimboh et al. (2001), which related the head at the source to the outflow at a node, would require calibration for every failure condition. Clearly, this is computationally expensive and there is also a question as to its accuracy.

An interesting observation from Tables 2 and 3 is that for a small increase in total source head, the outflow in demand Node 2 changes from 0 to 25 L/s. This is due to the fact that for one additional unit of water to flow from the source node to Node 2, the head loss is the lowest among all nodes. Once the outflow commences in Node 2, most of the additional units of water will flow out of Node 2, hence fulfilling its demand rapidly. This has the effect of creating a sharp increase in the total source head versus total supply curve. Thus, the relationship between total source head and total supply to the network is rather complex. The actual curve of this relationship would depend on the network layout, the nodal demands, and the prevailing network condition, as in whether there is any pipe breakage or fire-fighting demand.

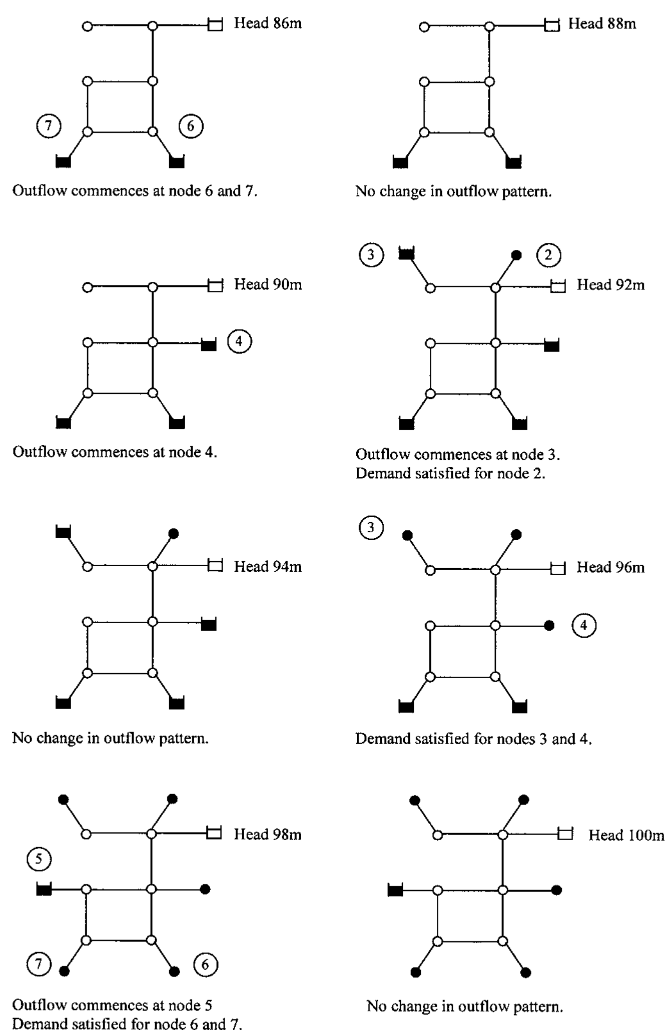

Fig. 8. Simplified diagrammatic illustration of data in Table 3

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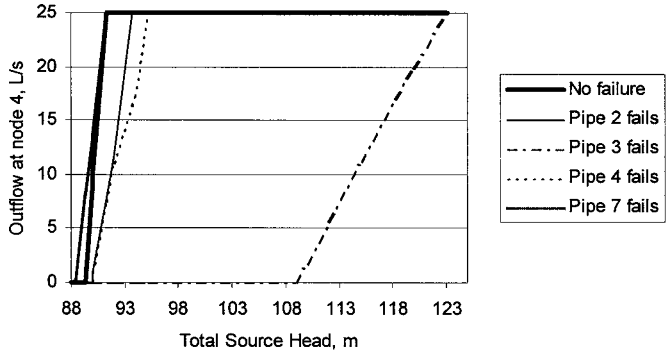

Fig. 9. Relationship of outflow at demand Node 4 and total source head for various network conditions

The total source head versus total supply to network is plotted for various pressure-deficient conditions in the looped network (Fig. 10). The curves in Fig. 10 reinforce the fact that the relationship between total source head and total supply is dependent on the prevailing network condition. Once again, the inadequacy of the parabolic curves that are used to describe the relationship between total source head and total network supply is exposed. The parabolic curves need to be calibrated for every pressure-deficient condition to maintain their accuracy.

Under the failure condition of Pipe Member 3, the performance of the network is most severely affected, as shown in Fig. 10. At the total source head of 100.00 m, the total network supply is only 102.75 L/s, which is about two thirds of the total demand of 150 L/s. The second most severe loss of supply to the network is the failure of Pipe Member 2. Furthermore, none of the curves for the cases of pipe failure gives a higher total network supply for the same total source head as compared to the curve of normal operating condition, which is the rational outcome.

Multiple-Source Networks

Normal Demands

The PDNA algorithm is equally applicable to multiple-source networks. The chosen multiple-source network has a total of two source nodes, nine demand nodes, and 14 pipes (Fig. 11). Nodal elevations and pipe diameters are shown in Fig. 11. The demand for all nodes under normal operating conditions is 25 L/s. All pipes are 1,000 m long, with a Hazen–Williams coefficient of 130. The head loss at the consumer connections is assumed to

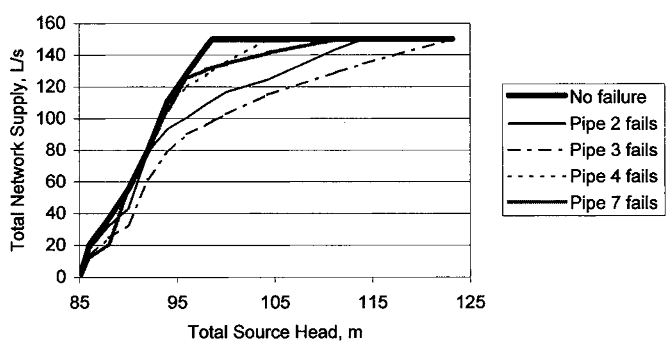

Fig. 10. Relationship of total network supply and total source head under various network conditions

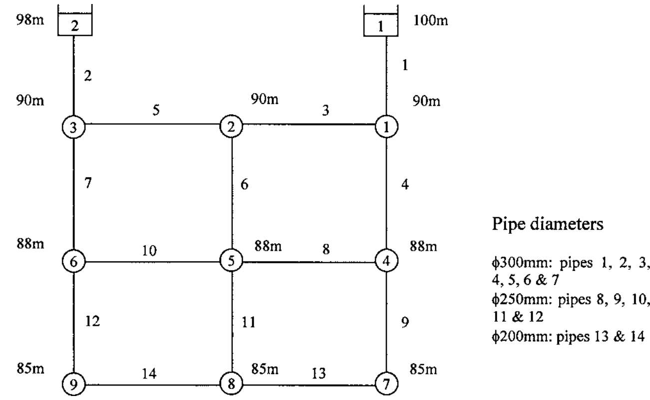

Fig. 11. Multiple-source network

be negligible and the minimum required head for outflow to commence at every demand node is equal to its elevation. Demand-driven analysis for the network showed that the nodal heads available are greater than their respective minimum required head under this normal operating condition.

Fire-Fighting Demand at Node 9

For an additional fire-fighting flow of 50 L/s at Node 9, the demand-driven analysis yields negative pressures at all demand nodes. The PDNA is used to solve for the flows in the pressure-deficient network for the total source heads ranging from 86.00 to 100.00 m in the same way as described previously. A simplified diagrammatic illustration of the flow pattern similar to Figs. 7 and 8, is shown in Fig. 12.

At a head of 100 m for source Node 1 and 98 m for source Node 2, there are three nodes where demands are not satisfied (Fig. 12). The outflows at Nodes 2, 6, and 9 are 4.63, 21.16, and 54.26 L/s, respectively. Clearly, Node 2 is the most severely affected by the fire-fighting flow occurring at Node 9 and is the last to commence. These results on the outflow at Node 2 are not readily anticipated by a mere inspection of the network data and its layout. From Fig. 12, it is again evident that the behavior of a water distribution network under pressure-deficient conditions is highly complex and interesting.

Conclusion

The purpose of this paper has been to introduce a novel solution methodology for water distribution networks under pressure-deficient conditions and show that this algorithm reveals some interesting and unanticipated network behaviors. The solution methodology is illustrated using a series network already described in the literature by Gupta and Bhave (1996).

The solution methodology, formalized algorithmically as the PDNA, does not involve any new equations and uses only the known hydraulic equations for pipe flows. No assumptions are required regarding the form and parameters of such as the parabolic pressure-dependent demand terms or emitter valves proposed elsewhere in the literature. The PDNA is presented in a form for coding into a computer program. It has been incorporated into the hydraulic network solver EPANET 2 and successfully applied to a single-source and multiple-source networks.

From the results obtained, the behavior of a water distribution network under pressure-deficient conditions is shown to be

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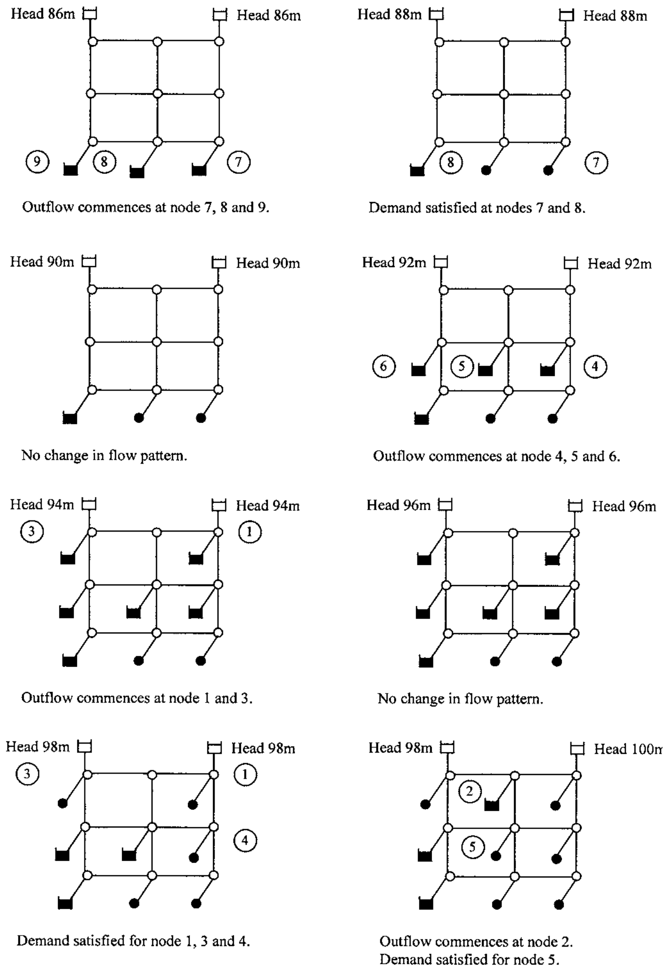

Fig. 12. Flow pattern for multiple-source network under fire-fighting flow at demand Node 9

complex and sometimes unexpected which accords with the views of other researchers (Tanyimboh and Tabesh 1997). Although the PDNA algorithm may not be as computationally efficient as solutions based on the use of such emitter valves, its advantage lies in providing this additional insight into network behavior under such conditions.

Further studies into the behavior of the water distribution network under pressure-deficient conditions are recommended, including networks which include pumps, in order to verify the statement "that deficiencies in pressure in water distribution systems are usually of limited areal extent" (Tanyimboh et al. 2001).

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Notation

The following symbols are used in this paper:

a_i = constant term at node i;
b_i = constant term at node i;
H_i = head at node i;
H_i^{des} = head at node i at which normal demand is satisfied;
$H_i^{\min}$ = head at node i at which outflow is zero;
K = constant term for informational entropy, which is set to unity in this paper;
K_i = resistance coefficient of pipe;
n = head-flow exponent;
Q_i = outflow at node; and
Q_i^{des} = demand at node i.

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