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Nico: An Environment for Mathematical Expression in Schools

Computer Science Tripos

Selwyn College

April 22, 2012

Proforma

Name:	Philip Yeeles
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Project Originator:	P. M. Yeeles (pmy22)
Supervisors:	Dr S. J. Aaron (sja55), A. G. Stead (ags46)

Original Aims of the Project

The aim of the project was to develop an application in the Clojure programming language which would allow users to express mathematical calculations using a graphical notation. The software was to be able to generate an abstract syntax tree from the graphical notation, evaluate it and pass the results back to the application in under 300ms. An extension to the project was to conduct a user study to evaluate the utility of the software.

Work Completed

I have successfully designed and implemented the application detailed in the previous section. That is, I have developed an application in which it is possible to express calculations using a graphical notation, that generates an abstract syntax tree from the language and that is able to parse the tree and return the results in

[1] This word count was computed by `detex diss.tex | tr -cd '0-9A-Za-z \n' | wc -w`

under 300ms. I have also conducted a user study to assess whether or not the software is actually of use with regard to mathematics education.

Special Difficulties

Learning the Clojure programming language.

Declaration of Originality

I, Philip Michael Yeeles of Selwyn College, being a candidate for Part II of the Computer Science Tripos, hereby declare that this dissertation and the work described in it are my own work, unaided except as may be specified below, and that the dissertation does not contain material that has already been used to any substantial extent for a comparable purpose.

Signed

Date April 22, 2012

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Acknowledgements

My thanks to Luke Church for his advice regarding user studies, and to Alistair Stead and Sam Aaron for their encouragement and patience.

Chapter 1

Introduction

The aim of this project has been to design and develop a notation and accompanying application to act as a learning aid for pre-algebra arithmetic by increasing visibility, reducing the number of hidden dependencies and making the flow of data obvious to the user. I have successfully developed such a system, intended initially for pupils in Year 5 (though extensible, through the creation of alternative question sets, to other age groups), and, as an extension, conducted a user study to assess its utility.

In this chapter, I will discuss my motivations for choosing this project, the pros and cons of the handwritten system it is attempting to augment, the technical challenges involved in developing such a system and related work that has previously been conducted with similar goals.

1.1 Motivations

My motivations behind this project lay in the limitations of the handwritten approach to solving mathematical problems that I had observed both in my own learning and in my own teaching experience. What follows is an evaluation of the pros and cons of the handwritten method of performing arithmetic calculations according to Blackwell and Green's "Cognitive Dimensions" framework [2], and a discussion of the properties a useful alternative notation should have.

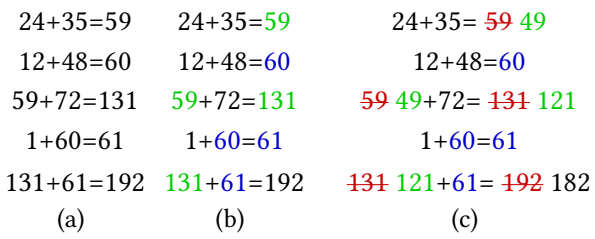

Figure 1.1: Illustrating the hidden dependencies and viscosity inherent in pre-algebra, handwritten arithmetic. The original calculation is shown in (a), has its otherwise-hidden dependencies highlighted in (b), and is altered slightly in (c).

1.1.1 Evaluation of the Handwritten Approach

There are a number of problems with handwritten, pre-algebra arithmetic that this project seeks to rectify. First of all, the fact that it is handwritten entails a high level of viscosity: it is difficult to make changes to a written calculation without sacrificing clarity. In particular, there is a lot of repetition viscosity involved in the modification of an existing piece of work; if a number is changed that is used in several calculations, then it is time- consuming to change it everywhere it appears in the working. If several calculations are dependent upon each other, then this entails a lot of knock-on viscosity in recalculating each stage after changing the number. This is exacerbated by the hidden dependencies between chained calculations in handwritten arithmetic (*Fig. 1.1*).

1.1.2 Requirements of a Replacement System

To improve upon the standard approach of listing the steps comprising a calculation, a system must acknowledge and try to overcome the drawbacks listed above. To this end, I have designed and developed a system that aims to eliminate in particular the many hidden dependencies of traditional, handwritten arithmetic.

1.2 Technical Challenges

Developing such an application comprises two main challenges: developing a backend that is capable of creating, storing, editing, deleting, evaluating, nesting and calculations, and a graphical, user-facing frontend that is able to render calculations into the devised notation, and allow the user to perform operations upon the notation that affect the underlying calculation.

I chose to use the Clojure language as it provided many features that would prove to be useful over the course of the application's development. As a dialect of LISP, Clojure is a homoiconic programming language – that is, a programming language in which code is represented as a data structure – which made passing around and performing operations upon calculations themselves, rather than just their results, considerably easier. A calculation can simply be represented as a piece of code, which can then be utilised as needed.

As the user experience is so crucial to the success of the application, it was also important that there be well-established GUI libraries available. Clojure runs on the Java Virtual Machine (JVM), which puts Java's considerable standard library at one's disposal, whilst still being able to program in a LISP. As I am familiar with Java and the Swing GUI libraries, it was advantageous to be able to leverage this knowledge in designing the application's interface.

1.3 Previous Work

There already exists a wide variety of educational software for mathematics, but much of this is in the form of "games", in which a series of mathematical problems to be solved is poorly disguised as a game – indeed, such problems would be more accurately said to be embedded into a game, rather than becoming the game themselves. Thus, the object becomes not to solve the problems, but to play the game that happens to surround the problems. Such software also does not often offer any means of solving the problems, other than the traditional pen-and-paper method (with a piece of paper next to the computer screen), or the mental approach. Hence, what the user is then presented with is essentially a game and a worksheet, awkwardly interleaved. In some cases, it is even possible for the user to simply press arbitrary buttons until they pass the questions, effectively removing the maths element of the game and replacing it with a series of short breaks in gameplay.

There also exist a few applications intended to represent calculations on a computer in novel ways. A relatively common approach to this has been to try to make on-screen calculations more like on-paper calculations. *Pi Cubed* takes this approach by trying to make complex calculations appear as they would be written in an exam or exercise book [6]. *Soulver*, conversely, tries to achieve this by simulating "back-of-the-envelope" calculations, whereby notes in English augment the calculation [7]. Another approach is that of the *Scrubbing Calculator* [11], which extends the *Soulver*-style environment by helping the user to solve equations by dragging values to increase and decrease them, showing how changing a value

affects the overall result. Values can be linked by dragging a line between them, which means that they are two instances of the same value – hence dragging one changes the value at every location in which it appears. This is a neat means of visualising equations, but it, too, is not intended for use in education, and still requires the user to be able to formulate some kind of equation. The *Scrubbing Calculator* is more a tool for facilitating algebraic understanding, as opposed to arithmetic understanding; indeed, it is inherently a **calculator**, and so does not encourage thinking about how to work out the arithmetic parts of a calculation manually.

1.4 Summary

Existing educational "games" for mathematics either have too much focus on being a game, rather than helping to learn mathematics, or are such that the mathematical element is circumventable. There exists software to aid in calculation and arithmetic by representing it clearly, but it is not intended for educational use, and often its purpose is to make on-screen calculations appear as one would handwrite them.

There is a niche for a tool for use in education that represents calculations in a visual manner, with a particular focus on making the method by which arithmetic problems are solved clear. My project aims to provide an environment in which the user can explore the many ways in which a problem can be solved using a novel graphical notation.

Chapter 2

Preparation

In this chapter I will discuss the work that was done prior to beginning the project proper. This includes learning the Clojure programming language and researching and auditioning graphical metaphors for calculation. This chapter comprises a requirements analysis, followed by an overview of the system architecture and a discussion of the additional tools used in the development of the project.

2.1 Requirements Analysis

2.1.1 Current System

2.1.2 Proposed System

2.1.2.1 Overview

2.1.2.2 Functional Requirements

To be an improvement upon the current system detailed above, the new system must satisfy the following properties:–

- lol

2.1.2.3 Non-Functional Requirements

The system must also satisfy a number of requirements outside of its basic functionality. These are listed below.

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2.1.2.4 Use Case

2.2 User Interface

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2.2.1 Prototyping

2.2.1.1 Low-Fidelity Prototyping

(a) (b)

(c) (d)

Figure 2.1: Empress Alexandra II Sexah

2.2.1.2 Preliminary Designs

Some preliminary designs for the application and graphical language follow. First is a combined flowgraph representation and Read-Evaluate-Print-Loop (REPL) design. The second design uses the Sierpiński triangle as a basis for the visual metaphor. The final design is the circle-based language that eventually became the basis for the rest of the project.

2.2.1.2.1 Flowgraph Metaphor

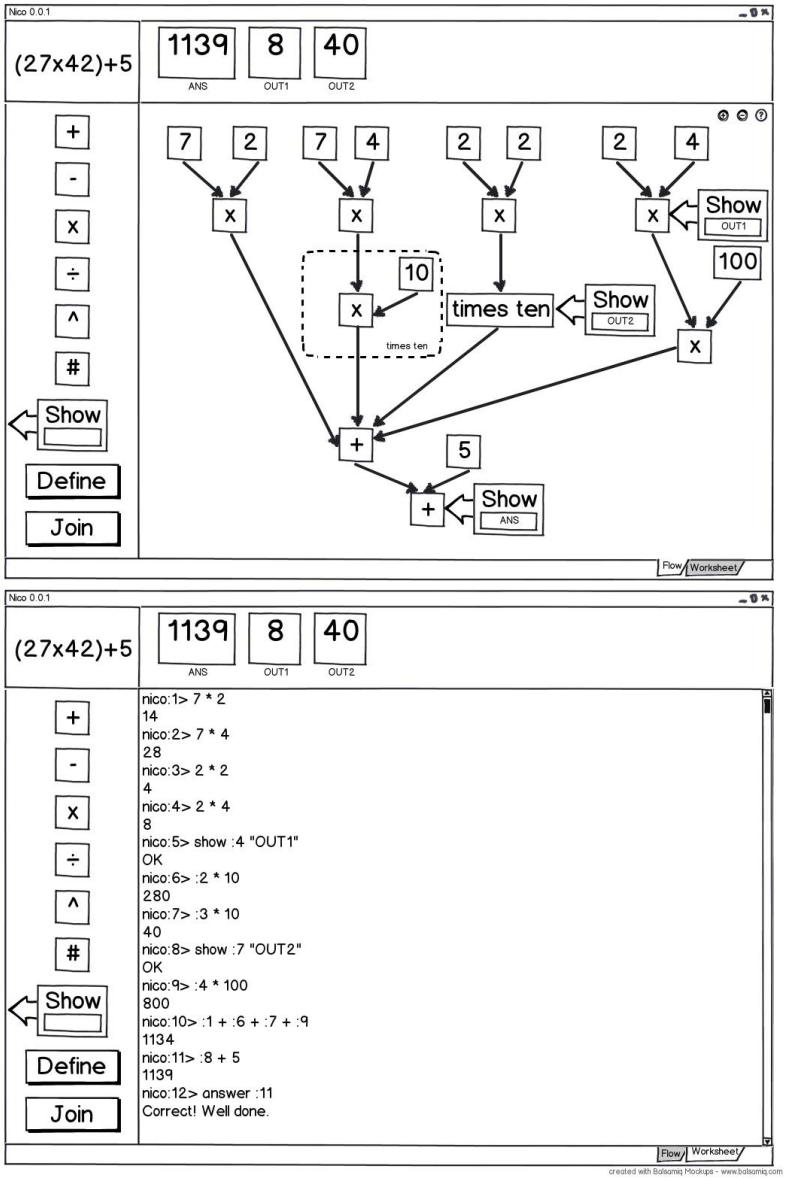

Figure 2.2: The prototype flowgraph-style language.

The flowgraph idea initially outlined in the project proposal and included in the low-fidelity prototypes above appears here as a more mature, revised prototype, showing two screens of a sample application. The top screen shows the proposed application manipulating the graphical language, whilst the bottom screen shows a REPL utilising a simple text-based language for users more comfortable with writing than the visual metaphor.

The application

Of all the prototypes shown here, this version of the graphical language is the most like a traditional programming language, in that it allows the user to define reusable functions, and includes functionality similar to print statements. The icons at the side of the application are used to select the desired function, from a set that includes addition, subtraction, multiplication, division, exponentiation, number input, Show, Define and Join. Addition, subtraction, multiplication, division and exponentiation are all dragged-and-dropped from the icons at the side to a point on the canvas, causing a box containing their respective functions to appear at that point. Number input (#) is similar, but the user inputs a number using the keyboard after the box is created. The box then contains that number. Show displays the output of the junction to which its arrow points, in the output box at the top of the screen with the label input by the user after placing it. If the box with the label does not yet exist, it is created next to the existing output boxes. Define and Join are not boxes to be dragged-and-dropped; they are modes of operation. When Define is activated, dragging the mouse on the canvas draws a box around sections of the diagram – the user is able to define functions by doing so, and by providing a label for the section of diagram that has been highlighted, a corresponding box can be dragged-and-dropped from the list of icons (as show in *Fig. 2.2* with the times ten function). When Join is active, dragging between two points on the canvas creates an arrow between them that means that the source of the arrow is used as an argument to the destination of the arrow. In defining functions, if arrows cross the boundary of the definition box and they are incoming, input is required to future uses of the function. One outgong arrow is allowed to indicate the output of the function. Using this notation, answers are submitted by Showing the output at a point in the calculation to the output box ANS.

The textual language improves upon the traditional means of handwriting arithmetic by revealing otherwise-hidden dependencies using references to line numbers. As the target audience is not yet required to have formally learnt (or, indeed, encountered) algebra, this notation includes a function `:`, which takes a single number *n* as an argument and returns the result of the calculation performed on line *n*. Answers are submitted using a function `answer` that takes a single number (shown here using a line reference that is resolved to a number) as an argument. Other functions include addition, subtraction, multiplication, division and exponentiation, all of which take two or more numerical arguments and are used in the familiar infix form. A function `define` is also included, although not shown here. Finally, there is the `show` function, which behaves similarly to a print statement. It takes a numerical argument *n* followed by a string argument `str`, and displays *n* in the output box named `str` at the top of the screen. If there exists no

such output box, it is created next to the existing boxes. Note that `answer n` is logically equivalent to `show n "ANS"`; the `answer` function was included here to increase clarity.

2.2.1.2.2 Sierpiński Triangle Metaphor

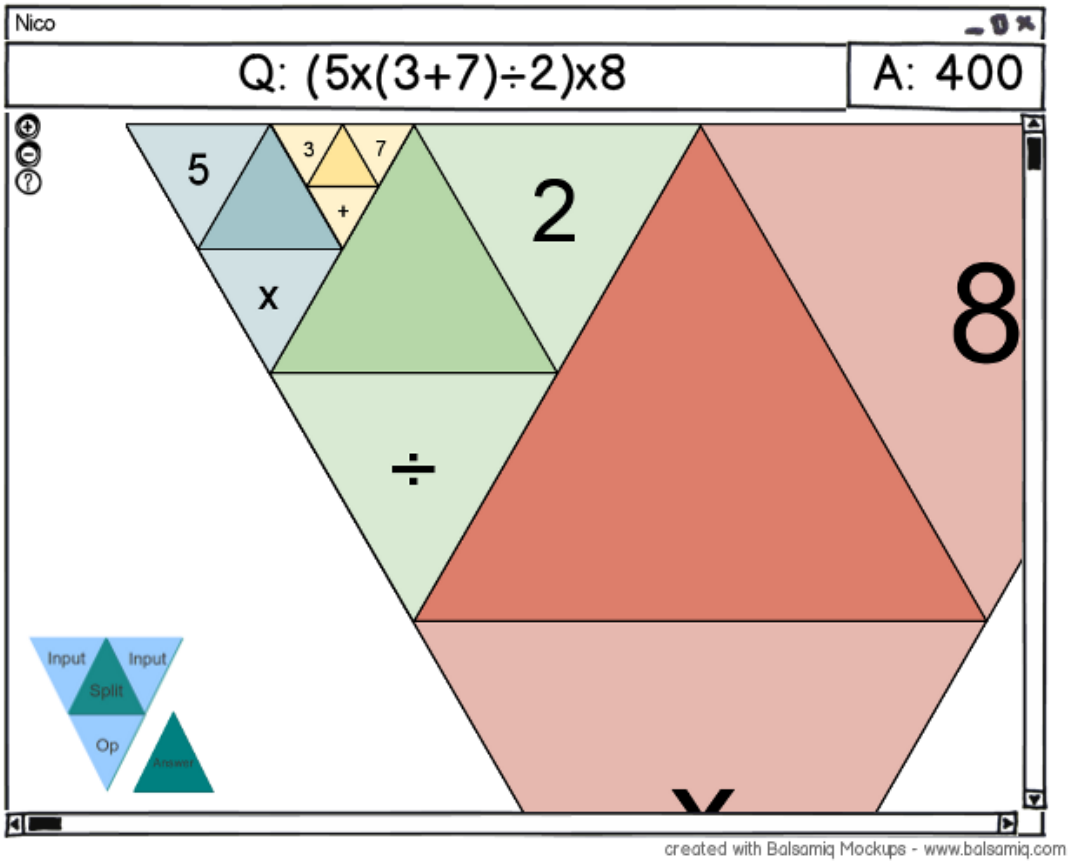

Figure 2.3: The prototype Sierpiński-triangle-based language.

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2.2.1.2.3 Circle Metaphor

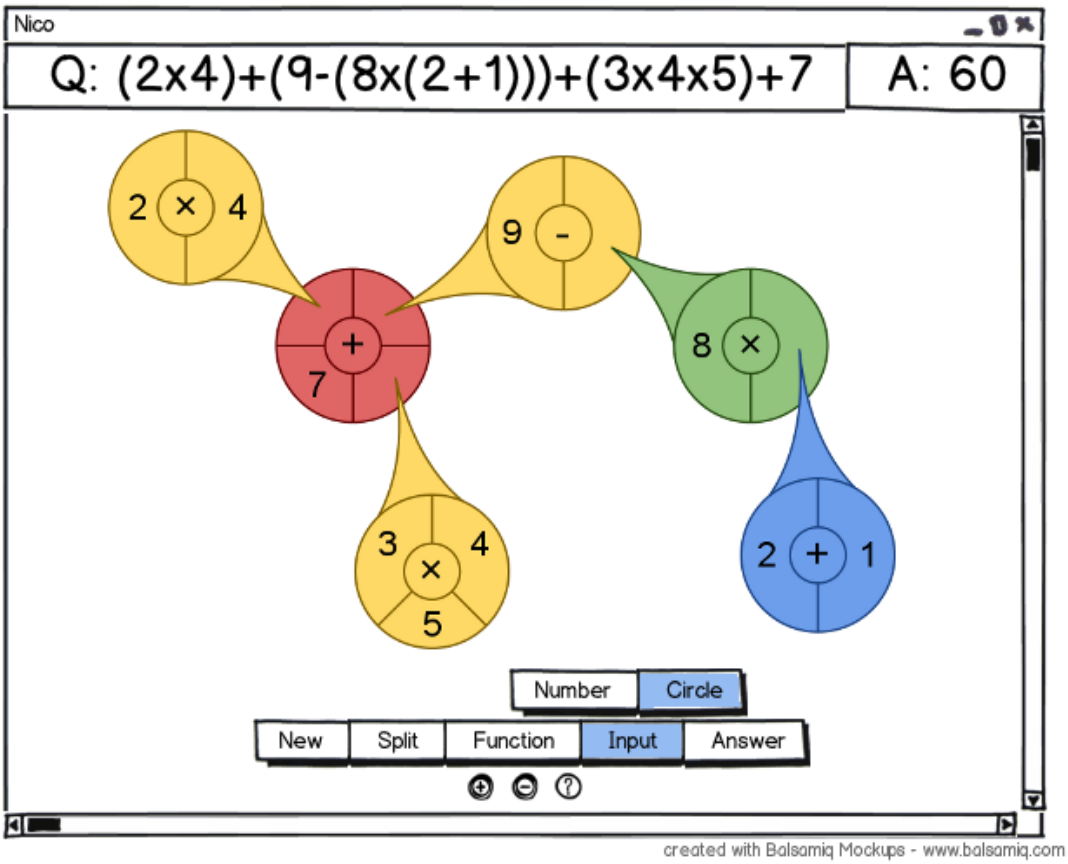

Figure 2.4: The prototype circle-based language.

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2.3 Additional Tools

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2.3.1 Third-Party Tools

What follows is a list of the third-party tools that were used in the development of the project.

- Ubuntu Linux 10.04, Arch Linux 2010.05, Microsoft Windows 7
- Clojure 1.2.0

- Leiningen 1.6.1.1
- OpenJDK 6
- Seesaw 1.3.1-SNAPSHOT
- swank-clojure 1.3.4-SNAPSHOT
- GNU Emacs 23.1.1
- A modified version of Overtone's Emacs configuration [9], including:–
  - SLIME/SWANK (revision as of 15/10/2009)
  - clojure-mode 1.11.5
  - undo-tree 0.3.3
- Git 1.7.0.4
- GitHub
- Balsamiq Mockups
- Google Docs

2.4 Summary

Chapter 3

Implementation

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3.1 Backend

3.2 User Interface

3.2.1 Metaphor

3.2.2 Application

3.3 Summary

Chapter 4

Evaluation

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4.1 Backend Testing

4.2 UI Evaluation

4.3 Language Evaluation

4.4 Summary

Chapter 5

Conclusions

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Bibliography

[1] R. Abraham. The Trouble with Math. *Educating the Whole Child for the Whole World: The Ross School Model and Education for the Global Era*, pages 125–137, 2010.

[2] A. F. Blackwell and T. R. G. Green. Cognitive dimensions of information artefacts: a tutorial, 1998.

[3] A. F. Blackwell and T. R. G. Green. Does metaphor increase visual language usability?, 1999.

[4] A. F. Blackwell and T. R. G. Green. A cognitive dimensions questionnaire optimised for users, 2000.

[5] A. F. Blackwell and T. R. G. Green. Notational systems – the cognitive dimensions of notations framework. *HCI Models, Theories, and Frameworks: Toward a Multidisciplinary Science*, 2003.

[6] Sunset Lake Software LLC. Pi Cubed | Sunset Lake Software. `http://www.sunsetlakesoftware.com/picubed`, 2012.

[7] Acqualia Software Pty. Ltd. Soulver | Acqualia. `http://www.acqualia.com/soulver/`, 2011.

[8] B. A. Myers. Taxonomies of visual programming and program visualization. *Journal of Visual Languages and Computing*, 1:97–123, 1990.

[9] samaaron and jlr. overtone/live-coding-emacs. `https://github.com/overtone/live-coding-emacs`, 2011.

[10] B. Victor. Kill Math. `http://worrydream.com/KillMath/`, 2011.

[11] B. Victor. Scrubbing Calculator. `http://worrydream.com/ScrubbingCalculator/`, 2011.

Appendix A

Project Proposal

The original project proposal follows.

Computer Science Part II Project Proposal

Nico: An Environment for Mathematical Expression in Schools

P. M. Yeeles, Selwyn College
Originator: P. M. Yeeles
18 October 2011

Special Resources Required

- PWF account
- SRCF account
- GitHub account
- Toshiba Satellite L500-19X (Intel Pentium T4300 2.0GHz, 4GB RAM, 500GB disk)
- Samsung NC10 Plus (Intel Atom 1.66GHz, 1GB RAM, 250GB disk)

Project Supervisors: Dr S. J. Aaron & A. G. Stead
Director of Studies: Dr R. R. Watts
Project Overseers: Dr J. A. Crowcroft & Dr S. Clark

Introduction

Discussions with local teachers have led me to hypothesise that educational software for mathematics could be used to reinforce learning by focussing on method, rather than on a numerical answer. My aim is to develop a problem-solving system aimed at pupils in year 5 in which the solution to a problem can be represented as a tree of operations – a block-based graphical language to describe mathematical method. The correctness of the solution is then assessed with respect to the structure of the tree. The application will be written in Clojure, using JavaFX 2 for the graphical elements, though if this becomes infeasible I will use either the Eclipse SWT or Swing with GUIFTW. This dissertation will determine whether Nico offers an improvement regarding pupils' ability to recall the correct method for answering mathematical problems. The success of the project will be gauged by whether or not the software is able to generate an abstract syntax tree in Clojure from the graphical language and evaluate such a tree, passing the results back to the graphical application and displaying this to user in less than 300ms[1]. As an extension, I will distribute Nico with anonymous feedback forms to local schools, to determine if the software is actually of use in the classroom.

Work that has to be done

The project breaks down into the following sections:–

1. Core system
    a. A syntax for questions and a means of loading them
    b. A set of basic functions available to the student
    c. A means of inputting an answer that can be evaluated on-the-fly
    d. A means of re-expressing the question to reflect how the student works (e.g. $12 \times 34 \Rightarrow (10 \times 34) + (2 \times 34)$)
    e. A method of validating the answer
    f. A means of tracking the current result of evaluating the method input so far
    g. A system of hints for students who may not know where to start
2. GUI
    a. A collection of drag-and-drop elements that can be used to construct a diagram representing how to solve the question

[1] *Interactive multimedia and next generation networks: Second International Workshop on Multimedia Interactive Protocols and Systems, MIPS 2004 Grenoble, France, November 2004, Proceedings (LNCS 3311)* by Roca and Rousseau has this to say on interactivity: "An abundance of studies into user tolerance of round-trip latency [...] has been conducted and generally agrees upon the following levels of tolerance: excellent, 0-300ms; good, 300-600ms; poor, 600-700ms; and quality becomes unacceptable [...] in excess of 700ms."

   b. A means of validating combinations of the drag-and-drop elements
   c. A means of defining functions
   d. A means of viewing documentation
3. Evaluation
   a. Test software on non-technical but mathematically-able subjects
   b. Evaluate the correctness of Nico's translations between diagram and code
4. Extensions
   a. Create and distribute questionnaires to test classes
   b. Collect and interpret data
   c. Create a tutorial mode for new users

Difficulties to Overcome

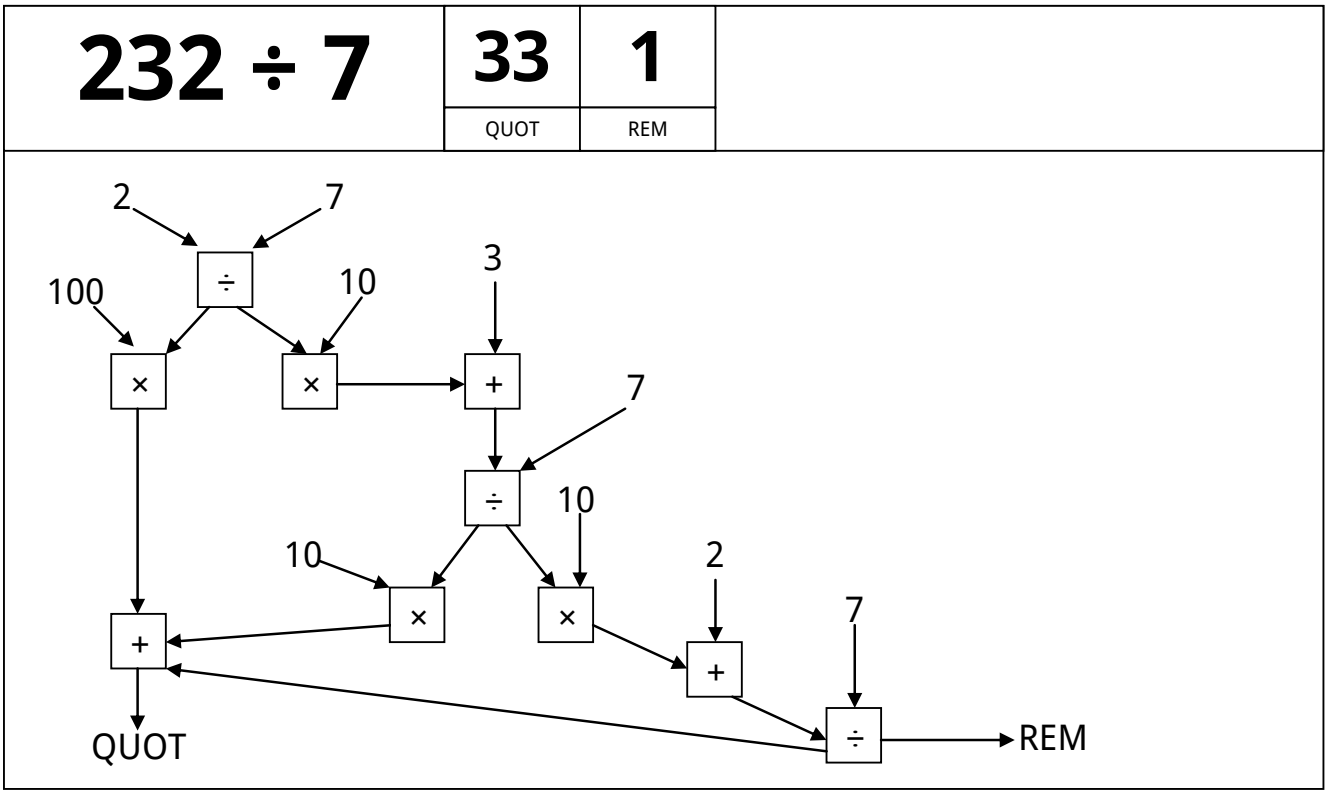
The following main learning tasks will have to be undertaken before the project can be started:

- Learn Clojure
- Become familiar with JavaFX 2
- Spend time designing the GUI and language

Starting Point

I have spent some months learning Clojure, and continue to do so. I have a good working knowledge of Java and experience teaching Mathematics and IT in Years 4 to 6. The first two years of the undergraduate course have familiarised me with Java and its libraries, and thanks to Clojure's interoperability I will be able to leverage these skills for this dissertation. My experience in schools has allowed me to develop the idea for this dissertation, and has given me an insight into what resources are useful in the classroom.

Below is a mockup of what I aim for Nico to look like. Notice how the method is expressed in the form of a flowchart, with outputs (in this case two) for the answer. Arrows show the direction of input and output, and the QUOT and REM boxes show the result of evaluating the functions being passed to them. Ideally, the question "232 ÷ 7" would also change to reflect how the student breaks down the question.



The above solution would auto-generate the following abstract syntax tree represented in Clojure code:–

QUOT is the output of:–

```
(+
  (*
    100
    (:quot
      (div
        2
        7)))
  (*
    10
    (:quot
      (div
        (+
          (*
            (:rem
              (div
                2
                7))
            10)
          3)
        7)))
  (:quot
    (div
      (+
        (*
          (:rem
            (div
              (+
                (*
                  (:quot
                    (div
                      2
                      7))
                  10)
                3)
              7))
            10)
          2)
        7)))
```

REM is the output of:–

```
(:rem
  (div
    (+
      (*
        (:rem
          (div
            (+
              (*
                (:rem
                  (div
                    2
                    7))
                10)
              3)
            7))
          10)
        2)
      7))
```

This assumes that we have a function `div` that takes two arguments `x` and `y` and returns an associative map `{:quot q :rem r}` such that `q` is the quotient of $x \div y$ and `r` is the remainder. Such a function will be included in the basic functions available to the user. Other functions of use would be addition, multiplication, subtraction, exponentiation, function definition and commenting (i.e. labels that are not evaluated), with options available in the question syntax (e.g. `:inhibit+ true`) to restrict arguments to a value of less than or equal to 10 (useful, for example, in questions on long multiplication, to prevent the student from simply giving `(* a b)` as the answer to $a \times b$). Hence a possible means of representing the question above could be:–

```
{:title “232 ÷ 7”
 :topic “arithmetic”
 :answer {:quot 33
          :rem 1}
 :inhibit+ false
 :inhibit- false
 :inhibit* false
 :inhibitdiv true}
```

Resources

This project requires little file space so my Toshiba PC’s disk should be sufficient. I plan to use the same PC as well as my Samsung PC to work on the project, and to back my files up to the PWF, the SRCF and GitHub. I will be using Git for version control.

Work Plan

Planned starting date is 27/10/2011.

October 2011

27/10/2011 - 10/11/2011
Work begins. Start covering the problems outlined in *Difficulties to Overcome*. Design the look and feel of the language and application.

November 2011

10/11/2011 - 24/11/2011
Design the question syntax. Implement the question interpreter. Implement the tree evaluator.

24/11/2011 - 08/12/2011
Implement the hints system. Begin work on the GUI.

December 2011

08/12/2011 - 22/12/2011
Finish the non-language section of the GUI. Begin implementing the graphical language.

29/12/2011 - 12/01/2012
Finish implementing the graphical language and its interpreter.

January 2012

12/01/2012 - 26/01/2012
Finish coding the core project. Begin extension work and evaluation.

26/01/2012 - 09/02/2012
Progress report written to be handed in by 03/02/2012. Preparation for presentation on 09/02/2012.

February 2012

09/02/2012 - 23/02/2012
Finish evaluation. Begin drafting the dissertation.

23/02/2012 - 08/03/2012
Finish extension work. Continue drafting the dissertation and evaluate extension work.

March 2012

08/03/2012 - 22/03/2012
Submit first draft of dissertation to supervisors by 16/03/2012. Begin redrafting on receipt of feedback.

22/03/2012 - 05/04/2012
Continuing redraft and resubmission of dissertation.

April 2012

05/04/2012 - 19/04/2012
Continuing redraft and resubmission of dissertation.

19/04/2012 - 03/05/2012
Dissertation complete 01/05/2012.

May 2012

03/05/2012 - 18/05/2012

Dissertation complete. Final edits, corrections. Binding and submission.