

Lesson 1

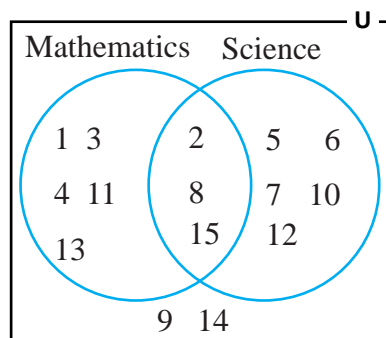
Sets

1.0 Review

Students from roll no. 1 to 15 in grade 10 are surveyed about whether they like Mathematics or Science. The information from the survey is presented in a Venn-diagram.

Discuss on the following questions based on the Venn-diagram alongside:

- Write the set of students who like Mathematics by listing method. Write the cardinal number of the set.
- Write the set of students who like Mathematics only by listing method. Write the cardinal number of the set.
- Write the set of students who like Science by listing method. Write the cardinal number of the set.
- Write the set of students who like Science only by listing method. Write the cardinal number of the set.
- Write the set of students who like both Mathematics and Science by listing method. Write the cardinal number of the set.
- Write down the set of students who do not like either Mathematics or Science by listing method. Write the cardinal number of the set.
- How many students were surveyed?



1.1 Cardinality of the Two Sets

Activity 1

An information obtained from the question asked among the students of grade 10 about whether they like coffee or tea. The following information was obtained.

- The number of students who like coffee is 15.
- The number of students who like tea is 10.
- The number of students who like both is 6.
- The number of students who dislike both is 5.

Discuss the following questions based on the above information:

- How shall the given information be shown in a Venn-diagram?
- How many students like coffee only?
- How many students like tea only?
- How many students were there in the class?

Here, $n(C)$ denotes the number of students who like coffee and $n(T)$ denotes the number of students who like tea. Similarly, $n_o(T)$ and $n_o(C)$ respectively denote the number of students who like tea only and coffee only.

Now, showing the information in a Venn-diagram,

First write the value of $n(C \cap T)$ in venn-diagram the since the number of students who like both is denoted by it.

Then, since the number of students who like coffee is $n_o(C) + n(C \cap T)$,

The number of students who like coffee only $n_o(C) = n(C) - n(C \cap T)$.

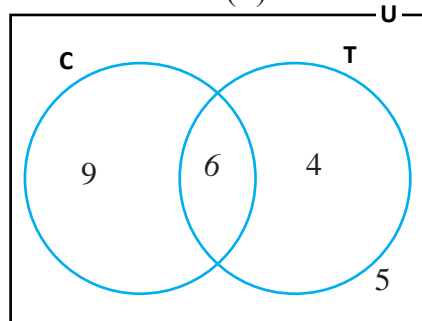
$$\text{i.e. } n_o(C) = 15 - 6 = 9$$

Likewise, in case of the number of students who like tea

$$\begin{aligned} \text{Let us insert the number of students who like tea only } n_o(T) &= n(T) - n(C \cap T) \\ &= 10 - 6 = 4. \end{aligned}$$

Now, let's insert the number of students who dislike tea or coffee $n(\overline{C \cup T}) = 5$

In this way, the total number of students in the class $n(U) = 9 + 6 + 4 + 5 = 24$



If A and B are overlapping sets,

- Total number of elements of both the sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- The number of elements in the set A only, $n_o(A) = n(A) - n(A \cap B)$.
- The number of elements in the set B only, $n_o(B) = n(B) - n(A \cap B)$.
- Total number of elements of both the sets, $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$.
- If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$
- If there are elements of A and B only in U, then $n(U) = n(A \cup B)$
- If there are elements other than A and B in U, then $n(U) = n(A \cup B) + n(\overline{A \cup B})$

Other terminologies:

At least one: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Or, $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$.

At most one: $n(\overline{A \cap B}) = n(U) - n(A \cap B)$.

Exactly one or only one: $n_o(A) + n_o(B) = n(A) + n(B) - 2 \times n(A \cap B)$.

Example 1

In a survey of 300 people of a community, it was found that 175 liked cricket and 150 liked football but 25 liked neither of them. Based on this, answer the following questions:

- Represent the above information in the Venn-diagram.
- Find the number of people who like both the games.
- Find the number of people who like exactly one game.

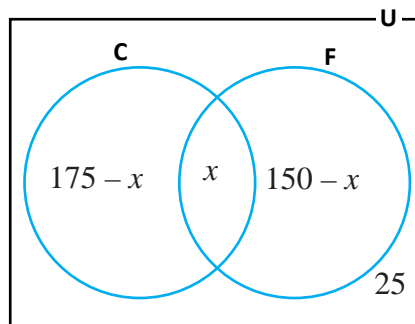
Solution

Let, C and F respectively denote the set of people who like cricket and football. Likewise, U denotes the set of total people.

According to the question,

$$n(U) = 300, n(C) = 175, n(F) = 150 \text{ and } n(\overline{C \cup F}) = 25$$

$$\text{Let, } n(C \cap F) = x$$



- The information is represented in the Venn-diagram alongside.
- From the Venn-diagram, it can be written as

$$n(U) = n_o(C) + n(C \cap F) + n_o(F) + n(\overline{C \cup F})$$

$$300 = (175 - x) + x + (150 - x) + 25$$

$$\text{or, } 300 = 175 - x + x + 150 - x + 25$$

$$\text{or, } 300 = 350 - x$$

$$\text{or, } x = 350 - 300$$

$$\therefore x = 50$$

$$\text{or, } n(C \cap F) = 50$$

\therefore The number of people who like both the games is 50.

Again,

$$\text{The number of people who liked only cricket, } n_o(C) = 175 - 50 = 125$$

$$\text{The number of people who liked only football, } n_o(F) = 150 - 50 = 100$$

$$\therefore \text{The number of people who liked exactly one game, } n_o(C) + n_o(F) = 125 + 100 = 225$$

Example 2

The result of a survey among 120 students of grade 10 is as follows:

30 like only Mathematics.

40 like only English.

10 like neither Mathematics nor English.

Based on this information, answer the following questions:

- Represent the above information in a Venn-diagram.
- Find the number of students who like both the subjects.
- Find the number of students who like at least one subject.

Solution

Let, M and E denote the set of students who like Mathematics and English respectively.

Likewise, U denotes the total students.

According to the question,

$$n(U) = 120, n_o(M) = 30, n_o(E) = 40 \text{ \& } n(\overline{M \cup E}) = 10$$

Let, $n(M \cap E) = x$

a) The information is represented in a Venn-diagram alongside.

b) From the Venn-diagram,

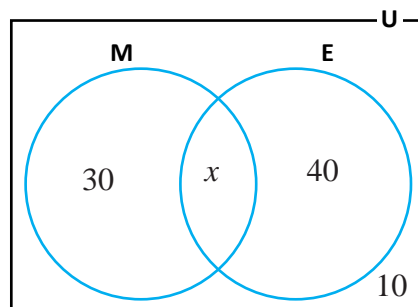
$$30 + x + 40 + 10 = 120$$

$$\text{or, } 80 + x = 120$$

$$\text{or, } x = 120 - 80 = 40$$

$$\therefore x = 40$$

$$\text{or, } n(M \cap E) = 40$$



\therefore The number of students who like both Mathematics and English is 40.

c) The number of students who like at least one subject $n(M \cup E) = 30 + 40 + 40 = 110$

Example 3

According to a survey among the SEE appeared students from a school, 75% were interested in studying science and 55% were interested in studying staff nurse but 5% denied to give information whilst 21 students were interested to study both science and staff nurse. Based on this information, answer the following questions:

- Show the above information in the Venn-diagram.
- Find the total number of students inquired in the survey.
- Find the number of students who were interested in studying only staff nurse.

Solution

Let, S and N denote the set of students interested to study science and staff nurse respectively.

Likewise, U denotes the total students.

According to the question,

Let,

$$n(U) = x,$$

$$n(S) = 75\% \text{ of } x = 0.75x,$$

$$n(N) = 55\% \text{ of } x = 0.55x,$$

$$n(S \cap N) = 21 \text{ and } n(\overline{S \cup N}) = 5\% \text{ of } x = 0.05x,$$

- The information is represented in a Venn-diagram alongside.
- From the Venn-diagram,

$$(0.75x - 21) + 21 + (0.55x - 21) + 0.05x = x$$

$$\text{or, } 1.35x - 21 = x$$

$$\text{or, } 1.35x - x = 21$$

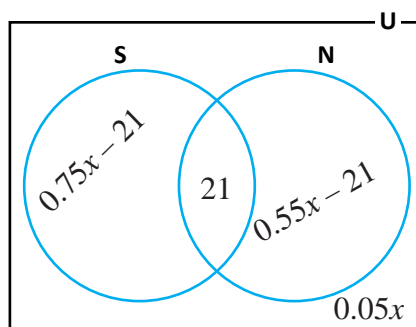
$$\text{or, } 0.35x = 21$$

$$\text{or, } x = \frac{21}{0.35} = 60$$

$$\therefore x = 60$$

$$\text{or, } n(U) = 60$$

\therefore The total number of students surveyed was 60.



- c) From the Venn-diagram,
 The number of students who were interested in studying only staff nurse
 $= 0.55x - 21$
 $= 0.55 \times 60 - 21$
 $= 33 - 21$
 $= 12$
 \therefore The number of students who were interested in studying only staff nurse was 12.

Example 4

In a survey of 300 foreign tourists visiting to Nepal, it was found that the ratio of the number of tourists who visited Pokhara and Lumbini was 2:3. Among them, 90 visited both the places and 60 visited neither Pokhara nor Lumbini. Based on this information, answer the following questions:

- Show the above information in a Venn-diagram.
- Determine the number of tourists who visited only one place.
- Find the number of tourists who visited at least one of the places.

Solution

Let, P and L denote the set of tourists who visited Pokhara and Lumbini respectively. Likewise, U denotes the total tourists.

According to the question,

$$n(U) = 300, n(P \cap L) = 90 \text{ and } n(\overline{P \cup L}) = 60$$

$$\text{Let, } n(P) = 2x, n(L) = 3x$$

- a) The information is represented in a Venn-diagram alongside.

- b) From the Venn-diagram,

$$(2x - 90) + 90 + (3x - 90) + 60 = 300$$

$$\text{or, } 5x - 30 = 300$$

$$\text{or, } 5x = 300 + 30$$

$$\text{or, } 5x = 330$$

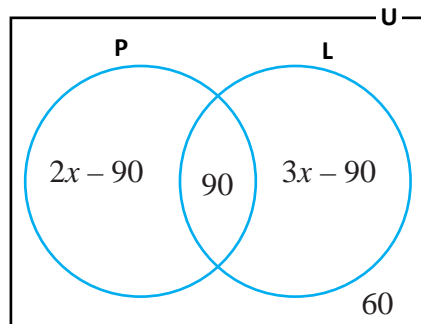
$$\text{or, } x = \frac{330}{5}$$

$$\therefore x = 66$$

$$\text{Thus, } n_o(P) = 2 \times 66 - 90 = 42 \text{ and } n_o(L) = 3 \times 66 - 90 = 108$$

$$\therefore \text{ The number of tourists who visited only one place } = 42 + 108 = 150$$

- c) The number of tourists who visited at least one place $= 300 - 60 = 240$



Example 5

In a survey among the 200 students studying in grade 10, it was found that the ratio of the number of students who likes Mathematics and English was 2:3. Among them, 30% like both of them but 15% like neither Mathematics nor English. Based on this information, answer the following questions:

- Represent the above information in a Venn-diagram.
- What is the difference between the number of students who like Mathematics and the number of students who like English? Find it.

Solution

Let, M and E denote the set of students who likes Mathematics and English respectively. Likewise, U denotes the total students.

According to the question,

$$n(U) = 200, n(M \cap E) = 30\% \text{ of } 200 = 60 \text{ and}$$

$$n(\overline{M \cup E}) = 15\% \text{ of } 200 = 30$$

$$\text{Let, } n_o(M) = 2x, n_o(E) = 3x$$

- The information is represented in a Venn-diagram alongside.
- From the Venn-diagram,

$$2x + 60 + 3x + 30 = 200$$

$$\text{or, } 90 + 5x = 200$$

$$\text{or, } 5x = 200 - 90$$

$$\text{or, } 5x = 110$$

$$\text{or, } x = \frac{110}{5}$$

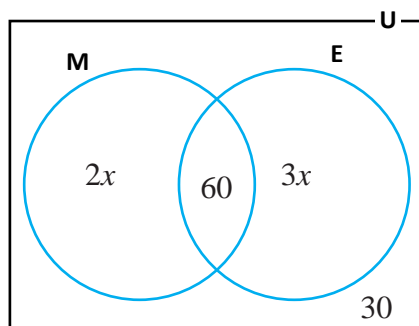
$$\therefore x = 22$$

Thus,

The number of students who likes Mathematics, $n(M) = 2x + 60 = 2 \times 22 + 60 = 104$

The number of students who likes English, $n(E) = 3x + 60 = 3 \times 22 + 60 = 126$

\therefore The difference between the number of students who like Mathematics and the number students who like English $= 126 - 104 = 22$

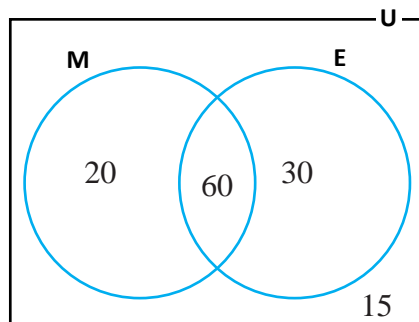


Exercise 1.1

1.
 - a) Present the cardinality of sets with examples and show it to your teacher.
 - b) For two sets A and B, $A \subset B$, find the values of $n(A \cup B)$ and $n(A \cap B)$.
 - c) If A and B are overlapping sets, state the formula for $n(A \cup B)$.
 - d) There are 12 and 8 elements in the sets A and B respectively, Find the minimum number of elements that would be in the set $n(A \cup B)$.

2. **In the given Venn-diagram, 80 people are in set M, 90 people are in set E and 15 people are not in both the sets. Determine the cardinality of following sets.**

- | | |
|------------------------------|-------------------|
| (a) $n_o(M)$ | (b) $n_o(E)$ |
| (c) $n(M)$ | (d) $n(E)$ |
| (e) $n(M \cup E)$ | (f) $n(M \cap E)$ |
| (g) $n(\overline{M \cup E})$ | (h) $n(U)$ |



3.
 - a) If $n(U) = 200$, $n_o(M) = 2x$, $n_o(E) = 3x$, $n(M \cap E) = 60$ and $n(\overline{M \cup E}) = 40$ find the value of x .
 - b) If $n(U) = 350$, $n(A) = 200$, $n(B) = 220$ and $n(A \cap B) = 120$, then find $n(A \cup B)$ and $n(\overline{A \cup B})$.
 - c) If $n(A) = 35$ and $n(\bar{A}) = 25$, then find the value of $n(U)$.
 - d) Out of two sets P and Q, there are 40 elements in P, 60 elements in $(P \cup Q)$ and 10 elements in $(P \cap Q)$. How many elements are there in Q? Find.
4.
 - a) **In a survey of 180 students of a school, 45 like Nepali only and 60 like English only but 15 like none of the subjects. Based on this information, answer the following questions:**
 - i) Show the above information in a Venn-diagram.
 - ii) Find the number of students who like both the subjects.
 - iii) Find the number of students who like at least one subject.
 - b) **In a survey among the 1200 students of a school, 100 like Mathematics only and 200 like Science only but 700 like neither of the subjects. Based on the information, answer the following questions:**
 - i) Show the above information in a Venn-diagram.
 - ii) Find the number of students who like both the subjects.

- iii) Find the number of students who likes at least one subject.
 - c) **In a survey among 60 students, 10 play football only and 20 play volleyball only but 12 play neither of the games. Based on the information, answer the following questions:**
 - i) Show the above information in a Venn-diagram.
 - ii) Find the number of students who play both the games.
 - iii) Find the number of students who play at least one game.
- 5. a) A survey was carried out among 900 people of a community. According to the survey, 525 read Madhupark, 450 read Yubamanch but 75 didn't read either of the newspapers. Using the information, answer the following questions:**
- i) Show the information in a Venn-diagram
 - ii) Find the number of people who read both the newspapers.
 - iii) Find the number of people who read only one newspaper.
- b) According to a survey among 150 people, 90 like modern songs, 70 like folk songs but 30 do not like either of the songs. Using the information, answer the following questions:**
- i) Show the information in a Venn-diagram
 - ii) Find the number of people who like both the songs.
 - iii) Find the number of people who like only modern songs.
- c) According to a survey among 360 players, 210 liked to play volleyball, 180 liked to play football but 30 liked to play neither of the games. Using the information, answer the following questions:**
- i) Show the information in a Venn-diagram
 - ii) Find the number of players who like to play both the games.
 - iii) Find the number of people who like to play only one game.
- 6. a) Out of the students who participated in an examination, 70% passed English, 60% passed Mathematics but 20% failed both the subjects and 550 students passed both the subjects. Based on the information, answer the following questions:**
- i) Show the above information in a Venn-diagram.
 - ii) Find the total number of students participated in the examination.
 - iii) Find how many students passed English only.

b) According to a survey of students who have appeared the examination of grade 10, 60% are interested to study Science, 70% are interested to study Management but 10% rejected in the interest to study both Science and Management whilst 400 students are interested to study both Science and Management. Based on this information, answer the following questions:

- i) Show the above information in a Venn-diagram.
- ii) Find how many students were participated in the survey.
- iii) Find the number of students who are interested to study Science only.

c) In a survey among people of a community, 65% ride motorcycle, 35% ride scooter but 20% ride both whereas 200 people ride both motorcycle and scooter. Using this information, answer the following questions:

- i) Show the above information in a Venn-diagram.
- ii) Find how many people were participated in the survey.
- iii) Find the number of people who ride motorcycles only.

7. a) Among 95 people of a community, it was surveyed that the ratio of the number of people who drink tea and coffee is 4:5, whereas 10 people drink tea but 15 do not drink either tea or coffee. Based on the information, answer the following questions:

- i) Show the information in a Venn-diagram.
- ii) Find the number of people who drinks exactly one of tea or coffee.
- iii) Find the number of people who drinks at least one; either tea or coffee.

b) In a survey of 64 students of a class, the ratio of number of students who like milk only and curd only is 2:1 whereas 16 like both based on this information, Answer the following questions:

- i) Show the above information in a Venn-diagram.
- ii) Find the number of students who like milk.
- iii) Find the number of students who like only one kind of drink.

c) In a conference of 320 participants, it was surveyed that 60 participants only sing and 100 only dance. If the number of people who do not do both is three times the number of people who do both. With the help of this information, answer the following questions:

- i) Show the above information in a Venn-diagram.
- ii) Find how many people do not do both genres.
- iii) Find the number of people who do one genre at most.

8. According to a survey of 200 people of a community, it was found that the ratio of the number of people who use laptop only and mobile only is 2:3, among them, 30% use both but 15% does not use both the gadgets. Based on this information, answer the following questions:

- i) Show the above information in a Venn-diagram.
 - ii) Find the number of people who uses laptop.
 - iii) Find how many people use one gadget at most.
9. Out of 300 players in a survey, one-third players play volleyball only. 60% of the remaining players play football only. But 60 players do not play both. Then, find the ratio of the number of players who play volleyball and football by using the Venn-diagram.
10. Among 65 players participated in a survey, 11 play volleyball only and 33 play cricket only. If the number of players who play cricket is the double of the number of players who play volleyball, find the number of players who play both and the number of players who does not play both by using Venn-diagram.
11. In a survey of 80 people, 60 like orange and 10 like both orange and apple. The number of people who likes orange is 5 times the number of people who likes apple. By using the Venn-diagram find the number of people who likes apples only and those who do not like both the fruits.

Project Work

Form group of five students each and go to different classes in your school. Find answers to the following questions:

Which of the following game do you like? (a) Cricket (b) Football (c) Cricket and football both (d) Others

Upon getting the informations, present it in the Venn-diagram and discuss it in the class.

Answers

1. (a) Show to your teacher. (b) $n(B)$, $n(A)$
(c) $n(A) + n(B) - n(A \cap B)$ or $n_0(A) + n_0(B) + n(A \cap B)$ (d) 12
2. (a) 20 (b) 30 (c) 80 (d) 90 (e) 110 (f) 60 (g) 15 (h) 125
3. (a) 20 (b) 300, 50 (c) 60 (d) 30
4. (a) (i) 60, (ii) 165 (b) (i) 200 (ii) 500 (c) (i) 18 (ii) 48
5. (a) (i) 150 (ii) 675 (b) (i) 40 (ii) 50 (c) (i) 60 (ii) 270
6. (a) (i) 1100 (ii) 220 (b) (i) 1000
(ii) 200 (c) (i) 1000 (ii) 450
7. (a) (i) 70 (ii) 80 (b) (i) 48 (ii) 48
(c) (i) 120 (ii) 280
8. (i) 104 (ii) 140
9. 6:7 10. 11, 10
11. 4, 6

1.2 Cardinality of Three Sets

Activity 2

The elements of three sets A, B and C are shown in the following two Venn-diagrams. Based on this, discuss the following questions:

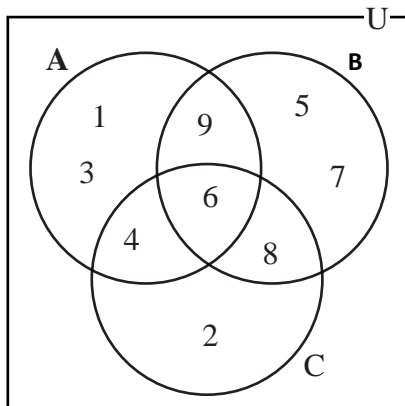


Figure 1

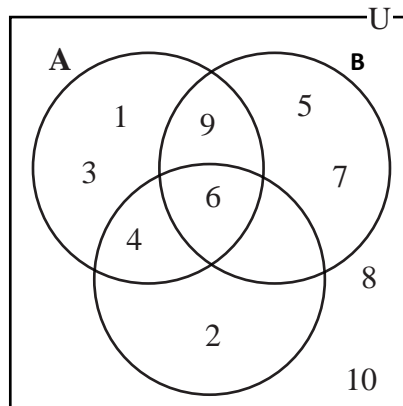


Figure 2

Observing the figure no. 1 and figure no. 2, write the elements of the sets A, B and C by listing method.

What are the cardinalities of each sets A, B and C in figure no. 1?

What are the cardinalities of each sets A, B and C in figure no. 2?

What are the values of $n(A \cup B \cup C)$ and $n(U)$ in figure no. 1?

What are the values of $n(A \cup B \cup C)$ and $n(U)$ in figure no. 2?

The conclusion from the discussion with friends can be shown below:

Figure 1	Figure 2
$A = \{1, 3, 4, 6, 9\} \therefore n(A) = 5$	$A = \{1, 3, 4, 6, 9\} \therefore n(A) = 5$
$B = \{5, 6, 7, 8, 9\} \therefore n(B) = 5$	$B = \{5, 6, 7, 9\} \therefore n(B) = 4$
$C = \{2, 4, 6, 8\} \therefore n(C) = 4$	$C = \{2, 4, 6\} \therefore n(C) = 3$
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $\therefore n(A \cup B \cup C) = 9$	$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$ $\therefore n(A \cup B \cup C) = 8$
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \therefore n(U) = 9$	$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $\therefore n(U) = 10$
From the above table, what is the relation between $n(A \cup B \cup C)$ and $n(U)$ in the figure no.1 and figure no. 2? And why?	

Activity 3

In a survey of students of a classroom, 40 students like orange, 35 like mango and 50 like banana. Along them, 15 like orange and mango, 20 like mango and banana, 25 like orange and banana, 5 like all the three fruits and 30 does not like either of the fruits. How shall the number of participants of the survey be found by using the Venn-diagram?

Here, O, M and B denote the set of students who like orange, mango and banana respectively.

At first, let us insert the number of students who likes all the three fruits $n(O \cap M \cap B) = 5$ and the number of students who does not like either of the fruits $n(\overline{O \cup M \cup B}) = 30$

After that, insert the number of students who likes exactly two fruits,

The number of students who likes orange and mango is $n(O \cap M) = 15$. Since 5 has already come in the number of students who likes all the three, insert the number of students who likes orange and mango only $n_0(O \cap M) = 15 - 5 = 10$. Again, since 5 has already come in the number of students who likes all the three, insert the number of students who likes mango and banana only, $n_0(M \cap B) = 20 - 5 = 15$ and the number of students who likes orange and banana only $n_0(O \cap B) = 25 - 5 = 20$.

Similarly,

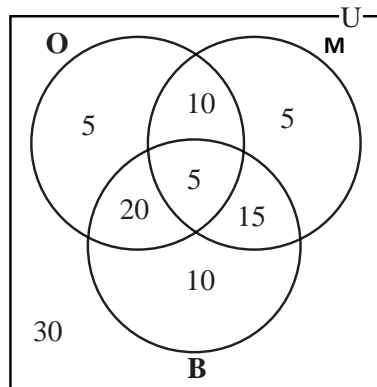
40 students like orange but 5 like all the three fruits; orange, mango and banana. Similarly, 10 like orange and mango only as well as 20 like orange and banana only. Thus, insert the number of students who likes orange only, $n_0(O) = 40 - (5 + 10 + 20) = 5$

35 students like mango but 5 like all the three fruits orange, mango and banana. Similarly, 10 likes orange and mango only as well as 15 like mango and banana only. Thus, insert the number of students who likes mango only, $n_0(M) = 35 - (5 + 10 + 15) = 5$

50 students likes banana but 5 likes all three fruits orange, mango and banana, 15 likes banana and mango only as well as 20 like orange and banana only. Thus, insert the number of students who like banana only, $n_0(B) = 50 - (5 + 15 + 20) = 10$.

Now, the total students $n(U) = 5 + 10 + 5 + 5 + 20 + 15 + 10 + 30 = 100$

\therefore The total number of participants is 100.

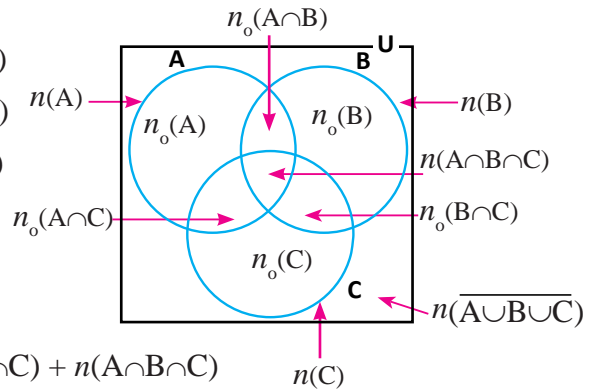


If A, B and C are overlapping sets, the following relations can be written from the Venn-diagram given alongside:

(a) $n_o(A \cap B) = n(A \cap B) - n(A \cap B \cap C)$

(b) $n_o(A \cap C) = n(A \cap C) - n(A \cap B \cap C)$

(c) $n_o(B \cap C) = n(B \cap C) - n(A \cap B \cap C)$



(d) $n(A) = n_o(A) + n_o(A \cap B) + n_o(A \cap C) + n(A \cap B \cap C)$

(e) $n(B) = n_o(B) + n_o(A \cap B) + n_o(B \cap C) + n(A \cap B \cap C)$

(f) $n(C) = n_o(C) + n_o(A \cap C) + n_o(B \cap C) + n(A \cap B \cap C)$

(g) $n(U) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) + n(A \cap B \cap C) + n(\overline{A \cup B \cup C})$

or, $n(U) = n(A \cup B \cup C) + n(\overline{A \cup B \cup C})$ where,

$$n(A \cup B \cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) + n(A \cap B \cap C)$$

From the Venn-diagram with three sets, the following relations can also be written:
If A, B and C are overlapping sets, then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Thus, $n(A \cup B \cup C) = n\{(A \cup B) \cup C\}$

$$= n(A \cup B) + n(C) - n\{(A \cup B) \cap C\}$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - n\{(A \cap C) \cup (B \cap C)\}$$

$$[\because (A \cup B) \cap C = (A \cap C) \cup (B \cap C)]$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - [n(A \cap C) + n(B \cap C) - n\{(A \cap C) \cap (B \cap C)\}]$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$[\because (A \cap C) \cap (B \cap C) = (A \cap B \cap C)]$$

$$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

If the sets are disjoint sets then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$

Example 1

If $n(U) = 120$, $n(A) = 48$, $n(B) = 51$, $n(C) = 40$, $n(A \cap B) = 11$, $n(B \cap C) = 10$, $n(A \cap C) = 9$, and $n(A \cap B \cap C) = 4$ then find $n(A \cup B \cup C)$ and $n(\overline{A \cup B \cup C})$. Also show the information in the Venn-diagram.

Solution

Here, given that

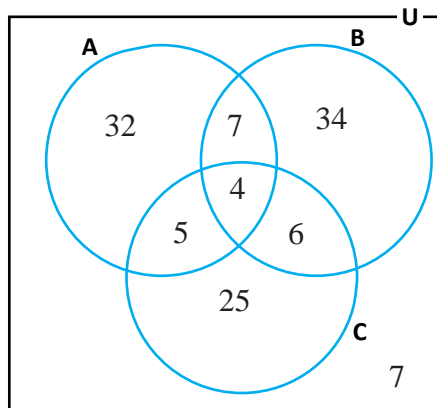
$$n(U) = 120, n(A) = 48, n(B) = 51, n(C) = 40, n(A \cap B) = 11, n(B \cap C) = 10, n(A \cap C) = 9, \text{ and } n(A \cap B \cap C) = 4$$

We know that

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) \\ &\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ &= 48 + 51 + 40 - 11 - 10 - 9 + 4 \\ &= 113 \end{aligned}$$

Again,

$$\begin{aligned} n(U) &= n(A \cup B \cup C) + n(\overline{A \cup B \cup C}) \\ \text{or, } 120 &= 113 + n(\overline{A \cup B \cup C}) \\ \text{or, } n(\overline{A \cup B \cup C}) &= 120 - 113 \\ \therefore n(\overline{A \cup B \cup C}) &= 7 \end{aligned}$$



The obtained information is shown in the Venn-diagram alongside.

Example 2

Among the 180 students who participated in SLC examination in 2071 from Nepal Madhyamik Vidhyaalaya, 86 passed in Science, 80 passed Maths and 76 passed in Nepali. Out of them, 26 passed in Science and Maths, 36 passed in Maths and Nepali as well as 32 passed in Science and Nepali but 20 did not pass all the subjects. Then,

- Show the given information in the Venn-diagram.
- Find the number of students who passed in all three subjects.

Solution

Here, $n(U)$, $n(M)$, $n(S)$ and $n(N)$ respectively denote the total number of students, the number of students who passed in Maths, the number of students who passed in Science and the number of students who passed in Nepali.

Here,

Total number of students $n(U) = 180$

The number of students who passed in Science, $n(S) = 86$

The number of students who passed in Maths, $n(M) = 80$

The number of students who passed in Nepali, $n(N) = 76$

The number of students who passed in Science and Maths, $n(S \cap M) = 26$

The number of students who passed in Maths and Nepali, $n(M \cap N) = 36$

The number of students who passed in Science and Nepali, $n(S \cap N) = 32$

The number of students who did not pass in any subject, $n(\overline{M \cup N \cup S}) = 20$

a) We know that,

$$n(U) = n(S) + n(M) + n(N) - n(S \cap M) - n(M \cap N) - n(S \cap N) + n(S \cap M \cap N) + n(\overline{S \cup M \cup N})$$

$$\text{or, } 180 = 86 + 80 + 76 - 26 - 36 - 32 + 20 + n(S \cap M \cap N)$$

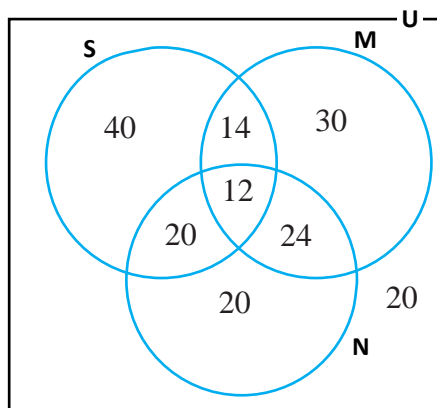
$$\text{or, } 180 = 168 + n(S \cap M \cap N)$$

$$\text{or, } n(S \cap M \cap N) = 180 - 168$$

$$\text{or, } n(S \cap M \cap N) = 12$$

Thus, the number of students who passed in all three subjects is 12.

b) Illustrating in the Venn-diagram,



Alternative Method

Suppose the number of students who passed in all three subjects $n(M \cap N \cap S) = x$

The information is shown in the Venn-diagram.

From venn-diagram,

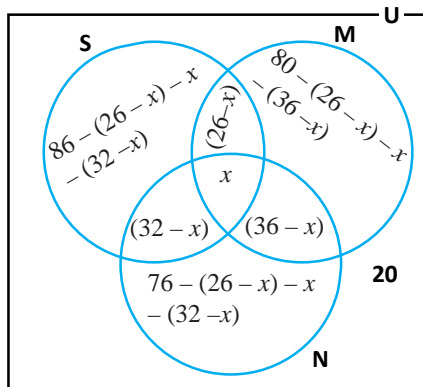
$$\{86 - (26 - x) - x - (32 - x)\} + (26 - x) + (32 - x) + x + (36 - x) + \{80 - (26 - x) - x - (36 - x)\} + \{76 - (36 - x) - x - (32 - x)\} + 20 = 180$$

$$\text{or, } (28 + x) + 94 - 2x + (18 + x) + (8 + x) + 20 = 180$$

$$\text{or, } 168 + x = 180$$

$$\text{or, } x = 180 - 168$$

$$\therefore x = 12$$



Thus the number of students who passed in all three subjects is 12.

Example 3

A school distributed medals for the students in different events of a competition. 36 got medals in dance, 12 in drama and 18 in music. If only 45 students got medals and 4 students got medals in all three events, then find the number of students who got medals in exactly two events.

Solution

Here, $n(A)$, $n(B)$ and $n(C)$ denote the number of students who got medals in dance, drama and music respectively. Then, we have

The number of students who got medals in dance, $n(A) = 36$

The number of students who got medals in drama, $n(B) = 12$

The number of students who got medals in music, $n(C) = 18$

The number of students who got medals in at least one event, $n(A \cup B \cup C) = 45$

The number of students who got medals in all three events, $n(A \cap B \cap C) = 4$

The number of students who got medals in exactly two events

$$n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) = ?$$

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\text{or, } 45 = 36 + 12 + 18 - n(A \cap B) - n(B \cap C) - n(A \cap C) + 4$$

$$\text{or, } 45 = 70 - n(A \cap B) - n(B \cap C) - n(A \cap C)$$

$$\text{or, } n(A \cap B) + n(B \cap C) + n(A \cap C) = 70 - 45$$

$$\therefore n(A \cap B) + n(B \cap C) + n(A \cap C) = 25$$

Now,

$$n_0(A \cap B) + n_0(B \cap C) + n_0(A \cap C)$$

$$= n(A \cap B) - n(A \cap B \cap C) + n(B \cap C) - n(A \cap B \cap C) + n(A \cap C) - n(A \cap B \cap C)$$

$$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 4 - 4 - 4$$

$$= 25 - 12$$

$$= 13$$

Alternatively,

$$\text{Let, } n_0(A \cap B) = a, n_0(B \cap C) = b, n_0(A \cap C) = c$$

We know that $a + b + c = ?$

The information is shown in the Venn-diagram alongside:

From the Venn-diagram, we have:

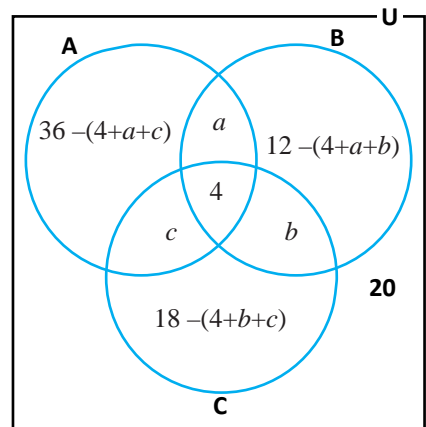
$$\{36 - (4 + a + c)\} + a + 4 + b + c + \{12 - (4 + a + b)\} + \{18 - (4 + b + c)\} = 45$$

$$\text{or, } (32 - a - c) + 4 + a + b + c + (8 - a - b) + (14 - b - c) = 45$$

$$\text{or, } 58 - a - b - c = 45$$

$$\text{or, } a + b + c = 58 - 45$$

$$\therefore a + b + c = 13$$

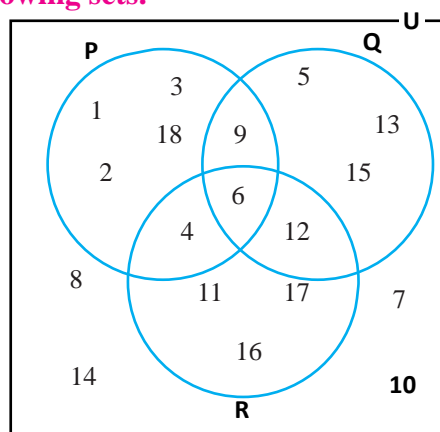


Hence, the number of students who got medals in only two events is 13.

Exercise 1.2

1. In the given Venn-diagram, the elements of sets P, Q and R are illustrated. Based on this, find the values of the following sets.

- (a) $n(P)$ (b) $n(Q)$
 (c) $n(P \cup Q \cup R)$ (d) $n_o(P)$
 (e) $n_o(R)$ (f) $n(P \cap R)$
 (g) $n(\overline{P \cup Q \cup R})$ (h) $n_o(P \cap Q)$
 (i) $n(P \cap Q \cap R)$



2. If $U = \{\text{positive integers less than } 30\}$,

$P = \{\text{multiples of 2 less than } 30\}$,

$Q = \{\text{multiples of 3 less than } 30\}$ and

$R = \{\text{multiples of 5 less than } 30\}$ then

Show the relation between the sets P, Q and R in a Venn-diagram and verify the following relations:

- (a) $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
 (b) $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)$
 (c) $n(P \cup Q \cup R) = n(P - Q) + n(Q - R) + n(R - P) + n(P \cap Q \cap R)$
3. (a) If $n(U) = 100$, $n(M) = 45$, $n(E) = 50$, $n(S) = 35$, $n(M \cap E) = 20$, $n(E \cap S) = 20$, $n(S \cap M) = 15$ and $n(M \cap E \cap S) = 5$, then find $n(\overline{M \cup E \cup S})$
 (b) If $n(U) = 105$, $n(A) = 40$, $n(B) = 35$, $n(C) = 30$, $n(A \cap B) = 15$, $n(B \cap C) = 12$, $n(A \cap B \cap C) = 6$ and $n(\overline{A \cup B \cup C}) = 30$, then find $n(A \cap C)$
 (c) If $n(U) = 120$, $n(M) = 50$, $n(E) = 40$, $n(S) = 45$, $n(M \cap E) = 15$, $n(E \cap S) = 15$, $n(S \cap M) = 15$ and $n(\overline{M \cup E \cup S}) = 15$, then find $n(M \cap E \cap S)$
 (d) If $n(A \cup B \cup C) = 105$, $n_o(A) = 25$, $n_o(B) = 25$, $n_o(C) = 15$, $n_o(A \cap B) = 15$, $n_o(A \cap C) = 10$ and $n(A \cap B \cap C) = 10$, then find $n_o(B \cap C)$
4. a) **Out of 90 students who participated in an examination, 43 passed in Science, 40 in Mathematics and 38 in Nepali. Among them, 13 passed in Science and Mathematics, 18 in Mathematics and Nepali as well as 16 passed in Science and Nepali. Using the information, answer the following questions:**
 i) Show the information in the Venn-diagram.
 ii) Find the number of students who did not pass in any subject.

- b) In a survey of a group, 60 like tea, 45 like coffee, 30 like milk, 25 like the coffee and tea, 20 like milk and tea, 15 like coffee and milk and 10 like all three drinks. Based on the information, answer the following questions:
- Show the information in a Venn-diagram.
 - Find how many people were surveyed.
- c) In a survey among 60 students, 23 played volleyball, 15 played basketball and 20 played cricket. If 7 played volleyball and basketball, 5 played basketball and cricket, 4 played volleyball and cricket but 15 played neither of the games. Based on this information, answer the following questions:
- Show the information in Venn-diagram.
 - Find how many students played all the three games.
 - How many played only volleyball and cricket.
5. Out of total students who participated in an examination, 40% passed in Science, 45% in Mathematics and 50% in Nepali. Similarly, 10% passed in Science and Mathematics, 20% in Mathematics and Nepali as well as 15% in Science and Nepal but 5% failed in all the three subjects. Based on the information, answer the following questions:
- Find the percentage of students who passed in all the three subjects.
 - find the percentage of students who passed in only one subject.
 - Find the percentage of students who passed in only two subjects.
 - Find the percentage of students who passed in at least one subject.
 - Show the information in a Venn-diagram
6. The following information was obtained from the survey on a questionnaire whether they read Yubamanch or Madhupark or Muna conducted among some people in a community:
- 30 read Yubamanch, 25 read Madhupark, 15 read both Yubamanch and Muna, 12 read both Yubamanch and Madhupark and 9 read Madhupark only, 11 read Muna only, 5 read Yubamanch and Madhupark only but 10 read neither of the newspapers. Based on this information, answer the following questions:
- Show the information in a Venn-diagram.
 - Find the total number of people who participated in the survey.

- c) Find the number of people who read exactly two newspapers.
- d) Find the number of people who read Muna.

7. In a survey among 90 people, who were asked which language film they like, 48 like Nepali, 40 like English, 31 like Hindi, 24 like Nepali and English, 19 like Hindi and English, 6 like of all the three languages and 21 did not like any. Then,

- a) How many people did like both Nepali and Hindi films?
- b) How many people did not like Hindi film?
- b) How many people did not like both Nepali and Hindi films?

Project work

It is informed from school administration that your class has to decide the destination of educational excursion to be organized by your school. For this, form groups of all students containing 5 members each. Collect the answers from all students of different classes by asking the following questions:

Which place do you like to go for your educational excursion?

(a) Pokhara (b) Lumbini (c) Kathmandu (d) Pokhara and Lumbini (e) Lumbini and Kathmandu (f) Pokhara and Kathmandu (g) Pokhara, Lumbini and Kathmandu (h) other places than these

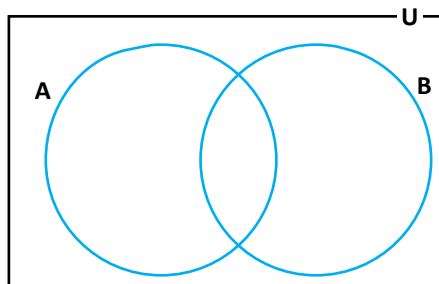
Find the number of students who like to visit only one place by illustrating the information obtained from the students answers in a Venn-diagram. Then, in turn, present the group work in the classroom.

Answers

- | | | | | | |
|----|-------------|-------------|------------|---------|-------|
| 1. | (a) 7 | (b) 6 | (c) 14 | (d) 4 | (e) 3 |
| | (f) 2 | (g) 4 | (h) 1 | (i) 1 | |
| 3. | (a) 20 | (b) 9 | (c) 15 | (d) 5 | |
| 4. | (a) (ii) 10 | (b) (ii) 85 | (c) (ii) 3 | (iii) 1 | |
| 5. | (a) 5% | (b) 60% | (c) 30%, | (d) 95% | |
| 6. | (b) 64 | (c) 17 | (d) 30 | | |
| 7. | (a) 13 | (b) 50 | (c) 24 | | |

Mixed Exercise

1. There are two overlapping sets A and B shown alongside in a Venn-diagram where $n_o(A) = 16 + x$, $n_o(B) = 5x$, $n(A \cap B) = y$ and $n(\overline{A \cup B}) = x$. Then, answer the following questions:



- Insert the above information by drawing a Venn-diagram.
 - If $n(A) = n(B)$, find the value of $n(\overline{A \cup B})$.
 - If $n(U) = 50$, find the ratio of $n(A \cap B)$ and $n(\overline{A \cup B})$.
2. A and B are the subsets of Universal set U such that $n(U) = 100$, $n(A - B) = 32 + x$, $n(B - A) = 5x$, $n(A \cap B) = x$ and $n(\overline{A \cup B}) = y$.

- Show the above information in a Venn-diagram.
- If $n(A) = n(B)$, find the value of $n(A \cap B)$.
- Find the value of $n(\overline{A \cup B})$.

By what percent $n(A \cap B)$ is more or less than $n(\overline{A \cup B})$? Find.

3. According to a survey of 93 women of a community, the number of women engaged in agriculture is 80 and that in sewing is 71 but the number of women engaged in other job is 10.
- Present the information in Venn-diagram by finding the cardinality of sets.
 - Find how many women were engaged in both agriculture and sewing.
 - By how many times the number of women engaged in agriculture only is more than the number of women engaged in sewing only? Calculate it.
4. According to a survey of 1000 farmers in a community, the number of farmers cultivating potatoes was 800 and the number of farmers cultivating tomatoes was 500 but 50 crops other than these.
- Show the information in Venn-diagram by finding the cardinality of sets.
 - Find the number of farmers who cultivate both.
 - Write the number of farmers cultivate potato only and that of tomato only in ratio.

- 5. In a survey of 400 people of a community, it was found that the ratio of the people having motorcycle license only and car license only was 5:3. Among them, one -fourth of the people had license of vehicles but 60 did not have any license.**

- Show the above information in a Venn-diagram.
- From the above information, how many people had license of each vehicle?
- Find the number of people who had license of motorcycle.

- 6. The information of the students of a school whether they like volleyball, football or cricket is as follows:**

- 30 like volleyball and football, 20 like volleyball and cricket as well as 35 like football and cricket.
 - 10 like all the three games; football, volleyball and cricket but 5 like neither of the games.
- Represent the given information in cardinality of sets.
 - Show the information in Venn-diagram.
 - Find the total number of students in the school.
 - What percentage of students like football only?

- 7. The following information from a survey of 45 people of different lingual group of a community is obtained:**

25 speak Nepal Bhasa, 23 speak Tamang and 15 speaks Maithili.

12 speaks Nepal Bhasa and Tamang, 5 speak Nepal Bhasa and Maithili as well as 10 speak Tamang and Maithili. 4 speak all the three languages.

Based on the information, answer the following questions:

- Show the above information in a Venn-diagram.
- Find how many people speak the language other than these languages; Nepal Bhasa, Tamang and Maithili.
- How many people speak only one language? Find.
- How many people speak both Nepal Bhasa and Tamang but do not speak the Maithili language?

Answers

1. (b) 4 (c) 3:2
2. (b) 8 (c) 12 (d) more than 50%
3. (b) 68 (c) 4
4. (b) 350 (c) 3:1
5. (b) 250 and 190 (c) 250
6. (c) 230 (d) 19.57%
7. (b) 5 (c) 21 (d) 8

Lesson 2

Compound Interest

2.0 Review

Discuss the following questions and present the conclusion in the classroom:

A person borrowed Rs. 10,00,000 for a business from a commercial bank.

- How much interest should he pay to the bank after 2 years at the rate of 8% per annum?
- What total sum of money should he pay after 2 years? What is the sum of money called?
- If he had borrowed the money for 5 years, how much interest should be paid at the same interest rate?

The conclusion drawn from the discussion is presented below:

The sum of money paid to the bank at the rate of 8% p.a. after 2 years is called simple interest.

The total sum of borrowed money and additional interest is called amount.

The following formula is used for the calculation of simple interest: Simple interest (SI) = $\frac{PTR}{100}$

2.1. Introduction of Compound Interest

2.1.1 Compound Interest Compounded Annually

Activity1

A teacher borrowed Rs. 5,00,000 for 2 years at the interest rate of 12% per annum from a commercial bank. He could not pay the interest at the end of the first year so that he had to pay interest of interest of the first year in the second year. In this situation, discuss the following questions and present the conclusion in the classroom:

- How much interest to be paid in the first year?
- What is the principal of the second year? Find it.
- How much interest of second year should be paid?

The conclusion from the discussion is mentioned below:

$$\begin{aligned}\text{The interest to be paid in the first year (I}_1\text{)} &= \frac{PTR}{100} = \frac{5,00,000 \times 1 \times 12}{100} \\ &= \text{Rs. 60,000}\end{aligned}$$

Since he could not pay the interest in the first year,

The principal for the second year (P_1) = Rs. 5,00,000 + Rs. 60,000 = Rs. 5,60,000

$$\text{Thus, the interest for the second year } (I_2) = \frac{P_1 T R}{100} = \frac{5,60,000 \times 1 \times 12}{100} \\ = \text{Rs. } 67,200$$

Hence, the total interest to be paid by the teacher = Rs. 60,000 + Rs. 67,200 = Rs. 1,27,200

Why is there difference between the interests for the first year and for the second year?

If the interest of a principal after every year or after certain period (yearly, half yearly or terminal) is calculated and added to the principal and again the interest is calculated, then the interest so obtained is called compound interest. The sum of the principal and compound interest is called compound amount.

Activity 2

Bishal and Badri borrowed Rs. 30,000 each from a bank for 3 years at the rate of 10% yearly with the promise that Bishal should pay simple interest and Badri should pay compound interest. Discuss the following questions based on the information:

- How much total interest should Bishal pay?
- How much total interest should Badri pay?
- Who should pay more interest?

Should be paid by Bishal (SI)	Should be paid by Badri (CI)
For the first year, Principal (P_1) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_1) = 1 year Interest (I_1) = $\frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000	For the first year, Principal (P_1) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_1) = 1 year Interest (I_1) = $(I_1) = \frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000
For the second year, Principal (P_2) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_2) = 1 year Interest (I_2) = $\frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000	For the second year, Principal (P_2) = Rs. 30,000 + Rs. 3,000 = Rs. 33,000 Rate of interest (R) = 10%, Time (T_2) = 1 year Interest (I_2) = $\frac{33,000 \times 10 \times 1}{100}$ = Rs. 33,00

<p>For the third year,</p> <p>Principal (P_3) = Rs. 30,000</p> <p>Rate of interest (R) = 10%, Time (T_3) = 1 year</p> <p>Interest (I_3) = $\frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000</p> <p>Total interest (I) = $I_1 + I_2 + I_3$ $= 3000 + 3000 + 3000$ $=$ Rs. 9,000</p>	<p>For the third year,</p> <p>Principal (P_3) = Rs. 33,000 + Rs. 3,300 $=$ Rs. 36,300</p> <p>Rate of interest (R) = 10%, Time (T_3) = 1 year</p> <p>Interest (I_3) = $\frac{36,300 \times 10 \times 1}{100}$ = Rs. 3,630</p> <p>v (I) = $I_1 + I_2 + I_3$ $= 3000 + 3300 + 3630$ $=$ Rs. 9,930</p>
---	---

Yeh! The compound interest is more than simple interest even at the same rate of interest. In compound interest, the principal of every year is equal to the sum of the interest and the principal of the preceding year. On which, the interest of interest is imposed.



While calculating simple interest, the principal is same for each year. But while calculating compound interest, the principals are different every year (the principal for the second year is the amount of the first year, the principal for the third year is the amount of second year, etc.). The compound interest of the same principal is more at the same rate and time than the simple interest of the same principal.

Activity 3

If a principal (P) is deposited at the rate of $R\%$ per annum for T years, then discuss the following questions:

- How much interest is at the end of the first year?
- How much interest is at the end of the second year?
- In this way, how much interest is at the end of the third year?
- How much interest is at the end of T years?

The interest at the end of the first year (I_1) = $\frac{P \times T \times R}{100} = \frac{P \times 1 \times R}{100} = \frac{PR}{100}$

The amount at the end of the first year (A_1) = $P_1 + I_1 = P + \frac{PR}{100} = P \left(1 + \frac{R}{100} \right)$

We know that,

The amount at the end of the first year = Principal for the second year

Thus, the principal of the second year (P_2) = $P\left(1 + \frac{R}{100}\right)$

$$\begin{aligned}\text{The interest of the second year } (I_2) &= \frac{P\left(1 + \frac{R}{100}\right) \times T \times R}{100} \\ &= \frac{P\left(1 + \frac{R}{100}\right) \times 1 \times R}{100} = P\left(1 + \frac{R}{100}\right) \times \frac{R}{100}\end{aligned}$$

$$\begin{aligned}\text{The amount at the end of the second year } (A_2) &= P\left(1 + \frac{R}{100}\right) + P\left(1 + \frac{R}{100}\right) \times \frac{R}{100} \\ &= P\left(1 + \frac{R}{100}\right) \left(1 + \frac{R}{100}\right) \\ &= P\left(1 + \frac{R}{100}\right)^2\end{aligned}$$

In this way, the amount at the end of second year = principal for the third year

$$\text{Thus, principal for the third year } (P_3) = P\left(1 + \frac{R}{100}\right)^2$$

$$\text{Interest of the third year } (I_3) = \frac{P_3 \times T \times R}{100} = \frac{P\left(1 + \frac{R}{100}\right)^2 \times 1 \times R}{100}$$

$$= P\left(1 + \frac{R}{100}\right)^2 \times \frac{R}{100}$$

$$\begin{aligned}\text{The amount at the end of the third year } (A_3) &= P_3 + I_3 \\ &= P\left(1 + \frac{R}{100}\right)^2 + \left(1 + \frac{R}{100}\right)^2 \times \frac{R}{100} \\ &= P\left(1 + \frac{R}{100}\right)^2 \left(1 + \frac{R}{100}\right) = P\left(1 + \frac{R}{100}\right)^3\end{aligned}$$

From the above calculation, what will be the compound amount (CA) at the end of T years?

$$\text{Hence, the compound amount at the end of T years } (CA) = P\left(1 + \frac{R}{100}\right)^T$$

Similarly, compound interest (CI) = Amount (CA) – Principal (P)

$$CI = P\left(1 + \frac{R}{100}\right)^T - P, \quad CI = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

Activity 4

How shall we calculate compound interest and compound amount of the following conditions?

- The rate of interest is different every year.
- Time in T years and M months.
- If the interest is to be calculated half yearly.
- If the interest is to be calculated terminally.

(a) The rate of interest is different every year

How can we calculate compound interest and compound amount in 3 years such that the rate of interest for the first year $R_1\%$, the second year $R_2\%$ and the third year $R_3\%$ respectively?

$$\text{Compound Amount (CA)} = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \text{ and,}$$

$$\text{Compound Interest (CI)} = P \left[\left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) - 1 \right]$$

(b) If Time in 'T' years and 'M' months is given, how shall compound interest and compound amount be computed?

$$\text{Compound Amount (CA)} = P \left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right)$$

$$\text{Compound Interest (CI)} = P \left[\left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right) - 1 \right]$$

(c) If the interest is to be calculated half yearly

If the interest is to be calculated half yearly, the rate of interest $R\%$ per annum being calculated as $\frac{R}{2}\%$ per semi-annual and Time (T) being calculated as $2 \times T$.

$$\text{Now, compound amount (CA) is } P \left(1 + \frac{R}{2 \times 100}\right)^{2T} = P \left(1 + \frac{R}{200}\right)^{2T}$$

$$\text{And compound interest CI is } P \left(1 + \frac{R}{2 \times 100}\right)^{2T} - P = P \left[\left(1 + \frac{R}{200}\right)^{2T} - 1 \right].$$

A financial institution releases the interest at the rate of $R\%$ yearly in the account of depositors two times, the first of Magh and the first of Shrawan every year. In this way, the interest of following 6 months is computed at the principal as the sum of the interest of the previous 6 months and the principal. This kind of interest is called half yearly compound interest.

What is the difference between yearly compound interest and half yearly compound interest? Discuss.

(d) If the interest is to be calculated terminally

If the principal and interest in a bank is to be computed terminally, then rate of interest $R\%$ is termed as $\frac{R}{4}\%$ per quarter and time (T) is termed as $4T$ per quarter. In this case,

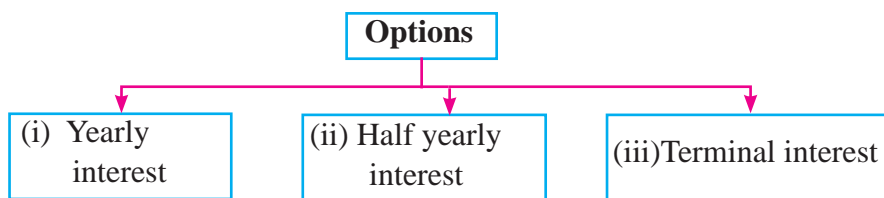
$$\text{Compound amount (CA)} = P \left(1 + \frac{R}{4 \times 100} \right)^{4T} = P \left(1 + \frac{R}{400} \right)^{4T}$$

$$\text{And compound interest (CI)} = P \left(1 + \frac{R}{4 \times 100} \right)^{4T} - P = P \left[\left(1 + \frac{R}{400} \right)^{4T} - 1 \right]$$

Terminal compound interest is also computed as the yearly compound interest and half yearly compound interest.

Activity 5

Your uncle has planned to invest Rs. 1,00,000 for a year at the rate of 15% compound interest per annum. He has been thinking about whether to invest it in yearly interest, half yearly interest or terminal interest to get more benefit. Which alternative would you suggest him to choose? Why? Discuss.



Here, principal (P) = Rs. 1,00,000, the rate of interest (R) = 15%, time (T) = 1 year

a) According to the first option,

$$\begin{aligned} \text{Compound interest (CI)} &= 1,00,000 \left[\left(1 + \frac{15}{100} \right)^1 - 1 \right] \\ &= 1,00,000 \left[\left(\frac{115}{100} \right) - 1 \right] = \text{Rs. } 15,000 \end{aligned}$$

b) According to the second option,

$$\text{Compound interest (CI)} = 1,00,000 \left[\left(1 + \frac{15}{200} \right)^{2 \times 1} - 1 \right] = \text{Rs. } 15,562.50$$

c) According to the third option,

$$\text{Compound interest (CI)} = 1,00,000 \left[\left(1 + \frac{15}{400} \right)^{4 \times 1} - 1 \right] = \text{Rs. } 15,865.04$$

Out of these three options, I would suggest him to invest according to the third option because the interest of this option is more than that of other two options. Accordingly, the interest is Rs. 865.04 more than the first option and Rs. 562.50 more than the second option.

While calculating the interest of the same sum at the same rate of interest at the same time, then terminal compound interest is more than half yearly compound interest and half yearly compound interest is greater than yearly compound interest.

Example 1

What will be the compound interest and compound amount of Rs. 2,000 at the interest rate of 12% p.a. in 2 years? Find the compound interest without using formula.

Solution

Here,

Principal (P_1) = Rs. 2,000

Rate of interest (R) = 12% p.a.

Time (T) = 2 years

At the end of the first year, simple interest (I_1) = $\frac{P_1 T R}{100} = \frac{2000 \times 1 \times 12}{100} = \text{Rs. } 240$

Principal for the second year (P_2) = Amount at the end of the first year
 $= P_1 + I_1 = \text{Rs. } (2000 + 240)$

$= \text{Rs. } 2,240$

Again, simple interest for the second year (I_2) = $\frac{2240 \times 1 \times 12}{100} = \text{Rs. } 268.8$

Thus, the compound interest at the end of 2 years (CI) = $I_1 + I_2 = 240 + 268.8 = \text{Rs. } 508.8$

Compound amount (CA) = $P_1 + CI = 2000 + 508.8 = \text{Rs. } 2,508.8$

Example 2

Find the compound interest and compound amount of the borrowed amount of Rs. 25,000 which is paid in exactly 3 years at the rate of yearly compound interest rate 12%.

Solution

Principal (P) = Rs. 25,000

Rate of interest (R) = 12% per year

Time (T) = 3 year

Compound interest and compound amount = ?

According to the formula,

$$\begin{aligned}\text{Compound interest (CI)} &= \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\ &= 25,000 \left[\left(1 + \frac{12}{100} \right)^3 - 1 \right] \\ &= 25,000 \left[\left(\frac{112}{100} \right)^3 - 1 \right] \\ &= 25,000 [1.404928 - 1] \\ &= 25,000 \times 0.404928 \\ &= 10123.20\end{aligned}$$

Again, compound amount (CA) = P + CI

$$= \text{Rs. } 25,000 + \text{Rs. } 10,123.20 = \text{Rs. } 35,123.20$$

Example 3

A man borrowed Rs. 32,000 from his friend at the rate of simple interest of 12.5% per annum. He lent the whole sum to a shopkeeper at the same rate of compound interest. How much more money will he get in 3 years? Find.

Solution

Here,

Principal (P) = Rs. 32,000

Rate of interest (R) = 12.5%

Time (T) = 3 year

Simple interest (SI) = ?

Compound interest (CI) = ?

$$\text{Case I : simple interest (SI)} = \frac{P T R}{100} = \frac{32000 \times 3 \times 12.5}{100} = \text{Rs. } 12,000$$

$$\begin{aligned}\text{Case II : compound interest (CI)} &= P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\ &= 32,000 \left[\left(1 + \frac{12.5}{100} \right)^3 - 1 \right]\end{aligned}$$

$$\begin{aligned}
&= 32,000 \left[\left(\frac{112.5}{100} \right)^3 - 1 \right] \\
&= 32,000 [1.423828125 - 1] \\
&= 32,000 \times 0.423828125 \\
&= \text{Rs. } 13562.50
\end{aligned}$$

∴ The more money received by the man = CI – SI = 13562.50 – 12000 = Rs. 1562.50

Example 4

Sameer decided to invest Rs. 5,000 at the rate of 8% per annum for 2 years. For this, he has two safe alternative. The first alternative is to get half yearly compound interest and the second alternative is to get yearly compound interest. If you were to suggest him, which alternative would you suggest? Write with reason.

Solution

Here,

Principal (P) = Rs. 5,000

Rate of interest (R) = 8%

Time (T) = 2 year

a) According to the first alternative,

$$\begin{aligned}
\text{Half yearly compound interest (CI}_1\text{)} &= P \left[\left(1 + \frac{R}{200} \right)^{2T} - 1 \right] \\
&= 5,000 \left[\left(1 + \frac{8}{200} \right)^{2 \times 2} - 1 \right] \\
&= 5,000 \left[\left(\frac{208}{200} \right)^{2 \times 2} - 1 \right] \\
&= 5,000 [1.16985856 - 1] \\
&= 5,000 \times 0.16985856 \\
&= \text{Rs. } 849.29
\end{aligned}$$

b) According to the second alternative,

$$\begin{aligned}
 \text{Yearly compound interest (CI}_2\text{)} &= P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\
 &= 5000 \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right] \\
 &= 5000 \left[\left(\frac{108}{100} \right)^2 - 1 \right] \\
 &= 5000 [(1.08)^2 - 1] \\
 &= 5000 (1.1664 - 1) \\
 &= 5000 \times 0.1664 \\
 &= \text{Rs. } 832
 \end{aligned}$$

The difference between half yearly compound interest and yearly compound interest is given by $CI_1 - CI_2 = \text{Rs. } 849.29 - \text{Rs. } 832 = \text{Rs. } 17.29$

\therefore Since half yearly compound interest is Rs. 17.29 more than yearly compound interest, I would suggest him to invest in first alternative.

Example 5

A twelve-grade student invest Rs. 10,000 for 2 years at the rate of yearly compound interest. If the compound interest in 1 year is Rs. 11,200

- (a) Find the rate of yearly compound interest.
 (b) Find the yearly compound amount at the end of the second year.

Solution

Here, (a) Principal (P) = Rs. 10,000

Compound amount at the end of the first year (CA) = Rs. 11,200

Time (T_1) = 1 year

Using the formula, $(CA) = P \left(1 + \frac{R}{100} \right)^T$

$$\text{or, } 11200 = 10000 \left(1 + \frac{R}{100} \right)^1$$

$$\text{or, } \frac{11200}{10000} = 1 + \frac{R}{100}$$

$$\text{or, } 1.12 = 1 + \frac{R}{100}$$

$$\text{or, } 0.12 \times 100 = R$$

$$\text{or, } R = 12\%$$

Thus, the rate of yearly compound interest is 12%.

(b) The compound amount at the end of the second year (CA) = ?

Time (T) = 2 year

$$\begin{aligned}\text{Compound amount (CA)} &= P \left(1 + \frac{R}{100} \right)^T \\ &= 10,000 \left(1 + \frac{12}{100} \right)^2 \\ &= 10,000 \left(\frac{112}{100} \right)^2 \\ &= \text{Rs. } 12,544\end{aligned}$$

Thus, the compound amount at the end of the second year = Rs. 12,544

Example 6

Find the compound interest and compound amount of Rs. 2,00,000 invested for 3 years such that the rate of interest for the first year is 8% p.a., for the second it is year 10% p.a. and for the third year it is 12%.

Solution

Here,

Principal (P) = Rs. 2,00,000

Time (T) = 3 year

Rate of interest for the first year (R_1) = 8%

Rate of interest for the second year (R_2) = 10%

Rate of interest for the third year (R_3) = 12%

Compound amount (CA) = ?

Compound interest (CI) = ?

According to the formula, compound amount

$$\begin{aligned}(\text{CA.}) &= P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right) \\ &= 2,00,000 \left(1 + \frac{8}{100} \right) \left(1 + \frac{10}{100} \right) \left(1 + \frac{12}{100} \right) \\ &= 2,00,000 \left(\frac{108}{100} \right) \left(\frac{110}{100} \right) \left(\frac{112}{100} \right) \\ &= \text{Rs. } 2,66,112\end{aligned}$$

Thus, compound interest (CI) = CA – P = Rs. 2,66,112 – Rs. 2,00,000 = Rs. 66,112

Example 7

A sum amounts to Rs. 14,520 in 2 years and Rs. 15,972 in 3 years at a certain rate of annual compound interest. Then,

- (a) Find the rate of compound interest.
- (b) Find what is the principal.

Solution

Here,

Rate of interest (R) = R% and Principal (P) = Rs. x

Case I:

Compound amount (CA₁) = Rs. 14,520

Time (T) = 2 years

Using the formula, compound amount (CA₁) = $P \left(1 + \frac{R}{100} \right)^T$

$$\text{Rs. } 14,520 = x \left(1 + \frac{R}{100} \right)^2 \dots\dots\dots (i)$$

Case II:

Compound amount (CA₂) = Rs. 15,972

Time (T) = 3 years

Using the formula, compound amount (CA₂) = $P \left(1 + \frac{R}{100} \right)^T$

$$\text{Rs. } 15,972 = x \left(1 + \frac{R}{100} \right)^3 \dots\dots\dots (ii)$$

Dividing equation (ii) by (i), we get

$$\text{or, } \frac{15,972}{14,520} = \frac{x \left(1 + \frac{R}{100} \right)^3}{x \left(1 + \frac{R}{100} \right)^2}$$

$$\text{or, } 1.10 = 1 + \frac{R}{100}$$

$$\text{or, } 1.10 - 1 = \frac{R}{100}$$

$$\text{or, } 0.10 \times 100 = R$$

$$\text{or, } R = 10\%$$

∴ Rate of interest (R) = 10% p.a.

Again, putting $(R) = 10\%$ in equation (i), we get

$$14520 = x \left(1 + \frac{R}{100} \right)^2$$

$$\text{or, } 14520 = x \left(1 + \frac{10}{100} \right)^2$$

$$\text{or, } 14520 = x \times 1.21$$

$$\text{or, } \frac{14520}{1.21} = x$$

$$x = 12000$$

\therefore Principal $(P) = x = \text{Rs. } 12,000$

Alternatively,

Compound amount of 2 years $(CA_1) = \text{Rs. } 14,520$

$$\text{or, } P \left(1 + \frac{R}{100} \right)^2 = \text{Rs. } 14,520$$

$$x \left(1 + \frac{R}{100} \right)^2 = \text{Rs. } 14,520 \dots\dots\dots(i)$$

Again, compound amount of 3 years $(CA_2) = \text{Rs. } 15,972$

$$\text{or, } P \left(1 + \frac{R}{100} \right)^3 = \text{Rs. } 15,972$$

$$\text{or, } x \left(1 + \frac{R}{100} \right)^3 = 15,972 \dots\dots\dots(ii)$$

From the equation (i) and (ii)

$$\text{or, } x \left(1 + \frac{R}{100} \right)^2 \left(1 + \frac{R}{100} \right) = 15,972$$

$$14,520 \left(1 + \frac{R}{100} \right) = 15,972 \text{ [from the equation (i)]}$$

$$\text{or, } \left(1 + \frac{R}{100} \right) = \frac{15,972}{14,520}$$

$$\text{or, } 1 + \frac{R}{100} = 1.10$$

$$\text{or, } \frac{R}{100} = 1.10 - 1$$

$$\text{or, } R = 0.10 \times 100$$

$$\therefore R = 10\% \text{ p.a.}$$

Now, putting $R = 10\%$ in the equation (i), we get

$$x \left(1 + \frac{10}{100} \right)^2 = \text{Rs. } 14,520$$

$$\text{or, } x \left(\frac{110}{100} \right)^2 = 14,520$$

$$\text{or, } x \times 1.21 = 14,520$$

$$\text{or, } x = \frac{14,520}{1.21}$$

\therefore Principal (P) = Rs. 12,000 and rate of interest (R) = 10% p.a.

Example 8

A person deposited Rs. 2,00,000 in a development bank for 2 years to get the half yearly compound interest at the rate of 10% per annum after deducting the 5% tax on the interest. But right after a year, bank has changed the policy and decided to accomplish the interest terminally at the same rate of interest.

- (a) Find the interest of the first year by deducting the tax.
- (b) What would be the interest of the second year after deducting the tax?
- (c) What is the difference between interests of the first year and second year after deducting the tax? Find.
- (d) After deducting the tax, by what percentage the interest of the first year differ from the interest of the second year?

Solution

Here, principal (P) = ₹. 2,00,000

Rate of interest (R) = 10% p.a.

- (a) For the first year, the half yearly compound interest

$$\begin{aligned} CI_1 &= P \left[\left(1 + \frac{R}{200} \right)^{2T} - 1 \right] \\ &= 200000 \left[\left(1 + \frac{10}{200} \right)^{2 \times 1} - 1 \right] \\ &= 2,00,000 \left[\left(\frac{210}{200} \right)^{2 \times 1} - 1 \right] \end{aligned}$$

$$= 2,00,000 [1.1025 - 1]$$

$$= 2,00,000 \times 0.1025 = \text{Rs. } 20,500$$

$$\text{After deducting 5\% tax, } CI_1 = \text{Rs. } 20,500 - \text{Rs. } 20,500 \times \frac{5}{100}$$

$$= \text{Rs. } 20,500 - \text{Rs. } 1,025 = \text{Rs. } 19,475$$

\therefore Interest of the first year after deducting tax is Rs. 19,475

$$(b) \text{ Compound amount after a year (CA) } = \text{Rs. } 2,00,000 + \text{Rs. } 19,475 = \text{Rs. } 2,19,475$$

Now, principal for the second year = compound amount of the first year = Rs. 2,19,475

$$\text{According to the quarterly compound interest, } (CI_2) = P \left[\left(1 + \frac{R}{400} \right)^{4T} - 1 \right]$$

$$= 2,19,475 \left[\left(1 + \frac{10}{400} \right)^{4 \times 1} - 1 \right]$$

$$= 2,19,475 [0.1038128906]$$

$$= 22,784.33$$

$$= 22,784.33$$

$$\text{Again, the interest after deducting 5\% tax } CI_2 = 22,784.33 - 22,784.33 \times \frac{5}{100}$$

$$= 22,784.33 - 1,139.21$$

$$= \text{Rs. } 21,645.12$$

\therefore The interest after deducting 5% tax is Rs. 21,645.12

$$(c) \text{ The difference in interests } = CI_2 - CI_1 = 21,645.12 - 19,475$$

$$= \text{Rs. } 2,170.12$$

$$(d) \text{ The difference of interests in percentage } = \frac{CI_2 - CI_1}{CI_1} \times 100\%$$

$$= \frac{2170.12}{19475} \times 100\% = 11.14\%$$

Thus, the interest of the second year differs by 11.14% than that of the first year.

Example 9

A commercial bank releases a loan of Rs. 52,500 to Babulal and Jibanlal at the rate of yearly 10% compound interest. If the compound amount paid by Babulal in 2 years is the same as the compound amount paid by Jibanlal in 3 years, how much loan did each of them borrow from the bank?

Solution

Let, the loan amount of Babulal (P_1) = Rs. x

The loan amount of Jibanlal (P_2) = Rs. $(52,500 - x)$

Here, the compound amount to be paid by Babulal in 2 years

$$\begin{aligned} CA_1 &= P_1 \left(1 + \frac{R}{100} \right)^T \\ &= x \left(1 + \frac{10}{100} \right)^2 \\ &= x \left(\frac{110}{100} \right)^2 \\ &= 1.21x \end{aligned}$$

Similarly, the compound amount to be paid by Jibanlal in 3 years

$$\begin{aligned} CA_2 &= P_2 \left(1 + \frac{R}{100} \right)^T \\ &= (52,500 - x) \left(1 + \frac{10}{100} \right)^3 \\ &= (52,500 - x) \left(\frac{110}{100} \right)^3 \\ &= (52,500 - x) 1.331 \\ &= 69,877.5 - 1.331x \end{aligned}$$

Now, according to the question,

$$\begin{aligned} CA_1 &= CA_2 \\ 1.21x &= 69,877.5 - 1.331x \\ \text{or, } 1.21x + 1.331x &= 69,877.5 \\ \text{or, } 2.541x &= 69,877.5 \end{aligned}$$

$$\therefore x = 27,500$$

Thus, Babulal borrowed Rs. 27,500 and Jibanlal borrowed Rs. $(52,500 - 27,500) =$ Rs. 25000.

Exercise 2.1

1. Define:

- (a) Yearly compound interest
- (b) Half yearly compound interest
- (c) Quarterly compound interest

2. (a) According to the yearly compound interest, if principal is P , yearly rate of interest is R and time is T years, write the formula to compute compound amount.

- (b) The compound interest CI of a sum P in T years at the rate of yearly compound interest is $R\%$, write the relation between P , T , R and CI .
- (c) Employee Provident Fund changes the rate of interest per annum according to the economic liquidity of the state. As per the given condition the compound amount of a sum P at the interest rate of $R_1\%$, $R_2\%$ and $R_3\%$ for the first, second and third years respectively is CA , then write the formula to find CA .

3. Without using formula, find the compound interest and compound amount for the following conditions:

- (a) Principal (P) = Rs. 10,000, Time (T) = 2 years and Rate of interest (R) = 6 % p.a.
- (b) Principal (P) = Rs. 64,000, Time (T) = 3 years and Rate of interest (R) = 6% p.a.
- (c) Principal (P) = Rs. 20,000, Time (T) = 2 years, Rate of interest for the first year (R_1) = 10% p.a. and the rate of interest for the second year (R_2) = 12% p.a.

4. (a) At what rate of compound interest per year will the compound interest of Rs. 100 in a year be Rs. 12? Write.

- (b) At what rate of compound interest per year will the compound interest of Rs. 200 in 2 years be Rs. 42? Find it.

5. (a) A farmer borrowed Rs. 20,000 from a co-operative to invest in poultry farm for 3 years at the rate of compound interest of 15% per year. Find the compound interest and compound amount of 3 years.

- (b) A teacher deposited Rs. 50,000 in a bank at the account of his daughter. If the bank provides yearly 10% interest, what will be the compound interest and compound amount in 3 years? Find.
- (c) Sabita deposited Rs. 1,50,000 in a bank. If the bank provides yearly 6% interest after 2 years 6 months,
 - (i) how much is the compound amount?
 - (ii) how much is the compound interest?

6. (a) Manisha deposited Rs. 50,000 in a bank at the rate of compound interest 8% p.a. If the bank provides half yearly compound interest, then find the compound interest and compound amount she receives after 2 years.
- (b) A bank provides quarterly compound interest. If Sunil deposited Rs. 50,00,000 for 1 year at the rate of 12% p.a. interest, then find the compound interest and compound amount.
7. (a) Karma Gurung deposited Rs. 80,000 in a bank at the rate of 8% compound interest 8% p.a. Find the difference between simple interest and compound interest of the sum in 2 years.
- (b) Find the difference between simple interest and yearly compound interest of a sum Rs. 7,500 at the rate of 12% p.a. interest in 3 years.
8. (a) Chhiring deposited Rs. 40,000 at the rate of 6% annual compound interest. Find the difference of yearly compound interest and half yearly compound interest of the sum in 2 years.
- (b) What is the difference between semi-annual compound interest and quarterly compound interest of Rs. 18,000 in a year at the rate of 12% compound interest p. a.? Find it.
- (c) You went to deposit Rs. 60,000 in a bank for 2 years. The information in the notice board of the bank is given below:

Notice for the interest of deposit collection	
It is notified that two types of fixed deposit accounts are going to be effective from 2079/01/01. Grab the opportunity that of fixed deposit in time.	
Fixed deposit (P)	Fixed deposit (Q)
Rate of half yearly compound interest 10% p.a.	Rate of yearly compound interest 12% p.a.

- (i) How much interest will be collected in account (P) after 2 years?
- (ii) How much interest will be collected in account (Q) after 2 years?
- (iii) After knowing the interest rates of both options, by which option will you deposit the money? And why?
9. (a) If the yearly compound interest of a sum in 2 years at the rate of 15% p.a. interest is Rs. 180 more than simple interest, then find the sum.
- (b) If the half yearly compound interest of a sum in a year at the rate of 10% interest p.a. is Rs. 40 more than the yearly compound interest of the same sum for the same period of time at the same rate of interest, then find the sum.

- (c) Suprim borrowed some money for 2 years at the rate of compound interest of 10% p.a. and immediately he lent the money at the same rate of half yearly compound interest for the same period of time. In this transaction, if he gained Rs. 2019.24, then find how much he borrowed?
- 10.** (a) In how many years, a sum of Rs. 1,00,000 amounts to Rs. 1,21,000 at the rate of compound interest 10% p.a.?
- (b) According to the compound interest, in how many years will the compound interest of Rs. 8,00,000 at the rate of 10% p.a. be Rs. 12,61,000? Find it.
- (c) At what rate of yearly compound interest, the sum of Rs. 700 amounts to Rs. 847 in 2 years?
- (d) At what rate of annual compound interest will the compound interest of Rs. 3,43,000 in 3 years be Rs. 1,13,533? Find.
- 11.** (a) At the rate of yearly compound interest, a sum will be Rs. 6,050 in 2 years and Rs. 6,655 in 3 years respectively then
(i) find the rate of the compound interest. (ii) find the sum.
- (b) At the rate of annual compound interest, a sum amounts to Rs. 10,580 in 2 years and Rs. 12,167 in 3 years respectively, then
(i) find the rate of the compound interest.
(ii) find the sum.
- (c) The compound interest of a sum at the rate of yearly compound interest in 1 year and 2 years are respectively Rs. 1,800 and Rs. 3,816, then find the rate of the interest and the sum.
- 12.** (a) A person deposited Rs. 5,00,000 interest received from a commercial bank for 2 years to get the half yearly compound interest at the rate of 10% per annum. 5% tax on the interest will be levied. But right after a year, the bank has changed the policy and decided to accomplish the interest quarterly at the same rate of interest.
(i) Find the interest of the first year by deducting the tax.
(ii) What would be the interest of the second year be after deducting the tax?
(iii) What is the difference between the interest of the first year and second year after deducting the tax? Find.
(iv) After deducting the tax, by what percentage the interest of the first year differ from the interest of the second year?
- (b) A person deposited Rs. 80,000 in a co-operative limited for 2 years to get the yearly compound interest at the rate of 15% per annum after deducting the 5% tax on the interest.

But right after a year, the bank has changed the policy and decided to accomplish the interest half yearly at the same rate of interest. After deducting tax, by what percentage the interest of the first year differ from the interest of the second year?

13. (a) Ram divided Rs. 41,000 in two parts and deposited in the bank account of two daughters at the rate of annual compound interest 5% for 2 years and 3 years respectively. If the compound amount received by them after 2 years and 3 years respectively are equal, find how much did each of them get?
- b) Divide Rs. 21,000 in two parts in such a way that the compound amount of the first part at the rate of 10% p.a. for 3 years is equal to the compound amount for 2 years. What sums are there in the first part and the second part? Find it.
14. a) According to the yearly compound interest, the compound interest of a sum for 1 year and 2 years are respectively Rs. 450 and Rs. 945, find the rate of interest and the sum.
- b) According to the yearly compound interest, the compound interests of a sum for 1 year and 2 years are Rs. 1800 and Rs. 3816 respectively. Find the rate of the interest and the sum.

Project work

Visit a co-operative or bank or financial institution in your surroundings. Get information about various plannings by visiting head of the institution or receptionist or any other who can give information. If you find the brochure of the institution, study the various schemes. If your father or mother or any other family member is intrested to deposit a sum of money, which scheme do you prefer? Why? Prepare a report with your reason and present the report in your classroom.

Answer

3. (a) Rs. 1,236 and Rs.11,236 (b) Rs. 12,225.02 and Rs. 76,225.02
(c) Rs. 4,640 and Rs. 24,640
4. (a) 12% (b) 10%
5. (a) Rs. 10,417.50 and Rs. 30417.50 (b) Rs. 16,550 and Rs. 66,550
(c) Rs. 1,72,753.50 and Rs. 22,753.50
6. (a) Rs. 58492.93 and Rs. 8492.93 (b) Rs. 5627,544.05 and Rs. 627, 544.05
7. (a) Rs. 512 (b) Rs. 336.96

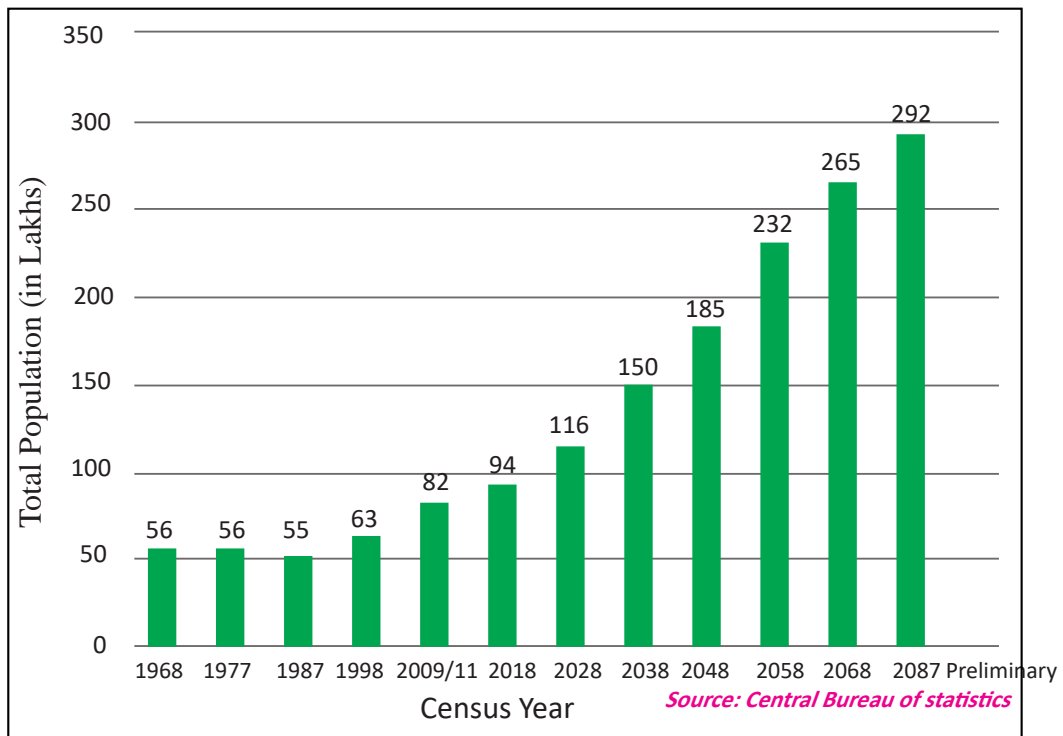
8. (a) Rs. 76.35 (b) Rs. 34.36 (c) (i) Rs. 12930.37
(ii) Rs. 15264 (iii) Rs. 2333.62 more interest in account Q
9. (a) Rs. 8,000 (b) Rs. 16,000 (c) Rs. 366717.82
10. (a) 2 year (b) 3 years (c) 10% (d) 10%
11. (a) (i) 10% (ii) Rs. 5,000
(b) (i) 15% (ii) Rs. 8,000 (c) 12% Rs. 15,000
12. (a) (i) Rs. 48,687.5 (ii) 54112.79
(iii) 5425.2936 (iv) 11.14% (b) 18.53%
13. (a) Rs. 21,000 and Rs. 20,000 (b) Rs.10,000 and Rs.11,000
14. (a) 10% and Rs. 4,500 (b) 12% and Rs.1,500

Lesson 3

Growth and Depreciation

3.0 Review

In Nepal, the census began in 1968 BS (1911) and since then, it has been conducted almost every 10 years. The initial result of the census from 1968 B.S. to 2078 B.S. is presented below. Discuss the following questions based on the table below:



- What was the population of Nepal in 1968 B.S.?
- How much did the population reach in the initial report of the 2078 B.S. census?
- According to the above table, discuss how much the population is increased or decreased in which years.
- With reference to the population of 2068 B.S., by what percentage has the population of 2078 B.S. increased?

- (e) With reference to the population of 1977 B.S., by what percentage was the population of 1987 B.S. decreased?
- (f) By how much was the populations increased or decreased in each census than that of the previous census? Compute.
- (g) In our daily life, what are other additional examples of such increment? Make a list.

3.1.1 Growth

Activity 1

According to the census 2078, the population of a municipality was 25,000. If the population increases by 2% every year,

- (a) What will be the population of the municipality in 2079 B.S.?
- (b) What will be the population of the municipality in 2080 B.S.?
- (c) Is the population growth similar to computing the compound interest?

Here, The population of the municipality in 2078 B.S. = 25,000

$$\begin{aligned}\text{The population of the municipality in 2079 B.S.} &= 25,000 + 2\% \text{ of } 25,000 \\ &= 25,000 + 25,000 \times \frac{2}{100} = 25,500\end{aligned}$$

$$\begin{aligned}\text{Again, the population of the municipality in 2080 B.S.} &= 25,500 + 2\% \text{ of } 25,500 \\ &= 25,500 + 25,500 \times \frac{2}{100} = 26,010\end{aligned}$$

The population in 2078 B.S. (P) = 25,000

Population growth rate (R) = 2%

Time (T) = 2 years

Population after 2 years (P_2) = ?

We know that, $P_T = P \left(1 + \frac{R}{100} \right)^T$

$$\begin{aligned}\text{or, } P_2 &= 25000 \left(1 + \frac{2}{100} \right)^2 \\ &= 25000 \times \left(\frac{51}{50} \right)^2 \\ &= 26,010\end{aligned}$$

Can we use the formula of compound interest in the above condition?



If external setup does not affect, the population of a place increases at a certain rate. This is called population growth rate and increased population is called growth population. The problem related to growth population can be solved by comparing with compound interest system.

$$\text{Population after T years } (P_T) = P \left(1 + \frac{R}{100} \right)^T$$

$$\text{Increased population after T years} = P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right]$$

Here, P = The population of the initial year

R = Population growth rate

T = Time

But in compound interest,

P denotes principal, R denotes rate and T denotes time.

Population growth is affected by migration and death.

If the population growth rate differs year by year, then the population after T years,

$$(P_T) = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right) \dots \left(1 + \frac{R_T}{100} \right). \text{ Where } R_1, R_2, R_3, \dots, R_T \text{ respectively represent the population growth rates of the first year, second year, third year } \dots \dots \dots T^{\text{th}} \text{ year's population growth.}$$

Example 1

The population of a city in 2078 B.S. was 50,000. If the annual population growth rate was 2% , what will be the population in 2080 B.S.? Calculate it.

Solution

Here, initial population of the city (P) = 50,000

Population growth rate (R) = 2% p.a.

Time (T) = 2080 B.S. – 2078 B.S. = 2 years

$$\begin{aligned} \text{We know that, } P_T &= P \left(1 + \frac{R}{100} \right)^T \\ &= 50,000 \left(1 + \frac{2}{100} \right)^2 \\ &= 50,000 \left(\frac{102}{100} \right)^2 = 50,000 \times 1.0404 = 52,020 \end{aligned}$$

Thus, the population in 2080 B.S. will be 52,020.

Example 2

Two years ago, the price of a sack of 25 kg jeera masino rice was Rs. 1,300. If the inflation rate was 5% p.a., by how much is the price increased now? Find it.

Solutions

Here, the initial price of jeera masino rice (P) = Rs. 1300

Increased rate of price (R) = 5% p.a.

Time (T) = 2 years

Increased price = ?

We know that,

$$\begin{aligned}\text{Increased price} &= P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\ &= \text{Rs. } 1300 \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right] \\ &= \text{Rs. } 1300 \left[\left(1 + \frac{105}{100} \right) - 1 \right] \\ &= \text{Rs. } 1300 \times 0.1025 \\ &= \text{Rs. } 133.25\end{aligned}$$

∴ The increased price of jeera masino rice is Rs. 133.25.

Example 3

If the price of a photocopy machine increases from Rs. 1,00,000 to Rs. 1,21,000 in 2 years, find the rate of yearly increment.

Solution

Here, the initial price of a photocopy machine (P) = Rs. 1,00,000

Time (T) = 2 years

The present price of the photocopy machine (P_T) = Rs. 1,21,000

Rate of price increment (R) = ?

We know that $P_T = P \left(1 + \frac{R}{100} \right)^T$

$$\text{or, Rs. } 1,21,000 = \text{Rs. } 1,00,000 \left(1 + \frac{R}{100} \right)^2$$

$$\text{or, } \frac{121000}{100000} = \left(1 + \frac{R}{100} \right)^2$$

$$\text{or, } \left(\frac{11}{10} \right)^2 = \left(1 + \frac{R}{100} \right)^2$$

$$\text{or, } \frac{11}{10} = 1 + \frac{R}{100}$$

$$\text{or, } \frac{11}{10} - 1 = \frac{R}{100}$$

$$\text{or, } \frac{11-10}{10} = \frac{R}{100}$$

$$\text{or, } \frac{1}{10} \times 100 = R$$

$$\therefore R = 10\%$$

Example 4

The number of students of a basic school is 500 now. In how many years will the number be 720 if the number of students increases by 20% p.a. every year?

Solution

Initial number of student (P) = 500

Increase rate (R) = 20% p.a.

Time (T) = T years

The number of students after T years (P_T) = 720

We know that, $P_T = P \left(1 + \frac{R}{100} \right)^T$

$$\text{or, } 720 = 500 \left(1 + \frac{20}{100} \right)^T$$

$$\text{or, } \frac{720}{500} = \left(1 + \frac{20}{100} \right)^T$$

$$\text{or, } \frac{720}{500} = \left(\frac{120}{100} \right)^T$$

$$\text{or, } \frac{36}{25} = \left(\frac{6}{5} \right)^T$$

$$\text{or, } \left(\frac{6}{5}\right)^2 = \left(\frac{6}{5}\right)^T$$

$$\Rightarrow T = 2 \quad [\because \text{indices are equal as the bases are same}]$$

Therefore, after 2 years, the number of students will be 720.

Example 5

The price of a plot of land is Rs. 15,97,200 per aana. If the rate of increase in price is 10% p.a., what was the price of the land per ropani before 3 years? Find it. (1 ropani = 16 aana)

Solution

Here, the price of a plot of land per aana (P_T) = Rs. 15,97,200

Time (T) = 3 years

Rate of increase in price (R) = 10% p.a.

Price of the land per aana before 3 years (P) = ?

We know that, $P_T = P \left(1 + \frac{R}{100}\right)^T$

$$\text{or, } 15,97,200 = P \left(1 + \frac{10}{100}\right)^3$$

$$\text{or, } 15,97,200 = P \left(\frac{110}{100}\right)^3$$

$$\text{or, } 15,97,200 = P \times 1.331$$

$$\text{or, } \frac{1597200}{1.331} = P$$

$$\therefore P = \text{Rs. } 12,00,000$$

Thus the price of land per aana (P) = Rs. 12,00,000

We know that, 16 aana = 1 ropani

\therefore The price of land per ropani = $16 \times \text{Rs. } 12,00,000 = \text{Rs. } 1,92,00,000$.

Example 6

The number of SEE appeared students from a district in 2076 B.S. was 50,000. If in the coming 3 years, the number increased by 5%, 6% and 4% respectively, find how many students will appear SEE in 2079 B.S.

Solution

Here, the number of the SEE appeared students before 3 years (P) = 50,000

Time (T) = 3 years

Rate of increase in the first year (R_1) = 5% p.a., rate of increase in the second year (R_2) = 6% p.a. and increase in the third year (R_3) = 4% p.a.

Number of the SEE appeared students after 3 years (P_T) = ?

$$\begin{aligned}\text{We know that, } (P_3) &= P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \\ &= 50,000 \left(1 + \frac{5}{100}\right) \left(1 + \frac{6}{100}\right) \left(1 + \frac{4}{100}\right) \\ &= 50,000 \left(\frac{105}{100}\right) \left(\frac{106}{100}\right) \left(\frac{104}{100}\right) \\ &= 57,876\end{aligned}$$

Therefore 57,876 students will appear in the SEE in 2079 B.S.

Example 7

The population of a municipality in 2078 B.S. was 1,00,000. In 2079 B.S., 8000 migrated there from other places and 500 died due to several circumstances. If the population increase rate is 2% p.a. every year, what will be the population in 2081 B.S.? Find it.

Solution

Here, case I,

Population in 2078 B.S. (P) = 1,00,000

Population increase rate (R) = 2% p.a.

Time (T) = 1 year

In-migration population (M_{in}) = 8,000

Death population (D) = 500

The population in 2079 B.S.

Since the population increases by 2% every year,

$$\begin{aligned}\text{We know that, } P_T &= P \left(1 + \frac{R}{100} \right)^T \\ &= 1,00,000 \left(1 + \frac{2}{100} \right)^1 \\ &= 1,00,000 \left(\frac{102}{100} \right)^1 \\ &= 1,02,000\end{aligned}$$

Now, the final population in 2079 B.S. = 1,02,000 + 8,000 + 500 = 1,09,500

Case II

Population in 2079 B.S. (P) = 1,09,500

Time (T) = 2 years

Population increase rate (R) = 2% p.a.

Population in 2081 B.S. (P_T) = ?

$$\begin{aligned}\text{We know that, } P_T &= P \left(1 + \frac{R}{100} \right)^T \\ &= 1,09,500 \left(1 + \frac{2}{100} \right)^2 \\ &= 1,09,500 \left(\frac{102}{100} \right)^2 \\ &= 1,13,924\end{aligned}$$

Therefore, the population of the municipality in 2081 B.S. will be 1,13,924.

Exercise 3.1

1.
 - (a) If the present population of a locality is P , the population after T years is P_T and yearly population growth rate is $R\%$, write the formula to find P_T .
 - (b) If the number of tigers of Chitwan National Park in 2079 B.S. is x and annual growth rate of tiger is $R\%$, then what will be the number of tigers after N years?
 - (c) The growth rate of foreign employment from Nepal of the first year, second year and third year are respectively $R_1\%$, $R_2\%$ and $R_3\%$. Write the formula to find the number of employees after 3 years.
2.
 - (a) If death rate is less than birth rate of a country, does the population of the country increase or decrease?
 - (b) One year ago, the price of a sack of 25 kg rice was Rs. 1500. As the price has increased 10% p.a., how much does the sack of 25 kg rice cost now?
3.
 - (a) According to the census 2021, the population of a city was 5,18,452. If the growth rate was 4.5% p.a., what will be the population of the city after 3 years?
 - (b) A landlord made an agreement with a businessman to increase the rent 5% per annum. If the rent of a shutter is Rs. 10,000 now, what will be the rent after 3 years?
 - (c) The growth rate of a bacteria in curd is 10% per hour. If the number of bacteria at 6 a.m. is 4×10^{11} , what will be the number of bacteria after 2 hours? Find.
4.
 - (a) The population of a rural municipality before 2 years was 28,500. If the population growth rate is 2% p.a., then by how much is the population increased in 2 years?
 - (b) Monthly fee of grade 10 of a school before 3 years was Rs. 6,500. If fees increase every year by 10% according to the rules and regulation of the school, then by how much has the fee increased in 3 years?
 - (c) A land costs Rs 6,00,000 at present. If the yearly increase rate of the price is 10%, then how much will increase in the price of the land in 2 years? Find.
5.
 - (a) The number of students studying in a university at present is 21,632. 2 years ago, the number of students studying in the university was 20,000. What was the yearly increment rate?

- (b) The population of a rural municipality at the end of 2018 A.D. was 40,000. If at the end of 2020 A.D., the population increased to 44,100, then find the yearly population growth rate.
- (c) 3 years ago, the price per liter of oil was Rs. 125. Now, the price has increased to Rs 216 per liter, then what is inflation rate?
6. (a) The population of a village is 13,310. Whilst the population growth rate of the village is 10% p.a., how many years ago was the population of the village 10,000?
- (b) At the beginning of Baisakh, the papaya plant was 4 meter high. If the growth rate of the plant is 4% per month, in how many months will the height of the plant be 4.3264 meter?
7. (a) The price of a plot of land is Rs. 15,97,200 per aana. If the rate of increase in the price is 10% p.a., what was the price of the land per ropani before 3 years? Find it. (1 ropani = 16 aana)
- (b) The price of a land is Rs.2,66,200 per aana. If the rate of increase in price is 10% p.a., what was the price of the land per ropani before 3 years? Find it.
8. (a) When urine of a patient was tested in a laboratory at 6 a.m., it was found that the number of bacteria 1×10^5 . After that, it was tested at 7 a.m., 8 a.m. and 9 a.m. and found that the increase rate per hour was 3%, 4% and 5% respectively. Find the total number of bacteria at 9 a.m. .
- (b) 3 years ago, the population of a city was 1,50,000. In the following 3 years, the population increased by 2% in the first year, 4% in the second year and 5% in the third year. Find the present population of the city.
9. (a) In an insurance company established 3 years ago, 1000 agents were trained by all the branches throughout the country. Customers should be increased according as the market competition. So that, from the very beginning, a policy “a group of every 5 agents should add 1 more agent every year” was implemented and the number of agents has been increased. How many agents are there in the company now?
- (b) A financial institution established at the beginning of 2078 has 200 marketers. After the expansion of market of the institution, a policy of a group of every 5 marketers should add 1 more new marketer every year was implemented and the number of marketer has been increased. Find how many marketers will be there at the end of 2079?

10. The population of a rural municipality is increasing by 10% every year. At the end of 2 years, the population reached to 30,000. If 5,800 migrated to the place finally, find the initial population.
11. After continuous inflation of American dollar at the rate of 5% p.a., in two years it becomes \$1 = Rs.120. Before 2 years, how much Nepalese currency was equal to \$1?
12. The population of a village before 2 years was 31,250. The population growth rate is 6% p.a.. One year ago, 625 migrated to other places. Find the number of present populations of the village.
13. A district had 3,75,000 population before 3 years. If 1480 migrated to the village at the end of 2 years and 2,750 died due to natural disaster and yearly population growth rate is 2%, then find the number of present populations of the district.
14. At the beginning of 2075 B.S., the population of a metropolitan city was 5,00,000. At the end of 2077 B.S, the population of the city was 6,65,500.
 - a) What was the population growth rate?
 - b) If the population increases in the same way, what will be the population of the city at the end of 2079?

Project work

By forming appropriate groups, every group should collect the data of present population from the respective ward office. From your municipality or ward office or other books or using brochures, find the increased population of the ward based on the data of census 2078. Prepare a report on how many migrated from other places, how many migrated to other places and how many died. Then present your report to the class.

Answers

- 1 and 2 show to your teacher.
3. (a) 5,91,640 (b) Rs. 15,208.75 (c) 4.84×10^{11}
4. (a) 1152 (b) Rs. 2151.5 (c) Rs. 1,26,000
5. (a) 4% (b) 5% (c) 20% 6. (a) 3 years (b) 2 month
7. (a) Rs. 1,92,00,000 (b) Rs. 32,00,000 8. (a) 1.1248×10^5
- (b) 1,67,070 9. (a) 1728 (b) 288 10. 20,000
11. 1 = Rs. 108.84 12. 34,450 13. 396,658 14. (a) 10% (b) 8,05,255

3.2 Depreciation

Activity 1

By dividing the students in appropriate groups, discuss the following questions:

- (a) If a farmer sells a tractor for Rs. 3,60,000 which was purchased for Rs. 5,50,000 before 2 years, why has the price decreased?
- (b) A cupboard can be bought for Rs. 15,000, less than the price of a new one from a second-hand shop. Why?
- (c) A photocopy machine was bought for Rs. 2,10,000 before some years. After using for sometimes, it costs Rs. 1,00,000 only now. Why?

From the above situations, the price of the tractor diminishes by Rs. 1,90,000 in (a). Similarly, in (b), it is known that the second-hand goods are cheaper than the new ones. In (c), after using the machinery goods, the price depreciates.

A product is prepared for a certain period. Its efficiency decreases in accordance with its increased uses. Thus, after using some machinery items for certain period of time, their cost diminishes at certain rate. This is called depreciation. The price that diminishes at a certain rate at certain rate during a certain period of time is called compound depreciation.

Activity 2

All the students Sit in appropriate groups.

Initial price of a good = V_o , similarly, yearly depreciation rate = $R\%$, time duration = T

Price of good after T years = V_T

In this condition, how can we find the price after T years? This has been discussed in groups and the conclusion is presented as below:

Price after 1 year (V_1) = $V_o - V_o$ or $R\%$

$$= V_o - V_o \times \frac{R}{100}$$

$$= V_o \left(1 - \frac{R}{100} \right)$$

For 2 years,

$$\begin{aligned}
 \text{Price after 2 years } (V_2) &= V_o \left(1 - \frac{R}{100}\right) - R\% \text{ of } V_o \left(1 - \frac{R}{100}\right) \\
 &= V_o \left(1 - \frac{R}{100}\right) - V_o \left(1 - \frac{R}{100}\right) \times \frac{R}{100} \\
 &= V_o \left(1 - \frac{R}{100}\right) \left(1 - \frac{R}{100}\right) \\
 &= V_o \left(1 - \frac{R}{100}\right)^2
 \end{aligned}$$

In this way, price after 3 years $(V_3) = V_o \left(1 - \frac{R}{100}\right)^3$

Like this,

(a) Price after T years $(V_T) = V_o \left(1 - \frac{R}{100}\right)^T$.

(b) Again, how can we find the depreciated price?

$$\begin{aligned}
 \text{Depreciated price} &= V_o - V_T \\
 &= V_o - V_o \left(1 - \frac{R}{100}\right)^T \\
 &= V_o \left[1 - \left(1 - \frac{R}{100}\right)^T\right]
 \end{aligned}$$

(c) Does the price of a good depreciate at the same rate every year? Discuss.

If $R_1\%$, $R_2\%$ and $R_3\%$ $R_T\%$ are respectively depreciating rate for first year, second year, third year T^{th} year, then price after T years

$$(V_T) = V_o \left(1 - \frac{R_1}{100}\right) \left(1 - \frac{R_2}{100}\right) \left(1 - \frac{R_3}{100}\right) \dots \left(1 - \frac{R_T}{100}\right).$$

Increases in growth and decreases in depreciation.



Example 1

3 years ago a book was published costing Rs. 200, is being sold in an exhibition at 5% rate of yearly depreciation. What is the price of the book this year?

Solution

Here, the initial price of the book $(V_o) = \text{Rs. } 200$

Rate of depreciation $(R) = 5\% \text{ p.a.}$

Time $(T) = 3 \text{ years}$

Solution

Present price of the book (V_T) = ?

$$\begin{aligned}\text{We know that, } V_T &= V_o \left(1 - \frac{R}{100}\right)^T \\ &= 200 \left(1 - \frac{5}{100}\right)^3 \\ &= 200 \left(\frac{95}{100}\right)^3 \\ &= 200 \times 0.857375 \\ &= \text{Rs. } 171.47\end{aligned}$$

Therefore, the book costs Rs. 171.47 this year.

Example 2

Seema admitted in BBA. She purchased a computer for Rs. 40,000 for her study. After using it for 2 years, if the price of the computer depreciates by Rs. 7,600, then find the rate of depreciation.

Solution

Here, the initial price of the computer (V_o) = Rs. 40,000

Rate of depreciation (R) = ?

Time (T) = 2 years

Depreciated price = Rs. 7,600

Now, the price after 2 years (V_T) = $V_o - \text{Rs. } 7,600 = \text{Rs. } 40,000 - \text{Rs. } 7,600 = \text{Rs. } 32,400$

$$\begin{aligned}\text{We know that, } V_T &= V_o \left(1 - \frac{R}{100}\right)^T \\ \text{or, } 32,400 &= 40,000 \left(1 - \frac{R}{100}\right)^2\end{aligned}$$

$$\text{or, } \frac{32400}{40000} = \left(1 - \frac{R}{100}\right)^2$$

$$\text{or, } \left(\frac{18}{20}\right)^2 = \left(1 - \frac{R}{100}\right)^2$$

$$\text{or, } \frac{18}{20} = 1 - \frac{R}{100}$$

$$\text{or, } \frac{R}{100} = 1 - \frac{18}{20}$$

$$\text{or, } \frac{R}{100} = \frac{2}{20}$$

$$\text{or, } R = \frac{2}{20} \times 100 = 10\%$$

Therefore, the rate of depreciation of the computer is 10% p.a.

Example 3

The price of a house is Rs. 20,00,000 now. If its price decreases by 10% every year, then in how many years will its price be Rs. 14,58,000?

Solution

Here, the present price of the house (V_0) = Rs. 20,00,000

Rate of depreciation (R) = 10%

Price after T years (V_T) = Rs. 14,58,000

Time (T) = ?

$$\text{We know that, } V_T = V_0 \left(1 - \frac{R}{100}\right)^T$$
$$\text{or, } 14,58,000 = 20,00,000 \left(1 - \frac{10}{100}\right)^T$$

$$\text{or, } \frac{1458000}{2000000} = \left(\frac{90}{100}\right)^T$$

$$\text{or, } 0.729 = (0.9)^T$$

$$\text{or, } (0.9)^3 = (0.9)^T$$

$$\Rightarrow T = 3 \text{ years}$$

Thus, 3 years later, the price of the house will be Rs. 14,58,000.

Example 4

A factory established with the capital of Rs. 4 crore gained Rs. 75 Lakhs in 3 years but its cost depreciated by 2.5% p.a. Then company was sold after 3 years. Now, calculate whether the factory is in profit or loss.

Solution

Here, initial price of investment (V_0) = Rs. 4,00,000

Rate of depreciation (R) = 2.5%

Time (T) = 3 years

Price after 3 years (V_3) = ?

$$\text{We know that, } V_T = V_0 \left(1 - \frac{R}{100}\right)^T$$
$$\text{or, } V_3 = 4,00,00,000 \left(1 - \frac{2.5}{100}\right)^3$$

$$= 4,00,00,000 \left(\frac{97.5}{100} \right)^3$$

$$= \text{Rs. } 3,70,74,375$$

Again, the profit gained by the factory after 3 years = Rs. 75,00,000

The total amount gained from the factory = Rs. 3,70,74,375 + Rs. 75,00,000 = Rs. 4,45,74,375

Total investment = Rs. 4,00,00,000

Thus, the profit while selling it in 3 years = Rs. 4,45,74,375 – Rs. 4,00,00,000 = Rs. 45,74,375

Example 5

When the price of a share of a finance company depreciates continuously for 2 years by 10% p.a. and it is sold for Rs. 25,920 then how many shares of Rs. 100 were sold? Find it.

Solution

Here, present value of the share of the finance (V_T) = Rs. 25,920

Depreciation rate (R) = 10%

Time (T) = 2 years

Initial price of shares of the finance (V_0) = ?

We know that, $V_T = V_0 \left(1 - \frac{R}{100} \right)^T$

$$\text{or, } 25,920 = V_0 \left(1 - \frac{10}{100} \right)^2$$

$$\text{or, } 25,920 = V_0 \left(\frac{90}{100} \right)^2$$

$$\text{or, } 25,920 = V_0 \times \frac{81}{100}$$

$$\text{or, } V_0 = \text{Rs. } \frac{25920 \times 100}{81}$$

$$\text{or, } V_0 = \text{Rs. } 32,000$$

Hence, the value of the share before 2 years (V_0) = Rs. 32,000

Two years ago, the price of a share = Rs. 100

$$\text{Total number of shares} = \frac{32000}{100} = 320$$

\therefore Two years ago, the finance company sold 320 shares of Rs. 100.

Exercise 3.2

1. a) If the initial price of a good is Rs. P and the rate of depreciation is R% p.a., then write the formula to find the price of the good after T years.
b) What does 'R' in $V_T = V_o \left(1 - \frac{R}{100}\right)^T$ represent value after 'T' years?
2. a) If Ram sold a watch which cost Rs. 5,000 after 1 year at 7% depreciation, then what will be the depreciated price?
b) A motorcycle is sold for Rs. 57,000 in 1 year after depreciating at the rate of 5% p.a. At what price was the motorcycle purchased for?
3. a) What will be the price of a cupboard of cost Rs. 16,800 after 2 years at the rate of 15% depreciation p.a. ?
b) The present price of a motorcycle of efficiency 125 c.c. produced in India is Rs. 2,50,000. If its cost depreciates every year at the rate of 4% p.a., what will be the price of motorcycle after using 3 years? Find it.
4. a) Sameer has bought a mobile phone for Rs. 30,000. Due to his household problem he has to sell it after using 2 years at 30% rate of depreciation. Find the depreciated amount.
b) As the laptop is convenient for online class, a mathematics teacher purchased a laptop for Rs. 96,000. It is depreciated at the rate of 15% every year. If he sells it after using it for 3 years, how much money is depreciated? Find it.
5. a) 3 years ago, a plot of land of 4 ropani in hilly region was purchased for Rs. 12,50,000. It can be sold now for Rs. 1,60,000 per ropani. By what percent per annum is the price of land depreciated? calculate it.
b) A man bought a watch for Rs. 5,000 and he sold it after using it for 3 years for Rs. 625 only. Find the annual rate of depreciation.
6. a) A press machine was purchased for Rs. 4,00,000 before some years and it is depreciated to Rs. 1,96,000 by reducing the price every year by 30%. How many years ago was the machine purchased?
b) A car costs Rs. 8,00,000. If its price reduces every year by 10%, in how many years will its price be Rs. 5,83,200?

7. a) After continue devaluation of the American dollar in 2 years by 5% p.a., American dollar \$ 1 = Rs.125 now, how much Nepali rupees was equal to \$ 1 before 2 years? Find it.
- b) 3 years ago, a small group of youths returned back from foreign employment and started a cow farm investing Rs. 2,80,000. Due to political instability of state, if the price of the farm depreciates by 5% p.a. then how much does the farm cost now?
8. a) A photocopy machine costs Rs. 5,00,000 now. If the machine depreciates by 15% in the first year and similarly depreciates by 10% and 5% in the second and third year respectively, then what will be the price of the machine in 3 years?
- b) A lamination machine is sold for Rs. 24,168 after depreciating it by 4% and 5% in the first and second year respectively. Find what the price of the machine before 2 years.
9. An entrepreneur purchased a heavy truck by investing Rs. 48,00,000. He earned Rs. 6,80,000 in 2 years. If it is depreciated by 10% every year and sold in 2 years, then find his loss or profit.
10. A bus owner bought a bus for Rs. 16,00,000 and conducted it in the Kathmandu - Baglung route for 3 years. He earned Rs. 5,10,000 only. If the value is depreciates every year by 5% and he sells it in 3 years, then find his loss or profit.
11. The price of a finance company listed in Nepal share market is falling down its price by 10% in 2 years. Your neighbour having the shares of the finance company sold his shares for Rs. 28,350. How many shares were sold by the company before 2 years at the rate of Rs. 100 per share?
- 12. The price of the share of a hydropower company is reducing by 10% p.a. If Sashi sells all his shares now and has the present value Rs. 7,10,775, then**
- a) What was the price of his shares before 2 years?
- b) He purchased the shares at the rate of Rs. 100 per share in IPO, how many shares did he buy before 2 years?
- 13. In certain rate of annual depreciation, the price of a good will be Rs. 10,240 and Rs. 8,192 in 2 years and 3 years respectively. Then,**
- a) Find the rate of depreciation.

b) What was the initial price of the good? Find it.

14. In a certain rate of yearly depreciation, the price of an article will be Rs. 5,41,500 and Rs. 5,14,425 in 2 and 3 years respectively. Then

a) What is the rate of depreciation?

b) What was the initial price of the article? Find it.

Project work

Make groups of suitable number of students. Visit the nearest shop or company or other intervention where second hand goods are sold. Ask the cost price of electric items like vehicle, photocopy machine or furniture, etc. After using the goods for a certain time, what is the present value? After purchasing the goods, how much profit or loss will be made? With the answers of the questions, find by what percent are the goods are depreciated? What percent of loss or profit can be made in selling the goods? Prepare a report and present in the classroom.

Answers

11. Show to your teacher. 2. (a) Rs. 350 (b) Rs. 60,000
3. (a) Rs. 12,138 (b) Rs. 2,21,184 4. (a) Rs. 15,300 (b) Rs. 37,044
5. (a) 20% (b) 50% 6. (a) 2 years (b) 3
7. (a) \$1 = Rs. 138.50 (b) Rs. 2,40,065 8. (a) Rs. 3,63,375 (b) Rs. 26,500
9. Rs. 2,32,000 shares 10. Rs.2,81,800 profit 11. 350 shares
12. (a) Rs. 8,77,500 (b) 8,775 shares 13. (a) 20% (b) Rs. 16,000
14. (a) 5% (b) Rs. 6,00,000

Lesson 4

Currency and Exchange Rate

4.0 Review

Make groups of suitable number of students and discuss the followings:

- You are going to visit abroad for an educational excursion. Can you buy goods and food items by spending Nepali rupees there?
- Sagun is a prestigious businessman. He imported goods from abroad countries. Can he pay the bills in Nepali rupees?
- Nepali people working abroad send money in Nepal. Do their family get the money in Nepali rupees? At what basis do they get the money in Nepali rupees of the foreign currency?

In the above mentioned conditions, wherever we go or conduct our business, we use the currency of the respective country. Family gets the Nepali currency if the money sent from different countries.

4.1 Currency and Exchange Rate

Activity 1

Make groups of suitable number of students. Study the exchange rate of currency of several countries issued by Nepal Rastra Bank on Bhadra 5, 2079, and answer the following questions:

Nepal Rastra Bank

Central Office, Thapathali, Kathamandu

Bhadra 5, 2079 (August 21, 2022)

Country	Currency	Unit	Buying rate (Rs.)	Selling rate (Rs.)	Symbol
India	Indian Rupees	1	160.00	160.15	₹
America	American Dollar	1	127.35	127.95	\$
Countries in European Union	Euro	1	128.13	128.73	€
UK	Pound Sterling	1	150.94	151.65	£

Switzerland	Swiss Frank	1	133.13	133.76	CHF
Australia	Australian Dollar	1	87.82	88.23	AUD\$
Canada	Canadian Dollar	1	98.08	98.54	CAD\$
Singapore	Singapore Dollar	1	91.68	92.11	SGD\$
Japan	Japanese Yen	10	9.32	9.37	¥
China	Chinese Yuan	1	18.70	18.79	¥
Saudi Arabia	Saudi Arabian Riyal	1	33.91	34.07	SAR رِيَال
Qatar	Qatarian Riyal	1	34.82	34.98	QAR رِيَال
Thailand	Thai Bhat	1	3.57	3.59	THB ฿
United Arab Emirates	United Arab Emirate Dihram	1	34.67	34.83	إ.د.
Malaysia	Malaysian Ringgit	1	28.45	28.58	MR
South Korea	South Korean wan	100	9.58	9.63	₩
Sweden	Swedish Corner	1	12.07	12.12	SEK kr
Denmark	Denish Corner	1	17.23	17.31	DKK kr
Hongkong	Hongkong Dollar	1	16.23	16.31	HKD \$
Kuwait	Kuwaiti Dinar	1	414.23	416.18	KD ك.د.
Bahrain	Bahrain Dinar	1	337.80	339.39	BD ب.د.

- Who determines the exchange rate of currency in our nation?
- If the currency of America is called dollar. What is the currency of China called?
- According to the exchange rate given above, why is selling rate more than the buying rate?
- What are buying rate and selling rate? Discuss.

The exchange rate of currency of a country to another country will be determined by the government or central bank of the country. Such rate is called foreign currency exchange rate. In our country, exchange rate with India is fixed and exchange rate with other countries are instable. In our country, the exchange rate is declared by Nepal Rastra Bank. The exchange rate given by bank or other financial institutions while purchasing the foreign currency is called buying rate. Similarly, the exchange rates fixed by bank or other financial institutions while selling the foreign currency is called selling rate.

Example 1

Based on the exchange rate given above, find the difference between buying and selling price of pound sterling 500.

Solution

According to the buying rate, 1 pound sterling = Rs. 150.94

Now, buying price of 500 pound sterling = Rs. 150.94×500 = Rs. 75,420

Again, according to the selling rate, 1 pound sterling = Rs. 151.65

Selling price of 500 pound sterling = Rs. 151.65×500 = Rs. 75,825

Hence, the difference between selling price and buying price = Rs. 75,825 – Rs. 75,420 = Rs. 405

Example 2

The exchange rate between the American Dollar and Nepali rupees for a specific day is \$ 1 = Rs. 126.35

- How many American Dollars can be exchanged for Rs. 85,500?
- How many rupees can be exchanged for \$ 3,000 ?

Solution

Here, a) \$ 1 = Rs. 126.35

Or, Rs. 126.35 = \$1

Or, Rs. 1 = \$ $\frac{1}{126.35}$

or, Rs. 85,500 = \$ $\frac{1}{126.35} \times 85,500$ = \$676.69

b) Again, \$ 1 = Rs. 126.35

Or, \$ 3000 = Rs. 126.35×3000 = Rs. 3,79,050

Example 3

Based on the above mentioned exchange rate, change the following currencies:

- 1 Canadian Dollar into Japanese Yen
- 250 Australian Dollar into Swiss Frank.

Solution

Here, a) 1 Canadian Dollar = Rs. 98.08

Or, Rs. 98.08 = 1 Canadian Dollar (According to buying rate)

$$\text{Or, Re. } 1 = \frac{1}{98.08} \dots\dots\dots\text{(i)}$$

Again, 10 Japanese Yen = Rs. 9.37

Or, Rs. 9.37 = 10 Japanese Yen (According to the selling rate)

$$\text{Or, Rs. } 1 = \frac{10}{9.37} \dots\dots\dots\text{(ii)}$$

From equation (i) and (ii), we have

$$\frac{1}{98.08} \text{ Canadian Dollar} = \frac{10}{9.37} \text{ Japanese Yen}$$

$$1 \text{ Canadian Dollar} = \frac{10 \times 98.08}{9.37} \text{ Japanese Yen} = 104.67 \text{ Japanese Yen}$$

Hence, 1 Canadian Dollar = 104.67 Japanese Yen.

Alternatively,

Let Canadian \$ 1 = x Japanese Yen

Now, we have,

Canadian Dollar \$1 = x Japanese Yen

10 Japanese Yen = Rs. 9.37

NRs. 98.05 = Canadian \$1

Multiply each values of the left hand side and similarly multiply each values of the right hand side. Then, we have;

$$1 \times 10 \times 98.05 = x \times 9.37 \times 1$$

$$\text{Or, } 10 \times \frac{10 \times 98.05}{9.37} = x$$

$$\text{Or, } x = 104.67$$

Hence, 1 Canadian Dollar = 104.67 Japanese Yen.

Yeh! While exchanging currency of other countries in Nepal, the currency of the country can be got after exchanging in Nepalese currency at first. The banks give Nepalese rupees for foreign currency at buying rate and foreign currency for Nepalese rupees at the selling rate to us.

1. Since starting with Canadian Dollar on the left hand side and taking Japanese Yen on the right hand side of first line, start with Japanese Yen in the second line.
2. Since the Nepali Rupees on right hand side of second line, write Nepali rupees on the left hand side and the Canadian Dollar on the right hand side of the third line because it is started with Canadian dollar.

b) $1 \text{ Australian Dollar} = \text{Rs. } 87.82$

Or, $\text{Rs. } 87.82 = 1 \text{ Australian Dollar}$

Or, $\text{Rs. } 1 = \frac{1}{87.82} \text{ Australian Dollar} \dots\dots\dots\text{(i)}$

Again, $1 \text{ Swiss Frank} = \text{Rs. } 133.76$

Or, $\text{Rs. } 133.76 = 1 \text{ Swiss Frank}$

Or, $\text{Rs. } 1 = \frac{1}{133.76} \text{ Swiss Frank} \dots\dots\dots\text{(ii)}$

From equation (i) and (ii), we get

$$\frac{1}{87.82} \text{ Australian Dollar} = \frac{1}{133.76} \text{ Swiss Frank}$$

or, $1 \text{ Australian Dollar} = \frac{1 \times 87.82}{133.76} \text{ Swiss Frank}$

Or, $250 \text{ Australian Dollar} = \frac{87.82}{133.76} \times 250 \text{ Swiss Frank} = 164.14 \text{ Swiss Frank}$

Hence, $250 \text{ Australian Dollar} = 164.14 \text{ Swiss Frank}$

Alternatively

Let, $250 \text{ Australian Dollar} = x \text{ Swiss Frank}$

$1 \text{ Swiss Frank} = \text{Rs. } 133.76$

$\text{Rs. } 87.82 = 1 \text{ Australian Dollar}$

$250 \times 1 \times 87.82 = x \times 133.76 \times 1$

Or, $x = \frac{250 \times 87.82}{133.76}$

$= 164.14 \text{ Swiss Frank}$

The alternative process of finding the values of unknown variables other than unitary method, ratio and proportion is chain rule.

Let, A, B and C are the currencies of different countries, then

If $A = B$, $B = C$ and $C = A$ then

$A \times B \times C = B \times C \times A$

Example 4

If American Dollar (\$) 500 = Pound Sterling (£) 390 and Nepali Rupees Rs. 7,547 = Pound sterling (£) 50, find how many American Dollars can be exchanged for Nepali Rupees. 10,308?

Let, American dollar \$ x = Nepali Rupees Rs. 10,308

And then write accordingly,

$$\text{\$ } 500 = \text{\pounds } 390$$

$$\text{\pounds } 50 = \text{Rs. } 7,547$$

$$\text{Rs. } 10,308 = \text{\$ } x$$

Using chain rule, we have

$$\text{or, } 500 \times 50 \times 10,308 = 390 \times 7547 \times x$$

$$\text{or, } \frac{500 \times 50 \times 10308}{7547 \times x} = x$$

$$\text{or, } x = 87.55$$

Hence \$ 87.55 can be exchanged for Nepali Rupees 10,308.

Example 5

A businessman exchanged Nepali Rupees 8,40,000 at the exchange rate of the Pound Sterling (£) 1 = Rs. 150. After 5 days, Nepali currency is deflated by 5% and then he exchanged Nepali currency into Pound Sterling. How much profit or loss does he get?

Solution

A businessman exchanged the Pound Sterling for Rs. 8,40,000

Rate of deflation = 5%

Loss or profit = ?

Now, Rs. 150 = £ 1

$$\text{Or, Rs. } 1 = \frac{1}{150}$$

$$\text{Or, Rs. } 8,40,000 = \text{\pounds } \frac{1}{150} \quad \text{Rs. } 8,40,000 = \text{\pounds } \frac{1}{150} \times 8,40,000 = \text{\pounds } 5,600$$

Since Nepali currency is deflated by 5% in 5 days, then new exchange rate is:

$$\text{\pounds } 1 = \text{Rs. } (150 - 5\% \text{ of } 150) = 150 - 150 \times \frac{5}{100} = \text{Rs. } 150 - \text{Rs. } 7.50 = \text{Rs. } 142.50$$

Again, he exchanged his Pound Sterling into Nepali Rupee.

Thus, £ 5,600 = Rs. $142.50 \times 5,600$ = Rs. 7,98,000

Here, Rs. 7,98,000 < Rs. 8,40,000 so he gets a loss.

Hence, loss amount = Rs 8,40,000 – Rs. 7,98,000 = Rs. 2,000

Example 6

A businessman of Nepali origin from Norway purchased 900 Pasmiana shawls at the rate of Rs. 4,000 in Kathmandu. He exported by paying 5% export tax and then at how much Euro should he sell all the shawls at the profit of 20%? (€1 = Rs. 130)

Solution

Here, the price of a Pasmiana shawl = Rs. 4,000

The cost of 900 shawls = Rs. $4,000 \times 900$ = Rs. 36,00,000

The cost price of the shawls including 5% export tax = Rs. 36,00,000 + 5% of Rs. 36,00,000

$$\begin{aligned} &= \text{Rs. } 36,00,000 + \text{Rs. } 36,00,000 \times \frac{5}{100} \\ &= \text{Rs. } 37,80,000 \end{aligned}$$

We know that, €1 = Rs. 130

$$\text{Or, Rs. } 130 = \text{€1}$$

$$\text{Or, Rs. } 1 = \text{€1}/130$$

$$\text{Or, Rs. } 37,80,000 = \frac{\text{€}}{130}$$

$$\begin{aligned} \text{or, Rs. } 37,80,000 &= \frac{\text{€}}{130} \times 37,80,000 \\ &= \text{€}29076.92 \end{aligned}$$

∴ The total cost price in euro = €29076.92

To sell at the profit of 20%

$$\begin{aligned} \text{The total selling price of 900 shawls} &= \text{€}29076.92 + \text{€}29076.92 \times \frac{20}{100} \\ &= \text{€}29076.92 + 5,815.38 \end{aligned}$$

Hence, the selling price of all the shawls = € 34,892.30

Alternatively

The cost of 900 Pasmina shawls with 5% export tax = Rs. 37,80,000

To sell with 20% profit,

The selling price of 900 shawls = Rs. 37,80,000 + 20% of Rs. 37,80,000

$$= \text{Rs. } 37,80,000 + \text{Rs. } 7,56,000$$

$$= \text{Rs. } 45,36,000$$

Now, Rs. 130 = €1

$$\text{Or, Rs. } 1 = \frac{\text{€}}{130}$$

$$\begin{aligned}\text{Rs. } 4536000 &= \frac{\text{€}}{130} \times 4536000 \\ &= 34892.30\end{aligned}$$

∴ The selling price of all the shawls = €34892.30

Exercise 4

1.
 - a) What is meant by money exchange?
 - b) Which of the buying rate or selling rate does a Nepali student use to exchange Nepali Rupees into American Dollars while s/he is going for abroad study in America? Write.
2. **By using the buying rate money exchange given above, change the following currency of different countries into Nepali currency:**
 - a) Indian Rupees 1425
 - b) American Dollar 2000
 - c) Pound Sterling Dollar 4672
 - d) Australian Dollar 672
 - e) Saudi Arabian Riyal 1851
 - f) Qatari Riyal 2225
 - g) South Korean Wan 58,230
 - h) Hongkong Dollar 4512
 - i) Malaysian Ringgit 6725
 - j) Chinese Yuan 3450
3. **By using the selling rate of money exchange, change Rs. 2,00,000 into the following currencies:**
 - a) Australian Dollar
 - b) American Dollar
 - c) Euro
 - d) United Arab Emirates Dirham

4. **According to the exchange rate declared by Nepal Rastra Bank on a specific day, the buying and selling rate for 1 American Dollar are Rs. 127.50 and Rs. 128 respectively,**
 - a) How many American Dollars can be exchanged for Rs. 81,280?
 - b) Find how many Nepali Rupees can you exchange with American Dollar 600?
5. **According to the exchange rate declared by Nepal Rastra bank on 23 August 2022, the buying and selling rate for Chinese Yuan 1 are Rs. 18.64 and Rs. 18.73 respectively,**
 - a) How many Chinese Yuan can be exchanged for Rs. 37,460?
 - b) How many Nepalese rupees can you exchange with Chinese Yuan 5000?
6. **Using the above-mentioned abovementioned, answer the following questions:**
 - a) How many United Emirates Dirhams are equal to 1 Euro?
 - b) How many Malaysian Ringgits are equal to 1 Qatari Riyal?
 - c) How many Canadian Dollars are equal to 200 American Dollar.
 - d) How many Thai Bhattas are equal to 200 Pound Sterling?
7. **Sobita went to Australia from Nepal for her higher studies. After completing her studies, she has been working as a secondary level teacher. She earns 37 Australian Dollar per hour. She teaches 5 hours a day except Sunday. If exchange rate of the Australian Dollar 1 = Rs. 90,**
 - a) How much salary does she have in Nepali Rupees for a week?
 - b) Find how much monthly salary does she have in Nepali Rupees?
8.
 - a) An American businessman exchanged American Dollar for NRs. 12,40,000 at the exchange rate of American Dollar 1 = NRs. 124. After a week, Nepali currency is inflated by 10%. At that time, he exchanged the dollar into Nepali currency then, how much loss or profit have? Find it.
 - b) A businessman buys goods from Thailand. For this, he exchanged Nepali currency Rs. 7,20,000 at the exchange rate 1 Thai Bhatt = NRs. 3.60. After a day, Nepali currency is deflated by 5%. Due to the inadequate circumstances of the country, he wishes not to buy the goods from Thailand. So that at that time of deflation, if he again exchanged the Thai Bhatt into Nepali currency, how much loss or profit will he have? Find it.

9. a) A Nepali went to visit the UAE. In that time, he purchased a branded television of 75 inch for 10,000 Dirham. The exchange rate of that day was UAE Dirham 1 = NRs. 34.50. He paid 20% custom tax and 13 VAT. His relative wishes to buy it. At what price should he sell at break-even? Find it.
- b) 20% profit can be gained at a good sold for Indian rupees 21,600. At what Nepalese rupees can it be sold to gain 25% profit?
10. a) A Nepali does business in Australia. He came to Nepal for his household work. He wished to take 500 Pasmina shawls while returning back. He purchased the shawls at the rate of Rs. 3,500 each. He took it in Australia by paying 5% export tax. He paid Australian Dollar 30 for transportation from the airport to his residence. He sold the shawls at the rate of Australian Dollar 80 for each, find the loss or profit percent he got. (Australian dollar 1 = NRs. 88.50)
- b) The fair of air ticket from Kathmandu to Bangkok of Nepal Airlines is Rs. 28,000. Again, if the same ticket can be bought for 8000 Thai Bhat for Thailand, what percentage is it cheaper to buy the ticket from which place? (Thai Bhatt 1 = NRs. 3.50)
11. a) If the American Dollar 176 = 100 Pound Sterling and Pound Sterling = Nepali Rupees 151, how many Nepali Rupees can be exchanged for 132 American Dollars?
- b) If I.C. 100 = NRs. 160 and 1 American Dollar = NRs. 120, how many American Dollars are equal to I.C. 7500?
- c) If Chinese Yuan 1 = NRs. 18.70 and UAE Riyal 1 = NRs. 33.91, then change the Chinese Yuan 5612 into the United Arab Emirates Riyal.

Project work

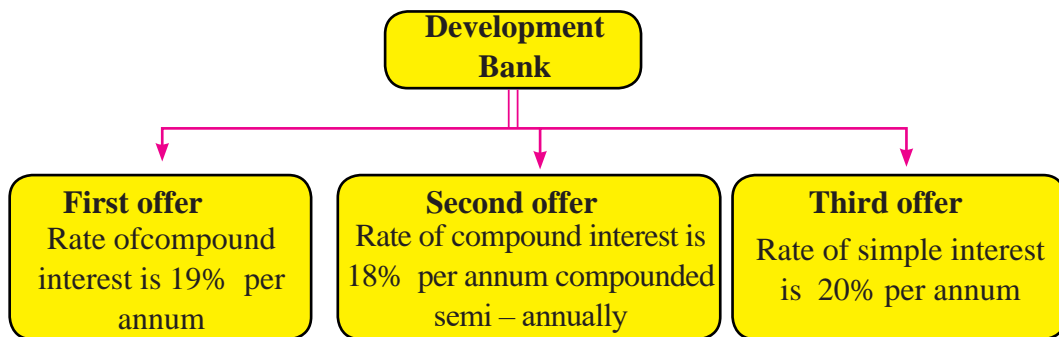
Make groups of suitable number of students. Collect exchange rates of the day searching from newspapers or the internet. Do you find difference in exchange rate of the day from the exchange rate of 2 days before? Besides, how much is the cost of gold per kilogram or per tola or per gram in the international market on that day? How much is in America? How much is in UK? How much is in Australia? How much is in the United Arab Emirates? On that day, in which city is the gold cheaper to buy in which city of which nation? Cheaper by how much. Prepare a report & present it to the class.

Answers

2. (a) Rs. 2280 (b) Rs. 254700 (c) Rs. 704724.48
(d) Rs. 59015.04 (e) Rs. 62767.41 (f) Rs. 77474.5
(g) Rs. 5578.43 (h) Rs. 73229.76
(i) Rs. 191326.25 (j) Rs. 64515
3. (a) Australian \$2266.8 (b) American \$1563.11
(c) Pound Sterling €1553.63 (d) Dirham 5742.17
4. (a) (\$)635 (b) Rs. 76,500
5. (a) Chinese Yuan 2000 (a) Rs. 93,200
6. (a) Dirham 3.68 (b) Malaysian Ringgit 1.22
(c) Canadian Dollar 258.47 (d) Thai Bhatt 8408.91
7. (a) 99,900 (b) 4,99,500
8. (a) Profit = Rs. 1,24,000 (b) Loss = Rs. 36,000
9. (a) Rs. 4,67,820 (b) NRs. 36,000
10. (a) Profit = 92.37% (b) 6.97% cheaper in Kathmandu
11. (a) Rs. 11,325 (b) Dollar 100
(c) UAE Riyal 3094.79

Mixed Exercise

1. **A family wants to deposit Rs. 4,00,000 in the account of a child aged. If who has just started grade 11. The money is intended for his/her study. If the money is deposited with the agreement that the amount with principal and interest would be drawn after he/she passed grade 12 at the age of 19. Then,**
 - a) How many times will the interest be calculated in 2 years according to the half yearly compound interest? Write.
 - b) Which of the following alternatives would you suggest to your parents to deposit the money and why? Justify with calculation.



2. **A sum of money amounts to Rs. 12,000 and Rs. 13,200 in 2 years and 3 years respectively at a certain rate of interest compounded annually.**
 - (a) Write the formula to find the compound interest.
 - (b) What should be the rate of the compound interest?
 - (c) What should be the principal?

3. **A man bought a land at Rs. 80,00,000 on 25th Baisakh of 2075 BS and started construction of a house on the same day. The construction of the house completed at the cost of Rs. 2,70,00,000. The price of land increased at the rate of 20% per year and the price of house decreased at the rate of 20% per year.**
 - (a) What does R indicate in the price after T years $(P_T) = P \left(1 + \frac{R}{100} \right)^T$?
 - (b) What will be the price of the land after 2 years?
 - (c) What will be the price of the house after 2 years?
 - (d) Will the prices of the land and house be the same after 2 years? If not, in how many years will the prices of the land and house be equal?

4. **Ram exchanged some Nepali rupees with American dollars at the exchange rate of \$1=Rs. 110. After 5 days, Nepali currency devalued against American dollars by 10% and he made a profit of Rs. 33,000 by exchanging the same dollars into Nepali currency again.**
 - (a) What is meant by exchange rate of currency?
 - (b) How much Nepalese rupees are equal to one American dollar (\$1) after devaluation of the Nepali currency?
 - (c) Find how much Nepali rupees did Ram exchange with American dollars at first?
 - (d) How much profit or loss would be there for him, if the Nepali rupees had revalued by 10% instead of devaluation of 10%?

5. The management committee of Nepal Bank Limited in its regular meeting has decided to change its annual policy slightly. According to the decision, the rate of interest for fixed deposit compounded semi annually is given below.

Depositing period	Rate of interest	Minimum deposit amount
Up to 6 months	6.75 %	Rs. 50,000
From 6 months to 1 year	7.25%	Rs. 50,000
From 1 year to 6 years	7.5%	Rs. 75,000

Sujit Thakur borrowed Rs. 2,00,000 from a cooperative one condition of paying it back in 2 years at the rate of simple interest of 5%. Immediately after borrowing, he deposited the same sum in a fixed deposit account of Nepal Bank Limited for the same duration.

- Which rate of interest should Sujit Thakur deposit the money in the fixed deposit?
- Find the compound amount that Sujit Thakur could receive in 2 years.
- Find the total amount to be paid to cooperative in 2 years.
- How much profit did Sujit Thakur earn in 2 years?

6. Anish has returned back Nepal from United Arab Emirate after 5 years. He earned 60,000 United Arab Emirate Dirhams and exchanged it with the Nepali rupees at the exchange rate of 1 Dirham = Rs. 34.83. He deposited the sum in the fixed deposit account of Nepal Bank for 1 year as per the following rate of interest compounded semi - annually.

Depositing Period	Rate of interest	Minimum deposit amount
Up to 6 months	9 %	Rs. 50,000
From 6 months to 1 year	10%	Rs. 50,000
From 1 year to 5 years	12%	Rs. 75,000

- Which body of government decides the exchange rate of currency in our country?
- How much Nepali rupees did Anish receive by exchanging 60,000 united Arab Emirate Dirhams?
- Which deposit offer of the bank is applicable for Anish? Give reason.
- How much amount will Anish receive from the bank after 1 year?

7. The price of a piece of land in Kathmandu was fixed at Rs. 60,00,000 at the end of 2020 AD. The price increased by 10% as the consequence of increasing buying pressure at beginning of 2021. But the price of land decreased by 4% due to the economic crisis at the end of 2022 AD.

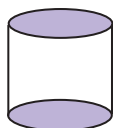
- (a) What does $P \left(1 + \frac{R}{100}\right)^T - P$ as per usual notation indicate?
- (b) What will be the price of the land at the end of 2022?
- (c) How much loss will be there to the person if the price decreased by 5% instead of decreasing by 4% in 2022 AD?
- 8. Kul Bahadur decided to go for foreign employment and lent a sum of Rs. 2,50,000 for 2 years at the interest rate of 15% compounded annually from a cooperative bank. After 1 year, he remitted 7,000 Ringgits to his home from the saving.**
- (a) Write the formula to find the compound amount.
- (b) What is the total amount to be paid after 1 year?
- (c) How much Nepali rupees are equal to Malaysian Ringgits 7,000?
- (d) How much total amount should he pay to clear all the debts finally?
- 9. A person lent a sum of Rs. 1,50,000 for 2 years at the interest rate of 10% per annum compounded annually. He paid Rs. 85,000 at the end of the first year.**
- (a) How much total amount should he pay to clear all the debts at the end of second year?
- (b) Find the total interest that he paid in two years.
- (c) If he cleared all the debts only after two years, how much more or less interest should have been paid?

Answers

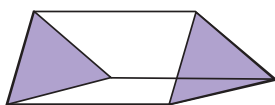
- (a) 4 times (b) As the first offer
- (a) $P \left[\left(1 + \frac{R}{100}\right)^T - 1 \right]$ (b) 10% (c) 9917.35
- (a) Growth rate (b) Rs. 1,15,20,000 (c) Rs. 1,72,80,000 (d) No, 3 year
- (b) Rs. 121 (c) Rs. 3,30,000 (c) Rs. 33,000 loss
- (a) 7.5 % (b) Rs. 2,31,125 (c) Rs. 2,20,000 (d) Rs. 31,125
- (a) Central Bank of Nepal (b) 20,89,800 (c) 10 % (d) Rs. 23,04,004.5
- (a) increase (b) Rs. 63,36,000 (c) Rs. 66,000 (loss)
- (b) Rs. 2,87,500 (c) Rs. 2,00,060 (d) Rs. 1,00,556
- (a) Rs. 88,000 (b) Rs. 23,000 (c) Rs. 8,500 more

5.0 Review

Form different groups of students of proper size and each group should take a solid object as given below. Discuss the following questions after observing the objects and also present it the classroom.



(i)



(ii)



(iii)



(iv)

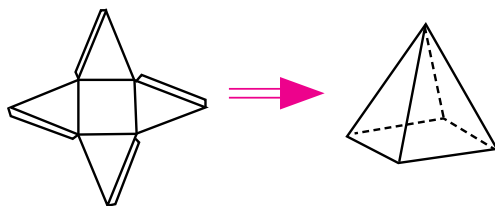
- What are the curved surface area and total surface area of the cylinder of diagram (i), if the radius of base is 7cm and height is 10cm?
- What is the volume of prism of diagram (ii), if the area of base is 24cm^2 and length is 8 cm?
- What is the radius of the sphere of diagram (iii) , if its volume is 616cm^3 ?
- What will be the ratio of the surface area of the sphere of diagram (iii) and the total surface area of hemisphere of diagram (iv) if their radii are equal?

5.1.1 Introduction of Pyramid

Activity 1

A figure of net of square based pyramid is given below. How can a pyramid be formed from a piece of paper?

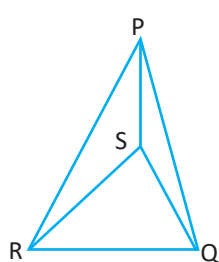
Take a rectangular piece of paper in pairs. Make a net of square based pyramid as given in the figure aside, and cut it with a scissor. Then, prepare a model of square based pyramid by joining all lateral surfaces with glue stick so that they all meet at a common vertex. Touch different parts of the model object and ask your friends the following questions.



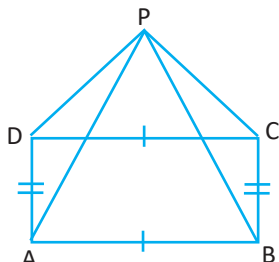
- How many surfaces are there?
- What are the shapes of the surfaces?
- How many vertices are there?
- In what way are the positions of surfaces are formed?

Activity 2

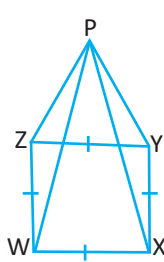
Form different groups containing proper number of students. In each group, take one solid object made from a paper or glass or wood. Discuss the following questions by observing the objects and also present it in your classroom.



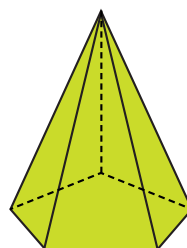
(i)



(ii)



(iii)



(iv)

- What are the shapes of the bases?
- Are there any surfaces other than bases? What are their shapes?
- How many vertices are there?
- Which surfaces are congruent?

The given solid objects are the images of pyramid. All the pyramids have polygonal base and besides this, there are also triangular lateral surfaces. The lateral surfaces of right pyramids with regular polygonal bases are congruent. The lateral surfaces have a common vertex. The vertical height is perpendicular to the surface of the base.

The three dimensional solid object having polygonal base and one common vertex to the lateral surfaces is called pyramid. Pyramids are given different names according to the types of polygonal base. For example: triangular based pyramid, square based pyramid.

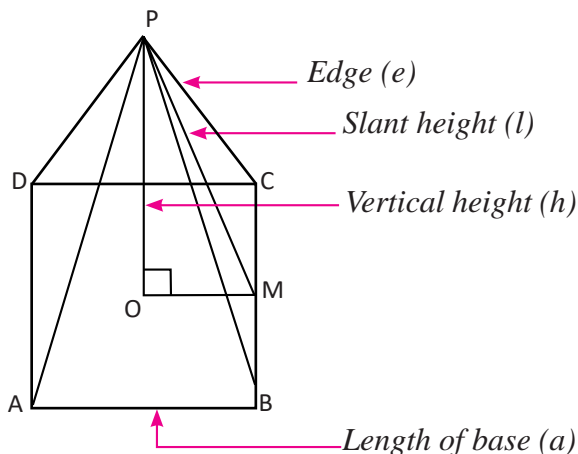
5.1.2 Different parts of pyramid and their relationship

Activity 3

A diagram of square based pyramid is given below. Study the given diagram and discuss the following questions:

Here, 'e', 'l', 'h' and 'a' denote the edge, slant height, vertical height and the length of edge of base respectively.

- What are the different parts of pyramid?
- What does the vertical height of a pyramid mean?
- What is slant height called?
- What is the edge of a pyramid?
- What is the relation between edge, slant height, vertical height and the length of edge of the base?



Now, from the diagram,

In the right angled triangle POM,

$$(PM)^2 = (PO)^2 + (OM)^2 \quad [\because \text{According to the Pythagoras theorem}]$$

$$l^2 = h^2 + \left(\frac{a}{2}\right)^2 \quad [OM = \frac{1}{2} AB = \frac{1}{2} a]$$

Similarly, in the right angled triangle PMC,

$$(PC)^2 = (PM)^2 + (MC)^2 \quad [\because \text{According to the Pythagoras theorem}]$$

$$e^2 = l^2 + \left(\frac{a}{2}\right)^2 \quad [MC = \frac{1}{2} BC = \frac{1}{2} a]$$

5.1.3 Surface Area of Pyramid

Activity 4

A pyramid and also its net are given in the diagram below, where ' l ' is the slant height and ' a ' is the length of the edge of base.

Now, discuss the following questions.

- How many triangular surfaces are there in the pyramid?
- Do all triangular surfaces have equal area?
- What are the shapes of surfaces in the pyramid and what are their area?

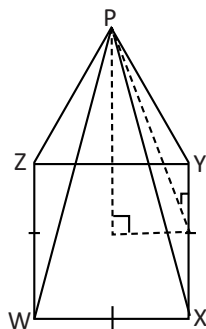


Diagram 1

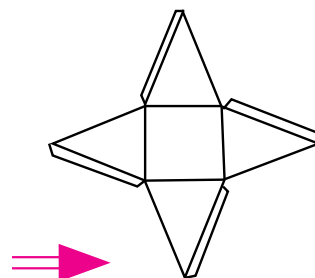


Diagram 2

Since, the base of the pyramid is square.

So, the area of base = area of square

$$= (\text{length of side})^2 = l^2$$

In the pyramid, the lateral surfaces are triangular.

Therefore, the area of one triangular surface = $\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times a \times l \quad [\because \text{The slant height of pyramid is the vertical height of the triangle}]$$

There are 4 triangular lateral surfaces with equal area in the pyramid.

So, the area of lateral surface area of pyramid = $4 \left(\frac{1}{2} \times a \times l \right)$

$$= 2al$$

Lateral surface area of square based pyramid (LSA) = $2al$

Total surface area (TSA) = Area of base + Area of lateral surface

$$= (a^2 + 2al) = a(a + 2l)$$

Example 1

Find the area of lateral and total surfaces of a square based pyramid if the length of side (a) = 8 cm and slant height (l) = 3 cm.

Solution

Here, the length of the side of base (a) = 8 cm

Slant height (l) = 3 cm

We know that the area of base = $a^2 = (8 \text{ cm})^2 = 64 \text{ cm}^2$

Lateral surface area (LSA) = $2al = 2 \times 8 \times 3 = 48 \text{ cm}^2$

Again, the total surface area = Area of base + Lateral surface area
 $= 64 \text{ cm}^2 + 48 \text{ cm}^2 = 112 \text{ cm}^2$

Hence, the surface area of the pyramid = 112 cm^2 .

Example 2

Find the total surface area of a square based pyramid with the length of the side of base is 10 cm and the length of the edge is 13 cm.

Solution

In the square based pyramid,

Length of the side of base (a) = 10 cm

Length of the edge (e) = 13 cm

Total surface area = ?

We know, $e^2 = l^2 + \left(\frac{a}{2}\right)^2$

$$\text{or, } (13)^2 = l^2 + \left(\frac{10}{2}\right)^2$$

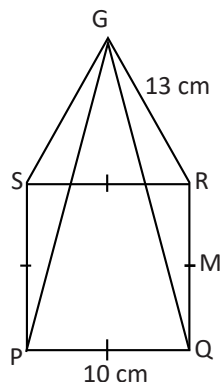
$$\text{or, } 169 = l^2 + 25$$

$$\text{or, } 169 - 25 = l^2$$

$$\text{or, } 144 = l^2$$

$$\therefore l = 12 \text{ cm}$$

$$\therefore \text{The slant height } (l) = 12 \text{ cm}$$



Again, we know that the area of base (A) = $a^2 = (10)^2 = 100 \text{ cm}^2$

Lateral surface area (LSA) = $2al = 2 \times 10 \times 12 = 240 \text{ cm}^2$

Then, the total surface area = Area of base (A) + Lateral surface area

$$= 100 + 240 = 340 \text{ cm}^2$$

Hence, the total surface of the pyramid is 340 cm^2 .

Example 3

Find the vertical height and the length of the edge of a square based pyramid if its total surface area is 144 cm^2 and slant height is 5 cm .

Solution

Here, in the square based pyramid,

Total surface area (TSA) = 144 cm^2

Slant height (l) = 5 cm

Vertical height (h) = ?

Length of the edge (e) = ?

We know that,

Total surface area = $a^2 + 2al$

$$\text{or, } 144 = a^2 + 2a \times 5$$

$$\text{or, } 144 = a^2 + 10a$$

$$\text{or } a^2 + 10a - 144 = 0$$

$$\text{or, } a^2 + (18 - 8)a - 144 = 0$$

$$\text{or, } a^2 + 18a - 8a - 144 = 0$$

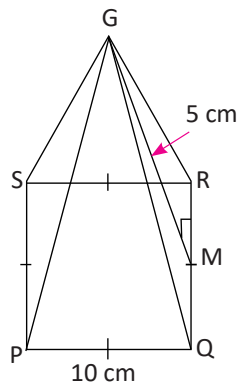
$$\text{or, } a(a + 18) - 8(a + 18) = 0$$

$$\text{or, } (a + 18)(a - 8) = 0$$

either, $a + 18 = 0 \therefore a = -18$ [\because length cannot be negative.

or, $a - 8 = 0 \therefore a = 8$ So, a cannot be equal to -18]

Therefore length of side of base (a) = 8 cm



$$\begin{aligned}\text{Again, } l^2 &= h^2 + \left(\frac{a}{2}\right)^2 \\ \text{or, } 5^2 &= h^2 + \left(\frac{8}{2}\right)^2 \\ \text{or, } 25 &= h^2 + 16 \\ \text{or, } 25 - 16 &= h^2 \\ \text{or, } 9 &= h^2 \\ h &= 3 \text{ cm}\end{aligned}$$

∴ The vertical height (h) = 3cm

$$\begin{aligned}\text{And, } e^2 &= l^2 + \left(\frac{a}{2}\right)^2 \\ \text{or, } e^2 &= 5^2 + \left(\frac{8}{2}\right)^2 \\ \text{or, } e^2 &= 25 + 16 \\ \text{or, } e^2 &= 41\end{aligned}$$

∴ $e = \sqrt{41}$ cm

Hence, the vertical height (h) and the length of the edge of the pyramid are 3 cm and $\sqrt{41}$ cm respectively.

Example 4

The figure given alongside is a square based pyramid. The length of base is 12 cm and the lateral surface area is 240 cm^2 . Find the slant height and vertical height.

Solution

Here, length of the base (a) = 12 cm

Lateral surface area (LSA) = 240 cm^2

Vertical height (h) = ?

Slant height (l) = ?

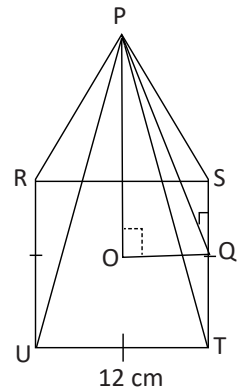
We know,

Lateral surface area = $2al$

$$\text{or, } 240 = 2 \times 12 \times l$$

$$\text{or, } \frac{240}{2 \times 12} = l$$

$$\therefore l = 10 \text{ cm}$$



$$\begin{aligned}\text{Now, } l^2 &= h^2 + \left(\frac{a}{2}\right)^2 \\ \text{or, } (10)^2 &= h^2 + \left(\frac{12}{2}\right)^2 \\ \text{or, } 100 &= h^2 + 36 \\ \text{or, } 100 - 36 &= h^2 \\ \text{or, } 64 &= h^2\end{aligned}$$

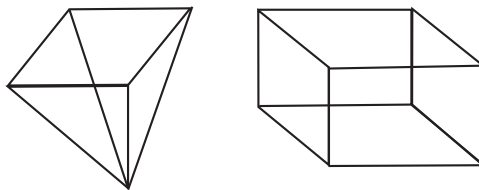
$$\therefore h = 8 \text{ cm}$$

Hence, the slant height (l) = 10 cm and the vertical height (h) = 8 cm.

5.1.4 Volume of Pyramid

Activity 1

Using card paper, construct a square based pyramid and a cube having equal base area and height. Construct the models so that we could keep sand or flour in them. Estimate how many times it should be poured from the pyramid into the cube to fill it up completely. Test whether your estimation is correct or not and also present the conclusion drawn to your classroom.



Here, we notice that the cube is filled with sand or flour after pouring 3 times from the pyramid.

Hence, $3 \times \text{Volume of the square based pyramid} = \text{volume of cube}$

Or, volume of the square based pyramid $= \frac{1}{3}$ volume of cube

$= \frac{1}{3} \text{ area of base} \times \text{height}$

If 'a' is the length of base and 'h' is the height of the square based pyramid, then its volume $= \frac{1}{3} \text{ area of base} \times \text{height}$

$$= \frac{1}{3} \times A \times h = \frac{1}{3} \times a^2 \times h$$

Example 5

Find the volume of square based pyramid in the given figure alongside.

Solution

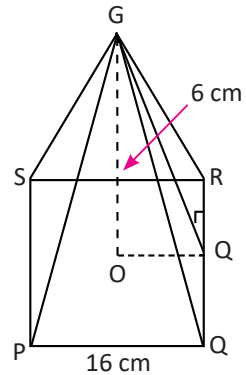
Length of the side of base (a) = 16 cm

Vertical height (h) = 6 cm

Volume of pyramid (V) = ?

We know, area of base (A) = $a^2 = (16)^2 = 256 \text{ cm}^2$

Now, volume of pyramid (V) = $\frac{1}{3} \times A \times h = \frac{1}{3} \times 256 \times 6 = 512 \text{ cm}^3$



Example 6

The volume of a square based pyramid is 384 cm^3 and the length of the side of base is 12 cm. Find the lateral surface area of the pyramid.

Solution

In the square based pyramid,

The volume of pyramid (V) = 384 cm^3

Length of the side of base (a) = 12 cm

Lateral surface area = ?

We know that, volume of the pyramid (V) = $\frac{1}{3} \times a^2 \times h$

$$\text{or, } 384 = \frac{1}{3} \times (12)^2 \times h$$

$$\text{or, } 384 = \frac{1}{3} \times 144 \times h$$

$$\text{or, } \frac{384}{144} \times 3 = h$$

$$\therefore h = 8 \text{ cm}$$

$$\text{Again, } l^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } l^2 = 8^2 + \left(\frac{12}{2}\right)^2$$

$$\text{or, } l^2 = 64 + 36$$

$$\text{or, } l^2 = 100 \text{ cm}^2$$

$$\therefore l = 10 \text{ cm}$$

Then, lateral surface area (LSA) = $2al = 2 \times 12 \times 10 = 240 \text{ cm}^2$.

Example 7

Find the volume of a square based pyramid if its total surface area is 96 cm^2 and the length of the side of base is 6 cm .

Solution

In the square based pyramid,

Total surface area (TSA) = 96 cm^2

Length of the side of base (a) = 6 cm

Volume of pyramid (V) = ?

According to the formula,

Total surface area (TSA) = $a^2 + 2al$

$$\text{or, } 96 = 6^2 + 2 \times 6 \times l$$

$$\text{or, } 96 - 36 = 12 \times l$$

$$\text{or, } 60 = 12 \times l$$

$$\text{or, } l = \frac{60}{12}$$

$$\therefore l = 5 \text{ cm}$$

$$\text{And, } l^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } 5^2 = h^2 + \left(\frac{6}{2}\right)^2$$

$$\text{or, } 25 = h^2 + 9$$

$$\text{or, } 25 - 9 = h^2$$

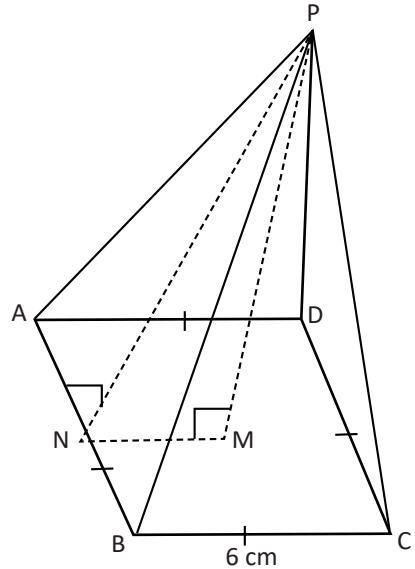
$$h^2 = 16$$

$$\therefore h = 4 \text{ cm}$$

Then, the volume of pyramid (V) = $\frac{1}{3}$ area of base \times height

$$= \frac{1}{3} \times 6^2 \times 4 = 48 \text{ cm}^3$$

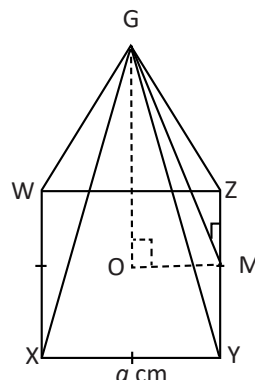
Hence, volume of the pyramid is (V) = 48 cm^3 .



Exercise 5.1

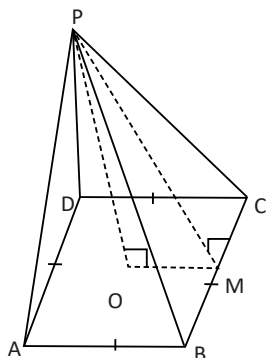
1. Study the given diagram of a square based pyramid and answer the following questions.

- What do GO and GM indicate in the pyramid?
- If $XY = a$ cm, then what is the length of OM?
- If the vertical height = h , slant height = l and the side of base = a , write their mathematical relation.
- What do the length GZ, GY, GX and GW represent? Are they equal to one another?

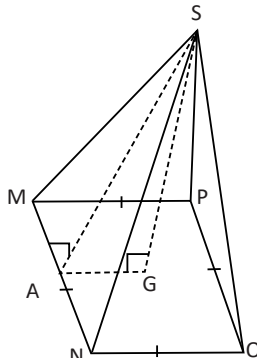


- Write the total surface area of a square based pyramid in which the side of base is ' m ' cm and slant height is ' n ' cm.
 - What is the lateral surface area of a square based pyramid if the length of the side of base is ' p ' cm and slant height is ' q ' cm?
 - Write the volume of square based pyramid in terms of r and h , if the length of the side of base is ' r ' cm and the vertical height is ' h ' cm.
 - Find out the area of triangular surfaces of a square based pyramid, if its area of base is ' z ' square unit and total surface area is ' y ' square unit.
- Find the volume of a square based pyramid in which the area of base is 64 cm^2 and height is 15 cm .
 - The total surface area of a square based pyramid is 285 cm^2 and the area of its triangular surfaces is 192 cm^2 . Find the area of base of the pyramid.
 - The diagrams of square based pyramids are given below:

- (i) If $PO = 8$ cm and $OM = 12$ cm, (ii) If $SA = 13$ cm and $NO = 10$ cm,

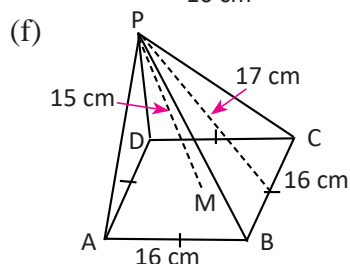
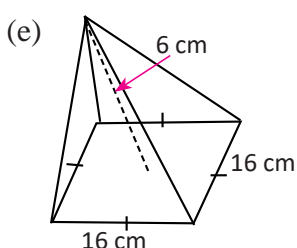
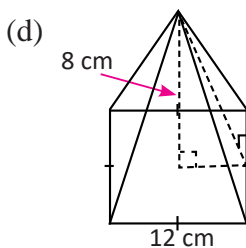
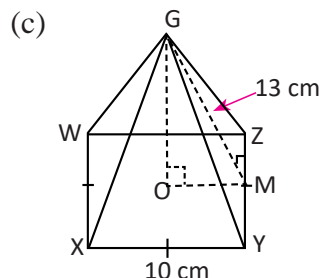
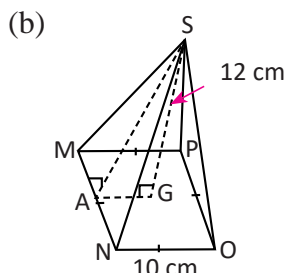
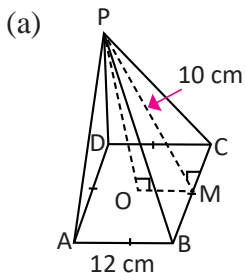


What are values of slant height and length of edge?



What are the values of vertical height and length of the edge?

4. Find the total surface area and volume of the square based pyramid from the given measurements.



- Sophiya has a pot with the shape of square based pyramid. She has fill it with water. Find how much water it can contain, if its height is 25 cm and the length of base is 30 cm.
- The total surface area of s square based pyramid is 800 cm^2 and its side of base is 16 cm. Find the area of triangular surfaces and volume of the pyramid.
- The total surface area of an aquarium with the shape of square based pyramid is 400 sq.inch. and its slant height is 15 inch. Then, find the volume of the aquarium.

8. A group of tourists visiting Egypt reached at a place where a pyramid was located. Their guide described that the area of the base of the pyramid is 3600 m^2 and its height is 50 m . On the basis of this information, a mathematics expert in the group was able to find its total surface area. Find the value that he obtained.

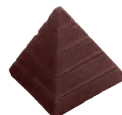
9. A hotel is planning to manage a tent with a shape of a square based pyramid for a group of tourists from or foreign country in which, the length of each side of base is 32 m and height is 50 m .

- (a) How much clothes in square meter is required to construct the tent?
(b) What will be the total cost to construct the tent if the cost of cloth per square metre is Rs. 500?

10. The figure given alongside is a telephone tower of Nepal Telecommunication Corporation (NTC) built in Asharam's field. The base of the tower is square shaped. Its slant height is 50 ft. and vertical height is 40 ft.



- (a) What is the area of the land occupied by the tower?
(b) If the rent of the field is Rs. 50 per sq.ft, find the total amount of rent for 20 years to be paid by NTC.
11. The figure given aside is a chocolate in the shape of a pyramid with a square base. The total surface area of the chocolate is 1920 cm^2 and length of the base is 30 cm . Find its vertical height.



Project work

Using chart paper, prepare the models of square based pyramids with different measurements. Measure the various parts of the pyramids and find out the value of following.

- (a) Area of base
(b) Area of triangular surfaces
(c) Total surface area
(d) Volume

Also, present the results in the classroom.

Answers

1. Show to your teacher.
2. (a) $(m^2 + 2mn) \text{ cm}^2$ (b) $2pq \text{ cm}^2$
(b) $\frac{1}{3} r^2 h \text{ cm}^3$ (c) $(y - z) \text{ sq. unit}$
3. (a) 320 cm^3 (d) 93 cm^2
(c) (i) $4\sqrt{13} \text{ cm}$ and $4\sqrt{22} \text{ cm}$ (d) $12 \text{ cm}, \sqrt{194}$
4. (a) 384 cm^2 and 384 cm^3 (b) 360 cm^2 and 400 cm^3
(c) 360 cm^2 and 400 cm^3 (d) 384 cm^2 and 384 cm^3
(e) 576 cm^2 and 512 cm^3 (f) 800 cm^2 and 1280 cm^3
5. 7.5 l 6. 544 cm^2 and 1280 cm^3
7. 471.40 inch^3 8. 10597.14 cm^2
9. 3359.84 cm^2 and Rs. 1679923.81
10. (a) 3600 sq.ft (b) Rs. 36,00,000 11. 8 cm

5.2.1 Introduction to cone

Activity 6

There is a cone of ice cream and a birthday cap in the figure.

Do they have same shapes? What are the shapes of these solid objects?

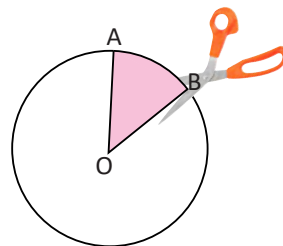
Cut the birthday cap with scissors as shown in the figure. Then, spread it on the table. Which shape that changed into?



The curved surface part of cone changes into sector after spreading it on the flat surface.

Activity 7

- Draw a circle on a rectangular piece of paper.
- Draw an angle $\angle AOB$ at the centre and cut the sector AOB as shown as in the figure.
- Now, bend the sector and join AO and BO.



Here, a cone with the circular base is formed in which the circumference of circular base is equal to arc AB.

Activity 8

A cone is shown in the figure given below where, the radius of the circular base is $OB = r$ cm. The line AO, joining the vertex A and the center O of circular base, is the height (h). And, the lines AC and AB are known as slant height (l).

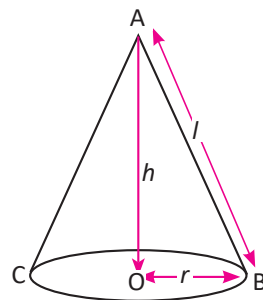
Now, discuss the relationship between the radius, height and slant height.

In the right angled triangle AOB,

$$(AB)^2 = (AO)^2 + (OB)^2 \quad [\because \text{From Pythagorus theorem}]$$

$$l^2 = h^2 + r^2$$

$$\therefore l = \sqrt{h^2 + r^2}$$



The slant height is equal to the square root of the sum of squares of height and radius of circular base, i.e. $l = \sqrt{h^2 + r^2}$.

Surface Area of Cone

Activity 9

Take a cone made from a paper. Cut it with scissor and spread it as shown in the figure (i). Now, the curved surface of cone is transformed into flat form as shown in the figure, which is a sector. And the length of this sector is equal to the circumference ($2\pi r$) of the circular base of the cone.

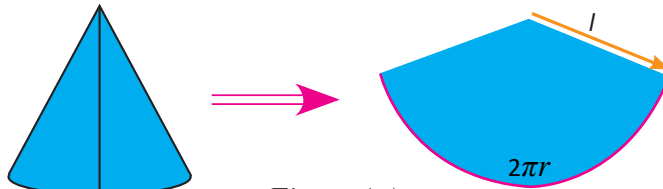


Figure (a)

Now, cut the sector into 4 equal pieces as shown in the figure (ii) and color two pieces with red.

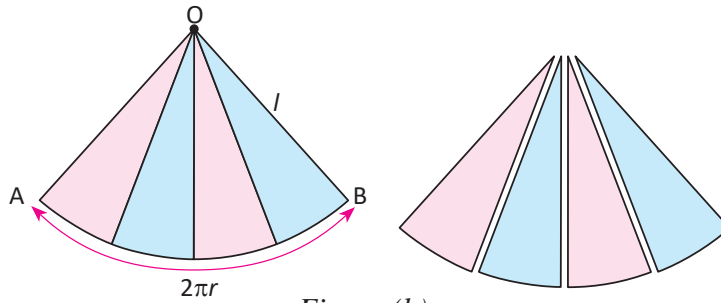


Figure (b)

Then, arrange two small sectors in the same directions and remaining two in opposite directions as shown in figure (iii).

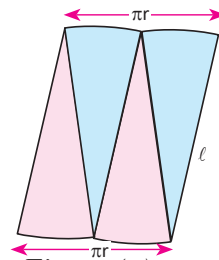


Figure (c)

Here,

If the radius of base of the cone is ' r ', slant height is ' h ' and the vertical height is ' h ', then,

$$\begin{aligned}
 \text{Curved surface area of the cone (LSA)} &= \text{Area of parallelogram} \\
 &= \text{length} \times \text{breadth} \\
 &= \pi r \times l = \pi r l
 \end{aligned}$$

$$\begin{aligned}
 \text{Total surface area of the cone (TSA)} &= \text{Area of base} + \text{Curved surface area (CSA)} \\
 &= \pi r^2 + \pi r l \\
 &= \pi r(r + l)
 \end{aligned}$$

If the radius of base of the cone is 'r', slant height is 'l' and the vertical height is 'h', then

$$\text{Curved surface area (CSA)} = \pi r \times l = \pi r l$$

$$\text{Total surface area (TSA)} = \pi r^2 + \pi r l = \pi r(r + l)$$

Example 1

Find the area of base and curved surface area of the cone given below.

Solution

Here, the vertical height of the cone (h) = 12 cm

Slant height of the cone (l) = 13 cm

Area of base of the cone = ?

Curved surface area of the cone = ?

Radius of base = r cm

In the figure,

From right angled triangle POQ,

$$PQ^2 = PO^2 + OQ^2 \quad [\because h^2 = p^2 + b^2]$$

$$\text{or, } 13^2 = 12^2 + r^2$$

$$\text{or, } 169 - 144 = r^2$$

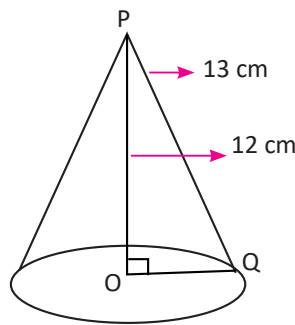
$$\text{or, } r^2 = 25$$

$$\therefore r = 5 \text{ cm}$$

We know that,

$$\text{The area of base} = \pi r^2 = \frac{22}{7} \times (5)^2 = 78.57 \text{ cm}^2$$

$$\text{Curved surface area of the cone} = \pi r l = \frac{22}{7} \times 5 \times 13 = 204.28 \text{ cm}^2$$



Example 2

Find the total surface area of a cone, if the diameter of base is 12 cm and the height is 8 cm.

Solution

Here, diameter of the base of the cone (d) = 12 cm

Radius of the base of the cone (r) = $\frac{d}{2} = \frac{12}{2} = 6$ cm

Vertical height of the cone (h) = 8 cm

Total surface area of the cone = ?

From the figure ,

$$l^2 = h^2 + r^2$$

$$\text{or, } l^2 = 8^2 + 6^2$$

$$\text{or, } l^2 = 64 + 36$$

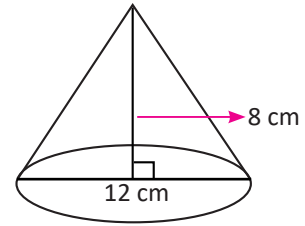
$$\text{or, } l^2 = 100 \quad \therefore l = 10 \text{ cm}$$

Now, the total surface area of the cone

$$= \pi r(r + l)$$

$$= \frac{22}{7} \times 6 \times (6 + 10)$$

$$= 301.71 \text{ cm}^2$$



Example 3

The sum of radius of base and slant height of a cone is 64 cm. If its total surface area is 2816 cm^2 , find its curved surface area.

Solution

Here, total surface area (TSA) = 2816 cm^2

By the question,

Radius of base (r) + slant height (l) = 64 cm

$$\therefore r + l = 64$$

Curved surface area of the cone (CSA) = ?

We know, total surface area of the a cone = $\pi r(r + l)$

$$\text{or, } 2816 = \frac{22}{7} \times r \times 64$$

$$\text{or, } 2816 \times 7 = r \times 22 \times 64$$

$$\text{or, } r = \frac{2816 \times 7}{22 \times 64}$$

$$\therefore r = 14 \text{ cm}$$

Putting the value of r in equation (i),

$$14 + l = 64$$

$$\text{or, } l = 64 - 14 = 50 \text{ cm}$$

Now, the curved surface area of the cone $= \pi r l = \frac{22}{7} \times 14 \times 50 = 2200 \text{ cm}^2$

Example 4

Find the length of circumference of base of a cone, if its total surface area and curved surface area are 1320 cm^2 and 704 cm^2 .

Solution

Here, total surface area (TSA) $= 1320 \text{ cm}^2$

Curved surface area (CSA) $= 704 \text{ cm}^2$

Length of circumference of base (C) $= ?$

We know,

Total surface area of a cone $= \text{Area of base} + \text{curved surface area}$

$$\text{or, } 1320 = \pi r^2 + 704$$

$$\text{or, } 1320 - 704 = \frac{22}{7} r^2$$

$$\text{or, } \frac{616}{22} \times 7 = r^2$$

$$\text{or, } r^2 = 196$$

$$\therefore r = 14 \text{ cm}$$

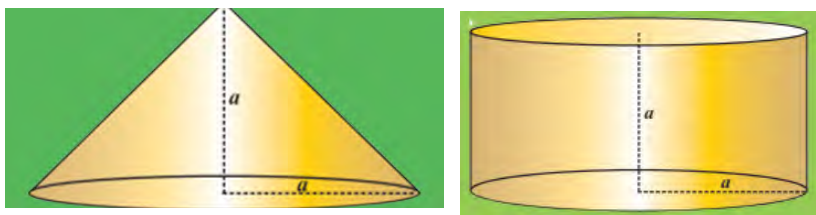
Now, the length of circumference (C) $= 2\pi r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm}$

Therefore, the length of circumference of the base of the cone is 88 cm.

5.2.2 Volume of Cone

Activity 10

Form different group of student of proper size. Using chart paper, make a cylinder with equal radius of base and height (suppose) in each group. Also, make a cone with the equal height and radius of the base as that of a cylinder.



Fill the cone with sand or dust of soil and pour it into the cylinder until it is filled up completely.

- (a) How many times it should be poured from the cone to fill up the cylinder?
- (b) Is it filled in three times?

Definitely, the cylinder will be filled up after pouring three times from the cone.

$$\begin{aligned}\text{So, volume of the cone (V)} &= \frac{1}{3} \text{ volume of cylinder} \\ &= \frac{1}{3} \times A \times h = \frac{1}{3} \pi r^2 h \quad [\text{Volume of a cylinder} = \pi r^2 h]\end{aligned}$$

Example 5

Find the volume of cone given aside.

Solution

Here, vertical height of cone (h) = 24 cm

Slant height of cone (l) = 26 cm

Volume of cone (V) = ?

In the figure,

Since, triangle MAN is a right angled triangle, from Pythagoras theorem,

$$AN = \sqrt{MN^2 - MA^2}$$

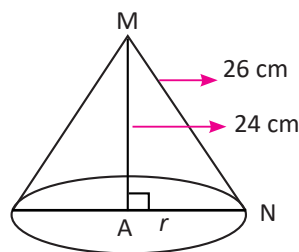
$$\text{or, } r = \sqrt{26^2 - 24^2}$$

$$\text{or, } r = \sqrt{676 - 576}$$

$$\therefore r = 10 \text{ cm}$$

We know,

$$\text{Volume of cone (V)} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (10)^2 \times 24 = 2514.28 \text{ cm}^3$$



Example 6

If the curved surface area of a cone is 1158.3 cm^2 and slant height is 19.5 cm , find its volume.

Solution

Here, curved surface area (CSA) = 1158.3 cm^2

Slant height (l) = 19.5 cm

Volume of cone (V) = ?

We know,

Curved surface area of cone = $\pi r l$

$$\text{or, } 1158.3 \text{ cm}^2 = \frac{22}{7} \times r \times 19.5 \text{ cm}$$

$$\text{or, } \frac{1158.3}{22 \times 19.5} \times 7 \text{ cm} = r$$

$$\text{or, } r = 18.5 \text{ cm}$$

$$\text{Now, height (h)} = \sqrt{l^2 - r^2} = \sqrt{(19.5)^2 - (18.5)^2} = 4.8 \text{ cm}$$

Again, we know that,

$$\text{Volume of cone is (V)} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (18.5)^2 \times 4.8 = 1796.25 \text{ cm}^3$$

Hence, the volume of cone (V) = 1796.25 cm^3

Example 7

The ratio of radius of base and height of a cone with volume 314.86 cm^3 is 5:12. Find its curved surface area and total surface area.

Solution

Here, the ratio of radius of base and height is 5:12.

Let radius (r) = $5x$ and height (h) = $12x$

$$\text{Volume (V)} = 314.86 \text{ cm}^3$$

Curved surface area = ?

Total surface area = ?

We know,

$$\text{Volume of cone (V)} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5x)^2 \times 12x$$

$$\text{or, } 314.86 \times 21 \text{ cm}^3 = 22 \times 25x^2 \times 12x$$

$$\text{or, } \frac{314.86 \times 21}{22 \times 25 \times 12} = x^3$$

$$\therefore x = 1.00060 \text{ cm} = 1 \text{ cm}$$

So, radius (r) = $5 \times 1 \text{ cm} = 5 \text{ cm}$ and height (h) = $12 \times 1 \text{ cm} = 12 \text{ cm}$

$$\begin{aligned}\text{Slant height (} l \text{)} &= \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (5)^2} \\ &= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{We know that the curved surface area (CSA)} &= \pi r l = \frac{22}{7} \times 5 \times 13 \\ &= 204.28 \text{ cm}^2\end{aligned}$$

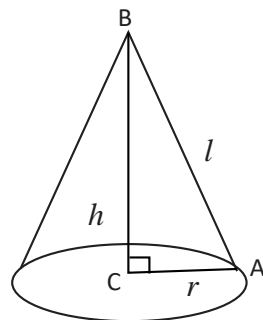
$$\begin{aligned}\text{Total surface area of cone (TSA)} &= \pi r(r + l) = \frac{22}{7} \times 5(5 + 13) \\ &= 282.85 \text{ cm}^2\end{aligned}$$

Exercise 5.2

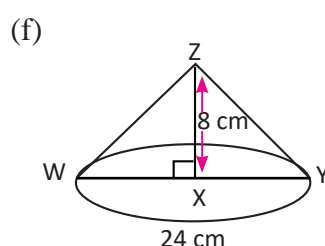
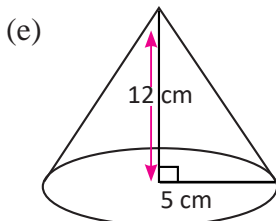
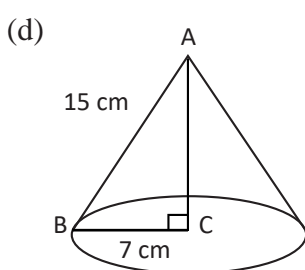
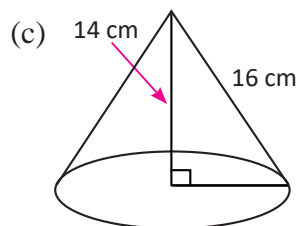
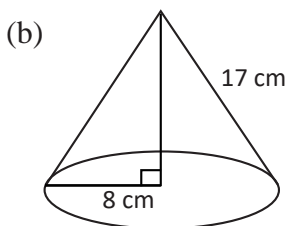
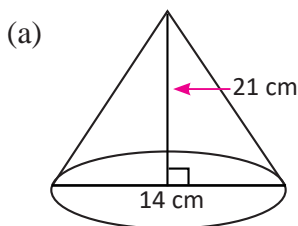
- 1. What is the shape of the surface of base in given figure? Answer the following questions on the basis of information given in the figure:**

- (a) What do the symbols l , h and r represent?
- (b) Write the mathematical relationship of l , h and r .
- (c) What is the value of h , if $l = 13 \text{ cm}$ and $r = 5 \text{ cm}$?

- 2.**
- (a) Write the curved surface area of a cone, if the radius of base is ' r ' cm and slant height is ' l ' cm.
 - (b) Write the total surface area of cone in terms of x and y , if the radius of base is ' x ' cm and slant height is ' y ' cm.
 - (c) What is the volume of a cone, if the diameter of base is ' p ' cm and vertical height is ' q ' cm.
- 3.**
- (a) Find the volume of a cone, if its curved surface area is 81 cm^2 and height is 15 cm .
 - (b) The total surface area of a cone is 250 cm^2 and the area of base is 118 cm^2 . Find its curved surface area.
 - (c) Find the volume of a cone if its slant height is 5 cm and radius of base is 4 cm .



4. Find the curved surface area, total surface area and volume of the following cones on the basis of given information.



- If the volume of a right circular cone is $100\pi \text{ cm}^3$ and height is 12 cm, find its slant height.
- Find the radius of base and height of conical shaped tent if its volume is 1232 cm^3 and the area of base is 154 cm^2 .
- The amount of plastic required to construct a conical shaped tent with slant height 14 cm is 77 m^2 . Find the area and perimeter of the base of tent.
- Find the volume, curved surface area and total surface area of a cone with the diameter of base 8 cm and vertical height 21 cm.
- The vertical height of a solid cone is three times the diameter of the base. Find its total surface area if the volume is $54\pi \text{ cm}^3$.
- A student studying in class 10 has prepared a cone made of wood as a task of his project work. The area of base and height of the cone are 154 m^2 and 14 m respectively, how much does it cost to color the surface of the cone excluding the base, if the cost per square meter is Rs. 1.50?
- A cone shaped pot is filled with water. The height of the pot is 21 cm and the diameter of base is 14 cm. When two solid cones with equal dimensions are immersed into the pot, one third of water flows outside from the pot. If so, what is the volume of each cone immersed in the pot?

12. Three solid cones having the same height and of radii 3 cm, 4 cm and 5 cm respectively are melted together and a big single solid cone is made. Find the diameter of the cone if its height is equal to the height of the small one.
13. A toy of conical shape is made from a wood and the diameter of its base is 10 cm long. It costs Rs. 880 to color its whole surface at the rate of Rs. 4 per square cm. Find the volume of the pot.

Project work

Using papers of different sizes, make cones of different dimensions. Measure the dimensions of the cones and find the area of the base, curved surface area, total surface area and volume of the cones. Also, present it in the classroom.

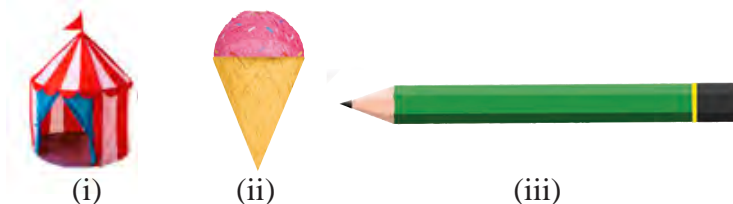
Answers

1. (a) $l^2 = h^2 + r^2$ (b) $h = 12$ cm
2. (a) πmn cm² (b) $\pi x(x + y)$ cm² (c) $\frac{1}{12} \pi p^2 q$ cm³
3. (a) 405 cm³ (b) 132 cm² (c) 16π cm³
4. (a) 486.99 cm², 640.99 cm², 1078 cm³
 (b) 136π cm², 200π cm², 320π cm³
 (c) 123.94π cm², 183.94π cm², 280π cm³
 (d) 330 cm², 484π cm², 681.01 cm³
 (e) 65π cm², 90π cm², 100π cm³
 (f) 156π cm², 300π cm², 240π cm³
5. 13 cm 6. 7 cm, 24 cm 7. 9.62 m², 11m
8. 352 cm³, 268.54 cm², 318.80 cm² 9. 200.34 cm² 10. Rs. 516.53
11. 179.67 cm³ 12. $10\sqrt{2}$ cm 13. 195.99 cm³

5.3.1 Combined Solid Object

Activity 11

Observe the following figures and discuss the questions given below.



(a) What shapes do the given figures contain?

(b) What types of solid objects are these?

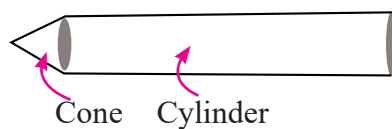
Here, (i) the tent contains a cone and a cylinder.



(ii) The ice cream contains a cone and a hemisphere.



(iii) The pencil contains a cone and a cylinder.



The combined solid objects contain two or more three dimensional objects.

5.3.2 Area and Volume of Combined Solid Objects

Activity 12

Discuss the formula to find the area and volume of combined solid objects in your groups.

1. Cylinder and Cone



The solid object combined with cone and cylinder is given in the figure. If the radius of the base is 'r', the height of the cylinder is ' h_1 ', the height of the cone is ' h_2 ' and slant height of the cone is ' l ', then,

The base area of the cone = area of circle = πr^2

Curved surface area of the cylinder = $2\pi r h_1$

Curved surface area of the cone = $\pi r l$

- (a) Total surface area = area of base + curved surface area of cylinder + curved surface area of cone

$$= \pi r^2 + 2\pi r h_1 + \pi r l$$

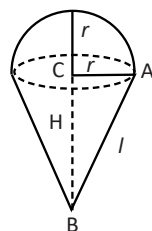
- (b) Volume (V) = volume of cylinder + volume of cone

$$= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2$$

$$= \pi r^2 \left(h_1 + \frac{1}{3} h_2 \right)$$

2. Cone and Hemisphere

A combined solid object containing the cone and hemisphere is shown in the figure. The vertical height of the conical part is ' h ', the slant height is ' l ' and the radius is ' r '.

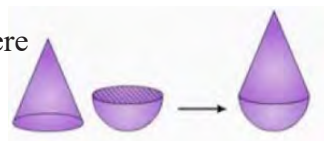


- (a) Total surface area (TSA) = curved surface area of cone + curved surface area of hemisphere

$$= \pi r l + 2\pi r^2 = \pi r (l + 2r)$$

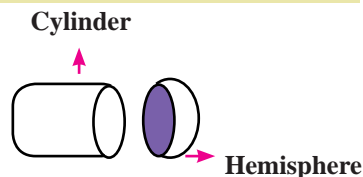
- (b) Volume (V) = volume of cone + volume of hemisphere

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r)$$



3. Cylinder and Hemisphere

A figure of solid object containing a cylinder and a hemisphere is given aside. The radius of the base of the object is 'r' and the height of the cylindrical part is 'h'.



- (a) Total surface area (TSA) = area of base + curved surface area of cylinder + curved surface area of hemisphere

$$= \pi r^2 + 2\pi r h + 2\pi r^2 = 2\pi r h + 3\pi r^2$$

- (b) Total volume = volume of cylinder + volume of hemisphere

$$= \pi r^2 h + \frac{2}{3} \pi r^3 = \pi r^2 \left(h + \frac{2}{3} r \right)$$

4. Cone and Cone

A figure of combined solid object containing two cones is given aside. Where,

Common radius of base = r

Vertical height of upper cone = h_1

Slant height of upper cone = l_1

Vertical height of lower cone = h_2

Slant height of lower cone = l_2

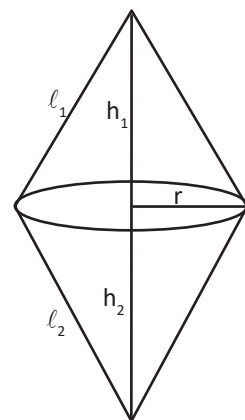
- (a) Total surface area (TSA) = curved surface area of upper cone + curved surface area of lower cone

$$= \pi r l_1 + \pi r l_2 = \pi r (l_1 + l_2)$$

- (b) Volume (V) = volume of upper cone + volume of lower cone

$$= \frac{1}{3} \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2$$

$$= \frac{1}{3} \pi r^2 (h_1 + h_2)$$



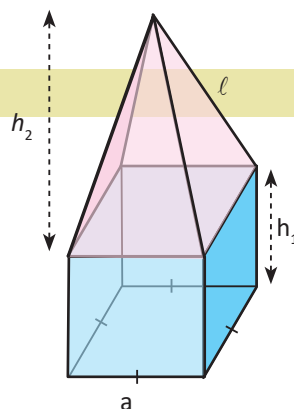
5. Prism and Pyramid

A figure of combined solid object containing a prism and a pyramid is given aside. The shape of the base of prism is square.

The different dimensions of solid are as follows:

Length of base = a

Height of prism = h_1



Height of pyramid = h_2

Slant height of pyramid = l

Area of base (A) = area of square = a^2

(a) Total surface area (TSA) = area of base + lateral surface area of prism + lateral surface area of pyramid

$$= a^2 + p \times h_1 + 2al \text{ (Where, } p \text{ is the perimeter of the base of prism)}$$

(b) Total volume (V) = volume of prism + volume of pyramid

$$= A \times h_1 + \frac{1}{3} Ah_2 = a^2 h_1 + \frac{1}{3} a^2 h_2 = a^2 (h_1 + \frac{1}{3} h_2)$$

Think!

Are the total surface and lateral surface area equal for all types of combined solid objects?



Example 1

A figure of a pencil is given aside. Find the total surface area and volume of the pencil.

Solution

Here, radius of the base (r) = 7 cm

Height of the cylinder (h_1) = 39 cm

Height of the cone (h_2) = 24 cm

Total surface area = ?

Volume = ?

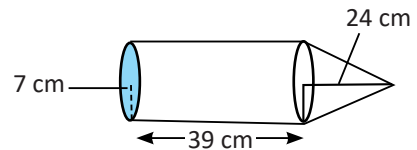
We know, $l^2 = h^2 + r^2 = (24)^2 + (7)^2 = 576 + 49 = 625 \text{ cm}^2$

Therefore, the slant height of the cone (l) = 25 cm

$$\begin{aligned} \text{Again, total surface area} &= \pi r^2 + 2\pi r h_1 + \pi r l \\ &= \pi r (r + 2h_1 + l) \\ &= \frac{22}{7} \times 7 (7 + 2 \times 39 + 25) = 22 \times 110 = 2420 \text{ cm}^2 \end{aligned}$$

Volume (V) = volume of cylinder + volume of cone

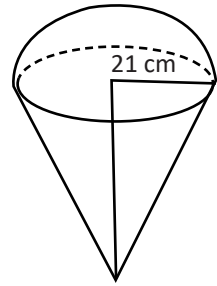
$$\begin{aligned} &= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 \\ &= \pi r^2 (h_1 + \frac{1}{3} h_2) = \frac{22}{7} \times (7)^2 (39 + \frac{1}{3} \times 24) = 7238 \text{ cm}^3 \end{aligned}$$



Example 2

A figure of ice cream is given aside. If the radius of the circular base is 21 cm and the volume of ice cream is 32340 cm^3 ,

- (a) Find the height of conical part.
- (b) Find the total surface area.



Solution

Here, radius of the base (r) = 21 cm

Volume of the cone with ice cream (V) = 32340 cm^3

- (a) We know,

$$\text{Volume of cone with ice cream (V)} = \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$\text{or, } 32340 = \frac{1}{3} \pi r^2 (h + 2r)$$

$$\text{or, } 32340 = \frac{1}{3} \times \frac{22}{7} \times (21)^2 (h + 2 \times 21)$$

$$\text{or, } \frac{32340 \times 21}{22 \times 441} = (h + 42)$$

$$\text{or, } 70 - 42 = h$$

$$\therefore h = 28 \text{ cm}$$

(b) Slant height (l)

$$\begin{aligned} &= \sqrt{h^2 + r^2} \\ &= \sqrt{(28)^2 + (21)^2} \\ &= \sqrt{1225} \\ &= 35 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Total surface area (TSA)} &= \pi r l + 2\pi r^2 \\ &= \pi r (l + 2r) \\ &= \frac{22}{7} \times 21 (35 + 2 \times 21) \\ &= 66 (35 + 42) = 66 \times 77 \\ &= 5082 \text{ cm}^2 \end{aligned}$$

Example 3

The solid object in the given figure contains two cones. On the basis of the given measurements, find its volume.

Solution

Here, the common diameter to the base to the both cones $(d) = 6 \text{ cm}$

Total height of the solid $= 20 \text{ cm}$

Let the height of cone of the left part $= h_1 \text{ cm}$

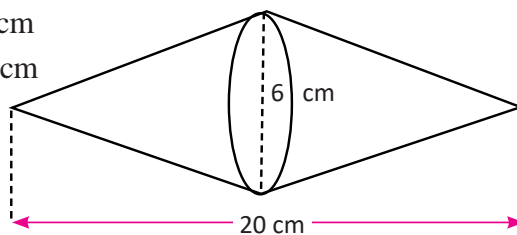
and the height of cone of the right part $= h_2 \text{ cm}$

$$\therefore h_1 + h_2 = 20 \text{ cm}$$

(a) Radius of base $(r) = \frac{d}{2} = \frac{6}{2} = 3 \text{ cm}$

Volume of solid $(V_1) = \frac{1}{3} \pi r^2 (h_1 + h_2)$

$$= \frac{1}{3} \times \frac{22}{7} \times (3)^2 (20) = 188.57 \text{ cm}^3$$



Example 4

A figure of stupa is given aside. The shape of its lower part is square based prism and the upper part is square based pyramid. Then, find its

(a) Volume

(b) Total surface area.

Solution

Here, total height of the stupa $= 5.5 \text{ m}$

Height of the prism $(h_1) = 5 \text{ m}$

Height of the pyramid $(h_2) = 5.5 - 5 = 0.5 \text{ m}$

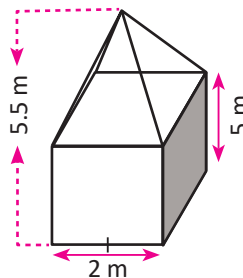
Length of the base of stupa $(a) = 2 \text{ m}$

(a) Area of base $(A_1) = a^2 = 2^2 = 4 \text{ m}^2$

Volume of the prism shaped part $(V_1) = A_1 \times h_1 = 4 \times 5 = 20 \text{ m}^3$

Volume of the pyramid shaped part $(V_2) = \frac{1}{3} A_1 \times h_2 = \frac{1}{3} \times 4 \times 0.5 = \frac{2}{3} \text{ m}^3$

Volume of the stupa $(V) = V_1 + V_2 = 20 + \frac{2}{3} = 20.67 \text{ m}^3$



(b) Perimeter of base $(P) = 4a = 4 \times 2 = 8 \text{ m}$
 Slant height of the pyramidal part $(l) = \sqrt{(h_2)^2 + (\frac{a}{2})^2}$
 $= \sqrt{(0.5)^2 + (\frac{2}{2})^2} = \sqrt{0.25 + 1} = \sqrt{1.25} \text{ m}$
 Lateral surface area of the prism shaped part $(A_2) = p \times h_1 = 8 \times 5 = 40 \text{ m}^2$
 Lateral surface area of the pyramid shaped part $(A_3) = 2al = 2 \times 2 \times \sqrt{1.25} = 4.47 \text{ m}^2$
 Total surface area of the stupa $(A) = A_1 + A_2 + A_3$
 $= 4 + 40 + 4.47$
 $= 48.47 \text{ m}^2$

Hence, the total surface area of the stupa is 48.47 m^2

Example 5

The given figure contains two square based pyramids with equal height. If the length of the base of each pyramid is 6 cm and the total volume is 96 cm^3 , find the height of each pyramid.

Solution

Here, the length of the base of each pyramid $(a) = 6 \text{ cm}$

Total volume of pyramids $(V) = 96 \text{ cm}^3$

Suppose, the height of each pyramid $= h$

Total volume of pyramids $(V_2) = \frac{1}{3}a^2h + \frac{1}{3}a^2h$

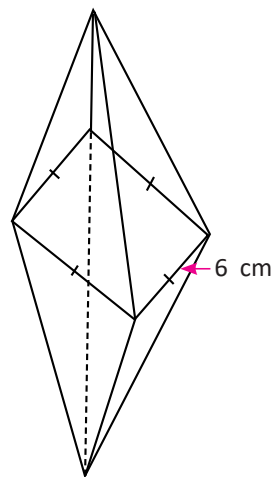
$$\text{or, } 96 = \frac{2}{3}a^2h$$

$$\text{or, } \frac{96 \times 3}{2} = (6)^2 \times h$$

$$\text{or, } \frac{96 \times 3}{2 \times 36} = h$$



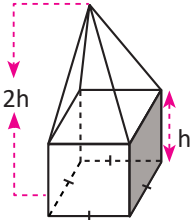
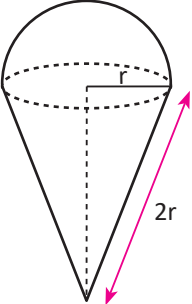
$$\therefore h = 4 \text{ cm}$$

Hence, the height of each pyramid (h) is 4 cm.

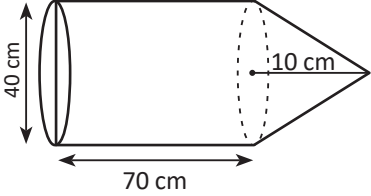
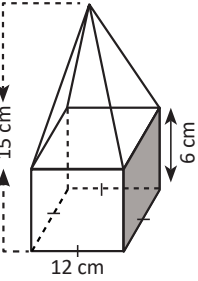
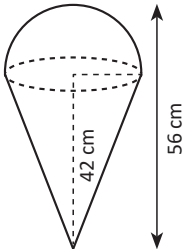
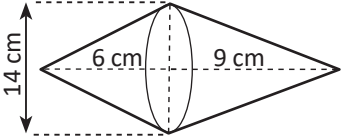


Exercise 5.3

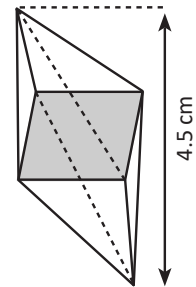
1. Answer the following questions on the basis of given information corresponding to the combined solid objects.

<p>(a) Total volume of combined solid (V) = 1050 cm^3 Volume of cylindrical part (V_1) = 748 cm^3</p>  <p>Volume of conical part (V_2) = ?</p>	<p>(b) Curved surface area of the conical part = 252 cm^2 Curved surface area of the cylindrical part = 272 cm^2 Area of the circular part = 154 cm^2</p>  <p>Total surface area = ?</p>
<p>(c) Area of the base = 36 cm^2 Volume of the prism shaped part (V_1) = 144 cm^3</p>  <p>Volume of the pyramid shaped part = ?</p>	<p>(d) Curved surface area of conical shaped part = 308 cm^2</p>  <p>Curved surface area of the hemispherical part = ?</p>

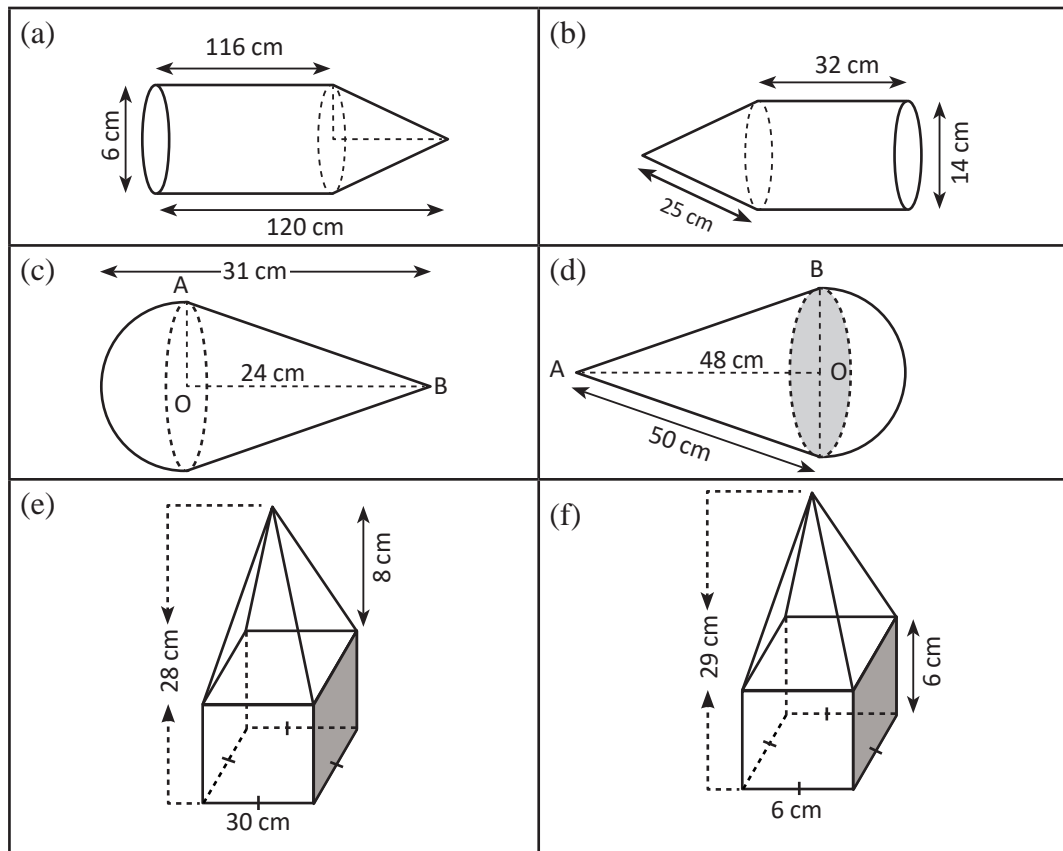
2. Find the volume of solid objects with given dimensions.

<p>(a)</p> 	<p>(b)</p> 
<p>(c)</p> 	<p>(d)</p> 

3. In the given figure of a crystal, the shaded part is square shaped of length 2.5 cm. The total height of the object is 4.5 cm. If the height of upper and lower pyramids are equal, find the total surface area and volume of the crystal.

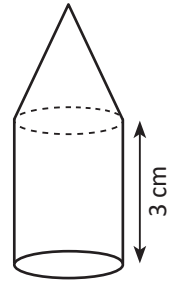


4. Find the total surface area of the given combined solids.

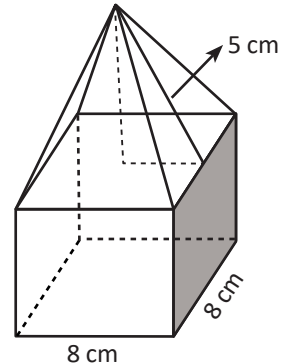


5. A toy contains conical and hemispherical parts of radius 14 cm. Find its total surface area if its total height is 49 cm.
6. A pyramid of height 12 cm is placed on the top of a solid of square based cuboid. If the base area of cuboid shaped solid is 100 cm^2 and its height is 10 cm, find
- (a) total volume and (b) total surface area of the combined solid.
7. A pyramid with vertical height 8 cm is placed on the top of a cubical solid. Find the total volume of the combined solid if the length of the base of cube is 12 cm.

8. (a) The solid object given in the figure contains a cone and a cylinder. The area of the base of cylinder is 100 cm^2 and its height is 3 cm. Find the total height of the solid if its total volume is 600 cm^3 .

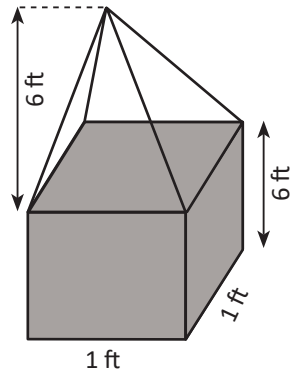


- (b) The upper part of a solid given aside is a pyramid with the slant height 5 cm. The lower part is a square based prism whose length of base is 8 cm. If the volume of solid is 448 cm^3 , find the height of the prism shaped portion.



9. **In the adjoining figure, a square based pyramid is placed on the top of a tower. The height of tower and pyramid are 6 ft. and the length of the base of tower is 1 ft.**

- (a) Find the lateral surface area of the pyramid shaped portion.
 (b) Find the total surface area of the tower that can be painted.



10. Ram received a spinning top (A toy) from his father as a gift on his sixth birthday. The lower part of the spinning top is conical and the upper part is hemispherical. The total height of the spinning top is 5 cm and diameter of hemisphere is 3.5 cm. Ram is planning to color it. Find the area of the surface that can be colored.

Project Work

Divide the students into various groups of proper size. Construct various types of solid objects using paper, wood, soil, bamboo, wire, sticks or any other locally available materials. Then, find the surface area and volume of such solids. Besides this, construct other new solids combining two or more such solids and also find their area and volume. Test whether the area and volume before and after combination are equal or not. Give reasons in both cases. Finally, prepare a report on the basis of your activities and findings, and also present it in your classroom.

Answers

1. (a) 302 cm^3 (b) 678 cm^2 (c) 48 cm^3 (d) 308 cm^2
2. (a) 92190.47 cm^3 (b) 1296 cm^3 (c) 14373.33 cm^3 (d) 770 cm^3
3. 9.375 cm^3 , 25.74 cm^2
4. (a) 2262.85 cm^2 (b) 2112 cm^2 (c) 858 cm^2
(d) 3432 cm^2 (e) 4320 cm^2 (f) 3504 cm^2 5. 2890.63 cm^2
6. (a) 1400 cm^3 (b) 760 cm^2 7. 2112 cm^3
8. (a) 12 cm (b) 6 cm
9. (a) $\sqrt{5} \text{ cm}^2$ (b) 26.23 cm^2 10. 39.55 cm^2

5.3.2 Cost Estimation

Activity 1

Form different groups each containing 3 students. Measure the length, breadth and height of your classroom. Also, measure the length, breadth and height of doors and windows. The classroom excluding doors and windows needs to be painted. Now, find the total cost of painting under the following cotation.

(a) The area of 6 m^2 can be painted with 1 liter color.

(b) The price of color is Rs. 200 per liter.

Activity 2

The length, breadth and height of a room are 14 ft., 12 ft. and 10 ft. respectively. There is a door of width 3 ft and height 6.6 ft. There is also a window of size $6 \text{ ft} \times 4 \text{ ft}$. The room has to be painted. There are two different conditions regarding the rate of painting and rate of colors. In which cotation can the room be painted in lower cost? Discuss in your group.

House owner provides colors and painting related materials	The painter himself provides colors and all the painting related materials
First cotation	Second cotation
Primer on wall and ceiling = Rs. 7 per sq.ft. Painting the walls and ceiling =Rs. 5 per sq.ft Enamel painting on doors and windows = Rs.12 per sq.ft Quantity of colors required = 7.2 liters Cost of colors = Rs 420 per liter	Primer on wall and ceiling = Rs.14 per sq.ft. Painting the walls and ceiling =Rs.30 per sq.ft Enamel painting on doors and windows =Rs.24 per square ft

According to the first cotation,

Area of a door (A_1) = $3 \times 6.6 = 19.8 \text{ ft}^2$

Area of a window (A_2) = $6 \times 4 = 24 \text{ ft}^2$

Area of 4 walls and ceiling excluding the door and window

$$\begin{aligned}(A_3) &= 2h(l + b) + l \times b - (A_1 + A_2) \\ &= 2 \times 10(14 + 12) + 14 \times 12 - (19.8 + 24) \\ &= (688 - 43.8) \text{ ft}^2 \\ &= 644.2 \text{ ft}^2\end{aligned}$$

Cost of primer for walls and ceiling (T_1) = $7 \times 644.2 = \text{Rs. } 4509.4$

Cost of painting on the door and window (T_2) = $12 \times (19.8 + 24) = \text{Rs. } 525.6$

Cost of painting on the 4 walls and ceiling (T_3) = $5 \times 644.2 = \text{Rs. } 3221$

Cost of colors required for painting (T_4) = $\text{Rs. } 7.2 \times 420 = \text{Rs. } 3024$

$$\begin{aligned}\text{Total cost of painting (T)} &= T_1 + T_2 + T_3 + T_4 \\ &= 4509.4 + 525.6 + 3221 + 3024 \\ &= \text{Rs. } 11280\end{aligned}$$

According to the second cotation,

Cost of primer for walls and ceiling (T_1) = $14 \times 644.2 = \text{Rs. } 9,018.8$

Cost of painting on the door and window (T_2) = $24 (19.8 + 24) = \text{Rs. } 1,051.2$

Cost of painting on the 4 walls and ceiling (T_3) = $30 \times 644.2 = \text{Rs. } 19,326$

Total cost of painting (T) = $T_1 + T_2 + T_3 = 9018.8 + 1051.2 + 19,326 = \text{Rs. } 29,396$

Therefore the cost of painting will be least using first cotation.

Example 1

If the inner length of a square based water tank is 3 m and height is 4 m, find how much water the tank may contain.

Solution

In the square based water tank,

Inner length (l) = 3 m

Inner breadth (b) = 3 m

Area of the base of the tank (A) = $l^2 = (3)^2 = 9 \text{ m}^2$

Volume of the tank (V) = $A \times h = 9 \times 4 = 36 \text{ m}^3$

Volume of tank (V) = volume of water

So, the volume of water (V) = 36 m^3

Again, we know that,

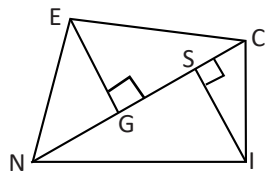
Amount of water in 1 cubic meter = 1000 l

$$\therefore 36 \text{ m}^3 = 36 \times 1000 \text{ l} = 36,000 \text{ l}$$

Hence, the amount of water that maybe contained in the tank is 36,000 l.

Example 2

The given figure NICE is a quadrilateral shaped land. The length between the corners N and C is 40 m. The length of perpendiculars from the corners E and I to the line NC are $EG = 10$ m and $IS = 15$ m respectively.



- Write the formula to find the area of quadrilateral.
- Find the area of the land.
- A male worker can finish the work of labeling the ground in 2 days and he receives wage at the rate of Rs 1500 per day. After then, a female worker plants the *dubo grass* (Bermuda grass) on it and she receives wages at the rate of Rs. 70 per square meter. Find the total cost of plantation.

Solution

In the quadrilateral NICE,

Length of perpendicular $EG = p_1 = 10$ m

Length of perpendicular $= p_2 = 15$ m

(a) The formula to find the area of quadrilateral $(A) = \frac{1}{2} \times d (p_1 + p_2)$

(b) Area of the ground $(A) = \frac{1}{2} \times 40 (10 + 15) = 20 \times 25 = 500 \text{ m}^2$

Amount to be paid to the female worker for plantation $= 70 \times 500 = \text{Rs. } 35,000$

Amount to be paid to the male worker for labeling the ground $= 1500 \times 2 = \text{Rs. } 3,000$

Therefore, the total cost of dubo plantation $= 35,000 + 3,000 = \text{Rs } 38,000$

Example 3

A rectangular room has length 14 ft, breadth 12 ft. and height 10 ft. There are two square shaped windows of length 3 ft each and two doors of size 6 ft \times 2 ft each

- Write the formula to find the area of 4 walls and ceiling.
- What is the total cost of carpeting the floor at the rate of Rs. 300 per square meter?
- What is the total cost of painting the 4 walls and ceiling excluding the doors and windows, if the rate of painting is Rs. 30 per square feet?
- If the given rate increases by one third, by how much does the total cost of painting increase?

Solution

In the rectangular room

Length (l) = 14 ft

Breadth (b) = 12 ft

Height (h) = 10 ft

Length of the square shaped window = 3 ft There are two doors of size 6 ft. \times 2 ft each.

(a) The formula to find out the area of 4 walls and ceiling (A) = $2h(l + b) + lb$

(b) Area of floor of the room $l \times b = 14 \times 12 = 168 \text{ ft}^2$

We know,

Area of carpet = area of floor of the room = 168 ft^2

Rate of carpet per square meter (R) = Rs. 300

Total cost of carpeting (T) = $300 \times 1168 = \text{Rs. } 50,400$

(c) Area of 2 doors (A_1) = $2(6 \times 2) = 24 \text{ ft}^2$

Area of 2 windows (A_2) = $2 \times (3)^2 = 18 \text{ ft}^2$

Now, area of 4 walls and ceiling excluding windows and doors;

$$\begin{aligned}(A) &= 2h(l + b) + lb - A_1 - A_2 \\&= 2 \times 10(14 + 12) + 14 \times 12 - 24 - 18 \\&= 20 \times 26 + 168 - 42 \\&= 520 + 128 \\&= 646 \text{ ft}^2\end{aligned}$$

Total cost of painting on 4 walls and ceiling at the rate of Rs. 30 per squared meter = $30 \times 646 = \text{Rs. } 19,380$.

(d) When the rate of painting increases by one third, the new rate = $30 + \frac{1}{3} \times 30 = \text{Rs. } 40$

Then, the total cost = $40 \times 646 = \text{Rs. } 25,840$

Increase in total cost = $25840 - 19380 = \text{Rs. } 6,460$

Hence, the total cost increases by Rs. 6,460.

Example 4

There are two pillars of height 10 ft each in the gate of a stadium. A pyramid with the same base and height 32 ft is placed on the top of each pillar. If the base of each pillar is 4 ft×4 ft then,

- draw two figures base on the given information.
- find the slant height of the pyramid.
- find the total surface area of pillars to be colored. Should we add the area of bases to find the total surface area for the purpose of painting? Give reason.
- how much does it cost to paint the pillars together with pyramids at the rate of Rs. 95 per squared feet?

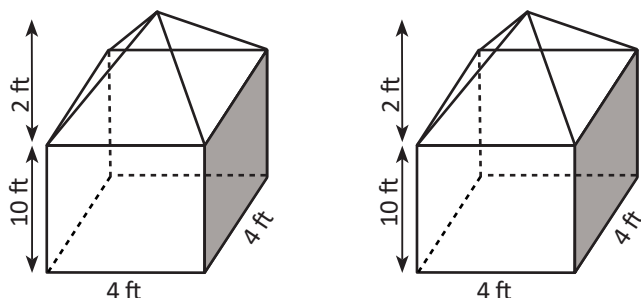
Solution

Here, the height of pillar (h_1) = 10 ft

Height of pyramid (h_2) = 2 ft

Since, the base of pillar is square shaped, its length (a) = 4 ft

(a)



(b) We know,

$$\text{Slant height of pyramid } (l) = \sqrt{(h_2)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{(2)^2 + \left(\frac{4}{2}\right)^2} = \sqrt{4 + 4} = 2.83 \text{ ft}$$

(c) Lateral surface area of prism (A_1) = perimeter of base (P) \times height (h_1)

$$= 4a \times 10 \text{ ft}$$

$$= 4 \times 4 \times 10 = 160 \text{ ft}^2$$

Lateral surface area of pyramid (A_2) = $2al = 2 \times 4 \times 2.83 = 22.64 \text{ ft}^2$

Therefore, the total surface area of a pillar with a pyramid = $A_1 + A_2$

$$= (160 + 22.64) = 182.64 \text{ ft}^2$$

Total surface area of two pillars containing pyramids = $2 \times 182.64 = 365.28 \text{ ft}^2$

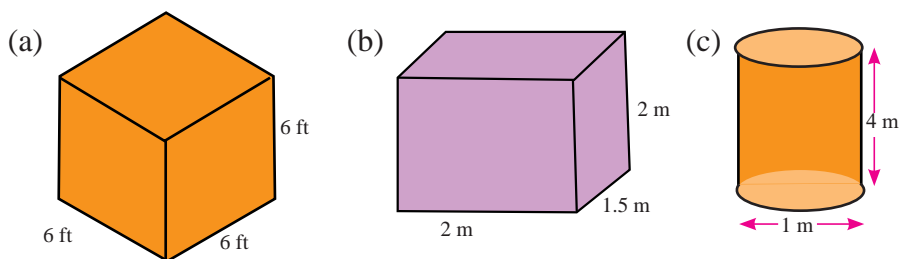
(The base area of pillar is not included to the total surface area as it is lying on the ground.)

(d) Rate of painting (R) = Rs. 95 per squared ft.

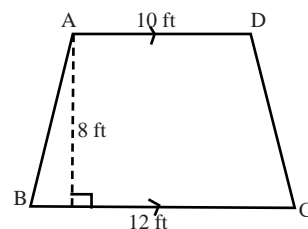
Then, the total cost of painting (T) = $R \times A = 95 \times 365.28 = \text{Rs. } 34,701.6$

Exercise 5.4

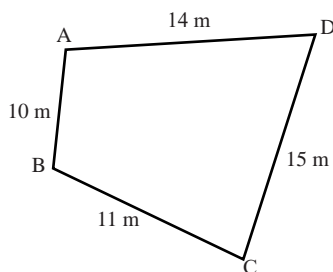
1. What is the maximum amount of water that can be contained in the tanks given below. ($1 \text{ ft}^3 = 28.317 \text{ l}$)



2. The parking area outside the National Insurance Company Limited Nepal is in geometric shape as shown in the figure. It is planning to pave the area with bricks. A brick occupies the area of 0.222 ft^2 and the cost of brick per piece is Rs. 16,



- (a) Find the area of the parking land.
 (b) How many bricks are needed to pave the whole parking area?
 (c) If 2 workers can complete the work of paving bricks in 3 days and the wage of a worker per day is Rs. 1200, how much does it cost to pave the bricks including the cost of bricks?
3. The figure given aside is a garden which is on the right side of a newly constructed temple. The management committee of temple has decided to fence around it with barbed wire rounding it 5 times. It can be completed by 3 workers in 2 days. The cost of wire per meter is Rs. 80 and the wage per worker per day is Rs. 1500,



- (a) Find the perimeter of the land.
 - (b) How much wire is required to fence the garden rounding it 5 times?
 - (c) Find the total cost of fencing the garden rounding it 5 times including the wages to the worker.
- 4. There is a cubical shaped room with length 9 ft which has two square shaped windows of length 2 ft each and two windows of size 5 ft×3 ft each.**
- (a) Find the area of the floor.
 - (b) Find the area of 4 walls excluding the windows and doors.
 - (c) Find the total cost of painting 4 walls including the window and doors at the rate of Rs. 350 per square meter.
 - (d) Another room having length, breadth and height each greater by 1 ft. than that of the first is also to be painted at the rate of Rs. 340 per square ft. Calculate by how much does the total cost increase or decrease while painting the 4 walls including the windows and doors.
- 5. It requires 60 concrete rings each of diameter 3.5 ft and height 1 ft. to construct a well. The cost of a ring is Rs. 1200. 2 workers can complete the work of keeping rings in 12 days and the wage of 1 worker is Rs. 1500 per day, (1 cubic foot = 28.317 liters)**
- (a) What is the cost of the rings only?
 - (b) How much does it cost to construct the well?
 - (c) Find the maximum amount of water that can be filled in the well.
 - (d) If the water level is seen at the 35th ring, how much water is there in the well? How many liters of water is needed to fill the well completely?
- 6. There are two square based pillars each of height 10 ft placed in the gate of a house. A pyramid of the same base and height 1 ft is placed on the top of each pillar and the length of the base of each pillar is 5 ft.**
- (a) Present the given information in a diagram.
 - (b) Find the slant height of the pyramid.
 - (c) What is the total cost of painting the pillars together with pyramids if the rate of painting is Rs. 94 per square feet?

- 7. There are two square based pillars each of height 6 ft in the gate of a house. A square based pyramid of same the base and height 1 ft is placed on the top of each pillar. The length of base of each pillar is 1 ft.**

- (a) Present the given information in a diagram.
- (b) Write the formula to find out the lateral surface area of a square based pyramid.
- (c) What is the slant height of the pyramid shaped part?
- (d) What is the cost of tiling per square feet if the total cost of tiling the pillars together with pyramids is Rs. 2729?

- 8. A circular tank with inner diameter 2.80 m and height 3 m is constructed in a nursery to collect rain water. If its upper part is conical shaped with vertical height 0.72 m,**

- (a) write the formula to find the volume of cylinder.
- (b) what is the maximum amount of water that can be contained in the tank?
- (c) find the total cost to fill up the whole tank if the cost of water is Rs. 1.85 per liter.

- 9. A pyramid shaped tent is formed using 8 equal triangular shaped pieces of clothes and the lengths of sides of each triangular piece of clothes are 5 m, 6 m and 6 m,**

- (a) Find the area of a piece of cloth.
- (b) Find the total area of clothes required for the tent.
- (c) If the cost of clothes per square meter is Rs. 600, find the total cost of tent construction.

Project work

1. Form different groups of students with proper size. Each group will visit a shop of construction materials and observe different types of water tanks made from plastic or metal. Identify their geometric shapes. Also, find the capacity of tanks in liters, prices and ratio of their capacity and price. Prepare a report including these details and also present it in your classroom.
2. You are planning to construct an underground water tank of capacity 30,000 liter at your home. The tank will be constructed in cuboid shape. What should be the length, breadth and depth of pit so that the tank may contain 30,000 liter water?

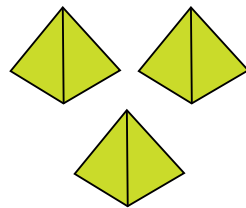
Answers

1. (a) 6116.47 *l* (b) 6,000 *l* (c) 3142.85 *l*
2. (a) 88ft² (b) 397 (at least) (c) Rs. 13,552
3. (a) 50 m (b) 250 m (c) Rs. 29,000
4. (a) 81 ft² (b) 286 ft² (c) Rs. 1,13,400 (d) Rs. 22,600
5. (a) ₹. 72,000 (b) Rs. 1,08,000 (c) 16,35, 06 *l* (d) 6813.78 *l*
6. (b) $\frac{\sqrt{29}}{2}$ (c) Rs. 48,109.51
7. (b) 2*al* (c) $\frac{\sqrt{5}}{2}$ ft. (d) Rs. 52
8. (a) $\pi r^2 h$ (b) 19958.4 *l* (c) Rs. 36923.04
9. (a) $\frac{5\sqrt{119}}{4}$ m² (b) 10 $\sqrt{119}$ m² (c) Rs. 65,452.27

Mixed Exercise

1. A student of grade 10 has submitted three paper made models of square based pyramids to his mathematics teacher as a project work. The length of base and slant height of each pyramid is 12 cm and 10 cm respectively.

- Write the formula to find out the volume of a pyramid.
- Find the vertical height of the pyramid.
- How much space is occupied by the given pyramids?
- Find the total cost of coloring each pyramid excluding the base if the rate of coloring is Rs. 80 per square cm.



2. If the length of the side of a square based pyramid is 16 cm and volume is 1280 cm^3 ,

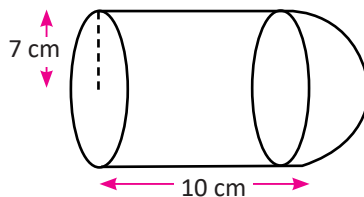
- Write the formula to find out the total surface area of the pyramid.
- Find the perimeter of the base.
- Find the slant height of the pyramid.
- Find the total surface area and the area of triangular surfaces. Which area is greater and by how much? Calculate the area and compare the result.

3. The lateral surface area of a square based pyramid is 540 cm^2 and its slant height is 15 cm.

- Write the formula to find out the total surface area of a pyramid.
- Find the length of its base.
- Find its volume.

4. The figure is a toy formed by combining a cylinder and hemisphere.

- What is the diameter of its base?
- Find the circumference of its base.
- Find the total surface area.
- Find the volume of the toy.



5. **A solid object contains a square based pyramid of slant height 5 cm on the top and a square based prism with length of base 8 cm on its bottom. The volume of object is 1024 cm^3 ,**
- (a) Draw the figure as per the information given in the question.
 - (b) Find the vertical height of the pyramid shaped part.
 - (c) Find by how much the vertical height of the pyramid shaped part is greater or less or equal to the height of prism shaped part.
 - (d) Find the total surface area of the object.
6. **The lower part of a tent is cylindrical and the upper part is hemispherical. The radius of both parts is the same. If the total height of the tent is 54 m and the height of the cylindrical part is 40 m,**
- (a) draw a figure as per the information given in the question.
 - (b) find the radius of the cylindrical part.
 - (c) find the total surface area of the tent.
 - (d) The cost of cloth per meter is Rs. 500, find the total cost to construct the tent.
7. **A solid object is formed with a hemisphere and cone combined together. The radius of the base of both parts is 7 cm. The total cost to color the object is Rs. 5148 at the rate of Rs. 6 per square cm.**
- (a) Find the total surface area of the combined solid.
 - (b) What is the slant height of the cone?
 - (c) Find the vertical height of the cone.
 - (d) What is the total height of the solid?
8. **A toy is formed combining a cylinder and a cone with the same base of diameter 10 cm. The length of the cylindrical part is 14 cm and vertical height of the conical part is 12 cm.**
- (a) Find the slant height of the cone.
 - (b) If the length of cylindrical part and the height of conical part are exchanged to form a new toy, which one costs more under the same rate of cost per square meter?
9. **A pyramid of vertical height 12 cm is placed on the top of a square based cuboid. If the area of base of cuboid shaped solid is 100 cm^2 and its height is 9 cm,**
- (a) What is the length of the base of of cuboid shaped object?

(b) Find the volume of the combined solid.

- 10. A cylindrical tank with radius 14 cm and height 40 cm is full of cement. A conical shaped pile of height 30 cm is formed while it is poured on the ground.**

- (a) Find the radius of the conical shaped pile of cement.
- (b) Find the surface area of the pile of cement excluding the base.
- (c) If the weight of 1 cm^3 cement is 2.5 gm, find the total weight of the cement.

- 11. There are two pillars with the base $6 \text{ ft} \times 6 \text{ ft}$ and height 8 ft of each in a stadium. A pyramid of height 4 ft is placed on the top of each pillar.**

- (a) Draw two figures as per the information given in question.
- (b) Write the formula to find the lateral surface area of a pyramid.
- (c) Find the slant height of the pyramid.
- (d) Find the surface area of the pyramid that need to be calculated for the painting purpose.
- (e) What will be the total cost to paint the pillars if the rate of cost of painting is Rs. 110 per square feet?

- 12. There are two cuboid shaped pillars of in the gate of a house, each of which has length 1 ft. breadth 1 ft. and height 6 ft. On the top of each pillar, a pyramid of the same base and height 1 ft is placed.**

- (a) Write the formula to calculate the lateral surface area of a prism.
- (b) What is the perimeter of the base of prism?
- (c) Find the slant height of the pyramid.
- (d) Find the area of pillars that need to be calculated for the painting purpose.
- (e) What will be the total cost to paint the pillars if the rate of cost of painting is Rs. 52 per square feet?

- 13. The diameter of concrete ring used in a well is 3.5 ft and its height is 1 ft. A ring costs Rs. 1200. If it requires 32 rings to construct the well,**

- (a) Find the total cost to construct the well.
- (b) Find the volume of water.
- (c) If the water level is seen at the eighteenth ring, find the volume of well above the water level.

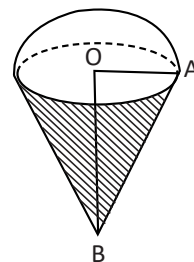
- 14. A solid object is formed combining two square based pyramids with the same base. The length of its base is 12 cm and the total height is 20 cm.**

- (a) Write the formula to calculate the lateral surface area of a pyramid.
- (b) What is the volume of that combined solid object?

- 15. Nandakishor has established an agricultural farm of cows and goats in his village after he returned from foreign employment. There is a water tank with combined shape of a cylinder and cone. The inner diameter and height of the tank are 1.4 m and 2.1 m respectively. The vertical height of the cone is 0.36 m.**

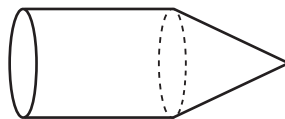
- (a) Write the formula to find the total surface area of a cone.
- (b) Find the perimeter of base of the tank.
- (c) How much water can be contained in the tank? Calculate in liters.

- 16. A combined solid object with volume $240\pi \text{ cm}^3$ is shown in the figure. The ratio of the slant height of the conical part and radius is 5 : 3.**



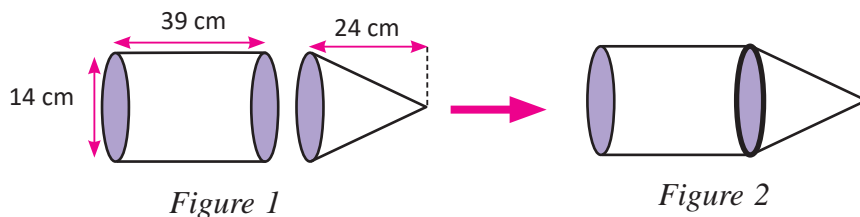
- (a) What are the different shapes contained in the solid?
- (b) Write the formula to calculate the curved surface area of the cone.
- (c) Find the total surface area of the given solid.
- (d) Are the total surface and curved surface area of the solid equal? Justify with reason.

- 17. The total surface area of a combined solid given in the figure is $840\pi \text{ cm}^2$. The ratio of slant height and vertical height of the conical part is 13 : 12 and the height of cylindrical part is 24 cm.**



- (a) What are the different shapes contained in the given solid?
- (b) Write the formula to find the perimeter of base of a cone.
- (c) Find the volume of the solid.
- (d) Is the total surface area of solid equal to its curved surface area? If not, calculate and compare the result.

18. There are a cylinder and a cone with the same base in the first figure. A solid shown in the second figure is formed combining the cylinder and cone of the first figure.



- Write the formula to find the area of a cone.
 - Find the surface area of solid objects of first figure separately and find their sum.
 - Is the sum of area of solids of the first figure equal to the total surface area of combined solid of the second figure?
 - Is the sum of volume of solids of the first figure equal to the volume of combined solid of the second figure? Justify with reasons.
 - Compare the total cost of painting the solids of the first and second figures if the rate of cost of each is Rs. 150 per square meter.
19. A toy contains a cylinder and a cone of the same base with a diameter of 10 cm. The length of the cylindrical part is 14 cm and height of the conical part is 12 cm.
- Write the formula to find the volume of a cone.
 - What is the slant height of the conical part?
 - A new toy is formed by exchanging the measurement of diameter and conical height. Which toy costs more to color its surface at the same rate?
20. A solid object is formed with the combination of a cylinder and a cone with the same radius. The height of the cylinder and slant height of the cone are 28 cm and 17 cm respectively. The total cost of coloring the solid at the rate of Rs 100 per square centimeter is Rs. 2851.20.
- Draw a figure using the information given in the question.
 - What is the shape of the cylinder?

(c) Find the height of the cone.

21. A water tank contains cylindrical and hemispherical parts. The total height of the tank is 20 m and the area of its base is 154 m^2 . If the tank is filled with water at the rate of 45 paisa per liter,

(a) How much water is contained in the volume of 1 m^3 ?

(b) What is the radius of its base?

Answers

1. (a) $\frac{1}{3} a^2 h$ (b) 8 cm (c) 432 cm^2 (d) Rs 57,600
2. (a) $a^2 + 2al$ (b) 64 cm (c) 17 cm
(d) 800 cm^2 , 544 cm^2 , greater by 256 cm^2
3. (a) $a^2 + 2al$ (b) 18 cm (c) 1296 cm^3
4. (a) 14 cm (b) 44 cm (c) 902 cm^2 (d) 2258.66 cm^3
5. (a) Show to your teacher (b) 3 cm (c) less by 12 cm (d) 624 cm^2
6. (a) Show to your teacher (b) 14 cm (c) 4752 cm^2 (d) Rs. 23,76,000
7. (a) 858 cm^2 (b) 25 cm (c) 24 cm (d) 31 cm
8. (a) 13 cm (b) The new toy costs more as its area is greater.
9. (a) 10 cm (b) 1300 cm^3
10. (a) 28 cm (b) 3611.21 cm^2 (c) 61,600 gm
11. (a) Show to your teacher (b) $2al$ (c) 5 cm (d) 252 cm^2 (e) Rs. 27,720
12. (a) ph (b) 4 ft. (c) $\frac{\sqrt{5}}{2} \text{ ft}$ (d) $(24 + \sqrt{5}) \text{ cm}^2$ (e) Rs. 1364.27
13. (a) Rs. 38,400 (b) 308 cm^3 (c) 134.75 cm^3
14. (a) $2al$ (b) 960 cm^2
15. (a) $\pi r(r + l)$ (b) 4.4 m (c) 3418.8 l
16. (a) cone and hemisphere (b) πrl (c) 414.86 cm^2
(d) Yes, because the area of both surfaces is same.
17. (a) cylinder and cone (b) $2\pi r$ (c) 10057.14 cm^3
(d) The total surface area is greater by 314.28 cm^2
18. (a) $\frac{1}{3} a^2 h$ (b) 2728 cm^2 (c) 308 cm^2
(d) Both have same volume 7238 cm^2 (e) Former costs more by 46200
19. (a) $\pi r^2 h$ (b) 13 cm (c) the second costs more
20. (a) Show to your teacher (b) circle (c) 15 cm
21. (a) $1 \text{ m}^3 = 1000 \text{ l}$ (b) $r = 7 \text{ m}$ (c) 13 m (d) Rs. 12,24,300

Lesson 6

Sequence and Series

6.0 Review

Observe the following sequences and answer the questions.

- (a) 2, 4, 6, 8, (b) 1, 3, 5, 7,
(c) 1, 4, 16, 64, (d) 24, 12, 6, 3,
(e) 5, 11, 8, 14 (f) 2, 6, 18, 54,

Questions

- (i) Identify the above sequences whether they are arithmetic or geometric.
(ii) Find the common difference or common ratio of the sequences.
(iii) Find the general term of the sequences.
(iv) Find the sixth and eight terms of the sequences.
(v) Express the sequences into series.

6.1 Means of Arithmetic Sequence

Activity 1

Study the following conversation.

Rita: How can the value of m be found, if 3, m , 7,are in arithmetic sequences?

Raju: The difference of each term to its preceding term of an arithmetic sequence is equal.

So, $m - 3 = 7 - m$

$$m - 3 = 7 - m$$

$$\text{or, } m + m = 7 + 3$$

$$\text{or, } m = \frac{7+3}{2} = 5$$

Rita: Yes. The second term is the average of the first and the third terms.

Anil: It can also be solved as follows.

First term $(a) = 3$

Second term $(t_2) = a + d$, where d is the common difference.

Third term $(t_3) = a + 2d$

$$\text{or, } 7 = 3 + 2d$$

$$\text{or, } 2d = 7 - 3$$

$$\text{or, } 2d = 4$$

$$\therefore d = 2$$

Putting the value of d in $t_2 = a + d$

$$m = 3 + 2 = 5$$

Rashmi : How can we find more than 1 unknown terms between two given terms?

For example: What are the values of x, y, z in the sequence $3, x, y, z, 19, \dots$?

Anil : It can be solved In the following way.

First term $(a) = 3$

Fifth term $(t_5) = 19$

Common difference $(d) = ?$

We know that,

$$t_5 = a + (5-1)d$$

$$\text{or, } 19 = 3 + 4d$$

$$\text{or, } 4d = 19 - 3$$

$$\text{or, } 4d = 16$$

$$\therefore d = 4$$

$$\text{Second term } (t_2) = x = a + d = 3 + 4 = 7$$

$$\text{Third term } (t_3) = y = a + 2d = 3 + 2 \times 4 = 11$$

$$\text{Fourth term } (t_4) = z = a + 3d = 3 + 3 \times 4 = 15$$

Rita : Yes, I knew that the three terms between 3 and 19 can be found taking 19 as the fifth term.

Anil : Yes, you are correct.

Total number of terms = number of means + 2

$$n = m + 2$$

(Finally, they reported their discussion to the mathematics teacher.)

Teacher: You are all correct.

The terms lying between the first and the last terms of an arithmetic sequence are called arithmetic means.

- (a) An arithmetic mean between the first and the last terms can be obtained by taking the arithmetic average of the first and the last terms.
- (b) If there are more than one arithmetic means between two terms, they can be obtained by calculating common difference (d).

Example 1

Find an arithmetic mean between the numbers 7 and 17.

Solution

Let m be the arithmetic mean between 7 and 17.

Then, 7, m , 17 are in arithmetic sequence.

First term (a) = 7

Third term (t_3) = 17

Arithmetic mean (m) = ?

$$\begin{aligned}\text{We know that } (m) &= \frac{a+b}{2} \\ &= \frac{7+17}{2} \\ &= 12\end{aligned}$$

Alternative method

Third term (t_3) = $a + 2d$

$$\text{or, } 17 = 7 + 2d$$

$$\text{or, } 2d = 17 - 7 = 10$$

$$\text{or, } d = 5$$

$$\text{Now, } (t_2) = a + d = 7 + 5 = 12$$

$$\therefore m = 12$$

Example 2

Find three arithmetic means between the terms 3 and 23.

Solution

The sequence with three arithmetic means is 3, m_1 , m_2 , m_3 , 23

First term (a) = 3

Number of means (m) = 3

Total number of terms (n) = $3 + 2 = 5$

Fifth term (t_5) = 23

We know that

$$\text{Fifth term } (t_5) = a + (5 - 1)d$$

$$\text{or, } 23 = 3 + 4d$$

$$\text{or, } 23 - 3 = 4d$$

$$\text{or, } \frac{20}{4} = d$$

$$\therefore d = 5$$

$$\text{Now, } m_1 = \text{second term} = a + d = 3 + 5 = 8$$

$$m_2 = \text{third term} = a + 2d = 3 + 2 \times 5 = 13$$

$$m_3 = \text{fourth term} = a + 3d = 3 + 3 \times 5 = 18$$

Hence, the three arithmetic means between 3 and 23 are 8, 13 and 18.

Example 3

Find 7 arithmetic means between two terms 3 and 43.

Solution

The arithmetic sequence with 7 means is $3, m_1, m_2, m_3, m_4, m_5, m_6, m_7, 43$.

First term (a) = 3

Number of means (m) = 7

Total number of terms (n) = $7 + 2 = 9$

Ninth term (t_9) = 43

We know, ninth term (t_9) = $a + (9 - 1)d$

$$\text{or, } 43 = 3 + 8d$$

$$\text{or, } 8d = 43 - 3 = 40$$

$$\text{or, } d = 5$$

Now, $m_1 = \text{second term} = a + d = 3 + 5 = 8$

$$m_2 = \text{third term} = a + 2d = 3 + 2 \times 5 = 13$$

$$m_3 = \text{fourth term} = a + 3d = 3 + 3 \times 5 = 18$$

$$m_4 = \text{fifth term} = a + 4d = 3 + 4 \times 5 = 23$$

$$m_5 = \text{sixth term} = a + 5d = 3 + 5 \times 5 = 28$$

$$m_6 = \text{seventh term} = a + 6d = 3 + 6 \times 5 = 33$$

$$m_7 = \text{eighth term} = a + 7d = 3 + 7 \times 5 = 38$$

6.2 Sum of Arithmetic Series

Activity 2

A shopping centre provides cash incentive to its employees according to their performance. One of the employees receives cash increment in his salary as given in the following table.

Year 2079	Baisakh	Jestha	Ashadh	Shrawan	Bhadra	Ashoj	Kartik
Increment in Salary (Rs.)	800	1000	1200	1400	1600	1800	2000

Study the information given in table and discuss the following questions.

- How much is the incentive amount increasing every month?
- What is the total amount of incentives received from Baisakh to Kartik of 2079 BS.?
- What will be the amount of the incentive in the month of Baisakh, 2080 B.S. if the same rate of growth continues?
- What will be the total amount of incentives that an employee receives upto Baisakh, 2080 BS?

The incentive amount is increased by Rs. 200 every month. The series of incentive amounts of 7 months is $800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000$.

If the total amount of incentives in seven months is denoted by S_7 ,

$$S_7 = 800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000 = 9800 \dots (i)$$

First term $(a) = 800$, Common difference $= 1000 - 800 = 200$

Number of term $(n) = 7$, seventh term $(t_7) = 2000$

The equation (i) can be expressed as

$$2S_7 = 2000 + 1800 + 1600 + 1400 + 1200 + 1000 + 800 = 9800 \dots (ii)$$

Adding the equations (i) and (ii),

$$S_7 = 2800 + 2800 + 2800 + 2800 + 2800 + 2800 + 2800$$

$$\text{or, } 2S_7 = 7 \times 2800$$

$$\text{or, } S_7 = \frac{7}{2} \times 2800$$

$$= 9800$$

Expressing the value 2800 in terms of first and seventh terms,

$$\text{or, } S_7 = \frac{7}{2} (800 + 2000)$$

$\therefore S_n = \frac{n}{2} (a + t_n)$ where a = incentive amount of first month, t_n = incentive amount of last month.

$$\text{Now, } S_n = \frac{n}{2} (a + t_n)$$

$$= \frac{n}{2} [a + a + (n - 1)d] = \frac{n}{2} [2a + (n - 1)d] \quad [\because t_n = a + (n - 1)d]$$

Total number of months upto Baisakh, 2080 BS is 13

$$\text{Incentive amount of 13}^{\text{th}} \text{ month } (t_{13}) = 800 + (13 - 1) 200$$

$$= 800 + 12 \times 200$$

$$= \text{Rs. } 3,200$$

Hence, the incentive amount received in Baisakh, 2080 BS is Rs. 3200.

The total amount of incentive received in 13 months (S_{13}) = ?

We know that,

$$S_n = \frac{n}{2} [(2a + (n - 1)d)]$$

$$= \frac{13}{2} [2 \times 800 + (13 - 1) 200]$$

$$= \frac{13}{2} [1600 + (12 \times 200)] = \frac{13}{2} [1600 + 2400] = \frac{13}{2} \times 4000$$

$$= 26000$$

Hence, the total amount of incentive received in 13 months = Rs. 26,000.

The sum of the first n terms of an arithmetic series is $S_n = \frac{n}{2} (a + t_n)$,
where first term = a , final term = t_n , common difference = d ,
number of terms = n , and sum of the first n terms = S_n

Again, substituting $t_n = a + (n - 1)d$

$$S_n = \frac{n}{2} (a + t_n) = \frac{n}{2} [a + \{a + (n - 1)d\}] = \frac{n}{2} [2a + (n - 1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n - 1)d]$$

Note : The final term can be denoted be either (t_n) or l .

Example 4

Find the sum of the first 20 terms of the series $1 + 2 + 3 + \dots$

Solution

Here, $1 + 2 + 3 + \dots$ is an arithmetic series.

First term (a) = 1

Common difference (d) = $2 - 1 = 1$

Number of terms (n) = 20

Sum of the first 20 terms (S_{20}) = ?

We know that,

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n - 1)d] = \frac{20}{2} [2 \times 1 + (20 - 1)1] \\ &= 10(2 + 19) \\ &= 10 \times 21 = 210 \end{aligned}$$

Example 5

Find the sum of the first 13 terms of the series $2 + 4 + 6 + \dots$

Solution

Here, $2 + 4 + 6 + \dots$ is an arithmetic series.

First term (a) = 2

Common difference (d) = $4 - 2 = 2$

Number of terms (n) = 13

Sum of the first 13 terms (S_{13}) = ?

We know that,

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n - 1)d] = \frac{13}{2} [2 \times 2 + (13 - 1)2] \\ &= \frac{13}{2} (4 + 24) \\ &= \frac{13}{2} \times 28 \\ &= 182 \end{aligned}$$

Example 6

Find the sum of the first 28 terms of the series $5 + 17 + 29 + \dots$

Solution

Here, $5 + 17 + 29 + \dots$ is an arithmetic series.

First term $(a) = 5$

Common difference $(d) = 17 - 5 = 12$

Number of terms $(n) = 28$

Sum of the first 28 terms $(S_{28}) = ?$

We know that,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{28} = \frac{28}{2} [2 \times 5 + (28 - 1)12]$$

$$= 14(10 + 324)$$

$$= 14 \times 334 = 4676$$

Example 7

An arithmetic series has the first term 2, the final term 29 and sum of its first n terms is 155. Find the number of terms and the common difference of the series.

Solution

First term $(a) = 2$,

Final term $(t_n) = 29$ and,

Sum of first n terms $S_n = 155$

Number of the terms $(n) = ?$

Common difference $(d) = ?$

We know that $S_n = \frac{n}{2} (a + t_n)$

$$\text{or, } 155 = \frac{n}{2}(2 + 29)$$

$$\text{or, } 310 = 31n$$

$$\text{or, } n = \frac{310}{31} = 10$$

Again, we have,

$$t_n = a + (n - 1)d$$

$$\text{or, } 29 = 2 + (10 - 1)d$$

$$\text{or, } 27 = 9d$$

$$\text{or, } d = \frac{27}{9} = 3$$

Hence, the common difference is 3.

Example 8

If the sum of certain number of terms in the arithmetic series $24+20+16+\dots\dots\dots$ is 72, find the number of terms.

Solution

Here, first term $(a) = 24$

Common difference $(d) = 20 - 24 = -4$

Sum of first n terms $S_n = 72$

Number of terms $(n) = ?$

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$72 = \frac{n}{2}[2 \times 24 + (n - 1)(-4)]$$

$$\text{or, } 144 = n(48 - 4n + 4)$$

$$\text{or, } 144 = 52n - 4n^2$$

$$\text{or, } 4n^2 - 52n + 144 = 0$$

$$\text{or, } n^2 - 13n + 36 = 0$$

$$\text{or, } n^2 - 9n - 4n + 36 = 0$$

$$\text{or, } n(n - 9) - 4(n - 9) = 0$$

$$\text{or, } (n - 9)(n - 4) = 0$$

$$\text{either, } n - 9 = 0 \quad \therefore n = 9$$

$$\text{or, } n - 4 = 0 \quad \therefore n = 4$$

Hence the total number of terms is either 4 or 9.

Example 9

Find the sum of the first 15 terms of an arithmetic series if its third term is 0 and the tenth term is 42.

Solution

Here, third term (t_3) = 0

Tenth term (t_{10}) = 42

Total number of terms (n) = 15

Sum of first 15 terms S_{15} = ?

We know that,

$$t_n = a + (n - 1) d$$

$$\text{or, } t_3 = a + (3 - 1) d$$

$$\text{or, } 0 = a + 2d$$

$$\text{or, } a + 2d = 0 \dots\dots\dots (i)$$

Also, $t_{10} = a + (10 - 1) d$

$$\text{or, } 42 = a + 9d$$

$$a + 9d = 42 \dots\dots\dots (ii)$$

Subtracting the equation (ii) from (i)

$$a + 2d = 0$$

$$a + 9d = 42$$

$$\underline{\quad \quad \quad}$$

$$-7d = -42$$

$$\text{or, } d = \frac{42}{7} = 6$$

$$\therefore d = 6$$

Substituting the value of d in equation (i),

$$a + 2 \times 6 = 0$$

$$a = -12$$

$$\begin{aligned} \text{Now, sum of the first 15 terms } S_{15} &= \frac{15}{2} [2 \times (-12) + (15 - 1)6] \\ &= \frac{15}{2} [-24 + 84] \\ &= 450 \end{aligned}$$

Hence, the sum of the first 15 terms is 450.

Example 10

The salary of a person is Rs. 40,000 at present and he receives a grade of Rs 1,000 on his monthly salary every year. What will be his total income in 6 years if the number of grades goes on increasing as the number of years increases? How much additional amount is required to earn Rs. 35,00,000 in 6 years?

Solution

Here, total income of the first year (a) = $40000 \times 12 = 4,80,000$

Increased amount every year (d) = $1000 \times 12 = 12,000$

Total income in 6 years (S_6) = ?

Number of years (n) = 6

We know that,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_6 = \frac{6}{2} [2 \times 480000 + (6 - 1) \times 12000]$$

$$= 3[960000 + 60000]$$

$$= 3 \times 10,20,000$$

$$= 30,60,000$$

Hence, the total income in 6 years is Rs. 30,60,000.

To earn Rs. 35,00,000 in 6 years, the required amount = $35,00,000 - 30,60,000 = 4,40,000$.

Exercise 6.1

1. (a) What are arithmetic means?
(b) If a , m , b are in arithmetic sequence, express m in terms of a and b .
(c) Find the mid value of the numbers 12 and 18.
2. Find the arithmetic mean between
(a) 6 and 10 (b) -2 and 2 (c) -4 and 8 (d) $(a + b)$ and $(a - b)$
3. Find arithmetic means in the following cases.
(a) 4 arithmetic means between 5 and 20
(b) 6 arithmetic means between 70 and 14
(c) 6 arithmetic means between 5 and -9

- 4. The following sequences are arithmetic sequences. From the sequences, find the value of x .**
- (a) $5, x, 9, \dots$
 - (b) $x + 1, x + 5, 3x + 1, \dots$
 - (c) $x + 2, 3x, 4x + 1, \dots$
- 5. Find the sum of the following arithmetic series.**
- (a) $7 + 11 + 15 + 19 \dots$, to 20 terms
 - (b) $4 - 1 - 6 - 11 - 16 - \dots$, to 7 terms
 - (c) $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \dots$, to 16 terms
 - (d) $5 + 10 + 15 + \dots + 65$
 - (e) $-64 - 48 - 32 - \dots + 32$
 - (f) The sum of first 10 odd numbers
 - (g) The sum of first 100 natural numbers
 - (h) The sum of natural numbers from 50 to 100
- 6. The first term, the last term and the sum of all the terms of an arithmetic series are given below. Find the common difference of the series.**
- (a) First term = 1, last term = 50 and sum of all terms = 204
 - (b) First term = 160, last term = 20 and sum of all terms = 440
 - (c) First term = 17, last term = $-\frac{99}{8}$ and sum of all terms = $\frac{407}{16}$
- 7. The common difference, number of terms and the sum of all the terms of an arithmetic series are given as follows. Find its first term.**
- (a) Common difference = -3, number of terms = 10 and sum of all terms = 325
 - (b) Common difference = 9, number of terms = 9 and sum of all terms = 108
 - (c) Common difference = 3, number of terms = 10 and sum of all terms = 155
- 8.**
- (a) How many terms of the series $4 + 10 + 16 + 22 + \dots$ has a sum 374?
 - (b) If the first term of an arithmetic series is 36 and its common difference is 9, how many terms will have a sum 540?
- 9.**
- (a) Find the sum of the first 16 terms of an arithmetic series, if its third term is -15 and eighth term is 10.
 - (b) Find the sum of the first 20 terms of an arithmetic series, if its fifth term is 10 and eleventh term is 22.

10. (a) If the sum of the first 6 terms of an arithmetic series is 75 and the sum of its first 12 terms is 390, find the sum of the first 20 terms.
- (b) If the sum of the first 7 terms of an arithmetic series is 21 and the sum of its first 12 terms is 126, then find the series.
11. (a) A worker in a garment factory produced 1000 Nepali cap (Dhaka Topi) at the first year and increased the production by 100 every year. Can he produce 15,000 caps in 10 years?
- (b) The monthly salary of a person appointed for a new job is Rs. 60,000 initially and it increases by Rs. 24,000 every year. In how many years he will earn the total amount of Rs.1 crore ? (The interest is not included in his earning)

Project work

Request your parents to deposit some amount on the occasion of your birthday and continue it for 5 years in such a way that the deposit amount for every year increases by the same value. Calculate the total amount deposited in 5 years and compare who will collect more amount in your group of classmates.

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1. (b) $m = \frac{a+b}{2}$ (c) 15 2. (a) 8 (b) 0 (c) 0 (d) a
3. (a) 8, 11, 14, 17 (b) 62, 54, 46, 38, 30, 22 (c) 3, 1, -1, -3, -5, -7
4. (a) 7 (b) 4 (c) 3
5. (a) 900 (b) -77 (c) 128 (d) 455
(e) -112 (f) 100 (g) 5050 (h) 3825
6. (a) 7 (b) -4 (c) $-\frac{47}{16}$
7. (a) 46 (b) -24 (c) 2
8. (a) 11 (b) 8
9. (a) 200 (b) 420
10. (a) 1050 (b) $-6 -3 + 0 + \dots$
11. (a) No, less by 500 (12) Nearly 12 years

6.3 Means of Geometric Sequence

Activity 1

How can the unknown terms of the following geometric sequences be found?
Discuss the solution in each of your pairs and present the conclusion.

(a) $3, m, 27, \dots$

(b) $2, m_1, m_2, m_3, 512, \dots$

(a) Since $3, m, 27, \dots$ are in geometric sequence, $\frac{3}{m} = \frac{m}{27}$

$$\frac{3}{m} = \frac{m}{27}$$

$$\text{or, } m^2 = 3 \times 27$$

$$\text{or, } m = \sqrt{(3 \times 27)} = 9$$

$$\text{Second term} = \sqrt{\text{first term} \times \text{third term}}$$

(b) $2, m_1, m_2, m_3, 512, \dots$ are in geometric sequence.

First term (a) = 2

Number of means (m) = 3

Total number of terms (n) = $m + 2 = 3 + 2 = 5$

Fifth term (t_5) = 512

We know that, $t_n = ar^{n-1}$

$$\text{or, } 512 = 2r^{5-1}$$

$$\text{or, } 256 = r^4$$

$$\text{or, } (4)^4 = r^4$$

$$\therefore r = 4$$

Then, second term (t_2) = $m_1 = ar = 2 \times 4 = 8$

Third term (t_3) = $m_2 = ar^2 = 2 \times 4^2 = 32$

Fourth term (t_4) = $m_3 = ar^3 = 2 \times 4^3 = 128$

The terms lying between the first and last terms of a geometric sequence are called geometric means.

(a) The single geometric mean between two terms is the square root of their product.

(b) Two or more than two geometric means can be calculated using the first term and common ratio. The common ratio of the sequence can be found taking the last term as n^{th} term.

Example 1

Find a geometric mean between two numbers 2 and 32.

Solution

If m is the geometric mean between 2 and 32, then the numbers 2, m , and 32 are in geometric sequence.

$$\text{First term } (a) = 2$$

$$\text{Third term } (t_3) = 32$$

$$\text{Mean } (m) = \text{second term} = ?$$

$$\text{We know, mean } (m) = \sqrt{(a \times t_3)} = \sqrt{(2 \times 32)} = 8$$

Hence, the geometric mean $(m) = 8$.

Example 2

Find 3 geometric means between two numbers, 5 and 405.

Solution

If m_1, m_2, m_3 are the 3 geometric means between 5 and 405, then 5, $m_1, m_2, m_3, 405$

$$\text{First term } (a) = 5$$

$$\text{Number of means } (m) = 3$$

$$\text{Total number of terms } (n) = 3 + 2 = 5$$

$$\text{Fifth term } (t_5) = 405$$

$$\text{We know that } t_5 = ar^{5-1}$$

$$\text{or, } 405 = 5r^{5-1}$$

$$\text{or, } 405 = 5r^4$$

$$\text{or, } 81 = r^4$$

$$\text{or, } 3^4 = r^4$$

$$\therefore r = 3$$

$$\text{Then, } m_1 = \text{second term} = ar = 5 \times 3 = 15$$

$$m_2 = \text{third term} = ar^2 = 5 \times 3^2 = 45$$

$$m_3 = \text{fourth term} = ar^3 = 5 \times 3^3 = 135$$

Hence, 3 geometric means between 5 and 405 are 15, 45 and 135.

6.4 Sum of Geometric Series

Activity 2

An employee working in a pharmaceutical industry had a salary of Rs. 10,000 initially. Because of the pandemic of Covid 19, the workers were highly demanded and it was decided to increase their salary by 10% every month. How much will he earn in 5 months if the salary increased by 10% every month? Discuss the problem.

Here, the monthly salary at the beginning = 10000

Salary of the second month = $10,000 + 10,000 \times 10\% = 11,000$

Salary of the third month = $11,000 + 11,000 \times 10\% = 12,100$

Salary of the fourth month = $12,100 + 12,100 \times 10\% = 13,310$

Salary of the fifth month = $13,310 + 13,310 \times 10\% = 14,641$

Now, the salary of 5 months in series is

$$S_5 = 10000 + 11000 + 12100 + 13310 + 14641 \dots \dots \dots (i)$$

$$\text{Common ratio } (r) = \frac{11000}{10000} = \frac{12100}{11000} = \frac{13310}{12100} = \frac{14641}{13310} = 1.1$$

Multiplying the equation (i) by 1.1

$$1.1 \times (S_5) = 11000 + 12100 + 13310 + 14641 + 16105.1 \dots \dots \dots (ii)$$

Subtracting the equation (i) from (ii),

$$\text{or, } (1.1 - 1) S_5 = 16105.1 - 10000$$

$$\text{or, } (1.1 - 1) S_5 = 10000 (1.1)^5 - 10000$$

$$\text{or, } (1.1 - 1) S_5 = 10000 \{ (1.1)^5 - 1 \}$$

$$\text{Or, } S_5 = \frac{10000[(1.1)^5 - 1]}{(1.1 - 1)}$$

It is in the form of $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where a is the first term, r is the common ratio and n is the total number of terms.

$$\text{or, } S_5 = 61051$$

Hence, the earning of the employee in 5 months is Rs. 61051.

The sum of the first n terms of a geometric sequence is given by $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where $r > 1$ and $S_n = \frac{a(1 - r^n)}{(1 - r)}$ जहाँ $r < 1$.

Here, first term = a , common ratio = r , number of terms = n and the sum of first n terms = S_n .

$$\begin{aligned} \text{Again, } S_n &= \frac{a(r^n - 1)}{(r - 1)} = \frac{ar^n - a}{(r - 1)} = \frac{ar^{n-1} \times r - a}{(r - 1)} \\ &= \frac{t_n r - a}{r - 1} \quad \because t_n = ar^{n-1} \end{aligned}$$

Example 3

Find the sum of the first 5 terms of the geometric series $1 + 3 + 9 + 27 + \dots$

Solution

Here, first term (a) = 1

Common ratio (r) = $\frac{3}{1} = \frac{9}{3} = 3$

Total number of terms (n) = 5

Here, $r > 1$, $S_n = \frac{a(r^n - 1)}{(r - 1)}$

$$\begin{aligned}\text{So, } S_5 &= \frac{1[(3^5 - 1)]}{(3 - 1)} \\ &= \frac{243 - 1}{2} = 121\end{aligned}$$

Hence, the sum of the first 5 terms of the series $1 + 3 + 9 + 27 + \dots$ is 121.

Example 4

Find the sum of the first 5 terms of the geometric series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$

Solution

Here, first term (a) = 2

Common ratio (r) = $\frac{1}{2}$

Number of terms (n) = 5

$$\begin{aligned}\text{We know that } r < 1, S_n &= \frac{a(1 - r^n)}{(1 - r)} \\ S_5 &= \frac{2\{1 - (\frac{1}{2})^5\}}{1 - \frac{1}{2}} = \frac{2(1 - \frac{1}{32})}{\frac{1}{2}} = 4(1 - \frac{1}{32}) = \frac{31}{8}\end{aligned}$$

Hence, the sum of the first 5 terms of the geometric series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$ is $\frac{31}{8}$.

Example 5

Find the sum of the geometric series $3 + 6 + 12 + 24 + \dots + 768$.

Solution

Here, first term (a) = 3

Common ratio (r) = $\frac{6}{3} = 2$

Last term (t_n) = 768

Since the first term and the last term are given, $S_n = \frac{t_n r - a}{r - 1}$

$$S_n = \frac{768 \times 2 - 3}{(2 - 1)} = 1536 - 3 = 1533$$

Hence, the sum of geometric series $3 + 6 + 12 + 24 + \dots + 768$ is 1533.

Example 6

Find the common ratio of a geometric series, if its first term is 7, last term is 448 and the sum of the terms is 889.

Solution

Here, first term (a) = 7

Last term (l) = 448

Total sum (S_n) = 889

The first and the last terms are given. So, $S_n = \frac{t_n r - a}{r - 1}$.

Therefore, $S_n = \frac{t_n r - a}{r - 1}$

$$\text{or, } 889 = \frac{448 \times r - 7}{r - 1}$$

$$\text{or, } 889r - 889 = 448r - 7$$

$$\text{or, } 889r - 448r = 889 - 7$$

$$\text{or, } 441r = 882$$

$$\text{or, } r = \frac{882}{441} = 2$$

Hence, the common ratio is 2.

Example 7

If the third and sixth terms of a geometric series are 27 and 729 respectively, find the sum of its first 10 terms.

Solution

Here, third term (t_3) = 27, sixth term (t_6) = 729

We know that $t_n = ar^{(n-1)}$

$$\text{or, } t_3 = ar^{3-1} = ar^2$$

$$\text{or, } 27 = ar^2 \dots \dots \dots \text{(i)}$$

$$\text{or, } t_6 = ar^{6-1} = ar^5$$

$$\text{or, } 729 = ar^5 \dots \dots \dots \text{(ii)}$$

Dividing the equation (ii) by (i)

$$\text{or, } \frac{729}{27} = \frac{ar^5}{ar^2}$$

$$\text{or, } 27 = r^3$$

$$\text{or, } 3^3 = r^3 \quad \text{or, } r = 3$$

$$\therefore r = 3$$

Hence, the common ratio is 3.

Again, putting the value of r in equation (i),

$$27 = a \times 3^2$$

$$\therefore a = 3$$

$$\text{Since } r > 1, S_n = \frac{a(r^n - 1)}{(r - 1)}$$

$$\text{or, } S_{10} = \frac{a\{(3)^{10} - 1\}}{(3 - 1)} = \frac{3(59049 - 1)}{2} = 88,572$$

Hence, the sum of first 10 terms is 88,572.

Example 8

How many terms of a geometric series $64 + 32 + 16 + \dots$ will have a sum $\frac{255}{2}$?

Solution

Here, first term $(a) = 64$

$$\text{Common ratio } (r) = \frac{32}{64} = \frac{1}{2}$$

$$\text{Sum of } n \text{ terms } S_n = \frac{255}{2}$$

Total number of terms $(n) = ?$

$$\text{Here, } r < 1, S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$\text{or, } \frac{255}{2} = \frac{64\{1 - (\frac{1}{2})^n\}}{(1 - \frac{1}{2})} = \frac{64\{1 - (\frac{1}{2})^n\}}{\frac{1}{2}} = 128 \{1 - (\frac{1}{2})^n\}$$

$$\text{or, } \frac{255}{256} = 1 - (\frac{1}{2})^n$$

$$\text{or, } (\frac{1}{2})^n = 1 - \frac{255}{256}$$

$$\text{or, } (\frac{1}{2})^n = \frac{1}{256}$$

$$\text{or, } (\frac{1}{2})^n = (\frac{1}{2})^8$$

$$\therefore n = 8$$

Hence, the number of terms to have a sum $\frac{255}{2}$ is 8.

Example 9

Hari borrowed Rs. 19,682 from Ram Naresh on condition that he would pay back in 9 installments. If the amount of each installment is three times of the former find the difference of first and last installments.

Solution

Here, common ratio (r) = 3 (3 times)

Sum of all the terms (S_n) = 19682

First term (a) = ?

Ninth term (t_9) = ?

Here, $r > 1$ So, $S_n = \frac{ar^n - 1}{r - 1}$

$$\text{or, } 19682 = \frac{a(3^9 - 1)}{(3 - 1)}$$

$$\text{or, } 19682 = \frac{a(19683 - 1)}{2}$$

$$\text{or, } 19682 = \frac{a \times 19682}{2}$$

$$\therefore a = 2$$

The first installment = Rs. 2

Again, the last installment (t_9) = $ar^{n-1} = 2 \times 3^{9-1} = 2 \times 3^8 = 13122$

Then, the difference of the first and last installments is $13122 - 2 = \text{Rs. } 13120$.

Exercise 6.2

1. (a) What is meant by geometric mean?
(b) If the positive numbers a , m and b are in geometric sequence, express m in terms of a and b .
(c) What is the geometric mean between 3 and 27?
2. Find the geometric mean between the given two terms.
(a) -4 and -64 (b) $\frac{1}{5}$ and 125 (c) 7 and 343
3. Find the geometric mean in the following cases.
(a) 4 geometric means between 6 and 192
(b) 3 geometric means between 5 and 405
(c) 3 geometric means between $\frac{9}{4}$ and $\frac{4}{9}$

4. Find the value of x from the following geometric series.
- $4, x$ and 9
 - $x, 4, 8$
 - $5, 25$ and $x + 1$
5. Find the sum of the following geometric series.
- $2 + 4 + 8 + 16 \dots$, to 6 terms
 - $\frac{1}{9} + \frac{1}{3} + 1 + \dots$, to 5 terms
 - $-\frac{1}{4} + \frac{1}{2} - 1 + \dots$, to 6 terms
 - $16 + 8 + 4 + \dots + \frac{1}{16}$
 - $1 + \frac{1}{3} + \frac{1}{9} + \dots + \frac{1}{729}$
6. **The first term, the last term and the sum of all the terms of geometric series are given below. Find their common ratio.**
- First term = 2 , last term = 486 and sum of all the terms = 728
 - First term = 5 , last term = 1215 and sum of all the terms = 1820
 - First term = 3 , last term = 768 and sum of all the terms = 1533
7. (a) Find the sum of the first 10 terms of a geometric series, if its second term is 4 and the seventh term is 128 .
- (b) Find the sum of the first 7 terms of a geometric series if its third term is 3 and the fifth term is 81 .
8. (a) How many terms of a geometric series $32 + 48 + 72 + \dots$ will have a sum 665 ?
- (b) How many terms of a geometric series $6 - 12 + 24 - 48 + \dots$ will have a sum -2046 ?
9. Sarita borrowed Rs. 43680 from her friend Garima on condition that she would pay back it in 6 installments. If the amount of each installment she paid was three times of the previous one, find the difference of the first and the last installments.

Project Work

Form two groups A and B of your friends in order to solve the mathematical problems assigned by your teacher. Your teacher assigns questions to the different groups for 1 week as given below.

Group A: 3 questions for the first day, after then the number of questions for each day is two times of the previous day.

Group B: 1 question for the first day and the number of questions for each day is three times of the previous day.

Submit the work to your teacher after a week and find how many questions are solved by each group. Discuss the problem and its relation with sequence and series.

Answer

1. (b) $m = \sqrt{a \times b}$ (c) 9
2. (a) -16 (b) 5 (c) 49
3. (a) 12, 24, 48, 96 (b) 15, 45, 135 (c) $\frac{3}{2}, 1, \frac{2}{3}$
4. (a) 6 (b) 2 (c) 124
5. (a) 126 (b) $\frac{121}{9}$ (c) $\frac{21}{4}$
(d) $31\frac{15}{16}$ (e) $1\frac{364}{729}$
6. (a) 3 (b) 3 (c) 2
7. (a) 2046 (b) 1093
8. (a) 6 (b) 10
9. Rs. 29,040

Lesson 7

Quadratic Equation

7.0 Review

A school is carpeting its office room which is rectangular in shape and the amount of the carpet used is 80 m^2 . Discuss in pair and answer the following questions.

- What is the length and breadth of the office room?
- What is the length and breadth of the room if the length is greater than its breadth by 2 meter?

If the breadth of the room is x , then its length will be $= x + 2$

Area of the room $= 80 \text{ m}^2$

$$(x + 2)x = 80$$

$$\text{or, } x^2 + 2x - 80 = 0$$

$$\text{or, } x^2 + 10x - 8x - 80 = 0$$

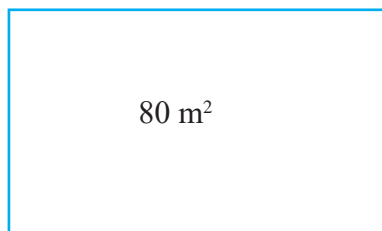
$$\text{or, } x(x + 10) - 8(x + 10) = 0$$

$$\text{or, } (x + 10)(x - 8) = 0$$

either $x + 10 = 0 \quad \therefore x = -10$, which is not possible.

or, $x - 8 = 0 \quad \therefore x = 8$

Here, the length of the room (l) $= x + 2 = 8 + 2 = 10 \text{ m}$, breadth of the room (b) $= x = 8 \text{ m}$



A quadratic equation is a second degree equation of one variable. It is in the form of $ax^2 + bx + c = 0$, where $a \neq 0$. There are two values of the variable satisfying the equation.

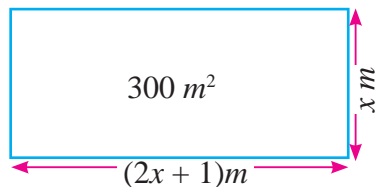
7.1 Solving Quadratic Equation

(a) Factorization Method

Activity 1

The area of a rectangular playground is 300 m^2 . If its length is greater than breadth by 1 m, discuss in your group and find its length and breadth.

Here, area of a playground $= 300 \text{ m}^2$



If breadth of the playground = x

then, length of playground = $2x + 1$

Now, area of rectangular playground = length \times breadth

$$300 = (2x + 1)x$$

$$\text{or, } 2x^2 + x - 300 = 0 \quad [\because \text{This is quadratic equation.}]$$

To find the value of x from above quadratic equation,

$$2x^2 + (25 - 24)x - 300 = 0$$

$$\text{or, } 2x^2 + 25x - 24x - 300 = 0$$

$$\text{or, } x(2x + 25) - 24(2x + 25) = 0$$

$$\text{or, } (2x + 25)(x - 24) = 0$$

If multiplication of two factors is zero, then one factor should be zero.

$$\text{either } (2x + 25) = 0 \quad \text{or } (x - 24) = 0$$

$$\text{If } 2x + 25 = 0$$

$$\text{then, } 2x = -25$$

$$x = -\frac{25}{2} \text{ is impossible}$$

$$\text{if, } x - 24 = 0$$

$$\text{or, } x = 24 \quad \therefore x = 24$$

Breadth of the playground (x) = 24 m, then length = $2x + 1 = 2 \times 24 + 1 = 49$ m

Example 1

Solve the equation and examine whether it is correct or not.

$$(a) \quad x^2 + 4x = 0$$

$$(b) \quad x^2 + 6x + 8 = 0$$

$$(c) \quad x^2 - 5x + 6 = 0$$

$$(d) \quad x^2 - x - 6 = 0$$

$$(e) \quad 2x^2 + 7x + 6 = 0$$

Solution

$$(a) \quad x^2 + 4x = 0$$

$$\text{or, } x(x + 4) = 0$$

$$\text{either } x = 0$$

$$\text{or, } x + 4 = 0$$

$$x = -4$$

$$\text{Thus, } x = 0, -4$$

When examined,

$$x = 0 \text{ is putting in } x^2 + 4x = 0$$

$$\text{LHS} = 0 + 4 \times 0 = 0 = \text{RHS}$$

When placed $x = -4$

$$\text{LHS} = (-4)^2 - 4 \times (-4) = 16 - 16 = 0 = \text{RHS}$$

(b) $x^2 + 6x + 8 = 0$

or, $x^2 + (4 + 2)x + 8 = 0$

or, $x^2 + 4x + 2x + 8 = 0$

or, $x(x + 4) + 2(x + 4) = 0$

or, $(x + 4)(x + 2) = 0$

either $x + 4 = 0$ $\therefore x = -4$

or, $(x + 2) = 0$ $\therefore x = -2$

When examined,

$x = -2$ putting $x^2 + 6x + 8 = 0$

LHS = $(-2)^2 + 6 \times (-2) + 8$

$4 - 12 + 8 = 0 = \text{RHS}$

$x = -4$ is placed in $x^2 + 6x + 8 = 0$

LHS = $(-4)^2 + 6 \times (-4) + 8$

$= 16 - 24 + 8 = 0 = \text{RHS}$

\therefore The roots of quadratic equations are $-2, -4$

(c) $x^2 - 5x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - (3 + 2)x + 6 = 0$

or, $x^2 - 3x - 2x + 6 = 0$

or, $x(x - 3) - 2(x - 3) = 0$

or, $(x - 3)(x - 2) = 0$

either, $(x - 3) = 0$ $\therefore x = 3$

or, $x - 2 = 0$ $\therefore x = 2$

When examined,

Putting $x = 2$

$x^2 - 5x + 6 = (2)^2 - 5 \times 2 + 6$

$= 4 - 10 + 6 = 0$

LHS = RHS

Again, when placed $x = 3$,

$(3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

LHS = RHS

\therefore The roots of quadratic equation are 2 and 3.

(d) $x^2 - x - 6 = 0$

or, $x^2 - (3 - 2)x - 6 = 0$

or, $x^2 - 3x + 2x - 6 = 0$

or, $x(x - 3) + 2(x - 3) = 0$

or, $(x - 3)(x + 2) = 0$

either, $(x - 3) = 0$ $\therefore x = 3$

or, $x + 2 = 0$ $\therefore x = -2$

When examined,

Putting $x = 3$,

LHS = $(3)^2 - 3 - 6$

$= 9 - 9 = 0 = \text{RHS}$

When placed $x = -2$

LHS = $(-2)^2 - 2 - 6$

$= 4 + 2 - 6 = 0 = \text{RHS}$

\therefore The roots of quadratic equation are 3 and -2.

(e) $2x^2 + 7x + 6 = 0$

or, $2x^2 + 7x + 6 = 0$

or, $2x^2 + (4 + 3)x + 6 = 0$

or, $2x^2 + 4x + 3x + 6 = 0$

or, $2x(x + 2) + 3(x + 2) = 0$

or, $(x + 2)(2x + 3) = 0$

either, $(x + 2) = 0$. $\therefore x = -2$

or, $2x + 3 = 0$. $\therefore x = -\frac{3}{2}$

\therefore The roots of $2x^2 + 7x + 6 = 0$ are -2 and $-\frac{3}{2}$.

When examined

Putting $x = -2$

$$\begin{aligned} \text{LHS} &= 2(-2)^2 + 7 \times (-2) + 6 \\ &= 8 - 14 + 6 = 0 = \text{RHS} \end{aligned}$$

Putting $x = -\frac{3}{2}$,

$$\begin{aligned} \text{LHS} &= 2 \times \left(-\frac{3}{2}\right)^2 + 7 \times -\frac{3}{2} + 6 \\ &= \frac{9}{2} - \frac{21}{2} + 6 = \frac{21 - 21}{2} \\ &= 0 = \text{RHS} \end{aligned}$$

(b) Solving quadratic equation by completing square

Activity 2

Solve the given quadratic equation

(a) $x^2 - 9 = 0$

(b) $x^2 - 5x + 6 = 0$

Solution

(a) $x^2 - 9 = 0$

or, $x^2 - 3^2 = 0$

or, $(x + 3)(x - 3) = 0$

Either $x + 3 = 0$ $\therefore x = -3$

or, $x - 3 = 0$ $\therefore x = 3$

$\therefore x = \pm 3$

We can also solve in this way.

$$x^2 - 9 = 0$$

or, $x^2 = 9$

or, $x^2 = 3^2$ [\because Here x^2 and 9 both are square.]

or, $x = \pm 3$

The roots of $x^2 = a^2$ form of quadratic equation is $x \pm a$.

$$(b) x^2 - 5x + 6 = 0$$

$$\text{or, } x^2 - 5x = -6$$

$$\text{or, } x^2 - 2 \cdot \frac{5}{2}x + \left(\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2 - 6 \quad [\because (a-b)^2 = a^2 - 2ab + b^2]$$

$$\text{or, } \left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 6 = \frac{25-24}{4} = \frac{1}{4}$$

$$\text{or, } \left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$$

$$\text{or, } \left(x - \frac{5}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$\therefore x - \frac{5}{2} = \pm \frac{1}{2}$$

Taking (+)ve sign,

$$x - \frac{5}{2} = \frac{1}{2} \quad \text{or, } x = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$$

Taking (-)ve sign,

$$x - \frac{5}{2} = -\frac{1}{2}$$

$$\text{or, } x = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$$

Therefore, the roots of x is 2 and 3.

Example 2

Solve by completing the square

$$(a) x^2 - 10x + 16 = 0$$

$$(b) x^2 - 7x + 12 = 0$$

$$(c) 2x^2 - 7x + 6 = 0$$

Solution

$$(a) x^2 - 10x + 16 = 0$$

$$\text{or, } x^2 - 2 \times x \times 5 + (5)^2 - (5)^2 + 16 = 0 \quad [\because (a-b)^2 = a^2 - 2ab + b^2]$$

$$\text{or, } x^2 - 2 \times x \times 5 + (5)^2 - 25 + 16 = 0$$

$$\text{or, } (x - 5)^2 - 9 = 0$$

$$\text{or, } (x - 5)^2 = 9$$

$$\text{or, } (x - 5)^2 = 3^2$$

$$\text{or, } x - 5 = \pm 3$$

Taking (+)ve sign,

$$x - 5 = 3 \quad \text{or, } x = 3 + 5 = 8$$

Taking (-)ve sign,

$$x - 5 = -3 \quad \text{or, } x = 5 - 3 = 2$$

$$\therefore x = 8, 2$$

$$(b) x^2 - 7x + 12 = 0$$

$$\text{or, } x^2 - 2 \cdot \frac{7}{2}x + \left(\frac{7}{2}\right)^2 + 12 - \left(\frac{7}{2}\right)^2 = 0$$

$$\text{or, } \left(x - \frac{7}{2}\right)^2 + 12 - \frac{49}{4} = 0$$

$$\text{or, } \left(x - \frac{7}{2}\right)^2 + \frac{48 - 49}{4} = 0$$

$$\text{or, } \left(x - \frac{7}{2}\right)^2 + \left(\frac{-1}{4}\right) = 0$$

$$\text{or, } \left(x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$\text{or, } \left(x - \frac{7}{2}\right) = \pm \frac{1}{2}$$

Taking (+)ve sign,

$$x - \frac{7}{2} = \frac{1}{2} \quad \text{or, } x = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$$

Taking (-)ve sign,

$$x - \frac{7}{2} = -\frac{1}{2} \quad \text{or, } x = \frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$$

$$\therefore x = 4, 3$$

$$(c) 2x^2 - 7x + 6 = 0$$

$$\text{or, } 2x^2 - 7x + 6 = 0$$

$$\text{or, } 2\left(x^2 - \frac{7}{2}x + 3\right) = 0$$

$$\text{or, } x^2 - \frac{7}{2}x + 3 = 0$$

$$\text{or, } x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + 3 = 0$$

$$\text{or, } x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 + 3 - \left(\frac{49}{16}\right) = 0$$

$$\text{or, } \left(x - \frac{7}{4}\right)^2 - \frac{1}{16} = 0$$

$$\text{or, } \left(x - \frac{7}{4}\right)^2 = \frac{1}{16}$$

$$\text{or, } \left(x - \frac{7}{4}\right)^2 = \left(\pm \frac{1}{4}\right)^2$$

$$\text{or, } \left(x - \frac{7}{4}\right) = \pm \frac{1}{4}$$

Taking (+)ve sign,

$$x - \frac{7}{4} = \frac{1}{4} \quad \text{or, } x = \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$$

Taking (-)ve sign, $x - \frac{7}{4} = -\frac{1}{4}$

$$\text{or, } x = \frac{7}{4} - \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$$

$$\therefore x = 2, \frac{3}{2}$$

(C) Solving quadratic equation by using formula

Activity 3

How shall we find the value of quadratic equation $ax^2 + bx + c = 0$?

Here, $ax^2 + bx + c = 0$

$$\text{or, } ax^2 + bx = -c$$

$$\text{or, } \frac{ax^2 + bx}{a} = -\frac{c}{a} \quad [\because \text{dividing both sides by } a]$$

$$\text{or, } x^2 + \frac{bx}{a} = -\frac{c}{a}$$

$$\text{or, } x^2 + 2 \times x \frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} \quad [\because \text{completing the square}]$$

$$\text{or, } \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\text{or, } \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\text{or, } \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\text{or, } \left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{4a^2}\right)$$

$$\text{or, } x + \frac{b}{2a} = \pm \sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$$

$$\text{or, } x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{or, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore, the roots of x are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Example 3

Solve the given quadratic equations by using formula.

$$(a) \quad x^2 - 5x + 6 = 0 \qquad (b) \quad x\left(x - \frac{2}{7}\right) = \frac{3}{49}$$

Solution

(a) Here, comparing $x^2 - 5x + 6 = 0$ to $ax^2 + bx + c = 0$, we get

$$a = 1, b = -5, c = 6$$

We know that,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1} \\ &= \frac{5 \pm \sqrt{25 - 24}}{2} \\ &= \frac{5 \pm 1}{2} \end{aligned}$$

$$\text{Taking (+)ve sign, } x = \frac{5+1}{2} = \frac{6}{2} = 3$$

$$\text{Taking (-)ve sign, } x = \frac{5-1}{2} = \frac{4}{2} = 2$$

Therefore, roots of x are 3 and 2.

$$(b) \quad x\left(x - \frac{2}{7}\right) = \frac{3}{49}$$

$$\text{Here, } x\left(x - \frac{2}{7}\right) = \frac{3}{49}$$

$$\text{or, } x^2 - \frac{2}{7}x - \frac{3}{49} = 0$$

$$\text{or, } 49x^2 - 14x - 3 = 0 \quad \text{Comparing } 49x^2 - 14x - 3 = 0 \text{ with}$$

$$ax^2 + bx + c = 0 \quad \text{we get}$$

$$a = 49, b = -14, c = -3$$

We know that,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 49 \times (-3)}}{2 \times 49} \\ &= \frac{14 \pm \sqrt{196 + 588}}{98} \end{aligned}$$

$$\text{Taking (+)ve sign, } x = \frac{14 + 28}{98} = \frac{42}{98} = \frac{3}{7}$$

$$\text{Taking (-)ve sign, } x = \frac{14 - 28}{98} = \frac{-14}{98} = -\frac{1}{7}$$

Therefore, roots of x are: $\frac{3}{7}$ and $-\frac{1}{7}$

Exercise 7.1

1. Which of the following are the quadratic equation? Write with reason.

- (a) $(x - 2)^2 + 1 = 2x - 3$ (b) $x(x + 1) + 8 = (x + 2)(x - 2)$
(c) $x(2x + 3) = x^2 + 1$ (d) $(x + 2)^3 = x^3 - 4$
(e) $x^2 + 3x + 1 = (x - 2)^2$ (f) $(x + 2)^3 = 2x(x^2 - 1)$

2. Solve by factorization method.

- (a) $x^2 - 3x - 10 = 0$ (b) $2x^2 + x - 6 = 0$ (c) $2x^2 - x + \frac{1}{8} = 0$
(d) $100x^2 - 20x + 1 = 0$ (e) $x^2 - 45x + 324 = 0$ (f) $x^2 - 27x - 182 = 0$

3. Solve by completing square.

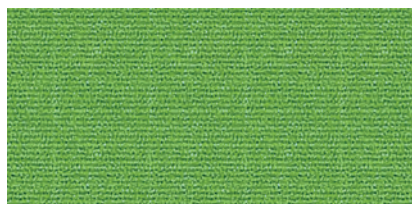
- (a) $x^2 - 6x + 9 = 0$ (b) $9x^2 - 15x + 6 = 0$ (c) $2x^2 - 5x + 3 = 0$
(d) $5x^2 - 6x - 2 = 0$ (e) $x^2 + \frac{15}{16} = 2x$ (f) $x^2 + \frac{2}{3}x = \frac{35}{9}$

4. Solve by using formula.

- (a) $x^2 - 9x + 20 = 0$ (b) $x^2 + 2x - 143 = 0$ (c) $3x^2 - 5x + 2 = 0$
(d) $2x^2 - 2\sqrt{2}x + 1 = 0$ (e) $x + \frac{1}{x} = 3$ (f) $\frac{1}{x} + \frac{1}{(x-2)} = 3,$
(g) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

5. Ramnaresh Mahato scored a total of 30 marks in two subjects VR. English and Mathematics in the first terminal examination of grade 10. If he scored 2 more marks in Mathematics and 3 fewer marks in English, then the product of his marks would be 210. Find his scores on both subjects.

6. A rectangular figure of a playground is given here. The length of the longer side of the playground is 30 m more than its shorter side but its diagonal is 60 m more than its shorter side.



- a) Find the length and breadth of the playground.
b) If $12\text{ m} \times 3\text{ m}$ size artificial grass turfs have to be placed on the ground, how many turfs are needed?
c) If the ground has to be fenced 4 times with a barbed wire costing Rs. 15 per meter, how much will be the cost?

Answers

1. (a) Yes (b) no (c) yes (d) yes (e) no (f) no
2. (a) $5, -2$ (b) $-2, \frac{3}{2}$ (c) $\frac{1}{4}, \frac{1}{4}$ (d) $\frac{1}{10}, \frac{1}{10}$ (e) $9, 36$ (f) $13, 14$
3. (a) $3, 3$ (b) $1, \frac{2}{3}$ (c) $1, \frac{3}{2}$ (d) $\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}$
(e) $\frac{3}{4}, \frac{5}{4}$ (f) $\frac{5}{3}, -\frac{7}{3}$
4. (a) $4, 5$ (b) $11, -13$ (c) $1, \frac{2}{3}$ (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(e) $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ (f) $\frac{4+\sqrt{10}}{3}, \frac{4-\sqrt{10}}{3}$ (g) $1, 2$
5. $12, 18$ or $13, 17$
6. (a) $120\text{ m}, 90\text{ m}$ or $13, 17$ (b) 300 (c) Rs. $25,200$

7.2 Word problems related to quadratic equation

Activity 4

Age of Sumitra is 12 years and her sister is 18 years now. How many years after will the product of their age be 280? How can we find it?

	Sumitra's age	Sumitra's sister's age	Product of their ages
Now	12	18	216
1 year later	13	19	247
2 years later	14	20	280

Here,

Now, Sumitra's age = 12 years

Sumitra's Sister's age = 18 years

x years later,

Sumitra's age = $12 + x$

Sumitra's sister's age = $18 + x$

According to the condition given,

$$(12 + x)(18 + x) = 280$$

$$\text{or, } 216 + 18x + 12x + x^2 = 280$$

$$\text{or, } x^2 + 30x + 216 - 280 = 0$$

$$\text{or, } x^2 + 30x - 64 = 0$$

$$\text{or, } x^2 + 32x - 2x - 64 = 0$$

$$\text{or } x(x + 32) - 2(x + 32) = 0$$

$$\text{or, } (x + 32)(x - 2) = 0$$

$$\text{Either } x + 32 = 0 \quad \therefore x = -32$$

$$\text{Or, } x - 2 = 0 \quad \therefore x = 2$$

Here, $x = -32$ is not a possible solution because age cannot be negative.

Therefore, $x = 2$

2 years later, the product of their age will be 280.

Example 4

If the sum of two positive numbers is 18 and their product is 77, then find these numbers.

Solution

Let these two numbers be x and y .

According to the question,

$$x + y = 18 \dots\dots\dots (i)$$

$$x \times y = 77 \dots\dots\dots (ii)$$

From equation (i) $y = 18 - x \dots\dots\dots (iii)$

Placing the value of y in equation (ii)

$$x(18 - x) = 77$$

$$\text{or, } 18x - x^2 = 77$$

$$\text{or, } 18x - x^2 - 77 = 0$$

$$\text{or, } x^2 - 18x + 77 = 0$$

$$\text{or, } x^2 - 11x - 7x + 77 = 0$$

$$\text{or, } x(x - 11) - 7(x - 11) = 0$$

$$\text{or, } (x - 11)(x - 7) = 0$$

$$\text{Either, } (x - 11) = 0 \quad \therefore x = 11$$

$$\text{Or, } (x - 7) = 0 \quad \therefore x = 7$$

Placing the value of x in equation (iii),

$$\text{If } x = 11 \text{ then } y = 18 - x = 18 - 11 = 7$$

$$\text{If } x = 7 \text{ then } y = 18 - x = 18 - 7 = 11$$

Therefore, two positive numbers are 7 and 11 or 11 and 7.

Example 5

If 11 is subtracted from the square of a positive integer, then the result is 38. Find the number.

Solution

Let the positive integer be x , then its square is x^2 .

According to the question, $x^2 - 11 = 38$

$$\text{or, } x^2 - 11 = 38$$

$$\text{or, } x^2 = 38 + 11$$

$$\text{or, } x^2 = 49$$

$$\text{or, } x^2 = (\pm 7)^2$$

$$\therefore x = \pm 7$$

But we need a positive integer, so $x = 7$ only.

Therefore, 7 is the required positive integer.

Example 6

If the product of two consecutive positive even numbers is 24, then find these numbers.

Solution

Let, the two consecutive even numbers be x and $x + 2$.

According to the question,

$$x \times (x + 2) = 24$$

$$\text{or, } x^2 + 2x - 24 = 0$$

$$\text{or, } x^2 + 6x - 4x - 24 = 0$$

$$\text{or, } x(x + 6) - 4(x + 6) = 0$$

$$\text{or, } (x + 6)(x - 4) = 0$$

$$\text{Either, } (x + 6) = 0 \quad \therefore x = -6 \quad [\because \text{This is a negative number}]$$

$$\text{Or, } x - 4 = 0 \quad \therefore x = 4$$

Therefore, the required two positive numbers are 4 and $4 + 2 = 6$.

Example 7

If the sum of a number and its reciprocal is $\frac{26}{5}$ then find the number.

Solution

Let, the number be x and the reciprocal of that number be $\frac{1}{x}$

According to the question,

$$x + \frac{1}{x} = \frac{26}{5}$$

$$\text{or, } \frac{x^2 + 1}{x} = \frac{26}{5}$$

$$\text{or, } 5x^2 + 5 = 26x$$

$$\text{or, } 5x^2 - 26x + 5 = 0$$

$$\text{or, } 5x^2 - 25x - x + 5 = 0$$

$$\text{or, } 5x(x - 5) - 1(x - 5) = 0$$

$$\text{or, } (5x - 1)(x - 5) = 0$$

$$\text{Either, } (5x - 1) = 0 \quad \therefore x = \frac{1}{5}$$

$$\text{Or, } x - 5 = 0 \quad \therefore x = 5$$

Therefore, the required numbers are 5 and $\frac{1}{5}$

Example 8

The sum of the two brothers' age is 34 and the product of their ages is 288, then find their present age.

Solution

Let the age of the elder brother and younger brother be x and y respectively.

According to question,

$$x + y = 34 \dots\dots\dots (i)$$

$$x \times y = 288 \dots\dots\dots (ii)$$

From equation (i) $y = 34 - x \dots\dots\dots (iii)$

Placing the value of y in equation (ii)

$$\text{or, } x(34 - x) = 288$$

$$\text{or, } 34x - x^2 = 288$$

$$\text{or, } x^2 - 34x + 288 = 0$$

$$\text{or, } x^2 - 16x - 18x + 288 = 0$$

$$\text{or, } x(x - 16) - 18(x - 16) = 0$$

$$\text{or, } (x - 16)(x - 18) = 0$$

$$\text{Either, } x - 16 = 0 \quad \therefore x = 16$$

$$\text{Or, } x - 18 = 0 \quad \therefore x = 18$$

Placing the value of x in equation (iii)

$$\text{If } x = 16 \text{ then } y = 34 - x = 34 - 16 = 18 \quad [\because \text{This is not possible.}]$$

$$\text{If } x = 18 \text{ then } y = 34 - x = 34 - 18 = 16$$

Therefore, the age of elder brother is 18 and the age of younger brother is 16.

Example 9

The product of the digits of a two digit number is 18. If 27 is added to the number, the places of digits are reversed. What is the number? Find it.

Solution

Let, the two digit number = $10x + y$ [\because where x is ten place and y is once place digit.]

According to the question,

$$x + y = 18$$

$$\text{or, } x = \frac{18}{y} \dots\dots\dots (i)$$

Again the second condition, $(10x + y) + 27 = 10y + x$

$$10x + y + 27 - 10y - x = 0$$

$$\text{or, } 9x - 9y + 27 = 0$$

$$\text{or, } 9(x - y + 3) = 0$$

$$\text{or, } x - y + 3 = 0 \dots\dots\dots (ii)$$

Placing the value of x from equation (i) to equation (ii)

$$\text{or, } \frac{18}{y} - y + 3 = 0$$

$$\text{or, } \frac{18 - y^2 + 3y}{y} = 0$$

$$\text{or, } y^2 - 3y - 18 = 0$$

$$\text{or, } y^2 - 6y + 3y - 18 = 0$$

$$\text{or, } y(y - 6) + 3(y - 6) = 0$$

$$\text{or, } (y - 6)(y + 3) = 0$$

$$\text{Either } y - 6 = 0 \quad \therefore y = 6$$

$$\text{Or, } y + 3 = 0 \quad \therefore y = -3$$

Placing the value of y in equation (ii)

$$\text{If } y = 6, \text{ then } x = \frac{18}{6} = 3.$$

$$\text{If } y = -3, \text{ then } x = \frac{18}{-3} = -6.$$

$$\text{If } y = 6 \text{ and } x = 3 \text{ then the number is } 10x + y = 10 \times 3 + 6 = 36.$$

$$\text{If } y = -3 \text{ and } x = -6 \text{ then the number is } 10x + y = 10 \times (-6) - 3 = -63.$$

Example 10

The present age of the father and his son is 42 years and 16 years respectively. Find how many years ago the product of their age was 272.

Solution

Let, x years ago, the age of the father and his son was $42 - x$ and $16 - x$ respectively.

According to the question,

x years ago, the product of their age = 272.

$$\text{or, } (42 - x)(16 - x) = 272$$

$$\text{or, } 672 - 42x - 16x + x^2 = 272$$

$$\text{or, } x^2 - 58x + 400 = 0$$

$$\text{or, } x^2 - 50x - 8x + 400 = 0$$

$$\text{or, } x(x - 50) - 8(x - 50) = 0$$

$$\text{or, } (x - 8)(x - 50) = 0$$

$$\text{Either, } x - 8 = 0 \quad \therefore x = 8$$

$$\text{O, } x - 50 = 0 \quad \therefore x = 50$$

Here, $x = 50$ years, which is impossible. Therefore, $x = 8$.

Hence, 8 years ago the product of the father and his son's age was 272.

Example 11

The length of the hypotenuse of a right angled triangle is 13m. If the difference of its other two sides is 7m, find the length of the remaining sides.

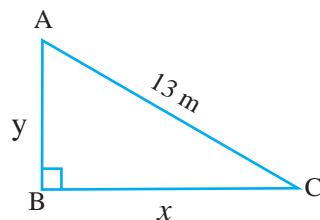
Solution

Here, $\triangle ABC$ is a right angled triangle where $\angle B = 90^\circ$ and hypotenuse (h) = $AC = 13\text{m}$

Let base (b) = $BC = x$ and perpendicular (p) = $AB = y$.

According to the question,

$$x - y = 7 \quad \text{or, } y = x + 7 \dots\dots\dots(i)$$



Now, in the right angled triangle ABC $h^2 = p^2 + b^2$.

$$\text{Therefore, } AC^2 = AB^2 + BC^2$$

$$\text{or, } 13^2 = (x + 7)^2 + x^2$$

$$\text{or, } 169 = x^2 + 14x + 49 + x^2$$

$$\text{or, } 2x^2 + 14x - 120 = 0$$

$$\text{or, } x^2 + 7x - 60 = 0$$

$$\text{or, } x^2 + 12x - 5x - 60 = 0$$

$$\text{or, } x(x + 12) - 5(x + 12) = 0$$

$$\text{or, } (x - 5)(x + 12) = 0$$

$$\text{Either, } (x - 5) = 0 \quad \therefore x = 5$$

$$\text{Or, } x + 12 = 0 \quad \therefore x = -12$$

Here, $x = -12$ is impossible because length of a base cannot be negative, therefore $x = 5$.

Hence, base (b) = BC = $x = 5$ m

and perpendicular (p) = AB = $y = 5 + 7 = 12$ m.

Hence, the remaining sides are 5m and 12m.

Example 12

The area of a rectangular land is 50m^2 and its perimeter is 90m. If the land is to be made square, by what percentage should it be reduced in length?

Solution

Let, the length and breadth of the rectangular land are x and y respectively.

According to question,

Area of a the rectangular land = 50m^2

$$\text{or, } xy = 500 \dots\dots\dots(i)$$

Perimeter of the rectangular land = 90m

$$\text{or, } 2(x + y) = 90$$

$$\text{or, } x + y = 45$$

$$\text{or, } y = 45 - x \dots\dots\dots(ii)$$

Now, place $y = 45 - x$ from equation (ii) in equation (i).

$$xy = 500$$

$$\text{or, } x(45 - x) = 500$$

$$\text{or, } 45x - x^2 = 500$$

$$\text{or, } x^2 - 45x + 500 = 0$$

$$\text{or, } x^2 - 25x - 20x + 500 = 0$$

$$\text{or, } x(x - 25) - 20(x - 25) = 0$$

$$\text{or, } (x - 25)(x - 20) = 0$$

$$\text{either, } (x - 25) = 0. \quad \therefore x = 25$$

$$\text{or, } x - 20 = 0 \quad \therefore x = 20$$

$$\text{If } x = 25, y = 45 - x = 45 - 25 = 20.$$

$$\text{If } x = 20, y = 45 - x = 45 - 20 = 25.$$

Therefore, the length and breadth of the rectangular land are 25m and 20m respectively.

If the land is to be made square, then the length and breadth of the land should be equal.

So, the length should be reduced by 5m.

$$\text{Therefore, percentage to be reduced in length} = \frac{5}{25} \times 100\% = 20\%$$

Example 13

Some students of grade 10 organized a picnic of Rs. 42,000 budget. They decided to collect equal money for picnic. But, 5 students were absent in the picnic day, so each student should collect Rs. 700 more. Based on this context, solve the problem given below.

- a) How many students attended the picnic?
- b) How much did each participant have to pay? Find it.

Solution

Let, the number of students = x and the amount to be paid by each participant = Rs. $\frac{42000}{x}$

Here, 5 students were absent.

Therefore, the number of students participated = $x - 5$

According to the question,

$$\frac{42000}{x-5} = \frac{42000}{x} + 700$$

$$\text{or, } \frac{42000}{x-5} - \frac{42000}{x} = 700$$

$$\text{or, } \frac{60}{x-5} - \frac{60}{x} = 1$$

$$\text{or, } 60x - 60x + 300 = x(x-5)$$

$$\text{or, } x^2 - 5x - 300 = 0$$

$$\text{or, } x^2 - 20x + 15x - 300 = 0$$

$$\text{or, } x(x-20) + 15(x-20) = 0$$

$$\text{or, } (x-20)(x+15) = 0$$

$$\text{Either, } (x-20) = 0 \quad \therefore x = 20$$

$$\text{Or, } (x+15) = 0 \quad \therefore x = -15$$

Here, x denotes the number of students, so $x = -15$ is impossible. Therefore, $x = 20$.

Hence,

a) The total number of students participated in picnic = $20 - 5 = 15$

b) The total amount of money per person = $\frac{42000}{x-5} = \frac{42000}{15} = \text{Rs. } 2800$

Exercise 7.2

1. If 11 is added to the square of a natural number, the sum is 36. Find the number.
2. If 11 is subtracted from the square of a number, the remainder number is 25. Find the number.
3. If 7 is subtracted from the double of a square of a positive number, the remainder is 91. Find the number.
4. If 2 is subtracted from the square of a natural number, the remainder is 7. Find the number.
5. If 11 is subtracted from the square of a number and the remainder is 89, find that number.
6. If 17 is subtracted from the square of a number, the remainder is 55. Find the number.
7. If 3 is subtracted from the double of the square of a positive number, the remainder is 285. Find the number.

8. If the sum of a number and its square is 72, find the number.
9. If the product of the two consecutive even numbers is 80, find the numbers.
10. If the product of the two consecutive odd numbers is 225, find the numbers.
11. If the sum of a number and their reciprocal is $\frac{10}{3}$, find the number.
12. If the sum of two natural numbers is 21 and sum of their square is 261, find the numbers.
13. If the age difference between two brothers is 4 years and the product of their ages is 221. Find their age.
14. The sum of the present age of two brothers is 22 and the product of their ages is 120. Find their present age.
15. The age difference between two sisters is 3 years and the product of their age is 180. Find their present age.
16.
 - (a) The present age of a father and his son is 40 years and 13 years respectively. Find how many years ago the product of their age was 198.
 - (b) The present age of a mother and her daughter is 34 years and 4 years respectively. Find how many years later the product of their age will be 400.
 - (c) The present age of a father and his son is 35 years and 1 year respectively. Find how many years later the product of their age will be 240.
 - (d) The present age of a husband and wife is 35 years and 27 years respectively. Find how many years ago the product of their age was 425.
17.
 - (a) The hypotenuse of a right angled triangle is 25m. If the difference between its other two sides is 17m, find the length of the remaining sides.
 - (b) The length of hypotenuse of a right angled triangle is double and 6m more than its shortest side. If the length of the remaining side is 2m less than hypotenuse, find the length of all sides.
 - (c) Calculate the length and breadth of a rectangular land whose area is 150m^2 and perimeter is 50m.
 - (d) Calculate the length and breadth of a rectangular land whose area is 54m^2 and perimeter is 30m.
 - (e) Calculate the area of a rectangle whose length is 24m and diagonal is 16m more than its breadth.

- (f) The area and perimeter of a rectangular land is 2000m^2 and 180m respectively. If the land is to be made square, by what percentage should it be reduced in length or breadth? Calculate.
18. The two digit number is equal to the four times the sum of their digits and three times the product of their digits. Find the number.
19. An institute made a plan to distribute 180 pencils equally to the students enrolled in grade one. On the pencil distribution day, 5 students were absent so that each students got 3 more pencils.
- (a) How many students enrolled in grade one?
- (b) How many pencil did each student receive in total?

Project work

Form three groups of students for preparing a volleyball court of your school. The first group made a volleyball court whose area is 128m^2 and perimeter is 48m . The second group made a volleyball court whose area is 162m^2 and perimeter is 54m . The third group made a volleyball court whose area is 200m^2 and perimeter is 60m . Discuss the dimension of volleyball court in group and conclude which court is suitable for playing volleyball. Present your conclusion in your class.

Answers

1. 5 2. ± 6 3. 7 4. 3 5. ± 10 6. ± 6
7. 12 8. 8 9. 8 and 10 or -10 and -8
10. 3 and 5 or -5 and -3 11. 3 and $1/3$ 12. 6 and 15
13. 17 years and 13 years 14. 12 years and 10 years 15. 15 years and 12 years
16. (a) 7 years (b) 6 years (c) 5 years (d) 10 years
17. (a) 24 m and 7m (b) 10m , 24m , 26m (c) 15 m and 10 m
- (d) 9 m and 6 m (e) 240 m^2 (f) less than 20%
18. 24 19. (a) 20 person (b) 12 pencil

8.0 Review

1) Change the given algebraic fractions into the lowest terms. Check with your friends whether they are correct or not.

(a) $\frac{xy}{x^2y}$

(b) $\frac{x-y}{x^2-y^2}$

(c) $\frac{a+3}{a^2+5a+6}$

(d) $\frac{a-2}{a^2-6a+8}$

(e) $\frac{a-6}{a^2-8a+12}$

(f) $\frac{a+2}{a^2-4a+12}$

2) Simplify the given fraction. Ask your friend to check your answer or not.

(a) $\frac{3}{5} + \frac{1}{5}$

(b) $\frac{2}{3} + \frac{1}{5}$

(c) $\frac{1}{4} + \frac{1}{6}$

(d) $\frac{a}{b} + \frac{2a}{b}$

(e) $\frac{3a}{b} - \frac{ab}{a}$

(f) $\frac{3}{xy} + \frac{2a}{xy^2}$

8.1 Simplification of Algebraic Fractions

Activity 1

Simplify the given algebraic fraction. Discuss with friends about the simplification process.

(a) $\frac{x}{x-y} - \frac{y}{x-y}$

(b) $\frac{x}{x-y} - \frac{y}{x+y}$

(c) $\frac{1}{a-b} - \frac{b}{a^2+b^2}$

When simplifying the above fraction, first confirm that denominators of the given fractions are same or different. If that denominators are same, only operation between numerator are done and a single denominator is placed. only and write denominator in once. If denominators are different, we have to find the LCM of the denominators.

For example:

(a) $\frac{x}{x-y} + \frac{y}{x-y}$ are like fraction.

$$\frac{x}{x-y} + \frac{y}{x-y} = \frac{x+y}{x-y} \text{ [adding the numerators and placing common denominator.]}$$

(b) $\frac{x}{x-y} + \frac{y}{x+y}$ are unlike fraction. Now to make denominator equal,

$$= \frac{x(x+y)}{(x-y)(x+y)} - \frac{y(x-y)}{(x+y)(x-y)}$$

[\because Multiplied denominator of one fraction to numerator and denominator of another fraction]

$$= \frac{x(x+y) - y(x-y)}{(x+y)(x-y)} = \frac{x^2 + xy - xy + y^2}{(x+y)(x-y)} = \frac{x^2 + y^2}{x^2 - y^2}$$

(c) $\frac{1}{a-b} - \frac{b}{a^2 - b^2}$

$$= \frac{1}{a-b} - \frac{b}{(a-b)(a+b)} \text{ are unlike fraction.}$$

Now, to make denominator same,

$$= \frac{1(a+b)}{(a-b)(a+b)} - \frac{b}{(a-b)(a+b)}$$

to make denominator equal

Denominator of the first fraction = $(a-b) \times (a+b)$

Denominator of the second fraction = $(a-b)(a+b) \times 1$

$$= \frac{a+b-b}{(a-b)(a+b)}$$

$$= \frac{a}{(a-b)(a+b)}$$

$$= \frac{a}{a^2 - b^2}$$

Alternatively,

Denominator of the first fraction = $(a-b)$

Denominator of the second fraction = $(a-b)(a+b)$

LCM = $(a-b)(a+b)$

Now to simplify,

$$= \frac{1}{(a-b)} - \frac{b}{(a-b)(a+b)}$$

$$= \frac{(a+b)-b}{(a-b)(a+b)}$$

$$= \frac{a}{(a-b)(a+b)}$$

$$= \frac{a}{a^2 - b^2}$$

[Placing the LCM of the denominator in denominator. LCM is divided by each term denominator and multiply to numerator of the same fraction.]

Example 1

Simplify: $\frac{x^2}{x+y} - \frac{y^2}{x+y}$

Solution

$$\begin{aligned}
 &= \frac{x^2 - y^2}{x+y} \\
 &= \frac{(x-y)(x+y)}{x+y} \\
 &= x - y
 \end{aligned}$$

Example 3

Simplify: $\frac{x+y}{x-y} + \frac{x-y}{x+y}$

Solution

$$\begin{aligned}
 &= \frac{x+y}{x-y} + \frac{x-y}{x+y} \\
 &= \frac{(x+y)^2 + (x-y)^2}{(x-y)(x+y)} \\
 &= \frac{x^2 + 2xy + y^2 + x^2 - 2xy + y^2}{x^2 - y^2} \\
 &= \frac{2(x^2 + y^2)}{x^2 - y^2}
 \end{aligned}$$

Example 5

Simplify: $\frac{1}{2a-3b} - \frac{a+b}{4a^2-9b^2}$

Solution

$$\begin{aligned}
 &= \frac{1}{2a-3b} - \frac{a+b}{4a^2-9b^2} \\
 &= \frac{1}{2a-3b} - \frac{a+b}{(2a-3b)(2a+3b)} \\
 &= \frac{(2a+3b)-(a+b)}{(2a-3b)(2a+3b)} \\
 &= \frac{(a+2b)}{4a^2-9b^2}
 \end{aligned}$$

Example 2

Simplify: $\frac{1}{x-y} - \frac{1}{x+y}$

Solution

$$\begin{aligned}
 &= \frac{1}{x-y} - \frac{1}{x+y} \\
 &= \frac{(x+y)-(x-y)}{x^2-y^2} \\
 &= \frac{x+y-x+y}{x^2-y^2} \\
 &= \frac{2y}{x^2-y^2}
 \end{aligned}$$

Example 4

Simplify: $\frac{a^3+1}{a^2-a+1} + \frac{a^3-1}{a^2+a+1}$

Solution

$$\begin{aligned}
 &= \frac{a^3+1}{a^2-a+1} + \frac{a^3-1}{a^2+a+1} \\
 &= \frac{(a+1)(a^2-a+1)}{a^2-a+1} + \frac{(a-1)(a^2+a+1)}{a^2+a+1} \\
 &= (a+1) + (a-1) \\
 &= 2a
 \end{aligned}$$

Calculating the multiplication of $4a^2 - 9b^2$

$$\begin{aligned}
 &= (2a)^2 - (3b)^2 \\
 &= (2a+3b)(2a-3b)
 \end{aligned}$$

Example 6

Simplify: $\frac{4x^2 + y^2}{4x^2 - y^2} - \frac{2x - y}{2x + y}$

Solution

$$\begin{aligned} &= \frac{4x^2 + y^2}{4x^2 - y^2} - \frac{2x - y}{2x + y} \\ &= \frac{4x^2 + y^2}{(2x - y)(2x + y)} - \frac{2x - y}{2x + y} \\ &= \frac{4x^2 + y^2 - (2x - y)^2}{(2x - y)(2x + y)} \\ &= \frac{4x^2 + y^2 - 4x^2 + 4xy - y^2}{4x^2 - y^2} \\ &= \frac{4xy}{4x^2 - y^2} \end{aligned}$$

Calculating the multiplication of $4x^2 - y^2$

$$\begin{aligned} &= (2x)^2 - (y)^2 \\ &= (2x + y)(2x - y) \end{aligned}$$

Example 7

Simplify: $\frac{x}{x - y} + \frac{x}{x + y} + \frac{2xy}{x^2 + y^2}$

Solution

$$\begin{aligned} &= \frac{x}{x - y} + \frac{x}{x + y} + \frac{2xy}{x^2 + y^2} \\ &= \frac{x(x + y) + x(x - y)}{x^2 - y^2} + \frac{2xy}{x^2 + y^2} \\ &= \frac{x^2 + xy + x^2 - xy}{x^2 - y^2} + \frac{2xy}{x^2 + y^2} \\ &= \frac{2x^2}{x^2 - y^2} + \frac{2xy}{x^2 + y^2} \\ &= \frac{2x^2(x^2 + y^2) + 2xy(x^2 - y^2)}{(x^2 - y^2)(x^2 + y^2)} \\ &= \frac{2x^4 + 2x^2y^2 + 2x^3y - 2xy^3}{(x^4 - y^4)} \end{aligned}$$

Example 8

Simplify: $\frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{x^2-y^2}$

Solution

$$\begin{aligned}
 & \frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{x^2-y^2} \\
 &= \frac{1}{2(x-y)} - \frac{1}{2(x+y)} - \frac{y}{(x-y)(x+y)} \\
 &= \frac{(x+y)-(x-y)-2y}{2(x-y)(x+y)} \\
 &= \frac{x+y-x+y-2y}{2(x-y)(x+y)} \\
 &= \frac{0}{2(x-y)(x+y)} \\
 &= 0
 \end{aligned}$$

Example 9

Simplify: $\frac{a-1}{a^2-4a+3} + \frac{a-2}{a^2-8a+12} + \frac{a-5}{a^2-8a+15}$

Solution

$$\begin{aligned}
 &= \frac{a-1}{a^2-4a+3} + \frac{a-2}{a^2-8a+12} + \frac{a-5}{a^2-8a+15} \\
 &= \frac{a-1}{(a-1)(a-3)} + \frac{a-2}{(a-6)(a-2)} + \frac{a-5}{(a-5)(a-3)} \\
 &= \frac{1}{(a-3)} + \frac{1}{(a-6)} + \frac{1}{(a-3)} \\
 &= \frac{a-6+a-3+a-6}{(a-6)(a-3)} \\
 &= \frac{3a-15}{(a-2)(a-3)} \\
 &= \frac{3(a-5)}{(a-2)(a-3)}
 \end{aligned}$$

$$\begin{aligned}
 & a^2 - 4a + 3 \\
 &= a^2 - 3a - 1a + 3 \\
 &= a(a-3) - 1(a-3) \\
 &= (a-3)(a-1) \\
 & a^2 - 8a + 15 \\
 &= a^2 - 5a - 3a + 15 \\
 &= a(a-5) - 3(a-5) \\
 &= (a-5)(a-3) \\
 & a^2 - 8a + 12 \\
 &= a^2 - 6a - 2a + 12 \\
 &= a(a-6) - 2(a-6) \\
 &= (a-6)(a-2)
 \end{aligned}$$

Example 10

Simplify: $\frac{pr^2 + q}{2r - 1} + \frac{pr^2 - q}{2r + 1} + \frac{4pr^3}{1 - 4r^2}$

Solution

$$\begin{aligned}
 &= \frac{pr^2 + q}{2r - 1} + \frac{pr^2 - q}{2r + 1} + \frac{4pr^3}{1 - 4r^2} \\
 &= \frac{pr^2 + q}{2r - 1} + \frac{pr^2 - q}{2r + 1} - \frac{4pr^3}{4r^2 - 1} \\
 &= \frac{pr^2 + q}{2r - 1} + \frac{pr^2 - q}{2r + 1} - \frac{4pr^3}{(2r - 1)(2r + 1)} \\
 &= \frac{(pr^2 + q)(2r + 1) + (pr^2 - q)(2r - 1) - 4pr^3}{(2r - 1)(2r + 1)} \\
 &= \frac{(2pr^3 + pr^2 + 2rq + q) + 2pr^3 - pr^2 - 2rq + q - 4pr^3}{4r^2 - 1} \\
 &= \frac{2q}{4r^2 - 1}
 \end{aligned}$$

Calculating the multiplication of

$$4r^2 - 1$$

$$= 4r^2 - 1$$

$$= (2r)^2 - (1)^2$$

$$= (2r - 1)(2r + 1)$$

Example 11

Simplify: $\frac{a-b}{a^2 - ab + b^2} + \frac{a+b}{a^2 + ab + b^2} - \frac{2a^3}{a^4 - a^2b^2 + b^4}$

Solution

$$\begin{aligned}
 &= \frac{a-b}{a^2 - ab + b^2} + \frac{a+b}{a^2 + ab + b^2} - \frac{2a^3}{a^4 - a^2b^2 + b^4} \\
 &= \frac{(a-b)(a^2 + ab + b^2) + (a+b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)(a^2 + ab + b^2)} - \frac{2a^3}{a^4 - a^2b^2 + b^4} \\
 &= \frac{a^3 - b^3 + a^3 + b^3}{(a^4 + a^2b^2 + b^4)} - \frac{2a^3}{a^4 - a^2b^2 + b^4} \\
 &= \frac{2a^3}{(a^4 + a^2b^2 + b^4)} - \frac{2a^3}{a^4 - a^2b^2 + b^4} \\
 &= \frac{2a^3(a^4 - a^2b^2 + b^4) - 2a^3(a^4 + a^2b^2 + b^4)}{(a^4 + a^2b^2 + b^4)(a^4 - a^2b^2 + b^4)} \\
 &= \frac{2a^7 - 2a^5b^2 + 2a^3b^4 - 2a^7 - 2a^5b^2 - 2a^3b^4}{(a^4 + a^2b^2 + b^4)(a^4 - a^2b^2 + b^4)} \\
 &= \frac{-4a^5b^2}{(a^8 + a^4b^4 + b^8)}
 \end{aligned}$$

Exercise 8.1

1. Change into the simplest form.

$$(a) \frac{x^2 - 5x}{x^2 - 25}$$

$$(b) \frac{x^2 - b^2}{(a+b)^2}$$

$$(c) \frac{x^2 - 5x + 6}{x^2 - 7x + 12}$$

2. Simplify:

$$(a) \frac{a}{a-b} + \frac{b}{b-a}$$

$$(b) \frac{1}{b-c} - \frac{b+c}{b^2-c^2}$$

$$(c) \frac{1}{m-n} + \frac{1}{m+n}$$

$$(d) \frac{m+n}{m-n} + \frac{m-n}{m+n}$$

$$(e) \frac{1}{m+n} + \frac{n}{m^2-n^2}$$

$$(f) \frac{3}{x^2-4} + \frac{1}{(x-2)^2}$$

$$(g) \frac{a^3+b^3}{a^2-ab+b^2} + \frac{a^3-b^3}{a^2+ab+b^2}$$

$$(h) \frac{4x^2+25y^2}{4x^2-25y^2} - \frac{2x-5y}{2x+5y}$$

$$(i) \frac{4x^3}{x^4+a^4} - \frac{8x^7}{x^8-a^8}$$

$$(j) \frac{x}{x-y} - \frac{x}{x+y} + \frac{2xy}{x^2+y^2}$$

$$(k) \frac{3}{a+3} + \frac{4}{a-3} + \frac{9a}{2(9-a^2)}$$

$$(l) \frac{1}{x+2y} - \frac{1}{x-2y} + \frac{2x}{4y^2-x^2}$$

$$(m) \frac{a}{(a-b)(a-c)} + \frac{b}{(b-a)(b-c)} + \frac{c}{(c-b)(c-a)}$$

$$(n) \frac{y-z}{x^2-(y-z)^2} + \frac{z-x}{y^2-(z-x)^2} + \frac{x-y}{z^2-(x-y)^2}$$

$$(o) \frac{x^2-(a-b)^2}{(x+b)^2-a^2} + \frac{a^2-(x-b)^2}{(x+a)^2-b^2} + \frac{b^2-(x-a)^2}{(a+b)^2-x^2}$$

$$(p) \frac{1}{p^2+7p+12} + \frac{2}{p^2+5p+6} - \frac{3}{p^2+6p+8}$$

$$(q) \frac{x+3}{x^2+3x+9} + \frac{x-3}{x^2-3x+9} - \frac{54}{x^4+9x^2+81} \quad (r) \frac{1}{x^2-5x+6} + \frac{2}{4x-x^2-3} - \frac{3}{x^2-3x+2}$$

$$(s) \frac{b+2}{1+b+b^2} - \frac{b-2}{1-b+b^2} - \frac{2b^2}{1+b^2+b^4} \quad (t) \frac{1}{1-b+b^2} - \frac{1}{1+b+b^2} - \frac{2b}{1-b^2+b^4}$$

$$(u) \frac{a+c}{a^2+ac+c^2} + \frac{a-c}{a^2-ac+c^2} + \frac{2c^3}{a^4+a^2c^2+c^4}$$

3. Simplify:

$$(a) \frac{1}{4(1-\sqrt{x})} - \frac{1}{4(1+\sqrt{x})} + \frac{2\sqrt{x}}{4(1-x)}$$

$$(b) \frac{1}{8(1-\sqrt{x})} - \frac{1}{8(1+\sqrt{x})} + \frac{2\sqrt{x}}{8(1-x)}$$

$$(c) \frac{1}{(a+1)^2} + \frac{1}{(a-1)^2} - \frac{2}{a^2-1}$$

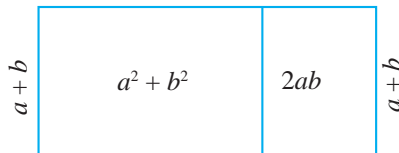
4. If $\frac{a}{2x+1} + \frac{1}{x+2} = \frac{4x+5}{2x^2+5x+2}$ what is the value of a? Find it.

5. If $\frac{a}{2x-3} + \frac{b}{3x+4} = \frac{x+7}{6x^2-x-12}$ what are the values of a and b? Find it

Project work

Divide a rectangular piece of paper into two parts from its length side making its breadth equal. Consider the area and breadth of first piece is $a^2 + b^2$ and $a + b$ respectively. Similarly, the area and breadth of second piece of paper is $2ab$ and $a + b$ respectively. Now, consider $a > b$,

- Find the total length of both pieces of paper. Express in terms of a and b .
- If $a = 5m$ and $b = 3m$, then find the relation of area, length and breadth of the rectangular piece of paper and present in your classroom.



Answers

- (a) $\frac{x}{a+5}$
 - (b) $\frac{a+b}{a-b}$
 - (c) $\frac{x-2}{x-4}$
- (a) 1
 - (b) 0
 - (c) $\frac{2m}{m^2-n^2}$
- (d) $\frac{2(m^2+n^2)}{m^2-n^2}$
 - (e) $\frac{m}{m^2-n^2}$
 - (f) $\frac{4(x-1)}{(x+2)(x-2)^2}$
- (g) $2a$
 - (h) $\frac{20xy}{4x^2-20y^2}$
 - (i) $\frac{4x^3}{a^4-x^4}$
 - (j) $\frac{4x^3y}{x^4-y^4}$
- (k) $\frac{5a+6}{2(a^2-9)}$
 - (l) $\frac{2}{2y-x}$
 - (m) 0
 - (n) 0
 - (o) 1
- (p) $\frac{1}{(p+2)(p+3)(p+4)}$
 - (q) $\frac{2(x-3)}{x^2-3x+9}$
 - (r) $\frac{4}{3x-x^2-2}$
- (s) $\frac{4}{1+b^2+b^4}$
 - (t) $\frac{-4b^3}{1+b^4+b^8}$
 - (u) $\frac{2(a+c)}{a^2+ac+c^2}$
- (a) $\frac{\sqrt{x}}{1-x}$
 - (b) $\frac{\sqrt{x}}{2(1-x)}$
 - (c) $\frac{4}{(a^2-1)^2}$
- $a = 2$
 - $a = 1, b = -1$

Lesson 9

Indices

9.0 Review

We studied about simplification of indices in the previous grades. Here, we discuss about exponential equation.

Fill in the blank spaces in the given table. Which value of x satisfies the given condition?

(a) $2^x = 2$

x	-3	-2	-1	0	1	2	3
2^x	$2^{-3} = \frac{1}{8}$

(b) $5^{x+1} = 125$

x	-3	-2	-1	0	1	2	3
5^{x+1}	$5^{-3+1} = \frac{1}{25}$

(c) $3^x = \frac{1}{9}$

x	-3	-2	-1	0	1	2	3
3^x	$3^{-3} = \frac{1}{27}$

9.1 Exponential Equations

Activity 1

How can we solve the given exponential equations? Discuss in groups.

(a) $2^x = 4$

(b) $3^{x-1} = 81$

(c) $3^{x+1} + 3^x = \frac{4}{27}$

(d) $3^x + \frac{1}{3^x} = 3\frac{1}{3}$

Put $x = 0, \pm 1, \pm 2, \pm 3, \dots$ in the given equations. The value that satisfies the equation of x is the value of ' x '.



Is there is another way to solve exponential equation other than putting $x = 0, \pm 1, \pm 2, \pm 3, \dots$ in given the exponential equation?



- (a) In $2^x = 4$ put $x = 0, \pm 1, \pm 2, \pm 3, \dots$ respectively. $x = 2$ satisfies the equation. Therefore, $x = 2$.

Alternatively,

Here, $2^x = 4$

or, $2^x = 2^2$

$\Rightarrow x = 2$

Oh! This method is short and easy. In equation, if the bases are equal then the indices should be equal. So, we should make equal bases on both sides.

(b) $3^{x-1} = 81$

or, $3^{x-1} = 3^4$

$\Rightarrow x - 1 = 4$

$\therefore x = 5$

Verification, $x = 5$

$3^{x-1} = 81$

LHS 3^{5-1}

$= 3^4 = 81$

$\therefore \text{LHS} = \text{RHS}$

So, $x = 5$ is correct.



(c) $3^{x+1} + 3^x = \frac{4}{81}$

or, $3^x \times 3^1 + 3^x = \frac{4}{81}$

or, $3^x(3 + 1) = \frac{4}{81}$

or, $3^x(4) = \frac{4}{81}$

or, $3^x = \frac{1}{81}$

or, $3^x = 3^{-4}$

$\Rightarrow x = -4$

Verification, $x = -4$

$3^{x+1} + 3^x = \frac{4}{81}$

LHS $3^{-4+1} + 3^{-4}$

$= 3^{-3} + 3^{-4}$

$= \frac{1}{3^3} + \frac{1}{3^4}$

$= \frac{1}{27} + \frac{1}{81}$

$= \frac{4}{81} = \text{RHS}$

The value of $x = -4$ is correct.

(d) $3^x + \frac{1}{3^x} = 3\frac{1}{3}$ What is the difference between this and previous exponential equation?

$$\text{or, } 3^x + \frac{1}{3^x} = 3\frac{1}{3}$$

$$\text{or, } \frac{(3^x)^2 + 1}{3^x} = \frac{10}{3}$$

$$\text{or, } 3 \times (3^x)^2 + 3 = 10 \times 3^x$$

$$\text{or, } 3 \times (3^x)^2 - 10 \times 3^x + 3 = 0$$

This is the quadratic equation of 3^x .

Now, putting the value of a in equation (i)

$$3^x = a \dots\dots\dots (i)$$

$$\text{Now } 3a^2 - 10a + 3 = 0$$

$$\text{or, } 3a^2 - 9a - a + 3 = 0$$

$$\text{or, } 3a(a - 3) - 1(a - 3) = 0$$

$$\text{or, } (a - 3)(3a - 1) = 0$$

$$\text{either, } (a - 3) = 0 \quad \therefore a = 3$$

$$\text{or, } (3a - 1) = 0 \quad \therefore a = \frac{1}{3}$$

Therefore, the value of x are 1 and -1.

$$\text{If } a = 3, 3^x = 3^1 \quad \Rightarrow x = 1$$

$$\text{If } a = \frac{1}{3}, 3^x = \frac{1}{3} = 3^{-1} \quad \Rightarrow x = -1$$

Example 1

Solve: $7^x = 49$

Solution

$$\text{Here, } 7^x = 49$$

$$\text{or, } 7^x = 7^2$$

$$\Rightarrow x = 2$$

Example 2

Solve: $4^{x-2} = 0.25$

Solution

Here, $4^{x-2} = 0.25$

$$\text{or, } (2)^{2(x-2)} = \frac{1}{4}$$

$$\text{or, } (2)^{2(x-2)} = \left(\frac{1}{2}\right)^2$$

$$\text{or, } (2)^{2(x-2)} = 2^{-2}$$

$$\text{or, } (2)^{2(x-2)} = 2^{-2}$$

$$\Rightarrow 2(x-2) = -2$$

$$\text{or, } x-2 = -1$$

$$\therefore x = 1$$

Example 3

Solve: $3^{5x-4} + 3^{5x} = 82$

Solution

Here, $3^{5x-4} + 3^{5x} = 82$

$$\text{or, } 3^{5x} \times 3^{-4} + 3^{5x} = 82$$

$$\text{or, } 3^{5x} \left(\frac{1}{81} + 1\right) = 82$$

$$\text{or, } 3^{5x} \left(\frac{82}{81}\right) = 82$$

$$\text{or, } 3^{5x} = 81$$

$$\text{or, } 3^{5x} = 3^4$$

$$\Rightarrow 5x = 4$$

$$\therefore x = \frac{4}{5}$$

Alternative Method

Here, $4^{x-2} = 0.25$

$$\text{or, } 4^{x-2} = \frac{1}{4}$$

$$\text{or, } 4^{x-2} = (4)^{-1}$$

$$\text{or, } (4)^{(x-2)} = (4)^{-1}$$

$$\Rightarrow (x-2) = -1$$

$$\text{or, } x = -1 + 2$$

$$\therefore x = 1$$

Example 4

Solve : $3^{x-1} + 3^{x-2} + 3^{x-3} = 13$

Solution

Here, $3^{x-1} + 3^{x-2} + 3^{x-3} = 13$

$$\text{or, } 3^x \times 3^{-1} + 3^x \times 3^{-2} + 3^x \times 3^{-3} = 13$$

$$\text{or, } \frac{1}{3} \times 3^x + \frac{1}{9} \times 3^x + \frac{1}{27} \times 3^x = 13$$

$$\text{or, } 3^x \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right) = 13$$

$$\text{or, } 3^x \left(\frac{9+3+1}{27}\right) = 13$$

$$\text{or, } 3^x \left(\frac{13}{27}\right) = 13$$

$$\text{or, } 3^x = 27$$

$$\text{or, } 3^x = 3^3$$

$$\Rightarrow x = 3$$

Example 5

Solve: $2^x + \frac{1}{2^x} = 2\frac{1}{2}$

Solution

Here, $2^x + \frac{1}{2^x} = 2\frac{1}{2}$

or, $2^x + \frac{1}{2^x} = \frac{5}{2}$

Let $2^x = a$(i)

so, $a + \frac{1}{a} = \frac{5}{2}$

or, $\frac{a^2+1}{a} = \frac{5}{2}$

or, $2(a^2 + 1) = 5a$

or, $2a^2 - 5a + 2 = 0$

or, $2a^2 - 4a - a + 2 = 0$

or, $2a(a - 2) - 1(a - 2) = 0$

or, $(a - 2)(2a - 1) = 0$

Either, $(a - 2) = 0 \quad \therefore a = 2$

Or, $(2a - 1) = 0 \quad \therefore a = \frac{1}{2}$

Substituting the value of a in eqⁿ(i), we get

$$a = 2 \text{ then } 2^x = 2^1 \quad \Rightarrow x = 1$$

$$a = \frac{1}{2} \text{ then } 2^x = \frac{1}{2} = 2^{-1} \Rightarrow x = -1$$

Hence, the values of x are 1 and -1.

Example 6

Solve: $5 \times 4^{x+1} - 16^x = 64$

Solution

Here, $5 \times 4^{x+1} - 16^x = 64$

or, $5 \times (4^x \times 4) - 4^{2x} = 64$

or, $20 \times 4^x - (4^x)^2 = 64$

Let $4^x = a$(i)

Hence, $20a - a^2 = 64$

$$\text{or, } a^2 - 20a + 64 = 0$$

$$\text{or, } a^2 - 16a - 4a + 64 = 0$$

$$\text{or, } a(a - 16) - 4(a - 16) = 0$$

$$\text{or, } (a - 4)(a - 16) = 0$$

$$\text{Either, } (a - 4) = 0 \quad \therefore a = 4$$

$$\text{or, } (a - 16) = 0 \quad \therefore a = 16$$

Now, putting the values of a in eqⁿ (i), we get

$$\text{If } a = 4 \text{ then } 4^x = 4^1 \quad \Rightarrow x = 1$$

$$\text{If } a = 16 \text{ then } 4^x = 16 = 4^2 \quad \Rightarrow x = 2$$

Hence, the values of x are 1 and 2.

Example 7

If $x^2 + 2 = 3^{\frac{2}{3}} + 3^{-\frac{2}{3}}$, then prove that $3x(x^2 + 3) = 8$

Solution

$$\text{Here, } x^2 + 2 = 3^{\frac{2}{3}} + 3^{-\frac{2}{3}}$$

$$\text{or, } x^2 = 3^{\frac{2}{3}} + 3^{-\frac{2}{3}} - 2$$

$$\text{or, } x^2 = \left(3^{\frac{1}{3}}\right)^2 - \left(3^{-\frac{1}{3}}\right)^2 - 2 \times 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}} \quad [\because 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}} = 1]$$

$$\text{or, } x^2 = \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)^2$$

$$\Rightarrow x = 3^{\frac{1}{3}} - 3^{-\frac{1}{3}} \quad \dots\dots\dots(i)$$

Cubing on both sides of eqⁿ (i)

$$\text{or, } x^3 = \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)^3$$

$$\text{or, } x^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{-\frac{1}{3}}\right)^3 - 3 \times 3^{\left(\frac{1}{3}\right)} \times 3^{-\left(\frac{1}{3}\right)} \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$$

$$\text{or, } x^3 = 3 - 3^{-1} - 3 \times 1 \times x$$

$$\text{or, } x^3 = 3 - \frac{1}{3} - 3x$$

$$\text{or, } x^3 = \frac{9 - 1 - 9x}{3}$$

$$\text{or, } 3x^3 = 8 - 9x$$

$$\text{or, } 3x^3 + 9x = 8 \text{ Proved.}$$

Exercise 9.1

1. Fill in the blank spaces in the following table and show it to your teacher.

(a)

x	-3	-2	-1	0	1	2	3
7^x

(b)

x	-3	-2	-1	0	1	2	3
5^{-x}

2. Solve and examine

(a) $3^x = 9$ (b) $5^{x-1} = 25$ (c) $\frac{1}{5^{2x-4}} = 125$
 (d) $4^{x-2} = 0.125$ (e) $\left(\frac{3}{5}\right)^x = \left(1\frac{2}{3}\right)^3$ (f) $2^x \times 3^{x+1} = 18$

3. Solve:

(a) $4^{\frac{1-x}{1+x}} = 4^{\frac{1}{3}}$ (b) $2^{x+4}\sqrt{4^{x+8}} = \sqrt[6]{128}$
 (c) $2^{x+1} + 2^{x+2} + 2^{x+3} = 448$ (d) $3^{x+1} - 3^x = 162$
 (e) $4^{x+1} - 8 \times 4^{x-1} = 32$ (f) $4 \times 3^{x+1} - 3^{x+2} - 3^{x-1} = 72$
 (g) $3^{x+2} + 3^{x+1} + 2 \times 3^x = 126$ (h) $2^x + 3^{x-2} = 3^x - 2^{x+1}$
 (i) $8^{x-1} - 23^{x-2} + 8 = 0$ (j) $\left(\frac{1}{4}\right)^{2-\sqrt{5x+1}} = 4 \times 2^{\sqrt{5x+1}}$

4. Solve:

(a) $5^x + \frac{1}{5^x} = 5\frac{1}{5}$
 (b) $7^x + \frac{1}{7^x} = 7\frac{1}{7}$
 (c) $9^x + \frac{1}{9^x} = 9\frac{1}{9}$
 (d) $4^x + \frac{1}{4^x} = 16\frac{1}{16}$
 (e) $5^x + 5^{-x} = 25\frac{1}{25}$
 (f) $81 \times 3^x + 3^{-x} = 30$

5. Solve:

- (a) $4 \times 3^{x+1} - 9^x = 27$
(b) $3 \times 2^{p+1} - 4^p = 8$
(c) $5^{2x} - 6 \times 5^{x+1} + 125 = 0$
(d) $2^{x-2} + 2^{3-x} = 3$
(e) $5^{x+1} + 5^{2-x} = 126$
(f) $3^{2y} - 4 \times 3^y + 3 = 0$
6. Solve $16^x - 5 \times 4^{x+1} + 64 = 0$. Prove that the value of x satisfies $5^x + \frac{125}{5^x} = 30$
7. a) If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, then prove that $3x(x^2 - 3) = 10$
b) If $x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$, then prove that $2x^3 + 6x - 3 = 0$

Answers

2. (a) 2 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{1}{2}$ (e) -3 (f) 1
3. (a) $\frac{1}{2}$ (b) 34 (c) 5 (d) 4 (e) 2 (f) 3 (g) 2 (h) 3
(i) 2 (j) 7
4. (a) ± 1 (b) ± 1 (c) ± 1 (d) ± 2 (e) ± 2 (f) 1, 3
5. (a) 1, 2 (b) 1, 2 (c) 1, 2 (d) 2, 3 (e) -1, 2 (f) 0, 1
7. 1, 2

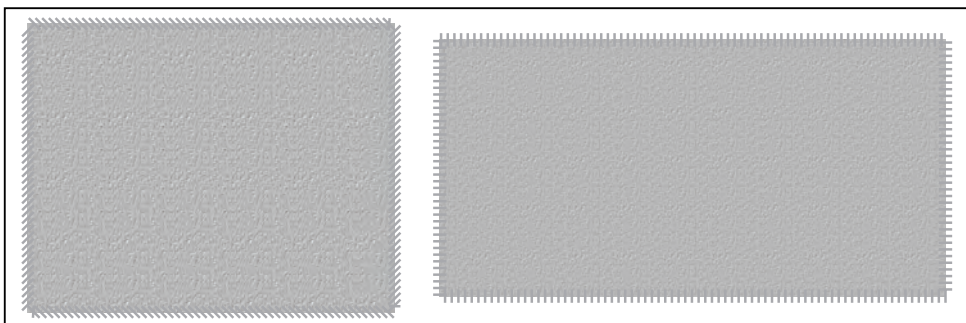
1. The commission of two employees of Nepal Pustak Pasal in five months is given below.

Months					
Name	Baishakh	Jestha	Ashadh	Shrawan	Bhadra
Employee A	Rs. 5000	Rs. 6000	Rs. 7000	Rs. 8000	Rs. 9000
Employee B	Rs. 2000	Rs. 3000	Rs. 4500	Rs. 6750	Rs. 10125

Answer the following questions based on the above table.

- Which employee received commission in arithmetic sequence? Write with reason.
 - What is the total amount received by employee A at the end of five months? Calculate using formula.
2. Bishal borrowed Rs. 45,000 from his friend, Sunil on the condition that he would pay the sum in 6 instalments. He continued to pay Rs. 1000 more in each instalment than the previous one. Similarly, Sita also borrowed Rs. 63,000 from her friend Om kumari on the condition that she would also pay in 6 instalments. But, she continued to pay double in each instalment than the previous instalment.
- Find, how much money do Bishal and Sita pay in the first installment.
 - Find the difference between the first and the last installments paid by Bishal and Sita.
 - In which installment do Bishal and Sita pay same amount of money? Calculate.
3. Two birds are migrated to a community forest on the first day. The birds that came the previous day had migrated with the double number of their friends the next day. If the birds continue to migrate at this rate, then
- How many birds will migrate on the tenth day? Find it.
 - How many birds in total will migrate in the ten days?
4. In Nabraj's piggy bank, his father deposited money from the 1st baisakh to the 7th baisakh. Each day, we deposited double the amount he had deposited the previous day. On the seventh day, there was a total of Rs. 635 in his piggy bank. Then,
- How much money did Nabraj's father deposit in Nabraj's piggy bank on the first day?
 - How many rupees were deposited on the seventh day?

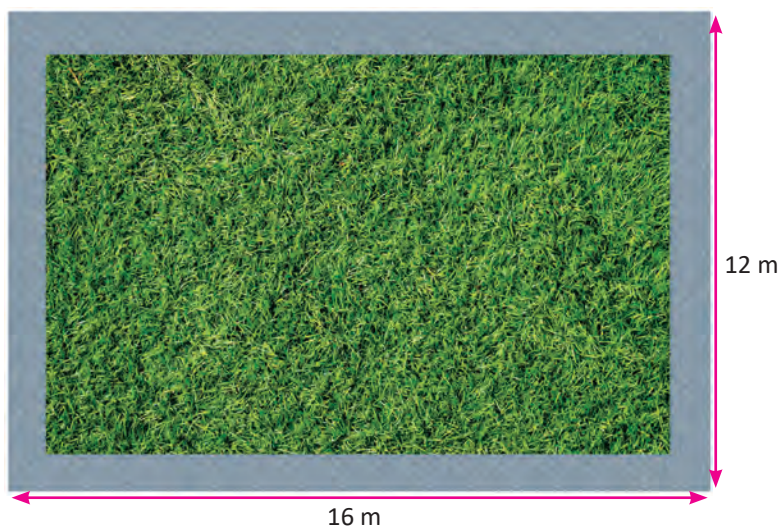
5. Sunil's father decided to deposit some amount of money on the occasion of Sunil's every birthday. Accordingly, Rs. 500 was deposited on the occasion of the first birthday, Rs. 1000 on the second birthday and Rs. 1500 on the third birthday. In this way, the deposit amount increased by Rs. 500 on every birthday.
- How much amount would be deposited on the occasion of Sunil's 16th birthday? Find it.
 - What is the total amount accumulated upto Sunil's 16th birthday?
 - How many birthdays would Sunil wait to collect Rs. 1 Lakh? Write with a reason .
6. Harisaran could not estimate the length of the barbed wire to fence his two rectangular field. The area of his both land is 360 m^2 . The difference between the length and breadth of the first land is 22m and the difference between the length and breadth of the second land is 9m.



Based on this, answer the following questions.

- Find the length and breadth of both fields.
 - Does it require equal length of wire to fence both of the plots?
 - Which plot of land costs more by how much to fence around with the wire that costs Rs. 10 per meter? Calculate.
7. Sita and Ram are wife and husband. The present age of Ram is 30 years and Sita is 25 years.
- What were Sita and Ram's ages x years ago?
 - If the product of their age was $500 x$ year ago, then find the value of x .
 - How many years later, the sum of their age will be 99? Calculate.

8. The first term of an arithmetic sequence is 2. If the sum of the next five terms is equal to four times the sum of the first five terms. Then,
- Find the common difference.
 - Prove that $t_{20} = -112$.
 - Find the sum of the first five terms.
9. A footpath of equal width has been constructed around 16m x 12m grassland as shown in the figure. The total area of the grassland and footpath is 320m^2 .



- Let the width of the footpath be x . Write the equation based on the above context.
 - Find the width of the footpath.
10. In a rectangular field, the longer side is 40m more than the shorter side but the diagonal of the field is 40m more than its longer side.
- Let the length of the shorter side be x and write the equation based on the above context.
 - Find the length of the shorter side, longer side and diagonal of the field.
 - Find how much it will cost to surround the land four times with a barbed fence at the rate of Rs. 10 per meter.
 - How many pieces of land of 20m x 15m can be prepared on that rectangular field? Calculate.

11. Ramesh and Sita are brother and sister. Ramesh's present age is 30 years and Sita's is 25 years.

- What were ramesh and sita's ages x years ago?
- If the product of their age was 644 x years ago, then find the value of x .
- How many years later will the product of their age be 864? Calculate.

12. Two cars leave a cross road at the same time. One is travelling towards the north and the other is travelling towards the west. When the car travelling towards the north has travelled a distance of 24 miles, the distance between the two cars was four miles more than three times the distance of the car travelling to the west.

- Write the equation based on above context.
- Find how far the car travelling to the west has travelled.
- Find the actual distance between the two cars.

13. A bus travelled a distance of 90km at the same speed. If the speed of the bus was 15km/hr more than the previous speed, the total travelling time would have been reduced by 30 minutes.

- Let the speed of the bus be x and write the equation based on above context.
- What was the initial speed of the bus? Find it.

14. Simplify

(a) $\frac{1}{a-b} - \frac{2b}{a^2-b^2}$

(b) $\frac{a-4}{a^2-4a+16} + \frac{a+4}{a^2+4a+16} + \frac{128}{a^4+16a^2+256}$

(c) $\frac{2a-6}{a^2-9a+20} - \frac{a-1}{a^2-7a+12} - \frac{a-2}{a^2-8a+15}$

(d) $\frac{a+b}{2ab}(a+b-c) + \frac{b+c}{2bc}(b+c-a) + \frac{c+a}{2ac}(c+a-b)$

15. Solve:

(a) $3^{x+2} + 3^{2-x} = 82$

(b) $\frac{3^{2x+1}}{3^x} = \frac{82}{9}$

16. Prove the following relations.

- (a) If $x = 1 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then prove that $x(x^2 - 3x - 3) = 1$
(b) If $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$, then prove that $x(x^2 - 9x + 8) = 12$
(c) If $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then prove that $x(x^2 - 6x + 18) = 22$

Answers

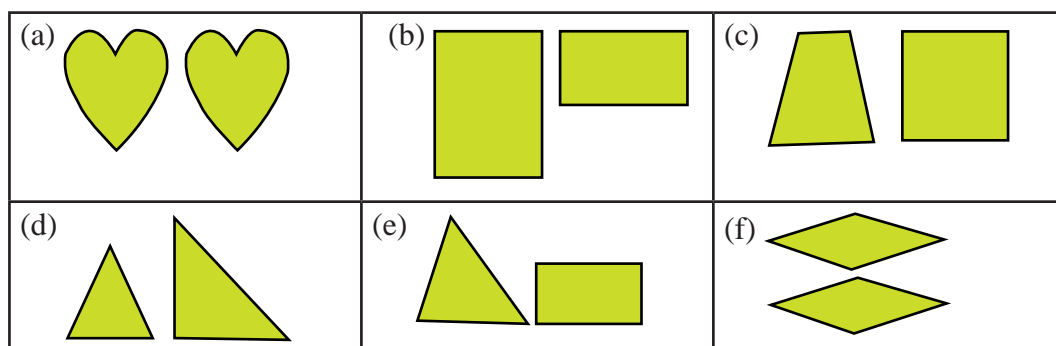
1. (a) staff A (b) Rs. 6000, Rs. 3000 (c) Rs. 8625
2. (a) Rs. 5000, Rs. 1000 (b) Rs. 4000, Rs. 22000 (c) Fourth
3. (a) Rs. 024 (b) Rs. 2046
4. (a) Rs. 5 (b) Rs. 320
5. (a) Rs. 8,000 (b) Rs. 68,000 (c) 20th years
6. (a) 20 m, 18 m and 24 m, 15 m (b) No, 76 m and 78 m
(c) second Rs. 20
7. (a) $(30 - x)$ years and $(25 - x)$ years (b) 5 years (c) 22 years
8. (a) -6 (c) -50
9. (a) $(16 + 2x)(12 + 2x) = 320$ (b) 2 m
10. (b) 120 m, 160 m and 200 m (c) Rs. 33600 (d) 64
11. (a) $30 - x$ and $25 - x$ (b) 2 years (c) 2 years
12. (b) 7 km (c) 25 km
13. (a) $\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$ (b) 45 km/h
14. (a) $\frac{1}{a+b}$ (b) $\frac{2(a-4)}{a^2-4a+16}$ (c) $\frac{5}{(a-3)(a-4)(a-5)}$ (d) 3
15. (a) ± 2 (b) ± 2 (c) $\frac{2(a-4)}{a^2-4a+16}$

Lesson 10

Triangle and Quadrilaterals

10.0 Review

Observe the given figures and discuss the questions given.



- Do the pair of pictures match when they are overlapped?
- Is the area of the pair of figures equal?
- Which pair of figures are congruent and which are not?
- Are all the figures having equal area congruent?

Discuss the above questions in groups and present the conclusion in your classroom.

10.1 Area of triangle and quadrilaterals

Activity 1

Observe the given figures and answer the questions.

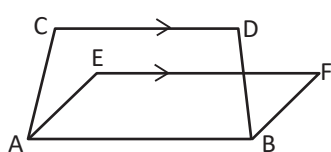


Figure (i)

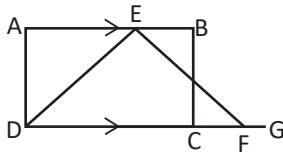


Figure (ii)

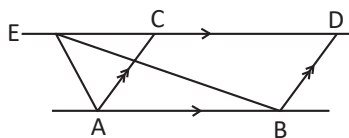


Figure (iii)

based on above figures,

- In which of the given figures are the quadrilaterals formed with the same base and different parallel lines? Name the quadrilaterals.
- In which of the given figures are the triangles and quadrilaterals formed with the different bases and the same parallel lines? Name the triangles and quadrilaterals.

- c. In figure (c), identify the triangles and quadrilateral standing on the same base and lying between parallel lines.

A. The relation of parallelogram standing on the same base and lying between the same parallel lines

Activity 2

In the given figure, what will be the area of parallelograms ABCD and ABEF? What is the relation between them?

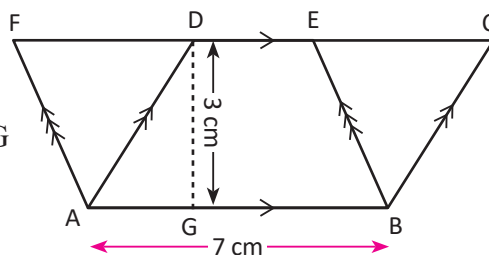
Here, the base and the height of both parallelograms are 7cm and 3cm respectively.

Hence, the area of parallelogram ABCD = base \times height

$$\begin{aligned} &= AB \times DG \\ &= 7 \times 3 \text{ cm}^2 \\ &= 21 \text{ cm}^2 \end{aligned}$$

Again, the area of parallelogram = AB \times DG

$$\begin{aligned} &= 7 \times 3 \text{ cm}^2 \\ &= 21 \text{ cm}^2 \end{aligned}$$



Hence, the area of parallelogram ABCD = area of parallelogram ABEF.

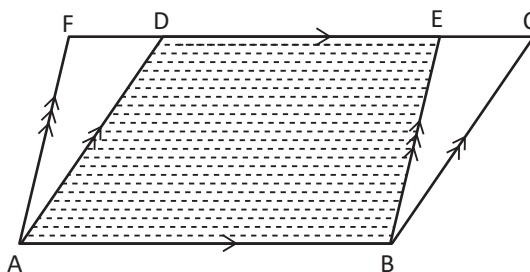
The area of parallelograms standing on the same base and lying between the same parallel lines are equal. Area of parallelogram ABCD = Area of parallelogram ABEF.

Theoretical Proof

Theorem 1

Parallelograms standing on the same base and lying between the same parallel lines are equal in area.

Given: parallelograms ABCD and ABEF are standing on the same base AB and lying between the same parallel lines AB and CF.s



To be proved: Area of parallelogram ABCD = area of parallelogram ABEF.

Proof

	Statements	Reasons
1.	In $\triangle ADF$ and $\triangle BCE$ i) $\angle ADF = \angle BCE$ (A) ii) $\angle AFD = \angle BEC$ (A) iii) $AD = BC$ (S))	1. (i) Being opposite sides of parallelogram ABCD. (ii) Corresponding angle being $AB \parallel BC$. (iii) Corresponding $AF \parallel BE$
2.	$\triangle ADF \cong \triangle BCE$	AAS axiom.
3.	Area of $\triangle ADF$ = Area of $\triangle BCE$	Congruent triangles are equal in area.
4.	Area of $\triangle ADF$ + area of trapezium ABED = area of $\triangle BCE$ + area of trapezium ABED	Adding the same trapezium on both sides.
5.	Area of parallelogram ABCD = Area of parallelogram ABEF.	From statement (4) (Whole part axiom)
Parallelograms standing on the same base and lying between the same parallel lines are equal.		

Proved.

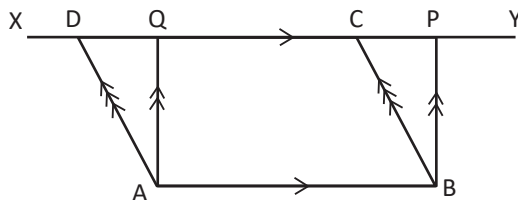
Example 1

In the given figure, $AB \parallel XY$. $ABPQ$ is a rectangle and $ABCD$ is a parallelogram. Prove that the area of parallelogram $ABCD$ = area of rectangle $ABPQ$.

Solution

Parallelogram $ABCD$ and rectangle $ABPQ$ are standing on the same base AB and lying between the same parallel lines $XY \parallel AB$.

To be proved: area of parallelogram $ABCD$ = area of rectangle $ABPQ$.



Proof

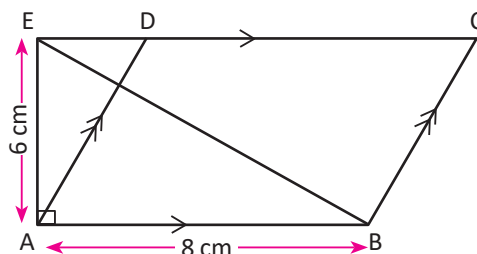
	Statements		Reasons
1.	Area of parallelogram ABCD = $AB \times AQ$	1.	Area of parallelogram = base \times height.
2.	Area of rectangle ABPQ = $AB \times AQ$	2.	Area of rectangle = length \times breadth
3.	Area of parallelogram ABCD = area of rectangle ABPQ.	3.	From statements (1) and (2)

Proved.

B. The relation of triangle and parallelogram standing on the same base and lying between the same parallel lines

Activity 3

Find the area of $\triangle ABE$ and parallelogram ABCD from the given figure. Discuss with your friends and find the relation between them.



Here,

$\triangle ABE$ is a right angled triangle with the base $AB = 8$ cm and the height $AE = 6$ cm.

Hence, the area of $\triangle ABE = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$.

Again, the base of parallelogram ABCD, $AB = 8$ cm and the height = $(AE) = 6$ cm

Hence, the area of parallelogram ABCD = base \times height = $8 \text{ cm} \times 6 \text{ cm} = 48 \text{ cm}^2$

Therefore, the area of $\triangle ABE = \frac{1}{2}$ area of parallelogram ABCD.

or, Area of parallelogram ABCD = $2 \times$ area of $\triangle ABC$

The area of a triangle is half of the area of a parallelogram standing on the same base and lying between the same parallels. Area of $\triangle ABE = \frac{1}{2}$ area of parallelogram ABCD.

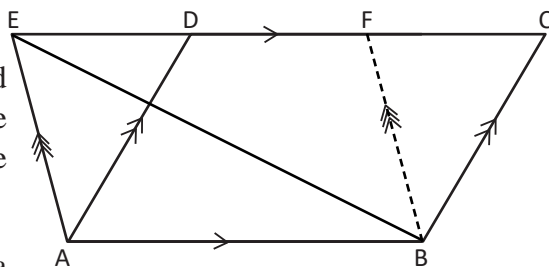
Theoretical Proof

Theorem 2

The area of a triangle is half of the area of a parallelogram standing on the same base and lying between the same parallels.

Given: Parallelogram ABCD and triangle ABE are standing on the same base AB and lying between the same parallels AB and EC.

To be proved: area of $\triangle ABC = \frac{1}{2}$ area of parallelogram ABCD



Construction: Draw $AE \parallel BF$. Now, ABFE is a parallelogram.

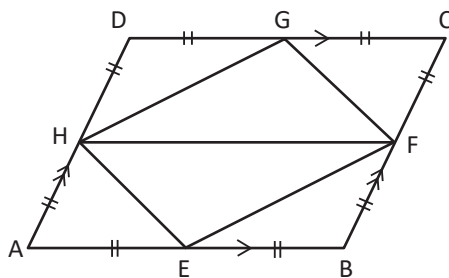
Proof:

	Statements		Reasons
1.	Area of parallelogram ABFE = area of parallelogram ABCD	1.	Both being on the same base AB and between the same parallels AB and CE.
2.	Area of $\triangle ABE = \frac{1}{2}$ area of parallelogram ABFE	2.	Diagonal EB bisects the parallelogram ABFE.
3.	Area of $\triangle ABE = \frac{1}{2}$ area of parallelogram ABCD	3.	From statements (1) and (2).
Hence, the area of a triangle is half the area of a parallelogram standing on the same base and lying between the same parallels.			

Proved.

Example 2

If E, F, G and H are the mid points of sides of the parallelogram ABCD. Then prove that the area of parallelogram EFGH = $\frac{1}{2}$ area of parallelogram ABCD.



Solution

Given: E, F, G and H are the mid points of sides AB, BC, CD and DA respectively of parallelogram ABCD given along sides.

To be proved: area of parallelogram EFGH = $\frac{1}{2}$ area of parallelogram ABCD.

Proof

	Statements	Reasons
1.	$AH = \frac{1}{2} AD$	H is the midpoint of AD
2.	$BF = \frac{1}{2} BC$	F is the midpoint of BC
3.	$AD = BC$	Opposite sides of parallelogram ABCD
4.	Hence, $AH = BF$ and $AH \parallel BF$	From statements (1), (2) and (3), $AD \parallel BC$
5.	Hence, ABFH being a parallelogram	By $AH \parallel BF$ and $AH = BF$
6.	Similarly, CDHF being a parallelogram	By $CF \parallel DH$ and $CF = DH$
7.	Now, the area of $\triangle HEF = \frac{1}{2}$ area of parallelogram ABFH	Both being on the same base HF and between the same parallel lines HF and AB.
8.	Again, the area of $\triangle HGF = \frac{1}{2}$ area of parallelogram CDHF	Both being on the same base HF and between the same parallel lines HF and CD.
9.	Area of $\triangle HEF$ + area of $\triangle HGF = \frac{1}{2}$ area of parallelogram ABFH + $\frac{1}{2}$ area of parallelogram CDHF or, Area of parallelogram HEFG = $\frac{1}{2}$ (area of parallelogram ABFH + area of parallelogram CDHF) \therefore or, Area of parallelogram HEFG = $\frac{1}{2}$ area of parallelogram ABCD	From statements (7) and (8) using addition law.
		<i>Proved.</i>

C. The relation of triangles standing on the same base and lying between same parallel

Activity 3

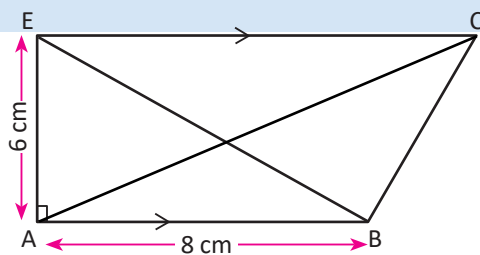
Find the area of $\triangle ABE$ and $\triangle ABC$ based on the given figure. Compare the area of triangles and find the relation.

Here, $\triangle ABE$ is a right angled triangle with the base $AB = 8$ cm and height $AE = 6$ cm (by $AB \parallel EC$)

$$\text{Area of } \triangle ABE = \frac{1}{2} \times \text{perpendicular} \times \text{base} = \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm} = 24 \text{ cm}^2$$

$$\text{Therefore, area of } \triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$$

Hence, the area of $\triangle ABE$ = the area of $\triangle ABC$



The area of triangles standing on the same base and lying between the same parallel lines are equal.

Theoretical Proof

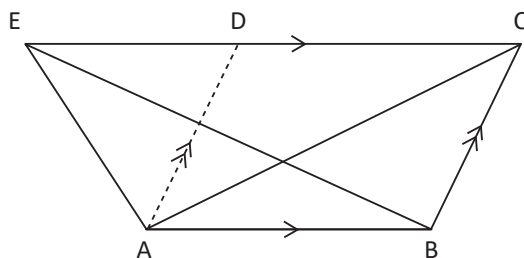
Theorem 3

The area of triangles standing on the same base and lying between the same parallel are equal.

Given: Triangles ABE and ABC are standing on base AB and lying between parallel lines EC and AB .

To be prove: The area of $\triangle ABE$ = the area of $\triangle ABC$

Construction: Draw $AD \parallel BC$. So that, $ABCD$ is a parallelogram.



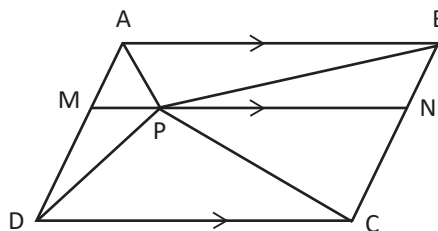
Proof

	Statements	Reasons
1.	Area of $\triangle ABC = \frac{1}{2}$ area of parallelogram ABCD	Diagonal AC bisects parallelogram ABCD.
2.	Area of $\triangle ABE = \frac{1}{2}$ area of parallelogram ABCD	$\triangle ABE$ and parallelogram ABCD are standing on the same base and lying between the same parallels.
3.	Area of $\triangle ABC =$ area of $\triangle ABE$	From statements (1) and (2) above
The area of triangles standing on same base and lying on the same parallel are equal.		

Proved.

Example 3

In the given figure, ABCD is a parallelogram. Prove that area of $\triangle APB$ + area of $\triangle PCD = \frac{1}{2}$ area of parallelogram ABCD.



Solution

Given: ACBD is a parallelogram.

To be Prove: Area of $\triangle APB$ + area of $\triangle PCD = \frac{1}{2}$ area of parallelogram ABCD.

Construction: Draw $MN \parallel CD$, where MN passes through the point P.

Here, ABNM and MNCD are parallelograms.

Proof

	Statements	Reasons
1.	Area of $\triangle APB = \frac{1}{2}$ area of parallelogram ABNM	Triangle and parallelogram standing on the same base AB and lying between AB//MN
2.	Area of $\triangle PCD = \frac{1}{2}$ area of parallelogram MNCD	Triangle and parallelogram standing on the same base AB and lying between CD//MN
3.	Area of $\triangle APB$ + area of $\triangle PCD = \frac{1}{2}$ area of parallelogram ABNM + $\frac{1}{2}$ area of parallelogram MNCD	Adding statements (1) and (2)
4.	Area of $\triangle APB$ + area of $\triangle PCD = \frac{1}{2}$ area of parallelogram ABCD	From statement (3) and by whole part axiom.

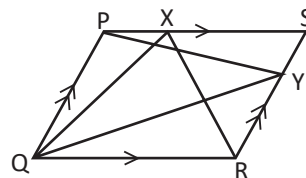
Proved.

Example 4

In the figure, PQRS is a parallelogram. In which X and Y are any two points on PS and SR respectively. Prove that area of $\triangle PQY = \text{area of } \triangle QRX$.

Solution

Given: PQRS is a parallelogram. X and Y are any two points on PS and SR respectively. PQY and QXR are two triangles.



To be Prove: area of $\triangle PQY = \text{area of } \triangle QRX$.

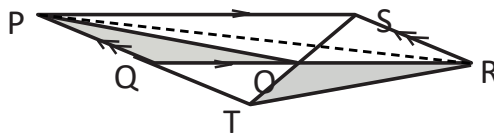
Proof

Statements		Reasons
1.	Area of $\triangle PQY = \frac{1}{2}$ area of parallelogram PQRS	The triangle PQY and the parallelogram PQRS standing on the same base PQ and between the same parallels PQ and SR.
2.	Area of $\triangle QRX = \frac{1}{2}$ area of parallelogram PQRS	$\triangle QRX$ and parallelogram PQRS are standing on the same base QR and between the same parallels QR and PS.
3.	Area of $\triangle PQY = \text{area of } \triangle QRX$	From statements (1) and (2) above.

Proved.

Example 5

In the given figure, PQRS is a parallelogram in which S is joined with any point O of side QR. SO and PQ are produced upto the point 'T'. Then, prove that the area of the $\triangle PQO = \text{the area of } \triangle RTO$.



Solution

Given: The point O is on the side QR in the parallelogram PQRS. PQ and SO are produced up to the point T.

To be Prove: area of $\triangle PQO = \text{area of } \triangle RTO$

Construction: Draw a diagonal PR in parallelogram PQRS.

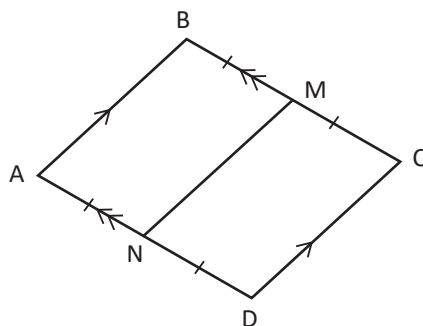
Proof

Statements		Reasons
1.	Area of $\triangle PRS$ = area of $\triangle PQR$	Diagonal PR bisects parallelogram PQRS.
2.	Area of $\triangle PRS$ = area of $\triangle TRS$	Standing on the same base SR and lying between SR//QR.
3.	Area of $\triangle PQR$ = area of $\triangle TRS$	From statements (1) and (2) above.
4.	Area of $\triangle POR$ = area of $\triangle SOR$	Triangles standing on the same base OR and lying between PS//QR.
5.	$\triangle PQR - \triangle POR = \triangle TRS - \triangle SOR$	(Equal axiom.) subtracting statement (4) from statement (3)
6.	Area of $\triangle PQO$ = area of $\triangle RTO$	Remainder axiom, from statement (5)

Proved.

Example 6

ABCD is a parallelogram. M and N are the mid points of BC and AD respectively. Prove that MN divides parallelogram ABCD into two equal parallelograms.



Solution

Given: ABCD is a parallelogram. M and N are the midpoints of BC and AD respectively.

To be Prove: area of parallelogram ABMN = area of parallelogram CDNM.

Proof

	Statements	Reasons
1.	$AD = BC$ and $AD \parallel BC$	Being opposite sides of a parallelogram.
2.	Again, $BM = MC$ and $AN = ND$	By M and N are midpoints of BC and AD respectively.
3.	$AN = BM$ and $AN \parallel BM$	From Statements (1) and (2).
4.	ABMN is a parallelogram	By $AN = BM$ and $AN \parallel BM$
5.	Again, $DN = CM$ and $DN \parallel CM$	Same as above
6.	CDNM is a parallelogram	By $DN = CM$ and $DN \parallel CM$
7.	area of parallelogram ABMN = area of parallelogram CDNM	Parallelogram standing on equal bases $AN = ND$ and lying between $BC \parallel AD$.

Proved.

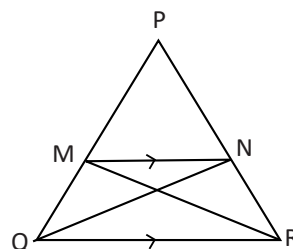
Example 7

In the given figure, $MN \parallel QR$, prove that area of $\triangle PQN$ = area of $\triangle PRM$.

Solution

Given: In triangle PQR, $MN \parallel QR$.

To be prove: area of $\triangle PQN$ = area of $\triangle PRM$.



Proof

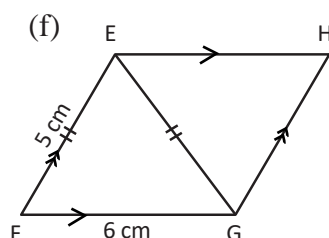
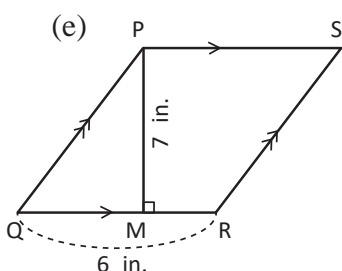
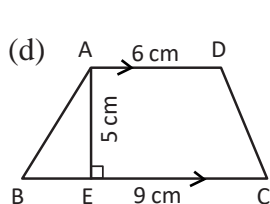
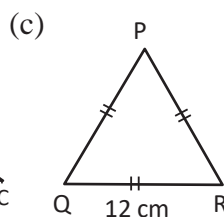
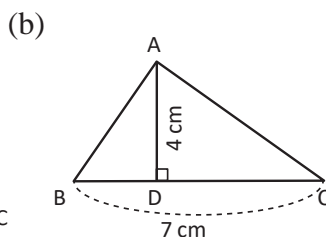
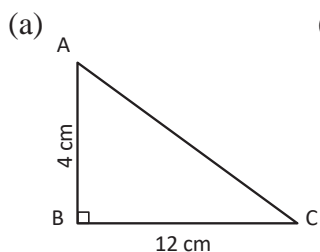
	Statements	Reasons
1.	Area of $\triangle MNQ$ = area of $\triangle MNR$	Triangles standing on the same base MN and lying between $QR \parallel MN$
2.	$\triangle PMN + \triangle MNQ = \triangle PMN + \triangle MNR$	Adding $\triangle PMN$ on both sides of statement (1)
3.	Area of $\triangle PNQ$ = area of $\triangle PRM$	From statement (2) [by whole part axiom]

Proved.

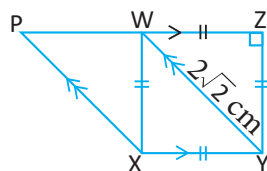
Exercise 10.1

A) Solve:

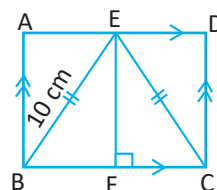
1. Find the area of given figures.



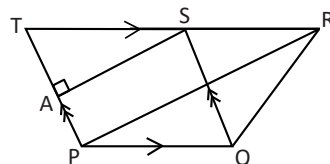
2. (a) In the given figure, WXYZ is a square. If $WY = 2\sqrt{2}$ cm then find the area of parallelogram PXYW.



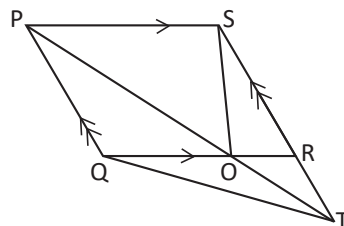
(b) The given figure, if $BE = EC$, $EF \perp BC$, $BE = 10$ cm and $AD = 16$ cm, find the area of parallelogram ABCD.



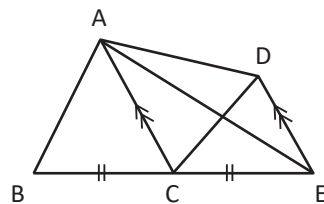
(c) The given figure, PQST is a parallelogram. If $SA \perp TP$, $SA = 8$ cm and area of $\triangle PQR = 64$ cm² then find the length of TP.



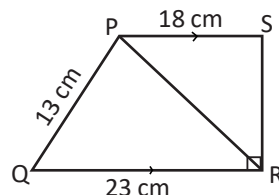
(d) In the given figure, PQRS is a parallelogram. Taking any point O on QR, join PO produced PO and SR to the point T. Q and T are joined. Proved that the area of $\triangle QOT =$ area of $\triangle ROS$.



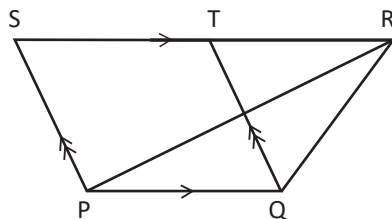
- (e) In the given figure, $AC \parallel DE$ and $BC = EC$. If, area of $\triangle ACE$ is 24cm^2 , then find the area of quadrilateral $ABCD$.



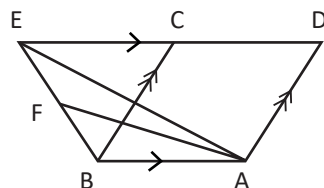
3. (a) In trapezium PQRS, $PS \parallel QR$, $PQ = 13\text{cm}$, $PS = 18\text{cm}$, $QR = 23\text{cm}$ and $SR \perp QR$, then find the area of $\triangle PSR$.



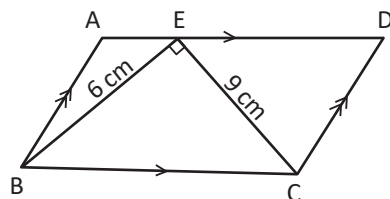
- (b) In the adjoining figure, $PQ \parallel SR$, $PS \parallel QT$, area of the trapezium PQRS is 95cm^2 and area of $\triangle QRT$ is 15cm^2 . Find the area of $\triangle RPQ$.



- (c) In the adjoining figure, $DE \parallel AB$, $AD \parallel BC$ and F is the midpoint of BE. If the area of $\triangle AFE$ is 12cm^2 then find the area of parallelogram ABCD.

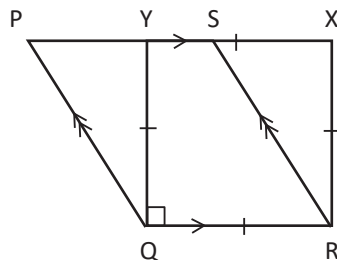


- (d) In the given parallelogram, ABCD, $\angle BEC = 90^\circ$, $BE = 6\text{cm}$, $CE = 9\text{cm}$, then find the area of parallelogram ABCD.

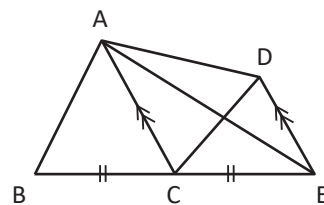


4. Prove that:

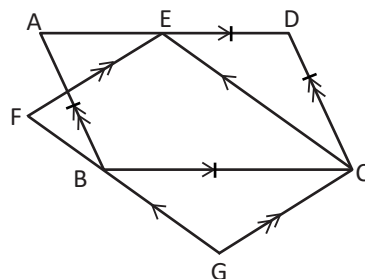
- (a) In the given figure, PQRS is a parallelogram and QRXY is a square. Prove that the area of the parallelogram PQRS and the area of square QRXY are equal.



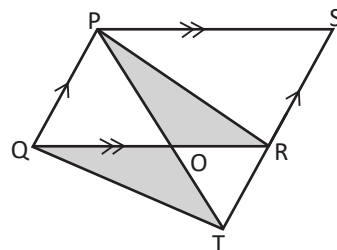
- (b) In the given figure, ABCD is a quadrilateral. Drawn PE so that $AC \parallel DE$. Prove that the area of quadrilateral ABCD = area of $\triangle ABE$.



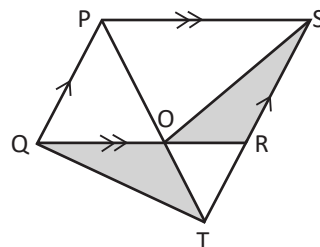
- (c) In the given figure, ABCD and EFGC are parallelograms. Prove that the area of parallelogram ABCD = area of parallelogram EFGC.



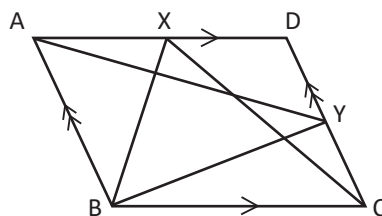
- (d) In the given figure, PQRS is a parallelogram and SR is extended upto T. Join P and T where PT intersects QR at O. Prove that area of $\triangle POR$ = area of $\triangle QOT$.



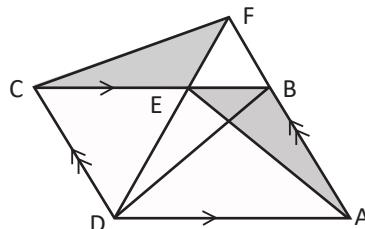
- (e) In the given figure, PQRS is a parallelogram in which SR is extended upto T and PT and ST are joined where PT meets QR at O. Prove that the area of $\triangle QOT$ = area of $\triangle ROS$.



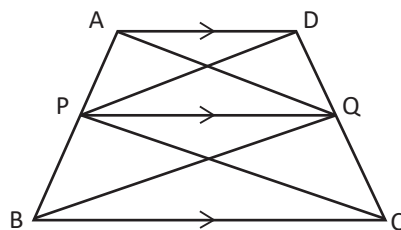
- (f) In the adjoining figure, ABCD is a parallelogram in which X and Y are any two points in which triangles XBC and YAB are made. Prove that area of $\triangle ABY$ = area of $\triangle ABX$ + area of $\triangle CDX$.



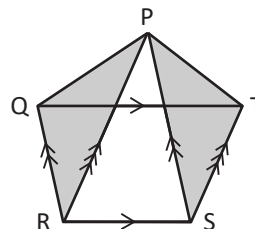
- (g) In the adjoining figure, ABCD is a parallelogram where E is any point on BC, DE and AB are produced upto F. Join C and F. Prove that area of $\triangle CEF$ = area of $\triangle ABE$.



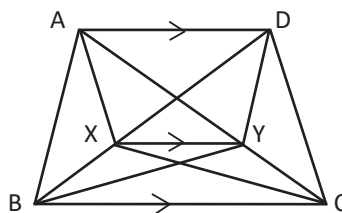
- (h) In the given figure, ABCD is a trapezium in which $AD \parallel PQ \parallel BC$. Prove that the area of $\triangle AQB$ and the area of $\triangle DPC$ are equal.



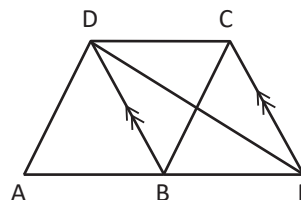
- (i) In the given figure, $QT \parallel RS$, $PR \parallel TS$ and $PS \parallel QR$. Prove that the area of $\triangle PQR$ = the area of $\triangle PTS$.



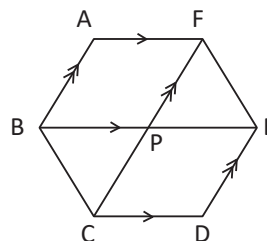
- (j) In the given figure, ABCD is a trapezium in which Y and X are any points on AC and BD respectively where $AD \parallel XY \parallel BC$. Prove that the area of $\triangle AXC$ and the area of $\triangle BYD$ are equal.



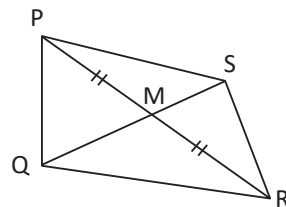
- (k) In the adjoining figure, ABCD is a quadrilateral in which $DB \parallel CE$. Prove that the area of $\triangle ADE$ is equal to the area of quadrilateral ABCD.



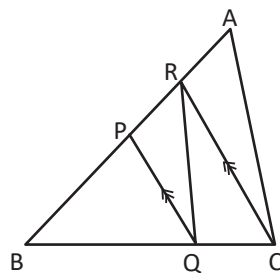
- (l) In the given figure, ABCDEF is a hexagon in which $AF \parallel BE \parallel CD$ and $AB \parallel CF \parallel DE$. If the area of parallelograms ABPF and CDEP are equal, then prove that $EF \parallel BC$.



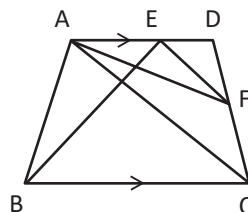
- (m) In the given figure, the diagonal QS of the quadrilateral PQRS bisects another diagonal PR. Prove that area of $\triangle PQS$ is equal to half the area of the quadrilateral PQRS.



- (n) In the given figure, P is midpoint of AB and Q is any point in BC. If $BC \parallel PQ$ then prove that area of $\triangle BQR = \frac{1}{2} \Delta ABC$ area of $\triangle ABC$.



- (o) In the given figure, $AD \parallel BC$. If area of $\triangle ABE$ and $\triangle ACF$ are equal then prove that $EF \parallel AC$.



Project work

Prepare the chart using different colors on graph paper or a square grid, Show the following relationships, which are standing on the same base and lying between the same parallel lines

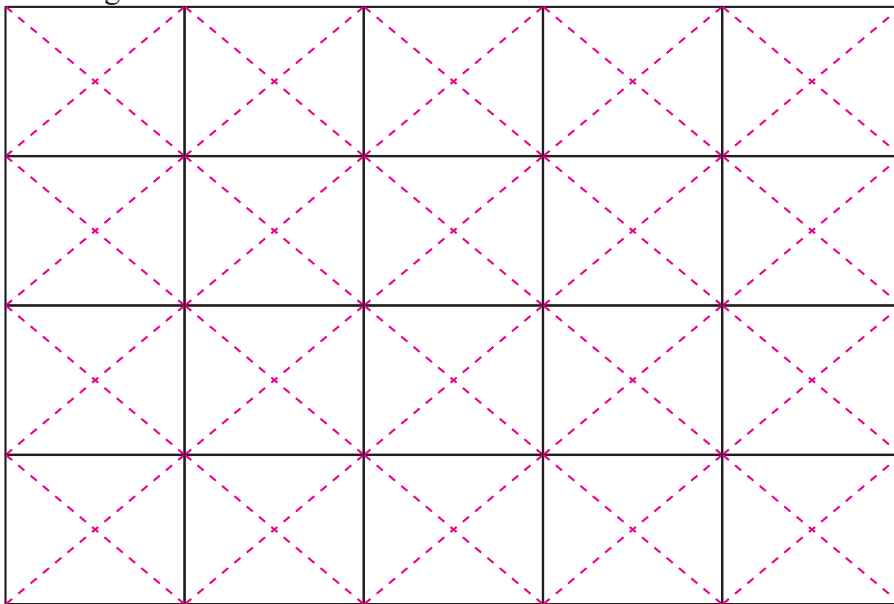
- Relation of parallelograms.
- Relation of parallelogram and triangle, and
- Relation of triangles.

Answers

- 24 cm^2
 - 21 cm^2
 - $36\sqrt{3} \text{ cm}^2$
 - 37.5 cm^2
 - 42 sq.inch
 - 24 cm^2
- 4 cm^2
 - 96 cm^2
 - 16 cm^2
 - Show it to the teacher.
 - 48 cm^2
- 138 cm^2
 - 40 cm^2
 - 48 cm^2
 - 54 cm^2
- Show it to the teacher.

11.0 Review

Draw a figure as given below in a square grid. Find and shade the following condition in the figure.



- The parallelogram standing on the same base and lying between the same parallels are equal in area.
- A triangle which is equal to one half of the area of a parallelogram standing on the same base and lying between the same parallels.
- A pair of triangles standing on the same base and lying between same parallels are equal in area.

11.1. Construction of triangle and quadrilaterals with equal areas

Steps:

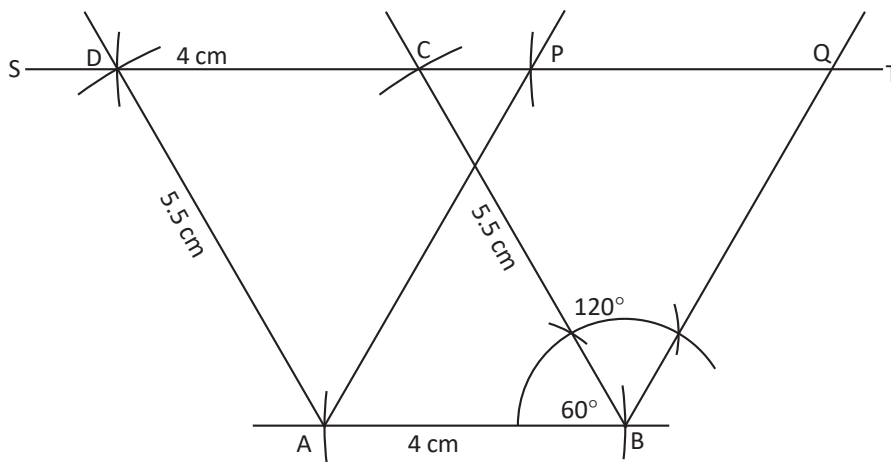
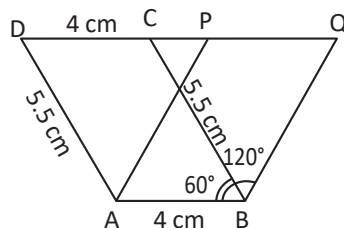
- Draw a rough sketch using a ruler and a pencil according to the given condition.
- Place the values on the rough sketch.
- Using a compass and a ruler, construct and name correct figures according to the given condition and data following the given steps.

(A) Construction of Parallelograms equal in area

Activity 1

Construct a parallelogram ABCD having $AB = 4\text{cm}$, $BC = 5.5\text{cm}$ and $\angle ABC = 60^\circ$. How can we construct a parallelogram whose area is equal to the area of parallelogram ABCD?

- (a) Parallelogram ABQP having an angle 120°
- (b) Parallelogram ABQP having one side 6 cm
- (a) **Parallelogram ABQP having an angle 120°**
 - i. Draw a rough sketch of parallelogram ABCD according to the given data.
 - ii. Draw $AB = 4\text{cm}$
 - iii. Draw an angle 60° at the point B. Again, take an arc of radius 5.5cm from the point B cutting the line from 60° at B and name the point C.
 - iv. Now, let us give name the point D where 5.5cm arc from A and 4cm from C are intersected.
 - v. Now, parallelogram ABCD is prepared after joining D with C and A with D.
 - vi. Extend DC up to T. Construct an angle 120° at point B and name the point Q where the line ST meets.
 - vii. Take an arc equal to AB and cut QS from Q and name the point P. Join A with P.



The area of parallelogram ABQP is equal to the parallelogram ABCD. How? Discuss with your friend.

Parallelograms ABCD and ABQP are standing on the same base and lying between the same parallels AB//ST are equal in area.

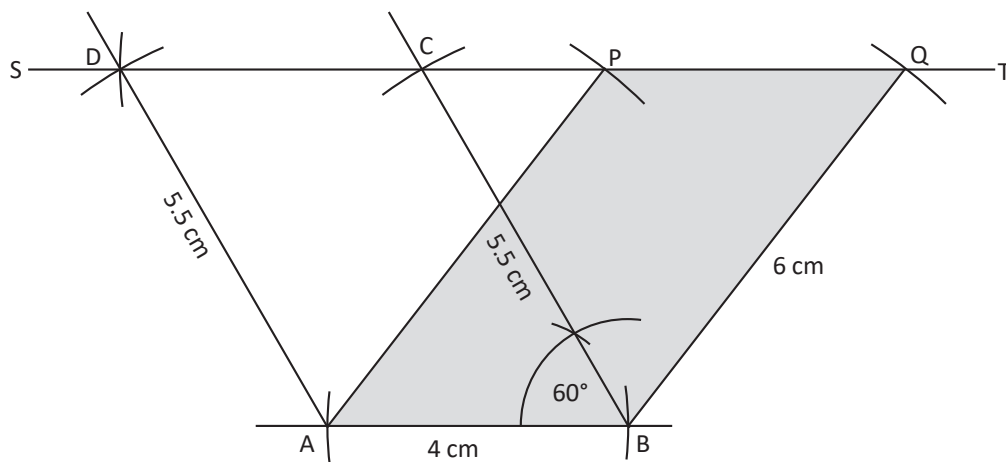
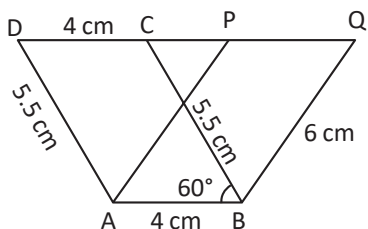
(b) Parallelogram ABQP having one side 6 cm

i. Let's draw a rough sketch of a parallelogram ABCD and place the value.

ii. Let's construct a parallelogram ABCD in which $AB = 4\text{ cm}$, $BC = 5.5\text{ cm}$ and $\angle ABC = 60^\circ$. Extend CD upto ST.

iii. Take an arc of radius 6 cm from point A on the line ST cutting it at a point and name it P. Again, take an arc of radius AP, from the point B on the line ST cutting it at a point and name it Q.

iv. Join A and P, B and Q by using a ruler and a pencil.



Why are the area of parallelograms ABQP and ABCD equal? Discuss with friends.

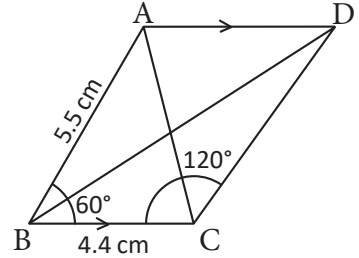
Parallelograms ABCD and ABQP standing on the same base AB and lying between the same parallels AB//ST are equal in area.

(B) Construction of triangles equal in area

Activity 2

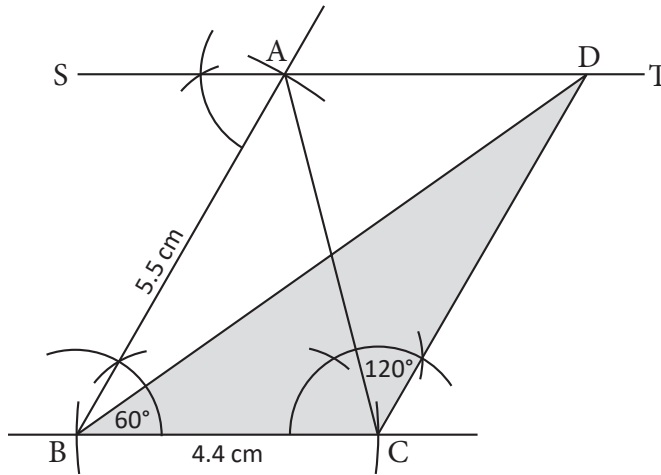
How can we construct a triangle which is equal in area with the triangle ABC having $\angle ABC = 60^\circ$, $BC = 4.4$ cm and $AB = 5$ cm?

- a) $\triangle ABC$ having an angle 120° .
- b) $\triangle DBC$ having a side 6.2 cm.



a) $\triangle ABC$ having an angle 120° .

- i. Let's draw a rough sketch of a triangle and place the values..
- ii. Construct a $\triangle ABC$ having $BC = 4.4$ cm, $\angle ABC = 60^\circ$ and $AB = 5.5$ cm.
- iii. Draw $BC \parallel ST$ from A making $\angle ABC = \angle BAS$.
- iv. Join D and C.

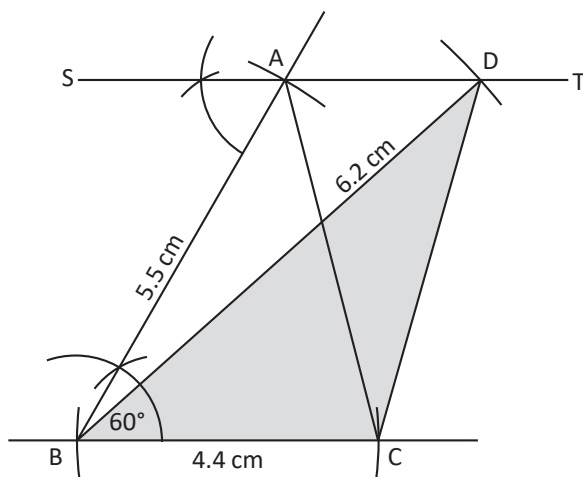
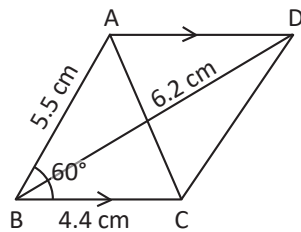


The area of the $\triangle DBC$ is equal to the area of the $\triangle ABC$. How? Discuss with your friends.

The triangles standing on the same base and lying between the same parallels $BC \parallel ST$ are equal in area.

b) A triangle DBC having one side 6.2cm

- Let's draw a rough sketch of $\triangle ABC$ as and place the values.
- Construct a triangle ABC in which $BC = 4.4\text{cm}$, $\angle ABC = 60^\circ$ and $AB = 5.5\text{cm}$.
- Draw $BC \parallel ST$ from A making $\angle ABC = \angle BAS$.
- Now, take an arc of radius 6.2cm and center B cutting ST at a point and name it D.
- Join D and C.



The area of $\triangle BCD$ is equal to the area of $\triangle ABC$. How? Discuss with your friends.

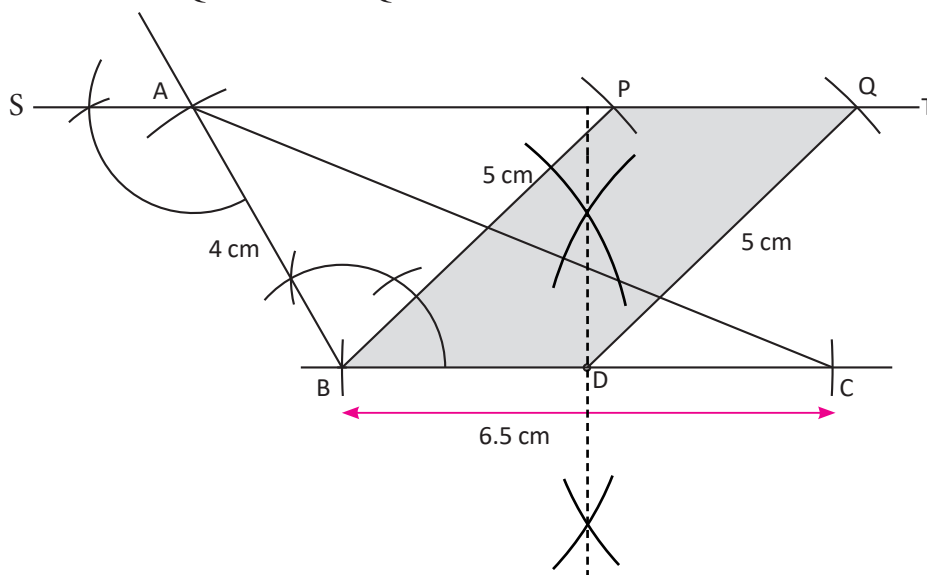
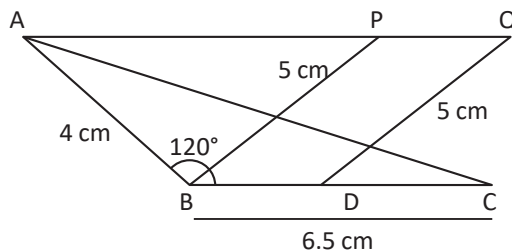
The triangle standing on the same base and lying between the same parallels $BC \parallel ST$ are equal in area.

(C) Construction of triangles and parallelograms equal in area

Activity 3

Construct a triangle ABC in which $AB = 4\text{cm}$, $BC = 6.5\text{cm}$ and $\angle ABC = 120^\circ$. How can we construct a parallelogram having a side $PB = 5\text{cm}$ and equal to the area of $\triangle ABC$?

- Let's draw a rough sketch of a triangle ABC and place the values.
- Construct a $\triangle ABC$ in which $AB = 4\text{cm}$, $BC = 6.5\text{cm}$ and $\angle ABC = 120^\circ$.
- Draw $ST \parallel BC$ from 'A' making $\angle ABC = \angle BAS$.
- Bisect BC and mark the midpoint D. Now, take an arc of radius 5cm from the point B on the line ST cutting it at a point and name it P.
- Take an arc of radius BP, from the point D on the line PT cutting it at a point and name it Q. Join D and Q.



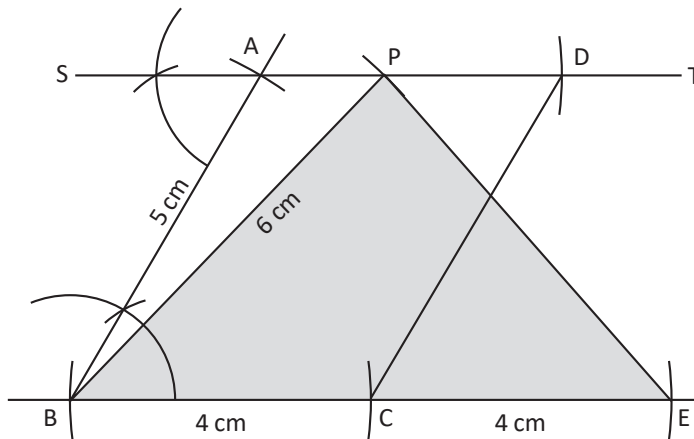
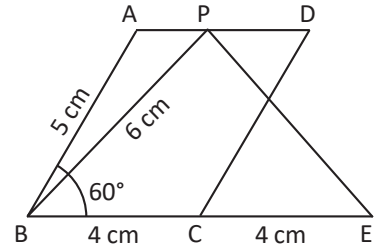
The area of the parallelogram PBD is Q equal to the area of the $\triangle ABC$. How? Discuss with your friend.

The area of a triangle is half of the area of the parallelogram standing on the same base and lying between the same parallels. If the parallelogram is constructed on the half of the base of triangle, then the area of the triangle is equal to the area of parallelogram.

Activity 4

Construct a parallelogram ABCD in which $AB = 5\text{cm}$, $BC = 4\text{cm}$ and $\angle ABC = 60^\circ$. How can construct a triangle PBE with a side $PB = 6\text{cm}$ and equal area to the parallelogram ABCD?

- Draw a rough sketch of parallelogram and place the values.
- Construct a parallelogram in which $AB = 5\text{cm}$, $BC = 4\text{cm}$ and $\angle ABC = 60^\circ$.
- Produce BC to E making $BC = CE$. Produce AD up to ST.
- Take an arc of radius 6.2cm from the point B on the line ST cutting it at a point and name it P.
- Join B and P, also P and E.



The area of the triangle BPE is equal to the area of the parallelogram ABCD. How? Discuss with your friends.

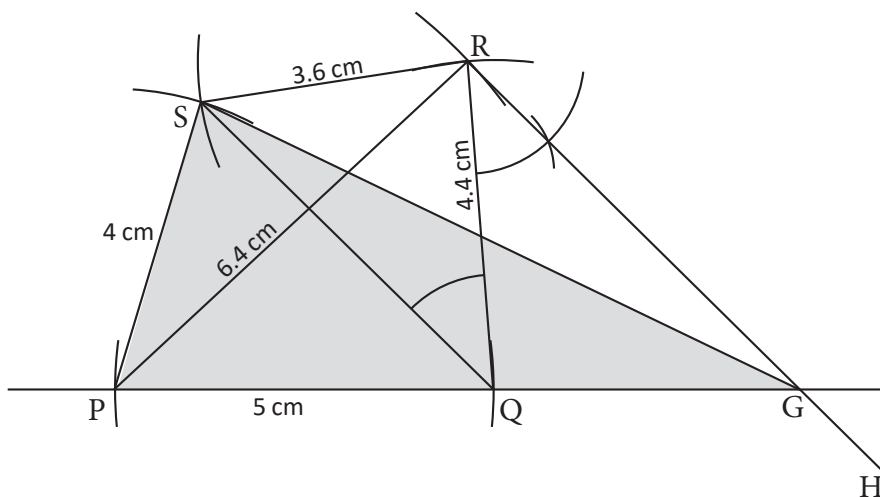
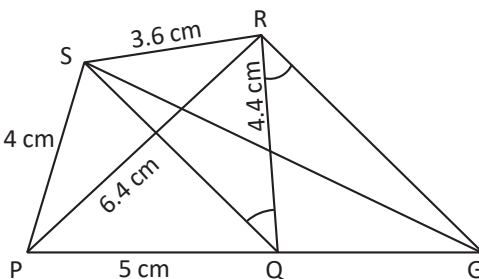
The area of a parallelogram is double of the triangle standing on the same base and lying between the same parallels. If the triangle base is double of the base of the parallelogram, then their area are equal.

(D) Construction of a triangle and a quadrilateral equal in area

Activity 5

Construct a quadrilateral PQRS in which $PQ = 5\text{cm}$, $PS = 4\text{cm}$, $QR = 4.4\text{cm}$, $RS = 3.6\text{cm}$ and diagonal $PR = 6.4\text{cm}$. How to construct a triangle PSG whose area is equal to the quadrilateral PQRS?

- Draw a rough sketch of quadrilateral PQRS according to the given data.
- Construct a quadrilateral PQRS in which $PQ = 5\text{cm}$, $PS = 4\text{cm}$, $QR = 4.4\text{cm}$, $RS = 3.6\text{cm}$ and diagonal $PR = 6.4\text{cm}$.
- Draw a diagonal SQ.
- Draw $SQ \parallel RH$ from R making $\angle SQR = \angle QRH$. Now, extend PQ and name the point H where the line meets RH. Join S and G.



The area of a triangle PSG is equal to the area of the quadrilateral PQRS. How? Discuss with your friends.

Area of $\triangle SQR$ = area of $\triangle SQG$,

Adding $\triangle PSQ$ on both sides [\because The triangles on the same base and same parallels $RH \parallel SQ$.]

Area of $(\triangle SQR + \triangle PSQ)$ = area of $(\triangle SQG + \triangle PSQ)$

\therefore area of quadrilateral PQRS = area of $\triangle PSG$.

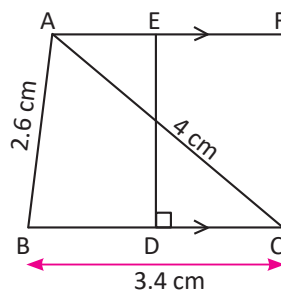
Example 1

Construct a triangle ABC in which $AB = 2.6\text{cm}$, $BC = 3.4\text{cm}$ and $CA = 4\text{cm}$.
Again construct a rectangle whose area is equal to $\triangle ABC$.

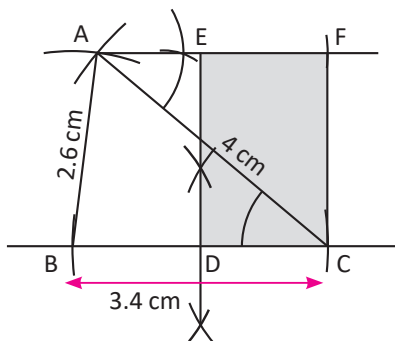
Solution

Here, $\triangle ABC$ having $AB = 2.6\text{cm}$, $BC = 3.4\text{cm}$ and $CA = 4\text{cm}$.

Rough sketch



The construction of required CDEF whose area is equal to the area of $\triangle ABC$ is below.



Hence, CDEF is the required rectangle whose area is equal to the area of $\triangle ABC$.

Exercise 11

1.
 - (a) Construct a parallelogram whose one angle is 45° and the area is equal to the parallelogram with $AB = 4$ cm, $AD = 6$ cm and $\angle BAD = 60^\circ$.
 - (b) Construct a parallelogram whose one angle is 75° and the area is equal to the parallelogram with $AB = 5$ cm, $AD = 6$ cm and diagonal $BD = 6$ cm.
 - (c) Construct a parallelogram whose diagonal is 7.2 cm and area is equal to the parallelogram whose diagonals are 6 cm and 4.8 cm and the angle between its diagonals is 30° .
 - (d) Construct the parallelogram $ABXY$ equal in area to the parallelogram $ABCD$ with $AB = 5$ cm, $AD = 6$ cm and diagonal $BD = 8$ cm.
 - (e) Construct a parallelogram whose one angle is 30° which equals area of a parallelogram $PQRS$ having $PQ = 4.2$ cm, $QR = 6$ cm and $\angle PQR = 60^\circ$.
2.
 - (a) Construct a triangle ABC in which $BC = 6.4$ cm, $AB = 5.6$ cm and $AC = 6$ cm. Then, construct another triangle having a side 7 cm and equal in area to the triangle ABC .
 - (b) Construct a $\triangle LMN$ in which $LM = 4.3$ cm, $\angle NLM = 30^\circ$ and $\angle LMN = 45^\circ$, then construct another triangle OLM having $OM = 7.5$ cm and which is equal in area to the $\triangle LMN$.
 - (c) Construct a triangle PQR in which $PQ = 4.5$ cm, $QR = 7$ cm and $PR = 6$ cm. Then, construct another triangle having a side 8 cm and which is equal in area to the triangle PQR .
 - (d) Construct triangle equal in area to the triangle ABC with $AB = 4.2$ cm, $BC = 5.2$ cm and $CA = 3.5$ cm.
 - (e) Construct a triangle MBC with one side 8 cm and whose area is equal to the area of a triangle ABC having $AB = 7.2$ cm, $BC = 5.9$ cm and $CA = 6.1$ cm.
3.
 - (a) Construct a triangle PQR in which $PQ = 6.5$ cm, $QR = 6$ cm and $PR = 5.5$ cm then construct a parallelogram $RSTI$ having $\angle TSR = 75^\circ$ and is equal in area to the triangle PQR .
 - (b) Construct a $\triangle ABC$ in which $AC = 5$ cm, $BC = 4.8$ cm and $\angle ABC = 45^\circ$. Then, construct a parallelogram $CDEF$ having $CD = 7.5$ cm and is equal in area to the $\triangle ABC$.
 - (c) Construct a $\triangle ABC$ in which $AB = 4$ cm, $BC = 3.2$ cm and $AC = 3.5$ cm. Then, construct a parallelogram $BXYE$ having $BE = 5$ cm and is equal in area to the $\triangle ABC$.

- (d) Construct a rectangle equal in area to the triangle XYZ having $XY = 4$ cm, $YZ = 6.8$ cm and $ZX = 6.5$ cm.
 - (e) Construct a triangle PQR in which $PQ = 7.1$ cm, $\angle RPQ = 60^\circ$ and $PR = 5.7$ cm. Then, construct a parallelogram having one side 7.5 cm and is equal in area to the triangle PQR.
- 4.
- (a) Construct a parallelogram in which $AB = 6$ cm, $BC = 4.5$ cm and $\angle DAB = 60^\circ$, then construct the $\triangle AEF$ having $FE = 7.5$ cm and is equal in area to the parallelogram.
 - (b) Construct a parallelogram ABCD in which $AB = 6$ cm, $BC = 4$ cm and $\angle BAD = 45^\circ$. Then, construct a triangle APQ having $\angle APQ = 60^\circ$ and is equal in area to the parallelogram ABCD.
 - (c) Construct a triangle equal in area to the parallelogram PQRS in which $PQ = 5$ cm, diagonal $PR = 6$ cm and diagonal $QS = 8$ cm.
 - (d) Construct a triangle equal in area to the parallelogram having $EF = 5$ cm, $GF = 4$ cm and $\angle EFG = 120^\circ$.
 - (e) Construct a triangle equal in area to the parallelogram IJKL in which $IJ = 5$ cm, $IK = 6$ cm and $JL = 8$ cm.
- 5.
- (a) Construct a quadrilateral PQRS in which $PQ = QR = 5.5$ cm, $RS = SP = 4.5$ cm and $\angle SPQ = 75^\circ$. Then, construct a triangle PST equal in area to the quadrilateral PQRS.
 - (b) Construct a quadrilateral ABCD in which $AB = 4.5$ cm, $BC = 5.5$ cm, $CD = 5.7$ cm and $DA = 4.9$ cm and diagonal $BD = 5.9$ cm. Then, construct a triangle DAE equal in area to the quadrilateral ABCD.
 - (c) Construct a $\triangle QRT$ equal in area to the quadrilateral PQRS in which $PQ = 5$ cm, $QR = 7$ cm, $RS = 4.5$ cm and $SP = 5.4$ cm, $QS = 6.5$ cm.
 - (d) Construct a rhombus PQRS in which the diagonals $PR = 6$ cm and $QS = 8$ cm. Then, construct a $\triangle PSA$ which is equal in area to the rhombus PQRS.
 - (e) Construct a triangle whose one angle is 60° and area is equal to the rectangle in which length is 6 cm and breadth is 4.5 cm.

Project work

Formulate groups with five students in each group. Using bamboo sticks, wheat straw or juice pipe, all the groups should make a sample of each of the above construction and paste it on the cardboard, and present it in the classroom.

12.0 Review

Experimentally verify the given statements.

- The perpendicular drawn from the centre of a circle bisects the chord.
- The line joining the centre of a circle to the midpoint of the chord is perpendicular to the chord.
- Chords which are equidistant from the centre of a circle are equal.

12.1 Central Angle and Inscribed Angle

Activity 1

Observe the given circles. O is the centre of the given circles. Based on this, discuss the questions below.

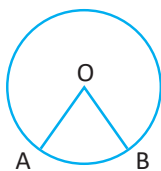


Figure (a)

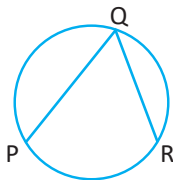


Figure (b)

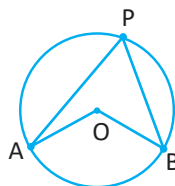


Figure (c)

- Where is the $\angle AOB$ formed? What is it called?
- Where is the $\angle PQR$ formed? What is it called?
- What is the difference between $\angle AOB$ and $\angle PQR$? Compare it.
- In figure (c), what are arc APB and AB called?

- The angle formed by two radii at the centre is called the central angle. In the given figure, $\angle AOB$ is called the central angle.
- The angle formed by joining two chords of a circle at the circumference is called the circumference (inscribed) angle. In the figure, $\angle PQR$ is called the circumference angle.
- If an arc is smaller than a semicircle, it is called a minor arc and if it is larger than a semicircle, it is called a major arc. Here, \widehat{APB} is a major arc and \widehat{AB} is a minor arc.

12.2 Relation between central angle and its corresponding arc

Activity 2

Draw circles with the centre O using a compass. Discuss the relationship between the central angles and its opposite arc.

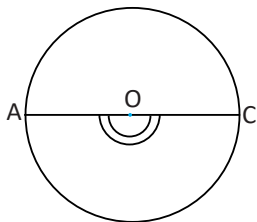


Figure 1

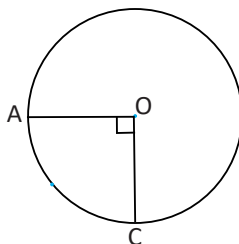


Figure 2

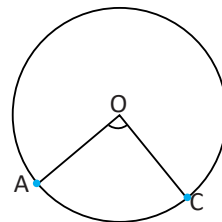


Figure 3

- (a) When the central angle is 180° , discuss what part of the circumference is the arc opposite to it.
- (b) When the central angle is one fourth of the circle, discuss what part of the circumference is the arc opposite to it.
- (c) When the central angle is one sixth of the circle, discuss what part of the circumference is the arc opposite to it.
- (d) Is there a direct relationship between the central angle and its opposite arc?

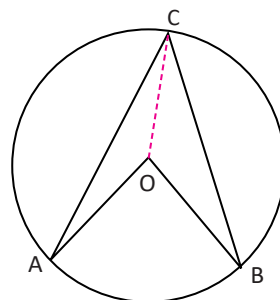
$\angle AOC \cong \widehat{AOC}$ read as arc AOC equals in degree measure. There is direct a relation between the central angle and its opposite arc. The symbol \cong is also read as equal influence.

12.3 Relation between inscribed angle and its corresponding

Activity 3

Draw a circle with centre O by using a compass and pencil where $\angle AOB$ is a central angle and $\angle ACB$ is a circumference angle. O and C are joined. Are the radii of the same circle OA, OB and OC equal?

Now, what types of triangles are triangle OAC and triangle OBC? Discuss which sides and angles of these triangles are equal.



Now, in an isosceles triangle OAC,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$\text{or, } 2\angle OCA = 180^\circ - \angle AOC \dots\dots\dots (i) \quad [\angle OAC = \angle OCA]$$

Again, in an isosceles triangle OBC,

$$\angle OCB + \angle OBC + \angle BOC = 180^\circ$$

$$\text{or, } 2\angle OCB = 180^\circ - \angle BOC \dots\dots\dots (ii) \quad [\angle OCB = \angle OBC]$$

Adding equation (i) and (ii), we get

$$2(\angle OCA + \angle OCB) = 360^\circ - (\angle AOC + \angle BOC)$$

$$\text{or, } 2\angle ACB = 360^\circ - \text{Reflex } \angle AOB$$

$$\text{or, } 2\angle ACB = \angle AOB$$

$$\text{or, } 2\angle ACB \cong \widehat{AB}$$

The relationship between the double the angle at the circumference and its opposite arc has the equal influence. It is denoted by $2\angle ACB \cong \widehat{AB}$

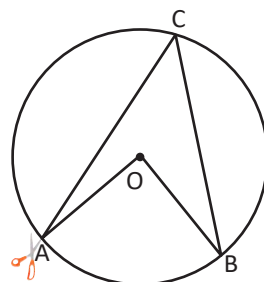
12.4 Relation between the central angle and inscribed angle

(a) Relation between the central angle and inscribed angle based on the same arc

Activity 4

(i) Using paper

Draw a central angle and inscribed angle in a chart paper as shown in the figure. Take out the central angle by cutting with a scissor. Now, fold the central angle making two equal parts and measure the inscribed angle and find the conclusion.



The central angle is double of the inscribed angle based on the same arc.
 $\angle AOB = 2\angle ACB$

(ii) Experimental verification

Draw the circles having different radii as shown in the figure.

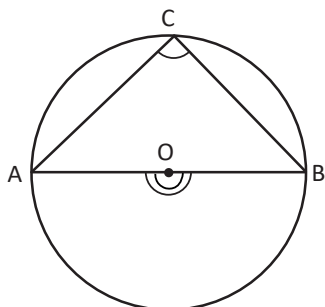


Figure 1

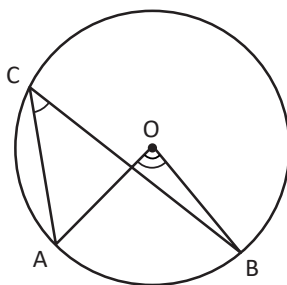


Figure 2

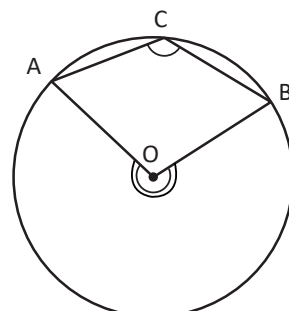


Figure 3

Measure the central angle AOB and the inscribed angle ACB based on the same arc AB of each circle and fill in the table below.

Figure No.	$\angle AOB$	$\angle ACB$	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

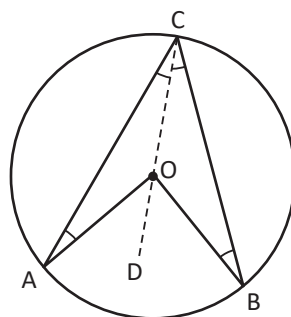
Theorem 1

The central angle is double of the inscribed angle based on the same arc.

Given: O is the centre of the circle. The central angle $\angle AOB$ and inscribed angle $\angle ACB$ are based on the same arc AB.

To be prove: $\angle AOB = 2\angle ACB$

Construction: Points C and O are joined and produced CO to the point D.



Proof

S.N.	Statement	Reason
1.	In $\triangle AOC$, (i) $\angle OAC = \angle OCA$ (ii) $\angle AOD = \angle OAC + \angle OCA$ (iii) $\angle AOD = \angle OCA + \angle OCA$ $= 2\angle OCA$	(i) $OA = OC$ (radii of the same circle), So base angles of isosceles \triangle . (ii) Exterior and opposite interior angle of $\triangle AOC$ (iii) from statement (i) and (ii)
2.	In $\triangle BOC$, $\angle BOD = 2\angle OCB$	Same as above facts and reasons.
3.	$\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$	From statements 1(iii) and (2) above.
4.	$\therefore \angle AOB = 2\angle ACB$	From statement (3), by whole part axiom.
Conclusion: The central angle is double of the inscribed angle based on the same arc.		

Proved.

The inscribed angle is half of the centre angle based on the same arc. That is, the central angle is twice the angle on the circumference. As in the above figure, $\angle AOB = 2\angle ACB$.

Activity 5

How to show the angle in the circumference of the semicircle is a right angle by experimental verification.

(ii) Experimental verification

Draw the circles having different radii as shown in the figures. The angle $\angle ACB$ is on the circumference based on the diameter. Write the conclusion by taking the value of $\angle ACB$ in the table below.

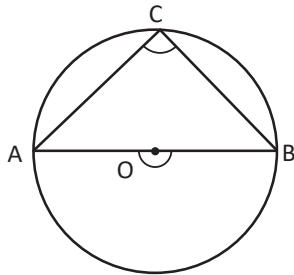


Figure 1

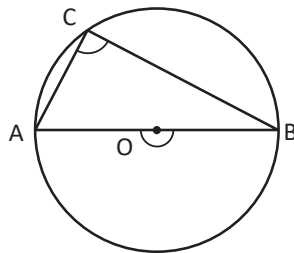


Figure 2

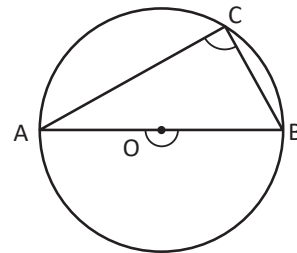


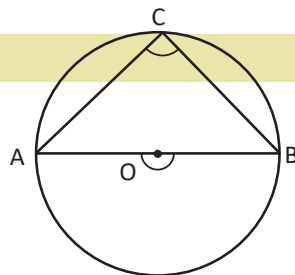
Figure 3

Measure the angle $\angle ACB$ of the semicircle of each circle and fill in the table below.

Figure No.	$\angle ACB$	Result
1.		
2.		
3.		
Conclusion:		

(ii) Theoretical proof

Given: O is the centre of a circle in which AOB is the diameter of the circle. $\angle ACB$ is the angle of the circumference based on the diameter.



To Prove: $\angle ACB = 90^\circ$

	Statement	Reason
1.	$\angle ACB = \frac{1}{2} \angle AOB$	Inscribed angle is half of the central angle standing on the same arc.
2.	$\angle AOB = 180^\circ$	Being $\angle AOB$ is a straight angle
3.	$\angle ACB = \frac{1}{2} \times 180^\circ = 90^\circ$	From statement (1) and (2) above
Conclusion: An angle in a semicircle is a right angle.		

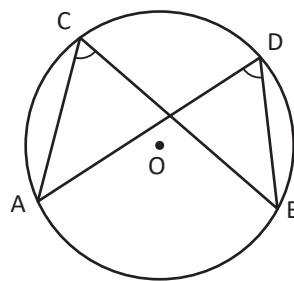
Proved.

(b) Relation between the angles in the circumference based on the same arc

Activity 6

(i) Using paper

On a chart paper, draw two angles in the circumference of the circle as shown in the figure. With the help of scissors, cut one angle from the circumference and fold it to the other angle. Write a conclusion based on this.



Inscribed angles are equal when they base on the same arc. $\angle ACB = \angle ADB$

(ii) Experimental verification

Draw the circles having different radii as shown in the figure. The angle $\angle ACB$ and $\angle ADB$ are the angles on the circumference based on the same arc AB.

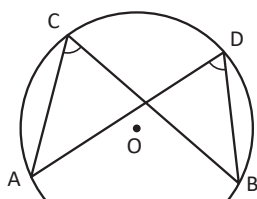


Figure 1

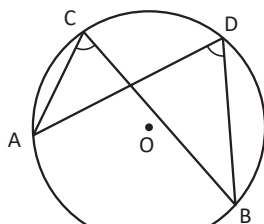


Figure 2

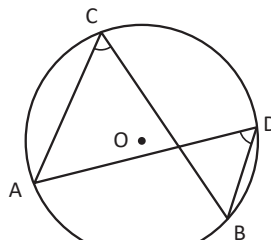


Figure 3

Measure the circumference angles $\angle ACB$ and $\angle ADB$ based on the same arc AB of each circle. Fill in the table below and write the conclusion.

Figure No.	$\angle ACB$	$\angle ADB$	Result
1.			
2.			
3.			
Conclusion:			

(iii) Theoretical proof

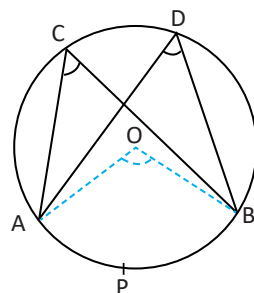
Theorem 2

The angles on the circumference of a circle, based on the same arc are equal.

Given: O is the centre of a circle in which inscribed angles $\angle ACB$ and $\angle ADB$ are based on the same arc AB.

To prove: $\angle ACB = \angle ADB$

Construction: Join centre O of the circle with the points A and B successively.



Proof

	Statements	Reasons
1.	$\angle AOB = 2\angle ACB$	The circumference angle and the central angle are based on the same arc APB.
2.	$\angle AOB = 2\angle ADB$	The circumference angle and the central angle are based on the same arc APB.
3.	$2\angle ACB = 2\angle ADB$ or, $\angle ACB = \angle ADB$	From statement (1) and (2)
Conclusion: The angles on the circumference of a circle, based on the same arc are equal.		

Proved.

12.5 The Relation between opposite angles of cyclic quadrilateral

(i) Experimental verification

Draw circles having different radii as shown in the figure. Draw the cyclic quadrilateral ABCD in each circle.

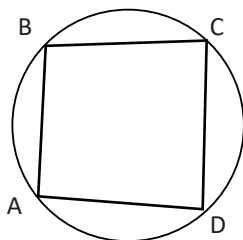


Figure 1

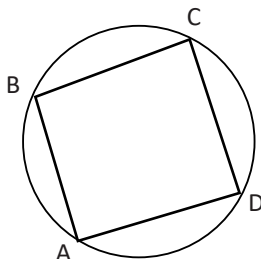


Figure 2

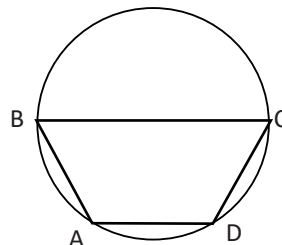


Figure 3

Measure the circumference angles and fill in the table below and write the conclusion.

Figure No.	$\angle DAB$	$\angle ABC$	$\angle BCD$	$\angle ADC$	$\angle DAB + \angle BCD$	$\angle ABC + \angle ADC$	Result
1.							
2.							
3.							
Conclusion:							

(ii) Theoretical proof

Theorem 3

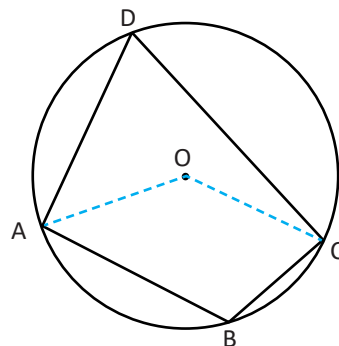
The opposite angles of a cyclic quadrilateral are supplementary.

Given: O is the centre of a circle. ABCD is a cyclic quadrilateral.

To prove: $\angle ABC + \angle ADC = 180^\circ$

$\angle BCD + \angle BAD = 180^\circ$

Construction: Join centre O of the circle with the points A and C successively.

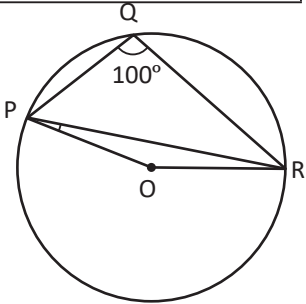


Proof

	Statements	Reasons
1.	Obtuse $\angle AOC = 2\angle ADC$	The central angle and the circumference angle based on the same arc APB.
2.	Reflex $\angle AOC = 2\angle ABC$	The central angle and the circumference angle based on the same arc ADC.
3.	$2\angle ADC + 2\angle ABC = \text{Obtuse } \angle AOC + \text{Reflex } \angle AOC$ or, $2(\angle ADC + \angle ABC) = 360^\circ$ or, $\angle ADC + \angle ABC = \frac{360^\circ}{2} = 180^\circ$ $\therefore \angle ADC + \angle ABC = 180^\circ$	From statement (1) and (2), the sum of the angles around the point O is 360°
4.	Similarly, $\angle DAB + \angle DCB = 180^\circ$	Similar as above.

Example 1

In the given figure, $\angle PQR = 100^\circ$ and the points P, Q and R are the circumference points of the circle with the centre O. What is the value of $\angle OPR$? Find.



Solution

According to the figure,

(i) Reflex angle $\text{POR} = 2 \times \angle \text{PQR} = 2 \times 100^\circ = 200^\circ$

[\because The central angle and the inscribed angle based on the same arc PR]

(ii) Reflex angle POR + obtuse angle POR = 360° [\because Sum of the angles around the point O]

$$200^\circ + \text{obtuse } \angle \text{POR} = 360^\circ$$

$$\angle \text{POR} = 360^\circ - 200^\circ = 160^\circ$$

(iii) Again, $\triangle \text{POR}$ is an isosceles triangle. So that, $\angle \text{OPR} = \angle \text{ORP}$

$$\angle \text{OPR} + \angle \text{ORP} + \angle \text{POR} = 180^\circ \quad [\because \text{The sum of the angles of a triangle}]$$

$$\text{or, } \angle \text{OPR} + \angle \text{OPR} + 160^\circ = 180^\circ \quad [\because \angle \text{OPR} = \angle \text{ORP}]$$

$$\text{or, } 2\angle \text{OPR} = 180^\circ - 160^\circ = 20^\circ$$

$$\text{or, } \angle \text{OPR} = \frac{20^\circ}{2} = 10^\circ$$

Example 2

In the adjoining figure, $\angle ABC = 74^\circ$ and $\angle ACB = 30^\circ$, then find the value of angle $\angle BDC$.

Solutions

Here,

- (i) In the triangle ABC, $\angle ABC + \angle ACB + \angle BAC = 180^\circ$

[\because Sum of interior angles of triangle.]

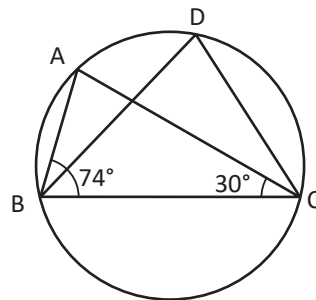
$$\text{or, } 74^\circ + 30^\circ + \angle BAC = 180^\circ$$

$$\text{or, } 104^\circ + \angle BAC = 180^\circ$$

$$\text{or, } \angle BAC = 180^\circ - 104^\circ = 76^\circ$$

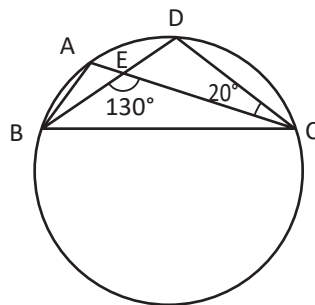
$$\therefore \angle BDC = 76^\circ$$

- (ii) $\angle BDC = \angle BAC$ [\because Inscribed angles standing on the same arc]



Example 3

In the adjoining figure, A, B, C and D are four points on the circumference of the circle. The chords AC and BD are intersecting at a point E. If $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$, find the value of $\angle BAC$.



Solution

- (i) $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$

$$\angle BEC + \angle CED = 180^\circ \text{ [}\because \text{The sum of the angles in a straight line]}$$

$$\text{or, } \angle CED = 180^\circ - \angle BEC = 180^\circ - 130^\circ = 50^\circ$$

- (ii) Again, $\angle EDC + \angle CED + \angle ECD = 180^\circ$ [\because The sum of the angles of a triangle]

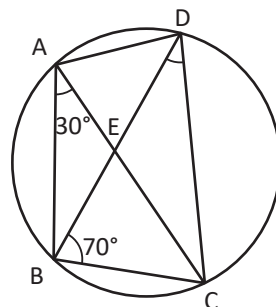
$$\text{or, } \angle EDC = 180^\circ - 50^\circ - 20^\circ = 110^\circ$$

- (iii) or, $\angle EDC = \angle BAC$ [\because Inscribed angles standing on the same arc BC]

$$\text{or, } \angle BAC = 110^\circ$$

Example 4

In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E. If $\angle DBC = 70^\circ$ and $\angle BAC = 30^\circ$, then find the value of $\angle BCD$. Also, if $AB = BC$, what is the value of $\angle ECD$?



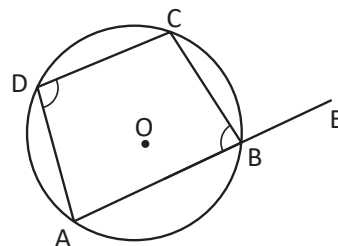
Solution

In a cyclic quadrilateral ABCD, the diagonals AC and BD are intersecting at a point E.

- (i) $\angle DAC = \angle DBC = 70^\circ$ [Inscribed angles standing on the same arc CD]
- (ii) $\angle DAB = \angle DAC + \angle BAC = 70^\circ + 30^\circ = 100^\circ$ [By whole part axiom]
- (iii) Again $\angle BCD + \angle DAB = 180^\circ$ [The sum of opposite angles of a cycle quadrilateral]
or, $\angle BCD + 100^\circ = 180^\circ$
or, $\angle BCD + 100^\circ = 180^\circ - 100^\circ = 80^\circ$
- (iv) Again $\angle BAC = \angle ACB = 30^\circ$ [Being $AB = BC$]
 $\angle BCD = \angle BCA + \angle ACD = 80^\circ$
or, $30^\circ + \angle ACD = 80^\circ$
or, $\angle ACD = 80^\circ - 30^\circ = 50^\circ$
 $\therefore \angle ACD = \angle ECD = 50^\circ$

Example 5

In the given figure alongside, ABCD is cyclic quadrilateral. If the side AB is produced to the point E then prove that $\angle ADC = \angle CBE$.



Solution

Given: ABCD is a cyclic quadrilateral of a circle with centre O. The side of cyclic quadrilateral AB is produced to the point E

To Prove: $\angle ADC = \angle CBE$

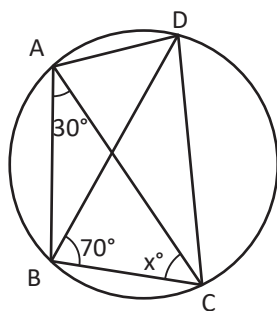
Proof:

	Statements	Reasons
1.	$\angle ADC + \angle ABC = 180^\circ$	The sum of opposite angles of a cyclic quadrilateral
2.	$\angle ABC + \angle CBE = 180^\circ$	Being straight angle
3.	$\angle ADC + \angle ABC = \angle ABC + \angle CBE$ or, $\angle ADC = \angle CBE$	From statement (1) and (2) above.

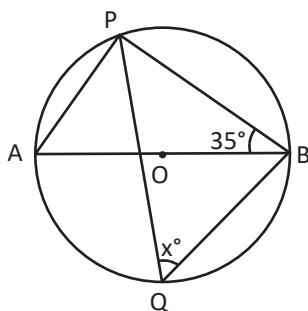
Proved.

Exercise 12

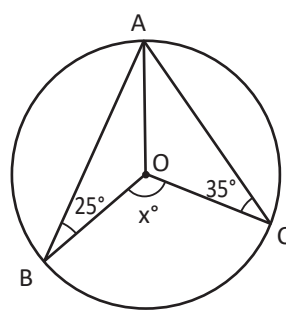
1. If O is the centre of the following circles, find the value of x .



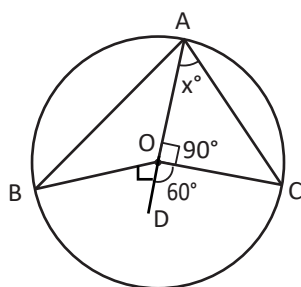
(a)



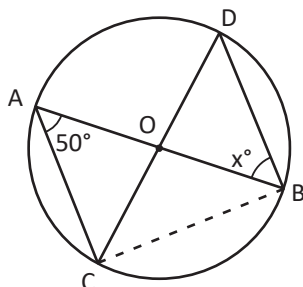
(b)



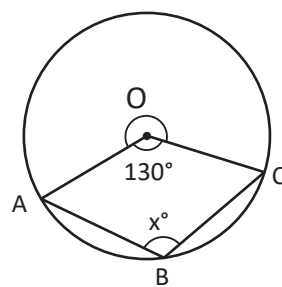
(c)



(d)

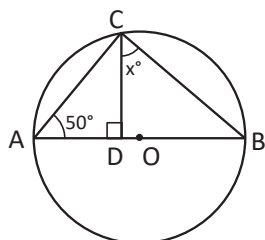


(e)

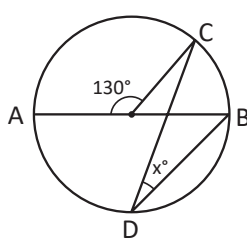


(f)

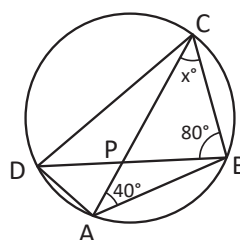
2. In the following figure, find the value of x .



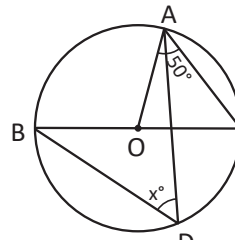
(a)



(b)

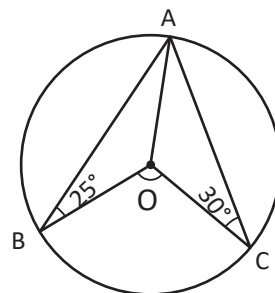


(c)

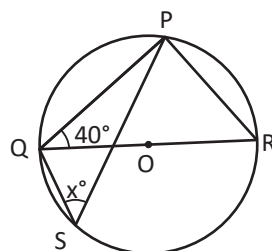


(d)

3. (a) In the adjoining figure, O is the centre of circle, $\angle OBA = 25^\circ$ and $\angle OCA = 30^\circ$ find the value of obtuse $\angle BOC$.

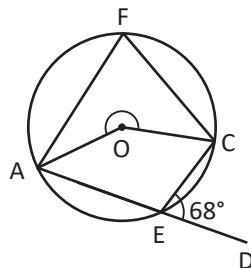


- (b) In the adjoining figure, O is the centre of the circle. If $\angle PQR = 40^\circ$ and $\angle PSQ = x^\circ$, find the value of x .

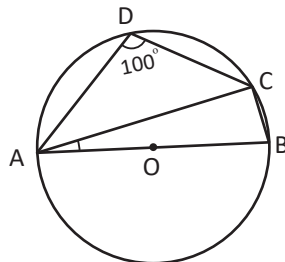


- (c) In the given figure, O is the centre of the circle. FAEC is a cyclic quadrilateral. If $\angle CED = 68^\circ$, then

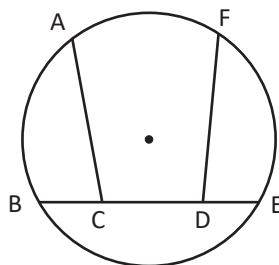
- Find the value of $\angle AFC$.
- Find the reflex $\angle AOC$.



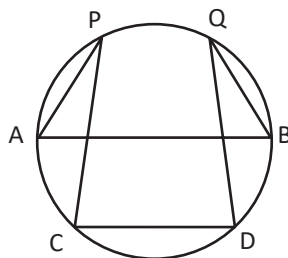
- (d) In the given figure, AOB is a diameter of the circle. If $\angle ADC = 100^\circ$, then find the value of $\angle BAC$.



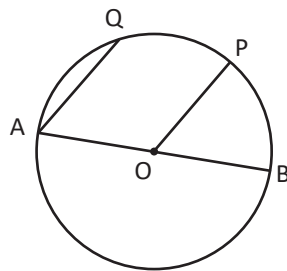
4. (a) In the adjoining figure, $BC = DE$ and $\widehat{AB} = \widehat{FE}$. Prove that $\angle ACB = \angle FDE$.



- (b) In the given figure, if $\angle APC = \angle BQD$, then prove that $AB \parallel CD$.

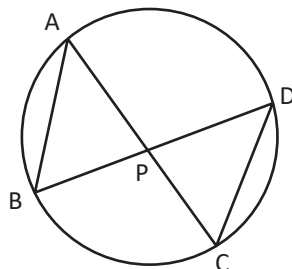


- (c) In the adjoining figure, O is the centre of the circle. If arc PQ = arc PB, then prove that $AQ \parallel OP$.

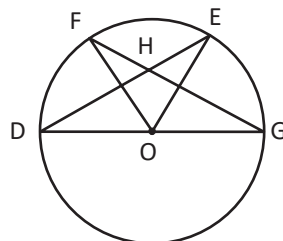


- (d) In the given figure, chords AC and BD are intersected at a point P. If $PB = PC$ then prove that:

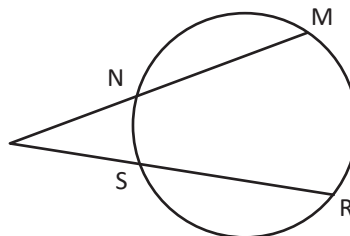
- (i) Chord AB = chord DC.
- (ii) Chord AC = chord BD.
- (ii) Arc ABC = arc BCD.



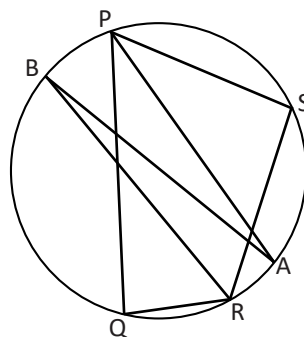
5. In the figure, O is the centre of the circle. If the chords DE and FG are intersected at a point H, prove that: $\angle DOF + \angle EOG = 2\angle EHG$.



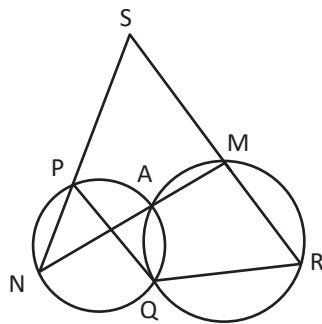
6. In the figure, chords MN and RS of the circle intersect externally at the point X. Prove that: $\angle MXR \overset{O}{=} \frac{1}{2}(\widehat{MR} - \widehat{NS})$.



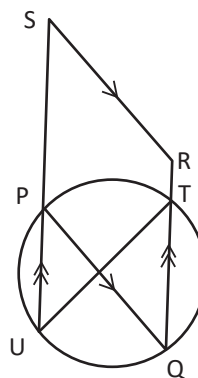
7. PQRS is a cyclic quadrilateral. If the bisectors of the $\angle QPS$ and $\angle QRS$ meet the circle at A and B respectively, prove that AB is a diameter of the circle.



8. In the given figure, NPS, MAN and RMS are straight lines. Prove that PQRS is a cyclic quadrilateral.



9. In the given figure, PQRS is a parallelogram. Prove that UTRS is a cyclic quadrilateral.



Practical work and project

- Make models of paper to show the relationship between the central angle and the inscribed angle, and the arcs and chords related to them. Present them in the classroom.
- Draw three pairs of equal circles ABP and CDQ having centres X and Y respectively. Join chords AB and CD making equal arcs AB and CD. Measure AB and CD and enter the result in a table.
 - Does chord AB = chord CD?
 - Are the angles subtended by chords AB and CD at the centre equal?
 - Is the angle subtended by the chord AB at the circumference of a circle half of the central angle? Fine thread or wire and tracing paper can be used for this work.

Answers

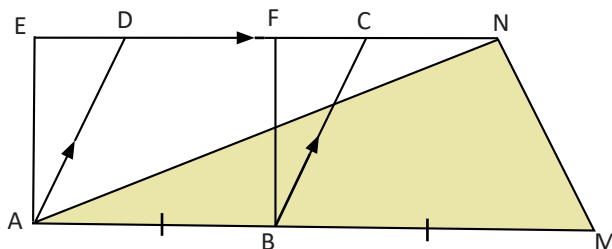
- (a) 80° (b) 55° (c) 120° (d) 75° (e) 50° (f) 115°
- (a) 50° (b) 25° (c) 60° (d) 50°
- (a) 110° (b) 50° (c) (i) 68° (ii) 136° (d) 10°

Show the answers from 4 to 9 to the teachers.

1. In the given figure, ABCD is a parallelogram and ABFE is a rectangle.

- (a) What is the relationship between the area of the parallelogram and rectangle? Write.

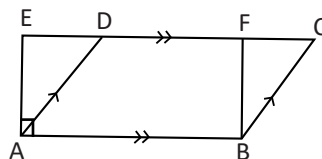
- (b) In the figure, if $AB = BM$, then write the relationship between the parallelogram ABCD and triangle AMN.



2. A parallelogram ABCD and a rectangle ABFE are on the same base AB and between the same parallel lines AB and EC.

- (a) What is the relationship between the area of the parallelogram ABCD and rectangle ABFE? Write it.

- (b) If the area of the rectangle ABFE is 35 cm^2 , what is the area of the parallelogram ABCD? Find.



- (c) Construct a parallelogram ABCD having the side $AB = 7 \text{ cm}$, $BC = 5 \text{ cm}$ and $\angle ABC = 120^\circ$. Construct a rectangle ABFE whose area is equal to the area of that parallelogram.

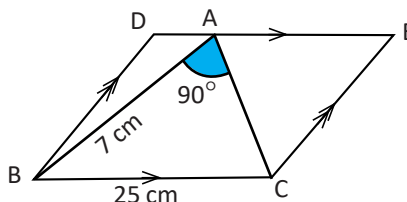
- (d) Are the triangle AED and triangle BFC congruent? Write with reason.

3. A parallelogram BCED and a triangle ABC are on the same base BC and between the same parallel lines BC and DE, where $\angle BAC = 90^\circ$, $AB = 7 \text{ cm}$ and $BC = 25 \text{ cm}$.

- (a) What is the measurement of AC? Find.

- (b) What is the area of parallelogram BCED? Find.

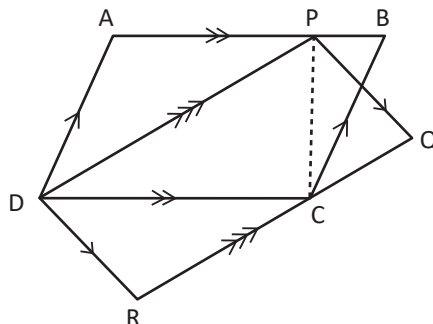
- (c) Theoretically prove that the relationship between the area of the parallelogram ABCD and triangle ABC.



- (d) Construct a triangle ABC, where $AC = 5 \text{ cm}$, $AB = 4 \text{ cm}$ and $\angle BAC = 45^\circ$. Also construct a parallelogram ADMN whose area is equal to the area of the triangle.

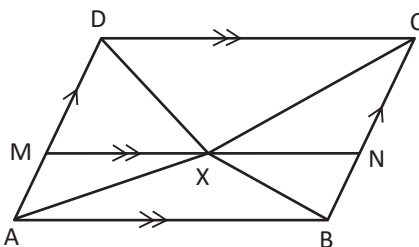
4. In the given figure, ABCD and PQRD are two parallelograms.

- Find the relation between the parallelograms ABCD and PQRD.
- If the base and height of the parallelogram ABCD are 8 cm and 7 cm respectively, find the area of the parallelogram PQRD.



5. In the given figure, ABCD is a parallelogram. X is a point inside it. If $MN \parallel AB$ then,

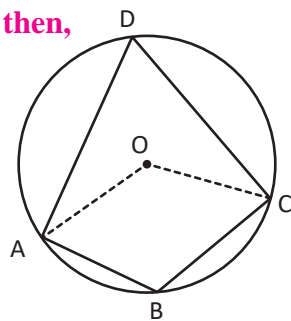
- Prove that the sum of the area of triangles XCD and XAB is equal to the half of the area of the parallelogram ABCD.
- What is the relationship among the triangle AXD, triangle BCX and parallelogram ABCD?



- Construct a parallelogram ABCD having $AB = 5$ cm, $BC = 4$ cm and $\angle ABC = 60^\circ$. Also construct a triangle PBE equal in area to the parallelogram having a side $PB = 5.6$ cm.
 - According to question (a) find the height of the parallelogram ABCD and then find the area of triangle PBE.
- Construct a parallelogram ABCD having $AB = 7$ cm, $BC = 5$ cm and $\angle ABC = 120^\circ$. Also, construct the rectangle ABFE equal in area to the parallelogram.
 - Find the side BF of the rectangle ABFE formed according to question no. (a) and also find the area of the parallelogram ABCD.
- If a circle with centre O has a central angle $\angle BOC$ and inscribed angle $\angle BDC$ based on the same arc BC, answer the following questions.**
 - Write the relation between $\angle BOC$ and $\angle BDC$.
 - Experimentally verify that the relationship between the central angle $\angle BOC$ and inscribed angle $\angle BDC$.
 - The measurement of the central angle is $(7x^\circ)$ and the inscribed angle is $(3x + 5)^\circ$, then find the value of x .

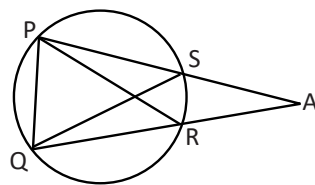
9. In the given figure, ABCD is a cyclic quadrilateral then,

- Write the relation of $\angle ABC$ and $\angle ADC$.
- Prove that $\angle ADC = \frac{1}{2} \angle AOC$.
- If $\angle ABC = 120^\circ$, what is the value of $\angle AOC$?



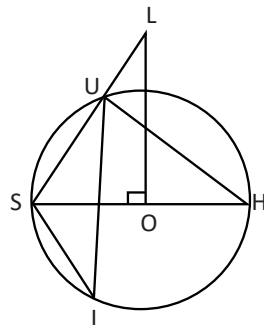
10. O is the centre of the given circle and P, Q, R and S are the circumference points on it. If $AP = AQ$ then,

- Write the name of the inscribed angles based on the arc PQ.
- If $\angle PSQ = 60^\circ$ then what is the measurement of the angle $\angle PRQ$?
- Prove that: $PR = QS$.



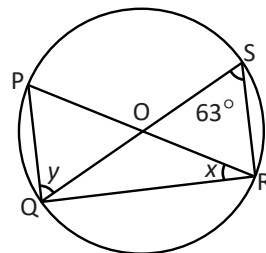
11. In the given figure, O is the centre of the circle and SH is a diameter. S, I, H and U are the circumference points and L is an external point Here LO is perpendicular to SH.

- What is the measurement of angle $\angle SUH$? Write with reason.
- Prove that: $\angle SIU = \angle OLS$.
- If $\angle USH = 50^\circ$, then what is the measurement of $\angle SIU$? Find.



12. In the given figure, O is the centre of circle, $\angle PQS = y$, $\angle QSR = 63^\circ$ and $\angle PRQ = x$.

- What is the measurement of angle $\angle PQR$? Write with reason.
- What is the value of angle $\angle POS$? Find.
- Prove that: $x + y = 90^\circ$.
- Prove that: $\triangle QOR$ is an isosceles triangle.



Answers

Show to your teacher.

Lesson 13

Statistics

13.0 Review

The marks obtained in Mathematics by 27 students of class 10 in the second terminal examination are given below. On the basis of this, answer the following questions.

25, 15, 30, 22, 27, 12, 25, 30, 22, 24, 15, 23, 19, 27, 28, 17, 19, 22, 25, 15, 14, 13, 28, 26, 18, 20, 22

- What is the average marks obtained in Mathematics?
- How many students have scored less and more than the average marks?
- Find the median, the first quartile and the third quartile on the basis of the data given above.
- What is the same mark obtained by maximum number of students? How many have got it? What is it called?

13.1 Mean

Activity 1

In a survey conducted among the students of class 10 of a community school about how much rupees they bring each day for their tiffin. The following results were obtained.

3 bring Rs. 25	6 bring Rs. 30	7 bring Rs. 35
4 bring Rs. 40	4 bring Rs. 45	1 bring Rs. 50

- Present the above data in table
- Find one day's average expenditure of the grade 10 students.

To find the average value (mean) from the data given above, the following formula is applied.

$$\begin{aligned}
 \text{Mean } \bar{X} &= \frac{(f_1 x_1 + f_2 x_2 + \dots + f_n x_n)}{f_1 + f_2 + \dots + f_n} \\
 &= \frac{\sum fx}{\sum f} \\
 &= \frac{\sum fx}{n}
 \end{aligned}$$

Activity 2

If the expansion of the raw data is too much, then its table will be so large. In that case, how to find the median?



If we have the data as you said, the mean can be calculated by making continuous series.



For example, the obtained marks of 40 students of class 9 of a school in Mathematics are as follows.

25, 10, 31, 22, 37, 42, 45, 37, 32, 34, 45, 40, 29, 27, 28, 17, 19, 22, 25, 33

15, 14, 13, 28, 36, 38, 41, 42, 39, 25, 24, 31, 21, 22, 25, 26, 35, 36, 39, 49

The median can be calculated by placing the above data in a continuous series.

- (a) The lowest obtained mark is 10 and the highest obtained mark is 49. We can make a table with the class interval of 10 as given below.

Description of the marks obtained by class 9 students in Mathematics

Marks obtained (X)	Tally bars	No. of students (f)
10 - 20		6
20 - 30		14
30 - 40		13
40 - 50		7
Total number of students		40

To find the mean of the continuous series first of all we have to find the mid value of each class interval.

$$\text{Mid-value (m)} = \frac{\text{Lower limit of a class interval} + \text{Upper limit of a class interval}}{2}$$

After that, as in the discrete series, mean is by placing 'm' in the place of x . calculated.

Description of the marks obtained by class 9 students in mid-value.

Marks obtained (X)	No. of students (f)	Mid value (m)	fm
10-20	6	$\frac{10+20}{2} = 15$	90
20-30	14	$\frac{20+30}{2} = 25$	350
30-40	13	$\frac{30+40}{2} = 35$	455
40-50	7	$\frac{40+50}{2} = 45$	315
	N = 40		$\Sigma fm = 1210$

$$\bar{X} = \frac{\Sigma fxm}{N} = \frac{1210}{40} = 30.25$$

The above table is called frequency table. The number of students of each class interval is called frequency of that class interval. This method to find the mean is called direct method.

Alternative method

Description of the marks obtained by class 9 students in Mathematics

Marks obtained X	No. of students f	Mid-value m	$d = m - A$	fd
10 – 20	6	$\frac{10+20}{2} = 15$	$15 - 25 = -10$	- 60
20 – 30	14	$\frac{20+30}{2} = 25$	$25 - 25 = 0$	0
30 – 40	13	$\frac{30+40}{2} = 35$	$35 - 25 = 10$	130
40 – 50	7	$\frac{40+50}{2} = 45$	$45 - 25 = 20$	140
	N = 40			$\Sigma fd = 210$

Let, assumed mean (A) = 25.

The deviation between mid-value and assumed mean = d

$$\begin{aligned}
 \text{Mean } \bar{X} &= A + \frac{\Sigma fd}{N} \\
 &= 25 + \frac{210}{40} \\
 &= 25 + 5.25 = 30.25
 \end{aligned}$$

The actual mean of the data can also be calculated by considering the assumed mean. The actual mean is calculated by considering any mid-value and any number as assumed mean. For this, the deviation between mid-value and assumed mean (d) should be found.

Example 1

If $\sum fm = 2700$ and $N = 50$, then find mean \bar{X} .

Solution

Here, $\sum fm = 2700$

$N = 50$

Mean $\bar{X} = ?$

$$\begin{aligned}\text{We know that } \bar{X} &= \frac{\sum fm}{N} \\ &= \frac{2700}{50} \\ &= 54\end{aligned}$$

Hence, mean (\bar{X}) = 54

Example 2

The weight of 100 students of Ganesh Secondary School is given in the table below. Find the mean weight of the students from this data.

Weight in (Kg)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	18	27	20	17	6

Solution

Here, the details of the weight of students

Weight in (Kg) (X)	No. of students (f)	Mid value (m)	fm
20 – 30	27	$\frac{20 + 30}{2} = 25$	675
30 – 40	20	$\frac{30 + 40}{2} = 35$	700
40 – 50	17	$\frac{40 + 50}{2} = 45$	765
50 – 60	6	$\frac{50 + 60}{2} = 55$	330
	$\Sigma f = N = 88$		$\Sigma fm = 2740$

We know that, mean $\bar{X} = \frac{\Sigma fm}{N} = \frac{2740}{88} = 31.14$

Hence, the mean weight of students $(\bar{X}) = 31.14$

Alternative method

Here, let the assumed mean (A) = 35

Description of the weight of students

Marks obtained (X)	No. of students (f)	Mid value (m)	$d = m - 35$	fd
10 – 20	18	15	-20	-360
20 – 30	27	25	-10	-270
30 – 40	20	35	0	0
40 – 50	17	45	10	170
50 – 60	6	55	20	120
	$N = 88$			$\Sigma fd = -340$

We know that, mean $(\bar{X}) = A + \frac{\Sigma fd}{N}$
 $= 35 + \frac{-340}{88}$
 $= 35 - 3.86 = 31.14$

Hence, the mean weight of the students $(\bar{X}) = 31.14$

Example 3

The assumed mean of any data (A) = 40, $\sum fd = 20$ and $N = 10$, then find the mean \bar{X} .

Solution

Here, $A = 40$,

$\sum fd = 20$ and

$N = 10$

mean (\bar{X}) = ?

$$\begin{aligned}\text{We know that, mean } (\bar{X}) &= A + \frac{\sum fd}{N} \\ &= 40 + \frac{20}{10} \\ &= 40 + 2 \\ &= 42\end{aligned}$$

Hence, the mean (\bar{X}) = 42

Example 4

The data prepared on the basis of the weight of the people of Janajagriti Tole are given in the table. Find the average weight (mean) based on the table.

Weight (Kg)	0-10	10-20	20-30	30-40	40-50	50-60
No. of people	10	18	25	20	12	5

Solution

Here, let assumed mean (A) = 25 and length of class interval (h) = 10

Description of the weight of people of Janajagriti Tole

Weight in Kg. (X)	No. of people (f)	Mid value (m)	$d = m - 25$	fd
0-10	10	5	-20	-200
10-20	18	15	-10	-180
20-30	25	25	0	0
30-40	20	35	10	200
40-50	12	45	20	240
50-60	5	55	30	150
	$\Sigma f = N = 90$			$\Sigma fd = 210$

We know that, mean $(\bar{X}) = A + \frac{\Sigma fd}{N}$

$$= 25 + \frac{210}{90}$$

$$= 25 + 2.33 = 27.33$$

Hence, the mean weight of the people $(\bar{X}) = 27.33$

Example 5

The data prepared based on the height of students of class 11 and 12 of Shanti Secondary School is given in the table. If the average height of the students $\bar{X} = 157.75$ cm, find the value of p.

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of students	2	5	8	p	7	5	9

Solution

Here, average height of the students $(\bar{X}) = 157.75$

Description of the height of the students of class 11 and 12 of Shanti

Secondary School

Height (cm) (X)	No. of students (f)	Mid-value (m)	fm
140-145	2	142.5	285
145-150	5	147.5	737.5
150-155	8	152.5	1220
155-160	p	157.5	157.5p
160-165	7	162.5	1137.5
165-170	5	167.5	837.5
170-175	3	172.5	517.5
	$\Sigma f = N = 30 + p$		$\Sigma fm = 4735 + 157.5 p$

We know that, mean $(\bar{X}) = \frac{\sum fm}{N}$

$$157.75 = \frac{4735 + 157.5p}{30 + p}$$

$$\text{or, } 4732.5 + 157.75p = 4735 + 157.5p$$

$$\text{or, } 157.75p - 157.50p = 4735 - 4732.5$$

$$\text{or, } 0.25p = 2.5$$

$$\text{or, } p = 10$$

Hence, the required value of $p = 10$.

Example 6

The number of people entering into the Balaju Park from 7 am to 8 am according to their age is given below. Construct a frequency table of class interval of 10 from the given data and find the average age of people visiting the park.

7, 22, 32, 47, 59, 16, 36, 17, 23, 39, 49, 31, 21, 24, 41, 12, 49, 21, 9, 8, 51, 36, 35, 18.

Solution

Here, tabulating the given data in the frequency distribution table

Height (cm) (X)	Tally Bars	frequency (f)	Mid-value (m)	fm
0-10		3	5	15
10-20		4	15	60
20-30		5	25	125
30-40		6	35	210
40-50		4	45	180
50-60		2	55	110
		$\sum f = N = 24$		$\sum fm = 700$

We know that, mean $(\bar{X}) = \frac{\sum fm}{N}$

$$= \frac{700}{24}$$

$$= 29.17$$

$$(\bar{X}) = 29.17$$

Hence, the average age of people $(\bar{X}) = 29.17$ years.

Exercise 13.1**1. Find the mean in the following condition:**

(i) 35, 36, 42, 45, 48, 52, 58, 60

(ii) 13.5, 14.2, 15.8, 15.2, 16.9, 16.5, 17.4, 19.3, 15.3, 15.9

(iii)

X	5	8	10	12	14	16
f	4	5	8	10	2	2

(iv) Details of the goals scored by players in the national football league.

Goals	12	13	14	15	16	17
Number of students	2	4	6	12	10	6

2. Find the mean of the following data by direct method and short-cut method.

(a) The details age of passengers travelling in a bus

Age (year)	0-10	10-20	20-30	30-40	40-50
No. of persons	5	9	15	7	4

(b) The details of the marks obtained by class 10 students in science.

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	1	4	10	8	7	5

(c) Details of daily wages of workers

Wages (Rs)	200-400	400-600	600-800	800-1000	1000-1200
No. of workers (Rs.)	3	7	10	6	4

(d) The details of the marks obtained by class 10 students in Mathematics.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	7	5	6	12	8	2

3. Find the unknown value of the following data.(a) $\bar{X} = 49$, $\sum fm = 980$, $N = ?$ (b) $\bar{X} = 102.25$, $N = 8$, $\sum fm = ?$ (c) $A = 100$, $\bar{x} = 90$, $\sum fd = ?$, $N = 10$ (d) $\bar{X} = 41.75$, $\sum fd = 270$, $N = 40$, $A = ?$

4. (a) In the given condition, if the mean (\bar{X}) is 32.5, find the value of k.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	k	35	15	10

- (b) In the given condition, if the mean (\bar{X}) is 46.2, find the value of p.

X	0-20	20-40	40-60	60-80	80-100
f	35	400	350	p	65

- (c) In the given condition, if the mean (\bar{X}) is 36.4, find the value of y.

Marks obtained	16-24	24-32	32-40	40-48	48-56	56-64
No. of students	6	8	y	8	4	2

- (d) In the given condition, the mean of daily expenditure (\bar{X}) is 264.67, find the value of the unknown frequency.

Daily expenditure	0-100	100-200	200-300	300-400	400-500	500-600
No. of students	20	30	?	20	18	12

5. Prepare a frequency distribution table from the given raw data and find the mean (\bar{X}).

- (a) 15, 51, 32, 12, 32, 33, 23, 43, 35, 46, 57, 19, 59, 25, 20, 38, 16, 45, 39, 40 (Class interval 10)
- (b) 25, 15, 24, 42, 22, 35, 34, 41, 33, 38, 54, 50, 36, 40, 27, 18, 35, 16, 51, 31, 23, 9, 16, 23, 31, 51, 7, 30, 17, 40, 60, 32, 50, 10, 23, 12, 21, 28, 37, 20, 58, 39, 10, 41, 13 (Class interval 5)

6. (a) Find the mean of the following data.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	14	10	8	10

- (b)
- | | | | | | | |
|-------------------|------|--------|---------|---------|---------|---------|
| Daily wages (Rs.) | 0-50 | 50-100 | 100-150 | 150-200 | 200-250 | 250-300 |
| No. of workers | 1 | 2 | 3 | 4 | 1 | 2 |

Project work

Ask the age of 100 people of your community and present it in the frequency distribution table with a suitable class interval. Present that frequency distribution table in histogram. Find the mean by direct method and short-cut method and present in classroom.

Answers

1. (a) 47 (b) 16 (c) 10.32 (d) 15.05
2. (a) 24 year (b) 43.86 (c) Rs. 706.67 (d) 28.75
3. (a) 20 (b) 818 (c) -100 (d) 35
4. (a) 25 (b) 150 (c) 12 (d) 50
5. (a) 34.5 (b) 31.29 6. (a) 30 (b) Rs. 155.77 (c) 34.5

13.2 Median

Activity 3

The marks obtained in Mathematics by class 10 students in the first terminal examination are given below.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,
28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

On the basis of the above data, solve the following questions and check your answer with your friends

- (a) What is the average mark of the students?
- (b) Find the median value of the student's marks by making individual series and discrete series.
- (c) Does the median value obtained from different series differ?

The median is the statistical value that divides the given data exactly into two parts.

Example 1

The marks obtained by class 8 students in Mathematics are given below. Find the median of the data.

Marks obtained	17	18	22	26	30	32
No. of students	3	4	8	10	7	5

Solution

Here,

Details of the marks obtained by class 8 students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
17	3	3
18	4	7
22	8	15
26	10	25
30	7	32
32	5	37
	$\Sigma f = N = 37$	

We know that,

$$\begin{aligned}\text{The position of median} &= \frac{N+1}{2}^{\text{th}} \text{ term} \\ &= \frac{37+1}{2}^{\text{th}} \text{ term} \\ &= 19^{\text{th}} \text{ term}\end{aligned}$$

From the above table, the value of the 19th item is 26. So, the median of the marks obtained by class 8 students is 26.

Activity 4

We were able to find the median from the data given in discrete series. Now, if the data is given in a continuous series, how to find the median?

Median from the continuous series of data

Like,

Marks obtained (X)	0-8	8-16	16-24	24-32	32-40	40-48	48-56
No. of students (f)	6	10	16	18	12	10	8

The median of the continuous series of data can be found in the following steps:

- Make a less than cumulative frequency table. (Less than the upper value of each class interval)
- Identification of the position of the median. The position of median = $\frac{N}{2}$ th item
- Finding the class interval of median. The class interval of the position of median is the median class interval.

(d) Finding the median by using the following formula.

$$\text{Median } (M_d) = L + \frac{\frac{N}{2} - cf}{f} \times h$$

Where, L = Lower limit of the median class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the median class

f = Frequency of the median class

h = Length (width) of class interval

Here, constructing the table for the median,

Marks obtained (X)	No. of students (f)	Marks	Less than cumulative frequency (cf)
0-8	6	Less than 8	6
8-16	10	Less than 16	6 + 10 = 16
16-24	16	Less than 24	16 + 16 = 32
24-32	18	Less than 32	32 + 18 = 50
32-40	12	Less than 40	50 + 12 = 62
40-48	10	Less than 48	62 + 10 = 72
48-56	8	Less than 56	72 + 8 = 80
	N = 80		

Total number of students (N) = 80

The position of median = $\frac{N}{2}$ th class = $\frac{80}{2}$ th class = 40 th class

40th item lies in the class interval (24 - 32).

Now, the lower limit of the median class (L) = 24

Cumulative frequency of the class preceding the median class (cf) = 32

Frequency of the median class (f) = 18

Length of class interval (h) = 32 - 24 = 8

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 24 + \frac{40 - 32}{18} \times 8$$

$$= 24 + \frac{64}{18}$$

$$= 24 + 3.56 = 27.56$$

Example 2

Find the median from the given data

Weight (Kg)	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	16	12	10	16	18	12

Solution: Here,

Details of the students' weight

Marks obtained (X)	No. of students (f)	Less than cumulative frequency	Cumulative frequency
20 - 30	16	Less than 30 = 16	16
30-40	12	Less than 40 = 16 + 12	28
40-50	10	Less than 50 = 16 + 12 + 10	38
50-60	16	Less than 60 = 16 + 12 + 10 + 16	54
60-70	18	Less than 70 = 16 + 12 + 10 + 16 + 18	72
70-80	12	Less than 80 = 16 + 12 + 10 + 16 + 18 + 12	84
	$\sum f = N = 84$		

Total number of students (N) = 84

The position of median = $\frac{N}{2}$ th item = $\frac{84}{2}$ th item = 42 th item

42th item lies in the class interval (50 - 60).

Now, the lower limit of the median class (L) = 50

Cumulative frequency of the class preceding the median class (cf) = 38

Frequency of the median class (f) = 16

Length of class interval (h) = 60 – 50 = 10

$$\begin{aligned}
 \text{We know that, median } (M_d) &= L + \frac{\frac{N}{2} - cf}{f} \times h \\
 &= 50 + \frac{42 - 38}{16} \times 10 \\
 &= 50 + \frac{40}{60} \\
 &= 50 + 2.5 = 52.5
 \end{aligned}$$

Example 3

The table given below is prepared based on the number of people voluntarily contributing to the public works in the village. If the median value of the given data is 93.6, find the value of the missing frequency 'y'.

Days (X)	0-30	30-60	60-90	90-120	120-150	150-180
No. of workers (f)	5	y	22	25	14	4

Solution: Here,

Table to find the cumulative frequency

Days (X)	No. of workers (f)	Less than cumulative frequency (cf)
0 - 30	5	5
30-60	y	5 + y
60-90	22	27 + y
90-120	25	52 + y
120-150	14	66 + y
150-180	4	70 + y
	N = (70 + y)	

Total number of workers $N = 70 + y$

Median $(M_d) = 93.6$

Median lies in the class interval (90-120)

Now, the lower limit of the median class (L) = 90

Cumulative frequency of the class preceding the median class (cf) = 27 + y

Frequency of the median class (f) = 25

Length of median class interval (h) = 120 - 90 = 30

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$\text{or, } 93.6 = 90 + \frac{\frac{70+y}{2} - (27+y)}{25} \times 30$$

$$\text{or, } 93.6 - 90 = \frac{70+y-2(27+y)}{2 \times 25} \times 30$$

$$\text{or, } 3.6 = \frac{70+y-54-2y}{50} \times 30$$

$$\text{or, } 3.6 = \frac{(16-y) \times 3}{5}$$

$$\text{or, } 3.6 \times 5 = 48 - 3y$$

$$\text{or, } 3y = 48 - 18$$

$$\text{or, } 3y = 30$$

$$\text{or, } y = 10$$

Hence, missing frequency (y) = 10

Activity 5

Below given is a frequency distribution table based on the height of trees in a garden.

Height (ft)	4-6	7-9	10-12	13-15	16-18	19- 21	22-24
No. of trees	2	3	10	7	4	3	2

(a) How to make the above data into continuous series?

(b) What is the median of the above data? Find.

In the data given here, the class intervals are not continuous. So, to make the class interval continuous, the correction factor should be found as follows.

$$\begin{aligned} \text{Correction factor} &= \frac{\text{Lower limit of the second class interval} - \text{Upper limit of the first class interval}}{} \\ &= \frac{7 - 6}{2} = 0.5 \end{aligned}$$

The correction factor is subtracted from the lower value and added to the upper value of each class interval to convert the class interval into continuous series.

Like, in the class interval = 4 - 6

Lower value 4 - 0.5 = 3.5 and upper value 6 + 0.5 = 6.5

Now, we should make class interval 3.5 - 6.5

Table to find the median

Height (cm) X	No. of trees (f)	Less than cumulative frequency (cf)
3.5 - 6.5	2	2
6.5 - 9.5	3	5
9.5 - 12.5	10	15
12.5 - 15.5	7	22
15.5 - 18.5	4	26
18.5 - 21.5	3	29
21.5 - 24.5	2	31
	N = 31	

The position of median = $\frac{N}{2}$ class

$$= \frac{31}{2} = 15.5^{\text{th}} \text{ class}$$

The class interval having 15.5th term = (12.5 - 15.5)

Now, the lower limit of the median class (L) = 12.5

Cumulative frequency of the class preceding the median class (cf) = 15

Frequency of the median class (f) = 7

Length of the median class interval (h) = 15.5 - 12.5 = 3

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 12.5 + \frac{15.5 - 15}{7} \times 3$$

$$= 12.5 + \frac{0.5 \times 3}{7}$$

$$= 12.5 + \frac{1.5}{7}$$

$$= 12.5 + 0.21 = 12.71$$

∴ Hence, the median height of trees = 12.71 ft.

Example 5

Construct a frequency distribution table with a class interval of 10 from the given data and find the median.

21, 9, 34, 42, 17, 54, 13, 38, 23, 39, 49, 29, 38, 44, 21, 42, 19, 7, 29, 8, 55, 36, 39, 13.

Solution

Frequency distribution table

Class interval (X)	Tally Bars	Frequency f	Less than cumulative frequency cf
0-10		3	3
10-20		4	3+4=7
20-30		5	7+5=12
30-40		6	12+6=18
40-50		4	18+4=22
50-60		2	22+2=24
		N = 24	

The position of median = $(\frac{N}{2})^{\text{th}}$ class

$$= (\frac{24}{2})^{\text{th}} = 12^{\text{th}} \text{ item}$$

The class interval having 12th term = (20 – 30)

Now, the lower limit of the median class (L) = 20

Cumulative frequency of the class preceding to median class (cf) = 7

Frequency of the median class (f) = 5

Length of median class interval (h) = 30 - 20 = 10

We know that, median (M_d) = $L + \frac{\frac{N}{2} - cf}{f} \times h$

$$= 20 + \frac{12-7}{5} \times 10$$

$$= 20 + 10$$

$$= 30$$

Exercise 13.2**1. Find the median from the given data.**

(a) 2.5, 4.5, 3.6, 4.9, 5.4, 2.9, 3.1, 4.2, 4.6, 2.2, 1.5

(b) 100, 105, 104, 197, 97, 108, 120, 148, 144, 190, 148, 22, 169, 171, 92, 100

(c)

Marks obtained	18	25	28	29	34	40	44	46
No. of students	3	6	5	7	8	12	5	4

(d)

Class interval (x)	102	105	125	140	170	190	200
Frequency (f)	10	18	22	25	15	12	8

2. Find the median from the given data.

(a)

Weight (Kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

(b)

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
Frequency	2	5	8	10	7	5	3

(c)

Expenditure (per day) Rs	less than 100	100-200	200-300	300-400	400-500	more than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

3. Find the missing frequency from the data given below.

(a) Median = 35

Marks obtained	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	k	4	5

(b) Median = 132.5

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	p	4	7	5

(c) Median = 39

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	?	220	70	60

4. Find the median of the following data.

(a)

Marks obtained	50-60	60-70	70-80	80-90	90-100
No. of students	2	3	6	5	4

(b)

Marks obtained	< 20	< 40	< 50	< 80	< 100
No. of students	9	23	43	55	60

(c)

Income (Rs)	< 600	< 700	< 800	< 900	< 1000
No. of workers	30	98	152	177	200

(d)

Temp (°c)	0-9	10-19	20-29	30-39	40-49
Days	8	10	20	15	7

5. (a) The marks obtained by 30 students in a class test are as follows.
22, 56, 62, 37, 48, 30, 58, 42, 29, 39, 37, 50, 38, 41, 32, 20, 28, 16, 43,
18, 40, 52, 44, 27, 35, 45, 36, 49, 55, 40

Construct a frequency distribution table with the class interval of 10 from the above data and find the mean median.

- (b) The height of 40 students of class 10 in cm is given below. Construct a frequency distribution table with the class interval of 5 from the data and find the mean and median.

142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149,
148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163,
149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155

Project work

Ask the ages of 100 people in your community. Find the median age by presenting the obtained data with a class interval of 10.

Answer

- | | | | |
|----------------|-----------------|---------------|----------|
| 1. (a) 3.6 | (b) 121 | (c) 34 | (d) 140 |
| 2. (a) 64.5 kg | (b) 157.5cm | (c) 246.15 lb | (d) 40.9 |
| 3. (a) 6 | (b) 3 | (c) 150 | |
| 4. (a) 78.33 | (b) 47 | (c) 703.70 | (d) 25.5 |
| 5. (a) 39, 40 | (b) 158, 155.42 | | |

13.3 Mode

Activity 6

The temperature of 20 days of a city is as follows find which temperature is repeated maximum.

70, 76, 76, 74, 70, 70, 72, 74, 78, 80, 74, 74, 78, 76, 78, 76, 74, 78, 80, 76

The maximum number of times repeated value of the given data is called mode.

Activity 7

Mode from continuous series

We can find the mode of grouped data in the following steps:

- Since, the number of repeated values is the mode, first find the class interval with the highest frequency.
- Finding the frequency of the model class is f_1 , the frequency of the class preceding the model class is f_0 , the frequency of the class succeeding the model class is f_2 .
- Finding the width of mode l class interval
- The following formula is using to find the mode

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where, L = Lower limit of the model class

f_1 = Frequency of the model class

f_0 = Frequency of the class preceding the model class

f_2 = Frequency of the class succeeding the model class

h = Width (size) of model class interval

Example 1

Find the mode from the data given below.

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Solution: Here,

Weight (kg)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	5	7	11	10	3	1

Here, the highest frequency is 11, so its corresponding class is 60 - 70.

Lower limit of the model class (L) = 60

Frequency of the model class (f_1) = 11

Frequency of the class preceding the model class (f_0) = 7

Frequency of the class succeeding the model class (f_2) = 10

Width (size) of model class interval (h) = 70 - 60 = 10

We know that, mode (Mo) = $L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 60 + \frac{11 - 7}{2 \times 11 - 7 - 10} \times 10$$

$$= 60 + \frac{4}{5} \times 10$$

$$= 60 + 8 = 68$$

Hence, mode (Mo) = 68

Exercise 13.3

1. Find the mode of the following data

(a) 29 cm, 34 cm, 29 cm, 26 cm, 55 cm, 34 cm, 35 cm, 40 cm, 34 cm, 56 cm

(b) 99 kg, 135 kg, 182 kg, 49 kg, 189 kg, 196 kg, 78 kg, 192 kg, 182 kg

2. Find the mode from given frequency table

(a)

Marks obtained	5	10	15	20	25	30	35	40	45
No. of students	2	6	7	9	11	5	15	2	3

(b)

Wages (Rs.)	50	75	100	125	150	175	200	225
No. of workers	8	12	17	29	30	27	20	11

3. Find the mode from the given frequency table

(a)

Marks obtained.	20-25	25-30	30-35	35-40	40-45	45-50
No. of students	2	5	8	6	4	5

(b)

Wages (Rs.)	100-110	110- 120	120-130	130-140	140-150	150-160
No. of workers	5	6	4	7	5	4

(c)

Age (yrs)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	50	70	100	300	220	150	70	60

Answer

- (a) 34 cm (b) 182 kg
- (a) 35 (b) 150 kg
- (a) 33 (b) 136 (c) 38.57

13.4 Quartiles

Activity 8

In class 9, we studied to find the first quartile and third quartile from the individual and discrete data. The data given below is the marks obtained in Mathematics by class 10 students of Janta Secondary School in the first terminal examination.

21, 23, 28, 14, 10, 18, 19, 29, 27, 25, 19, 17, 18, 20, 21, 17, 15, 16,

28, 23, 24, 17, 16, 19, 14, 24, 23, 27, 14, 15, 21, 24, 26, 24, 18

From the above data, find the value of the first quartile and the third quartile by making individual series and discrete series. Is the value of the first quartile and the third quartile obtained from different series also different? Discuss in the group of two.

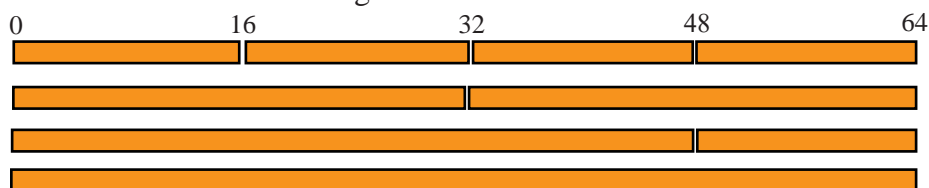
Activity 9

How to find the first quartile and the third quartile from the data given in continuous series? Do the following activities in groups:

- (a) Take any 4 sticks of length 64 cm.

- (b) Divide first stick into 4 equal parts. Find the length of each piece. 16 divide by 64 into 4 equal parts. This is the first quartile.
- (c) Let's divide the second stick into 2 equal parts and find the length of each piece.
- (d) Divide the third stick into 4 equal parts such that 3 parts in one side and one part on another side.

This can be shown in the figure as follows.



As seen in the picture above, when divided into 4 equal pieces, 16 cm pieces are formed. When divided into 2 equal parts, 32 cm pieces are formed. It is also called median. Similarly, if you add 3 equal pieces out of 4 equal pieces, the total length is 48 cm. 16 is the first quartile (Q_1), 32 is the second quartile (Q_2) and 48 is the third quartile (Q_3).

For grouped series

- (a) Find the less than cumulative frequency.
- (b) The position of first quartile $Q_1 = \frac{N}{4}^{\text{th}}$ item and the position of the third quartile $Q_3 = (\frac{3N}{4})^{\text{th}}$ item
- (c) Look at the cumulative frequency for (Q_1) in the class interval with cumulative frequency equal to or greater than $\frac{N}{4}$ and for (Q_3) in the class interval with cumulative frequency exactly greater than $\frac{3N}{4}$.
- (d) After that, use the following formula:

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times h$$

Here,

L = Lower limit of the first quartile (Q_1) class

N = Total numbers of data

cf = Cumulative frequency of the class preceding the first quartile (Q_1) class

f = Frequency of the first quartile class

h = Length (width) of class interval

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times h$$

Where, L = Lower limit of the third quartile class

N = Total numbers of data

cf = Cumulative frequency of the class preceding to the third quartile class

f = Frequency of the third quartile class

h = Length (width) of class interval

Example 1

Find the first quartile (Q_1) and third quartile (Q_3) from the following data.

Age (yrs)	20	25	28	30	32	35	42	46
No. of workers	2	8	12	10	14	7	5	1

Solution

Details of workers' age

Workers age X	No. of workers (f)	cf
20	2	2
25	8	10
28	12	22
30	10	32
32	14	46
35	7	53
42	5	58
46	1	59

$$\begin{aligned}
 \text{The position of the first quartile} &= \frac{N+1}{4}^{\text{th}} \text{ term} \\
 &= \frac{59+1}{4}^{\text{th}} \text{ term} \\
 &= \frac{60}{4} = 15^{\text{th}} \text{ term}
 \end{aligned}$$

The corresponding value of the 15th item is 28. So, the first quartile (Q_1) is 28.

$$\begin{aligned}
 \text{Again, the position of the third quartile} &= \frac{3(N+1)}{4}^{\text{th}} \text{ term,} \\
 &= \frac{3(59+1)}{4}^{\text{th}} \text{ term} = \frac{180}{4} = 45^{\text{th}} \text{ term}
 \end{aligned}$$

The corresponding value of the 45th item is 32. So the third quartile (Q_3) is 32.

Example 2

The marks obtained in Mathematics by class 7 students in are given below. Find the first quartile (Q_1) and the third quartile (Q_3) from the data.

Marks obtained (X)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (f)	2	8	15	14	10	8	3

Solution

Details of the mark obtained by students

Marks obtained (X)	No. of students (f)	Cumulative frequency (cf)
10-20	2	2
20-30	8	10
30-40	15	25
40-50	14	39
50-60	10	49
60-70	8	57
70-80	3	60
$\Sigma f = N = 60$		

The position of the first quartile (Q_1) = $\frac{N}{4}$ th item = $\frac{60}{4}$ th item = 15th item

The class interval having the 15th item is (30 - 40)

Now, lower limit of the first quartile class interval (L) = 30

Cumulative frequency of the class preceding the first quartile class (cf) = 10

Frequency of the first quartile class (f) = 15

Length (width) of first quartile class interval (h) = 40 - 30 = 10

$$\begin{aligned}\text{We know that, first quartile } (Q_1) &= L + \frac{\frac{N}{4} - cf}{f} \times h \\ &= 30 + \frac{15 - 10}{15} \times 10 \\ &= 30 + \frac{50}{15} \\ &= 30 + 3.34 = 33.34\end{aligned}$$

The position of the third quartile = $\frac{3N}{4}$ th item = $\frac{3 \times 60}{4}$ th item = 45th item

The class interval having the 45th item is (50 - 60).

Now, lower limit of the third quartile class interval (L) = 50

Cumulative frequency of the class preceding the third quartile class (cf) = 39

Frequency of the third quartile class (f) = 10

Length (width) of the third quartile class interval (h) = 60 - 50 = 10

$$\begin{aligned}\text{We know that, third quartile } (Q_3) &= L + \frac{\frac{3N}{4} - cf}{f} \times h \\ &= 50 + \frac{45 - 39}{10} \times 10 \\ &= 50 + \frac{60}{10} \\ &= 50 + 6 = 56\end{aligned}$$

Hence, the first quartile(Q_1) = 33.34 and the third quartile (Q_3) = 56

Example 3

In the following table workers' incomes are given. Find Q_1 , Q_2 and Q_3 from the data.

Income (in thousands)	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of workers	10	15	40	55	30	25	5

Solution

Details of workers' income

Income (in thousands) (X)	No. of workers (f)	Cumulative frequency (cf)
0-5	10	10
5-10	15	25
10-15	40	65
15-20	55	120
20-25	30	150
25-30	25	175
30-35	5	180

Total number of workers (N) = 180

The position of the first quartile = $\frac{N}{4}$ th item = $\frac{180}{4}$ th item = 45th item

The class interval having the 45th item is (10 – 15)

Now, lower limit of the first quartile class interval (L) = 10

Cumulative frequency of the class preceding the first quartile class (cf) = 25

Frequency of the first quartile class (f) = 40

Length (width) of the first quartile class interval (h) = 15 - 10 = 5

$$\begin{aligned}\text{We know that, first quartile } (Q_1) &= L + \frac{\frac{N}{4} - cf}{f} \times h \\ &= 10 + \frac{45 - 25}{40} \times 5 \\ &= 10 + \frac{100}{40} \\ &= 10 + 2.5 = 12.5\end{aligned}$$

Again, the position of the second quartile or median = $\frac{N}{2}$ th item = $\frac{180}{2}$ th item = 90th item

The class interval having 90th item is (15 - 20)

Now, lower limit of the median class (L) = 15

Cumulative frequency of the class preceding the median class (cf) = 65

Frequency of the median class (f) = 55

Length (width) of median class interval (h) = 20 - 15 = 5

$$\begin{aligned}\text{We know that, second quartile or median } (Q_2) &= L + \frac{\frac{N}{2} - cf}{f} \times h \\ &= 15 + \frac{90 - 65}{55} \times 5 \\ &= 15 + \frac{25}{11} \\ &= 15 + 2.27 = 17.27\end{aligned}$$

Now, the position of the third quartile = $\frac{3N}{4}$ th item = $\frac{3 \times 180}{4}$ th item = 135th item

The class interval having the 135th item is (20 - 25).

Now, lower limit of the third quartile class (L) = 20

Cumulative frequency of the class preceding the third quartile class (cf) = 120

Frequency of the third quartile class (f) = 30

Length (width) of third quartile class interval (h) = 25 - 20 = 5

We know that, third quartile (Q_3) = $L + \frac{\frac{3N}{4} - cf}{f} \times h$

$$= 20 + \frac{135 - 120}{30} \times 5$$

$$= 20 + \frac{15}{6} = 20 + 2.5$$

$$= 22.5$$

Hence, the first quartile (Q_1) = 12.5, the second quartile or median (Q_2) = 17.27 and the third quartile (Q_3) = 22.5.

Example 4

Students' expenditure on tiffin in one week and the number of students are given in If the upper quartile of the data is 460, then find the value of p.

Expenditure (Rs.)	100-200	200-300	300-400	400-500	500-600
No. of students	15	18	P	20	17

Solution

Table to find quartile

Expenditure (Rs) (X)	No. of students (f)	Cumulative frequency (cf)
100-200	15	15
200-300	18	33
300-400	p	33 + p
400-500	20	53 + p
500-600	17	70 + p
	N = 70 + p	

Total number of students (N) = 70 + p

Since, third quartile (Q_3) = 460

The third quartile class interval is (400 - 500).

Now, lower limit of the third quartile class (L) = 400

Cumulative frequency of the class preceding the third quartile class (cf) = 33 + p

Frequency of the third quartile class (f) = 20

Length (width) of the third quartile class interval (h) = 500 - 400 = 100

We know that, third quartile (Q_3) = $L + \frac{\frac{3N}{4} - cf}{f} \times h$

$$\text{or, } 460 = 400 + \frac{\frac{3(70+p)}{4} - (33+p)}{20} \times 100$$

$$\text{or, } 460 - 400 = \frac{210 + 3p - 132 - 4p}{4 \times 20} \times 100$$

$$\text{or, } 60 = \frac{78 - p}{4} \times 5$$

$$\text{or, } 78 - p = \frac{60 \times 4}{5} = 48$$

$$\text{or, } 78 - 48 = p$$

$$\text{or, } 30 = p$$

$$\therefore p = 30$$

Hence, the missing frequency (p) = 30

Exercise 13.4

1. Find the value of Q_1 and Q_3 from the data given below:

(a) 10, 12, 14, 11, 22, 15, 27, 14, 16, 13, 25

(b)

Marks obtained	42	48	49	53	56	59	60	65	68	70
No. of students	2	3	5	8	9	11	7	8	6	4

2. Find the value of Q_1 and Q_3 from the data given below:

(a)

Age (yrs)	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of students	5	12	25	26	24	28	20	15

(b)

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	12	13	11	7

(c)

Height (cm)	100-110	110-120	120-130	130-140	140-150	150-160	160-170
No. of students	3	4	9	15	20	14	7

(d)

Wages (Rs.)	100-150	150-200	200-250	250-300	300-350	350-400
No. of workers	6	11	21	34	25	22

(e)

Marks obtained	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	8	12	15	14	12	9	10

(f)

Time (minutes)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	3	10	6	4	2

(g)

Time (minutes)	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	2	5	8	6	4	5

3. (a) If $Q_1 = 8$, what is the value of k ?

Age(yrs)	0-6	6-12	12-18	18-24	24-30	30-36
No. of pupils	9	6	5	k	7	9

- (b) If $Q_1 = 31$, what is the missing frequency?

Class interval	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	5	?	8	7	6

- (c) If $Q_1 = 51.75$, what is the value of q ?

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	8	10	14	q	3	1

4. Find the value of Q_1 and Q_3 from the data given below:

(a)

Height (cm)	<125	<130	<135	<140	<145	<150	<155
No. of students	0	5	11	24	45	60	72

(b)

Weight (lbs)	110-119	120-129	130-139	140-149	150-159	160-169	170-179	180-189
Frequency	5	7	12	20	16	10	7	3

(c)

Expenditure (per day)	Less than 100	100-200	200-300	300-400	400-500	More than 500
Frequency	22	34	52	20	19	13

(d)

Marks obtained	less than 20	less than 40	less than 60	less than 80	less than 100
No. of students	21	44	66	79	90

5. (a) The data given below represent the marks obtained by 30 students in an internal examination. Find the first and third quartiles by tabulating the data taking a class interval of 10.

42, 65, 78, 70, 62, 50, 72, 34, 30, 40, 58, 53, 30, 34, 51, 54, 42, 59, 20, 40, 42, 60, 25, 35, 35, 28, 46, 60, 47, 52

- (b) The number of eggs produced everyday in a chicken farm is given below. Find the first and third quartiles by tabulating the data taking a class interval of 20.

32, 87, 17, 51, 99, 79, 64, 39, 25, 95, 53, 49, 78, 32, 42, 48, 59, 86, 69, 57, 15, 27, 44, 66, 77, 92

Project work

Ask and write the total marks obtained by the 100 students of classes 9 and 10 in an internal examination out of 100 full marks.

- (a) Construct the frequency distribution table with a suitable class interval of the given data.
 (b) Prepare the more than and less than cumulative frequency table by using the given data.
 (c) Prepare a report of all the work in sequential order and present it in the classroom.

Answer

1. (a) 12, 22 (b) 53, 65
 2. (a) 7.74, 13.73 (b) 42.08, 64.09 (c) 131.33, 152.14 (d) 230.35, 334.5
 (e) 40, 98.33 (f) 18.3, 37.5 (g) 30.31, 40.63
 3. (a) 8 (b) 10 (c) 16
 4. (a) 137.69, 148 (b) 136.16, 159.5 (c) 152.94, 360 (d) 20.45, 61.15
 5. (a) 37.5, 59.28 (b) 38, 61.57

Lesson 14

Probability

14.0 Review

Make a suitable group and discuss the following questions, and find the answers.

- (a) What is the probability of getting even or prime number when a die is thrown one time?
- (b) What is the probability of getting an ace or face cards when a card is drawn from a well shuffled pack of cards?
- (c) What is the probability of getting both heads when two coins are tossed together?

On the basis of the above questions

- (a) Write the sample space of each experiment.
- (b) Write each event.
- (c) Find the probability of each event.
- (d) Find what each event is.

Each group should work in group and present your task to the class.

14.1 Principles of Probabilities

(a) Mutually Exclusive

Activity 1

When a dice is tossed once, let's write the event where

- (a) Even number or odd number
- (b) Let's write the event where there is an even number or prime number.

Here, the set of numbers that appears on the top of the dice $(S) = \{1, 2, 3, 4, 5, 6\}$

Let, the sset of even numbers that appears on the top (A) ,

Set of odd numbers that appears on the top (B) and

Set of prime numbers that appears on the top (C)

$$A = \{2, 4, 6\}$$

$$B = \{1, 3, 5\}$$

$$C = \{2, 3, 5\}$$

From the above example, we can see that event A and event B do not have their common element. So that event A and event B can not occur at the same time. Therefore, A and B are called mutually exclusive events.

Again, in events A and C, 2 is the common element. When we get 2 after rolling a die it might be even number or prime number. So that, there is a probability of getting event A which is equal to getting event C.

Similarly, what events happen in B and C? Discuss and write.

In an experiment, if the occurrence of any one event excludes the occurrence of the other event, then such events are called mutually exclusive events. In the same way, when one event occurs, another event may also occur, which is not a mutually exclusive event. In the above example events A and B are mutually exclusive events but events A and C are not mutually exclusive event.

(b) Addition Law of Probability

Activity 2

Write the sample space (S) when a die is rolled. The event of even number is (A), the event of odd number is (B), event of prime number is (C) and write its cardinality and probability. Based on this, answer the questions asked below and reach in conclusion.

(a) What is the probability of getting even number or odd number?

(b) What is the probability of getting even number or prime number?

The possible outcomes when a die is thrown $(S) = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Events	Cardinality	Probability
Getting even number $(A) = \{2, 4, 6\}$	$n(A) = 3$	$p(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$
Getting odd number $(B) = \{1, 3, 5\}$	$n(B) = 3$	$p(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$
Getting prime number $(C) = \{2, 3, 5\}$	$n(C) = 3$	$p(C) = \frac{n(C)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(a) Getting even number or odd number = $\{2, 4, 6\}$ or $\{1, 3, 5\}$

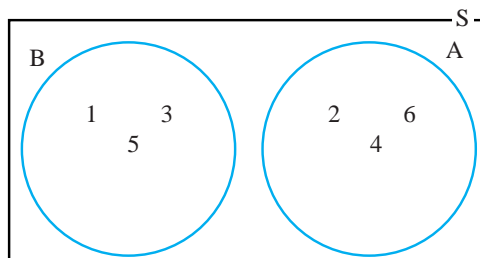
$$(A \cup B) = \{1, 2, 3, 4, 5, 6\}$$

$$\therefore n(A \cup B) = 6$$

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{6}{6} = 1$$

$$\text{Again, } P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore P(A \cup B) = P(A) + P(B)$$



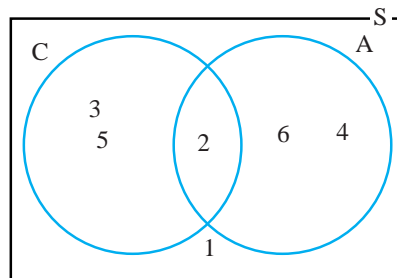
(b) Getting even number or prime number
= $\{2, 4, 6\}$ or $\{2, 3, 5\}$

$$(A \cup C) = \{2, 3, 4, 5, 6\} \therefore n(A \cup C) = 5$$

$$P(A \cup C) = \frac{n(A \cup C)}{n(S)} = \frac{5}{6}$$

$$\text{Here, } (A \cap C) = \{2\} \therefore n(A \cap C) = 1$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{1}{6}$$



If A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$ and if A and C are not mutually exclusive event $P(A \cup C) = P(A) + P(C) - P(A \cap C)$. This is called addition law of probability.

Example 1

What is the probability of getting that a name beginning with W or a name beginning with T when one of the name of days is taken from a week.

Solution

Here, $S = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$;
 $n(S) = 7$

Let, $A = \{\text{Days starting from W}\} = \{\text{Wednesday}\}$; $n(A) = 1 \therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$

$B = \{\text{Days starting from T}\} = \{\text{Tuesday, Thursday}\}$; $n(B) = 2 \therefore P(B) = \frac{n(B)}{n(S)} = \frac{2}{7}$

The probability of getting the names starting from W or T = $P(A \cup B) = ?$

A and B are mutually exclusive events so that,

$$P(A \cup B) = P(A) + P(B) = \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$$

Hence, the probability of getting the names starting from W or T = $\frac{3}{7}$.

Example 2

A bag contains 4 black, 6 yellow and 5 red balls of same shape and size.

- (a) If a ball is drawn randomly, find the probability of getting either black or red ball.
- (b) Find the probability of getting either yellow or red ball when a ball is drawn randomly.

Solution

Here, total number of balls $n(S) = (4 + 6 + 5) = 15$

Number of black ball $n(B) = 4$, probability of getting black ball $P(B) = \frac{n(B)}{n(S)} = \frac{4}{15}$

Number of yellow ball $n(Y) = 6$, probability of getting yellow ball $P(Y) = \frac{n(Y)}{n(S)} = \frac{6}{15}$

Number of red ball $n(R) = 5$, probability of getting red ball $P(R) = \frac{n(R)}{n(S)} = \frac{5}{15}$

- (a) Probability of getting either black or red ball $P(B \cup R) = ?$

Events B and R are mutually exclusive events, so by addition law of probability;

$$P(B \cup R) = P(B) + P(R) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$$

Hence, the probability of getting either black or red ball is $\frac{3}{5}$

- (b) Probability of getting either yellow or red ball $P(Y \cup R) = ?$

Events Y and R are mutually exclusive events, so by addition law of probability;

$$P(Y \cup R) = P(Y) + P(R) = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

Hence, the probability of getting either yellow or red ball is $P(Y \cup R) = \frac{11}{15}$.

Example 3

From the set of 20 cards numbered from 1 to 20, a card is drawn randomly. Find the probability of getting a number that is either divisible by 4 or by 3.

Solution

Here, total number of number cards $n(S) = 20$

Let A be the event of getting a number divisible by 4 and B be the event of getting a number divisible by 3.

Now, $A = \{4, 8, 12, 16, 20\}$

$$n(A) = 5, \quad \therefore P(A) = \frac{n(A)}{n(S)} = \frac{5}{20}$$

$$B = \{3, 6, 9, 12, 15, 18\} \text{ and } n(B) = 6, \quad \therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{20}$$

$A \cap B = \{12\}$; (12 is in both events A and B)

$$n(A \cap B) = 1; \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{20}$$

$$P(A \cup B) = ?$$

Here, events A and B are not mutually exclusive events so by addition law of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{20} + \frac{6}{20} - \frac{1}{20}$$

$$= \frac{5+6-1}{20}$$

$$= \frac{10}{20}$$

$$= \frac{1}{2}$$

Hence, the probability of getting a number is either divisible by 4 or by 3 is $\frac{1}{2}$.

Example 4

From a well-shuffled pack of 52 playing cards, a card is drawn randomly. Find the probability of getting the king, queen or a jack.

Solution

Here, total number of cards $n(S) = 52$

Let K, Q and J be the events of getting the king, queen and jack respectively.

$$\text{Now, No. of king } n(K) = 4; \quad P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{No. of queen } n(Q) = 4; \quad P(Q) = \frac{n(Q)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{No. of jack } n(J) = 4; \quad P(J) = \frac{n(J)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Events K, Q and J are mutually exclusive events, so by addition law of probability;

$$P(K \cup Q \cup J) = P(K) + P(Q) + P(J).$$

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}$$

Hence, the probability of getting the king, a queen or jack is $\frac{3}{13}$.

Exercise 14.1

1. Find out whether the given events are mutually exclusive or not.

- (a) When a coin is tossed, A = getting the head (H) and B = getting the tail (T).
- (b) When a dice is rolled, P = getting an even number and Q = getting an odd number die.
- (c) A card is drawn from well-shuffled pack of cards, F = getting a face card and A = getting a spade.
- (d) A card is drawn from well-shuffled pack of cards, T = getting 10 and A = getting an ace.
- (e) A bag contains 5 white, 8 green and 7 blue balls. A ball is drawn at random, G = getting green ball and B = getting blue ball

2. Find the probability of the given events.

- (a) Getting at least one head (H) when two coins are tossed
- (b) Getting prime number when a die is rolled
- (c) Getting a face card when a card is drawn from a well-shuffled pack of cards.
- (d) According to the English Months, a boy is born in a month with 30 days.
- (e) Getting a white ball when a ball is drawn from a bag containing 4 white, 7 green and 5 blue balls.

3. What is the probability of the following events? Find.

- (a) A bag contains 6 red, 5 yellow and 7 blue identical balls. If a ball is drawn randomly from the bag, it may be either, red or blue ball.
- (b) Three coins are tossed together, getting all three are head (H) or all three are tail.
- (c) When a die is rolled, getting prime number or 4.
- (d) When a card is drawn, from a well-shuffled pack of cards, 10 card or ace card (A).
- (e) When a card is drawn from a well-shuffled pack of cards, face card or spade cards.

4. Find the probability of the following events.

- (a) When a letter is drawn from MATHEMATICS, getting the letter M or T.
- (b) When a letter is drawn from STATISTICS, getting the letter S or T.

- (c) When a letter is drawn from RHODODENDRON, getting the letter O or D.
- (d) Out of 15 students of a class, 8 students opted English, 9 students Mathematics and 4 students opted both subjects. When a student is selected at random, what is the probability of getting Mathematics or English?

Answer

1. Show it to your teacher.
2. (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{13}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$
3. (a) $\frac{13}{18}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{2}{13}$ (e) $\frac{11}{26}$
4. (a) $\frac{4}{11}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{13}{15}$

14.2 Independent and Dependent Events

Activity 3

Compare the probabilities obtained from the following two cases

A bag contains 5 red, 7 green and 4 blue identical balls.

- (i) First condition (with Replacement)
 - (a) What is the probability that the first ball is red?
 - (b) If the same ball (red ball) is placed in the same bag and the second ball is drawn, what is the probability that the second ball will be red?
- (ii) Second condition (Without replacement)
 - (a) What is the probability that the first ball is red?
 - (b) If the same ball (red ball) is not replaced in the same bag and the second ball is drawn, what is the probability that the second ball will be red?

Comparing the probabilities obtained from the above two conditions

(i) First condition (With replacement)	(ii) Second condition (Without replacement)
<p>Number of red balls $n(R) = 5$</p> <p>Probability that the first ball is red $P_1(R)$</p> $= \frac{n(R)}{n(S)}$ $= \frac{5}{16}$ <p>Now, there are 15 balls in the bag. The first drawn ball is replaced, so that there are again 16 balls in the bag.</p> <p>The probability that the second ball is red $P_2(R) = \frac{n(R)}{n(S)} = \frac{5}{16}$. It's the same as first.</p>	<p>Number of red balls $n(R) = 5$</p> <p>Probability that the first ball is red $P_1(R)$</p> $P_1(R) = \frac{n(R)}{n(S)} = \frac{5}{16}$ <p>Now, there are 15 balls in the bag. The first drawn ball is not replaced, so that there are again 15 balls in the bag.</p> <p>The probability that second ball is red $P_2(R) = \frac{n(R)}{n(S)} = \frac{4}{15}$. It is different than the first.</p>
The occurrence of the first event does not affect the occurrence of the second event. So, these are independent events.	The occurrence of the first event affects the occurrence of the second event. So, these are dependent events

In two or more events, the occurrence or non-occurrences of any one event does not affect the occurrence or non-occurrences of any other events are called independent events. Similarly, in two or more events, the occurrence of any one event affects the occurrences of any other events are called dependent events.

Example 1

When a coin and a die are tossed at the same time, what is the event that head (H) comes up on the coin and 4 on the dice?

Solution

When a coin is tossed and a die is rolled simultaneously, we can get head (H) or tail (T) in the coin and we can get any numbers from 1 to 6 in the die. So, the event obtained in the dice does not affect the event we get in the coin. Hence, it is an independent event.



Example 2

Two cards are drawn from a well-shuffled deck of 52 playing cards one after another without replacement. What is the event that both cards are the king (K)?

Solution

Here, let K be the event of the king card in a sample space.

There are 4 king cards in a deck of 52 playing cards.

Therefore, $n(K) = 4$ and $n(S) = 52$

Probability of getting the king $P_1(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

If card 'K' obtained from the first case is not replace there are only 51 cards.

Now, in the second case, $n(S) = 51$ and $n(K) = 3$

Probability of getting the king second time $P_2(K) = \frac{n(K)}{n(S)} = \frac{3}{51} = \frac{1}{17}$. In this case, the probability of getting the king second time depends on the probability of getting the king in the first time. Hence, these are dependent events.

14.3 Multiplication Principle of Probability

Activity 4

Form pairs and solve the given problems.

When a coin is tossed and a cubical die is rolled,

- (a) Write the possible sample space.
- (b) What is the probability of getting the head (H) in a coin and 4 in a die?
- (c) What types of events are these?

When a coin is tossed and a die is rolled simultaneously, we can get head (H) or tail (T) in a coin and we can get any numbers from 1 to 6 in a die. So, the events obtained in a die does not affect the event we get in a coin. Hence, these are independent events.

For a coin, sample space

$$(S_1) = \{H, T\} \quad \therefore n(S_1) = 2$$

$$\text{For a cubical die, sample space } (S_2) = \{1, 2, 3, 4, 5, 6\} \quad \therefore n(S_2) = 6$$

When a coin is tossed and a die is thrown, then the possible outcomes are $(S) = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\} \therefore n(S) = 12$.

Now, what is the probability of getting H in a coin and 4 in a die?

Let, A be an event of getting H in the coin, $\therefore n(A) = 1$

Let, B be an event of getting 4 in the die. $\therefore n(B) = 1$

H in the coin and 4 in the die $(A \cap B) = \{(H, 4)\}$; $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S_1)} = \frac{1}{2} \quad \text{and} \quad P(B) = \frac{n(B)}{n(S_2)} = \frac{1}{6}$$

$$\text{Similarly, } P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{12}$$

Can you get a conclusions from the above solution?



$$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \cap B)$$

If two events A and B are independent events then, $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$.

Example 3

A coin is tossed and a spinner with three colors; green, blue and red is spun together. What is the probability of getting 'T' in the coin and the needle of the spinner stopping on green?

Solution

Here, sample space of the coin $(S_1) = \{H, T\}$

$$\therefore n(S_1) = 2$$

$$\text{Probability of getting T on the coin } P(T) = \frac{n(T)}{n(S_1)} = \frac{1}{2}$$

Let G be the event of green. Similarly, the sample space of the spinner $(S_2) = \{\text{green, blue, red}\} \therefore n(S_2) = 3$

$$\text{Probability of the needle of the spinner stopping on green } P(G) = \frac{n(G)}{n(S_2)} = \frac{1}{3}.$$

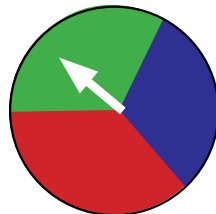
Probability of getting 'T' in the coin and the needle of the spinner stopping on green $P(T \cap G) = ?$

Since, getting 'T' in the coin and the needle of the spinner stopping on green is independent event, $P(T \cap G) = P(T) \times P(G) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$

Alternative method

The possible outcomes of a coin and spinner $S = \{(H, \text{Green}), (H, \text{Blue}), (H, \text{Red}), (T, \text{Green}), (T, \text{Blue}), (T, \text{Red})\} \therefore n(S) = 6$

The event of getting 'T' in the coin and the needle of the spinner stopping on green $(A) = \{(T, \text{Green})\} \therefore n(A) = 1$ Probability of getting 'T' in the coin and the needle of the spinner stopping on green $P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}.$



Example 4

Two cards are drawn randomly in succession with a replacement from a well-shuffled pack of 52 cards. Find the probability of getting the first king card (K) and the second an ace card (A).

Solution

Here, there are 4 king cards in a deck of 52 cards. 'S' be the sample space.

Now, $n(S) = 52$, $n(K) = 4$

The probability of getting the first card king is $P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

If the first drawn card is replaced, there are 52 cards again. Number of ace cards also 4.

Now, $n(S) = 52$, $n(A) = 4$

The probability of getting the second card ace is $P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

Probability of getting the first king card (K) and the second an ace card (A) = $P(K \cap A) = ?$

Since, K and A are independent events;

We know that, $P(K \cap A) = P(K) \times P(A) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Hence, the probability of getting the first king card (K) and the second an ace card (A) is $= \frac{1}{169}$.

Example 5

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. Find the probability of getting the first king card (K) and the second an ace card (A).

Solution

Here, there are 4 king cards in a deck of 52 cards. Let, 'S' be the sample space.

Now, $n(S) = 52$, $n(K) = 4$

The probability of getting the first card king is $P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

If the second card is drawn without replacement of the first drawn card, so there are $52 - 1 = 51$ cards remaining. But, number of ace card are still 4.

Now, $n(S_1) = 51$, $n(A) = 4$

The probability of getting the second card ace is $P(A) = \frac{n(A)}{n(S_1)} = \frac{4}{51}$

Probability of getting the first king card (K) and the second an ace card (A) = $P(K \cap A) = ?$

Since, K and A are dependent events;

We know that, $P(K \cap A) = P(K) \times P(A) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

Hence, the probability of getting the first king card (K) and the second an ace card (A) is $= \frac{4}{663}$.

Example 6

A bag contains 5 blue and 6 red marbles. Two marbles are drawn randomly one after another, what is the probability of getting the first one red and the second one blue?

(a) Second marble is drawn with replacing the first drawn marble

(b) Second marble is drawn without replacing the first drawn marble

Solution

Here, let, 'B' and 'R' be the events of red and blue marbles respectively. 'S' be the total number of balls in the bag.

Now, $n(B) = 5$, $n(R) = 6$ and $n(S) = 5 + 6 = 11$

Probability of getting a blue marble $P(B) = \frac{n(B)}{n(S)} = \frac{5}{11}$

Probability of getting a red marble $P(R) = \frac{n(R)}{n(S)} = \frac{6}{11}$

a) The second marble is drawn with a replacement of the first drawn marble

Probability of getting the first one red and the second one blue $P(B \cap R) = ?$

Since, B and R are independent events;

We know that, $P(R \cap B) = P(R) \times P(B) = \frac{6}{11} \times \frac{5}{11} = \frac{30}{121}$

Hence, the probability of getting the first one red and the second one blue $= \frac{30}{121}$.

(b) Second marble is drawn without replacement of the first drawn marble

The probability of getting the first marble red $P(R) = \frac{6}{11}$

The second marble is drawn without replacement of the first drawn marble.

So, there are $11 - 1 = 10$ marbles remaining. But the number of blue marbles are 5.

Now, $n(S_1) = 10$, $n(B) = 5$

The probability of getting the second blue marble $P(B) = \frac{5}{10}$

Probability of getting the first red marble and second blue marble $P(R \cap B) = ?$

Since, the first drawn marble is not replaced, so R and B are dependent events;

We know that, $P(R \cap B) = P(R) \times P(B) = \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

Exercise 14.2

1. When a coin is tossed and a dice is rolled simultaneously. What is the probability of getting tail (T) on coin and 3 on dice?
2. **A box contains 2 green, 3 red and 5 black balls of same shape and size. Two balls are drawn randomly and replaced. Then after, another ball is drawn. Find the probabilities of getting following balls.**
 - (a) Both of them are of the same color.
 - (b) Both of them are of the different color.
 - (c) At least one ball is red or black
3. **A box contains 2 green, 3 red and 5 black balls of same shape and size. Two balls are drawn randomly and not replaced. Then after, another ball is drawn. Find the probabilities of getting following balls.**
 - (a) Both of them are of the same color.
 - (b) Both of them are of the different color.
 - (c) At least one ball is red or black
4. A bag contains 7 red and 8 yellow balls of same shape and size. Two balls are drawn randomly one after another. Find the probabilities of getting both balls are red or yellow. (First drawn ball is not replaced in a bag)
5. **Two cards are drawn randomly from a well shuffled deck of 52 cards in succession without replacement.**
 - (a) What is the probability of getting both are ace cards?
 - (b) What is the probability of getting one is ace card or other is king card?
6. A bag contains one red, one green and one black marble of the same shape and size. Two marbles are drawn randomly and not replaced. Then after, another marble is drawn. Find all the probabilities.

Project work

Where is the use of probability in our daily life? Find. Prepare an article about its positive use and present it in the classroom.

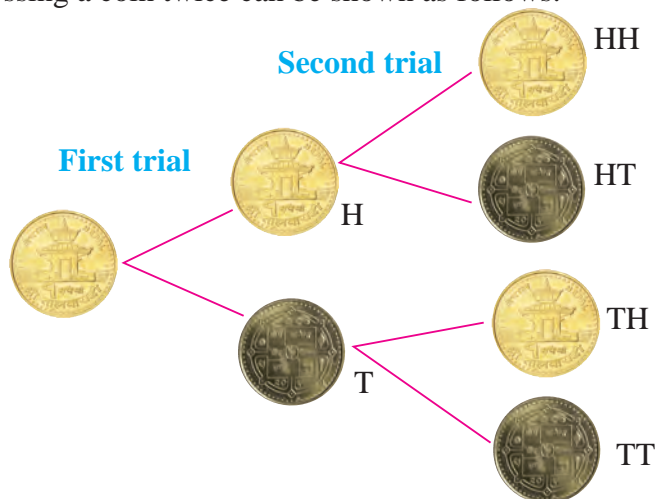
Answers

1. $\frac{1}{12}$
2. (a) $\frac{12}{25}$ (b) $\frac{31}{50}$ (c) $\frac{13}{20}$
3. (a) $\frac{14}{45}$ (b) $\frac{31}{45}$ (c) $\frac{57}{90}$
4. $\frac{7}{15}$
5. (a) $\frac{1}{221}$ (b) $\frac{8}{663}$
6. $\frac{1}{6}$

14.3 Tree Diagram

Activity 5

Make a list of the events that occur when a coin is tossed two times. For example; the results of tossing a coin twice can be shown as follows.



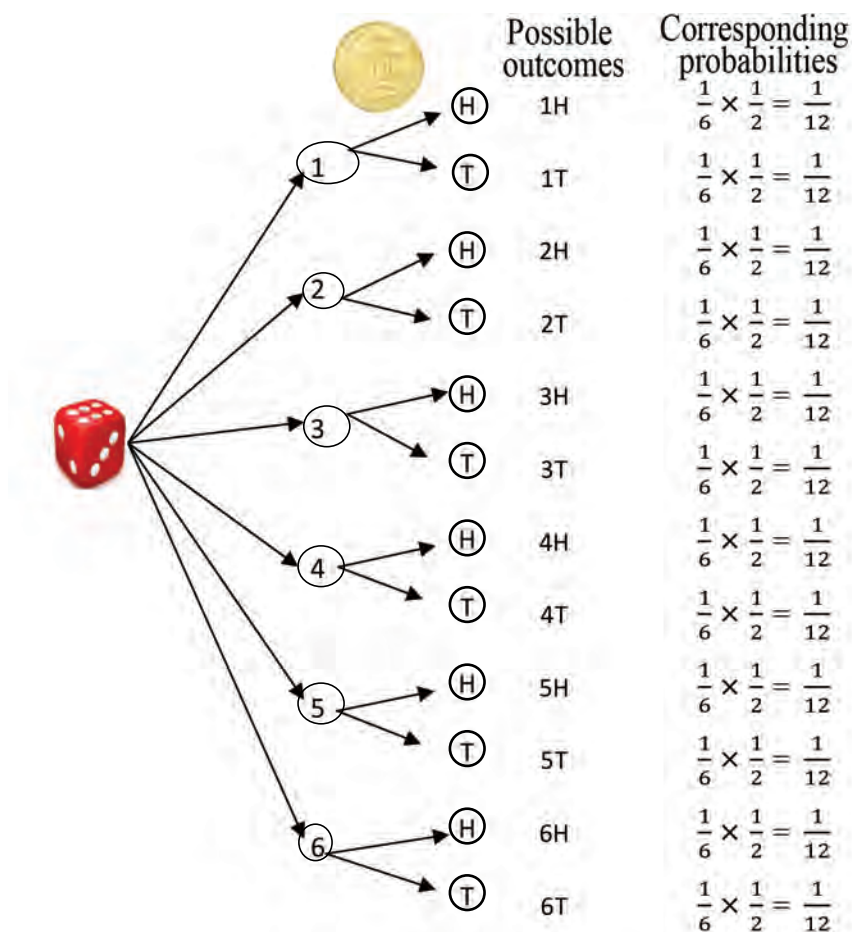
A picture like the above, which presents the results from a random experiment is called tree a diagram. Every branch of a tree diagram shows the probability of respective event. It gives all the possible outcomes and their probabilities. Like, from the above experiment sample space $S = \{ HH, HT, TH, TT \}$.

Example 1

Prepare a tree diagram showing the results and probabilities that may come when a die is rolled and a coin is tossed simulation?

Solution

When a dice is rolled and a coin is tossed simultaneously the following tree diagram showing the events and their probabilities:



Example 2

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. Draw a tree diagram to represent the probabilities of getting a red or black card.

Solution

Here,

Two cards are drawn randomly from a well-shuffled deck of 52 cards in succession without replacement. The following tree diagram shows the events and their probabilities:

- (b) The sample space (S) = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} $\therefore n(S) = 8$

Let A be the event of getting at least 2 heads then, (A) = {HHH, HHT, HTH, THH}
 $\therefore n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Hence, the probabilities of getting at least two heads is $\frac{1}{2}$.

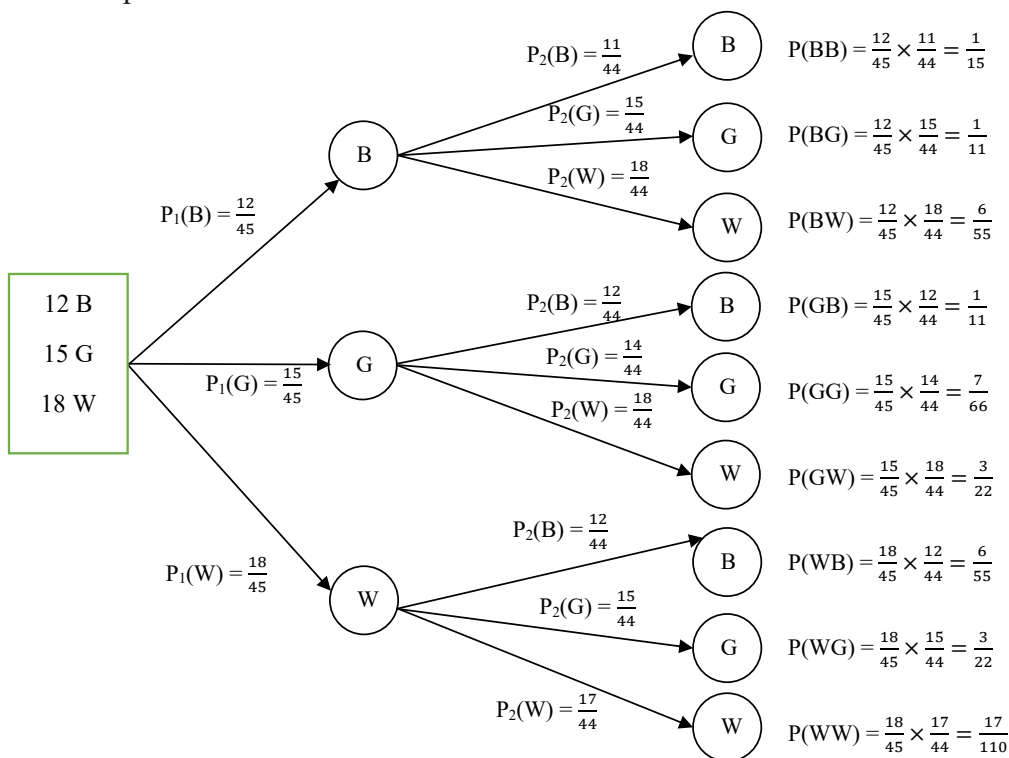
Example 4

A bag contains 12 blue, 15 green and 18 white identical balls. Two balls are drawn one after another without replacement. Show the probabilities of all the outcomes. By using a tree diagram, find the probabilities of the following.

- Both balls are blue.
- First ball is white and the second ball is green.
- One ball is blue and the other ball is white.

Solution

A bag contains 12 blue, 15 green and 18 white identical balls. Two balls are drawn one after another without replacement. The following tree diagram shows the events and their probabilities:



Now, from the tree diagram,

- (a) The probability of both balls are blue $P(BB) = \frac{1}{15}$
- (b) The probability of the first ball is white and the second ball is green $P(WG) = \frac{3}{22}$
- (c) The probability of one ball is blue and the other ball is white,
 $P(BW) + P(WB) = \frac{6}{55} + \frac{6}{55} = \frac{12}{55}$

Exercise 14.3

1. **A coin is tossed three times. Draw a probability tree diagram to show all the possible outcomes. Find the probabilities of the following.**
 - (a) All three are tails (T)
 - (b) At least two heads (H)
 - (c) Three tails (T)
2. **A spinner with three colors viz red, blue and brown is spun and a coin is tossed together. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.**
 - (a) The spinner can land on red and head (H) on coin.
 - (b) The spinner can land on brown and tail (T) or head (H) on coin.
3. **A coin is tossed and a die is rolled one after another. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.**
 - (a) Head (H) on the coin and even number on the dice.
 - (b) Tail (T) on the coin and square number on the dice.
4. Prepare a tree diagram to show the probability of getting a card (spade club, diamond and heart) from a well-shuffled pack of 52 cards and when a coin is tossed. Draw a probability tree diagram to show all the possible outcomes. By using a probability tree diagram, find the probabilities of the following.
 - (a) Red on the card and head (H) on the coin.
 - (b) Black on the card and tail (T) on the coin.
5. A bag contains 7 red and 5 green marbles. Three marbles are drawn one after another; (i) with replacement (ii) without replacement
Draw a probability tree diagram to show all the possible outcomes.

Project work

Take a deck of 52 cards. Three cards are drawn one after another at random without replacement. Out of these three cards; draw probability diagram for the following.

- (a) All three cards are spade.
- (b) Only two cards are spade.
- (c) Only one card is spade.
- (d) All three cards are not spade.

Answer

1. $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ 2. (b) $\frac{1}{6}$ (b) $\frac{1}{3}$
3. (a) $\frac{1}{4}$ (b) $\frac{1}{6}$ 4. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ 5. Show it to your teacher.

Mixed Exercise

1. The heights (in cm) of 50 people are given below.

Height (in cm)
125, 137, 155, 149, 122, 128, 133, 144, 115, 118, 142, 145, 151, 157, 159, 160, 165, 162, 156, 158, 155, 141, 147, 149, 148, 159, 154, 155, 166, 168, 169, 172, 174, 173, 176, 161, 164, 163, 149, 150, 154, 153, 152, 164, 158, 159, 162, 157, 156, 155

- (a) Construct a frequency distribution table with the class interval 10 of the given data.
- (b) Find the mean from the above frequency distribution table.
- (c) Show the histogram of frequency distribution table prepared in (a).
- (d) What is the median value of the above data?

2. The table given below represents the weight (in kg) of students of a school.

Weight (in kg)
18, 20, 13, 24, 35, 34, 56, 45, 33, 23, 24, 56, 33, 22, 26, 35, 39, 44, 42, 47, 46, 48, 55, 51, 44, 40, 47, 49, 34, 31, 28, 29, 35, 39, 28, 48, 51, 50, 47, 23, 19, 27, 57, 42, 33, 23, 38, 36, 45, 45, 37, 29, 27, 22, 28, 36, 35, 57, 54, 40, 50, 30, 29

- (a) Construct a frequency distribution table with the class interval 5 of the given data.
- (b) Prepare a less than and more than cumulative frequency table.
- (c) Find the difference of mode and median.

3. In a survey conducted in a community related to the job holder's age obtained the following data: :

Age (year)	0-15	15-30	30-45	45-60	60-75
No. of person	5	6	10	6	3

- (a) In the above table, which age group has the maximum number of employees?
- (b) What is the average age of the employees?
- (c) Angel said that mean and median lie on the same class interval. Is it right?
4. The details of monthly electricity consumption by 40 households of Thaha Municipality are as follows:

Consumption unit	10-20	20-30	30-40	40-50	50-60	60- 70	70- 80
No. of household	5	6	8	9	7	4	1

- (a) From the above table, prepare the less than and more than cumulative frequency table.
- (b) On the basis of the answer of (a), draw the cumulative frequency curve.
- (c) Do the mean and median lie on the same class interval?
5. The data given below represents the number of patients admitted in a hospital in a week.

Age (yrs)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of patients	4	5	8	13	12	8	4	9	7

- (a) From the above data, find the first and third quartiles.
- (b) Which value of the given data divides the number of the patients into two equal parts?
- (c) Niruta said that mode and median lie on the same class interval. Is she correct right?
6. The given data represents the monthly expenditure (in thousands) of the families of Lamapatan

Expenditure(000)	10-20	20-30	30-40	40-50	50-60
No. of family	3	2	6	5	4

- (a) From the above data, write the class interval of mode.
- (b) From the above data, what is the expenditure of the maximum family?
- (c) From the above data, find the first and third quartiles.

- 7. The marks obtained in Mathematics by the students of class 10 in the second terminal examination are given below in the table.**

Marks obtained	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
No. of students	2	4	5	3	2	4

- (a) How many students of class 10 obtained the marks less than 20?
- (b) Construct the cumulative frequency table on the basis of the given table.
- (c) Find the values which divide the data given in the above table into 4 equal parts.
- (d) What is the maximum number of students who obtained the marks less than the median, based on the median?
- 8. Three coins are tossed together then,**
- (a) Write the possible outcomes
- (b) Find the probability of getting only one head.
- (c) Find the probability of getting at least 2 tails (T).
- (d) Find the probability of not getting head (H).
- (e) Find the probability of getting 2 heads (H) or not to get H.
- 9. A card is drawn randomly from a well-shuffled deck of 52 cards then,**
- (a) What is the probability of getting an ace?
- (b) Find the probability of getting a spade or diamond.
- (c) Find the probability of getting a spade or ace.
- (d) Find the probability of getting a club or face card.
- 10. A coin is tossed and a spinner with 4 colours (blue, green, red, purple) is spun together:**
- (a) Draw a probability tree diagram to show all the possible outcomes. Find the possible outcomes of this experiment separately and together.
- (b) Find the probability of getting the head (H) in the coin and landing the needle at green or blue.
- (c) Find the probability of getting head (H) in a coin and landing the needle at green.
- 11. Two cards are drawn randomly from a well-shuffled deck of 52 cards one after another, then,**
- (a) If the first drawn card is replaced and drawn another card, what is the probability of getting both cards are spade?
- (b) If the first drawn card is not replaced and second card is drawn, what is the probability of getting the first card spade and the second card heart?
- (c) Show the probability tree diagram for (a) and (b) separately.

- 12. A bag contains 7 red and 8 yellow balls of the same shape and size:**
- (a) If the balls are drawn one after another (without replacement), find the probability of getting both balls are red.
 - (b) If two balls are drawn one after another (with replacement), find the probability of getting both balls are red.
 - (c) Ramila said that both of the above conditions are independent. Is she correct? Write.
- 13. A bag contains one red, one green and one black ball of the same shape and size. A ball is drawn randomly and without replacing the ball, another ball is drawn from the bag. Then,**
- (a) Show the probability tree diagram for the events.
 - (b) Find the probability of the events.
- 14. A coin is tossed and a die is thrown then,**
- (a) Draw a probability tree diagram to show all the possible outcomes.
 - (b) By using the probability tree diagram, find the probabilities of the following.
 - (i) Head (H) in the coin and even number in the dice.
 - (ii) Tail (T) in the coin and square number in the dice.
 - (c) Are the events of (b) independent? Write with reason.
- 15. A card is drawn randomly from a well-shuffled deck of 52 cards (spade, club, diamond and heart) and a coin is tossed.**
- (a) Draw the probability tree diagram to show all the possible outcomes.
 - (b) On the basis of probability tree diagram, write the sample space.
 - (c) By using a probability tree diagram, find the probabilities of the following:
 - (i) Red in cards and H on the coin.
 - (ii) Black in cards and tail (T) in the coin.
 - (d) Bikash said that the events obtained from the cards and the coin is dependent. Is that statement, correct? Write.
- 16. A bag contains 7 red, and 5 green marbles. 2 marbles are drawn randomly one after other: (a) With replacement (b) Without replacement. Draw a probability tree diagram to show all the possible outcomes in the following conditions.**
- (i) On the basis of the tree diagram, what is the probability of getting both marbles red?
 - (ii) What is the probability of getting both marbles green?

- (iii) Is the probability of getting first red and the second green equal to the probability of getting the first green and the second red? (A marble is drawn and not replaced before drawing second.)

17. From the number cards number from 1 to 20, a card is drawn at random,

- Make a list of number cards divisible by 3 and divisible by 5.
- What is the probability of getting a number card having a number divisible only by 3 or divisible only by 5?
- Kopila said that the events in question (a) are not mutually exclusive events. Is Kopila's statement correct? Write with reason.
- What is the probability of getting a number card having a number either divisible by 3 or by 5?

18. In a bag, there are 5 red and 3 blue balls. A ball is drawn randomly and replaced before drawing the second ball then,

- Find the probability of getting both blue balls.
- What is the probability that none of them are blue?
- Show both the above probabilities in a tree diagram.

Answer

1 - 7 Show them to your teacher.

8. (a) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

- (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{1}{2}$

9. (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{4}{13}$ (d) $\frac{11}{26}$

10. (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ 11. (a) $\frac{1}{16}$ (b) $\frac{13}{204}$ (c) Show it to your teacher

12. (a) $\frac{1}{5}$ (b) $\frac{49}{225}$ (c) no

13. show them to your teacher

14. (b) (i) $\frac{1}{4}$ (ii) $\frac{1}{6}$ (iii) हृत्

15 - 16 show to your teacher.

17. (a) $M_3 = \{3, 6, 9, 12, 15, 18\}$, $M_5 = \{5, 10, 15, 20\}$ (b) $\frac{2}{5}$

- (c) yes (d) $\frac{9}{20}$

18. (a) $\frac{9}{64}$ (b) $\frac{25}{64}$

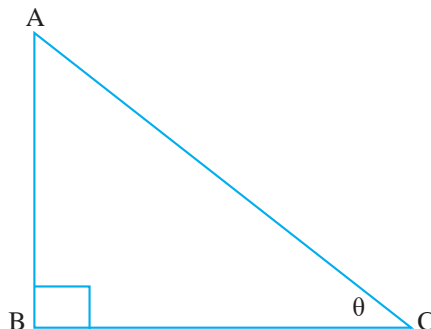
Lesson 15

Trigonometry

15.0 Review

A right angled triangle ABC is given. Based on this, discuss on the following questions.

- What is the area of the right angled triangle ABC?
- Find the trigonometric ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$.
- State the relationship between the sides of a right angled triangle.? What is this relation called?
- If $AB = 6$ cm and $AC = 10$ cm, then find the trigonometric ratios.
- What are the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$. When $\theta = 30^\circ$, $\theta = 45^\circ$, $\theta = 60^\circ$ and $\theta = 90^\circ$.



Values	0°	30°	45°	60°	90°
\sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
\cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
\tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

15.1 Angle of Elevation and Angle of Depression

Activity 1

Ramnaresh is looking at the tree on the school ground and seems to be thinking about something. At the same time, his Mathematics teacher arrives.

Teacher: What happened to you, Ramnaresh? Why are you staring at a tree continuously?



Ramnaresh: Sir Namskar. Sir, how tall is this tree? What is the distance between the tree and the place where I am standing? What is the height of the flagstaff of the school? Sir, I am thinking about how to find all these things.

Teacher: Wait Ramnaresh, you have asked how many questions. Let's go to the classroom, make a picture of a question, and discuss it with your friends. (Both go to the classroom.)

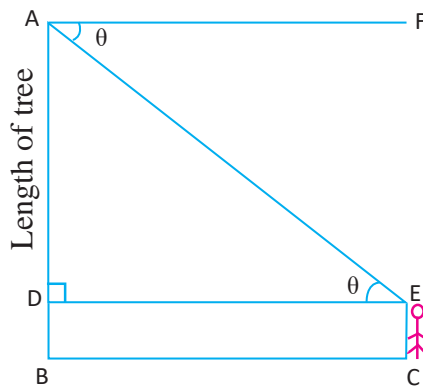
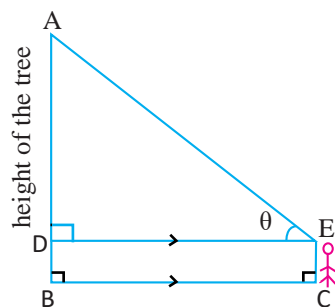
Teacher: Ramnaresh, the first question you asked was how tall the tree on the ground is, Right? Look at the picture on the whiteboard. Let x be the angle formed when we look at the top of the tree. In the figure, $BD = CE$. Now, if we can find the value of AD , we can find the height of the tree. A right-angled triangle ADE is formed. Let's try to remember how we derive the trigonometric ratios, in class 9.

Omkumari: What is the name of the angle, formed by the line of sight while looking at the top of the tree? How can we find it?

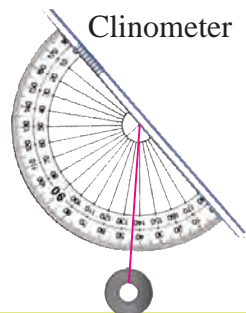
Teacher: Omkumari, first of all, we have to draw an imaginary line from our eye level parallel to the ground (DE in the picture). When we look at the top of the tree, an angle is formed by the line of our sight (EA in the figure) with the line parallel to the ground. This is called an angle of elevation. In the figure, $\angle AED$ is the angle of elevation.

Ramnaresh: Sir, even if you look from the top of the tree (point A in the picture) to point E , the angle is equal, isn't it? What is it called?

Teacher: Yes, you are right Ramnaresh. In this situation too, first we have to draw an imaginary line parallel to the ground from the top of the tree at the height of the top of the tree (AF in the figure). Yes, the angle made by our line of sight (AE in the figure) with the same line is the angle made by us from the top of the tree. This is called an angle of depression. In this figure, $\angle FAE$ is the angle of depression. Now, in which condition are the angle of elevation and the angle of depression formed in the following statements?



- Looking at the top of a house from the ground
- Looking at the top of a tree from the ground
- Looking at the top of a tower from the ground
- Looking at a vehicle from the roof of a house
- Observing the floor from the top of Dharahara
- Observing the ground from the top of a school building



When an observer observes any object from a lower place to an upper place, the angle formed between the line of our sight and the line parallel to the ground is called the angle of elevation. In the figure, $\angle AED$ is called the angle of elevation. Similarly, when an observer observes any object from the upper place to the lower place, then the angle formed between the line of our sight and the horizon is called an angle of depression. In the figure, $\angle FAE$ is called the angle of depression. The instrument used to measure these angles is called a clinometer.

Ramnaresh: Sir, which is bigger, the angle of elevation formed when we look at the top of the tree or the angle of elevation formed when we look at the middle of the tree?

Teacher: Find out yourself. How the angle made by the line of sight changes when you look at the mid of the tree and top of the tree?

Ramnaresh: Looking at the top of the tree, angle made by the line of sight was greater than when looking at the mid-part of the tree. It seemed that the angle of elevation formed when looking at the top of the tree is larger than the angle of elevation formed when looking at the middle of the tree.

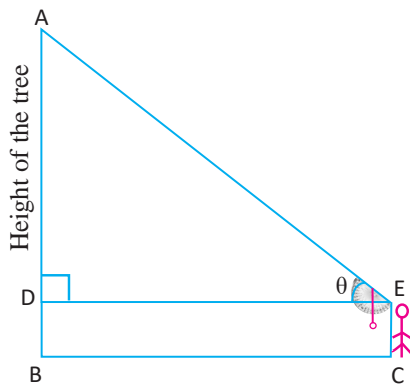
Teacher: Ramnaresh, what you said is correct. The angle of elevation formed when we look at the top of the tree is greater than the angle of elevation formed when we look at the middle of the tree. Similarly, the angle of depression formed when we look from the top of the tree is bigger when we look at a shorter distance than when we look from a farther distance. Let's do this much for today. We will do the rest the the question tomorrow.

Activity 2

(Next day in class)

Omkumari: Yesterday, we were talking about finding the height of trees, sir. How to find the height of the tree in the lawn?

Teacher: In the right triangle ADE, if we know the value of $\angle AED$ and the distance



of the tree from us i.e. (ED), then we can find the height of the tree. Let's look at the top of the tree with a Clinometer. The angle of elevation of the top of the tree is found to be 45° . If the height of your eyes (CE) = 1.6 m, the angle of elevation $\angle AED = 45^\circ$ and the distance from you to the bottom of the tree is $BC = DE = 30$ m then,

From the right angled triangle ADE

$$\tan 45^\circ = \frac{AD}{30}$$

$$\text{or, } 1 = \frac{AD}{30}$$

$$\text{or, } AD = 30 \text{ m}$$

Example 1 Hence, the total height of the tree = $30 + 1.60 = 31.60$ m.

A man of 1.80 meter high observes the angle of elevation at the top of tree and found it to be 45° . If the distance of the man and the tree is 20 meter, find the height of the tree.

Solution

When the man observes to the top of the tree, the angle between the line of sight and the line parallel to the ground is 45° .

In the figure, the height of the man is 1.80 meter and the total height of the tree = $(x + 1.80)$ meter

In the right angled triangle ABC

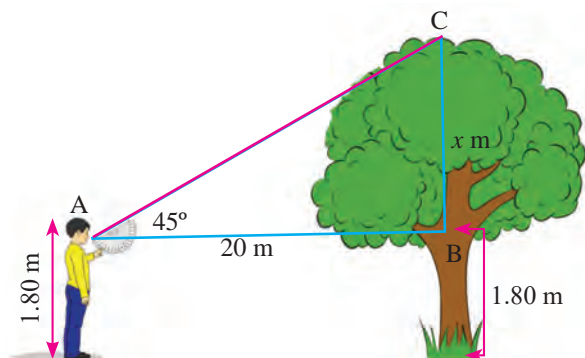
We have,

$$\tan 45^\circ = \frac{x}{20}$$

$$\text{or, } 1 = \frac{x}{20}$$

$$\text{or, } x = 20 \text{ m}$$

Hence the height of tree = $20 + 1.80 = 21.80$ meter.



Example 2

A man finds the angle of elevation of the top of a tower to be 60° . The height of tower is 140 m and the distance between man and tower is x m. Find the value of x .

Solution

Here,

Height of tower (AC) = 140 m

The distance between man and tower (BC) = x m

The angle of elevation $\angle ABC = 60^\circ$

In right angled triangle ACB,

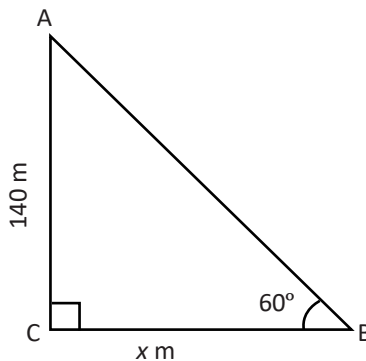
We have, $\tan 60^\circ = \frac{AC}{BC}$

$$\text{or, } \sqrt{3} = \frac{140}{x}$$

$$\text{or, } x\sqrt{3} = 140$$

$$\text{or, } x = \frac{140}{1.732} \quad \therefore BC = 80.83 \text{ m}$$

Hence, the distance between man and tower is 80.83



Example 3

A tree 18 m high is broken by the wind so that its top touches the ground and makes an angle of 30° with the ground. Find the length of broken part of the tree.

Solution

Let, the height of tree (AB) = 18 m

The length of broken part of tree (AD) = CD = x m

The height of remaining part of tree (BD) = $(18 - x)$ m

The angle making by broken part of tree = $\angle DCB = 30^\circ$

Now,

In right angled triangle CBD,

We have, $\sin 30^\circ = \frac{BD}{CD}$

$$\text{or, } \frac{1}{2} = \frac{18-x}{x}$$

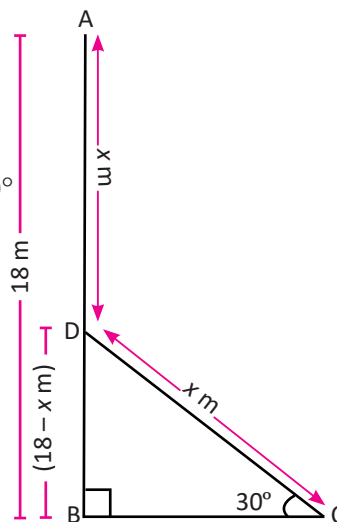
$$\text{or, } x = 36 - 2x$$

$$\text{or, } x + 2x = 36$$

$$\text{or, } 3x = 36$$

$$\text{or, } x = 12 \text{ m}$$

Hence, the length of broken part of the tree is 12 m



Example 4

The distance between a tower and a house is one third of the height of the tower. If the height of the tower is 60 m and the angle of depression from the top of the tower to the house is 45° , find the height of the house.

Solution

Let, the height of the tower (AB) = 60 m

DE be the height of the house

BE be the distance between the tower and house.

The distance between the tower and house;

$$BE = CD = 60 \times \frac{1}{3} = 20 \text{ m}$$

We know that, $DE = BC$ and by alternate angle $= \angle FAD = 45^\circ$

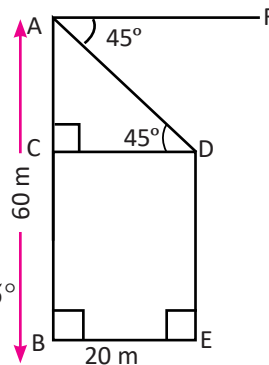
Now, in the right angled triangle ACD,

$$\text{We have, } \tan 45^\circ = \frac{AC}{CD}$$

$$\text{or, } 1 = \frac{AC}{20} \quad \text{or, } AC = 20 \text{ m}$$

$$\text{Again, } BC = DE = AB - AC = 60 - 20 = 40 \text{ m}$$

Hence, the height of the house is 40 m.



Example 5

A man 1.2 m tall observes the angle of elevation at the top of a tower and finds it to be 60° . If the height of the tower is 53.2 m, find the distance between the tower and the man.

Solution

Let, the height of the tower (AB) = 53.2 m

The height of the man (DE) = 1.2 m

BE be the distance between the tower and the man

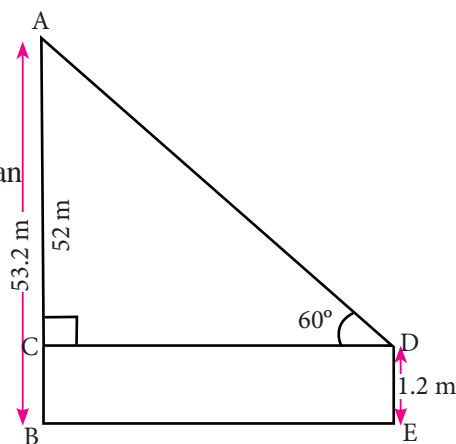
The angle of elevation $\angle ADC = 60^\circ$

We know that, $DE = BC$ and $BE = CD$

(\therefore Being opposite sides of a rectangle)

Now, in the right angled triangle ACD,

$$\text{We have, } \tan 60^\circ = \frac{AC}{CD}.$$



$$\text{or, } \sqrt{3} = \frac{AB - BC}{CD}$$

$$\text{or, } \sqrt{3} = \frac{53.2 - 1.2}{CD}$$

$$\text{or, } \sqrt{3} = \frac{52}{CD}$$

$$\text{or, } \sqrt{3} \times CD = 52$$

$$\text{or, } CD = \frac{52}{\sqrt{3}}$$

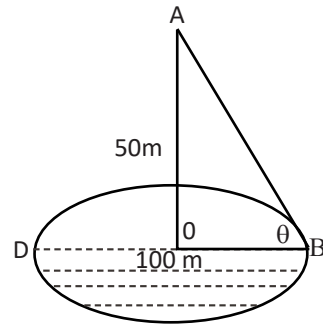
$$\text{or, } CD = \frac{52}{1.732}$$

$$\text{or, } CD = 30.02 \text{ m}$$

Hence, the distance between the tower and the man is 30.02 m.

Example 6

The diameter of a circular pond is 100 m. A pole is fixed at the centre of the pond and the height of the pole above the water surface is 50 m, what is the angle of elevation to the top of the pole observed from the edge of the pond? Find it.



Solution

Let, the diameter of the pond (BD) = $d = 100 \text{ m}$

The radius of the pond (OB) = $d/2 = 100/2 = 50 \text{ m}$

AO = 50 m be the height of the pole above the water surface.

The angle of elevation from B to A $\angle ABO = \theta$

Now, in right angled triangle AOB,

$$\text{We have, } \tan \theta = \frac{OA}{OB} = \frac{50}{50} = 1$$

$$\text{or, } \tan \theta = \tan 45^\circ$$

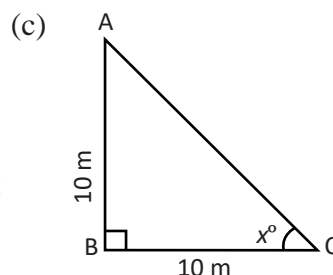
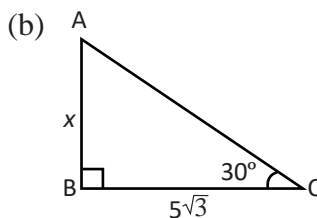
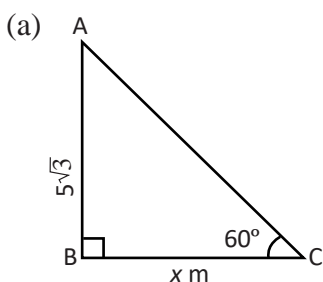
$$\therefore \theta = 45^\circ$$

Hence, the angle of elevation to the top of pole observed from the edge of the pond is 45° .

Exercise 15

1. (a) Define the angle of elevation and depression with example.
(b) In a right angled triangle, which trigonometric ratio is related to both perpendicular and base?
(c) In a right angled triangle, which trigonometric ratio has the relation with perpendicular and hypotenuse?

2. Find the value of x on the given right-angled triangles.



3. (a) The height of a tower is 60 m and the distance between a man and a tower is x m. A man finds the angle of elevation of the top of a tower to be 30° . Find the value of x .
(b) The height of tower is 12 m and the distance between man and tower is 12 m. The man finds the angle at elevation of the top of the tower to be x° . Find the value of x .
(c) The height of a tower is x m and the distance between a man and the tower is 12 m. The man finds the angle of elevation at the top of the tower to be 45° . Find the value of x .
4. (a) A tree of 14 m high is broken by the wind so that its top touches the ground (not separated from the main stem and makes an angle of 60° with the ground. Find the length of the broken part of the tree.
(b) A tree is broken from the middle part by the wind so that it's top touches the ground (not separated from the main stem and makes an angle of 60° with the ground. If the length of the broken part of the tree is 7.5 m, find the height of the tree before it was broken.
(c) A tree is broken at the middle part by the wind so that it's top touches the ground (not separated from the main stem and makes an angle of 30° with the ground. The length of the broken part of the tree is 30 m.

- (i) Find the height of the tree before it was broken.
 - (ii) How far does it meet the ground level from the base of the tree?
5.
 - (a) A man of 1.7 m height observes the angle of elevation at the top of a tower and finds it to be 60° . If the distance between the tower and the man is 30 m, find the height of the tower.
 - (b) A man of height 2 m is flying a kite from the roof of a house of 33.6 m. If the length of the string of the kite is $90\sqrt{2}$ m and it makes an angle of 45° with the horizon, find the height of kite from the ground level.
 - (c) A 1.5 m tall man observes the angle of elevation at the top of a tree of 51.5 m height and finds it to be 45° , find the distance between the tree and the man.
6.
 - (a) The distance between a tower and a man is 20 m. The height of the tree is 36.5 m. If the angle of elevation from the eye of the man to the top of the tower is 60° , find the height of the man.
 - (b) From the top of a house of 30 ft high, the angle of depression at the top of a tree is 30° . If the distance between the house and the tree is $10\sqrt{3}$ m, find the height of the tree.
 - (c) The height of tower is 60 m and the distance between the tower and house is 35 m. If a man observes the angle of depression from the top of tower to the top of house is 45° , find the height of house.
7.
 - (a) The diameter of a circular pond is 90 m. A pole is fixed at the centre of the pond and height of the pole above the water surface is 45 m, what is the angle of elevation of the top of pole observed from the edge of the pond? Find it.
 - (b) The diameter of a circular pond is 130 m. A pole is fixed at the centre of the pond. A person finds the angle of elevation of the top of the pole observed from the edge of the pond is 45° . What is the height of the pole above the water surface? Find it.
 - (c) At the centre of a circular pond, there is a pole of 11.62 m height above the surface of water. From a point on the edge of the pond, a man of 1.62 m height observed the angle of elevation at the top of the pole and found it to be 30° . Find the diameter of the pond.
8.
 - (a) On the occasion of a festival, Ramesh is flying a kite. The thread of the kite makes an angle of 30° with the horizon. If the length of the thread is 120 m and the Ramesh's height is 1.5 m, find the height of the kite from the ground.

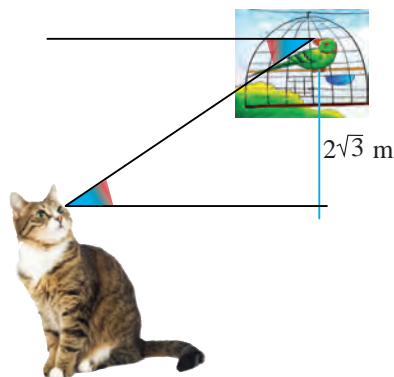
- (b) 1.5 m tall Ramsharan is flying a kite from the roof of a house of 9 m height. The string of the kite makes an angle of 30° with the horizon. If the height of the kite from the ground is 58 m, what is the length of the string. Find it.
- (c) A man of height 2 m is flying a kite from the roof of a house of 32 m high. If the length of the string of the kite is $66\sqrt{2}$ m and makes an angle of 60° with the horizon, work out to calculate the height of the kite from the ground?
9. The length of the shadow of a pole of 20 m high at 2 pm is $20\sqrt{3}$ m. on the meantime. Find the length of the shadow of the tower with the height $25\sqrt{3}$ m?

10. A tree 25 m high is just in the one corner of the ground of a school. A man of height 1.2 m is sitting on another corner of the ground. The distance between the man and the tree is 23.8 m.

- (a) What is the angle of elevation at the top of the tree made by the man?
- (b) Does the angle of elevation increase when the height of the tree is increases? Write with reason.
- (c) Is the angle of elevation whether increases or decreases when the distance between the man and tree is decreases. Write with reason.

11. In the given figure two buildings 20 m and 32 m are shown. The distance between them is 12 m.

- (a) What is the name of the the angle formed with the horizontal line when the parrot looks at the cat?
- (b) Find the angle in degree with the horizontal line when the parrot looks at the cat?
- (c) What is the distance between the pole and cat?
- (d) Does the angle the parrot makes when looking at the cat decrease as the cat moves towards the pole? Write with reason.



12. The circumference of a circular pond is 176 m. A pole is fixed at the centre of the pond. From a point on the edge of the pond, a man of 1.6 m tall observed the angle of elevation of the top of the pole and found it be 45° .

- (a) Find the distance between the man and the pole.
- (b) What is the height of the pole above the water surface? Find it.
- (c) How much less should the height of the pole above the water surface such that it would have made the angle of elevation of 30° ?

13. A man of height 1.2 m is flying a kite from the roof of a house 8.8 m high. If the length of the string of the kite is 180 m and makes an angle of 30° with the horizon,

- (a) Represent the given relationship in a diagrammatical form.
- (b) What is the height of the kite in meter from the ground? Find it.
- (c) What is the check distance between the man and the kite? Find it.

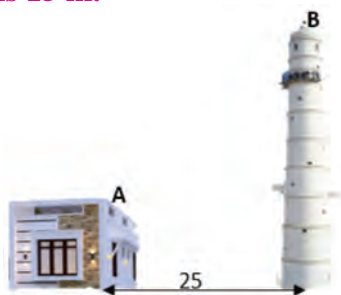
14. In the figure, two buildings 20 m and 32 m high are shown. The distance between them is 12 m.



- (a) What type of angle is formed when observed from the roof of tall building to the roof of the short building?
- (b) Explain with reason the relation between the angle that forms when you observe the top of the tall building from the top of the short building and the top of the short building from the top of the big building.
- (c) Calculate the angle in degree, when an observer observes from the roof of the small building to the roof of big building.
- (d) If the ladder is put from the top of the small building to the top of the big building, what should be the length of the ladder? Calculate it.

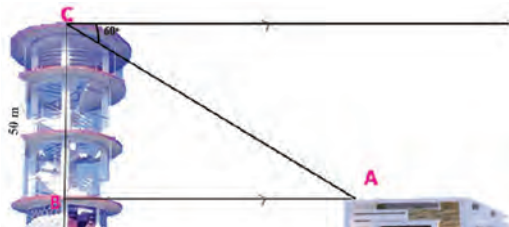
15. The tower and the house are on the same ground level and distance between them is 25 m. The height of the house is 15 m.

- (a) The angle of elevation from a point A of the roof of the house to the top of tower B is 45° , what is the height of the tower?
- (b) From the figure, how the angle of elevation changes if the height of the tower increases. The angle of depression of a house 20 m to the east of the tower is 60° when observed from top of the tower of height 50 m.



16. From the top of the 50 m high tower an angle of 60° is formed when looking at the roof of a house 20 m to the east of the tower.

- What is the angle formed when observed from a point A on the roof of the house to the top of the view tower?
- What is the height of the part BC of the tower? Find it.
- What is the height of the house? Find it.
- What distance does she have to come down from the top of the tower to observe the terrace (top) of the house such that the angle of depression of 45°



Project work

Make a group of friends and find out the things in the highest and lowest areas at around your house. Find out practically the angles with respect to their height and distance or the height with respect to their angles and distance. Discuss the result obtained thus and find out the conclusion, and present it in the class.

Answer

- | | | |
|-----------------------|--------------------|---|
| 2. (a) 5 m | (b) 5 m | (c) 45° |
| 3. (a) $60\sqrt{3}$ m | (b) 45° | (c) 12 m |
| 4. (a) 7.5 m | (b) 14 m | (c) 45 m, $15\sqrt{3}$ m |
| 5. (a) 53.6 m | (b) 125 m | (c) 50 m |
| 6. (a) 1.86 m | (b) 20 ft | (c) 25 m |
| 7. (a) 45° | (b) 65 m | (c) 11.55 m |
| 8. (a) 61.5 m | (b) 95 m | (c) 100 m |
| 9. 30° , 75 m | 10. (a) 45° | 11. (a) 30° (b) 6 m |
| 12. (a) 28 m | (b) 29 m | (c) 11.83 m |
| 13. (a) 100 m | (c) $90\sqrt{3}$ m | 14. (a) and (b) Show ot to your teacher |
| (a) 45° | (d) $12\sqrt{2}$ m | 15. (a) 40 m |
| 16. (a) 60° | (b) 34.64 m | (c) 15.36 m (d) 14.64 m |

