1 - (a). $A = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \\ 2 & -1 & 3 & 1 \\ 3 & -1 & 4 & 3 \end{pmatrix}$ perform Gaussian elimination.

Input:
$E_1 : 2x_1 + x_3 + 2x_4 = a_{1,5}$
$E_2 : x_1 + x_2 + 2x_4 = a_{2,5}$
$E_3 : 2x_1 - x_2 + 3x_3 + x_4 = a_{3,5}$
$E_4 : 3x_1 - x_2 + 4x_3 + 3x_4 = a_{4,5}$

STEP 2-3. diagonal element가 모두 0이 아니므로 row exchange는 필요없다.

STEP 4. diagonal element 밑의 element 들을 소거해주기.

Let $m_{ji} = a_{ji}/a_{ii}$, then $E_j = E_j - m_{ji} \cdot E_i$

① $E_2 = E_2 - \frac{1}{2} \cdot E_1$ ② $E_3 = E_3 - \frac{2}{2} \cdot E_1$ ③ $E_4 = E_4 - \frac{3}{2} E_1$

$$\begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 2 & -1 & 3 & 1 \\ 3 & -1 & 4 & 3 \end{pmatrix} \begin{pmatrix} a_{1,5} \\ a_{2,5} - \frac{1}{2} a_{1,5} \\ a_{3,5} \\ a_{4,5} \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & 2 & -1 \\ 3 & -1 & 4 & 3 \end{pmatrix} \begin{pmatrix} a_{1,5} \\ a_{2,5} - \frac{1}{2} a_{1,5} \\ a_{3,5} - a_{1,5} \\ a_{4,5} \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & \frac{5}{2} & 0 \end{pmatrix} \begin{pmatrix} a_{1,5} \\ a_{2,5} - \frac{1}{2} a_{1,5} \\ a_{3,5} - a_{1,5} \\ a_{4,5} - \frac{3}{2} a_{1,5} \end{pmatrix}$$

Step 2. diagonal element 중 0이 된 것이 있으므로 가장 가까운 것과 row exchange 해줌.

④ $E_3 \leftrightarrow E_4$

Step 4 다시 진행.

⑤ $E_3 = E_3 - \frac{-1}{1} E_2$

⑥ $E_4 = E_4 - \frac{-1}{1} E_2$

$$\left(\begin{array}{cccc} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & \frac{5}{2} & 0 \\ 0 & -1 & 2 & -1 \end{array}\right) \left(\begin{array}{c} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{4,5} - \frac{3}{2}a_{1,5} \\ a_{3,5} - a_{1,5} \end{array}\right)$$

$$\left(\begin{array}{cccc} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & 2 & 1 \\ 0 & -1 & 2 & -1 \end{array}\right) \left(\begin{array}{c} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{4,5} - \frac{3}{2}a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{3,5} - a_{1,5} \end{array}\right)$$

$$\left(\begin{array}{cccc} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & \frac{1}{2} & 0 \end{array}\right) \left(\begin{array}{c} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{4,5} - \frac{3}{2}a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{3,5} - a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} \end{array}\right)$$

Step 2. diagonal element 중 0이 된 것이 있으므로 가장 가까운 것과 row exchange 해줌.

⑦ $E_3 \leftrightarrow E_4$

Step 4 다시 진행

⑧ $E_4 = E_4 - \frac{2}{\frac{1}{2}} E_3$

$$\left(\begin{array}{cccc} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 2 & 1 \end{array}\right) \left(\begin{array}{c} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{3,5} - a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{4,5} - \frac{3}{2}a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} \end{array}\right)$$

$$\left(\begin{array}{cccc} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{c} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ a_{3,5} - a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} = -\frac{3}{2}a_{1,5} + a_{2,5} + a_{3,5} \\ a_{4,5} - \frac{3}{2}a_{1,5} + a_{2,5} - \frac{1}{2}a_{1,5} - 4a_{3,5} + 4a_{1,5} + 4a_{2,5} - 2a_{1,5} \end{array}\right)$$

$$= 5a_{2,5} - 4a_{3,5} + a_{4,5}$$

Step 8 Set $x_n = a_{n,n+1}/a_{nn}$. *(Start backward substitution.)*

last form. $\Rightarrow$ $\left(\begin{matrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{matrix}\right) \left(\begin{matrix} a_{1,5} \\ a_{2,5} - \frac{1}{2}a_{1,5} \\ -\frac{3}{2}a_{1,5} + a_{2,5} + a_{3,5} \\ 5a_{2,5} - 4a_{3,5} + a_{4,5} \end{matrix}\right)$

(9) $X_1 = a_{1,n+1}/2 = \frac{1}{2}a_{1,n+1} = \frac{1}{2}a_{1,5}$

(10) $X_2 = a_{2,n+1}/1 = a_{2,n+1} = a_{2,5} - \frac{1}{2}a_{1,5}$

(11) $X_3 = a_{3,n+1}/\frac{1}{2} = 2a_{3,n+1} = -3a_{1,5} + 2a_{2,5} + 2a_{3,5}$

(12) $X_4 = a_{4,n+1}/1 = a_{4,n+1} = 5a_{2,5} - 4a_{3,5} + a_{4,5}$.

Step 9 For $i = n-1, \dots, 1$ set $x_i = \left[a_{i,n+1} - \sum_{j=i+1}^{n} a_{ij}x_j\right] \Big/ a_{ii}$.

(13) $X_3 = \left[a_{3,n+1} - \sum_{j=4}^{4} a_{3j}X_j\right] \Big/ a_{33} = \left[-\frac{3}{2}a_{1,5} + a_{2,5} + a_{3,5} - 0\right] \Big/ \frac{1}{2} = -3a_{1,5} + 2a_{2,5} + 2a_{3,5}$

(14) $X_2 = \left[a_{2,n+1} - a_{23}\cdot X_3 - a_{24}\cdot X_4\right] \Big/ a_{22} = a_{2,5} - \frac{1}{2}a_{1,5} - (-\frac{1}{2})\cdot(-3a_{1,5} + 2a_{2,5} + 2a_{3,5})$

$- 1\cdot(5a_{2,5} - 4a_{3,5} + a_{4,5}) = -2a_{1,5} - 3a_{2,5} + 5a_{3,5} - a_{4,5}$

(15) $X_1 = \left[a_{1,n+1} - a_{12}\cdot X_2 - a_{13}\cdot X_3 - a_{14}\cdot X_4\right] \Big/ a_{11} = \frac{1}{2}\Big(a_{1,5} - 0 - (-3a_{1,5} + 2a_{2,5} + 2a_{3,5})$

$- 2(5a_{2,5} - 4a_{3,5} + a_{4,5})\Big) = (\frac{1}{2} + \frac{3}{2})a_{1,5} + (-1-5)a_{2,5} + (-1+4)a_{3,5} - a_{4,5}$

$= 2a_{1,5} - 6a_{2,5} + 3a_{3,5} - a_{4,5}$

Step 10

output $\left(\begin{array}{l} X_1 : 2a_{1,5} - 6a_{2,5} + 3a_{3,5} - a_{4,5} \\ X_2 : -2a_{1,5} - 3a_{2,5} + 5a_{3,5} - a_{4,5} \\ X_3 : -3a_{1,5} + 2a_{2,5} + 2a_{3,5} \\ X_4 : 5a_{2,5} - 4a_{3,5} + a_{4,5}. \end{array} \right)$

1-(b)

$$A = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \\ 2 & -1 & 3 & 1 \\ 3 & -1 & 4 & 3 \end{pmatrix}$$

new $A = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & \frac{5}{2} & 0 \end{pmatrix}$

Let $B = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & \frac{5}{2} & 0 \end{pmatrix}$

new $B = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$

$A = L_1 \cdot$ new A.

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \frac{3}{2} & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & \frac{5}{2} & 0 \end{pmatrix}.$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \frac{3}{2} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$\text{Let } C = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{new } C = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ \frac{3}{2} & -1 & \frac{4}{3} & 1 \end{pmatrix}$$

$$\| $$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ \frac{3}{2} & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \end{pmatrix}$$

계산과정.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \frac{3}{2} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ \frac{3}{2} & -1 & \frac{4}{3} & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L \cdot U$$

2. -(a) $\begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}$

(i) if det $= 0$, singular.

$\det\begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix} = 6 - 1 = 5 \neq 0$. nonsingular.

(ii) if $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}| \ (i \neq j)$, strictly diagonally dominant. (SDD)

(first row) (second row)

$|-2| > |1|$, $|-3| > |1|$. SDD.

(iii) if $X^t A X > 0$, positive definite.

$$(x_1 \ \ x_2)\begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (x_1 \ \ x_2)\begin{pmatrix} -2x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix} = x_1(-2x_1 + x_2) + x_2(x_1 - 3x_2)$$

$$= -2x_1^2 + 2x_1x_2 - 3x_2^2$$

$$= -(x_1 + x_2)^2 - 2x_2^2 - x_1^2 < 0$$

not positive definite.

2-(b) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{pmatrix}$

i) if det $=0$, singular.

(j=3으로 고정,)

$$\det\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{pmatrix} = \sum_{i=1}^{3} a_{ij}A_{ij} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

$$= 0 + 2\cdot A_{23} + 4\cdot A_{33} = 2\cdot(-1)^{2+3}M_{23} + 4\cdot(-1)^{3+3}M_{33}$$

$$= -2M_{23} + 4M_{33} = -2\cdot\det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} + 4\cdot\det\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$

$$= -2\cdot(4-1) + 4\cdot(6-0) = -6 + 24 = 18 \neq 0.$$

nonsingular.

ii) (first row) $|2| > |1| + |0|$, (second row) $|3| > |2| + |0|$, (third row) $|4| > |1| + |2|$

SDD.

2-(b) (iii) $X^t A X = (x_1 \ x_2 \ x_3)\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (x_1 \ x_2 \ x_3)\begin{pmatrix} 2x_1 + x_2 \\ 3x_2 + 2x_3 \\ x_1 + 2x_2 + 4x_3 \end{pmatrix}$

$$= \left(x_1(2x_1 + x_2) + x_2(3x_2 + 2x_3) + x_3(x_1 + 2x_2 + 4x_3) \right)$$

$$= 2x_1^2 + x_1x_2 + 3x_2^2 + 4x_2x_3 + x_1x_3 + 4x_3^2$$

$$= \left(x_1 + \tfrac{1}{2}x_2\right)^2 + \left(x_1 + \tfrac{1}{2}x_3\right)^2 + \tfrac{11}{4}x_2^2 + 4x_2x_3 + \tfrac{15}{4}x_3^2$$

$$= \left(x_1 + \tfrac{1}{2}x_2\right)^2 + \left(x_1 + \tfrac{1}{2}x_3\right)^2 + \left(\tfrac{3}{2}x_2 + \tfrac{4}{3}x_3\right)^2 + \tfrac{5}{4}x_2^2 + \tfrac{71}{36}x_3^2 > 0.$$

positive definite

2-(c)

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{pmatrix} \qquad (j = 3 \text{ 고정})$$

(i)
$$\det\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{pmatrix} = \sum_{i=1}^{3} a_{ij}A_{ij} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

$$= 0 + 2\cdot A_{23} + 2\cdot A_{33} = 2\cdot(-1)^{2+3}M_{23} + 2\cdot(-1)^{3+3}M_{33} = -2M_{23} + 2M_{33}.$$

$$= -2\cdot\det\begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix} + 2\cdot\det\begin{pmatrix} 2 & -1 \\ -1 & 4 \end{pmatrix} = -2\cdot(4-0) + 2(8+1) = -8+18 = 10 \neq 0$$

<u>non singular</u>.

(ii) (first row) (second row) (third row)

$|2| > |1| + |0|$, $|4| > |-1| + |2|$, $|2| \not> |2| + |0|$, <u>not SDD</u>.

(iii)
$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}\begin{pmatrix} 2x_1 - x_2 \\ -x_1 + 4x_2 + 2x_3 \\ 2x_2 + 2x_3 \end{pmatrix} = \left(x_1(2x_1 - x_2) + x_2(-x_1 + 4x_2 + 2x_3) + x_3(2x_2 + 2x_3)\right)$$

$$= 2x_1^2 - 2x_1x_2 + 4x_2^2 + 4x_2x_3 + 2x_3^2 = (x_1 - x_2)^2 + x_1^2 + \left(\tfrac{3}{2}x_2 + \tfrac{4}{3}x_3\right)^2 + \tfrac{3}{4}x_2^2 + \tfrac{2}{9}x_3^2 > 0$$

<u>Positive definite</u>.

2-(d)

$$\begin{pmatrix} 2 & 3 & 1 & 2 \\ -2 & 4 & -1 & 5 \\ 3 & 7 & 1.5 & 1 \\ 6 & -9 & 3 & 7 \end{pmatrix}$$

(i) $\det\begin{pmatrix} 2 & 3 & 1 & 2 \\ -2 & 4 & -1 & 5 \\ 3 & 7 & 1.5 & 5 \\ 6 & -9 & 3 & 7 \end{pmatrix}$ (j=4로) $= \sum_{i=1}^{4} a_{ij}A_{ij} = 2\cdot A_{14} + 5A_{24} + 5A_{34} + 7A_{44}$

$= -2M_{14} + 5M_{24} - 5M_{34} + 7M_{44} = -2\cdot\det\begin{pmatrix} -2 & 4 & -1 \\ 3 & 7 & 1.5 \\ 6 & -9 & 3 \end{pmatrix} + 5\cdot\det\begin{pmatrix} 2 & 3 & 1 \\ 3 & 7 & 1.5 \\ 6 & -9 & 3 \end{pmatrix}$

$- 5\cdot\det\begin{pmatrix} 2 & 3 & 1 \\ -2 & 4 & -1 \\ 6 & -9 & 3 \end{pmatrix} + 7\cdot\det\begin{pmatrix} 2 & 3 & 1 \\ -2 & 4 & -1 \\ 3 & 7 & 1.5 \end{pmatrix}$

i=1로,

$= -2\left(\sum_{j=1}^{3} a_{ij}A_{ij} = -2A_{11} + 4A_{12} - A_{13}\right) + 5\left(\sum_{j=1}^{3} a_{ij}A_{ij} = 2A_{11} + 3A_{12} + A_{13}\right)$

$- 5\left(\sum_{j=1}^{3} a_{ij}A_{ij} = 2A_{11} + 3A_{12} + A_{13}\right) + 7\left(2A_{11} + 3A_{12} + A_{13}\right) = \left(4M'_{11} + 8M'_{12} + 2M'_{13}\right)$

$+ \left(10M''_{11} - 15M''_{12} + 5M''_{13}\right) + \left(-10M'''_{11} + 15M'''_{12} - 5M'''_{13}\right) + \left(14M''''_{11} - 21M''''_{12}\right.$
$\left. + 7M''''_{13}\right)$

$= 4\cdot(21 + 13.5) + \cancel{8(9-9)} + 2(-27-42) + 10(21+13.5) \cancel{-15(9-9)} + 5(-27-42)$

$-10(18-24) \cancel{+15(-6+6)} - 5(18-24) + 14(6+7) \cancel{-21(-3+3)} + 7(-14-12)$

$= \cancel{138} - \cancel{138} + 1560 - 345 + 60 + 30 + \cancel{182} - \cancel{182} = 1305 \neq 0.$ nonsingular

$$\begin{pmatrix} 2 & 3 & 1 & 2 \\ -2 & 4 & -1 & 5 \\ 3 & 7 & 1.5 & 1 \\ 6 & -9 & 3 & 7 \end{pmatrix}$$

(ii) (first row)

$|2| \not> |3| + |1| + |2|$. not SDD

(iii) $x^tAx = (x_1 \ x_2 \ x_3 \ x_4)\begin{pmatrix} 2x_1+3x_2+x_3+2x_4 \\ -2x_1+4x_2-x_3+5x_4 \\ 3x_1+7x_2+1.5x_3+x_4 \\ 6x_1-9x_2+3x_3+7x_4 \end{pmatrix}$

$= x_1(2x_1+3x_2+x_3+2x_4)+x_2(-2x_1+4x_2-x_3+5x_4)+x_3(3x_1+7x_2+1.5x_3+x_4)+x_4(6x_1-9x_2+3x_3+7x_4)$

$= 2x_1^2+x_1x_2+4x_1x_3+8x_1x_4+4x_2^2+6x_2x_3-4x_2x_4+1.5x_3^2+4x_3x_4+7x_4^2.$

... 계산 불가능하므로.

Theorem
A symmetric matrix A is positive definite if and only if each of its leading principal submatrices has a positive determinant.

Theorem 을 이용해서 submatrices 가 positive determinant 를 가지는지 확인해보면,

① $\det A_1 = \det(2) = 2 > 0$. ② $\det A_2 = \det\begin{pmatrix} 2 & 3 \\ -2 & 4 \end{pmatrix} = (8+6) = 14 > 0.$

$(i = 1, 2, 3)$

③ $\det\begin{pmatrix} 2 & 3 & 1 \\ -2 & 4 & -1 \\ 3 & 7 & 1.5 \end{pmatrix} = \sum_{j=1}^{3} a_{ij}A_{ij} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$$\begin{pmatrix} 2 & 3 & 1 & 2 \\ -2 & 4 & -1 & 5 \\ 3 & 7 & 1.5 & 1 \\ 6 & -9 & 3 & 7 \end{pmatrix}$$

$= 2A_{11} + 3A_{12} + A_{13} = 2M_{11} - 3M_{12} + M_{13}$

$= 2\cdot(6+7) - 3\cdot(-3+3) + (-14-12) = 26 - 26 = 0. \not> 0$

$\therefore$ not positive definite.