

Question 1

(a):

$$m \frac{dv}{dt} = -mg + c_1 v$$

If v is positive, (moving up), the drag term $c_1 v$ would be positive and thus accelerating, this cannot be a drag force. If v is negative (moving down), the drag term $c_1 v$ would also be negative and again, be causing an acceleration, and not a drag, (a) represents neither situation.

(b):

$$m \frac{dv}{dt} = -mg - c_1 v$$

If v is positive, the drag term would be negative, thus it would be resisting the direction of motion, and thus be a drag force. If v is negative, the drag term would be positive, thus also resisting the direction of motion. (b) Represents a ball that could be traveling up or down.

(c):

$$m \frac{dv}{dt} = -mg + c_2 v^2$$

No matter if the ball is moving up or down, the drag term is positive. The only time this can represent a drag force is if the ball is moving down, and the positive value would be resisting the motion. Down only.

(d):

$$m \frac{dv}{dt} = -mg - c_2 v^2$$

No matter if the ball is moving up or down, the drag term is negative. The only time this can represent a drag force is if the ball is moving up, and the negative value would be resisting the motion. Up only.

Question 2

(a): The terminal velocity of an object occurs when the sum of the forces from gravity and drag = 0:

$$\begin{aligned}0 &= mg - c_2 v^2 \\ mg &= c_2 v^2\end{aligned}$$

Solving for v gives:

$$\begin{aligned}\frac{mg}{c_2} &= v^2 \\ v_{term} &= \sqrt{\frac{mg}{c_2}}\end{aligned}$$

(b): Terminal velocity for an object subject to both drag forces is a bit more complicated. The same holds true however, that terminal velocity occurs when the sum of the gravity force and drag forces equals 0:

$$0 = mg - c_1 v - c_2 v^2$$

Defining a few terms to make this cleaner:

$$k_1 = \frac{c_1}{m}, \quad k_2 = \frac{c_2}{m} \quad k_2 v^2 + k_1 v - g = 0$$

Quadratic:

$$\begin{aligned}a &= k_2, \quad b = k_1, \quad c = g \\ v &= \frac{-k_1 \pm \sqrt{(k_1)^2 + 4k_2 g}}{2k_2}\end{aligned}$$

Question 3

$$\left| \frac{c_2 v^2}{c_1 v} \right| = \frac{0.22 v |v| D^2}{(1.55 * 10^{-4}) v D} = (1.4 * 10^3) |v| D$$

(a): I will assume that the quadratic term dominates over the linear term when the ratio is 100 or higher. Solving for v:

$$100 = (1.4 * 10^3) |v| D$$

$$|v| = \frac{100}{(1.4 * 10^3) D} \approx 7.14 \frac{m}{s}$$

Speeds below $7.14 \frac{m}{s}$ will be dominated by the linear drag and speeds above will be dominated by the quadratic drag.

(b): The velocity of the ball

$$m \frac{dv}{dt} = -mg + c_1 v D + c_2 v^2 D^2$$