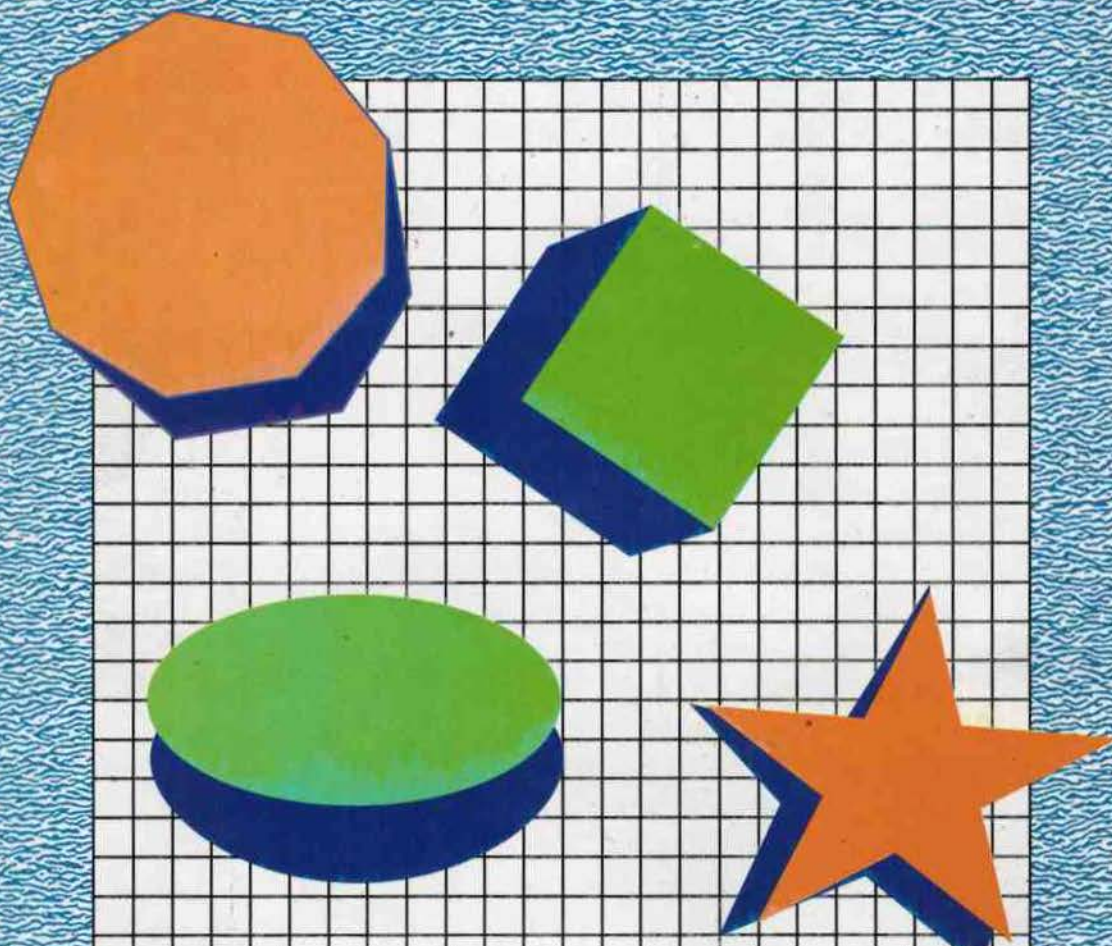
8

OUR

New Edition

Mathematics



Grade 8

Our Mathematics

Grade 8

This English version Mathematics has been translated by
Janak Educational Material Centre, Ltd.

Publisher

Government of Nepal
Ministry of Education
Curriculum Development Centre
Sano Thimi, Bhaktapur, Nepal

First Colour Print Edition (English Version)	: 2061
Reprint	: 2062/2063/2064/2065
Second & Revised Edition	: 2066/2067
Reprint	: 2069

JEMC welcomes any suggestions regarding the textbook.

Printed & Distributed By:
Nepal Sahitya Prakashan Kendra
Kathmandu, Nepal Tel: 4435856, 4411652, 4417709
Fax: 977-1-4420990, Email: nspk@mail.com.np

You can exchange this book from the nearest local book distributor if any technical mistakes are found.
-Publisher

Acknowledgements

The following people have provided special contribution in developing Nepali version *Ganit* textbook of grade eight.

Mr Balkrishnaman Singh
Prof Dr Siddhi Prasad Koirala
Mr Lok Bilas Pant
Mr Khamba Rai
Mrs Indira Joshi
Mr Manaraja Rajopadhyaya
Dr Hira Maharjan
Mr Krishnabhakta Tuladhar
Mr Shuvadarshan Acharya
Mr Lekhnath Poudyal
Mrs Jwala Nepali
Mr Koji Takahasi

Language Edit (Nepali) :

Mr Bharat Nepali Pyakurel
Mr Hari Gautam

Design :

Mr Tarjan Rai

About the book

Revision and improvement in different sectors of education are being done to catch up with the ever-changing scenario. Some of the national objectives of education are; foster democratic norms and values in students, inhibit the feelings of national unity and integrity, develop human resources required for nation building, develop competency to survive successfully in the competitive society. In order to achieve these objectives the curriculum and textbooks have been reviewed and revised.

An attempt has been made to incorporate the exercises related to the day-today life along with their experiment and examples. Almost all exercises are designed in such a way that they can be taught by using locally availably materials. Originally written in 2053 BS by Dr. Santoshman Maskey, Mr. Harinarayan Upadhaya and Ms. Sungma Tuladhar, this book has thoroughly been revised by Dr Siddhi Koirala, Mr. Bhoj Raj Sharma, Mr Shalikram Bhusal, Mr. Barun Baidhay, Ms. Indira Aryal and Ms Nirmala Gautam in line with the revised curriculum. Mr Shiva Prasad Satyal, Mr. Jagannath Awa, Mr,Gopal Prasad Adhikari, Mr.Bharat Nepali Pakuryal, Mr. Mukti Bhandari and Mr. Shyam Prasad Acharya also contributed in the revision work. Suggestions form the teachers' workshop and the subject committee comprising Mr. Shivaram Neupane, Mr. Mukunda Sharma and Tank Lal Gaire were also taken into account while revising the book. The language editing was done by Mr. Shanbhu Dahal and Mr. Bishnu Adhikari, and its lay-out design and cover page design were done by SP Sharma and Tarjan Rai respectively. The CDC heartly extends its gratitude to all those who were involved in the development of this book.

Since a textbook is only a tool for teaching learning process, an experienced teacher can teach the subject matter specified by the curriculum successfully by utilizing different resources. However, most of our classroom teachings are found entirely based on the textbook only. In this context, possible endeavor has been done to bring this book up to the standard. Despite our sincere effort, there might be some errors both in language and the subject matter. Therefore, Curriculum Development Centre invites constructive suggestions form the valued readers.

Government of Nepal
Ministry of Education
Curriculum Development Centre
Sano Thimi, Bhaktapur

Preface

Now that the overwhelming majority of people in Nepal question the quality of education, it is, indeed, desirable to do something about it. One of the major tasks of the government is to provide quality education to all the people. In this context, Curriculum Development Center (CDC) is the authorized institution in the country to design and develop textbooks and teachers' guides used throughout the kingdom to meet this challenging need. Likewise, Janak Education Materials Centre (JEMC) also plays an equally crucial role by providing the textbooks to all the public schools across the country. To cater to the needs of both private and public schools, the JEMC has come one step forward by translating the authorized version of Nepali books into English. The Centre is confident that it will be able to provide English version books in different subjects to the learners as reference materials step-by-step.

JEMC really feels proud of accomplishing a substantial job of translating public school textbooks into English for English medium learners across the country.

This book is translated by Maheswor Prasad Upadhayaya from Nepali version *Hamro Ganit* of Grade 8 We are highly grateful to Dhyan Bahadur Karki and Bhim Bahadur Basnet who were involved in subject- matter and language editing. The JEMC invites positive suggestions from all concerned to make the book ever better.

We would like to express our sincere thanks to the teachers of some renowned schools of 5 Development Regions who have assisted in evaluation of the proposed translated lessons and all the others who have also contributed to the preparation of these books.

Finally, we would also like to express our gratitude to the CDC for giving us an opportunity to translate the government textbooks into English in order to cater the needs of the pupils with English school background.

Date: 2062, Baishakh

Janak Education Materials Centre Ltd.
Sano Thimi, Bhaktapur

Thankful

We are thankful to the following experts, schools and their headteachers.

Mr Manaraja Rajopadhyaya, C D C, Sano Thimi
Mr Sungma Tuladhar, St. Xavier School, Lalitpur
Mr Tarun Kumar Lal, Bhudhanilkantha School, Kathmandu
Mr Dhurba Narayan Chaudhari, G E M S, Sanepa, Lalitpur
Mr Keshav Acharya, Birendra Sainik M Vidhalaya, Bhaktapur
Mr Raghubar Shrestha, E P S, Min Bhawan, Kathmandu
Mr Buddha Saran Manandhar, Siddhartha Banasthali, Balaju, Kathmandu
Mr Nhuchhe Bahadur Khatakho, Basu M Vidhalaya, Bhaktapur
Mr Hari Govinda Shrestha, Adarsha M Vidhalaya, Thimi, Bhaktapur
Mr Ram Hada, Shree Padma M Vidhalaya, Bhaktapur

We are very much grateful to the following people who were involved in subject matter, evaluation and language editing.

Mr Naresh Gautam, Ministry of Education
Mr Manaraja Rajopadhyaya, C D C, Sano Thimi
Mr Pramod Kumar Shah, C D C, Sano Thimi
Mr Dhurba Narayan Chaudhari, G E M S, Sanepa, Lalitpur

CONTENTS

UNIT 1

SETS

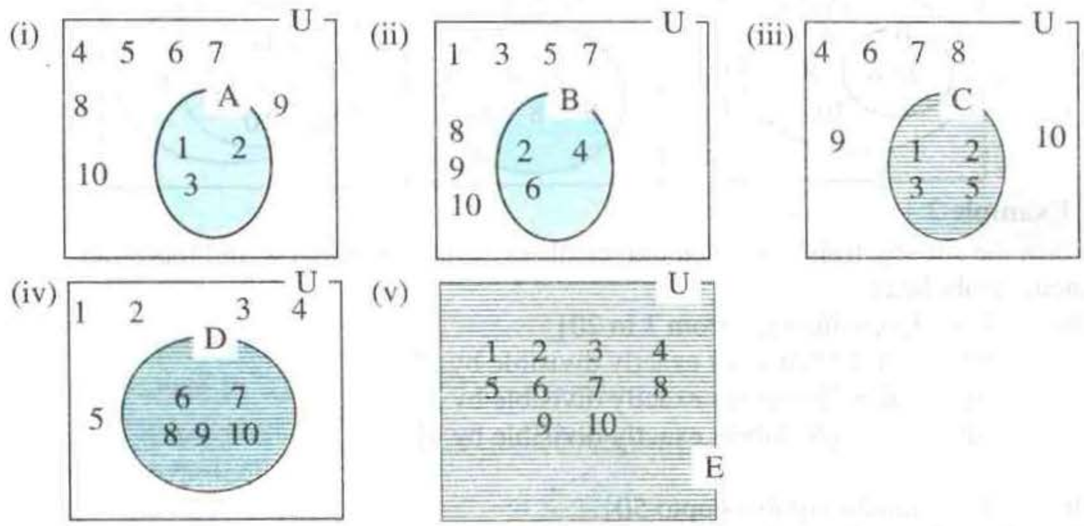
1.1. Proper and Improper Subsets

Let us form different subsets from the Universal set
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
For Example,

A = {1, 2, 3}, B = {2, 4, 6}, C = {1, 2, 3, 5}
D = {6, 7, 8, 9, 10} and E = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Here, subsets A, B, C and D do not contain all the elements of the Universal set U but the subset E contains all the elements of U.

Subsets A, B, C and D are the proper subsets of the Universal set U and subset E is an improper subset of U. A proper subset is represented by the symbol $\subset$ whereas an improper subset is represented by the symbol $\subseteq$. Thus, the relations between the universal set U and the subsets A, B, C, D and E can symbolically be written as:
(i) $A \subset U$, (ii) $B \subset U$, (iii) $C \subset U$, (iv) $D \subset U$ and (v) $E \subseteq U$
The above relations can be represented by Venn-diagrams as follows:



In order to form the subsets, it is not necessary that we always have the universal set. We can form the subsets from the set A or another set like A and can establish the above relation.

Thus if a subset is formed, from a universal set U or other sets A, B, C, D etc, taking only some elements but not all, then the subset is called a proper subset. The subset formed from a set is called an improper subset if it contains all the elements of the set.

Example 1

Classify the following subsets formed from the universal set as proper or improper and represent them by Venn-diagram.

(a) U = {a, e, i, o, u}

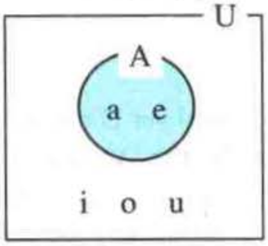
A = {a, e}, B = {e, o, u}, C = {a, e, i, o, u}

(b) A = {2, 4, 6, 8, 10}

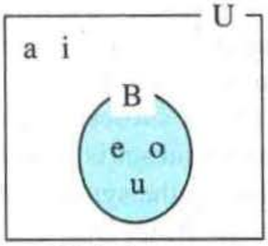
B = {2, 6}, C = {2, 4, 6, 8, 10}, D = {2, 4, 8, 10}

Solution:

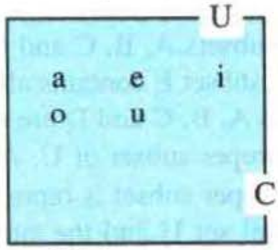
(a) A is proper



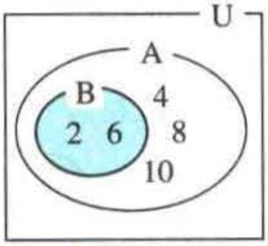
B is proper



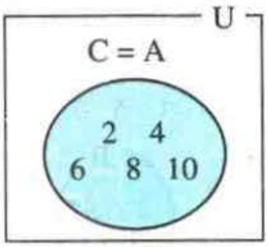
C is improper



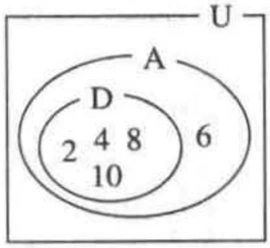
(b) (B) is proper



(C) is improper



(D) is proper



Example 2

Form the subsets from the given universal set as indicated below and represent them symbolically.

(a) U = {Even numbers from 1 to 20}

(i) A = {Numbers exactly divisible by 2}

(ii) B = {Numbers exactly divisible by 3}

(iii) C = {Numbers exactly divisible by 4}

(b) S = {Square numbers upto 50}

(i) A = {Squares of 2, 4 and 6 }

(ii) B = {Squares of 3, 4 and 7}

(iii) C = {Square numbers from 0 to 50}

Solution:

(a) Here, (i) $A \subseteq U$ (ii) $B \subset U$ (iii) $C \subset U$

(b) Here, (i) $A \subset S$ (ii) $B \subset S$ (iii) $C \subseteq S$

EXERCISE 1 [A]

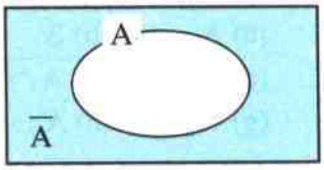
1. If U = {1, 3, 5, 7, 9}, classify the following subsets as proper or improper and represent them by Venn diagrams.
   (i) E = {1, 3, 5}   (ii) F = {1, 3, 5, 7, 9}   (iii) G= {3, 5, 7}
2. Form the subsets proper and improper from the given universal set as indicated below and represent them symbolically.
   M = {Factors of 18}
   (i) A = {Factors of 2}
   (ii) B = {Numbers which exactly divide 18}
   (iii) C = { Factors of 3}
3. If A = {1, 2, 3, 4, 5}, B = {2, 3, 4} and C = { 1, 2, 5}, determine the relations between A and B, A and C. Express the relations by Venn diagram.
4. Given that B = {Whole numbers from 3 to 15, exactly divisible by 3}.
   Construct 2 proper and 1 improper subsets from B.
   Can we form more than one improper subsets from B?

1.2. Complement of a Set

Let U={Tourists coming in Nepal} and A = {Tourists climbing the mountains of Nepal}. Here, set A is the subset of the universal set U. The tourists who do not go climbing cannot be the members of set A though they are the members of set U. The set formed by the members who do not go climbing is known as the complement of set A. It is denoted by $\bar{A}$.

(?) Can you derive the following relations using this Venn- diagram?

$U = A \cup \bar{A}$ and $A \cap \bar{A} = \phi$



The complement of set A is a set which contains the element of the universal set U which is not in A. It is denoted by $\overline{A}$

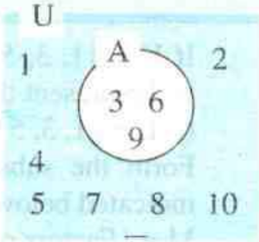
Example 1

If U = {1, 2, 3,....,10} and A = {3, 6, 9} form each of the following set and represent them by Venn-diagrams.

(a) $\overline{A}$ (b) $A \cup \overline{A}$ (c) $A \cap \overline{A}$ (d) $\overline{\overline{A}}$

Solution :

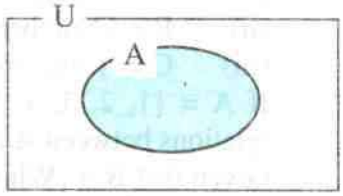
(a) $\overline{A} = \{1, 2, 4, 5, 7, 8, 10\}$ in the adjoining figure, the shaded portion represents ?



$\overline{A}$

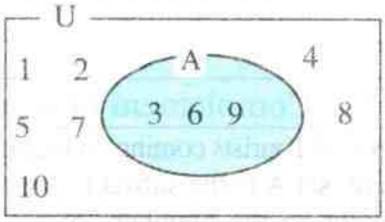
(b) $A \cup \overline{A} = \{3,6,9\} \cup \{1, 2, 4, 5, 7, 8, 10\}$
$= \{1, 2, 3, ..., 10\}$
In the adjoining figure, shaded region represents $A \cup$?

(c) $A \cap \overline{A} = \{3, 6, 9\} \cap \{1, 2, 4, 5, 7, 8, 10\}$
$= \phi$
Since $A \cap \overline{A}$ is an empty set, it can not be shown by shading.



$A \cap \overline{A}$

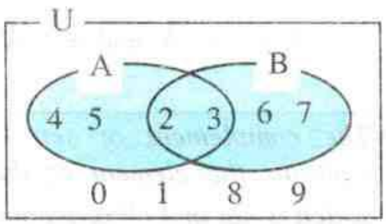
(d) From (a) we have,
$\overline{A} = \{1, 2, 4, 5, 7, 8, 10)$
Hence, $\overline{\overline{A}}$ = {elements of U not in $\overline{A}$)
$= \{3, 6, 9\} = A$
Shaded portion represents ?
$\therefore A = \overline{\overline{A}}$.
Here, $\overline{\overline{A}}$ represents the complement of ? .



$A \cup \overline{A}$

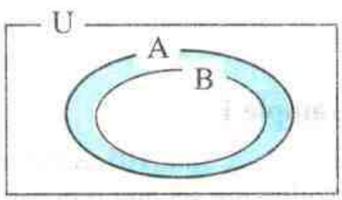
EXERCISE – 1 [B]

1. From the adjoining Venn- diagram, prepare a list that represents the following sets.
(a) A (b) B (c) $\overline{A}$
(d) $\overline{B}$ (e) $\overline{A \cap B}$ (f) ? $\cap \overline{B}$
(g) $\overline{A \cup B}$ (h) $\overline{A} \cup \overline{B}$ (i) U (j) $\overline{U}$



2. Using the result of (1), verify the following.
(a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

3. (a) Copy the given diagram separately, and represent the following sets by shading
(i) $\overline{A}$ (ii) $\overline{B}$
(iii) $\overline{A \cap B}$ (iv) $\overline{A \cup B}$
(b) Under what condition $\overline{A \cap B} = \overline{B}\ \overline{A}$



4. If the universal set $U = \{1, 2, 3, ..., 10\}$, $A = \{$factors of 6$\}$,

B = {factors of 8}, and C = {Factors of 9), then

(a) Represent the relationship among U, A, B and C by Venn-diagram.

(b) Form the following sets, using the sets above.

(i) $\overline{A}$ (ii) $\overline{B}$ (iii) $\overline{C}$ (iv) $\overline{A \cup B}$ (v) $\overline{B \cup C}$ (vi) $\overline{A \cup C}$

(vii) $\overline{A \cap B}$ (viii) $\overline{A \cap C}$ (ix) $\overline{B \cap C}$ (x) $\overline{A \cup B \cup C}$

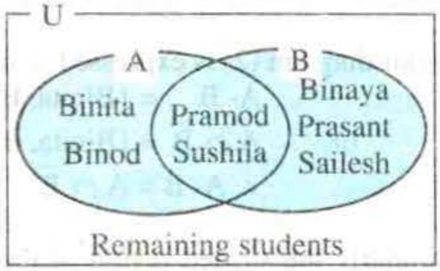
(xi) $\overline{A \cap B \cap C}$

5. If the universal set U = {1, 2, 3, 4, 5}, A = {2, 4}, B = {1, 3, 5} and C = {2, 3}, then form the following sets and represent them by Venn-diagrams.

(i) $\overline{A}$ (ii) $\overline{B}$ (iii) $\overline{C}$ (iv) $\overline{\overline{A}}$ (v) $\overline{\overline{B}}$ (vi) $\overline{\overline{C}}$

1.3. Difference of Sets

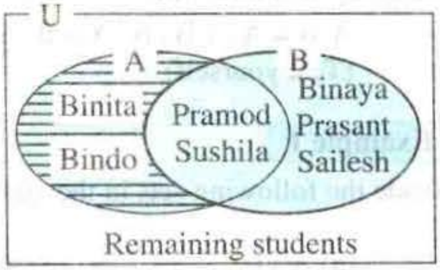
In the Venn-diagram, on the right side, U represents all the students of the class, set A represents the students who like badminton and set B represents the students who like football.



Venn diagram (1)

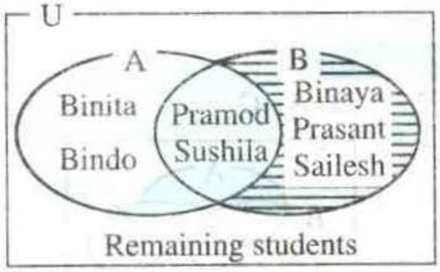
Now, study the three sets that follow:

(a) Shaded region of Venn-diagram (2) represents the set of students who like badminton only.



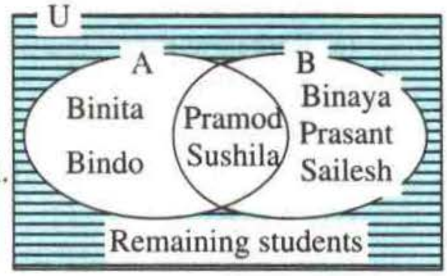
Venn diagram (2)

(b) Shaded region of Venn-diagram (3) represents the set of students who like to play football only.



Venn diagram (3)

(c) Shaded region of Venn-diagram (4) represents the set of students who do not like to play badminton as well as football.



Venn diagram (3)

(?) How can these sets be expressed in a mathematical sentence?

(?) Study Venn-diagram (2) properly and express the shaded portion in words.

If the overlapped portion of set B is eliminated from set A then the set is called A difference B and is denoted by A-B and is read as 'A minus B'.

Venn-diagram (2) is expressed in words as follows

$A - B = \{\text{Binita, Binod}\}$

or, $A \cap \overline{B} = \{\text{Binita, Binod}\}$,

$\therefore A - B = A \cap \overline{B}$

Similarly the shaded region of the Venn-diagrams (3) and (4) is denoted by B-A and U – (A∪B) and, if these sets are expressed in terms of complementary sets, then they will look like.

$A - B = A \cap \overline{B}$, $B - A = B \cap ?$, $U - (A \cup B) = \overline{A \cup B}$.

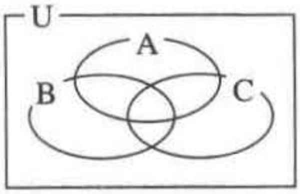
(Test yourself)

Example 1

Shade the following sets in the adjoining Venn-diagram.

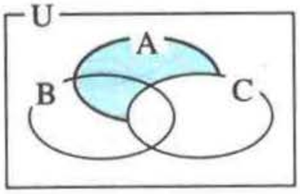
(a) $A - C$ (b) $A - (B \cup C)$

(c) $A - (B \cap C)$ (d) $(A \cup B) - C$

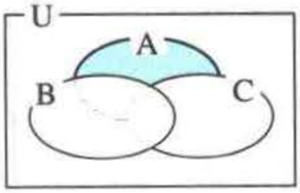


Solution :

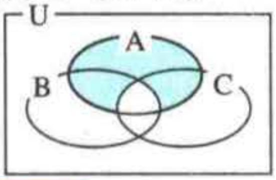
(a) $A - C$



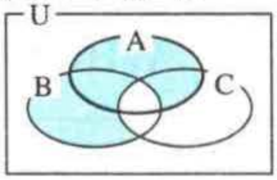
(b) $A - (B \cup C)$



(c) $A - (B \cap C)$



(d) $A \cup B) - C$

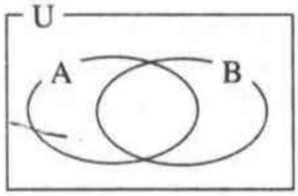


Example 2

In the Venn-diagram given to the right.

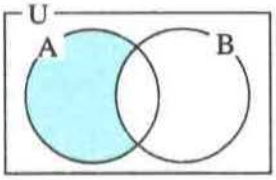
(a) Shade the region representing A- B

(b) Can we write $A - B = \overline{B} - ?$?

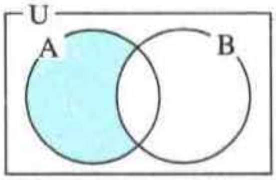


Solution :

(a) Let eliminate the overlapping region of A from B (left)



Venn-diagram representing A-B



Venn-diagram representing $\overline{B}$ - ?

(b) Yes, we can. $\therefore A - B = \overline{B} - \bar{A}$

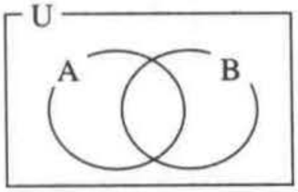
EXERCISE 1 [C]

1. In the adjoining Venn-diagram, shade the region representing

   (a) $A - B$ (b) $A - \overline{B}$

   (c) $\overline{A} - B$ (d) $A - (A \cap B)$

   (e) $(A \cup B) - A$



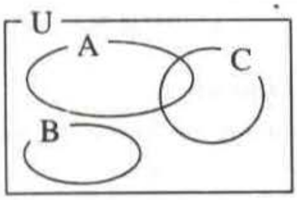
2. Which of the sets in QNo. 1 are equal?

3. Shade the region representing the following sets in the Venn-diagram in the right.

   (a) $A - B$ (b) $B - C$

   (d) $(A \cup B) - C$ (d) $A - (B \cup C)$

   (e) $A - (B \cap C)$



4. Observe the shaded Venn-diagram of Q. N. 3 and determine whether the following statements are true or false.

   (a) $(B - C) \subset \{(A \cup B) - C\}$ (b) $\{A - (B \cup C)\} \subset (A - B)$

   (c) $\{(A \cup B) - C\} \subset \{(A - (B \cap C)\}$ (d) $(A - B) = \{A - (B \cap C)\}$

UNIT - 2

WHOLE NUMBER

2.1. Addition and subtraction of Binary and Quinary numbers

2.1 (a) Addition and subtraction of binary numbers

In binary system only two digits 0 and 1 are used to write the numbers. Therefore it is known as binary number system or base two system. Conversions of the numbers from binary (base two) to denary (base ten) and vice versa are shown in the examples below.

Example 1

Convert the following numbers into binary system.

(a) 107_{10} (b) 135_{10}

Solution :

Here

(a) 107_{10}

Dividing 107 by 2 successively

2	107	remainder
2	53	1
2	26	1
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

$\therefore 107_{10} = 1101011_2$

(b) 135_{10}

Dividing 135 by 2 successively.

2	135	remainder
2	67	1
2	33	1
2	16	1
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$\therefore 135_{10} = 10000111_2$

Example 2

Convert the following numbers into denary (base 10) system.

(a) 111_2 (b) 1010_2

Solution :

(a) Here, $111_2 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 4 + 2 + 1 = 7.$

(b) Here, $1010_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 8 + 0 + 2 + 0 = 10$

EXERCISE 2 [A]

1. Convert the following denary numbers into binary.
   (a) 405 (b) 535 (c) 63 (d) 1234
2. Transform the following base – 2 numbers into decimal (base –10) numbers.
   (a) 1110 (b) 11111 (c) 110010 (d) 10101

(i) Addition

There are only two numerals 0 and 1 in the binary system it is easy to remember the basic rules of addition in this system. Addition facts are tabulated in the chart below.

+	0	1
0	0	1
1	1	10

Example 3

(a) Add 11_2 and 10_2 (b) Find the sum of 101_2 and 111_2.

Solution :

(a) Here,
$$\begin{array}{r} 11_2 \\ \underline{+\,10_2} \\ 101_2 \end{array}$$
[∴ $1+1 = 10_2$, hence we write down 0 and carryover 1]

(b) Here,
$$\begin{array}{r} 101_2 \\ \underline{+\,111_2} \\ 1100_2 \end{array}$$
[$1+1 = 10_2$, write down 0 and carryover 1.]
[$1+1+1= 11_2$ write down 1 and carryover 1]

(ii) Subtraction

Subtraction facts are shown in the table below. Here the column numbers (subtrahends) are subtracted from the row-numbers (minuends). Study the subtraction table.

	← Minuend —			
← Subtrahend —	-	10_2	1	0
	0	10_2	1	0
	1	1	0	
	10_2	0		

Example 4

(a) Subtract 11_2 from 101_2 (b) Take away 101_2 from 110_2

Solution :

(a) Here, $\begin{array}{r} 101_2 \\ -\ 11_2 \\ \hline 10_2 \end{array}$

(b) Here, $\begin{array}{r} 110_2 \\ -\ 101_2 \\ \hline 1_2 \end{array}$ [When we borrow 1 from round position we will have 1_2 we will 10_2 at unit position and $10_2 - 1_2 = 1_2$]

EXERCISE 2 [B]

1. Find the sum of the following binary numbers.

(a) $\begin{array}{r} 101_2 \\ +\ 10_2 \\ \hline \end{array}$	(b) $\begin{array}{r} 110_2 \\ +\ 1_2 \\ \hline \end{array}$	(c) $\begin{array}{r} 1001_2 \\ +\ 110_2 \\ \hline \end{array}$	(d) $\begin{array}{r} 1010_2 \\ +\ 101_2 \\ \hline \end{array}$
(e) $\begin{array}{r} 101_2 \\ +\ 11_2 \\ \hline \end{array}$	(f) $\begin{array}{r} 111_2 \\ +\ 10_2 \\ \hline \end{array}$	(g) $\begin{array}{r} 1010_2 \\ +1011_2 \\ \hline \end{array}$	(h) $\begin{array}{r} 101_2 \\ +\ 1011_2 \\ \hline \end{array}$
(i) $\begin{array}{r} 1011_2 \\ +\ 101_2 \\ \hline \end{array}$	(j) $\begin{array}{r} 1101_2 \\ +101_2 \\ \hline \end{array}$	(k) $\begin{array}{r} 10111_2 \\ +\ 1_2 \\ \hline \end{array}$	(l) $\begin{array}{r} 10111_2 \\ +10001_2 \\ \hline \end{array}$

2. Find the difference of the following base-two numbers.

(a) $\begin{array}{r} 11_2 \\ -\ 10_2 \\ \hline \end{array}$	(b) $\begin{array}{r} 11_2 \\ -\ 1_2 \\ \hline \end{array}$	(c) $\begin{array}{r} 101_2 \\ -\ 1_2 \\ \hline \end{array}$	(d) $\begin{array}{r} 1011_2 \\ -\ 10_2 \\ \hline \end{array}$
(e) $\begin{array}{r} 101_2 \\ -\ 11_2 \\ \hline \end{array}$	(f) $\begin{array}{r} 101_2 \\ -\ 10_2 \\ \hline \end{array}$	(g) $\begin{array}{r} 11001_2 \\ -\ 1011_2 \\ \hline \end{array}$	(h) $\begin{array}{r} 1000_2 \\ -\ 1_2 \\ \hline \end{array}$
(i) $\begin{array}{r} 1000_2 \\ -\ 10_2 \\ \hline \end{array}$	(j) $\begin{array}{r} 1001_2 \\ -\ 101_2 \\ \hline \end{array}$	(k) $\begin{array}{r} 1001_2 \\ -\ 111_2 \\ \hline \end{array}$	(l) $\begin{array}{r} 10000_2 \\ -\ 1_2 \\ \hline \end{array}$

3. Simplify :

(a) $110_2 + 101_2$ (b) $1010_2 - 101_2$

(c) $101_2 + 100_2 - 10_2$ (d) $1101_2 - 110_2 + 1001_2$

(e) $1110_2 - 101_2 + 110_2$ (f) $11_2 + 1011_2 - 1110_2$

2.2 (b) Addition and Subtraction of Quinary System

There are only five numerals 0,1, 2, 3 and 4 in the quinary system. Hence it is called base five or quinary system. Conversion of the numbers from quinary to denary and denary to quinary are shown in the examples below. Let us review the conversion of number from one system into another as given in examples below.

Example 5

Convert the following numbers into base five numbers.

(a) 465_{10} (b) 2504_{10}

Solution:

(a) Here, 465_{10}

Dividing 465 by 5,

5	465	remainder
5	93	0
5	18	3
5	3	3
	0	3

$\therefore 465_{10} = 3330_5$

(b) Here, 2504_{10}

Dividing 2504 by 5

5	2504	remainder
5	500	4
5	100	0
5	20	0
5	4	0
	0	4

$\therefore 2504_{10} = 40004_5$

Example 6

Convert the following into base – ten numbers.

(a) 14_5 (b) 342_5

Solution :

(a) Here ,

$$\begin{aligned} 14_5 &= 1 \times 5^1 + 4 \times 5^0 \\ &= 5 + 4 \\ &= 9 \\ \therefore 14_5 &= 9_{10} \end{aligned}$$

(b) Here,

$$\begin{aligned} &342_5 \\ &= 3 \times 5^2 + 4 \times 5^1 + 2 \times 5^0 \\ &= 3 \times 25 + 4 \times 5 + 2 \times 1 \\ &= 75 + 20 + 2 \\ &= 97 \\ \therefore\ & 352_5 = 97_{10} \end{aligned}$$

Example 7

Change the following into binary number.

(a) 342_5 (b) 2023_5

Solution :

(a) $342_5 = 3 \times 5^2 + 4 \times 5^1 + 2 \times 5^0$
$= 75 + 20 + 2$
$= 97_{10}$

Now, 97 is converted into base two system.

		remainder
2	97	
2	48	1
2	24	0
2	12	0
2	6	0
2	3	0
2	1	1
	0	1

$\therefore 97_{10} = 1100001_2$
Therefore, $342_5 = 1100001_2$

(b) Here, $2023_5 = 2 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 3 \times 5^0$
$= 250 + 0 + 10 + 3 = 263_{10}$

Now, 263_{10} is converted to base two system as follows.

		remainder
2	263	
2	131	1
2	65	1
2	32	1
2	16	1
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$\therefore 263_{10} = 100001111_2$
Here, $2023_5 = 100001111_2$

Example 8

Convert the following into binary or quinary system.

(a) 1011_2 (b) 423_5

Solution :

(a) First, we convert the given number into base - 10 system and then into base - 5 system.

Here, $1011_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 0 + 2 + 1 = 11_{10}$

Now, 11_{10} is changed into quinary system as follows.

5	11	remainder
5	2	1
	0	2

i. e. $11_{10} = 21_5$

Hence, $1011_2 = 21_5$

(b) Here, at first 423_5 is changed into denary system.

$423_5 = 4 \times 5^2 + 2 \times 5^1 + 3 \times 5^0$

$= 100 + 10 + 3 = 113_{10}$

Now, 113_{10} is converted into binary system as follows.

2	113	remainder
2	56	1
2	28	0
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

i. e. $113_{10} = 1110001_2$

Hence, $423_5 = 1110001_2$

EXERCISE 2 [C]

1. Convert the following into decimal system.
   (a) 201_5 (b) 344_5 (c) 4201_5 (d) 2304_5
2. Change the following decimal number into quinary numbers.
   (a) 600 (b) 548 (c) 726 (d) 2468
3. Convert the following binary numbers into quinary and quinary into binary.
   (a) 11001_2 (b) 3424_5 (c) 11100_2 (d) 4321_5

(i) Addition:

Addition facts related to base five system are displayed in the table below study the table and try to understand the facts.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10_5
2	2	3	4	10_5	11_5
3	3	4	10_5	11_5	12_5
4	4	10_5	11_5	12_5	13_5

Example 9

(a) Add 23_5 and 42_5 (b) Find the sum of 342_5 and 134_5

Solution:

(a) Here
$$\begin{array}{r} 23_5 \\ +\,42_5 \\ \hline 120_5 \end{array}$$
[∴ 3 + 2 = 10_5, hence write down 0 and carryover 1]

(b) Here,
$$\begin{array}{r} 342_5 \\ +\,134_5 \\ \hline 1031_5 \end{array}$$
[From the table above, we notice that carryovers at the positions 5^0, 5^1, 5^2 are respectively 1, 1 and 1]

(ii) Subtraction:

Subtraction facts for quinary system are shown in the table below. Study the table carefully.

-	10_5	4	3	2	1	0
0	10_5	4	3	2	1	0
1	4	3	2	1	0	
2	3	2	1	0		
3	2	1	0			
4	1	0				
10_5	0					

Example 10

(a) Subtract 23_5 from 42_5 (b) Take away 24_5 from 142_5

Solution:

(a) Here,

$$\begin{array}{r} 42_5 \\ \underline{-\ 23_5} \\ 14_5 \end{array}$$

***Explanation** : In the position of 5^0, we have to subtract 3 from 2, Which is not possible. Hence we borrow 1 from the place of 5^1 so that we have $5+2 = 7_{10} = 12_5$ in 5^0 – place and hence difference in $7 - 3 = 4$ and the remainder at the place of 5^1 is $4 - 1 = 3$.*

(b) Here

$$\begin{array}{r} 142_5 \\ \underline{-\ 24_5} \\ \underline{113_5} \end{array}$$

EXERCISE 2 (D)

1. Find the sum.

(a) $\begin{array}{r} 432_5 \\ \underline{+\ 23_5} \end{array}$ (b) $\begin{array}{r} 444_5 \\ \underline{+\ 1_5} \end{array}$ (c) $\begin{array}{r} 314_5 \\ \underline{+30_5} \end{array}$ (d) $\begin{array}{r} 1042_5 \\ \underline{+321_5} \end{array}$

(e) $\begin{array}{r} 1003_5 \\ \underline{+\ 321_5} \end{array}$ (f) $\begin{array}{r} 402_5 \\ \underline{+213_5} \end{array}$ (g) $\begin{array}{r} 1432_5 \\ \underline{+\ 4243_5} \end{array}$ (h) $\begin{array}{r} 4132_5 \\ \underline{+\ 444_5} \end{array}$

(i) $\begin{array}{r} 3214_5 \\ \underline{+3431_5} \end{array}$ (j) $\begin{array}{r} 1001_5 \\ \underline{+\ 4444_5} \end{array}$ (k) $\begin{array}{r} 3042_5 \\ \underline{+\ 434_5} \end{array}$ (l) $\begin{array}{r} 3214_5 \\ \underline{+\ 4243_5} \end{array}$

2. Find the difference.

(a) $\begin{array}{r} 342_5 \\ \underline{-\ 211_5} \end{array}$ (b) $\begin{array}{r} 403_5 \\ \underline{-342_5} \end{array}$ (c) $\begin{array}{r} 333_5 \\ \underline{-124_5} \end{array}$ (d) $\begin{array}{r} 234_5 \\ \underline{-144_5} \end{array}$

(e) $\begin{array}{r} 440_5 \\ \underline{-\ 123_5} \end{array}$ (f) $\begin{array}{r} 313_5 \\ \underline{-240_5} \end{array}$ (g) $\begin{array}{r} 312_5 \\ \underline{-\ 222_5} \end{array}$ (h) $\begin{array}{r} 314_5 \\ \underline{-\ 234_5} \end{array}$

UNIT 3

SQUARE ROOT AND CUBE ROOT

3.1. Square and Square Root

Study the facts given below.

(i) If a number is multiplied by itself, the product obtained is called the square of that number. For example: $4 \times 4 = 16$

$\therefore$ 16 is the square of 4.

Similarly, $5 \times 5 = 25$

Here, 25 is the square of 5.

(ii) If a number is factorized into two equal (same) factors then each of them is called the square root of the number. For example, 4 is the square root of 16 because $16 = 4 \times 4$

Similarly, 3 is the square root of 9. Because $9 = 3 \times 3$. Mathematically, square root is denoted by the symbol $\sqrt{\ }$. For example, square root of 64 is written as $\sqrt{64}$ and $\sqrt{64} = \sqrt{8^2} = 8$.

Example 1

Find the square root of 576.

Solution:

$$\text{Here, } \sqrt{576} = \sqrt{2\times2\times2\times2\times2\times2\times3\times3} = \sqrt{2^2\times2^2\times2^2\times3^2}$$
$$= 2\times2\times2\times3 = 24$$
$$\therefore \text{ square root of } 576 = 24.$$

To find the square root by division method

If the number, whose square root is to be found, is very large or it is difficult to factorize it, then division method is useful for finding the root.

In order to find the square root of a number by division method, following steps are suggested. Study the example given below carefully.

Step I: Mark a bar over each pair of the number, whose root is to be found from the right. For example in 15376, first pair from right is 76. Mark a bar over 76 i.e. $\overline{76}$. Mark a bar over second pair i. e. $\overline{53}$ and so on. The last number may be a pair or a single.

1	$1\overline{53}\ \overline{76}$	124
+1	-1	
22	×53	
+2	-44	
244	976	
+4	976	
248	×	

Step II : Consider the left most number, it is of either pair digits or single digit. Determine the nearest root of the number. The square root of the number is less or equal to the number. Here the left most number is of single digit 1. Its root is 1. Write down 1 as the division and quotient.

Step III: Write down the product of the divisor and quotient just below the number considered in Step II and subtract it. On the divisor side, write down 1(divisor) just below it and add them. Here it is $1 + 1 = 2$

Step IV: Now copy down the next pair. In the example above it is 53. Divide the first digit of 53 by the first digit of the divisor (here, it is 2). Since $2 \times 2 = 4 < 5$, we set 2 with 2 so that we have 22 as the new divisor and 2 with 1 as quotient (so that the quotient is 12) and again write down the new 2 just below the second 2 is the divisor and find the product mentally. Here it is $22 \times 2 = 44$. This product is written down below the second dividend 53 and subtract. On the left side add 2 and get the two digits of divisor as 24.

Step V : Again copy down the next pair to the right of I so that it is 976. Divide the first digit of 976 by first digit of 24 (the third divisor) i. e. $9 \div 2 = 4$. Write down 4 to the right of 24 so that the third divisor is 244. Repeat the problem division as unless the remainder is zero or the division is impossible.

Example 2

Find the square root of 46656.

Solution:

	216
2	$\overline{4}\ \overline{66}\ \overline{56}$
+2	-4
41	×66
+1	-41
426	2556
+6	2556
432	×

$\therefore$ the reqd. square root = 216

Example 3

Find the square root of 525625.

Solution:

	725
7	$\overline{52}\ \overline{56}\ \overline{25}$
+7	-49
142	×356
+2	-284
1445	$72\overline{25}$
+5	-7225
1450	×

$\therefore \sqrt{525625} = 725$

EXERCISE 3 [A]

1. Find by factorization method, the square root of the following.

(a) 256	(b) 625	(c) 169	(d) 676
(e) 484	(f) 3364	(g) 5625	(h) 1024
(i) 2025	(j) 3249	(k) 1225	(l) 6561

2. Find the square root of the following by division method.

(a) 7225	(b) 15376	(c) 181476	(d) 63504
(e) 28224	(f) 15129	(g) 165649	(h) 64009
(i) 11664	(j) 2916	(k) 4489	(l) 9025
(m) 95481	(n) 1811716	(o) 651249	(p) 819024

3. Simplify.

(a) $\sqrt{2^2 \times 3^2}$	(b) $\sqrt{2^2 \times 3^2 \times 4^4}$	(c) $\sqrt{3^4 \times 5^4}$	(d) $\sqrt{3^4 \times 5^2}$
(e) $\sqrt{18}$	(f) $\sqrt{50}$	(g) $\sqrt{80}$	(h) $\sqrt{1008}$
(i) $\sqrt{1^3 + 2^3 + 3^3}$	(j) $\sqrt{10^2 - 6^2}$	(k) $\sqrt{10^2 + 44}$	(l) $\sqrt{8^2 + 6^2}$

3.2. Cube and Cube Root

Study the facts about cube and cube root given below.

(i) The product of three equal (same) numbers is called the cube of the number.
For example, $4 \times 4 \times 4 = 64$.
Here, 64 is the cube of 4.
Similarly, $3 \times 3 \times 3 = 27$
Hence, 27 is the cube of 3.

Example 4

Find the cube of 11.

Solution :

Here,

Cube of 11 $= 11 \times 11 \times 11$
$= 1331.$

$\therefore$ Cube of 11 $= 1331.$

(ii) Any one of the three equal factors of a number is called the cube root of the number.

For example

$8 = 2 \times 2 \times 2$

Here, Cube root of 8 is 2 because product of three 2 is 8.

$$\begin{aligned}\text{Similarly, } 64 &= \overline{2\times2} \times \overline{2\times2} \times \overline{2\times2}\\ &= 4 \times 4 \times 4\end{aligned}$$

Here, 4 is the cube root of 64.

Mathematically, cube root is written as $\sqrt[3]{}$

$$\begin{aligned}\text{i e. Cube root of } 64 &= \sqrt[3]{64}\\ &= \sqrt[3]{4^3}\\ &= 4.\end{aligned}$$

Example 5

Find the cube root 5832.

Solution :

Here,

Factorizing 5832,

We get

$$\begin{aligned}5832 &= \overline{2\times2\times2} \times \overline{3\times3\times3} \times \overline{3\times3\times3}\\ \text{or } \sqrt[3]{5832} &= 2 \times 3 \times 3\\ \sqrt[3]{5832} &= 18.\end{aligned}$$

EXERCISE 3 [B]

1. Find the cube of the following.

(a) 3	(b) 9	(c) 11	(d) 13
(e) 72	(f) 181	(g) 108	(h) 115
(i) 143	(j) 620	(k) 700	(l) 650

2. Find the cube root of the following numbers.

(a) 1728	(b) 4096	(c) 9261	(d) 10648
(e) 3375	(f) 2744	(g) 46656	(h) 91125
(i) 21952	(j) 64000	(k) 729000	(k) 32768000

UNIT 4

RATIONALIZATION

4.1. (a) Rationalization of the Denominator.

If the denominator of a fraction contains $\sqrt{\ }$ sign then the $\sqrt{\ }$ sign of the denominator can be eliminated multiplying the numerator and denominator by the same suitable number. This process is known as the rationalization of the denominator.

Example 1

Rationalize the denominator of the following fractions.

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{\sqrt{2}}{\sqrt{3}}$ (c) $\frac{6}{\sqrt{3}}$ (d) $\frac{3}{\sqrt{24}}$

Solution :

(a) $\frac{2}{\sqrt{5}} = \frac{2\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{2\sqrt{5}}{5}$

(b) $\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times\sqrt{2}} = \frac{\sqrt{6}}{3}$

(c) $\frac{6}{\sqrt{3}} = \frac{6\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

(d) $\frac{3}{\sqrt{24}} = \frac{3}{2\sqrt{6}} = \frac{3\sqrt{6}}{2\sqrt{6\times\sqrt{6}}} = \frac{3\sqrt{6}}{12} = \frac{\sqrt{6}}{4}$

EXERCISE – 4 [A]

1. Rationalize the denominators of the following.

(a) $\frac{4}{\sqrt{7}}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\frac{\sqrt{2}}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{6}}$

(e) $\frac{8}{\sqrt{6}}$ (f) $\frac{15}{\sqrt{10}}$ (g) $\frac{3}{2\sqrt{3}}$ (h) $\frac{6}{\sqrt{12}}$

4.2 (b) Addition and subtraction of numbers involving radicals ($\sqrt{\ }$) sign.

The expression involving the radicals of the same number can be simplified in a way similar to the simplification of the algebraic expression.

Example 2

Simplify

(a) $5\sqrt{2} + 3\sqrt{2}$ (b) $\sqrt{5} - 2\sqrt{5}$

(c) $4\sqrt{5} + 5\sqrt{2} - 3\sqrt{5} - 2\sqrt{2}$

Solution :

(a) $5\sqrt{2} + 3\sqrt{2}$

$= (5 + 3)\sqrt{2}$

$= 8\sqrt{2}$

(b) $\sqrt{5} - 2\sqrt{5}$

$= (1-2)\sqrt{5}$

$= -\sqrt{5}$

(c) $4\sqrt{5} + 5\sqrt{2} - 3\sqrt{5} - 2\sqrt{2}$

$= (4-3)\sqrt{5} + (5-2)\sqrt{2}$

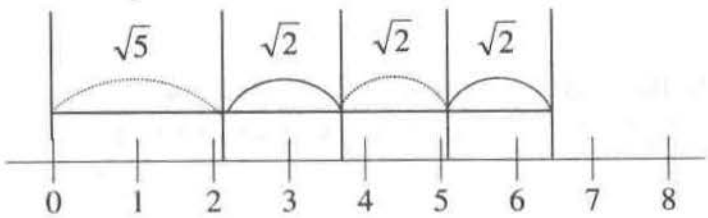
$= \sqrt{5} + 3\sqrt{2}$

In the examples above the answer of (c) $\sqrt{5} + 3\sqrt{2}$ cannot be simplified further, still it is a number.

From the table, we have, $\sqrt{2} = 1.414$ and $\sqrt{5} = 2.236$

$\therefore \sqrt{5} + 3\sqrt{2} = 2.236 + 3 \times 1.414 = 2.236 + 4.242 = 6.478$

In a number line, it can be shown as:



Example 3

Simplify

(a) $\sqrt{18}+\sqrt{8}$ (b) $3\sqrt{12}-\sqrt{63}-\sqrt{27}$

Solution :

(a) $\sqrt{18}+\sqrt{8}$

$=3\sqrt{2}+2\sqrt{2}$

$=5\sqrt{2}$

(b) $3\sqrt{12}-\sqrt{63}-\sqrt{27}$

$=6\sqrt{3}-3\sqrt{7}-3\sqrt{3}$

$=3\sqrt{3}-3\sqrt{7}$

Example 4

Simplify

(a) $3\sqrt{2}+\dfrac{4}{\sqrt{2}}$ (b) $\dfrac{6}{\sqrt{3}}-2\sqrt{5}-\dfrac{5\sqrt{3}}{4}$

Solution :

(a) $3\sqrt{2}+\dfrac{4}{\sqrt{2}}$

$=3\sqrt{2}+\dfrac{4\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$=3\sqrt{2}+\dfrac{4\sqrt{2}}{2}$

$=3\sqrt{2}+2\sqrt{2}$

$=5\sqrt{2}$

(b) $\dfrac{6}{\sqrt{3}}-2\sqrt{5}-\dfrac{5\sqrt{3}}{4}$

$=\dfrac{6\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}-2\sqrt{5}-\dfrac{5\sqrt{3}}{4}$

$=\dfrac{6\sqrt{3}}{3}-2\sqrt{5}-\dfrac{5\sqrt{3}}{4}$

$=2\sqrt{3}-2\sqrt{5}-\dfrac{5\sqrt{3}}{4}$

$=\dfrac{3}{4}\sqrt{3}-2\sqrt{5}$

EXERCISE – 4 [B]

1. Simplify :

(a) $6\sqrt{6}+2\sqrt{6}$ (b) $4\sqrt{5}-\sqrt{5}$ (c) $-7\sqrt{3}+2\sqrt{3}$

(d) $3\sqrt{2}-5\sqrt{2}$ (e) $-2\sqrt{7}-4\sqrt{7}$ (f) $-2\sqrt{10}+8\sqrt{10}$

2. Simplify:

(a) $2\sqrt{10}-6\sqrt{10}+7\sqrt{10}$ (b) $\sqrt{2}+2\sqrt{3}-5\sqrt{2}+\sqrt{3}$

(c) $3\sqrt{7}-3-2\sqrt{7}+2$ (d) $2\sqrt{3}+3\sqrt{2}-4\sqrt{3}+5\sqrt{2}$

3. Simplify:

(a) $\sqrt{125}-\sqrt{45}$ (b) $\sqrt{72}-\sqrt{50}+\sqrt{32}$ (c) $4\sqrt{7}+\sqrt{49}-3\sqrt{28}$

(d) $2\sqrt{8}-5\sqrt{2}+3\sqrt{12}$ (e) $3\sqrt{50}-7\sqrt{18}+4\sqrt{8}-6\sqrt{2}$

(f) $\sqrt{20}-2\sqrt{32}+\sqrt{18}-2\sqrt{45}$

4. Simplify (rationalize the denominator if necessary).

(a) $\frac{\sqrt{5}}{2}-\frac{1}{\sqrt{5}}$ (b) $\frac{21}{\sqrt{7}}-\frac{5\sqrt{7}}{2}$ (c) $\sqrt{48}-\frac{6}{\sqrt{3}}+5\sqrt{3}$

(d) $\sqrt{3}+\sqrt{27}-\frac{12}{\sqrt{3}}$ (e) $\frac{6}{\sqrt{2}}-\frac{24}{\sqrt{12}}+2\sqrt{48}-3\sqrt{8}$

(f) $-2\sqrt{20}+\frac{42}{\sqrt{28}}+\frac{60}{\sqrt{45}}+2\sqrt{175}$

4.3. Scientific Notation of the Numbers

Sometimes we need to write huge numbers and sometimes very small numbers, for example, the distance between the Sun and the earth in metres is written as 160000000000 m. similarly, we study about electrons. The mass of an electron is written as 0;00000000000000000000000000000091 gm. Such numbers are difficult to read and write. Hence these numbers are written in simplified forms as the product of a number between 0 and 10 and a number in integral powers of 10. In the examples above the distance between the sun and the earth is written as 1.6×10^{11} m and the mass of an electron as 9.1×10^{-31} kg.

Example 1

Write down the number 310000000 in scientific notation.

Solution :

Here, 310000000 $= 31 \times 10^7$ (31 is followed by 7 zeros)
$= 3.1 \times 10^1 \times 10^7$
$= 3.1 \times 10^8$ $(10^1 \times 10^7 = 10^{1+7})$

Alter, Here, 310000000 = 3.1×10^8 [put a decimal point after first digit from the left and multiply the number by 10 to the power number of digits to the right of decimal point]

Example 2

Write down in scientific notation.

(a) 3760000 (b) 0.00000529

Solution :

(a) Here, 3760000 $= 376 \times 10000$

$= \frac{376 \times 100}{100} \times 10^4$

$= \frac{376}{100} \times 10^2 \times 10^4$

$= 3.76 \times 10^6$

$\therefore$ 3760000 $= 3.76 \times 10^6$

(b) Here, 0.00000529 $= \frac{0.00000529}{1000000} \times 1000000$

$= \frac{0.00000529 \times 10000000}{10^6} = \frac{5.29}{10^6}$

$= 5.29 \times 10^{-6}$

Alter,
Here, $0.00000529 = 5.29 \times 10^{-6}$

EXERCISE 4 [C]

1. Express the following in scientific notation.
   (a) 6200000 (b) 380000000 (c) 32800000
   (d) 360000 (e) 276 (f) 0.000052
   (g) 0.00000083 (h) 0.000000099

2. Convert the following number in general form
   (a) 3.8×10^6 (b) 2.5×10^4 (c) 3.72×10^7 (d) 2.5×10^5
   (e) 4.46×10^7 (f) 2.5×10^{-5} (g) 3.7×10^{-7} (h) 2.8×10^{-9}
   (i) 3.15×10^{-10} (j) 1.4×10^{-5}

3. Simplify the following and give your answer in scientific notation.
   (a) $2.34 \times10^2 + 2.35 \times10^2$ (b) $10^5 + 10^7$
   (c) $10^{-4} + 10^{-6}$ (d) $2.4 \times 10^2 + 8 \times 10^3$

4. Find the difference. Give your answer in scientific notation.
   (a) $3.75 \times 10^3 - 3.75 \times 10^2$ (b) $2.2 \times 10^2 - 3.4 \times 10^2$
   (c) $3.7 \times 10^6 - 2.4 \times 10^4 - 2.4 \times 10^4$ (d) $6.75 \times 10^5 - 6.78 \times 10^6$

5. Simplify:

(a) $(3 \times 10^5)(3 \times 10^4)$ (b) $(7 \times 10^5)(10 \times 10^3)$

(c) $(7.2 \times 10^7 + 2.3 \times 10^7) - (8 \times 10^2 - 2.5 \times 10^3)$

(d) $\dfrac{(6 \times 10^{-3})(7 \times 10^6)}{3 \times 10^{-7}}$ (e) $\dfrac{15000 \times 0.000004}{0.005}$

(f) $\dfrac{0.348 \times 0.002}{0.00058 \times 0.03}$ (g) $5.78 \times 10^2 - 6.19 \times 10^{-3} + 5.67 \times 10^{-3}$

6. **Express the numbers used in the following statements in scientific notation.**

(a) Population of Nepal, at present, is expected as 25 000000.

(b) Area of Asia continent is 42700000sq. km.

(c) The distance of the planet mars from our earth is 66600000000 meter.

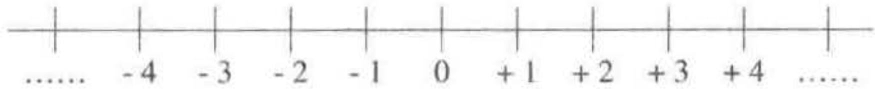
(d) 1sq. m. = $\dfrac{1}{10000}$ sq. mm.

(e) 1 sq. km. = $\dfrac{1}{1000000000000}$ sq. m.

UNIT 5

RATIONAL AND IRRATIONAL NUMBERS

5.1. (a) Rational and Irrational Numbers



Choose any two numbers from the number line representing the set of integers and add, subtract and multiply them separately. Determine whether the result is an integer or not. What conclusion did you derive from it?

The sum, difference and product of two integers are also the integers.

For example, $2 + 3 = 5$, $2\text{-}3 = -1$, $2 \times 3 = 6$ etc. Here 5, -1 and 6 are integers.

Now, choose any two integers as before and divide one by another. Let the integers be 15 and 8. On division, $15 \div 8 = 1\frac{7}{8}$ and $8 \div 15 = \frac{8}{15}$

Are these two numbers integers?

Here, either of the numbers is not exactly divisible by the other. Hence, to include the division operation within the family, the set of rational numbers have been developed.

Rational number can be defined as follows.

If a and b are two integers and $b \neq 0$, then the number of the form $\frac{a}{b}$ is called a rational numbers. The number that cannot be expressed in this form is called an irrational number.

The Set of rational numbers is denoted by Q. In the definition above, if b = 1, $\frac{a}{b} = \frac{a}{1} = a$.

Hence, the set of integers is the member of set Q, or, the set of integers Z is in the set Q. Mathematically we write $Q \supset Z$ or $Z \subset Q$.

Rational and irrational numbers can be expressed as decimal fractions in the following ways.

(i) Terminating decimal, for example, $\frac{2}{5} = 0.4$, $\frac{1}{4} = 0.25$

(ii) Non-terminating and repeating decimal: for example,

$\frac{1}{3} = 0.3$, $\frac{3}{7} = 0.4285713$ etc.

(iii) Non – terminating and non-repeating decimal : for example.

$\sqrt{2} = 1.41421356$.............

$\sqrt{5} = 2.23620679$...........

Non-repeating and non-terminating decimal numbers are called irrational numbers.

If a natural number n is not a perfect square then $\sqrt{n}$ is an irrational number. Furthermore, though a is rational, the number $a\sqrt{n}$ is an irrational number. An irrational number when multiplied or divided by a rational number gives an irrational number.

Example 1

Which of the following are irrational number ?

(a) - 0.7 (b) $\sqrt{6}$ (c) $\sqrt{36}$ (d) $-\frac{\sqrt{3}}{5}$ (e) $5\sqrt{3}$

Solution :

(a) $-0.7 = -\frac{7}{10}$, hence it is a rational number.

(b) In $\sqrt{6}$, 6 is not perfect square, hence it is an irrational number.

(c) In $\sqrt{36}$, 36 is a perfect square, hence it is a rational number.

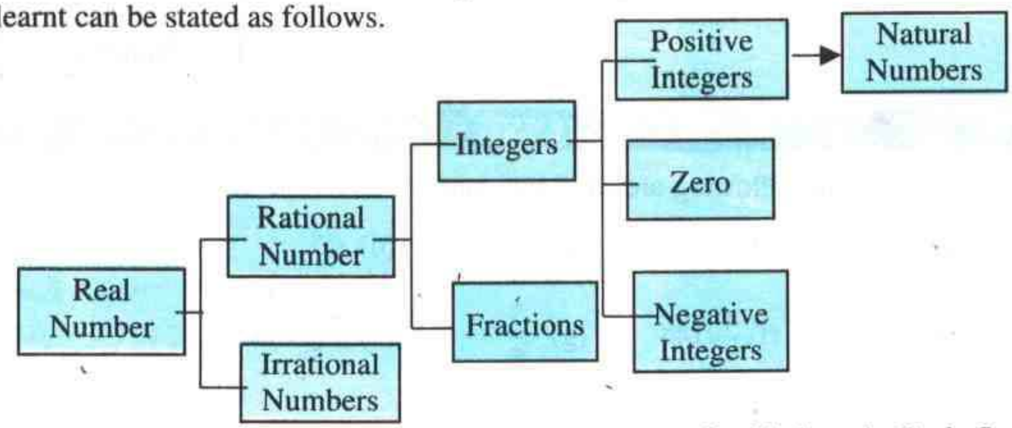
(d) In $-\frac{\sqrt{3}}{5}$, 3 is not a square number. Hence it is an irrational number.

(e) In $5\sqrt{3}$, $\sqrt{3}$ is an irrational number. Hence $5\sqrt{3}$ an irrational number.

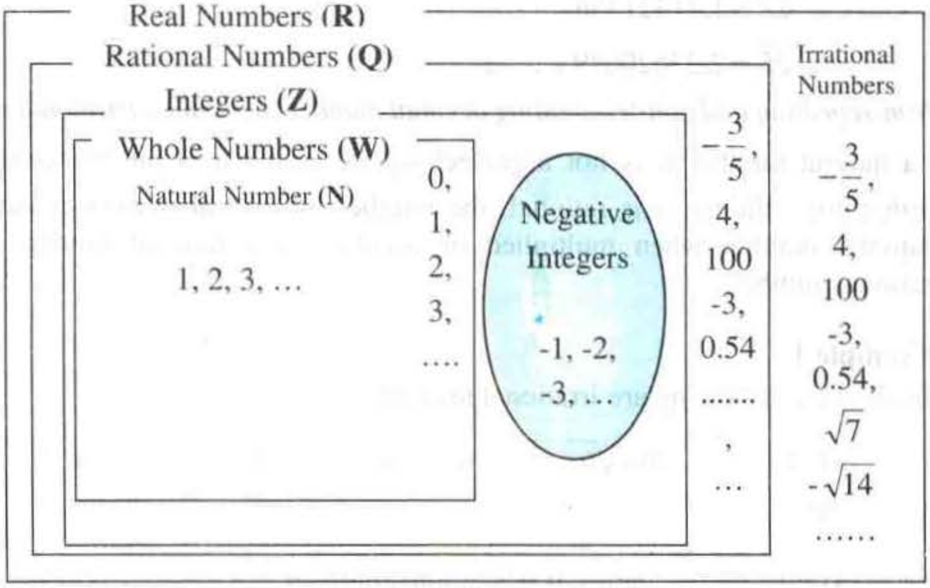
Now, in order to represent all the points in a number line, we have to consider the union of the sets of rational number. Such a set is known as the set of real number.

The union of the set of rational number and the set of irrational number is called the set of real numbers.

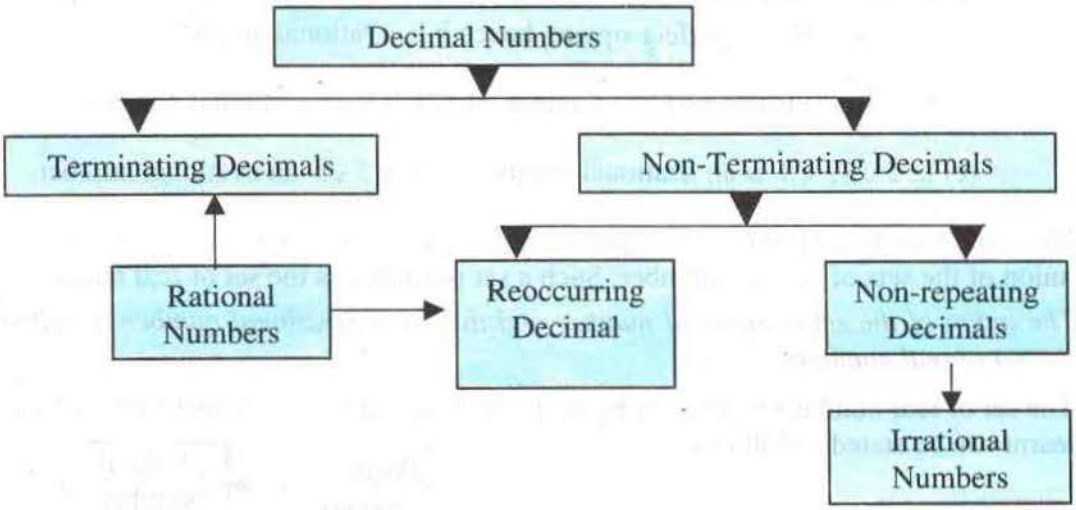
The set of real numbers is denoted by R. The relation of set of number that we have learnt can be stated as follows.



The relation of set numbers in Venn – diagrams can be shown as follows.



The relation when expressed in decimal number, we have the following chart.



EXERCISE 5 [A]

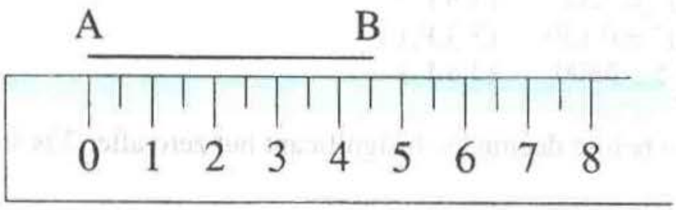
1. Which of the following are Irrational number ?

(a) $\frac{22}{7}$ (b) - $\sqrt{2}$ (c)- $\sqrt{4}$ (d) $\sqrt{5}$

(e) $\sqrt{7}$ (f) - $\sqrt{144}$ (g) - $\frac{3}{5}$ (h) $\frac{\sqrt{10}}{5}$

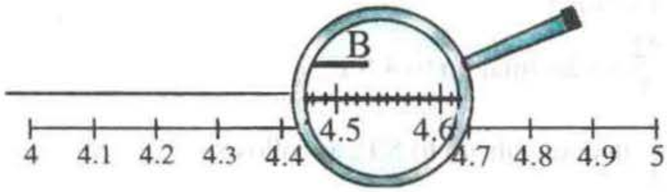
2. Identify the following statements as True or False.
   (a) Every real number is a rational number.
   (b) Every rational number is a real number.
   (c) Each real number is either rational or irrational.
   (d) Each irrational number is non-terminating decimal number.
   (e) Non- terminating and repeating decimal number is an irrational number.
   (f) Every non –terminating decimal number can be converted into a rational number.

5. 1 (b) Introduction to Significant Figure



[?] How long is the ling segment AB in the figure above?

Length of line segment AB is in between 4 cm and 5cm. Hence, it is measured in 10^{th} fraction of a cm is 4.5 cm. If we divide the section (portion) between 4.5 cm and 4.6 cm into 10 equal parts, then the point B is found to be quite close to 4.52 cm.



In this case the length of AB when measured into 100^{th} part of a cm., is 4.52 cm. Again the portion between 4.52 and 4.53 can be divided into 10 equal parts. In each division, length of AB is found to be differed after some places of decimal. Thus we conclude that the measure of length is not accurate. In such a case, the measurement is approximated. Among the various methods of approximation of measurements, one of the method in significant figures.

For Example,

AB = 4.5 cm (accurate measure in 2 significant figures)
AB = 4.52 cm (accurate measure in 3 significant figures)
Significant figure is abbreviated as S.F.

Consider some more examples.

The number of S.F. in 0.205 is 3

The number of S.F. in 0.025 is 2

Because in 0.025 zero after the decimal stands only for the position but it does not have any significant value. Such a zero is called an in significant figure.

Example 1

Round off up to 3 significant figures (S.F).

(a) 2.316 and (b) 0.1302 (c) 5762.5 (2 S.F.)

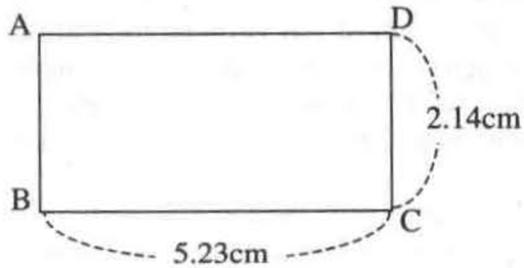
Solution :

(a) 2.316 = 2.32 (3 S.F.)
(b) 0.1302 = 0.130 (3 S.F.)
(c) 5762.5 = 5800 (2 S.F.)

[Here, zero before decimal is insignificant but zero after 3 is significant]

EXERCISE 5 [B]

1. Round off the following numbers to the significant figures claimed against it.
   (a) 1.055 (3 S.F.) (b) 0.2708 (3 S.F.)
   (c) 0.0047 (1 S.F.) (d) 27.9 (2 S.F.)
   (e) 0.0473 (2 S.F.) (f) 143. 528 (4S.F.)
   (g) 256.21 (2S.F.)
2. Express $\frac{22}{7}$ in decimal up to 4 S.F.
3. Express $\frac{2}{3}$ in decimals up to S.F. as follows
   (a) 2 S.F. (b) 3 S.F. (c) 4 S.F.
4. Find area of rectangle ABCD
   (a) in 4 S.F.
   and (b) in 3 S.F.



UNIT 6

RATIO, PROPORTION AND PERCENTAGE

6.1. Ratio

Study the examples given below.

(a) Ram has Rs 50.
(b) Sita is 4 feet tall.
(c) Madhav secured 90 marks in mathematics in class 7.
(d) Pokhara is 200 km far from Kathmandu.
(e) Mohan has Rs 250.
(f) Geeta's height is 36 inches.
(g) Keshav stood first obtaining 95 marks in mathematics in class 7.
(h) Janakpur is 400 km far from Kathmandu.

In the examples above which pair of the statements are comparable?
Objects having identical nature can be compared and hence they are paired as follows.

(i) a and e (ii) b and f (iii) c and g (iv) d and h

(i) Here, the ratio of money possessed by Ram and Mohan is 50 to 250. In other words, the ratio of rupees Ram and Mohan have $= \frac{50}{250}$ (comparison by division method).

Ratio of the amounts Ram and Mohan have is also denoted as 50: 250. Ratio is usually expressed in lowest form. Therefore, ratio of the rupees Ram and Mohan have $\frac{50}{250} = \frac{1}{5}$

$\therefore$ Ratio of the sum Ram and Mohan have $= \frac{1}{5}$

Ratio does not have any unit. If the value of one quantity is known, then with the help of ratio of the quantities, value of other quantity (-ies) can be determined.

Here, ratio of rupees Ram and Mohan have $= \frac{1}{5}$

If Ram has Rs 50 then $\frac{\text{Ram's amount}}{\text{Mohan's amount}} = \frac{1}{5}$

Or, $\frac{\text{Rs. 50}}{\text{Mohan's amount}} = \frac{1}{5}$

$\therefore$ Mohan's amount $= \text{Rs } 50 \times 5 = \text{Rs } 250.$

Similarly, if the amount Mohan possesses is known, we can find the amount which Ram possesses.

i.e. $\dfrac{\text{Ram's amount}}{\text{Mohan's amount}} = \dfrac{1}{5}$

Or, $\dfrac{\text{Ram's amount}}{\text{Rs. 250}} = \dfrac{1}{5}$

Or, Ram's amount $= \dfrac{\text{Rs. 250}}{5}$

$\therefore$ Ram has Rs 50

(ii) In the pair b and f, the statements are of identical character. Therefore, ratio of the heights of Sita and Geeta $= \dfrac{4\text{ft.}}{36\text{in.}}$

In order to find the ratio, units of the objects should be identical. Now, ratio of heights of Sita and Geeta $= \dfrac{48\text{in.}}{36\text{in.}} = \dfrac{4\text{ in.}}{3\text{ in.}}$

Or, the ratio = 4 : 3 (When reduced to lowest form).

$\therefore \dfrac{\text{Sita's height}}{\text{Geeta's height}} = \dfrac{4}{3}$ ($\therefore$ ratio has no unit)

Now, find the ratio of the objects in the pairs (ii) and (iv).

Example 1

Determine the ratio of the quantities given below.

(a) 75 cm and 1m (b) 2 kg and 250 gm

Solution :

In order to determine the ratio of quantities, units of the quanities should be made identical.

(a) Here, 1m = 100cm.

$\therefore$ Ratio of 75 cm and 1 m $= \dfrac{75\text{cm}}{100\text{cm}} = \dfrac{3}{4}$ or 3:4

(b) Here 2 kg = 2000gm

$\therefore$ Ratio of 2 kg and 250 gm $= \dfrac{2000\text{gm}}{250\text{gm}} = \dfrac{8}{1}$ or 8:1

Example 2

Two quantities are in the ratio 3:4. If 5 is subtracted from the both, the ratio will be 5:7, find the quantities.

Solution :

Let the quantities be 3x and 4x

By the question, $\frac{3x - 5}{4x - 5} = \frac{5}{7}$

$$\text{Or, } 7(3x - 5) = 5(4x - 5)$$
$$\text{Or, } 21x - 35 = 20x - 25$$
$$\therefore x = 10$$

$\therefore$ required quantities are $3x = 3 \times 10 = 30$

$4x = 4 \times 10 = 40$

Example 3

Three friends jointly started a business with an investment of Rs. 1,17,000. If they have invested in the ratio 5:4:3, how had each invested?

Solution :

Here, Rs 1,17,000 is divided in the ratio 5:4:3.

Let each has invested 5x, 4x and 3x respectively.

By the question, $5x + 4x + 3x = \text{Rs } 1{,}17{,}000$

$$\text{Or, } 12x = \text{Rs } 1{,}17000$$
$$\therefore x = \text{Rs.}\frac{1{,}17{,}000}{12} = \text{Rs.}9750$$

$$\therefore 5x = 5 \times \text{Rs } 9750 = \text{Rs } 48{,}750$$
$$4x = 4 \times \text{Rs } 9750 = \text{Rs } 39{,}000$$
$$3x = 3 \times \text{Rs } 9750 = \text{Rs } 29{,}250$$

Hence, their investment was respectively Rs 48, 750, Rs 39,000 and Rs 29, 250.

EXERCISE - 6 [A]

1. Find the ratio of the quantities (first to second) and reduce it to lowest term.
   (a) 32 : 56 (b) 120 cm : 2 m (c) 450 gm : 1 kg
   (d) 2 kg : 750 gm (e) 375ml : 1l. (f) 15 sec : 1 hr
2. Find the value of x in each case:
   (a) $\frac{x}{3} = \frac{5}{15}$ (b) $\frac{2}{x} = \frac{10}{25}$ (c) $\frac{3}{5} = \frac{x}{15}$ (d) $\frac{7}{9} = \frac{28}{x}$
3. If a:b = 3 : 4 and b:c = 5 : 7, then
   (a) What is the value of a ?
   (b) What is the value of c?
   (c) Using the value of a and c above, find a:c.
4. Breads and biscuits are distributed in the ratio 5:3 for tiffin in a school. If 100 breads were served, find the quantity of biscuits required.

5. A map was drawn to the scale 1 : 20000. If the distance between two places in the map is 10cm, what is the actual distance (in km) between two places?
6. The ratio of annual income and expenditure of a family is 7 :5. Find the saving of the family if it has an income of Rs. 77, 000.
7. Two numbers are in the ratio 5:7. If 3 is added to both of them. The ratio is found to be 4:5. What are the numbers?
8. Sheela, Kailash and Shiva are 12, 10 and 8 years old respectively. If Rs. 60 is divided among them in the ratio of their ages, how much will each get?
9. Three partners invested in the ratio 3:5:7 and purchased a bus for Rs. 6,00,000. How much did each invest ?
10. B spent double of A and C spent double of B. Altogether they spent Rs. 490. How much did each spend ?
11. 198 students appeared in the S.L.C. Examination from a school. Among them the ratio of the students who passed in the first and the second division was 2:3. 60 Students were passed in the third division and 78 failed.
    (a) How many students passed in the first and the second division each?
    (b) Find the ratio of the students who passed in the first, second, and third division as well as failure.

Proportion

Ganesh secured 84 out of 100 in mathematics and 42 out of 50 in health and physical.

Therefore the ratio of score in mathematics is $\frac{84}{100}$ whereas in health and physical is $\frac{42}{50}$.

Are the ratios equal?

Here, $\frac{84}{100} = \frac{21}{25}$ and $\frac{42}{50} = \frac{21}{25}$

The reduced forms of both ratios are same (i. e. $\frac{21}{25}$)

Hence the ratios $\frac{84}{100}$ and $\frac{42}{50}$ are equal.

Therefore $\frac{84}{100}$ and $\frac{42}{50}$ are in proportion;

Or, 84, 100, 42, 50 are proportional.

Among these 4 numbers, $84 \times 50 = 100 \times 42$.

i. e the product of extremes (outer terms) is equal to the product of means (inner terms).

Therefore if $\frac{a}{b} = \frac{c}{d}$ then ad = bc.

Example 1

First three terms of the proportional are respectively 3,4 and 6. Find the fourth term.

Solution :

Let the required term be x, then

$$\frac{3}{4}=\frac{6}{x}$$

Or, $3x = 24$

Or, $x = \frac{24}{3} = 8.$

$\therefore$ the fourth term is 8.

Example 2

Two quantities are in the ratio 3:4. If 5 is subtracted to both, the new ratio will be 5:7. Find the quantities.

Solution :

Let the quantities be 3x and 4x.

By the question, $\frac{3x-5}{4x-5}=\frac{5}{7},$

Or, $7(3x-5) = 5(4x-5)$

Or, $21x - 35 = 20x - 25$

Or, $21x - 20x = 35 - 25$

Or, $x = 10$

$\therefore$ required quantities are $3x = 3 \times 10 = 30$

and $4x = 4 \times 10 = 40.$

Example 3

If the cost of painting 4 walls having area 150 sq.m. is Rs 300, what will be the cost of painting for the walls having an area of 200 sq.m.?

Solution :

Let the required cost be x

Since the costs are proportional to area, we have

$$\frac{150}{300}=\frac{200}{x},$$

Or, $150x = 200 \times 300$

Or, $x = \frac{200 \times 300}{150}$

$\therefore x = 400$

Hence, req. cost of painting the walls = Rs 400.

EXERCISE 6 [B]

1. Determine whether the given four numbers are proportional or not.
   (a) 2 cm, 3 cm, 4 cm, 6 cm,   (b) 5sec, 15 sec, 1 sec, 3 sec
   (c) 5 kg, 3kg, 15 kg, 25 kg   (d) 3 *l.*, 6 *l.*, 10 *l.*
2. Find the fourth term if first three terms of the proportionals are.
   (a) 3, 4, 6   (b) 5, 7, 10   (c) 16, 4, 4   (d) 7, 9, 14.
3. Find the value of x for the following:
   (a) $x : 3 = 5 : 15$ (b) $2 : x = 10 : 25$
   (c) $3 : 5 = x : 15$ (d) $7 : 9 = 28 : x$
4. Breads and biscuits are distributed in the ratio 6:4 in a school. If 120 breads are served, find the quantity of biscuits required.
5. If 15 chocolates cost Rs 105, how much will 20 chocolates cost?
6. If a bus travels 120 km in 3 hours, how long will it take to travel 200 km at the same speed?
7. How much will it cost to paint the walls having an area of 180 sq.m. if the cost of painting 4 walls having the area 120 sq.m. is Rs 180?
8. 12 men can plant the paddy in a ropani of field in 10 hours. How many men are required to plant the same in 6 hours only?
9. If the gravitational ratio of the Moon to the Earth is 1:6, what is the weight of a man on the Moon if he weighs 72 N on the Earth?
10. 340 students appeared in the S.L.C. Examination from a school. Among them the ratio of the students who passed in the first and the second division was 3:5. 34 students were passed in the third division and 58 failed.
    (a) How many students passed in the first and the second division each?
    (b) Find the ratio of the students who passed in the first, second and third division.

6.3. Percentage

6.3.1. Introduction

Study the examples below.

(i) 10% of 100 =10   (ii) 10% of 150 = 15
(iii) 10% of 220 =22   (iv) 10% of 25 = 2.5

Examples above suggest that 10% (presents) in different amounts. Though the percentage number is same, amounts depending on the numbers are different. Therefore, percentage is related to the given number.

Let us have some other examples.

(i) 10% of 100 = 10   (ii) 5% of 200 = 10
(iii) 20% of 50 = 10   (iv) 25% of 40 = 10

Here the resulting amounts are 10 in each cases but the percentages are different. Therefore percentage is related to the given number. Percentages always do not represent the same number. It only indicates a relative measure. Hence percentage alone cannot be considered as a quantitative number.

6.3.2. (a) To find the given percentage of the given number

Example 1

A man earns Rs 6000 a month and spends as follows.

(i) On food 40% (ii) On education 20%

(iii) On clothes 15% (iv) On others 10%

How much does he save monthly?

Solution :

Here, expenses on different items are :

(i) On food $= 40\%$ of Rs. 6000

$= \frac{40}{100} \times \text{Rs } 6000 = \text{Rs } 2400$

(ii) On education $= 20\%$ of Rs $6000 = \frac{20}{100} \times \text{Rs } 6000$

$= \text{Rs } 1200$

(iii) On clothes $= 15\%$ of Rs $6000 = \frac{15}{100} \times \text{Rs } 6000 = \text{Rs } 900.$

(iv) On others $= 10\%$ of Rs $6000 = \frac{10}{100} \times \text{Rs } 6000 = \text{Rs } 600.$

Therefore, total expenses $= \text{Rs } (2400 + 1200 + 900 + 600)$

$= \text{Rs } 5100$

$\therefore$ Saving = income – expense = Rs (6000-5100)

$= \text{Rs } 900.$

Alternate method:

Total percentage expense $= (40 + 20 + 15 + 10)\% = 85\%$

Now, percentage saving $= (100 - 85)\% = 15\%$

$\therefore$ Actual saving $= 15\%$ of Rs $6000 = \frac{15}{100} \times \text{Rs } 6000$

$= \text{Rs } 900$

Example 2

Ramesh purchased a pant in the Dashain market, marked Rs 325 at a discount of 15%. How much did it cost?

Solution :

Marked price of the pant = Rs 325

Discount p.c. = 15%

Now, actual discount = 15% of Rs 325

$= \frac{15}{100} \times \text{Rs } 325 = \text{Rs } 48.75$

Amount paid by Ramesh = Marked price of pant –discount

= Rs 325 - Rs 48.75 = Rs 276.25

6.3.2 (b) To find one quantity as percentage of another.

Example 3

What percent of Rs 1500 is Rs 300?

Solution :

Let the required percentage is $x\%$

By the question $x\%$ of Rs 1500 = Rs 300

Or, $\frac{x}{100} \times \text{Rs}1500 = \text{Rs } 300$

Or, $15x = 300$

$\therefore \; x = 20$

$\therefore$ required percentage = 20%

Example 4

Last year Ram's salary was Rs 7000. This year his salary is Rs 7500. By what percent has his salary increased?

Solution :

Here, increase in his salary = Rs 7500 - Rs 7000

= Rs 500

Let his salary has increased be $x\%$

By the question ,

$x\%$ of Rs 7000 = Rs 500

Or, $\frac{x}{100} \times 700 = 500$

Or, $70x = 500$

$\therefore \; x = \frac{50}{7} = 7\frac{1}{7}.$

$\therefore$Required percentage = $7\frac{1}{7}\%$

Example 5

On the auspicious occasion of new year, a shop displayed the price list as shown below. Find the discount percentage in each items.

Article	Price	Price after discount
Jean Pant	Rs. 520	Rs. 442
Shirt	Rs. 225	Rs. 198
Shoes	Rs. 625	Rs. 450

Solution :

Here,

(a) Discount in Jean's pant
= (Marked) price – price after discount
= Rs 520 - Rs 442 = Rs 78

Now, out of Rs 520, amount of discount is Rs 78.

$\therefore$ Out of Rs.1, amount of discount = $\frac{\text{Rs } 78}{520}$ (amount will be less, hence divided)

$\therefore$ Out of Rs 100, amount of discount = $\frac{\text{Rs } 78}{520} \times 100 = \text{Rs } 15$

(amount will be more, hence multiplied)

$$\therefore \text{Discount} = \frac{15}{100} = 15\%$$

(b) Discount in Shirt
= marked price – price after discount
= Rs 225 - Rs 198 = Rs 27.

Now, out of Rs 225, amount of discount is Rs 27.

$$\text{Hence percent discount} = \frac{\text{discount}}{\text{marked price}} \times 100\%$$

$$= \frac{27}{225} \times 100\% = 12\%$$

[Note : Discount percentage is calculated on original (or, marked) price]

(c) Finally, discount in shoes = Rs 625 - Rs 450 = Rs 175

$$\text{Now, discount percentage} = \frac{\text{actual discount}}{\text{original price}} \times 100\%$$

$$= \frac{175}{625} \times 100\% = 28\%$$

EXERCISE 6 [C]

1.
   (a) What is the sum whose 15% is Rs 300?
   (b) What is the amount whose 17% is Rs 340?
   (c) Find the number of days whose 12% is 48 days.
   (d) Find the population whose 25% is 600 people.
   (e) Determine the number of students whose 35% is 70.
   (f) What is the amount of liquid whose 10% is 200ml?
   (g) What is the length whose 8% is 4 m?

2. 190 students, on a day, arrived in time whereas 5% were late in the school assembly. How many students came late? How many students were altogether in the school?

3. Among 1200, only 960 voters cast their votes in a VDC election.
   (a) What percent of the total voters cast their votes?
   (b) What percent of the voters did not cast their votes?
   (c) If 25% of those who did not cast their votes had gone abroad, what is their number ?

4. Out of 700 students in a school. Weekly attendance in percentage was as follows.

   On Sunday 92% On Monday 89%, On Tuesday 93%
   On Wednesday 95% On Thursday 96% On Friday 83%

   Find the number of students attending school on each day.

5. Monthly income of a family is Rs 7000. It spends different items as follows:
   On food 30%, on clothes 18%, on education 25%
   On medicines 5% and on others 12%
   (a) What is the expenditure on each item?
   (b) What percentage of the income does the family save each month?
   (c) What sum does the family save per month?

6. 65% of the total population of Ramghat Pokhara are adults and the rest are children. If the number of children is 7000, then
   (a) What is the total population of that place?
   (b) What are the numbers of adults there?

7. The population of Nepal in 2058 B.S was 2, 35,00,000. If the annual growth rate is 2% find the population in 2060 B.S.

8. If Bhagawati bought a radio for Rs 1200 at a discount of 25% then what was the selling price without discount?

9. A book-distributor (dealer) offers 30% discount on his publications. While purchasing his publications, a book-seller gained a discount of Rs 90000, what was the labeled price of the books purchased?

10. The number of tourists visiting Nepal was 1,20,000 in 1993 A.D. In 1994, it was increased by 10% and in 1995 it was decreased by 10%. How many tourists visited Nepal in 1995 A.D.?

11. Shiva joined a job in 2050 B.S. His starting salary was Rs 8000 per month. In 2051, his salary increased by 10%. Again he got an increase of 15% in 2052 B.S, How much did he receive monthly in 2052 B.S.?

12. On the occasions of Vijaya Dashami, price list of same articles with sale price displayed in a ready made shop was as follows.

Articles	Price	Discount	Sale Price
Pant	Rs. 345	15%	Rs. 290
Shirt	Rs. 175	20%	Rs. 145
Track Suit	Rs. 445	18%	Rs. 365

From the above sale chart find which one would be cheaper; the articles bought at discount percent or on sale price. Calculate the difference.

UNIT 7

PROFIT AND LOSS

In business articles are purchased and sold. If the selling price of the article is more than the cost price, then there is gain or profit. If the cost price of the article is more than the selling price, there is a loss. Usually profit and loss are concerned with trade and commerce, we often face these situations in our daily life. In mathematical terms, profit and loss are calculated using the formulae given below.

Profit = Selling Price - Cost Price
$\therefore$ P = S.P. - C.P.

And, Loss = Cost Price - Selling Price.
$\therefore$ L = C.P. - S.P.

In order to be acquainted with relative position of the business, profit and loss are calculated and expressed as percentages. Following formulae are used to calculate profit and loss percents.

$$\text{Profit percent} = \frac{\text{Actual Profit}}{\text{Cost Price}} \times 100\%$$

$$\text{Loss percent} = \frac{\text{Actual Loss}}{\text{Cost Price}} \times 100\%$$

Example 1

Ram Prasad purchased a motorbike for Rs 65000 and after one month he sold it at a loss of Rs 7500. For what price did he sell the bike?

Solution :

Here, Cost Price of the Bike (C.P.) = Rs 65000
Loss (L) = Rs 7500
Selling Price (S.P.) = ?
We know the formula, (L) = C.P. - S.P.
or S.P. = Rs 65000 – Rs 7500
= Rs 57500
$\therefore$ Required Selling Price = Rs 57, 500.

Example 2

Ram's father bought a milking buffalo for Rs 12000 and sold it for Rs 15500. Did he gain or lose? By how much ? Calculate in percentage?

Solution :

Here, Cost Price of the Buffalo (C.P.) = Rs 12000

Selling Price (S.P.) = Rs 15500

Since S.P. < C.P., there is profit.

$\therefore$ Profit (P) = S.P. - C.P. = Rs 15500 - Rs12000

= Rs 3500.

Further, we know profit percent $= \dfrac{\text{Actual Profit}}{\text{Cost Price}} \times 100\%$

$$= \frac{3500}{12000} \times 100\% = \frac{175}{6}\% = 29\frac{1}{6}\%$$

Example 3

Uma's mother purchased a Japanese Saree for Rs 780. After two months she intended to have the next new and hence she sold it for Rs 500 to her friend. Find her loss percent.

Solution :

Here, Cost Price of Saree (C.P.) = Rs 780

Selling Price of Saree (S.P.) = Rs 500

Since S.P. < C.P., there is loss.

And, Loss (L) = C.P. - S.P. = Rs 780 – Rs 500

= Rs 280

By the question,

$$\text{loss percent} = \frac{\text{Real loss}}{\text{Cost price}} \times 100\%$$

$$= \frac{280}{780} \times 100\% = \frac{1400}{39}\% = 35\frac{35}{39}\%$$

Example 4

Sita's elder brother was a business man. He brought a watch for Rs 700 from a town and sold it at a profit of 20%. How much did he gain? At what price did he sell the watch?

Solution :

Here, Cost Price of the watch (C.P.) = Rs 700

Profit percent = 20%

We know,

$$P\% = \frac{\text{Real profit}}{\text{Cost price}} \times 100\%$$

Or, Real Profit × 100 = P × Cost Price

Or, Real Profit = $\frac{P}{100} \times$ Cost price

$\therefore$ Real Profit = $\frac{20}{100} \times$ Rs 700 = Rs 140.

$\therefore$ Her brother made a Profit of Rs 140.

Further, S.P. = C.P. + P = Rs 700 + Rs 140 = Rs 840.

EXERCISE – 7

1. Shyam Sanker Sharma sold a goat-kid for Rs 1075 to Sanee's mother which he had bought for Rs 1250. Did he gain or lose and by how much?
2. Laxmi Bhatta, in this Tihar, bought caps at the rate of Rs 150 each and sold for Rs 200 each. What percent did he gain in each cap?
3. Price list displayed at Sani Gurung friend's shop was read as cement for sale, Rs 313 per bag. One day Sani's friend Geeta with her father went there. If Geeta's father purchased cement for Rs 300 a bag, at what rate of discount did he purchase the cement?
4. A copy costs Rs 12.50. If it is sold at a profit of 10%, what profit is made by selling 3 dozen copies?
5. If an article purchased at the rate of Rs 950 per quintal is sold at the rate of Rs 1054.50 per quintal. How much will be gained in 10 ton? [10 quintals = 1 ton]
6. What is the total selling price of 30 dozen calculators when sold at a discount of 10%. If it costs Rs 500 each?
7. He purchased a goat for Rs 4500 is sold at a profit of 20%. What will be its selling price?
8. What percent profit will one get if a goat bought for Rs 480 is sold for Rs 600?
9. Thule Rai sold a he-buffalo at a profit of 60% which he had bought 2 years ago on Rs 800. What was the selling price of the buffalo?
10. Kamal lost a book priced Rs 600 Which he had borrowed from Salikram. After one year Salikram charged him the price of the book along with a penalty of 20%. How much did he pay?
11. Ram bought a bicycle for Rs 3000 and sold it for Rs 3450. How much did he gain? What percent did he gain?
12. Krishna had purchased a TV - set for. Rs 25000. After 2 years he sold it at a loss of Rs 5000. What was its elling price? What percent did he lose?
13. An auto - merchant bought a second hand motorbike and sold it for Rs 23400 gaining there by 17%. What was the cost price of the motorbike?
14. Govinda purchased 10 dozen of copies from the factory and sold it to a stationer at a profit of 5%. The stationer sold it at a profit of 5% for Rs 1386.
    (a) What was the cost price of the copies for the stationer?
    (b) What was the cost price of the copies for Govinda?

UNIT 8

UNITARY METHOD

While solving the problems using unitary method first we find the cost of unit (one) article, and then the cost of required number of articles. Study the table below.

Direct Variation	
Number of Pens	**Cost**
5	Rs. 25
10	Rs. 50
4	Rs. 20
x	?

Indirect Variation	
Number of Workers	**Number of days Required**
2	15
10	3
3	10
y	?

From the first table, if 5 pens cost Rs 25, how can we say that 10 pens cost Rs 50? In order to find to cost of 10 pens we need to know the cost of 1 pen or the unit cost. Study the process shown below:

Cost of 5 Pens $= \text{Rs } 25$

$\therefore$ Cost of 1 Pen $= \dfrac{\text{Rs } 25}{5} = \text{Rs } 5$

$\therefore$ Cost of 10 Pens $= \text{Rs.}5 \times 10 = \text{Rs } 50$

And Cost of 4 Pens $= \text{Rs.}5 \times 4 = \text{Rs } 20$

Similarly, Cost of x Pens $= \text{Rs. } 5 \times x = \text{Rs } 5x$.

Here when the number of pens decreases the cost of pens also deceases and the number of pens increases, the cost increases. Therefore, the variation (or the change) is direct.

Direct Variation: *In between the two quantities, when one increases the other also increases and when one decreases other also decreases then the relation between the quantities is known as a direct variation.*

Now from the second table,

2 men (works) can do a piece of work in 15 days.

$\therefore$ 1 man can do the same in 15×2 days = 30 days.

$\therefore$ 10 men can do same work in $\dfrac{30}{10}$ days = 3 days.

$\therefore$ 3 men can do the same work in $\dfrac{30}{3}$ days = 10 days

Similarly, y men can do the same work in $\dfrac{30}{y}$ days

Here as the number of men (workers) increases the number of days required to complete the work decreases and as the number of men decreases the number of days increases. Therefore the variation is indirect or inverse.

***Indirect Variation :** In between the two quantities if one increases the other decreases and as one decreases the other increases then the relation among the quantities is called an indirect variation.*

Example 1

If 30 pens cost Rs 600 how much will 4 dozen pens cost?

Solution :

Here, the cost of pens is to be found hence we place the cost quantity at the end of the statement.

Since 30 pens cost Rs 600

$\therefore$ 1 pen will cost Rs $\frac{600}{30}$ = Rs 20

$\therefore$ 4 dozen (or $4 \times 12 = 48$) pens will cost Rs 20×48 = Rs 360

$\therefore$ Required price of the pens = Rs 960.

Alternate method

Here let the required cost be x.

We tabulate the in formations provided in the problem as follows.

Number of Pens	Cost
30	Rs. 600
4 dozen = $4 \times 12 = 48$	Rs. x (let)

Now the number of pen and the cost are directly varied, hence

$$\frac{30}{48} = \frac{600}{x}$$

Or, $$\frac{x}{600} = \frac{48}{30}$$

Or, $$x = \frac{48 \times 600}{30} = \text{Rs.}960$$

$\therefore$ Required Price = Rs 960.

[Note : In direct variation if 39 is in denominator so will be 600 and if 48 is in denominator so will be x].

Example 2

If 15 men can do a piece of work in 12 days how many men will complete the work in 18 days?

Solution :

Here let the number of men required be y.

Data provided are tabulated below.

Number of men	Working days
15	12
y	18

Since the relation between the number of men and the days required to do the work is indirectly varied we have,

$$\frac{15}{y} = \frac{18}{12}$$

Or, $\frac{y}{15} = \frac{12}{18}$

Or, y $= \frac{12 \times 15}{18} = 10.$

[Note : the variation is indirect hence if 15 is in the numerator then 18 will be in the numerator and if 18 is in the numerator then 12 will be in the denominator.]

∴ Number of men required = 10.

EXERCISE 8

1. If 3 dozen pencils cost Rs 90, how much will 30 pencils cost?
2. If 60 Kg of Mansuli rice costs Rs 1320, how much will 5 quintals of rice cost? (1 quintal = 100 kg)
3. If 200 Kg of ration lasts 3 months for 4 boys, how many boys will consume 650 kg. of ration in the same period?
4. If Eurika can buy 40 copies for Rs. 500, how much will she need to buy 6 dozen copies?
5. If 20 agro workers can dig a farm in 240 hours, how many workers will be required to dig the farm in 60 hours?
6. If 30 men working 6 hours a day can level up a road, how many men can do the same work in 10 hours a day?
7. An aero plane completes a journey in 480 minute travelling at the rate of 400 km per hour, how long will it take to complete the journey flying at the rate of 24 km per hour?
8. If 6 men can do a job in 7 days, how long will it take to do the same for 14 men ? How many men can complete the job in 2 days?
9. Daily income of 5 workers is Rs 1500.
   (a) How many workers will earn Rs 600?
   (b) And, if a worker spends Rs 150 daily, how much will be saved by 17 workers?
10. One can travel 45 km. by a motorbike on 1 liter of petrol,
    (a) How far will it travel on 4 liter of petrol?
    (b) How much petrol will be needed to travel 585 km. ?

UNIT 9

SIMPLE INTEREST

If you deposit Rs 500 in the saving account of Rastriya Banijya Bank, the bank after 1 year will return you Rs 523.75 or will deposit the amount in your account. Here,

(a) The sum deposited in the bank is known as the principal (P). Here, the sum of Rs 500 deposited in the bank is the principal.

(b) The sum returned by the Bank is known as an amount (A). In other words, amount is the principal accompanied by interest. Here Rs 523.75 is the amount.

(c) An extra sum provided by the bank over the deposited sum is known as an interest (I). Here the extra sum is Rs 523.75 – Rs 500 = Rs 23.75 is the interest.

(d) The bank provides interest at a fixed rate. Here, the bank provides Rs 23.75 as an interest on the sum of Rs 500 in 1 year. Therefore the interest on Rs 100 in 1 year $= \frac{23.75}{500} \times 100\% = 4.7\%$ is known as rate or rate of interest (R).

(e) The period during which the sum is kept in the bank is known as time of deposit or time (T). Here time is 1 year, mathematical words and symbols used in the problem of simple interest are.

Words	**Symbols**
Principal	P
Amount	A
Interest	I
Rate (of interest)	R
Time	T

Example 1

Nirmala deposited a sum of Rs P for T years at the rate of R% per year in Ratriya Banijya Bank, Sanothimi. How much will she get as an interest? Calculate the interest using unitary method.

Solution :

Here, R% per year means

Interest on Rs 100 for 1 year $= R$

$\therefore$ Interest on Rs 1 for 1 year $= \frac{R}{100}$

$\therefore$ Interest on Rs 1 for T years $= \frac{T \times R}{100}$

$\therefore$ Interest on Rs P for T years = $\frac{P \times T \times R}{100}$

Therefore, formula for calculating interest is $I = \frac{P \times T \times R}{100}$.......(I)

Now, derive the following formulae using the process above.

(1) $P = \frac{I \times 100}{T \times R}$ (2) $T = \frac{I \times 100}{P \times R}$ (3) $R = \frac{I \times 100}{P \times T}$ %

9.1. Amount

Goma deposited Rs 2500 in the saving account at Rastriya Banijya Bank, Bardiya. After one year, she asked for the statement. Bank provided her a statement in which it was mentioned: She has Rs 26125 in her account.

Here, Sum deposited by Goma in the saving account
Principal (P) = Rs 25000
Existing Sum (A) = Rs 26125
Extra Sum (I) = Existing sum - Deposited sum
= Rs 26125 - Rs 25000 = Rs 1125

Goma noticed that the existing sum Rs 26125 was the total of the sum deposited Rs 25000 and the interest Rs 1125.

Sum (total) of principal and interest is known as amount.
Therefore, A = P + I ....(II)

From the formula, A = P + I, if any two among A, P and I be provided, the third can be found as follows.

(a) P = A - I
(b) I = A - P

But if I (interest) is not given and P is to be found using T, R and A, then we need an extra formula. This extra formula can be calculated by using the formulae (I) and (II) above.

We know,

$A = P + I$

Or, $A = P + \frac{P \times T \times R}{100}$

$$\text{Or, } A = \frac{100P + PTR}{100}$$

$$\text{Or, } P\,(100+TR) = 100 \times A$$

$$\text{Or, } P = \frac{100 \times A}{100 + TR}$$

Therefore, formula for P (without I) is

$$P = \frac{A \times 100}{100 + TR} \text{......(III)}$$

Example 2

If a sum of Rs 900 amounts to Rs 990 in 2 years, what sum will amount to Rs 762.50 in 5 years at the same rate?

Solution :

According to the first condition,

Time (T) = 2 years
Principal (P) = Rs 900
Amount (A) = Rs 990
Interest (I) = ?
Rate of Interest (R) = ?

We know,

Interest (I) = A - P = Rs 990 - Rs 900 = Rs 90

$$\text{And (R)} = \frac{I \times 100}{P \times T} = \frac{90 \times 100}{900 \times 2} \%\text{per year} = 5\%\text{per year}$$

From the second condition,

Rate of interest (R) = 5% per year [From above]
Principal (P) = ?
Time (T) = 5 years
Amount (A) = Rs 76.250

We know,

$$\text{Principal (P)} = \frac{A \times 100}{100 + TR} (\because \text{I is not given})$$

$$= \frac{\text{Rs } 762.50 \times 100}{100 + 5 \times 5}$$

$$= \text{Rs. } 610$$

$\therefore$ Required principal = Rs 610.

EXERCISE 9

1. Find the amount if Rs. 6000 is lent out at the rate of $12\frac{1}{2}\%$ per annum for 10 months.
2. Bhuwan had given Rs. 1800 to Sanju for 2 year as a loan Sanju returned Rs. 2222 him. At what rate did she clear the debt?
3. What sum should be deposited to get an interest of Rs. 40 at the rate of 5% per year in 3 months. How much will be the amount?
4. What sum will amount to Rs. 1500 at the rate of 5% per annum in 4 years?
5. How long will a sum of Rs. 2500 take to amount to Rs. 3250 at the rate of 5% per year?
6. Shiva lent a sum to Rampyari at the rate of 8 percent per annum. After 5 years she returned Rs. 10,080 to Shiva and cleared the debt what sum had Shiva lent her?
7. How much should be deposited to get an amount of Rs. 56610 in 4 months at the rate of 6% per annum?
8. Anuradha lent out a sum to Krishna at the rate of 12% p.a. for 18 months. She received back a sum of Rs. 708. What sum did she lend out to Krishna?
9. At what rate per annum a sum of Rs. 25000 amounts to Rs. 26250 in 6 months?

UNIT 10

STATISTICS

10.1. Frequency Table

In statistics most often the data are collected in large scales and are analysed. In such cases it is difficult to tabulate each of the variates along with their frequencies. In order to over come this difficulty data collected is arranged in some fixed classes with definite intervals and a frequency table is formed. Study the example below.

Example 1

Marks secured by 40 students studying in class 8 at Argharajasthal Secondary School, Arghakhanchi in the final examination in mathematics of full marks 100 are as follows. 56, 53, 66, 74, 78, 71, 58, 65, 45, 73, 68, 49, 51, 52, 77, 57, 68, 48, 65, 60, 63, 82, 74, 67, 40, 44, 59, 75, 60, 75, 66, 70, 69, 39, 76, 54, 69, 72, 65, 61 study the situations bared on the data illustrated above.

10.1.1.Use of Tally marks

By observation, we find that the least score in the undraped raw data above is 39 and the highest score is 82 Therefore taking an internal of 10, we can arrange the data in 6 classes 30-40, 40 –50, 80 –90. let us classify the data into the classes using tally marks. How many scores fall into each of the classes?

Class (Score)	Tally Marks	Frequency
30-40	I	1
40-50	𝍸	5
50-60	𝍸 III	8
60-70	𝍸 𝍸 III	14
70-80	𝍸 𝍸 I	11
80-90	I	1

The left number in each class of the grouped classes is known as the lower limit of the class and the right number is known as the upper limit in it, for example, in the class 30-40, 30 is the lower limit and 40 is the upper limit but the score 40 is not included in this class. Therefore the class 30-40 means it includes the scores from 30 to below 40. If x belongs to the class 30 – 40 then $30 \leq x < 40$. However the upper limit of the largest class is included in the class in the frequency table occurrence of any score or variate in the entire data is represented by vertical line (1) which is known as tally mark. If a variate occurs once we draw one tally mark 1. If there are five scores lying in a class then the fifth mark crosses the previous 4 tallies. For example : 𝍸

10.1.2. Cumulative Frequency Table

In the frequency table above, when frequencies of consecutive classes are added, we will get a new frequency table as shown below.

Class Scours	Cumulative Frequency	
Scores below 40 (or, <40)	1	= 1
Scores below 50 (or, <50)	1 + 5	= 6
Scores below 60 (or, <60)	6 + 8	= 14
Scores below 70 (or, <70)	14 + 14	= 28
Scores below 80 (or, <80)	28 + 11	= 39
Scores upto 90 (or, >90)	39 + 1	= 40

Thus when frequencies of the consecutive classes are added, the sum of frequencies is known as cumulative frequency *(f)*. It is denoted by (c.f.). To find cumulative frequency (c.f.) proceed as follows

1. Arrange the given data in ascending or increasing order.
2. Add the frequency of the former class to the following class successively.

Therefore, cumulative frequency of a class is the sum of the frequencies of that class and the previous classes.

EXERCISE 10.1.

1. Represent the following ungrouped data in a frequency table taking 2 as the class interval-using tally-marks.
   17, 11, 21, 12, 23, 9, 27, 18, 23, 18, 16, 20, 23, 20, 19,
   24, 22, 22, 14, 19, 21, 17, 11, 24, 22, 23, 9, 9, 27, 16.
2. Daily wages (in Rs) of 30 workers are given below. Construct a frequency table taking 10 as the class interval.
   41, 61, 63, 41, 22, 32, 51, 42, 21, 32,
   41, 30, 62, 71, 33, 81, 81, 73, 30, 75,
   51, 27, 24, 27, 30, 71, 81, 75, 48, 59.
   Hence represent the above data in a cumulative frequency table.
3. Weaver Hari Maya income (in Rs) received as the wages for 30 days from a carpet factory are tabulated below. Using a class interval of 10, construct a frequency distribution table.
   40, 31, 25, 37, 48, 52, 57, 63, 42, 35, 32, 31, 25, 37, 48, 65, 32, 45, 42, 53, 61, 65, 37, 41, 39, 60, 65, 42, 48, 50.
   Also construct a cumulative frequency table.
4. Taking class interval 2, express the following data in a frequency table.
   14, 16, 15, 13, 14, 16, 13, 15, 16, 15, 13, 17,
   14, 17, 15, 13, 15, 14, 16, 16, 16, 17, 12, 8
   6, 8, 9, 12, 15, 8, 5, 14, 5, 15, 17, 10,
   18, 16, 15, 14, 8, 16, 15, 14, 15, 13, 9, 7,
   Also construct a cumulative frequency table.

10.2. Arithmetic mean

Marks secured in Math of full marks 100 by the students of class 8 in two terminal examinations are as follows

First Terminal	: 65 53 71 49 73 85 95 63 78 98
Second Terminal	: 54 61 95 93 75 76 86 69 79 89

On the basis of the score table above, can you say in which terminal examination students showed better performance?

Variates having similar characters can be compared finding their average value. The average is also known as arithmetic mean.

Here, averages of both terminals are computed.

Average marks in the first terminal

$$= \frac{65+53+71+49+73+85+95+63+78+98}{10}$$

$$= \frac{730}{10} = 73$$

Average marks in the second terminal

$$= \frac{54+61+95+93+75+76+86+69+79+89}{10}$$

$$= \frac{777}{10} = 77.7$$

On this basis, it can be said that students performed better in the second terminal rather than in the first.

- Average is also known as arithmetic mean. Arithmetic mean ($\overline{X}$) is the quotient when sum of the quantities is divided by number of quantities or observations.
- Therefore, an average of a distribution is called mean.

  Arithmetic mean $(\overline{X}) = \dfrac{\text{Sum of quantities}}{\text{Number of quantities}}$

- Statistically, if $X_1, X_2, X_3, \ldots\ldots, X_n$ be n observations (or items) then their arithmetic mean $\overline{X} = \dfrac{X_1 + X_2 + \ldots.. + X_n}{n}$. In brief, $\overline{X} = \dfrac{\Sigma X}{n}$, where Σ means the sum and is read as sigma.

If the number of items (variates) are many and the variates are repeated frequently then the method for finding an average discussed above becomes lengthier. In this case following rule is to be used.

$$\text{Mean } (\overline{X}) = \frac{\Sigma fx}{\Sigma f} \text{ or } \frac{\Sigma fx}{N}$$

Here f, represents the frequency of the variate Σfx represents the sum of product of f and x in each classes and Σf represents the sum of frequencies. Σf is also denoted by N, the total number.

Example 1

The scores obtained by 30 students in a test out of full marks 50 are given below.

10, 20, 15, 10, 15, 5, 0, 40, 50, 35, 30, 30,
15, 15, 0, 20, 25, 25, 20, 25, 30, 30, 25, 30,
30, 25, 30, 30, 10, 10, 0

Construct a frequency table and find the arithmetic mean.

Solution :

Given data is represented in the frequency table below

Score x	Tally	Frequency f	fx
0	III	3	0
5	II	2	10
10	IIII	4	40
15	III	3	45
20	III	3	60
25	~~IIII~~	5	125
30	~~IIII~~ II	7	210
35	I	1	35
40	I	1	40
50	I	1	50
		$N = \Sigma f = 30$	$\Sigma fx = 615$

By the formula,

$$\text{Mean } \overline{X} = \frac{\Sigma fx}{N} = \frac{615}{30} = 20.5$$

∴ The required arithmetic mean = 20.5

Example 2

In a school time span (in minutes) of telephone calls during a day was recorded as follows.

3, 5, 12, 2, 8, 9, 4, 3, 2, 4, 5, 7, 6, 9, 7, 5, 4, 3, 3, 4, 5, 2, 1, 10

(a) What is the average time span of telephone calls ? (upto 2 decimal places)

(b) If the rate of calls per minute is Rs 4, how much is to be paid against the calls on the day ?

Solution :

Here,

(a) $\Sigma x = 3 + 5 + 12 + 2 + 8 + 9 + 4 + 3 + 2 + 4 + 5 + 7 + 6 + 9 + 7 + 5 + 4 + 3 + 3 + 4 + 5 + 2 + 1 + 10 = 123$ minutes

Number of telephone calls (N) =24

Therefore, average time span or the arithmetic mean

$$\overline{X} = \frac{\Sigma x}{N} = \frac{123}{24} \text{ minutes} = 5.125 \text{ minutes}$$

(b) Total expense on calls $= 123 \times \text{Rs } 4 = \text{Rs } 492$

To find average of the combined group when the averages of the groups are known.

If averages of n, elements is $\overline{X}_1$ and averages of n_2 elements is $\overline{X}_2$ then the averages of the combined or the entire group $\overline{X}$ is given by

$$\overline{X} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$

Example 3

In a marathon from DANDAKHANI to PAHIGAUN, averages speed of 25 runners led by Krishna was 8.70 km/hr and that of 35 runner led Shyam was 9.50 km/hr is the averages speed of the runner ?

Solution :

Here,

Number of runners is the team led by Krishna $(n_1) = 25$

Averages speed of the team $\overline{X}_1 = 8.70$km/hr

And, number of runner in Shayam's team $n_2 = 35$

Averages speed of the team $\overline{X}_2 = 9.50$ km/hr

Now if $\overline{X}$ be that averages speed of any runner then

$$X = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2} = \frac{25 \times 8.70 + 35 \times 9.50}{25 + 35}$$

$$= \frac{217.5 + 332.5}{60}$$

$$= \frac{550}{60}$$

$$= 9.16 \text{ km/hr}$$

Hence , average speed of the runner =9.16 km/hr

EXERCISE - 10.2

1. Find the arithmetic mean for the following data
   (a) 3, 5, 4, 7, 6, 8, 7, 13, 10, 5, 6, 7,
   (b) 15cm, 23cm, 10cm, 12cm ,18cm, 19cm, 8cm,
   (c) Rs 15, Rs 21, Rs 37, Rs 18, Rs 22, Rs 31,
   (d) 12kg, 9kg, 6kg, 3kg, 18kg, 21kg, 22kg,
   (e) 15cm, 55cm, 32cm, 28cm, 40cm, 49cm,
2. A businessman did 15 telephone calls during a day. The span of each call in minutes are as follows
   4, 8, 3, 12, 3, 6, 2, 2, 3, 3, 5, 11, 9, 3, 10
   (a) What are the average span ?
   (b) If the calls are charged at the rate of Rs 3 per minute, what was the total charge on calls on the day ?
3. The sales for ten days in a book shop were as follows

Rs 2035,	Rs 1752,	Rs 1567,	Rs 2570,	Rs 2035,
Rs 2578,	RS 1896,	Rs 1538,	RS 3025,	Rs 1965

   (a) What was the average sale per day ?
   (b) On the basis of average sale find
      (i) The sale in a month
      (ii) The sale in a year.
   (c) If the profit is 30% on total sale then what is the profit during a year ?
4. The average weight of 25 boys is 45.6 kg and that of 32 girls is 39.9 kg. Find their average weight.
5. The average age of 20 boys and 10 girls is 13 yrs. If the average age of the boys is 40.2 yrs. What is the average age of the girls ?
6. The average height of X boys and 40 girls is 140.2cm. If the average height of the boys is 140.375cm and that of the girls is 139.8cm then how many boys are there ?
7. From the following frequency distribution find the arithmetic mean

(a)

Duration of Calls (min.) (x)	1	2	3	4	5
Number of Telephone Calls (f)	5	10	13	8	4

(b)

x	10	12	14	16	18	20	22
f	3	5	9	14	12	5	2

(c)

x	5	6	7	8	9	10	11	12
f	3	10	14	20	22	16	10	5

10.3. Median

Median is the value of the variate which exactly lies in the middle position when the data is arranged in the increasing (or decreasing) order. For example in the sequence of data 1,3,5 the value of the variate in the middle position is 3. Here median of this sequence is 3.

Example 1

In a test of ten marks the scores of 9 students are 4, 5, 3, 6, 7, 4, 5, 9, 8. Find the median.

Solution :

Arranging the given data in the increasing order

3, 4, 4, 5, 5, 6, 7, 8, 9,

Here value of the variate in the middle position = 5

$\therefore$ Median =5

Alternative method

Here, N =9

Position of the median $= \frac{N+1}{2}$ th term $= \frac{9+1}{2}$ th term $= \frac{10}{2}$ th term

$= 5^{th}$ term

$\therefore$ The median is at 5^{th} position

$\therefore$ Median = 5

[Note: In order to find the median the data can be arranged either in the ascending (increasing) or descending (decreasing) order.]

Example 2

Find the median for the following data.

5, 7, 3, 4, 8, 5, 7, 9

Solution :

Arranging the data in descending order.

9, 8, 7, 7, 5, 5, 4, 3

Here, variate in the middle position is not only one. In this case, the median is calculated as the average of middle variates, hence,

$$\text{Median} = \frac{5+7}{2} = \frac{12}{2} = 6$$

Alternate method

Here, N= 8

Now, Position of the median $= \frac{N+1}{2}$th term $= \frac{8+1}{2}$th term

$= \frac{9}{2}$th term = 4.5th term

Or, Thus the median lies at 4.5^{th} position.

It lies between 7 and 5

$\therefore$ Median $= \frac{5+7}{2} = \frac{12}{2} = 6$

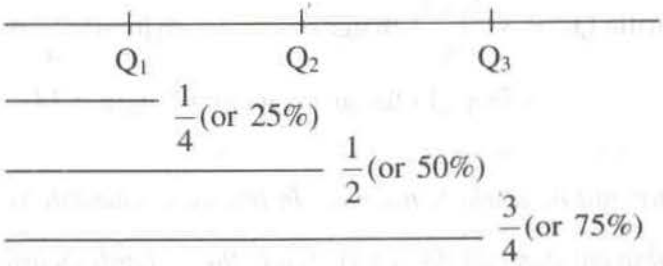
EXERCISE 10.3

1. Find the median and arithmetic mean for each of the following data.
   (a) 7, 9, 7, 11, 10
   (b) Rs 5, Rs 4, Rs 8, Rs 6, Rs 1, Rs 5, Rs 8, Rs 5, Rs 3,
   (c) 1 kg, 3kg, 5kg, 1kg, 1kg, 3kg, 14kg,
   (d) 4cm, 6cm, 2cm, 5cm, 9cm, 4cm
   (e) 6, 8, 7, 6, 9, 6, 7, 8, 6, 6, 9, 6
2. In a test of 10 marks 9 students scored as follows: 9, 5, 8, 4, 2, 3, 1, 6, 7
   (a) Find the median.
   (b) How many students are above and below the median ?

10.4. Quartiles

Median, one of the measures of central tendency, divides the whole group into halves. This concept is extended to another measure which divides the entire Imps (or observations) or the frequency table into 4 quarters. This type of measure is known as quartile.

In order to divide a distribution into 4 quarters we need three points. Therefore we have three quartiles. Among these the first quartile is known as lower quartile, the second is median and the third is called upper quartile. These quartiles are respectively denoted by Q_1, Q_2 and Q_3 The quartiles Q_1, Q_2, and Q_3 cover respectively 25%, 50% and 75 % of the distribution. Following figure is used to illustrate it.



To find the quartiles of ungrouped data proceed as follows.

1. At first arrange the data either in ascending or descending order of magnitudes.
2. Find the value of the quartile variates using the formula,

   $(Q_i) = \frac{i(N+1)}{4}$ th item for i=1, 2, 3

   i.e. When i = 1, we have the first quartile Q_1 as

   $Q_1 = \frac{1(N+1)}{4}$ th variate

   Similarly second quartile $Q_2 = \frac{2(N+1)}{4}$ th the variate

   Third quartile $Q_3 = \frac{3(N+1)}{4}$ th item

Example 1

Find the quartiles Q_1, Q_2 and Q_3 for the distribution 62, 50, 78, 66, 74, 71, 80

Solution :

Here, arranging the data in ascending order

50, 62, 66, 71, 74, 78, 80

Number of variates N = 7

(a) First quartile $Q_1 = \frac{N+1}{4}$ th item $= \frac{7+1}{4}$ th item $= \frac{8}{4}$ th item

$= 2^{nd}$ item = 62 [$\therefore$ In the array second variate value = 62]

$\therefore Q_1 = 62$

(b) Second quartile $Q_2 = \frac{2(N+1)}{4}$ th item $= \frac{2(7+1)}{4}$ th item

$= \frac{16}{4}$ th item = 4th item = 71 [$\therefore$ In the array, 4^{th} item = 71]

$\therefore Q_2 = 71$

(c) Third quartile $Q_3 = 3\frac{(N+1)}{4}$ th item $= \frac{3(7+1)}{4}$ th item $= \frac{24}{4}$ th item $= 6^{th}$ item

$= 78$ [$\therefore$ In the array above 6^{th} item = 78]

Note: $\frac{i(N+1)}{4}$ *may not be a whole number. In this case, quartile is calculated with the help of two adjacent items in the array. Study the example below.*

Example 2

Scores obtained by 30 students of class 8 in mathematics are as follows.

14, 19, 26, 34, 37, 39, 42, 44, 44, 44, 45, 47, 48, 49, 50,
51, 58, 68, 70, 71, 75, 84, 91, 91, 97, 98,

Find the quartiles Q_1, Q_2 and Q_3, from this data.

Solution :

Here the given data is already in ascending order. Number of variates N = 30

$\therefore Q_1 \quad = \frac{1(N+1)}{4}$ th variate $= \frac{30+1}{4}$ th variate $= \frac{31}{4}$ th variate $= 7.75$th

Variate $= 7^{th}$ variate + (8^{th} Variate – 7^{th} variate) × 75%
$= 42 + (44\text{-}42) \times 0.75$
$= 43.5$

$\therefore Q_2 \quad = \frac{2(N+1)}{4}$ th variate $= \frac{2(30+1)}{4}$ th variate $= \frac{62}{4}$ th variate
$= 15.5^{th}$ variate $= 15^{th}$ variate + (16^{th} Variate – 15^{th} variate) × 50%
$= 49 + (50 - 49) \times 0.5$
$= 49.5$

And, $Q_3 = \frac{3(N+1)}{4}$ th variate $= \frac{3(30+1)}{4}$ th variate $= \frac{93}{4}$ th variate
$= 23.25^{th}$ variate = 23 rd variate + ($24^{th} - 23^{rd}$) variate × 25%
$= 75 + (75\text{-}75) \times 0.25$
$= 75$

$\therefore Q_1 = 43.5$, $Q_2 = 49.5$ and $Q_3 = 75$.

EXERCISE 10.4

1. Find the first, second and third quartiles from the following data.
   (i) 52, 50, 68, 56, 64, 61, 72,
   (ii) 2, 3, 4, 5, 6, 6, 6, 7, 8, 9, 10, 11, 12, 12, 13
   (iii) 20, 40, 60, 100, 120, 140, 160
   (iv) 2, 3, 4, 3, 5, 6, 3, 4, 4, 5, 7, 6, 5, 4, 5, 3, 5, 6
2. Find Q_1, Q_2 and Q_3 from the scores distribution of 30 students of class 8 in mathematics.
   12, 17, 24, 32, 35, 37, 40, 42, 42, 42, 43, 45, 46, 47,
   48, 49, 56, 64, 68, 68, 69, 73, 73, 73, 82, 89, 89, 95, 96
3. Pocket money (in Rs) possessed by 12 students are given below. Find the quartiles Q_1, Q_2 and Q_3.
   11, 13, 15, 17, 19, 21, 23, 12, 16, 18, 20, 22.

10.5. Mode

While collecting the information for some purpose sometimes the same information occurs repeatedly. For instance the sizes of the shoes worn by the students of class 5 were collected and it was found that 10 students were wearing the following sizes.

4, 3, 4, 3, 3, 4, 3, 3, 4, 3.

This data is arranged in ascending order as:-

$$\underbrace{3, 3, 3, 3, 3}_{\text{5 Times}} \quad \underbrace{4, 4, 4, 4}_{\text{4 Times}} \quad \underbrace{5}_{\text{1 Times}}$$

Here the size of shoes used by many students = 3

$\therefore$ Mode = 3.

Mode is the value of the variate which occurs mostly or maximum number of times.

EXERCISE 10.5

1. Find the Mode for each of the following.
   (a) 6, 3, 5, 6, 3, 1, 6
   (b) Rs 5, Rs 3, Rs 5, Rs 2, Rs 3, Rs 3, Rs 7, Rs 6
   (c) 17cm, 5cm, 16cm, 17cm, 5cm, 5cm, 6cm, 12cm, 5cm
   (d) 13kg. 12kg. 13kg. 12kg. 12kg. 13kg. 15kg. 13kg. 15kg.
   (e) 10cm, 15cm, 12cm, 10cm, 12cm, 11cm, 12cm, 9cm
2. A factory produces nail of 3cm. The measure of 15 sample nail are as follows.
   3cm, 2.9cm, 2.5cm, 2.5 cm, 2.9cm, 3.1cm, 2.5cm, 3.1cm, 2.9cm, 3cm, 2.5cm, 2.5cm, 3cm, 3cm, 2.5cm
   Find the mode for this data.

10.6 Range of the Data

When the data is analysed on the basis of arithmetic mean sometimes two data may have same mean and lead toward the same conclusion but the conclusion drawn on the basis of their characteristics can be different. For example, scores obtained by 10 student in 2 test are as follows.

Roll no of students :	I	II	III	IV	V	VI	VII	VIII	IX	X
First test :	3	2	3	4	2	5	4	3	2	2
Second test :	1	2	1	1	0	0	1	2	10	12

Here the average score of both tests is 3. But on this basis, it is not proper to say that the standard of the students on both tests is same. In such cases, it will be proper to study the range of the data. Range of the data means the difference betweens the highest value variate and the lowest value variate

In the example above, scores in the first test spans from 2 to 5 therefore the difference of the highest and the lowest score is 5 - 2 = 3 similarly in the second test the difference is 10 - 1 = 9 Although the average score in both tests are the same, the nature of the span of the score is different.

Range = Highest value (H) – lowest value (L)

Example 1

Find the range,

6, 8, 14, 5, 2, 7, 22, 8, 4

Solution :

Here,

Highest value of the variate (H) = 22

Lowest value of the variate (L) = 2

∴ Range = H - L = 22 - 2 = 20

EXERCISE 10.6

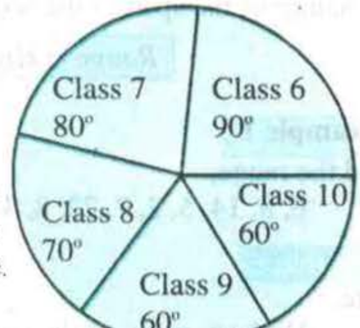
1. Heights of 10 students of class 8 are as follows :
   98cm, 110cm, 95cm, 112cm, 115cm,
   105cm, 111cm, 97cm, 110cm, 120cm,
   (a) What is the height of the tallest student ?
   (b) What is the height of the shortest student ?
   (c) What is the range ?
2. The money spent on the Tiffin by class 5 students are as follows. Rs 15, Rs 18, Rs. 12, Rs. 10, Rs. 17, Rs. 22, Rs. 20, Rs. 27.
   (a) What is the highest expense?
   (b) What is the lowest expense?
   (c) How much is the range?
3. Find the range for each of the following.
   (a) 1, 2, 0, 1, 3, 0, 2, 1, 3, 5
   (b) 5cm, 15 cm, 12cm, 7cm, 2cm, 18cm, 6 cm, 24 cm
   (c) 15 kg, 7kg, 5 kg, 7 kg, 20 kg, 50 kg, 20 kg, 12 kg
   (d) 5l, 7l, 2l, 6l, 1l, 3l, 8l

10.7. Pie Chart

Class wise distribution of 180 students studying from class 6 to 10 in Jeevan Jyoti secondary school is given in the table below.

Class	6	7	8	9	10
Number of Students	45	40	35	30	30

Data given above can also be represented in another form as shown below.



Thus if the numerical data are displayed through the sector's of a circle then the figure is known as pie-chart or pie- diagram.

How to construct a pie-chart ?

We know the measure of the central angle of a circle is 360^0. Therefore, the items in the data should the divided into 360^0 parts. Hence all the students (i.e. 180) are to be represented by 360^0.

Therefore, 180 students $=360^0$

$$\therefore 1 \text{ student } = \frac{360^0}{180^0} = 2^0$$

Here, central angle of 2^0 represents 1 student. On this basis we can find the equivalent central angle in degrees representing the students of each class i.e.

1 student = 2^0

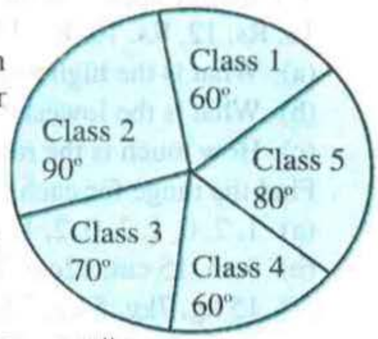
$\therefore$ 45 students of class $6 = 2^0 \times 45 = 90^0$

Similarly, 40 students of class $7 = 2^0 \times 40 = 80^0$

Proceeding in this way we can convert the number of students in each class into equivalent degrees and can represent the data in the pie-chart.

Example 7

Following pie-chart represents class wise distribution 720 girl students in a school, study the chart ar answer the questions.



1. How many girls are in the class one ?
2. How many girls are studying in class five ?

Solution :

In the given pie-chart 720 girls are represented by 360^0 hence

$360^0 = 720$ girls

$\therefore 1^0 = \frac{720}{360} = 2$ girls

(i) Angle representing girls in class $1 = 60^0$

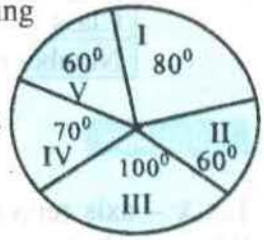
$\therefore$ Number of girls $= 2 \times 60^0 = 120$

(ii) Similarly angle representing class $5 = 80^0$.

$\therefore$ Girls in class 5 = $2 \times 80 = 160$

EXERCISE 10.7

1. Study the adjoining pie – chart and answer the following questions.



   (a) Which class has maximum number of students ?
   (b) If 1^0 represents 2 students how many students are there in class 4 ?
   (c) What is the total number of students ?

2. Following the table represents the students studying in and verbal school, coming 2 school by different vehicles

Vehicle	On Foot	By Bus	By Bicycle	By Car
Number of Students	25	15	40	20

Represent this data by pie-chart

3. Following table displays the percentages of different gases in the atmosphere, represent the data by a pie-chart

Gas	Nitrogen	Oxygen	Others
Percentage	78	20	2

4. Table below shows the level wise number of the teachers in a school. Express the data in a pie-chart

Level	Secondary	Lower Secondary	Primary
Number of Teachers	6	12	18

10.8. Line Graph

If the given data in the form of ordered pair (variate, frequency) are plotted as points on a graph and are joined successively to form a line then it is known as line graph.

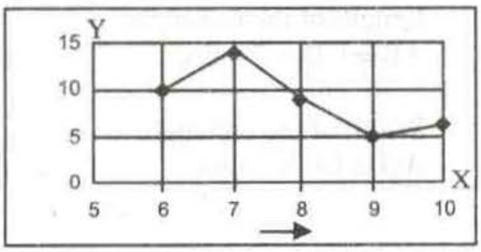
Example 8

Express the following data by a line graph :

Class	6	7	8	9	10
Number of Benches	10	14	8	4	6

Solution :

Let us represent the variates (classes) along x –axis and the number of benches along y-axis now we plot the points in the graph and join the



consecutive points, the graph looks like:

Example 9

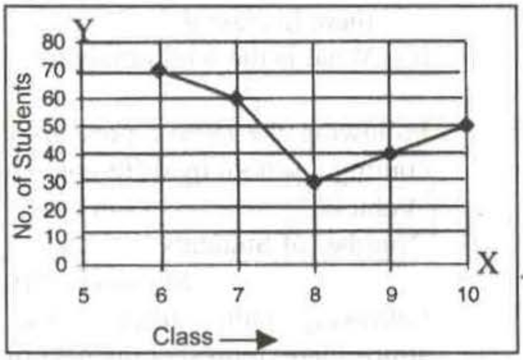
Number of class wise students is given in the table below.

Class	6	7	8	9	10
Number of Students	70	60	30	40	50

Sketch a line graph that fits the given data.

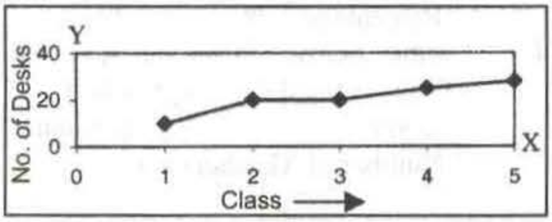
Solution :

Let x – axis represent the class and the y – axis represent the number of students, corresponding numbers are plotted and the points are joined successively to form the line graph as shown in the graph below.



Example 10

Express the data represented by the adjoining line graph in the form of frequency table.



Solution :

Given line graph is represented in tabular form as follows.

Class	1	2	3	4	5
Number of Desks	10	20	20	25	28

Example 11

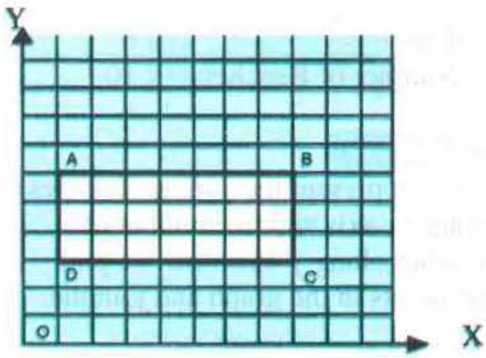
Find the length and width of the rectangle shown in the graph.

Solution :

By observation we find

Length of the rectangle
AB = CD = 7 units.

Width of the rectangle
AD = BC = 3 units.



EXERCISE 10.8

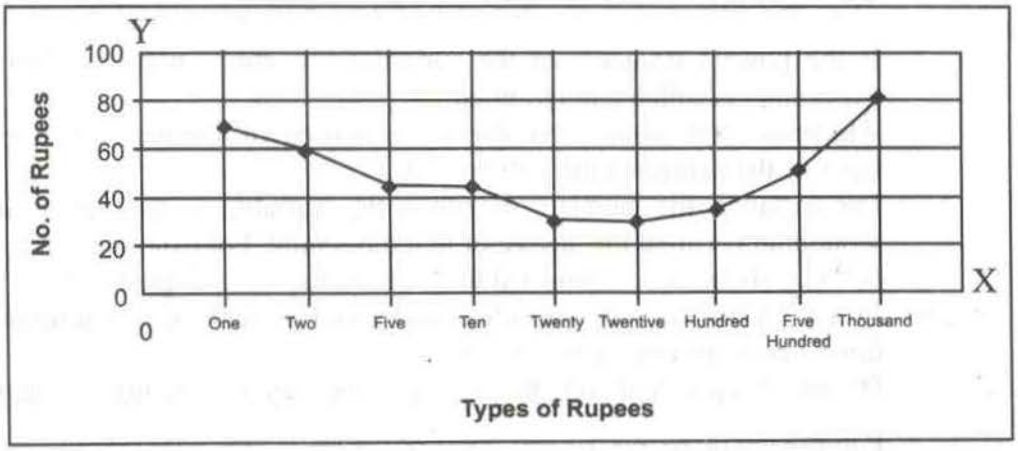
1. Represent the following data in the form of line graph.

Class	6	7	8	9	10
Number of Students	50	75	80	80	50

2. Table below represents the number of reference books read by the students of different classes in Shree Jeevanjyoti Secondary School. Sketch a line graph for the data.

Class	1	2	3	4	5	6	7	8	9	10
No. of Reference books	10	12	9	16	16	25	26	24	18	17

3. Following ling graph displays the quantity of notes of different denominations possessed by a merchant. Represent the data in tabular form and answer the questions below.



(a) Notes of which denominations are maximum?
(b) Which notes are equal in number?

4. Sketch the graph for the data given and hence find the area of the rectangle.

x	7	7	11	11
y	5	7	5	7

UNIT 11

ALGEBRAIC EXPRESSION

11. 1 (a) Polynomials

- Algebraic term expressed as the product of number and variable is called monomial expression. For example: $3x$, $\sqrt{2}\ xy$, $-x^2y$ etc.
- Algebraic terms expressed as the sum (or, difference) of monomials is called a polynomial expression.
  For example, $3x + 2$, $x^2 - 5x + 4$, $4ab^2 - ab + 3ab$ etc.
- The polynomials which contain only two terms are called binomials and which contain three terms are called trinomials. For example $\frac{1}{2}x + yz, 4a + \sqrt{5}$ etc. are binomials and $3x^2 + 2x - 1$, $a^4 + a^3 + a^2$ etc. are trinomials but $x^3 + \frac{3}{x^2}$ is not a polynomial, because the variable x in the second term has negative index -2 $\left(\text{i.e. } \frac{3}{x^2} = 3x^{-2}\right)$

  If the powers (indices) of the variables are non – negative then the expression is called a polynomial.

  Algebraic expressions can also be classified in another way, on the basis of the degree of the polynomial.

  For instance the number of times the variable is multiplied in a monomial is called the degree of that monomial. For example.
- In $7x^3$, variable x is multiplied three times, hence its degree is 3.
- In x^2y^5, variable x is multiplied twice and variable y is multiplied 5 times hence, its degree is $2 + 5 = 7$.

  Degree of a polynomial is the degree of the term having highest degree.

  Polynomial of degree 1 : $3x - 8a$, $\frac{7}{2}z + 3$ etc.

  Polynomial of degree 2 : $4d^2$, $9x + 7xy$, $\sqrt{2}m^2 + \sqrt{3}m - 5$ etc

  Polynomial of degree 3: $6x^3$, $3xyz$, $-4mn^2 + 1$ etc.

Example 1

Write the degree of each polynomials below .

(a) $3x^3 - 6x^2 + 5x + 6$

(b) $6x^5 - 3x^4 + 8x^3 - 6x^5 - 3x^3$

(c) $3x^4y - x^2y^2 + xy$

Solution :

(a) Here, $3x^3 - 6x^2 + 5x + 6$

Highest power of the variable in the polynomial is 3 hence its degree is 3.

(b) $6x^5 - 3x^4 + 8x^3 - 6x^5 - 3x^3$

$= 6x^5 - 6x^5 - 3x^4 + 8x^3 - 3x^3$ (First the expression should be simplified)

$= 3x^4 + 5x^3$

Here, degree of polynomial is 4.

(c) $3x^4y - x^2y^2 + xy$

The polynomial contains two variable, hence adding the power of variables in each term.

In the first term : $4 + 1 = 5$ In the second term : $2 + 2 = 4$

In the third term : $1 + 1 = 2$

Here, 5 is the highest, hence the degree of polynomial is 5.

Example 2

If $x = 2$ find value of $5x^3 + x^2 - 10x$.

Solution :

Here, Given polynomial

$= 5x^3 + x^2 - 10x$

$= 5 \times 2^3 + 2^2 - 10 \times 2$ (putting $x = 2$)

$= 5 \times 8 + 4 - 20 = 24$

EXERCISE 11 [A]

1. Which of the following expressions are the polynomials, identify.

   (a) $3x^2 + 11x - 4$ (b) $-8x^2y^2$ (c) $\sqrt{x+6}$

   (d) $\sqrt{2}x - xy - 8y^2$ (e) $\dfrac{3x-4y+5z}{2}$ (f) $\dfrac{3y^3-4}{y}$

2. Classify each of the polynomials on the basis of terms.

   (a) $9x + 2y + 3z$ (b) $-2x^2 + y^2$ (c) $-5x$

   (d) $3x + 1$ (e) $p + q + 1$ (f) $4a^3 + 3b^2 - 2c$

3. Write the degree of each polynomials.

   (a) $4x - 2xy + y$ (b) $5x^3 - 7x - 2x + 3$ (c) $2p - p^2 + p^3 - 2p$

   (d) $x^2 + 2 + 3x^3 - 5 + 9xy^4$ (e) $x^3y^3 + 2y^2 + 3y - 7y^2 + 15y - 10$

4. Find the value of

   (i) (a) $5x^2 + 6x - 3$ (b) $-x^2 + 3x + 16$, if $x = 4$

   (ii) $3x^2 - 2xy + y^2$, if $x = -1, y = 5$

   (iii) $2a^2 - ab - 3b^2 + 7$, if $a = 3, b = -2$

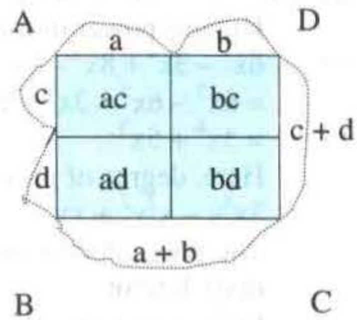
   (iv) $4x^2y - 2x^2 + y^2 - 7$, if $x = -2, y = 1$

11.1 (b) Multiplication of Binomials

Let us study the figure in the side in order to find the product $(a + b)(c + d)$

Here, $(a + b)(c + d)$ denotes the area of rectangle ABCD having the length $a + b$ and the width $c + d$.

As shown in the figure, the area of rectangle ABCD is the sum of areas of 4 rectangles formed inside.



Therefore, ($(a + b)(c + d) = ac + ad + bc + bd$.

From the above discussion we conclude.

The product of two binomials is an expression which is the sum of product obtained by multiplying both terms of a binomial by the terms of another separately, for example. $(a+b)(c+d) \quad = \frac{ac}{1}+\frac{ad}{2}+\frac{bc}{3}+\frac{bd}{4}$

Example 1

Multiply

(a) $(x + 3)(x + 7)$ (b) $(x + a)(x + b)$

(c) $(p - 5)(2p - 3)$ (c) $(x - 3y)(2x + y)$

Solution :

Here,

(a) $(x + 3)(x + 7)$

$= x(x + 7) + 3(x + 7)$

$= x^2 + 7x + (x + 7)$

$= x^2 + 10x + 21$

(b) $(x + a)(x + b)$

$= x(x + b) + a(x + b)$

$= x^2 + bx + ax + ab$

$= x^2 + (a + b)x + ab.$

(c) $(p - 5)(2p - 3)$

$= 2p^2 - 3p - 10p + 15$

$= 2p^2 - 13p + 15$

(d) $(x - 3y)(2x + y)$

$= 2x^2 + xy - 6xy - 3y^2$

$= 2x^2 - 5xy - 3y^2$

EXERCISE 11 [B]

1. Expand :

(a) $(x + 2)(x + 3)$ (b) $(p - 1)(p + 3)$ (c) $(a - 2)(a - 4)$

(d) $(r + 6)(r - 8)$ (e) $(c + 3)(c + 8)$ (f) $(x - 12)(x + 8)$

2. Find the product

(a) $(a + 3)(4a - 3)$ (b) $(2x - 5)(3x - 7)$

(c) $(x - 3y)(2x + 7y)$ (d) $(a + 3b)(2a + 3b)$

(e) $(9x - 2y)(7x + 8y)$ (f) $(7x + 2k)(3x - k)$

(g) $(11x - 12y)(11x + 12y)$ (h) $(4p - 5q)(5q - 2p)$

(i) $(3p - 4y)(5p - 8y)$ (j) $(ax + by)(cx + dy)$

3. Simplify

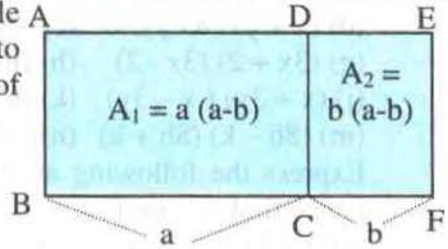
(a) $(a + 4)(a + 9) + (a + 3)(a + 6)$ (b) $(x + 3)(x - 2) - (x + 5)(x - 2)$

(c) $3(x + 5)(x - 4) - (2x - 3)(x - 2)$

(d) $2(p - 3)(2p - 3) - 4(2p - 5)(p + 6)$

11.1. (c) Multiplication of the sum and difference of two terms.

In how many ways can the area of rectangle ABFE, in the adjoin figure be found ? Try to find. Is area of ABFE the sum of areas of rectangle ABCD and CDEF?



Here, Area of ABCD (A_1) $= a(a-b)$ ....(i)

Area of CDEF (A_2) $= b(a-b)$ ....(ii)

Area of ABFE $= (a + b)(a - b)$ ....(iii)

From (i), (ii) and (iii) we have,

Area of ABFE = area of ABCD + area of CDEF

or, $(a+b)(a-b) = a(a-b) + b(a-b)$

$= a^2 - b^2$

$\therefore (a + b)(a - b) = a^2 - b^2$

Example 1

Find the product

(a) $(x + y)(x - y)$ (b) $(m + n)(m - n)$

Solution :

(a) $(x + y)(x - y) = x^2 - xy + xy - y^2 = x^2 - y^2$

(b) $(m + n)(m - n) = m^2 - mn + mn - n^2 = m^2 - n^2$

From the discussion above.

$(a + b)(a - b) = a^2 - b^2$
= square of first term – square of second term

Example 2

Find the product

(a) $(3m + 2n)(3m - 2n)$ (b) 297×303

Solution :

(a) $(3m + 2n)(3m - 2n)$
$= (3m)^2 - (2n)^2$
$= 9m^2 - 4n^2$

(b) 297×303
$= (300 - 3)(300 + 3)$
$= 300^2 - 3^2 = 90000 - 9 = 89991$

EXERCISE - 11 [C]

1. Find the product

(a) $(a + 2)(a - 2)$ (b) $(x - 3)(x + 3)$ (c) $(4 + q)(4 - p)$
(d) $(x + y)(x - y)$ (e) $(m + n)(m - n)$ (f) $(a - b)(a + b)$
(g) $(3x + 2)(3x - 2)$ (h) $(m - 9)(m + 9)$ (i) $(4 + 5x)(4 - 5x)$
(j) $(x + 3y)(x - 3y)$ (k) $(k + 2m)(k - 2m)$ (l) $(5p - t)(5p + t)$
(m) $(8h - k)(8h + k)$ (n) $(10x + 11y)(10x - 11y)$ (o) $(8m - 9n)(8m + 9n)$

2. Express the following as the sum and the difference of two number and hence find the product

(a) 9×11 (b) 51×49 (c) 58×62
(d) 78×82 (e) 47×53 (f) 75×85

11.1 (d) Multiplication of Polynomials

Multiplication of polynomials is similar to the multiplication of a binomial by another binomial. We multiply every terms of a polynomial by each term of another polynomial. Study the examples below.

Example 1

Find the products

(a) $(a + b)(a + b + c)$ (b) $(x - 9)(x - 9 + m - n)$

Solution :

Here,

(a) $(a + b)(a + b + c) = a((a + b + c) + b(a + b + c)$
$= a^2 + ab + ac + ab + b^2 + bc$
$= a^2 + 2ab + b^2 + ac + bc.$

(b) $(x - y)(x - y + m - n) = x(x - y + m - n) - y(x - y + m - n)$
$= x^2 - xy + mx - nx - xy + y^2 - my + ny$
$= x^2 - 2xy + y^2 + mx - nx - my + ny.$

Example 2

Find the products

(a) $(p + q + r)(a + b + c + d)$ (b) $(a + b + c + d)(m - n + r + s)$

Solution :

Here,

(a) $(p + q + r)(a + b + c + d) = p(p + b + c + d) + q(a + b + c + d) + r(a + b + c + d)$
$= pa + pb + pc + qa + qb + qc + qd + ra + rb + rc + rd.$

(b) $(a + b + c + d)\ (m - n + r + s)$
$= a(m - n + r + s) + b(m - n + r + s) + c(m - n + r + s) + d(m - n + r + s)$
$= ma - na + ra + sa + mb - nb + rs + sb + mc - nc + rc + sc + md - nd + rd + sd.$

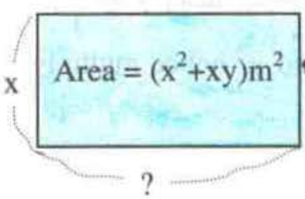
EXERCISE – 11 [D]

1. Find the products.
   (a) $(a + b)(m + n + r)$ (b) $(p - 2)(q + r + 3)$
   (c) $(x - y)(3a - 2 + c)$ (d) $(x - 3)(2a - b - c)$
   (e) $(a - b)(2a - b - 3c)$ (f) $(3x - 6y)(3x - 2y + z)$
   (g) $(a + b)(x - y - 2a + 3b)$ (h) $(p - q)(2x - 3y - a + 2b)$
   (i) $(m - 2n)(x - y - 2a + 3b)$ (j) $(3x - 2y)(3x - y + a + b)$
   (k) $(P + q)(2p + 3q - r - 25)$ (l) $(10a - 9b)(a + b - 2ab - 3bc)$
2. Find the products of each of the following.
   (a) $(a + b - c)(a - b + c)$ (b) $(2a - 3b + c)(a - 2b - 2c)$
   (c) $(m - n - 2)(m - 2n + 3)$ (d) $(2p - 3q + r)(3r - 2q - p)$
   (e) $(2a - 3b - c)(3b - 2c - 5a)$ (f) $(3x - y - 2z)(4x - 3y - z)$
   (g) $(2x - y + a)(x - y - 2a + b)$ (h) $(2a + 3b - c)(a - b - 2c + d)$
   (i) $(x - 2y + 3z)(2x - y + 4z - a)$ (j) $(10m + 9n + s)(m - 20n - 3s - p)$
   (k) $(6p - 7q - r)(2p + 3q + 4r + s)$ (l) $(10a + 12b - 15c)(5a - 6b + 7c - 8d)$

11.2. Division of Polynomials

11.2 (a) Dividing a Polynomial by a Monomial

Study the example below. The area of a rectangle is $(x^2 + xy)\ m^2$ and its one side measures xm, what is the measure of other side?



$$l \times x = x^2 + xy$$

$$\text{or } l = \frac{x^2 + xy}{x} = \frac{x^2}{x} + \frac{xy}{x} = x + y.$$

$\therefore$ Length = other side = $(x + y)$ m.

While dividing a poly nominal by a monomial, each term of the polynomial should be divided by the monomial respectively.

Example 1

Divide: $8x^5 - 20x^2$ by $4x^2$

Solution :

Here by actual division we have,

$$\begin{array}{r|l} & 2x^3 - 5 \\ 4x^2 & 8x^5 - 20x^2 \\ & \pm 8x^5 \\ \hline & -20x^2 \\ & \pm 20x^2 \\ \hline & \times \end{array}$$

$\therefore$ Quotient $= 2x^3 - 5$

Alternate method :

$$\frac{8x^5 - 20x^2}{4x^2} = \frac{8x^5}{4x^2} - \frac{20x^2}{4x^2}$$
$$= 2x^3 - 5$$

Example 2

Simplify : $\dfrac{(3x-2y)(x+y)+y(x-y)}{3x}$

Solution :

$$= \frac{3x^2 + 3xy - 2xy - 2y^2 + xy - y^2}{3x}$$

$$= \frac{3x^2 + 2xy - 3y^2}{3x}$$

Alternate method :

$$= \frac{3x^2}{3x} + \frac{2xy}{3x} - \frac{3y^2}{3x}$$

$$= 3 + \frac{2y}{3} - \frac{y^2}{x}$$

By actual division, we have

$$\begin{array}{r|l} & x + \frac{2}{3}y - \frac{y^2}{x} \\ 3x & 3x^2 + 2xy - 3y^2 \\ & -3x^2 \\ \hline & +2xy \\ & \pm 2xy \\ \hline & -3y^2 \\ & \pm 3y^2 \\ \hline & \times \end{array}$$

[$\therefore$ instead of 3 we need 2, hence multiply by $\frac{2}{3}$]

[$\therefore$ instead of x we need y^2 hence multiply by $\frac{y^2}{x}$]

$\therefore$ Quotient $= x + \frac{2}{3}y - \frac{y^2}{x}$

EXERCISE 11 [E]

1. Simplify

(a) $\dfrac{10x-15y+30}{5}$ (b) $\dfrac{x^4-2x^3+x^3}{x}$

(c) $\dfrac{5x^6-3x^5+5x^3}{x^3}$ (d) $\dfrac{4p^3q^2+45p^2q+60pq^2}{4pq}$

(e) $\dfrac{5x^2y+10x^2y^2+5xy^2}{5xy}$ (f) $\dfrac{30p^2q^2+45p^2q-60pq^2}{15pq}$

(g) $\dfrac{15x^5+15x^4-30x^2}{-3x}$ (h) $\dfrac{21x^5y^3-14x^4y^4-28x^3y^5}{-7x^2y^3}$

(i) $\dfrac{16x^3y^2-4x^2y^2}{4x^2y^2}$ (j) $\dfrac{27x^3y^3-18x^2y^2}{9x^3y^3}$

2. Simplify

(a) $\dfrac{2x(3x+5)+(2x+4)(2x-4)}{2}$ (b) $\dfrac{2y(5y+7)+(2y-3)(y+8)}{3}$

(c) $\dfrac{2x(3x+7)+x(4x+1)}{5x}$ (d) $\dfrac{3x(2x-3)-x(3x+3)}{3x}$

3. The area of a rectangle is $(12x^2+9x)$ sq. units. If the length of its one side is $3x$ units what will be the length of the other side?

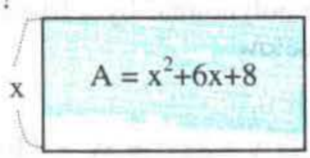
4. The area of a rectangle is $(56x^2y-30xy^2)$ sq. units and its one side measures $2xy$ units, find the measure of the other side.

11.2 (b) Dividing a Polynomial by a Binomial

The area of a rectangle is (x^2+6x+8) sq. units and is its one side measures $(x+4)$ units. How can we find the length of other side?

Example 1

Divide : x^2+6x+8 by $x+4$



Solution: *(Method I)*

By direct division,

$$
\begin{array}{r|l}
x+4 & x^2+6x+8\,(\,x+2 \\
 & \underline{\pm x^2 \pm 4x} \\
 & \quad +2x+8 \\
 & \quad \underline{\pm 2x \pm 8} \\
 & \quad\quad \times
\end{array}
$$

(Method II)

$$\frac{x^2+6x+8}{x+4} = \frac{(x+2)(x+4)}{x+4} = x+2$$

Check;

$(x+4)(x+2)$

$= x^2+2x+4x+8$

$= x^2+6x+8$

$\therefore$ Measure of other side of the rectangle $= x+2$

Example 2

Divided : $2x^2 + 13x - 26$ by $x+8$

Solution :

By actual division

$$x+8)\quad 2x^2+13x-26(2x-3$$
$$\underline{-2x^2 \pm 16x}$$
$$-3x-26$$
$$\underline{\pm 3x \pm 24}$$
$$-2 \quad \text{(Remainder)}$$

Check :

$(x + 8)(2x - 3) = 2x^2 + 13x - 24$

and $(x + 8)(2x - 3) - 2$

$= 2x^2 + 13x - 24 - 2$

$= 2x^2 + 13x - 26.$

Therefore, in the division of algebraic expressions

Dividend = divisor × quotient + remainder

Where degree of divisor > degree of remainder.

EXERCISE 11 [F]

1. Find the quotients.

(a) $(x^2 + 7x + 12) \div (x + 3)$
(b) $(x^2 - 10x + 21) \div (x - 7)$
(c) $(x^2 + 3x - 28) \div (x - 4)$
(d) $(x^2 + 4x - 32) \div (x + 8)$
(e) $(2x^2 + 9x - 5) \div (2x - 1)$
(f) $(3y^2 - 5y - 28) \div (3y + 7)$
(g) $(3x^2 - 16xy + 5y^2) \div (x - 5y)$
(h) $(6x^2 + xy - 2y^2) \div (2x - y)$
(i) $(2x^2 + 13xy + 15y^2) \div (x + 5y)$
(j) $(9x^2 + 9xy - 10y^2) \div (3x - 2y)$
(k) $(12x^2 - 18xy - 42y^2) \div (x + y)$
(l) $(15x^2 - 2xy - 79y^2) \div (3x - 7y)$
(m) $(14x^2 + 19xy - 5y^2) \div (2x + 3y)$
(n) $(16x^2 + 24xy + 9y^2) \div (4x + 3y)$
(o) $(25x^2 - 60xy + 36y^2) \div (5x - 6y)$
(p) $(x^3 - 2x^2 - 11x - 20) \div (x - 5)$
(q) $(2x^3 - 5x^2 - 24x - 18) \div (2x + 3)$
(r) $(6x^3 - 4x^2 - 13x - 12) \div (3x + .4)$

11.2 (c) Dividing a polynomial by a Trinomial

Division of a polynomial by a trinomial is similar to that of by a binomial. Study the example below.

Example 1

Divide : $x^3 + 4x^2y + 3xy^2 + 3y + x$ by $x^2 + xy + 1$.

Solution :

Here, actual division is shown below.

$$x^2+xy+1)x^3+4x^2y+3xy^2+3y+x(x+3y$$
$$\underline{-x^3 \pm x^2y \pm x}$$
$$+3x^2y+3xy^2+3y$$
$$\underline{\pm 3x^2y \pm 3xy^2 \pm 3y}$$
$$\times$$

Check : $(x^2 + xy + 1)(x + 3y) = x^3 + x^2y + 3x^2y + 3xy^2 + 3y + x$
$= x^3 + 4x^2y + 3xy^2 + 3y + x$

EXERCISE – 11 [G]

1. Find the quotients of :
   (a) $(a^3 + 6a^2 + 7a + 10) \div (a^2 + a + 2)$
   (b) $(2a^3 + 5a^2 + 3a + 2) \div (2a^2 + a + 1)$
   (c) $(x^3 + 5x^2 + 5x + 4) \div (x^2 + x + 1)$
   (d) $(3x^3 + 7x^2 + 11x + 5) \div (3x^2 + 2x + 1)$
   (e) $(5x^3 - 11x^2 + 3x - 2) \div (5x^2 - x + 1)$
   (f) $(10y^3 + 28y^2 - 5y + 3) \div (10y^2 - 2y + 1)$
   (g) $(10y^3 - 3y^2 - 7y + 3) \div (5y^2 + y - 3)$
   (h) $(6p^3 - 4p^2 + 7p + 3) \div (2p^2 - 2p + 3)$
   (i) $(15q^4 + 34q^3 + 15q^2 + 6q + 10) \div (5q^3 - 3q^2 + 2)$
   (j) $(a^4 + 3a^3 + 4a^2 + 3a + 1) \div (a^2 + a + 1)$

11.3. Laws of Indices

In class 7, you have learnt the following rules.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $a^0 = 1$ where $a \neq 0$.

Here m and n are positive integers.
Now study the examples below.

Example 1

Simplify.
(a) $a^2 \div a^5$ (b) $x^3 \div x^4$

Solution :

(a) Here $a^2 \div a^5 = \dfrac{a^2}{a^5} = \dfrac{a \times a}{a \times a \times a \times a \times a} = \dfrac{1}{a^3}$

Using laws of indices this is written as : $a^2 \div a^5 = a^{2-5} = a^{-3}$

(b) Here, $x^3 \div x^4 = \dfrac{x^3}{x^4} = \dfrac{x.x.x}{x.x.x.x.} = \dfrac{1}{x}$ using laws of indices, this is written as

$x^3 \div x^4 = x^{3-4} = x^{-1}$

Here in the example (a) $\dfrac{1}{a^3} = a^{-3}$ and in the example (b) $\dfrac{1}{x} = x^{-1}$

Therefore, ($\dfrac{1}{a^m} = a^{-m}$ where $a \neq o$ and m is an integer.)

Example 2

Simplify :

(a) $a^{-2} \times a^4 \div a^5$ (b) $x^2y^3 \div x^{-3y} \times xy^2$

Solution :

(a) Here $a^{-2} \times a^4 \div a^5 = a^{-2+4-5} = a^{-3} = \frac{1}{a^3}$

(b) Here, $x^2y^3 \div x^{-3}y \times xy^2 = x^{2-(-3)+1}y^{3-1+2}$
$= x^6y^4$

Example 3

Simplify :

(a) $(a^2)^3$ (b) $(xy)^3$

Solution :

(a) Here, $(a^2)^3 = a^2 \times a^2 \times a^2 = a^{2+2+2}$
$= a^{2\times3} = a^6$

(b) $(xy)^3 = xy \times xy \times xy = x^3y^3$

From this example, we conclude

$(a^{m)\,n} = a^{mn}$
$(ab)^m = a^mb^m$
Where $a \neq 0$, $b \neq 0$ m and n are integers.

Example 4

(a) $\left(\frac{p}{q}\right)^3$ (b) $\left(\frac{a^2b}{c^3}\right)^2$ (c) $\{2(x^2y)^3\,x^{-3}\}^2$

Solution :

(a) $\left(\frac{p}{q}\right)^3 = (pq^{-1})^3 = p^3q^{-3} = \frac{p^3}{q^3}$

(b) $\left(\frac{a^2b}{c^3}\right)^2 = (a^2bc^{-3})^2 = a^{2\times2}\,b^2\,c^{-3\times2} = a^4b^2c^{-6} = \frac{a^4b^2}{c^6}$

(c) $\{2(x^2y)^{3x-3}\}^2 = \{2x^6y^3x^{-3}\}^2 = \{2x^3y^3\}^2 = 4x^6y^6$

Example 5

Simplify:

(a) $(-3a^2b)^3 \times (2a^{-3}b)^2$ (b) $(2x^{m+2}y^{3m}) \times (-4x^{-2m+1}y^{m-4})$

Solution :

(a) $(-3a^2b)^3 \times (2a^{-3}b)^2$

$= - 27 a^6b^3 \times 4a^{-6}b^2$

$= (-27 \times 4) \times a^{6-6} \times b^{3+2}$

$= - 108b^5$ (because $a^{6-6} = a^0 = 1$)

(b) $(2x^{m+2}y^{3m}) \times (-4x^{-2m+1}y^{m-4})$

$= - 8 \times x^{(m+2) +(-2m+1)} \times y^{3m+(m-4)}$

$= - 8x^{-m+3} y^{4m-4}$

EXERCISE 11 [H]

1. Simplify using the law of indices:
   (a) $a^2 \times a^3$ (b) $3p^2 \times (-4)p^4$ (c) $x^5 \times x^{-3}$
   (d) $2y^{-4} \times (-3y^{3})$ (e) $-5z^2 \times (-2z^{-3})$ (f) $kx^{2m} \times hx^{3-m}$

2. Simplify using the law of indices.
   (a) $p^4 \div p^3$ (b) $x^{-2} \div x^3$ (c) $z^{-4} \div z^{-5}$
   (d) $- 24a^2 \div 6a^3$ (e) $- 12x^3 \div (-8x^{-3})$ (f) $q^{m-2} \div q^{3m-5}$

3. Simplify using the law of indices.
   (a) $(x^3)^4$ (b) $(pq)^2$ (c) $(x^2yz^3)^4$
   (d) $(\frac{2x}{yz})^3$ (e) $(p^2q^{-2}x^3)^{-2}$ (f) $(-3a^{-3}b^2c^{-4})^3$

4. Simplify using the law of indices.
   (a) $(8mn)^2 \times (-m^3n^2)$ (b) $3a^3b \times (-a^3b^2)^3$
   (c) $2x^py^{3q+1} \times (-4x^{2p}y^{2-q})$ (d) $(-2xy^2z^{-1})^2 \times (3x^2y^3z)^{-1}$

5. If $a = 2$, $b = -1$ and $c = 3$, find the value of each of the following.
   (a) a^{-1} (b) b^{-1} (c) $a^{-3}c$
   (d) $3a^2b^{-2}$ (e) $4a^{-2}b^2$ (f) $6a^0b^5x^{-2}$
   (g) $(a^2b)^{-2} (ab^3c^2)^2$ (h) $a^bb^cc^a$ (i) $a^{-b}b^{-c}c^{-a}$

11.4. Factorization

You have learnt the following types of problems,

(a) $x (x - 4) = x^2 - 4x$

(b) $(x +2) (x +3) = x^2 + 5x + 6$

(c) $(3p + \frac{1}{2}) (3p - \frac{1}{2}) = 9 p^2 - \frac{1}{4}$

The problems above, when viewed from the right side is seen that a polynomial expression is converted into the product of factors. Here x and x - 4 are the factors (multiplier and multiplicant) of $x^2 - 4x$, $(x + 2)$ and $(x + 3)$ are the factors of $x^2 + 5x + 6$, and $(3p + \frac{1}{2})$ and $(3p - \frac{1}{2})$ are the factors of $9p^2 - \frac{1}{4}$. Converting a polynomial expression into the product of factors is called the factorization therefore factorization is the reverse of expansion in the example above,

Factorization

$$x^2 - 4x = x(x - 4)$$
$$x^2 + 5x + 6 = (x + 2)(x + 3)$$
$$9p^2 - \frac{1}{4} = (3p + \frac{1}{2})(3p - \frac{1}{2})$$

11. 4 (a) Factorization of a Polynomial having a Monomial Factors

If each terms of a polynomial includes a common factor then the expression can be factorized keeping that factor as a multiplier for example $ma + mb + mc = m(a + b + c)$. Here, m is the common factor (c.f.) study the example below.

Example 1

Factorize each of the following expression :

(a) $2x^2 + 12x$ (b) $6mx - 9my$

Solution :

(a) Here the common factor of $2x^2$ and $12x$ is $2x$, therefore,

$$2x^2 + 12x = 2x.\,x + 2x.\,6 = 2x(x + 6)$$

(b) Here the common factor of 6mx and 9my is 3m

$$\therefore 3mx - 9my = 3m\times 2x - 3m\times 3y$$
$$= 3m(2x - 3y)$$

EXERCISE 11 [I]

1. Factorize each of the following expression.

(a) $5x + 10$ (b) $6a + 12$ (c) $4x + 10$ (d) $9a - 12$ (e) $n^2 - 6n$
(f) $x2 + 8x$ (g) $16m^2 - 24m$ (h) $4p^2 + 16p$ (i) $12xy + y$
(j) $4xy - y$ (k) $21xy + 7x$ (l) $3y^2 - 3y$

2. (a) $4x^2 - 8x + 16$ (b) $2m^2 + 4m + 8$ (c) $4p^3 - 6p^2 + 8p$
(d) $-4x + 8$ (e) $-10x - 5$ (f) $-p + 5p^2$

(g) $-x^2 - 5x$ (h) $-16x^2 + 8x$ (i) $-y^2 + 5y$
(f) $-x^3 - 2x^2 + 8x$ (k) $60x^4 - 4mx^2 + 12x$
(l) $30x^2y^2 + 35x^2y - 40y^2$ (m) $15p^2q + 9pq - 24pq^2$

11.4 (b) Factorization by Grouping

The terms of a polynomial may contain more than one common factor in this case, terms containing like common factors are collected together or grouped and then the polynomial is factorized.

Factorize the following

For Example,

$ma + mb + na + nb$
$= (ma + mb) + (na + nb)$
$= m(a+b) + n(a+b)$
$= (a+b)(m+n)$

Terms having like common factors are grouped

In the first and second terms b is c.f. and in the 3rd and 4th terms 2 is c.f. hence they are grouped then both terms have $a + 3$ as common factor.

Example 1

Factorize : $ab + 3b + 2a + 6$

$ab + 3b + 2a + 6$
$= (ab + 3b) + (2a + 6)$
$= b(a + 3) + 2(a + 3)$
$= (a + 3)(b + 2)$

Alternate method

Here, $ab + 3b + 2a + 6$
$= ab + 2a + 3b + 6$
$= (ab + 2a) + (3b + 6)$
$= a(b + 2) + 3(b + 2)$
$= (b + 2)(a + 3)$

Example 2

Factorize

(a) $p^2 + ab + ap + bp$ (b) $ab - ac - b + c$

Solution :

(a) $p^2 + ab + ap + bp$
$= (p^2 + ap) + (ab + bp)$
$= p(p + a) + b(a + p)$
$= (a + p)(p + b)$

(b) $ab - ac - b + c$
$= (ab - ac) - (b - c)$
$= a(b - c) - 1(b - c)$
$= (b - c)(a - 1)$

EXERCISE – 11 (J)

1. Factorize

(a) $x(x+4)+5(x+4)$
(b) $4(x-3)+x(x-3)$
(c) $4x(x+2)+3(x+2)$
(d) $x(2x-1)-3(2x-1)$
(e) $p(p+q)+r(p+q)$
(f) $r(r+3)-2(r+3)$
(g) $a(a-b)+a-b$
(h) $xy+5y+2x+10$
(i) $pq+3p+5q+15$
(j) $x^2-xy+2x-2y$
(k) $xy+x+y+1$
(l) $a^2+3b+3a+ab$
(m) $p^2-15q-5p+3pq$
(n) $-x-y+1+xy$
(o) $p^2+6pq-2pq-12q^2$
(p) $a^2b+ab^2c+ca+bc^2$

11. 4 (c) Factorization of the trinomial of the form x^2+px+q

From expansion formula, we have.

$x^2+(a+b)x+ab=(x+a)(x+b)$

Let $x^2+(a+b)x+ab=x^2+px+q$

From above it can be said that the factorization of x^2+px+q takes the form $x^2(a+b)x+ab=(x+a)(x+b)$. Hence to factorize x^2+px+q. we have to set.

$a+b=p$ and $ab=q$

Example 1

Factorize :

(a) $x^2+7x+12$
(b) $x^2-14x+24$
(c) $x^2+6x-40$
(d) $x^2-18x-63$

Solution :

ab = 12	a + b = 7
1 × 12	×
2 × 6	×
3 × 4	✓
-1 × -12	×
-2 × -6	×
-6 × -4	×

(a) Here, comparing $x^2+7x+12$ with

$x^2+(a+b)x+ab$ we get

$a+b=7$, and $ab=12$

or, $3+4=7$ and $3x4=12$

Hence, $x^2+7x+12=x^2+(3+4)x+12$

$=(x+3)(x+4)$

ab = 24	a + b = -19
1 × 24	×
2 × 12	×
3 × 8	×
4 × 6	×
-1 × -24	×
-2 × -12	✓
-3 × -8	×
-4 × -6	×

(b) Here, comparing $x^2-14x+24$ with

$=x^2+(a-b)x+ab$ we get

$=a+b=-14$ and $ab=24$

or, $-2+(-12)=-14$ and $(-2)(-12)=24$

Hence, $x^2-14x+24$

$=x^2+(-2-12)x+24$

$=x^2-2x-12x+24$

$=(x-2)(x-12)$

(c) Here, for $x^2 + 6x - 40$ two numbers whose sum is 6 and product -40 are -4 and 10
Hence,

$$x^2 + 6x - 40$$
$$= x^2 + (10 - 4)x - 40$$
$$= x^2 + 10x - 4x - 40$$
$$= (x - 4)(x + 10)$$

(d) For $x^2 - 18x - 63$ two numbers whose sum is -18 and product is 63 are 3 and -21
Hence,

$$x^2 - 18x - 63$$
$$= x^2 + (3 - 21)x - 63$$
$$= x^2 + 3x - 21x - 63$$
$$= (x + 3)(x - 21)$$

EXERCISE 11 (K)

1. Find the value of a and b for each of the following.
   (a) $ab = 12, a + b = 7$ (b) $ab = 6, a + b = 5$
   (c) $ab = 32, a + b = -12$ (d) $ab = -18, a + b = -7$
   (e) $ab = 30, a + b = 13$ (f) $ab = -6, a + b = -1$
2. Factorize:
   (i) $x^2 + 7x + 10$ (ii) $x^2 + 7x + 12$ (iii) $x^2 + 11x + 30$
   (iv) $x^2 - 8x + 15$ (v) $x^2 - 12x + 35$ (vi) $x^2 - 12x + 27$
   (vii) $x^2 + 7x - 18$ (viii) $x^2 + 4x - 21$ (ix) $y^2 + 4y - 32$
   (x) $x^2 - 13x + 30$ (xi) $y^2 - y - 30$ (xii) $p^2 - 7p - 18$
   (xiii) $y^2 + 15y + 56$ (xiv) $m^2 - 4m - 5$ (xv) $x^2 - 41x + 40$
   (xvi) $p^2 + 29p - 30$ (xvii) $x^2 - 10x - 39$ (xviii) $x^2 - 16x + 39$
   (xix) $x^2 - 42x - 43$ (xx) $x^2 - 12x + 36$ (xxi) $p^2 + 22p + 121$
   (xxii) $22 - 9x - x^2$ (xxiii) $15 + 2x - x^2$ (xxiv) $8 - 7p - p^2$
   (xxv) $19 + 18x - x^2$ (xxvi) $42 - 11p - p^2$ (xxvii) $x^2 - 3xy + 2y^2$
   (xxviii) $x^2 + 11xy + 30y^2$ (xxix) $p^2 - pq - 6p^2$ (xxx) $x^2 + 3xy - 4y^2$

11.4. (d) Factorization of the Trinomial of the Type $px^2 + qx + r$.

Study the examples below.

Example 1

Factorize :

(a) $3x^2 + 8x + 4$ (b) $5a^2 - 8a - 4$.

Solution :

Here, $3x^2 + 8x + 4$

Hence we need to determine two numbers a and b such that $a.b = 4 \times 3$ and $a + b = 8$.
Since $6 + 2 = 8$ and $6 \times 2 = 12$ the expression

$$3x^2 + 8x + 4 = 3x^2 + (6 + 2)x + 4$$
$$= 3x^2 + 6x + 2x + 4$$
$$= (x + 2)(3x + 2)$$
$$\therefore 3x^2 + 8x + 4 = (x + 2)(3x + 2)$$

Here, $5a^2 - 8a - 4.$
$= 5a^2 - (10 - 2)\ a - 4$
$= 5a^2 - 10a + 2a - 4$
$= 5a\ (a - 2) + 2\ (a-2)$
$= (a - 2)\ (5a + 2)$
$\therefore\ 5a^2 - 8a - 4 = (a - 2)\ (5a + 2)$

[$\therefore$ The numbers whose product is -20 and difference is -8 are -10 and +2]

EXERCISE 11 (L)

1. Factorize the following.

(a) $3x^2 + 4x + 1$ (b) $2x^2 + 5x + 3$ (c) $3x^2 - 4x + 1$
(d) $4m^2 - 8m + 3$ (e) $4x^2 + 4x - 15$ (f) $15x^2 - 13x + 2$
(g) $12y^2 - 25y - 7$ (h) $10x^2 - 3x - 1$ (i) $12x^2 - 32x + 5$
(j) $50a^2 - 25a + 3$ (k) $60b^2 - b - 10$ (l) $27n^2 - 30n + 8$
(m) $56p^2 - 6 - 5p$ (n) $21x^2 + 25x - 4$ (o) $15x^2 - 2 - x$
(p) $16q - 15 - 4q^2$ (q) $120p^2 - 26p - 3$ (r) $224t^2 - 4tp - 4p^2$
(s) $150s^2 - 60st - 3t^2$ (t) $64m^2 + 16mn - 15n^2$
(u) $192n^2 - 44ns - 15s^2$ (v) $120a^2 + 38ab - 5b^2$

11. 4 (e) Factorization of a Trinomial of the form $a^2 + 2ab + b^2$

We know, the expansion formula.

$a^2 + 2ab + b^2 =$ formulas $(a + b)^2$

and $a^2 - 2ab + b^2 = (a - b)^2$

We can apply these formulas for factorization as well.

Example 1

What should be filled in the blanks in order to make the expression a perfect square?

(a) $x^2 + \ldots\ldots\ldots\ldots + 9$ (b) $y^2 - \ldots\ldots\ldots\ldots + 25$

Solution :

(a) Here as $9 = 3^2$ according to the formula.
$x^2 + \ldots\ldots\ldots\ldots + 3^2$ should be of the form $(x + 3)^2$

(b) Here as $25 = 5^2$ given expression $y^2 - \ldots\ldots\ldots\ldots + 25$
Should be of the form $(y-5)^2$
and $(y - 5)^2 = y^2 - 10y + 25$, hence the missing term must be -10y

Example 2

Factorize :

(a) $x^2 + 8x + 16$ (b) $x^2 - 12\ x + 36$

Solution :

(a) Since $8 = 2 \times 4$ and $16 = 4^2$

$$x^2 + 8x + 16 = x^2 + 2 \times 4 \times x + 4^2$$
$$= (x + 4)^2$$

$$x^2 + 8x + 16 = x^2 + 2 \times 4 \times x + 4^2 = (x + 4)^2$$
$$a^2 + 2\,a\,b + b^2 \qquad = (a + b)^2$$

(b) Since $12 = 2 \times 6$ and $36 = 6^2$

$$x^2 - 12x + 36 = x^2 - 2 \times 6 \times x + 6^2$$
$$= (x - 6)^2$$

$$x^2 - 12x + 36 = x^2 - 2 \times x \times 6 + 6^2 = (x - 6)^2$$
$$a^2 - 2\,a\,b + b^2 \qquad = (a - b)^2$$

EXERCISE – 11 (M)

1. What should be filled in the blanks in order to make the expression a perfect square?

   (a) $x^2 + ... + 9y^2$ (b) $y^2 - .... + 49$ (c) $9x^2 + ... + 16y^2$

   (d) $16y^2 + ... + 1$ (e) $x^2 + ... + (15)^2$ (f) $m^2 + ... + 100$

2. Factorize:

   (a) $x^2 + 10x + 25$ (b) $x^2 + 12x + 36$ (c) $y^2 - 8x + 16$

   (d) $x^2 - 14x + 49$ (e) $x^2 + 18x + 81$ (f) $P^2 - 24p + 144$

   (g) $9x^2 - 24x + 16$ (h) $16p^2 + 8p + 1$ (i) $25x^2 - 80x + 64$

   (j) $49q^2 + 14q + 1$ (k) $9x^2 - 66x + 121$ (l) $144 + 24x + x^2$

   (m) $36 - 60y + 25y^2$ (n) $25x^2 + 30x + 9$ (o) $3x^2 - 6x + 3$

11.4 (e) Factorization of Difference of Two Squares:

We know the expansion formula.

$a^2 - b^2 = (a + b)(a - b)$ It can be applied to factorize

Example 1

(a) $p^2 - 25$ (b) $9x^2 - 16y^2$

Solution :

(a) as $25 = 5^2$

$$p^2 - 25 = p^2 - 5^2$$
$$= (p + 5)(p - 5)$$

$$x^2 - 25 = (p + 5)(p - 5)$$
$$a^2 - b^2 = (a + b)(a - b)$$

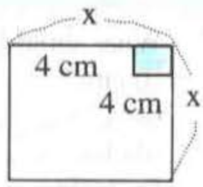
(b) As $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2$$
$$= (3x + 4y)(3x - 47)$$

$9x^2 - 16y^2 = (3x + 4y)(3x - 4y)$
$a^2 - b^2 = (a + b)(a - b)$

EXERCISE – 11 (N)

1. Factorize :
   (a) $x^2 - 36$ (b) $m^2 - 64$ (c) $y^2 - 49$
   (d) $p^2 - q^2$ (e) $a^2 - 121$ (f) $1 - p^2$
   (g) $81 - x^2$ (h) $100 - x^2$ (i) $225 - n^2$
   (j) $9x^2 - 100$ (k) $25 - 49y^2$ (l) $256 - 121y^2$
   (m) $3y^2 - 27$ (n) $5x^2 - 80$ (o) $2b^2 - 72$
   (p) $48 - 3y^2$ (q) $5a^3 - 20ab^2$ (r) $m^3 - 4m$
2. (a) Find the area of shaded portion in the adjoining figure?
   (b) If x = 8 cm, What is the actual area of shaded portion?



3. Find the area of the shaded portion in the adjoining figure, if $\pi = \frac{22}{7}$, R = 7cm, r = 14 cm.

4. If n is a natural number and n – 2 is another natural number less than n, prove that the difference of square of these number is an even number.

11.5 (a) Highest Common Factor (H.C.F.)

Expressions

Example 1

Find the highest degree polynomial expression that exactly divides the expressions $x^2y - 4x^3y^3$ and $x^3y^2 + 2x^2y^3$

Solution :

Factoring given expression,

First expression $= x^3y^2 + 2x^2y^3$
$= x^2y^2(x + 2y)$

Second expression $= x^5y - 4x^3y^3$
$= x^3y(x^2 - 4y^2)$
$= x^3y(x + 2y)(x - 2y)$

The common factors in these expressions is:

$x, x^2, y, xy, x^2y, x(x + 2y), x^2(x+2y), y(x+2y), xy(x+2y)$.

Each of these common factors exactly divide both expressions and the highest (degree) of these is $x^2y\ (x+2y)$
$\therefore$ H.C.F. $= x^2y\ (x + 2y)$

H.C.F. of given expressions is the product of maximum number of common factors.

Example 2

Find H.C.F. of $2x^2 - 6x$ and $x^2 - x - 6$

Solution :

First expression $= 2x^2 - 6x = 2x\ (x - 3)$
Second expression $= x^2 - x - 6 = x^2 - 3x + 2x - 6 = x(x - 3) + 2(x - 3)$
$= (x - 3)\ (x + 2)$
Common factors in these expressions are : $x - 3$
$\therefore$ H.C.F. $= x - 3$

EXERCISE – 11 (O)

1. Find the H.C.F.
   (a) $3x - 6$ and $x - 2$
   (b) $3x + y$ and $12x + 4y$
   (c) $2x + 14$ and $3x + 21$
   (d) $3y - 24$ and $4y - 32$
   (e) $3x - 6y$ and $4x - 8y$
   (f) $3x + 9$ and $5x + 15$
   (g) $2a^2 + 6ac$ and $4a^2c + 12ac^2$
   (h) $2x - 10$ and $x^2 - 25$
   (i) $3a^2 - 6ab$ and $2ac - 4bc$
   (j) $12m^2 + 6m$ and $8m + 4$
   (k) $x^2 - 3x$ and $x^2 - 9$
   (l) $x^3y - xy^2$ and $x^2 + xy$
   (m) $x + 2$ and $x^2 + 4x + 4$
   (n) $x^2 - 5x$ and $x^2 - 6x + 5$
   (o) $x^3 - x$ and $x^2 - x - 2$
   (p) $x^2 + 8x$ and $x^2 + x + 6$
   (q) $x^2 - 2x - 15$ and $x^2 - x - 20$
   (r) $x^2 + 5x + 6$ and $x^2 + x - 6$
   (s) $x^2 + 2x - 15$ and $x^2 - 7x + 12$
   (t) $x^2 + 6x - 7$ and $x^2 + 5x - 14$
   (u) $x^2 + 4x + 4$, $x^2 + 7x + 10$ and $x^2 - x - 6$
   (v) $x^2 + 2x - 3$, $x^2 - 3x + 2$ and $x^2 - 1$

11.5 (b) Lowest Common Multiple (L.C.M.) of Polynomials.

Example 2

Find the least (degree) expression exactly divisible by $2x^2y$ and $3xy^3$

Solution :

Here the coefficient of the expression divisible by $2x^2y$ and $3xy^3$ is the common multiples of 2 and 3 = 6, 12, 18 .....and 6 is the least.
Expression divisible by x^2 and x is the common multiples of x^2 and x and are x^2, x^3, x^4 ......among these x^2 is the least expression divisible by y and y^3 is the common

multiples of y and y^3. They are y^3, y^4, y^5 ......and among these y^3 is the least. Therefore least expression exactly divisible by $2x^2y$ and $3xy^3$ is $6x^2y^3$.

This problem can be solved easily by factorization method. Study the example below.
Factors of $2x^2y = 2. x. x. y$
Factors of $3xy^3 = 3. x. y. y. y$.
Common factors = x.y ....(i)
From (i) and (ii) we have $x.y.2.3.x.y.y = 6x^2y^3$
Which is the least common multiple.

Least common multiple of two expressions = common factors × remaining factors

Example 2

Find the L.C.M. of $x^2 - y^2$ and $x^2 + 2xy + y^2$

Solution :

First expression = $x^2 - y^2 = (x + y)(x - y)$
Second expression = $x^2 + 2xy + y^2 = (x + y)^2$
Common factor in these expressions is : $x + y$ ....(i)
Remaining factors in these expressions are $(x - y)(x + y)$ ....(ii)

From (i) and (ii) we have $(x + y)(x - y)(x + y)$.
$\therefore$ L.C.M. = $(x + y)^2(x - y)$

EXERCISE 11 (P)

1. Find the L.C.M.

(a) $4x$ and $8x^2$	(b) $4y$ and 6
(c) $5xy$ and $10x^2$	(d) $4m^2n^3$ and $6m^4n^2P$
(e) $5x$ and $3x^2 + x$	(f) $6x^2$ and $8x^2 + 2x$
(g) $m + 4$ and $m^2 + 4m$	(h) $4x - 20$ and $6x - 30$
(i) $5x + 10$ and $4x + 8$	(j) $4x^2 - 9$ and $2x + 3$
(k) $4x^2 - 100$ and $x - 5$	(l) $4 - x^2$ and $4 + 2x$
(m) $4(x - 1)$ and $x^2 + x - 2$	(n) $x^2 - 4$ and $x^2 + 3x + 2$
(o) $a^2 + a - 6$ and $a^2 - 9$	(p) $x^2 - 6x + 8$ and $x - 4$
(q) $3x - 21$ and $x^2 - 4x - 21$	(r) $x^2 - 2x - 3$ and $x + 2$
(s) $x^2 - 25$ and $x^2 - 9x + 20$	(t) $x^2 - 2x - 3$ and $x + 2$

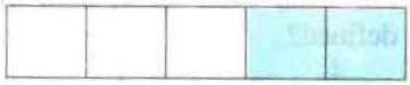
(u) $x^2 + 6x + 8$, $x^2 + 9x + 20$ and $x^2 + 7x + 10$

[Hint (for you):]

L.C.M. = Common factors in 3 expressions × Common factors in 2 expressions × remaining factors in all expressions.

11.6 Addition and Subtraction of Rational Expressions

The shaded region of the adjoining figure represents the fraction $\frac{2}{5}$



Numbers expressed in the form of fraction. e.g. $\frac{2}{5}, \frac{6}{11}, \frac{4}{1}$ etc, are rational numbers. Each rational number can be expressed in the form of a fraction.

If a and b be polynomials then $\frac{a}{b}$ is called rational expression.

Some examples of rational expressions are

$$\frac{7}{x}, \frac{a+b}{c}, \frac{4}{m+n}, \frac{1}{x-y}, \frac{2x^2-11x-6}{x-6} \ etc.$$

If the denominator of a rational expression is 0 (zero), then the expression is called undefined or the meaningless rational expression. For example if x = 0, then $\frac{7}{x}$ is undefined.

If x = y then, $\frac{3}{x-y}$ is undefined.

If x = 6 then, $\frac{2x^2-11x-6}{x-6}$ in not defined.

Example 1

For what value of y, $\frac{7}{y-8}$ is undefined?

Here, If y - 8 = 0 then the $\frac{7}{y-8}$ becomes zero

So, y = 8, then it becomes undefined.

Example 2

Simplify :

(a) $\frac{(x-3)^2}{2x-6}$ (b) $\frac{x^2-5x+6}{x^2-3x+2}$

Solution :

(a) $\frac{(x-3)^2}{2x-6} = \frac{(x-3)^2}{2(x-3)} = \frac{x-3}{2}$, (b) $\frac{x^2-5x+6}{x^2-3x+2} = \frac{(x-3)(x-2)}{(x-2)(x-1)} = \frac{x-3}{x-1}$

EXERCISE – 11 (Q)

1. For what value of x, each of the following rational expressions are not defined?

(a) $\frac{4}{x}$ (b) $\frac{1}{x-7}$ (c) $\frac{x}{x+9}$ (d) $\frac{x+y}{x-6}$

2. Simplify

(a) $\frac{m^2+4m}{m^2-6m}$ (b) $\frac{x^2+xy}{x^2-xy}$ (c) $\frac{3x+9}{x^2-9}$

(d) $\frac{a^2-b^2}{a+b}$ (e) $\frac{x^2-25}{2x-10}$ (f) $\frac{x^2-6x+9}{x-3}$

(g) $\frac{x-2y}{x^2-4y^2}$ (h) $\frac{x^2-3x+2}{x^2-4}$ (i) $\frac{x^2+x-6}{x+3}$

(j) $\frac{x+7x^2}{1-49x^2}$ (k) $\frac{(x-6)^2}{x^2-36}$ (l) $\frac{x^2+x-12}{12-4x}$

(m) $\frac{-p^2+p+12}{p^2-10p+24}$ (n) $\frac{s^3-12s^2+20s}{s^2+6s-16}$

11.7. (b) Addition and Subtraction of rational expressions having same denominator.

While adding or subtracting the rational expressions having same denominator, the numerators are added or subtracted and the sum or difference is taken as the numerator of the result and the denominator is kept unchanged.

Example 2

Simplify :

(a) $\frac{3}{x-7}+\frac{4}{x-y}$ (b) $\frac{9}{2x}-\frac{3}{2x}$

Solution :

(a) $\frac{3}{x-y}+\frac{4}{x-y}=\frac{3+4}{x-y}=\frac{7}{x-y}$

(b) $\frac{9}{2x}-\frac{3}{2x}=\frac{9-3}{2x}=\frac{6}{2x}=\frac{3}{x}$

Example 2

Simplify:

(a) $\frac{3x-2}{x-5}+\frac{3}{x-5}$ (b) $\frac{a^2}{a-4}-\frac{16}{a-4}$

Solution :

(a) Here $\frac{3x-2}{x-5}+\frac{3}{x-5}=\frac{3x-2+3}{x-5}=\frac{3x+1}{x-5}$

(b) Here $\frac{a^2}{a-4}-\frac{16}{a-4}=\frac{a^2-16}{a-4}=\frac{(a+4)(a-4)}{a-4}=a+4$

EXERCISE 11 (R)

1. Simplify :

(a) $\frac{x}{5}+\frac{3x}{5}$ (b) $\frac{7}{x}+\frac{4}{x}$ (c) $\frac{x}{3y}+\frac{2x}{3y}$

(d) $\frac{3}{t+2}+\frac{4}{t+2}$ (e) $\frac{-1}{x-1}+\frac{x}{x-1}$ (f) $\frac{2a}{a+2}+\frac{1}{a+2}$

2. Simplify:

(a) $\frac{7x}{9}-\frac{4x}{9}$ (b) $\frac{6x}{x}-\frac{x}{5}$ (c) $\frac{9}{y}-\frac{5}{y}$

(d) $\frac{6}{y-3}-\frac{2y}{y-3}$ (e) $\frac{3}{x+2}-\frac{1}{x+2}$ (f) $\frac{a+b}{a^2+1}-\frac{b}{a^2+1}$

3. Simplify:

(a) $\frac{3m+7}{m+1}-\frac{2m+1}{m+1}$ (b) $\frac{x+6}{x^2-1}-\frac{5}{x^2-1}$

(c) $\frac{y-10}{y^2-9}+\frac{7}{y^2-9}$ (d) $\frac{x+4}{x^2-25}+\frac{x+6}{x^2-25}$

(e) $\frac{x}{x^2-4}-\frac{2}{x^2-4}$ (f) $\frac{m^2}{m+2}-\frac{4}{m+2}$

(g) $\frac{x^2}{x-3}-\frac{9}{x-3}$ (h) $\frac{x^2}{x+4}-\frac{16}{x+4}$

(i) $\frac{x^2}{x+1}+\frac{2x+1}{x+1}$ (j) $\frac{3x^2}{x+4}+\frac{15x+12}{x+4}$

(k) $\frac{m^2}{m-1} - \frac{2m-1}{m-1}$

(l) $\frac{x^2}{x+2} + \frac{4x+4}{x+2}$

(m) $\frac{5x^2}{4-x} - \frac{35x-60}{4-x}$

(n) $\frac{2p^2}{p+3q} + \frac{2pq-12q^2}{p+3q}$

(o) $\frac{m^4}{4(m+3)^2} + \frac{81-18m^2}{4(m+3)^2}$

(p) $\frac{12xy}{xz+yz} + \frac{3x^2+9y^2}{xz+yz}$

11.7. (c) Addition and subtraction of rational expressions having unequal denominators.

Example 1

Out of Rs. x which is his monthly income. Ramesh spends Rs. $\frac{x}{2}$ on food and Rs. $\frac{x}{3}$ on rent. How much does he spend altogether on food and house rent?

Solution :

Here, Expense on food = Rs. $\frac{x}{2}$

Expense on rent = Rs. $\frac{x}{3}$

$\therefore$ Total expense = Rs. $\frac{x}{2}$ + Rs $\frac{x}{3}$

= Rs. $\left(\frac{x}{2}+\frac{x}{3}\right)$

Here, denominators are different hence in order to make same, we find L.C.M. of 2 and 3.

L.C.M. of 2 and 3 = 2 x 3

$$\therefore \frac{x}{2}+\frac{x}{3} = \frac{x.3}{2.3}+\frac{x.2}{3.2} = \frac{3x+2x}{6} = \frac{5x}{6}$$

Hence, Ramesh's expense = Rs. $\frac{5x}{6}$

Example 2

Here, L.C.M. of $4x^2$ and $6x = 12x^2$

Now, $\frac{3}{4x^2}+\frac{5}{6x} = \frac{3\times3}{3\times4x^2}+\frac{5\times2x}{6x\times2x} = \frac{9+10x}{12x^2}$

Example 3

Simplify :

(a) $\frac{4}{x+2}-\frac{3}{x-2}$ (b) $\frac{7}{y-1}-\frac{7}{y(y-1)}$

Solution :

(a) $\frac{4}{x+2}-\frac{3}{x-2}=\frac{4(x-2)}{(x+2)(x-2)}-\frac{3(x+2)}{(x-2)(x+2)}$

$$=\frac{4x-8-3x-6}{(x+2)(x-2)}=\frac{x-14}{x^2-4}$$

[Because L.C.M. of x + 2 and x –2 = (x +2) (x –2)]

(b) $\frac{7}{y-1}-\frac{7}{y(y-1)}$

$$=\frac{y\times 7}{y(y-1)}-\frac{7}{y(y-1)}$$

$$=\frac{7y-7}{y(y-1)}=\frac{7(y-1)}{y(y-1)}=\frac{7}{y}$$

[Because L.C.M. of (y-1) and y(y-1) = y(y-1)]

Example 4

Simplify : $\frac{1}{x^2+x-2}-\frac{1}{x^2-x-6}+\frac{1}{x^2-4x+3}$

Solution :

Here,

$$=\frac{1}{x^2+x-2}-\frac{1}{x^2-x-6}+\frac{1}{x^2-4x+3}$$

$$=\frac{1}{(x+2)(x-1)}-\frac{1}{(x-3)(x+2)}+\frac{1}{(x-1)(x-3)}$$

$$=\frac{(x-3)-(x-1)+(x+2)}{(x-1)(x+2)(x-3)}$$

$$=\frac{x-3-x+1+x+2}{(x-1)(x+2)(x-3)}$$

$$=\frac{x}{(x-1)(x+2)(x-3)}$$

EXERCISE 11 (S)

1. Simplify:

(a) $\frac{x}{3}+\frac{x}{15}$ (b) $\frac{y}{6}+\frac{y}{9}$ (c) $\frac{x}{3}+\frac{4}{x}$

(d) $\frac{2}{3y}+\frac{3}{4y}$ (e) $\frac{2}{x}+3$ (f) $\frac{x}{y}+4$

2. Simplify:

(a) $\frac{7}{x}-\frac{x}{3}$ (b) $\frac{2}{m}-\frac{3}{mn}$ (c) $\frac{3}{2y}-\frac{1}{8y}$

(d) $\frac{x}{y}-3$ (e) $\frac{5}{2x+3}+\frac{1}{3}$ (f) $\frac{5}{x-2}+\frac{2}{x+2}$

3. Simplify:

(a) $\frac{x}{7x-7}+\frac{1}{x-1}$ (b) $2+\frac{2y+1}{6}$ (c) $\frac{3}{2(m+n)}-\frac{5}{3(m+n)}$

(d) $\frac{x+7}{x-7}-\frac{x}{7-x}$ (e) $\frac{1}{x+2}-\frac{1}{x-5}$ (f) $\frac{y}{3(y+3)}+\frac{2}{3(y-3)}$

(g) $\frac{8}{5(x-5)}+\frac{5}{6(x-5)}$ (h) $\frac{5x+20}{x^2+4x}+1$

(i) $\frac{x-2}{x+2}+\frac{x-2}{x^2-4}$ (j) $\frac{7x+3}{x^2-9}-\frac{3}{x-1}$

(k) $\frac{x+2}{x^2+x}-\frac{3}{x^2-x-2}$ (l) $\frac{2}{x-1}+\frac{2x-5}{x^2+2x-3}$

(m) $\frac{2x-1}{x^2+4x}-\frac{x-2}{x^2+2x-8}$ (n) $\frac{2}{x+1}+\frac{2x}{x-1}-\frac{x^2+3}{x^2-1}$

(o) $\frac{2}{x^2+3x+2}+\frac{5x}{x^2-x-6}-\frac{x+2}{x^2-x-2}$

(p) $\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}$

11.6 (b) Multiplication and Division of Rational expressions

You have learnt the factorizations of algebraic expression in previous lesson. Here you will learn the simplification of rational expressions.

Study the example below.

Example 1

Simplify : $\frac{x^2+5x+6}{x^2-4}\times\frac{x^2+x-6}{x^3-9x}$

Solution :

Here, $\frac{x^2+5x+6}{x^2-4}\times\frac{x^2+x-6}{x^3-9x}$

$=\frac{(x+3)(x+2)}{(x+2)(x-2)}\times\frac{(x+3)(x-2)}{x(x^2-9)}$

$=\frac{(x+3)(x+3)}{x(x+3)(x-3)}$

$=\frac{x+3}{x(x-3)}$

Factorization:

(1) x^2+5x+6

$= x^2+3x+2x+6$

$= x(x+3)+2(x+6)$

$= (x+3)(x+2)$

(2) x^2+x-6

$= x^2+3x-2x-6$

$= x(x+3)-2(x+3)$

$= (x+3)(x-2)$

Example 2

Simplify : $\frac{x^2-x-2}{x^2-3x}\div\frac{x^2+3x+2}{x-3}$

Solution :

Here, $\frac{x^2-x-2}{x^2-3x}\div\frac{x^2+3x+2}{x-3}$

$=\frac{(x+1)(x-2)}{x(x-3)}\div\frac{(x+1)(x+2)}{x-3}$ (Numerators are factorized)

$=\frac{(x+1)(x-2)}{x(x-3)}\div\frac{x-3}{(x+1)(x+2)}$

$=\frac{x-2}{x(x+2)}$

EXERCISE – 11 (T)

1. Simplify the following:

(a) $\frac{x^2-y^2}{6x+6y}\times\frac{2x}{3}$

(b) $\frac{-x^2+4x-4}{xy-3y}\times\frac{4x-12}{x-2}$

(c) $\frac{a^2-b^2}{a^2+2a+ab+2b}\times\frac{a+2}{a+3}$

(d) $\frac{a^2+10a+24}{a^2+2a-8}\times\frac{a-3}{a+6}$

(e) $\dfrac{a^2+3a-10}{a^2-5a+6} \times \dfrac{ax-3x}{ay+5y}$

(f) $\dfrac{x^2-11x+30}{x^2-3x-10} \times \dfrac{5x+10}{x^2-8x+12}$

(g) $\dfrac{x^2+4x-12}{x^2-5x+6}$

(h) $\dfrac{x^2+3x}{x^2+x-2} \times \dfrac{x^2-4}{x+4}$

(i) $\dfrac{x^2+2x-15}{(x-2)} \div \dfrac{3(x^2+4x-5)}{x^2-3x+2}$

(j) $\dfrac{x^2+12x+27}{x^2+x-6} \div \dfrac{x^2+4x-45}{9(x^2-4x-5)}$

(k) $\dfrac{x^2+8x+15}{xy-x+5y-5} \div \dfrac{3y+2y+xy+6}{xy-x+2y-2}$

(l) $\dfrac{xy+35-5x-7y}{xy-2x-7y+14} \div \dfrac{y^2-2y-15}{y^2-4}$

(m) $\dfrac{a^2+4a-12}{a^2-5a+6} \div \dfrac{a^2-3a-18}{a^2-9}$

(n) $\dfrac{b^2-8b+15}{b^2-14b+45} \div \dfrac{b^2+2b-15}{b^2-8b-9}$

(o) $\dfrac{x^2-9x+18}{x^2-x-6} \div \dfrac{x^2-4x-12}{x^2+3x+2}$

(p) $\dfrac{y^2-7y+12}{y^2-2y-8} \div \dfrac{y^3-3y^2}{x^2y-5x^2}$

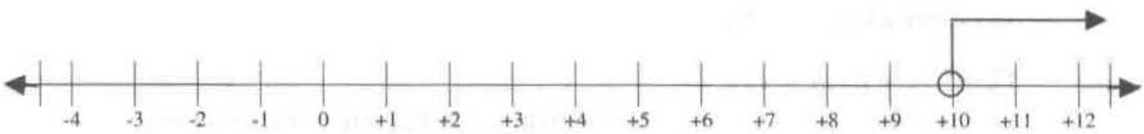
UNIT - 12

EQUATION, INEQUALITY AND GRAPH

12.1 Equation and Inequality

Before solving the mathematical problems given in words they should be translated into mathematical language. For example, what is the number to which when 4 is added makes 20 ? In order to solve this problem, we assume the required number to be x and we write the problem in mathematical language as $x + 4 = 20$.

It is an open mathematical sentence because its truth depends on the value of x. If $x = 1$ then $x+4 = 20$ is a false sentence and if $x = 16$ then $x + 4 = 20$ is a true sentence. Therefore this is an equation. Similarly if the mathematical sentence includes any of the signs : is greater than (>), is less than (<) is greater or equal to ($\geq$) is less or equal to ($\leq$) then the sentence is called an inequality or in equation (The signs are known as signs of trichotomy) for example consider a statement: Ramesh has more than Rs 10. If we wish to write it in mathematical language it will be of the form. If R denotes the amount of money Ramesh has, than $R > 10$. In the inequation $R > 10$ the value of $R = 11, 12, 13$ .....the set of values of R can be written as $R = \{11, 12, 13.....\}$ In number line it can be represented as follows.



12.2. Solution of Inequalities in one Variable.

In class VII, you have learnt the following.

Properties of Inequality

1. If $A < B$, then $A + C < B + C$ and $A - C < B - C$
2. If $A < B$ and $C > 0$ then $AC < BC$ and $\frac{A}{C} < \frac{B}{C}$
3. If $A < B$ and $C < 0$ then $AC > BC$ and $\frac{A}{C} > \frac{B}{C}$

***Note :** According to the properties above if both sides of an inequality is multiplied or divided by a negative number, the sign of the inequality is reversed (i. E., Changed to > (or<) from < (or >)).*

Example 1

Solve the following inequalities and represent in a number line.

(a) $-4x \le 36$ (b) $7 - 3x > 10$

(c) $3x + 2(2x-9) \le x$ (d) $\frac{3}{4}x - \frac{1}{2} > \frac{2}{3}x$

Solution :

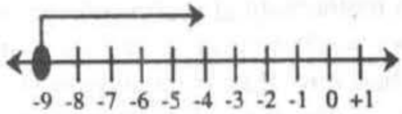
(a) Here, $-4x \le 36$

Or, $\frac{-4x}{-4} \ge \frac{36}{-4}$ [$\therefore$ Sign of inequality is reversed when divided by a negative number]

Or, $\therefore x \ge -9$

In set notation

$x \subset \{-9, -8, -7, \ldots\}$



(b) $7 - 3x > 10$

Or, $7 - 3x - 7 > 10 - 7$ [$\therefore$ Sign of inequality is reversed when divided by a negative quantity]

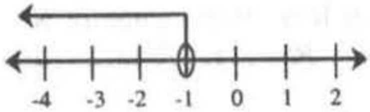
Or, $-3x > 3$

Or, $\frac{-3x}{-3} < \frac{3}{-3}$

$\therefore x < -1$

In set notation, $x \subset \{-1, -2, -3 \ldots\ldots\}$

In number line, it is represented as



(c) Here, $3x + 2(2x - 9) \le x$

Or, $3x + 4x - 18 \le x$ [If the expression contains brackets then brackets should be operated first]

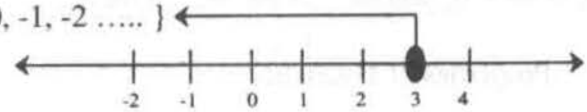
Or, $3x + 4x - x \le 18$

Or, $6x < 18$

$\therefore x < 3.$

In set notation, $x \subset \{3, 2, 1, 0, -1, -2 \ldots\}$

In number line

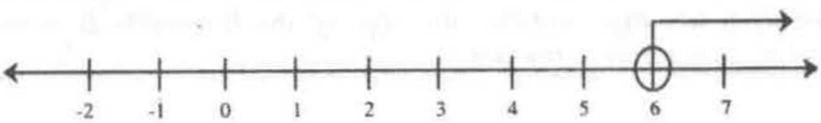


(d) Here $\frac{3}{4}x - \frac{1}{2} > \frac{2}{3}x$ [If the inequality contains rational expressions, simplify it to clear the rational expressions]

Or, $\frac{3}{4}x - \frac{2}{3}x > \frac{1}{2}$

Or, $\frac{9x - 8x}{12} > \frac{1}{2}$ Or, $9x - 8x > ½ \times 12$ Or, $x > 6.$

In set notation, $x \subset \{7,8,9, \ldots\ldots\}$ In number line,



Example 2

Represent the inequality $-2 < x \leq 3$ in set notation and in number line.

Solution :

The inequality $-2<x \leq 3$ should be divided into two in equations as follows.

$-2 < x$ or $x > -2$

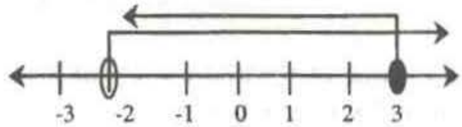
And $x \leq 3$

Here $-2 < x$ can be expressed in set notation as

$x \subset \{-1, 0, 1, 2 \ldots\}$

And $x \leq 3$ can be expressed as: $x \subset \{3,2,1,0\ldots\ldots\}$

In number line it in represented as:



Example 3

A copy which costs Rs. 15 and some pens each worth Rs. 8 are to be purchased. How many pens at most Can be purchased cost for Rs. 100?

Solution :

Let the required number of pens be x.

Total cost of 1 copy and x pens = Rs $(15 + 8x)$.

Since total cost should not exceed Rs. 100, we have,

$15 + 8x \leq 100$

Or, $8x \leq 100-15$

Or, $x \leq \frac{85}{8}$

$\therefore x \leq 10\frac{5}{8}$

As x is the number of pens it is a whole number and the maximum value of x will be 10 . Hence at most 10 pens can be purchased .

Example 4

If in the equation, $y = 3x - 2$, and $x \leq -3$ What will be the value of y?

Solution :

Here, $y = 3x - 2 \leq 3(-3) - 2$ $[\because x \leq -3]$

Or, $y \leq -11$

EXERCISE – 12 (A)

1. Solve the following inequalities and draw the line graph.

(i) $7x - 2(x-3) < 16$	(ii) $3(x-1) - x < 5$
(iii) $x - (4x - 1) < -8$	(iv) $5 - 2(x + 3) > -1$
(v) $2(x + 5) < 4 + 5x$	(vi) $3 + 2(2x - 3) < 3x + 4$
(vii) $\frac{3}{2}x - \frac{8}{3} > \frac{x}{6}$	(viii) $\frac{x-2}{3} > \frac{3x-1}{4}$
(ix) $\frac{x+4}{6} - \frac{x}{3} \geq x - 4$	(x) $3 - \frac{1}{6}x \geq x - \frac{1}{2}$
(xi) $0.3x > 0.8 - 0.1x$	(xii) $0.7 + 0.6 \leq 0.5x + 1$
(xiii) $-3 < x \leq 3$	(xiv) $-4 \leq x \leq -1$

2. If $x > -2$ what is the value of y when $y = 4 - 6x$?
3. Fro the given equation $5x + 7y + 9 = 0$.
   (a) What is the value of y if $x \leq 8$?
   (b) What is the value of x if $y > -7$?
4. Ram ordered 3 packets of biscuits which cost Rs. 12 per packet and some cups of tea each worth Rs 3. If he had Rs.80 only, how many cups of tea at most did he order?
5. When 9 is to be taken out from 5 times of a number the difference is greater than – 3. Find the least integer satisfying this statement.
6. Ball pens each worth Rs 8 and pencils each worth Rs 3 were purchased altogether 15 in number. If you have only Rs. 100, how many ball pens at most can be purchased?
   (a) If the number of ball pens purchased is x, how many pencils can be purchased?
   (b) Using x, what will be the total cost?
   (c) Express the relation by an inequality?
   (d) Solving the inequality find the maximum number of ball pens that can be purchased.

12.3 Graphical Solution of Simultaneous Equations in two Variables.

Shiva gave Rs 9 to Sheela and Kailash saying divide it among you. Now, how much will each get?

For their it Kailash spends twice as much Sheela spends how much do they spend each?

Being based on the statements shares of each are tabulated below. Study the tables.

Table according to the first statement.

Sheela's Share	0	1	2		4	5		7		
Kailash's Share	9	8	7	6			3		1	0

From the table above we notice that the sum of shares of Sheela and Kailash's be x and y then $x + y = 9$

Here, x and y both are the variables hence $x + y = 9$ is called an equation in two variables.

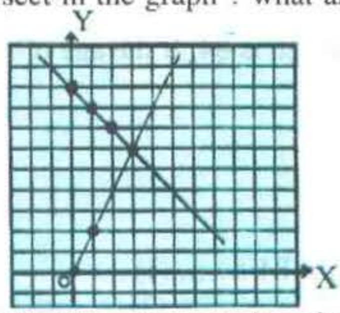
Now table according to second condition is as follows.

Sheela's Share	0	1	2	3		5
Kailash's Share0	0	2		6	8	

Second condition when translated into mathematical language becomes $2x = y$. Why ? Discuss.

If the equations representing both of the statements are expressed in a graph sheet, it looks as shown below.

In which point the lines intersect in the graph ? what are the ordinates of the point of intersection ?



The point of intersection of the lines representing simultaneous equations in two variables in the solution of the equation. Here the solution of the equations $x + y = 9$ and $y = 2x$ is $x =3$ and $y= 6$ or (3,6). Does the point (3,6) satisfy both of the equation? Check them.

Example 1

Solve Graphically : $x + y = 10$, $2x - y = -1$.

Solution :

Here, $x + y = 10$ ....(i)

$2x - y = - 1$ ....(ii)

x	0	3	5	9
y	10	7	5	1

From equation (i) $y = 10 - x$.

Table for equation (a) is

x	0	2	5	-3
y	1	5	11	-5

From equation (2) $y = 2x + 1$.

Table for equation (2) is

Sketching the line is the graph,

From the graph we find point of in trisection of the lines is (3,7).

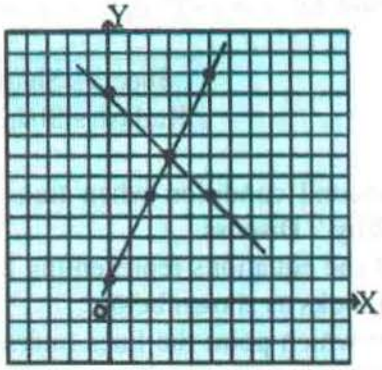
Cheek : $x + y = 10$ $2x + 1 = y$

Or $3 + 7 = 10$ or $2 \times 3 + 1 = 7$

Or $10 = 10$ or $7 = 7$

Since point of intersection (3,7) satisfies both equation

$\therefore$ Required solution is $(x, y) = (3, 7)$



Example 2

Sum of ages of father and his son is 84. If the age of father is thrice of his son's age find their ages.

Solution :

Let age of father be x and

Age of son be y

By the question, $x + y = 84$ ....(1)

and $x = 3y$ ....(2)

Table for eq^n (1) is given by $y = 84 - x$.

x	20	50	60
y	64	34	24

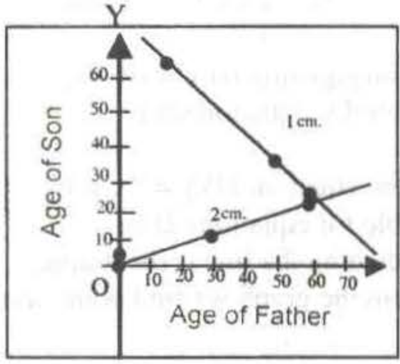
From equation (2) $y = \dfrac{x}{3}$ and the table is

x	0	30	60	45
y	0	10	20	15

Both lives are sketched in the graph shown below.

By observation, point of intersection is

$x = 63$ and $y = 21$.



Therefore father's age

= 63 yrs and son's age = 21 years.

EXERCISE – 12 (C)

1. Solve graphically the following equations and cheek.

   (a) $2x - y = 5$, $x - y = 1$ (d) $3x + y = 7$, $x = 2y$

   (c) $x + y = 2$, $3x - y = 10$ (d) $2x - y = -5$, $x + y = -1$

   (e) $-x - y = -5$, $2x-y = -1$ (f) $3x + y = 8$, $2x + y = 7$

   (g) $4x + y = 2$, $3x - 2y = 7$ (h) $2x + y = 4$, $x + 2y = -1$

   (i) $3x + 2y = 1$, $2x - 3y = 5$ (j) $4x + 2y = 2$, $x - 3y = 11$

   (k) $2x + 2y = 0$, $5x + y = -8$

2. Translate the following word problems into equation and solve graphically.

   (a) Find two numbers whose sum is 21 and the difference is 1.

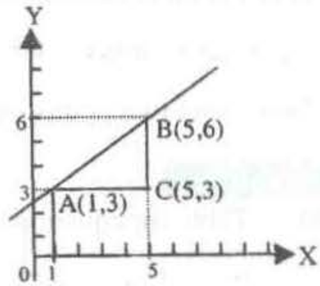
   (b) The difference of two numbers is 5. If one is twice of the other, find the numbers.

   (c) The difference of two numbers is 15. If twice of the greater is equal to the 5 times of the smaller number what are the numbers?

   (d) Age of a father is double of the age of his son. Difference of their ages is 25 years. What are their ages?

(e) Ramesh is 5 years older than Ravi. After 5 years Ramesh's age will be twice of Ravi's present age. Find their present ages.

12. 4 (a) Slope of a Straight Line

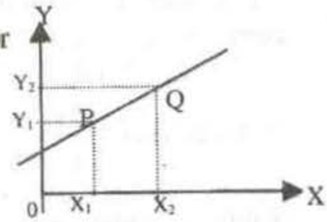


A line passes through A (1,3) and B (5,6).
The line drawn through A parallel to x-axis and the line through B parallel to y-axis intersect at C (5,3)

Here, $AC = 5 - 1 = 4$

$BC = 6 - 3 = 3$

AC is the difference of abscissa (x – co-ordinates) of A and B where BC is the difference of ordinates (y- co-ordinates) of A and B.
This means, line AB has changed its direction (from points A to B) by 4 units along x-axis and 3 units along y-axis.



If a line passes thought P (x_1, y_1) and Q(x_2, y_2), then for PQ, difference of abscissas $= x_2 - x_1$ of ordinates $= y_2 - y_1$. The ratio of the difference of ordinates to the difference of abscissas (taken in order) is known as its slope.
It is denoted by m.

$$\text{Slope of a line} = \frac{\text{difference of ordinates}}{\text{difference of abscissas}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

A line passes through L (6,2) and M (-2,6).

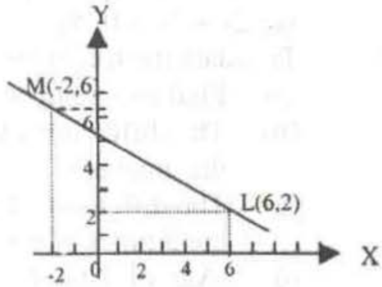
(i) Draw the graph of the line.

(ii) Find the difference of ordinates and abscissas for the line LM.

(iii) Find the slope of LM.

Solution :

(i) The graph is shown in the adjoining figure.



(ii) Here, $(x_1, y_1) = L\ (6,2)$

$(x_2, y_2) = M\ (-2,6)$

For LM difference of ordinates

$y_2 - y_1 = 6 - 2 = 4$

Difference of abscissas

$x_2 - x_1 = -2 - 6 = -8$

(iii) Slope of LM $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4}{-8} = \dfrac{-1}{2}$

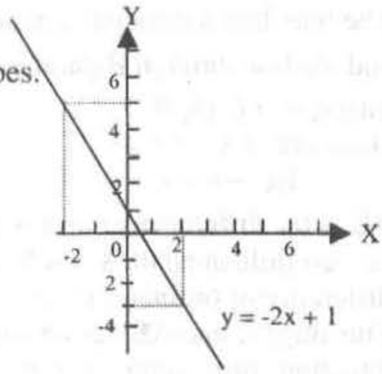
Note : Graph of a straight line can be sketched with the help of two points.

Example 2

Make the table showing the relationship between x and y for the equations

(i) $y = -2x + 1$ and (ii) $y = \dfrac{1}{2}x - 2$ and draw the graph.

Choose any two points on the lines and find their slopes.



Solution :

(i) Table for equation $y = -2x + 1$

x	-2	-1	0	1	2	.....
y	5	3	1	-1	-3	.....

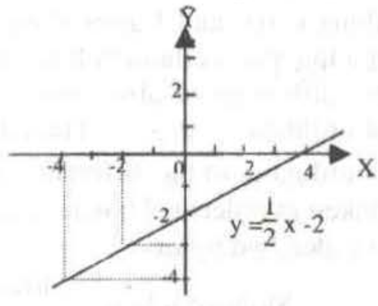
Graph of this line is shown in the right.

(-1, 3) and (2, -3) on the line are considered.

Here, $(x_1, y_1) = (-1, 3)$

$(x_2, y_2) = (2, -3)$

$\therefore$ Slope (m) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6}{3} = -2$



(ii) Table for equation $y = \dfrac{1}{2}x - 2$

x	-4	-2	0	2	4	.....
y	-4	-3	-2	1	0	.....

Graph of this line is shown in the right side.
Consider the points (-4, -4) and (0, -2) on the line.

Here, $(x_1, y_1) = (-4, -4)$
$(x_2, y_2) = (0, -2)$

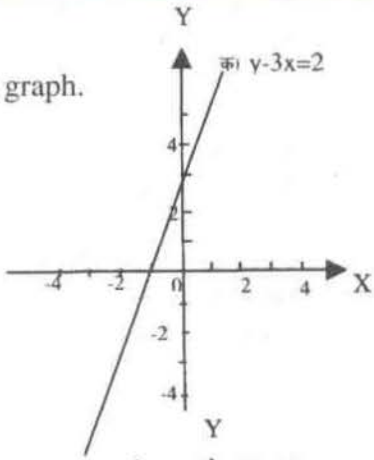
$\therefore$ Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-(-4)}{0-(-4)} = \dfrac{2}{4} = \dfrac{1}{2}$

In the example above let us find the slope of the line considering other two points for, (i) consider the points (-2, 5) and (0, 1) so that slope (m) = $\dfrac{1-5}{0-(-2)} = \dfrac{-4}{2} = -2$ (which is same as above) For (ii) Consider the points (-2,-3) and (4,0) so that m = $\dfrac{0-(-3)}{4-(-2)} = \dfrac{3}{6} = \dfrac{1}{2}$ (which is same as above).

Here, for line (i) the slope is always – 2 and for line (ii) the slope is always $\dfrac{1}{2}$

Thus, the slope of a line will be the same whichever the points we consider.
Therefore,

Slope of a line is always the same for any two points. The slope of line y = mx + c is m.



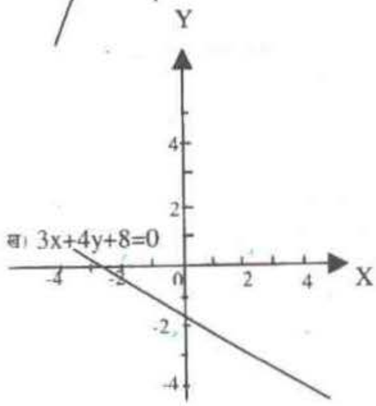
Example 3

Find the slope of each of the line below and draw the graph.
(i) $y - 3x = 2$ (ii) $3x + 4y + 8 = 0$

Solution :

(i) Here $y - 3x = 2$
Or, $y = 3x + 2$ $\therefore$ Slope = 3
[The equation when compared to $y = mx + c$ gives $m = 3$]

(ii) Here $3x + 4y + 8 = 0$
Or, $4y = - 3x - 8$
Or, $y = -\dfrac{3}{4}x - \dfrac{8}{4}$
$\therefore$ Slope (m) = $-\dfrac{3}{4}$



The graph of the lines are shown below.
(lines are sketched as in example 2)

x	0	4
y	2	-5

In the example (i) slope is positive and the line is inclined to the right whereas in (ii) slope is negative and the line is inclined to the left.

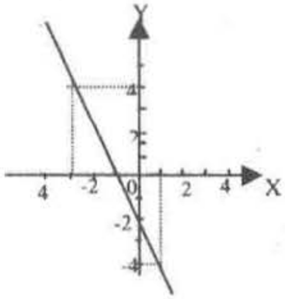
Therefore, the relation between the slope and position of the line can be traced as:

Slope of the Line	The Graph
Positive	Right UP
Negative	Left Up

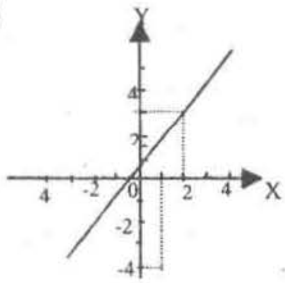
EXERCISE – 12 (C)

1. Find the slope of the lines below.

(a)



(b)



2. Find the slope of the line passing through the following two points.

(i) A (3, 9), B (6,3) (ii) C (9, 8), D (7, 2)

(iii) E (-3, -1), F(-1,1) (iv) K(1, -2), L(3, -6)

(v) M (-8, -1), N (-5,6) (vi) U (0,0), V(3-2)

(vii) W (0,6), X (7,0) (viii) P (3,1), Q (5,1)

3. Sketch the graph of the lines in Q.N. 2, identifying whether it is inclined to right up or left up.

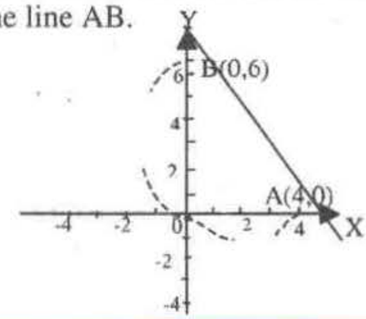
4. Find the slope of the lines below and draw the graph.

(a) $y = 3x + 6$ (b) $y = -2x - 3$

(c) $x + y = 1$ (d) $4y - 2x = 8$

12.4 (b) Intercepts of a Straight Line

In the adjoining graph, line AB cuts x – axis at the point A (4,0) so that OA = 4 units and y – axis at B (0, 6) so that OB = 6 units. Here OA and OB are respectively known as x – intercept and y – intercept of the line AB.



Hence,
For AB, x intercept = 4
and y intercept = 6.

Example 1

Find x – intercept and y- intercept the line $y = 2x + 4$ and draw its graph.

Solution :

Here, we know, the ordinate of any point on x – axis is o and the abscissa of any point on y – axis is o. Hence, if we put $y = 0$ in the equation of a line, we get x – intercept and if we put $x = 0$, we get y – intercept. Now putting $y = 0$ in the equation $y = 2x + 4$

We get $0 = 2x + 4$

Or, $x = -2$

x-Intercept = - 2

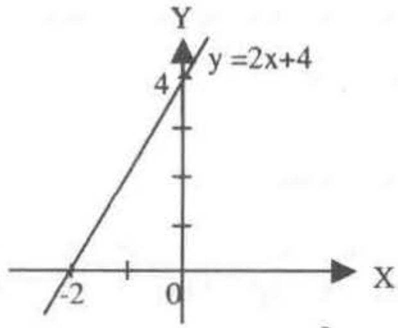
Similarly, putting $x = 0$

In the equation

$y = 2x + 4$

We get y-intercept=4, $y = 2.0 + 4 = 4$

The graph is shown in the right side.



Example 2

Sketch the graph of $3x + 2y = 6$ and find the intercepts of the line.

Solution :

Here the given equation is $3x + 2y = 6$.

Putting $y = 0$ in this equation

We get $3x + 2.0 = 6$

Or, $3x = 6$, or $x = 2$

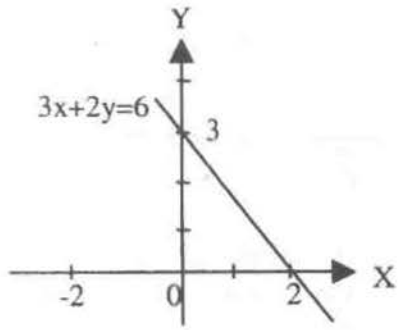
$\therefore$ x- Intercept = 2

Similarly, putting $x = 0$ in the Equation, we get

$3.0 + 2y = 6$, or $y = 3$

$\therefore$ y – intercept = 3

The graph is shown in the right side.



> ***Note :** To find the intercepts of a line $ax + by + c = 0$, we should put $y = 0$ (to find x - intercept) and $x = 0$ (to find y - intercept) respectively in the equation.*

Example 3

Find at which points the line $y = -\dfrac{3}{2}x - 6$ cut the x-axis and draw the graph of the line.

Solution :

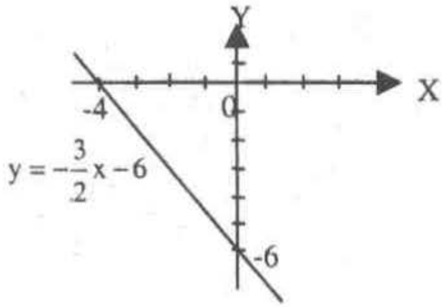
Here, given line is : $y = -\frac{3}{2}x - 6$

Putting y = 0 in the equation,

$$0 = -\frac{3}{2}x - 6$$

Or, x = -4

$\therefore$ x intercept = - 4



Similarly, putting x = 0 in the equation,

$$y = -\frac{3}{2}.0 - 6$$

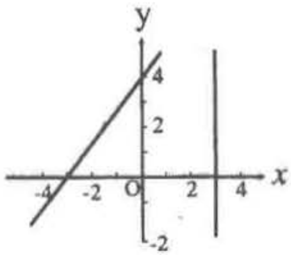
$\therefore$ y = - 6

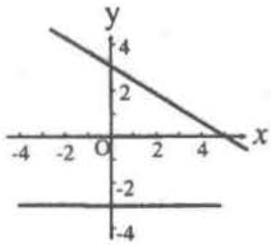
$\therefore$ y – intercept = - 6

Hence, the line cuts x – axis at (-4,0) and y – axis at (0, -6). Plotting and joining these points, the graph of the line looks as shown above.

EXERCISE – 12 [D]

1. Find x – intercept and y – intercept of each line.





2. Find x – intercept for each of the lines represented by the equations.
   (i) $y = 2x + 4$ (ii) $y = -x - 3$
   (iii) $y + x - 2 = 0$ (iv) $3x + 2y - 6 = 0$
3. Find y – intercept for each of the lines.
   (i) $y = 3x + 1$ (ii) $y = -5x - 6$
   (iii) $x - y + 4 = 0$ (iv) $2x - 3y - 4 = 0$

4. Find the points at which each of the line cuts x and y axes and draw the graph plotting the points.
   (i) $y = x + 2$ (ii) $y = -3x + 6$
   (iii) $y = \frac{1}{2}x + 3$ (iv) $y = -\frac{4}{5}x - 8$
   (v) $2x + y = 4$ (vi) $3x - 2y + 12 = 0$

12.5 Solution of Quadratic Equation by Factorization.

Equation in one variable of degree 2 is known as a quadratic equation for example.

(i) $x^2 + 2x^{+1} = 0$ (ii) $2x^2 - 5x + 2 = 0$

(iii) $x^2 = 36$ (iv) $7x^2 - 175 = 0$

A quadratic equation which contains the term having the power both 1 and 2 of the variable is known as mixed quadratic equation or affected quadratic equation. In the examples above, equations (i) and (ii) are mixed quadratic equation.

The quadratic equation which does not contain the first degree term is known as pure quadratic equation, equations (iii) and (iv) are pure quadratic equations in the examples above.

To find the value of the variable in the equation is to solve the equation In a linear equation of one variable, value of the variable is only one but in quadratic equation (of one variable) the variable has two values. These values are known as the root of the equation.

Here we will learn to solve a quadratic equation by factorization. Study the examples below.

Example 1

Solve: $x^2 - 5x + 6 = 0$,

Solution :

Here, $x^2 - 5x + 6 = 0$

Or, $x^2 - (3+2)x + 6 = 0$

Or, $x^2 - 3x - 2x + 6 = 0$, or $x(x-3) - 2(x-3) - 0$

Or, $(x - 3)(x-2) = 0$

If $x - 3 = 0$, then $x = 3$. If $x - 2 = 0$ then $x = 2$

Therefore $x = 2$ or 3.

> ***Note :** Find the value of x if $xy = 0$ then the product of x and y is 0. Hence either x or y or both should be equal to zero. if $x = 0$ or $y = 0$*

Example 2

Find the value of x if $9x^2 - 64 = 0$.

Solution :

Here, $9x^2 - 64 = 0$ or $(3x)^2 - 8^2 = 0$

Or, $(3x + 8)(3x - 8) = 0$

If $3x + 8 = 0$ then $3x = -8$, $\therefore x = \frac{-8}{3}$, if $3x - 8 = 0$ then $3x = 8$ $\therefore x = \frac{8}{3}$

Therefore, $x = \pm\frac{8}{3}$ or $\pm 2\frac{2}{3}$

EXERCISE 12 [E]

1. Solve the following quadratic equations.

(a) $x^2 + x = 0$ (b) $x^2 - x = 0$

(c) $x^2 + 3x + 2 = 0$ (d) $x^2 - x - 2 = 0$

(e) $x^2 + x - 2 = 0$ (f) $x^2 - 7x + 12 = 0$

(g) $x^2 + 3x - 4 = 0$ (h) $x^2 - 3x - 4 = 0$

(i) $x^2 - 9 = 0$ (j) $x^2 - x - 12 = 0$

(k) $x^2 - x - 20 = 0$ (l) $x^2 - 7x + 12 = 0$

(m) $x^2 + 3x - 10 = 0$ (n) $x^2 - 9x + 18 = 0$

(o) $x^2 - x - 30 = 0$ (p) $x^2 - 2x - 3 = 0$

(q) $x^2 + 8x + 16 = 0$ (r) $x^2 - 4x + 4 = 0$

(s) $x^2 - 25 = 0$ (t) $x^2 + 10x + 25 = 0$

(u) $x^2 - 36 = 0$ (v) $x^2 - 12x + 36 = 0$

2. Find the value of x in the following equations.

(a) $2x^2 + 3x = 0$ (b) $2x^2 - 3x = 0$

(c) $3x^2 + 2x - 1 = 0$ (d) $2x^2 + 3x - 1 = 0$

(e) $3x^2 + 5x - 2 = 0$ (f) $3x^2 - 8x - 3 = 0$

(g) $3x^2 - 5x + 2 = 0$ (h) $3x^2 - 4x - 4 = 0$

(i) $3x^2 + 8x - 3 = 0$ (j) $2x^2 - x - 3 = 0$

(k) $2x^2 - 5x + 3 = 0$ (l) $9x^2 + 9x + 2 = 0$

(m) $9x^2 - 4 = 0$ (n) $16x^2 - 1 = 0$

(o) $16x^2 - 8x + 1 = 0$ (p) $16x^2 + 1 + 8x = 0$

(q) $16x^2 - 9 = 0$ (r) $16x^2 - 24x + 9 = 0$

(s) $6x^2 + 7x + 2 = 0$ (t) $6x^2 + x - 2 = 0$

(u) $6x^2 - 7x + 2 = 0$ (v) $10x^2 - 7x + 1 = 0$

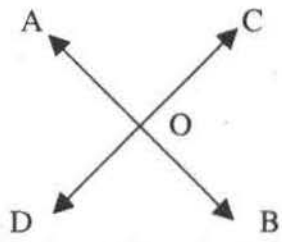
UNIT 13

ANGLES AND PARALLEL LINES

13.1. Angles

1. Vertically Opposite Angles

In the figure lines AB and CD intersect at the Point O.
Now, use your protractor and complete the table below.



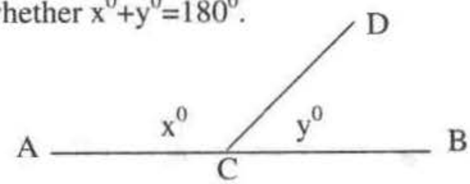
Angle	$\angle AOD$	$\angle COB$	$\angle AOC$	$\angle BOD$
Measurement				

Is $\angle AOD$ equal to $\angle COB$? Is $\angle AOC$ equal to $\angle BOD$?
What do you conclude from this experiment ?

Conclusion : If two straight lines intersect at a point then vertically opposite angles are equal.

2. Adjacent Angles

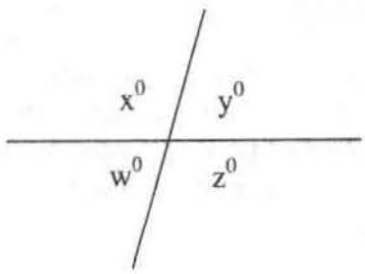
In the figure, line segment CD is joined to an interior point of line segment AB to form the adjacent angles which measure x^0 and y^0.
Measure x^0 and y^0 and determine whether $x^0+y^0=180^0$.



Conclusion : Sum of the pair of adjacent angles formed when a straight line cuts another line is 180^0.

In the figure, is

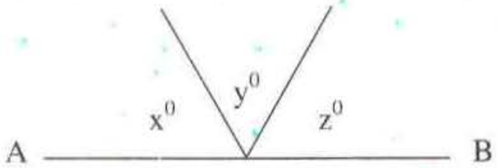
x^0+y^0 equal to 180^0 ?
y^0+z^0 equal to 180^0 ?
w^0+z^0 equal to 180^0 ?
x^0+w^0 equal to 180^0 ?
Measure the angles and determine.



3. Straight Angle

In the figure, x^0, y^0 and z^0 are the adjacent angles formed on the same side (upward) of AB. Measure the angles x^0, y^0 and z^0.

Is $x^0 + y^0 + z^0 = 180^0$?



Conclusion : Sum of angles formed at any point and on the same side of a line is 180^0.

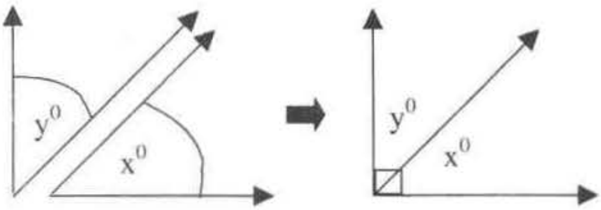
4. Complementary Angles

If sum of two angles is 90^0 or one right angle then each of the angles are known as complementary angles. 60^0 and 30^0 are complements of each other.

In the figure,

$x^0 + y^0 = 90^0$.

Therefore, x and y are the complements of each other.



5. Supplementary Angles

If sum of two angles is 180^0 then the angles are called supplementary angles. 60^0 and 120^0 are supplements of each other. Similarly, 100^0 and 80^0 are supplements of each other.

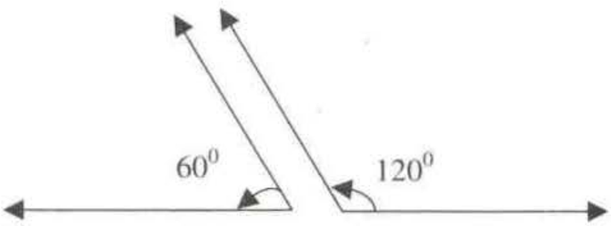


Figure (a)

If the supplementary angles lie on a same straight line then they are adjacent angles. In the figure (a) above, if the angles of 60^0 and 120^0 are constructed on a straight line, the figure looks like as in figure (b). In this condition the angles of 60^0 and 120^0 are adjacent angles.

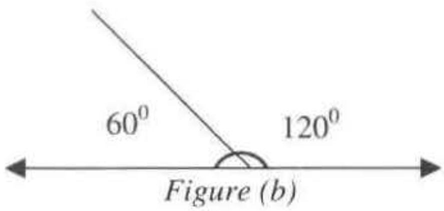
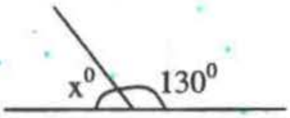
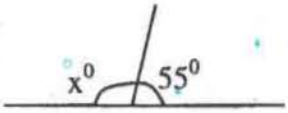


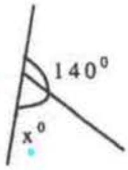
Figure (b)

EXERCISE 13 [A]

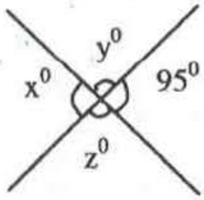
1. Find the value of x in each of the following.

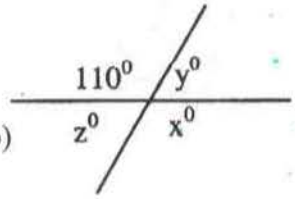
(a) 

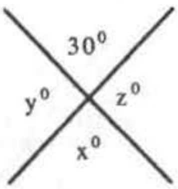
(b) 

(c) 

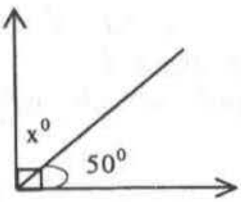
2. Find the value of x^0, y^0 and z^0 on the basis of the figures given below.

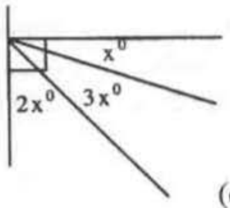
(a) 

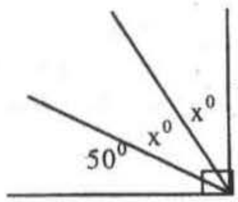
(b) 

(c) 

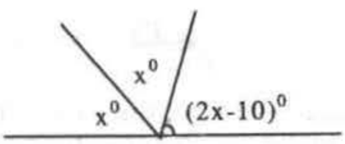
3. Determine the value of x from the following figures.

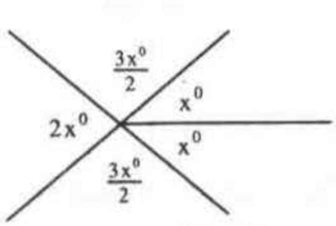
(a) 

(b) 

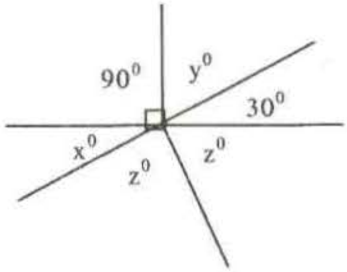
(c) 

4. Calculate the value of x^0, y^0, z^0, a^0, b^0 and c^0 from the following.

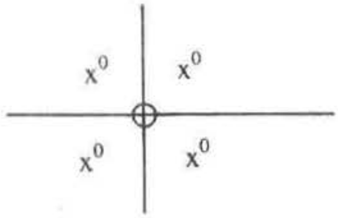
(a) 

(b) 

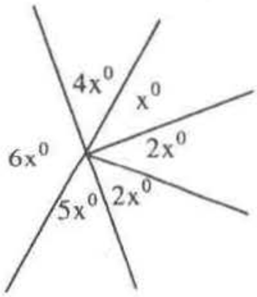
(c)



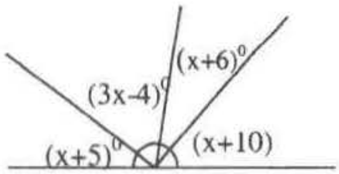
(d)



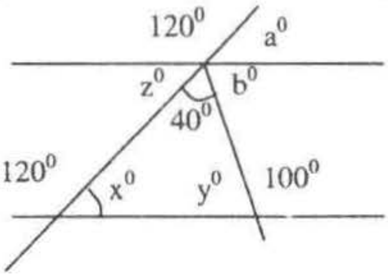
(e)



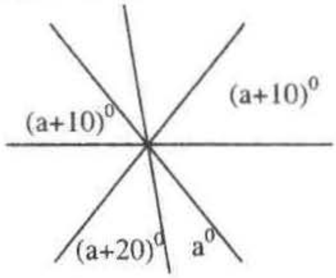
(f)



(g)



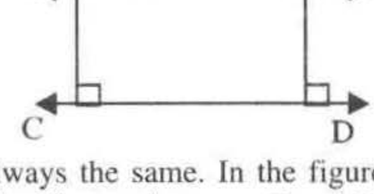
(h)



5. Determine the complements and supplements of the angles 30^0, 40^0, 45^0 and 80^0.
6. If $9x^0$ and $6x^0$ are complements of each other, find the angles.
7. Find the value of x if $(2x + 20)^0$ and $(3x - 40)^0$ form a straight angle.

13.2. [A] Parallel Lines

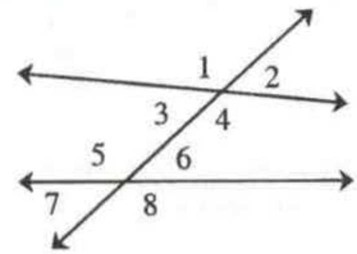
1. Two straight lines are known as parallel lines if they do not intersect or meet how far produced on either sides lines AB and CD shown in the adjoining figure are parallel. It is written as AB||CD.



2. The distance between two parallel lines is always the same. In the figure above, distance between A and C is equal to the distance between B and D, i.e., AC = BD.

13.2. [B] Angles formed by a transversal with the parallel lines

In the figure, angles formed by a transversal, when it intersects a pair of lines are denoted by 1, 2, 3, 4, 5, 6, 7 and 8. Among these, angles 1, 2, 7 and 8 which lie outside the pair-lines are known as exterior angles and the angles 3, 4, 5 and 6 which lie inside the pair lines are called interior angles.



(i) Alternate Angles

When a transversal cuts a pair of lines then the interior non-adjacent angles lying on the either side of the transversal are known as alternate angles. In the figure above, angles 4 and 5, 3 and 6 are alternate angles.

(ii) Co-interior Angles

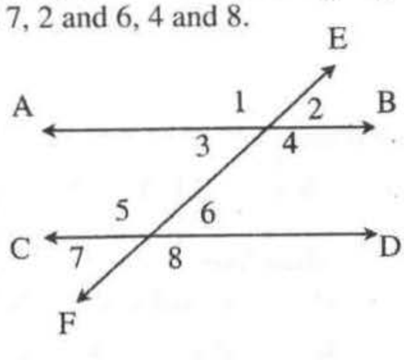
When a transversal line cuts a pair of lines then the interior angles on the same side of the transversal are called co-interior or consecutive interior angles. Angles 3 and 5, 4 and 6 in the figure above are co-interior angles.

(iii) Corresponding Angles

If a pair of lines is intersected by a transversal line, then the pair of non-adjacent interior and exterior angles formed on the same side of it are called corresponding angles. In the figure above, a pair of corresponding angles are 1 and 5. Similarly other pairs are 3 and 7, 2 and 6, 4 and 8.

Activity 1

Using set-squares draw a pair of parallel lines AB and CD and intersect them by a transversal EF. Name the angles as 1, 2, 3, 4, 5, 6, 7 and 8 as shown in the adjoining figure. Measure the angles using a protractor and complete the table below.



		Corresponding angles	Alternate angles	Consecutive interior angles
$\angle 1 = \ldots$	$\angle 5 = .$	$\angle 1 = \angle 5 = \ldots.$	$\angle 3 = \angle 6 = \ldots.$	$\angle 3 + \angle 5 = \ldots.$
$\angle 2 = \ldots$	$\angle 6 = \ldots$	$\angle 3 = \angle 7 = \ldots.$		
$\angle 3 = \ldots$	$\angle 7 = \ldots$	$\angle 2 = \angle 6 = \ldots.$	$\angle 4 = \angle 5 = \ldots.$	$\angle 4 + \angle 6 = \ldots.$
$\angle 4 = \ldots$	$\angle 8 = \ldots$	$\angle 4 = \angle 8 = \ldots.$		

What conclusions can you draw from the table above ? Write down.

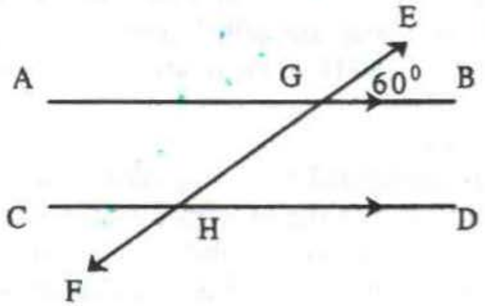
Conclusion : When parallel lines are cut by a transversal,
then the
(i) Corresponding angles are equal.
(ii) Alternate angles are equal.
(iii) Sum of the co-interior angles is 180^0.

Condition of Lines being Parallel :
Two lines, when cut by a transversal, are parallel if any one of the following conditions hold :

(i) If corresponding angles are equal.
(ii) If alternate angles are equal.
(iii) If sum of the co-interior angles is 180^0.

Example 1

In the adjoining figure, AB||CD and EF is a transversal line. If $\angle EGB = 60^0$, find the measure of all remaining angles.



Solution:

Here, Given : $\angle EGB = 60^0$

Therefore,

(i) $\angle EGA = 180^0 - 60^0 = 120^0$ - (straight angle)

(ii) $\angle AGH = 60^0$ - (vertically opposite to $\angle EGB$)

(iii) $\angle EGB = 120^0$ - (vertically opposite to $\angle EGA$)

(iv) $\angle GHD = \angle AGH = 60^0$ (Being alternate angles)

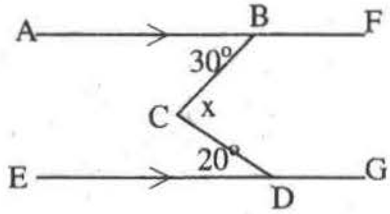
(v) $\angle DHF = 120^0$ - (corresponding to $\angle BGH$)

(vi) $\angle CHF = \angle AGH = 60^0$ - (corresponding angles)

(vii) $\angle CHG = \angle BGH = 120^0$ - (corresponding angles)

Example 2

Find the value of x in the given figure.



Solution:

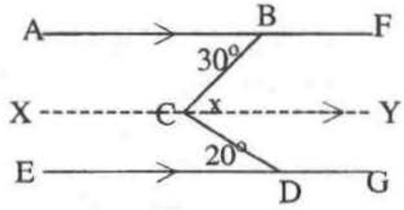
Construct : We draw a line through C parallel to AF or EG.

Now, $\angle BCY = \angle ABC$ ($\therefore$ AF||XY and hence alternate angles are equal)

$\therefore \angle BCY = 30^0$

$\angle YCD = \angle CDE$ ($\therefore$ XY||EG and hence alternate angles are equal)

$\therefore \angle YCD = 20^0$



Now, $x = \angle BCY + \angle YCD$

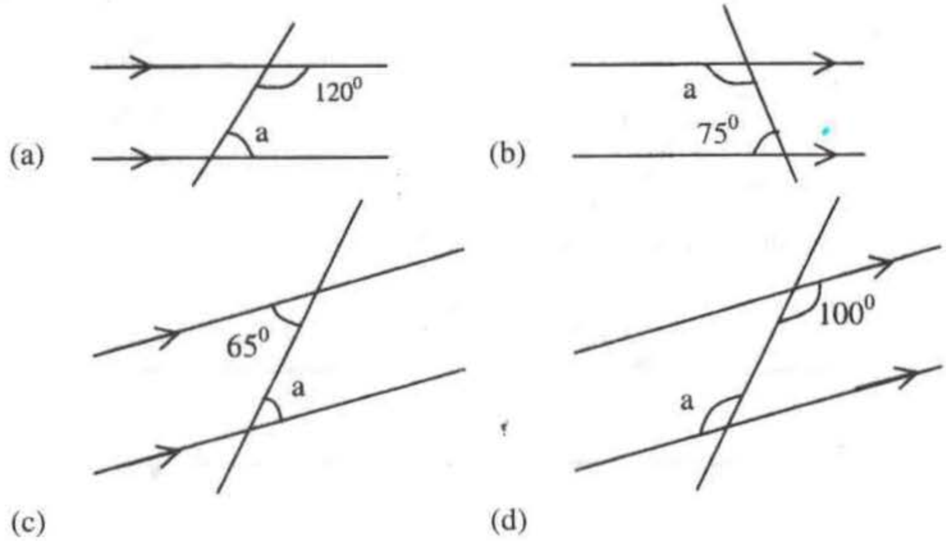
$= 30^0 + 20^0 = 50^0$

$\therefore x = 50^0$

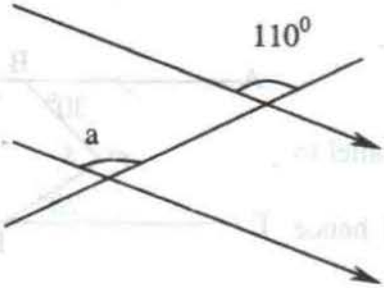
Note:- The problem above can also be solved by producing BC to meet EG or DC to meet AF.

EXERCISE 13 [B]

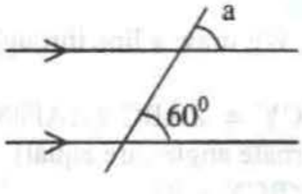
1. In each of the following figures, two parallel lines are cut by a transversal. Determine the value of the unknown angle a.



(e)

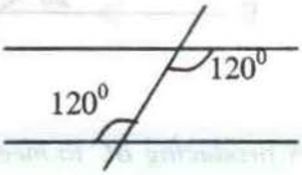


(f)

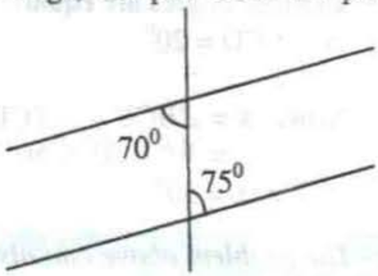


2. Determine whether in each of the followings, the pair lines are parallel. Give reason.

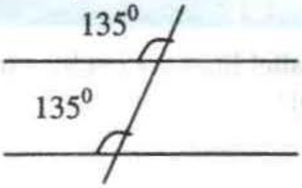
(a)



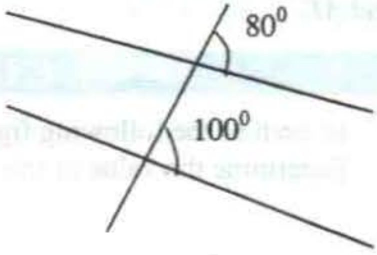
(b)



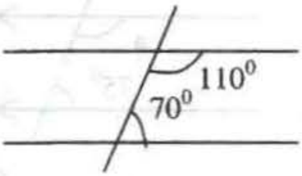
(c)



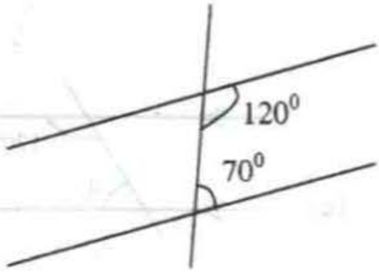
(d)



(e)

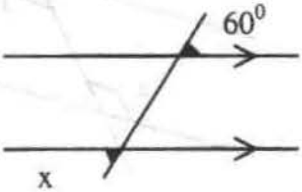


(f)

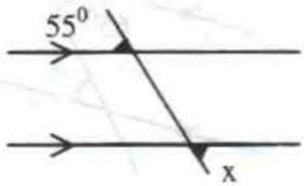


3. (i) Find the value of x in each of the following.

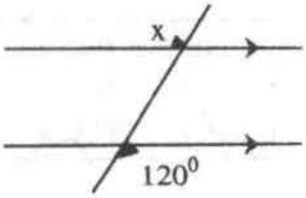
(a)



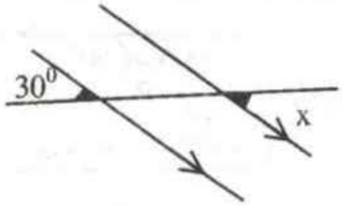
(b)



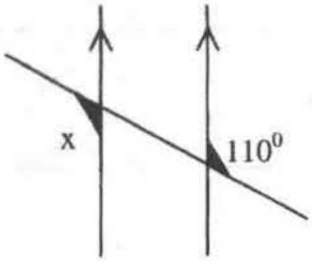
(c)



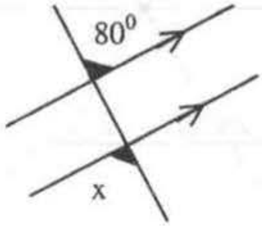
(d)



(e)



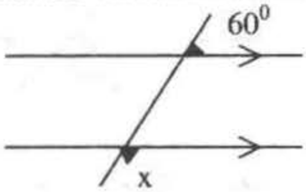
(f)



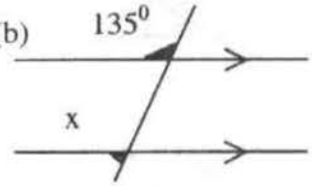
(ii) In the figure above, if the marked angles are known as exterior alternate angles write a statement that establishes a relation between them.

4. (i) Find the value of x in each of the following.

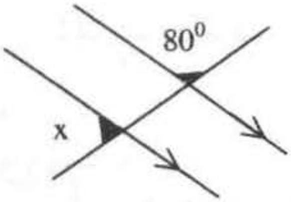
(a)



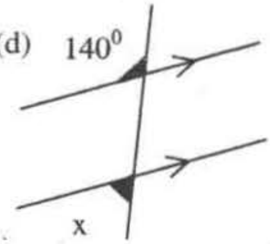
(b)



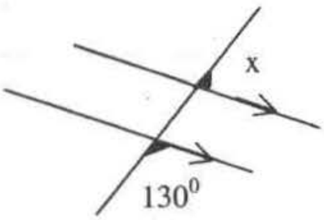
(c)



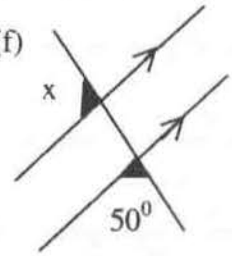
(d)



(e)



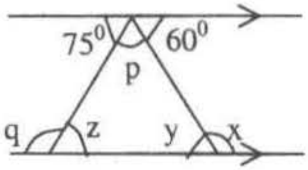
(f)

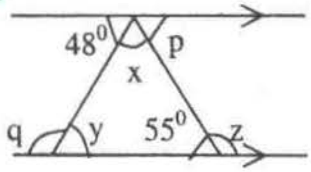


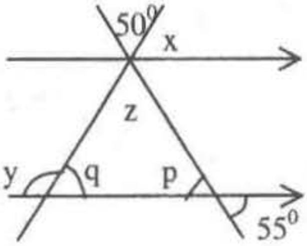
(ii) What is the sum of x and the given angle ?

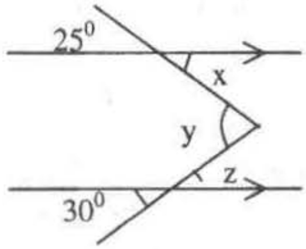
(iii) If the marked angles are called co-exterior angles, write a sentence that states the relationship between them.

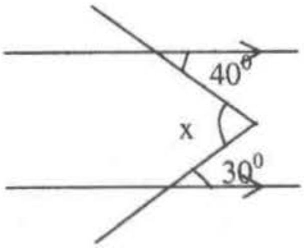
5. Find the value of p, q, x, y and z in each of the following cases.

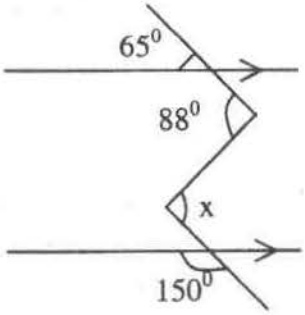
(a)


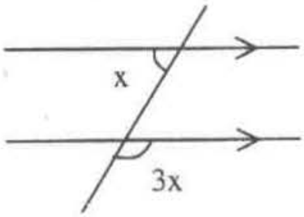
(b)


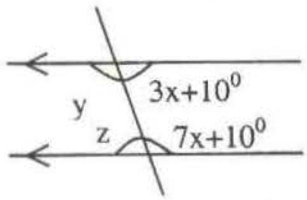
(c)


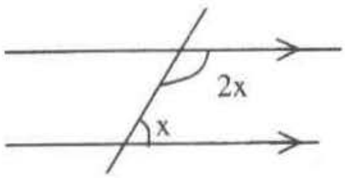
(d)


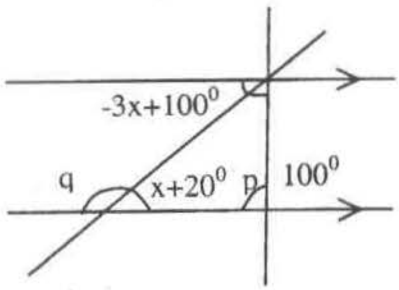
(e)


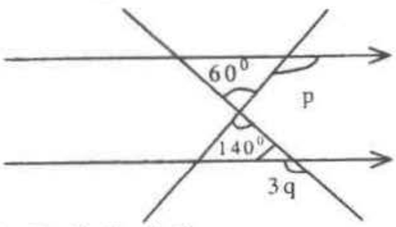
(f)


(f)


(g)


(h)


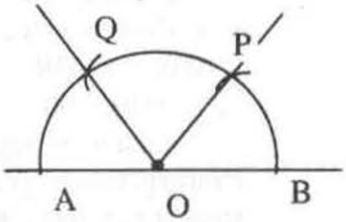
(i)


(j)


13.3. Construction of Angles

You have learnt to construct certain angles in the previous class. You will learn some more about the construction of angles in this class.

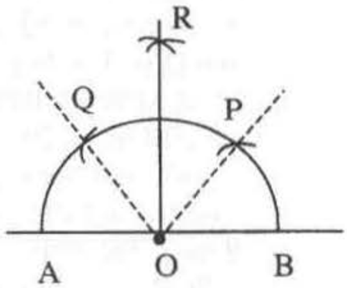
(a) Construction of angle measuring 75^0. Construct according to the steps given below:



STEP-1:

At a point O on a line, draw a semicircle with arc equal to radius OA. Taking the same arc, from B cutoff and arc at P on the semicircle and from P cutoff an arc at Q. Join O, P and O, Q so that

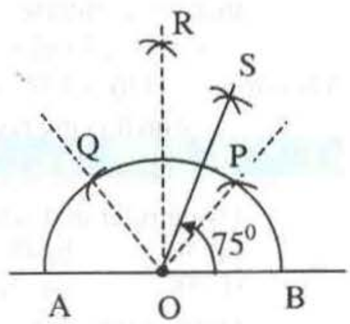
$\angle POB = \angle POQ = \angle QOA = 60^0$.



STEP-2:

From P, cutoff an arc at R and from Q cutoff at R the same arc.

Joint R, O so that $\angle BOR = \angle ROA = 90^0$.



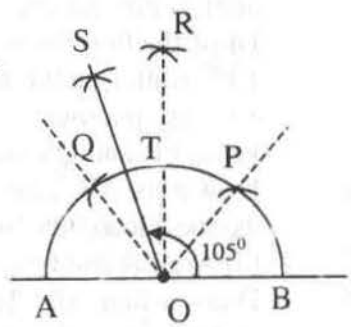
STEP-3:

Proceeding as in step-2, draw the bisector SO of $\angle ROP = 30^0$.

Therefore, $\angle ROS = \angle SOP = 15^0$

and $\angle BOS = \angle BOP + \angle POS$
$= 60^0 + 15^0$
$= 75^0$

(b) Construction of angle measuring 105^0.
Repeat the steps 1 and 2 above.
In order to bisect $\angle ROQ$, cutoff the same arc from Q and T so that they intersect at S, so that $\angle SOQ = \angle ROS = 15^0$.



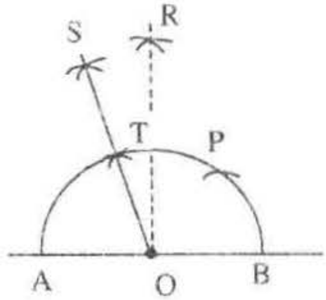
Therefore,

$\angle BOS = 90^0 + 15^0 = 105^0$
which is the required angle.

(c) Construction of angles measuring 135^0.

Repeat the steps 1 and 2 above.

To bisect $\angle ROA$, taking an arc more than half of AT cutoff the same arcs from A and T so that they intersect at S. Join S, O so that $\angle AOS = \angle SOR = 45^0$.

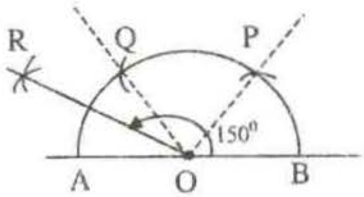


Therefore, $\angle SOB = 90^0 + 45^0 = 135^0$ which is the required angle.

(d) Construction of angles of measuring 150^0.

Repeat the step 1 above.

To bisect $\angle AOQ$, cutoff the same arcs from A and Q so that they intersect at R Join R, O.

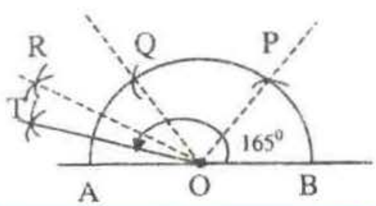


Now, $\angle AOR = \angle ROQ = 30^0$

Therefore, $\angle BOR = 120^0 + 30^0 = 150^0$ which is the required angle.

(e) Construction of angle measuring 165^0.

Repeat the steps in 7.2. above and construct $\angle AOR = \angle ROQ = 30^0$ above. To bisect $\angle AOR$, cutoff the same arc from A and S so that they intersect at T. Join T, O so that $\angle AOT = \angle TOR = 15^0$.



Therefore, $\angle TOB = 150^0 + 15^0 = 165^0$ which is the required angle.

EXERCISE 13 [C]

1. Using ruler and compass, construct the following angles.
   (a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0 (e) 120^0
   (f) 75^0 (g) 105^0 (h) 135^0 (i) 150^0 (j) 165^0
2. Draw a line AB, mark two points P and Q on AB. Construct 75^0 angle on the right side of P and Q and above AB. Draw lines PR and QS, making 75^0 angles. PR and QS are parallel, why ?
3. Draw the line AB twice separately. Take two points P and Q on it. Construct 135^0 angles on the left of P and Q and above AB. Lines PR and QS making 135^0 are parallel. Give reason. Construct 105^0 angle on other line similarly. Lines PR and QS making 105^0 are parallel. Give reason.
4. Draw a line AB. Take two points P and Q on it and draw 90^0 angles at P and Q. Name the lines PR and QS. Now answer the following on the basis of your construction.
   (a) Are PR and QS perpendicular to AB ? (b) Are PR and QS parallel ?
5. Draw a line AB. Take two points on it at P, on the left, draw a line PR making an angle of 150^0 and at Q, on the right, draw a line QS making a 150^0 angle. Is PR parallel to QS ? If not, why ?

UNIT 14

TRIANGLE

Activity 1

a) Draw a triangle ABC on a sheet of paper as shown in the figure and cut it by the scissors.

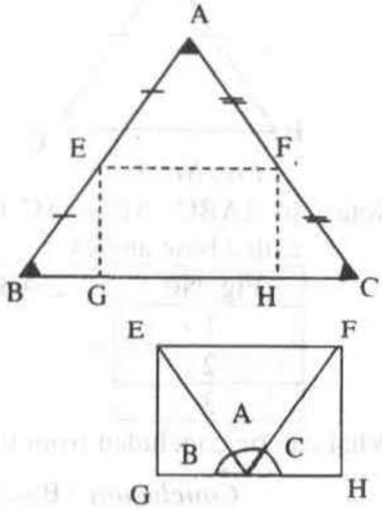
b) Fold back the triangle along the dot line EF as shown in the figure.

c) Fold back also along the dot lines EG and FH.

d) Now your triangular piece of paper has reduced to quadrilateral EFGH. Is it ?

e) Thus ∠A, ∠B and ∠C are now combined without overlapping and have formed a straight angle.

What can you conclude from this experiment ?



Conclusion : Sum of the three angles of a triangle is 180^0.

Verification of sides and angles of triangles

Experiment 1 - Verification of sum of angles of a triangle.

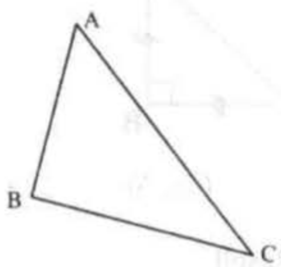


Fig. No. 1

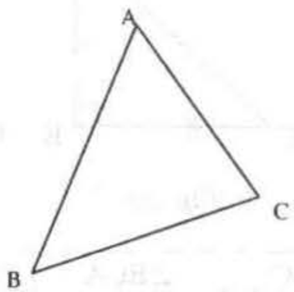


Fig. No. 2

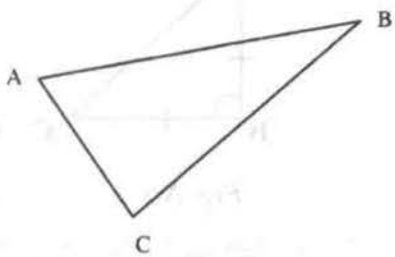


Fig. No. 3

Draw 3 different triangles as shown above, in your exercise book. Measure the angles ABC, BCA and CAB using protractor and complete the table below.

Fig.No.	∠BAC	∠ABC	∠ACB	∠BAC+∠ABC+∠ACB
1				
2				
3				

What did you conclude from the table above ? Does your conclusion match with the following statement.

Sum of Inner angles of a triangle is 180°

Experiment 2 - Verification of base angles of an isosceles triangle.

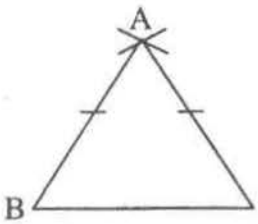


Fig. No. 1

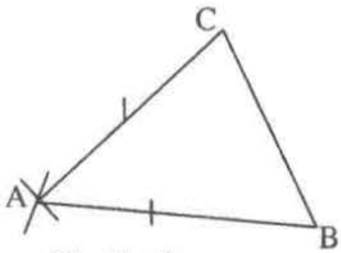


Fig. No. 2

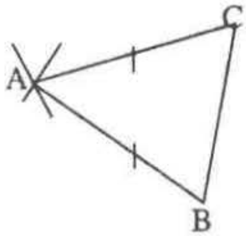


Fig. No. 3

Note: In ΔABC, $AB = AC$ hence angles B and C opposite to AB and AC are called base angles.

Fig. No.	$\angle ABC$	$\angle ACB$	Result
1			
2			
3			

What can be concluded from the observations above ?

Conclusion : Base angles of an isosceles triangle are equal.

Experiment 3 - Verification of base angles of an isosceles right triangle.

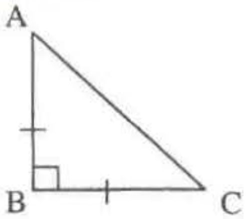


Fig. No. 1

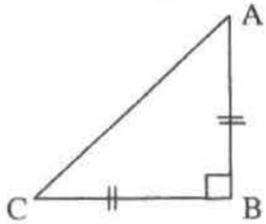


Fig. No. 2

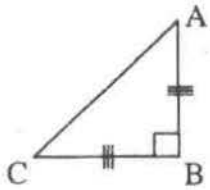


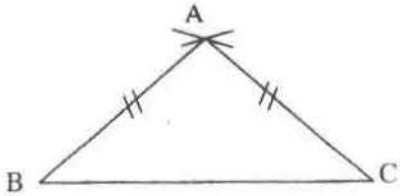
Fig. No. 3

Fig.No.	$\angle BAC$	$\angle BCA$	Result
1			
2			
3			

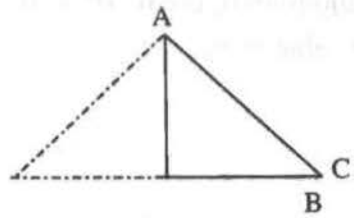
Conclusion : Each of the base angles of an isosceles right triangle is 45^0.

Activity 2

1. Construct an isosceles ΔABC as shown in the adjoining figure. Cut out the triangle by the scissors.



2. Fold back the triangle through A so that B and C coincide.

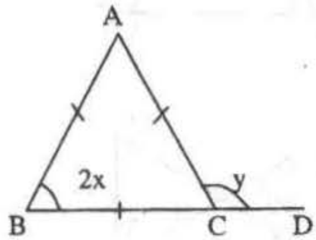


3. Are $\angle B$ and $\angle C$ equal ?
What can you conclude from this experiment ?

Conclusion : Base angles of an isosceles triangle are equal.

Example 1

Find the values of x and y from the adjoining figure.



Solution :

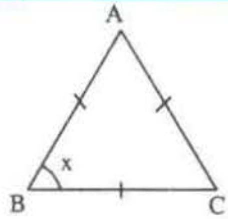
Here,

1. $\angle ABC + \angle BAC + \angle ACB = 180^0$ (Sum of angles of a triangle is 180^0)
or, $2x + 2x + 2x = 180^0$ (Angles of an equilateral triangle are equal)
or, $6x = 180^0$
$\therefore x = 30^0$

2. $\angle ACB + \angle ACD = 180^0$ (Straight angle)
or, $2x + y = 180^0$
or, $2 \times 30^0 + y = 180^0$
or, $60^0 + y = 180^0$
or, $y = 180^0 - 60^0$
$\therefore y = 120^0$.

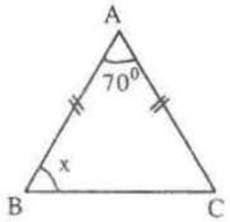
EXERCISE 14

1. In the adjoining figure if AB = BC = CA, find the value of x.

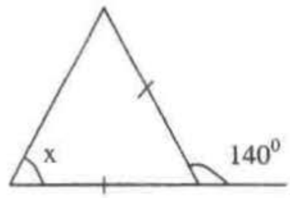


2. Find the values of x and y from the following figures.

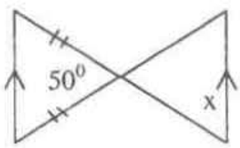
(a)



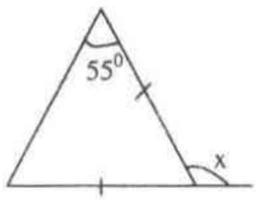
(b)



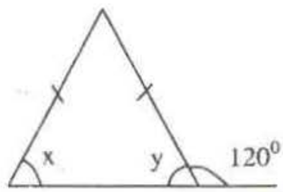
(c)



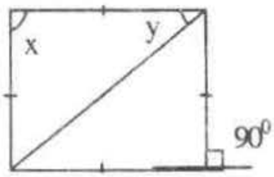
(d)



(e)



(f)



3. Construct an equilateral triangle with a side 6cm. Measure the angles and complete the table below.

Angle	$\angle A$	$\angle B$	$\angle C$	Result
Measurement				

What can be concluded from this observation ?

4. The line joining the vertex and the midpoint of the base of an isosceles triangle is perpendicular to the base. Verify experimentally.

5. Angles of an equilateral triangle are equal. Verify experimentally.

UNIT 15

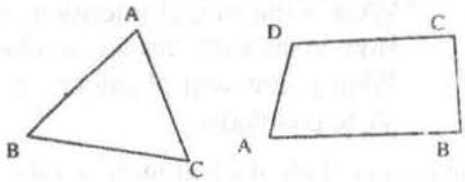
INTERIOR AND EXTERIOR ANGLES OF A REGULAR POLYGON

1. Regular Polygon

Polygon in which all the angles and sides are equal is called a regular polygon. Equilateral triangle and square are regular polygons.

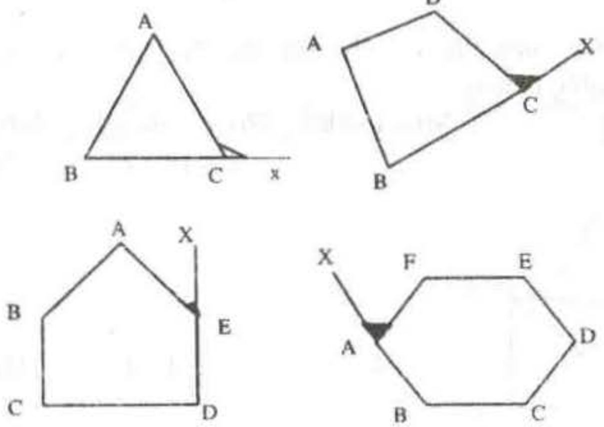
2. Interior Angles of Polygon

Angles formed by the sides of the polygon inside it are called interior angles of the polygon.



3. Exterior Angles of a Polygon

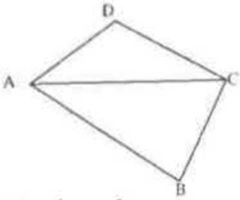
In the figure below, a side in each of the polygons is produced in one direction upto x and the angles formed when the side is produced are shaded.

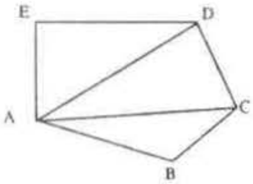


A side of polygon when produced forms an angle outside the polygon. Such an angle is called exterior angle.

4. Interior Angles of a Polygon and their Measurement

Activity 1





Draw the quadrilateral and pentagon as shown above in your exercise book. Join the diagonals A, C, A, D.

Now, answer the following questions.

a) Into how many triangles the diagonal AC has divided the quadrilateral ABCD ?

b) What is the sum of interior angles of quadrilateral ABCD ?

c) How many triangles are combined together to make the pentagon ABCDE ?

d) What is the sum of interior angles of the pentagon ABCDE ? Discuss and try to conclude.

Can we conclude the following on the basis of the activities above ?

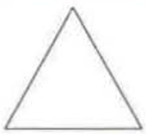
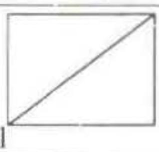
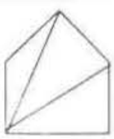
(i) Sum of interior angles of a quadrilateral $= 2 \times$ (sum of interior angles of a triangle.)

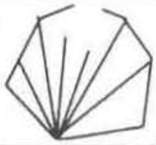
(ii) Sum of interior angles of a pentagon $= 3 \times$ (sun of interior angles of a triangle.)

Similarly, as above, what is the sum of interior angles of hexagon, heptagon ?

Activity 2

Observations made and the conclusion drawn from the activity-1 above are displayed in the table below.

Polygon	No. of sides	No. of triangle	Sum of interior angles
Triangle 	3	$1 = (3 - 2)$	$180^0 = 180^0 \times (3 - 2)$
 Quadrilateral	4	$2 = (4 - 2)$	$360^0 = 180^0 \times (4 - 2)$
Pentagon 	5	$3 = (5 - 2)$	$540^0 = 180^0 \times (5 - 2)$
Hexagon 	6	?	?

Heptagon 	7	?	?
......	....	....	....
 N-gon	n	n-2	$180^0 \times (n - 2)$

From the activities above, we conclude :

Sum of interior angles of a polygon = $180^0 \times (n - 2)$
Sum of interior angles of a polygon = $180^0 \times$ (number of sides-2)

5. Measure of Interior Angles of a Regular Polygon

If the number of sides of a regular polygon is n then the sum of interior angles = $180^0 \times (n - 2)$.
Further, the number of sides of a polygon is equal to the number of angles in it. Therefore a regular polygon having n equal sides has n equal interior angles and

each interior angle $= \dfrac{180^0 \times (n-2)}{n}$.

Thus, if an interior angle of a regular polygon is denoted by x then,

$$x = \frac{180^0 \times (n-2)}{n}$$

Example 1

What is the measure of an interior angle of a regular hexagon ?

Solution :

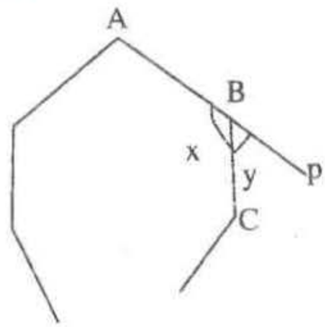
Here, the number of sides of a regular hexagon.

If the interior angle is x then

$$x = \frac{180^0 \times (n-2)}{n} = \frac{180^0 \times (6-2)}{6} = \frac{180^0 \times 4}{6} = 120^0.$$

$\therefore$ required interior angle = 120^0.

6. Measure of an Exterior Angle of a Regular Polygon



A side AB of a regular polygon is produced to P and exterior ∠PBC is formed as shown in the figure.
Let, ∠PBC = y and interior angle adjacent to it be x so that ∠ABC = x and

$x + y = 180^0$

(sum of adjacent angles is a straight angle)

$\therefore y = 180^0 - x$

$= 180^0 - \frac{180^0 \times (n-2)}{n}$

$= \frac{180^0 n - 180^0 n + 360^0}{n}$

$\therefore y = \frac{360^0}{n}$

Therefore, Exterior angle of a regular polygon $y = \frac{360^0}{n}$

Example 2

What is the measure of an exterior angle of a regular octagon ?

Solution :

Here, the number of sides in an octagon n $= 8$

Exterior angle of regular octagon y $= \frac{360^0}{n}$

$= \frac{360^0}{8} = 45^0$

$\therefore$ Required exterior angle $= 45^0$

EXERCISE 15 [A]

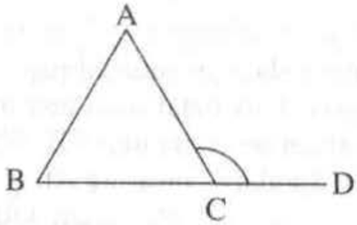
1. What is the measure of an interior angle of the following regular polygons ?
   (a) Pentagon (b) Heptagon (c) Octagon
   (d) Nonagon (e) Decagon (f) Dodecagon
2. Find, by the using the formula, the measure of an exterior angle of the following regular polygons.
   (a) Pentagon (b) Heptagon (c) Octagon
   (d) Nonagon (e) Decagon (f) Dodecagon
3. Side BC of ΔABC is produced to D.
   (a) Using protractor, measure ∠A and ∠B and find the sum.

(b) Measure ∠ACD

(c) Is ∠A + ∠B = ∠ACD ?

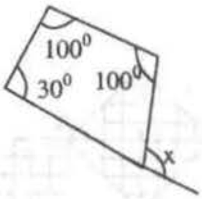
(d) What can you conclude from this activity ?

(e) If side AB is produced to E, sum of which angles will be equal to ∠CBE ?

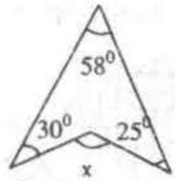


4. Find the value of x from the following polygons.

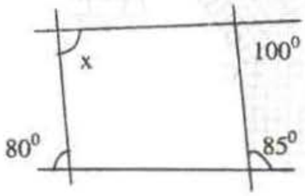
(a)



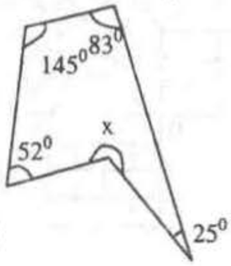
(b)



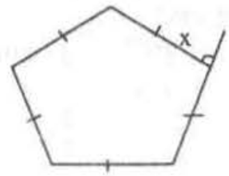
(c)



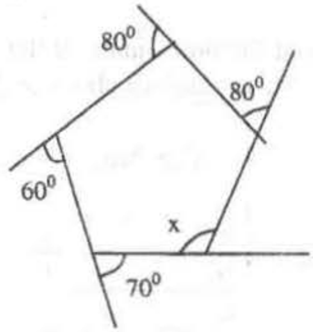
(d)



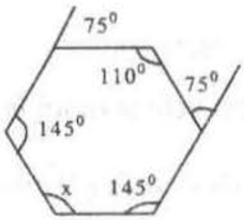
(e)



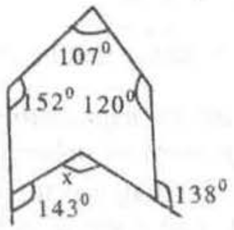
(f)



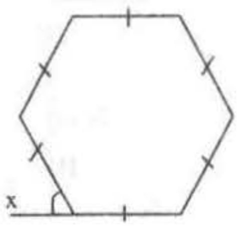
(g)



(h)



(i)



15.2. Pythagoras Theorem

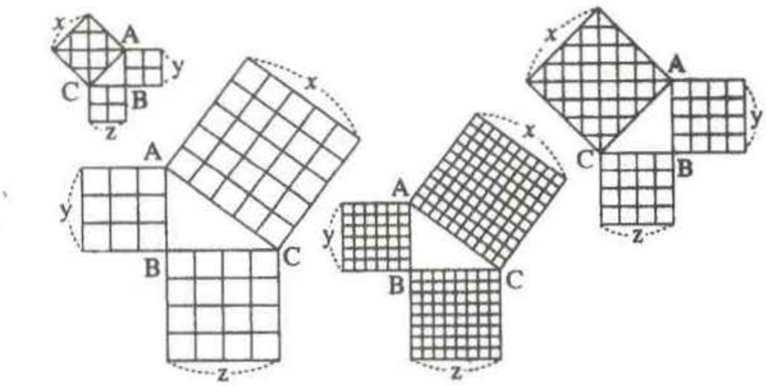
Take a sheet of squared paper. Take the point A at the end of the 4th square along x-axis (horizontal line) and the point B at the end of the 3rd square along y-axis (vertical line) and join AB. What type of triangle is formed ? Measure the length of AB. Similarly, measure AB if,

(i) OA = 4 cm, OB = 4 cm

(ii) OA = 4 cm, OB = 5 cm

(iii) OA = 4 cm, OB = 6 cm

Activity 1



Count the unit square of the squares formed on the sides of the right ΔABC in the figure (a), (b), (c) and (d) above and fill in the table as given below in your exercise book.

Fig. No.	Area of Square on the side			Sum area y + z
	AC (x)	AB (y)	BC (z)	
a.				
b.				
c.				
d.				

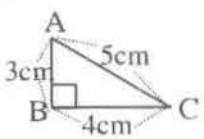
What conclusion can be drawn from this experiment ?

Square formed on the hypotenuse of a right angled triangle is equal in area to the sum of squares formed on other two sides.

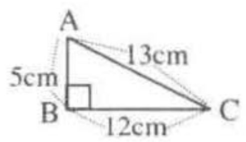
This is known as Pythagoras theorem. The Greek mathematician, Pythagoras found it out and established it about 2500 years ago.

Activity 2

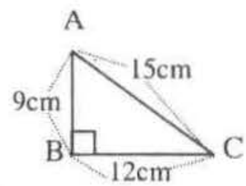
(a)



(b)



(c)

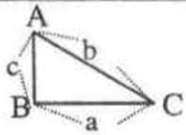


Draw the right angled triangles in your exercise book as given above and fin the square of each side and complete the following table in your exercise book.

Right angled ΔABC	AB	BC	CA	AB^2	BC^2	CA^2	AB^2+BC^2	Conclusion
(a)								
(b)								
(c)								

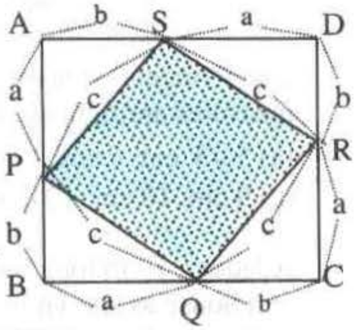
Can you justify the following statement on the basis of table above ?

In any right angle triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.
In the figure $b^2 = a^2 + c^2$.



Proof of Pythagoras theorem

In the adjoining figure, ABCD is a square having the length of a side a + b consider four points P, Q, R and S, taking one in each respectively and joining them, we can prove that
$\Delta APS \cong \Delta DSR \cong \Delta CRQ \cong \Delta BQP$.
Therefore, the shaded portion PQRS is also a square. Let the length of its side be c.



Now, area of square PQRS = c^2 .... (i) but area of PQRS

= are of square ABCD - sum of the areas of 4 congruent triangles

$$= (a + b)^2 - \frac{1}{2} \times 4 \times ab = a^2 + 2ab + b^2 - 2ab$$

$$= a^2 + b^2 \text{ .... (ii)}$$

From (i) and (ii), we get

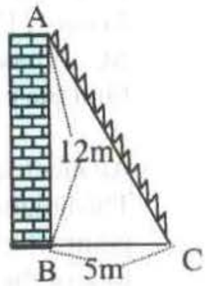
$a^2 + b^2 = c^2$ (proved)

Example 1

Find the length of the ladder in the adjoining diagram.

Solution :

Here, the wall is erected vertically on the ground. Hence, the wall, the ladder and the ground make a right angled ΔABC in which AB = 12 m, BC = 5 m and AC = ?



By Pythagoras theorem, we have,

$AC^2 = AB^2 + BC^2 = 12^2 + 5^2 = 144 + 25 = 169$

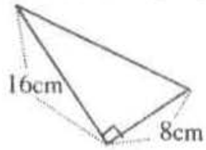
$AC = \sqrt[+]{169} = \pm 13$

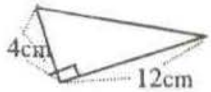
$\therefore$ The length of the ladder = 13 (the length can't be negative)

EXERCISE 15 [B]

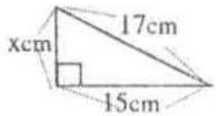
1. Find the length of hypotenuse of the following right triangles.

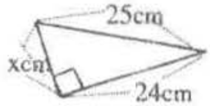
(a)

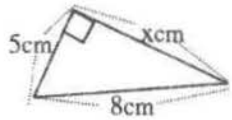

(b)


(c)


2. Find the value of x in each of the following right triangles.

(a)


(b)


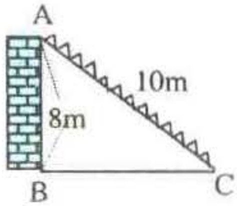
(c)


3. Use Pythagoras theorem, determine whether the triangles having following measures are right angled.

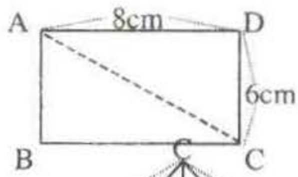
(a) $5\sqrt{3}$ cm, 5 cm, 10 cm (b) 8 cm, 7 cm, 10 cm

(c) 9 cm, 10 cm, 12 cm (d) 9 cm, 40 cm, 41 cm

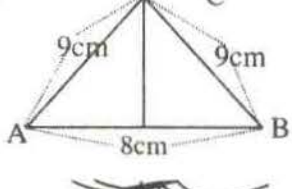
4. A ladder, 10 m long, is leaning against the wall of a house as shown in the adjoining figure. If the ladder touches the wall at a height of 8 m, how far from the wall will it touch the ground ?



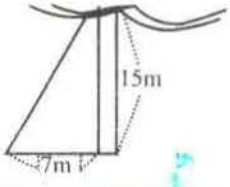
5. Length and width of a field are respectively, 8 m and 6 m. Shiva reached at C walking through B whereas Kailash reached at C directly from A ? Who walked through short route ? By how much the distance walked by them differs ?



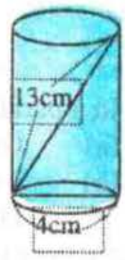
6. In the adjoining figure ABC is an isosceles triangle. From A, AD is drawn perpendicular on BC. If base BC = 8 cm and AB = AC = 9 cm. Find the measure of height AD.



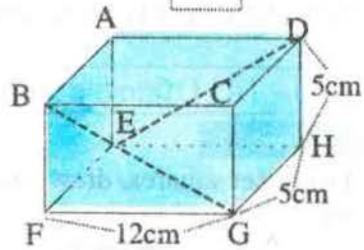
7. An electric post is supported by a stretched iron coil. The height of the post is 15 m and the distance of the point where the coil touches the ground from the foot of the post is 7m. How long is the coil used the support the post ?



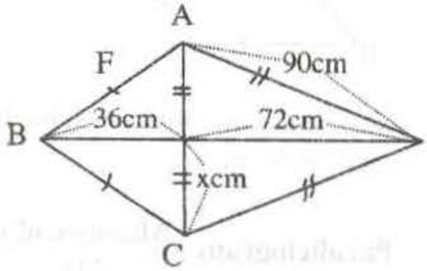
8. A pencil 13 cm long is kept inside a cylindrical glass as shown in the figure. Find the height (inner) of the glass, if its diameter is 4 cm.



9. In the adjoining rectangular parallelepiped ABCD, EFGH, length is 12 cm, width is 5cm and height is 5 cm. Find the diagonals of the faces ABFE, EFGH and AEHD.

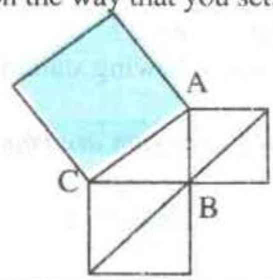


10. Find the value of x and y from the given figure of the kite.

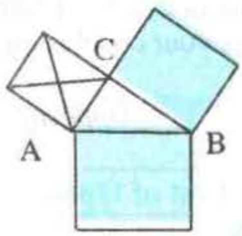


11. Draw the following figures (a) and (b) in a sheet of paper. Cut out the squares on sides AB and BC in triangular pieces of ΔABC in figure (a) and check whether the pieces fit into the square on AC. If it fit, draw the figure of the piece shapes on the way that you set. Repeat the process for the figure (b).

(a)

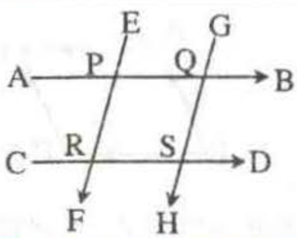


(b)



15.3. Properties and Construction of Parallelograms

In the figure, a quadrilateral PQSR is formed. Here, two pairs of parallel lines AB||CD and EF||GH intersect each other. PQSR is known as a parallelogram. In the parallelogram PQSR, PQ||SR and PS||QR. Why ? You have learnt the following definition of a parallelogram in the previous class.



A quadrilateral whose opposite sides are parallel is called a parallelogram.

Properties of a Parallelogram :

In a parallelogram,

(a) Opposite sides are equal.

(b) Two pairs of opposite angles are equal.

(c) Diagonals bisect each other.

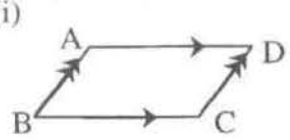
Now, let us verify these properties by experiment.

15.3. [A] Opposite side Test of a Parallelogram

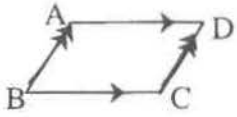
Activity 1

Using set squares, draw parallelograms in your exercise book as shown below.

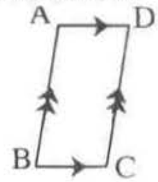
(i)



(ii)



(iii)



Now, measure the sides by the ruler and complete the following table in your exercise book.

Parallelogram	Measure of opposite sides		Measure of opposite sides		Result
	AD	BC	AB	CD	
(i)					
(ii)					
(iii)					

What did you conclude from the table above ?

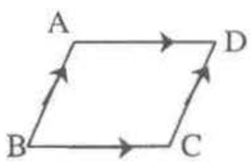
Does your conclusion match with the following statement ?

Opposite sides of a parallelogram are equal.

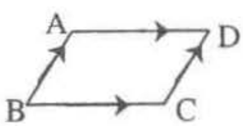
15.3. [B] Test of Opposite Angles of a Parallelogram

Activity 2

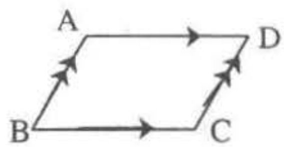
Using set squares, draw a parallelogram in your exercise book.



(i)



(ii)



(iii)

Now, measure the opposite angles using protractor. Complete the table below in your exercise book.

Parallelogram	Measure of opposite angles		Measure of opposite angles		Result
	∠A	∠C	∠B	∠D	
(i)					
(ii)					
(iii)					

What did you conclude from the table above ?
Does your conclusion match with the following ?

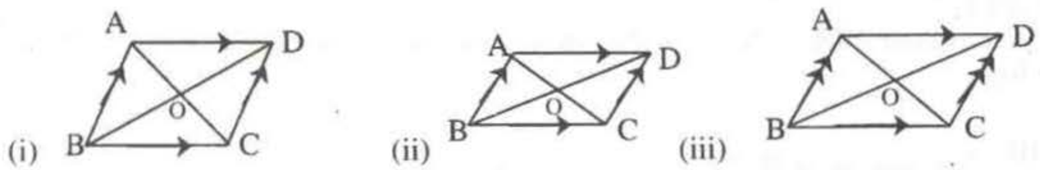
Opposite angles of a parallelogram are equal.

15.3. [C] Test of Diagonals of a Parallelogram

In parallelogram ABCD, AC and BD are joined. AC and BD are called the diagonals of the parallelogram.

A ivity 3

Using set square draw, three parallelograms in your exercise book.



Now, measure the diagonal-segments by the ruler and complete the following table in your exercise book.

Parallelogram	Diagonal segments of AC		Diagonal segments of BD		Result
	AO	CO	BO	DO	
(i)					
(ii)					
(iii)					

What conclusion did you draw ?

Does your conclusion match with the following ?

Diagonals of a parallelogram bisect each other.

15.3. [D] Conditions of a Parallelogram

We should different properties of a parallelogram. Now, let us think under what conditions a quadrilateral is a parallelogram.

Conditions of a parallelogram

A quadrilateral is a parallelogram if any of the following hold :

(i) opposite sides are parallel.
(ii) opposite sides are equal.
(iii) opposite angles are equal.
(iv) diagonals bisect each other.
(v) one pair of opposite sides are equal and parallel.

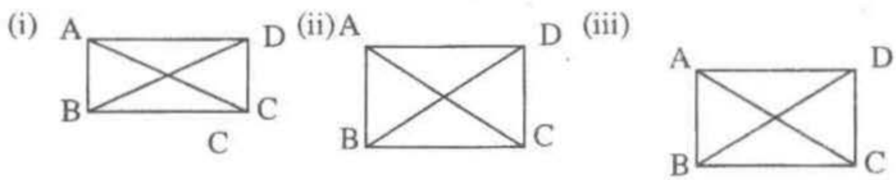
15.4. [A] Rectangle

You have already learnt, in the previous class, what the rectangle is. Rectangle is a parallelogram in which all the angles are 90^0. Since the rectangle is a parallelogram, it possesses all the properties of a parallelogram. Besides these, the diagonals of a rectangle are also equal.

Verification of diagonals of rectangle

Activity 4

Using set-squares draw the rectangles of different sizes, as shown below, in your note-book.



Now, measure the diagonals AC and BD and fill in the table as shown below.

Fig.No.	AC	BD	Result
(i)			AC = BD
(ii)			AC = BD
(iii)			AC = BD

What can you conclude from the table above ?
Does your conclusion match with the following statement ?

Diagonals of a rectangle are equal.

15.4. [B] Square

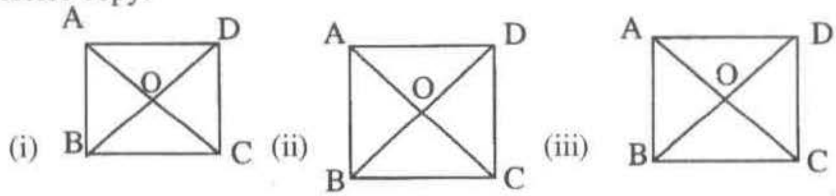
In class 7 you have learnt the following definition of a square.

Rectangle whose adjacent sides are equal is called a square. All the properties of a rectangle hold good for a square. Furthermore, the diagonals of a square bisect each other at right angles.

Verification of diagonals of square

Activity 5

Using set squares draw the squares of different sizes as shown below in your exercise copy.



Now, measure the segments of the diagonals using ruler and angles between diagonals by protractor and complete the table shown below.

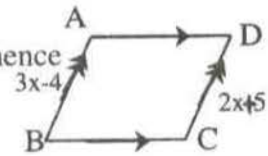
Fig.No.	AO	CO	BO	DO	∠AOB	∠COD	∠AOD	∠BOC	Result
(i)									
(ii)									
(iii)									

What can you conclude from the table above ?
Does it match with the following statement ?

Diagonals of a square bisect each other at right angles.

Example 1

Find the length of AB in the adjoining parallelogram.



Solution:

We know, opposite sides of a parallelogram are equal, hence

$AB = CD$

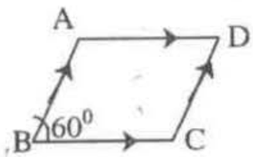
or, $3x - 4 = 2x + 5$, or, $3x - 2x = 5 + 4$, or, $x = 9$

$\therefore$ $AB = 3x - 4 = 3 \times 9 - 4 = 27 - 4 = 23$

$\therefore$ $AB = 23$ cm

Example 2

If $\angle B = 60^0$ in the parallelogram ABCD.
Find the remaining angles.



Solution :

Hence, $\angle D = 60^0$ ($\angle B = 60^0$ and $\angle B = \angle D$)

$\angle C = 180^0 - 60^0$ ($\angle B + \angle C = 180^0$, co-interior angle)

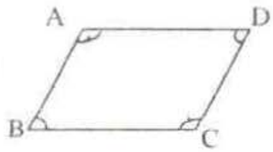
$= 120^0$

$\therefore \angle A = 120^0$ (angle opposite to $\angle C$)

Example 3

In quadrilateral ABCD if $\angle A = \angle C$ and $\angle B = \angle D$ then ABCD is a parallelogram.

Solution :



$\angle A + \angle B + \angle C + \angle D = 360^0$
(sum of interior angles of a quadrilateral)

or, $\angle A + \angle B + \angle A + \angle B = 360^0$
($\therefore \angle A = \angle C, \angle B = \angle D$)

or, $2\angle A + 2\angle B = 360^0$

$\therefore$ $\angle A + \angle B = 180^0$

$\therefore$ AD//BC (Sum of co-interior angles is 180^0)

Similarly, $\angle C + \angle B + \angle C + \angle B = 360^0$

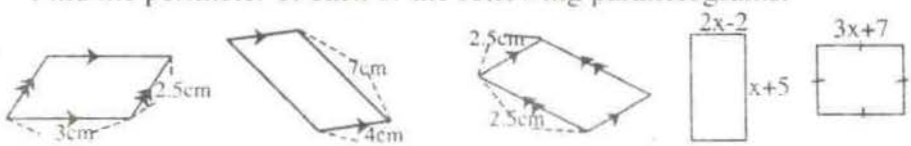
or, $\angle C + \angle B = 180^0$

$\therefore$ AB//DC (sum of co-interior angles)

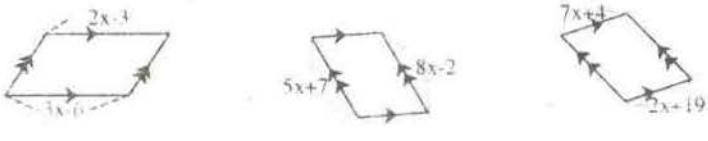
$\therefore$ ABCD is a parallelogram. (opposite sides are parallel)

EXERCISE 15 [C]

1. Find the perimeter of each of the following parallelograms.



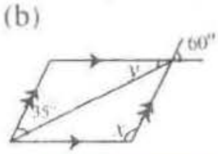
2. Find the value of x in each of the following parallelograms.



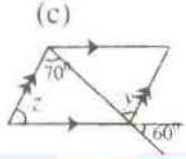
3. Find the value of unknown angles x, y and z from the following:

(a)

(b)

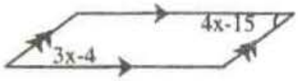


(c)

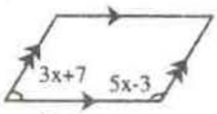


4. Find the interior angles of the parallelogram in each of the following cases:

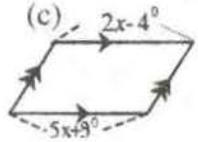
(a)



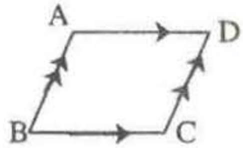
(b)



(c)

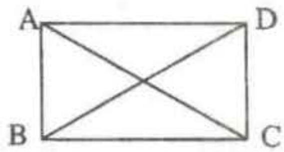


5. Study the presentation about parallelogram ABCD below and give reason for each of the statements.

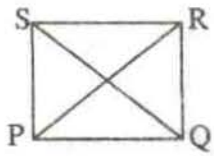


Statements Known	Reasons
(i) $\angle A = 180^0 - \angle B$	(i)
(ii) $\angle C = 180^0 - \angle B$	(ii)
(iii) $\angle A = \angle C$	(iii)

6. In the adjoining figure a rectangle is drawn. Measure its
   (a) Opposite sides and angles and tabulate the measures.
   (b) Diagonal segments and tabulate the measurement.
   (c) What can you conclude from the table ? Write down your conclusion.



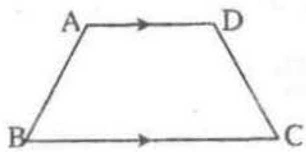
7. A square PQRS is shown in the figure. Draw this square in your exercise book. Measure its opposite sides, angles and diagonals and tabulate the measurements. Does a square satisfies all the properties of a rectangle and of a parallelogram ?



15.5. Construction of Trapezium

A quadrilateral in which a pair of opposite sides are parallel is called a trapezium.

In the adjoining quadrilateral, pair of opposite sides AD and BC are parallel, hence ABCD is a trapezium.



Example 1

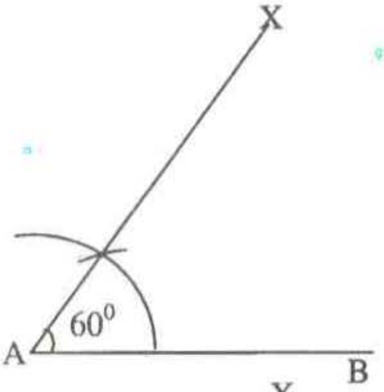
Construct a trapezium in which
$AB = 6cm, BC = 3$ cm, $\angle DAB = 60^0$, $\angle BCD = 90^0$ and AD∥BC.

Solution:

Method of construction is described below.

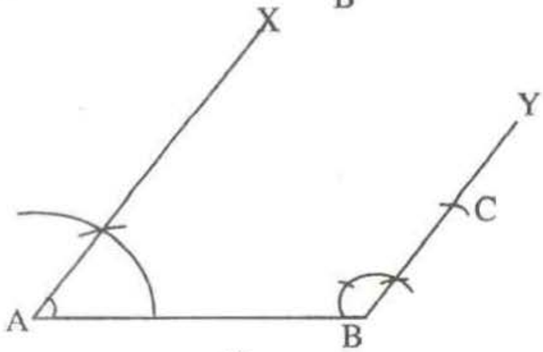
Step 1 :

Draw a segment AB, 6 cm long and at A draw an angle of 60^0. Produce AX such that $\angle BAX = 60^0$.



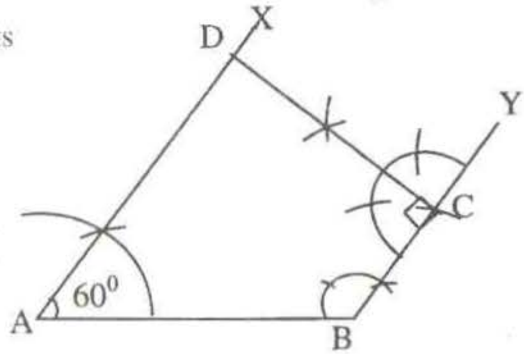
Step 2 :

Since AX||BY, draw BY such that $\angle ABY = 120^0$. Cutoff an arc of 3 cm at C on BY such that BC = 3 cm.



Step 3 :

At C, draw an angle of 90^0. The line making 90^0 angle with BC at C meets AX at D.



Here, ABCD is the required trapezium.

Example 2

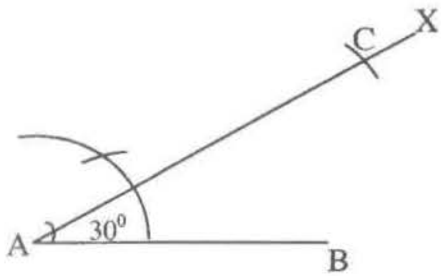
Construct a trapezium ABCD in which AB = 5 cm, diagonal AC = 7 cm, $\angle CAB = 30^0$, CD = 8 cm., and AB || DC.

Solution:

Method of construction is described below.

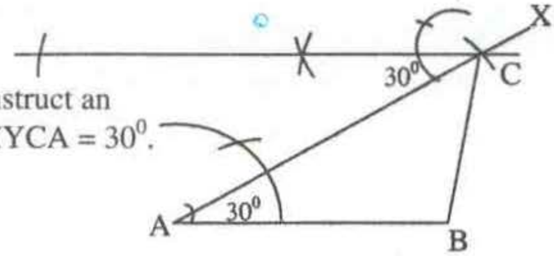
Step 1 :

Draw a segment AB of 5 cm. At A, make an angle of 30^0 and draw a line AX such that $\angle XAB = 30^0$. Along AX, cutoff an arc of 7 cm at C such that AC = 7cm.



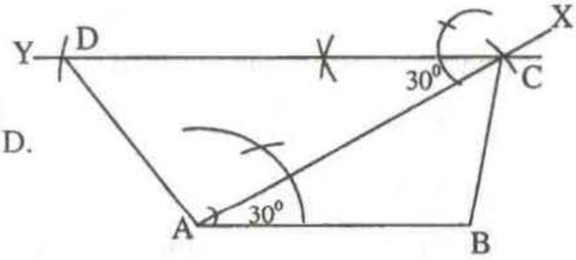
Step 2 :

Join B to C. Since AB ∥ DC, at C, construct an angle of 30^0 and draw CY such that $\angle YCA = 30^0$.



Step 3 :

Cut off an arc of 8cm from C along CY such that CD = 8 cm. Join A to D. ABCD is the required trapezium.



Example 3

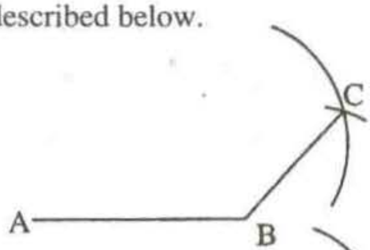
Construct a trapezium in which AB = 4 cm. Diagonal AC = 7 cm., AD=BC=4.5 cm and AB ∥ CD.

Solution :

Stepwise detail for constructing the trapezium is described below.

Step 1 :

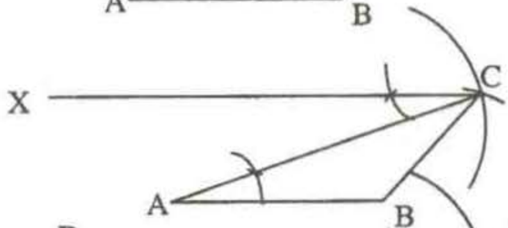
Draw a line AB of 4 cm. Draw an arc of 7 cm from A and that of 3.5 cm from B so that they intersect at C



Step 2 :

Join C to A and B.

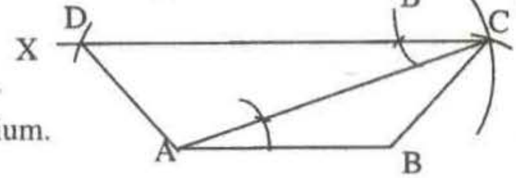
At C, construct $\angle ACX$ such that $\angle ACX = \angle BAC$.



Step 3 :

From A, cutoff an arc of 4.5 cm on CD.

Join A to D. ABCD is the required trapezium.



EXERCISE 15 [D]

1. Construct the trapezium ABCD such that :

   (i) $AB = 5cm$, $BC = 3.5$ cm, $\angle DAB = 75^0$, $\angle BCD = 60^0$ and $AD \| BC$

   (ii) $AB = 6cm$, $BC = 4$ cm, $\angle DAB = 90^0$, $\angle BCD = 60^0$ and $AD \| BC$

   (iii) $AB = 6cm$, $BC = 5$ cm, $\angle DAB = 120^0$, $\angle BCD = 75^0$ and $AD \| BC$

2. Construct a trapezium in which

   (i) $PQ = 5cm$, diagonal $PR = 8cm$, $\angle QPR = 45^0$, $RS = 7cm$, $PQ \| SR$

   (ii) $PQ = 4cm$, diagonal $PR = 6cm$, $\angle QPR = 30^0$, $PS = 5cm$, $PQ \| SR$

   (iii) $PQ = 3cm$, diagonal $PR = 8cm$, $\angle QPR = 75^0$, $PS = 4cm$, $PQ \| SR$

3. Construct a trapezium ABCD in which

   (i) $AB = 3.5$ cm, diagonal $AC = 7cm$, $AD = BC = 4cm$. and $AB \| CD$.

   (ii) $AB = 6$ cm, diagonal $AC = 8cm$, $AD = BC = 5cm$. and $AB \| CD$.

   (iii) $AB = 5.5$ cm, diagonal $AC = 7.5cm$, $CD = 7cm$, $BC = 4cm$. and $AB \| CD$.

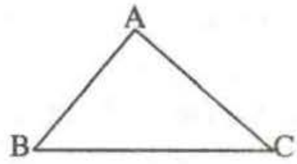
UNIT 16

CONGRUENT AND SIMILAR TRIANGLES

16.1. Test of Congruent of Triangles

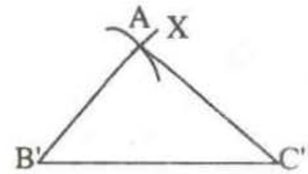
Activity 1

In how many different ways given ΔABC can be constructed ? Try it. Some of the methods are given below.



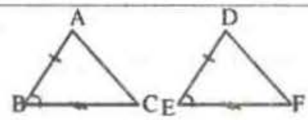
Method 1

Draw a segment B'C' equal in measure to BC, and at B', draw $\angle XB'C'$ equal to $\angle ABC$ cutoff $B'A' = BA$ along B'X. Joint A'C'. Measure the corresponding parts of ΔABC and $\Delta A'B'C'$ and determine whether they are equal.



Here, using two sides and the angle between them, it is possible to construct a triangle congruent to the given triangle. Hence this fact can be used to test the congruency of two triangles.

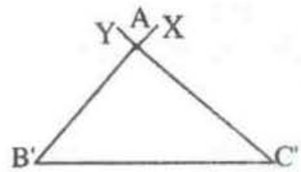
In two sides of a triangle and the angle included are equal to the two sides and the angle between them of the other triangle then the triangles are congruent. It known as SAS (side angle side) axiom.



$\Delta ABC \cong \Delta DEF$

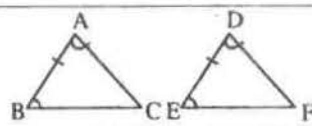
Method 2

Draw a segment B'C' equal in measure to BC.
At B' draw an angle $\angle XB'C' = \angle B$ and at C' draw $\angle YC'B' = \angle C$, Let them intersect at A'. Now measure the corresponding parts of ΔABC and $\Delta A'B'C'$.



Here, using one side and two angles on it, it is possible to draw a triangle congruent to the given triangle. Hence it can be used as a fact to test the congruency of two triangles.

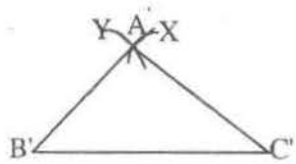
One side of a triangle and the angles at the ends of that side are equal to a side and the angles on it of another triangle, then the triangles are congruent. It is known as ASA (angle-side-angle) axiom.



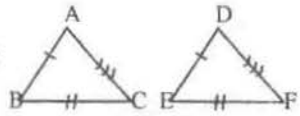
$\Delta ABC \cong \Delta DEF$

Method 3

Draw B'C' making equal to BC. From B' cutoff an arc equal to AB and from C' cutoff an arc equal to AC so that they intersect at A'. Join A' to B' and C' respectively. Measure the corresponding parts of ΔABC and ΔA'B'C' and determine whether they are equal.

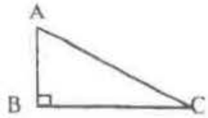


Here, using three sides a triangle can be drawn which is congruent to the given one. Hence this fact can be used to test the congruency of the triangles.

> In two triangles, if three sides of one triangle are equal to the three sides of the other separately, then they are congruent. It is known as SSS (side-side-side) fact.
>
> 
>
>
> $\Delta ABC \cong \Delta DEF$

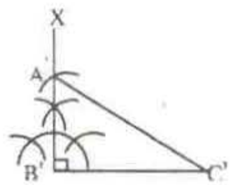
Activity 2

Can you construct a triangle congruent to right ΔABC by the method different from that in the activity 1 ? Study the procedure below.

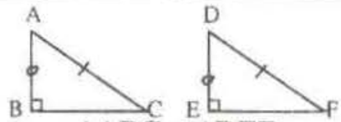


Method 4

Draw B'C' making equal to BC. At B' draw a 90^0 angle and cutoff C'A' = CA on B'X. Join A'C'. Measure the corresponding parts of ΔABC and ΔA'B'C' and determine whether they are equal.

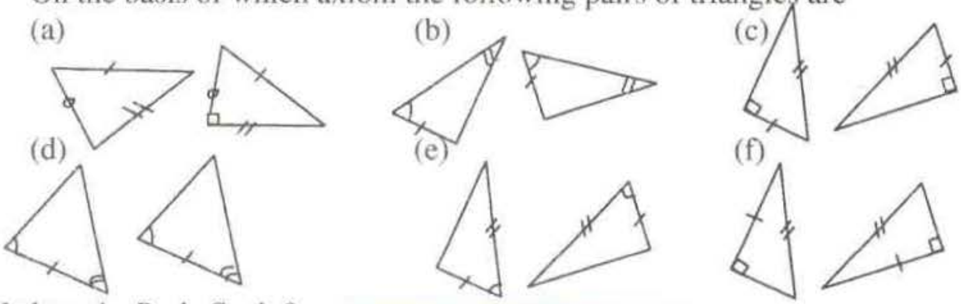


Here, using right angle, hypotenuse and a side, it was possible to construct a triangle congruent to the given one. Hence, this fact can be used as a test of congruency.

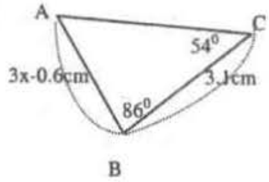
> If hypotenuse and a side of a right triangle are equal to the hypotenuse and one side of the other right triangle, then the triangles are congruent. This fact is known as R.H.S. in short.
>
> 
>
>
> $\Delta ABC \cong \Delta DEF$

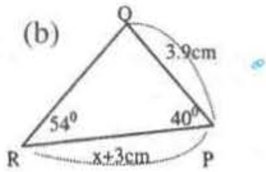
EXERCISE 16 [A]

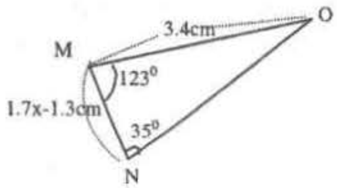
1. On the basis of which axiom the following pairs of triangles are

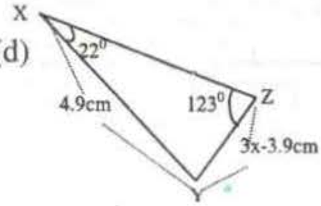


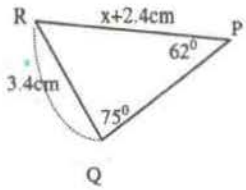
2. If the pairs of triangles given below are congruent, find the value of x and the unknown sides and angles.

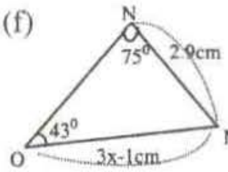
(a)


(b)


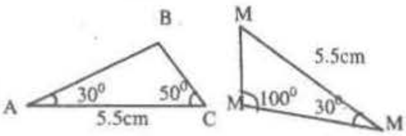
(c)


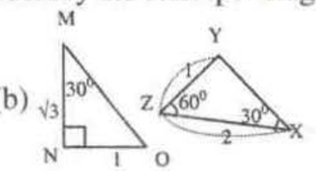
(d)


(e)


(f)


3. If the following pairs of triangles are congruent, identify the corresponding sides.

(a)


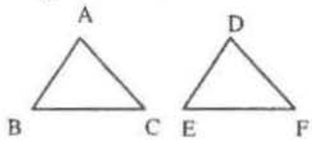
(b)


4. What should be added to the following conditions so that ΔABC and ΔDEF are congruent ?

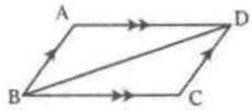
a) $AB = DE, \angle B = \angle E$

b) $\angle A = \angle D, \angle C = \angle F$

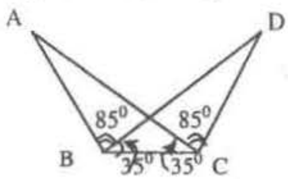
c) $AC = DF, BC = EF$



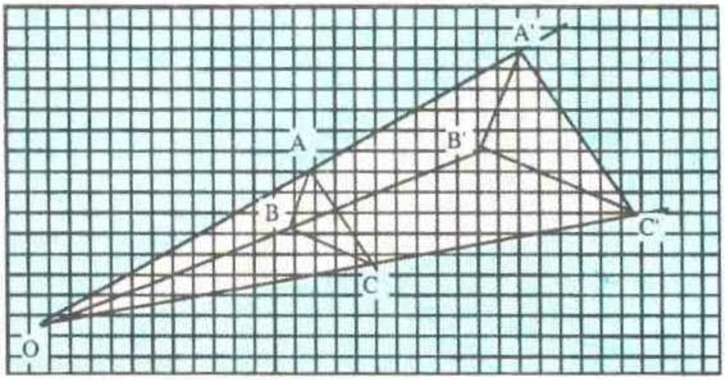
5. In the quadrilateral ABCD, AD||BC and AB||CD. BD is the diagonal. For what reason, $\Delta ABD \cong \Delta CDB$.



6. According to which axiom $\Delta ABC \cong \Delta DBC$?



16.2. Similar Triangles



In the diagram above, considering the point O as the center and doubling ΔABC a new triangle ΔA'B'C' is formed. Here, ΔABC when double, will be congruent to ΔA'B'C' and ΔA'B'C' when halved, will be congruent to ΔABC.

In order words, two figures are called similar figures if a figure when enlarge or reduced (multiplied by some number) is congruent to another.

ΔABC and ΔA'B'C' above are similar triangles. We denote it by writing ΔABC~ΔA'B'C' where the symbol ~ is read as 'is similar to', here, corresponding vertices of the vertices A, B and C are A', B' and C' respectively.

Measure the sides of the similar triangles above. Draw the table below in your exercise book and complete it.

ΔABC			ΔA'B'C'		
AB =	BC =	CA =	A'B' =	B'C' =	C'A' =
∠A =	∠B =	∠C =	∠A' =	∠B' =	∠C' =

From this table, we have in ΔABC, ΔA'B'C'

$$\frac{A'B'}{AB} = 2, \qquad \frac{B'C'}{BC} = 2 \quad \text{and} \quad \frac{C'A'}{CA} = 2$$

$$\text{and } \angle A = \angle A', \qquad \angle B = \angle B' \quad \text{and } \angle C = \angle C'$$

Therefore,

In similar triangles the corresponding angles and ratio of the corresponding sides are equal.

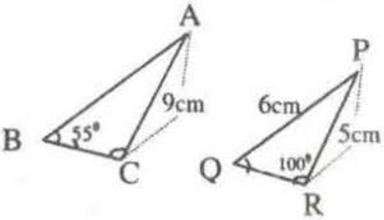
Example 1

In the adjoining figure. $\Delta ABC \sim \Delta PQR$, find the length of side AB and measure $\angle C$ and $\angle Q$.

Solution:

Since the ratio of corresponding sides are equal in the similar triangles, we have,

$$\frac{PQ}{AB} = \frac{PR}{AC}$$



If we let AB = x cm. then it reduces to $\frac{6}{x} = \frac{5}{9}$

or, $5x = 9 \times 6$

or, $x = 10.8$

$\therefore$ AB = 10.8 cm.

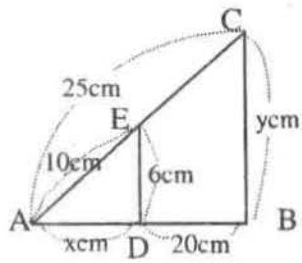
Further, corresponding angles of similar triangles are equal. hence.

$\angle B = \angle Q$ and $\angle C = \angle R$

$\therefore \angle Q = 55^0$ and $\angle C = 100^0$

Example 2

In the figure, $\Delta ABC \sim \Delta ADE$ and AB = 20 cm, AC = 25 cm., AE = 10 cm. and DE = 6 cm. find the value of x and y.



Solution :

Here, in Δ^s ABC and ADE, side corresponding to BC is DE and their ratio is $\frac{BC}{DE}$; side corresponding to AB is AD and their ratio is $\frac{AB}{AD}$; similarly AC and AE are corresponding sides and their ratio is $\frac{AC}{AE}$.

$\Delta ABC \sim \Delta ADE$, hence corresponding sides are proportional or, $\frac{BC}{DE} = \frac{AB}{AD} = \frac{AC}{AE}$

or, $\frac{y}{6} = \frac{20}{x} = \frac{25}{10}$

From first and third ratios we have, $\frac{y}{6} = \frac{25}{10}$

$\therefore$ y = 15 cm.

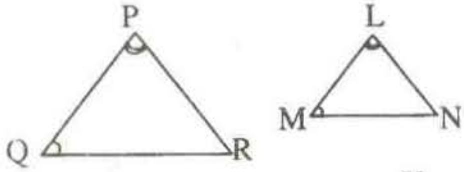
From second and third ratios, we get, $\frac{20}{x} = \frac{25}{10}$

$\therefore$ x = 8 cm.

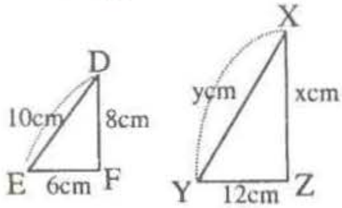
EXERCISE 16 [B]

1. In the given figure, $\Delta PQR \sim \Delta LMN$. Identify corresponding angles and sides then fill in the blanks below:

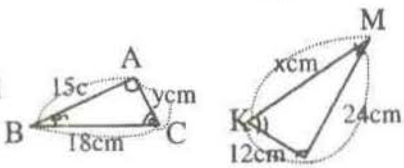
$$\frac{PQ}{LM} = \frac{QR}{\ldots\ldots}, \frac{PR}{LN} = \frac{\ldots\ldots}{MN}, \frac{\ldots\ldots}{LM} = \frac{PR}{LN}$$



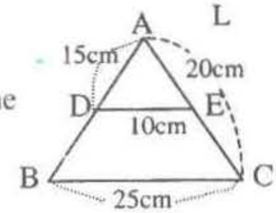
2. In the figure given $\Delta DEF \sim \Delta XYZ$ and $DE = 10$ cm, $EF = 6$cm, $DF = 8$ cm and $YZ = 12$cm, find the value of x and y.



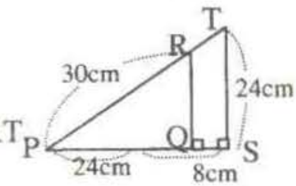
3. In the adjoining figure, $\Delta ABC \sim \Delta KLM$ and $AB = 15$ cm, $BC = 18$ cm, $LM = 24$ cm and $KL = 12$cm, find the value of x and y.



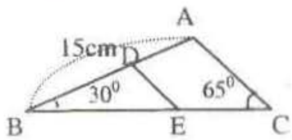
4. In the figure $\Delta ABC \sim \Delta ADE$ and $AD = 15$ cm, $DE = 10$ cm, $BC = 25$ cm and $AC = 20$cm find the measure of AB and AE.



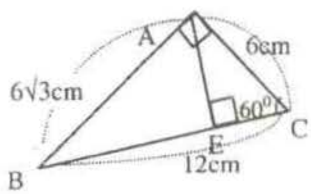
5. In the adjoining figure if $\Delta PQR \sim \Delta PST$, and $PR = 30$ cm, $PQ = 24$ cm, $QS = 8$ cm and $ST = 24$ cm, then find the measure of RQ and RT



6. In the figure, $\Delta ABC \sim \Delta EBD$, $\angle B = 30^0$, $\angle C = 65^0$, $AC:DE = 5:3$ and $AB = 15$ cm, find the measure of $\angle BDE$ and BE.



7. In the figure, $\Delta ABC \sim \Delta EBA \sim \Delta EAC$, $\angle C = 60^0$, $AB = 6\sqrt{3}$ cm, $BC = 12$ cm and $CA = 6$cm, find $\angle BAE$, AE, BE and CE.



16.3. Test of Similarity

In the figure given, $\Delta ABC \sim \Delta A'B'C'$ and ratios of corresponding sides are 2, or, $A'B' = 2AB$, $B'C' = 2BC$, $C'A' = 2CA$ and $\angle A = \angle A'$, $\angle B = \angle B'$ and $\angle C = \angle C'$.

Activity 1

ΔDEF is constructed (in the right side) in which $\angle E = \angle B'$, $EF = 2a$ and $\angle F = \angle C'$. Here, in Δ^{es} A'B'C' and DEF, angle-side-angle are equal, hence $\Delta A'B'C' \cong \Delta DEF$. Therefore, as $\Delta ABC \sim \Delta A'B'C'$, $\Delta DEF \sim \Delta ABC$. Here, in Δ^{es} ABC and DEF, two pairs of corresponding angles are equal.

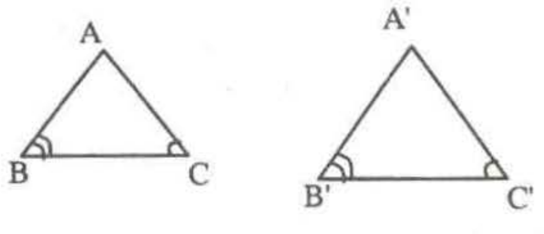
Activity 2

ΔDEF is constructed in the right side in which $DE = 2c$, $\angle E = \angle B'$ and $EF = 2a$. Here in Δ^{es} A'B'C' and DEF, side-angle-side are equal, hence $\Delta A'B'C' \cong \Delta DEF$. Therefore, $\Delta DEF \sim \Delta ABC$.

Here, in Δ^{es} ABC, DEF, two pairs of corresponding sides are proportional and angles included by the sides are equal.

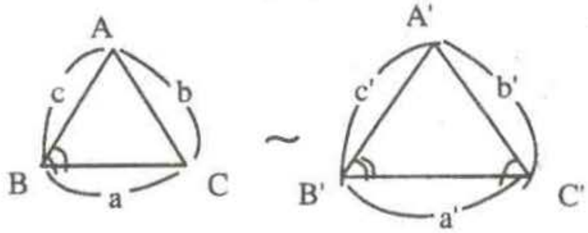
Condition of similarity of triangles if any of the following holds, then the triangles are similar.

a) Two pairs of angels are equal.



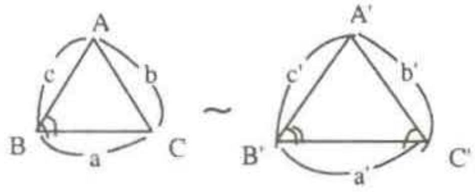
$\angle B = \angle B'$
$\angle C = \angle C'$

b) Two pairs of sides are proportional and included angles are equal.



$\angle B = \angle B'$
$\frac{a'}{a} = \frac{c'}{c}$

c) Three pairs of sides are proportional.



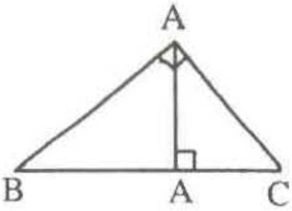
$$\frac{a'}{a}=\frac{b'}{b}=\frac{c'}{c}$$

Example 1

In the right angled ΔABC, $\angle A = 90^0$. From vertex A, perpendicular AD is drawn on base BC. Prove that

(a) $\Delta ABC \sim \Delta DBA$

(b) $\dfrac{BC}{BA}=\dfrac{BA}{BD}$



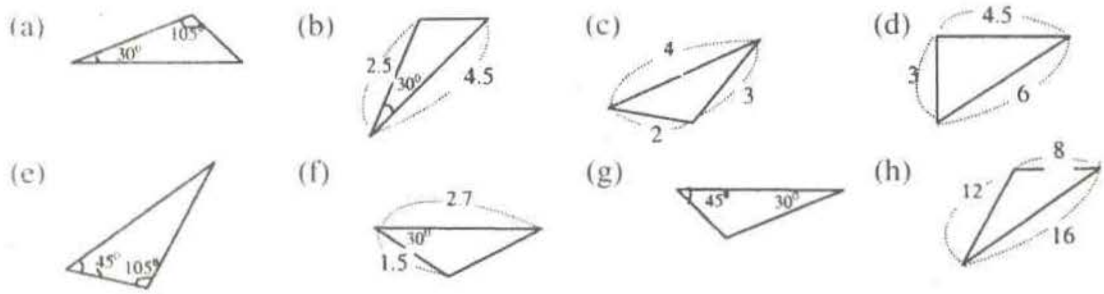
Solution :

(a) In ΔABC and ΔDBA, $\angle BAC = \angle BDA = 90^0$
$\angle ABC = \angle DBA$ (common angles)
$\therefore \Delta ABC \sim \Delta DBA$ ($\therefore$ two pairs angles are equal)

(b) In similar ΔABC and ΔDBA, corresponding sides are proportional, hence
$\dfrac{BC}{BA}=\dfrac{BA}{BD}$

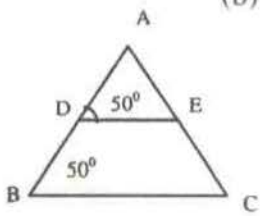
EXERCISE 16 [C]

1. From the figures given below, find the pairs of similar triangles and give reasons.

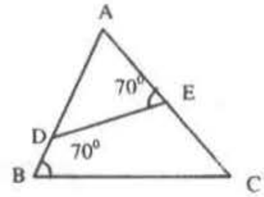


2. For each of the figures below, using the symbol ~, write the pair of similar triangles with reasons.

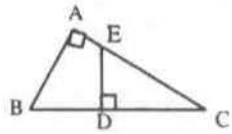
(a)



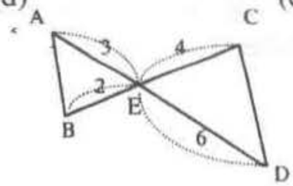
(b)



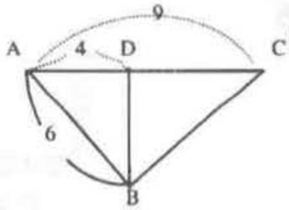
(c)



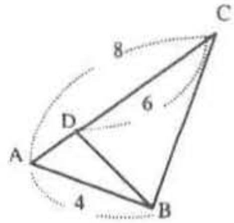
(d)



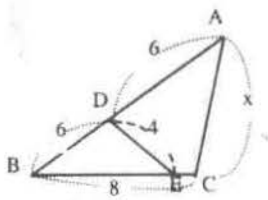
(e)



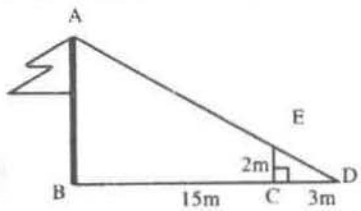
(f)



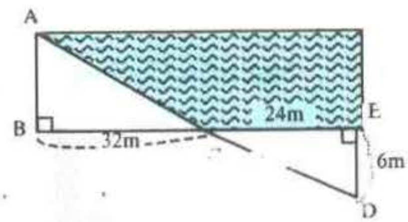
3. In the adjoining figure,
   (a) Prove : $\Delta ABC \sim \Delta EBD$
   (b) Find the value of x



4. In the figure given
   (a) Prove : $\Delta ABD \sim \Delta ECD$
   (b) Find the height of the flag AB.



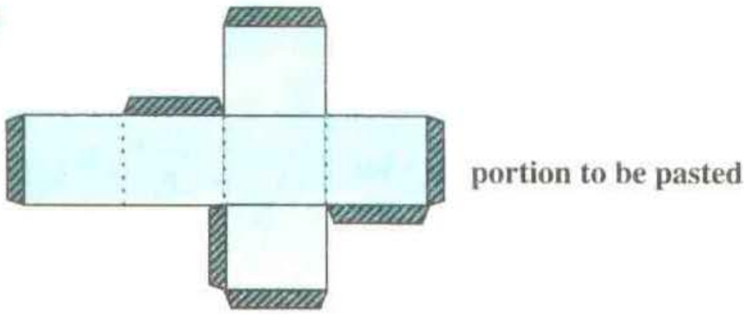
5. In the figure given
   (a) Prove : $\Delta ABC \sim \Delta DEC$
   (b) Find the width of the river.



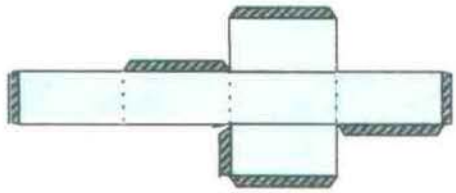
SOLIDS

In the previous classes you have studied the method of constructing the nets and the corresponding solids. Here you will study about some more solids and their nets.

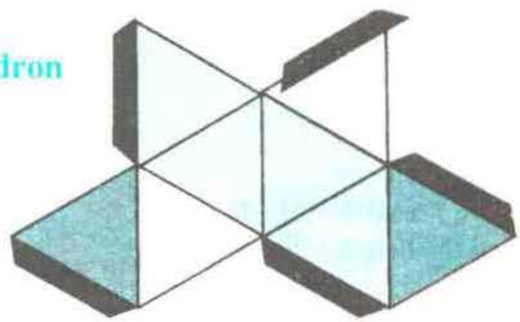
1. Net of a Cube



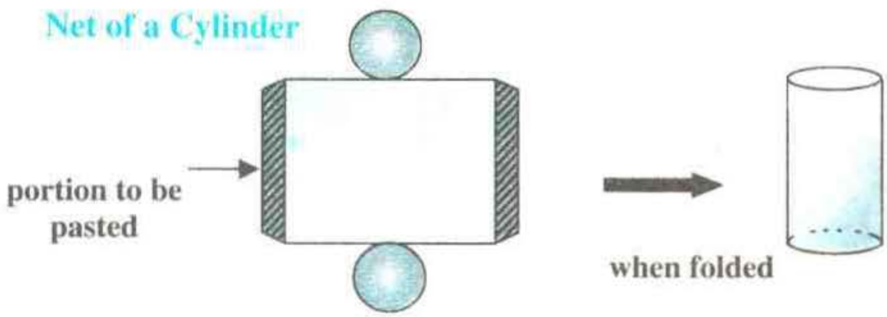
2. Net of Cuboid



3. Net of an Octahedron

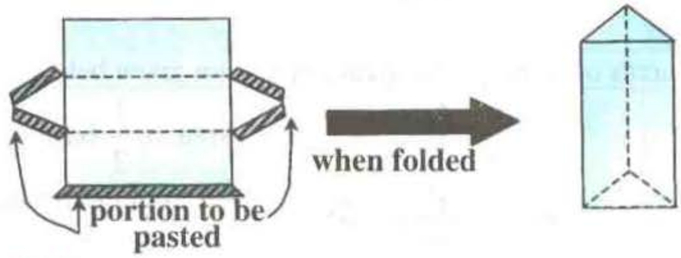


4. Net of a Cylinder



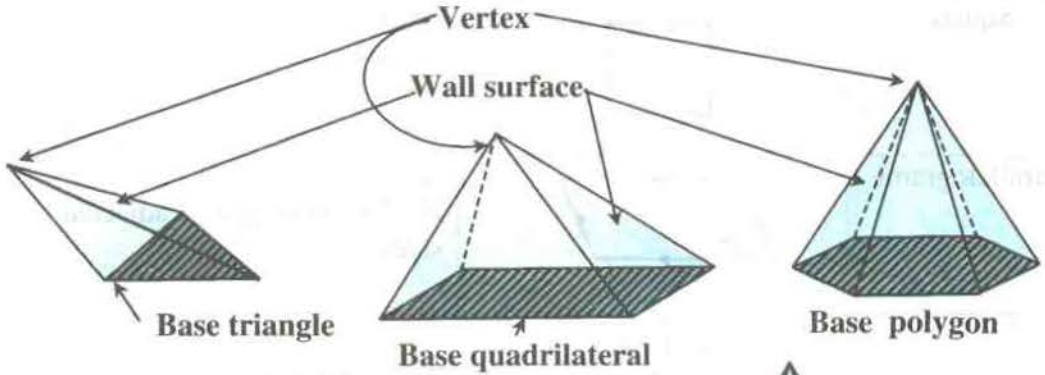
5. Net of Triangular Prism

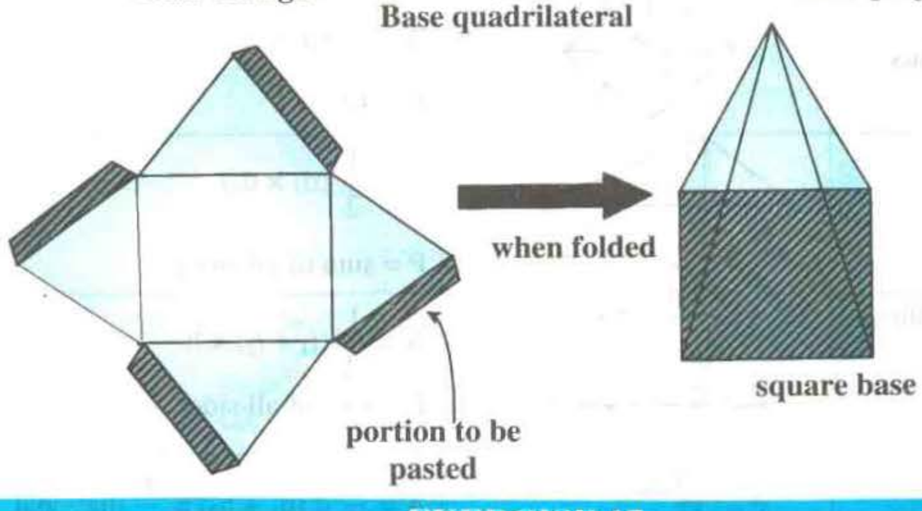
A three dimensional solid in which lateral surfaces are congruent triangles and other three surfaces are suitable rectangle is known as a triangular prism. A sample of its net along with the solid is shown in the figure below.



6. Net of a Pyramid

A three dimensional solid with a base (triangular or quadrilateral or polygonal) and suitable surfaces with a common vertex is known as a pyramid. Some samples of the pyramids and net are shown in the figure below.





EXERCISE 17

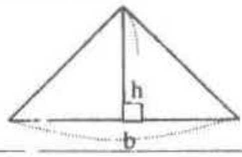
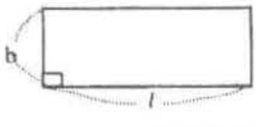
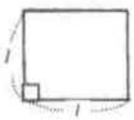
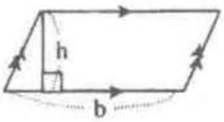
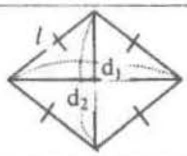
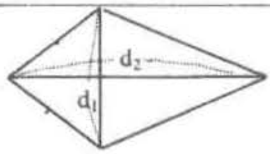
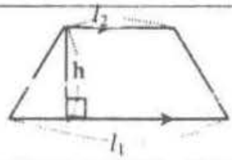
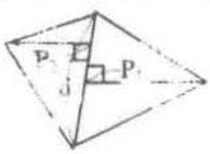
1. Trace the nets of cube, cuboid, octahedron, cylinder, triangular prism and pyramids shown above. Cut it out. Fold it properly and paste to form the solids.

UNIT 18

PERIMETER, AREA AND VOLUME

18.1. Revision

Perimeters and areas of some geometrical objects are given below.

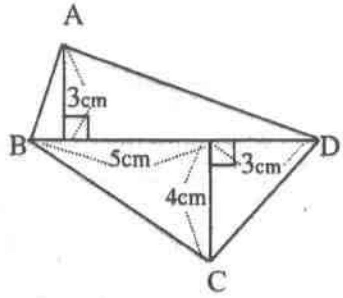
Triangle		Area $= \frac{1}{2}$ base $\times$ height P = Sum of all sides
Rectangle		Area = length $\times$ breath $A = l \times b$ $P = 2(l + b)$
Square		$A = l^2$ $P = 4l$
Parallelograms		$A = b \times h$ P = 2 (sum of pair of adjacent sides)
Rhombus		$A = \frac{1}{2}(d_1 \times d_2)$ $P = 4l$
Kite		$A = \frac{1}{2}(d_1 \times d_2)$ P = sum of all sides
Trapezium		$A = \frac{1}{2}(l_1 + l_2) \times h$ P = sum of all sides
Quadrilateral		$A = \frac{1}{2} d (p_1 + p_2) = \frac{1}{2}$ diagonal (sum of perpendiculars) P = sum of all sides

Example 1

Find the area of the quadrilateral in the adjoining figure.

Solution :

Here,



Area of quadrilateral ABCD

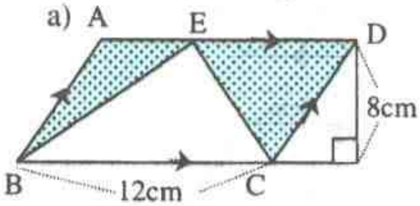
$= \frac{1}{2}$ diagonal × sum of perpendiculars

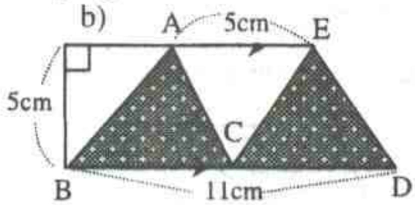
$= \frac{1}{2}\ (5 + 3)\text{ cm} \times (3 + 4)\text{ cm}$

$= \frac{1}{2} \times 8 \times 7\text{ cm}^2 = 28\text{ cm}^2$

Example 2

Find the area of shaded region in the following figures.

a) 

b) 

Solution :

(a) Area of the shaded region

$= \square ABCD - \Delta EBC$

$= 12 \times 8 - \frac{1}{2} \times 12 \times 8 = 48\text{ cm}^2$

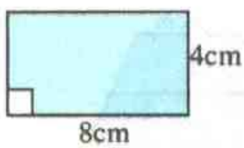
(b) Area of the shaded region

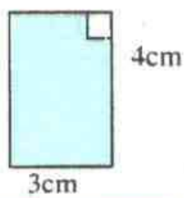
= trapezium ABDE - ΔAEC

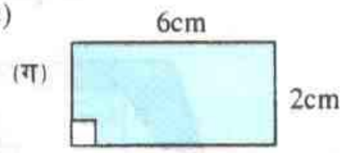
$= \frac{1}{2} \times (5+11) \times 5 - \frac{1}{2} \times 5 \times 5 = 27.5\text{ cm}^2$

EXERCISE 18 [A]

1. Find the areas and perimeters of the rectangles given below.

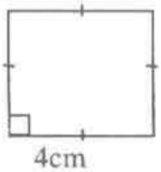
a) 

b) 

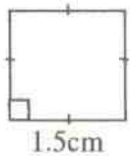
c) 

2. Calculate the area and perimeter of the squares given below:

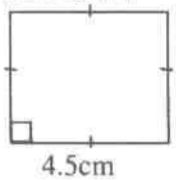
a)



b)

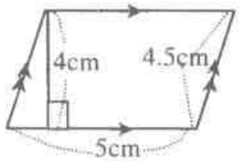


c)

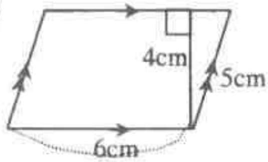


3. Compute the area and perimeter of the parallelograms given below.

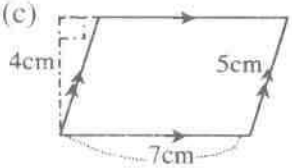
(a)



(b)



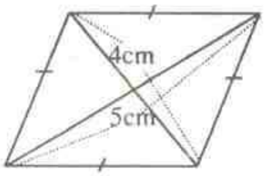
(c)



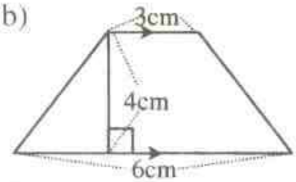
(d)

4. Find the area of the quadrilaterals.

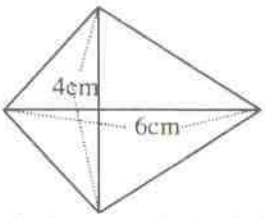
(a)



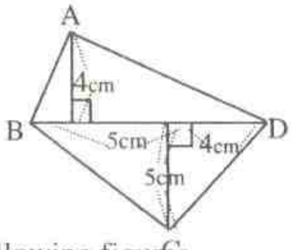
(b)



(c)

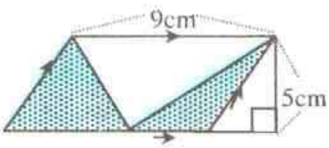


(d)

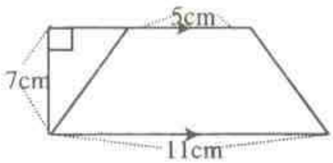


5. Find the area of shaded regions in the following figures.

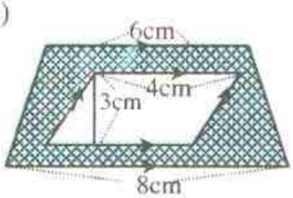
(a)



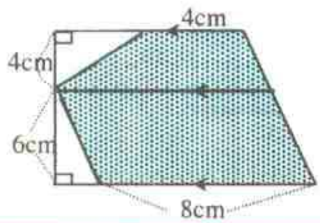
(b)



(c)

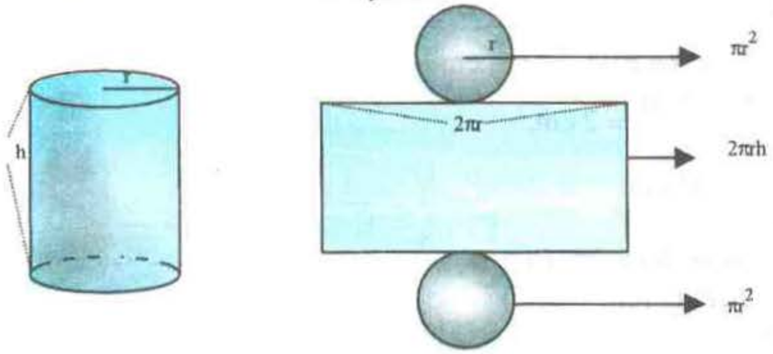


(d)



18.2. Surface Area of Cylinder and Triangular Prism

(a) Total Surface Area of a Cylinder



r : radius of the base.

h : height of the cylinder.

A cylinder when cut out, as shown in the figure above, is separated into rectangular and circular pieces. Rectangular piece separated into rectangular and circular pieces. Rectangular piece is also known as the curved surface of the cylinder.

Here, length of rectangle = circumference of the circle = $2\pi r$

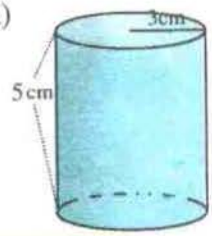
Therefore,

> *Total surface are of the cylinder (A) = area of rectangle + 2 × area of circle.*
> $= 2\pi rh + 2\pi r^2$
> $\therefore A = 2\pi r\,(h + r)$

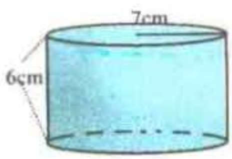
Example 1

Find the total surface area of the following cylinders.

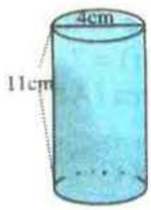
(a)



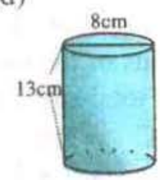
(b)



(c)



(d)



Solution :

(a) Here, height of cylinder (h) = 5 cm

radius of the base (r) = 3 cm

Total surface area (A) = $2\pi r\,(h + r)$

$= 2 \times 3.14 \times 3\text{ cm}\,(5\text{cm} + 3\text{ cm})$

$= 2 \times 3.14 \times 3\text{ cm} \times 8\text{ cm}$

$= 150.72\text{ cm}^2$

(b) Similarly, $A = 2\pi r\,(h + r)$

$= 2 \times 3.14 \times 7\text{ cm}\,(6\text{cm} + 7\text{cm})$

$= 571.72\text{ cm}^2$

(c) Here, diameter of the base (d) = 4 cm

$\therefore$ radius (r) $= \dfrac{d}{2} = \dfrac{4\text{cm}}{2} = 2$ cm.

and height (h) = 11 cm.

By formula,

Total surface area $(A) = 2\pi r\,(h + r)$

$= 2 \times 3.14 \times 2\text{ cm}\,(11\text{cm} + 2\text{cm})$

$= 163.28\text{ cm}^2$

(d) Similarly,

radius (r) $= \dfrac{d}{2} = \dfrac{8}{2} = 4$ cm.

height (h) = 13 cm.

$A = 2\pi r\,(h + r)$

$= 2 \times 3.14 \times 4\text{cm}\,(13\text{cm} + 4\text{cm})$

$= 427.04\text{ cm}^2$

Example 2

(i) If the circumference of the base of a cylinder is 31.4 cm and height is 7cm. What will be its total surface area ?

(ii) The total surface area of a cylinder is 628 cm^2 and radius of its base is 4 cm, what will be its height ?

Solution :

(i) Here, circumference of the base C = 31.4 cm.

radius of the base (r) = ?

and total surface area (A) = ?

We know, $C = 2\pi r$

or, 31.4 cm. $= 2 \times 3.14 \times r$

$\therefore\ r = \dfrac{31.14\text{cm}}{2\times 3.14} = 5$ cm

and $A = 2\pi r\,(r + h) = 2 \times 3.14 \times 5\text{ cm.}\,(5\text{cm} + 7\text{cm})$

$= 376.8\text{ cm}^2$

(ii) Here, total surface area $(A) = 628\text{ cm}^2$

radius of the base (r) = 4 cm

height of the cylinder (h) = ?

We know, $A = 2\pi r(r + h)$

or, $628 \text{ cm}^2 = 2 \times 3.14 \times 4 \text{ cm } (4\text{cm} + h)$

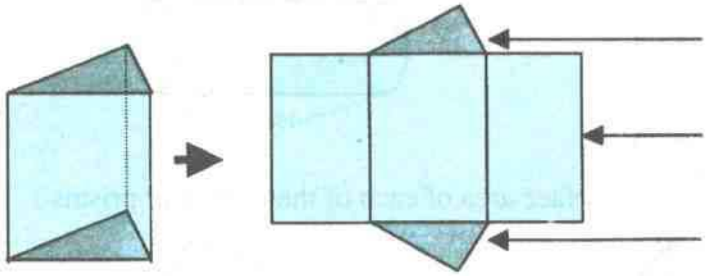
or, $4 \text{ cm} + h = \dfrac{628\text{cm}^2}{25.12\text{cm}}$

or, $4 \text{ cm} + h = 25 \text{ cm}$

or, $h = 25 \text{ cm} - 4 \text{ cm}$

$\therefore$ $h = 21 \text{ cm}$

(b) Total Surface Area of a Triangular Prism. **(Area)**



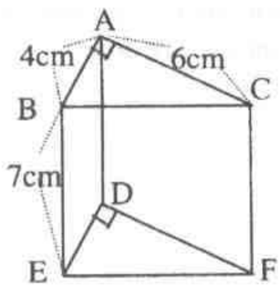
A triangular prism, when cutout as shown in the figure above, is separated into rectangular and triangular pieces. Therefore,

Total surface area of triangular prism
= area of rectangle + 2 area of base triangle.
= (perimeter of base triangle × height of prism) + 2 × (area of base triangle)
Here, area of rectangle
= Perimeter of base triangle × height

Example 3

Find the total surface area of the prism shown in the adjoining figure.

Solution :



From the adjoining figure,

In ΔABC,

$BC = AB^2 + AC^2 = 4^2 + 6^2$

$= 16 + 36 = \sqrt{52} \text{ cm} = 2\sqrt{13} \text{ cm}$

Therefore,

Total surface area

= perimeter of the base triangle × height + 2 × area of base triangle

$= (4 + 6 + 2\sqrt{13}) \times 7 + 2 \times \dfrac{1}{2} \times 4 \times 6 = (94 + 14\sqrt{13}) \text{ cm}^2$

EXERCISE 18 [B]

1. Find the total surface area of the cylinder in each of the following cases.

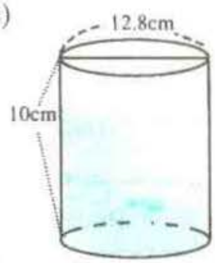
(a) height = 20 cm
radius = 7 cm

(b) length = 6 cm
diameter = 3.5 cm

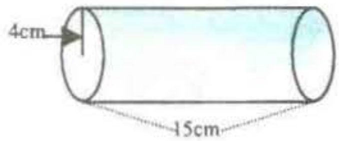
(c) length = 15 cm
radius = 4 cm

(d) height = 15.2 cm
diameter = 14.5 cm

(e)

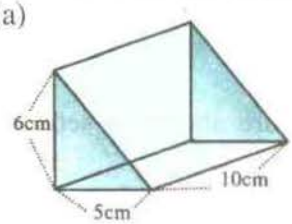


(f)

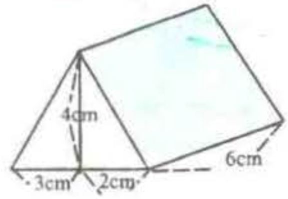


2. Determine the total surface area of each of the triangular prisms :

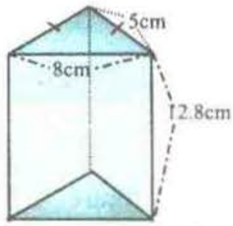
(a)



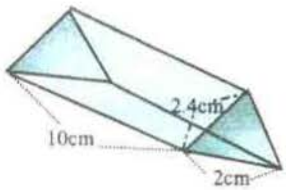
(b)



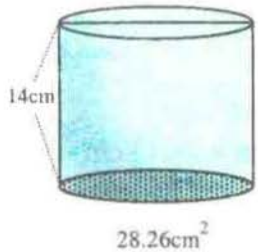
(c)



(d)



3. Base area of a cylinder can is 28.26 cm^2 and its height is 14 cm. Find its total surface area.



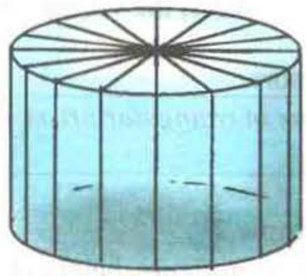
4. The outer circumference of a piece of iron-pipe is 15.7 cm and its length is 84 cm.
(a) What is its diameter ?
(b) What is the area of its curved surface ?

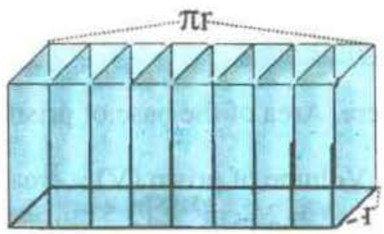
5. The length of a bamboo pipe is 15 cm and its base area is 50.24 cm^2, what is its curved surface area ?
6. The curved surface area of a cylindrical tank is 301.44 m^2. If the radius of its base is 6m, what is its height ?
7. The radius of a culvert is 40 cm and its curved surface is 37.68 cm^2. What will be its length ?
8. Circumference of a cylindrical electric bulb is 12.8 cm and its length is 80 cm. What will be its curved surface area ?

18.3. Volume of Cylinder and Triangular Prism

Activity I

Take a cylindrical solid (For the experiment, it is better to have a model of wet soil). Cut it into pieces and combine them to form a cuboid as shown in the figure.





Here, height of cylinder = height of cuboid = h,
radius of cylinder = width of the cuboid = r,

$\frac{1}{2}$ of circumference = length of cuboid = πr

and, volume of cylinder = volume of cuboid

$= \text{length} \times \text{width} \times \text{height}$

$= \pi r \times r \times h$

$= \pi r^2 h$

$= \text{area of base} \times \text{height}$

Therefore,

Volume of Cylinder = area of base × height

$= A \times h$

$= \pi r^2 h$

Example 1

The circumference of the base of a cylinder is 43.96 cm. If its height is 14cm, what will be its volume ?

Solution :

Here, circumference of the cylinder (c) = 43.96 cm
height of the cylinder (h) = 14 cm
volume of cylinder (V) = ?

We know, $C = 2\pi r$

or, $\quad 43.96 \text{ cm} = 2 \times 3.14 \times r$

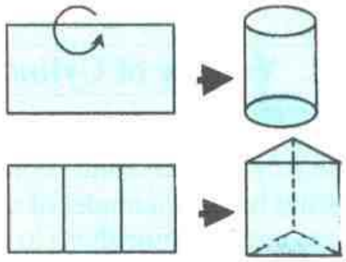
or, $\quad r = \dfrac{43 \times 96\text{cm}}{2 \times 3.14} = 7 \text{ cm}$

and, $\quad V = \pi r^2 h = 3.14 \times (7\text{cm})^2 \times 14 \text{ cm}$

$\quad = 2154.04 \text{ cm}^3$

Activity 2

Take two pieces of paper of same size and shape it into as shown in the figure.



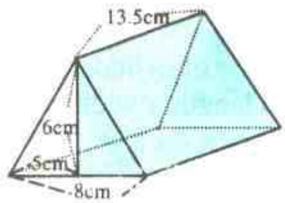
Are the structures of the formulae of the volumes of cylinder and prism alike ? Similar to the volume of the cylinder, the volume of prism is base × height.

Therefore,

Volume of triangular prism V = area of base × height
$= A \times h$

Example 2

In the given figure, height of prism = 13.5 cm
height of triangular base = 6 cm
base of triangle = 8 cm
What will be its volume ?



Solution :

Here, Area of the base of prism $= \dfrac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$

$\therefore$ Volume of prism (V) = area of base × height
$= 24 \text{ cm}^2 \times 13.5 \text{ cm} = 324 \text{ cm}^3$

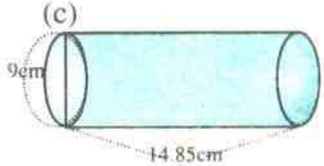
EXERCISE 18 [C]

1. Find the volume of the cylinders :

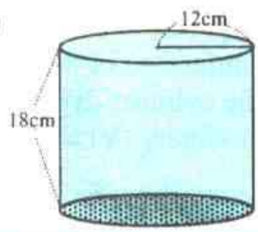
(a) length = 15 cm
diameter = 6 cm

(b) height = 12.8 cm
radius = 7 cm

(c)

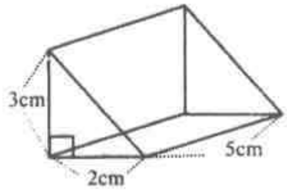


(d)

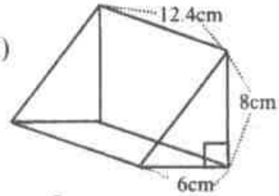


2. Find the volume of the following triangular prisms.

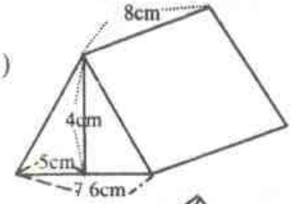
(a)



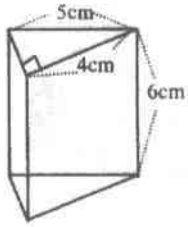
(b)



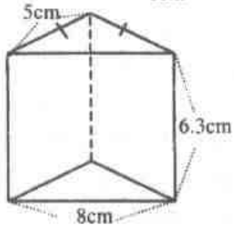
(c)



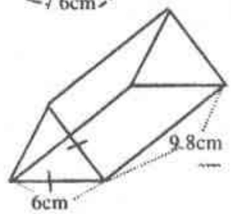
(d)



(e)



(f)



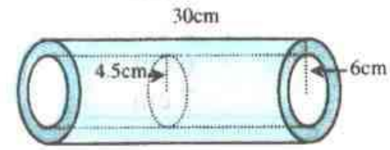
3. The curved surface are of a cylindrical vessel is 8792 cm^2 and length is 7 cm, what will be its volume ?

4. If the diameter of a cylindrical tank is 14m and the curved surface area is 263.76 m^2, then how much water (in m^3) will fill in the tank ? If the tank is filled by a pipe that flows water at the rate of 1.2m^3/min., how long will it take to fill in the tank ?

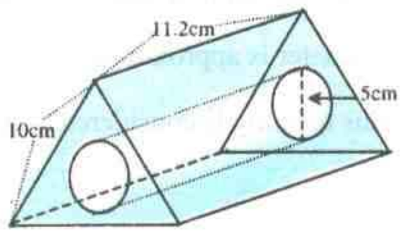
5. While digging a cylinder well, 452.16 m^3 soil, was taken out. If the depth of the well is 4 m, what is the area of its base ?

6. The diameter of a cylindrical bucket is 0.5 m and volume 98125 cm^3, what is its height ?

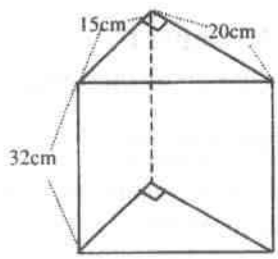
7. The diameter of kerosene drum is 1 m. If its capacity is 942 liters, find the height of the drum. (1000 cm^3 = 1 liter)



8. If the inner radius of a 30 cm long bamboo pipe is 4.5 cm and its outer radius is 6cm, what will be the volume of bamboo wood in it ?



9. There is a cylindrical hole of diameter 5 cm in a wooden triangular prism. The base of prism is an equilateral triangle with side 10 cm. If the length of prism is 11.2 cm, what will be the volume of wood in it ?



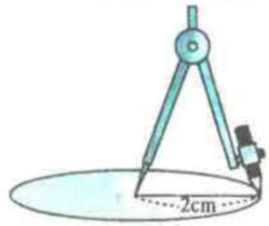
10. A prismic can is as shown in the adjoining figure.
    (a) How much water will be filled in it ?
    (b) If 2.7 liter water is poured in it, what will be the depth of water in it ?

UNIT 19

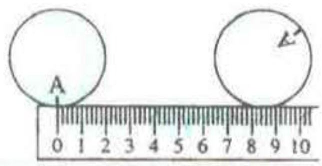
CIRCLE

19.1. Circumference of Circle

Activity 1



Draw a circle of radius 2 cm on a chart paper (card-board) sheet and cut it out by scissors. Mark a point A on the border and erect it vertically at O as shown in the figure and roll it over the scale unit the point A touches the scale. The length of the scale to this point is equal to the circumference (denoted by C). Now, divide the circumference C by the diameter-d. How much is it ? Repeat this experiment with the circular shapes with different diameters and fill in the table as shown below.



Diameter of Circle	Circumference of Circle	$\frac{\text{circumference}}{\text{diameter}} = \left(\frac{c}{d}\right)$
2 cm	6.28 cm	$\frac{6.28}{2} = 3.14$
3 cm	9.42 cm	
4 cm	12.56 cm	
5 cm		
6 cm		
7 cm		

From the table above, we find that the ratio of the circumference of a circle to its diameter is approximately 3.14 or $\frac{22}{7}$ or nearest to it. In all circles, $\frac{c}{d} = 3.14$. Hence this quantity is considered as a constant and is denoted by the Greek alphabet π (pie).

$$\textit{Therefore, } \frac{c}{d} = \pi \textit{ or, } c = \pi d = 2\pi r$$

Example 1

What is the circumference of a circle having the diameter 14 cm ?

Solution :

Here, diameter (d) = 14 cm
circumference (C) = ?

We know,

$$C = \pi d = 3.14 \times 14 \text{ cm} = 43.96 \text{ cm}$$

$$\text{Again, } C = \pi d = \frac{22}{7} \times 14 \text{ cm} = 44 \text{ cm.}$$

Note : Here, 43.96 cm when rounded off at unit place equals 44 cm. The value of C obtained by putting $\pi = \frac{22}{7}$ and 3.14 are quite close or approximately equal, hence we can put either of the values $\frac{22}{7}$ or 3.14 for π.

Example 2

If the circumference of a circle is 6.28 cm then how long is its radius ? (use $\pi = 3.14$)

Solution :

Here, circumference of the circle (C) = 6.28 cm

radius (r) = ?

We know, $C = 2\pi r$

or, $6.28 \text{ cm} = 2 \times 3.14 \times r = \frac{6.28}{2\times3.14} = 1 \text{ cm}$

$\therefore$ radius = 1 cm

EXERCISE 19 [A]

(Use $\pi = 3.14$ in each problems unless stated otherwise)

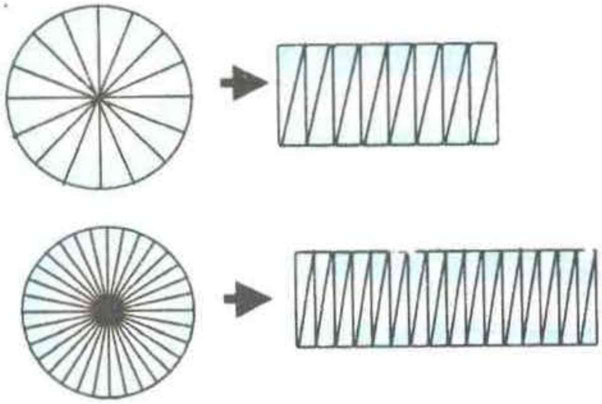
1. Find the circumference of the circle in each of the following cases.
   (a) diameter = 4 cm (b) radius = 1.5 cm
   (c) diameter = 5 cm (d) radius = 3 cm
   (e) diameter = 7 cm (f) radius = 5 cm
2. Find the radius of each of the following circle.
   (a) circumference = 47.1 cm (b) circumference = 56.52 cm
   (c) circumference = 34.54 cm (d) circumference = 21.98 cm
   (e) circumference = 9.42 cm (f) circumference = 37.68 cm
3. If the diameter of a saucer is 14 cm, what will be its circumference ? (use $\pi = \frac{22}{7}$)
4. A horse is tethered to a stake by a 6.5 m long rope. If the horse completes one round making the rope stretched, how far does it move ?
5. The radius of a bicycle wheel is 50 cm. If it completes 500 cycles how far will it go ? If a distance of 1 km is to be traveled, how many cycles has it to complete ?
6. A motor wheel has rolled over a distance of 1 km 256 m in 400 cycles. What is its diameter ?
7. Find the diameter of the foot of a tree if it needs 5.809 m long rope to tie it around.

19.2. Area of a Circle

1. Formula for area of a circle

Activity 1

Take a circular piece of paper and cut it into equal pieces and combine the pieces as shown in the figure below.



Make the pieces smaller. The pieces when combined, as shown above will make a rectangular shape. In this rectangle,

length = half of circumference = $\frac{1}{2} \times 2\pi r = \pi r$ and

breadth= radius of the circle = r

If the radius is r then the area of the circle = area of rectangle

$= \text{length} \times \text{breadth}$

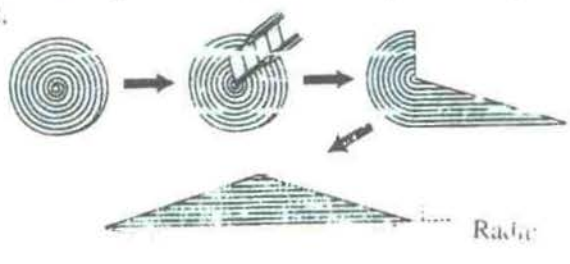
$= \pi r \times r$

$= \pi r^2$ $\qquad \therefore$ area of circle (A) = πr^2

2. Alternative method

Activity 2

Fix a nail on a plane wooden surface. Reel the thread around it and form a circular shape as shown in the figure. Cut out the shape and set it into the triangular shape as shown below.



Here, base of the triangle = circumference of the circle = $2\pi r$
height of the triangle = radius of circle = r.
and area of circle = area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 2\pi r \times r = \pi r^2$$

Example 3

What is the area of the circle whose radius is 7 cm ?

Solution :

Here, radius of circle r = 7 cm

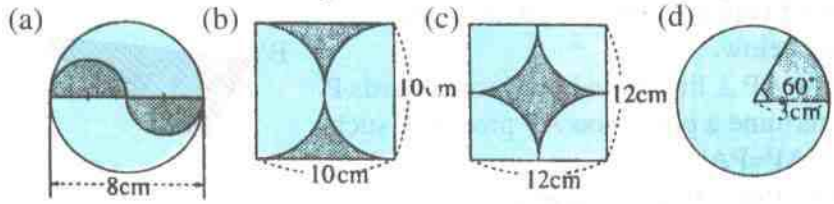
$\therefore$ area of circle $A = \pi r^2 = 3.14 \times (7\text{ cm})^2$
$= 153.86\text{ cm}^2$

EXERCISE 19 [B]

1. Find the area of the circle in each of the following :

(a)		(b)	
	radius = 8 cm		diameter = 4 cm
	radius = 7 cm		diameter = 2.8 cm
	radius = 6.5 cm		diameter = 18 cm
	radius = 5.6 cm		diameter = 14 cm
	radius = 5 cm		diameter = 30 cm

2. The bottom of a milk can is circular and its diameter is 14 cm. What surface will it occupy on the table ?
3. The bottom of a funnel is circular and its diameter is 6 cm. What will be the area of the bottom ?
4. Find the area of the shaded portion in each of the following.



5. If the area of a circle is 78.5 cm^2, find its
   (a) radius and (b) circumference
6. Circumference of a circle I 62.8 cm., find its radius and area.
7. A goat tethered to a stake by a rope has grazed the meadow over an area of 12.5 m^2 around it stretching the rope.
   (a) What is the length of the rope ?
   (b) What is the perimeter of the meadow ?

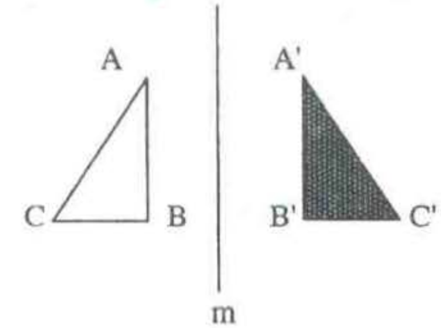
UNIT 20

TRANSFORMATION

20.1. Reflection

Reflection is a transformation. You have learnt elementary concepts of reflection in previous classes. Here you will learn some more about reflection.

Reflection is a transformation about a fixed line called an axis. In the figure, ΔABC is reflected about the line m (which is called the axis of reflection) and an image $\Delta A'B'C'$ is formed.



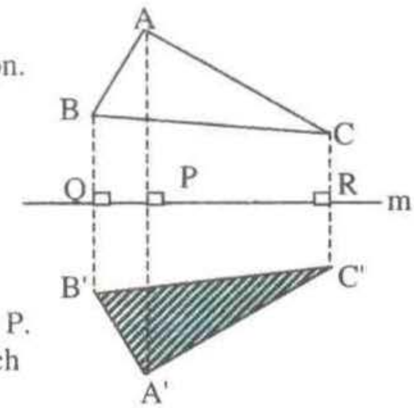
On the basis of this example, you can notice the following facts.

If any geometrical object is reflected then

- the object and its image lie on equal or same distance from the axis of reflection.
- object is congruent to its image.
- object and image are reverse of each other.

Example 1

In the adjoining figure, m is the axis of reflection. Reflect ΔABC about the line m geometrically and sketch its image.



Solution:

Method of reflecting the object about an axis is described below.

(i) Draw $AP \perp$ line m and produce towards P. Determine a pint A' on AP produced such that AP=PA'.

(ii) Draw $BQ \perp$ line m and produce it towards Q. Mark a point B' on BQ produced such that BQ = QB'.

(iii) Similarly, draw $CR \perp$ line m and mark C' such that CR = RC'.

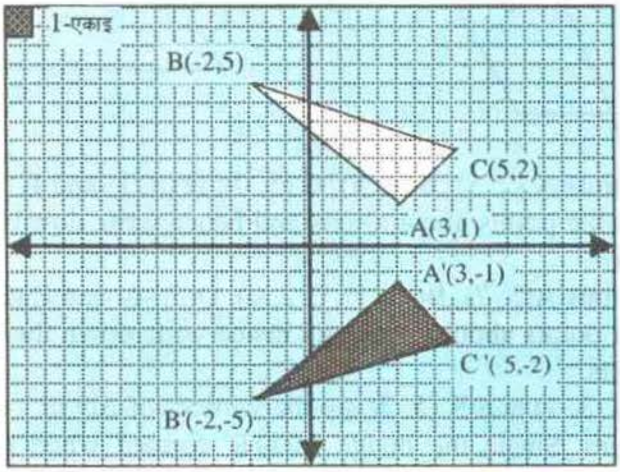
(iv) Join the points A', B' and C' successively so that Δ A'B'C' is the required image.

Example 2

A (3, 1), B (-2, 5) and C (5, -2) are the vertices of ΔABC. Plot the points in the graph. Reflect ΔABC about x-axis and hence write down the vertices of image triangle.

Solution :

Here ΔABC is reflected about x-axis and shown in the graph below.

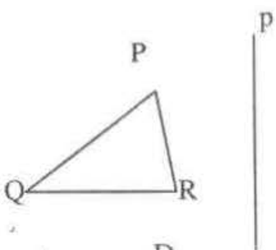


ΔA'B'C' in the image of ΔABC when reflected on x-axis. From the graph, the vertices of the image are found to be: A'(3, -1), B'(-2, -5) and C' (5, -2)

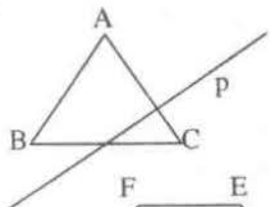
EXERCISE 20 [A]

1. Copy the following objects in your exercise book and reflect them about the given line P and hence shown the images geometrically.

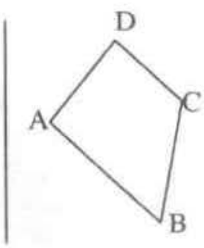
(a)



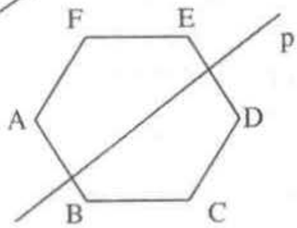
(b)



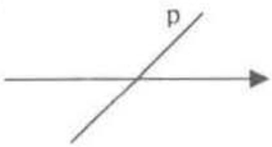
(c)



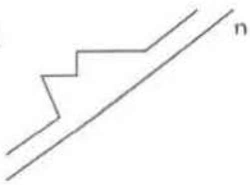
(d)

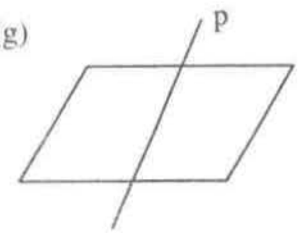


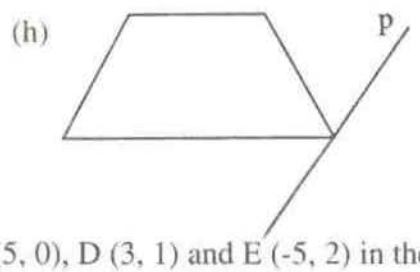
(e)



(f)



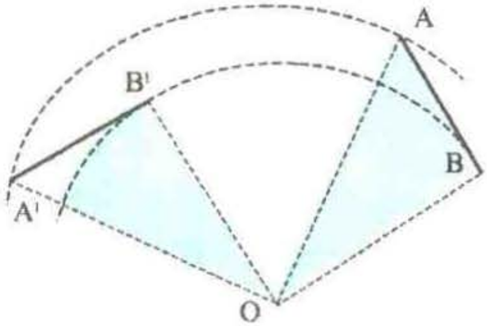




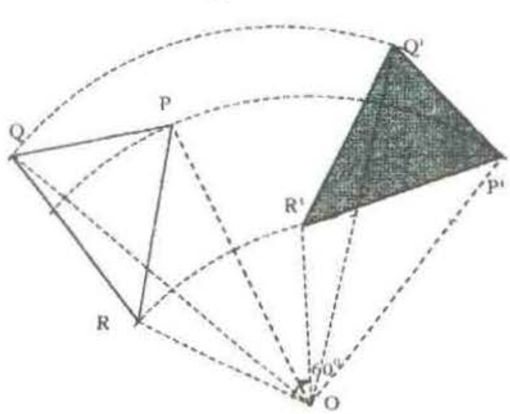
2. Plot the points A (2, 5), B (1, 5), C (5, 0), D (3, 1) and E (-5, 2) in the graph and reflect the points about x-axis and hence write down the coordinates of the images.
3. Plot the points A(5, 7), B(-3, 1), C(2, 0), D(3, -6) and E (5, -7) in the graph. Reflect the points about y-axis and hence write down the coordinates of the image points.
4. P(-1, 3), Q (3, 1) and R (5, 2) are the vertices of ΔPQR - sketch the triangle in the graph. Shade the image ΔP'Q'R' formed when ΔPQR is reflected about x-axis and hence write down the coordinates of the image points.
5. Sketch the quadrilateral WXYZ with vertices W (3, 1), X (2, 5), Y (1, 7) and Z (-3, 4) in the graph. Reflect it on Y-axis. Shade the image.

20.2. Rotation

Rotation is a transformation of an object in a direction about an angle. In the adjoining figure, segment AB is rotated in anticlockwise direction about the point O about an angle.



A'B' is the image of segment AB. In the figure, ΔPQR is rotated through 60^0 about the point O in clockwise or negative direction and ΔP'Q'R' is the image of ΔPQR.



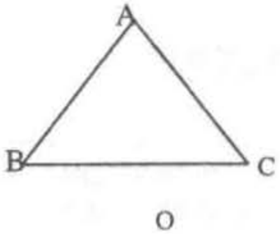
Measure the sides of ΔPQR and ΔP'Q'R' verify.

On the basis of the example above, can you conclude the following facts ?
If the point, angle and direction of rotation are known (or given) then the object can be rotated and its image can be determined.

- Rotation in clockwise direction is known as negative rotation and rotation in anticlockwise direction is known as positive rotation.
- In rotation, each element of the object is transformed in the same direction and by the same distance.
- In rotation, object and image are congruent. Study the examples given below.

Example 1

Rotate the triangle ABC given in the adjoining figure in negative direction about the point O through an angle of 60^0 and sketch the image.



Solution :

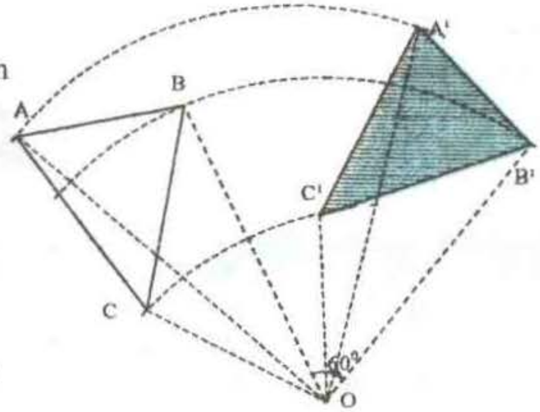
Method of rotating an object is described below.

(i) Join O to A. From O, taking OA as radius draw an arc through A in clockwise direction such that $\Delta AOA' = 60^0$.

(ii) Join O, B. Taking OB as radius, from O, draw an arc through B in clockwise direction such that $\Delta BOB' = 60^0$.

(iii) Similarly join O,C. Draw an arc CC' from O such that $\Delta COC' = 60^0$.

(iv) Join A', B' and C' successively. $\Delta A'B'C'$ is the image of ΔABC rotated about the point O is clockwise direction through me angle of 60^0.

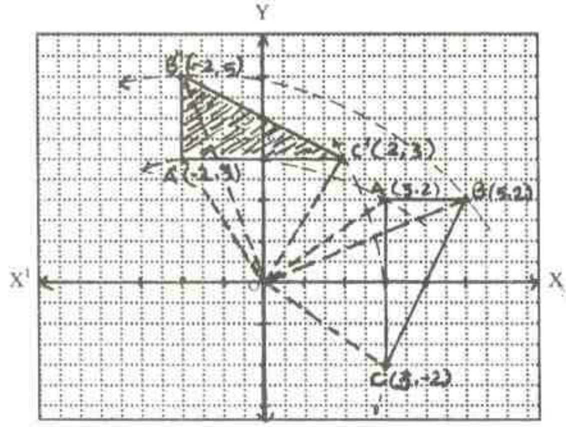


Example 2

Sketch ΔABC with vertices A (3, 2), B (5, 2) and C (3, -2) in the graph. Rotate ΔABC about the origin O through 90^0 in positive direction. And hence write down the vertices of $\Delta A'B'C'$.

Solution :

Here, ΔABC is sketch in the graph as shown below. ΔABC is rotated about the point O in anticlockwise direction through an angle of 90^0 and ΔA'B'C' is the image.



Object and image are shown in the graph.

Vertices of image are A' (-2, 3), B' (-2, 5), C' (2, 3).

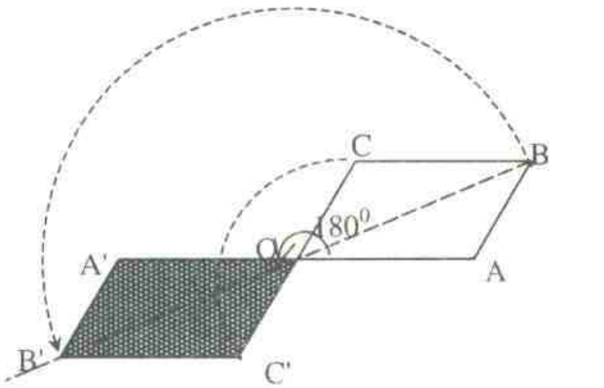
Example 3

Rotate the quadrilateral OABC is the given figure about the point O through an angle of 180^0. Draw the image formed by rotation.

Solution :

Here, the image formed by rotating the given quadrilateral OABC is shown in the figure below.

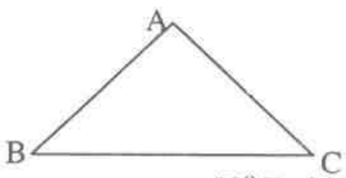
Here, OA'B'C' is the image of quadrilateral O ABC.



EXERCISE 20 [B]

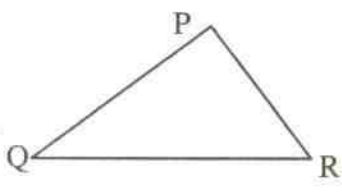
1. Copy the figures given below in your exercise book. Rotate them about the point O through the given angle in the given direction. Draw the image formed by rotation.

(a)



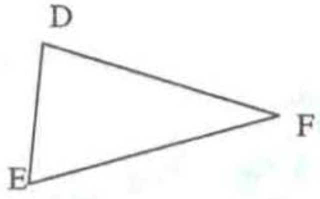
90^o Positive

(b)



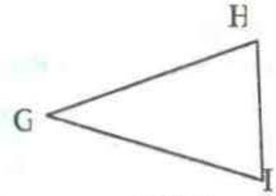
90^o Negetive

(d)



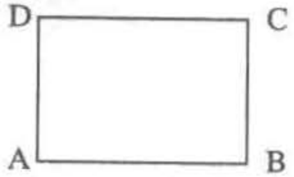
60° **Positive**

(e)



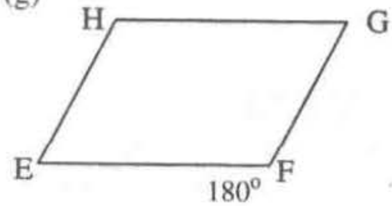
120° **Negative**

(f)



120° **Negative**

(g)

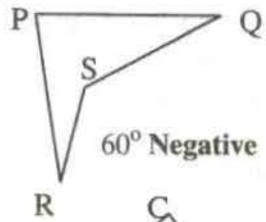


180° **Positive**

(h)

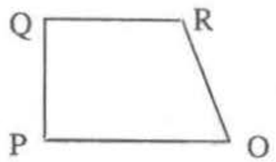
180° **Negative**

(i)



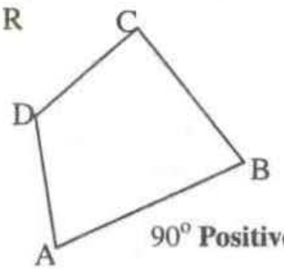
60° **Negative**

(j)



120° **Positive**

(k)



90° **Positive**

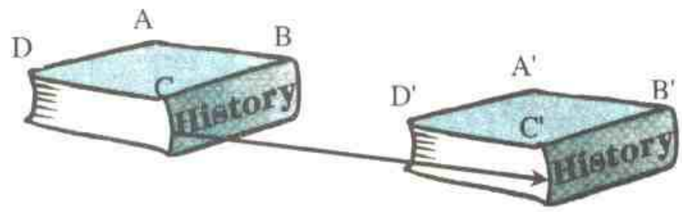
2. Rotate the segment AB joining the points A (2, 3) and B (-1, 7) about origins O through 90^{0} in anticlockwise direction. Write down the vertices of the image formed by rotation.

3. P (3, 1), Q (5, 0) and R (3, -1) are the vertices of a triangle. Sketch ΔPQR in graph and rotate it about origin O through 90^{0} in clockwise direction. Label the image so formed and hence write down the coordinates of its vertices.

4. A (1, 5), B (5, 5), C (6, -5) and D (1, -2) are the vertices of a quadrilateral ABCD. Plot the points in the graph and rotate the quadrilateral about origin O through an angle of 180^{0}. Write down the coordinates of the vertices of the image.

20.3. Displacement

In the figure, there is a history book at the position ABCD. The book is pushed forward to rest at the position A'B'C'D'. What is the relation between AA', BB', CC' and DD' ?

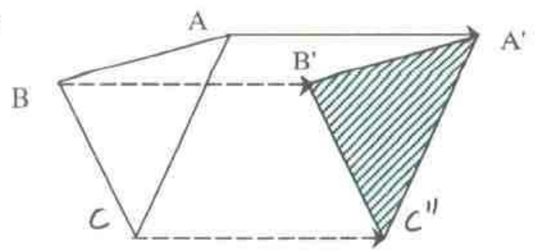


Activity 1

(i) Place a set square on your copy and trace it. Name the tracked triangle as ΔABC.

(ii) Place the set square as in the previous position and push it forward.

(iii) Again trace the set square to form $\Delta A'B'C'$.



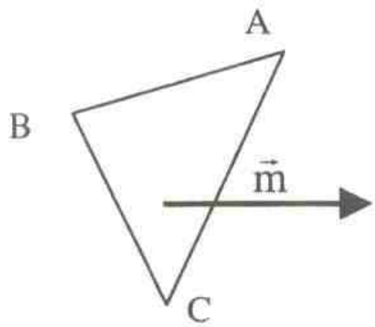
(iv) Now, join AA', BB' and CC'. Measure the segments.
Are AA' = BB' = CC' ?
Are AA' ∥ BB' ∥ CC' ?

Thus, if each point of an object on a place surface is moved along the same distance and same direction then the transformation is called displacement or translation. Therefore, for displacement, the direction and the distance should be mentioned. A quantity having magnitude and direction is called a vector.

In displacement, Object and image are congruent study the examples below.

Example 1

Displace ΔABC given in the figure by the vector $\vec{m}$.



Solution :

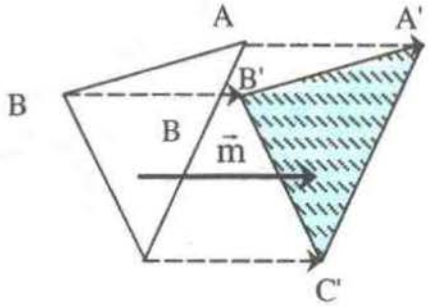
Method of displacing ΔABC is described below.

(i) Through A, draw AA' such that AA' ∥ m and AA' = m.

(ii) Though B, draw BB' ∥ m and BB' = m.

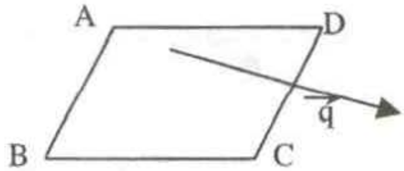
(iii) Similarly, through C, draw CC' ∥ m, CC' = m.

(iv) Join A', B', C' respectively. Now, ΔA'B'C' is the displaced image of ΔABC.



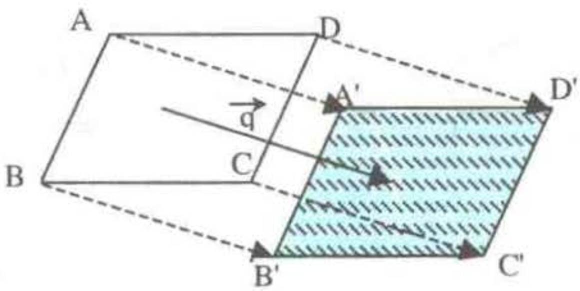
Example 2

Displace quadrilateral ABCD given in the adjoining figure by the vector $\vec{q}$.



Solution :

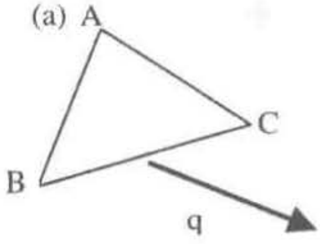
Here, quadrilateral ABCD is displaced by the vector $\vec{q}$ and shown in the figure below.



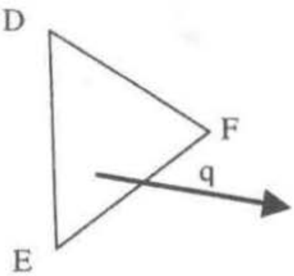
EXERCISE 20 [C]

1. Copy the figures given below in your exercise book and transform them by the displacement of vector $\vec{q}$. Shade the image formed by displacement.

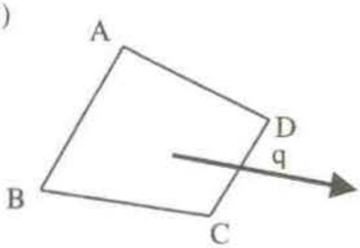
(a)



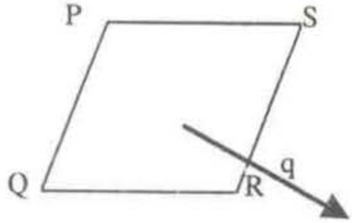
(b)



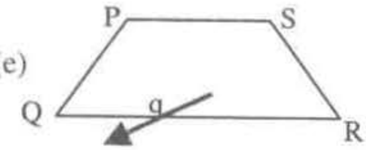
(c)



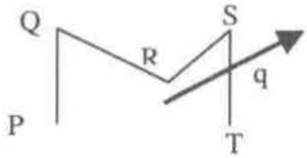
(d)



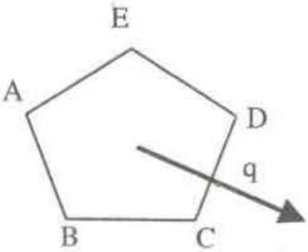
(e)



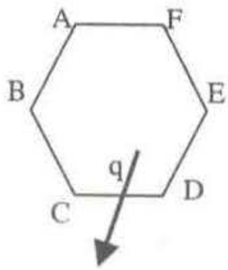
(f)



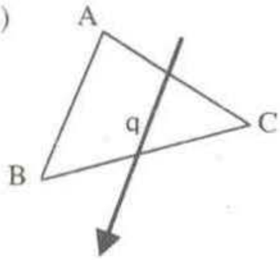
(g)



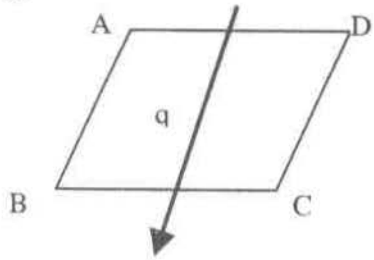
(h)



(i)



(j)



UNIT 21

BEARING AND SCALE DRAWING

21.1. Bearing

In the adjoining figure, different directions as shown by a compass are sketched. Study the figure and determine the angles between the directions (in 3 digits).

(i) North (N) and Northeast (NE) = 045^0

(ii) North (N) and East (E)

(iii) North (N) and Southeast (SE)

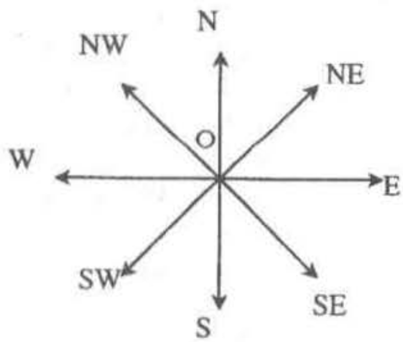
(iv) North (N) and South (S)

(v) North (N) and Southwest (SW)

(vi) North (N) and West (W)

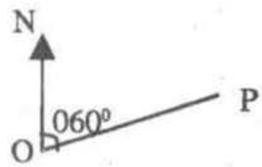
(vii) North (N) and Northwest (NW)

(viii) North (N) and North (N)



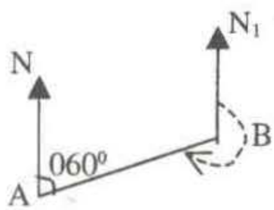
Which direction is common in these questions ?

Considering the line directed towards the north as an initial line (or the base line), the process of expressing the distance of any point in clockwise (negative) direction in 3 digits is known as three figure bearing or compass bearing. In the figure, the bearing of P from O is denoted by 060^0. Study the example below.



Example 1

If the bearing of B from A is 060^0, what will be the bearing of A from B ?



Solution :

From the figure,

bearing of A from B = 360^0 - interior $\angle$N, BA

$= 360^0 - 120^0 = 240^0$

EXERCISE 21 [A]

1. Express each of the following compass bearings in term of an angle,

(a) north east (NE)	(b) north-north-east (NNE)
(c) north west (NW)	(c) north-north-west (NNW)

2. An aeroplane was flying in the bearing of 095^0 from Kathmandu. After flying over some distance it changed its direction and flew along the bearing of 300^0. By what angle has the aeroplane changed its direction ?

3. The bearing of Kathmandu from Pokhara is 090^0. What is the bearing of Pokhara from Kathmandu ?

4. In the adjoining figure, some places of Central Development Region are shown. Trace the map in your exercise book and find the 3 digit bearing of the following places from Kathmandu.

(a) Gosainkund

(b) Tatopani

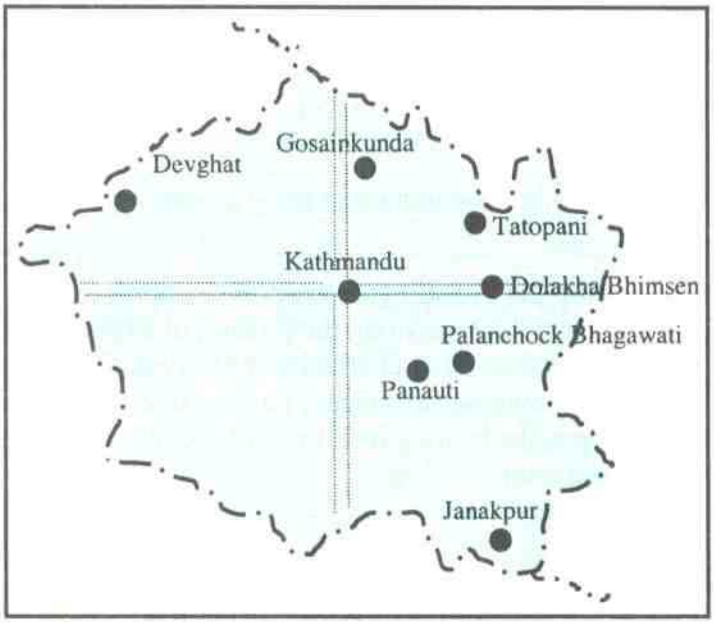
(c) Devghat

(d) Dolakha Bhimsen

(e) Palanchowk - Bhagawati

(f) Panauti

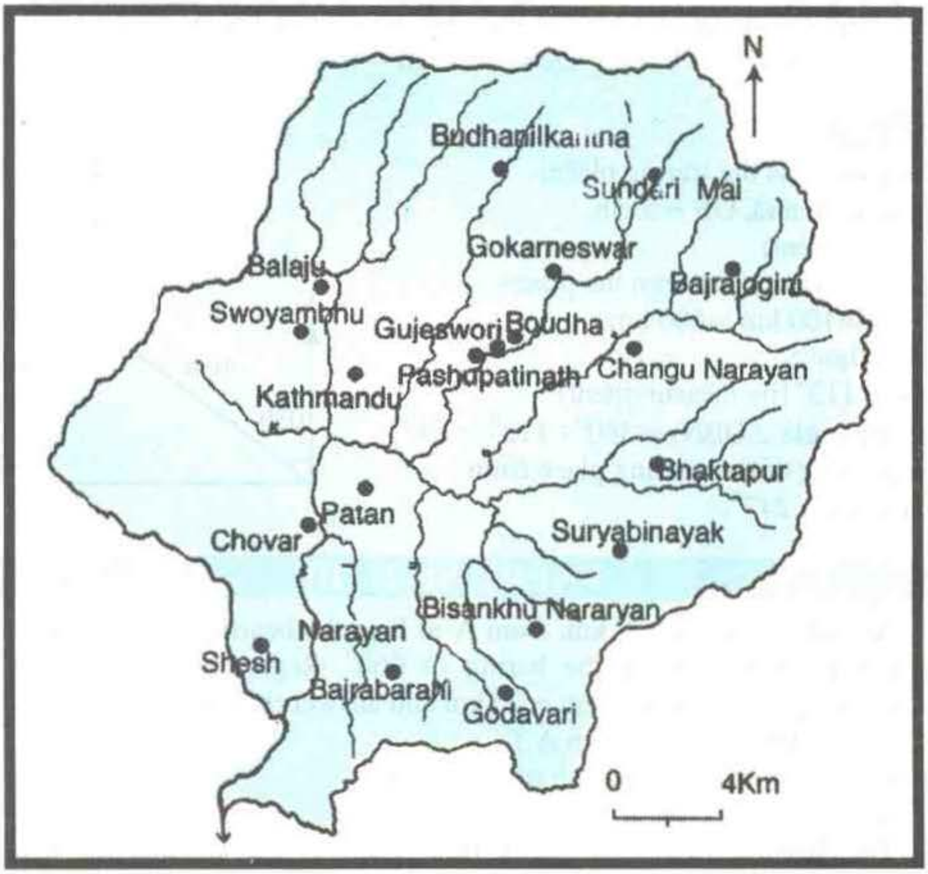
(g) Janakpur



21.2. Scale Drawing

(a) Introduction to scale drawing

Kathmandu Valley (Religious Places)



In the map above some important religious places in Kathmandu valley are sketched in a scale of 1 cm = 2 km (1:2000000). Measure the distance between the places below (in the map) in cm. scale and find the actual distance between them

(i) Balaju and Budhanilkantha = 4 cm (map - distance)
$\therefore$ actual distance = 4×2 km = 8 km

(ii) Balaju and Kathmandu

(iii) Patan and Kathmandu

(iv) Suryavinayak and Changunarayan

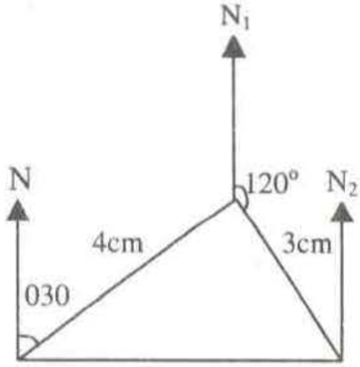
(v) Pashupatinath and Gokarneshwor

(b) Scale drawing with bearing

Example 2

If an aeroplane flies 400 km in the bearing of 030^0 and then 300 km in the bearing of 120^0, find the distance between the two places and the bearing of the starting place from the least place using the scale 1 cm = 100 km.

Solution :



In the figure, O is the staring place, last place is B. and, OB = 5 cm. (by measurement)

Therefore, distance between the places

$= 5\times100$ km = 500 km.

From the figure,

$\angle OBN_2 = 113^0$ (by measurement)

$\therefore$ exterior angle $\angle OBN_2 = 360^0 - 113^0 = 247^0$

Hence, bearing of the starting place from the last place $= 247^0$.

EXERCISE 21 [B]

1. An aeroplane flies 80 km. from A to B in the bearing of 080^0 and then 100 km from B to C in the baring of 060^0. Represent this information by drawing in a scale of 1 cm = 20 km and answer the following questions :
   (a) How far is C from A ?
   (b) What is the bearing of C from A ?
   (c) What is the bearing of A from C ?
2. The distance of Suryabinayak from Patan is 9.6 km and the distance of Godavary from Suryabinayak is 6.8 km. Drawing in a scale of 1 cm = 2 km, find
   (a) the distance of Godavary from Patan.
   (b) the bearing of Godavary from Patan.
   (c) the bearing of Patan from Godvary.
   [Hint: use the map of Kathmandu valley in page 181]
3. From C, a surveyor observes two places, named A and B, 120 km apart and finds $\angle ABC = 65^0$ and $\angle BAC = 55^0$ using a suitable scale, represent it by a drawing and find the following :
   (a) the distance from A to C. (b) the distance from B to C.
4. A motor is at S, a place 200 km east of T. Traveling a distance of 300 km to the due west of S, it reached at U. Express this information by scale - drawing and find
   (a) the distance from T to U. (b) the bearing of T from U.

ANSWER SHEET

UNIT 1: SET

Exercise 1 (A)

1. (i) $E \subset U$ (ii) $F \subseteq U$ (iii) $G \subset U$
   Show the veen-diagram to the teacher.
2. (i) $A \subset M$ (ii) $B \subseteq M$ (iii) $C \subset M$
3. (i) $B \subset A$ (ii) $C \subset A$
4. $B = \{3, 6, 9, 12, 15\}$, proper subsets $\{3, 6, 9\}$, $\{12, 15\}$ etc.
   Improper subset $\{3, 6, 9, 12, 15\}$

Exercise 1 (B)

1. (a) $A = \{2, 3, 4, 5\}$ (b) $B = \{2, 3, 6, 7\}$ (c) $\overline{A} = \{0, 1, 6, 7, 8, 9\}$
   (d) $\overline{B} = \{0, 1, 4, 5, 8, 9\}$ (e) $\overline{A \cap B} = \{0, 1, 4, 5, 6, 7, 8, 9\}$
   (f) $\overline{A} \cap \overline{B} = \{0, 1, 8, 9\}$ (g) $\overline{A \cup B} = \{0, 1, 8, 9\}$
   (h) $\overline{A} \cup \overline{B} = \{0, 1, 4, 5, 6, 7, 8, 9\}$ (i) $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
   (j) $\overline{U} = \phi$
2. (a) $= \{0, 1, 8, 9\} = \overline{A \cup B} = \overline{A} \cap \overline{B}$
   (b) $\overline{A \cap B} = \{0, 1, 4, 5, 6, 7, 8, 9\} = \overline{A} \cup \overline{B}$
3. (a) Show your work to the teacher. (b) $A \supset B$ or $A \cap B = B$ or $A \cap \overline{B} = B$
4. (a) Show your work to the teacher.
   (b) i) $\overline{A} = \{4, 5, 7, 8, 9, 10\}$ ii) $\overline{B} = \{3, 5, 6, 7, 9, 10\}$
   iii) $\overline{C} = \{2, 4, 5, 6, 7, 8, 10\}$ iv) $\overline{A \cup B} = \{5, 7, 9, 10\}$
   v) $\overline{B \cup C} = \{5, 6, 7, 10\}$ vi) $\overline{A \cup C} = \{4, 5, 7, 8, 10\}$
   vii) $\overline{A \cap B} = \{3, 4, 5, 6, 7, 8, 9, 10\}$ viii) $\overline{A \cap C} = \{2, 4, 5, 6, 7, 8, 9, 10\}$
   x) $\overline{A \cup B \cup C} = \{5, 7, 10\}$ xi) $\overline{A \cap B \cap C} = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
5. (a) $\overline{A} = \{1, 3, 5\}$ (b) $\overline{B} = \{2, 4)$ (c) $\overline{C} = \{1, 4, 5\}$
   (d) $\overline{\overline{A}} = \{2, 4\}$ (e) $\overline{\overline{B}} = \{1, 3, 5\}$
   (f) $\overline{\overline{C}} = \{2, 3\}$ Show the Venn-diagram to the teacher.

Exercise 1 (C)

1. Show your work to the teacher.
2. $A - B = A - (A \cap B)$
3. Show your work to the teacher.
4. (a) True (b) False (c) False (d) True

UNIT 2: WHOLE NUMBER

Exercise 2 (A)

1.	(a) 110010101	(b) 1000010111	(c) 111111	(d) 10011010010
2.	(a) 14	(b) 31	(c) 50	(d) 21

Exercise 2 (B)

1.	(a) 111_2	(b) 111_2	(c) 1111_2	(d) 1111_2
	(e) 1000_2	(f) 1001_2	(g) 10101_2	(h) 10000_2
	(i) 10000_2	(j) 10010_2	(k) 11000_2	(l) 101000_2
2.	(a) 1_2	(b) 10_2	(c) 100_2	(d) 1001_2
	(e) 10_2	(f) 11_2	(g) 1110_2	(h) 111_2
	(i) 110_2	(j) 100_2	(k) 10_2	(l) 1111_2
3.	(a) 1011_2	(b) 1111_2	(c) 111_2	(d) 10000_2
	(e) 1111_2	(f) 0		

Exercise 2 (C)

1.	(a) 51	(b) 99	(c) 551	(d) 329
2.	(a) 4400_5	(b) 4143_5	(c) 10401_5	(d) 34333_5
3.	(a) 100_5	(b) 111101001_2	(c) 103_5	(d) 1001001010_2

Exercise 2 (D)

1.	(a) 1010	(b) 1000	(c) 344	(d) 1413
	(e) 1324	(f) 1120	(g) 11230	(h) 10131
	(i) 12200	(j) 11000	(k) 4031	(l) 13012
2.	(a) 222	(b) 110	(c) 4001	(d) 444
	(e) 3204	(f) 3414	(g) 2403	(h) 243
	(i) 442	(j) 2443	(k) 404	(l) 2334
3.	(a) 131_5	(b) 11_5	(c) 204_5	(d) 40_5
	(e) 312_5	(f) 23_5	(g) 40_5	(h) 30_5

UNIT 3: SQUARE ROOT AND CUBE ROOT

Exercise 3 (A)

1.	(a) 16	(b) 25	(c) 13	(d) 26
	(e) 22	(f) 58	(g) 32	(h) 75
	(i) 45	(j) 57	(k) 35	(l) 81
2.	(a) 85	(b) 124	(c) 426	(d) 252
	(e) 168	(f) 123	(g) 407	(h) 253
	(i) 108	(j) 54	(k) 67	(l) 95
	(m) 309	(n) 1346	(o) 807	(p) 905
3.	(a) 6	(b) 96	(c) 225	(d) 45
	(e) $3\sqrt{2}$	(f) $5\sqrt{2}$	(g) $4\sqrt{5}$	(h) $12\sqrt{7}$
	(i) 6	(j) 8	(k) 12	(l) 10

Exercise 3 (B)

1.	(a) 27	(b) 729	(c) 1331	(d) 2197
	(e) 373248	(f) 5929741	(g) 1259712	(h) 1520875
	(i) 2924207	(j) 238328000	(k) 343000000	(l) 274625000
2.	(a) 12	(b) 16	(c) 21	(d) 22
	(e) 15	(f) 14	(g) 36	(h) 45
	(i) 28	(j) 80	(k) 90	(l) 320

UNIT 4: RATIONALIZATION

Exercise 4 (A)

1.	(a) $\frac{4\sqrt{7}}{3}$	(b) $\frac{\sqrt{6}}{2}$	(c) $\frac{\sqrt{10}}{5}$	(d) $\frac{\sqrt{6}}{3}$
	(e) $\frac{4\sqrt{6}}{3}$	(f) $\frac{3\sqrt{10}}{2}$	(g) $\frac{\sqrt{3}}{2}$	(h) $\sqrt{3}$

Exercise 4 (B)

1.	(a) $8\sqrt{6}$	(b) $3\sqrt{5}$	(c) $-5\sqrt{3}$	(d) $-2\sqrt{2}$
	(e) $-6\sqrt{7}$	(f) $6\sqrt{10}$		
2.	(a) $3\sqrt{10}$	(b) $3\sqrt{3} - 4\sqrt{2}$	(c) $\sqrt{7} - 1$	(d) $8\sqrt{2} - 2\sqrt{3}$
3.	(a) $2\sqrt{5}$	(b) $5\sqrt{2}$	(c) $7 - 2\sqrt{7}$	(d) $6\sqrt{3} - \sqrt{2}$
	(e) $-4\sqrt{2}$	(f) $-4\sqrt{5} - 5\sqrt{2}$		
4.	(a) $\frac{3\sqrt{5}}{10}$	(b) $\frac{\sqrt{7}}{2}$	(c) $7\sqrt{3}$	(d) 0
	(e) $4\sqrt{3} - 3\sqrt{2}$	(f) $13\sqrt{7} - 8\sqrt{5}$		

Exercise 4 (C)

1.	(a) 6.2×10^6	(b) 3.8×10^8	(c) 3.28×10^7	(d) 3.6×10^5
	(e) 2.76×10^2	(f) 5.2×10^{-5}	(g) 8.3×10^{-7}	(h) 9.9×10^{-7}
2.	(a) 3800000	(b) 25000	(c) 37200000	(d) 250000
	(e) 44600000	(f) 0.000025	(g) 0.00000037	(h) 0.0000000028
	(i) 0.000000000315	(j) 0.000014		
3.	(a) 4.69×10^2	(b) 1.01×10^7	(c) 1.01×10^{-4}	(d) 8.24×10^3
4.	(a) 3.375×10^3	(b) -1.2×10^2	(c) 3.676×10^6	(d) -6.105×10^6
5.	(a) 9×10^9	(b) 5×10^9	(c) 9.50017×10^7	(d) 1.4×10^{11}
	(e) 1.2×10	(f) 4×10	(g) 5.78×10^2	
6.	(a) 2.5×10^7	(b) 4.27×10^7	(c) 6.66×10^{10}	(d) 10^{-4}
	(e) 10^{-12}			

UNIT 5: RATIONAL AND IRRATIONAL NUMBERS

Exercise 5 (A)

1. (b) $-\sqrt{2}$ (d) $\sqrt{5}$ (e) $\sqrt{7}$ (h) $\frac{\sqrt{10}}{5}$
2. (a) False (b) True (c) True (d) True (e) False (f) False

Exercise 5 (B)

1. (a) 1.06 (b) 0.271 (c) 0.005 (d) 28
   (e) 0.047 (f) 143.5 (g) 260.00
2. 3.143
3. (a) 0.67 (b) 0.667 (c) 0.6667
4. (a) 11.19 cm^2 (b) 11.2 cm^2

UNIT 6: RATIO, PROPORTION AND PERCENTAGES

Exercise 6 (A)

1. (a) $\frac{4}{7}$ (b) $\frac{3}{5}$ (c) $\frac{9}{20}$ (d) $\frac{8}{3}$ (e) $\frac{3}{8}$ (f) $\frac{1}{240}$
2. (a) 1 (b) 5 (c) 9 (d) 36
3. (a) $a = \frac{3b}{4}$ (b) $c = \frac{7b}{5}$ (c) 15:28
4. 60 Pcs. 5. 2 km 6. Rs. 22000
7. 5 and 7 8. Rs. 24, Rs. 20 and Rs. 16
9. Rs. 120000, Rs. 200000 and Rs. 280000
10. A - Rs. 70, B - Rs. 140 and C - Rs. 280
11. (a) First - 24 and Second - 36 persons (b) 4:6:10:13

Exercise 6 (B)

1. (a) Proportional (b) Proportional (c) Non-proportional (d) Non-proportional
2. (a) 8 (b) 14 (c) 1 (d) 18
3. (a) 1 (b) 5 (c) 9 (d) 36
4. 80 5. Rs. 140 6. 5 hours 7. Rs. 270
8. 20 men 9. 12 N 10. (a) 93: 155 (b) 93:155:34

Exercise 6 (C)

1. (a) Rs. 2000 (b) Rs. 2000 (c) 400 days (d) 2400 persons
   (e) 200 persons (f) 2 liter (g) 50 m
2. 10 persons, 200 persons 3. (a) 80% (b) 20% (c) 60 persons
4. Sunday - 644, Monday - 623, Tuesday - 651, Wednesday - 665
   Thursday - 672, Friday - 581

5. (a) Food - Rs. 2100, Cloth - Rs. 1260, Education - Rs. 1750, Medicines - Rs. 350 Others - Rs. 840 (b) 10% (c) Rs. 700
6. (a) 20000 persons (b) 13000 persons 7. 24449400 persons
8. Rs. 1600 9. Rs. 300000 10. 118800 persons 11. Rs. 10120
12. Pant is cheaper in sale price by Rs. 3.25
Shirt is cheaper in discount price by Rs. 5
Tracksuit is cheaper in discount price by Rs. 0.10

UNIT 7: PROFIT AND LOSS

Exercise 7

1. Loss Rs. 175	2. Profit = $33\frac{1}{3}\%$	3. Discount = 4.15%
4. Rs. 45	5. Rs. 10450	6. Rs. 162000
7. Rs. 5400 (SP)	8. 25% Profit	9. Rs. 1280
10. Rs. 720	11. Profit Rs. 450, 15%	12. Loss = 20%
13. Rs. 20000	14. (a) Rs. 1257.14	(b) Rs. 1320

UNIT 8 : UNITARY METHOD

Exercise 8

1. Rs 45	2. Rs 1100	3. 13 person	4. Rs. 900
5. 80 person	6. 18 person	7. 800 minute	8. 3 day, 21 day,
9. 2 person, Rs 2550	10. (i) 180 km (ii) 13 litre		

UNIT 9: SIMPLE INTEREST

Exercise 9

1. Rs 6625	2. 12% per year	3. P = Rs 3200, A = Rs 3240
4. Rs 1250	5. 6 years	6. Rs 7200
7. Rs 55,500	8. Rs 600	9. 10% per annum

UNIT 10: STATISTICS

Exercise 10.1

Show your works to the teacher.

Exercise 10.2

1. (a) 6.75 (b) 15 cm (c) Rs. 24 (d) 13 kg (e) 36.5 m
2. (a) 5.6 min. (b) Rs. 252
3. (a) Rs. 2096.10 (b) (i) Rs. 62883 (ii) Rs. 765076.50

(c) Rs. 229522.95 4. 42.4 kg. 5. 10.6 years 6. 32 boys

7. (a) 2.9 minutes (b) 16 (c) 8.61

Exercise 10.3

1. (a) Median = 9; Mean = 8.8 (b) Median = Rs. 5 and Mean = Rs. 5
   (c) Median = 3 kg.; Mean = 4 kg. (d) Median = 4.5 cm; Mean = 5 cm
   (e) Median = 6.5; Mean = 7
2. (a) 5 (b) 4/4 students

Exercise 10.4

1. (i) $Q_1 = 52$, $Q_2 = 61$ and $Q_3 = 68$ (ii) $Q_1 = 5$, $Q_2 = 7$ and $Q_3 = 11$
   (iii) $Q_1 = 40$, $Q_2 = 100$ and $Q_3 = 140$ (iv) $Q_1 = 3$, $Q_2 = 4$ and $Q_3 = 4$
2. $Q_1 = 41.5$, $Q_2 = 47.5$ and $Q_3 = 73.25$
3. $Q_1 = 14.5$, $Q_2 = 17.5$ and $Q_3 = 20.25$

Exercise 10.5

1. (a) 6 (b) Rs. 3 (c) 5 cm (d) 13 kg. (e) 12 cm **2.** 25 cm.

Exercise 10.6

1. (a) 120 cm (b) 95 cm (c) 25 cm 2. (a) Rs. 27 (b) Rs. 10 (c) Rs. 17
3. (a) 5 (b) 22 cm (c) 45 kg. (d) 7 liter

Exercise 10.7

1. (a) Class 3 (b) 280 student (c) 1440 student
2. Show your work to the teacher.
3. Show your work to the teacher.
4. Show your work to the teacher.

Exercise 10.8

1. Show your work to the teacher.
2. Show your work to the teacher.
3. (a) 80 notes of 1000 (b) 45 each of 5^5 and 10^5; 30 each of 20^5 and 25^5
4. 8 sq. unit

UNIT 11: ALGEBRAIC EXPRESSIONS

Exercise 11 (A)

1. (a), (b), (d) 2. (a) Trinomial (b) Binomial
   (c) Monomial (d) Binomial (e) Trinomial (f) Trinomial
3. (a) 2 (b) 3 (c) 3 (d) 5 (e) 6
4. (a) (i) 101, (ii) 12 (b) 38 (c) 19 (d) 2

Exercise 11 (B)

1. (a) $x^2 + 5x + 6$ (b) $p^2 + 2p - 3$ (c) $a^2 - 6a + 8$
(d) $r^2 - 2r - 48$ (e) $c^2 + 11c + 24$ (f) $x^2 - 4x - 96$
2. (a) $4a^2 + 9a - 9$ (b) $6x^2 - 29x + 35$ (c) $2x^2 + xy - 21y^2$
(d) $2a^2 + 9ab + 9b^2$ (e) $63x^2 + 58xy - 16y^2$ (f) $21x^2 - kx - 2k^2$
(g) $121x^2 - 144y^2$ (h) $-8p^2 + 30pq - 25q^2$ (i) $15p^2 - 44py + 32y^2$
(j) $acx^2 + adxy + bcxy + bdy^2$
3. (a) $2a^2 + 22a + 54$ (b) $-2x + 4$ (c) $x^2 + 10x - 66$ (d) $-4p^2 - 46p + 138$

Exercise 11 (C)

1. (a) $a^2 - 4$ (b) $x^2 - 9$ (c) $16 - p^2$ (d) $x^2 - y^2$ (e) $m^2 - n^2$ (f) $a^2 - b^2$
(g) $9x^2 - 4$ (h) $m^2 - 81$ (i) $16 - 25x^2$ (j) $x^2 - 9y^2$ (k) $k^2 - 4m^2$ (l) $25p^2 - t^2$
(m) $64h^2 - k^2$ (n) $100x^2 - 121y^2$ (o) $64m^2 - 81n^2$
2. (a) 99 (b) 2499 (c) 3596 (d) 6396 (e) 2491 (f) 6375

Exercise 11 (D)

1. (a) $am + an + ar + bm + bn + br$ (b) $pq + pr + 3p - 2q - 2r - 6$
(c) $3ax - 2x + cx - 3ay + 2y - cy$ (d) $2ax - bx - cx - 6a + 3b + 3c$
(e) $2a^2 - 3ab - 3ac - b^2 + 3bc$ (f) $9x^2 - 24xy + 3xz + 12y^2 - 6yz$
(g) $ax - ay - 2a^2 + ab + bx - by + 3b^2$
(h) $2px - 3py - ap + 2bp - 2qx + 3qy + aq - 2bq$
(i) $mx - my - 2am + 3bm - 2nx + 2ny + 4an - 6nb$
(j) $9x^2 - 9xy + 3ax + 3bx + 2y^2 - 2ay - 2by$
(k) $2p^2 + 5pq - pr - 25p + 3q^2 - qr - 25q$
(l) $10a^2 + ab - 20a^2b - 30abc - 9b^2 -18ab^2 + 27b^2c$
2. (a) $a^2 - b^2 - c^2 + 2bc$ (b) $2a^2 - 7ab - 3ac + 6b^2 + 4bc - 2c^2$
(c) $m^2 - 3mn + m + 2n^2 + n - 6$ (d) $5pr - 2p^2 - 11qr + 6q^2 + pq + 3r^2$
(e) $-10a^2 - 9b^2 + 2c^2 + 21ab + 3bc + ac$
(f) $12x^2 + 3y^2 + 2z2 - 13xy - 11xz + 7yz$
(g) $2x^2 - 3xy - 3ax + 2bx + y^2 + ay - by - 2a^2 + ab$
(h) $2a^2 - 5ac + 2ad + ad - 3b^2 - 5bc + 3bd + 2c^2 - cd$
(i) $2x^2 + 2y^2 + 12z2 + 10xz - ax + xy - 11yz + 2ay - 3az$
(j) $10m^2 - 180n^2 - 3s^2 - 191mn - 29ms - 10mp\ 47ns - 9px - ps$
(k) $12p^2 - 21q^2 + 4r^2 + 4pq + 26pr + 6ps + 25qr - 7qs - rs$
(l) $50a^2 - 72b^2 - 105c^2 - 80ad + 174bc - 96bd - 5ac + 120cd$

Exercise 11 (E)

1. (a) $2x - 3y + 6$ (b) $x^3 - 2x^2 + x$ (c) $5x^3 - 3x^2 + 5$
(d) $p^2 + 2pq + 3q^2$ (e) $x + 2xy + y$ (f) $2pq + 3p - 4q$
(g) $-5x^4 - 5x^3 + 10x$ (h) $-3x^3 + 2x^2y + 4xy^2$
(i) $4x - 1$ (j) $3 - \frac{2}{x}$

2. (a) $35x^2 + 5x - \frac{9}{2}$ (b) $4y^2 + 9y - 8$ (c) $42x + 3$ (d) $x - 4$

3. $4x + 3$ unit 4. $28x - 15y$ unit

Exercise 11 (F)

1. (a) $x + 4$ (b) $x - 3$ (c) $x + 7$ (d) $x - 4$ (e) $x + 5$
(f) $y - 4$ (g) $3x - y$ (h) $3x + 2y$ (i) $2x + 3y$ (j) $3x + 5y$
(k) $12x - 42y$ (l) $Q = 5x + 11y, R = -2y^2$ (m) $Q = 7x - y, R = -2y^2$
(n) $4x + 3y$ (o) $5x - 6y$ (p) $x^2 + 3x + 4$ (q) $x^2 - 4x - 6$
(r) $Q = 2x^2 - 4x + 1$ or $R = -16$

Exercise 11 (G)

1. (a) $a + 5$ (b) $a + 2$ (c) $x + 4$ (d) $x + 5$ (e) $x - 2$
(f) $y + 3$ (g) $2y - 1$ (h) $3p + 1$
(i) $3q + 5$ (j) $a^2 + 2a + 5$

Exercise 11 (H)

1. (a) a^5 (b) $-12p^6$ (c) x^2 (d) $-6y^{-1}$ (e) $10z^{-5}$ (f) khx^{m+3}

2. (a) P (b) x^{-5} (c) z (d) $-4a^{-1}$ (e) $\frac{3}{2}x^6$ (f) q^{3-2m}

3. (a) x^{12} (b) p^2q^2 (c) $x^8y^4z^{12}$ (d) $8x3y^{-3}z^{-3}$
(e) $p^{-4}q^4x^{-6}$ (f) $-27a^{-9}b^6c^{-12}$

4. (a) $-64m^5n^4$ (b) $-3a^{12}b^7$ (c) $-8x^{3p}y^{2a+3}$ (d) $\frac{4}{3}yz^{-3}$

5. (a) $\frac{1}{2}$ (b) -1 (c) $\frac{3}{8}$ (d) 12 (e) 1
(f) $-\frac{2}{3}$ (g) $\frac{81}{4}$ (h) $-\frac{9}{2}$ (i) $-\frac{2}{9}$

Exercise 11 (I)

1. (a) $5(x + 2)$ (b) $6(a + 2)$ (c) $2(2x + 5)$ (d) $3(3a - 4)$ (e) $n(n - 6)$
(f) $x(x + 8)$ (g) $8m(2m -2)$ (h) $4p(p + 4)$ (i) $y(12x + 1)$ (j) $y((4x - 1)$
(k) $7x(3y + 1)$ (l) $3y(y - 1)$

2. (a) $4(x^2 - 2x + 4)$ (b) $2(m^2 + 2m + 4)$ (c) $2p(2p^2 - 3p + 4)$
(d) $-4(x - 2)$ (e) $-5(2x + 1)$ (f) $-p(1 - 5p)$
(g) $-x(x + 5)$ (h) $-8x(2x - 1)$ (i) $-y(y - 5)$
(j) $-x(x^2 + 2x - 8)$ (k) $4x(15x3 - mx + 3)$
(l) $5y(6x^2y + 7x^2 - 8y)$ (m) $3pq(5p + 3 - 8q)$

Exercise 11 (J)

(a) $(x + 4)(x + 5)$ (b) $(x - 3)(x + 4)$ (c) $(x + 2)(4x + 3)$ (d) $(2x - 1)(x - 3)$
(e) $(p + q)(p + r)$ (f) $(r + 3)(r - 2)$ (g) $(a - b)(a + 1)$ (h) $(x + 5)(y + 2)$
(i) $(p + 5)(q + 3)$ (j) $(x + 2)(x - y)$ (k) $(x + 1)(y + 1)$ (l) $(a + 3)(a + b)$
(m) $(p - 5)(p + 3q)$ (n) $(1 - x)(1 - y)$ (o) $(p + 6q)(p - 2q)$ (p) $(a + bc)(ab + c)$

Exercise 11 (K)

1. (a) 3, 4 (b) 2 , 3 (c) -4 , -8 (d) -9 , 2
   (e) 3 , 10 (f) -3 , 2
2. (i) $(x + 2)(x + 5)$ (ii) $(x + 3)(x + 4)$ (iii) $(x + 5)(x + 6)$
   (iv) $(x - 3)(x - 5)$ (v) $(x - 5)(x - 7)$ (vi) $(x - 3)(x - 9)$
   (vii) $(x + 9)(x - 2)$ (viii) $(x + 7)(x - 3)$ (x) $(x - 3)(x - 10)$
   (xi) $(y + 5)(y - 6)$ (xii) $(p + 2)(p - 9)$ (xiii) $(y + 7)(y + 8)$
   (xiv) $(m + 1)(m - 5)$ (xv) $(x - 1)(x - 40)$ (xvi) $(p + 30)(p - 1)$
   (xvii) $(x + 3)(x - 13)$ (xviii) $(x - 3)(x - 13)$ (xix) $(x + 1)(x - 43)$
   (xx) $(x - 6)^2$ (xxi) $(p + 11)^2$ (xxii) $-(x + 11)(x - 2)$
   (xxiii) $-(x + 3)(x - 5)$ (xxiv) $-(p + 8)(p - 1)$
   (xxv) $-(x + 1)(x - 19)$ (xxvi) $-(p + 14)(p - 3)$
   (xxvii) $(x - y)(x - 2y)$ (xxviii) $(x + 5y)(x + 6y)$
   (xxix) $(p + 2q)(p - 3q)$ (xxx) $(x + 4y)(x - y)$

Exercise 11 (L)

1. (a) $(3x + 1)(x + 1)$ (b) $(2x + 3)(x + 1)$ (c) $(3x - 1)(x - 1)$
   (d) $(2m - 1)(2m - 3)$ (e) $(2x - 3)(2x + 5)$ (f) $(3x - 2)(5x - 1)$
   (g) $(4y - 1)(3y + 7)$ (h) $(5x + 1)(2x - 1)$ (i) $(6x - 1)(2x - 5)$
   (j) $(10a - 3)(5a - 1)$ (k) $(12b - 5)(5b + 2)$ (l) $(9x - 4)(3x - 2)$
   (m) $(7p - 2)(8p + 3)$ (n) $(7x - 1)(3x + 4)$ (o) $(5x - 2)(3x + 1)$
   (p) $(2q - 5)(3 - 2q)$ (q) $(12p + 1)(10p - 3)$ (r) $(16t - 2p)(14t - 2p)$
   (s) $(10s - 3t)(15s + t)$ (t) $(18m - 3n)(8m + 5n)$ (u) $(16s + 3s)(12 - 5s)$
   (v) $(10a - b)(12a + 5b)$

Exercise 11 (M)

1. (a) 6xy (b) 14 (c) 24xy (d) 8y (e) 30x (f) 20m
2. (a) $(x + 5)^2$ (b) $(x + 6)^2$ (c) $(y - 4)^2$ (d) $(x - 7)^2$ (e) $(x + 9)^2$
   (f) $(p - 12)^2$ (g) $(3x - 4)^2$ (h) $(4p + 1)^2$ (i) $(5x - 8)^2$ (j) $(7q + 1)^2$
   (k) $(3x - 11)^2$ (l) $(12 + x)^2$ (m) $(6 - 5y)^2$ (n) $(5x + 3)^2$ (o) $3(x - 1)^2$

Exercise 11 (N)

1. (a) $(x + 6)(x - 6)$ (b) $(m + 8)(m - 8)$ (c) $(y + 7)(y - 7)$
   (d) $(p + q)(p - q)$ (e) $(a + 11)(a - 11)$ (f) $(l + p)(l - p)$
   (g) $(9 + x)(9 - x)$ (h) $(10 + x)(10 - x)$ (i) $(15 + n)(15 - n)$
   (j) $(3x + 10)(3x - 10)$ (k) $(5 + 7y)(5 - 7y)$ (l) $(16 + 11y)(16 - 11y)$

(m) $3(y + 3)(y - 3)$ (n) $5(x + 4)(x - 4)$ (o) $2(b + 6)(b - 6)$
(p) $3(4 + y)(4 - y)$ (q) $5a(a + 2b)(a - 2b)$ (r) $m(m + 2)(m - 2)$

2. (a) $x^2 - 16$ (b) 48 sq.unit 3. 147.84 cm^2

4. $n^2 - (n - 9)^2 = 4(n - 1)$. It is an even number

Exercise 11 (O)

1. (a) $(x - 2)$ (b) $(3x + y)$ (c) $(x + 7)$ (d) $(y - 8)$ (e) $(x - 2y)$
(f) $(x + 3)$ (g) $2a(a + 3c)$ (h) $(x - 5)$ (i) $(a - 2b)$ (j) $2(2m + 1)$
(k) $(x - 3)$ (l) $x(x + y)$ (m) $(x + 2)$ (n) $(x - 5)$ (o) $(x + 1)$
(p) $(x + 8)$ (q) $(x - 5)$ (r) $(x - 3)$ (s) $(x - 3)$ (t) $(x + 7)$
(u) $(x + 2)$ (v) $(x - 1)$

Exercise 11 (P)

1. (a) $8x^2$ (b) $12y$ (c) $10x^2y$ (d) $12m4n^3p$ (e) $5x(3x + 1)$
(f) $6x^2(4x + 1)$ (g) $m(m + 4)$ (h) $12(x - 5)$ (i) $20(x + 2)$
(j) $(2x + 3)(2x - 3)$ (k) $4(x + 5)(x - 5)$ (l) $2(2 + x)(2 - x)$
(m) $4(x + 2)(x - 1)$ (n) $(x + 2)(x - 2)(x + 1)$ (o) $(a + 3)(a - 3)(a - 2)$
(p) $(x - 2)(x - 4)$ (q) $3(x + 3)(x - 7)$ (r) $(x + 1)(x + 2)(x - 3)$
(s) $(x - 4)(x + 5)(x - 5)$ (t) $(x + 2)(x + 3)(x - 2)$
(u) $(x + 2)((x + 4)(x - 4)(x - 5)(x + 5)$

Exercise 11 (Q)

1. (a) 0 (b) 7 (c) -9 (d) 6

2. (a) $\frac{m+4}{m-6}$ (b) $\frac{x+y}{x-y}$ (c) $\frac{3}{x-3}$ (d) $(a - b)$ (e) $\frac{x+5}{2}$
(f) $x - 3$ (g) $\frac{1}{x+2y}$ (h) $\frac{x-1}{x+1}$ (i) $(x - 2)$ (j) $\frac{x}{1-7x}$
(k) $\frac{x-6}{x+6}$ (l) $\frac{-(x+4)}{4}$ (m) $\frac{-(p-3)}{p-6}$ (n) $\frac{s(s-10)}{s+8}$

Exercise 11 (R)

1. (a) $\frac{4x}{5}$ (b) $\frac{11}{x}$ (c) $\frac{x}{y}$ (d) $\frac{7}{t+2}$ (e) 1 (f) $\frac{2a+1}{a+2}$

2. (a) $\frac{x}{3}$ (b) x (c) $\frac{4}{y}$ (d) -2 (e) $\frac{2}{x+2}$ (f) $\frac{a}{a^2+1}$

3. (a) $\frac{m+6}{m+1}$ (b) $\frac{1}{x-1}$ (c) $\frac{1}{y-3}$ (d) $\frac{2}{(x-5)}$ (e) $\frac{1}{x+2}$
(f) $m - 2$ (g) $x + 3$ (h) $(x - 4)$ (i) $x + 1$
(j) $3(x + 1)$ (k) $m - 1$ (l) $x + 2$ (m) $5(3 - x)$

(n) $2(p - 2q)$ (o) $\frac{(m-3)^2}{4}$ (p) $\frac{3(x+3y)}{z}$

Exercise 11 (S)

1. (a) $\frac{2x}{5}$ (b) $\frac{5y}{18}$ (c) $\frac{x^2+12}{3x}$ (d) $\frac{17}{12y}$ (e) $\frac{2+3x}{x}$ (f) $\frac{x+4y}{y}$

2. (a) $\frac{21-x^2}{3x}$ (b) $\frac{2n-3}{mn}$ (c) $\frac{12y-1}{8y}$ (d) $\frac{x-3y}{y}$
   (e) $\frac{12-2x}{3(2x+3)}$ (f) $\frac{3x+14}{(x+2)(x-2)}$

3. (a) $\frac{x+7}{7(x-1)}$ (b) $\frac{2y+13}{6}$ (c) $\frac{-1}{6(m+n)}$ (d) $\frac{2x+7}{x-7}$
   (e) $\frac{-7}{x^2-5x-10}$ (f) $\frac{x^2-y+6}{3(y^2-9)}$ (g) $\frac{73}{10(x-5)}$ (h) $\frac{x+5}{x}$
   (i) $\frac{x-3}{x+2}$ (j) $\frac{4x-6}{x^2-9}$ (k) $\frac{x-4}{x(x-2)}$
   (l) $\frac{11}{(x-1)(x+3)}$ (m) $\frac{x-1}{x(x+4)}$ (n) $\frac{x+5}{x+1}$
   (o) $\frac{4x-5}{(x+1)(x-3)}$ (p) 0

Exercise 11 (T)

1. (a) $\frac{x(x-y)}{9}$ (b) $\frac{-4(x-2)}{y}$ (c) $\frac{a-b}{a+3}$ (d) $\frac{a-b}{a-2}$
   (e) $\frac{x}{y}$ (f) $\frac{5}{x-2}$ (g) $\frac{x-1}{x-3}$ (h) $\frac{x-2}{x(x+4)}$
   (i) $\frac{x-3}{3}$ (j) $\frac{9(x+1)}{x-2}$ (k) $\frac{x+2}{y+2}$ (l) $\frac{y+2}{y+3}$
   (m) $\frac{a+6}{a-6}$ (n) $\frac{b+1}{b+5}$ (o) $\frac{x-3}{x-6}$ (p) $\frac{x^2(y-5)}{y^2(y+2)}$

UNIT 12 : EQUATION, INEQUALITY AND GRAPH

Exercise 12 (A)

1. (a) $x < 2$ (b) $x < 4$ (c) $x > 3$ (d) $x < 0$ (e) $x > 2$
   (f) $x < 7$ (g) $x > 2$ (h) $x < -1$ (i) $4 \geq x$ (j) $3 \geq x$

(k) $x \geq 2$ (l) $x \leq x$ (m) $-3 < x, x \leq 3$ (n) $-4 \leq x, x \leq 1$

Show the number-lines to the teacher.

2. $y < 10$ 3. (a) $y \geq -7$ (b) $x < 8$
4. 14 5. 2
6. (a) $15 - x$ (b) $8x + 3(15 - x)$ (c) $8x + 3(15-x) \leq 100$ (d) 11

Exercise 12 (B)

1. (a) (4, 3) (b) (2, 1) (c) (3, -1) (d) (-2, 1) (e) (-6, 11) (f) (1, 5)
   (g) (1, 2) (h) (3, -2) (i) (1, -1) (j) (2, -3) (k) (-2, 2)
2. (a) 11 and 10 (b) 11 and 5 (c) 25 and 10
   (d) Age of father = 50 years and Age of son = 10 years
   (e) Age of Ramesh = 15 years and age of Ravi = 10 years

Exercise 12 (C)

1. (a) -2 (b) $\frac{7}{6}$
2. (a) -2 (b) 3 (c) 1 (d) -2 (e) $\frac{7}{3}$ (f) $-\frac{2}{3}$ (g) $-\frac{6}{7}$ (h) 0
3. (a) Left up (b) Right up (c) Right up (d) Left up (e) Right up
   (f) Left up (g) Left up
4. (a) 3 (b) -2 (c) -1 (d) $\frac{1}{2}$ (Show the graph works to your teacher)

Exercise 12 (D)

1. (a) x - intercept : -3, y - intercept : -4 (b) x - intercept : -3, y - intercept : no
   (c) x - intercept : 5, y - intercept : 3 (d) x - intercept : no, y - intercept : 3
2. (a) -2 (b) -3 (c) 2 (d) 2
3. (a) 1 (b) -6 (c) 4 (d) $-\frac{4}{3}$
4. (a) x - axis (-2, 0), y - axis (0, 2) (b) x - axis (2, 0), y - axis (0, 6)
   (c) x - axis (-6, 0), y - axis (0, 3) (d) x - axis (-10, 0), y - axis (0, -8)
   (e) x - axis (2, 0), y - axis (0,4) (f) x - axis (-4, 0), y - axis (0, 6)
   (Show the graphs to your teacher)

Exercise 12 (E)

1. (a) 0, -1 (b) 0, 1 (c) -2, -1 (d) 2, -1 (e) -2, 1
   (f) 3, 4 (g) - 4, 1 (h) 4, -1 (i) ± 3 (j) 4, -3
   (k) 5, - 4 (l) 3, 4 (m) -5, 2 (n) 3, 6 (o) 6, -5
   (p) 1, -3 (q) - 4 (r) 2 (s) ± 5 (t) -5
   (u) ± 6 (v) 6
2. (a) 0, $-\frac{3}{2}$ (b) 0, $\frac{3}{2}$ (c) -1, $\frac{1}{3}$ (d) -1, $-\frac{1}{2}$ (e) -2, $\frac{1}{3}$

(f) $3, -\frac{1}{3}$ (g) $1, \frac{2}{3}$ (h) $2, -\frac{2}{3}$ (i) $-3, \frac{1}{3}$ (j) $-1, \frac{3}{2}$

(k) $1, \frac{3}{2}$ (l) $-\frac{3}{2}, -\frac{2}{3}$ (m) $\pm\frac{2}{3}$ (n) $\pm\frac{1}{4}$ (o) $\frac{1}{4}$

(p) $-\frac{1}{4}$ (q) $\pm\frac{3}{4}$ (r) $\frac{3}{4}$ (s) $-\frac{2}{3}, -\frac{1}{2}$ (t) $-\frac{2}{3}, \frac{1}{2}$

(u) $\frac{2}{3}, \frac{1}{2}$ (v) $\frac{1}{2}, \frac{1}{5}$

UNIT 13 : ANGLES

Exercise 13 (A)

1. (a) 50^0 (b) 125^0 (c) 40^0
2. (a) $x = 95^0, y = 85^0, z = 85^0$ (b) $x = 110^0, y = 70^0, z = 70^0$
   (c) $x = 30^0, y = 150^0, z = 150^0$
3. (a) 40^0 (b) 15^0 (c) 20^0
4. (a) 47.5^0 (b) 51.4^0 (c) $x = 30^0, y = 60^0, z = 75^0$ (d) 90^0 (e) 180^0
   (f) 27.2^0 (g) $x = 60^0, y = 80^0, z = 60^0, a = 60^0, b = 80^0$ (h) 35^0
5. Complementary $60^0, 50^0, 45^0, 10^0$
   Supplementary $150^0, 140^0, 135^0, 100^0$
6. $180^0, 72^0$ 7. 40^0

Exercise 13 (B)

1. (a) 60^0 (b) 105^0 (c) 65^0 (d) 100^0 (e) 110^0 (f) 60^0
2. (a) parallel (b) parallel (c) parallel (d) non-parallel (e) parallel
   (f) non-parallel
3. (i) (a) 60^0 (b) 55^0 (c) 120^0 (d) 30^0 (e) 110^0 (f) 80^0
   (ii) If a transversal line intersects two parallel lines then the interior alternative angles are equal.
4. (i) (a) 120^0 (b) 45^0 (c) 100^0 (d) 40^0 (e) 50^0 (f) 130^0
   (ii) 180^0
   (iii) If a transversal intersects two parallel lines then the sum of co-interior angles is 180^0.
5. (a) $p = 45^0, q = 105^0, x = 120^0, y = 60^0, z = 75^0$
   (b) $p = 55^0, q = 132^0, x = 77^0, y = 48^0, z = 125^0$
   (c) $p = 55^0, q = 75^0, x = 75^0, y = 150^0, z = 50^0$
   (d) $x = 25^0, y = 55^0, z = 30^0$
   (e) 70^0 (f) 53^0 (g) 45^0 (h) $x = 160^0, y = 122^0, z = 58^0$
   (i) $p = 140^0, q = 33.3^0$

Exercise 13 (D)

1. Show your construction works to the teacher.
2. Parallel because corresponding angles are equal.
3. Parallel, (corresponding angles are equal)
4. (a) Perpendicular (b) Parallel
5. Not parallel, sum of co-interior angles are not 180^0.

UNIT 14 : TRIANGLES

Exercise 14

1. $x = 60^0$
2. (a) 55^0 (b) 70^0 (c) 65^0 (d) 110^0 (e) $x = 60^0$, $y = 60^0$ (f) $x = 90^0$, $y = 45^0$
3. Angles of an equilateral triangle are equal and each angle is 60^0.

UNIT 15 : INTERIOR AND EXTERIOR ANGLES OF REGULAR POLYGONS

Exercise 15 (A)

1. (a) 108^0 (b) 128.57^0 (c) 135^0 (d) 140^0 (e) 144^0 (f) 150^0
2. (a) 72^0 (b) 51.43^0 (c) 45^0 (d) 40^0 (e) 36^0 (f) 30^0
3. (a) $\angle A = 70^0$, $\angle B = 58^0$, Sum $= 128^0$ (b) $\angle ACD = 128^0$
   (c) Yes, it is
   (d) If one side of a triangle is produced, then the exterior angle formed is equal to the sum of non-adjacent interior angles.
   (e) $\angle CEB = \angle A + \angle ACB$
4. (a) 50^0 (b) 113^0 (c) 85^0 (d) 235^0 (e) 72^0 (f) 110^0 (g) 110^0
   (h) 98^0 (i) 60^0

Exercise 15 (B)

1. (a) 13 cm (b) $8\sqrt{5}$ cm (c) $4\sqrt{10}$ cm
2. (a) 8 cm (b) 7 cm (c) 6.24 cm
3. (a) Right angled triangle (b) Non-right angle triangle
   (c) non-right angled triangle (d) Right angled triangle
4. 6 cm. 5. Kailash, 4 m
6. $AD = \sqrt{65}$ cm 7. $\sqrt{274}$ cm 8. $3\sqrt{17}$ cm
9. $BE = 5\sqrt{2}$ cm, EG = 13 cm., DE = 13 cm
10. $x = 54$ cm, $y = 18\sqrt{13}$ cm
11. Show your work to the teacher.

Exercise 15 (C)

1. (a) 11 cm (b) 22 cm (c) 10 cm (d) $2x + 6$ (e) $12x + 28$
2. (a) 3 (b) 3 (c) 3
3. (a) $x = 108^0$, $y = 108^0$, $z = 72^0$ (b) $x = 120^0$, $y = 25^0$
   (c) $x = 50^0$, $y = 70^0$, $z = 50^0$
4. (a) 29^0, 151^0 (b) 73^0, 107^0 (c) 46^0, 134^0
5. (i) Sum of co-interior angles = 180^0,
   (ii) Sum of co-interior angles = 180^0
   (iii) From (i) and (ii), Equal substitution ......
6. Show your work to the teacher.
7. Show your work to the teacher.

Exercise 15 (D)

Show your works to the teacher.

UNIT 16 : CONGRUENT AND SIMILAR TRIANGLES

Exercise 16 (A)

1. (a) SSS (b) SAA (c) RHS (d) ASA (e) SAS (f) RHS
2. (a) $x = 1.5$ cm, $\angle A = 40^0$, $\angle Q = 86^0$, AB = 3.9 cm,
   CA = RP = 4.5 cm, QR=3.1cm
   (b) $x = 2$ cm, $\angle M = 22^0$, $\angle Z = 35^0$, OM = 4.9 cm,
   NO = YZ = 2.1 cm, XY = 3.4cm
   (c) $x = 1.7$ cm, $\angle R = 43^0$, $\angle M = 62^0$, RP = OM = 4.1 cm NO = 3.4cm
3. (a) MO = AC, ON = BC, MN = AB
   (b) YZ = NO, XY = MO, XZ = MN
4. (a) BC = EF or $\angle A = \angle D$ (b) AC = DF (c) AC = DF or $\angle C = \angle F$
5. ASA **6.** ASA

Exercise 16 (B)

Show your answer to the teacher.

Exercise 16 (C)

1. (a) (e) and (g), (b) and (f), (c) and (h)
2. (a) $\Delta ABC \sim \Delta ADE$ (b) $\Delta ABC \sim \Delta AED$
   (c) $\Delta ABC \sim \Delta DCE$ (d) $\Delta ABE \sim \Delta DCE$
   (e) $\Delta ABC \sim \Delta ADB$ (f) $\Delta ABC \sim \Delta ADB$
3. (a) Show your work to the teacher. (b) 24 cm
6. (a) Show your work to the teacher. (b) $x = 4.8$ cm, $y = 4.5$ cm
7. Show your work to the teacher.

UNIT 17 : SOLIDS

Exercise 17

Show your works to the teacher.

UNIT 18 : PERIMETER, AREA AND VOLUME

Exercise 18 (A)

1. (a) $32\ cm^2$, 24 cm (b) $12\ cm^2$, 14 cm (c) $12\ cm^2$, 16 cm
2. (a) $16\ cm^2$, 16 cm (b) $2.25\ cm^2$, 6 cm (c) $22.25\ cm^2$, 18 cm
3. (a) $20\ cm^2$, 19 cm (b) $24\ cm^2$, 22 cm (c) $28\ cm^2$, 24 cm
   (d) $28\ cm^2$, 24 cm
4. (a) $10\ cm^2$ (b) $18\ cm^2$ (c) $12\ cm^2$ (d) $40.5\ cm^2$ (e) $42\ cm^2$
5. (a) $22.5\ cm^2$ (b) $38.5\ cm^2$ (c) $23\ cm^2$ (d) $72\ cm^2$

Exercise 18 (B)

1. (a) $1186.92\ cm^2$ (b) $85.1725\ cm^2$ (c) $477.28\ cm^2$
   (d) $1022.1485\ cm^2$ (e) $659.1488\ cm^2$ (f) $477.28\ cm^2$
2. (a) $(140 + 10\sqrt{61})\ cm^2$ (b) $(80 + 12\sqrt{5})\ cm^2$ (c) $254.4\ cm^2$
   (d) $67.84\ cm^2$
3. $320.28\ cm^2$ 4. (a) 5 cm (b) $1318.8\ cm^2$
5. $376.8\ cm^2$ 6. 8 cm 7. 15 cm 8. $1024\ cm^2$

Exercise 18 (C)

1. (a) $423.9\ cm^3$ (b) $1969.408\ cm^3$ (d) $8138.058\ cm^3$
   (d) $8138.88\ cm^3$
2. (a) $15\ cm^3$ (b) $297.6\ cm^3$ (c) $152\ cm^3$ (d) $36\ cm^3$
   (e) $75.6\ cm^3$ (f) $88.2\sqrt{3}\ cm^3$
3. $879200\ cm^3$ 4. $923.16\ cm^3$, 796.3 min or 13 hrs. 16 mins. 18 secs.
5. $113.04\ m^2$ 6. 50 cm. 7. 120 cm or 1 m 20 cm
8. $1483.65\ cm^3$ 9. $(88.2\sqrt{3} - 219.8)\ cm^3$
10. (a) 4.81 (b) 18 cm

UNIT 19 : CIRCLE

Exercise 19 (A)

1. (a) 12.56 cm (b) 9.42 cm (c) 15.7 cm (d) 18.84 cm
   (e) 21.98 cm (f) 31.4 cm
2. (a) 7.5 cm (b) 9 cm (c) 5.5 cm (d) 3.5 cm
   (e) 1.5 cm (f) 6 cm
3. 44 cm 4. 40.82 cm 5. 157 m 319 Wheels
6. 1 m 7. 1.85 cm

Exercise 19 (B)

1. (a) 200.92 cm^2 (b) 12.56 cm^2 (c) 153.86 cm^2
 (d) 6.1544 cm^2 (e) 132.665 cm^2 (f) 254.34 cm^2
 (g) 98.4704 cm^2 (h) 153.86 cm^2 (i) 78.5 cm^2
 (j) 706.5 cm^2
2. (a) 153.86 cm^2 3. 28.26 cm^2
4. (a) 12.56 cm^2 (b) 21.5 cm^2 (c) 30.96 cm^2 (d) 4.71 cm^2
5. (a) 5 cm (b) 31.4 cm **6.** 10 cm, 314 cm^2
7. (a) 2 m (b) 12.56 cm

UNIT 20 : TRANSFORMATION

Exercise 20 (A)

1. Show your works to the teacher.
2. A^1 (2, -5), B^1 (1, 5), C^1 (5, 0), D^1 (3, 01), E^1 (-5, -2)
3. A^1 (-5, 7), B^1 (3, 1), C^1 (-2, 0), D^1 (-3, -6), E^1 (-5, -7)
4. P^1 (-1, -3), Q^1 (3, -1), R^1 (5, -2)
5. W^1 (-3, 1), X^1 (-2, 5), Y^1 (-1, 7), Z^1 (3, -4)

Exercise 20 (B)

1. Show your works to the teacher.
2. A^1 (-2, 2), B^1 (7, -1) 3. P^1 (1, -3), Q^1 (2, -5), R^1 (-1, 3)
4. A^1 (-1, -5), B^1 (-5, -5), C^1 (-6, 5), D^1 (-1, 2)

Exercise 20 (C)

Show your works to the teacher.

UNIT 21 : BEARING AND SCALE DRAWING

Exercise 21 (A)

1. (a) 045^0 (b) 22.5^0 (c) 315^0 (d) 337.5^0
2. 205^0 3. 270^0
4. (a) 003^0 (b) 067^0 (c) 282^0 (d) 084^0 (e) 111^0
 (f) 127^0 (g) 148^0

Exercise 21 (B)

1. (a) 178 km (b) 68^0 (c) 248^0
2. (a) 9 km (b) 45^0 (c) 325^0
3. (a) 126 km (b) 114 km (c) 325270^0
4. (a) 270^0 (b) 90^0

Appendix 1

Table of Square Root = ? to be checked