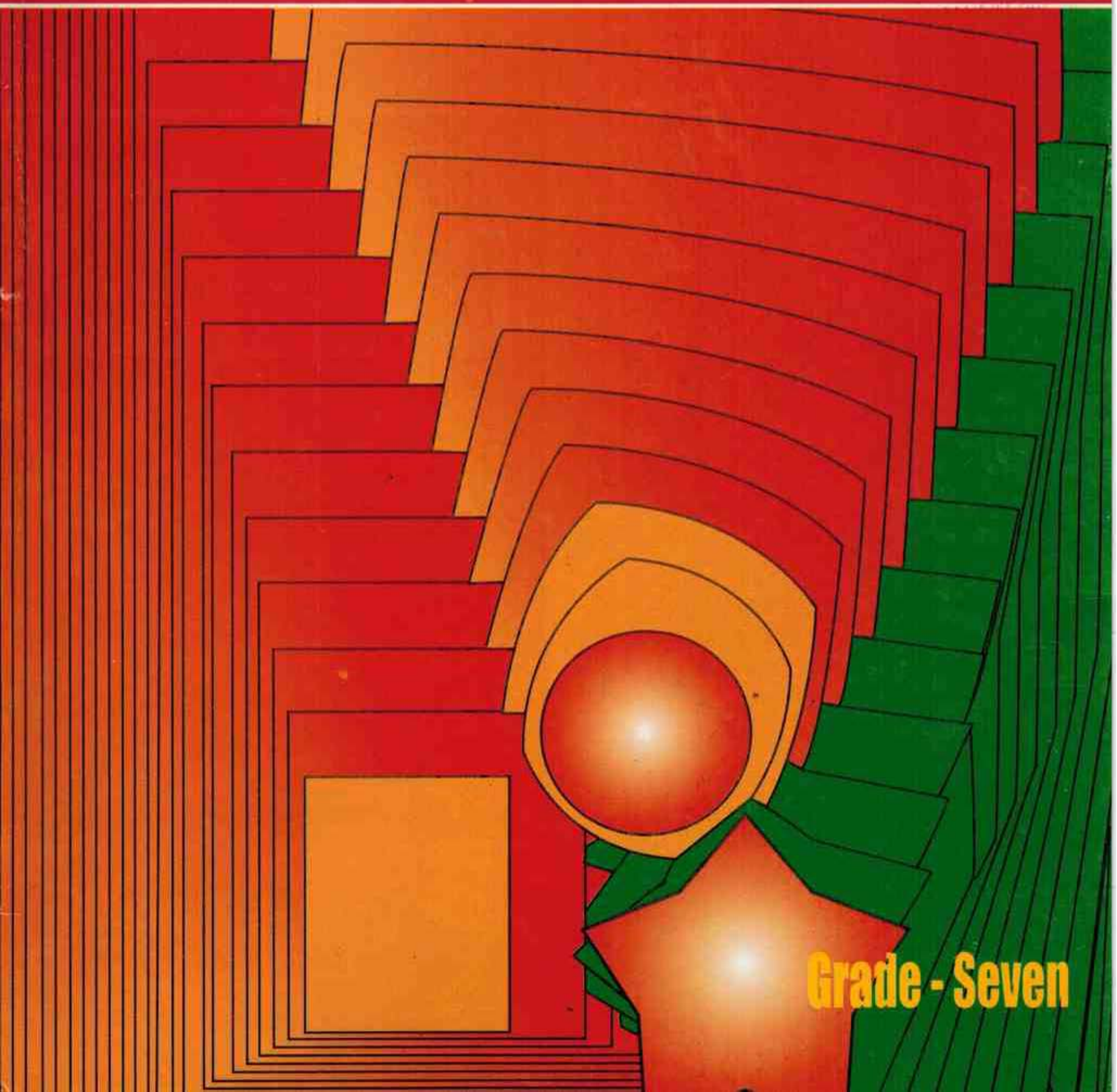
7

OUR

New Edition

Mathematics



Our Mathematics

Grade 7

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Preface

Now that the overwhelming majority of people in Nepal question the quality of education, it is, indeed, desirable to do something about it. One of the major tasks of the government is to provide quality education to all the people. In this context, Curriculum Development Centre (CDC) is the authorized institution in the country to design and develop textbooks and teachers' guides to be used throughout the kingdom to meet this challenging need. Likewise, Janak Education Materials Centre (JEMC) also plays an equally crucial role by printing and distributing the textbooks to all the public schools across the country. To cater for the needs of both private and public schools, JEMC has come one step ahead by translating the authorized version of Nepali books into English. The Centre is confident that it will be able to provide English version books in different subjects to the learners step-by-step.

With completion of the third phase, the Centre really feels proud of accomplishing a substantial job of translating school textbooks into English for English medium learners across the country. The Centre also looks forward to working in a similar manner in order to serve the needy pupils in the days to come.

This book is translated by Krishnadev Chaudhary, Dhurba Narayan Chaudhary, Birsingh Chhetri from Nepali version *Hamro Ganit* of Grade 7. We are highly grateful to Sungma Tuladhar, Arunkiran Pradnan who were involved in subject matter and language editing. The JEMC invites positive suggestions from all the concerned to make the book even better.

We would like to express our sincere thanks to the teachers of some renowned schools of 5 Development Regions who have assisted in evaluating the proposed translated lessons and all the others who have also contributed to the preparation of these books.

Finally, we would also like to express our gratitude to CDC for giving us an opportunity to translate the government textbooks into English in order to cater for the needs of the pupils of English medium school.

JEMC
Sano Thimi, Bhaktapur

CONTENTS

1 Set

1.1 Review of Set

Let's review the concept of set learnt in the previous class.

a) Set: A set is a collection of well-defined objects. The collection of fruits in the Fig. No. 1.1 is a set. The fruits in the collection are the elements of the set. There is one or more than one fruit in this collection, but one element of one kind is sufficient to be in a set.

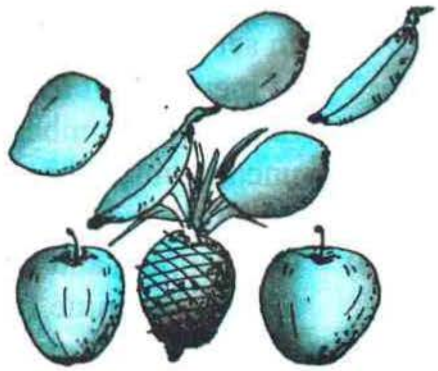


Fig.No. 1.1

b) Methods of defining a Set:

A set can be represented by any one of the following methods.

i) Description Method:
A set can be represented by describing the property of the elements of the set.
For example, V = {the vowels of the English alphabet}
In this method, set V is expressed by describing the elements of the set.

ii) Listing Method:
In this method, a set is represented by listing its elements within braces, separated by commas.
For example V = {a, e, i, o, u} or V = {i, a, o, u, e}
When listing the elements of set, the order does not matter.

iii) Diagrammatic Method:
A set is represented by enclosing the elements of the set in a circle.
For example, a, e, i, o, and u is represented by a diagram.

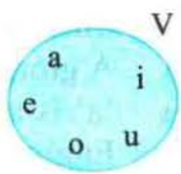


Fig.No. 1.2

c) Membership of a Set:

In a set of domestic animals of a cow, a dog and a rabbit, a dog is a member but a cat is not. In set language, it is stated in this way:

dog $\in$ {cow, dog, rabbit}

cat $\notin$ {cow, dog, rabbit}

d) Finite and Infinite Sets:

In set E = {0, 2, 4, 6}, there are four elements. The number of members of this set can be counted. It is a finite set. The number of members of certain sets can not be counted. It is uncountable. For example, a set of natural numbers, $N = \{1,2,3,..............\}$. This type of set is called an infinite set.

e) Overlapping and Disjoint Sets:

Set A, the prime factors of 6; Set B, the prime factors of 9 and Set C, the multiples of 5 less than 20 are represented by the listing method as shown below.

$A = \{1, 2, 3, 6\}$
$B = \{1, 3, 9\}$
and $C = \{5, 10, 15\}$

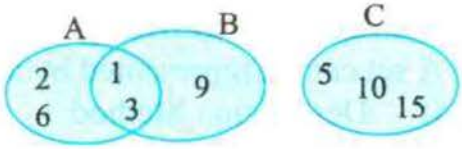


Fig. No. 1.3

Here, the elements 1 and 3 belong to both the sets A and B. Therefore, 1 and 3 are common members of both the sets. A and B are overlapping sets. But the elements of C do not belong to A and B. Therefore, A and C are disjoint sets. Similarly, B and C are disjoint sets.

Exercise 1.1 (Revision Exercise)

1. State which of the following statements about sets are true and which are false.

a) A group of tall people is a well defined set.

b) 'a' is an element of the set of the first three letters of the English alphabet.

c) 44 belongs to the set of numbers exactly divisible by 8.

d) S represents SAARC countries and India $\notin$ S.

e) If A is the set of the factors of 2 and B, the factors of 3, then A and B are disjoint sets.

f) The set of numbers exactly divisible by 6 is a finite set.

g) The set of natural numbers less then 1000 is an infinite set.

h) The set of districts of Nepal is not a well-defined set.

i) If A is the set of the first five letters of the English alphabet and V, the set of vowels then A and V are overlapping sets.

2. Express the following sets by the listing method.

a) W = Set of days of the week.

b) E = A set of whole numbers less than 20 and exactly divisible by 2.

c) O = A set of whole number less than 20 and exactly divisible by 3.

3. Find which sets in Q.N.2 are overlapping and which are non-overlapping.

4. Express the overlapping sets in Q.N.2 by a Venn-diagram

5. If $P = \{1,2,3,4,5\}$, express the following sets by the listing method.

a) The set formed by subtracting 1 from each element of P.

b) The set obtained by multiplying, each member of P by 3.

c) The set of even numbers of P.

d) The set obtained by adding 1 to odd numbers of P.

e) The set of prime factors of 4 in P.

1.2. Sub- Sets:

A set of a mango, an apple and a banana is shown in Fig. No. 1.4 Shiva formed the following sets with the members of this set.

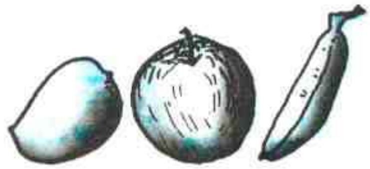
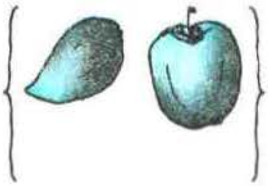


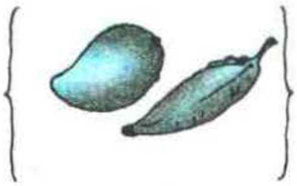
Fig.No. 1.4

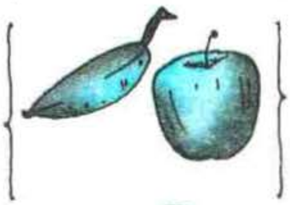
B=  = {mango}. It is a singleton set.

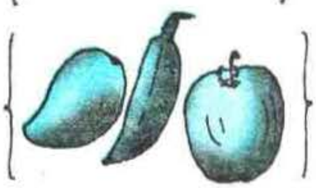
C=  = {apple}. It is a singleton set.

D=  = {banana}. It is a singleton set.

E=  = {mango and apple}. It is a set of two members.

F=  = {mango and banana}. It is a set of two members.

G=  = {banana and apple}. It is a set of two members.

H=  = {mango, banana and apple}. It is a set of three members.

I= = { }. It is a null set.

Fig. No. 1.5

Shiva was puzzled at this. Shiva named all these newly formed sets as subsets of Set A. In addition, set A is named the super set of all these sets. Can you make more sets than Shiva using the elements in Fig. No.1.5?

Answer the following questions:

a) Do all the elements of set B belong to set A as well?
b) Do all the elements of set E also belong to set A?
c) Do all the elements of set H belong to set A, too?
d) Is set I, a subset of set A?

What did you learn from the above discussion ?

a) If all the elements of set P belong to set Q as well, then P is the subset of Q and Q is called super set of P. It can be written as $P \subset Q$ and $Q \supset P$.
b) If any set P is a subset of another set Q and $n(P) < n(Q)$, then P is called the proper subset of Q and it is written as $P \subset Q$ or $Q \supset P$. In the example above $E \subset A$ or $A \supset E$.
c) If any set P is a subset of another set Q and $n(P) = n(Q)$ or $n(P) = 0$, then P is called Improper subset of Q.
This implies that every set P is a subset of itself and null set is a subset of every set. Hence, $P \subset P$ and $\Phi \subset P$

Exercise 1.2

1. **In the figure given below, G is a set of some geometrical shapes.**

G= 

Now, list all the possible subsets ot G for the following.

a) The set of three-sided figures in G.
b) The set of four-sided figures in G.
c) The set of curve lined figures in G.
d) The set formed by figures made of curved lines and straight lines.
e) All the possible sets of two members in G.

2. How many subsets are formed in Q.N. 1 (e) ?

3. If P = {1, 2, 3, 4, 5, 6, 7}, make the following subsets and write them using the signs $\subset$ $\supset$.

a) Set Q of prime numbers from the members of P.
b) Set R of composite numbers from the members of P.
c) Set S of odd numbers from the members of P.
d) Set T of even numbers from the members of P.
e) Set U of the prime factor of 6 from the members of P.

4. Write all possible subsets of set P= {a}
5. Write all possible subsets of set Q= {a, b}
6. Write all possible subsets of set R= {a, b, c}
7. Write all possible subsets of set S = {a, b, c, d}
8. Fill in the table below on the basis of the answers of question nos. 4-7.

Set	No. of elements	No. of subsets
{ a }		
{ a, b }		
{ a, b, c }		
{ a, b, c, d }		

1.3. Universal Set:

Activity 1

Consider the set of whole numbers W = { 0, 1, 2, 3, 4, ...} and construct the following sets.

a) Set E of even numbers.
b) Set O of odd numbers.
c) Set S of square numbers.
d) Set C of cube numbers.
e) Set P of prime numbers.
f) Set N of counting numbers.

What relation did you observe between Set W and the other sets from A to F ?

Activity 2

Consider the Set S of your class-mates = { Students of Class 7}
Now, construct the following sets.

a) The set of your friends' names that starts with the letter 'A'.
b) The set of your friends' names that starts with the letter 'S'
c) The set of your friends' names that starts with the letter 'Z'.
d) The set of your friends who have completed 13 years.

e) The set of your friends who have not completed 13 years.

What relation did you find between Set W and S the sets from 'a' to 'e'.

The set that includes all kinds of sets that can come under discussion is called a Universal set. It is denoted by U. Sets W and S of Activities (1) and (2) are Universal sets for those discussions.

Example 1

Students of a class are discussing.

Narendra - " It includes all the multiples of 3 except 0".

Manoj- "And numbers like 1.5, $\frac{3}{7}$ are not included, are they?

Sunita - "Yes, but all the prime numbers are included".

Now, what may be the Universal set for their discussion?

According to them, it seems that the Universal set includes all the numbers except 0, decimals and fractions. Therefore, the set of natural numbers is the topic of their discussion.

Exercise 1.3

1) **Consider, Universal set U = whole numbers less than 100; then answer the following questions:**
   a) List, by the listing method, the set (M_6) and n(M_8).
   b) List, by the listing Method, the set M_8, multiples of 8.
   c) Compare n(M_6) and n(M_8).
   d) Show the relation between the set of M_2, multiples of 2 and M_6.
   e) Show the relation between the set of M_{24}, the multiples of 24 and M_6 and the relation between M_{24} and M_8.

2. **Consider the following discussion among the students.**

Ramila - "Considering M_3, $M_3 = \{0, 3, 6, 9\}$"
Bijay - "Only 2, 4, 6, 8, 10 are even numbers."
Sanjay - "11 is not included, is it?
Laxmi - "Yes and proper fraction is also not included."

Now, what may be the Universal set for their discussion?

1.4. Venn Diagram

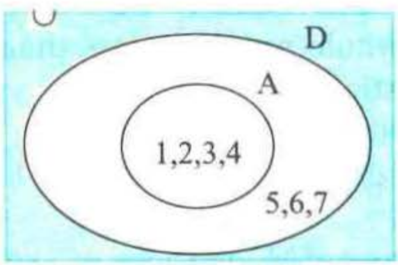
The diagrammatic representation of sets or different relationships between sets is called Venn Diagram. A rectangular region is used for Universal set while for others circular region is used. In other words each element of a set is shown within a circular region. Also, those circular regions should be equal to each other. All types of sets can be illustrated with the following example. For example,

N= {1, 2, 3, 4, ...}. is a set of natural numbers Let's suppose a set of whole numbers as a Universal set. In addition the different kinds of sets and subsets which can be constructed from it have been presented in a Venn Diagram in this way:

(A) = {1, 2, 3, 4}   (B) = {2, 3, 4, 5}
(C) = {6, 7, 8, 9}   (D) = {1, 2, 3, 4, 5, 6, 7}
(E) = {1, 2, 3, 4}.

Subsets A, B, C, D, E are constructed from the Universal set N respectively. Now, let's present them in order by Venn- Diagram.

(i) In subsets A and D, the subset A is also a subset of D. Thus,



$A \subset D$

Fig. No. 1.6

(ii) The subsets A and E are equal to each other. Thus,

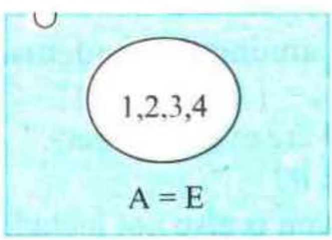


Fig. No. 1.7

(iii) Subset B and C do not have common elements. Therefore, they are disjoint sets.

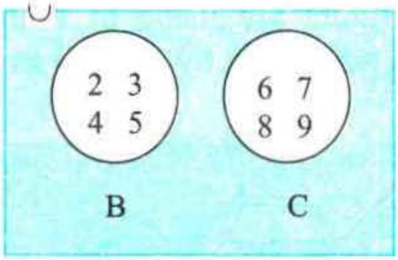


Fig. No. 1.8

(iv) Subsets A and B have some common elements. Therefore, they are overlapping sets.

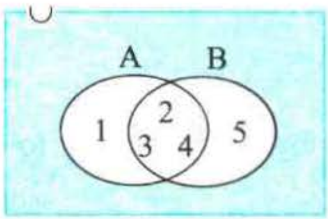


Fig. No. 1.9

Exercise 1.4

1) Show the following sets in a Venn diagram.

A = {a, b, c, d} B ={c, d, e, f, g}
C = {f, g, h, i, j} D = {c, d, e, f, g}

1.5. Union of Sets

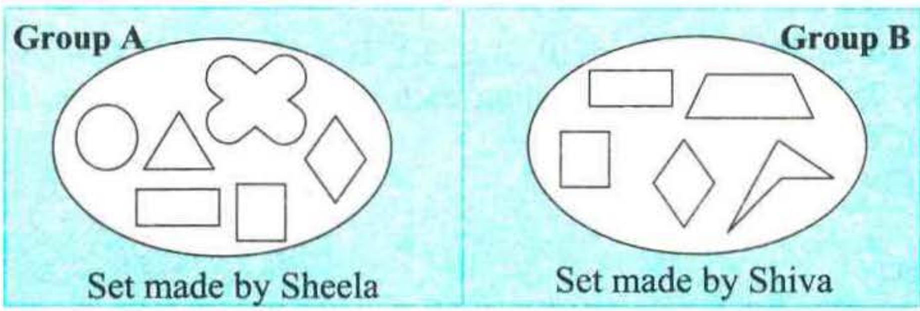
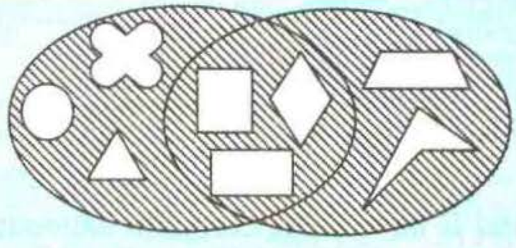


Fig. No. 1.10

In the figure above, each element S of set A of geometrical figures made by Sheela has a line symmetry. In set B, constructed by Shiva, the members of every geometrical figure are made of four sides. Combining these two sets Krishna has made a set by shading as shown in the figure on right side



Set made by Krishna

Fig. No. 1.10

Answer the following questions:

a) Do all the elements of the set made by Shiva belong to the set made by Krishna?.

b) Do all the elements of the set made by Sheela belong to the set made by Krishna?

c) What types of shapes are included in the shaded part of the Venn-diagram which was constructed by Krishna to represent a set.

Here, the set made by Krishna is called Union of sets A and B.

The members of either A, B or both sets are included in the union of set A and B and this is denoted by $A \cup B$. It is read as A union B.

Example-1

If U = {0, 1, 2,..., 10}; A = {0, 1, 2, 3,}, B = {2, 3, 4, 5} C = {3, 4}, D ={6, 7, 8} then by constructing each of the following sets, show them in Venn- diagram.

a) $A\cup B$ b) $A\cup D$ c) $B\cup C$

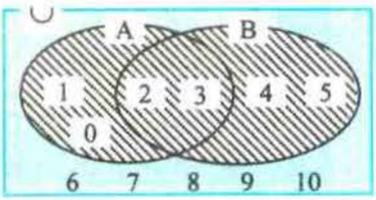


Fig. No. 1.12

Answer:-

a) $A\cup B = \{0,1,2,3\} \cup \{2,3,4,5\}$
$= \{0,1,2,3,4,5\}$

The set denoting $A \cup B$ is shown in the figure, by the shaded part. (Diagram)

b) $A \cup B = \{0,1,2,3\} \cup \{6,7,8\}$
$= \{0, 1, 2, 3, 6, 7, 8\}$

The set representing $A \cup B$ is shown in the figure by the shaded part.

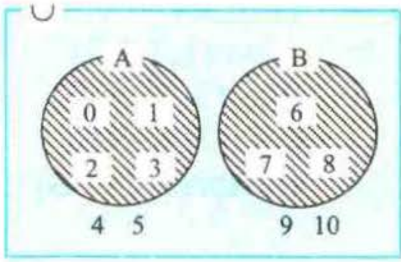


Fig. No. 1.13

c) $B \cup C = \{2, 3, 4, 5\} \cup \{3, 4\}$
$= \{2, 3, 4, 5\}$

The set representing $B \cup C$ is shown in the figure by the shaded part.

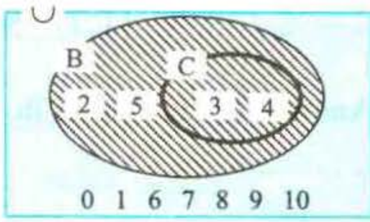


Fig. No. 1.14

Example -2

If A = {prime factors of 8} and B = { prime factors of 12}.

a) Write the sets A and B by the listing method.

b) Prove that $A \cup B = B \cup A$.

c) Prove that $A \cup A = A$.

Answer :-

a) $A\{1,2,4,8\}$ and $B\{1,2,3,4,6,12\}$

b) $A \cup B = \{1,2,4,8\} \cup \{1,2,3,4,6,12\} = \{1,2,3,4,6,8,12\}$

$B \cup A = \{1,2,3,4,6,12\} \cup \{1,2,4,8\} = \{1,2,3,4,6,8,12\}$

$\therefore A \cup B = B \cup A$. Hence proved

c) $A \cup A = \{1,2,4,8\} \cup \{1,2,4,8\} = \{1,2,4,8\} = A$

$\therefore A \cup A = A$, Hence proved

Example-3

U = $\{1,2,3,4,5......10\}$

A = $\{1,3,5,7\}$

B = $\{2, 3, 5, 7\}$

C = $\{3, 6, 9\}$

a) Express $(A\cup B) \cup C$. b) Show $(A\cup B)$ by Venn-diagram.

Answer (a)

$$(A\cup B) \cup C = [\{13,5,7\} \cup [\{2,3,5,7\}] \cup \{3,6,9\}$$
$$= \{1,2,3,5,7\} \cup \{3,6,9\}$$
$$= \{1,2,3,5,6,7,9\}.$$

Answer (b) is shown in the Venn-diagram,

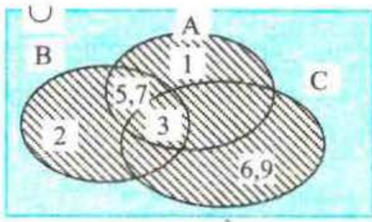


Fig. No. 1.15

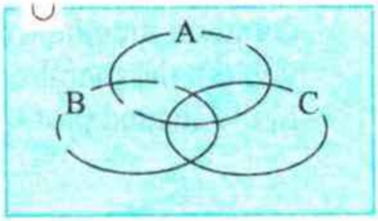
Exercise 1.5

1. **If Universal set $U = \{0, 1, 2....10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{3, 4, 5\}$ then**
   a) Show the relation of the sets U, A and B by a Venn-diagram.
   b) List the following sets on the basis of question (1.a):-
   i) $A\cup B$ ii) $B\cup A$ iii) $A\cup A$ iv) $B\cup B$
   c) Prove on the basis of the question (1.b):-
   i) $A\cup B = B\cup A$ ii) $A\cup A = A$ iii) $B\cup B = B$

2. **If U =}0,1,2,3,4,5} and A ={2,3,4} then**
   a) List $A\cup U$, and show by a Venn- diagram.
   b) Can $A\cup U = U$ be written?

3. If A = {Prime factors of 6} and B is a null set, then find $A\cup B$.

4. **Draw separate Venn- diagrams like the figure on the right and show by shading the unions.**



a) $A \cup B$ b) $B \cup A$ c) $B \cup C$ d) $C \cup B$

e) $A \cup C$ f) $C \cup A$ g) $(A \cup B) \cup C$ h) $A \cup (B \cup C)$.

5. **If A = {1, 2, 3, 4, 5}, B = {2, 4, 6} and C = {3, 6, 9} then**
   a) From the following sets , show them in Venn-diagram.
      i) $(A \cup B) \cup C$ ii) $A \cup (B \cup C)$

   b) Can $(A \cup B) \cup C = A \cup (B \cup C)$ be written?.

1.6. Intersection of Sets

(A) BADMINTON	(B) FOOTBALL
Sushil	Prmod
Binita	Binay
Susheela	Sushil
Pramod	Prashant
Binod	Shailesh

In the above diagram, the names of students interested in playing badminton and football are given.

Answer the following questions.

a) Supposing that set A is the list of the students interested in playing badminton and set B, the students interested in playing football, a Venn diagram is prepared as shown in the figure. Which student belongs to which part?

b) Which students like both the games?

c) Who are those students who only like to play badminton?

d) The shaded part denotes the intersection of sets A and B, which include the common members of A and B. Can you list the members who are included in the shaded part?

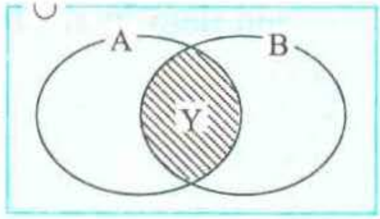


Fig. No. 1.16

Only common members of A and B are included in the intersection of A and B. It is denoted by A∩B. It is read as A intersection B.

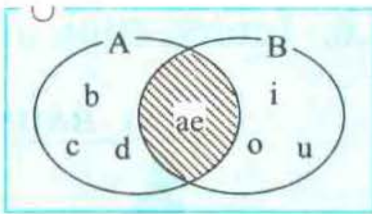
Example 1

If U = { letters of English alphabet}, A = {a,b,c,d,e}, B = {a,e,i,o,u} and C= {o, u}, then from the following sets, make Venn-diagrams.

a) A∪B b) B∪C c) C∪A

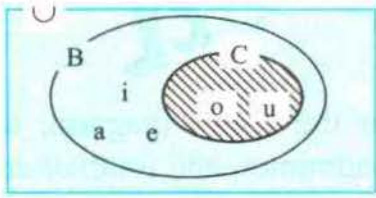
Answer:

a) A ∩ B = {a,b,c,d,e} ∩ {a,e,i,o,u}
= {a, e}.



The shaded part of the figure denotes A∩B

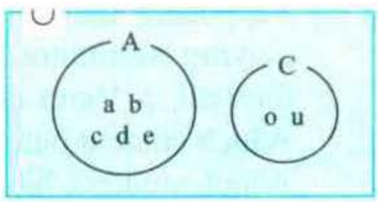
Fig. No. 1.17



b) B∩C = {a,e,i,o,u} ∩ {o,u}
= {o,u}
= C

The shaded part of the figure denotes B∩C

Fig. No. 1.18



c) C∩A = {o, u} {a, b, c, d, e}
= ϕ
It is a null set.

Fig. No. 1.19

Example 2

If Universal set, U = {O, 1, 2,….9}, A = {2, 4, 6, 8}, B = {1, 2, 4, 8} and C = {1, 2, 3, 6}, then list the following sets.

a) $A \cap B$ b) $B \cap C$ c) $A \cap B \cap C$ Also show them in a Venn-diagram.

Answer :

a) $A \cap B = \{2,4,6,8\} \cap \{1,2,6,8\}$

$\therefore A \cap B = \{2,4,8\}$

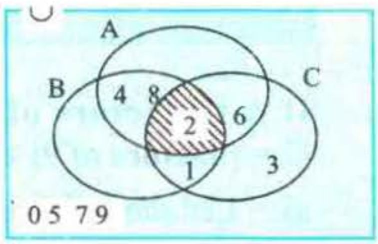
b) $B \cap C = \{1,2,4,8\} \cap \{1,2,3,6\}$

$\therefore B \cap C = \{1,2\}$

c) $A \cap B \cap C = \{2,4,6,8\} \cap \{1,2,4,8\} \cap \{1,2,3,6\}$

$\therefore A \cap B \cap C = \{2\}$

This is shown in a Venn-diagram



$A \cap B \cap C = \{2\}$

Fig. No. 1.20

Exercise 1.6

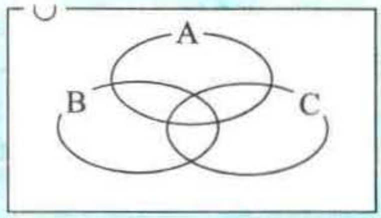
1) **If Universal set U = {1, 2, 3…..10}, A = {1, 3, 5, 7, 9} and B = {3, 6, 9} then,**
   - a) Show the relation of the sets U, A and B in a Venn-diagram.
   - b) List the following sets on the basis of question (1) a):
     i) $A \cap B$ ii) $B \cap A$ iii) $A \cap A$ iv) $B \cap B$
   - c) Prove these on the basis of question (1) b):
     i) $A \cap B = B \cap A$ ii) $A \cap A = A$ iii) $B \cap B = B$

2) **If M = {1, 2, 3, 4} and N = {1, 2, 3, 4, 5, 6} then**
   - a) Find $M \cap N$ and $N \cap M$ then show it in a Venn-diagram.
   - b) Can $M \cap N = N \cap M$ be written?
   - c) Which set denotes $M \cap N$?

3) If R denotes the set of red roses and W denotes the set of white roses then show R∩W in a Venn-diagram.

4) If P = {0,1,2} and Q is a null set then what type of set does P∩Q denote?

5)

Draw the figure at the side in 7 places, show by shading A∩B, B∩C, C∩A, (A∩B)∩C, U∩A, U∩B, U∩C.	Diagram: 

6) If A ={ factors of 6}; B = { factors of 8} and C = { factors of 9} then

a) List the following sets and show them in Venn-diagrams.

i) (A∩B) ∩C ii) A∩(B∩C)

b) Can (A∩B) ∩ C=A∩ (B∩C) be written?

2. Whole Numbers and Integers

2.1. Naming of Numbers in International System

Do you know?

i) According to the census of 2048 B.S., the population of Nepal is approximately 2,00,00,000.

ii) The live telecast of the World Cup football tournament of 1994 A.D. was watched in the continent of Asia by approximately 1,35,67,89,643 people on T.V. The commas used to express numbers or to write and express in words are based on the following place value table that follows the national system.

Kharabs		Arbas		Crores		Lakhs		Thousands				
Tens	Unit	Tens	Unit	Tens	Unit	Tens	Unit	Tens	Unit	Hundreds	Tens	Unit
10^{12}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	10^{0}
					2	0	0	0	0	0	0	0
			1	3	5	6	7	8	9	6	4	3

In order to express number, most countries of the world put commas in front of the last three digits and infront of every three digits before them, and are named respectively as thousands, mullions, billions.

Billions			Millions			Thousands			Hundred	Tens	Ones
	Tens	Ones		Tens	Ones		Tens	Ones			
10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	10^{0}
3	2	0	5	7	9	5	0	1	6	2	9

According to the international system the number shown in the table is written as
320, 579, 501, 625 or 320, 529, 501, 629 and in words as "three hundred twenty billion, ….. six hundred twenty-nine.

Example 1

Write the number 1257403652 in International System.

a) Using commas   b) In expanded form

Solution:

a) Using commas of the number 1257403652 is written as 1,257,403,652.

b) The expanded form of 1257403652 is
$1 \times 10^9 + 257 \times 10^6 + 403 \times 10^3 + 652 \times 10^0$
$= 1 \times 1000000000 + 257 \times 1000000 + 403 \times 1000 + 652 \times 1$

Exercise 2.1

1) **Use commas to express each of the following numbers in the National system and in the International system. Also write in words.**

a) 305674 b) 3596876 c) 12579640312
d) 3000050201 e) 75792402361

2) **Express each of the following number in the expanded form according to the international system.**

a) Sixteen crore eighty five lakh, three hundred and fifty three.
b) Thirteen arba and sixty.
c) Seventeen arba sixty lakh, three thousand and one.
d) Eighty eight crore three lakh two thousand and fifteen.

3) **National Method :**

a) How many lakhs of national system is equivalent to one million in international system?.
b) How many lakhs of national system is equivalent to one billion in international system?.

4) **Write the following numbers in figures**

a) Seven million four hundred seventy thousand and five hundred.
b) Fifteen billion, three hundred million, three hundred twenty two thousand, eight hundred and sixteen.
c) One hundred thirteen billion, two hundred ten million, seven hundred and twenty eight.
d) One hundred billion, one hundred million, one hundred thousand, one hundred and ten.
e) Nine billion seven million, three thousand and seven.

5) **The maximum distance between the sun and different planets are given below.**

Mercury	69,800,000 km.	Venus	108,900,000 km.
Earth	152,100,000 km.	Mars	249,200,000 km.
Jupiter	816,000,000 km.	Saturn	1,508,800,000 km.
Uranus	3,008,100,000 km.	Neptune	4,544,900,000 km.
Pluto	7,388,000,000 km.		

Write these distances in words in :-

a) Nepali b) English

2.2. Introduction to Square Root

Copy the table given on the right in an exercise book and write the square numbers from 0 to $\pm$ 10.

Numbers	Square Number
0	0
±1	$1=(\pm1)^2$
±2	$4=(\pm2)^2$
±3	$9=(\pm3)^2$
±4	$16=(\pm4)^2$
±5	-
±6	-
±7	-
±8	-
±9	-
±10	-

Now write the answers to the following questions.

a) Can you write the square numbers of whole numbers from 0 to $\pm$10 from the table ?

b) How many square numbers are there from 0 to 100?.

From the table on the right as $3^2=(-3)^2 = 9$, so the square number of 3 and -3 is 9. To put it another way, the square root of 9 is 3 and -3. The symbol $\sqrt{}$ is used to represent the square root of any number and while using the symbol $\sqrt{}$ only the positive number is used under the symbol.

$\boxed{\sqrt{a} \quad -\sqrt{a}}$ $\xrightarrow{\text{Square}}$ $\xleftarrow[\text{Square root}]{}$ $\boxed{\left(\sqrt{a}\right)^2 = \left(-\sqrt{a}\right)^2 = a}$

Similarly, the square roots of 2 are $\sqrt{2}$ and $-\sqrt{2}$ and the square roots of 5 are $\sqrt{5}$ and $-\sqrt{5}$. Hence, these can be written respectively as $\pm\sqrt{2}$and $\pm\sqrt{5}$.

Example 1

Find the square root of each of the following numbers.

a) 144 b) $\frac{9}{25}$ c) 0.04 d) 0

Solution:

a) $144=(12)^2 = (-12)^2$. Therefore, the square root of 144 is ±12.

b) $\frac{9}{25}=\left(\frac{3}{5}\right)^2 = \left(-\frac{3}{5}\right)^2$. Therefore, the square root of $\frac{9}{25}$ is $\pm\frac{3}{5}$.

c) $0.04 = (0.2)^2 = (-0.2)^2$ Therefore, the square root of 0.04 is ±0.2.

d) $0 = 0^2$ (0 has no positive or negative therefore the square root of 0 is 0 only.

Exercise 2.2

1. **Find the square of each of the following numbers.**

a) 5 b) 7 c) 11 d) 12 e) 15
f) 18 g) 22 h) 47

2. **Find by the method of factors the square root of the each of the following numbers:**

a) 25 b) 49 c) 121 d) 169 e) 289
f) 625 g) 1764 h) 2704

3. **Express each of the following given numbers in the form $16 = 4^2$**

a) 1 b) 4 c) 9 d) 25 e) 36
f) 49 g) 64 h) 81 i) 100 j) 121
k) 144 l) 169

4. **Find the value of :**

a) 1^2 b) 2^2 c) 0^2 d)12^2 e) 20^2
f) 50^2 g) 100^2 h) 125^2 i) 140^2 j) 200^2
k) 500^2 l) 1000^2

5. **What is the square number of each of the following numbers ?**

a) 1 b) 2 c) 3 d) 4 e) 9
f) 10 g) 15 h) 25 i) 100 j) 125
k) 300

6. **Find the square root of :**

a) 25 b) 36 c) 64 d) 81 e) 121
f) 324 g) 1225 h) 1764 i) 3025 j) 5184

7. **Simplify**

a) $\sqrt{18}$ b) $\sqrt{80}$ c) $\sqrt{1008}$ d) $\sqrt{845}$ e) $\sqrt{1536}$
f) $\sqrt{476}$

8. **Simplify**

a) $\sqrt{1+3+5+7}$ b) $\sqrt{5^2-3^2}$
c) $\sqrt{1^3+2^3+3^3}$ d) $\sqrt{15^2-14^2}$

2.3. Square Root of Perfect Square Numbers

In figure no 2.1, in a field 64 cabbage plants are planted in a square form. How many plants have been planted in each side ?

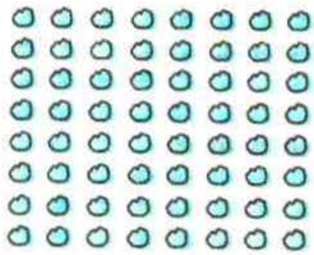


Fig. No. 2.1

Here, $64 = 8 \times 8$. Therefore, on every side 8 plants have been planted. Here, 64 is a perfect square number and the square root of 64 is 8.

Of the two like factors of a perfect square number, one is called the square root of the number.
e.g., $a^2 = a \times a$ or $a^2 = -a \times -a$, so the square root of a^2 is a or -a. It is written as $\sqrt{a^2} = \pm a$. Here, $\sqrt{\ }$ means square root.

Example 1

Find the square root of 324.

Solution:

Here,

Factorizing $\sqrt{324}$

$$324 = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

2	324
2	162
3	81
3	27
3	9
	3

Therefore, $\sqrt{324}$

$= \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt{2^2 \times 3^2 \times 3^2}$

$= \sqrt{(2 \times 3 \times 3)^2}$

$= \sqrt{(\pm 18)^2}$ (Taking only one of the

$\therefore = \sqrt{324} = \pm 18$ two same factors.)

Checking : $18 \times 18 = 324$
or $-18 \times -18 = 324$

Example 2

Find the value of $\sqrt{49} \times \sqrt{144}$

Solution:

$\sqrt{49} \times \sqrt{144} = \sqrt{7^2} \times \sqrt{12^2} = 7 \times 12 = 84$

In this example,

$84 = \sqrt{84^2} = \sqrt{7056} = \sqrt{49 \times 144}$ $\therefore \sqrt{49} \times \sqrt{144} = \sqrt{49 \times 144}$

Therefore, for any two positive numbers a and b, $\sqrt{a} \quad \sqrt{b} = \sqrt{ab}$

Note : $\sqrt{a} \times \sqrt{b}$ is also written as $\sqrt{ab}$ and a $\times$ $\sqrt{b}$ is also written as a$\sqrt{b}$.

Example 3

Find the value of $\frac{\sqrt{169}}{\sqrt{64}}$

Solution :

$$\sqrt{\frac{169}{64}} = \sqrt{\frac{13^2}{8^2}} = \frac{13}{8}$$

In this example

$$\frac{\sqrt{169}}{\sqrt{64}} = \frac{\sqrt{13^2}}{\sqrt{8^2}} = \frac{13}{8}$$

$$\therefore \frac{\sqrt{169}}{\sqrt{64}} = \frac{\sqrt{169}}{\sqrt{64}}$$

Therefore, for any two positive numbers a and b. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Example 4

Simplify

a) $\sqrt{8} \times \sqrt{12}$ b) $\frac{\sqrt{64}}{\sqrt{24}}$

Solution :

a) $\sqrt{8} \times \sqrt{12} = \sqrt{8 \times 12} \times = \sqrt{96} = \sqrt{4^2 \times 6}$
$= \sqrt{4^2} \times \sqrt{6} = 4\sqrt{6}$

b) $\frac{\sqrt{64}}{\sqrt{24}} = \frac{\sqrt{64}}{\sqrt{24}} = \sqrt{\frac{8}{3}} = \sqrt{\frac{2^2 \times 2}{3}} = \sqrt{2^2} \times \sqrt{\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$

Example 5

In a tree planting program, each person planted as many saplings as the total number of participants. If the total number of saplings planted is 1296, then what is the number of participants?

Solution:-

Here, no. of participants = No. of plants planted by each participant.

Therefore, here No. of participants = square root of 1296.

Here,

$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Now, $\sqrt{1296}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt{2^2 \times 2^2 \times 3^3 \times 3^3}$

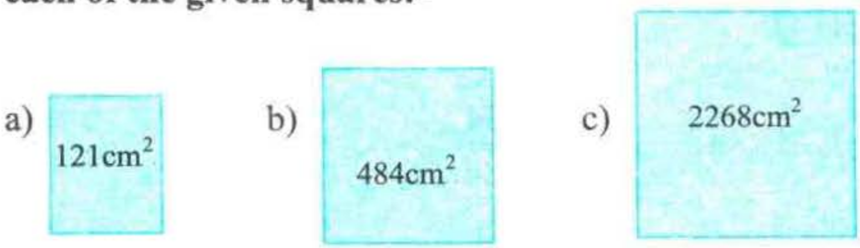
$= \pm 36$

Expression $\therefore \sqrt{1296} = \pm 36$

But the expression no. of participants=-36 is not true. Therefore, the no. of participants=36.

Exercise 2.3

1) **Find the length of the side of each square from the area of each of the given squares:-**



2) **Find the length of the side and the perimeter of the following square shaped objects.**
   a) A mat of $1m^2$ area
   b) A window of $2025cm^2$ area
   c) A field of $625m^2$ area

3) **Find the product of : -**

a) $\sqrt{15} \times \sqrt{5}$ b) $2\sqrt{2} \times 3\sqrt{2}$

c) $2\sqrt{21} \times \sqrt{14}$ d) $3\sqrt{14} \times \sqrt{28}$

e) $2\sqrt{20} \times \sqrt{45}$ f) $\sqrt{20} \times \sqrt{72} \times \sqrt{6}$

g) $\sqrt{14} \times 2\sqrt{6}$ h) $\sqrt{7} \times 2\sqrt{21}$

i) $\sqrt{15} \times \sqrt{24}$ j) $\sqrt{32} \times \sqrt{66}$

k) $\sqrt{39} \times \sqrt{52}$ l) $\sqrt{63} \times \sqrt{27} \times \sqrt{35}$

4) **Simplify:-**

a) $\dfrac{\sqrt{3} \times \sqrt{20}}{\sqrt{2} \times \sqrt{5}}$ b) $\sqrt{\dfrac{252}{273}}$ c) $\dfrac{\sqrt{27}}{3}$

d) $\sqrt{45} \div \sqrt{45}$ e) $\sqrt{72} \div \sqrt{30}$ f) $\sqrt{176} \div \sqrt{264}$

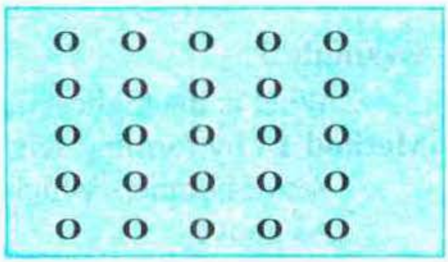
g) $\sqrt{\dfrac{56}{294}}$ h) $\sqrt{\dfrac{48}{72}}$ i) $\dfrac{\sqrt{270}}{\sqrt{234}}$ j) $\dfrac{\sqrt{875}}{\sqrt{1125}}$

5) **What is the least number by which each of the following numbers should be multiplied to make it a perfect square number?**

a) 72 b) 108 c)125 d) 363 e) 1008

f) 192 g) 243 h) 845 i) 588 j) 3528

6) When arranging 25 marbles in the form of a square there are 5 marbles along the length and 5 marbles along the breadth. What is the number of additional marbles required to form a square shape with 6 marbles along the length and 6 along the breadth?



7) How many students should be placed along the length and breadth when 1156 students are arranged in the form of square, during a school prayer assembly ?

8) When equal number of saplings are planted along the length and the breadth of a field, 15625 saplings are required. What is the number of saplings planted in each row?

9) What is the number of saplings required if 81 saplings are planted along the length and 81 along the breadth in the form of a square ?

10) When arranging in a square form, a general kept 64 soldiers in each row. If there are extra 129 soldiers, then..
   a) How many soldiers are there ?
   b) To arrange all of them in a square form, what should be the number of soldiers in each row?

2.4. Highest Common Factor

The set of factors of 12.

$F_{12} = \{1, 2, 3, 4, 6, 12\}$

Similarly the factors of 18,

$F_{18} = \{1, 2, 3, 4, 6, 9, 18\}$

Now, the set of common factors of 12 and 18 = $\{1, 2, 3, 6\}$

In the set of common factors, the highest common factor= 6. Therefore, the highest common factor (H.C.F)=6. Divide the given numbers by 6 and see what happens.

The highest factor of the common factors of given natural numbers is called the highest common factor. In the short form, the highest common factor is written as H.C.F.

Example 3

What is the highest number which divides 12 and 30 exactly ?

Method 1 - by forming sets

Set of numbers which divides 12 exactly

As above

$F_{12} = \{1, 2, 3, 4, 6, 12\}$

The set of numbers which divides 30 exactly.

$F_{30} = \{1, 2, 3, 4, 5, 6, 10, 15, 30\}$

The set of common factors = $\{1, 2, 3, 6\}$

Highest common factor = $\{6\}$

Therefore, H.C.F. = 6

Method - 2 By Prime Factorization

By prime factorization of 12, $12 = 2 \times \boxed{2} \times \boxed{3}$

By prime factorization of 30, $30 = \boxed{2} \times \boxed{3} \times 5$

Common factors = 2×3

Therefore, H.C.F. = 6

H.C.F. = Product of the common factors

Method 3 - By division Algorithm

Of the given numbers, dividing the greater one by the smaller one, Keeping these operations in a place

12) 30(2
  24
  6 Remainder

Dividing the divisor by the remainder

6) 12(2
  12
  ×

12) 30(2
  24
  6) 12(2
    12
    ×

Therefore H.C.F. = 6

Example 4

What is the highest number which divides 18,27 and 48 exactly ?

Solution :

Here, the required number is H.C.F. of 18,27 and 48.

Therefore, 18) 27(1
  18
  9) 18(2
    18
    ×

$\therefore$ Highest number that divides 18 and 27 exactly = 9. Now, dividing the remaining number 48 by 9.

9)48(5
  45
  3) 9(3
    9
    ×

$\therefore$ Highest number dividing 9 and 48 exactly = 3. Therefore, the highest number dividing 18,27 and 48 exactly = 3.

Example 5

What is the highest number of children to whom 35 apples, 75 oranges and 105 bananas can be distributed equally and what is the number of each fruit that each will get ?

Solution :

Here, the required number is the H.C.F. of 35,75,105

Therefore,

```
35) 75(2                          5) 105(21
    70                               10
    5) 35(7                           5
       35                             5
        ×                             ×
```

Therefore, the highest no. of children to whom the fruits are distributed equally = 5 and each will get $35 \div 5 = 7$ apples, $75 \div 5 = 15$ oranges and $105 \div 5 = 21$ bananas.

c) The relation between H.C.F. and L.C.M. of two numbers. Determining the H.C.F. and L.C.M. of 12 and 18.

The H.C.F. of 12 and 18 = 6

```
12) 18(1
   -12
    6) 12(2
      -12
        ×
```

And, the L.C.M. of 12 and 18 = $6 \times 2 \times 3 = 36$

$$\begin{array}{r|l} 6 & 12, 18 \\ \hline & 2, 3 \end{array}$$

Here H.C.F. × L.C.M. = $6 \times 36 = 216$

and, 1st number × second number = $12 \times 18 = 216$

$\therefore$ H.C.F. × L.C.M. = the 1st number × the 2nd number.

Example 6

The L.C.M. and H.C.F. of two numbers are respectively 240 and 16 and if one number is 48 then what is the other number ?

Solution :

Now, H.C.F. × L.C.M. = 1st No. × 2nd No.

$16 \times 240 = 48 \times$ 2nd No.

$\therefore$ 2nd number $= \dfrac{16 \times 240}{48} = 80$

Exercise 2.4

1. **Find the H.C.F. of the following given numbers by forming set of factors.**

   a) 6, 9 b) 4, 6 c) 8, 12 d) 8, 16
   e) 9, 12 f) 12, 15 g) 15, 20 h) 18, 22

2. **Find the H.C.F. of the following given number by prime factorization method.**

   a) 16, 40 b) 27, 36 c) 18, 45 d) 72, 80
   e) 24, 60 f) 52, 104 g) 64, 80 h) 54, 81

3. **Find the H.C.F. of the following given numbers by Division algorithm method**

   a) 24, 36 b) 42, 54 c) 12, 32 d) 50, 75
   e) 48, 72 f) 35, 56 g) 64, 80 h) 99, 165

4. What is the greatest number of people to whom 180 apples and 270 oranges can be distributed equally ? How many fruits of each kind will each person get ?
5. What is the greatest number of children to whom 225 bananas, 250 guavas and 300 peaches can be distributed equally ? How many fruits of each kind will each child get ?
6. If the L.C.M. of two numbers are 60, H.C.F. is 5 and one of the numbers is 15 then what is the other number ?

7. If the product of two numbers is 216 and H.C.F. is 6, then what is the L.C.M. ?
8. There are 208 apples in a basket and 247 apples in another basket. What is the highest number of apples that can be picked out at a time from each basket so that both the baskets will be empty at the same time ?
9. A vessel has 30 liters of milk and another has 5.5 liters. What is the highest capacity in liters of the container that can empty each vessel ?
10. A rectangular court-yard is 21 m long and 9 m wide. What is the length of the biggest square marble needed to pave it with the square marbles of the same size ?

2.5. Lowest Common Multiple

Make a set of multiples of 4 and 6:
The set of multiples of 4:

$$M_4 = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...\}$$
$$M_6 = \{6, 12, 18, 24, 30, 36, 42, ...\}$$

Now, the set of common multiples of numbers 4 and 6 = {12, 24, 36, ...} The element 12 of this set of these numbers 4 and 6 is called the lowest common multiple. In short, the Lowest Common Multiple is written as L.C.M. Divide each common multiple by 4 and 6 separately. What conclusion do you make from this ?

Divide the Lowest Common Multiple 12 of 4 and 6 also by 4 and 6 separately.

> The lowest common multiple of two or more than two natural numbers is the smallest natural number exactly divisible by those numbers. In abbreviation, the Lowest Common Multiple is written as L.C.M.

Example 1

Find the lowest common multiple of 8 and 12.

Solution :

Method - I By forming a set :

The set of multiples of 8

$M_8 = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...\}$

The set of multiples of 12

$M_{12} = \{12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...\}$

Now, the set of common multiples of 8 and 12 = {24, 48, 72, ...}

Here, the smallest common multiple = 24

Therefore, Lowest Common Multiple (L.C.M.) = 24.

Method - II By Prime Factorization :

Factors of the following numbers.

Here, while finding prime numbers that divide 8 and 12 exactly,

$8 = \boxed{2} \times \boxed{2} \times 2$ Here, common factors = 2×2

$12 = \boxed{2} \times \boxed{2} \times 3$ Remaining factors = 2×3

$12 = \text{Common Factors} \times 2 \times 3$

Therefore, L.C.M. = $2 \times 2 \times 2 \times 3 = 24$

Only for two numbers:

L.C.M. of the numbers = Common Factors × Remaining Factors.

Method - III Short-Cut Method

Dividing exactly 8 and 12 continuously by prime numbers.

Common Factors

$$\begin{array}{r|l} 2 & 8,\ 12 \\ \hline 2 & 4,\ 6 \\ \hline & 2,\ 3 \end{array}$$

Remaining Factors

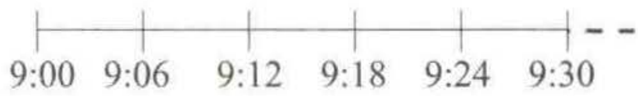
Therefore, L.C.M. = $2 \times 2 \times 2 \times 3 = 24$

Example 2

From a bus stop, buses depart to west, every 6 minutes, to east every 7 minutes and to south every 8 minutes. If buses depart from the bus-stop at 9.00 am in these three directions, at what time will buses from the same bus-stop will leave next time at the same time ?

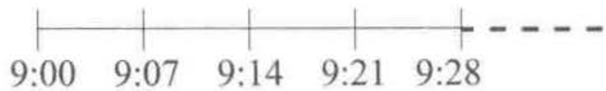
Solution : Representing according to the question by figure :-

Towards West

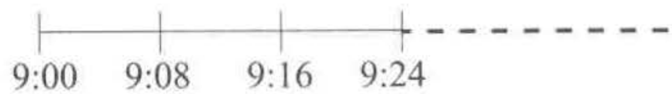


(Here, departure time is represented)

Towards East



Towards South



Now, to find the L.C.M. of 6, 7 and 8,

$$\begin{array}{r|l} 2 & 6, 7, 8 \\ \hline & 3, 7, 4 \end{array} \qquad \text{L.C.M.} = 2 \times 3 \times 7 \times 4 = 168$$

Therefore, next time, buses will depart in the three directions at 168 minutes past 9 o'clock or after 2 hours 48 minutes.
2 hours 48 minutes means 11:48 a.m.

Exercise 2.5

1. **Find the Lowest Common Multiple of each of the following set of numbers by writing sets of multiples.**

a) 3, 5	b) 4, 6	c) 6, 8
d) 8, 10	e) 8, 12	f) 6, 7
g) 9, 12	h) 10, 12	i) 6, 9
j) 12, 16		

2. **Find, by the division method, the Lowest Common Multiple of each of the following given numbers.**

a) 6, 9	b) 9, 12	c) 10, 14	d) 14, 20
e) 20, 24	f) 24, 30	g) 24, 36	h) 21, 28
i) 6, 9, 12	j) 20, 24, 30	k) 21, 28, 42	l) 18, 24, 30

3. Three bells ring at the intervals of 20 minutes, 24 minutes and 30 minutes respectively. If they ring at once at 10 o' clock in the morning, then, after what time will they ring at the same time again ?

4. Of the three corporations, the meeting of the first corporation is conducted every 6 weeks, the meeting of the second corporation every 9 weeks and the meeting of the third corporation every 12 weeks. If all the corporations conducted the meeting at once on 1st Baisakh, 2052, then after how many weeks, will the second meeting be conducted at the same time ?

5. What is the least number of students for a community work in class7 while making group of 5, then 7 and finally 10 in each so that no students are left out ?

6. A motorcycle is to be refueled after covering every 800 km., mobil is to be changed after every 1000 km. and servicing is to be done after every 1500 km. After performing all these three works at the same time, after traveling what distance, should all these three works be performed at the same time again ?

2.6. Binary Number System

Hindu Arabic Number System consists of ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In this system, as the base of a number is ten, each whole number is expressed in powers of ten.

For example

653 = 6 hundreds + 5 tens + 3 ones

$= 6 \times 10^2 + 5 \times 10^1 + 3 \times 10^0$

$12067 = 1\text{ten thousand} + 2\text{ thousand} + 0\text{ hundred} + 6\text{ tens} + 7\text{ ones}$
$= 1 \times 10^4 + 2 \times 10^3 \times 0 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$

Thus, the Hindu Arabic Number System is called the Decimal Number System. At present, in the age of science and technology, computer can solve difficult mathematical problems in a second. The operation to put on circuits of a computer on and off is denoted by only two symbols 1 and 0. The system which uses only 1 and 0 is called Binary System.

As numbers in decimal system are representation numbers in powers of ten, numbers in binary system are represented in powers of two.

For instance $101 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$1001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1\times2^0$ etc.

To make it clear 101 and 1001 (Numbers of Binary System) are written also as 101_2 and 1001_2 respectively.

To convert 101_2 and 1001_2 in decimal system.

$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 1 + 4 + 0 \times 2 + 1 \times 1$
$= 5$

$1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1\times2^0$
$= 1 \times 8 + 0 \times 4 + 6 \times 2 + 1 \times 1$
$= 9$

or, $101_2 = 5$ and $1001_2 = 9$

Now look at the table below:

Decimal Number	-	Grouping of binary base 2^4	2^3	2^2	2^1	2^0	Binary Numerials
0						×	0
1						□	1_2
2					□□	×	10_2
3					□□	□	11_2
4				□□□□	×	×	100_2
5				□□□□	×	□	101_2
6				□□□□	□□	×	110_2
7				□□□□	□□	□	111_2
8			□□□□□□□□	×	×	×	1000_2
9			□□□□□□□□	×	×	□	1001_2
10			□□□□□□□□	×	□□	×	1010_2
⋮			⋮	⋮	⋮	⋮	⋮
25		□□□□□□□□□□□□□□□□	□□□□□□□□	×	×	□	11001_2
⋮		⋮	⋮	⋮	⋮	⋮	⋮

One square (□) represents 1 in Decimal Number System.

Now, write the answers of the following questions;

a) In the above table, in which numbers, the digits representing the number has increased by one ?

b) In this system, how does the place value of the numbers increase ?

Example 1

Write the given numbers 1101_2, 10110_2, 100000_2, 111111_2 in a place value table.

Solution :

The place value table of binary number system.

	2^5	2^4	2^3	2^2	2^1	2^0
1101_2			1	1	0	1
10110_2		1	0	1	1	0
100000_2	1	0	0	0	0	0
111111_2	1	1	1	1	1	1

Example 2

Evaluate the given numbers of Example 1 in Decimal Number System.

Solution :

a) 1101_2 $= 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
$= 1\times8 + 1\times4 + 0\times2 + 1\times1$
$= 13$

b) 10110_2 $= 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$
$= 1\times16 + 0\times8 + 1\times4 + 1\times2 + 0\times1$
$= 22$

c) 100000_2 $= 1\times2^5 + 0\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0$
$= 1\times32 + 0\times16 + 0\times8 + 0\times4 + 0\times2 + 0\times1$
$= 32$

d) 111111_2 $= 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$
$= 1\times32 + 1\times16 + 1\times8 + 1\times4 + 1\times2 + 1\times1$
$= 63$

Example 3

Convert 58 to binary number system.

Solution :

Dividing till the quotient is O and writing.
the remainders from the bottom to the top.

$58_{10} = 111010_2$

Lets us check;

$$111010_2 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$$
$$= 1\times32 + 1\times16 + 1\times8 + 0\times4 + 1\times2 + 0\times1$$
$$= 58_{10}$$

$$\begin{array}{c|ll} 2 & 58 & \\ \hline 2 & 29 & \rightarrow 0 \\ \hline 2 & 14 & \rightarrow 1 \\ \hline 2 & 7 & \rightarrow 0 \\ \hline 2 & 3 & \rightarrow 1 \\ \hline 2 & 1 & \rightarrow 1 \\ \hline & 0 & \rightarrow 1 \end{array}$$

Exercise 2.6

1. **Convert each of the following numbers from binary to decimal form.**

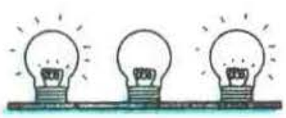
a) 101	b) 111	c) 100	d) 110
e) 1110	f) 1010	g) 1011	h) 10101
i) 11111	j) 101001	k) 110010	l) 100001

2. **Convert the following numbers from decimal form to binary form.**

a) 135	b) 127	c) 107	d) 529
e) 345	f) 572	g) 809	h) 100
i) 1234	j) 5742	k) 1324	l) 1001

3. If glowing bulb represents digit 1 of binary system and non-glowing bulls represents digit 0 of binary systems
Then,

   a) What binary number is represented by series of bulbs of right and what is its value in decimal system ?



   b) Make a series of bulbs to represent a binary number to represent 125 of decimal system.

4. Sitting students represent digit 0 of binary number system and standing students represent digit 1 of binary number system. While forming a number to represent the presence of students one day of class VIII, a model is formed as shown in the figure on right, then, what is the number of students present in the class ?

2.7. Quinary Number System

The base of a number is ten in decimal number system and the base of a number is two in binary number system. Similarly, the number system with base five of a number is called Quinary Number System. This system consists of only five digits 0, 1, 2, 3 and 4.

Also, in quinary number system, the numbers are represented in powers of five as in decimal system, powers of ten and as powers of two in binary number system. For example.

$$12 = 1 \times 5^1 + 2 \times 5^0\ ;\ \ 234 = 2 \times 5^2 + 3 \times 5^1 + 4 \times 5^0 \text{ etc.}$$

These 12 and 234 (Numbers of quniary number system) are made clear also by writing respectively 12_5 and 234_5.

To convert 12_5 and 234_5 to decimal form,

$$\begin{aligned} 12_5 &= 1 \times 5^1 + 2 \times 5^0 \\ &= 1 \times 5 + 2 \times 1 \\ &= 7_{10} \end{aligned}$$

$$\begin{aligned} 234_5 &= 2 \times 5^2 + 3 \times 5^1 + 4 \times 5^0 \\ &= 2 \times 25 + 3 \times 5 + 4 \times 1 \\ &= 69_{10} \end{aligned}$$

or $12_5 = 7_{10}$ and $234_5 = 69_{10}$

Example 6

Prepare a place value table of quinary number system and show the numbers 400_5, 4310_5 and 22243_5 in a place value table.

Place Value table of quinary number system.

	5^4	5^3	5^2	5^1	5^0
400_5			4	0	0
4310_5		4	3	1	0
22243_5	2	2	2	4	3

Example 7

Write the value of the numbers given in Example 6 in decimal form.

Solution :

a) $400_5 = 4\times5^2 + 0\times5^1 + 0\times5^0$
$= 4\times25 + 0\times5 + 0\times1$
$= 100_{10}$

b) $4310_5 = 4\times5^3 + 3\times5^2 + 1\times5^1 + 0\times5^0$
$= 4\times125 + 3\times25 + 1\times5 + 0\times1$
$= 580_{10}$

c) $22243_5 = 2\times5^4 + 2\times5^3 + 2\times5^2 + 4\times5^1 + 3\times5^0$
$= 4\times625 + 2\times125 + 2\times25 + 4\times5 + 3\times1$
$= 1573$

Example 8

Convert 567 to quinary number form.

Solution :

$\overline{0}$ 4
$5\overline{)4}$ 2
$5\overline{)22}$ 3
$5\overline{)113}$ 2
$5\overline{)567}$

Divide until the result is 0 and writing the remainders from the top to the bottom, it can be written as $567 = 4232_5$

Let us check ;

$$4232_5 = 4\times5^3 + 2\times5^2 + 3\times5^1 + 2\times5^0$$
$$= 4\times125 + 2\times25 + 3\times5 + 2\times1$$
$$= 567$$

Exercise 2.7

1. **Convert the following quinary numbers to decimal form.**

a) 123_5	b) 321_5	c) 213_5
d) 432_5	e) 1024_5	f) 2001_5
g) 4203_5	h) 1000_5	i) 3024_5
j) 3004_5	k) 2400_5	l) 12004_5

2. **Convert the following decimal numbers to quinary form.**

a) 147	b) 721	c) 432
d) 579	e) 608	f) 500
g) 728	h) 1000	i) 1002
j) 5072	k) 9095	l) 9999

3. **Convert each of the following numbers from binary to quinary form and quinary to binary form.**

a) 1010_2	b) 1001_2	c) 1000_2
d) 1111_2	e) 2340_5	f) 3000_5
g) 4321_5	h) 1020_5	

2.8. Cube and Cube Root

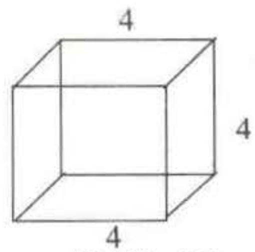


Fig.No. 2.2

Measure a small cubical paper box as given in figure. Find its length, breadth and height. Suppose the measure of each side i.e. l, b, h is respectively 4, 4, 4 cm. or, in total it is 64 cubic cm. $64 = 4 \times 4 \times 4$, Thus, here 64 is a cube number and the cube root of that cube number is 4.

One of the equal factors of a perfect cube number is called the cube root of that number.
For example :
$a^3 = a\times a\times a$ or a is the cube root of a^3 and it is written as $\sqrt[3]{a^3} = a$.

Example 1

Find the cube root of 125.

Solution :

Here, factorizing 125,

$$125 = 5 \times 5 \times 5$$

Therefore, $= \sqrt[3]{125}$

$= \sqrt[3]{5\times5\times5}$ (Cube root of 125)

$= 5$ (Taking only one from a group of three same factors)

Example 2

When the length, breadth and height of cubical box are multiplied, the total volume is 27 cubic cm. What is the length of the box ?

Solution :

Here, to find the factors of 27

$$27 = 3 \times 3 \times 3$$

Therefore, $\sqrt[3]{3\times3\times3}$

$= 3$

$\therefore$ 3 in cube root of 27 and 27 is a cube number of 3.

Exercise 2.8

1. **Express each of the following numbers in the form $8 = 2^3$**

   a) 1 b) 64 c) 125 d) 27

2. **Evaluate**

   a) 4^3 b) 6^3 c) 3^3 d) 8^3

3. **Find the cube root of**

   a) 125 b) 64 c) 216 d) 243

2.9. Inte

2.9.1. Introduction of Integers

Take any two numbers from a set of whole numbers, W = {0, 1, 2, 3, 4, 5, ...}. Suppose those numbers are 3 and 4. Now, the sum of 3 and 4 is 7 and product is 12. Showing these two operations on a number line:

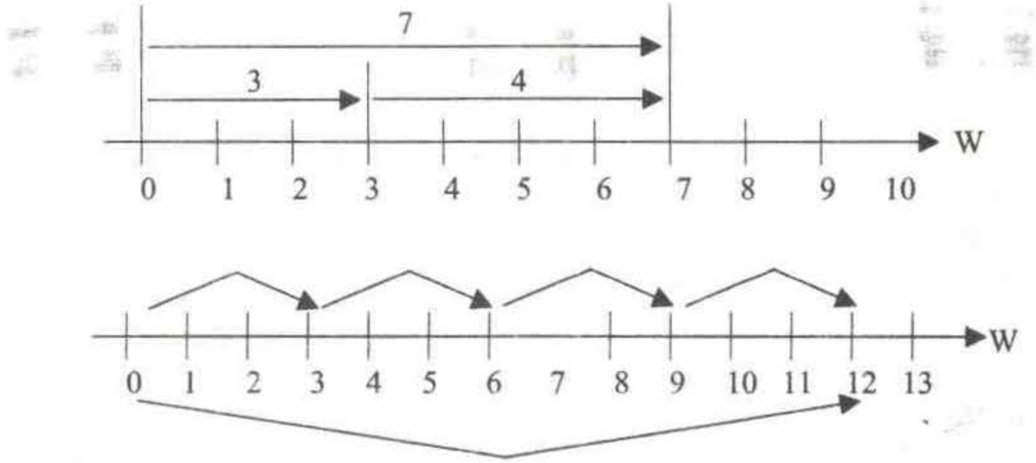


Fig. No. 2.3

Here, number 7 representing the sum and the number 12 representing the product are both whole numbers. Similarly, the sum or product of other members of whole numbers is also a whole number. But what happens then it is subtracted ?

Could you say 3 - 4 = ? Let us see this on a number line.

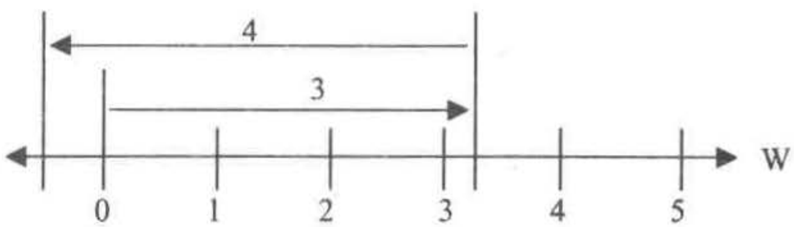


Fig. No. 2.4

From number line 3 - 4 means a number 1 unit less than 0. Write this as - 1. Similarly, 3 - 5= -2 (2 units less than 0_; 3 - 6 = -3 etc. In this manner the number line is extended towards left from 0.

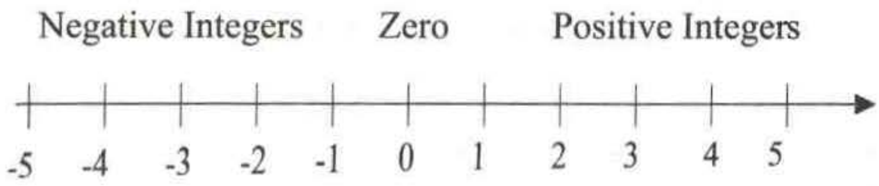


Figure No. 2.5

This set of numbers Z = {... -2, -1, 0, 1, 2, ...} is called the set of integers and subtraction operation is defined on the set of integers. On a number line, the position of 0 is called Point of Reference. The numbers on the right of the point of reference are positive. The set of these numbers, Z^+ = {+1, +2, +3, +4, ...} is called the set of Positive Integers. The numbers on the left of the point of reference are negative. The set of these numbers, Z^- = {-1, -2, -3, -4, ...} is called the set of Negative Integers. Thus,

> Set of integers means the set of numbers consisting of positive integers negative integers and 0 (Zero).

2.9.2. Comparing Integers

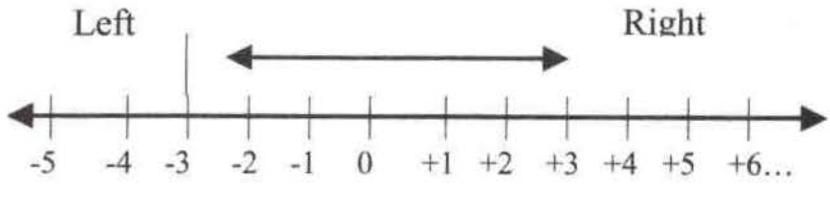


Figure No. 2.6

Take an integer on number line (Fig. No. 2.6). Suppose that the integer is -2. Now, what can you say about the number -3 on the left and the number -1 on the right of -2 ? Could you write the following expression on a number line ?

$$-5 < -4 < -3 < -2 < -1 < 0 < +1 < +2 < +3$$

On a number line the numbers on the right of an integer is greater than that integer and the number on the left is less than that integer.

And,

If a and b be two integers , then only one of the following three facts is possible $a > b$, $a < b$, $a = b$. For example $a = -4$ and $b = -5$ then $a > b$.

Discuss about the above facts by considering any two numbers. Therefore, the facts $a > b$, $a < b$ and $a = b$ are called law of trichotomy.

2.9.3. Opposite of an Integer

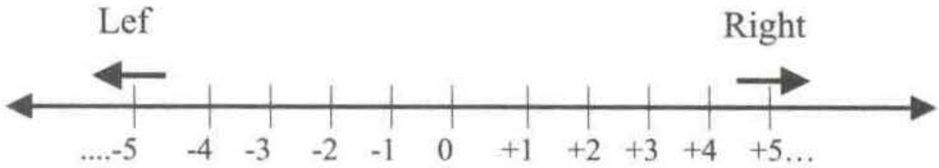


Fig. No. 2.7

Integer +3 represents a number 3 units on the right of the point of reference then -3 represents a number 3 units on the left of the point of reference. Here, integers +3 and -3 are called opposite integers of each other.

An integer at the same distance in opposite directions to any integer at a certain distance from the point of reference is called opposite of that integer.

2.9.4. Absolute Value of Integer

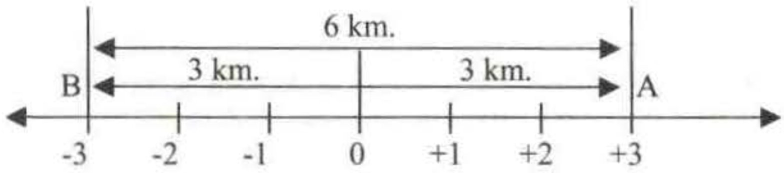


Fig. No. 2.8

In Fig. no. 2.8, point A is at 3 km. on the right of the point of reference. Therefore, we write +3 to represent the position of A and to represent the position of B 3 km. on the left, is written -3. Now, answer the following questions :

a) How far is A from the point of reference ?
b) How far is B from the point of reference ?
c) What is the distance between A and B in Kilometers ?

If we ignore the direction then the numerical value of both +3 and -3 is 3 and in the above example the distance between 3 km. to the right and 3 km. to the left is 6 km. or the absolute value of $^{+}3$ and $^{-}3$ is 3.
We write it as $|{}^{+}3| = |{}^{-}3| = 3$

a) The absolute value of any integer means the numerical value of that integer. Therefore $|{}^{+}a| = |{}^{-}a| = a$
b) Zero 0 is neither positive nor negative. Thus $|0| = 0$ (Zero).

Exercise 2.9

1. Answer the following questions on the basis of the number line.

a) On which side of the point of reference do the numbers less than 0 lie ?
b) On which side of the point of reference do the numbers grater than 0 lie ?
c) On which side of a given number does a number 1 unit less than the given number lie?
d) On which side of a given number does a number 1 unit more than the given number lie?

e) Of the two numbers - 6 and -5, which is greater ?
f) How many integers are there between -5 and 3 ?

2. Write a number 3 units to the left on the basis of a number line ?
a) 5 b) 2 c) 0 d) -1 e) -3

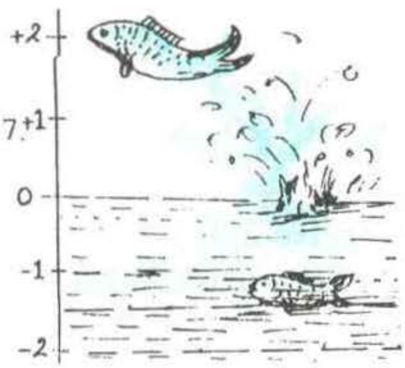
3. Put the symbols '>' or '<' between the following two numbers.

a) +7 ☐ -3 b) +3 ☐ +5
c) -3 ☐ -2 d) -5 ☐ -7
e) -5 ☐ +2 b) +5 ☐ -5

4. How many integers are there between -13 and +3 ?

5. Write the opposite integer of each of the following given integers.
a) - 13 b) +7 c) -4
d) -2 e) +5 f) +3

6. Write the absolute value of each of the following given integer.
a) - 9 b) +7 c) -2
d) 0 e) -5



7. A fish was 1 m. below the water level of a pond. If the fish jumped 2 m above the water level, then how many meters has the fish jumped altogether?

8. Hari is at a place 4 km. east and Ram is at a place 2 km. west of a statue.
a) Using integers show this information on a number line.
b) What is the distance between Ram and Hari ? Find on the basis of absolute value of integers.

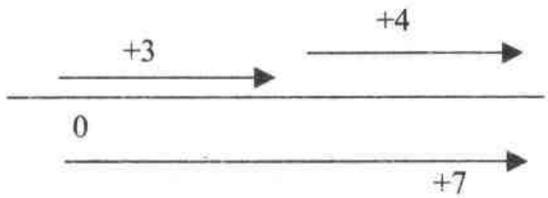
2.10. Addition and Subtraction of Integers

2.10.1. Addition of Integers

Look at the following example related to addition of integers.

Example 1

$$(+3) + (+4) = +(3+4)$$
$$= +7$$



Example 2

$$(-3) + (-4) = -(3+4)$$
$$= -7$$

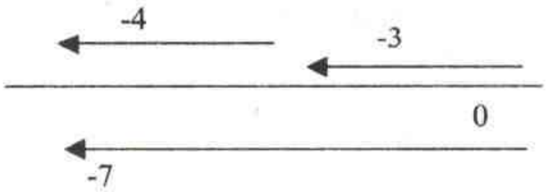


Figure No. 2.9

Based on Examples 1 and 2

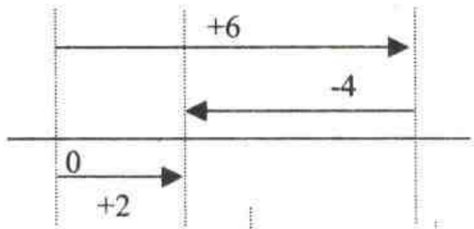
If the sign of integers to be added is the same then the common sign is put in only place to the sum of the absolute value of the integers.

For Examples $(+a) + (+b) = +(a+b)$

$(-a) + (-b) = -(a+b)$

Example 3

$$(+6) + (-4) = +(6-4)$$
$$= +2$$



Example 4

$$(+3) + (-4) = -(4-3)$$
$$= -1$$

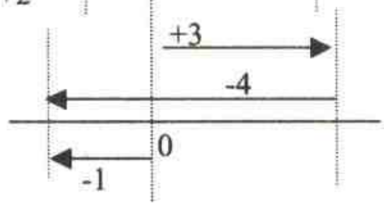


Fig.No. 2.10

Example 5

$(^{+}5) + (^{-}5) = 0$

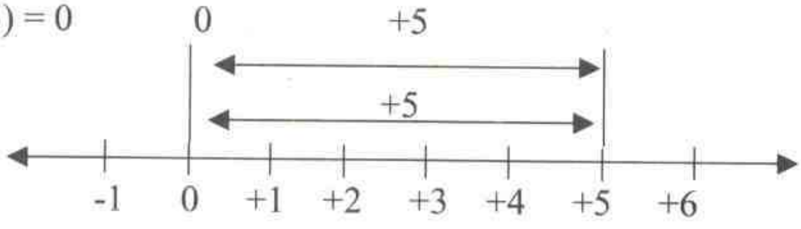


Fig. No. 2.11

The sum of same quantity of positive and negative integers is zero. Here, 0 is called identity of element of Addition. +5 and -5 are called inverse quantity of each other. Similarly, +a and -a are inverse quantity of each other.

Example 6

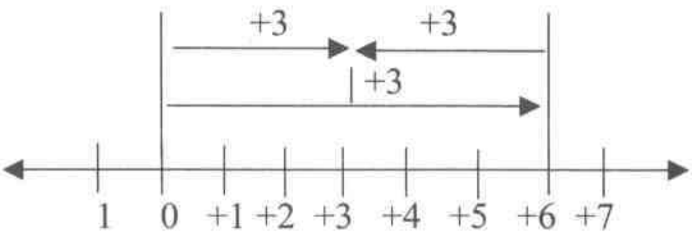
Find the sum :

a) +6 and +3 b) -3 and +6

Solution :

To represent on a number line.

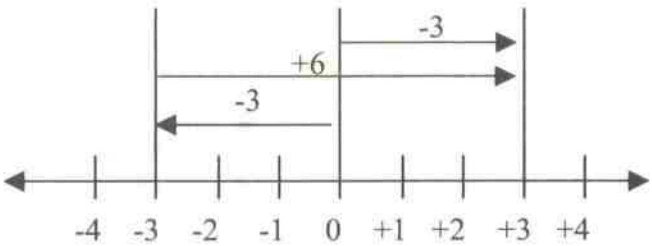
a)



$(+6) + (-3) = +3$

Fig. No. 2.12

b)



$(-3) + (+6) = +3$

Fig. No. 2.13

From the above example :

When finding the sum of integers, integers kept in any order give the same result. This law is called the commutative law. For example; $a + b = b + a$, where a and b are integers.

Example 7

Find the sum of +3, -4 and +5

Solution :

Here, $(+3) + (-4) + (+5)$	$(+3) + (-4) + (+5)$
or $= [(+3) + (-4)] + (+5)$	$= (+3) + [(-4) + (+5)]$
$= (-1) + (+5)$	$= (+3) + (+1)$
$= +4$	$= +4$

From the above example:

When adding three integers, first add any two integers and then the third integer to obtain the sum. In relation to the addition of integers, this law is called the associative law. For example,
$(a+b) + c = a + (b+c) = (a+c) + b$ where, a, b and c are integers.

Example 8 : Add

(a) $(-60) + (+40)$ b) $(-30) + (-20)$

Solution :

a) Here, integers have different sign. Therefore, after determining the absolute value of integers, smaller one is subtracted from bigger one.
$|-60| = 60$; $|+40| = 40$ and $60 - 40 = 20$
Therefore, $(-60) + (+40) = -(60 - 40) = -20$

b) Here, $(-30) + (-20) = -(30 + 26) = -50$

2.10.2. Subtraction of Integers

Look at the following example related to the subtraction of integers.

Write a correct number in the following blank spaces $\square$.

Example 1

Write a correct number in the following $\square$.

a) $\square + (+5) = +2$ b) $\square + (-5) = +2$

Here, the number in $\square$ of (a) can be determined by the following subtraction.

(Explain equation)
$+ (+2) - (+5) =$
The required number according to the figure on the right,
$+ (+2) + (-5) = \square$ can be determined also by addition

Therefore,
$(+2) - (+5) = (+2) + (-5) = -3$

Similarly, subtraction can be done by converting to addition.
Here, "to subtract +5" is the same as "to add -5".

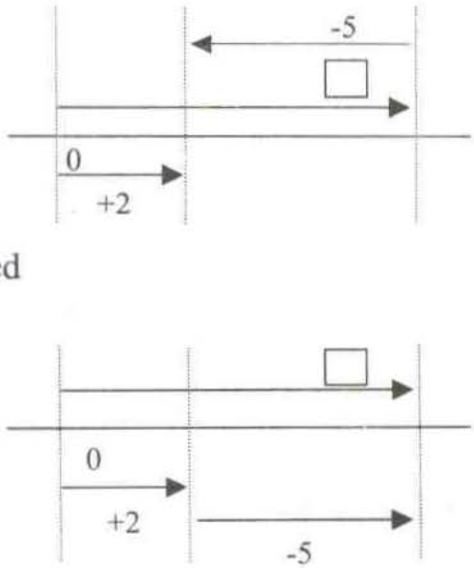


Fig. No. 2.14

b) The number in $\square$ of (b) can be determined by the following subtraction. $(+2) + (+5) = \square$
The required number in $\square$ according to
the figure on the right,
$(+2) + (+5) = \square$ can be determined
also by addition.

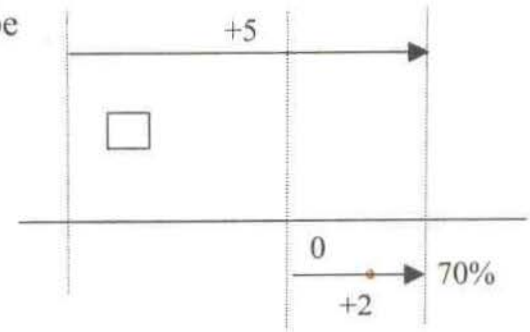


Fig.No. 2.15

Therefore,
(+2) - (-5) = (+2) + (+5) = +7
Here, "to subtract -5" and "to add +5" in same.

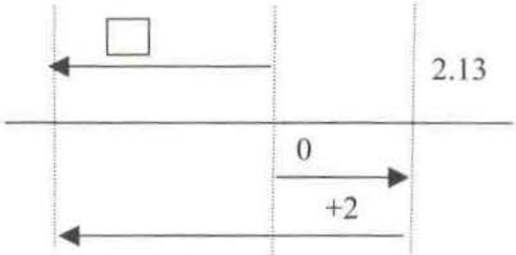


Fig. No. 2.15

Therefore,
(+a) - (+b) - (+a) + (-b)
(+a) - (-b) - (+a) + (+b)

2.10.3. Simplification :

Look at the example

Example 1

6 - 9 = (+6) - (+9) = (+6) + (-9) = -3

6 - 9 = (+6) - (+9) : In difference form
= (+6) + (-9) : In sum form

In this, 6 - 9 represents the difference of positive numbers.
But, 6 - 9 = (+6) + (-9), it is also the sum of 6 and -9.

Example 2

4 - 7+ 9 - 5 ……….. (1) writing as the sum of integers.
As (1) is 4 + (-7) + 9 + (-5) ………. (2), it is the sum of 4, -7, 9 and -5.

Therefore, when brackets and the addition signs are removed from problem 2, it is reduced to problem 1.

Example 3

Simplify the following problem without bracket :

a) 9 + (-4) - 8 - (-6)   b) -17 - (-25) + 3 + (-14)

Solution :

a) $9 + (-4) - 8 - (-6)$
$= 9 - 4 - 8 + 6$
$= 5 - 8 + 6$
$= -3 + 6$
$= 3$

b) $-17 - (-25) + 3 + (-14)$
$= -17 + 25 + 3 - 14$
$= 8 + 3 - 14$
$= 11 - 14$
$= -3$

Exercise 2.10

1. **Decide whether the following statements about integers are true or false.**
   a) Zero is a negative number.
   b) The integers on the left of the origin are negative.
   c) Of the numbers on the left of the origin, the number nearer to the origin is greater than the farther one.
   d) 0 is the smallest integer.
   e) The sum of two integers is always an integer.
   f) The difference of two integer is always an integer.
   g) 0 is called the identity element of addition.
   h) $(+a) - (-b) = -(-b) - (+a)$ is true.

2. **Show the following operations of integers on a number line.**
   a) $(+5) + (+3)$
   b) $(+6) + (-4)$
   c) $(-4) - (+3)$
   d) $(-5) + (-4)$
   e) $(-2) - (-3)$
   f) $(+3) + (-3)$
   g) $(+5) - (+3)$
   h) $(-8) + (+6)$
   i) $(+5) - (-7)$

3. **Fill in the table below with addition and subtraction:**

a)

+	-2	-1	0	+1	+2
-2					
-1					
0					
+1					
+2					

b)

-	-2	-1	0	+1	+2
-2					
-1					
0					
+1					
+2					

4) **Fill in the blanks with suitable integers and state the direction.**

a) East 2 + West 3 = ...
b) West 3 + West 2 = ...
c) West 4 + East 4 = ...
d) West 5 + East 2 = ...
e) East 4 + ... = East 2
f) East 5 + ... = East 7
g) Below 3 + Above 4 = ...
h) Below 3 + Above 4 = ...
i) Right 6 + Left 4 = ...
j) .... + Left 3 = Right 1

5. **An ant moved 3 cm. towards east from the origin and then 2 cm. west.**

a) Show this information on a number line.
b) How far has the ant reached from the origin ?

6. If the sum of two integers is -119 and if the greater integer is 177, then what is the smaller integer ?

7. The difference of two integers is -17. If one is +2 then what is the other ?

8. The minimum temperature of Kathmandu on 3rd Poush, 2051 is -2^0c and maximum, 12^0C.

a) What is the difference between the maximum and minimum temperature ?
b) If the minimum temperature forecast for that day is expected to be 2^0, then what is the difference in the minimum temperatures ?

9. **Simplify using associative law.**

a) (-20) + (+60) - (-30)
b) (+70) + (-25) - (-65)
c) (-45) + (+25) + (-20)
d) (+45) + (-146) + (+209)

10. **Subtract and decide whether it illustrates the commutative law or not.**

a) (+5) - (+2) and (+2) - (+5)
b) (-7) - (-2) and (-2) - (-7)
c) (-5) - (+2) and (+2) - (-5)

11. Simplify :

a) $-21 + 5 + (-32) + 7$ b) $-4 + 14 + 25 + (-52)$

c) $-13 + (+7) - 8 + 14 - 40$ d) $3 - 8 - 11 + 40 - 21 + 5 - 32$

2.11. Multiplication and Division of Integers

2.11.1. Multiplication of Integers

Look at the following examples related to multiplication of integers.

Example 1

A man is going towards east at the speed of 60 m/min. Answer the following questions supports the distance covered towards east as positive and towards west as negative.

a) Position of the man after 5 minutes.

b) Position of the man before 5 minutes.

Solution :

Solving this problem on a number line.

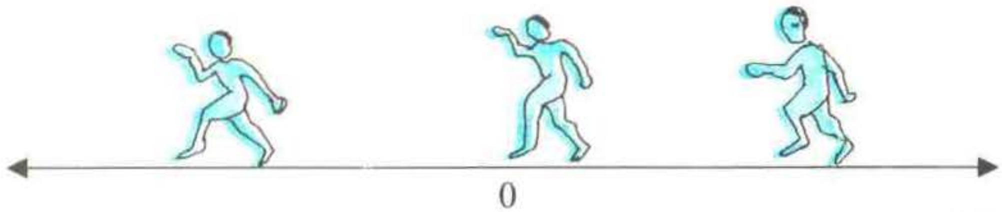


Figure No. 2.16

a) Position after 5 minutes = $(+60\text{m} / \text{min}) \times (+5\text{min.})$

$= +300$ m or 300 m East

b) Before 5 minutes means

-5 minutes = $(+60\text{m} / \text{min}) \times (-5\text{min.})$

Therefore,

Position before 5 minutes = -300 m (West)

From the above example.

The product of two positive integers is positive for example $+60 \times +5 = +300$. The product of one positive and another negative integer is negative. For example $(+60) \times (-5) = -300$

Example 2

A person is moving towards west at the rate of 60 m/min. Answer the following questions assuming that the distance covered towards west as negative and towards east as positive.

a) Position of the man after 5 minutes.
b) Positive of the man before 5 minutes.
Let us find the answer of this problem also by number line.

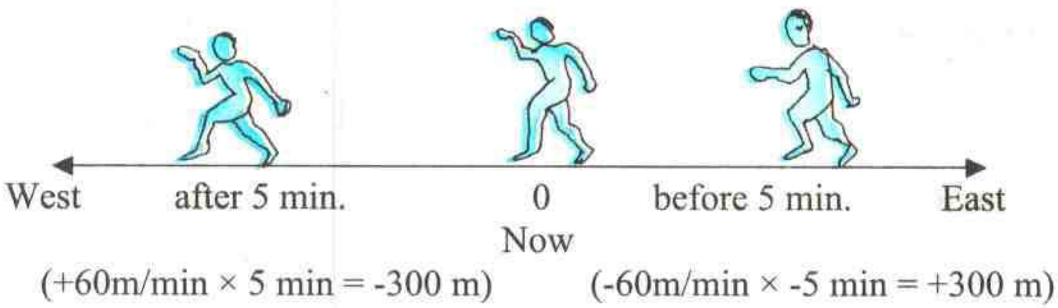


Fig. No. 2.17

a) After 5 minutes means, the time is positive.
The man is moving towards west (Negative direction). That's why the speed in negative.
Therefore, the position of the man after 5 minutes.
$= -60\text{ m / min.} \times (+5\text{ min.})$
$= -300$ m or 300 m West

b) Before 5 minutes means, the time is negative.
Therefore, the position before 5 minutes
$= -60\text{ m/min.} \times (-5\text{ min.})$ Here, also the speed is negative.
$= +300$ m or 300 m East

Therefore, from example 2,

The product of two negative integers is positive. For example - $(-60) \times (-5) = +300$. The product of one negative and another positive integer is negative. For example $(-60) \times (+5) = -300$.

Example 3

Find the product

a) 4×7 b) $(-4) \times (-7)$

Learn by heart
$(+) \times (+) \supset (+)$
$(-) \times (-) \supset (+)$

Solution :

a) $4 \times 7 = (+4) \times (+7)$
$= +28$
$= 28$

b) $(-4) \times (-7) = +28$
$= 28$

Example 4

a) $3\ (-5)$ b) $(-3)\ (5)$

Learn by heart
$(+) \times (-) \supset (-)$
$(-) \times (+) \supset (-)$

Solution :

a) $3\ (-5) = (+3) \times (-5) = -15$

b) (-3(5 $= (-3) \times (+5) = -15$

b) The commutative law is true in multiplication operation of integers.
$(-5) \times (+3) = (+3) \times (-5) = -15$

Example 5

$(+3) \times (+1) = +3$ and $(-3) \times (+1) = -3$

The product of any integer a and +1 is a. Similarly, the product of +1 and a is a. Or $a \times (+1) = a = (+1) \times a$. Here, +1 is called identity element of multiplication.

Example 6

Multiply

$(+2) \times (-3) \times (-5)$

Here, $(+2) \times (-3) \times (-5)$
$= [(+2) \times (-3)] \times (-5)$
$= (-6) \times (-5)$
$= +30$

First,
Finding the product of
+2 and -3

$$\begin{aligned}\text{Here, } & (+2) \times (-3) \times (-5) \\ &= (+2) \times [(-3) \times (-5)] \\ &= (+2) \times (+15) \\ &= +30\end{aligned}$$

First,
Finding the product of -3 and -5

Thus,

$$[(+2) \times (-3)] \times (-5) = (+2) \times [(-3) \times (-5)]$$

Why is the answer same from both the methods ?

When finding the product of any three integers first the product of any two integers is determined and then multiplied by the remaining integer. Or $(a\times b) \times c = a \times (b\times c)$, where a, b, c are integers. This law of multiplication is called associative property.

Example 7

Multiply

a) $(+5) \times [(+7) + (+2)]$ b) $(+5) \times (+7) + (+5) \times (+2)$

Solution :

a) Here, $(+5) \times [(+7) + (+2)]$
$= +5 \times +9 = +45$

b) Here, $(+5) \times (+7) + (+5) \times (+2)$
$= (+35) + (+10) = 45$

Thus, $(+5) \times [(+7) + (+2)] = (+5) \times (+7) + (+5) \times (+2)$

For any integers a, b and c; $a \times (b+c) = a\times b + a\times c$. This law of multiplication is called distributive law.

Example 8

Find the product :

a) $12 \div 4$ b) $(-12) \div (-4)$

Solution :

a) $12 \div 4 = (+12) \div (+4)$
$= +3$
$= 3$

Learn by heart
$(+) \div (+) \supset (+)$
$(-) \div (-) \supset (+)$

b) $(-12) \div (-4) = +3$
$= 3$

Example 9

Find the product

a) $12 \div (-4)$ b) $(-12) \div 4$

Solution :

a) $12 \div (-4) = (+12) \div (-4)$
$= -3$

b) $(-12) \div 4 = (-12) \div (+4)$
$= -3$

Learn by heart
$(+) \div (-) \supset (-)$
$(-) \div (+) \supset (-)$

2.11.2. Division of Integers

Multiplication and Division are Inverse Operations of each other. The laws of sign of integers is same for the multiplication and division,

Now, look at the following examples :

a)	$(+5) \times (+3) = +15$	Therefore,	$(+15) \div (+5) = +3$
		and	$(+15) \div (+3) = +5$
b)	$(+5) \times (-3) = -15$	Therefore,	$(-15) \div (+5) = -3$
		and	$(-15) \div (-3) = +5$
c)	$(-5) \times (-3) = +15$	Therefore,	$(+15) \div (-5) = -3$
		and	$(+15) \div (-3) = -5$

a) When a positive integer is divided by a positive integer, the quotient is positive.
b) When a positive integer is divided by a negative integer, the quotient is negative.
c) When a negative integer is divided by a negative integer, the quotient is positive.

Exercise 2.11

1. **Multiply**

a) $(-3) \times (+5)$ b) $(+7) \times (-4)$ c) $(+6) \times (+8)$
d) $(-8) \times (-2)$ e) $(-16) \times (-45)$ f) $(-17) \times (+13)$

2. **Multiply the integers, using the associative law of multiplication by two methods.**

a) $(+5) \times (+3) \times (+2)$ b) $(+3) \times (+5) \times (-4)$
c) $(-7) \times (+6) \times (-2)$ d) $(-14) \times (-10) \times (+12)$
e) $(-13) \times (-4) \times (-25)$ f) $(-16) \times (-13) \times (-5)$

3. **Simplify by using the distributive law of multiplication.**

a) $(+5) \times [(-3) + (+6)]$ b) $(-2) \times [(+3) - (-5)]$
c) $(-2) \times [(-2) + (-3)]$ d) $[(-15) + (+12)] \times (+6)$
e) $[(-5) + (-5)] \times (+7)$ f) $(-3) \times [(-3) + (-3)]$

4. **Fill in the multiplication table below.**

×	-2	-1	0	1	2
-2					
-1					
0					
1					
2					

5. On the basis of table of Q.No. 4, write 5 examples of commutative law of multiplication.

6. **Evaluate :**

a) $(-12) \div (+3)$ b) $(-15) \div (+5)$
c) $(-16) \div (+4)$ d) $(-24) \div (-8)$
e) $(-42) \div (-14)$ f) $(-96) \div (-24)$

7. The product of two integers is +15. If one of them is -5, then what is the other ?
8. By what quantity -9 should be multiplied to make the product +36 ?
9. The product of two integers is -21 and if one integer is 7 then what is the other ?

2.12. Order of Operation and Simplification Including Brackets

Learn by heart the following rules of simplification.

a) In a mixed problem of addition, subtraction and multiplication, first work for multiplication.
b) First of all, do the division operation in solving a problem including addition, subtraction and division operations.
c) First do division in a problem involving multiplication and division or while simplifying from left to right work for the sign which comes first.
d) In a problem involving different types of brackets, operate the small brackets ( ), Middle bracket { }, and big bracket [ ] in order.

Look at the following examples :-

Example 1

Simplify

$6 - [6 - 5\{3 - 2 \div (3 - 2)\}]$

Here,

$$\begin{aligned} & 6 - [6 - 5\{3 - 2 \div (3 - 2)\}] \\ = \ & 6 - [6 - 5\{3 - 2 \div 1\}] \\ = \ & 6 - [6 - 5\{3 - 2\}] \\ = \ & 6 - [6 - 5 \times 1] \\ = \ & 6 - [6 - 5] \\ = \ & 6 - 1 \\ = \ & 5 \end{aligned}$$

Example 2

Simplify

$6 + [2 - 7 \{13 - 14 \div (4 - 6)\}]$

Here,

$$
\begin{aligned}
& 6 + [2 - 7 \{13 - 14 \div (4 - 6)\}] \\
= \; & 6 + [2 - 7 \{13 - 14 \div (- 2)\}] \\
= \; & 6 + [2 - 7 \{13 + 7\}] \\
= \; & 6 + [2 - 7 \{20\}] \\
= \; & 6 + [2 - 140] \\
= \; & 6 + [-138] \\
= \; & 6 - 138 \\
= \; & - 132
\end{aligned}
$$

Example 3

What is the number when 23 is subtracted from 1 part out of 5 parts of 50 and the result is multiplied by -6 ?

Solution :

Here; translating the given problem to the mathematical statement:

$\{(50 \div 5) - 23\} \times (-6)$

Now, simplifying

$$
\begin{aligned}
& \{10 - 23\} \times (-6) \\
= \; & (-13) \times (-6) \\
= \; & 78
\end{aligned}
$$

Exercise 2.12

1. $20 - \{8 - (15 + 2)\}$
2. $17 - \{19 - 2\,(1 - 3)\}$
3. $-9 + [17 - \{4 + (5 - 2\}]$
4. $[-10 \div \{20 - 3\,(7 - 2\}] - 6$
5. $16 \div \{6 + (17 - 19) - 8\} + 4$
6. $4\,[190 - \{7 - 8\,(9 - 2)\}]$
7. $11 \times 11 \div [-11 \div \{12 - (13 - 12)\}]$

8. What is the number when 3 is subtracted from 1 part out of 6 parts of 4 times 15 is multiplied by 7 ?

9. What is the number when 5 is added to the product of 3 and one fourth of 16 ?

10. What is the number when 30 is subtracted from the product of 5 and 7 and 7 multiplies the difference to which 4 is added ?

11. What is the number when 2 is subtracted from the product of 11 and 6 divided by 22 and the result multiplied by 9 ?

12. What is the number when the sum of one-fourth of 72 itself is added to the sum of one-eight of 72 and 1 ?

3 Rational Numbers

3.1. Introduction :

i) Does the sum of two whole numbers belong to the set of whole numbers ?

ii) Does the difference of two whole numbers belong to a number of Z?

iii) Is the product of any two whole numbers a whole number ?

On the basis of the previous chapter, the sum, the difference and the product of two whole numbers are whole numbers and all belong to the set of integers of Z. Now, take two whole numbers and divide one by the other. Let the whole numbers be 2 and 3. Dividing one by other, we get $2 \div 3 = \frac{2}{3}$ or $3 \div 2 = \frac{3}{2}$. Does $\frac{2}{3}$ or $\frac{3}{2}$ belong to the set of whole numbers ? No, they do not belong to the set of whole numbers so they can not be called integers. Again, let's choose 4 and 8. Dividing one by the other, we get $4 \div 8 = \frac{4}{8} = \frac{1}{2}$ or $8 \div 4 = \frac{8}{4} = 2$. Here $\frac{1}{2}$ and 2 are the quotients, $\frac{1}{2}$ is not a whole number whereas 2 is a whole number.

Therefore, when an integer is divided into another integer, the quotient is not always an integer. (That is why, the division of integers has no meaning for the set of a ....)

Now, we will develop a set of numbers so that the operation of addition, subtraction, multiplication and division will be meaningful. The set of numbers consisting of numbers formed by division of any two integers leads to the formation of such a set. Such type of number is called a rational number. The answers $\frac{2}{3}$, $\frac{3}{2}$, $\frac{1}{2}$, 2 that we obtained above are all rational numbers. 2 means 2 is divided by 1 or $\frac{2}{1}$. So all the whole numbers are rational numbers. Even though 0 (Zero) is an integer, division by 0 is not permissible. The definition of a rational numbers is given below.

If a and b are two whole numbers such that $b \neq 0$, then the numbers expressed in the form of $^{a}/_{b}$ are rational numbers.

The set of rational numbers is denoted by the letter Q of the English alphabet. So Q = $\{-3, -2, -1, 0, 1, 2, 3, \ldots, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \ldots\}$. Therefore $Z \subset Q$. In other words, the set of integers is a subset of the set of rational numbers.

3.2 Rational number and Decimal number.

$\frac{1}{2}$ and $\frac{1}{3}$ are rational numbers. These numbers in decimal form are $\frac{1}{2} = 0.5$ and $\frac{1}{3} = 0.3333 \ldots\ldots = 0.\dot{3}$. In this way when converting rational numbers into decimal we can express them as terminating and non-terminating and recurring decimal numbers. Therefore, the decimal form of rational numbers are always terminating or non-terminating and recurring.

Again, $\sqrt{4}$ means the square root of 4. Its value is 2. So $\sqrt{4}$ is a rational number. So, if the square roots of any integers are integers, they are also rational numbers. Like $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{64}$, $\sqrt{100}$ etc. But $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{10}$ are not rational numbers.

Example 1

Decide whether the following numbers are rational or not.

a) $-\frac{3}{4}$ b) 0.2 c) -0.5 d) $\sqrt{6}$ d) $\frac{3}{2}$ e) $\sqrt{\frac{1}{4}}$

Solution :

a) $-\frac{3}{4}$ can be written as -3 ÷4. Here, -3 and 4 are both integers so the quotient is a rational number

b) As 0.2 is the terminating decimal number so it is a rational number. It can also be written as $\frac{2}{10}$ and since it is the quotient of the whole numbers 2 and 10, it is a rational number.

c) -0.5 is also a terminating decimal number; so it is a rational number.

d) The value of $\sqrt{6}$ is not an integer or the square root of 6 is not an integer, so it is not a rational number.

e) $\frac{3}{2}$ is the quotient of integers 3 and 2, so $\frac{3}{2}$ is a rational number.

f) $\sqrt{\frac{1}{4}}=\frac{\sqrt{1}}{\sqrt{4}}=\frac{1}{2}$ because the square root of 1 is 1 and that of 4 is 2. Now $\frac{1}{2}$ is formed by the division of integers, so it is a rational number.

Example 2

Convert the following numbers into decimals and check whether they are rational or not ?

a) $\frac{4}{5}$ b) $\frac{4}{15}$ c) $\frac{2}{3}$ d) $\frac{1}{9}$

Solution :

a) $\frac{4}{5}$, now 4 is divided by 5

$$\begin{array}{l} 5)\,40\,(\,0.8 \\ \;\;\underline{40} \\ \;\;\;\times \end{array}$$

$\frac{4}{5}=0.8$. It is a terminating decimal number.

Therefore, $\frac{4}{5}$ is a rational number.

b) $\frac{4}{16}$ when reduced into the lowest form.

$\frac{4}{16} = \frac{1}{4}$ Now dividing

4) 10 (0.25
  8
  20
  20
  ×

It is a terminating decimal number.

$\therefore \frac{4}{16}$ is a rational number.

c) $\frac{1}{9}$ when divided

9) 10 (0.111.......
  9
  10

$\frac{1}{9}$ 0.111 ....=0.1 It is a non-terminating but a recurring decimal number.

$\therefore \frac{1}{9}$ is a rational number.

Exercise 3

1. Which of the following numbers are rational numbers ?

a) $\frac{22}{7}$ b) $-\sqrt{14}$ c) $-\frac{3}{5}$ d) $-\frac{\sqrt{10}}{2}$ e) $\sqrt{5}$

f) $-\sqrt{144}$ g) $-\sqrt{2}$ h) 0.666 .. i) 0.125 j) $\sqrt{\frac{9}{16}}$

2. State whether the following statements are true or false.

a. Every integer is a rational number.
b. Every rational number is an integer.
c. A non-terminating decimal number is a rational number.
d. When a rational number is converted into decimal it is either terminating or non-terminating and recurring.
e. Natural numbers are not rational numbers.
f. Even and odd numbers are the rational numbers.

4 Fraction and Decimal

4.1 Word Problems of fraction

In pervious classes, we learnt the four basic rules of fractions. We also learnt to simplify fractions. Now we are gong to learn about fractions that we come across in our daily life problems. The problems that come in our daily life do not always involve integers only. In many cases, fractional numbers seem to appear very often. Study the problems given below and try to solve it.

Ram cut an apple into three equal pieces and ate two pieces. Similarly Krishna cut an apple of same size into four equal pieces and ate three pieces. Who ate more ? How to show in fraction the parts of the apple eaten by Ram ?

How to show in fraction the parts of the apple eaten by Krishna?

Now, $\frac{2}{3}$ the part of the apple is eaten by Ram and $\frac{3}{4}$ of the apple is eaten by Krishna. What should be done to show who has eaten more ? What should be done to the denominator of a fraction to find the bigger or smaller fraction or to compare them ?

To equalize the denominator of $\frac{2}{3}$ and $\frac{3}{4}$, L.C.M. of the denominators should be determined and here 12 is the L.C.M.

Now, make the denominator of both the fractions 12

$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Here $\frac{8}{12} < \frac{9}{12}$

Therefore, the fraction $\frac{9}{12}$ represents the bigger part of the apple. Meaning that the one having $\frac{3}{4}$ part has eaten more so Krishna has eaten more apple than Ram.

While solving word problems involving fractions first express the statement in fraction, then using operations of fraction, find the solution.

Example 1

Prem and Gita surveyed their village. According to that survey, out of five equal parts of children of Prem's village, only three parts of children had taken vaccination while out of ten equal parts of children of Gita's village, seven parts of children were found vaccinated. If the number of children of both villagers is equal, whose village has more vaccinated children ?

Answer :

While writing in a fraction, $\frac{3}{5}$ of the children of Prem's village were vaccinated. The vaccinated children in Gita's village is $\frac{7}{10}$.

To find the greater one, both the fractions should be written with equal denominators. The L.C.M. of 5 class and 10 is 10.

$$\frac{3}{5}=\frac{3\times 2}{5\times 2}=\frac{6}{10}<\frac{7}{10}$$

Now, in $\frac{6}{10}$ and $\frac{7}{10}$, $\frac{6}{10}<\frac{7}{10}$

Therefore, more children of Gita's village were vaccinated.

Example 2

A tank can hold one thousand liters of water. If the tank is one fourth full, how many liters of water are there in the tank ?

Answer:

In fraction, the water contained the tank is $\frac{1}{4}$. The tank can hold 1000 liters of water. Therefore, at present the water in the tank is $\frac{1}{4}$ of 1000 liters.

$\therefore$ Water in the tank $=\frac{1}{4}\times 1000\ l\ = 250$ liter

Exercise 4.1

1. There are 40 children in a village. If 24 children are vaccinated, then what fraction of children are vaccinated ? What part of children are not vaccinated ?
2. There are ten classes in a school, from class 1 to class 10, Classes One to Five, Six to Eight and are primary, lower secondary and secondary levels respectively. What part is the primary, lower secondary and secondary level of total classes ?
3. Hari, Gopal and Shyam bought a sugar cane. Hari, Gopal and Shyam ate $\frac{5}{12}, \frac{1}{3}$ and $\frac{1}{4}$ part respectively. Who did eat less and who did eat more ?
4. Ramesh had some liter of milk. He drank $\frac{7}{8}$ part of it. Leaving $\frac{51}{2}$ liters. How many liters of milk did he have at first ?
5. $\frac{1}{3}$ of the students of a school are boys. If number of girls is 140 then (a) How many students are these ? (b) What is the number of boys ?
6. Geeta spent $\frac{9}{10}$ of the money she had and saved Rs. 20 a) How much did she spend ? b) How much did she have at first?
7. When $3\frac{1}{2}$ of a number is multiplied by $4\frac{1}{3}$ and their product is 7, What is the number ?
8. Sharada solved $\frac{9}{10}$ questions in an exam and $\frac{5}{6}$ were correct. If there were 100 questions then how many did she answer correctly ?
9. Radha feeds $\frac{3}{8}$ packet of 'Lito' to her sister daily. For how many days will 24 packets of Lito last ?

4.2. Terminating, Non-Terminating and Recurring Decimals

Example 1

Convert the following fractions into decimals :

a) $\frac{3}{4}$ b) $\frac{2}{9}$ c) $4\frac{1}{7}$ d) $3\frac{4}{5}$

Solution:

To convert a fraction into a decimal we divide the numerator of the fraction by its denominator.

a) $\frac{3}{4} = 0.75$ b) $\frac{2}{9} = 0.222 \ldots.$

c) $4\frac{1}{7} = 4.1428 \ldots$ d) $3\frac{4}{5} = 3.8$

Now, what do you know ?

- Decimals such as 0.75, 3.8 etc are called terminating decimals.
- 0.222 …, 4.1428 …. etc are called non-terminating of non-terminating decimals. The numerator of a fraction which represents such type of non-terminating decimal numbers is not exactly divisible by the denominator.
- 0.222 …. etc is called a recurring decimal. The numerator of the fraction which denotes such type of decimal numbers is not exactly divisible by the denominator and the same number is repeated in the quotient.

Example 2

Convert each of the following fractions into decimals and then classify each of them as terminating, non-terminating or recurring decimal.

a) $\frac{2}{5}$ b) $1\frac{1}{3}$ c) $5\frac{2}{7}$

Solution :

a) Converting the fraction $\frac{2}{5}$ into decimal, $\frac{2}{5} = 0.4$. It is a terminating decimal.

b) Converting the fraction $1\frac{1}{3}$ into decimal, $1\frac{1}{3}=1.333$ ..., the same number 3 repeatedly comes again and again in this decimal number and hence the decimal number contains a never-ending chain of 3's. So it is a non-terminating and recurring decimal. Here, we write $1\frac{1}{3} = 1.\bar{3}$ to mean that 3 comes again and again in $1\frac{1}{3} = 1.333$ ....

c) Converting $5\frac{2}{7}$ into decimal. $5\frac{2}{7} = 5.2857142857142857142 8$ ...., 285714 comes repeatedly in the decimal number and hence the decimal number is never ending. So it is a non-terminating and recurring decimal. Here, the number 285714 comes repeatedly in $5\frac{2}{7} = 5.285714285714285$ ...., so it is written and $5.\overline{285714}$

Example 3

Multiply each of the following recurring decimal by 10, 100 and 1000.

a) $0.\bar{3}$ b) $0.\overline{147}$

Solution:

a) $0.\bar{3} \times 10 = 0.\overline{33} \times 10 = 3.\bar{3}$

$0.\bar{3} \times 100 = 0.\overline{333} \times 100 = 33.\bar{3}$

$0.\bar{3} \times 1000 = 0.\overline{3333} \times 1000 = 333.\bar{3}$

b) $0.\overline{147} \times 10 = 0.147\overline{147} \times 10 = 1.47\overline{147}$

$0.\overline{147} \times 100 = 0.147\overline{147} \times 100 = 14.7\overline{147}$

$0.\overline{147} \times 1000 = 0.147\overline{147} \times 1000 = 147.\overline{147}$

Example 4

Convert each of the following decimal numbers into fractions :

a) 0.25 b) $0.\bar{3}$ c) $0.\overline{132}$

Solution :

a) $0.25 = \frac{0.25 \times 100}{100} = \frac{25}{100} = \frac{1}{4}$

b) Let, $x = 0.\bar{3}$ .... (i)

$\therefore$ $10x = 3.\bar{3}$ .... (ii)

Subtracting (i) from (ii)

$$\begin{array}{r} 10x = 3.\bar{3} \\ \underline{(-)x = 0.\bar{3}} \\ 9x = 3.0 \end{array}$$

$$\therefore x = \frac{3}{9} = \frac{1}{3}$$

c) Let, $x = 0.\overline{132}$ .... (i)

$\therefore 1000x = 132.\ .\overline{132}$ .... (ii)

Subtracting (i) from (ii),

$$\begin{array}{rl} 1000x = & 132.\ .\overline{132} \\ \underline{(-)\ x =} & \underline{0.\ .\overline{132}} \\ 999x = & 132.000 \end{array}$$

$$x = \frac{132}{999}$$

$$\therefore \quad = \frac{3 \times 44}{3 \times 333} = \frac{44}{333}$$

4.3. Fraction, Decimal and Number line

Fractions and decimals can be shown on a number line. Look at the diagram below. What fraction and decimal number should be written in place of A, B, C, D and E. Write them.

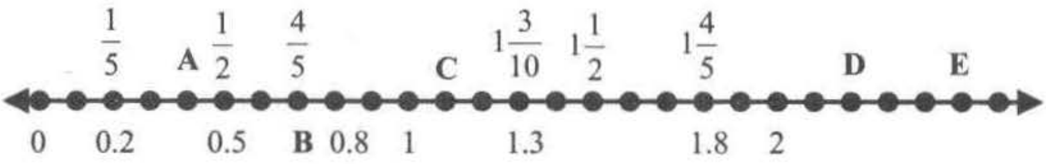


Fig. No. 4.1

Negative fractions and decimals can be shown by extending from 0 to the left the number line as shown in Fig. 4.1. Study the diagram.

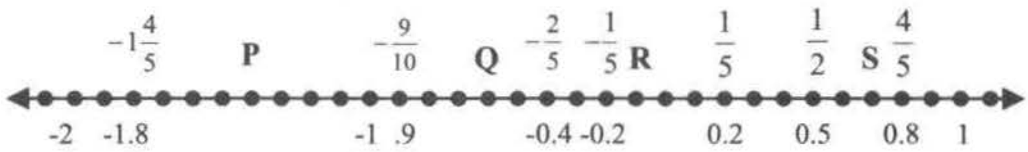


Fig. No. 4.2

What numbers should be written in places of P, Q,R and S on the number line; write them in fractions and decimals.

Exercise 4.2

1. **Find the decimal number representing each of the following fraction by division operation and find out whether the decimal number is terminating, non-terminating or recurring.**

   a) $\frac{2}{3}$ b) $\frac{5}{8}$ c) $\frac{3}{5}$ d) $\frac{5}{7}$

   e) $\frac{7}{8}$ f) $5\frac{5}{6}$ g) $3\frac{1}{16}$ h) $2\frac{4}{11}$

2. **Reduce each of the following decimals to fractions.**

a) 0.25 b) 1.37 c) 7.025 d) 9.001

e) 0.6 f) $3.\overline{16}$ g) $0.\overline{23}$ g) $0.\overline{135}$

3. **Show the following numbers on a number line.**

a) 0.5 b) $-\frac{1}{3}$ c) $2\frac{3}{4}$ d) -1.2 e) $-1\frac{3}{10}$

4.4 Four Fundamental Operations with Decimals.

4.4.1 Addition and Subtraction of Decimals. Look at the examples below.

Example 1

Do the calculation

a) 3.254 + 16.37 b) 14.24 – 90.863

Solution

In addition and subtraction of decimals in the vertical from, decimal points should be placed one below the other in a line.

a)
$$\begin{array}{r} 3.254 \\ +16.370 \\ \hline 19.624 \end{array}$$

Addition of fraction is also done according to the place value as in whole numbers. Putting 0 in thousandths place.

b) Here, if the number to be subtracted is greater than the number from which subtraction is to be done, it can be done as follows. Therefore, a – b = - (b – a) in which b is greater than 'a'.

Therefore, 14.24 – 90.863

= - (90.863 – 14.24)

$$= \begin{array}{r} 90.863 \\ -14.240 \\ \hline 76.623 \end{array}$$ (Putting O in thousandths place)

or - 76.623

$\therefore 14.24 - 90.863 = -76.623$

4.4.2 Multiplication and Division of Decimals

Example 2

Multiply

a) 5.038×1.27 b) $0.259 \times (-0.0406)$

Solution

a) 5.038×1.27

Writing in vertical form

$$\begin{array}{r} 5.038 \\ \times\ 1.27 \\ \hline 352666 \\ 10076 \\ 5038 \\ \hline 6.39826 \end{array}$$

5.038 .... 3 places of decimal

1.27 2 places of decimal

6.39826 There should be adding 5 places after the decimal point

So, $5.038 \times 1.27 = 6.39826$

b) When a positive number is multiplied by a negative number, the product is a negative number.

Writing in vertical form.

$$\begin{array}{r} 0.259 \\ \times 0.0406 \\ \hline 1554 \\ 105154 \\ \hline 0.0105154 \end{array}$$

0.259 .... 3 place of decimal

0.0406 ....4 place of decimal

Adding

0.0105154 ...... There are 7 places after decimal point (So, 7 digits are made by putting 0)

$\therefore 0.259 \times (-0.0406) = -\ 0.0105134$

Example 3

Divide

a) $2.928 \div 0.03$ b) $-45.6 \div 0.8$ c) $1.032 \div (-0.24)$

Solution

a) $2.928 \div 0.03$, when multiplying both the divisor and dividend by 100, the decimal point disappears. Therefore

$$2.928 \times 100 = 292.8$$
$$0.03 \times 100 = 3$$

Now, dividing

$$\begin{array}{l} 3)\ 292.8\ (97.6 \\ \quad \underline{27} \\ \quad\ 22 \\ \quad\ \underline{21} \\ \qquad 18 \\ \qquad \underline{18} \\ \qquad \times \end{array}$$

$\therefore\ 2.928 \div 0.03 = 97.6$

b. Here, when a negative number is divided by a positive number, the quotient is a negative number.
Writing as absolute values

$0.8\overline{)45.6} =$

(Multiply by 10) (Multiply by 10)

$$\begin{array}{r} 57 \\ 8\overline{)456} \\ \underline{40} \\ 56 \\ \underline{56} \\ \times \end{array}$$

So, $-45.6 \div 0.8 = -57$

c. Here, when a positive number is divided by a negative number, the quotient is a negative number.

Writing the absolute value in the vertical form.

$0.24\overline{)1.032}$

(Multiply by 10) (Multiply by 10)

$$\begin{array}{r} 4.3 \\ 24\overline{)103.2} \\ \underline{96} \\ 72 \\ \underline{72} \\ \times \end{array}$$

$\therefore 1.032 \div 0.24 = -4.3$

Know the following facts related to division of decimal.

- When dividing a decimal number by a whole number after dividing the digit put a decimal point in the quotient after decimal.
- When dividing a decimal numbering a decimal number, multiply both the divisor and the dividend by 10 or power as 10 to remove the decimal point from the divisor. Only after this divisor 5 should be done.

Example 4

If Ram earns Rs. 375.85 daily, then what is his income for 12 days ?

Solution :

Here, Ram's 12 days income is 12 times his daily income so,

$$\begin{array}{lr} \text{Rs.} & 375.85 \\ & \times \quad 12 \\ \hline & 75170 \\ & 375850 \\ \hline & \text{Rs. } 4510.20 \end{array}$$

Therefore, 12 days income of Ram = Rs. 4510.20

Example 5

If a road 24 km. 640 m long is constructed in 11 days then what is the length of the road constructed per day ?

Solution :

Here, when the total length of the road 24 km. 640 m. is divided by 11 the total number of days, then the length of the road constructed in 1 day is obtained :

So,

$$\begin{array}{r@{\,}l} & 2.240 \\ 11\overline{)} & \overline{24.640} \\ & 22 \\ \hline & 26 \\ & 22 \\ \hline & 44 \\ & 44 \\ \hline & 0 \end{array} \qquad (24 \text{ km. } 640 \text{ m} = 24.640 \text{ km.})$$

Therefore, the length of the road constructed per day = 2.240 km. or 2km. 240 m.

Example 6

Simplify

$$\{(-3.1)^2 - 2.5\} \div (-0.3) + 10\frac{4}{5}$$

Solution :

$$\{(-3.1)^2 - 2.5\} \div (-0.3) + 10\frac{4}{5}$$
$$= \{(-3.1)^2 - 2.5\} \div (-0.3) + 10\frac{4}{5}$$
$$= 7.11 \div (-0.3) + 10.8$$
$$= -23.7 + 10.8$$
$$= -12.9$$

Example 7

Simplify

$$(-5.4) \times \{24 \div (-16) + 9.6\} \div \frac{27}{10}$$

Solution

$$(-5.4) \times \{24 \div (-16) + 9.6\} \div \frac{27}{10}$$

$$= (-5.4) \times \{-15 + 9.6\} \div \frac{27}{10}$$

$$= (-5.4) \times \{-54\} \div \frac{27}{10}$$

$$= 29.16 \times \frac{10}{27}$$

$$= \frac{291.6}{27} = 10.8 \text{ Answer}$$

Exercise 4.3

1) **Add**

a) $3.012 + 4.257$ b) $17.05 + 16.953$

c) $40.527 + 7.027 + 0.592$ d) $7.542 + 39.407 + 305.2$

2) **Do the sums**

a) $14.57 - 12.659$ b) $-17.05 + 16.953$

c) $345.987 - 317.058$ d) $-6.752 - 4.249 + 11$

e) $3.57 - 7.425 + 13.65$ f) $40.527 - 7.027 - 35.592$

3) **Find the product of**

a) 3.045×5 b) 15.373×14

c) 16.504×27 d) 3.479×1.5

e) $(-5.005) \times 2.7$ f) $-9.046 \times (-2.6)$

g) 56.875×1.25 h) 79.235×4.56

i) $46.025 \times (-1.28)$

4) **Find the quotient of**

a) $7.71 \div 0.3$ b) $4.136 \div 0.02$
c) $-59.84 \div 3.4$ d) $38.745 \div (2.7)$
e) $31.008 \div 9.6$ f) $-13.095 \div (-0.15)$
g) $72.512 \div (-0.016)$ h) $128.73 \div 0.015$
i) $-80.35 \div (-3214)$

5) **Simplify**

a) $3.7 \times 0.9 - 7.6$
b) $(74.5 - 0.7) \times 0.35$
c) $(4.67 - 13.12) \times (-5.6)$
d) $(4.8 \div 0.24) \times 0.572$
e) $-5.7 + \{4.25 \times (-2.4) + 6.2\}$
f) $(-38.72 - 68.38) \div (20.35 + 3.45)$
g) $8.16 \div \{(-1.6 \times 2.532) + 1.5012\}$ h) $\frac{3}{4} \div (0.25 - 0.5) + 3$
i) $\left(\frac{2}{3} - \frac{4}{5}\right) \times (1.4 - 5) \div 0.8$

6) **If a rectangular court-yard is 45.75 m. long and 30.45 m. wide,**

a) What is the perimeter of the court-yard ?
b) What is the area of the court-yard ?

7) **If a rectangular field of area 308.5726 m^2 is 22.54 m long then,**

a) What is its breadth ?
b) What is its perimeter ?

8) The circumference of the wheel of a car is 2.75m. How many revolutions are made by the wheel in covering 22 km ?

9) The diameter of a coin is 1.025cm. How many coins identical to this coin must be placed in a row to cover a distance of 15.37 m ?

5 Percentage, Ratio and Proportion

5.1 Percentage:

Ram obtained 45 marks out of full marks of 100. It is written as $\frac{45}{100}$.

Now $\frac{45}{100}$ is written in percent as 45 percent or 45%. The symbol % denotes percent. A fraction with a denominator 100 is called percent.

Example 1

There were 8 apples in a basket. If two apples were bad (rotten) then what percentage of apples were rotten ?

Solution

Number of rotten apples = 2

Total no. of apples in the basket = 8

Now,

$$\begin{aligned}\frac{2}{8} &= \frac{2}{8} \times 100\% \\ &= \frac{1}{4} \times 100\% \\ &= 25\%\end{aligned}$$

By alternative method:

$$\frac{2}{8} = \frac{1}{4} = 0.25 = 0.25 \times 100\% = 25\% \qquad (0.25 \times 100 = 25)$$

Example 2

Convert 75% to a fraction

Solution

Here, according to the definition of percent:

$$75\ \% = \frac{75}{100} = \frac{3 \times 25}{4 \times 25} = \frac{3}{4}$$

In the above examples -

- $\frac{2}{8}$ is a fraction. To convert it to percent multiply by 100%.
- $\frac{2}{8} = 0.25$ is a decimal fraction and $0.25 = \frac{25}{100}$.

It is a fraction which represents 25%.

Therefore, the relation of fraction, decimal and percent can be shown as follows:

Fraction → Decimal Fraction → Fraction → Percentage

$\frac{2}{8}$ → 0.25 → $\frac{25}{100}$ → 25%

Rule :

a) When converting a percent to a fraction divide by 100 and remove symbol %.

b) When converting a fraction to a percent, multiply by 100 and write the symbol%.

Example 3

There were 12 eggs in a basket. If 75% of eggs were broken :

a) How many eggs were broken ?

b) How many eggs were in good condition?

Solution

a) Here, 75% of 12

$= 12 \times \frac{75}{100}$ (While converting % to a fraction, divide by 100)

$= 12 \times \frac{3}{4} \qquad \left(\frac{75}{100} = \frac{3}{4}\right)$

$= 3 \times 3 = 9$ eggs were broken.

b) If 9 eggs out of 12 eggs were broken, the remaining eggs = 12 - 9 = 3 eggs are in good condition.

Example 4

If 2 men out of 7 men employed for a piece of work left the job, then what percent of the men left the job ?

Solution :

When expressing the men who left the job as a fraction $\frac{2}{7}$.

To find in percentage = $\frac{2}{7} = \frac{2}{7} \times 100\%$

$= \frac{200}{7}\% = 28\frac{4}{7}\%$

Exercise 5.1

1. **Convert each of the following fractions to percent.**

   a) $1\frac{3}{5}$ b) $2\frac{7}{8}$ c) $3\frac{1}{20}$ d) $2\frac{3}{25}$ e) $1\frac{5}{16}$

2. **Convert the following decimal fractions to percent.**

   a) 0.23 b) 1.2 c) 0.35 d) 12.5 e) 0.2

3) **Change each of the following percents to fractions.**

   a) $55\frac{1}{2}\%$ b) $57\frac{2}{3}\%$ c) $60\frac{1}{4}\%$ d) $70\frac{2}{5}\%$ e) $90\frac{3}{5}\%$

4) **Find the value:**

   a) 25% of Rs. 12 b) 10% of 1 hour
   c) 5% of 20 Kg d) 20% of 5 liters
   e) 40% of 2 km f) 25% of 30 cm

5) If Bishal stood first securing 80% in an examination of full marks of 60, then what mark is obtained by Bishal ?

6) **If in a class of 36 students, 9 are absent then,**
   a) What percentage are present ?
   b) What percentage of students are absent ?

7) **If there are 12 girls and 18 boys in a class then,**
   a) What percentage are girls ?
   b) What percentage are boys ?

8) **If Pawan spent 37% of his Rs. 2500, then**
   a) How much did Pawan spend ?
   b) How much is left with him ?

9) **If 5 students out of 40 students fail in an examination,**
   a) What percentage have failed ?
   b) What percentage have passed ?

10) What interest is received by depositing Rs. 500 in a bank for 1 year at the interest rate of 9% per annum ?

11) Ram gave 55% of Rs. 500 to his son Hari and the rest to another son Santosh. Now, how much did each receive ?

12) **If Namuna has spent 37% of her Rs. 2500 and deposited 60% of the remainder in a bank,**
   a) How much has she spent ?
   b) What amount is deposited in the bank ?
   c) What amount is left with her ?

5.2 Ratio

5.2.1 Introduction to Ratio

In Mathematics test of full marks of 20, Shila obtained 12 marks and Kailash obtained 18 marks. How to compare the marks obtained by Shila and Kailash ?

One method to compare is also to find the difference of their marks. The difference of marks of Kailash and Sheela is 18 - 12 = 6 marks. This method of comparison is not effective because the difference of 10 and 4 is also 6. The next method of comparison is the marks obtained by one is divided by the marks obtained by the other.

Here, the marks obtained by Kailash is $\frac{18}{12}$ or $\frac{3}{2}$ times the marks obtained by Shila.

Shila's marks is $\frac{18}{12}$ or $\frac{3}{2}$ times Kailash's marks.

The method of comparing two quantities by dividing one quantity by another same kind of quantity is called finding the ratio of those two quantities. The ratio of marks obtained by Kailash and Sheela is represented by a fraction $\frac{3}{2}$ or 3:2 and reads as ratio of 3 and 2.

Here, in a ratio $\frac{3}{2}$, the first term or numerator is 3 and the second term or denominator is

The ratio of two quantities of the same kind a and b is $\frac{a}{b}$ or a:b, where 'a' in the ratio of a and b, a is the first term (antecedent) and 'b' the second term (consequent).

A ratio is also a fraction so all the laws of fraction are also true for ratios.

e.g.: $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$ ....etc are equal fractions

Example 1

A ribbon is 63 cm long, 28 cm of the ribbon is painted red and the rest is painted blue. What is the ratio of red and blue colour ?

Solution :

The length of the ribbon painted red = 28 cm
The length of the ribbon painted blue = (63 – 28)cm = 35cm
Therefore, the ratio of the parts painted red and
blue colours = $\frac{28cm}{35cm} = \frac{7\times4}{7\times5} = \frac{4}{5}$

Here, $\frac{4}{5}$ has no unit because a ratio is determined from the quantities of the same kind. So, its has no unit.

Example 2

The ratio of ages of a son and the father is 1:4 . If the age of father is 48 years then what is the age of the son ?

Solution

Here, $\frac{\text{Son's age}}{\text{Father's age}} = \frac{1}{4}$

or, $\frac{\text{Son's age}}{\text{48 years}} = \frac{1}{4}$

Therefore, Son's age = $\frac{1}{4} \times 48\text{years}$

$= 12$ years

The age of the son is 12 years

5.1.2 Comparing fractions

See the example below:

Example 3

Krishna secured 24 marks in health education of full marks 30 and 15 marks in Geography of full marks of 20. Now, in which subject has Krishna done better ?

Krishna obtained marks in health education = $\frac{24}{30}=\frac{4}{5}$

Krishna obtained marks in geography = $\frac{15}{20}=\frac{3}{4}$

To know in which subject Krishna has done better is to compare the ratios = $\frac{4}{5}$ and $\frac{3}{4}$ and determine which one is greater.

Here, L.C.M. of 5 and 4 = 20.

Therefore, $\frac{4}{5}=\frac{4\times4}{5\times4}=\frac{16}{20}$

and $\frac{3}{4}=\frac{3\times5}{4\times5}=\frac{15}{20}$

Here, $\frac{16}{20}>\frac{15}{20}$ or $\frac{4}{5}>\frac{3}{4}$

So, Krishna has done better in health education.

Exercise 5.2.1

1) **Reduce each of the following ratios in its lowest terms.**

a) $\frac{12}{18}$ b) $\frac{25}{70}$ c) $\frac{26}{78}$ d) $\frac{64}{80}$

e) $\frac{51}{68}$ f) $\frac{56}{49}$

2) **Write the ratios of each of the following and reduce them to lowest terms.**

a) 75 cm and 1 m b) 3 days and 3 weeks

c) 6 hours and 1 day d) 3 months and 1 year

e) 250 g and 2 kg f) 750 ml and 1 litre

3) **Compare each of the following ratios**

a) 3:5 and 7:8 b) 2:3 and 4:5

c) 3:7 and 7:11 d) 3:7 and 5:9

e) 5:9 and 7:11 f) 3:4 and 8:9

4) **If, out of 260 students of a school, 182 are boys, then find the following ratios of :**
a) Boys and the total students
b) Girls and the total students
c) Boys and Girls

5) **If the ratio of two numbers is 3:4 and the second number is 24, then ....**
a) What is the first number ?
b) What is the ratio of the difference of two numbers and of their sum ?

6) The ratio of father's and his son's age is 4:2. If the age of the son is 14 years then what is the age of the father ?

7) **If Shiva secured 16 marks out of 20 marks in English, 18 marks out of 25 marks in Science and 25 marks out of 30 marks in Mathematics then**
a) In which subject English or Science has he done better?
b) In which subject Science or Mathematics has he done better?
c) In which subject has he done the best ?

5.2.3. Concept of Proportion

Ram has spent Rs 9 out of Rs 12 he had. Similarly, Shyam has spent Rs 15 out of Rs. 20. Now, the ratio of Ram's expenditure and the money he had with him = $\frac{9}{12}$ or $\frac{3}{4}$

The ratio of Shyam's expenditure and the money he had with him

$$= \frac{15}{20} \text{ or } \frac{3}{4}$$

Therefore, $\frac{9}{12} = \frac{15}{20}$

If the two ratios are equal, then the terms of those ratios are proportional. Here, 9 is the first term, 12 is the 2^{nd} term, 15 is the 3^{rd} term and 20 is the 4^{th} term.

Of the four quantities a, b, c and d of the same kind of quantities and if the ratio of a and b is equal to the ratio of c and d then a, b, c and d are said to be in proportion and it is denoted by $\frac{a}{b}=\frac{c}{d}$ or a:b::c:d

Example 1

If the three terms of a proportion are 1, 2, 3 then what is the fourth term ?

Solution

If x is the fourth term, then

$$\frac{1}{2}=\frac{3}{x} \qquad \left(\begin{array}{l} If\ \frac{a}{b}=\frac{c}{d},\ then \\ a\times d=b\times c \end{array} \right.$$

or $1 \times x = 2 \times 3$

Therefore, $x = 6$

Example 2

If 5 pens cost Rs. 75 then what is the cost of 2 pens ?

Solutions :

Taking the ratio of pens and the ratio of costs,

$\frac{5}{2}=\frac{Rs.\ 75}{Rs.\ x}$ [Supposing, cost of 2 pens = Rs. x]

or $5 \times x = 75 \times 2$

Therefore, $x = \frac{75\times 2}{5} = 30$

Cost of pens =Rs 30

Exercise 5.2.2

1) **Determine whether or not the following four numbers are in proportion.**

a) 3, 4, 9, 12	b) 1, 3, 5, 15	c) 4, 5, 8, 12
d) 7, 9, 21, 27	e) 5, 7, 20, 27	f) 4, 7, 12, 21

2) **Of the 4 numbers which are proportion in proportion the first three numbers are as follows. Find the fourth term.**

a) 1, 4, 5	b) 3,7, 9	c) 16, 12, 4
d) 6, 9, 4	e) 12, 20, 6	f) 27, 45, 18

3) **The ratio of the length and breadth of a rectangular field is 5:3 and lenght is 60m.**
   a) What is the length ?
   b) What is the perimeter ?

4) If 6 kilos of apples cost Rs 240 then what is the cost of $2\frac{1}{2}$ kilos ?

5) In a scale drawing of a house, the length is shown as 5cm and the breadth as 3 cm. actual length is 15 m. then what is the actual breadth ?

6) Kailash traveled a distance of 180 km on a Honda Motorcycle on 3 liters of petrol. How much petrol is required to travel 150 km. farther ?

5.3 Profit and Loss

In business, a businessman pays certain money to buy goods. The money paid is called the cost price. A businessman sells goods to customers by adding a certain profit on his cost price. The price at which goods are sold is called the selling price. It can not be said that there is always a profit in business. If the selling price is greater than the cost price, there is a profit and if the selling price is less than the cost price, there is a loss. Know the following facts about profit and loss.

- Profit = Selling Price - Cost Price
- In abbreviation (P) = S.P. - C.P. (S.P. > C.P.) (Profit)
- Loss = Cost Price - Selling Price.
- In abbreviation (L) = C.P. - S.P. (S.P. < C.P.) (Loss)

Example 1

If a tape recorder is bought for Rs. 4300 and sold for Rs. 4500. What is the profit or loss ?

Solution :

Here Cost Price (C.P.) = Rs. 4300

Selling Price (S.P.) = Rs. 4500

Here; S.P. > C.P. Therefore, there is a profit.

$\therefore$ Profit (P) = S.P. - C.P.

= Rs. 4500 - Rs. 4300 = Rs. 200

Example 2

If Rs. 375 is lost on selling a cycle bought for Rs. 3250, then what is the selling price ?

Solution :

Here, Cost Price (C.P.) = Rs. 3250

Loss (L) = Rs. 375

Selling Price (S.P.) = ?

We know,

Loss (L) = C.P. - S.P.

or, Rs. 375 = Rs. 3250 - S.P.

$\therefore$ Selling Price (S.P.) = Rs. 3250 - Rs. 375

= Rs. 2875.

Example 3

A businessman bought 12 dozen pencils at the rate of Rs. 1.75 per pencil and sold them at the rate of Rs. 2 each, then what is the profit ?

Solution :

Cost Price (C.P.) = Not of objects × Rate per price
= 12 × 12 × Rs. 1.75
= 144 × Rs. 1.75
= Rs. 252

Selling Price (S.P.)= No. of objects × Rate of selling price per piece
= 144 × Rs. 2
= Rs. 288

Therefore, Profit (P) = S.P. - C.P.
= Rs. 288 - Rs. 252
= Rs. 36

Exercise 5.3

1. **In each of the following cases, find what the profit or loss is.**

	Cost Price	Selling Price
a)	Rs. 140	Rs. 175
b)	Rs. 300	Rs. 270
c)	Rs. 250.50	Rs. 300
d)	Rs. 500	Rs. 475.75
e)	Rs. 375.50	Rs. 325.25
f)	Rs. 1275.90	Rs. 1300.10

2. If 50 oranges are bought at the rate of Rs. 2.50 each and sold at the rate of Rs. 2.75 each, then what is the profit ?
3. If Bishal Electronics gained Rs. 115 on selling a radio bought for Rs. 1785, then what is the selling price of the radio ?
4. If there is a loss of Rs. 50 on selling a bag bought for Rs. 587, then what is the selling price ?
5. If sugar bought at the rate of Rs. 2200 per quintal is sold at the rate of Rs. 24.50 per kg., what is the profit or loss ? Find out.
6. If 1 dozen copies is bought for Rs. 207 and sold at the rate of Rs. 18 per copy, what is the profit or loss ? Find it.

7. If there is a profit of Rs. 100, on selling 20 copies bought for Rs. 500, then at what rate per copy should they be sold ?

8. A businessman bought 360 radio sets of the same kind. 65% of them were sold at the rate of Rs. 1540 each and the rest at the rate of Rs. 1420 each. If there is a profit of Rs. 42480, then what is the cost price of each radio ?

5.3.1. Problems Involving Percentage on Profit and Loss

Example 4

Sabitri bought 1000 eggs for Rs 3500. 200 eggs are rotten. On selling the rest of the eggs at the rate of Rs 3.60 per egg, what is the profit or loss ? Find in percent.

Solution.

No. of eggs bought = 1000

No. of rotten eggs = 200

$\therefore$ remaining no. of eggs = 1000 - 200

= 800.

Now,

Cost price (C.P.) = Rs 3,500

Selling price (S.P.) = Remaining no. of eggs × Selling price of each egg

= 800 X Rs. 3.60

= Rs 2,880.

Here, C.P. > S.P.

Therefore, there is loss

Loss = C.P. - S.P.

= Rs 3,500 - Rs 2,880

= Rs 620

Now,

$$\text{Loss }\% = \frac{\text{Actual Loss}}{\text{C.P.}} \times 100\%$$

$$= \frac{\text{Rs } 620}{\text{Rs}3500} \times 100\% = \frac{124}{7}\%$$

$$= 17\frac{5}{7}\%$$

Formula to determine the profit percent and loss percent

$$\text{Profit }\% = \frac{\text{Actual Profit}}{\text{C.P.}} \times 100\%$$

$$\text{Loss }\% = \frac{\text{Actual Loss}}{\text{C.P.}} \times 100\%$$

Example 5

A businessman sold a T.V. set for Rs 4000 and made a profit of 25% from it. What is the cost price ?

Solution

Supposing the cost price as 100%, profit is 25%.

Therefore,

S.P. = 125% of C.P.

Now,

$$\text{Rs. } 4000 = \frac{125}{100} \times \text{C.P}$$

$$\therefore \text{C.P.} = \text{Rs. } \frac{4000 \times 100}{125}$$

$$= \text{Rs. } 3200$$

Exercise 5.3.1

1) **Find the profit or loss percent for each of the following situations:**

	Cost Price (Rs.)	Selling Price (Rs.)
a)	500	575
b)	1800	2000
c)	760	660
d)	450	540
e)	380	304
f)	600	540

2) If a fruit seller bought apples at the rate of Rs 60 per dozen and sold them at the rate of Rs 5.50 per apple, then what is his profit in percent ?

3) A stationery shopkeeper bought 100 dot pens at the rate of Rs 4 per dot pen. 20 dot pens are spoilt. The rest of the dot pens are sold at the rate of Rs 4.75 each. What is his profit or loss in percent ?

4) A businessman bought 100 bulbs at the rate of Rs 16.50 each, out of which 5 bulbs are bad. If the rest of the bulbs are sold at the rate of Rs 18 each then what is his profit or loss in percent ?

5) A book seller made a profit of 30% by selling books. He made a profit of Rs 2600, then, how much did he pay for the books ?

6) A watch seller made a profit of 20% on selling a watch for Rs 1,260.

   a) What is the cost price ?

   b) To make a profit of 30%, at price should it be said ?

7) At what price should an umbrella bought for Rs 180 be sold to make a profit of 20% ?

8) A fruit seller bought 120 oranges for Rs 500 and sold 80 oranges at the rate of Rs 5 each. If the remaining oranges were sold at the rate of Rs 4.50 each, then what is the profit or loss in percent ?

5.4 Simple Interest

If you deposit Rs 300 in a bank in a saving account then after 1 year the bank returns Rs 324 to you.

Here,

a) Sum deposited in the bank is called the principal (P). Here, Rs 300 is the principal.

b) The total sum returned by the bank is called the amount (A). Here, Rs 324 is the amount.

c) Bank added a certain sum to the sum deposited. This added sum is called the interest (I). Here, interest = Rs 324 - Rs 300 = Rs 24.

d) Bank pays interest at a certain rate.
Here the bank paid Rs 24 as interest on Rs 300 deposited in a saving account.

Rate per annum (Rate - R) = $\frac{24}{300} \times 100\% = 8\%$

Therefore, 8% is the interest rate per annum

e) The period for which a sum is deposited in a bank is called Time (T). Here, time is 1 year. Know the following terms used in Simple Interest

a) Principal (P)	d) Rate (R)
b) Amount (A)	e) Time (T)
c) Interest (I)	

Formula for Simple Interest
Example 1

Hari deposited Rs P in a bank for T years at the interest rate of R% per annum then what is his interest I ?

Here, R% per annum means

Interest on Rs 100 for 1 year $= R$

Interest on Re 1 for 1 year $= \frac{R}{100}$

$\therefore$ Interest on Re 1 for T years $= \frac{T \times R}{100}$

$\therefore$ Interest on Rs P for T years $= \frac{P \times T \times R}{100}$

Therefore,

Formula to find Interest

$$I = \frac{P \times T \times R}{100}$$

Example 2

Find the simple interest and the amount on Rs 5000 for 9 months at 10% per annum.

Solution :

Here,

Principal (P) = Rs 5000

Time (T) = 9 months $= \frac{9}{12}$ years $= \frac{3}{4}$ years

Rate of Interest (R) = 10% per annum

Interest (I) = ?

(Here, why months are expressed in years ?)

Now,

$$\text{Interest (I)} = \frac{P \times T \times R}{100} = \frac{5000 \times \frac{3}{4} \times 10}{100} = \text{Rs } 375$$

Therefore,

Interest = Rs 375

Amount (A) = P + I

= 5000 + 375

= 5375

Therefore,

Amount (A) = Rs 5375

Example 3

What sum should be lent to get Rs. 200 in interest in 2 years at the interest rate of 8% per annum ?

Solution

Here, Interest (I) = Rs 200

Principal (P) = ?

Time (T) = 2 years

Rate of Interest (R) = 8% per annum

By,

$$\text{Interest (I)} = \frac{P \times T \times R}{100}$$

$$\text{or } 200 = \frac{P \times 2 \times 8}{100}$$

$$\therefore P = \text{Rs } 1250$$

Therefore, Principal Rs 1250.

Example 4

Shiva got Rs 2232 as amount on a loan of Rs 1800 at the rate of 12% per annum. Then for how long was the loan given ?

Solution :

Here, Principal (P) = Rs 1,800

Time (T) = ?

Rate of Interest (R) = 12% per annum

Amount = Rs 2232

Here, Interest (I) = A - P

= Rs 2232 - Rs 1800

= Rs 432

$$\text{Therefore, (I)} = \frac{P \times T \times R}{100}$$

$$\text{or } 432 = \frac{1800 \times T \times 12}{100}$$

$$\therefore T = 2 \text{ years}$$

Therefore, Time = 2 years

Example 5

At what rate percent per annum will Rs 1250 amount to Rs 1580 in 4 years ?

Solution :

Here, Principal (P) $= \text{Rs } 1{,}250$

Time (T) $= 4$ years

Rate of Interest (R) $= ?$

Amount $= \text{Rs } 1500$

Here, Interest (I) $= A - P$

$= \text{Rs } 1500 - \text{Rs } 1250$

$= \text{Rs } 250$

Therefore, (I) $= \dfrac{P \times T \times R}{100}$

or 250 $= \dfrac{1250 \times 4 \times R}{100}$

$\therefore$ R $= 5$

Therefore, Interest Rate $= 5\%$ per annum.

Example 6

What sum will amount to Rs 3250 in 6 years at 5% per annum ?

Solution :

Here, Interest (I) $=$ Amount (A) - Principal (P) $= \text{Rs } (3250 - P)$

Principal (P) $= ?$

Time (T) $= 6$ years

Rate of Interest (R) $= 5\%$ per annum

Now, by formula

$$3250 - P = \frac{P \times T \times R}{100}$$

$$\text{or} \quad 3250 - P = \frac{P \times 6 \times 5}{100}$$

$$\text{or} \quad \frac{13P}{10} = 3250$$

$$\therefore P = \frac{10}{13} \times 3250 = \text{Rs } 2500$$

Therefore,

Principal $= \text{Rs } 2500.$

Exercise 5.4

1) **Find the simple interest**

	Principal	Interest Rate	Time
a)	Rs 350	7% per annum	3 years
b)	Rs 720	$8\frac{1}{3}$% per annum	5 years
c)	Rs 210.52	5% per annum	5 years
d)	Rs 1500	12% per annum	3 years 6 months

2) Find the simple interest and amount of Rs 2500 for 6 years at the rate of 9% per annum.

3) Find the simple interest and amount of Rs 868.50 for 16 months at $3\frac{1}{3}$% per annum ?

4) **Find the rate percent per annum**

	Principal	Interest	Time
a)	Rs 300	Rs 37.50	5 years
b)	Rs 825	Rs 82.50	3 years
c)	Rs 900	Rs 450	5 years
d)	Rs 250	Rs 40	8 months

5) At what rate percent will Rs 400 amount to Rs 600 in 2 years ?

6) At what rate percent will Rs 850 amount to Rs 1275 in 5 years ?

7) **Find the time**

	Principal	Interest	Rate (Per annum)
a)	Rs 475	Rs 57	8%
b)	Rs 1680	Rs 238	10%
c)	Rs 251.25	Rs 40.20	12%
d)	Rs 1700	Rs 850	10%

8) In what time will Rs. 2400 amount to Rs. 3000 at 10% per annum ?

9) Krishna deposited Rs. 30,000 in a savings account at the rate of 8% per annum. In how many years later will the bank return Rs. 77,200 in total to Krishna ?

10) **Find the principal**

	Rate per annum	Time	Interest
a)	6%	5 years	Rs. 300
b)	12%	6 years	Rs. 144.72

11) What amount should be deposited by Kailash in a bank in a savings account at 8% per annum for 3 years to receive interest of Rs. 288?

6 Unitary Method

6.1 Direct Variation

The quantity and the price of potatoes are shown in the following table. Copy the table in an exercise book and fill in the blanks.

Quantity of potatoes (Kg.)	1	2	3	4	6	8	?	?	20
Price (Rs.)	12	24	36	48	?	?	144	192	240

Here, what has happened to the prices when the quantity of potatoes is 2 times, 3 times, ...?

Here, the relation between the quantity of potatoes and its price is shown in graph as follows:

Now,

Give the answer to the following questions from the graph.

a. What is the price of 4 kg of potatoes?

b. What is the price of 2.5 kg of potatoes?

c. How many kilos of potatoes can you buy for Rs.30?

d. What type of equation is formed if the graph given on the right is presented by an equation?

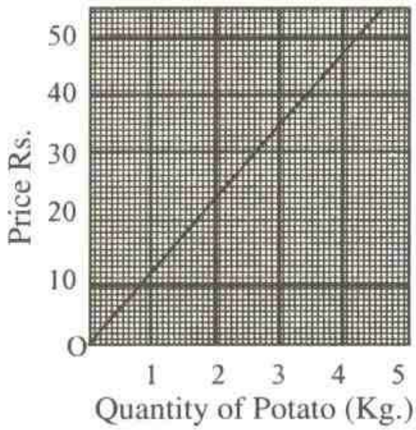


Fig.No. 6.1

Thus, if one quantity increases or decreases in a certain ratio, the other quantity also increases or decreases in the same ratio then such type of quantities are said to be in direct variation.

Example-1

If the price of 10 meters of cloth is Rs.300, answer the following questions on the basis of the following graph.

a) What is the price of 5 meters of cloth?

b) How many meters of cloth can you buy for Rs.120?

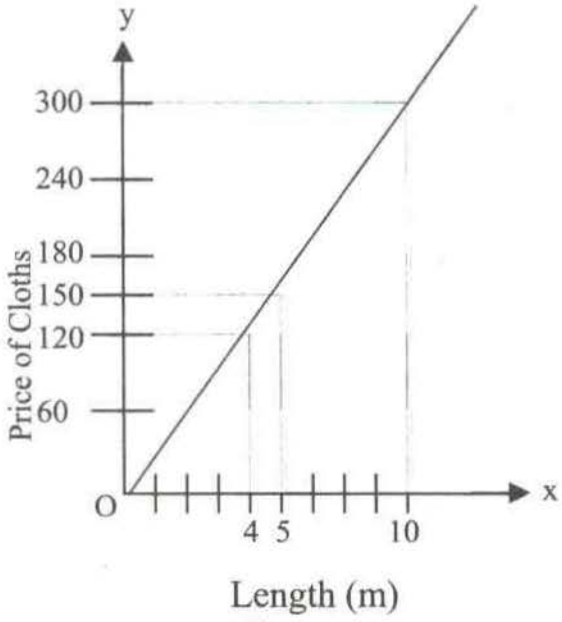


Fig... ... ..

Solution

a) When a perpendicular `is drawn on the price line from the point of intersection of the straight line and the perpendicular drawn on the length line at 5, it meets the price line at 150. So the price of 5m, of cloth is Rs.150.

b) The perpendicular drawn on the length line from the point of intersection of the straight line and the perpendicular drawn on the price line at 120 meets the length line at 4, So 4m of cloth can be bought for Rs.120.

Excerise-6.1

1. **Do these relations relate to the problems of direct variation ?**
   a) Number of pens and the price of pens
   b) Cow and its legs
   c) Orange trees and oranges
   d) Age of a human and his/her height

2. **Draw the graph of direct variation from the following table.**

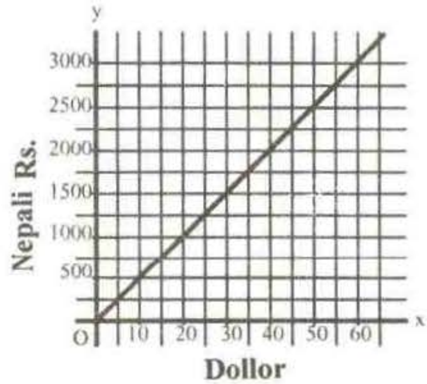
a)

Quantity of apples (kg)	1	2	3	4	5
Price (Rs)	12	24	36	48	60

b)

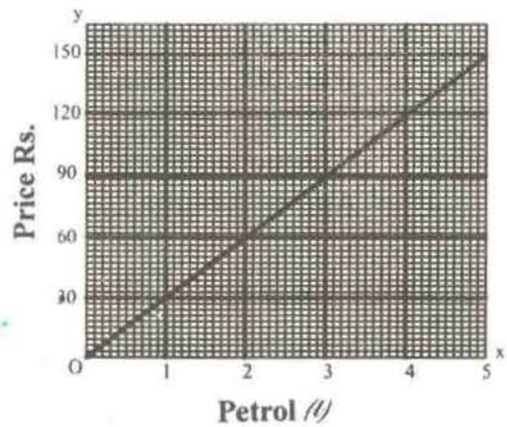
No.of workers	1	2	3	10	20
Wages (Rs)	85	170	255	850	1700

3. **The exchange rate of the U.S. dollar and the Nepali rupee is shown in the following graph. Now, give the answer of the following questions.**



   a) How many N.Rs. do you get when you exchange \$ 15?
   b) How many N.Rs. is equal to \$50?
   c) How many N.Rs. is equal to \$35?
   d) How many U.S. dollar can be exchanged with N.Rs 2750?
   e) Find the equation which represents the graph on the right.
   f) How many N.Rs. do you get for \$250? Find out from the equation.

4. **The quantity (in litre) of petrol and its price (in Rs.) are given in the following graph. Now, give the answer of the following questions.**



   a) What is the price of 3.5 litres of petrol?
   b) How many litres of petrol can you buy for Rs.120?
   c) Find out the equation which represents the graph on the right.
   d) How many litres of petrol can you buy for Rs.870?

6.2 Indirect Variation

a) The speed per hour of a vehicle and the time taken by it to cover a distance of 240km are given in the following table. Copy the table in an exercise book and fill in the blanks.

Speed (Km/h)	....	10	20	30	40	?	80	?	20	....
Time (h)	....	24	12	?	6	4	3	2.4	?	....

b) Here, what has happened to the time when the speed is 2 times, 3 times,....................?
The tables above are represented by the graph on the right.

c) In the figure on the right, has the vehicle taken more or less time as its speed increases ?

d) What is the required speed of the vehicle to cover the distance in 10 hours ?

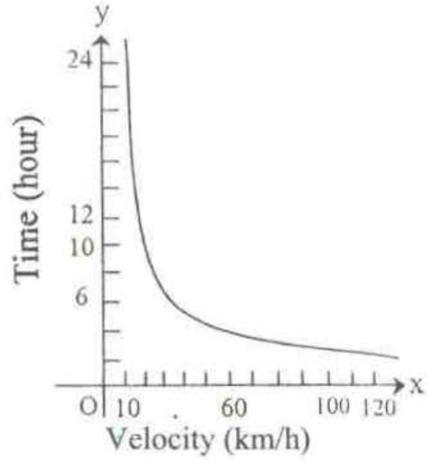


Fig. No. 6.3

Thus, if one quantity increases (or decreases) and the other quantity also decreases (or increases) in the same ratio, then they are said to be in inverse proportion.

Example-1

The area of a rectangle is 36 cm^2 .

a) Show the given graph in a table.

b) If the measurement of one side is 9cm, find the measurement of the other side.

Solution:

a) Here, from the given graph

Measurement of one side (cm)	2	4	6	9	12	18
Measurement of another side (cm)	18	9	6	4	3	2

b) When the measurement of one side is 9cm, the measurement of the other side is 4cm.

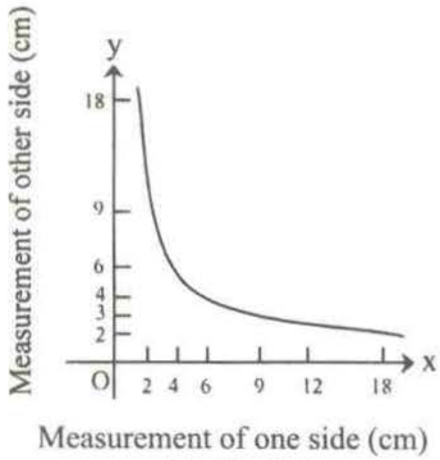


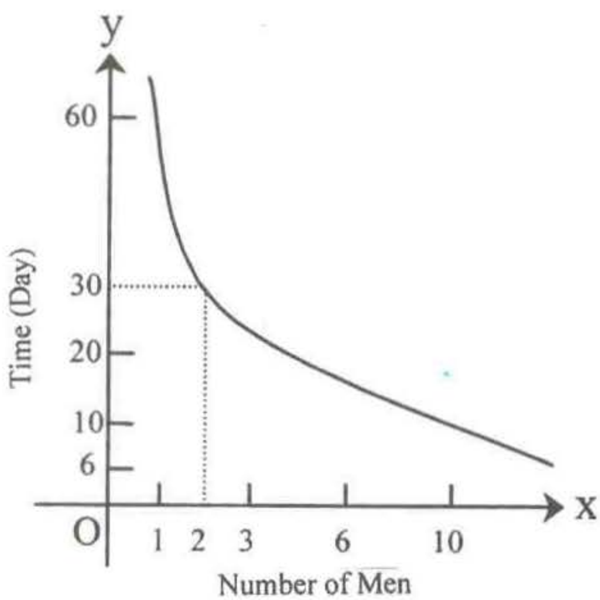
Fig.No. 6.4

Example 2

10 men do a piece of work in 6 days.

a) In how many days can 2men do the same work?

b) Show the given graph in a table.



Solution

a) The perpendicular drawn from the point of intersection of the curve from which a perpendicular is drawn at 2 has met the time line at 30. Therefore, 2 men can do the work in 30 days.

b) Showing the given graph is in the table.

No.of man	1	2	3	6	10
Time (days)	60	30	20	10	6

Exercise-6.2

1. **Identify whether each of the following examples is a problem of inverse variation or not.**

   a) Number of workers and day.
   b) The capacity of pipe to fill water and time.
   c) Hand and the number of fingers.
   d) The length and the breadth of the rectangles of equal areas.

2. Draw graphs from the following table that show the relation of inverse variation.

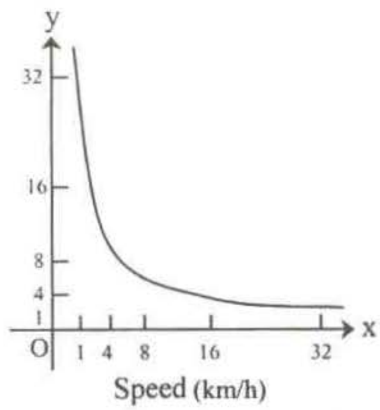
a)

X	1	2	3	4	6	8	12	24
Y	24	12	8	6	4	3	2	1

b)

X	1	2	4	5	8	10	20	40
Y	40	20	10	8	5	4	2	1

3. **The speed and the time taken to cover a distance of 32km are shown in the graph. Answer the following questions.**



   a) How many hours are required to cover the distance at a speed of 4 km/hr?
   b) At what speed can the distance be covered in 2 hours?
   c) Find out the equation to represent the graph on the right.
   d) How many hours are required to cover the distance at the speed of 10 km/hr?

7 Statistics

7.1. Bar Diagram

Example 1

The information gathered by interviewing some students about the subjects which they want to study after they pass the S.L.C. is shown in a bar graph in Fig.No7.1. Study the bar graph carefully and answer the following questions.

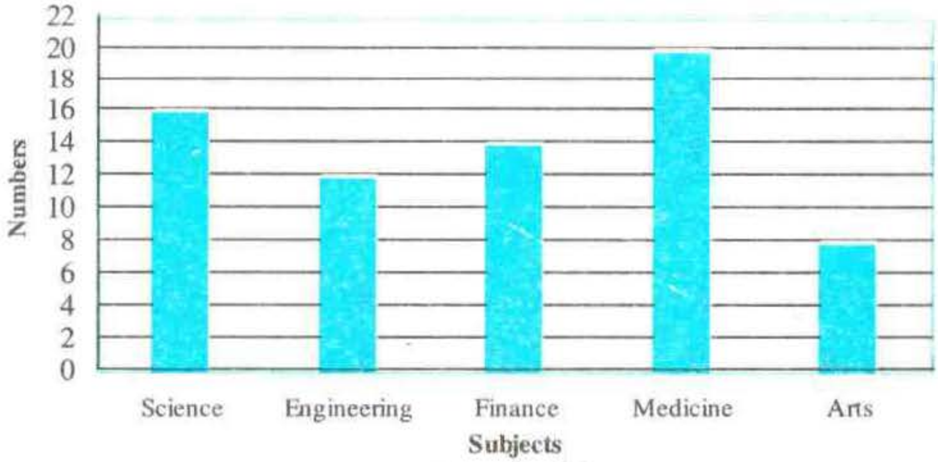


Fig. No. 7.1

a) What does the height of the bar in the graph represent?
b) Are the distances between two adjacent bar in the graph equal?
c) Why has bar graph been used to show the information ?
d) Which subject is preferred most by the students to study?
e) Which subject is least preferred by the students to study ?
f) What percent of students of the total number of students like to study science?

Remember the following points about the bar graph.

- When the information is presented in a bar graph.
- It is easier to understand and compare.
- It is easy to make a bar graph.
- The width of bars should be equal while making a bar graph and the length of bar stands for the number of things. The distance between the two bars should be the same.

Example 2

The information of the students from classes one to five of Mangalodaya Primary School is shown in a bar graph in Fig.No 7.2. Study the bar graph carefully and answer the following questions.

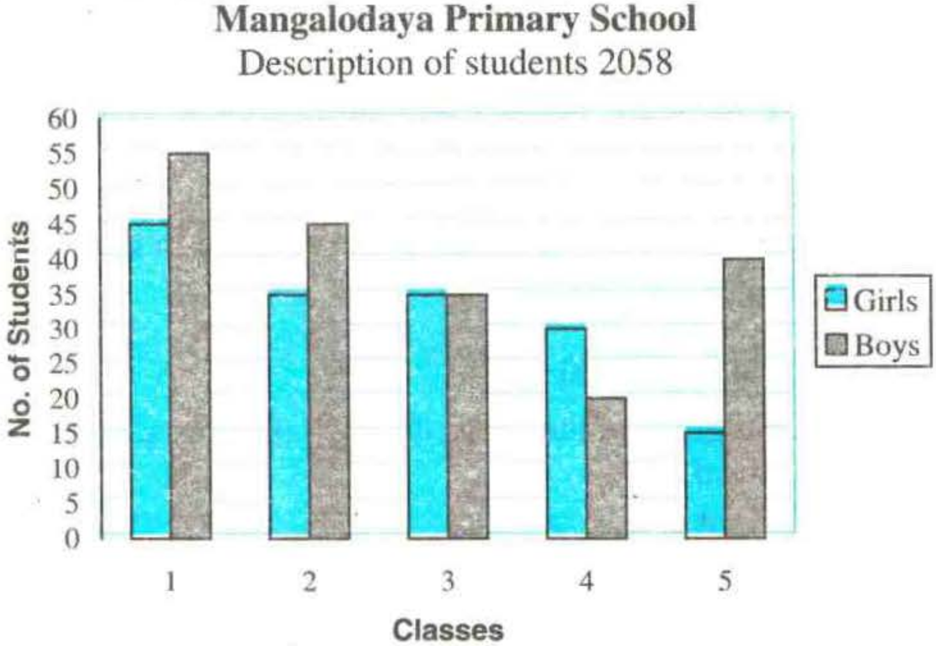


Fig. No. 7.2

a) What is the bar graph about?

b) What is the difference between the bar graphs in Fig.No-7.1 and Fig.No-7.2?

c) How many students are represented by a box of the vertical line in the bar graph?

d) Which is the subject preferred by most students to study ?

e) Which is the subject least preferred by students to study ?

f) Which class has more girls than boys ?

g) What percentage of girls are there out of the total number of students?

The graph which shows more than one data related to each other is called the multiple bar graph. Two data related to each other are shown in the bar graph of Fig.No-7.2. This graph is called a multiple bar graph. While constructing a multiple bar graph too, we make the width of the bars equal and represent the number by the length.

Exercise 7.1

1. **Manisha obtained the following marks in S.L.C. examination of 2058B.S.**

Subject	Nepali	English	Maths	Science	Voc.Agri	Opt.Maths	Opt.Geography
Marks Obtained	55	60	85	75	65	90	45

Draw the bar graph of the above information in graph paper by taking 1 sq. box=10marks in the vertical line.

2. **It rained for 5 days in Pokhara as recorded in the table :**

Days	Sun	Mon	Tue	Wed	Thurs
Rain	135	120	80	90	100

Draw a bar graph of the above data by taking 1 sq. box=10mm in the vertical axis.

3. **The number of students from classes 6 to 10 of Himalaya Secondary School is shown in the following table.**

Class	6		7		8		9		10	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
No. of Students	10	22	16	28	20	18	14	36	16	20

Draw a multiple bar graph of the above information by taking 1 sq.room= 4 students in the vertical axis.

4. **The tourist who came to Nepal are divided into two types as: those from India and from other Western countries. The two types of tourists who came to Nepal from 1989 to 1993 are recorded in the table below:**

Year	1989		1990		1991		1992		1993	
	In	Wes	In	Wes	In	Wes	In	Wes	In	Wes
No. of tourists (In thousand)	12	15	20	22	25	23	28	24	30	24

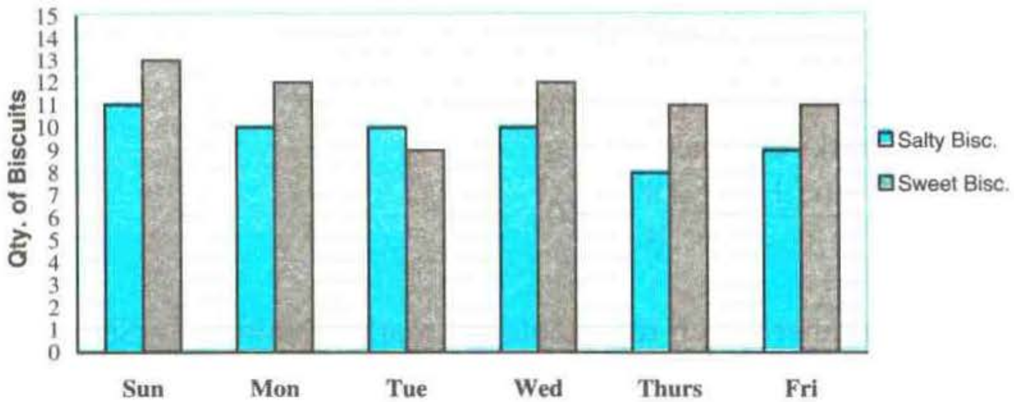
Draw a multiple bar graph of the above information by taking 1 sq. box = 2000 tourists in the vertical axis.

5. **The students come to school on foot or by using vehicles as shown in the following table.**

Way of coming to school	On foot		By Bus		By Car		By Motorcycle		By Cycle	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
No. of students	25	55	40	80	30	20	10	30	25	15

Express the above information by preparing a multiple bar graph.

6. The production of two types of biscuits (sweet and salty) produced by Biratnagar Biscuit Factory in a certain week is shown in the following multiple bar graph.



Now answer the following questions:

a) On which day is the production of biscuits maximum?

b) On which day is the production of biscuits minimum?

c) On which day is the production of salty biscuits more than the production of sweet biscuits?

d) On which two days of the week are the number of biscuits produced the same?

e) How much salty and sweet biscuits are produced on Thursday?

7.2 Finding the Mean from Ungrouped data:

7.2.1 Arithmetic Mean or Average

Shyam and Hari obtained the following marks in Mathematics in three terminal examinations of 100 marks each.

Marks obtained by Shyam	65	50	80
Marks obtained by Hari	60	65	55

Now answer the following questions:

a) What is the one number that can represent the marks obtained by Shyam and another number that can represent the marks obtained by Hari?

b) Who has done well in mathematics in terminal examinations Shyam or Hari?

Here,

A number which represents the marks obtained by Shyam

$$= \frac{65 + 50 + 80}{3} = \frac{195}{3} = 65$$

and the number which represents the marks obtained by Hari

$$= \frac{60 + 65 + 55}{3} = \frac{180}{3} = 60$$

Here, the number which represents the marks obtained by Shyam and Hari in all three terminal examinations is called the arithmetic mean or Average. On the basis of the average marks of all the three examinations, Shyam does well in Mathematics examination. What are the two merits in finding the average in this way ? Write about the merits after discussion in class.

From the above example-

$$\text{Arithmetic Mean} = \frac{\text{Total Quantity}}{\text{Number of items}}$$

Example : Arithmetic Mean of 3, 2, 6, 3, 2, 1, 0, 7 is

$$\frac{3 + 2 + 6 + 3 + 2 + 1 + 0 + 7}{8} = \frac{24}{8} = 3$$

Example 1

The mistakes made in a spelling contest by 30 students of a class of a school are recorded below.

1 , 2 , 3 , 2 , 5 , 4 , 5 , 2 , 3 , 2 ,
5 , 0 , 2 , 2 , 5 , 1 , 2 , 3 , 5 , 4 ,
2 , 2 , 5 , 5 , 2 , 2 , 2 , 3 , 4 , 5

Find the arithmetic mean (average spelling mistake) from the above data.

Solution:

1st Method

Here, adding all the spelling mistakes,

1+2+3+2+5+4+5+2+3+2+5+0+2+2+5+1+2+3+5+4+2+2+5+5+2+2+2 +3+4+5=90

$$\therefore \text{Arithmetic Mean} = \frac{\text{Total Sum}}{\text{No. of Students}} = \frac{90}{30} = 3$$

Hence, average mistakes or arithmetic mean=3.

2nd method

To express the given data in the frequency table

Mistake (x)	Frequency (f)	$x \times f = fx$
0	1	$0 \times 1 = 0$
1	2	$1 \times 2 = 2$
2	12	$2 \times 12 = 24$
3	4	$3 \times 4 = 12$
4	3	$4 \times 3 = 12$
5	8	$5 \times 8 = 40$
Total	30	90

$$\therefore \text{Arthmetic mean } (\bar{x}) = \frac{\text{Total Sum } (\sum fx)}{\text{No. of students } (\sum f)}$$

or, $$\bar{x} = \frac{\sum fx}{\sum fx} = \frac{90}{30} = 3$$

Example 2

The marks out of 20 obtained by the students in an examination of Maths are shown in the following frequency table.

Marks (x)	5	8	10	12	15	18	20
No. of students (f)	2	5	8	10	12	2	1

Find the arithmetic mean from the table.

Solution:

The given table is written in a vertical form

Marks (x)	Frequency (f)	$x \times f = fx$
5	2	$5 \times 2 = 10$
8	5	$8 \times 5 = 40$
10	8	$10 \times 8 = 80$
12	10	$12 \times 10 = 120$
15	12	$15 \times 12 = 180$
18	2	$18 \times 2 = 36$
20	1	$20 \times 1 = 20$
Total	40	486

$$\therefore \text{Arithmetic mean} = \frac{\text{Total Sum}}{\text{No. of Students}} = \frac{486}{40} = 12.15$$

Exercise 7.2

1. Find the arithmetic mean of each of the following sets of data: -

a) 3,5,7,9,11   b) 2,4,6,8,10

c) 3,2,3,4,5,1   d) 5,8,12,15,14,12

e) $1\frac{1}{2},\ 2\frac{1}{4},\ 1\frac{1}{4},\ 3\frac{1}{4},\ 2\frac{1}{2},\ 1\frac{1}{4}$

2. **Find the arithmetic mean of each of the following frequency tables**

a)

Marks (x)	frequency (f)	$x \times f$
2	2	
4	5	
6	8	
8	3	
10	2	
Total		

b)

Marks (x)	frequency (f)	$x \times f$
1	2	
3	3	
5	10	
7	4	
9	1	
Total		

c)

Marks (x)	frequency (f)	$x \times f$
12	5	
15	11	
18	15	
20	13	
22	6	
Total		

d)

Marks (x)	frequency (f)	$x \times f$
40	5	
45	9	
50	15	
55	13	
60	3	
Total		

3. **Construct a frequency table for each of the following sets of data and find the arithmetic mean: -**
   a) 0, 5, 4, 3, 6, 5, 2, 2, 1, 4, 7, 2, 2, 6, 5.
   b) 12, 10, 9, 10, 8, 11, 5, 11, 8, 10, 7, 14, 11, 13, 11
   c) 15, 20, 12, 21, 21, 14, 9, 9, 9, 22, 22, 20, 14, 20, 15
   d) 1, 3, 0, 2, 2, 5, 4, 4, 1, 0, 3, 3, 3, 4, 5, 2, 5, 2, 2, 1
   e) 9, 9, 12, 10, 11, 9, 9, 10, 12, 12, 12, 10, 9, 9, 8, 9

4. **Find the arithmetic mean for each set of data given in the following frequency tables: -**

a)

x	15	17	19	21	23	25
f	2	5	12	10	4	3

b)

x	2	5	8	12	15
f	3	7	12	2	1

c)

x	4	5	6	7	8	9	10
f	1	2	5	14	9	6	3

d)

x	10	20	30	40	50	60	70
f	2	5	10	16	10	9	3

5. **The students of Class 7 obtained the following marks (out of 10) in a class test.**

   3 , 2 , 0 , 1 , 1 , 1 , 5 , 7 , 9 , 8 , 7 , 9 , 8 , 7 , 5 ,

   3 , 2 , 0 , 3 , 1 , 2 , 5 , 7 , 8 , 7 , 9 , 3 , 5 , 9 , 7

   Construct frequency table on the basis of above data and find the arithmetic mean. Find the number of the students who obtained marks above the mean and below the mean.

6. **In a survey of the number of children in 50 houses of a certain ward in Kathmandu, the following data are obtained.**

   1,2,2,2,3,4,2,1,1,2,1,2,3,2,1,2,1,3,3,2,2,

   1,0,4,3,2,2,1,3,1,2,3,2,2,3,1,1,2,2,3,2,4,3,

   0,2,0,3,2,2,2

   a) Find the arithmetic mean on the basis of the above data.

   b) How many houses have more than 8 children ?

   c) How many families have fewer children than the average? (The family which has no children is also included)

8 Algebraic Expressions

8.1. Power and Exponents

Meaning, definition and example of algebraic expressions.

$2+2+2+2 = 4\times2 = 8$
but $2\times2\times2\times2 = ?$
How can it be written in short form ?

Fig. No. 8.1

Look at the following pattern for repeated multiplication of the same factor and fill in the blanks:

$2\times2 = 4$	$2^2 = 4$	(Product of two 2's)
$2\times2\times2 = 8$	$2^3 = 8$	(Product of three 2's)
$2\times2\times2\times2 = 16$	$2^4 = 16$	(Product of four 2's)
$2\times2\times2\times2\times2 = 32$	$2^5 = 32$	(Product of five 2's)
: :	: :	(:)
: :	: :	(:)

$2\times2\times2\times$ ..... factors of 10 2's $= 2^{\square} = \square$ (..........................)

: : : : (:)

: : : : (:)

$2\times2\times2\times$ .... factors of 100 2's $= 2^{\square} = \square$ (..........................)

We use the exponent to represent the process of repeated multiplication of the same number. 2^{100} represents the product of 100 2's. Here, 2 is the base and 100 is the exponent.	100 Exponent 2 Base (Read 100 as the exponent of 2)

In this way the product of repeated multiplication of the same number can be expressed in exponent form.

e.g.	$3 \times 3 = 3^2$	Product of two 3's (base -3, exponent -2)
	$3 \times 3 \times 3 = 3^3$	Product of three 3's (base -3, exponent -3)
	$3 \times 3 \times 3 \times 3 = 3^4$	Product of four 3's (base -3, exponent -4)
	: :	: : :
	$\underset{(n\ 3\text{'s})}{3 \times 3 \times 3 \times \ldots\ldots \times 3} = 3^n$	Product of n^{th} 3's (base -3, exponent -n)

Similarly,

$a = a^1$	Product of one a (base -a, exponent -1)
$a \times a = a^2$	Product of two a (base -a, exponent -2)
$a \times a \times a = a^3$	Product of three a (base -a, exponent -3)
$a \times a \times a \times a = a^4$	Product of four a (base -a, exponent -4)
: :	: : :
$a \times a \times a \times \ldots\ldots \times a = a^n$	Product of n a's (base -a, exponent -n)

In a^n, a is called the base and n is called the exponent. a^n is called the power of a. Here, base a can also be a fractional or negative number.

Example 1

Express the following continued product in exponential form :

a) $(-3) \times (-3) \times (-3) \times (-3) \times (-3)$

b) $\left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

c) $(3x) \times (3x) \times (3x) \times (3x) \times (3x) \times (3x) \times (3x)$

Solution :

a) $(-3) \times (-3) \times (-3) \times (-3) \times (-3)$

$= (-3)^5$ (Product of five (-3)'s)

b) $\left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

$= \left(\frac{2}{5}\right)^4$ (Product of four $\left(\frac{2}{5}\right)$'s)

c) $(3x) \times (3x) \times (3x) \times (3x) \times (3x) \times (3x) \times (3x)$
$= (3x)^7$ (Product of seven (3x)'s)

Example 2

Find the product of :

a) $2^5 \times 3^3$ b) $\left(\frac{5}{6}\right)^3 \times \left(\frac{2}{3}\right)^2$ c) $(3x)^2 \times (2y)^4$

Solution :

a) $2^5 \times 3^3$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 32 \times 27$

$= 864$

b) $\left(\frac{5}{6}\right)^3 \times \left(\frac{2}{3}\right)^2 \quad = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{2}{3} \times \frac{2}{3}$

$= \frac{500}{1944}$

c) $(3x)^2 \times (2y)^4$

$= 3x \times 3x \times 2y \times 2y \times 2y \times 2y$

$= 3 \times 3 \times x \times x \times 2 \times 2 \times 2 \times 2 \times y \times y \times y \times y$

$= 144x^2y^4$

Example 3

Express 600 by using the powers of 10.

Solution :

Here,

$600 = 6 \times 100$

$= 6 \times 10 \times 10$

$= 6 \times 10^2$

Example 4

Prime factorizing write 432 in exponential form

Solution :

Here,

$$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= 2^4 \times 3^3$$

Exercise 8.1

1. Express each of the following continued products in exponential form:

a) $2\times2\times2\times2\times2$ b) $3\times3\times3\times3\times3\times3$

c) $4\times4\times4\times4$ d) $(-5)\times(-5)\times(-5)\times(-5)\times(-5)$

e) $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)$ f) $\left(-\frac{3}{7}\right)\times\left(-\frac{3}{7}\right)\times\left(-\frac{3}{7}\right)$

g) $(2.5)\times(2.5)\times(2.5)\times(2.5)$ h) $x\times x\times x\times x\times x\times x$

i) $2y\times2y\times2y\times2y$ j) $\frac{2}{p}\times\frac{2}{p}\times\frac{2}{p}\times\frac{2}{p}$

2. Express each of the following powers as repeated multiplication :

a) 5^3 b) 3^7 c) 4^4 d) $(-5)^3$ e) $(-7)^6$

f) $\left(\frac{2}{3}\right)^3$ g) $\left(-\frac{1}{3}\right)^4$ h) $(2x)^5$ i) $\left(\frac{3x}{2}\right)^4$ j) $\left(\frac{4y}{5z}\right)^5$

3. Express each of the following numbers by using powers of 10:

a) 300   b) 3,000   c) 30,000   d) 30,00,000

e) 5,000   f) 90,000   g) 25,000   h) 5,70,000

4. Find the value of :

a) 3×10^2 b) 5×10^4 c) 8×10^3 d) 9×10^5 e) 2×10^7

5. Find the value of :

a) 5^3 b) 6^4 c) 7^3 d) 3^7 e) 3×5^3 f) $2^2\times6^4$

g) $7^2 \times 7^3$ h) $2^3 \times 3^2$ i) $\left(\frac{3}{4}\right)^2$ j) $\left(\frac{5}{7}\right)^4$ k) $\left(\frac{2}{3}\right)^5$

l) $\left(\frac{3}{4}\right)^2 \times \left(\frac{5}{7}\right)^3$

6. **Simplify**

a) $a^2 \times a^3$ b) $b^4 \times b^4$ c) $x^3 \times x^4$

d) $(2x)^7 \times (x)^3$ e) $(3y)^2 \times (5y)^3$ f) $(4z)^3 \times (5z)^2$

7. **Which one is greater :**

a) 2^3 or 3^2 b) 2^5 or 5^2 c) 3^4 or 4^3

d) 10^2 or 2^{10} e) 7^2 or 2^7 f) 0^3 or 3^0

8. **Express each of the following as prime factors and write them in exponential form :**

a) 625 b) 864 c) 1256 d) 1296 e) 1728

8.2. Laws of Indices

8.2.1. Multiplication of the powers having the same base :

Look at the following pattern of the process of multiplication of the powers having the same base :

$$2^1 \times 2^1 = 4 = 2^2 = 2^{1+1}$$
$$2^1 \times 2^2 = 8 = 2^3 = 2^{1+2}$$
$$2^1 \times 2^3 = 16 = 2^4 = 2^{1+3}$$
$$2^1 \times 2^4 = 32 = 2^5 = 2^{1+4}$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

Therefore,

$$2^1 \times 2^n = 2^{1+n} = 2^{n+1}$$

The indices are added and the base remains the same When multiplying powers having the same base. Therefore, $a^m \times a^n = a^{m+n}$

8.2.2. Division of powers having the same base :

Look at the following pattern of division of powers having the same base.

$$2^2 \div 2^1 = \frac{2\times2}{2} = 2^1 = 2^{2-1}$$

$$2^3 \div 2^1 = \frac{2\times2\times2}{2} = 2^2 = 2^{3-1}$$

$$2^4 \div 2^1 = \frac{2\times2\times2\times2}{2} = 2^3 = 2^{4-1}$$

$$2^5 \div 2^1 = \frac{2\times2\times2\times2\times2}{2} = 2^4 = 2^{5-1}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\therefore 2^n \div 2^1 = \frac{2\times2\times2\times......\times2}{2} = 2^{n-1}$$

When dividing powers having the same base, the base remains the same and the exponent of divisor is subtracted from the exponent of dividend. That is $a^m \div a^n = a^{m-n}$, where $a \neq 0$, $m > n$, m and n both are positive integers.

8.2.3. Meaning of Zero Index

Look at the following examples

(The quotient of the number divided by the same number is 1)

$$2^3 \div 2^3 = \frac{2\times2\times2}{2\times2\times2} = 1$$

By using law of indices -

$$2^3 \div 2^3 = 2^{3-3} = 2^0 = 1$$

If the index of any number is zero then its value is equal to 1. That is, $a^0=1$ where $a\neq0$

Example 1

Multiply : $3p^2 \times 6p^3$

Solution :

$$\begin{aligned} 3p^2 \times 6p^3 &= 3 \times 6 \times p^2 \times p^3 \\ &= 18 \times p^{2+3} \\ &= 18\, p^5 \end{aligned}$$

Example 2

Simplify : $\dfrac{x^4 \times x^3}{x^5}$

Solution :

$$\frac{x^4 \times x^3}{x^5} = \frac{x^{4+3}}{x^5} \quad \text{(indices are added when multiplying)}$$

$$= \frac{x^7}{x^5}$$

$$= x^{7-5} \quad \text{(The indices are subtracted when dividing)}$$

$$= x^2$$

Exercise - 8.2

1. **Simplify each of the following problems by using the laws of indices and express the answer in index form :**

a) $2^3 \times 2^2$ b) $5^7 \times 5^4$ c) $(-2)^4 \times (-2)^6$

d) $\left(\frac{3}{2}\right)^5 \times \left(\frac{3}{2}\right)^7$ e) $\left(-\frac{1}{4}\right)^3 \times \left(-\frac{1}{4}\right)^6$ f) $\left(-\frac{4}{5}\right)^{11} \times \left(-\frac{4}{5}\right)^{13}$

g) $x^2 \times x^3$ h) $p^3 \times p^6$ i) $m^{10} \times m^{12}$

j) $\left(\frac{x}{y}\right)^5 \times \left(\frac{x}{y}\right)^2$ k) $\left(\frac{p}{q}\right)^7 \times \left(\frac{p}{q}\right)^2$ k) $\left(-\frac{y}{z}\right) \times \left(-\frac{y}{z}\right)^5$

2. **Simplify by using the laws of indices :**

a) $2^4 \div 2$ b) $3^5 \div 3^2$ c) $4^5 \div 4^2$

d) $5^8 \div 5^3$ e) $\left(\frac{2}{3}\right)^7 \div \left(\frac{2}{3}\right)^2$ f) $7^{10} \div 7^3$

g) $\frac{2^3 \times 2^8}{2^5}$ h) $\frac{5^{10}}{5 \times 5^2}$ i) $\frac{7^{10} \times 7^2}{7^6}$

j) $x^5 \div x^2$ k) $y^7 \div y^4$ l) $(2z)^6 \div (2z)^4$

m) $\frac{x^5 \times x^7}{x^8}$ n) $\frac{p^3 \times p^5}{p^4}$ o) $\frac{(3p)^7 \times (3p)^9}{(2p)^{11}}$

3. **What is the value of each of the following powers ?**

a) P^0 b) $(3p)^0$ c) $(x)^0$

d) $\left(\frac{1}{2}\right)^0$ e) $\left(\frac{3}{4}\right)^0$ f) $\left(-\frac{4}{5}\right)^0$

4. **Find the value of :**

a) x^7, when $x = 2$ b) y^3, when $y = 6$

c) z^4, when $z=4$ d) $\frac{x^2 + y^2}{x + y}$, when $x = 6$ and $y = 2$.

e) $\frac{x^6 \times x^7 \times x^3}{x^{12}}$, when $x = 5$

f) $\frac{x^2 + 2xy + y^2}{x + y}$, when $x = 4$ and $y = 5$.

8.3. Addition and Subtraction of Algebraic Expressions.

8.3.1. Algebraic Expression :

When 4 is added to x, the sum is x+4. When 2 is subtracted from y, the difference is y-2. When P is divided by 3, the result is $\frac{P}{3}$. When 3 is multiplied by x, the product is 3x. Here, the mathematical statement

$x+4$, $y-2$, $\frac{P}{3}$, $3x$ etc all are examples of expressions. Here, the mathematical operations (+, -, ÷, ×) are used with the variables or constants. The mathematical statements formed by the use of the signs of mathematical operations between variables and constants are called algebraic expressions. For examples: $6a^2b$, $7x^2+3y^2$, $xy^2+3xy-y^3$, $\frac{x^3+xy+y^3}{x^3+y^3}$ etc. are all algebraic expressions. In these examples, $6a^2b$ has only one term. It is called monomial expression. Similarly, $7x^2+3y^2$ and $xy^2+3xy-y^3$ are respectively called a binomial and a trinomial because the first expression has two terms and the second expression has 3 terms.

8.3.2. Addition and subtraction of Algebraic Expressions :

Example 1

Add : $4x^2y$, $-3x^2y$, $7x^2y$ and $-2x^2y$.

Solution :

Here, all the terms used in the expression are alike.

Therefore, to add these terms

$$
\begin{aligned}
& 4x^2y + (-3x^2y) + 7x^2y + (-2x^2y) \\
= \ & 4x^2y - 3x^2y + 7x^2y - 2x^2y \\
= \ & 4x^2y + 7x^2y - 3x^2y - 2x^2y \\
= \ & 11x^2y - 5x^2y \\
= \ & 6x^2y
\end{aligned}
$$

Example 2

Simplify by grouping like terms.

$5x^3 - 3x^2 + 2x^3 + 7x^2 + y2 - 2y^2 + 1$

Solution :

$$
\begin{aligned}
& 5x^3 - 3x^2 + 2x^3 + 7x^2 + u2 - 2y^2 + 1 \\
= \ & 5x^3 + 2x^3 - 3x^2 + 7x^2 + y^2 - 2y^2 + 1 \\
= \ & 7x^3 + 4x^2 - y^2 + 1
\end{aligned}
$$

Example 3

Add :

$3x + 4,\ 4x+2y,\ -3x - 4y - 3$

Solution :

1st method :

Adding $3x+4,\ 4x+2y,\ -3x - 4y - 3$

$$
\begin{aligned}
&= 3x + 4 + (4x+2y) + (-3x -4y - 3)\\
&= 3x + 4 + 4x + 2y - 3x - 4y - 3\\
&= 3x + 4x - 3x + 2y - 4y + 4 - 3\\
&= 4x - 2y + 1
\end{aligned}
$$

2nd Method :

Arranging the expressions to be added vertically by writing like terms one above the other in a line.

$$
\begin{array}{rrrr}
 & 3x & + 4 & +3 \\
 & 4x & + 2y & \\
(+) & -3x & - 4y & - 3 \\
\hline
 & 4x & - 2y & +1
\end{array}
$$

Example 4 **Add :**

$a^2 + 3a^2b + 4ab^2,\ -2a^2 + 4a^2b - 2ab^2$ and $3a^2 - 4a^2b - 7ab^2$

Solution :

Writing vertically the expressions to be added :

$$
\begin{array}{rrrr}
 & a^2 & + 3a^2b & + 4ab^2 \\
 & -2a^2 & + 4a^2b & - 2ab^2 \\
(+) & 3a^2 & - 4a^2b & - 7ab^2 \\
\hline
 & 2a^2 & + 3a^2b & - 5ab^2
\end{array}
$$

Example 5

(i) Subtract 3a from 7a (ii) Subtract $-5p^2q^2$ from $3p^2q^2$

Solution :

(i) Subtract 3a from 7a

$$7a - (+3a)$$
$$= 7a - 3a$$
$$= 4a$$

(ii) Subtract $-5p^2q^2$ from $3p^2q^2$

$$3p^2q^2 + 5p^2q^2$$
$$= 3p^2q^2 + 5p^2q^2$$
$$= 8p^2q^2$$

Subtracting the problems of example - 5 by the vertical method.

i)
$$\begin{array}{r} 7a \\ \underline{-\ 3a} \\ 4a \end{array} \quad (7 - 3 = 4)$$

ii)
$$\begin{array}{r} 3p^2q^2 \\ \underline{(+)\ -5p^2q^2} \\ 8p^2q^2 \end{array}$$

Example 6

Subtract $3a^2x - 3ax^2$ from $5a^2x + 8ax^2 - 4$

Solution :

Here, subtracting

$$3a^2x - 3ax^2 \text{ from } 5a^2x + 8ax^2 - 4$$
$$= (5a^2x + 8ax^2 - 4) - (3a^2x - 3ax^2)$$
$$= 5a^2x + 8ax^2 - 4 - 3a^2x - (- 3ax^2)$$
$$= 5a^2x + 8ax^2 - 4 - 3a^2x + 3ax^2$$
$$= (5a^2x - 3a^2x) + (8ax^2 + 3ax^2) - 4$$

Subtracting the problem of example - 6 by the vertical method.

$$\begin{array}{r} 5a^2x + 8a^2x - 4 \\ 3a^2x - 3a^2x \\ \underline{-\quad +\quad} \\ 2a^2x + 11ax^2 - 4 \end{array}$$

Example 7

What should be added to x + 2y + z so that the result is z ?

Solution :

Here,

the required expression = A (let's suppose)

$\therefore$ x + 2y + z + A = z, why ?

or, A = z - (x + 2y + z)

= z – x – 2y – z = – x – 2y *Answer*

Example 8

Find the area of the following rectangle.

Solution :

	Length	= 5x cm
	Breadth	= 3y cm
	Area	= ?
$\therefore$	Area	= 5x cm × 3y cm
		= 15 xy cm^2

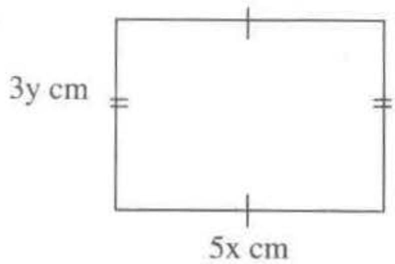


Fig No. 8.2

Example 9

Find the volume of the given solid cuboid

Solution :

Length	= 3a cm
Breadth	= 2b cm
Height	= 5c cm
Volume	= ?
Volume	= length × breadth × height
	= 3a cm × 2b cm × 5c cm
	= 30 abc cm^3

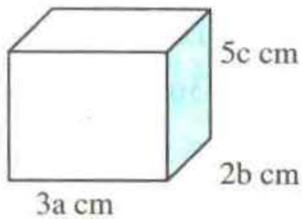


Fig No. 8.3

Exercise 8.3

1. **Identify whether the following algebraic expressions are monomials, binomials or trinomials:**

   a) $3x^2$ b) $5x + 7$ c) $3x^2 + 4x + 5$ d) $\frac{4x + 2}{5}$

   e) $4x^2 + \frac{7}{2}xy$ f) $8x^2 + 5x + 7$

2. **Add :**

a) $3a^2, 4a^2, 7a^2$
b) $3a^2b, 5a^2b, -4a^2b$
c) $4x^2y, 7x^2y, -x^2y$
d) $4ax^2, -7ax^2, 15ax^2$

3. **Subtract the second expression from the first expression:**

a) $5mn, 3mn$
b) $12pq, 7pq$
c) $13m^2n, 9m^2n$
d) $-15x^3y, 3x^3y$

4. **Simplify by grouping like terms together**

a) $3x^2 + 4x^2 - 7x^2 + 9x^2$
b) $15ab^2 + 17ab^2 - 13ab^2 - 4ab^2$
c) $17p^2q - 13pr^2 + 9p^2q + 12pr^2$
d) $-2x^3 + 3x^2 + 5x^3 - x^2 + 4$

5. **Find the sum of the expressions by both the vertical and horizontal methods :**

a) $a + 3b + 5c$ and $5a - 4b + 3c$
b) $3a^2 - 5b^2 + c^2$ and $-2a^2 + 3b^2 - 10c^2$
c) $4x^2y + 7xy^2 - 9$ and $7x^2y - 3xy^2 + 4z^2$
d) $2x^3 + 3y^3 - 10z^3, 5x^3 - 7y^3 + 12z^3$ and $-6x^3 + 3y^3 + 2z^3$
e) $7m^3 + 3mn^2 - 4mn, 8m^3 + 3mn^2 - 5mn$ and $-6m^3 - 4mn^2 + 6mn$

6. **Find the difference of the expressions by both the vertical and horizontal methods:**

a) $-3a + 2b - 5c$ from $2a + b - 8c$
b) $-3a^2 + 2ab - 4b^2$ from $2a^2 + 3ab - 5b^2$
c) $a^2 - b^2 + abc$ from $3a^2 - b^2 - 3abc$
d) $5xy - 7$ from $x^2y^2 + 7xy + 12$
e) $2x^3 - 4x^2 + 7x + 5$ from $x^3 - 3x^2 + 6x + 7$

7

a) What should be subtracted from $8a$ to make it $6a$?
b) What should be added to $5a$ so that the result is $3a$?
c) hat should be added to $-2x + 5y - 4z$ so that the result is $x + y + z$?
d) What should be added to $3x^2 + 5x - 6$ to make it x^2 ?
e) What should be subtracted from $a^2 + 2ab + b^2$ so that the result is $2ab$?

8 **Simplify :**

a) $\frac{1}{2}a + \frac{3}{4}a + 2a$ b) $\frac{2}{5}ab - \frac{3}{5}ab + \frac{7}{10}ab$

c) $0.5a^2b + 0.7a^2b + 1.25\,a^2b$ d) $-1.5x + 2.5y - 3z - 2x - 3.5y + 1.5z$

e) $0.75x^3 + 0.25xy - 0.5y^3 - 0.25x^3 + 1.5y^3$

9 a) What is the result if $-1.67a + 3.15b + 6.75c$ is added to $0.33a - 1.25\,b + 3.25c$?

b) What is the result if $3.69a^2b - 4.5ab^2 + 6.79\,bc$ is added to $2.36\,a^2b + 3.56ab^2 - 4.76bc$?

c) When $-3.67p^2 + 6.45\,pq + 2.31q^2$ is subtracted from $6.57p^2 + 3.56pq + 4.69q^2$, what is left ?

d) When $-2.2x^2y + 3.25xy^2 + 1.75$ is subtracted from $3.15x^2y + 5.65xy^2$, what is left ?

8.4 Multiplication of Algebraic Expressions

8.4.1 Review

When a monomial is multiplied by another monomial we should first multiply the coefficients of the expressions and then multiply the variables of the expressions.

Example 1

What is the area of a square of length x cm ? What is the area of the square when x = 4 cm ?

Solution :

Here,

Length of the square = x cm

Area (A) = ?

From formulae,

Area (A) = Length × Breadth

$= x\text{ cm} \times x\text{ cm}$

$= x^2\text{ cm}^2$

If $x = 4$ cm, then $x^2 = (4\text{cm})^2 = 16\text{cm}^2$

$\therefore$ Area of the Square (A) = 16 cm^2

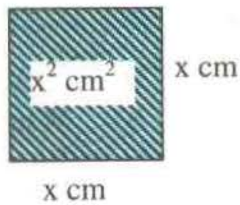


Fig.No. 8.4

Example 2

Multiply : $3x^2y \times 4x^3$

Solution:

Here, $3x^2y \times 4x^3$

$= 3 \times 4 \times x^2 \times x^3 \times y$

$= 12x^5y$

Exercise 8.4.1

1. **Multiply**

a) $3a \times 4b$ b) $5a \times 7b$ c) $1.5a \times 2.4b$

d) $x^2 \times x^5$ e) $2y^3 \times y^4$ f) $3.2xy^2 \times 1.5\ x^2y$

g) $-p \times p^3$ h) $-2p \times 3pq$

i) $-7m^2n \times 2.5mn^2$ j) $(-2ab)\ (-3ab)$

k) $(-3m^2n^3)\ (-2m^2n)$

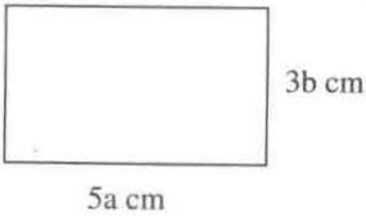
l) $(-6.5z^2)\ (-1.4z^3)$ m) $5x^2 \times 12x^3 \times 2x$

n) $3m^2 \times 4mn \times 5n^2$

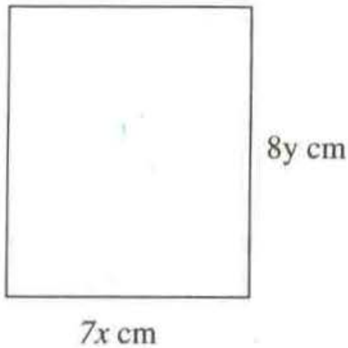
o) $(-3ab) \times 2b^2c \times (-6c^2a)$

2. **Find the area of each the following rectangles :**

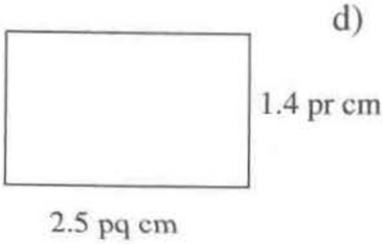
a)



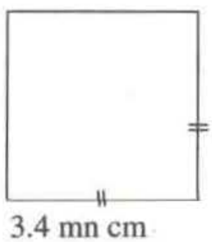
b)



c)

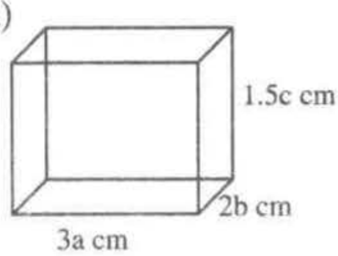


d)

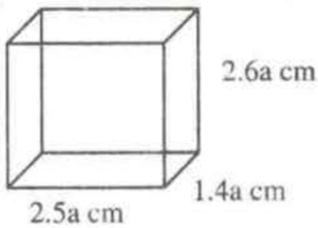


3. **Find out the volume of each the following solid cuboids:**

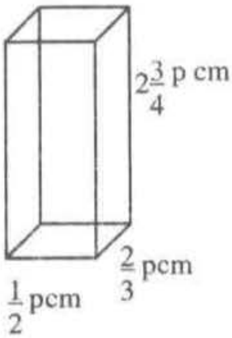
a)



b)



c)



[Volume of a Solid Cuboids = length × breadth × height]

8.4.2 Multiplication of a Binomial by a Binomial

What is the area of a rectangle of length (a + b) cm and breadth (x + y) cm ?

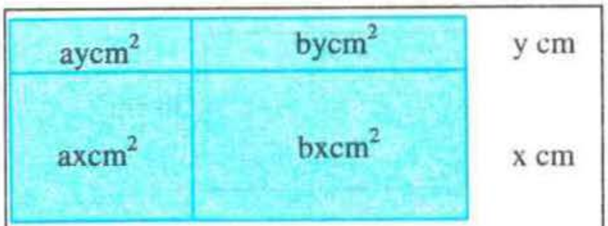


Fig. No. 8.5

In the figure, the rectangle is divided into two parts a cm and b cm along the length and into two parts x cm and y cm along the breadth. So that it is in four parts.

From the figure, the area of the rectangle = (ax + ay + bx + by) cm^2

Think it as

$(a + b)\ (x + y) = a(x + y) + b(x + y)$, why ?

$= ax + ay + bx + by$

$= (ax + ay + bx + by)$

$\therefore$ area of the rectangle = $(ax + ay + bx + by)\ cm^2$

When a binomial expression is multiplied by another binomial expression we first multiply each term of the second expression by each term of the first expression separately and then add all the products.

$$(a + b)\ (x + y) = (ax + ay + bx + by)$$

Example 1

Multiply $(7x - 3y) \times (5x + 4y)$

Solution :

Here,

$$\begin{aligned} & (7x - 3y)\ (5x + 4y) \\ &= x\ (5x + 4y) - 3y\ (5x + 4y) \\ &= 35x^2 + 28xy - 15xy - 12\ y^2 \\ &= 35x^2 - 13xy - 12y^2 \end{aligned}$$

Example 2

Find the product of : $(3.5a + 2.5b)\ (1.4a - 2.6b)$

Solution :

Here,

$$\begin{aligned} & (3.5a + 2.5b)\ (1.4a - 2.6b) \\ &= 3.5a(1.4a - 2.6b) + 2.5b(1.4a - 2.6b) \\ &= 4.9a^2 - 9.1ab + 3.5ab - 6.5b^2 \\ &= 4.9a^2 - 5.6ab - 6.5b^2 \end{aligned}$$

Example 3

If the length and breadth of a rectangle are $(3x + y)$ and $(4x - 3y)$ respectively, what is its area ?

Solution :

Here,

Length of rectangle (l) $= 3x + y$
Breadth of rectangle (b) $= 4x - 3y$
Area of rectangle (A) $= ?$

$$\begin{aligned} A &= l \times b \\ &= (3x + y) \times (4x - 3y) \\ &= 3x(4x - 3y) + y(4x - 3y) \\ &= 12x^2 - 9xy + 4xy - 3y^2 \\ &= 12x^2 - 5xy - 3y^2 \text{ sq. unit.} \end{aligned}$$

Exercise 8.4.2

1. **Find the product of :**

a) $x(2x - y)$ b) $(a - 2b) \times 3c$ c) $4x(2x - 3y)$

d) $(3a^2 + 2b^2) \times 5c$ e) $\frac{3}{5}x(x^2 + y^2)$ f) $a\left(\frac{1}{2}a + \frac{1}{4}b\right)$

2. **Simplify :**

a) $2x^2(3x - 2) + 2x(2x + 3)$ b) $\frac{1}{2}a(a - 2) + 2a\left(\frac{3}{4}a - 1\right)$

c) $(3x + 4)y + 2x(y - 5)$ d) $7x - 3(2 - y) + 2(y - 3x)$

e) $x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$

3. **Multiply**

a) $(a + b)(a + b)$ b) $(a - b)(a - b)$

c) $(a + b)(a - b)$ d) $(x + y)(2x + 3y)$

e) $(2x + 3y)(2x - 3y)$ f) $(m^2 + n^2)(2m - 3n)$

g) $(5p + 6q)(6q - 5p)$ h) $\left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)$

i) $\left(\frac{1}{2}x + \frac{3}{4}y\right)\left(\frac{3}{4}x + \frac{1}{2}y\right)$ j) $\left(\frac{2x}{y} + \frac{2y}{x}\right)\left(\frac{x}{2y} - \frac{y}{2x}\right)$

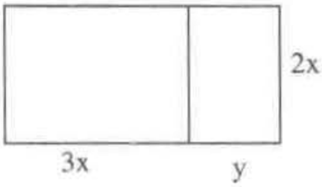
k) $(1.5x^2 + 2.6y^2)(2x^2 + 3y^2)$ l) $(2.5p - 3.6q)(3.4p + 4.5q)$

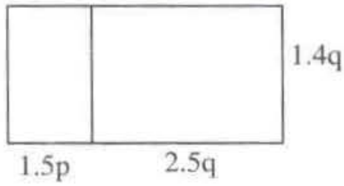
4. Find the product of $(x^2 - 4y^2)$ and $(x + 5y)$. If $x = 1$ and $y = 2$, what is the numerical value of the product ?

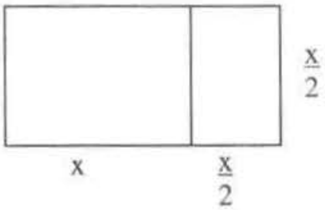
5. Find the product of $(5x + 2)$ and $(x + 1)$. If $x = -2$, what is the value of the product ?

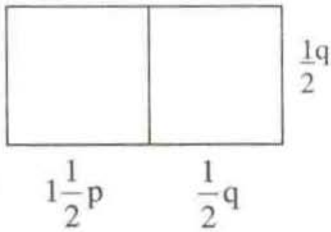
6. Find the product of $(y - x)$ and $(2x + y)$. Also find its value of by taking $x = 2$ and $y = 3$.

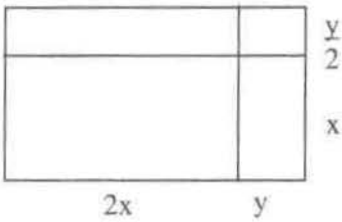
7. Find the area of each of the following rectangular objects.

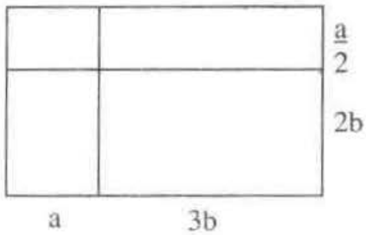
a)


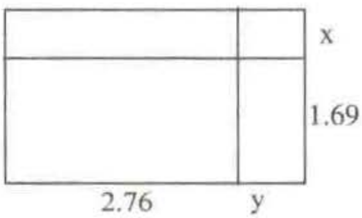
b)


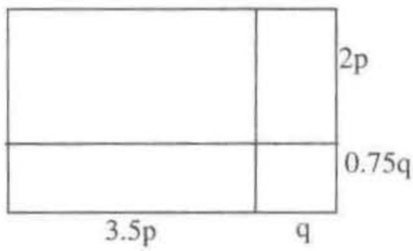
c)


d)


e)


f)


g)


h)


8.4.3 Multiplication of Trinomial by a Binomial.

What is the area of a rectangle of length (a+b+c)cm and breadth (x+y) cm ?

In the figure, the length of the rectangle is divided into 3 parts, a cm., b cm and c cm. and the breadth is divided into two parts x cm and y cm. As a result the whole rectangle is divided into six parts.
ax+bx+cx+ay+by+cy

	(a + b + c) cm			
x cm	ax	bx	cx	(x+y)
y cm	ay	by	cy	cm
	a cm	bcm	c cm	

Fig. No. 8.6

Working it in this way,

$$
\begin{aligned}
& (a + b + c)(x + y) \\
&= a(x + y) + b(x + y) + c(x + y) \\
&= ax + bx + cx + ay + by + cy
\end{aligned}
$$

$\therefore$ area of the rectangle = $(ax + bx + cx + ay + by + cy)\ cm^2$

When a trinomial expression is multiplied by a binomial expression, we first multiply each term of the second expression by each term of the first expression separately and then add all the products.

Example 1

Multiply : $(x + y + z) \times (a + b)$

Solution :

Here, $(x + y + z) \times (a + b)$

$$
\begin{aligned}
&= x(a + b) + y(a + b) + z(a + b) \\
&= ax + bx + ay + by + az + bz
\end{aligned}
$$

Example 2

Multiply $(x^2 + y - 1) \times (2x - 7)$

Solution :

Here, $(x^2 + y - 1) \times (2x - 7)$

$$
\begin{aligned}
&= x^2(2x - 7) + y(2x - 7) - 1(2x - 7) \\
&= 2x^3 - 7x^2 + 2xy - 7y - 2x + 7 \\
&= 2x^3 - 7x^2 - 2x + 2xy - 7y + 7
\end{aligned}
$$

Example 3

If the length and the breadth of a rectangle are $(3x + y + 2)$ cm and $(3x - 5y)$ cm respectively, find its area.

Solution:

Here,

Length of rectangle (l) $= (3x + y + 2)$ cm

Breadth of rectangle (b) $= (3x - 5y)$ cm

Area (A) = ?

Now, $A = l \times b$

$$= (3x + y + 2) \times (3x - 5y)$$
$$= 3x(3x - 5y) + y(3x - 5y) + 2(3x - 5y)$$
$$= 9x^2 - 15xy + 3xy - 5y^2 + 6x - 10y$$
$$= (9x^2 - 5y^2 - 12xy + 6x - 10y)\ cm^2.$$

Exercise 8.4.3

1. **Find the product of :**

   a) $(x + y + z) \times (3x - y)$ b) $(x + 2y + 4) \times (a - b)$

   c) $(2x - 3y + 4z) \times (a - b)$ d) $(2a + 3b + 4c) \times (a^2 + b^2)$

2. **Multiply :**

   a) $(x^2 + 4x - 3) \times (3x + 4)$ b) $(1 + 4y + 3y^2) \times (2 - 3y)$

   c) $(4x^2 + 5x + 1) \times (x + 3)$ d) $(x^2 + ax + a^2) \times (x - a)$

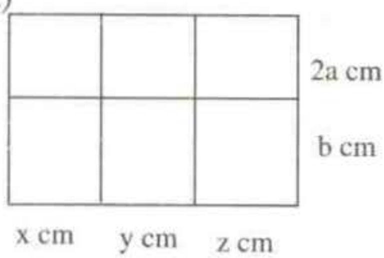
   e) $(x^2 - xy + y^2) \times (x + y)$ f) $(a^2 + ab + b^2) \times (a - b)$

3. a) Find the product of $(x^2 - 3x + 7) \times (a \times 4)$. If $x = 2$, what is the value of the product ?

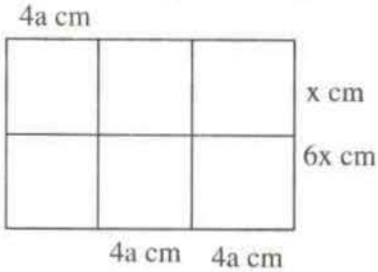
   b) Find the product of $(5x^2 + xy + y^2) \times (2x + y)$. If $x = 3$ and $y = 2$, what is the value of the product ?

4. **Find the area of each of the following rectangular objects :**

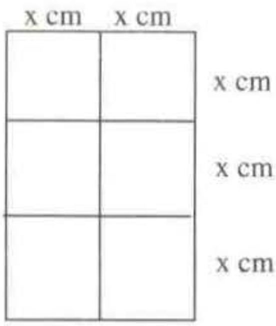
a)



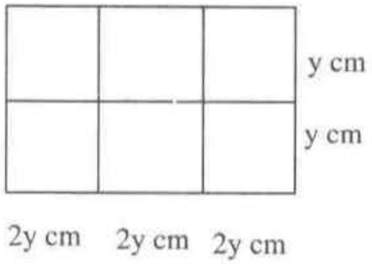
b)



c)



d)



8.4.4 Product of $(a + b)^2$

What is the area of a square of length $(a + b)$ cm and breadth $(a + b)$ cm ?

In the figure, $AB = AE + EB = a + b$

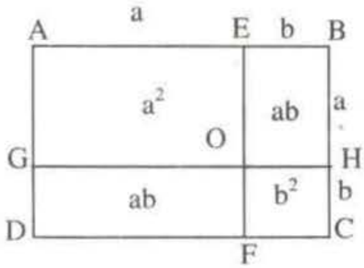


Figure No. 8.7

Similarly, $BC = BH + HC = a + b$

$\therefore$ area of the square ABCD

$= (a + b)(a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Square ABCD = Square AGOE + rectangle EOHB + square OFCH + rectangle GOFD

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$\therefore (a+b)^2 = a^2 + 2ab + b^2$

[It is called the square of binomial expression]

8.4.4.1 Square of a Binomial

a) In the figure, the length 'a' unit of a square is increased by length 'b' unit. Now, the area of the large square can be written in two ways.

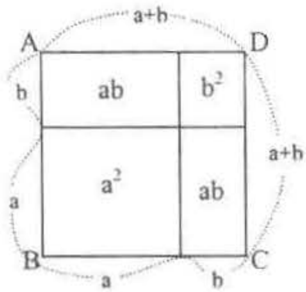


Fig.No. 8.8

i) Area of the large square

$= (a + b)(a + b)$

$= (a + b)^2$

ii) Area of the large square as the sum of the areas of separate rectangles $= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

Therefore, from (i) and (iii),

$$(a + b)^2 = a^2 + 2ab + b^2$$

b) In the figure, the length 'a' unit of a square is decreased by length 'b' unit

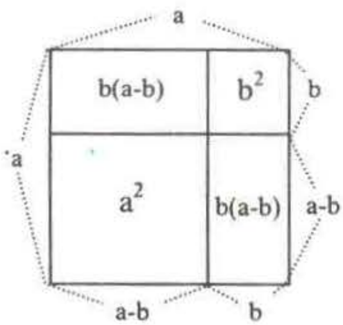


Fig.No. 8.9

Now,

(i) Area of a big square $= a^2$

(ii) Area of a big square as the sum of the area of separate rectangles.

$= (a - b)^2 + b(a - b) + b(a - b) + b^2$

$= (a - b)^2 + 2b(a - b) + b^2$

$= (a - b)^2 + 2ab + b^2$

From (i) and (ii)

$$(a - b)^2 + 2ab - b^2 = a^2$$

$$\therefore (a - b)^2 = a^2 - 2ab + b^2$$

From the discussion above

$$\boxed{\begin{array}{l}(a + b)^2 = a^2 + 2ab + b^2 \\ (a - b)^2 = a^2 - 2ab + b^2\end{array}}$$

Example 1

Find the square of each of the following each binomial expressions.

a) $x + 3$ b) $2x - y$

Solution :

a) $(x + 3)^2 = x^2 + 2 \times x \times 3 + (3)^2$
$= x^2 + 6x + 9$

b) $(2x - y)^2 = (2x)^2 - 2 \times 2x \times y + (y)^2$
$= 4x^2 - 4xy + y^2$

Example 2

Expand

a) $(2x - 3y^2)^2$ b) $(a + b + c)^2$

Solution :

a) $(2x - 3y^2)^2 = (2x)^2 - 2 \times 2x \times 3y^2 + (3y^2)^2$
$= 4x^2 - 12xy^2 + 9y^4$

b) In $(a + b + c)^2$, supposing $a + b = A$,
$(a + b + c)^2 = (A + C)^2$
$= A^2 + 2AC + C^2$
$= (a + b)^2 + 2(a + b)c + c^2$
$= a^2 + 2ab + b^2 + 2ac + 2bc + c^2$
$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Exercise - 8.4.4

1. **Expand**

a) $(a + 3)^2$ b) $(y - 4)^2$ c) $(2x + 3)^2$
d) $(4 - x)^2$ e) $(m + n)^2$ f) $(3p - 2q)^2$
g) $(9x + 4y)^2$ h) $(4a - 5b)^2$ i) $(-8s + 3t)^2$

2. **Expand**

a) $(x^2 - 2y)^2$ b) $(pq + rs)^2$ c) $(a^2b + b^2c)^2$
d) $(-2p^3 + 3q^4)^2$ e) $\left(x^2 - \frac{1}{x}\right)^2$ f) $\left(3a^2 - \frac{1}{6a^3}\right)^2$

g) $(x + 2y - 3z)^2$ h) $(-8a - 5b + 7c)^2$

i) $\left(-p^2 - \frac{1}{p^2q^2} + q^2\right)^2$ j) $\{(x + y)^2\}^2$

3. a) If $x + \frac{1}{x} = 3$, find out the value of $x^2 + \frac{1}{x^2}$.

   b) If $a - \frac{1}{a} = 5$, find out the value of $a^2 + \frac{1}{a^2}$.

   c) If $p - \frac{1}{p} = 3$, find out the value of $\left(p + \frac{1}{p}\right)^2$ and $p^2 + \frac{1}{p^2}$.

   d) Simplify : $(2x + 3y)^2 + (6x - 5y)^2$.

   e) Simplify : $2(m - 3)^2 - 2(m - 5)(m + 6)$

4. **Find the product of :**

   a) $(a + b) \times (a^2 - ab + b^2)$ b) $(a - b)(a^2 + ab + b^2)$

5 a) If $x + \frac{1}{x} = 4$, prove that : $x^2 + \frac{1}{x^2} = 14$.

   b) If $y - \frac{1}{y} = 4$, prove that : $y^2 + \frac{1}{y^2} = 18$.

6. **Simplify :**

   a) $(a + b)^2 + (a - b)^2$ b) $(a + b)^2 - (a - b)^2$

   c) $(a + b)^2 + (b - a)^2$ d) $(a - b)^2 - (a + b)^2$

8.4.5 Product of $(a + b)^3$

Here,

$(a + b)^3 = $ (How much ?)

or, $(a + b)^3 = (a + b)^2 (a + b)$

$= (a^2 + 2ab + b^2)(a + b)$

$= a^2 (a + b) + 2ab(a + b) + b^2 (a + b)$

$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

By the same method,

$(a - b)^3 = $ (How much ?)

$(a - b)^3 = (a^2 - 2ab + b^2)(a - b)$
$= a^2(a - b) - 2ab(a - b) + b^2(a - b)$
$= a^3 - a^2b - 2a^2b + 2ab^2 + ab^2 - b^3$
$= a^3 - 3a^2b + 3ab^2 - b^3$

$\therefore (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Thus, from the discussion above

1. (i) $(a+b)^3 = a^2+3a^2b+3ab^2b^3$ $= a^3+b^3+3ab(a+b)$ ii) $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

2. (i) $(a-b)^3 = a^3 - 3a^2b+3ab^2-b^3$ $= a^3-b^3-3ab(a-b)$ ii) $a^3-b^3 = (a-b)^3 + 3ab(a-b)$

Example 1

Find the cube of :

a) $x + 3$ b) $4x + 3y$

Solution :

a) $(x + 3)^3$
$= (x)^3 + 3(x)^2(3) + 3(x)(3)^2 + (3)^3$
$= x^3 + 9x^2 + 27x + 27$

b) $(4x + 3y)^3 = (4x)^3 + 3(4x)^2(3y) + 3(4x)(3y)^2 + (3y)^3$
$= 64x^3 + 9y \times 16x^2 + 12x \times 9y^2 + 27y^3$
$= 64x^3 + 144x^2y + 108xy^2 + 27y^3$

Example 2

If $a + b = 5$ and $ab = 6$, find out the value of $a^3 + b^3$

Solution :

Here, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
or, $(5)^3 = a^3 + b^3 + 3 \times 6 \times 5$
or, $125 = a^3 + b^3 + 90$
or, $a^3 + b^3 = 125 - 90$
$\therefore a^3 + b^3 = 35$

Next Method :

Here, $a^3 + b^3$

$= (a + b)^3 - 3ab(a + b)$

$= (5)^3 - 3 \times 6 \times 5$

$= 125 - 90 = 35$

Example 3

If $a + \frac{1}{a} = P$, prove that : $a^3 + \frac{1}{a^3} = P^3 = p^3 - 3p$

Solution :

Here,

L.H.S. $= a^3 + \frac{1}{a^3}$

$= \left(a + \frac{1}{a}\right)^3 - 3(a)\left(\frac{1}{a}\right)\left(a + \frac{1}{a}\right)$

$= (P)^3 - 3p$

$= P^3 - 3p =$ R.H.S. proved.

Example 4

Find the cube of :

a) a - 2 b) 5 m - 4n

Solution :

Here,

a) $(a - 2)^3$

$= (a)^3 - 3(a)^2(2) + 3(a)(2)^2 - (2)^3$

$= a^3 - 6a^2 + 12a - 8$

b) $(5m - 4n)^3$

$= (5m)^3 - 3(5m)^2(4n) + 3(5m)(4n)^2 - (4n)^3$

$= 125\ m^3 - 12\ n \times 25\ m^2 + 15\ m \times 16n^2 - 64\ n^3$

$= 125\ m^3 - 300\ m^2n + 240\ mn^2 - 64n^3$

Example 5

What is the cube of a - b- c ?

Solution :

Here,

$(a-b-c)^3$

$= [(a-b) - c]^3$

$= (a-b)^3 - 3(a-b)^2 (c) + 3 (a-b) (c)^2 - (c)^3$

$= a^3 - 3a^2b + 3ab^2 - b^3 - 3(a^2 - 2ab + b^2) c + 3 (a-b)c^2 - c^3$

$= a^3 - 3a^2b + 3ab^2 - b^3 - 3a^2c + 6abc - 3b^2c + 3ac^2 - 3bc^2 - c^3$

$= a^3 - b^3 - c^3 - 3a^2b + 3ab^2 - 3a^2c + 3ac^2 - 3b^2c - 3bc^2 + 6abc$

Exercise 8.4.5

1. **Find the cube of :**

a) $a + 3$	b) $2x + 1$	c) $3a + b$	d) $3x + 4y$
e) $xy + yz$	f) $a+b+2c$	g) $3a + 5b$	h) $x^2 + xy$
i) $x - 2$	j) $2x - 1$	k) $2 - 3a$	l) $3 - 4a$
m) $2a - 3b$	n) $2x - 5y$	o) $2a - b - c$	p) $2x - 3y - z$

2. a) If $a + b = 6$ and $ab = 7$, what is the value of $a^3 + b^3$?
   b) If $a - b = 7$ and $ab = 8$, find the value of $a^3 - b^3$.

3. a) If $a + b = 2$, what is the value of $a^3 + b^3 + 6ab$?
   b) If $x - y = 3$, find out the value of $x^3 + y^3 - 9xy$?
   c) If $a = 3$, find out the value of $64 - 144a + 108a^2 - 27a^3$

4. a) If $a + \frac{1}{a} = 3$, prove that : $a^3 + \left(\frac{1}{a}\right)^3 = 18$.
   b) If $x + \frac{1}{x} = 4$, what is the value of $x^3 + \left(\frac{1}{x}\right)^3$?
   c) If $x - \frac{1}{x} = 3$, what is the value of $x^3 - \left(\frac{1}{x}\right)^3$?
   d) If $ax - \frac{1}{ax} = 5$, what is the value of $a^3x^3 - \frac{1}{a^3x^3}$?
   e) If $a - \frac{1}{a} = P$, find out the value of $a^3 - \frac{1}{a^3}$.

5. **Simplify :**

a) $a^3 - b^3 + (a+b)^3$ b) $(a^3+ab) + (2a^2b + b^3) + (a-b)^3$

c) $(x-y)^3 + 3xy^2 + (x^3+y^3) + x^2y$ d) $(x^3+y^3) - (x+y)^3 - 3xy$

8.5. Division of Algebraic Expressions

8.5.1. Division of a monomial by monomial :

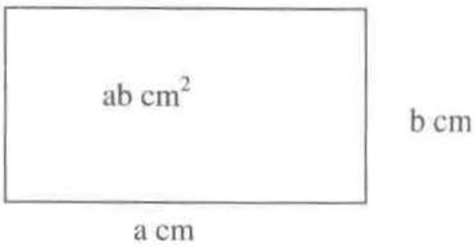


Fig.No. 8.10

The area of a rectangle of length a cm and breadth b cm is ab cm^2. Here, to calculate the area of a rectangle, its length is multiplied by its breadth. If a = 3cm and b = 2cm, what is the area of the rectangle ? That is why the area of rectangle = $3\times2 = 6\ cm^2$.

Let's consider the following questions from the above example.

a) What is the breadth of the rectangle, when the length is 3cm and the area $6cm^2$?

b) What is the length of the rectangle, when the area is $6cm^2$ and the breadth is 2cm ?

Here, for the calculation of the breadth, the area is divided by the length when the area and the length of the rectangle are given.

Therefore, the answer to the question (a), $6cm^2 \div 3cm = 2cm$ or, $\frac{6cm^2}{3cn} = 2cm$. In the same way, for the calculation of the length, the area is divided by the breadth when the area and the breadth are given.

Therefore, the answer to the question (b), $6cm^2 \div 2cm = 3cm$, or $\frac{6cm^2}{2cm} = 3cm$. On the basis of the discussion above.

a) Multiplication and division are inverse operations of one another.

b) Every multiplication operation is denoted by two equivalent division operations. That is why, $ab \div a = b$ and $ab \div b = a$ are the two equivalent division operations of the multiplication operation $a \times b = ab$

Here,

In $ab \div a = b$, ab is called the dividend, a is called the divisor and b is called the quotient. But in $ab \div b = a$, ab is called the dividend, b is called the divisor and a is called the quotient.

Example 1

Write two equivalent operations representing $3x \times 2y = 6xy$ and find the quotient in each case.

Solution :

The two equivalent division operations which represent the multiplication operation $3x \times 2y = 6xy$ are as follows :

a) $6xy \div 3x = \dfrac{6xy}{3x} = \dfrac{3 \times 2 \times x \times y}{3 \times x} = 2y$

b) $6xy \div 2y = \dfrac{6xy}{2y} = \dfrac{3 \times 2 \times x \times y}{2 \times y} = 3x$

Example 2

Divide : $5x^2y^3$ by $3xy$

Solution :

Here,

$$5x^2y^3 \div 3xy = \frac{5x^2y^2}{3xy} = \frac{5 \times x \times x \times y \times y \times y}{3 \times x \times y} = \frac{5xy^2}{3}$$

or,

$$5x^2y^3 \div 3xy = \frac{5x^{2-1}y^{3-1}}{3} = \frac{5xy^2}{3}$$

8.5.2. Dividing a Binomial or a Trinomial by a Monomial :

Look at the following examples

Example 3

Divide $3a^3b^2 - 2a^2b^3$ by a^2b^2

Solution :

Here,

$$(3a^3b^2 - 2a^2b^3) \div a^2b^2$$
$$= \frac{3a^3b^2 - 2a^2b^3}{a^2b^2}$$
$$= \frac{3a^3b^2}{a^2b^2} - \frac{2a^2b^3}{a^2b^2}$$
$$= 3a^{3-2}b^{2-2} - 2a^{2-2}b^{3-2}$$
$$= 3a^1b^0 - 2a^0b^1$$
$$= 3a - 2b \qquad [\therefore b^0 = 1, a^0=1]$$

Example 4

Divide

$9x^5-4x^4a-2x^3a^2$ by $3x^3$ and express the quotient with fractional coefficients.

Solution :

Here, $(9x^5-4x^4a-2x^3a^2) \div 3x^3$

$$= \frac{9x^5}{3x^3} - \frac{4x^4a}{3x^3} - \frac{2x^3a^2}{3x^3}$$
$$= \frac{3\times3\times x^2\times x^3}{3\times x^3} - \frac{4\times x\times x^3\times a}{3\times x^3} - \frac{2\times x^3\times a^2}{3\times x^3}$$
$$= 3x^2 - \frac{4xa}{3} - \frac{2a^2}{3}$$

Exercise 8.5

1. **Write two equivalent division operations which represent each of the following multiplication operations and find the quotient :-**

a) $p \times q = pq$

b) $m \times n = mn$

c) $2a \times 4b = 8ab$

d) $2x^2 \times 3x = 6x^3$

e) $3a^2b \times 2b = 6a^2b^2$

f) $3p^3q^2 \times 4p^2q^3 = 12p^5q^5$

g) $16y^2z^4 \times 4y^3z = 64y^5z^5$

2. **Divide**

a) $12x^3 \div 2x^2$

b) $15y^4 \div \frac{3y^3}{5}$

c) $16z^6 \div 0.8z^4$

d) $18a^2b^2 \div (-6ab)$

e) $-25pq^4 \div \frac{5q^3}{4}$

f) $36x^7y^2 \div 1.2x^3y$

g) $-5m^4n^2 \div (-2m^4n^2)$

h) $\frac{16y^4z^3}{5} \div \left(-\frac{8}{15}y^2z^2\right)$

3. **Find the length of the unknown side of the rectangle for each of the following conditions :**

a) area $6xy^2$ sq. unit and length $3xy$ unit.

b) area $7p^2q$ sq. unit and breadth pq unit.

c) area $12m^3n^2$ sq. unit and length $4m^2n$ unit.

d) area $21y^4z$ sq. unit and breadth $3z$ unit.

e) area $56m^2n$ sq. unit and length $7mn$ unit.

4. **Divide :**

a) $(2a^3b + 3ab^3) \div ab$

b) $(6a^4b^2 - 9a^2b^4) \div 3a^2b^2$

c) $(-2x^4 + 8x^3) \div \frac{2}{5}x^2$

d) $(1.2x^4y^2 - 9x^5y) \div (-3x^3y)$

e) $(6x^2y^4 + 3x^3y^3) \div 3x^2y^2$

f) $(x^2 - 25x + 5xy) \div (-5x)$

g) $(-9x^3 + 6x^2 + x) \div 3x$

h) $(-x^3 + \frac{14x^2}{15} - 19x) \div 7x$

5. **The area of a rectangle is $2a^2 + 4ab$ square units.**
   a) If the breadth is 2a unit, what is its length ?
   b) Write an expression which represents the perimeter of the rectangle.
   c) If a =3 cm and b = 2cm, what is the perimeter of the rectangle ?

6. **The area of a square field is $64x^2y^2$**
   a) What is its length ?
   b) If x = 2m and y = 3m, what is its perimeter ?

8.6. Simplification :

Example 1

Simplify :

$(x+y)(x-y) - x(x-y)$

Solution :

Here,

$$\begin{aligned}&(x+y)(x-y) - x(x-y)\\ &= x^2 - xy + xy - y^2 - x^2 + xy\\ &= -y^2 + xy = xy - y^2\end{aligned}$$

Example 2

Simplify :

$$\frac{a^2}{bc} \times \frac{ab^2}{a}$$

Solution :

Here,

$$\begin{aligned}&\frac{a^2}{bc} \times \frac{ab^2}{a}\\ &= \frac{a \times a}{b \times c} \times \frac{a \times b \times b}{a}\\ &= \frac{a \times a \times b}{c} = \frac{a^2 b}{c}\end{aligned}$$

Example 3

Simplify :

$$\frac{5x+4}{x+2}-\frac{4x+2}{x+2}$$

Solution :

Here,

$$\frac{5x+4}{x+2}-\frac{4x+2}{x+2}$$

$$= \frac{5x+4-(4x+2)}{x+2}$$

$$= \frac{5x+4-4x-2)}{x+2}=\frac{x+2}{x+2}=1$$

Example 4

If the sides of a triangle are (2x-3y) cm, (3x+4) cm and (4y+7) cm respectively,

a) What is the perimeter of the triangle ?

b) What is the perimeter of the triangle if x=2 cm. and y=3 cm ?

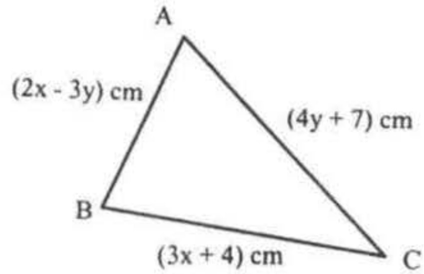


Fig. No. 8.11

Solution :

a) P = Perimeter of a triangle ABC

$\therefore$ P = AB + BC + CA

= (2x-3y) cm + (3x+4) cm + (4y + 7) cm

= (2x - 3y + 3x + 4 + 4y + 7) cm

= (5x + y + 11) cm.

b) If x = 2 cm and y = 3cm, then

P = $(5 \times 2 + 3 + 11)$ cm

= 24 cm

Exercise 8.6

1. **Simplify :**

a) (a+b) (a-b) +a (a-b) b) 3 (p+q) - 2(p+3q) + 3(p+5)

c) 4(x-y) - 3(y-z) + 4(z-x) d) (x+3) (x-5) + 15 (x+1)

e) (y-7) (x-y) +6 (x-2y)

2. **Simplify :**

a) $\frac{10ab}{15bc}$ b) $\frac{12xy}{16yz}$ c) $\frac{18pq}{6pqr}$

d) $\frac{9x^2y}{5xy^2}$ e) $\frac{48p^2q}{64pq^2}$ f) $\frac{5(m+n)}{5m+5n}$

g) $\frac{3a^2b}{5ab}\times\frac{5ac}{12abc}$ h) $\frac{p^2q}{qr^3}\times\frac{p^4}{r^5}\times\frac{r^6}{p^7}$ i) $\frac{x}{x+5}\times\frac{3(x+5)}{5x^2}$

j) $\frac{(a+b)(a-b)}{a^2-b^2}$

3. **Simplify :**

a) $\frac{5x}{x+1}-\frac{4x-1}{x+1}$ b) $\frac{x+5}{x-5}-\frac{10}{x-5}$ c) $\frac{6x-1}{x-1}-\frac{5x}{x-1}$

d) $\frac{x^2+2x}{x^2+y^2}+\frac{y^2-2x}{x^2+y^2}$ e) $\frac{x+3}{x+2}-\frac{x+1}{x+2}$ f) $\frac{3x+1}{2x+1}-\frac{x-2}{2x+1}$

4. **If the length and the breadth of a rectangle are (3x+y) cm and (3x-y) cm respectively.**

a) What is its perimeter ?
b) What is its area ?
c) What are its perimeter and area if x = 2 and y =3 ?

5. **If the side of a square is (5a-3b) cm.**

   a) What is its perimeter ?
   b) What is its area ?
   c) Find the actual perimeter and area if $a = 1$ and $b=1$

6. **If the length of a rectangle is x cm and its breadth is 3cm less than its length,**

   a) What is its perimeter ?
   b) What is its area ?
   c) What are its perimeter and area if $x = 7$cm ?

7. If the area of a rectangle is $(9x^2 + 12xy)$ cm^2 and length is 3x cm., what is its breadth ?

8. **In question no. 7, if $x = 3$ and $y = 2$**

   a) Find the actual perimeter.
   b) What is the actual area ?

9 Equation and Inequality

9.1 Trichotomy property of integers in a number line

Shiva has written a number greater than 2 by using the sign of trichotomy. The table prepared by him looks like this :

$3 > 2$

$4 > 2$

$5 > 2$

$6 > 2$

$7 > 2$

$8 > 2$

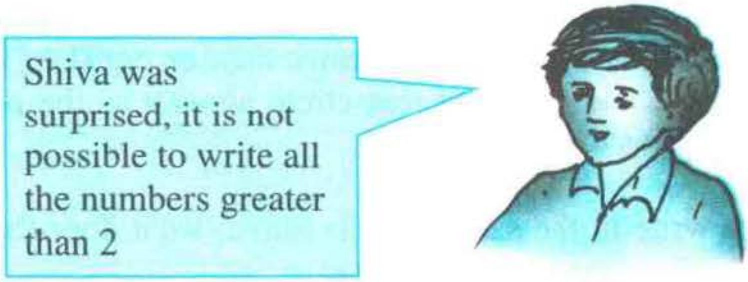


Fig.No. 9.1

Shiva has tried to solve this question by using a number line. He circled 2 because 2 is not in the table of the numbers greater than 2. He coloured up the part of the number line which lies to the right side of 2 because numbers greater than z lie to the right of 2.

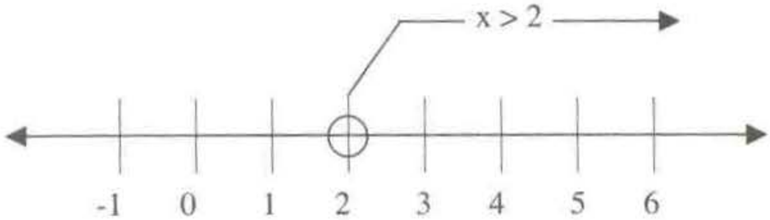


Fig. No. 9.2

He used the variable x to denote any number which lies in the coloured part of the number line. He wrote the solution of his problem as $x>2$.

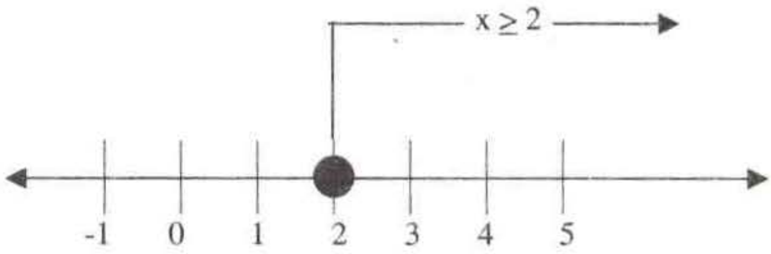


Fig. No. 9.3

Similarly, Shiva wrote $x \geq 2$ to denote the number greater than 2 or equal to 2 and read "x is greater than or equal to 2". He circled the point at 2 and coloured that circle as well as the part of the number line that lies to the right of 2.

If we write in the same way as Shiva, what does the coloured part of the number line below represent ?

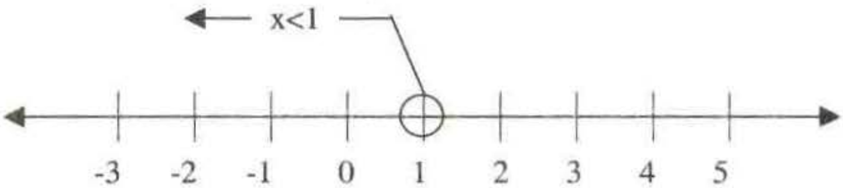


Fig. No. 9.4

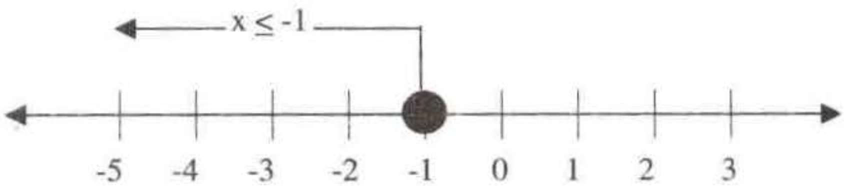


Fig. No. 9.5

In the Fig.no. 9.4, 1 is kept in a circle and, leaving the circle, the part to the left of 1 is coloured. So the numbers less than 1 which are represented by x in the coloured part of the number line is represented by $x<1$. Similarly, in the fig.no9.5, $x \leq 1$ represents the coloured part in the number line

According to the mathematical statement, x≤ 2 represents the opposite of x≥2 and x≥ represents the opposite of x≤1.
Here

$x \ngeq$ means x is not equal or greater than 2.

$x \nleq -1$ indicates x is not less than or equal to -1.

Study a) and b) as given below :-

a) If a and b are two integers in which a>b and c is another integer, then

Addition axiom $(a + c) > (b + c)$

Subtraction axiom $(a - c) > (b - c)$

Multiplication axiom $ac > bc$, where c is a positive number.

Division axiom $\frac{a}{c} > \frac{b}{c}$, where $c \neq O$ and c is a positive number.

$ac < bc$, where c is a negative number.

$\frac{c}{a} < \frac{b}{c}$, where $c \neq O$ and c is a negative.

If both the sides of a mathematical sentence which includes the trichotomy symbols > or < is multiplied or divided by a negative whole number, then the trichotomy symbols(> or <) in the sentence are changed.

b) If two integers a and are such a=b and other any integer is c, then

$(a+c) = (b+c)$	equality addition axiom.
$(a-b) + (b-c)$	equality subtraction axiom.
$ac=bc$	equality multiplication axiom.
$\frac{a}{c} = \frac{b}{c}$, where $c \neq o$	equality division axiom.

Exercise 9.1

1. **Write the correct sign (>, < or =) between the integers given below.**

a) 3.....5	d) 3....3	g) -6.....-7
b) 3.....-5	e) -7.... -8+1	h) -5.....2
c) -3......-5	f) -7.....-6	i) -8....-1

2. **State whether the following trichotomy statements are true or false.**

a) $3 > 2$	d) $7 < 4$	g) $-7 > -6$
b) $-5 < -2$	e) $5 > 6$	h) $7 > -7$
c) $-6 > -2$	f) $-6 < -4$	i) $-7 < -9$

3. **Write the negation statement of each of the following statements.**
   a) 3 is an odd number.
   b) Pokhara is the capital of Nepal.
   c) 281 a prime number.
   d) 120 is exactly divisible by 5.
   e) Earth is a star.
   f) 16 is a square of 4.
   g) If a, b, c are the three sides of a triangle, then $(a + b) > c$.
   h) If a, b, c are three integers and $a > b$, then

      i) $a+c > b+c$ ii) $a-c > b-c$ iii) $ac < bc$ iv) $\frac{a}{c} < \frac{b}{c}$

   i) 8 is a factor of 123
   j) The numbers which are divisible by 2 are even numbers.

4. **Construct separate number lines for each of the following inequality and show the solutions on the number lines by colouring.**

a) $x > 1$	e) $x < -2$	i) $x > 7$
b) $x > 5$	f) $x < 5$	j) $x < -5$
c) $x > -3$	g) $x > 2$	k) $x < -10$
d) $x < -5$	h) $x > -2$	l) $x < 4$

5. **Decide whether the following statements are true or false according to the law of trichotomy.**

3 and 5 are two whole numbers and-7 is another whole number, then

a) $3 + (-7) = 5 + (-7)$
b) $3 - (-7) = 5 - (-7)$
c) $3 \times (-7) = 5 \times (-7)$
d) $3 + (-7) > 5 + (-7)$
e) $3 - (-7) > 5 - (-7)$
f) $3 \times (-7) > 5 \times (-7)$
g) $5 \times (-7) < 3 \times (-7)$
h) $3 \div (-7) > 5 \div (-7)$
i) $3 + (-7) < 5 + (-7)$
j) $3 \div (-7) < 3 \div (-7)$

9.2. Formation of linear Equation in one variable and problems.

9.2.1.Linear Equation in one variable

Mathematical open sentence containing equal sign '=', like x+4=7 has only one variable x. The exponent of the variable x is 1.An equation like this is called a linear equation. If x=3 then x+4=7 is a true statement. Therefore x=3 is called the solution of the equation x+4=7. Some examples of linear equations are as follows:-

$\square + 7 = 3(4 - \square)$, $\square$ is the variable.

$x + 3\frac{1}{2} = 7$, x is the variable.

$y - 4.5 = 6.9$, y is the variable.

$3.5z = 7$, z is the variable.

$\frac{W}{5} = 3\frac{1}{5}$, w is the variable.

9.2.2 Solution of Linear Equation of one Variable:-

Example 1

Solve

a) $5x + \frac{1}{2} = 8$ b) $3m - 4 = m + 2$

Solution:

a) $5x + \frac{1}{2} = 8$

or, $5x + \frac{1}{2} - \frac{1}{2} + 8 - \frac{1}{2}$ (subtracting $\frac{1}{2}$ from the both sides)

or, $5x = \frac{15}{2}$

or, $5x \times \frac{1}{5} = \frac{15}{2} \times \frac{1}{5}$ (multiplying both sides by $\frac{1}{5}$.)

or, $x = \frac{3}{2}$

or, $x = 1\frac{1}{2}$ or 1.5

Checking,

$5x + \frac{1}{2} = 8$

or $5 \times \frac{3}{2} + \frac{1}{2} = 8$

or, $\frac{15}{2} + \frac{1}{2} = 8$

or, $\frac{15+1}{2} = 8$

or, $\frac{16}{2} = 8$

or, $8 = 8$, which is true.

b) $3m - 4 = m + 2$

or, $3m - 4 + 4 = m + 2 + 4$ (adding 4 on the both sides)

or, $3m = m + 6$

or, $3m - m = m + 6 - m$ (subtracting m on the both sides)

or, $2m = 6$

or, $2m \times \frac{1}{2} = 6 \times \frac{1}{2}$ (multiplying both sides by $\frac{1}{2}$.)

or, $m = 3$

Exercise - 9.2.1

1) **Solve each of the following equations, which involve more than one mathematical operation by using equal axiom and check your answer.**

a) $2x - 5 = 9$

b) $5x = -3x - 4$

c) $\frac{3m}{2} = 7 + 2m$

d) $3y + 1 = \frac{y}{4} + \frac{31}{9}$

f) $4.5x - 3 = \frac{1}{2}x + 1$

g) $2y = y + 4$

h) $-8x + 4 = x - 8$

i) $2x + 3 = 0.5x - 1.5$

j) $1.8 + 2z = -1.2 + 3.2z$

k) $0.4p + 8 = -6 - \frac{3}{10p}$

2. **Solve**

a) $10(y-6) = 4 - 6y$

b) $3y + 4 = 2(y + 11)$

c) $5(w - 4) = 3(8 - w)$

d) $\frac{7}{2}(p-1) = 4-3p$

e) $3(m - 2) = 5(2 - m)$

f) $2(x+3) = 8-3(x-4)$

g) $2x - 1 - (1 - 3x) = \frac{7}{2}(\frac{1}{3} - x)$

h) $= \frac{1}{2}(3 - 5x) = \frac{1}{4}(-12x - x)$

i) $3.4-2(3x - 1) = 0.6(2x - 9)$

j) $2.3x\{4.5 + 2(3.2x - 1.5)\} = 2.5x + 5.7$

9.2.3. Leading problem of linear equation of one variable:-

Example 1

There are 28 students in a class and there are 4 more boys than the girls.

a) Write an equation which represents the numbers of students.

b) What are the numbers of boys and girls?

Solution:

a) Suppose the number of girls $= x$

Here , the numbers of boys $= x + 4$.

The total number of students is 28.

Here, boys + girls = 28.

Or, $x + 4 + x = 28$

Or, $2x + 4 = 28$,which is the required equation.

b) To solve,

$2x + 4 = 28$

Or, $2x + 4 - 4 = 8 - 4$, why?

Or, $2x = 24$

Or, $2x \times \frac{1}{2} = 24 \times \frac{1}{2}$

$\therefore \quad x = 12$

Hence, the number of girls =12 and

the number of boys $= x + 4$

$= 12 + 4 = 16$

Example 2

The length of a rectangle is 4cm longer than its breadth. If the perimeter of the rectangle is 16cm,

a) What are the length and breadth of the rectangle?

b) What will be the area of the rectangle?

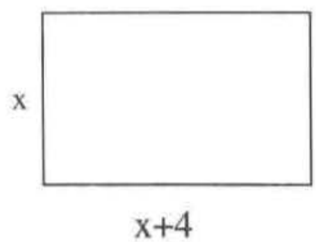


Fig.No.9.6

Solution

a) The breadth of the rectangle (b) = x cm

$\therefore$So length of the rectangle (l) = x+4cm.

According to question,

Perimeter (p) = 2 (l + b) = 16

or, $2(x + 4 + x) = 16$

or, $2(2x + 4) = 16$

or, $4x + 8 = 16$

or, $4x + 8 - 8 = 16 - 8$

or, $4x = 8$

or, $4x \times \frac{1}{4} = 8 \times \frac{1}{4}$

$\therefore$ $x = 2cm$

and $x = 4 = (2 + 4)$ cm = 6 cm.

hence, length(l) = 6 cm.

breadth (b) = 2 cm

b) Area (A) $= l \times b$

$= 6\ cm \times 2cm$

$= 12cm^2$

Example 3

Neelu is 3 years older than Mahesh. 5 years ago Neelu was twice the age of Mahesh. What are the present ages of Neelu and Mahesh?

Solution:-

Suppose the present age of Mahesh $= x$.

Then, the present age of Neelu $= x + 3$.

5years ago, Mahesh's age $= x - 5$.

5years ago, Neelu's $= x + 3 - 5 = x - 2$

According to question,

5years ago, Neelu's age $= 2 \times$ Mahesh's age

or, $x - 2 = 2(x - 5)$

or, $x - 2 = 2x - 10$

or, $x - x - 2 = 2x - x - 10$, why ?

or, $-2 = x - 10$

or, $-2 + 10 = x - 10 + 10$, why

or, $8 = x$

Hence, the present age of Mahesh = x = 8 year and the present age of Neelu = x + 3 = 8 + 3 = 11years.

Exercise- 9.2.2

1) The total cost of 8 copies of the same type and a pen is Rs.55. What is the cost of a copy?

2) There are 30 eggs in a box. Two eggs are left after using equal number of eggs everyday for 7 days. At what rate are the eggs eaten every day?

3) The total number of students in a school is 400. The number of boys is 100 more than the number of girls, what are the numbers of boys and girls?

4) Shiva has two packets of sweets. The number of sweets in one packet is double the number of sweets in the other packet. The total number of sweets in the two packets is 45. How many sweets are there in each packet ?

5) **The numbers of girls in a class is two-third of the number of boys. If the total number of the students is 40:-**
   a) Write down an equation representing the numbers of boys and girls.
   b) What are the numbers of boys and girls?

6) **In a school the number of students present is 210 on a day when one third of the total number of students was absent.**
   a) What is the total number of students in the school?
   b) How many students are absent?.

7) The sum of two consecutive numbers is 19. What are these two consecutive numbers? (Hint- suppose one number be x and other be x+1 or x-1).

8) The sum of three consecutive numbers is 60. What will be these three numbers?

9) One number is double of another and the sum of these two numbers is 99. What are these two numbers?.

10) **The breadth of a rectangle is 3cm. less than its length. If the perimeter of this rectangle is 18cm.**
   a) Write an equation for its perimeter.
   b) What are the measurements of length and breadth?

9.3. Solving Inequalities using Number Line

9.3.1 Introduction to inequalities

Mathematical open sentences containing equal (=) sign and the variable x or y or other letters is called an equation. $3x + 4 = 2$, $y = 2(y - 4)$ etc, are examples of equation.

If the mathematical open sentence contains the sign of trichotomy like 'greater than (>)', 'less than (<)', 'greater than or equal to (≥)', 'less than or equal to (≤)', then such a mathematical sentence is called an inequality.

For example $x > 2$, $x \leq N$ are inequality.

The values of inequality $x > 2$ are infinite $x = 2.01, 2.1, 3, 4, 5, 6$...etc, are the solutions of $x > 2$. We use a number line to show all these values of the inequality.

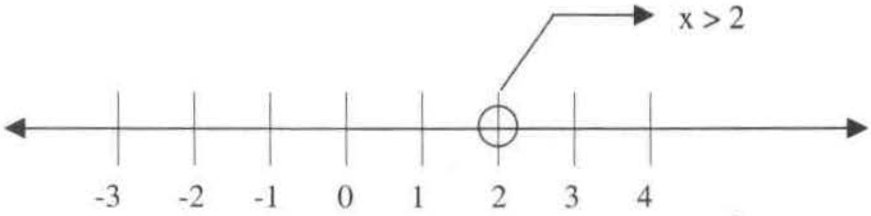


Fig. No. 9.7

On the number line all the numbers greater than 2 are on the right side of 2. 2 is not a solution of $x > 2$. Therefore, the place where 2 is written is circled and the arrow sign just above the circle represents all the numbers to the right side of 2 except 2 which are the solutions of the inequality $x > 2$. The following process to represent the inequality on a number line should be studied carefully and remembered.

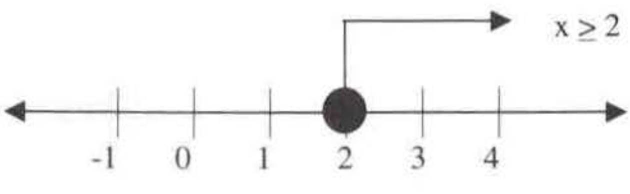


Fig. No. 9.8

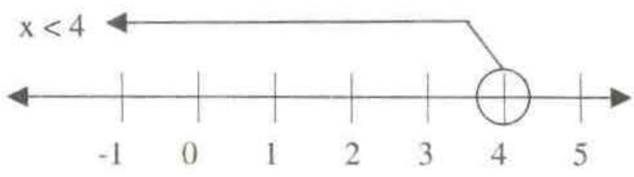


Fig. No. 9.9

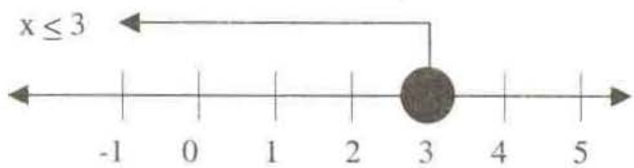


Fig. No. 9.10

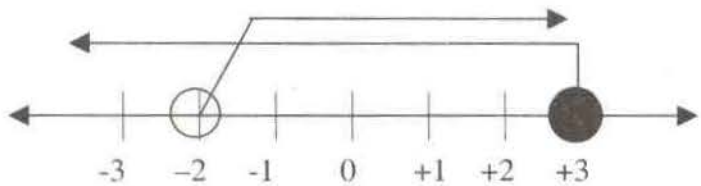


Fig. No. 9.11

X is greater than -2 and equal to 3 or less than 3. It is written as $x > -2$, and $x \leq 3$. These two in equations are combined, and the coloured part of the number line in Fig. No 9.11 is represented by $-2 < x \leq 3$.

9.3.2. Solution of Inequalities

Example 1

Solve the inequality $x + 7 > 9$ and show the solution on the number line.

Solution:

Here,

$x + 7 > 9$

or, $x + 7 - 7 > 9 - 7$(subtracting 7 from both sides)

Or, $x > 2$

Showing the solution on the number line.

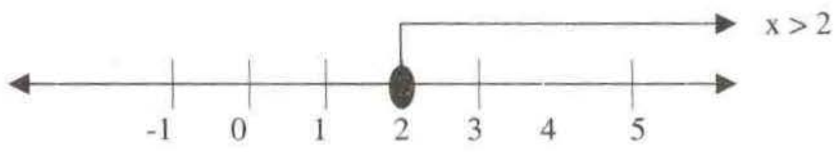


Figure No 9.12.

The arrow sign above the circle on the number line represents $x > 2$.

Example 2

Solve the inequality $-8x \leq 12$ and show the solution on the number line.

Solution:-

Here, $-8x \leq 12$

Or, $(-\frac{1}{8}), \times (-8x) \geq 12 \times (-\frac{1}{8})$ (both sides of the inequality is multiplied by $-\frac{1}{8}$, so the sign $\leq$ be changed in$\geq$.)

Or, $x \geq -\frac{3}{2}$.

To Showing $x \geq -\frac{3}{2}$ on the number line,

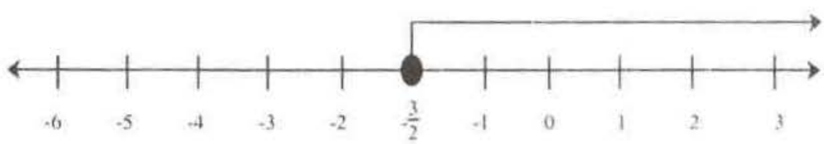


Figure No.9.13.

The coloured part on the number line represents $x \geq -\frac{3}{2}$.

Example 3

If 5 is added to two times a number then the sum is always greater than or equal to 9.

a) Write down an inequality to represent this statement.
b) Solve the inequality and show the solution on the number line.

Solution

a) Suppose, the required number is x.
According to question.
$2x + 5 \geq$ which is the required inequality.

b) Solving the inequality $2x + 5 \geq 9$.
$2x + 5 - 5 \geq 9 - 5$ (subtracting 5 from both sides)
or, $2x \geq 4$
or, $\frac{2x}{2} \geq \frac{4}{2}$ (dividing both sides by 2)

Solution of $x \geq 2$ on the number line

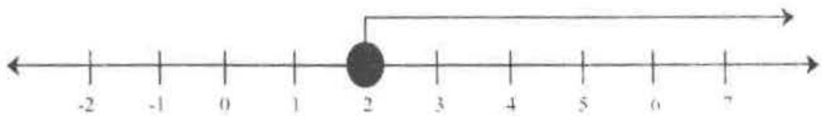


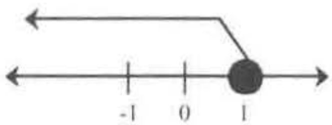
Figure No. 9.14.

The coloured part on the number line represents $x \geq 2$.

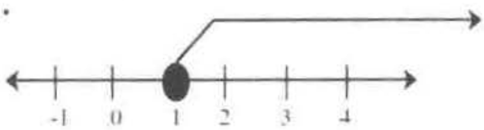
Exercise 9.3

1. Write an inequality shown in each of the number line.

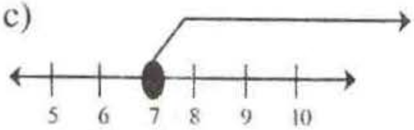
a)



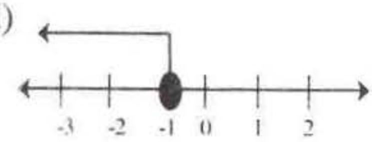
b)



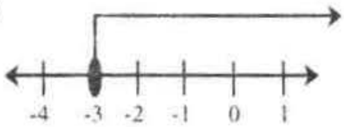
c)



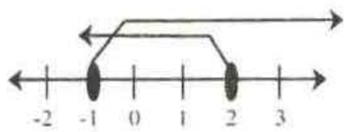
d)



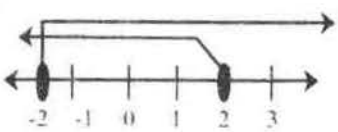
e)



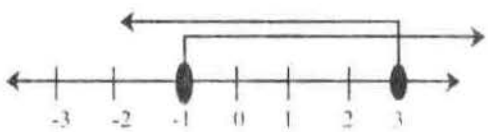
f)



g)



h)



2. Draw number lines to show each of the following inequalities.

a) $x > 1.$

b) $X > -3$

c) $x \leq 4$

d) $x \geq -2$

e) $x < \frac{3}{5}$

f) $x < \frac{1}{2}$

g) $x \leq -\frac{3}{4}$

h) $x \leq 2.3$

i) $x > -3.5$

j) $-4 \leq x < 7$

k) $\frac{7}{2} \leq x - 1$

l) $1.7 \leq x < 2.6$

3. Solve each of the inequalities given below and show the solution on number lines.

a) $x + 4 > 7$　　b) $x + 3 < -6$　　c) $x - 2 \geq -1$

d) $2.2 - x \leq 1.5$　　e) $\frac{3}{4} > \frac{5}{4} - x$　　f) $2x + 7 \leq 5$

4. **Solve each of the following inequalities and show the solutions on number lines.**

a) $2x + 1 < 9$ b) $3x + 15 > 0$ c) $2x - 1 \leq 3$

d) $12 - 4x < 4$ e) $30 - 5x > 10$ f) $15 - 8x \leq 2x - 15$

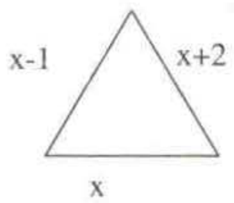
g) $3(x+1) \geq 5x + 15$ h) $\frac{3}{2}x + 2 < \frac{1}{4} - 2x$

i) $4.3 - x \geq -4.1 - 4.5x$ j) $1.3x - 4(0.7 - 0.3x) < -2.4x + 2.1$

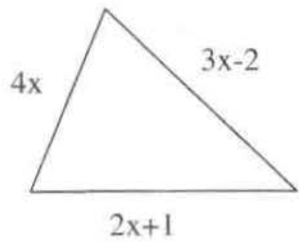
k) $\frac{3}{4}(5-7x) + \frac{7}{4}x \geq \frac{1}{2}(3x + \frac{5}{2})$

5. **"The sum of the lengths of any two sides of a triangle is greater than the length of third side". This statement is called a triangle inequality. Write down 3 triangle inequalities for each triangle given below and solve each of the inequalities.**

a)



b)



6. If 3 is subtracted from twice a number , the remainder will be less than 5". Write down an inequality to represent this statement and show the solution on the number line.

7) **If 4 is added to two- third of a number, then the sum is greater than7.**

a) Write down an inequality to represent this statement and solve it.

b) Show the solution on the number line.

c) What is the smallest whole number that satisfies the inequality?

10 Co-ordinates

10.1 Plotting points in all the quadrants.

In the fig no 10.1, two number lines x^1ox and yoy^1 are intersecting each other perpendicularly at a point 0. Here, the horizontal line x^1ox is called the x-axis, the vertical line yoy^1 is called the y-axis and the point 0 is called the origin. O is also called the point of reference. Study the figure at the right and answer the following questions..

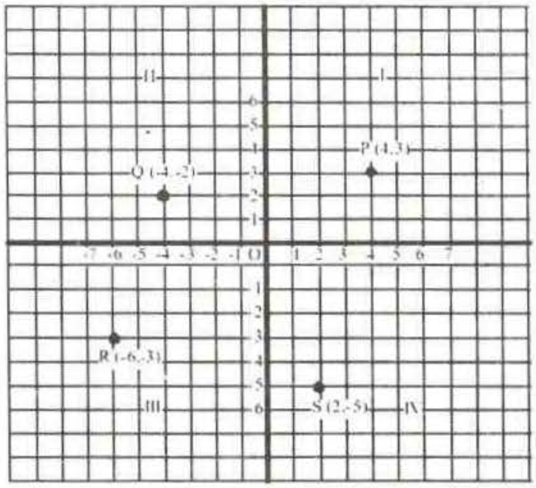


Fig.No. 10.1

a) How are the numbers to the right side of the origin on the x-axis written?

b) How are the numbers to the left side of the origin on the x-axis written?

c) How are the numbers above the origin on the y-axis written?

d) How are the numbers below the origin on y-axis written?

The numbers to the right side and above the origin are positive. Therefore the co-ordinates of the point P on the region XOY or the quadrant is (4.3).

The numbers to the left side of the origin are negative and above the origin are positive. Therefore, the co-ordinates of the point Q on the region X^1OY or the second quadrant is (-4,2).

The numbers to the left side and below the origin are negative. Therefore, the co-ordinates of the point R on the region X^1OY^1 or the third quadrant is (-6,-3).

The numbers to the right side of the origin are positive and below the origin are negative. Therefore, the co-ordinates of the point S on the region XOX^1 or the fourth quadrant is (2,-5).

Example 1

Plot the points A(3,3), B(2,2), C(1,1), O(0.0), D(-1,-1), E(-2.-2) on a sheet of graph paper. Join all of these points in order. What is the figure formed?

Solution

When the points A(3,3), B(2,2),C(1,1), O(0,0), D(-1.-1) and E(-2,-2) are joined, a straight line AE is formed.

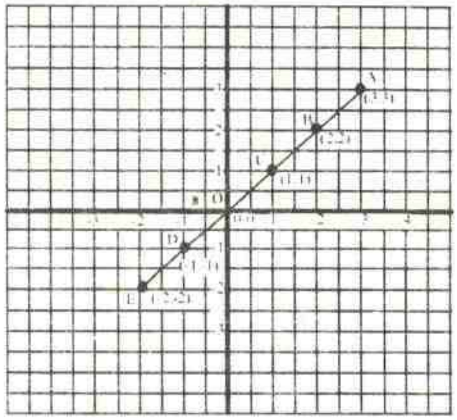


Fig.No. 10.2

To form the table of co-ordinates

x	-2	-1	0	1	2	3
Y	-2	-1	0	1	2'	3

Example 2

The point A (-3,2), B(1,2), C(1,-1) ,and D are the vertices of a rectangle.

a) Plot these points on graph paper and find the co-ordinates of D.

b) Calculate the area of the rectangle ABCD.

Solution

The co- ordinates of the point D is (-3,-1) from the graph.

Area of the rectangles ABCD

$=AB \times BC$

$= 4$ units $\times$ 3 units

$=12$ square units.

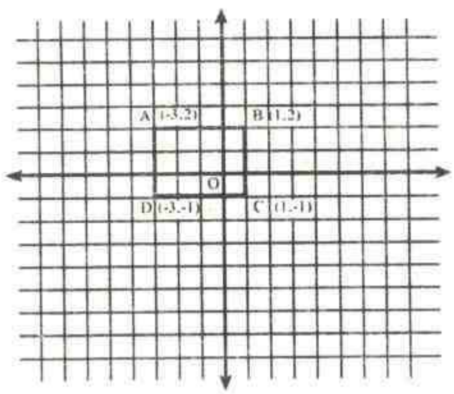
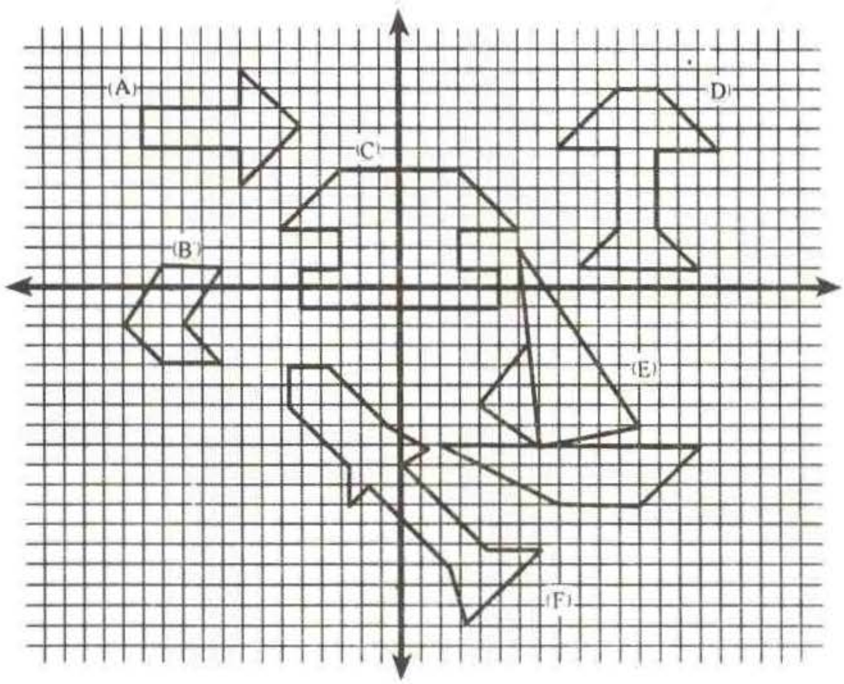


Fig. No. 10.3

Exercise 10.1

1. **Plot each of the following points on a sheet of graph paper.**
   A(1,1), B(-1,2), C(7,0), D(3,-2), E(7,-2), F(0,5), G(-5,0), H(4,4), I(-3,-4), J(5, $4\frac{1}{2}$), K(3,-3), L(0,-4), M(2,6), N(-1,6), O($4\frac{1}{2}$,6), P($1\frac{1}{2}$, - $4\frac{1}{2}$).
2. In Q. No.1, which points are equidistance from the X –axis and the Y-axis?
3. **Plot each of the following points on a sheet of graph paper and join them in order, Also write the name of the figure so formed.**
   A(-4,-3), B(-2,1), C(1,7), D(4,2), E(1,3), F(5,-1), G(-2,1).
4. Plot the points A (1,4), B(-3,-3),C(5,-1), D(-5-,3) and E(2,-5) on a sheet of graph paper and join them in order with the help of a ruler. Write the name of the figure formed.
5. **Write the co-ordinates of the vertices of the following each figures.**



6. **Plot the points A (7, 0), B (7, 2), C(7, 7), D(7, -3), E(7, -5), F(7, -10) on a sheet of graph paper and join them in order.**
   a) What is the figure formed ?
   b) Write down a mathematical statement which satisfies all these points:

7. **Plot the following two sets of points on the same graph paper taking the same origin and join the points of each set in order.**
   a) (-2,3), (-1,3),(0,3), (1,3),(2,3)
   b) (3,-5),(3,-2),(3,0),(3,5), (3, 4)
   c) What are the co-ordinates of the point where the graph of two sets intersect each other.

8. **Plot the points A (2,5),B(-2,-2) and C(6,-2) on graph paper.**
   a) Write down the co-ordinates of the mid-point D of BC.
   b) What is the length of AD and of BC?
   c) Calculate the area of triangle ABC by using the formula of area of triangle $\frac{1}{2} \times$ base $\times$ height ?

9. **Points A(3,0), B(-1,4),C(-1,-2) are the vertices of a parallelogram. The fourth point D is opposite to the point B. Plot these points on the graph paper.**
   a) Find the co-ordinates of point D.
   b) If the diagonals AC and BD intersect each other at point E, what are the co-ordinates of E?

10. **Points P(4,3),Q(-2,3)R(-2,-2) and S are the vertices of a rectangle. The fourth points S is opposite to the point Q. Plot these points on graph paper and answer the following questions.**
   a) Write down the co-ordinates of S.
   b) Calculate the area of rectangle PQRS.

10.2. Reflection and Rotation of Geometrical Shapes using Co-ordinates.

10.2.1. Concept of Reflection:

In the figure No.10.4, Hari is eating his morning meal in front of a mirror. Hence, Hari looks at his reflection in the mirror. The shape seen in the mirror is called the image of Hari. In figure 10.6, if the letter A of the English alphabet is folded along the dotted line, it will be divided into two equal parts. Therefore, each half is the image or the reflection of the other half. Here, the dotted line is called the axis of reflection.

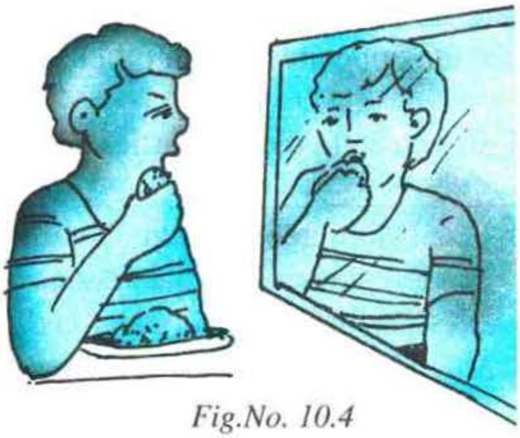


Fig.No. 10.4

In the figure No.10.5, Hari (object) and the image are equidistant from the mirror. In fig no.10.6, the hole in the letter A and its image are equidistant from the axis of reflection.

Axis of Reflection

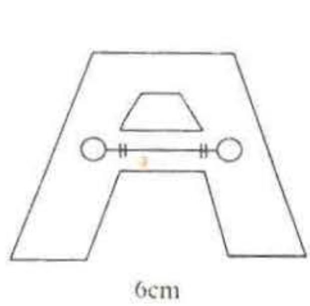


Fig.No. 10.5

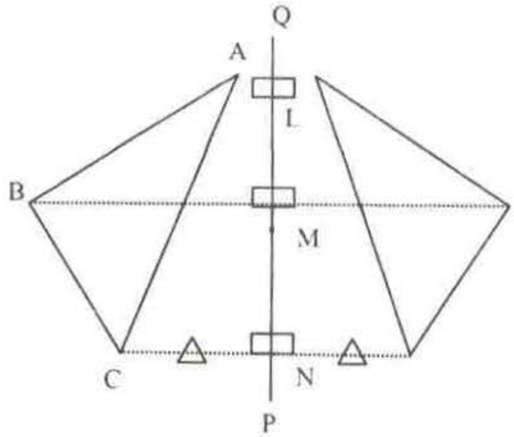


Fig.No. 10.6

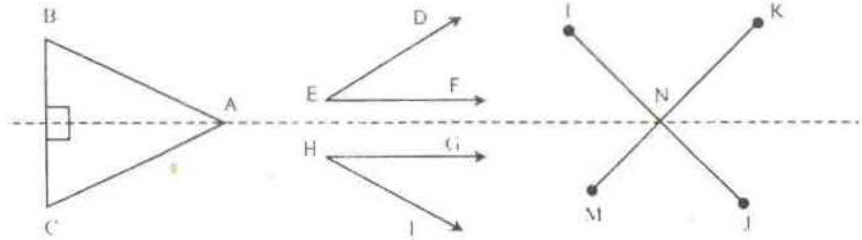
In figure No. 10.6 , when a triangle ABC is reflected on the axis of reflection PQ, then the image $A^1B^1C^1$ is formed .Here , the lines AA^1, BB^1 and CC^1 after joining the points A and A^1, B and B^1 and C and C^1 respectively are perpendicular to the axis of reflection PQ . Each as a point and its image are equidistant that the axis of reflection ,$AL=A^1L$,$BM=B^1M$ and $CN = C^1N$ and area or size of triangle ABC and its image triangle $A^1B^1C^1$ are the same but the object and image are the opposite to each other .

Can you state the properties common to the examples given above ?

1. If any geometrical shape is reflected , then the object and its image are at equal distances from the axis of reflection .
2. The areas of an object and its image are equal .
3. The object and the image are laterally inverted to each other

Exercise 10.2.1

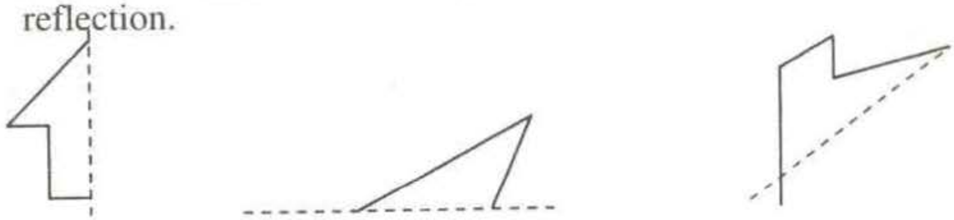
1. **Some geometrical shapes and dotted line are given in the following figures.**



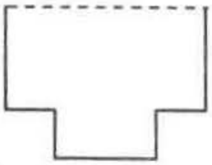
Now, draw the image formed after reflection of the following objects in the dotted line .

(a) A (b) B (c) AB (d) F (e) M
(f) K (g) KM (h) $\angle$ DEF (i) EF (j) BXA

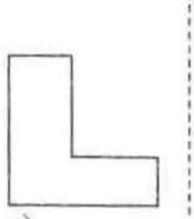
2. Draw the images that will be formed after reflection of each of the following geometrical objects in the dotted line, the axis of reflection.



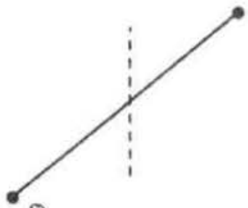
a) b) c)



d)

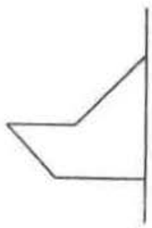


e)



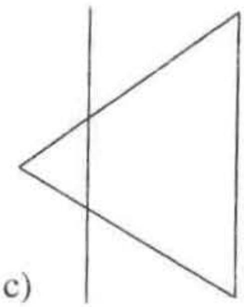
f)

3. The dotted line represents the axis of reflection in the graph paper. Show by constructing the images that will be formed after reflection of each of the following objects in the axis of reflection.

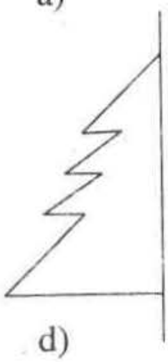


a)

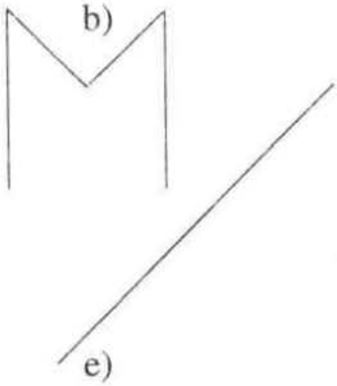
b)



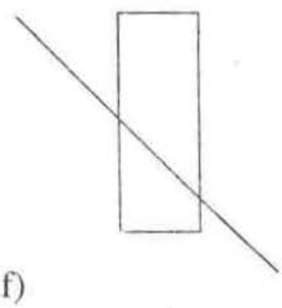
c)



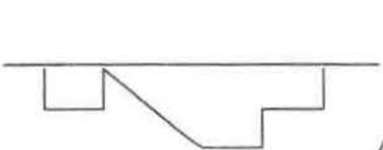
d)



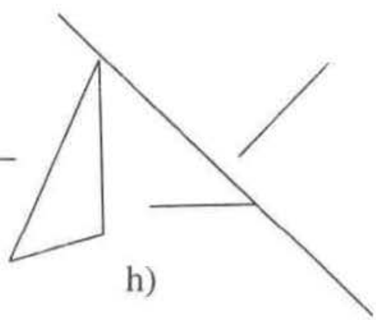
e)



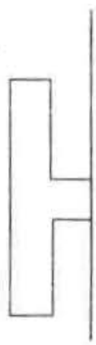
f)



g)



h)



i)

10.2.2. Concept of Rotation

On the way to school Krishna was telling his friend Raju about his work.

1. Opening and closing the tap.
2. Brushing teeth, opening and closing the cap of a toothpaste tube.
3. Opening and closing the door.
4. Unlocking and locking the padlock with a key.

At that time both of them had to cross the road so they looked to the right left and right and crossed the road. Both of them looked back because someone was calling them. Krishan said , "Again the same work is repeated ." Raju was surprised at this but he went on listening.

Can you say, what was common in this talk ? Can you give the example of other such work ?

In all these works , there is a rotation about an axis of an object.

When rotating such objects care must be taken to rotate just the right amount. To do this, it is necessary to find out how much to rotate the objects and how much the objects can be rotated. For example, the turning the hands of a clock to adjust time, and the turning of the volume knob of a radio to control sound.

This type of turning of objects is called the rotation . The direction of rotation based on the movement of the hands of a clock. If the rotation is in the direction of the hands of clock, then it is called negative rotation.

If the rotation is opposite to the movement of the hand of the clock , then the rotation is called positive rotation.

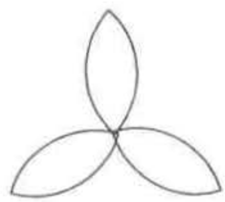


Fig.No. 10.7

The black dot in the middle in Fig .No 10.7 is the centre of rotation. Now, when every object is rotated about the centre of rotation through 360^0, then every object comes to the same state. Therefore, the rotation of 360 or a complete turn can be taken as the unit of rotation.

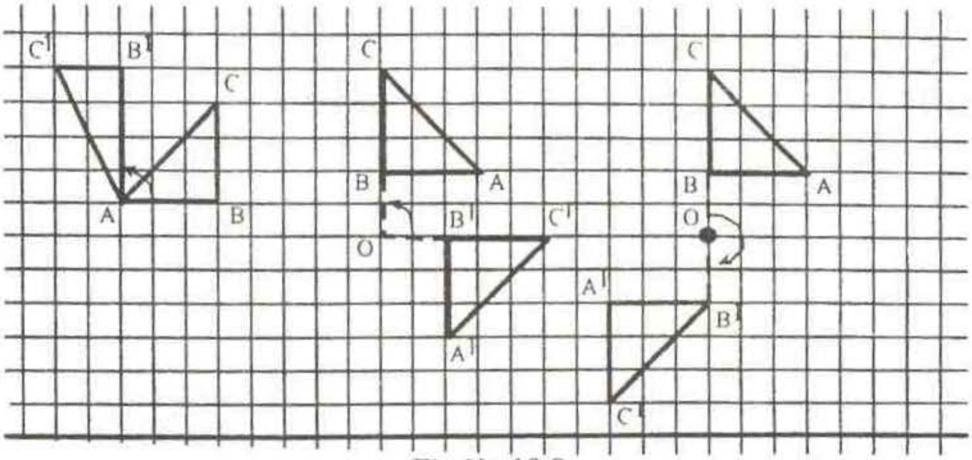


Fig.No.10.8

In Fig. No .10.8 (a) When ΔABC is rotated about the centre of rotation A through a quarter turn or 90^0 in the positive direction , then $\Delta A^1B^1C^1$ is formed. $\Delta A^1B^1C^1$ is called the image of ΔABC. Similarly, in Fig. No 10.8 (b) and (c), ΔABC is rotated through a quarter turn or 90^0 and half turn or 180^0 in the negative direction respectively , then $\Delta A^1B^1C^1$ is formed. Here also $\Delta A^1B^1C^1$ is the image of ΔABC.

The rotation of 180^0 is also called the point reflection .

Trace the given ΔABC in Fig.No.10.8 on a thin piece of paper or type paper. Where is the position of triangle formed after rotation according to the above rotation form the point A or O when the object is placed over the given ΔABC ?

The images figure of ΔABC after rotation concides with $\Delta A^1B^1C^1$ or $\Delta A^1B^1C^1$.

Example 5

Draw the image formed after a rotation the shape given below about the fixed point 0 through 90^0 and 180^0 in the positive direction .

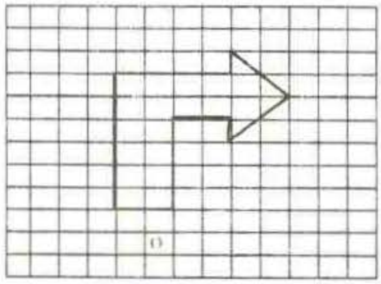
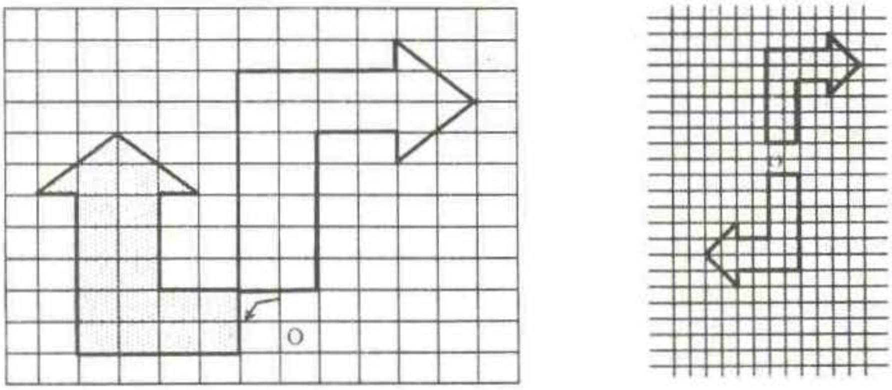


Fig. No. 10.9

Solution :

When the object given in Fig.No .10.9 is rotated about the fixed point 0 through 90^0 and 180^0 in the positive direction , then the images so formed are shown by colouring in Fig.No 10.10 (a) and (b) respectively ,



(a) (b)

Fig.No. 10.10

Now , trace the shape in Fig.No 10.9 on a thin piece of paper and rotate the figure about the point 0 and verify Example 5.

Example 6

Draw the image formed after rotating each of the given shapes about the given point through a half turn or 180^0.

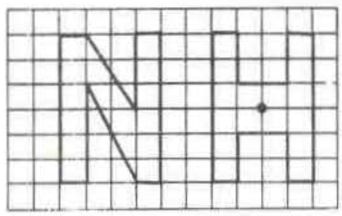


Fig.No. 10.11

Solution :

When the given shapes are rotated about the given point through 180^0 (half turn), the position of the shape is as in Fig. 10.11.

Exercise -10.2.2

1. **The minute hand of a clock completes one rotation in each hour. What parts of a complete rotation will it make for the following times ?**

   (a) 2 hours (b) $\frac{1}{2}$ hour (c) 15 minutes (d) 45 minutes .

2. **What time is represented by the following rotations of the hands of a clock?**
   (a) Quarter turn of the minute hand
   (b) One complete rotation of the hour hand
   (c) Quarter turn of the hour hand
   (d) Half turn of the second hand .

3. **The wheel which is used to make clay pots turn 45 times in a minute, how many times will it turn in the following times ?**
   a) Half minute b) Ten minutes
   c) Four seconds d) One second

10.3 Function Machine and Relation Between the Variables X and Y

Sheela has gone with her brother Shiva to see the flour grinding machine. Sheela and Shiva saw a machine there. When any food grain is put in part of the machine, One flour comes out in another part of the machine. Here, the function of a machine can be expressed as follows.

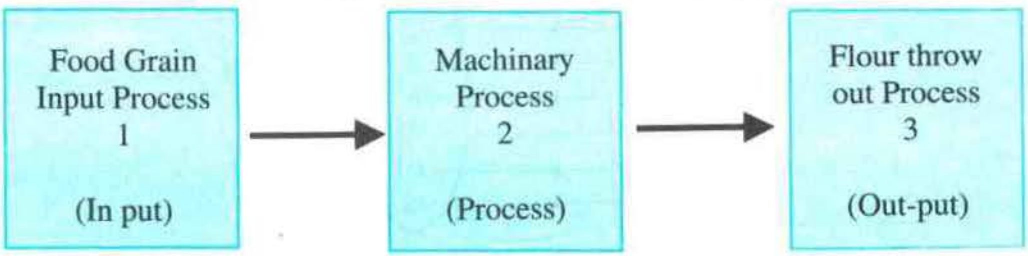


Fig. No. 10.12

When they returned, they wondered about, 'how does the machine work?' Any food grain put in the machine the 'out-put' is always flour. How is it possible ? Discussing in this way they reached home. Shiva thought about it the whole night and told Sheela about it the next morning. Sheela, we can also make a similar kind of machine in mathematics. This kind of machine is called a function machine. Shiva showed to Sheela the machine he had made.

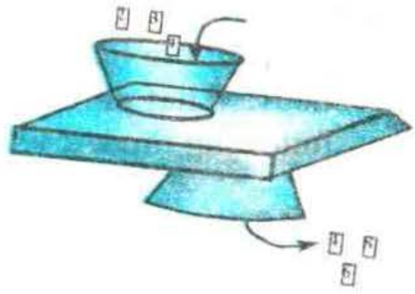


Fig.No. 10.13

Sheela inserted cards numbered 2, 3, 4 in the machine and those cards came out with the numbers of 4, 6, 8 printed on them. Sheela was surprised. To explain how the function machine works, Shiva made a table like this.

Input numbers in machine	0	1	2	3	4	5	6
Output numbers from machine	0	2	4	6	8	10	12

On the basis of the above table, Shiva told Sheela, "Here the machine multiplies any number multiplied by 2 and sends the returned as out put. To make it still clear, the members of set of number put through the machine is called X set and output number from the machine is set y and the relation between the elements of two sets can be shown by making an arrow diagram .

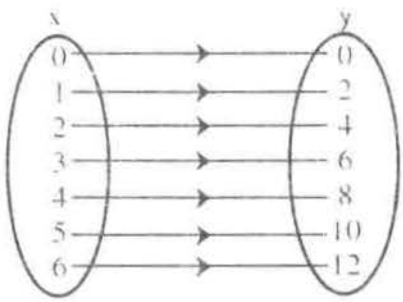


Fig No -10.14

Rule - Multiply by 2

Here, the rule for the arrow diagram made by Shiva and the rule for the function machine is the same. The elements of set X is multiplied by 2 then the product is the elements of set Y. Here, any number can be included in set X. The elements of set Y are depended on the number which are taken from set X. Therefore the elements of set X are independent variables and the elements of set Y are dependent variables. The relation between these variables can also be shown on graph paper

Input numbers in machine	0	1	2	3	4	5	6
Output numbers from machine	0	2	4	6	8	10	12

A graph is drawn by taking the elements of set X along the X-axis and the elements of sets Y along the Y-axis, then the relation between x and y is expressed as a straight line in the graph.

Here, in the mathematical statement, the relation between X and Y can be written as $y = 2x$.

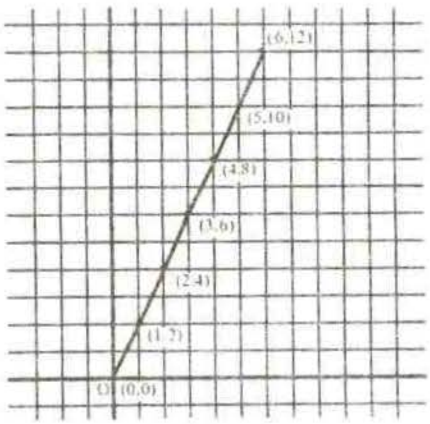


Fig.No. 10.15

Example 1

In the figure, a function machine is shown. The function of the machine is such that it adds 2 to any numbers put into it .

a) Write down the numbers in the output in the form of a table when the numbers from 1 to 6 are put into the machine.

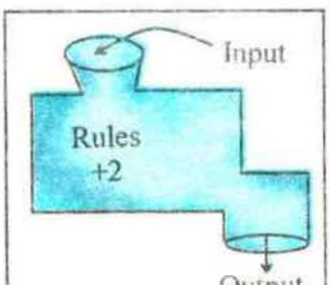


Fig.No. 10.16

b) Write down the relation between the numbers x which are input numbers in machine and y which are output numbers from the machine.

c) Draw an arrow diagram to show the relation between the numbers x and y

d) Show the relation between the numbers x and y graphically

Solution :

a)

Input numbers in machine	0	1	2	3	4	5	6
Output numbers from machine	0	2	4	6	8	10	12

b) $y = x + 2$

c)

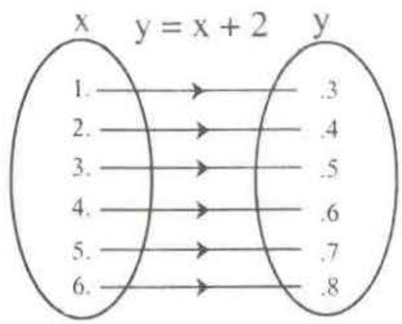


Fig.No. 10.17

d)

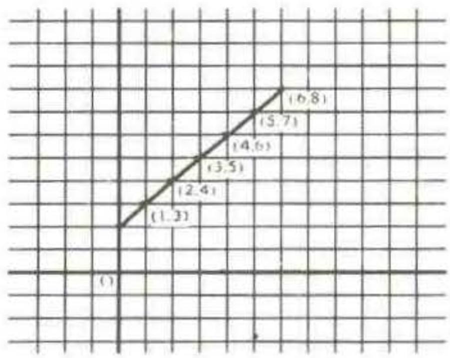
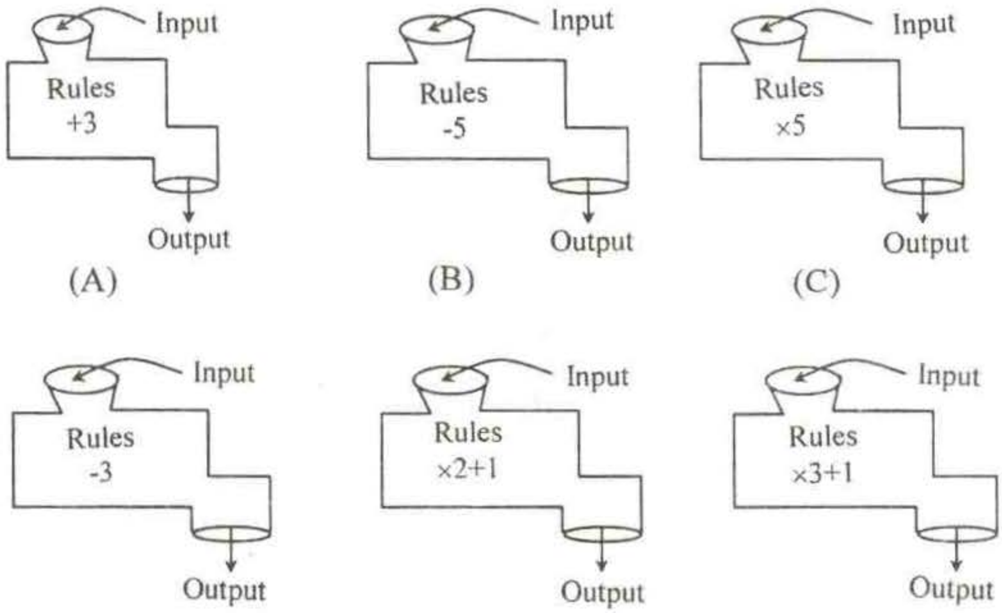


Fig.No. 10.18

Exercise 10.3

1. Express the out- put by making a table when the set of numbers form 0 to 10 are put into each of the following function machines.



2. For each of the function machines in Q.NO .1,write down the relation between x which represents input numbers and y which represents output numbers in a mathematical statement

3. Draw an arrow diagram to represent the relation of Q .No.2

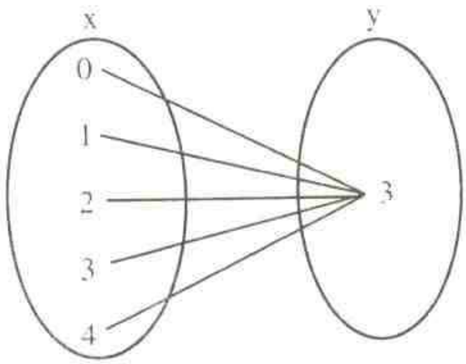
4. Express the relation of Q .No 2 by drawing a graph.

5. x denotes the number of pigeons and y denotes the number of legs of pigeons in the given table.

x	1	2	3	4	5	6
y	2	4	6	8	10	12

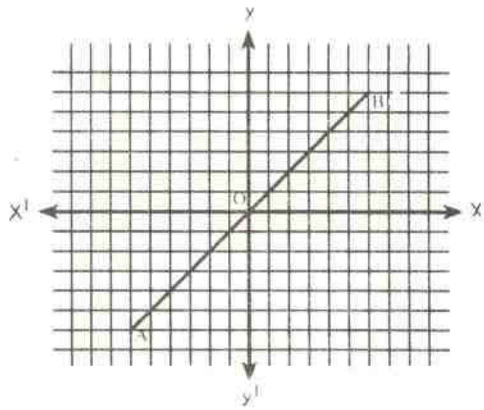
a) Write down this relation in a mathematical statement.
b) Draw an arrow diagram to represent this relation.
c) Draw a graph to represent this relation.

6. The relation between the variables X and Y is shown in the following figure.

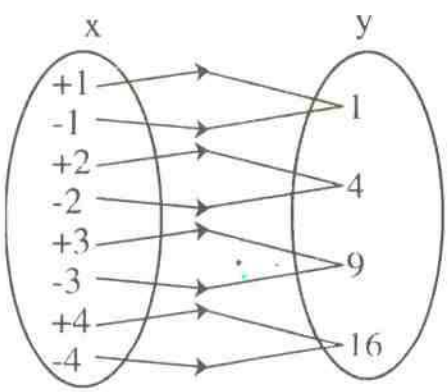


Draw the graph to represent this relation.

7. In the graph alongside the relation between the variables X and Y is represented by a straight line AB. Write this relation in mathematical statement.



8. Express the relation between X and Y represented by an arrow diagram in the mathematical sentence.



11 Geometrical Shapes and Measurement

11.1. Pair of Angles

11.1.1. Vertically Opposite Angles

In Figure No. 11.1 the straight lines AB and CD interest at O. Can you say? how many angles are formed.

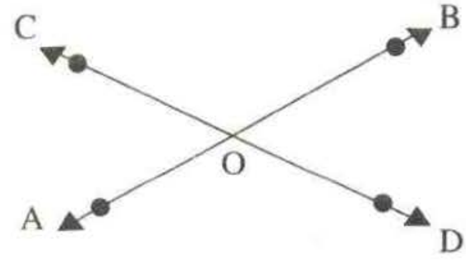


Fig.No. 11.1

The straight lines AB and CD intersect at O. The angles AOC and BOD so formed are called vertically opposite angles. Similarly, the angles COB and AOD are also vertically opposite angles.

Verification of Vertically Opposite Angles

Experiment-1:

Take a piece of rectangular paper and fold it as shown in the figure.

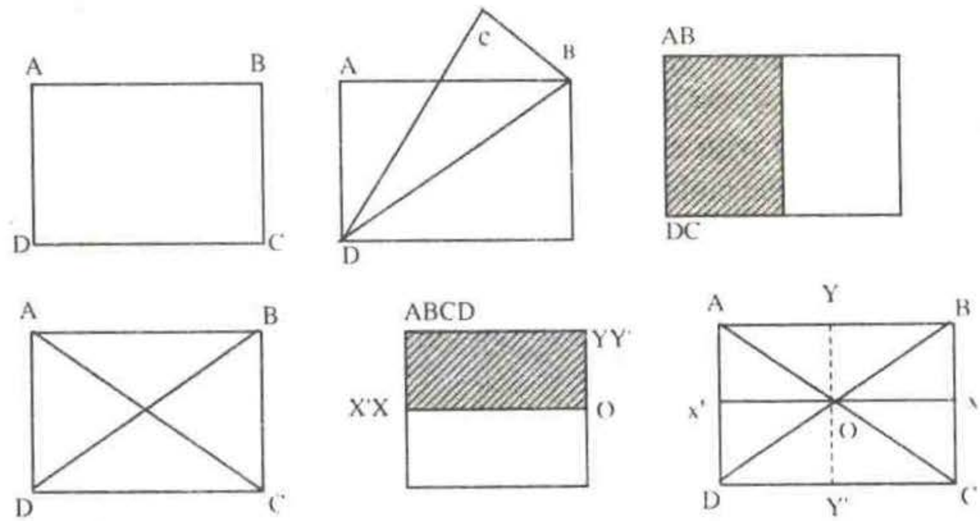


Fig.No. 11.2

By folding the rectangle ABCD of Fig. 11.2 along YOY_1,

$O \rightarrow O$

$B \rightarrow A$ and $C \rightarrow D$

$OB \rightarrow OA$ and $OC \rightarrow OD$

Therefore $\angle BOC \rightarrow \angle AOD$

or, $\angle BOC = \angle AOD$

Similarly, folding rectangle ABCD along XOX_1,

$\angle AOB = \angle DOC$ can be shown.

Conclusion

Vertically opposite angles formed by two intersecting straight lines are equal

Experiment : 2

Draw two line segments AB and CD intersecting at O in your exercise copy. Measure the vertically opposite angles AOD and COB; AOC and BOD with a protractor and record them in the table given below.

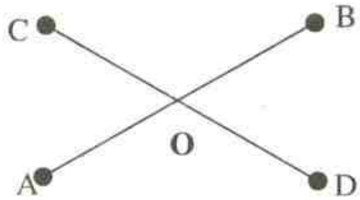


Fig.No. 11.3

Angle	$\angle AOD$	$\angle COB$	$\angle AOC$	$\angle BOD$
Measurement				

Conclusion

Vertically opposite angles formed by two intersecting straight lines or line segments are equal.

11.1.2. Adjacent Angles

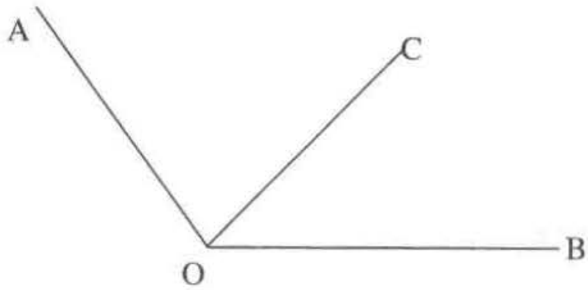


Fig.No. 11.4

In Figure 11.4, there are three line segments at O. ∠AOC and ∠COB are formed. O and OC are the common vertex and the side of ∠AOC and ∠BOC respectively. Therefore, angles having a common vertex and a common side are called adjacent angles.

Verification of Adjacent Angles :

Experiment No. 1

At any point O on a straight line AB draw a line OC. Measure the adjacent angles AOC and COB with a protractor and record them in the table given below.

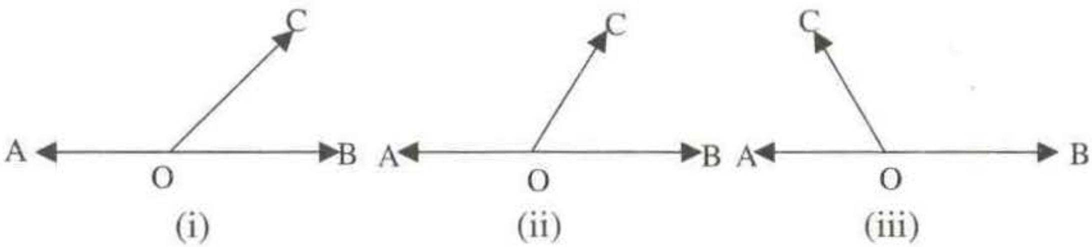


Fig.No. 11.5

Angle / Fig.	∠AOC	∠COB	∠AOC+∠COB
(i)			
(ii)			
(iii)			

Conclusion

1] The sum of adjacent angles formed on a straight line is equal to 180^0 (two right angles).
2] If the adjacent angles are not formed in a straight line, then their sum is less or more than 180^0.

Experiment No. 2

Take a rectangular piece of paper and go on folding as shown in the figure.

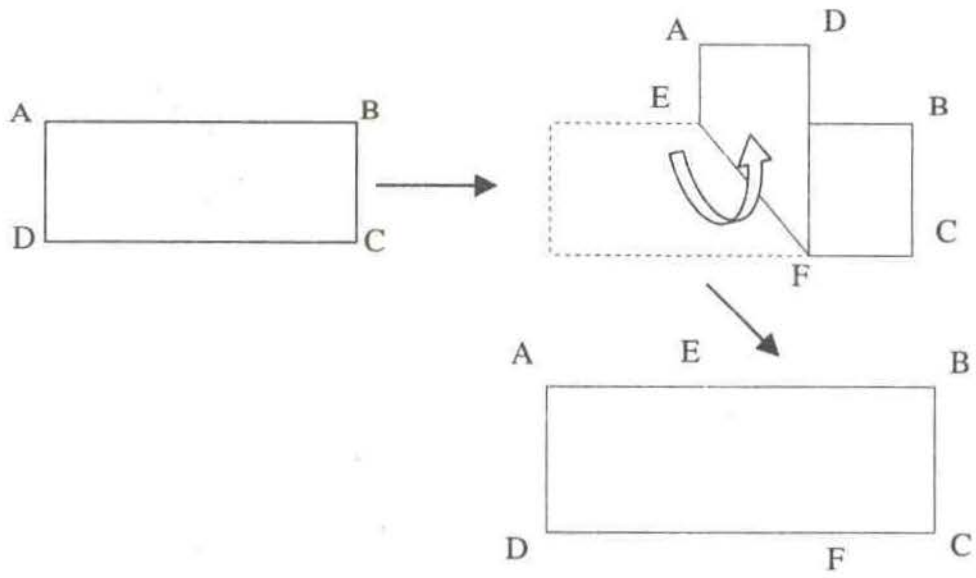


Fig.No. 11.6

By folding rectangle ABCD of figure number 11.6 in EF,

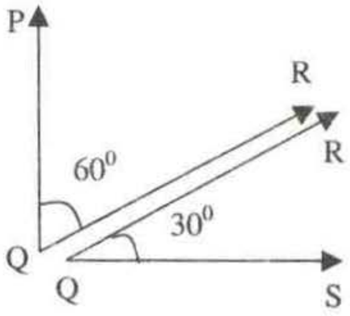
$\angle AEF + \angle FEB = \angle AEB$
$\angle DFE + \angle EFC = \angle DFC$

but $\angle AEB = \angle DFC = 180^0$ (straight angles)

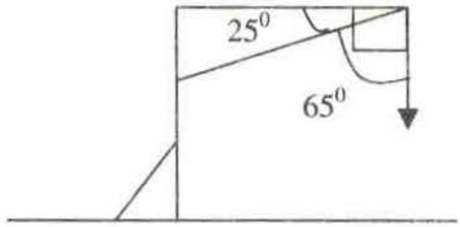
Therefore,

The sum of adjacent angles formed by a line drawn at any point of the straight line is equal to 180^0.

11.1.3. Complementary Angles



(a)



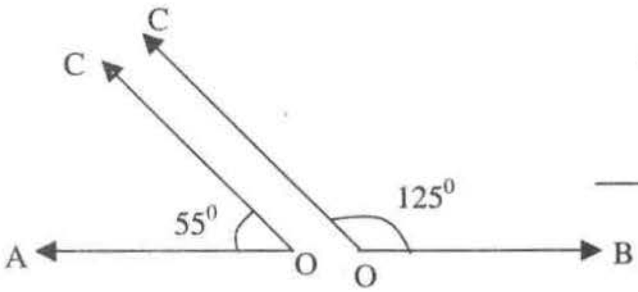
(b)

Fig. No. 11.7

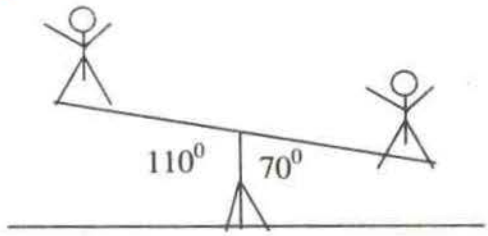
In Fig. No. 11.7, the sum of given angles in each figure is 90^0.

If the sum of two angles are 90^0 (one right angle) then they are complementary angles. Angles 30^0 and 60^0 are complementary to each other.

11.1.4. Supplementary Angles



(a)



(b)

Fig.No. 11.8

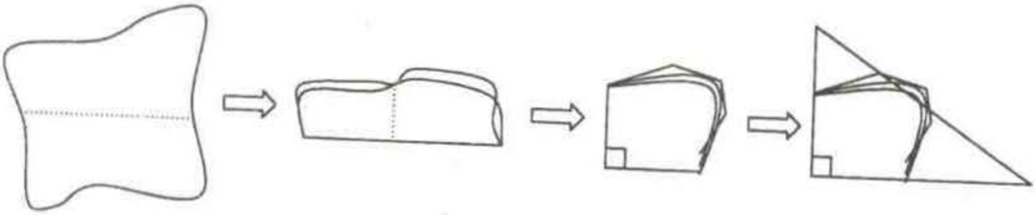
In Fig. No. 11.8, the sum of two angles in each figure (a) and (b) is equal to 180^0.

If the sum of two angles are equal to 180^0, then they are called supplementary angles. Angles 60^0 and 120^0 are supplementary angles.

11.1.5. Verification of Angles at a Point :

Experiment 1

Take a piece of paper. Now fold it as shown in the figure. Measure the angle formed with a set square. Check whether it is a right angle or not.



Now, open the folded paper and observe the formed angles. What is the measurement of each angle? What is their sum ? What did you get from this ? Express in your own words.

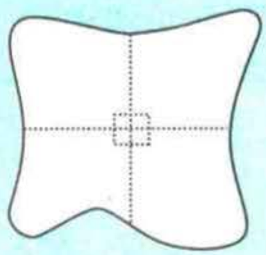


Fig.No. 11.9

The sum of angles formed at a point is equal to four right angles (360^0)

Experiment No. 2

Draw a line AB in your exercise copy. At point O above AB divide the straight angle AOB into 4 parts. Similarly, below AB divide the straight angle AOB into 3 parts. Now, answer the questions given below:

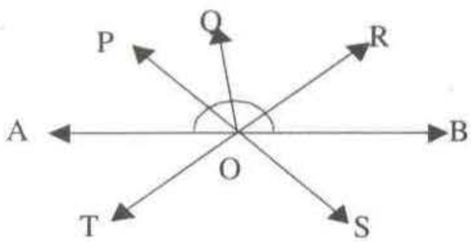


Fig.No. 11.10

a) How many angles are formed at O ?

b) What is the sum of angles AOP, POQ, QOR and ROB ?

c) What is the sum of angles AOT, TOS and SOB ?

d) What is the sum of all angles at O ?

(Answer the question "d" on the basis of answers in "b" and "c")

What is the conclusion of the experiment ?

The sum of angles formed at a point is 360^0

Example 1

What are the complementary and supplementary angles of 50^0 ?

Answer : The complementary angle of the $50^0 = 90^0 - 50^0 = 40^0$

The supplementary angle of the $50^0 = 180^0 - 50^0 = 130^0$

Example 2

In Figure No. 11.11, lines AB and CD intersect at O and form $\angle AOC = 35^0$. What are the values of x, y and z ?

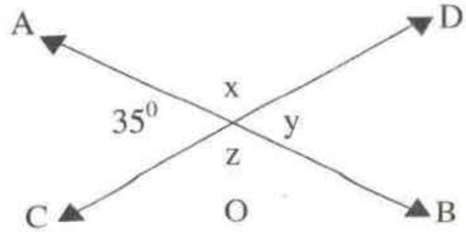


Fig.No. 11.11

Answer:

Here, $x + 35^0 = 180^0$: straight angle

$x = 180^0 - 35^0 = 145^0$

$y = \angle AOC = 35^0$ (vertically opposite angles)

$z = x = 145^0$ (vertically opposite angles)

Example 3

What is the value of y in Fig. 11.12 ?

Answer: $y + 90^0 + 75^0 + 60^0 + 65^0$

$= 360^0$ (Why ?)

or, $y + 290^0 = 360^0$

$\therefore y = 360^0 - 290^0 = 70^0$

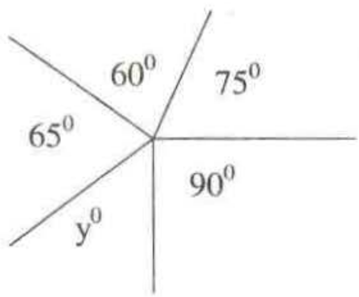


Fig.No. 11.12

Example 4

What is the value of y in Fig. No. 11.13 ?

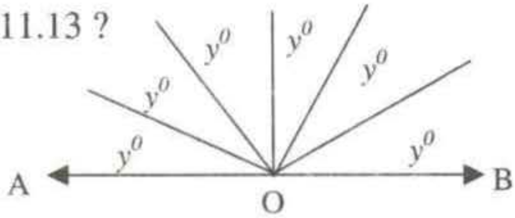


Fig.No. 11.13

Answer: Here, $y + y + y + y + y + y = 180^0$ (Why ?)

or, $6y = 180^0$

$$y = \frac{180^0}{6} = 30^0$$

Exercise 11.1.1.

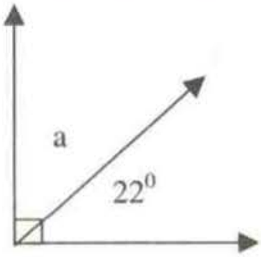
1. Write the measures of the complementary angles of the angles given in the table below.

Angle	10^0	22^0	30^0	35^0	47^0	53^0	60^0	67^0	$72\frac{1}{2}^0$	$83\frac{1}{4}^0$
Complementary Angle										

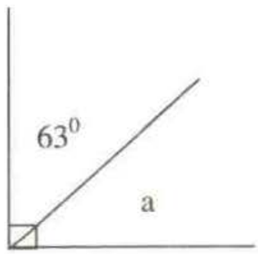
2. Write the measures of the supplementary angles of the angles given in the table below.

Angle	35^0	45^0	50^0	65^0	78^0	85^0	95^0	120^0	$135\frac{1}{2}^0$	$167\frac{1}{2}^0$
Supplementary Angle										

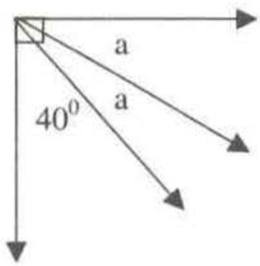
3. What are the values of angles a, b, c, d in each of the figures given below.



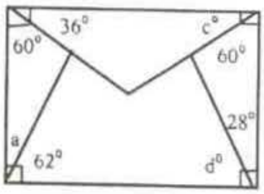
(a)



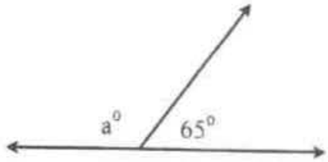
(b)



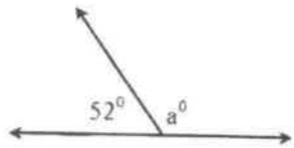
(c)



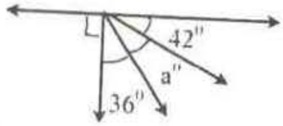
(d)



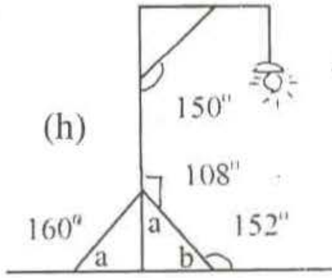
(e)



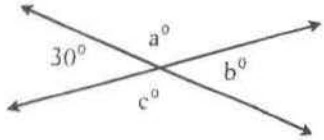
(f)



(g)

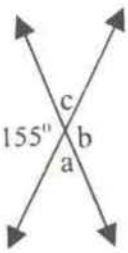


(h)

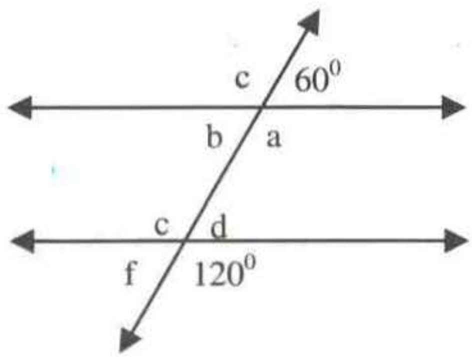


(i)

(j)



4. (a) What are the values of a, b, c, d, e and f in the given figure?
   b) What is the sum of a and d ?
   c) What is the sum of b and c ?
   d) Can we say a and d are supplementary angles ?
   e) Can we say b and c are supplementary angles ?



11.1.6. Angles made by a transversal with the lines

In the figure alongside, EF cuts lines AB and CD at G and H respectively. Here, EF is called a transversal.

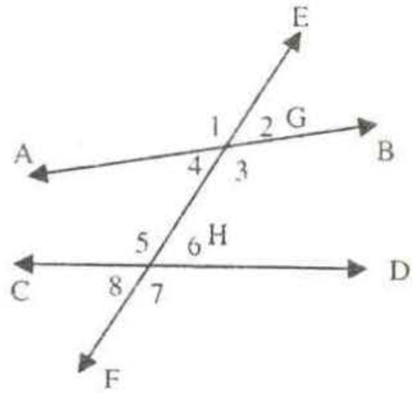


Fig.No. 11.14

a) **Exterior angles :**
Angles 1, 2, 8 and 7 in the given figure are called exterior angles because those angles are formed outside the line segment AB and CD.

b) **Interior angles :**
Angles 3, 4, 5 and 6 in the given figure are called interior angles because those angles are formed inside the line segment AB and CD.

c) **Corresponding Angles :**
Angles 2 and 6 of the given figure or $\angle$EGB and $\angle$GHD lying on the same side of the transversal EF are exterior and interior non-adjacent angles respectively. The angles EGB and GHD are called corresponding angles.

How many similar types of corresponding angles can be written ? Write down.

A pair of an exterior and an interior nonadjacent angles lying at the same side of the transversal are called corresponding angles.

d) **Co-interior angles :**
In the above figure, angles 3 and 6 or $\angle$BGH and $\angle$GHD are interior angles lying on the same side of the transversal. These angles are called co-interior angles. Write down another pair of co-interior angles from the above figure.

When a transversal cuts two line segments, a pair of interior angles lying on the same side of the transversal are called co-interior angles.

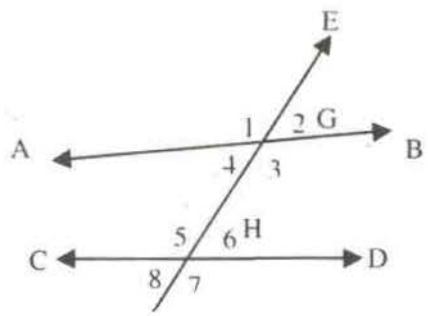
e) **Alternate Angles :**

In the above figure angles 4 and 6 or ∠AGH and ∠GHD are interior non-adjacent angles are called alternate angles. Write down another pair of alternate angles from the above figure.

When a transversal cuts two line segments, a pair of non-adjacent interior angles lying on both sides of the transversal are called alternate angles.

Exercise 11.1.2.

1. **The transversal EF cuts lines AB and CD at G and H respectively and the angles formed are denoted by 1, 2, 3 ....**



a) Write down the numbers and names that denote 4 pairs of corresponding angles.

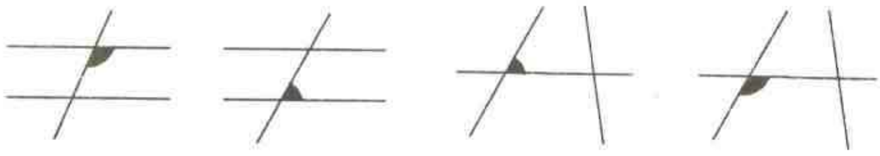
b) Write down the numbers and names that denote two pairs of co-interior angles.

c) Write down the numbers and names that denote two pairs of alternate angles.

2. **Copy the given figures in exercise copy and write angles corresponding to the coloured angles.**

(a) (b) (c) (d)

3. **Copy the given figures in exercise copy and write the co-interior angles of the coloured angles.**



(a) (b) (c) (d)

4. **Copy the figures given in question no. 3 and write the alternate angles of coloured angles.**

11.2. Geometric Construction

11.2.1. Drawing the Perpendicular Bisector of a Line Segment.

The line which bisects the given line segment perpendicularly is called the perpendicular bisector.

Let PQ be a given line segment

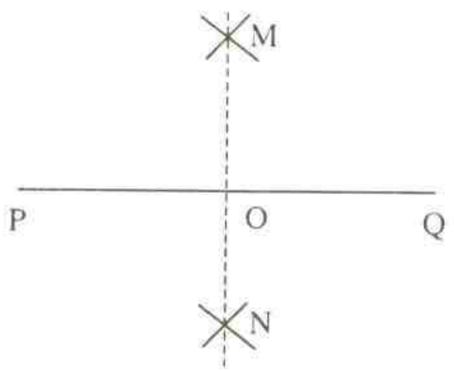


Fig. No. 11.15

Method of Construction :

a) Fixing the needle point of the compass at P, an arc equal to more than half of PQ draw two arcs above and below PQ.

b) Without changing the length of the previous arc and keeping the needle point of the compass fixed at Q draw two arcs above and below PQ to cut the previous arcs at M and N.

c) Join M and N which cuts PQ at O. Measure PO and QO. MN is required perpendicular bisector.

11.2.2. Transferring Angles :

Drawing an angle equal to the given angle at given point on a line is called transferring angle. Let PQR be a given angle. This angle should be constructed so that the vertex Q falls on S.

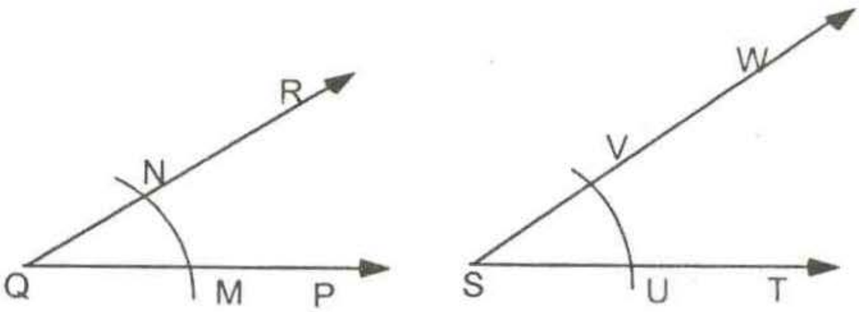


Fig. No. 11.16

Method of Construction :

a) From the point S draw a line ST.

b) Fixing the needle of the compass at Q; draw an arc of any measurement cutting QP and QR respectively at M and N. Without changing the measurement of the arc draw an arc to cut ST at U.

c) Keeping the compass needle at M, measure the length of arc MN. Keeping the compass needle fixed at U and without changing the measurement of the arc draw an arc to cut the previous arc at V.

d) Draw a line SW passing through S and V.
In this way $\angle TSW$ is constructed equal to $\angle PQR$.

11.2.3. Constructing Angles with Given Measures

(i) Method of constructing an angle of 15^0.

a) Draw a line PQ

b) Construct an angle 60^0 at P.

c) Bisect $\angle QPR$. Name the bisector SP. $\angle SPQ = 30^0$. Check it by measuring.

d) Again, bisect $\angle SPQ$ and name the bisector PT.

e) Measure the angle QPT, what is its measure ?

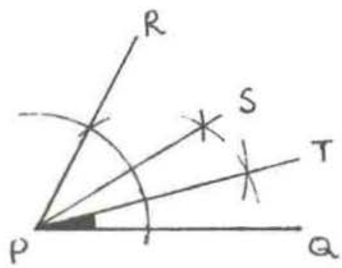


Fig. No. 11.17

ii) Method of Constructing a 45^0 angle.

a) Draw a line PQ.

b) At point P, construct a 90^0 angle and name it as $\angle QPR$.

c) Bisect $\angle RPQ$ and name the bisector SP.

d) $\angle SPQ = \angle PPS = 45^0$

Check it by measuring

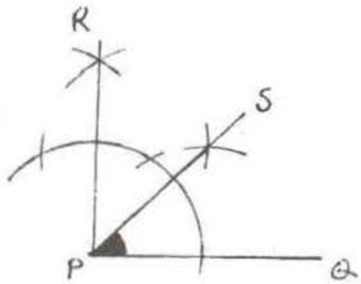
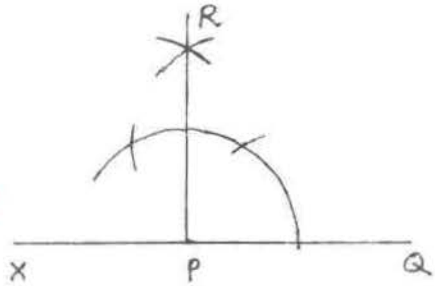


Fig.No. 11.18

iii) Method of constructing an angle of 135^0

a) Draw a line XPQ.

b) At P, construct an angle of 90^0. Name the line RP which makes a 90^0 angle with PQ. (135^0 is equal to the sum of 90^0 and 45^0)



That is why 45^0 or the half of 90^0 should be added to 90^0. If the 45^0 angle is constructed on a straight line then its adjacent angle becomes 135^0, because 45^0 and 135^0 are supplementary angles)

c)

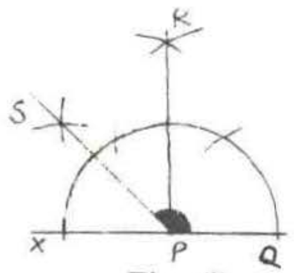


Fig. (i)

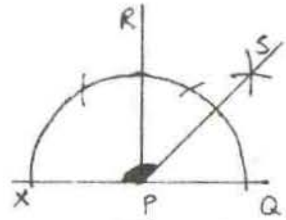


Fig. (ii)

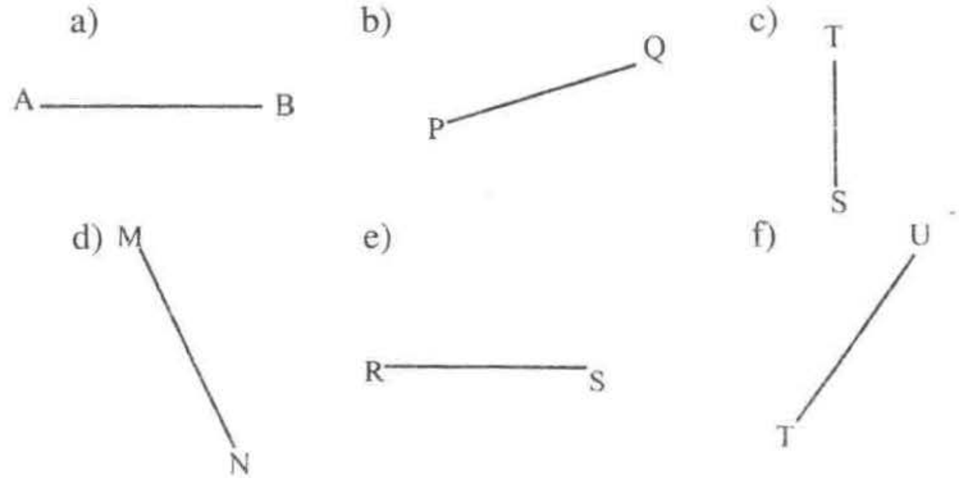
Fig.No. 11.19

In the given construction, 90^0 and 45^0 are added to make $\angle SPQ = 135^0$ in the first figure.

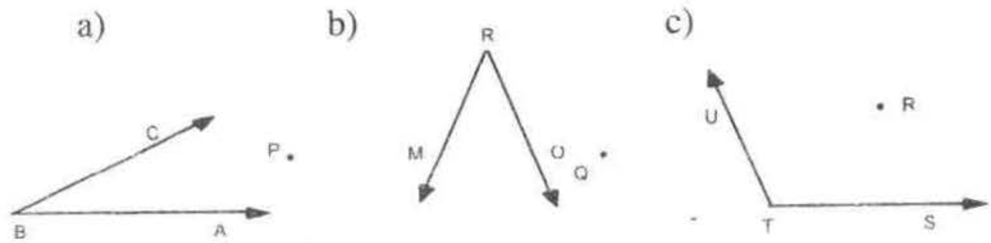
In the second figure $\angle SPQ$ is constructed equal to 45^0. So its adjacent $\angle XPS$ becomes 135^0.

Exercise 11.2

1. Copy the given line segments and draw the perpendicular bisectors in each case.

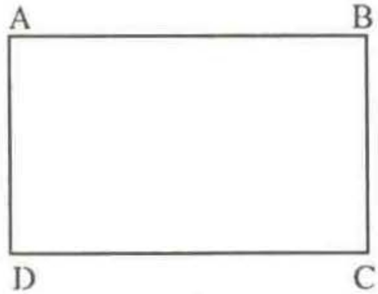


2. Transfer the given angle to the given point.

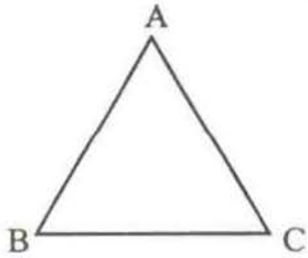


3. Draw any three angles. Bisect each.

4. a) Draw a rectangle ABCD. Draw the perpendicular bisectors of sides AB and CB.
   b) Do the perpendicular bisectors of AB and BC also bisect CD and AD ? Measure them.



5. a) Draw the perpendicular bisectors of sides AB, BC and CA in ΔABC.
   b) Do all these perpendicular bisectors intersect at a point ?



6. Construct angles of given measurements.

a) 30^0 b) 45^0 c) 60^0
d) 90^0 e) 120^0 f) 135^0

7. Construct angles of 60^0 at each end point of the given line segment AB. What figure is formed ?

A ———————————— B

8. Construct as indicated different angles at the end points of the line segment AB of Q.No.7.

a) 60^0 at A, 135^0 at B b) 45^0 at A, 30^0 at B
c) 90^0 at A, 60^0 at B.

11.3.Construction of Triangles, Parallelograms, Rectangles, Rhombus and Kites.

11.3.1. Construction of Triangle

A triangle can be constructed under the following conditions.

a) If the measurements of three sides are given

Example 1

Construct ΔABC in which AB = 4 cm. BC = 5cm and AC = 6cm

Answer :

Method :

a) Draw a line segment AB equal to 4 cm. Draw an arc equal to 6 cm from A and another arc equal to 5 cm from B. They cut each other at C. Join A and C; B and C. The required ΔABC is constructed.

b) If the measurement by two sides and the angle formed of them are given.

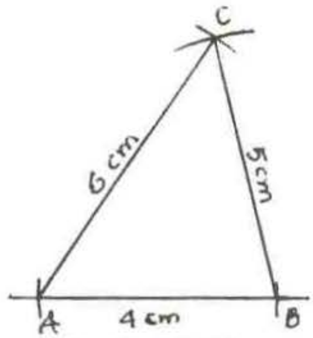


Fig.No. 11.20

Example 2

Construct ΔABC in which AB = 6cm, AC = 5cm and $\angle A = 60^0$

Answer :

Method

Draw a line segment AB of length 6 cm. Construct $\angle BAC = 60^0$ at A. Take AC = 5cm. Join B and C. The required ΔABC is constructed.

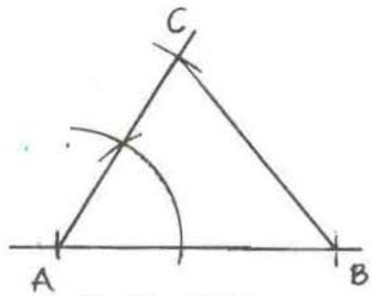


Fig.No. 11.21

If the measurement of a side and the angles formed on this side are given.

Example 3

Construct ΔABC in which $\angle A = 30^0$, $\angle B = 45^0$ and AB = 7cm.

Answer :

Draw a line segment AB = 7cm. Construct a 30^0 angle at A and a 45^0 angle at B. AC and BC cut each other at C. The required ΔABC is constructed.

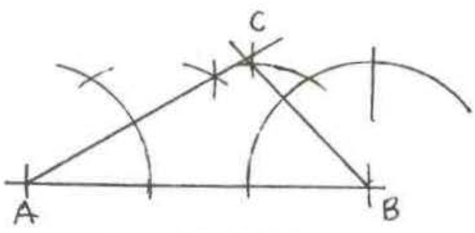


Fig. No. 11.22

Exercise 11.3.1

1. **Construct ΔPQR from the data given below.**

   a) $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$cm

   b) $PQ = 6$ cm, $QR = 5$ cm and $RP = 10$ cm

   c) $PQ = 7$ cm, $QR = 5$ cm and $RP = 8$ cm

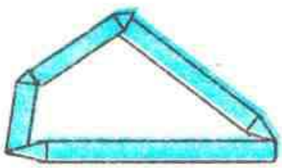
2. **Construct ΔABC from the given data.**

   a) $AB = 5$cm, $BC = 6$ cm and $\angle B = 45^0$

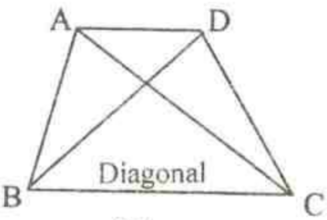
   b) $AB = AC = 6.5$cm and $\angle A = 120^0$

   c) $AC = 4.5$cm, $BC = 3.5$ cm and $\angle C = 15^0$

11.3.2. Quadrilaterals :



(a)



(b)

Fig. No. 11.23

When the points of four pencils are joined in order as shown in Fig. No. 11.23 (a) a closed shape is formed. This is called a quadrilateral. In quadrilateral ABCD the 4 vertices are A, B, C, D, and the 4 sides are AB, BC, CD and AD. [Fig. No. 11.23 (b)]

The sum of the 4 interior angles of a quadrilateral is equal to 4 right angles (i.e. 360^0). The line joining the opposite vertices of a quadrilateral is called a diagonal. A quadrilateral has 2 diagonals. [Fig. 11.23 (b)].

Types of Quadrilaterals

The teacher gave pencils of different sizes to Ramesh, Anju and Sudip and asked them to make all possible types of quadrilaterals. Ramesh, Anju and Sudip made the following type of quadrilaterals.

Quadrilateral made by Anju ***Quadrilateral made by Ramesh***

Quadrilateral made by Sudip

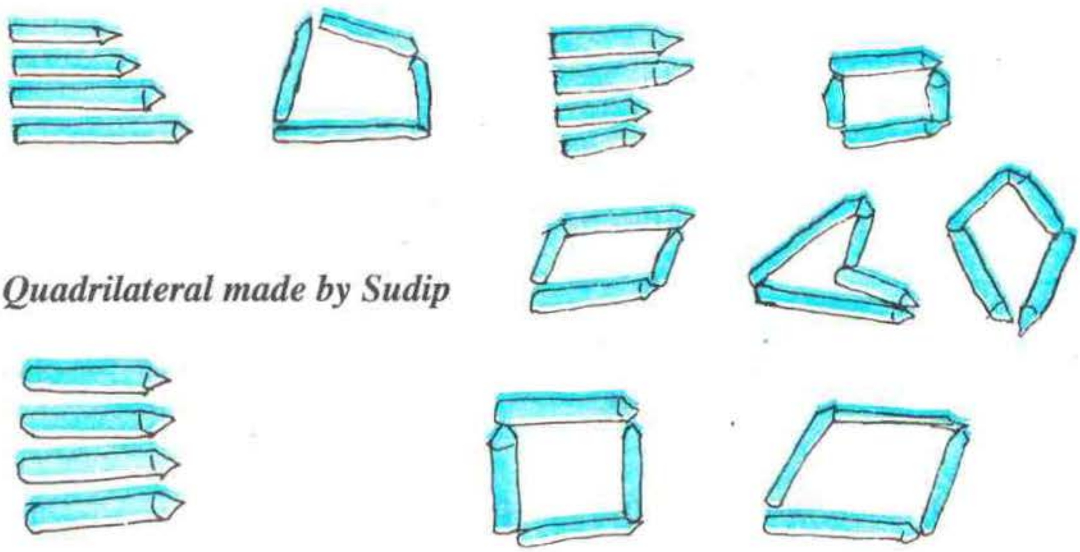
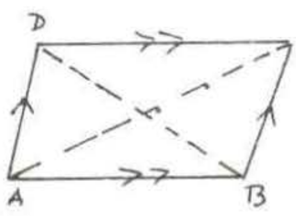


Fig. No. 11.24

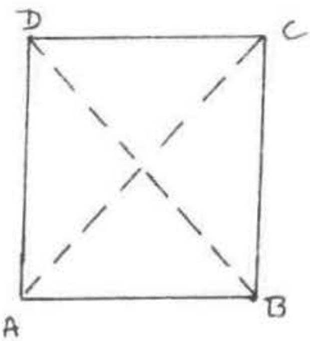
The teacher classified the quadrilaterals made by Ramesh, Anju and Sudip as follows.

1. Parallelograms

A



B



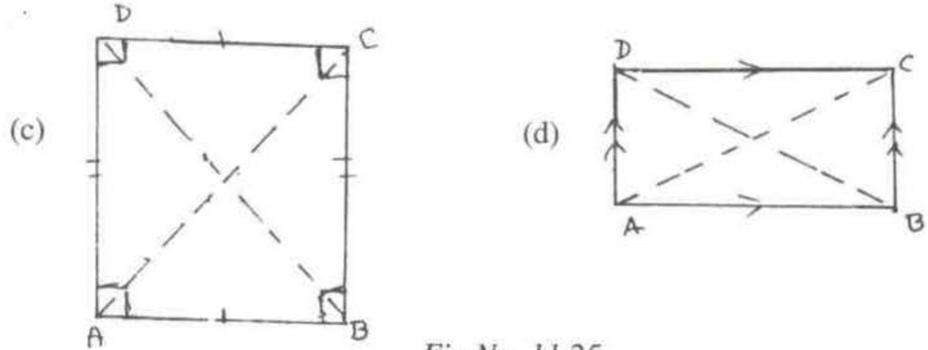


Fig.No. 11.25

A quadrilateral having the opposite sides parallel is called a parallelogram. The properties of a parallelogram are as follows :

1. The opposite angles and sides are equal.
2. Diagonals bisect each other.

Verification of the properties of a parallelogram

Experiment No. 1

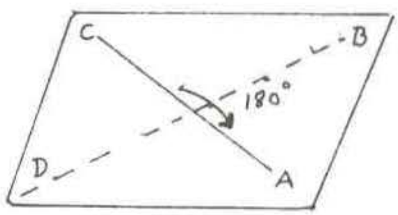


Fig.No. 11.26

Trace the parallelogram shown in Figure 11.26 on a transparent-sheet. Put the tip of the pencil at point O, the point of intersection of the diagonals, rotate the parallelogram through at 180^0. Observe what happens.
Rotating the parallelogram ABCD around O through 180^0,

$$D \rightarrow B \text{ and } A \rightarrow C$$

Therefore, $DA \rightarrow BC$ or $DA = BC$

Again, $A \rightarrow C$ and $B \rightarrow D$

Therefore, $AB \rightarrow CD$ or $AB = CD$

The opposite sides of a parallelogram are equal

Rotating the parallelogram ABCD through 180^0 around O.

$$\angle A \rightarrow \angle C \quad \text{or} \quad \angle A = \angle C$$

$$\text{and } \angle B \rightarrow \angle D \text{ or } \angle B = \angle D$$

The opposite angles of a parallelogram are equal

Now, rotating parallelogram ABCD at 180^0 around O.

$$O \rightarrow O \text{ and } A \rightarrow C$$

Therefore, $OA \rightarrow OC$ or $OA = OC$

Again, $O \rightarrow O$ and $B \rightarrow D$

Therefore $OB \rightarrow OD$ or $OB = OD$

The diagonals of a parallelogram bisect each other.

Construction of Parallelogram

a) If the measurements of the adjacent sides and angle between them are given.

Example 1

Construct a parallelogram in which the adjacent sides are 6.5 cm and 4.6 cm and angle between them is 60^0.

Answer :

Step 1

- Draw a line AX,

- Construct 60^0 angle at A, i.e. $\angle XAY = 60^0$

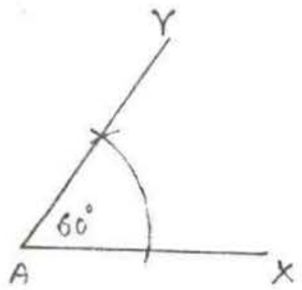


Fig. No. 11.27

Step 2

Draw an arc AD equal to 4.6 cm on AY and another arc AB equal to 6.5 cm on AX.

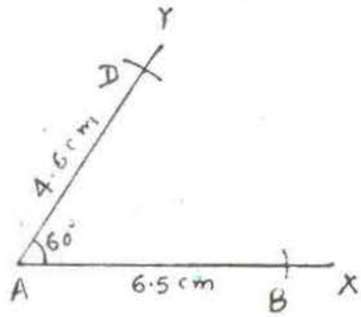


Fig. No. 11.28

Step 3

Draw an arc equal to 4.6 cm from B and another arc 6.5 cm from D. These arcs cut at C. Join DC and BC.

The required parallelogram is ABCD.

b) If the base, a diagonal and angle between them are given.

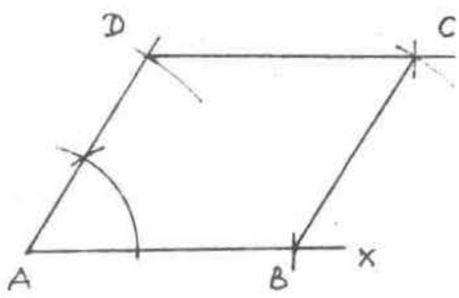


Fig. No. 11.29

Example 2

Base $= 5$ cm.
Diagonal $= 6$ cm

The angle between the base and diagonal is 30^0. Construct a parallelogram having above measurements.

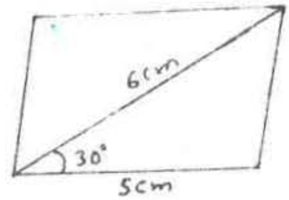


Fig. No. 11.30

Answer :

Step 1

- Draw a straight line AX
- Construct an angle 30^0 at A i.e. $\angle XAY = 30^0$

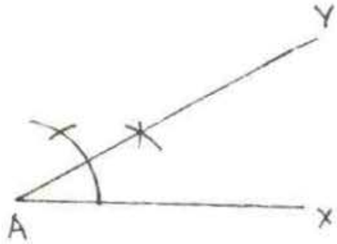


Fig. No. 11.31

Step 2

Take AB = 5cm on AX
AC = 6 cm on AY. Join B and C.

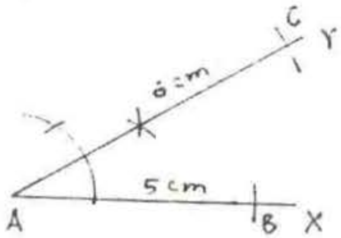


Fig. No. 11.32

Step 3

From C, draw an arc equal to AB (=5cm) and from A draw another arc equal to BC. These two arcs cut each other at D. Joint D and C and D and A. The required parallelogram is ABCD.

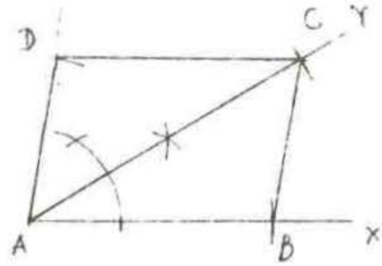


Fig. No. 11.33

c) If the length of diagonals and the angle between them are given.

Example 3

The lengths of diagonals are 5.4 cm and 6 cm. The angle between them is 30^0.

Construct a parallelogram having above measurements.

Answer :

Step 1

Draw a line AC = 5.4 cm. Find O the mid point of AC by drawing the perpendicular bisector of AC.

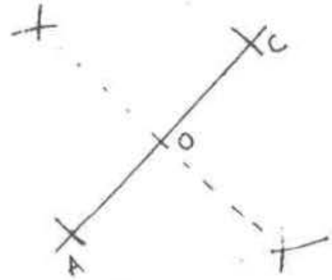


Fig. No. 11.34

Step 2

At O on OC draw a line XOX^1 so that the 30^0 angle is formed at O.

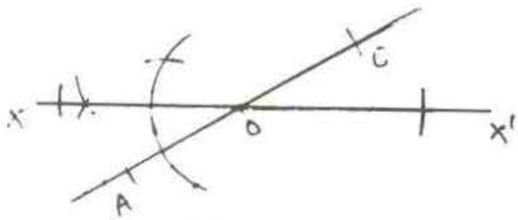


Fig. No. 11.35

Step 3

With the help of a compass, cut off OB = OD = 3cm by fixing the tip of the compass at O, so that B is on OX^1 and D is on OX. Join A, B, C and D serially. The required parallelogram is ABCD.

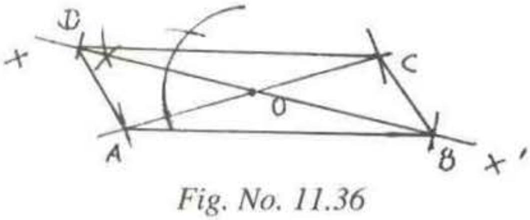


Fig. No. 11.36

Exercise 11.3.2

1. **Construct a parallelogram ABCD from the following data.**
   a) AB = 5 cm AD = 4 cm and $\angle A = 30^0$
   b) AB = 3.52 cmAD = 5.5 cm and $\angle A = 75^0$
   c) AB = 6 cm AD = 4.2 cm and $\angle A = 60^0$
   d) AB = 5.4 cm AD = 3.2 cm and $\angle A = 45^0$

e) AB = 6.3 cm AD = 4.8 cm and $\angle A = 120^0$

2. **Construct parallelograms ABCD from the following data.**

	Base	Diagonal	Angle between base and diagonal
a	5 cm	7 cm	45^0
b	3.7 cm	4.5 cm	15^0
c	4 cm	3 cm	60^0
d	6 cm	7 cm	30^0
e	4.5 cm	6 cm	15^0

3. **Construct parallelograms ABCD from the following data.**

Length of diagonal	*Angle between diagonals*
a) 5 cm, 6 cm	75^0
b) 4 cm, 9 cm	60^0
c) 8.5 cm 7.5 cm	45^0
d) 6.5 cm 5.8 cm	120^0
e) 9 cm 8.6 cm	150^0

2. Verification and Construction of Rectangle

Definition of a rectangle : If all the angles of a parallelogram are 90^0, then it is called a rectangle. The rectangle has all the properties of a parallelogram and its diagonals are equal.

1. Verification of the opposite sides of a rectangle:

Experiment 1

Take a rectangular piece of paper and go on folding it as shown in the figure.

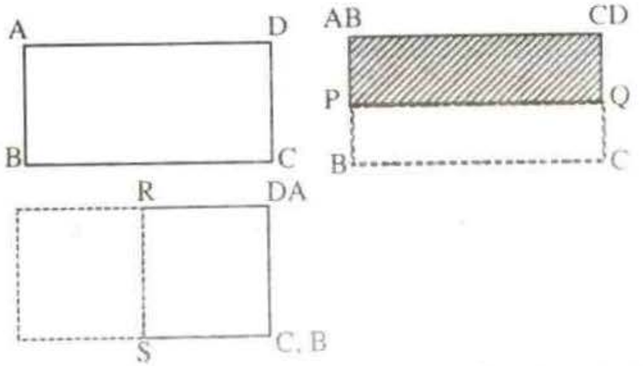


Fig. No. 11.37

By folding rectangle ABCD along PQ, B $\rightarrow$A and C $\rightarrow$ D Therefore BC $\rightarrow$AD or BC = AD

By folding rectangle ABCD along RS A $\rightarrow$ D and B $\rightarrow$ C

Therefore AB $\rightarrow$ DC or AB = DC

The opposite sides of a rectangle are equal

2. Verification of the diagonals a rectangle:

Experiment 2

In Fig. 11.38, diagonals AC and BD of rectangle ABCD intersect at O. Trace the rectangle ABCD in a tracing paper and rotate the figure through 180^0.

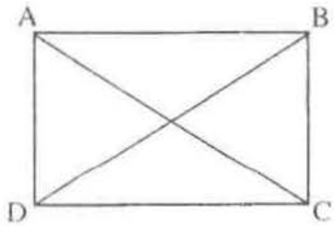


Fig. No. 11.38

Then O $\rightarrow$ O and A $\rightarrow$ C
Therefore OA $\rightarrow$ OC or OA = OC
Again O $\rightarrow$ O and B $\rightarrow$ D
Therefore OB $\rightarrow$ OD or OB = OD

Experiment 3

Measure the segments of the diagonals of the rectangle given in Fig. 11.38 with the help of a scale and record the measurement in table given below. Make a conclusion.

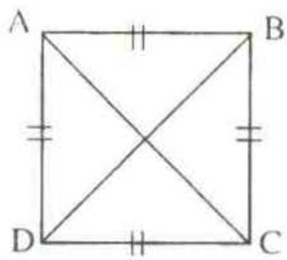
Segments of diagonals	OA	OC	OB	OD	AC	BD
Measurement						

a) The diagonals of rectangle bisect each other.

b) The diagonals of a rectangle are equal.

3. Square :

A rectangle having adjacent sides equal is called a square. It possesses all the properties of a rectangle.



The diagonals of square bisect each other at 90^0

Construction of Rectangle :

Example 1

Construct a rectangle ABCD in which AB = 5 cm., and BC = 3 cm.

Methods :

Draw a line AB = 5 cm. construct 90^0 angles at A and B using a compass and a ruler. Take BC = 3 cm. and AD = 3 cm. Join D and C. The required rectangle is ABCD.

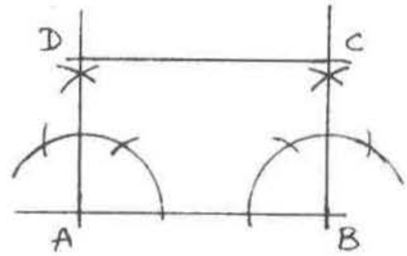


Fig.No. 11.39

Example 2

Construct a rectangle in which the lengths of diagonals are 6 cm. and the angle between them is 60^0.

Solution :

Process :

Take a line segment PR of 6 cm. At O the mid point of PR, construct an angle of 60^0. Cut off OS = OQ = 3 cm. Join P, Q, R, S, The required rectangle is ABCD.

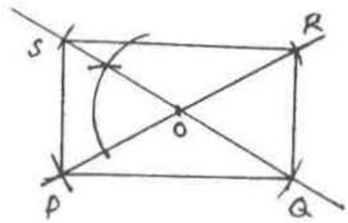


Fig.No. 11.40

Exercise 11.3.3

1. **Using a compass and a ruler construct rectangles with the following measurements.**
   a) Sides 6.5cm and 4cm.
   b) Sides 3.6cm and 2.5cm.
   c) Sides 5.7cm and 4.5cm.

2. Construct rectangles from the following data.

a) Length of diagonals 24cm.
   Angle between diagonals = 30^0.

b) Length of diagonals = 6 cm.
   Angle between diagonals = 45^0.

c) Length of diagonals = 5 cm.
   Angle between diagonals = 60^0

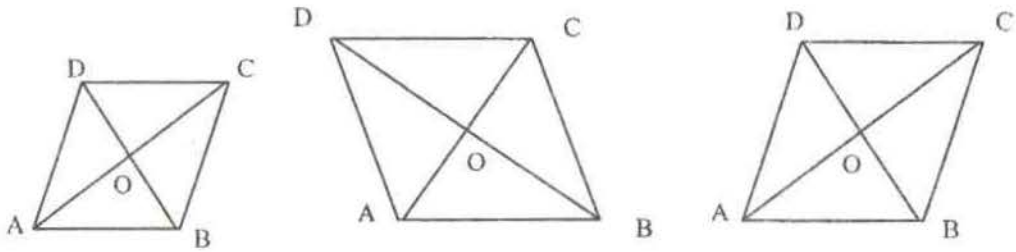
4. Rhombus :

A quadrilateral having the adjacent sides equal is called a rhombus. It has all the properties of a parallelogram.

> The diagonals of a rhombus bisect each other at right angles.
> The diagonals of a rhombus bisect the vertex angle.

Experiment 4

Measure the following parts of the given rhombuses and complete the table.



(a) (b) (c)

Fig.No. 11.41

Fig. No.	AB	BC	CD	DA	AC	BD	OA	OB	OC	OD	∠AOB	∠BOC	∠COD	∠DOA	∠ADO	∠CDO	∠DAO	∠BAO	
a																			
b																			
c																			

Write down the conclusion

Construction of Rhombus :

Construction 1

Construct a rhombus in which sides are 4 cm and the angle between two adjacent sides is 30^0.

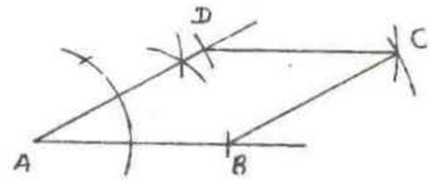


Fig.No. 11.42

Process :

Draw a line segment AB = 4 cm. Using a compass and a ruler construct an angle 30^0 From A cut off AD = 4 cm. as shown. From B and D draw arcs equal to 4 cm. which cut at C. Join B and C; D and C. The required rhombus is ABCD.

Construction 2

Construct a rhombus of diagonals 5 cm and 4 cm.

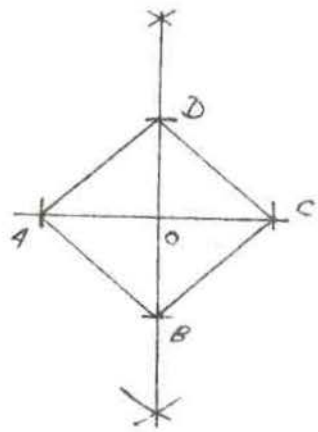


Fig. No. 11.43

Process :

Draw a diagonal AC = 5 cm. Draw the perpendicular bisector of AC and name the mid-point O. The half of another diagonal $^{4\text{ cm}}/_2$= 2 cm. Cutoff OB = OD=2 cm from O on the perpendicular bisector. Join A, B, C, and D. The required rhombus is ABCD.

5. Kite :

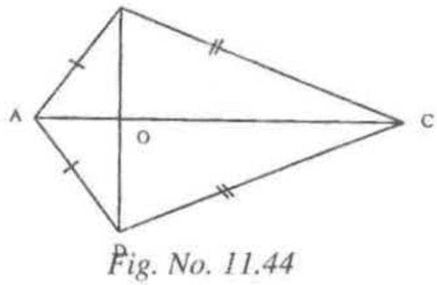


Fig. No. 11.44

A quadrilateral in which the adjacent sides AB and AD, and BC and DC arc equal is called a kite.

Measure the following parts of the kite given above and fill in the table.

AB	AD	BC	CD	AO	CO	BO	DO	∠AOB	∠AOD	∠COD	∠BOC

a) Do the diagonals bisect each other ?

b) Which diagonal bisects which one ?

c) Do the diagonals cut each other perpendicularly ?

Construction of Kite

Construction 1

Construct a kite in which the unequal sides are 3 cm and 5 cm and the angle between them is equal to 120^0.

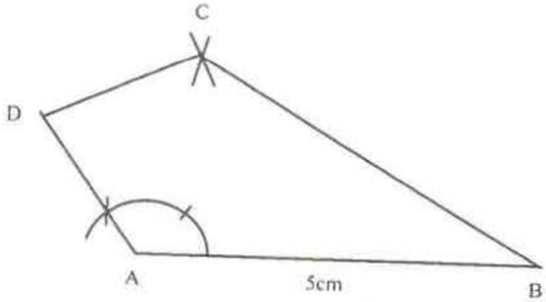


Fig. No. 11.45

Process :

Take AB = 5 cm. At A, construct angle 120^0 using a compass and a ruler. Cut off AD = 3 cm from A and cut off BC = 5 cm. from B and again cut off CD = 3 cm from D. These arcs cut at C. Join B and C; C and D. The required kite is ABCD.

Construction 2

Construct a kite in which unequal sides are 2.5 cm and 6 cm and the angle between them is 90^0.

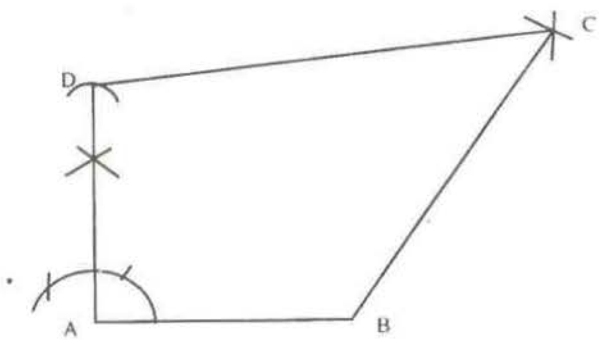


Fig. No. 11.46

Process :

Draw AB = 2.5 cm. Construct a 90^0 angle at A. Using a compass and a ruler. Cutoff AD = 2.5 cm.

Draw an arc equal to 6 cm from D and B, these arcs cut at C. Join B and C; C and D. The required kite is ABCD.

Construction 3

Construct a kite whose the diagonals are 5 cm. and 4 cm. long.

Process :

Draw a line AC = 5 cm.. Take a point O in AC. Construct a 90^0 angle at O. Cut OD = 2 cm. and OB = 2 cm. Joint A, B, C and D. The required kite is ABCD.

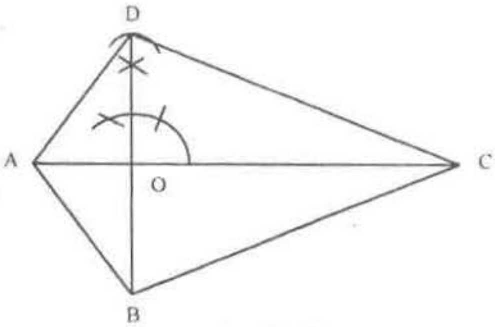


Fig. No. 11.47

Exercise 11.3.4.

1. Using compass and ruler construct rhombuses with the given measurements.

a) Sides = 5.5 cm and angle between the adjacent sides = 45^0

b) Sides = 4.5 cm and angle between the adjacent sides = 60^0

c) Sides = 6.3 cm. and angle between the adjacent sides = 135^0

2. Construct rhombuses with.

a) Diagonals 7 cm and 6 cm.

b) Diagonals 6.5 cm and 5.2 cm.

c) Diagonals 6.2 cm. and 5.4 cm.

3. Construct a kite given.

a) Unequal sides 3.5 cm and 6 cm. and angle between them 120^0

b) Unequal sides 2.8 cm and 9 cm and angle between them 90^0.

c) Unequal sides 3 cm. and 5.5 cm and angle between 3 cm side 120^0.

d) Unequal sides 1.8 cm and 4.8 cm and angle between 4.8 cm 15^0

e) Diagonals 3.6 cm and 4.8 cm.

f) Diagonals 8 cm and 6.2 cm

11.4. Areas of Triangles and Quadrilaterals

11.4.1. Are of Triangle

Activity 1

Study the given figure

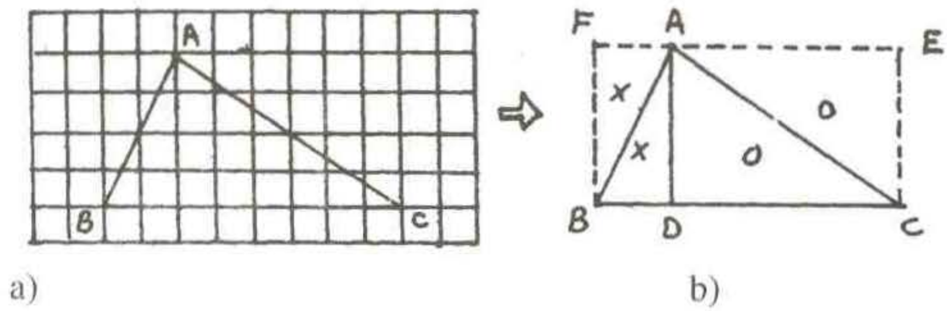


Fig. 11.48

Now answer the following questions :

a) What is the area of ΔABC ? Answer the question by counting the number of squares in Figure No. 11.48 (a)

b) In Figure No. 11.4 (b), how is rectangle BCEF constructed from triangle ABC ?

c) What are the length and the breadth of rectangle BCEF ?

d) What is the area of rectangle BCEF ?

e) What is the relation between the areas of rectangle BCEF and triangle ABC ?

Activity 2

Take a triangular piece of paper. Continue folding and cutting it as shown in the figure.

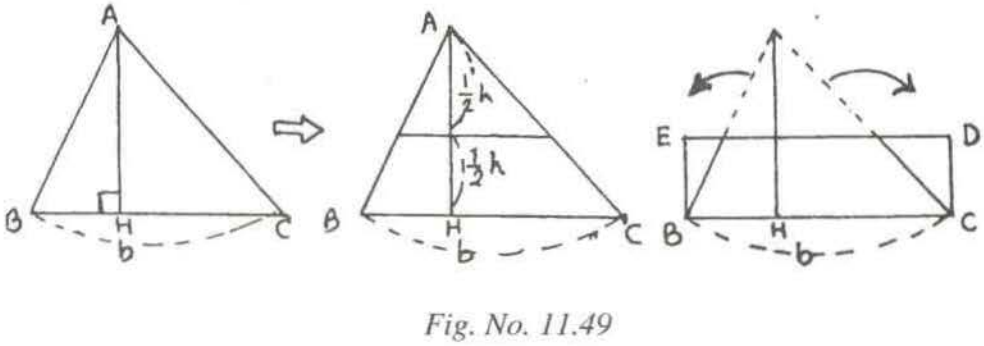


Fig. No. 11.49

Area of ΔABC = ? $\therefore$ Area of $\Delta ABC = \frac{1}{2}$

$\therefore$ Area of ΔABC = Area of rectangle BCDE.

By adjusting the pieces of paper obtained from the piece of triangular paper of Figure No. 11.49, rectangle BCDF is formed. Therefore, the area of ΔABC and rectangle BCDF are equal or

$$\text{Area of } \Delta ABC = BC \times BE \ (A = l \times b) \qquad = BC \times \frac{1}{2} AH$$

$$= b \times \frac{1}{2} h \ (BE = \frac{1}{2} AH) \qquad = \frac{1}{2} \text{ Base} \times \text{Height.}$$

Therefore Area of $\Delta ABC = \frac{1}{2}$ Base $\times$ Height $= \frac{1}{2} \times b \times h$

11.4.2. Area of a parallelogram:

Take a piece of paper having the shape of a parallelogram. Cut it as shown in the figure and adjust the pieces. A rectangle is formed.

The area of a rectangle and area of a parallelogram are equal. From this experiment also, area of a parallelogram = Base × Height. Try yourself.

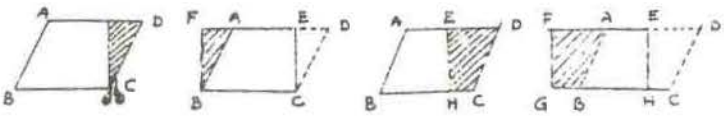


Fig. No. 11.50

Area of parallelogram

A = Base × Height = b×h

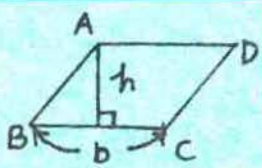


Fig.No. 11.51

Example 1

Calculate the area of a triangle the base and height of which are 32 cm and 12 cm respectively.

Answer :

Base of triangle (b)= 32 cm

Height (h) = 12 cm

Area (A) = ?

Now,

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 32 \text{ cm} \times 12 \text{ cm}$$

$$= 16 \text{ cm} \times 12 \text{ cm} = 192 \text{ cm}^2$$

Example 2

The area of a triangle and the area of a parallelogram are equal. The base and the height of the parallelogram are 32 cm and 16 cm respectively. If base of the triangle is 64 cm., calculate its height.

Here,

Base of the parallelogram (b) = 32 cm

Height (h) = 16 cm

Area (A) = ?

Now, A = b × h

$= 32 \text{ cm} \times 16 \text{ cm} = 512 \text{ cm}^2$

According to the question if the base and the height of the triangle are b_1 and h_1 then,

Area of triangle = Area of parallelogram

or, $\frac{1}{2} \times b_1 h = 512 \text{ cm}^2$ b_1= base h_1 = height b_1= 64 cm

or, $\frac{1}{2} \times 64 \text{ cm} \times h_1 = 512 \text{ cm}^2$

$$\text{or, } h_1 = \frac{512}{32} = 16 \text{ cm}$$

$\therefore$ The required height is 16 cm.

Example 3

In the given figure ABCD, BF = BC. Find the area of the shaded part.

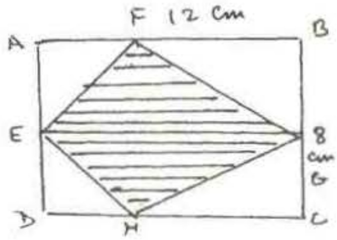


Fig. No. 11.52

Answer

Join EG. Draw HI⊥EG and FJ⊥EG.

Mark the equal parts with the same sign.

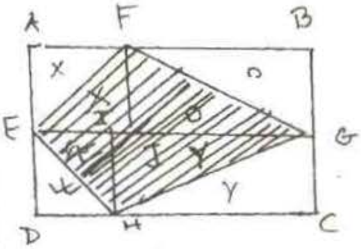


Fig. No. 11.53

In the figure, the area of the shaded part

$$= \frac{1}{2} \text{ the area of the rectangle ABCD.}$$

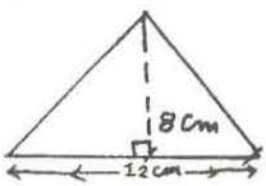
$$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm} = 48 \text{ cm}^2$$

The required area $= 48 \text{ cm}^2$

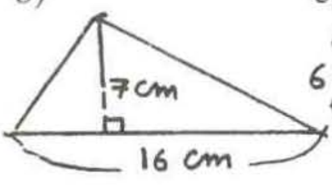
Exercise 11.4

1. Find the area of each triangle.

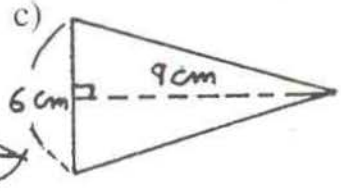
a)



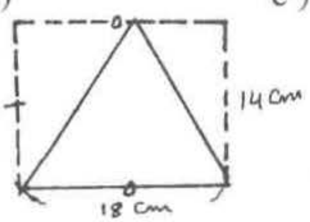
b)



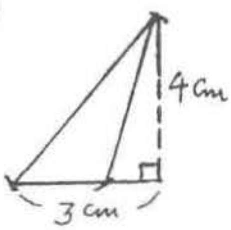
c)



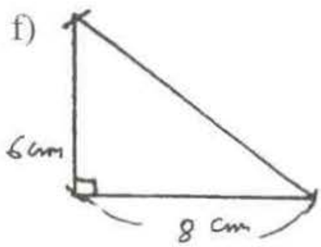
d)



e)

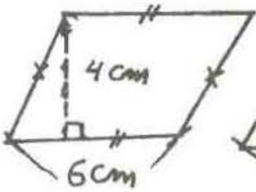


f)

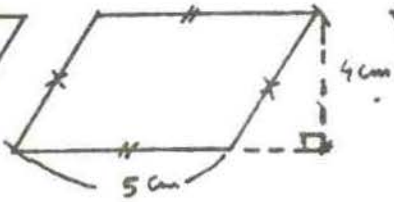


2. **Calculate the area of each quadrilateral.**

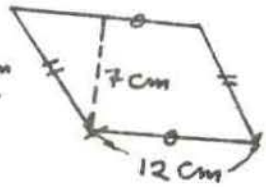
a)



b)

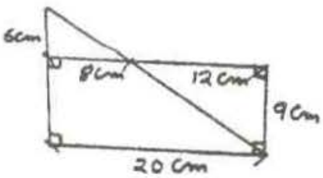


c)

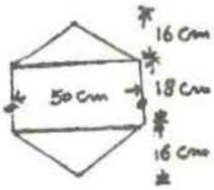


3. **Find the area of each figure**

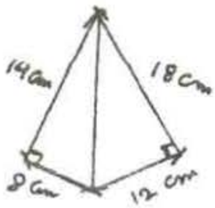
a)



b)



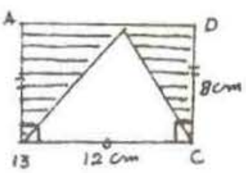
c)



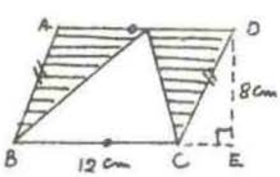
4. a) The base of a triangle is 9.5 cm and the height is 3.6 cm. Find the area of the triangle.
   b) The base and the perpendicular of a right angle triangle area are 6 cm and 8 cm respectively. Calculate its area.
   c) If the area of a triangle is 18 cm^2 and its height 4cm., Find its base.
   d) If the area of a triangle is 22 cm^2 and its base 10 cm, find its height.
   e) The diagonals of a rhombus are 12 cm and 8 cm. Find its area.

5. **Find the area of the shaded part in each figure.**

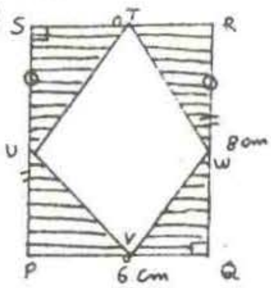
a)



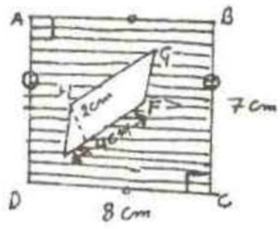
b)

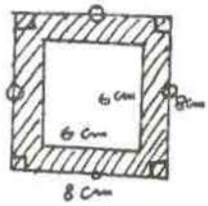


c)

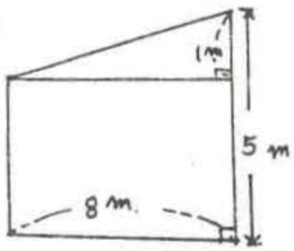


d) e)



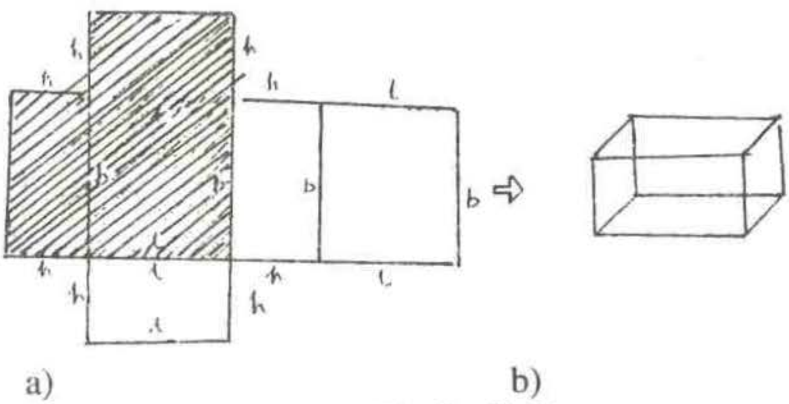


6. The floor of a room is covered with a carpet. The length and the breadth of the room are 5 m and 4 m respectively. Find the area of the carpet. (The area of the floor and the area of the carpet are equal.)
7. The length and the breadth of a rectangular piece of paper are 12 cm and 9 cm respectively. Without changing the length, by how much should the breadth be reduced to make the area equal to 96 cm^2 ?
8. The figure shows the front part of the wall of a house along the breadth. Calculate the area of the wall.



11.5. Surface Area and Volume of a Cuboid and a Cube.

11.5.1.Surface Area of a Cuboid and a Cube



a) b)

Fig. No. 11.54

Study the above figure carefully and answer the following questions.

a) What is the relation between figure no. 11.54 a) and b)

b) How many rectangles are used to make the net of the cuboid ?

c) Are the sizes of all the rectangles the same ?

d) How many rectangles have the area equal to $l \times b$?

e) How many rectangles have the area equal to $b \times h$?

f) How many rectangles have the area equal to $l \times h$?

g) What part of the net is represented by the shaded portion ?

Here, the area of the shaded portion = $l \times b + b \times h + l \times h$ sq. unit.
Therefore, the area of the whole net (A) = $2 (l \times b + b \times h + l \times h)$ sq. unit.
Similarly, a cuboid having the length, breadth and the height equal is called a cube.

Therefore the surface are of a cube = $2(a \times a + a \times a + a \times a)$

$= 2(3a^2)$

$= 6a^2$

Therefore,

Surface area of cuboid (A) = $2 (l \times b + b \times h + l \times h)$ sq. unit.
Surface area of cube (A) = $6a^2$

Example 1

Find the total surface area of a cuboid having length of 12 cm, breadth of 8 cm and height of 5 cm.

Answer :

Here,

the length of cuboid (l)	= 12 cm
breadth (b)	= 8 cm
height (h)	= 5 cm

The total surface
area (A) $= 2 (l \times b + b \times h + lh)$

$= 2 (12 \text{ cm} \times 8 \text{ cm} + 8 \text{ cm} \times 5 \text{ cm.} + 12 \text{ cm} \times 5 \text{ cm})$

$= 2 (96 + 40 \text{ cm}^2 + 60 \text{ cm}^2)$

$= 2 \times 196 = 392 \text{ cm}^2$

Example 2

Find the total surface area of a cube having one edge of 10 cm.

Answer :

Here,

One edge (a) $= 10$ cm

Total surface Area $= 6a^2$

$= 6 \times (10\text{cm})^2$

$= 6 \times 100 \text{ cm}^2$

$= 600 \text{ cm}^2$

Example 3

Find the length of the side of a cube, the surface area of which is equal to 150 cm^2

Answer:

Here,

The surface area of cube (A) $= 150 \text{ cm}^2$

Length of side (a) $= ?$

From formula $6a^2 = A$

or, $6a^2 = 150$

or, $a^2 = \frac{150}{6} = 25$

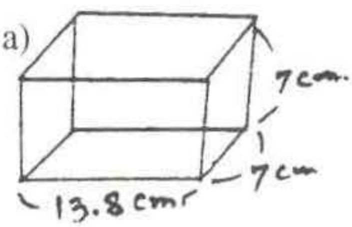
or, $a^2 = 5^2$

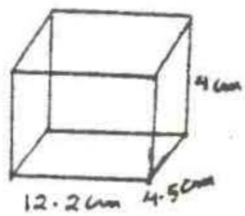
or, $a = 5$ cm

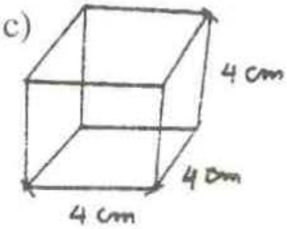
$\therefore$ length of the side of a cube $= 5$ cm.

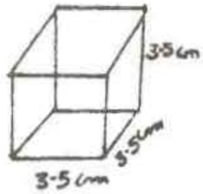
Exercise 11.5.1

1. **Find the surface area of the solids given below.**

a) 

b) 

c) 

d) 

2. Find the surface area of the cuboids given below.

a) $l = 7$ cm b = 5 cm h = 4 cm
b) $l = 12$ cm b = 10 cm h = 2.5 cm
c) $l = 15$ cm b = 4.3 cm h = 9 cm
d) $l = 4.2$ cm b = 3.6 cm h = 6.8 cm

3. Find the surface of the cubes given below

a) a = 5 cm b) a = 6 cm
c) a = 3.5 cm d) a = 5.2 cm

4. If a box has length of 120 cm, breadth of 80 cm and height of 60 cm.

a) What is the surface area of the box ?

b) When the box is kept on a table, how much of the table top surface will it cover ?

5. A box of chalk has length of 10 cm, breadth of 8 cm and height of 6 cm.

a) Draw a diagram which represents the net of the box of chalk.
b) What is the surface area of the net ?
c) The lid of the box is lost. Find the surface area of the box without its lid.

6. A match box has a length of 4.5 cm, a breadth of 3 cm. If its surface area is 49.5 cm^2 find its height.

7. When a cubical box is kept on top of a table, it covers 2.25 m^2 area. Find the total surface area of the box.

11.5.2. Volume of a Cuboid and a Cube

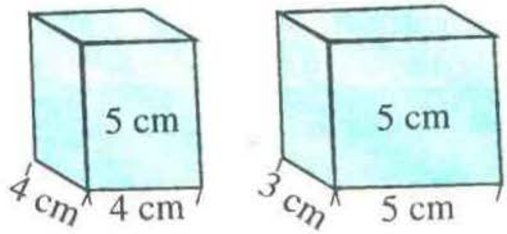


Fig. No. 11.55

In Fig. No. 11.55m sugar is kept in both the cubiod boxes. Study the figure carefully and answer the following questions.

a) Which packet contains more sugar ?
b) Which packet has a larger base ?
c) Which packet has a greater height ?
d) Which packet can hold more amount of sugar ?

A cube having length of 1 cm breadth of 1 cm and height of 1 cm is taken as the unit of volume.

The volume of the cube is 1 cubic cm ($1cm^3$). How many 1 cm^3 cubes can be fitted in the above packet ?

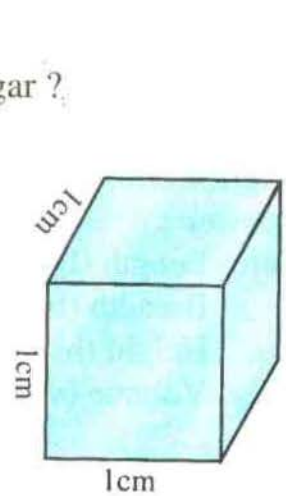


Fig. No. 11.56

16 packets of cubes can be fitted in one layer of packet (a) 15 packets of cubes can be fitted in one layer of packet (b). The number of these cubes can be calculated by multiplying length and breadth. In the above example 4×4=16 and 5×3=15 both the packets are 5 cm high. Thus, both the packets have five layers each. Therefore, packets (a) contains 16×5=80 unit cubes and packet (b) contains 15×5=75 unit cubes. Therefore, we can say that packet (a) contains more sugar.

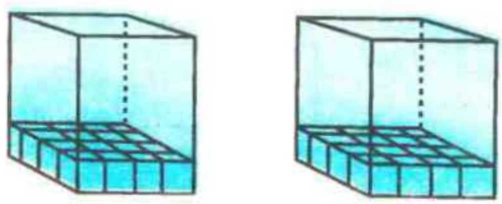


Fig. No. 11. 57

In both the examples above, the product of length, breadth and height gives the same result. Therefore, the volume of cuboid is calculated by multiplying its length, breadth and height.

Volume of cuboid $V = l \times b \times h$
$V = A \times h$
where A = Area of Base.

Length, breadth and height are equal in a cube.
Therefore, taking $l = b = h = a$
Volume of a cube $V = a^3$

Example-1

Find the volume of a cuboid having length 8 cm, breadth 3 cm and height 4 cm.

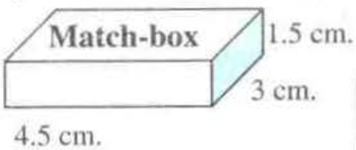
Answer :

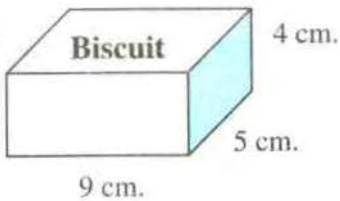
Here, Length (l) = 8 cm.
Breadth (b) = 3 cm.
Height (h) = 4 cm.
Volume (v) = ?

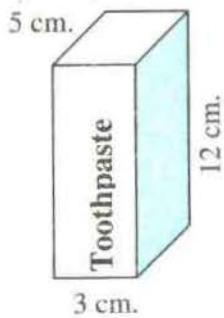
From formula, $V = l \times b \times h$
$= 8 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm}.$
$= 96 \text{ cm}^3$

Exercise 11.5.2

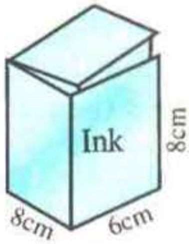
1. **Find the volume of each given solid objects.**

a)


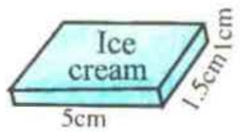
b)


c)


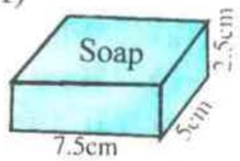
d)



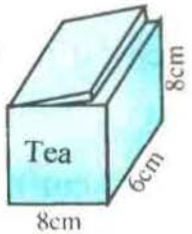
e)



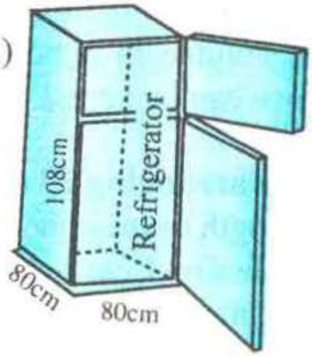
f)



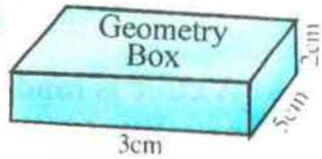
g)



h)



i)



2. Find the volume of the cuboids given below.

a) l = 12 cm	b = 8 cm.	h = 5cm
b) l = 15 cm	b = 5 cm.	h = 6cm
c) l = 4 cm	b = 2.5 cm.	h = 5.2cm
d) l = b = 5 cm	h = 8 cm	
e) l = h = 12 cm	b = 7.5 cm.	h = 5cm

3. Find the volume of the cube one side of which is given below.

a) 6 cm. b) 5.2 cm. c) 7.5 cm d) 8.2 cm.

4. If 1 litre = 1000 cm3, Find out how much water (in liters) the rectangular tanks given below can hold.

a) l = 2 m	b= 1/4 m	h = 4 m
b) l = 8 m	b= 3/4 m	h = 5 m
c) l = 6 m	b= $1^3/_4$ m	h = 7 m
d) 1 = 50 m	b= 30 m	h = 8 m

5. **A petrol tank has length of 4 m., breath of 3 m. and height of 5 cm.**
   a) What is the capacity of the tank in cubic meters?
   b) How many liters of petrol can this tank hold ?
   c) If a full tank of petrol is sold in 15 days then what is the average of petrol sold in a day ?

6. **A rectangular piece of wood has length of 4 cm., breadth of 2 mm and height of 2 mm.**
   i) What is the volume of the wood ?
   ii) What is the volume of a box in which 50 pieces of the wood as above can be kept ?

7. **A cube is made by arranging 27 unit cubes ($1\ cm^3$).**
   a) What is the length of the cube ?
   b) How many cubes of 1 $cm.^3$ should be added to increase the length of the cube by 1 cm.?

8. A tea packet is in the shape of a cuboid. Its base area is 72 cm^2 and height 8 cm. Find its volume.
9. The floor area of a class room is 48 m^2 and its height is 3 m. Find the volume of the room.
10. The volume of a packet of milk is 576 cm^3. If its length and breadth are 8 cm. and 6 cm. respectively, what is the height ?
11. The base area and volume of a rectangular tank are $6m^2$ and $54m^3$ respectively. What is the height of the tank ?

11.6. Congruency and Similarity

11.6.1. Introduction to Congruency Figures.

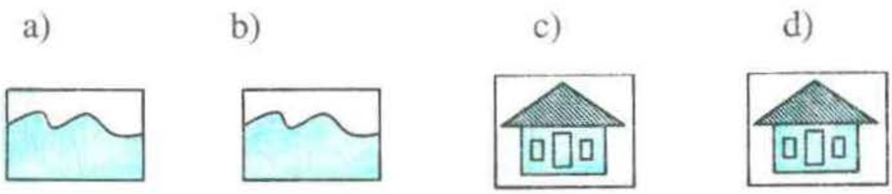


Fig. 11.58

In the above figures, (a) and (b) have the same shape and size. Similarly (c) and (d) have the same shape and size. Figures having the same shape and size are called congruent figures.

Fig. 11.59

Measure the sides and the angles of triangles ABC and DEF and complete up the table below.

AB	DE	BC	EF	AC	DF	∠A	∠B	∠C	∠D	∠E	∠F	

Are the three sides of ΔABC equal to the three sides of ΔDEF separately ?

Are the three angles of ΔABC equal to the three angles of ΔDEF separately ?

Here, ΔABC and ΔDEF are of the same shape and size. Therefore, ΔABC and ΔDEF are congruent.

It is written as ΔABC ≅ ΔDEF

11.6.2. Introduction to similar figures

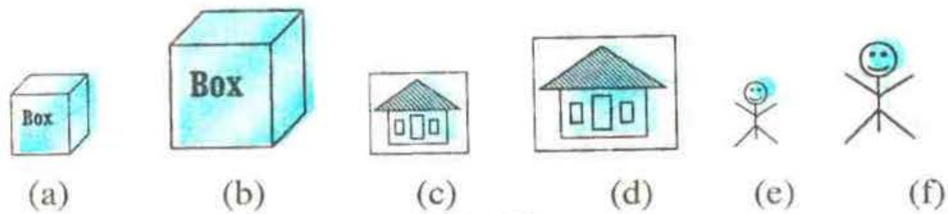


Fig. 11.60

The above figures, (a) and (b) have the same shape but they do not have the same size. Therefore, they are not congruent. Are the figures (c) and (d); and (e) and (f) congruent ?

Therefore, the figures having the same shape but different measures are called similar figures.

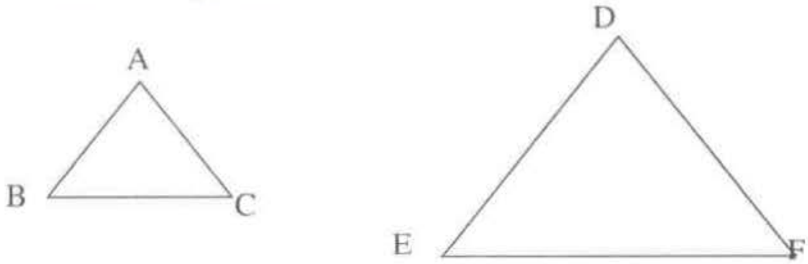


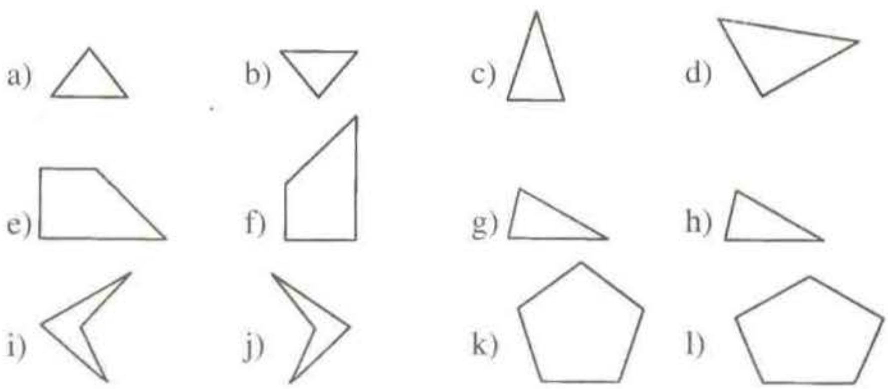
Fig. 11.61

Measure the sides of ΔABC and ΔDEF. Are all the sides of ΔABC equal to the sides of ΔDEF ? Are all the angles of ΔABC equal to the angles of ΔDEF ?

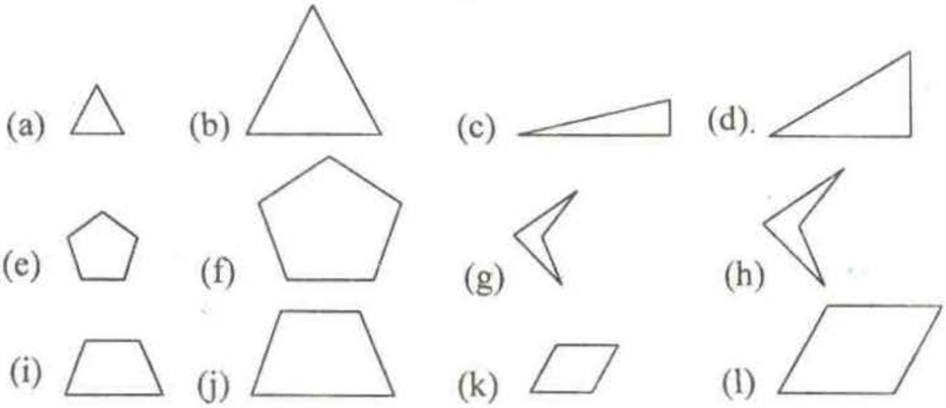
If all the angles of ΔABC are equal to all the angles of ΔDEF separately but the sides are not equal then ΔABC is said to be similar to ΔDEF. It is written as ΔABC~ΔDEF.

Exercise 11.6

1. State which given figures are congruent.



2. **State which given figures are similar ?**



3. **Find out whether the following statements are true or false.**
   a) All the equilateral triangles are congruent.
   b) All squares are congruent.
   c) All equilateral triangles are similar.
   d) Equilateral triangles having 3 cm. sides 5 cm are congruent.
   e) Isosceles triangles of sides 5 cm are congruent.
   f) Equilateral triangles of 4 cm. and 5 cm. are similar.
   g) All squares are similar.

11.7. Circle

11.7.1. Introduction to different parts of a circle.

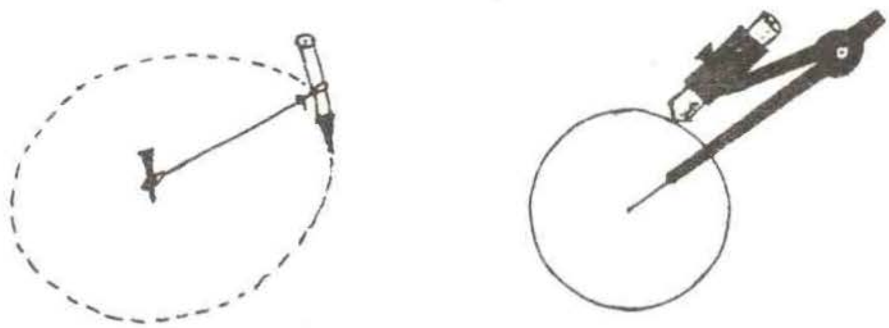


Fig. No. 11.62

Place a piece of paper on a table. Tie one end of a piece of thread round the thumb tack and fix it at the center part of the paper. Tie a pencil at the other end of the thread. Rotate the pencil thread keeping the thread stretched (Fig. No. 11.62 (a)). At another part of the paper, fix the tip of the compass and rotate the pencil. (Fig. No. 11.62). What difference and similarity do you find in Figure No. 11.62 (a) and (b) ? Here both the figures are called circles. The point where thumb tack or the needle of the compass is fixed is called the center of the circle. The length of the thread from the thumb tack to the pencil or the distance between the needle of the compass and the pencil is called the radius and the boundary line formed by the pencil is called the circumference.

- The line segment from the centre to the circumference of a circle is called radius. OA is the radius in the given figure.
- The line which joins any two points of circumference of a circle is called the chord. DC is a chord in the given figure.
- The chord passing through the center is called diameter. In the figure BE in a diameter.
- The region between two radii and an arc is called a sector. In the figure, BOC in a sector.
- Half of a circle BCDE is called a semi circle.

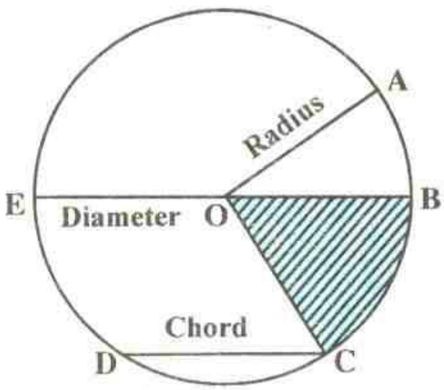
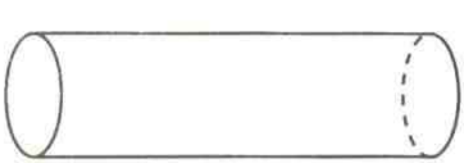


Fig. No. 11.63

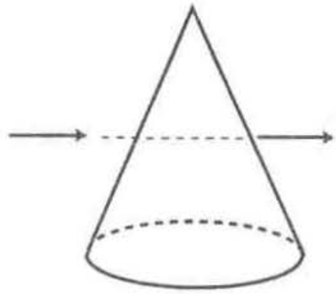
Exercise 11.7

1) The upper surface and lower surface of a drum which are shown in the figure have circular surfaces. Write down ten such examples which are found in nature.



2) Take a lemon. Cut it into two equal parts. What type of shape is formed ?

3) If you cut a shape as shown in the figure, parallel to the horizontal surface, what type of shape will be formed at the surface cut ?



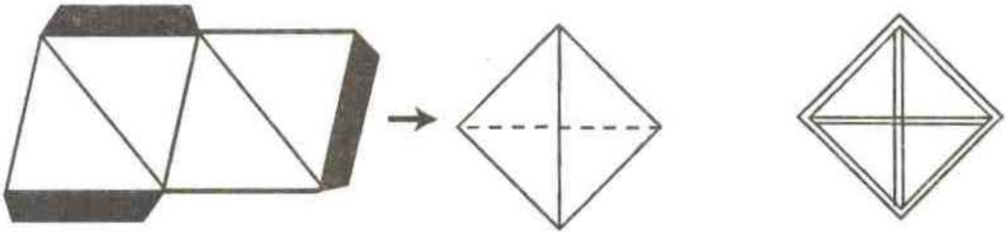
11.8. Nets and Skeleton Models of Regular Solids.

11.8.1. Introduction :

You must have already noticed that the structure of a modern house is constructed at first. The final shape of the building is completed according to this structure. Similarly, the structures of solid objects can be constructed by using drinking straws of coca-cola or wheet straws. By folding paper the surface of a solid object and its model can be constructed. The nets and the models of a regular polyhedron are given below as examples.

a) Tetrahedron

Each surface of a tertrahedron is made of an equilateral triangle. There are four surfaces in it. It is a regular solid object. Observe its net and skeleton model in Fig. No. 11.64.

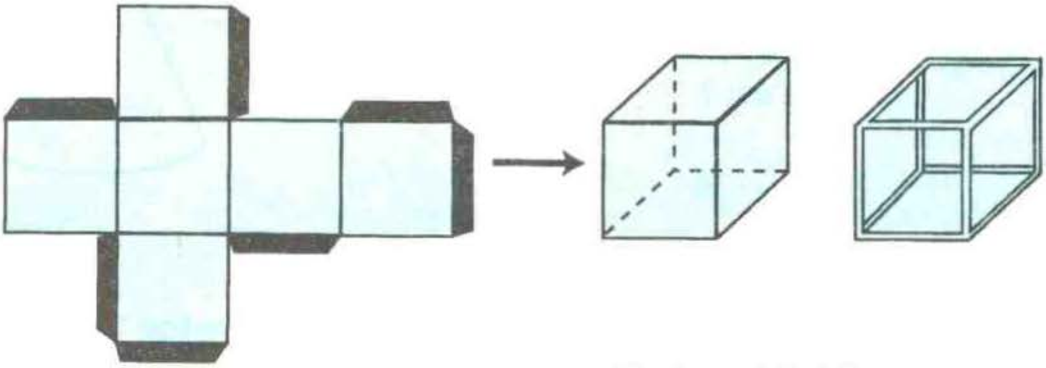


Net Model Skeleton Model

Fig. No. 11.64

b) Cube or Regular Hexahedron

Cube is a regular solid object. It has six square surfaces. Its net and skeleton model are given in Fig. No. 11.65.

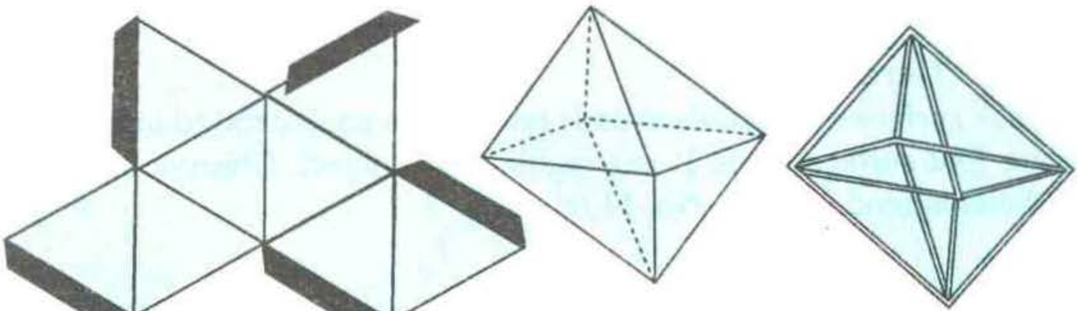


Net Model Skeleton Model

Fig. No. 11.65

c) Octahedron :

Each surface of the octahedron are like equilateral triangles and it has eight surfaces. The net and skeleton model of octahedron are given in Fig. No. 11.66.



Net Model Skeleton Model

Fig. 11.66

From each net and skeleton model of the regular polyhedron, the following facts are obtained.

a) The faces of regular polyhedron are similar.

b) The line segment which joins two faces of a regular polyhedron is called an edge. Such edges are equal.

c) Three or more edges and No. of regular polyhedron form a vertex.

d) The number of edges, the number of faces and the number of vertices of a regular polyhedron are given in the table below.

Regular polyhedron	**Vertex 'V'**	**Edges E**	**Faces (F)**	**Shape of Faces.**
Tetrahedron	4	6	4	Triangle
Hexahedron	8	12	6	Square
Octahedron	6	12	8	Triangle

In all polyhedrons V - E + F = 2. It is called Euler's formula. Justify this fact in the table below.

d) Cylinder

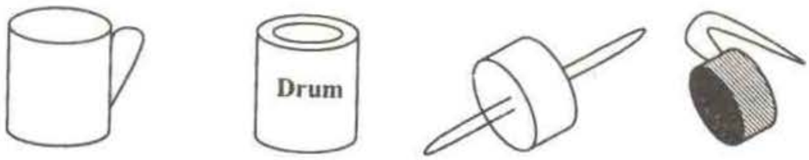


Figure No. 11.67

- ❑ What are the shapes of the above objects ?
- ❑ What is the shape of the bases ? What does the surface look like ?
- ❑ Can you give more examples ?

Objects having two circular bases and a curved surface is called a cylindrical object.

Take a rectangular piece of paper

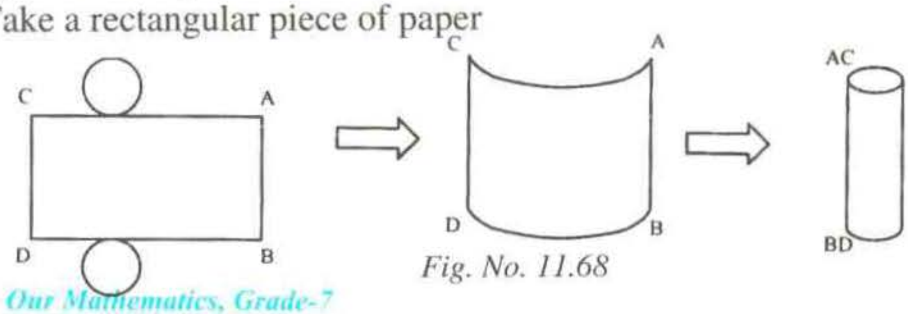


Fig. No. 11.68

As shown in the fig., roll the paper around so that it fits along the circumference of the two equal circles.

e) Cone

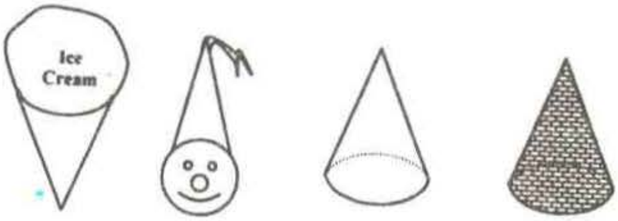


Fig. No. 11.69

- ❑ What is the shape of bases in the above objects ? What does the surface look like ?
- ❑ An object having a circular base and a curved surface meeting at a point is called a cone.

Cut a circle as shown in the figure. Cut out a part of the circle from the center. Join OA and OB. Has a cone been constructed ? Is the vertex of the cone O ? Can a circle be fitted on the base of the cone ?

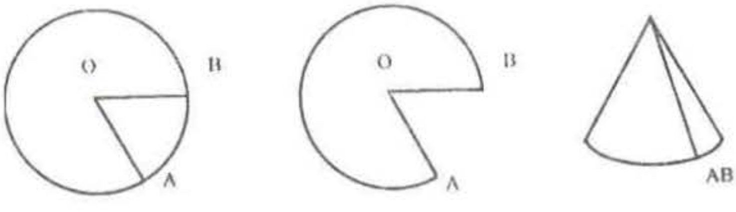


Fig.No. 11.70

Exercise 11.8

1. Introduce a tetrahedron, a cube and an octahedron.
2. Prepare a skeleton model of a tetrahedron, a cube and an octahedron as given in Question No. 1 either from thread or 5 cm. long straws or drinking straws.
3. Prepare cones and cylinders from paper.

11.9. Symmetry and Tessellation

11.9.1. Rotational Symmetry

Experiment 1

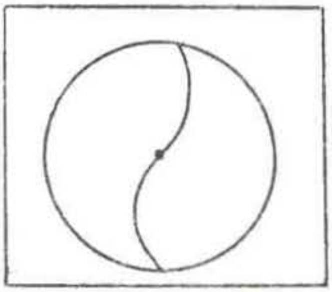


Fig.No. 11.71

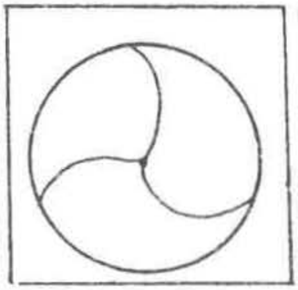


Fig. No. 11.72

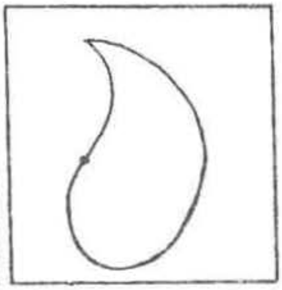


Fig. No. 11.73

Trace the shape in Fig. No. 11.71 on a sheet of thin paper. Place this congruent shape over the Fig. No. 11.71 and press a pencil through its center. Now rotate the top shape until it completely covers in Fig. No. 11.71 again. In this state, what part of a whole rotation is rotated ?

Again, from this state rotate it while it completely covers the shape. What part of a whole rotation is rotated this time ? Is the Fig. in thin paper in the same state as in the beginning?

Trace the shape of Fig. No. 11.72 on a thin piece of paper and repeat the above process. Rotate it until it completely covers Fig. No. 11.72. What part of rotation is rotated ? How may times does it completely cover the Fig. No. 11.72 in one complete rotation ?

Even though this is not a line symmetry there is some kind of uniformity in rotation. This is called the rotational symmetry of these figures.

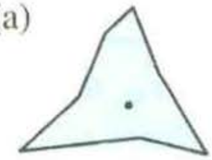
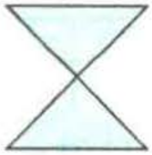
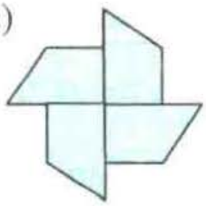
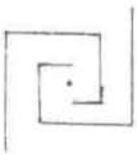
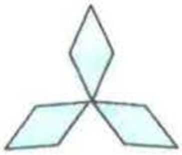
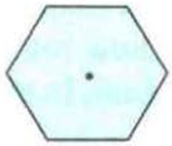
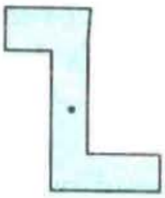
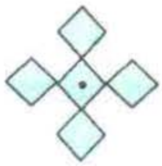
The figure traced out in the thin paper covers Fig. No. 11.71 twice in one rotation. This is called order 2 rotational symmetry. Figure traced in this paper covers Fig. 11.72, 3 times in one rotation. It is called order 3 rotational symmetry.

Shapes like the Fig. No. 11.73 has no rotational symmetry. If this shape is rotated about a point, can you find what part it covers in one rotation. What is the order of its rotational symmetry ?

What is the order of the rotational symmetry of it ?

Exercise 11.9.1.

1. **Find the order of the rotational symmetry of the figures in the given points using thin paper.**

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)


2. **Prepare the geometric design of the rotational symmetry in the given order.**
   a) Three   b) Four   c) Six   d) Five

11.9.2. Line and Rotational Symmetry.

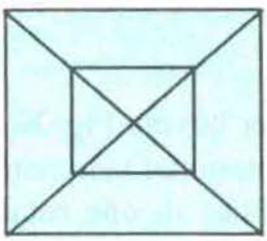
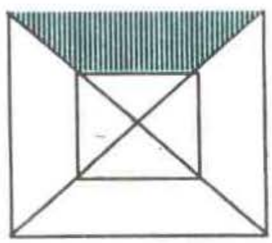
a)

b)


Fig. 11.74

Rotational symmetry of Figure 11.74 (a) belongs to order - 4. There is also a line symmetry in this figure. How many axes of line symmetry are there?

Experiment 2

One part of Fig. No. 11.74 (b) is shaded. What is the order of the rotational symmetry of this figure ? How many axes of line symmetry are there ? Draw this figure in exercise book. Now, which part of this figure should be shaded so that there are two axes of line symmetry and order 2 rotational symmetry? Now shade the third part of the figure in such a way that there is only one axis of line symmetry and the order of rotational symmetry decreases.

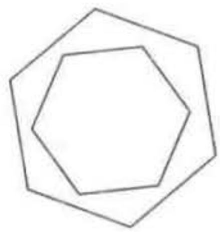
Can you shade the 4th part in such a way that the order of the rotational symmetry is 2 ? Now how many axes of line symmetry are there ?

Therefore,

There may not be an axis of line symmetry of geometrical figures. But the order of line symmetry of geometrical figures can never be less than 1.

Exercise 11.9.2

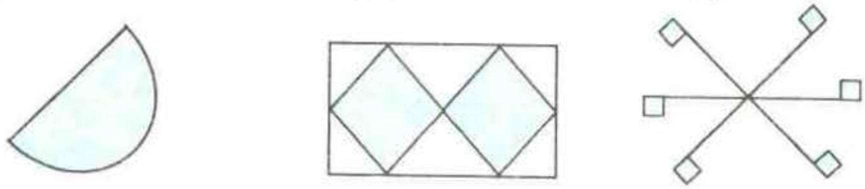
1. **Find out the order of rotational symmetry and the axis of line symmetry.**



2. **Write in the symmetry of the given figures in the table below.**

Fig.	Axes of line symmetry	Order of rotational symmetry
a		
b		
c		
d		
e		
f		

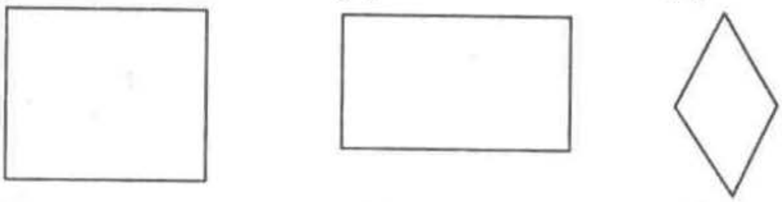
(a) (b) (c)



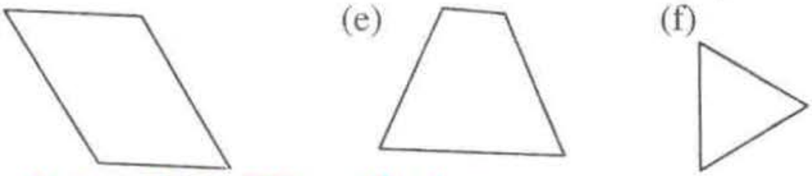
(d) (e) (f)

3. **Write the symmetry of figures given below in the table as shown in Figure No. 2.**

(a) (b) (c)



(d) (e) (f)



11.9.3. Extension of Tessellations

Concept of Tessellation

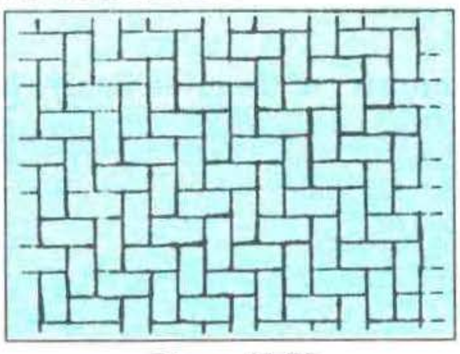


Figure 11.75

You must have already seen an ordinary brick or Telya bricks paved in the garden path, corridor, chowk etc. A chowk or garden may be paved as shown in Fig. No. 11.75.

Answer the following questions by studying the figure.

a) How are bricks arranged to pave the surface ?
b) What is the size of the rectangular brick which are used to pave the surface ?
c) Can the surface be paved without leaving space or not ?
d) Is there any geometric pattern or design in the construction of the path ?
a. What can you say about the process of paving the surface on the basis of the above questions ?

The process of covering plane surfaces without having any gaps or without overlapping, with one or more than one kind of identical geometrical shapes is called tessellation or tilling.

Tessellation is naturally formed when we are decorating or covering a surface or floor. For example, honey bees construct their hives on the basis of tessellation. The bee hive is formed with regular hexagons as in Fig. No. 11.76. Tessellation can also be done with equilateral triangles and square tiles. But tessellation can not be done only with regular octagonal tiles.

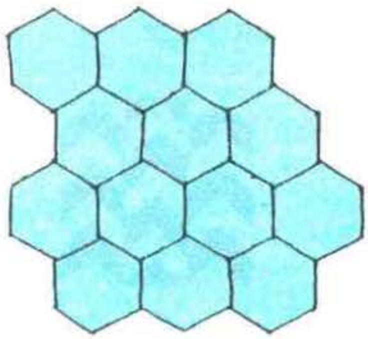


Fig. No. 11.76

Types of Tessellations

Study carefully the tessellation in Fig. No. 11.77.

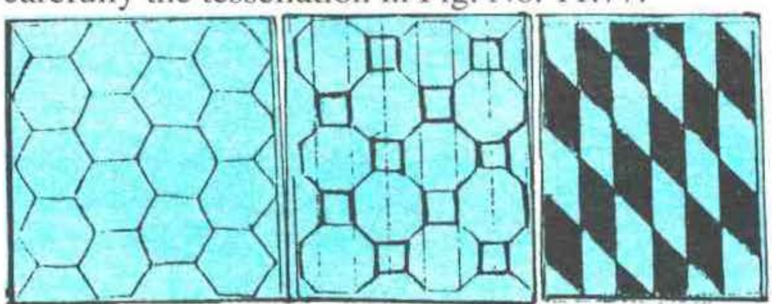


Fig. No. 11.77

Now answer the following questions.

a) How many types and geometric patterns are used in Fig. No. 11.77. (a).

b) Are both the patterns used in Fig. No. 11.77 (b) regular or irregular ?

a) The tessellation in which only one type of geometric pattern is used is called the regular tessellation. Such tessellations can only be constructed with equilateral triangle, squares and regular hexagons.

b) The tessellation in which two or more types of geometric patterns are used is called the semi-regular tessellation.

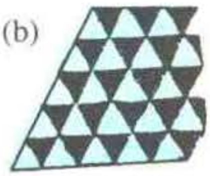
c) The tessellation formed by using irregular shapes is called the irregular tessellation. Similar triangles or quadrilaterals can be used for tessellation of a surface.

Exercise 11.9.3.

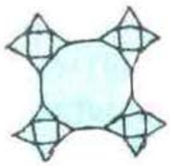
1. **Trace each tessellation on paper and extend it to cover the whole page.**

(a)

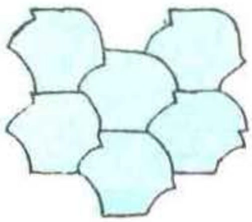
(b)



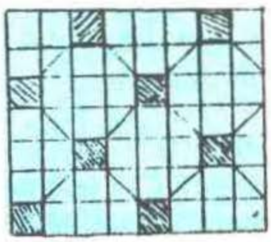
(c)



(d)

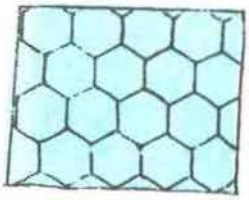


(e)

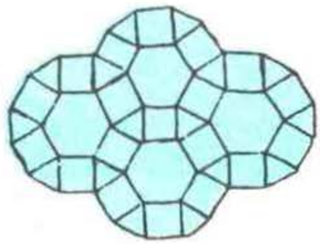


(f)

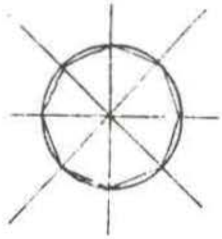
(g)



(h)



2. In Question No 1, which tessellations are regular, semi regular and irregular ?
3. Construct 10 identical octagons as shown in the figure. Cut it and tessellate it.



11.10. Bearing and Scale Drawing
11.10.1. Bearing

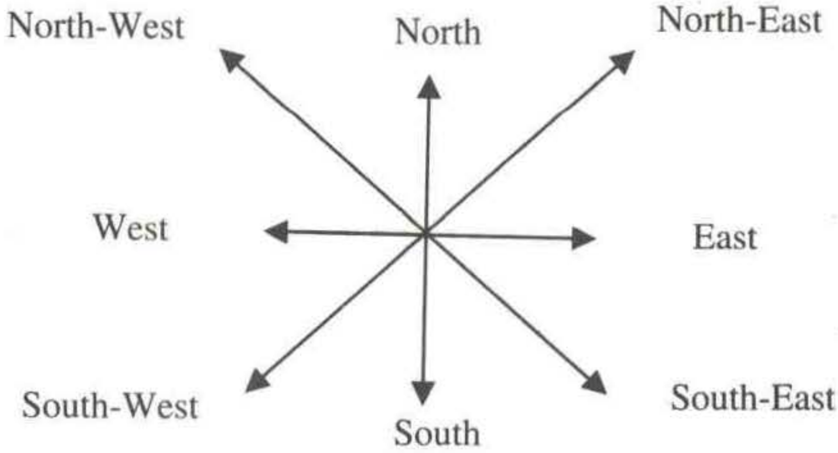


Fig.No. 11.78

The figure shown above shows the directions seen in a compass.

Example 1

Study the figure below and answer the following questions :

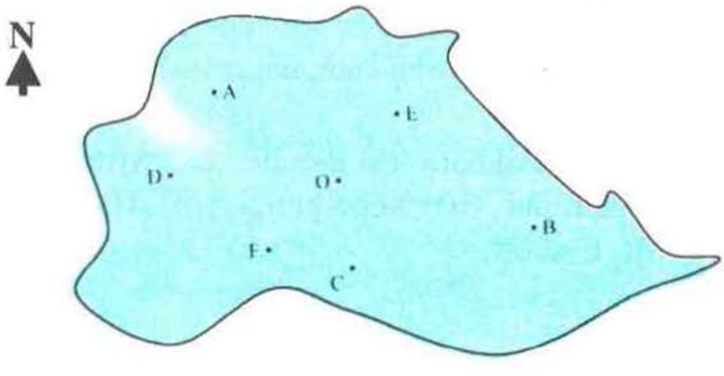


Figure No. 11.79

a) What is the bearing of A from O ? Similarly, write the bearing of B, C,D, E, and F,

Answer :

a) A is of O north-west.

b) B is of O south-east, C in south, D in west, E north-east and F of O south west.

Exercise 11.10.1

1. Study the map of Nepal given below and answer the following questions.

a) Assuming Kathmandu as the center write the bearing of (i) Mt. Dhaulagiri (ii) Mahendranagar (iii) Birgunj and (iv) Ilam.

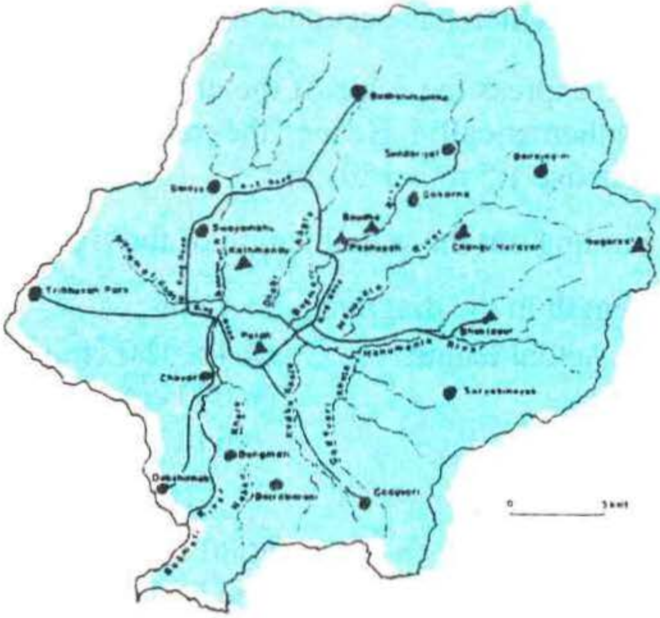
b) Assuming Pokhara as the center, write the bearing of (i) Bhadrapur (ii) Nepalgunuj (iii) Baitadi (iv) Butwal (v) Mt. Everest.

11.10.2. Scale Drawing

The picnic and tourist spots of the Kathmandu valley.

In Fig. 11.80, the picnic and tourist spots of the Kathmandu Valley are shown. If 1 cm equals about 5 km. in the figure, estimate the distance between the following places :

a) Patan and Bhaktapur.
b) Budhanilkantha and Nagarkot
c) Kathmandu and Bhaktapur
d) Godawari and Sundarijal
e) Dakshinkali and Gokarna



Legend
Δ Toursim Spot
• Picnic and Tourism

Fig. No. 11.80

Discuss how we draw put the shapes of an elephant and an amoeba (that we cannot see with our naked eyes) on the same page. We use a scale to outline the figures of too big and too small things. The floor plan diagram of a house has, for example, been shown in the following. If 1 cm = 1.5 m is assumed in making this diagram, answer the following questions.

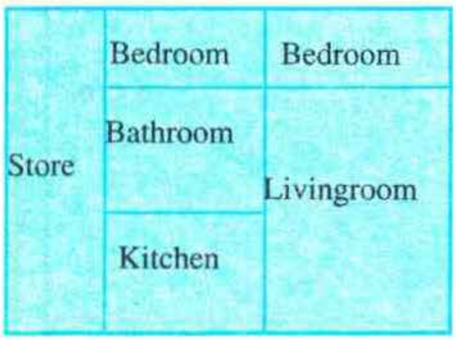


Fig.No. 11.81

a) What is the length of the store room ?

b) What is the length of the bedroom ?

c) What is the length of the kitchen ?

d) What is the length of the living room ?

e) What is the total area of the house?

f) What is the area of the bathroom ?

g) Which has occupied more space: the store room or the kitchen ?

We express the scale of the above diagram on the basis of proportion, mathematically. Hence, the scale of Fig. No. 11.81 is 1:150. It is because 1.5 m = 150 cm.

Finding out the actual length of the sitting room in the above example.

Length in the diagram $= 4$ cm.

$\therefore$ Actual length $= 4 \times 150$ cm.

$= 600$ cm

$= 6$ m [100 cm = 1 m]

Example 1

A lawn is 120 m long and 80 m wide. Assuming 10 m = 1 cm on the scale, draw the outline of the lawn.

Answer :

Here, 10 m = 1 cm

Therefore, 120 m = 12 cm and 80 m = 8 cm

Now, on drawing the outline of the lawn

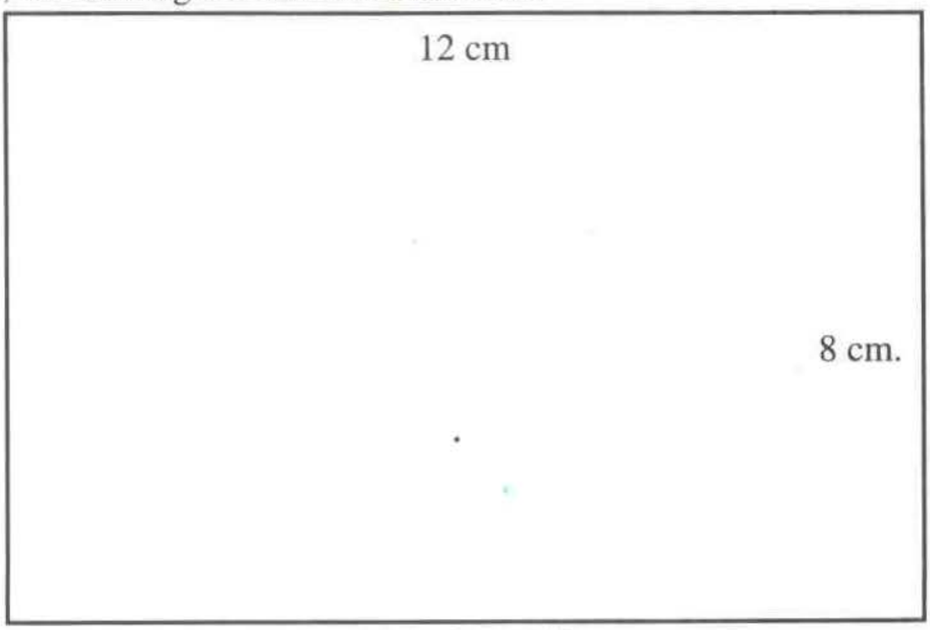


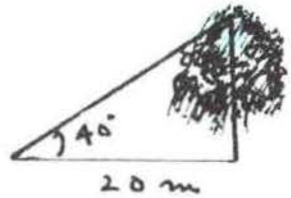
Figure No. 11.82

Exercise 11.10.2

1] Assuming the scale of 2 m = 1 cm on the scale, draw the outline of the floor of a rectangular house, the actual length and breadth of which are 16 m and 10 m respectively.

2] Make a scale drawing of a theater which is 55 m long and 30 m broad. Assume that 5 m = 1 cm on the scale.

3] Make a scale drawing of the picture at the side, assuming 5 m = 1 cm on the scale and find the actual height of the tree (Use the protector to construct 40^0).

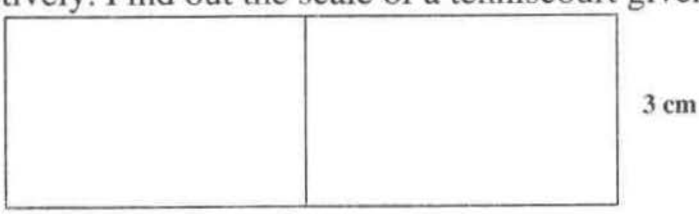


4] Imagine that an aeroplane from airport A flew 300 km due nort to B and from B it flew 400 km due east to C. In order to determine the distance of the plane from A, assume 100 km = 1 cm on the scale and make a scale drawing Measure AC.

5] A map has been drawn, assuming 1.5 km. = 1 cm. on the scale. Now find out the actual distances of the following :
a) 5 cm, b) 4.5 cm c) 9.3 cm.

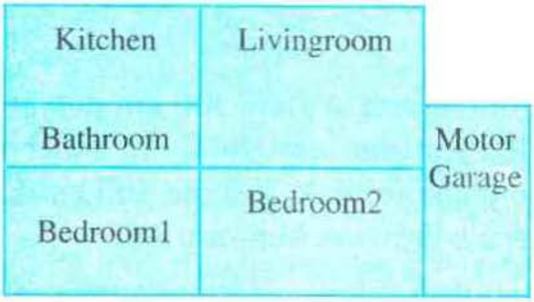
6] If the tree in the diagram has been drawn on the scale of 1:500, what is the actual height of the tree ?

7] The length and breadth of a field are 30 m and 15 m respectively. Find out the scale of a tenniscourt given below.



8] What is the actual length of the car of the picture if it is drawn on the scale of 1 cm = 1.2 m?

9] The plan of a house shown in the picture is drawn on the scale of 1:200. Measuring with a ruler find the actual length and breadth of the house and answer the following questions.

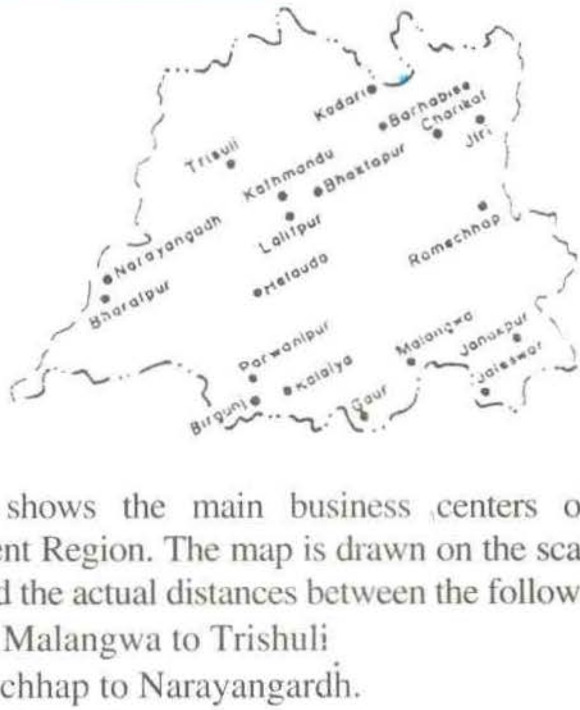


a) How long are the length and breadth of the kitchen ?

b) What is the area of the garage ?

c) Which is bigger - the garage or the bed room 1 ?

10]



The map shows the main business centers of the Centeral Development Region. The map is drawn on the scale 11 cm equals 40 km. Find the actual distances between the following places.

a) From Malangwa to Trishuli

b) Ramechhap to Narayangardh.

c) From Jiri to Parwanipur

d) From Khadwari to Kathmandu

e) From Bhaktapur to Charikot.

ANSWER SECTION

> Direction to the teacher and students Answers of the exercises which are not given in the answer section should be done by the students and shown to the teacher and the teacher should check them.

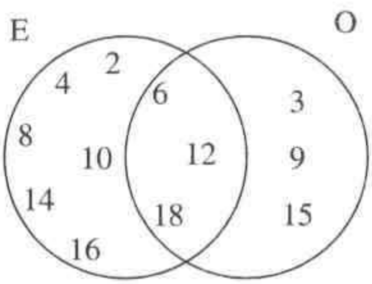
Exercise 1.1

1. a) False b) True c) False d) False (e) False
   f) False g) False h) False i) True

2. (a) W = {Sun, Mon, Tues, Wed, Thu, Fri, Sat}
   (b) E = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18}
   (c) O = {0, 3, 6, 9, 12, 15, 18}

3. i) Overlapping set E and O ii) Disjoint set W and E; W and O.

4. Diagram



5. (a) {0, 1, 2, 3, 4}
   (b) {3, 6, 9, 12, 15}
   (c) {2, 4}
   (d) {2, 4, 6}
   (e) {1, 2, 4}

Exercise 1.2

1. a) {Δ} b) {□, ◺} c) {O, ⌓} d) {⌓}
   e) {Δ, □}, {Δ, O}, {Δ, ⌓}, {□, ◺}, {□, O}, {□, ⌓}, {□, ◺}, {O, ⌓}, {O, ◺}, {⌓, ◺}

2. 10 Subsets

3. (a) Q = {2, 3, 5, 7} (b) R = {4, 6} (c) S = {1, 3, 5, 7}
   (d) T = {2, 4, 6} (e) U = {1, 2, 3, 6}

   (a) (T) ⊂ P or P ⊂ T (b) (T) R ⊂ P (c) (T) S ⊂ P
   (d) (T) T ⊂ P (e) (T) U ⊂ P

4. Φ, {a}   5. Φ,. {a}, {b}, {a, b}

6. Φ, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}

7. All that of 6 along with {b, c, d} {c, d, a}, {d, a, b}, {a, b, c, d}, {d} {a, d} {b, d} and {c, d}

8.

Sets	No. of Members	No. of Sub-sets
{a}	1	2
{a, b}	2	4
{a, b, c}	3	8
{a, b, c, d}	4	16

Exercise 1.3

1. (a) $M_6 = \{6, 12, 18, 24, 30, 36 \ldots\ldots\ldots\ldots\ldots 90, 96\}$
   (b) $M_8 = \{8, 16, 24, 36, 40 \ldots\ldots\ldots\ldots\ldots\ldots 88, 96\}$
   (c) $n(M_6) > n(M_8)$
   (d) $M_2 \supset M_6$
   (e) $M_6 \supset M_{24}$, $M_8 \supset M_{24}$

2. Set of whole numbers smaller than 11
   Show the answers to Exercise 1.4 to the teacher.

Exercise 1.5

1. (a) Show to the teacher
   (b) (i) $A \cup B = \{1, 3, 4, 5, 7, 9\}$ (ii) $B \cup A = \{1, 3, 4, 5, 7, 9\}$
   (iii) $A \cup A = \{1, 3, 5, 7, 9\}$ (iv) $B \cup B = \{3, 4, 5\}$
   (c) (i) $A \cup B = \{1, 3, 4, 5, 7, 9\} = B \cup A$
   (ii) $A \cup A = \{1, 3, 5, 7, 9\} = A$ (iii) $B \cup B = \{3, 4, 5\} = B$

2. (a) $A \cup U = \{0, 1, 2, 3, 4, 5\}$, Show the Venn diagram to the teacher
   (b) Can be done .

3. $A \cup B = \{1, 2, 3, 6\} \cup \Phi = \{1, 2, 3, 6\}$
4. Show it to the teacher

5. (a) (i) $(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 9\}$ (ii) $A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 9\}$
   Show the Venn diagram to the teacher
   (b) Can be done

Exercise 1.6

1. a) Show it to the teacher
   b) (i) $A \cap B = \{3, 9\}$ (ii) $B \cap A = \{3, 9\}$
      (iii) $A \cap A = \{1, 3, 5, 7, 9\}$ (iv) $B \cap B = \{3, 6, 9\}$
   c) (i) $A \cap B = \{3, 9\} = B \cap A$
      (ii) $A \cap A = \{1, 3, 5, 7, 9\} = A$ (iii) $B \cap B = \{3, 6, 9\} = B$

2. a) $M \cap N = \{1, 2, 3, 4\}$, $N \cap M = \{1, 2, 3, 4\}$
      Show the Venn Diagram to the teacher
   b) Can be done .
   c) Set M expresses $M \cap N$ or $N \cap M$.

3. Show it to the teacher.
4. $P \cap Q = \{0, 1, 2\} \cap \Phi = \Phi$
5. Show it to the teacher

6. a) (i) $(A \cap B) \cap C = \{1\}$ (ii) $A \cap (B \cap C\} = \{1\}$
   b) Can be written.

Exercise 2.1

1. a) 305,674 → Three hundred five thousand and six hundred seventy four.
   b) 3,596,876 → Three million five hundred ninety six thousand eight hundred seventy six.
   c) 12,579,640,312 → Twelve billion five hundred seventy nine million six hundred forty thousand and three hundred twelve.
   d) 3,000,050,201 → Three billion fifty thousand and two hundred one.
   e) 75,792,402,361 → Seventy five billion seven hundred ninety two million four hundred two thousand and three hundred sixty one .

2. a) $168 \times 1000000 + 500 \times 1000 + 353 \times 1$
   b) $13 \times 1000000000 + 000 \times 1000000 + 000 \times 1000 + 060$
   c) $17 \times 1000000000 + 006 \times 1000000 + 003 \times 1000 + 001 \times 1$
   d) $880 \times 1000000 + 302 \times 1000 + 015$

3.	a)	Ten lakhs	b) One thousand lakhs
4.	a)	74,70,500	(Seventy four lakh seventy thousand five hundred.)
	b)	15,30,03,22,816	(Fifteen arab thirty crore three lakh twenty two thousand eight hundred sixteen.)
	c)	1,13,29,07,00,028	(One Kharab thirteen arab twenty one crore seven lakh twenty eight.)
	d)	1,00,10,01.00,110	(One Kharab ten crore one lakh one hundred ten.)
	e)	9,00,70,03,007	(Nine arab seventy lakh three thousand and Seven.)

5.	A	B
Mercury:	Six crore ninety eight lakh km.	Sixty nine millions and eight hundred thousand km.
Venus:	Ten crore eight nine Lakh km.	One hundred eight million and nine hundred thousand km.
Earth:	Fifteen crore twenty one Lakh km.	One hundred fifty two million and one hundred thousand km.
Mars:-	Twenty four crore ninety two lakh km.	Two hundred forty nine million and two hundred thousand km.
Jupiter:	Eighty one crore sixty lakh km	Eight hundred and sixteen million km.
Saturn:	One arab fifty crore eighty eight lakh km.	One billion five hundred eight million and eighty hundred thousand km.
Uranus:	Three Arab eight one Lakh km.	Three billion eight million and one hundred thousand km.
Nepto:	Four Arab fifty four core Forty nine lakh km.	Four billion, five hundred forty four million and nine hundred thousand (km)
Pluto:	Seven Arab thirty eight core Eighty lakh km.	Seven billion three hundred eighty eight million km.

Exercise 2.2

1. a) 25 b) 49 c) 121 d) 144
   e) 225 f) 324 g) 484 w) 2209

2. a) ±5 b) ±7 c) ±11 d) ±13
   e) ±17 f) ±25 g) ± 42 h) ±52

3. a) 1^2 b) 2^2 c) 3^2 d) 5^2
   e) 6^2 f) 7^2 g) 8^2 h) 9^2
   i) 10^2 j) 11^2 k) 12^2 l) 13^2

4. a) 1 (b) 4 (c) 0 (d) 144
   e) 400 (f) 2500 (g) 10000 (h) 15625
   (i) 19600 (j) 40000 (k) 250000 (l) 1000000

5. (a) 1 (b) 4 (c) 9 (d) 16
   (e) 81 (f) 100 (g) 225 (h) 625
   (i) 10000 (j) 15625 (k) 90000

6. (a) 5 (b) 6 (c) 8 (d) 9 (e) 11
   (f) 18 (g) 35 (h) 42 (i) 55 (j) 72

7. a) $3\sqrt{2}$ b) $4\sqrt{5}$ c) $12\sqrt{7}$ d) $13\sqrt{5}$ e) $16\sqrt{6}$ f) $2\sqrt{119}$

8. a) 4 b) 4 c) 6 d) $\sqrt{29}$

Exercise 2.3

1. (a) 11cm (b) 22cm (c) $18\sqrt{7}$ cm
2. (a) 1m,4m (b) 45cm ,180 cm (c) 25m,100m

3. (a) $5\sqrt{3}$ (b) 12 (c) $14\sqrt{6}$ (d) $42\sqrt{2}$
   (e) 60 (f) $24\sqrt{15}$ (g) $4\sqrt{21}$ (h) $14\sqrt{3}$
   (i) $6\sqrt{10}$ (j) $8\sqrt{33}$ (k) $26\sqrt{3}$ (l) $63\sqrt{15}$

4. (a) $\sqrt{6}$ (b) $\frac{2\sqrt{3}}{\sqrt{13}}$ (c) $\sqrt{3}$ (d)1 (e) $\frac{2\sqrt{3}}{\sqrt{5}}$
   (f) $\frac{2}{\sqrt{6}}$ (g) $\frac{2}{\sqrt{12}}$ (h) $\frac{\sqrt{2}}{\sqrt{3}}$ (i) $\frac{\sqrt{15}}{\sqrt{13}}$ (j) $\frac{\sqrt{7}}{3}$

5.	a) 2	b)3	c) 5	d) 3	e) 7
	f) 3	g) 3	h) 5	i) 3	j) 2

6. 11 7. 34 8. 125 9. 6561

10. a) 4225 persons b) 65 persons

Exercise 2.4

1. (a) 3 (b)2 (c) 4 (d) 8 (e) 3 (f) 3 (g) 5 (h)2
2. (a) 4 (b) 9 (c) 9 (d) 8 (e) 12 (f) 52 (g) 8 (h)27
3. (a) 12 (b) 6 (c) 4 (d) 25 (e) 24 (f) 7 (g)16 (h)33
4. 70 persons , 3 apples , 4 oranges
5. 25 persons , 9 bananas , 10 guaves , 12 pears
6. 20 only
7. 36 only
8. If we take out at the rate of 16 from the first basket and 19 from the second basket , both the baskets will be empty in the thirteenth time
9. 5 liter
10. A square stone with the length of 3 m

Exercise 2.5

1.	(a) 15	(b)12	(c) 24	(d)40	(e)24
	(f) 42	(g)36	(h)60	(i) 18	(j) 48

2.	(a) 36	(b) 36	(c) 70	(d) 140	(e) 120	(f) 120
	(g) 72	(h)84	(i) 36	(j)120	(k) 84	(l)360

3. 12 o'clock
4. After 36 weeks
5. 70 persons
6. 12000km

Exercise 2.6

1.	(a) 5	(b) 7	(c) 4	(d) 6	(e) 14	(f) 10
	(g)11	(h) 21	(i) 31	(f) 41	(k)50	(l) 64

2.	(a) 10000111	(b) 1111111	(c) 1101011
	(d) 1000010001	(e) 101011001	(f) 1000111100
	(g) 1100101001	(h) 1100100	(i) 10011010010
	(j)1011001101110	(k) 10100101100	(l) 1111101001

3. a) 100 (b)show it to the teacher
4. 18

Exercise 2.7

1. (a) 38 (b) 86 (c) 58 (d) 117 (e) 139 (f) 251
   (g) 553 (h)125 (i) 389 (j) 379 (k) 350 (l) 879

2. (a) 1042 (b) 10341 (c) 3212 (d) 4304
   (e) 4413 (f) 4000 (g) 10403 (h) 13000
   (i) 13002 (j) 130242 (k) 242340 (l) 304444

3. (a) 20_5 (b) 14_5 (c) [illegible]$_5$ (d) 30_5 (e) 101011001_2
   (f) 101110111_2 (g) 1001001010_2 (h) 10000111_2

Exercise 2.8

1. (a) 1^3 (b) 4^3 (c) 5^3 (d) 3^3

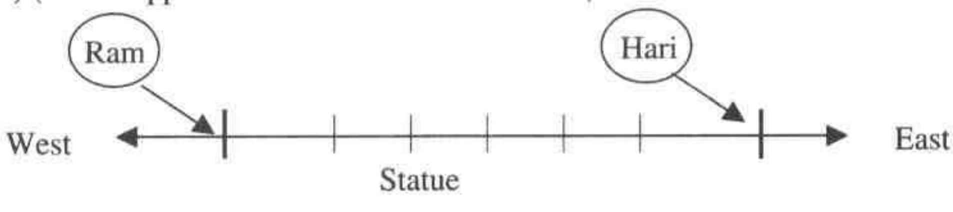
2. (a) 64 (b) 216 (c) 27 (d) 512

3. (a) 5 (b) 4 (c) 6 (d) 7

Exercise 2.9

1. (a) West (b) East (c) West (d) East
   (e) -5 (f) -4,-3,-2,-1,0,1,2 (there are 7)
2. (a) 2 (b) -1 (c) -3 (d) -4 (e) -6
3. (a) > (b) < (c) < (d) > (e) < (f) >
4. -12, -11, -10, -9, - 8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2
5. (a) +13 (b) -7 (c) +4 (d) +2 (e) -5
6. (a) 9 (b) 7 (c) 2 (d) 0 (e) 53 (f) 3
7. 3 m

8. a) (If we suppose east as + then west will be)



km - 2 , - 1 , 0 , 1 , 2 , 3 , 4 km

b) 6 km.

Exercise 2.10

1. (a) Wrong (b) Right (c) Right (d) Wrong
   (e) Right (f)Right (g) Right (h) Wrong
2. Show the answer to the teacher.
3. Show the answer to the teacher.

4. (a) 1west (b) 5 west (c) 0 point of origin
   (d) 3 west (e) 2 west (f) 2 east
   (g) 7down (h) 1 up (i) 2 right (j) 4 right

5. (a) show the answer to the teacher (b) 1cm
6. -296
7. -15 or + 19
8. (a)14^0 (b) -10^0
9. (a) + 70 (b) + 110 (c) -40 (d) + 108
10. (a) + 3, -3 (b) -5, + 5 (c) -7, + 7
11. (a) -41 (b) -17 (c) -40 (d) -24

Exercise 2.11

1. (a) -15 (b) -28 (c) + 48 (d) + 16 (e) + 720 (f) -221
2. (a) + 30 (b)-60 (c) + 84 (d) + 1680 (e) -1300 (f) -1040
3. a) + 15 (b) -16 (c) + 10 (d) -18 (e) -70 (f) + 18
4. Show it to the teacher

5. (i) $-2 \times -1 = -1 \times -2$
   (ii) $-1 \times +1 = +1 \times -1$
   (iii) $+1 \times +2 = +2 \times +1$
   (iv) $+2 \times -1 = -1 \times +2$
   (v) $0\times +2 = +2 \times 0$
   Besides these other answer are possible

6. (a) -4 (b) -3 (c) -4 (d) + 3 (e) + 3 (f) + 4
7. -3 8. -4 9. -3

Exercise 2.12

1. 29 2. -6 3. 1 4. -8 5. 0 6. 956
7. -121 8. 49 9. 17 10. 39 11. 9 12. 100

Exercise 3

1. a) $-\sqrt{2}$ b) $-\sqrt{5}$ c) $-\frac{\sqrt{10}}{2}$
2. (b), (c), (d), (f) True, (a), (e) False

Exercise 4.1

1. $\frac{3}{5}, \frac{2}{5}$ 2. $\frac{1}{2}, \frac{3}{10}, \frac{1}{5}$ 3. $\frac{5}{14}, \frac{1}{4}$ 4. $6\frac{3}{8}$ *ltr*. ltr.
5. (a) 420 (b) 280
6. (a)180 (b) 200
7. $\frac{6}{13}$ 8. $83\frac{1}{3}$ 9. 9

Exercise 4.2

1. (a) 0.6 Infinite revision (b) Finite at 0.625
   (c) Finite at 0.6 (d) 0.714285 Infinite revision
   (e) Finite at 0.875 (f) 5.83 Infinite revision
   (g) Finite at 3.0625 (h) 2.36 Infinite revision

2. (a) $\frac{1}{4}$ (b) $\frac{137}{100}$ or $1\frac{37}{100}$ (c) $\frac{281}{40}$ or $7\frac{1}{40}$ (d) $\frac{9001}{1000}$ or $9\frac{1}{1000}$
   (e) $\frac{3}{5}$ (f) $\frac{313}{99}$ (g) $\frac{23}{29}$ (h) $\frac{15}{111}$

3. Show the answer to the teacher

Exercise 4.3

1. (a) 7.269 (b) 34.003 (c) 48.146 (d) 352.149
2. (a) 1.911 (b) -0.097 (c) 28.929 (d) -0.001
   (e) 9.795 (f)-2.092
3. (a) 15.225 (b) 215.306 (c) 445.608 (d) 5.2185
   (e) -13.5135 (f) 23.5196 (g) 71.09375
   (h) 361.3116 (i) -58.912
4. (a) 25.7 (b) 206.8 (c) -17.6 (d) -14.35 (e) 3.23
   (f) 127.3 (g) -4532 (h) 8582 (i) 25

5. (a) -4.27 (b) 25.83 (c) 47.32 (d) 11.44 (e) -9.7
   (f) -4.5 (g) -3.2 (h) 0 (i) $\frac{3}{5}$ or 0.6

6. (a) 151.8m (b) 1379.3625m^2
7. (a) 13.69m (b) 72.46m
8. 8000 revelation
9. 1500

Exercise 5.1

1. (a) 160% (b) 287.5% (c) 305% (d) 212% (e) 131.25%
2. (a) 23% (b) 120% (c) 35% (d) 1250% (e) 20%
3. (a) $\frac{111}{200}$ (b) $\frac{173}{300}$ (c) $\frac{141}{400}$ (d) $\frac{176}{250}$ (e) $\frac{453}{500}$
4. (a) Rs. 3 (b) 6minutes (c) 1kg
   (d) 1 liter (e) 0.8km (f) $7\frac{1}{2}cm.$
5. (a) 480
6. (a) 75% (b)25%
7. (a) 40% (b) 60%
8. (a) Rs 925 (b)Rs 1575
9. (a) 12.5% (b) 87.5%
10. Rs 45
11. Rs 275 to Hari and Rs.225 to Santosh
12. (a) Rs 925 (b) Rs 945 (c) Rs. 630

Exercise 5.2.1

1. (a) $\frac{2}{3}$ (b) $\frac{5}{14}$ (c) $\frac{1}{3}$ (d) $\frac{4}{5}$ (e) $\frac{3}{4}$ (f) $\frac{8}{7}$
2. (a) $\frac{3}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{4}$ (d) $\frac{1}{4}$ (e) $\frac{1}{8}$ (f) $\frac{3}{4}$
3. (a) $3:5 < 7:8$ (b) $2:3 < 4:5$ (c) $3:7 < 7:11$
   (d) $3:7 < 5:9$ (e) $5:9 < 7:11$ (f) $3:4 < 8:9$
4. (a) 7:10 (b)3:10 (c)7:3
5. (a) 18 (b)1:7
6. 35 years
7. (a) English (b) Math (c) Math

Exercise 5.2.2

1. (a) yes (b) yes (c) No (d) yes (e) No (f) yes
2. (a) 20 (b) 21 (c) 3 (d) 6 (e) 10 (f) 30
3. (a) 100m (b) 320m
4. Rs.100
5. 9 m
6. 2.5 liter

Exercise 5.3

1. (a) Profit, Rs. 35 (b) Loss, Rs. 30 (c) Profit, Rs. 49.50
(d) Loss, Rs. 24.25 (e) Loss, Rs. 50.25 (f) Profit, Rs. 24.20

2. Rs. 12.50 3. Rs. 1900 4. Loss Rs. 537
5. Profit Rs. 2.50 kg. 6. Profit Rs. 9 7. Rs. 30
8. Rs. 1380

Exercise 5.3.1

1. (a) Profit, 15% (b) Profit, $\frac{100}{9}$ % (c) Loss, $\frac{500}{38}$ %
(d) Profit, 20% (e) Loss, 20% (f) Loss, 10%
2. 10% 3. Loss 5% 4. Profit $\frac{40}{11}$ %
5. Rs. 2,000
6. (a) Rs. 1050 (b) Rs. 1365
7. Rs. 216 8. loss 16%

Exercise 5.4

1. (a). Rs. 73.50 (b) Rs. 300 (c) Rs. 52.63 (d) Rs. 630
2. Rs. 1350 and Rs. 3850 3. 38.60 and Rs. 907.10
4. (a) 2.5% (b) $3\frac{1}{3}$ % (c) 10% (d) 24%
5. 25% 6. 10%
7. (a) 1 year 6 months (b) 1 year 5 months (c) 1 year4 months (d)5yeaRs.
8. 2 year 6 months 9. 19 year Rs. 8 months
10. (a) Rs. 1000 (b)Rs. 201 11. Rs. 1200.

Exercise 6.1

1. (a) yes (b) yes (c) No (d) No
2. Show the graph to the teacher.
3. (a) Rs. 750 (b) Rs. 2500 (c) Rs. 1750 (d) $ 55
(e) y = 50x (f) Rs. 12500
4. (a) Rs. 105 (b) 4 liter (c) y=30x (d) 29 liter

Exercise 6.2

1. (a) Yes (b) Yes (c) No (d) No
2. Show the graph to the teacher
3. (a) 8 hours (b) 16km/hr (c) $y = \frac{32}{x}$ (d) 3.2 hours

Exercise 7.1

Show the graphs 1-5 to the teacher.

6. (a) Sunday (b) Tuesday and Thursday (c) Tuesday
   (d) Monday and Wednesday, Tuesday and Thursday
   (e) Salty – 8 thousand, Sweet-11 thousand

Exercise 7.2

1. (a) 7 (b) 6 (c) 3 (d) 11 (e) 12
2. (a) 5.8 (b) 4.9 (c) 17.74 (d) 50
3. Show the frequency table and the mean to the teacher.
   (a) 3.6 (b) 10 (c) 16.2 (d) 2.6 (e) 10
4. (a) 20 (b) 7.04 (c) 7.45 (d) 42
5. Show the frequency table to the teacher.
   Mean is 4.8 no. of students who got marks below the mean = 13 no. of students who got marks above the mean = 17
6. (a) median=2 (b) 14 family (c) 14

Exercise 8.1

1. (a) 2^5 (b) 3^6 (c) 4^4 (d) $(-5)^5$ (e) $\left(\frac{1}{4}\right)^3$
   (f) $\left(-\frac{3}{7}\right)^3$ (g) $(2.5)^4$ (h) x^6 (i) $(2y)^4$ (j) $\left(\frac{2}{p}\right)^4$

2. (a) $5 \times 5 \times 5$ (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
   (c) $4 \times 4 \times 4 \times 4$ (d) $(-5) \times (-5) \times (-5)$
   (e) $(-7) \times (-7) \times (-7) \times (-7) \times (-7) \times (-7)$ (f) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
   (g) $\left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right) \times \left(-\frac{1}{3}\right)$ (h) $2x \times 2x \times 2x \times 2x \times 2x$
   (i) $\frac{3x}{2} \times \frac{3x}{2} \times \frac{3x}{2} \times \frac{3x}{2}$ (j) $\frac{4y}{5z} \times \frac{4y}{5z} \times \frac{4y}{5z} \times \frac{4y}{5z} \times \frac{4y}{5z}$

3. (a) 3×10^2 (b) 3×10^3 (c) 3×10^4
   (d) 3×10^6 (e) 5×10^3 (f) 9×10^4
   (g) 25×10^3 (h) 57×10^4

4. (a) 300 (b) 50000 (c) 8000 (d) 900000 (e) 20000000

5. (a) 125 (b) 1296 (c) 343 (d) 2187 (e) 375

(f) 5184 (g)16807 (h)72 (i) $\frac{9}{16}$ (j) $\frac{625}{2401}$

(k) 32243 (l) $\frac{1125}{5488}$

6. (a) a^5 (b) b^8 (c) x^7 (d) $128x^{10}$ (e) $1125y^5$ (f) $1600z^5$
7. (a) 3^2 (b) 2^5 (c) 3^4 (d) 2^{10} (e) 2^7 (f) 3^{10}
8. (a) 5^4 (b) 2^5x^3 (c) $2^3 \times 157$ (d) $2^4 \times 3^4$ (e)$2^6 \times 3^3$

Exercise 8.2

1. (a)2^5 (b) 5^{11} (c) $(-2)^{10}$ (d) $\left(\frac{3}{2}\right)^{12}$ (e)$\left(-\frac{1}{4}\right)^9$

   (f) $\left(-\frac{4}{5}\right)^{24}$ (g) x^5 (h) p^9 (i) m^{22} (j) $\left(\frac{3}{4}\right)^7$

   (k) $\left(\frac{p}{d}\right)^9$ (l) $\left(-\frac{y}{z}\right)^6$

2. (a) 2^3 (b) 3^3 (c) 4^3 (d) 5^5 (e) $\left(\frac{2}{3}\right)^3$

   (f) 7^7 (g) 2^6 (h) 5^7 (i)7^6 (j) x^3

   (k) y^3 (l)$(2z)^2$ (m) x^4 (n) p^4 (o) $\frac{3^{16}}{2^{11}}\ p^5$

3. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 1
4. (a) 128 (b) 216 (c) 256 (d) 5 (e) 625 (f) 9

Exercise 8.3

1. (a) Monomial (b) Binomial (c) Trinomial
   (d) Monomial (e) Binomial (f) Trinomial
2. (a) $14a^2$ b) $4a^2b$ (c) $10x^2y$ (d) $12ax^2$
3. (a) 2mn (b) 5pq (c) $4m^2n$ (d) $-18x^3y$
4. (a)$9x^2$ (d)$15ab^2$ (c) $26p^2d - pr^2$ (d) $3x^3 + 2x^2 + 4$
5. (a) $6a - b + 8c$ (b) $a^2 - 2b^2 - 9c^2$ (c)$11x^2y + 4xy^2 + 4z^2 - 9$
   (d) $x^3 - y^3 + 4z^3$ (e) $9m^3 + 2mn^2 - 3mn$

6. (a) $5a - b - 3c$ (b) $5a^2 + ab - b^2$ (c) $2a^2 - 4abc$
(d) $x^2y^2 + 2xy + 19$ (e) $-x^3 + x^2 - x + 2$

7. (a) $2a$ (b) $-2a$ (c) $3x - 4y + 5z$
(d) $-2x^2 - 5x + 6$ (e) $a^2 + b^2$

8. (a) $\frac{13}{4}a$ (b) $\frac{1}{2}ab$ (c) $2.5a^2b$ (d) $-3.5x - y - 1.5z$ (e) $0.5x^3 + 0.25xy + y^3$

9. (a) $-1.34a + 1.9b + 10c$ (b) $6.05a^2b - ab^2 + 2.03bc$
(c) $10.24p^2 - 2.89pq + 2.38q^2$ (d) $5.35x^2y + 2.4xy^2 - 1.75$

Exercise 8.4.1

1. (a) $12ab$ (b) $35ab$ (c) $3.6ab$
(d) x^7 (e) $2y^7$ (f) $4.8x^3y^3$
(g) $-p^4$ (h) $-6p^2q$ (i) $-17.5m^3n^3$
(j) $6a^2b^2$ (k) $6m^4n^4$ (l) $9.1z^2$
(m) $120x^6$ (o) $60m^3n^3$ (p) $36a^2b^3c^3$

2. (a) $(15ab)cm^2$ (b) $(56xy)cm^2$ (c) $(3.5p^2qr)cm^2$ (d) $(11.56m^2n^2)cm^2$

3. (a) $9abc\ cm^3$ (b) $9.1a^3\ cm^3$ (c) $\frac{11}{12}p^3\ cm^3$

Exercise 8.4.2

1. (a) $2x^2 - xy$ (b) $3ac - 6bc$ (c) $8x^2 - 12xy$
(d) $15a^2c + 10b^2c$ (e) $\frac{3}{5}x^3 + \frac{3}{5}xy^2$ (f) $\frac{1}{2}a^2 + \frac{1}{4}ab$

2. (a) $6x^3 + 6x$ (b) $2a^2 - 3a$ (c) $5xy - 10x + 4y$ (d) $x + 5y - 6$ (e) 0

3. (a) $a^2 + 2ab + b^2$ (b) $a^2 - 2ab + b^2$ (c) $a^2 - b^2$
(d) $2x^2 + 5xy + 3y^2$ (e) $4x^2 - 9y^2$ (f) $2m^3 + 2mn^2 - 3m^2n - 3n^3$
(g) $36q^2 - 25p^2$ (h) $\frac{x^2}{4} - \frac{y^2}{9}$ (i) $\frac{3}{8}x^2 + \frac{13}{16}xy + \frac{3}{8}y^2$
(j) $\frac{x^2}{y^2} - \frac{y^2}{x^2}$ (k) $3x^4 + 9.7x^2y^2 + 7.8y^4$ (l) $8.5p^2 - 0.99pq - 16.2q^2$

4. $x^3 + 5x^2y - 4xy^2 - 20y^3$; 135 5. $5x^2 - 3x - 2$; 24 6. $y^2 + yx - 2x^2$; 7

7. (a) $6x^2 + 2xy$ (b) $3.5q^2 + 2.1pq$ (c) $\frac{3}{4}x^2$
(d) $\frac{1}{4}q^2 + \frac{3}{4}pq$ (e) $2x^2 + 2xy + \frac{y^2}{2}$ (f) $\frac{a^2}{2} + 3\frac{1}{2}ab + 6b^2$
(g) $2.76x + 1.69y + xy + 4.6644$. (h) $7p^2 + 4.625pq + 0.75q^2$

Exercise 8.4.3

1. (a) $3x^2 + 2xy + 3xz - yz - y^2$ (b) $ax - bx + 4a + 2ay - 2by - 4b$
(c) $2ax - 3ay + 4az - 2bx + 3by - 4bz$ (d) $3a^3 + 3a^2b + 4a^2c + 2ab^2 + 4b^2c + 3b^3$

2. (a) $3x^3 + 16x^2 + 7x - 12$ (b) $-9y^3 - 6y^2 + 5y + 2$ (c) $4x^3 + 17x^2 + 16x + 3$
(d) $x^3 - a^3$ (e) $x^3 + y^3$ (f) $a^3 - b^3$

3. (a) $x^3 + x^2 - 5x + 28$ $\rightarrow 30$ (b) $10x^3 + 7x^2y + 3xy^2 + y^3$ $\rightarrow 172$

4. (a) $(2ax + 2ay + 2az + bx + by + bz)\ cm^2$ (b) $84ax\,cm^2$
(c) $6x^2cm^2$ (d) $12y^2cm^2$

Exercise 8.4.4

1. (a) $a^2 + 6a + 9$ (b) $y^2 - 8y + 16$ (c) $4x^2 + 12x + 9$
(d) $16 - 8x + x^2$ (e) $m^2 + 2mn + n^2$ (f) $9p^2 - 12pd + 4d^2$
(g) $81x^2 + 72xy + 16y^2$ (h) $16a^2 - 40ab + 25b^2$ (i) $643^2 - 48st + 9t^2$

2. (a) $x^4 - 4x^2y + 4y^2$ (b) $p^2q^2 + 2pqrs + r^2s^2$ (c) $a^4b^2 + 2a^2b^3c + b^4c^2$
(d) $4p^6 - 12p^3q^4 + 9q^8$ (e) $x^4 - 2x + \frac{1}{x^2}$ (f) $9a^6 + 1 + \frac{1}{36a^2}$
(g) $x^2 + 4xy + 4y^2 - 6xz - 12y^2 + 9z^2$
(h) $64a^2 + 80ab + 25b^2 - 112ac - 70bc + 49c^2$
(i) $p^4 + \frac{2}{p^2} + \frac{1}{p^4}q^4 - 2p^2q^2 - \frac{2}{p^2} + q^4$ (j) $x^4 + 6x^2y^2 + y^4 + 4x^3y + 4xy^3$

3 (a) 7 (b) 27 (c) 13,11 (d) $40x^2 - 48xy + 34y^2$ (e) $-14m + 78$

4. (a) $a^3 + b^3$ (b) $a^3 - b^3$

6. (a) $2(a^2 + b^2)$ (b) $4ab$ (c) $2(a^2 + b^2)$ (d) $-4ab$

Exercise 8.4.5

1. (a) $a^3 + 9a^2 + 27a + 27$ (b) $8x^3 + 12x^2 + 6x + 1$
(c) $27a^3 + 27a^2b + 9ab^2 + b^3$ (d) $27x^3 + 108x^2y + 144xy^2 + 64y^3$
(e) $x^3y^3 + 3x^2y^3z + 3xy^3z^2 + y^3z^3$
(f) $a^3 + 3a^2b + 3ab^2 + b^3 + 6a^2c + 12abc + 6b^2c + 12ac^2 + 12bc^2 + 8c^3$

(g) $27a^3 + 135a^2b + 225ab^2 + 125b^3$ (h) $x^6 + 6x^4y + 12x^2y^2 + 8y^3$

(i) $x^3 - 6x^2 + 12x - 8$ (j) $8x^3 - 12x^2 + 6x - 1$

(k) $8 - 36a + 54a^2 - 27a^3$ (l) $27 - 108a + 144a^2 - 64a^3$

(m) $8a^3 - 36a^2b + 54ab^2 - 27b^3$ (h) $8x^3 - 60x^2y + 150xy^2 - 125y^3$

(0) $8a^3 - 12a^2b + 6ab^2 - b^3 - 12a^2c + 12abc - 3b^2c + 6ac^2 - 3bc^2 - c^3$

(p) $8x^3 - 36x^2y + 54xy^2 - 27y^3 - 12x^2z + 36xyz - 27y^2z + 6xz^2 - 9yz^2 - z^3$

2. (a) 90 (b)511 3. (a) 8 (b) 27 (c) -125

4. (b) 52 (c)36 (d)140 (e)$p^3 + 3p$

5. (a) $2a^3 + 3ab(a + b)$ (b) $2a^3 - a^2b + 3ab^2 + ab$

(c) $2x^3 - 2x^2y + 6xy^2$ (d) $-3x^2y - 3xy^2 - 3xy$

Exercise 8.5

1. (a) p= $\frac{pq}{q}$ d= $\frac{pq}{p}$ (b) m= $\frac{m\times n}{n}$, n= $\frac{m\times n}{m}$

(e) 2a= $\frac{8ab}{4b}$, 4b= $\frac{8ab}{2a}$ (d) $2x^2$= $\frac{6x^3}{3x}$; 3x= $\frac{6x^3}{2x^2}$

(e) $3a^2b$= $\frac{6a^2b^2}{2b}$, 2b= $\frac{6a^2b^2}{3a^2b}$ (f) $3p^3q^2$= $\frac{12p^5q^5}{4p^2q^3}$; $4p^2q^3$= $\frac{12p^5q^5}{3p^3q^2}$

(g) $16y^2z^4$= $\frac{64y^5z^5}{4y^3z}$; $4y^3z$= $\frac{64y^5z^5}{16y^2z^4}$

2. (a) 6x (b) 25y (c) $20z^2$ (d) -3ab (e) -20pq (f)$30x^4y$ (g) $\frac{5}{2}$ (h)$-6y^2z$

3. (a) 2y (b) 7p (c) 3mn (d)7y4 (e) 8m

4. (a) $2a^2 + 3b^2$ (b) $2a^2-3b^2$ (c) $-5x^2 + 20x$ (d) $-0.4xy + 3x^2$

(e) $2y^2 + xy$ (f) $-\frac{1}{5}$ x + 5-y (g) $-3x^2 + 2x + \frac{1}{3}$ (h)- $\frac{x^2}{7} + \frac{2}{15}x - \frac{19}{7}$

5. (a) (a + 2b) (b) 2(3a + 2b) (c) 26cm

6. (a) 8xy (b) 192m

Exercise 8.6

1. (a) $2a^2-ab-b^2$ (b) $4p-3q+15$ (c) $-7y+7z$ (d) x^2+13x (e) $xy-x-5y-y^2$

2. (a) $\frac{2a}{3c}$ (b) $\frac{3x}{4z}$ (c) $\frac{3}{r}$ (d) $\frac{9x}{5y}$ (e) $\frac{3p}{4q}$ (f) 1

   (g) $\frac{a}{4b}$ (h) $\frac{1}{pr^2}$ (i) $\frac{3}{5x}$ (j) 1

3. (a) 1 (b) 1 (c) 1 (d) 1 (e) $\frac{2}{x+2}$ (f) $\frac{2x+3}{2x+1}$
4. (a) 12x cm (b) $9x^2 - y^2$ - cm^2 (c) 24cm, $27cm^2$
5. (a) 4(5a-3b)cm (b) $(5a-3b)^2$ cm^2 (c) 32cm, $64cm^2$
6. (a) 2(2x-3)cm (b) $x^2-3x cm^2$ (c) 22cm, $28cm^2$
7. (3x + 4y)cm 8. (a) 52cm (b) $153cm^2$

Exercise 9.1 show the answer to the teacher

Exercise 9.2.1

1. (a) 7 (b) 4 (c) -1 (d) $\frac{4}{3}$ (e) -14 (f) -3 (g) $\frac{8}{9}$ (h) 2.5 (i) 0 (j) -20
2. (a) 4 (b) 18 (c) $\frac{11}{2}$ (d) $\frac{15}{13}$ (e) 2 (f) $\frac{14}{5}$ (g) $\frac{25}{66}$ (h) 2 (i) 1.5 (j) $-\frac{12}{11}$

Exercise 9.2.2

1. Rs.. 5 2. 4
3. 250 boys ,150girls 4. 15 and 30
5. (a) x boys , $\frac{2}{3}$ x girls, $x + \frac{2}{3}x = 40$ (b) 24 boys 16 girls
6. (a) 315 (b) 105
7. 9 and 10 8. 19,20 and 21
9. 33 and 66 10. (a) 2(2x-3)=18 (b) 6cm and 3cm

Exercise 9.3 show the answer to the teacher.

Exercise 10.1

5. (a) (-7,7), (-7,9), (-7,11), (-7,13), (-12, 9) (-12,11), (-4,10)
   (b) (-7,1), (-7,-5) , (-10,1), (-10,-5), (-12,-2)
6. (b) x = 7
8. (a) D (2,-2) (b) AD = 7, BC = 8 (c) ABC = 28 sq. unit
9. (a) D(3, -6) (b) E (1,-1)
10. (a) S(4,-2) (b) Rectangle ABCD=30 sq. unit

Exercise 10.2.1

1. (a) A (b) C (c)AC (d) G (e) L (f) J
   (g) JL (h) $\angle$IHG (i) HG (j) ΔCXA

Exercise 10.2.2

1. (a) Two revolutions (b) half revolution
   (c) One fourth revolution (d) Three fourths revolution
2. (a) 15 minutes (b) 12 hours
   (c) 3 hours (d) 30 Seconds
3. (a) $22\frac{1}{2}$ Revolutions (b) 450 Revolutions
   (c) 3 Revolutions (d) $\frac{3}{4}$ Revolution

Exercise-10.3

1. (a)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	3	4	5	6	7	8	9	10	11	12	13

(b)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	-5	-4	-3	-2	-1	0	1	2	3	4	5

(c)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	0	5	10	15	20	25	30	35	40	45	50

(d)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	-3	-2	-1	0	1	2	3	4	5	6	7

(e)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	1	3	5	7	9	11	13	15	17	19	21

(f)

No Put in the machine(x)	0	1	2	3	4	5	6	7	8	9	10
No Produced in the machine(y)	1	4	7	10	13	16	19	22	15	18	31

2. (a) $y = x + 3$ (b) $y = x-5$ (c) $y = 5x$ (d) $y = x-4$
   (e) $y = 2x + 1$ (f) $y = 3x + 1$

5. (a) $y = 2x$ 7. (b) $y = x$ 8. $y = x^2$

Geometry

Exercise 11.1.1

1. $80^0, 68^0, 50^0, 55^0, 43^0, 37^0, 30^0, 23^0, 17\frac{1^0}{2}, 6\frac{3^0}{4}$

2. $145^0, 135^0, 130^0, 115^0, 102^0, 95^0, 85^0, 60^0, 44\frac{1^0}{2}, 12\frac{1^0}{2}$

3. (a) 68^0 (b) 27^0 (c) 25^0 (d) $60^0, 28^0, 30^0, 62^0$
   (e) 115^0 (f) 128^0 (g) 30^0 (h) $30^0, 28^0, 30^0, 72^0$
   (i) $150^0, 30^0, 150^0$ (j) $25^0, 155^0, 25^0$
4. (a) $120^0, 60^0, 120^0, 60^0, 120^0, 60^0$ (b)180^0
   (c) 180^0 (d) yes (e) yes

Exercise 11.1.2

1. (a) 1 and 5, 2and 6, 3and 7, 4and 8 (b) 3 and 6, 4 and 5
   (c) 4and 6, 3and 5 Show 2,3 and 4 to the teacher

Exercise 11.2

1. 1,2,3, and 4 (a) Shot it to the teacher (b)Bisects
5. (a) Show it to the teacher (b) Intersects
6. Show it to the teacher 7. Triangle
8. Show to the teacher

Show all the figures of Exercise 11.31,11.3.2, 11.3.3 and 11.3.4 to the teacher

Exercise 11.4

1. (a) $48cm^2$ (b)$56cm^2$ (c) $27cm^2$ (d) $126cm^2$ (e) $6cm^2$
   (f) $24cm^2$2. (a) $24cm^2$ (b) $20cm^2$ (c) $84cm^2$

3. (a) $204cm^2$ (b) $1700cm^2$ (c) $164cm^2$
4. (a) $17.1cm^2$ (b) $24cm^2$ (c) 9cm (d) 4.4cm (e) $96cm^2$
5. (a) $48cm^2$ (b) $48cm^2$ (c) $24cm^2$ (d) $48cm^2$ (e) $28cm^2$
6. $20cm^2$ 7. 1cm 8. $44cm^2$

Exercise 11.5.1

1.	(a) $484.4cm^2$	(b) $243.4cm^2$	(c) $96cm^2$	(d) $73.5cm^2$	
2.	(a) $166cm^2$	(b) $350cm^2$	(c) $476.4cm^2$	(d) $136.32cm^2$	
3.	(a) $64cm^2$	(b) $216cm^2$	(c) $73.5cm^2$	(d) $162.24cm^2$	
4.	(a) $43200cm^2$	(b) $9600cm^2$			

5. (a) Show it to the teacher (b) $376cm^2$ (c) $296\ cm^2$
6. 1.5cm
7. $13.5cm^2$

Exercise 11.5.2

1. (a) $20.25cm^3$ (b)$180cm^3$ (c)$180cm^3$ (d) $384cm^2$ (e) $7.5cm^2$
   (f) $93.75cm^3$ (g)$384cm^3$ (h)$1152000cm^3$ (i)$288cm^3$

2.	(a) $480cm^3$	(b) $450cm^3$	(c) $52cm^3$	(d) $200cm^3$	(e) $1080cm^3$
3.	(a) $216cm^3$	(b) $140.6cm^3$	(c) $421.9cm^3$	(d) 551.4	
4.	(a) 2,000 lt.	(b) 30000 lt.	(c) 73,500 lt.	(d) 1,20,00,000 lt.	
5.	(a) $60m^3$	(b) 60,000 lt	(c) 4,000 lt		
6.	(a) $160mm^3$	(b) $8000mm^3$			
7.	(a) 3cm	(b) 37			
8.	$576cm^3$	9.$144cm^3$	10.12cm	11.9m	

Exercise 11.6

1. Show 1and 2 to the teacher
2. wrong , wrong , Right ,Right , Wrong , Right , Right

Exercise 11.7

1. Show it to the teacher
2. Circle 3. Circle

Exercise 11.8 Show the answer to the teacher

Exercise 11.9.1

1. 3,2,4,2,3,6,2,4 2. Show to the teacher

Exercise 11.9.2

1. 6 Linear Symbolically circular , symmelry grades -6

2.

Figure	Linear Similarity	Grade of Circle Similarity
a	1	1
b	2	2
c	0	6
d	0	1
e	3	6
f	4	4

3.

Figure	Linear Similarity	Grade of Circle Similarity
a	4	4
b	2	1
c	2	2
d	0	2
e	1	1
f	3	3

Exercise 11.9.3

1. Show to the teacher.
2. b and f regular, c, e and h semi regular, the rest irregular.
3. Show to the teacher

Exercise 11.10.1 show the answer to the teacher.

Exercise11.10.2

4. Show 1,2,3 and 4 to the teacher.
5. (a) 7.5 km
   (b) 6.75 km
   (c) 13.95 km.
6. Show to the teacher
7. 1:500 8.3m

Show 9 and 10 to the teacher