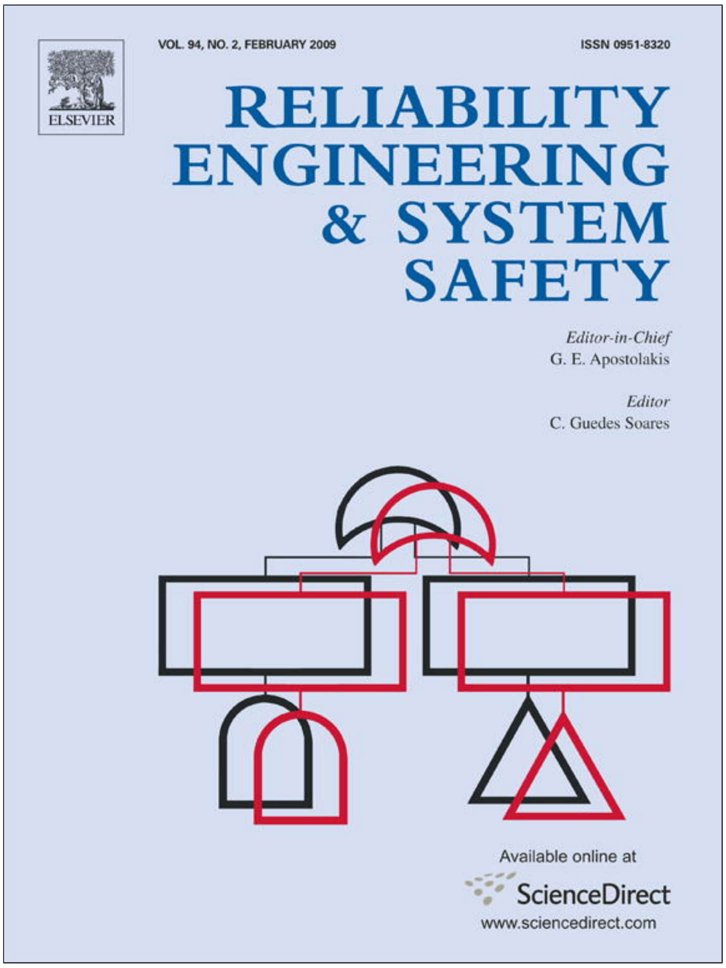
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Spatial distribution of water supply reliability and critical links of water supply to crucial water consumers under an earthquake

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A B S T R A C T

This paper describes a process to characterize spatial distribution of water supply reliability among various consumers in a water system and proposes methods to identify critical links of water supply to crucial water consumers under an earthquake. Probabilistic performance of water supply is reflected by the probability of satisfying consumers' water demand, Damage Consequence Index (DCI) and Upgrade Benefit Index (UBI). The process is illustrated using a hypothetical water supply system, where direct Monte Carlo simulation is used for estimating the performance indices. The reliability of water supply to consumers varies spatially, depending on their respective locations in the system and system configuration. The UBI is adopted as a primary index in the identification of critical links for crucial water consumers. A pipe with a relatively large damage probability is likely to have a relatively large UBI, and hence, to be a critical link. The concept of efficient frontier is employed to identify critical links of water supply to crucial water consumers. It is found that a group of links that have the largest UBI individually do not necessarily have the largest group UBI, or be the group of critical links.

1. Introduction

The basic function of a water supply system is to deliver water from sources to customers. Water flows from sources to customers through a network of pipelines, pumps, valves, and other appurtenances, and it is also stored in tanks and reservoirs to accommodate fluctuations in demand due to varying rates of usage or fire protection. Pipelines, pumps, valves, storages, and the supporting infrastructures together comprise a water supply system. Observations from past earthquakes, such as the 1906 San Francisco earthquake [1,2], the 1994 Northridge earthquake [3], and the 1995 Hyogoken-Nanbu (Kobe) earthquake [4], have clearly demonstrated that, in the event of an earthquake, water supply systems may sustain various kinds of damage, such as pipeline ruptures or leakages, and result in reduction of water delivery capability. Compromising a water supply system after an earthquake not only impairs fire-fighting capacity, but also disrupts residential, commercial, and industrial activities, and threatens the economic well-being, security, and social fabric of the communities they serve.

Seismic risk of water supply systems has been investigated extensively [5–13], and modeling procedures have been developed to evaluate the seismic system serviceability and reliability of several water supply systems, such as the San Francisco auxiliary water supply system [2,8,14], the Memphis, Tennessee water supply system [10,13,15], and the Los Angeles water supply system [11,12]. However, research is rare on spatial distribution of water supply reliability among various customers within a system.

In an event of a major earthquake, some water consumers, such as acute care facilities and/or hospitals, play vital roles in emergency response and must remain operational to rein in the losses. These water consumers are referred herein as crucial water consumers. A survey of the residents in high seismic risk communities [16] showed that water pipeline systems and major hospitals are two of the most important infrastructure elements that must remain operational in the event of a major earthquake. Those surveyed also indicated that they are more willing to invest in the seismic mitigation of these infrastructure elements. A key step in the seismic mitigation of water supply to these crucial water consumers is to identify critical links that significantly affect their water supply under an earthquake.

This paper develops procedures to characterize the spatial distribution of seismic water supply reliability among various customers within a system and proposes methods to identify critical links of water supply to crucial water consumers under an earthquake. The paper starts with definitions of several

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probabilistic measures, followed by descriptions of a hypothetical water supply system and simulation procedure used in this work. With the aid of Monte Carlo simulations, the spatial distribution of seismic water supply reliability is characterized. The concept of efficient frontier is then employed to identify critical links of water supply to crucial consumers. As an illustration, critical links of a water consumer are identified using the proposed method.

2. Reliability measurement and conditional probability quantities

A consumer of a water supply system is represented as a Node i with water demand Q_i in a hydraulic network model. The water supply reliability for the consumer i can be measured by the probability, $P(Q_i)$, of Q_i satisfied. $P(Q_i)$ can be estimated for each consumer of a water supply system, and its spatial distribution reflects the spatial variation of water supply reliability among various consumers within the system.

Two probabilistic measures associated with the water demand at Node i, Damage Consequence Index (DCI_{ij}) and Upgrade Benefit Index (UBI_{ij}), are defined to measure the impact of a Pipe j on the reliability of water supply to a consumer i and to identify critical links that significantly affect the $P(Q_i)$. The DCI_{ij} for Pipe j is defined to reflect the consequence from damaging the pipe. It is expressed as

$$\mathrm{DCI}_{ij} = \frac{P(Q_i) - P(Q_i|L_j)}{1 - P(Q_i)} \tag{1}$$

in which $P(Q_i|L_j)$ is the conditional probability of Q_i satisfied, given that Pipe j is damaged. DCI_{ij} is the percent reduction of $P(Q_i)$, given that Pipe j is damaged, and its relative value is a measure of Pipe j's impact on the reliability of water supply to consumer i.

The UBI_{ij} for Pipe j is expressed as

$$\mathrm{UBI}_{ij} = \frac{P_{\mathrm{upgrade}_j}(Q_i) - P(Q_i)}{1 - P(Q_i)} \tag{2}$$

in which $P_{\mathrm{upgrade}_j}(Q_i)$ is the conditional probability of Q_i satisfied in a system where Pipe j is "upgraded". The term "upgrade" herein means that the probability of pipe damage, given an earthquake occurs, is negligible compared to its value before upgrade. In Monte Carlo simulations (see later), this is enforced by setting the damage occurrence probability to be practically zero. UBI_{ij} is the percent increase of $P(Q_i)$, given that Pipe j is upgraded, and it ranges from 0 to 1. The relative value of UBI_{ij} is a measure of the Pipe j's impact on the reliability of water supply to consumer i. The extreme value of $\mathrm{UBI}_{ij} = 0$ indicates that the upgrade of Pipe j has no impact on the reliability of water supply to consumer i, as opposed to $\mathrm{UBI}_{ij} = 1$, which implies that upgrade of the Pipe j guarantees the water supply to consumer i. Because UBI_{ij} describes the benefits to consumer i that result from the upgrade of Pipe j, it corresponds directly to the context of seismic mitigation. UBI is used as the primary index in this work, and critical links in seismic mitigation are those with relatively large UBI values. The meaning of DCI, on the other hand, is complementary to UBI but less direct since it is related to the consequence of damage. Nevertheless, it is shown later that in the context of the problem herein the two indices are related.

Spatial distribution of water supply reliability and identification of critical links using DCI and UBI are illustrated in this work using a hypothetical water supply system, which is described in the next section.

3. Hypothetical water supply system

Fig. 1 shows a hydraulic network model of the hypothetical water supply system, which is the same example system used in Users Manual of the hydraulic analysis software EPANET [17]. The system contains 92 junctions and 117 pipes representing about 66 km of pipelines in a service area of about 10 km^2, 59 consumer demand nodes with a total demand of 40,634 l/min (10,736 gallon/min), three storage tanks, two reservoirs representing a river and a lake, and two pumps that extract water from the river and lake. Water flow starts from the river in the north of the system or the lake in the north-west of the system, and it follows a general trend from north to south and west to east. The main water source of the system is the river, which provides about 80% of the water.

Table 1 summarizes the demand node information, including node ID, elevation, and water demand. The largest consumer demand is 17,076 l/min at Node 203. The 117 pipes in the system have diameters varying from 200 mm (8 in.) to 760 mm (30 in.). As shown in Fig. 1, most of the pipes with diameters of 610 mm (24 in.) or above are used to deliver water from the river, the main water source, to Node 203, the largest demand node in the system. In this study, the pipes with diameters of 610 mm (24 in.) or above are assumed to be steel pipes, as opposed to cast iron pipes that are assumed for the remaining pipes with relatively small diameters.

4. Monte Carlo simulations with GIRAFFE

Seismic performance of the hypothetical water supply system is evaluated using Monte Carlo simulations in conjunction with a special hydraulic analysis computer program, Graphical Iterative Response Analysis of Flow Following Earthquakes, or GIRAFFE [11,12]. GIRAFFE incorporates a special algorithm for the treatment of negative node pressures in a heavily damaged water supply system to provide a more realistic assessment of the seismic performance [8]. The algorithm has been used to simulate the flow and pressure distributions of the San Francisco Auxiliary Water Supply System after the 1989 Loma Prieta earthquake [8] and those of the Los Angeles Water Supply System after the 1994 Northridge earthquake [11,12]. Similar algorithm was also described by Tanaka [9] and Hwang et al. [10], and was applied to the Memphis water supply system [10].

To facilitate Monte Carlo simulations, a MATLAB [18] code was developed to repeatedly execute GIRAFFE to perform hydraulic analysis in a deterministic manner. The MATLAB code has four key functions: first, it simulates the occurrence of pipe damage and generates the pipe damage file for the deterministic hydraulic simulation in GIRAFFE; second, it collects GIRAFFE input files, such as hydraulic system definition file and pipe damage file; third, it performs Monte Carlo simulations by repeatedly carrying out GIRAFFE deterministic runs with different pipe damage files of random damage configurations; fourth, it collects the Monte Carlo simulation results and calculates the probability quantities of interests.

The occurrence of damage in each pipe was simulated using a Poisson process with a damage rate, λ. The value of λ for steel pipes (with diameters 610 mm or above) and cast iron pipes (with diameters smaller than 610 mm) are assumed to be 0.0178 and 0.1254 damage per kilometer of pipe length, respectively. These values correspond to a peak ground velocity (PGV) of 50 cm/s in accordance with the λ versus PGV regressions developed from observations of the 1994 Northridge earthquake [19,20]. It is assumed that the seismic risk is uniform over the approximately 10 km^2 area represented by the hypothetical water

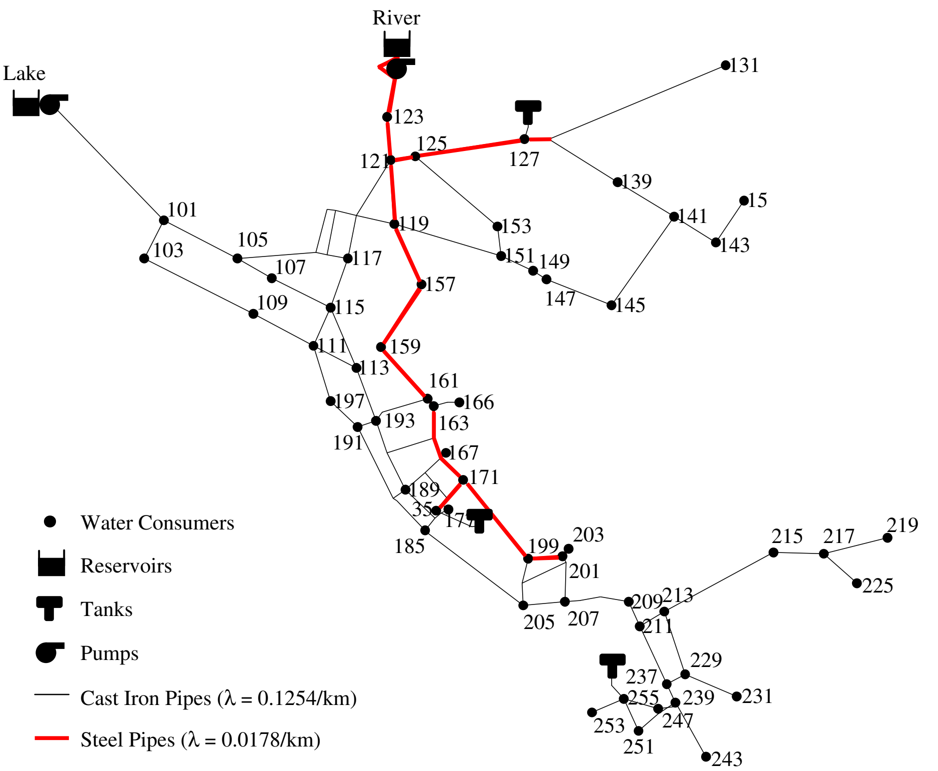

Fig. 1. Schematic view of the hypothetical water supply system.

Table 1
Summary of demand node information and $P(Q_i)$

Node ID	Elevation (m)	Demand (l/min)	$P(Q_i)$	Node ID	Elevation (m)	Demand (l/min)	$P(Q_i)$
15	9.75	1000	0.616	167	−1.52	55	0.994
35	3.81	6503	0.934	171	−1.22	149	0.990
101	12.80	719	0.775	177	2.44	220	0.996
103	13.11	504	0.682	185	4.88	97	0.942
105	8.69	512	0.806	189	1.22	409	0.994
107	6.71	207	0.869	191	7.62	310	0.799
109	6.19	876	0.841	193	5.49	270	0.916
111	3.05	537	0.950	197	7.01	65	0.831
113	0.61	76	0.955	199	−0.61	452	0.971
115	4.27	197	0.942	201	0.03	169	0.964
117	4.15	446	0.938	203	0.61	17,076	0.934
119	0.61	667	0.981	205	6.40	247	0.794
121	−0.61	158	0.992	207	2.74	263	0.928
123	3.35	4519	0.979	209	−0.61	3	0.932
125	3.35	173	0.992	211	2.13	33	0.913
127	17.07	67	0.988	213	2.13	53	0.938
131	1.83	162	0.771	215	2.13	349	0.772
139	9.45	22	0.941	217	1.83	92	0.729
141	1.22	37	0.917	219	1.22	156	0.677
143	−1.37	24	0.871	225	2.44	86	0.644
145	0.30	105	0.879	229	3.20	243	0.928
147	5.64	32	0.836	231	1.52	62	0.871
149	4.88	103	0.890	237	4.27	59	0.941
151	10.21	547	0.780	239	3.96	169	0.957
153	20.18	167	0.577	243	4.27	16	0.879
157	3.99	196	0.956	247	5.49	266	0.950
159	1.83	156	0.993	249	5.49	91	0.844
161	1.22	60	0.994	251	9.14	206	0.928
163	1.52	36	0.988	253	10.97	153	0.972
166	−0.61	10	0.960				

supply network, and shaking attenuation or locate site effects (if any) are not included in the analysis. Under the Poisson model, the distance from the pipe origin to the first pipe damage or the distance between two consecutive pipe damages can be simulated by an exponential random variable, D_i, expressed as

$$D_i = -\frac{1}{\lambda}\ln(U) \tag{3}$$

where U is a random variable uniformly distributed on [0,1]. For a pipe with length D and damage rate λ, a series of exponential random numbers, $D_1, D_2, \ldots$, are generated using Eq. (3) until their sum exceeds D. The number of damage locations on the pipe, n, is the largest integer such that $\sum_{i=1}^{n} D_i \leqslant D$. The kth ($k = 1, 2, \ldots, n$) damage in the pipe occurs at a distance of $\sum_{i=1}^{k} D_i$ from the pipe origin.

In hydraulic analysis of the hypothetical water supply system, it is assumed that pipe damage can be modeled by disconnecting the pipe at the location of damage and allowing water to flow out from the system (i.e., pipe break) [11]. Although this simplification is sufficient for illustrating the methodology proposed in this work, it should be recognized that the pipe damage in real water supply systems includes both pipe break (complete disconnection) and pipe leak (additional outflow from the damaged pipe). This simplification should be properly accounted for in the simulations when applying the proposed methods to real water supply systems. This work focuses on seismic response of the pipeline system, and the three storage tanks, two reservoirs, and two pumps in the hypothetical system are assumed intact in the simulations.

5. Spatial distribution of water supply reliability

Fig. 2 shows spatial distribution of $P(Q_i)$ estimated using 10,000 Monte Carlo samples. As summarized in Table 1, the $P(Q_i)$ varies from 0.577 to 0.996, and the $P(Q_i)$ for the largest water consumer, Node 203, is 0.934. The 59 water consumers in Fig. 2 are color-coded according to their respective $P(Q_i)$ values. The water consumers with $P(Q_i)<0.75$ are shown in red, and those with $0.75 \leqslant P(Q_i) < 0.95$ and $P(Q_i) \geqslant 0.95$ are illustrated in yellow and green, respectively. In Fig. 2, six out of 59 water consumers are shown in red, and most red nodes (i.e., Nodes 15, 217, 219, and 225) are located in the far end of the system. In addition, 21 out of 59 water consumers are illustrated in green, and most green nodes are located immediately downstream of water storage tanks or along the large-diameter trunk lines (i.e., steel pipes in Fig. 2).

The reliability of water supply to consumers varies spatially, depending on their respective locations in the system and system configuration. When water consumers are located in the far end of the system and away from water sources, water has to travel a relatively long distance. Therefore, the chance of disruption is relatively high when the water is conveyed from the sources to the consumers. This leads to relatively low $P(Q_i)$ at Nodes 15, 217, 219, and 225. In contrast, when the water consumers are located immediately downstream of the water storage tanks or along the large-diameter trunk lines, the water consumers are close to the water sources. Therefore, the chance of disruption is small when the water is conveyed from the sources to the water consumers, resulting in relatively high $P(Q_i)$ at Nodes 119, 121, 123, 125, 127, 157, 159, 161, 163, 166, 167, 171, 177, 189, 199, 201, 203, 239, 247, and 255.

Fig. 2 also shows that the $P(Q_i)$ is relatively low for those water consumers that have much higher elevation when compared with their neighboring water consumers. Consider, for example, Node 153 which has the lowest $P(Q_i)$ of 0.577 in the system. Although Node 153 is located close to water sources or storage tanks (refer to Fig. 2), it has an elevation of 20.18 m (refer to Table 1). This elevation is significantly larger than those of its neighboring nodes, such as elevation of 3.35 and 10.21 m at Nodes 125 and 151, respectively. The equivalent pressures of the elevation differences between Node 153 and Nodes 125 and 151 are about 100 and 170 kPa, respectively. This implies that water supply to Node 153 requires an additional water pressure of 100 or 170 kPa when compared with Nodes 125 or 151. When the water supply system is disrupted by the earthquake-induced damage, the water pressure in the system will decrease. Therefore, the chance that the pressure is not sufficient to deliver water to high elevation

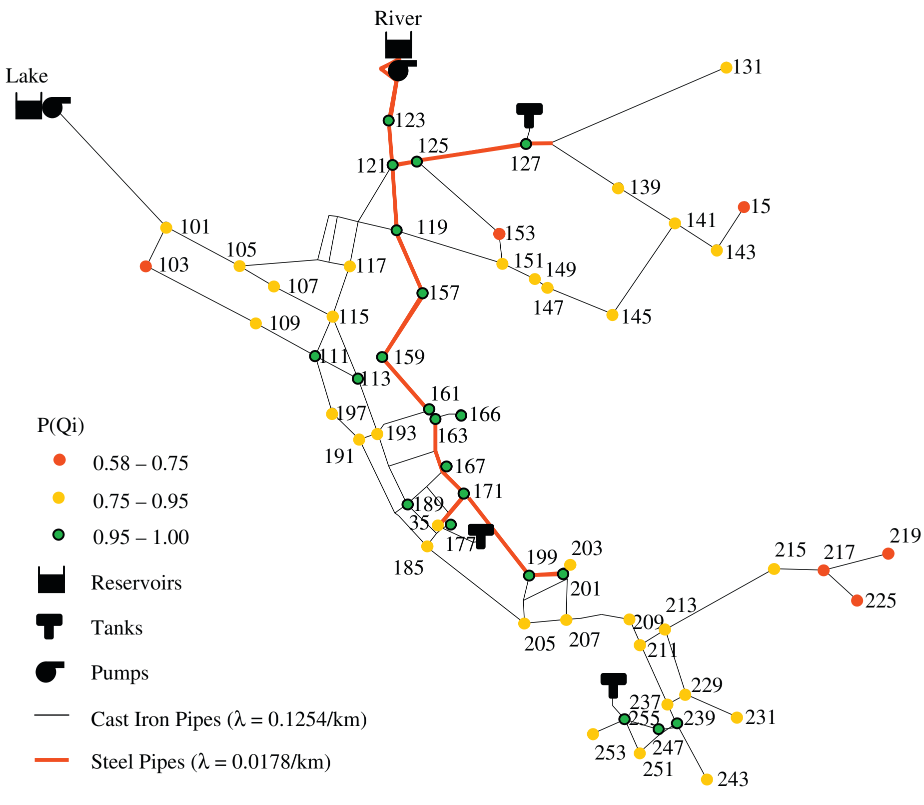

Fig. 2. Spatial distribution of water supply reliability.

area (e.g., Node 153) is higher than that for low elevation area (Nodes 125 and 151).

6. Critical links of water supply to crucial water consumers

A key step in the seismic mitigation of water supply to crucial water consumers is to identify critical links that significantly affect their water supply under an earthquake. A Pipe j is considered as a critical link of water supply to Node i, when the upgrade of Pipe j has relatively significant impact to the reliability of water supply to Node i (i.e., UBI_{ij} is relatively large). This section proposes a method to identify critical links with the aid of efficient frontier. As an illustration, critical links of water supply to Node 203 are identified using the proposed method.

6.1. Upgrade analysis using conditional samples

For a given water consumer i (e.g., Node 203), a Pipe j has a UBI_{ij} that can be estimated from a separate set of Monte Carlo samples in which the damage occurrence probability of the upgraded Pipe j is set to 0, i.e., $P_{\mathrm{upgrade}}(L_j) = 0$. As a complicated water supply system might contain a large number of pipes, identification of critical links in such system involves a large number of separate Monte Carlo runs for estimating UBI of each pipe. Consider, for example, the hypothetical water supply system used in this work, which contains 117 pipes. Then, totally 117 separate Monte Carlo runs are needed to estimate 117 UBI for the respective 117 pipes. As critical links are those with relatively large UBI, they are identified by comparing these 117 UBI.

In the context of the problem herein, repeated Monte Carlo runs for different upgrade configurations can be avoided by noting that Monte Carlo samples under the upgraded scenarios can be obtained from the conditional samples of a single Monte Carlo run for the nominal scenario, i.e., without upgrade provision. In the run for assessment of spatial variation of water supply reliability, a subset of the samples in which the given Pipe j is observed intact can be treated as an equivalent set of Monte Carlo samples that the given Pipe j is upgraded. Consider, for example, the 10,000 Monte Carlo samples described in Section 5. Pipe 321 is found intact in 9933 out of the 10,000 samples. These 9933 samples can be treated as an equivalent set of Monte Carlo samples in which Pipe 321 is upgraded such that no damage occurs in it. UBI for Pipe 321 can then be estimated from these 9933 samples using the following equation:

$$\mathrm{UBI}_{ij} = \frac{n_2/m_2 - n_1/m_1}{1 - n_1/m_1} \tag{4}$$

in which m_1 is the number of all Monte Carlo samples under the nominal scenario, n_1 is the number of Monte Carlo samples in m_1 that Q_i is satisfied, m_2 is the number of Monte Carlo samples that no damage occurs in Pipe j, and n_2 is the number of Monte Carlo samples in m_2 that Q_i is satisfied.

Table 2 summarizes the ten largest UBIs for Node 203 obtained from both a single run of nominal scenario (using conditional samples) and multiple runs of the upgraded scenarios. Generally, they compare well to each other. Theoretically, the third column in Table 2 should be identical to the second column. The apparent difference is due to statistical error in the estimates. The consistency shows that a single Monte Carlo run of nominal scenario for assessment of spatial variation of water supply reliability can be used to estimate the UBI for pipes in the system, resulting in identification of critical links of water supply to crucial consumers.

Table 2 also indicates that, when the damage probability after upgrade is 10% of the nominal damage probability, i.e., $P_{\mathrm{upgrade}}(L_j)/P_{\mathrm{nominal}}(L_j) = 0.1$, the UBIs from a single run and multiple runs are also comparable. This is a close approximation to the second column in Table 2, because the probability of damage after upgrade is subject to 10% of its nominal value (instead of a clear zero). For pipeline upgrade that can significantly reduce the damage probability, saying less than 10% of its nominal probability, this simulation technique is applicable that uses a single simulation run to replace a large number of repeated runs for different upgrade scenarios.

Table 2
Comparison of 10 largest UBIs from single and multiple Monte Carlo runs for Node 203

Pipe ID	UBI from single run (using conditional samples)	UBI from multiple runs	
		$P_{\mathrm{upgrade}}(L_j) = 0$	$P_{\mathrm{upgrade}}(L_j)/P_{\mathrm{nomial}}(L_j) = 0.1$
229	0.28	0.28	0.26
177	0.16	0.16	0.14
321	0.09	0.09	0.09
187	0.09	0.07	0.07
175	0.06	0.08	0.07
183	0.05	0.07	0.06
231	0.05	0.05	0.05
329	0.04	0.05	0.02
201	0.04	0.05	0.06
179	0.03	0.02	0.02

For a given water consumer (e.g., Node 203), each individual pipe has a UBI, and so does a group of pipes. The technique of using conditional samples is readily extended to estimate UBI for a group of pipes and to construct efficient frontier, which is described in the next section. For the pipe groups along the efficient frontier illustrated in the next section, the ratio of pipe group UBI from a single run to that from multiple runs falls within a narrow range between 0.96 and 1.04 and has a mean of 0.99.

6.2. Efficient frontier

Upgrade decisions often need to address cost–benefit concerns. Naturally, the larger the amount of resources allocated for upgrade, the larger the increase in system reliability and UBI. A diagram of benefit versus cost provides a convenient means for making upgrade decisions under limited resources. A plot of UBI versus the number of upgraded links in the system is developed to identify the critical links. Note that for a given number of links to upgrade, the resulting benefit, in term of UBI, depends strongly on which link is to be upgraded. Therefore, the plot of UBI versus the number of upgraded links contains families of discrete points that correspond to different upgrade configurations at a given number of links to upgrade. For a given number of links to upgrade, the upgrade configuration (characterized by a group of links to be upgraded) that results in the highest UBI and locates in the upper bound of the plot corresponds to the set of critical links. Hence, the set of critical links can be identified from the upper bound of the plot of UBI versus the number of upgraded links, and they are the most efficient choice to upgrade. This leads to the concept of "efficient frontier".

Fig. 3 shows the UBI efficient frontier for Node 203 with the number of upgraded links increasing from one to ten. The UBI efficient frontier increases nonlinearly from 0.28 with upgrade of one link to 0.93 with upgrade of ten links. Fig. 4 shows spatial distribution of the ten critical links (i.e., Pipes 229, 177, 321, 187, 175, 329, 231, 183, 179, and 191) that are selected from the UBI efficient frontier. The ten critical links are located along the trunk lines that deliver water from the main water source (i.e., the river) to the Node 203.

The group of critical links is shown to occur in a recursive fashion. Consider, for example, the group of ten critical links determined from group UBI efficient frontier includes the group of nine critical links from the group UBI efficient frontier and one additional link. The order of adding critical links is shown in Figs. 3 and 4 (i.e., Pipes 229, 177, 321, 187, 175, 329, 231, 183, 179, and 191), and it is found that the order does not necessarily follow the decreasing order of the UBI for each individual pipe (i.e., Pipes 229, 177, 321, 187, 175, 183, 231, 329, 201, and 179). For example, the sixth largest UBI occurs in Pipe 183. However, the efficient frontier in Fig. 3 shows that the sixth pipe added to the critical link group is Pipe 329. The UBI values for pipe groups that contain pipes with the largest individual UBI value are shown in Fig. 3, and

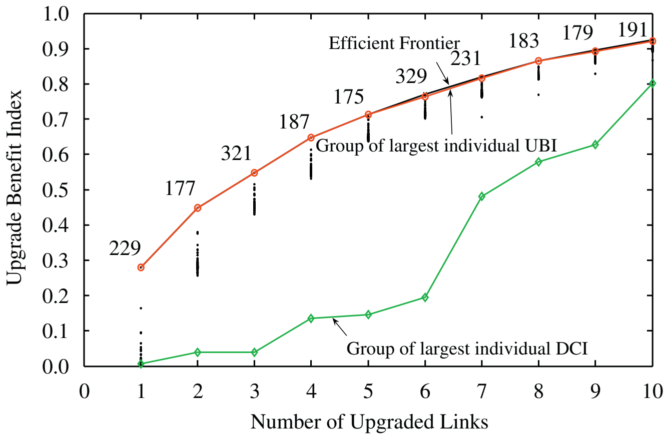

Fig. 3. Efficient frontier for Node 203.

they are slightly less than the UBI efficient frontier, particularly when the number of pipes is six, seven, nine, or ten. A group of links that have the largest UBI individually do not necessarily have the largest group UBI, or form part of the efficient frontier.

7. DCI and UBI relationship

As mentioned before, DCI and UBI are based on two different considerations, i.e., reduction of $P(Q_i)$ due to pipe damage and increase of $P(Q_i)$ as a result of pipe upgrade, respectively. Nevertheless, they are related through an identity that can be derived from the total probability theorem:

$$P(Q_i) = P(Q_i|L_j)P(L_j) + P(Q_i|\overline{L_j})P(\overline{L_j}) \tag{5}$$

in which $P(L_j)$ and $P(\overline{L_j})$ are probability of Pipe j damaged and intact, respectively. Dividing both sides by $P(Q_i)$:

$$1 = \frac{P(Q_i|L_j)}{P(Q_i)}P(L_j) + \frac{P(Q_i|\overline{L_j})}{P(Q_i)}P(\overline{L_j}) \tag{6}$$

Replacing the unity on the left-hand side by $P(L_j) + P(\overline{L_j})$ and rearranging gives:

$$\frac{P(Q_i|\overline{L_j}) - P(Q_i)}{1 - P(Q_i)} = \frac{P(Q_i) - P(Q_i|L_j)}{1 - P(Q_i)}\frac{P(L_j)}{P(\overline{L_j})} \tag{7}$$

That is

$$\mathrm{UBI}_{ij} = \mathrm{DCI}_{ij}\frac{P(L_j)}{P(\overline{L_j})} \tag{8}$$

Eq. (8) shows that UBI_{ij} is a product of DCI_{ij} and the odds of Pipe j damage, i.e., $P(L_j)/P(\overline{L_j})$. For a given Node i, the DCI_{ij} reflects the effect of system characteristics, such as system connectivity, redundancy, and damage probabilities of pipes other than the given Pipe j, while the odds signifies the influence of the damage

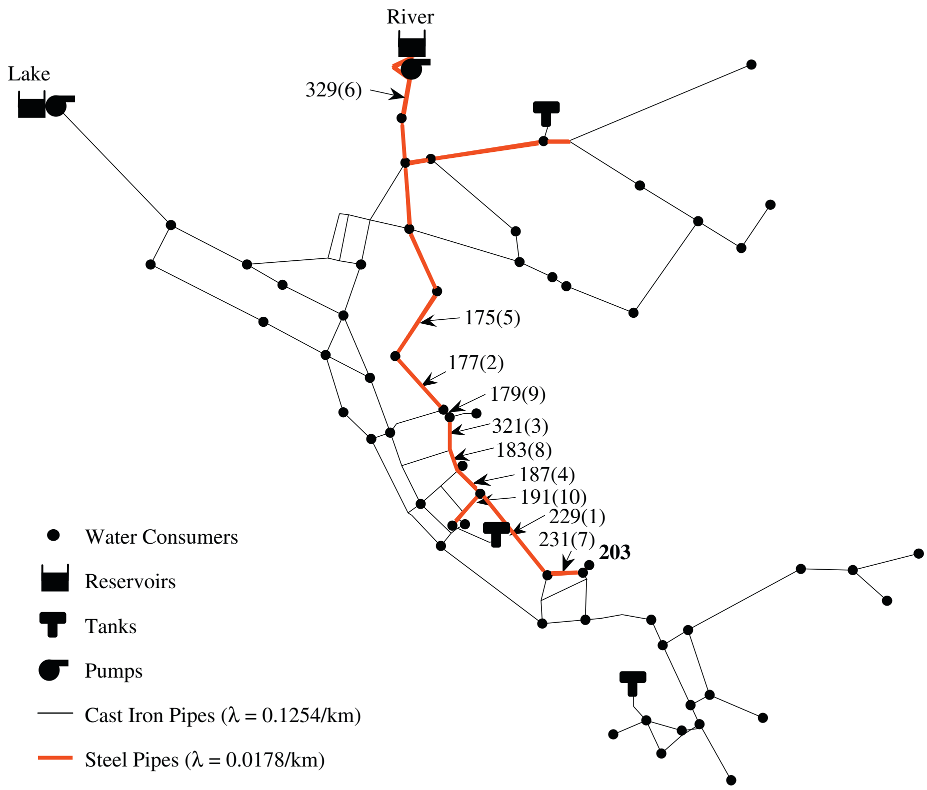

Fig. 4. Spatial distribution of critical links from UBI efficient frontier for Node 203 (number in the parenthesis after the pipe ID indicates the order of adding the critical links to the critical link group).

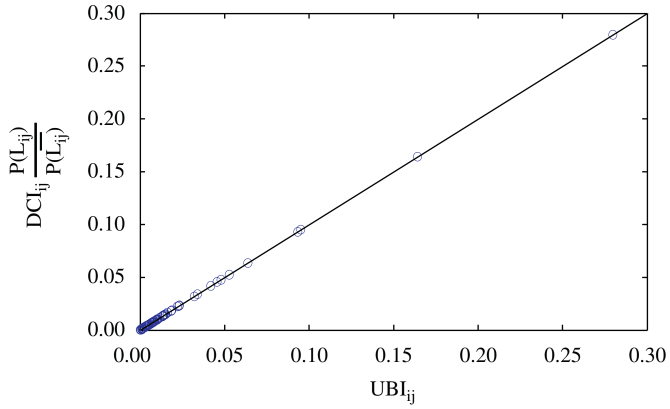

Fig. 5. UBI_{ij} and DCI_{ij} relationship obtained from Monte Carlo simulations.

probability of this given Pipe j. The equation suggests that, as far as upgrading benefit to Node i is concerned, both the consequence of damage (DCI) and the likelihood of damage (the odds) should be factored in. For example, for two pipes with equal DCI, the one with a higher odds of damage should be upgraded first. On the other hand, among the pipes that have the same odds of damage, the one with a high damage consequence has a higher upgrade priority. These deductions from Eq. (8) are quite intuitive. Note that the odds is a nonlinear function of $P(L_j)$, and it increases indefinitely when $P(L_j)$ approaches 1. Therefore, when the odds is close to 1, UBI_{ij} tends to be large and less sensitive to DCI_{ij}; the corresponding Pipe j is likely to be a critical link.

Similar to UBI_{ij}, the DCI_{ij} for Node i and Pipe j can be estimated from Monte Carlo simulations as

$$DCI_{ij} = \frac{n_1/m_1 - n_3/m_3}{1 - n_1/m_1} \tag{9}$$

in which m_3 is the number of Monte Carlo samples that damage is observed in Pipe j, and n_3 is the number of Monte Carlo samples in m_3 that Q_i is satisfied. Fig. 5 plots the right-hand side of Eq. (8) for all Pipe j as a function of its corresponding UBI_{ij} from the Monte Carlo simulations. It is evident that all data points fall along the diagonal line of the figure, verifying the identity in Eq. (8).

The UBI values for pipe groups that contain pipes with the largest individual DCI are also shown in Fig. 3. The UBI values are shown to be far lower than the UBI efficient frontier. This verifies that, when upgrading pipes for a given Node i, using UBI as a primary index to identify critical links provides more significant increases of $P(Q_i)$ than using DCI. UBI is a more appropriate index than DCI in identifying critical links in seismic mitigation.

8. Conclusions

This paper describes a process to characterize the spatial distribution of seismic water supply reliability among various consumers within a system and proposes methods to identify critical links of water supply to crucial water consumers under an earthquake. The reliability of water supply to a consumer i with water demand Q_i can be measured by the probability, $P(Q_i)$, of Q_i satisfied. $P(Q_i)$ can be estimated for each customer of a water supply system, and its spatial distribution reflects the spatial variation of water supply reliability among various consumers within the system. Two conditional probability quantities of $P(Q_i)$, DCI_{ij} and UBI_{ij}, are proposed to measure the impact of Pipe j on the reliability of water supply to a consumer i and to identify critical links that significantly affect water supply to the consumer i.

The reliability of water supply to consumers varies spatially, depending on their respective locations in the system and system configuration. When water consumers are located in the far end of the system and away from water sources, their water supply reliability is relatively low. In contrast, when water consumers are located immediately downstream of the water storage tanks or along the large-diameter trunk lines, they are close to the water sources, and hence, have relatively high reliability of water supply. It was also found that the reliability is relatively low for those water consumers that have much higher elevation when compared with their neighboring water consumers. When the water supply system is disrupted by the earthquake-induced damage, the water pressure in the system will decrease. Therefore, the chance that the pressure is not sufficient to deliver water to high elevation area is higher than that for low elevation area.

Both DCI and UBI have been estimated to identify critical links of water supply to crucial water consumers (e.g., acute care facilities and/or hospitals), who play vital roles in emergency response and must remain operational to rein in the losses in an event of a major earthquake. Although DCI and UBI are based on two different considerations, they are interrelated such that, for a water consumer i, UBI_{ij} is a product of DCI_{ij} and the odds of Pipe j damage. For the given water consumer i, DCI_{ij} reflects the effect of system characteristics, such as system connectivity, redundancy, and damage probabilities of pipes other than the given Pipe j, while the odds of Pipe j damage signifies the influence of the damage probability [$P(L_j)$] of the given Pipe j. UBI is shown to be the primary index in the identification of critical links for crucial water consumers, and critical links are pipes with relatively large UBI values.

The concept of efficient frontier has been employed to identify group of critical links of the system under an earthquake. Although the order of adding critical link to the group of critical links does not necessarily follow the decreasing order of the UBI for each individual pipe, the group of critical links is shown to occur in a recursive fashion. It is also found that a group of links that have the largest UBI individually do not necessarily have the largest group UBI or be the group of critical links.

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