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Parameter Estimation Assignment

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Q 1) For a Normal distribution
Mean = θ_1
Variance = θ_2

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

likelihood function :

$$L(x_1, x_2, x_3, \dots x_n) = \prod_{i=1}^{n} f(x)$$

$$L(x_1, x_2 \dots x_n) = \left(\frac{1}{2\pi\theta_2}\right)^{\frac{n}{2}} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

Taking natural log on both sides

$$\ln L(x) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Taking differentiation w.r.t θ_2 & $\theta_1 = 0$

$$\frac{\partial}{\partial\theta_2}\ln L(x) = -0 - \frac{n}{2}\left(\frac{1}{\theta_2}\right) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2(-1)(\theta_2$$

$$0 = -\left(\frac{n}{2}\right)\left(\frac{1}{\theta_2}\right) + \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta_1)^2(\theta_2)^{-2}(-1)$$

$$\left(\frac{n}{2}\right)\left(\frac{1}{\theta_2}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{\theta_2}\right)^2 \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\boxed{\theta_{2}\_MLE = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

Taking differentiation w.r.t θ_1

$$\frac{\partial}{\partial \theta_1} \ln L(X) = -0 - \cancel{0}\, 0 - \frac{1}{2\theta_2} 2 \sum_{i=1}^{n} (x_i - \theta_1) (-1)$$

$$0 = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

$$0 = \sum x_i - n\theta_1$$

$$\theta_1 = \frac{\sum x_i}{n}$$

$$\boxed{\theta_{1}\_MLE = \frac{\sum x_i}{n}}$$

Q2) For binomial distribution $B(m, p)$

$$f(x) = {}^{m}C_{x}\, p^{x} (1-p)^{m-x}$$

Likelihood function:

$$L(p) = \prod_{i=1}^{m} {}^{m}C_{x_i}\, p^{x_i} (1-p)^{m-x_i}$$

Taking natural log on both sides

$$\ln L(p) = \cancel{\sum_{i=1}^{n} \left[\ln {}^{m}C_{x_i} \right]}$$

$$\ln p \left(\sum_{i=1}^{m} x_i \right) + \ln(1-p)\left(m - \sum_{i=1}^{n} x_i\right)$$

$$= n\left(\bar{x} \ln p + (1-\bar{x}) \ln(1-p)\right)$$

Differentiating w.r.t p

$$\frac{\partial}{\partial p} \ln L(p) = n\left(\frac{\bar{x}}{p} - \frac{1-\bar{x}}{1-p}\right) = n\left(\frac{\bar{x}-p}{p(1-p)}\right) = 0$$

$\therefore$ $p\_MLE = \bar{x}$

In question $p = \theta$

$\therefore$ $\boxed{\theta\_MLE = \bar{x}}$