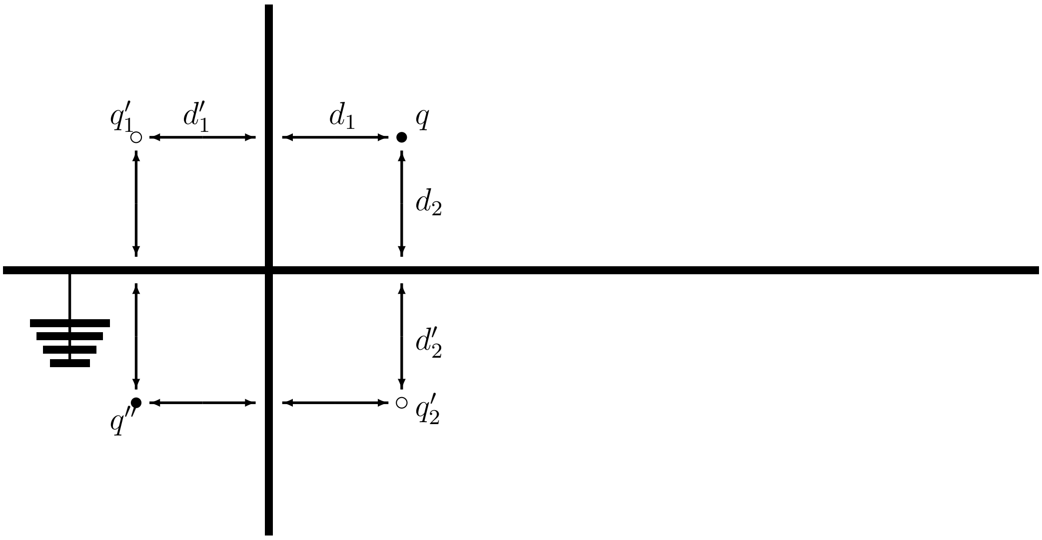
Question 1



The infitnite plates will have induced charges on them, each independently will act like an imaginary charge q' with the same charge but opposite sign as q. With both of the plates together, the induced charges on one plate will effect the charge q and also the imaginary charges q_1' and q_2'. This interaction will be like having another imaginary charge q'' in the lower left corner shown in the modified diagram. This charge should be equal and opposite to the first imaginary charge, and thus $q'' = q$ Thus to calculate the force exerted on q, a calculation can be done as if negative charges q_1' and q_2' as well as a positive charge q'' are placed in a quadrapole configuration.

The force on q is equal to the sum of the three forces: F_1 from q_1', F_2 from q_2' and F_3 from q''. Using equation (1) and substituting the proper charge in for Q, the three forces can be found. Adding them together will give the total force $\mathbf{F_T}$.

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r} \tag{1}$$

$$\mathbf{F_1} = \frac{1}{4\pi\epsilon_0}\frac{q(-q)}{(2d_1)^2}\hat{x} \tag{2}$$

$$\mathbf{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q(-q)}{(2d_2)^2}\hat{y} \tag{3}$$

$$\mathbf{F_3} = \frac{1}{4\pi\epsilon_0}\frac{q(q)}{(2d_1)^2}\frac{\sqrt{2}}{2}\hat{x} + \frac{1}{4\pi\epsilon_0}\frac{q(q)}{(2d_2)^2}\frac{\sqrt{2}}{2}\hat{y} \tag{4}$$

$$\mathbf{F_T} = \frac{\sqrt{2}-2}{32(d_1)^2\pi\epsilon_0}q^2\hat{x} + \frac{\sqrt{2}-2}{32(d_2)^2\pi\epsilon_0}q^2\hat{y} \tag{5}$$

Q.1 cont I am guessing that the far field potential goes like a $\frac{1}{r^3}$ at values of $r >> 2d$, much like a quadrupole would behave.

Question 2

(a)

Prove the Green's reciprocity theorem:

$$\int_{allspace} \rho_1 \phi_2 d^3r = \int_{allspace} \rho_2 \phi_1 d^3r$$

Evaluating the integral $\int \mathbf{E_1} \cdot \mathbf{E_2} d^3r$ substituting $\mathbf{E_1} = -\vec{\bigtriangledown}\phi_1$ by parts:

$$\int -\vec{\bigtriangledown}\phi_1 \cdot \mathbf{E_2} d^3r = (-\phi_1 \mathbf{E_2})\big|_{allspace} - \int -\phi_1 \vec{\bigtriangledown} \cdot \mathbf{E_2} d^3r \quad (6)$$

$$\int -\vec{\bigtriangledown}\phi_1 \cdot \mathbf{E_2} d^3r = \frac{1}{\epsilon_0} \int \phi_1 \rho_2 d^3r \quad (7)$$

Note that: $(-\phi_1 \mathbf{E_2})\big|_{allspace} = 0$.
Then evaluating the same integral substituting $\mathbf{E_2} = -\vec{\bigtriangledown}\phi_2$ again by parts:

$$\int \mathbf{E_1} \cdot -\vec{\bigtriangledown}\phi_2 d^3r = (-\phi_2 \mathbf{E_1})\big|_{allspace} - \int \phi_2 \vec{\bigtriangledown} \cdot \mathbf{E_1} d^3r \quad (8)$$

$$\int \mathbf{E_1} \cdot -\vec{\bigtriangledown}\phi_2 d^3r = \frac{1}{\epsilon_0} \int \phi_2 \rho_1 d^3r \quad (9)$$

Thus:

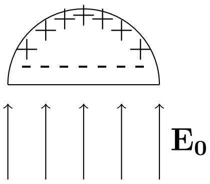
$$\int \phi_1 \rho_2 d^3r = \int \phi_2 \rho_1 d^3r \quad (10)$$

(b)

If putting a charge Q on A generates a potential V_{AB} on B and another V_{AA} on A, wouldn't generating a charge Q on B generate V_{BA} on a AND V_{BB} on B? For the same reason? Thus, wouldn't $V_{BB} + V_{BA} = V_{AB} + V_{AA}$?

Question 3

The hemisphere is neutral to begin with, when placed in an electric field $\mathbf{E_0}$ the hemisphere will polarize. I am not sure if the polarization will be as displayed here, or a more dificult polarization with more negative charge twords the outer rim of the flat surface of the sphere.



The boundary conditions of the hemisphere are $(i) V = 0$ when $r = R$ and $0 \leq \theta \leq \frac{\pi}{2}$, $(ii) V = 0$ when $0 \leq r \leq R$ and $\theta = \frac{\pi}{2}$ and finally $(iii) V = -E_0 r\ cos\ \theta + C$ when $r >> R$. Unlinke the sphere problem, I don't see an easy way to state that the potential is 0 for an entire plane and thus C cannot be eliminated right away.

Starting with the phi angle symetric version of the spherical Laplace equation:

$$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + \frac{B_l}{r^{l+1}})P_l(cos\ \theta) \tag{11}$$

Boundary conditions are used to solve for A_l and B_l. $P_l(cos\ \theta)$ is the Legandre polynomial for $cos\ \theta$.

From boundary condition (i):

$$0 = A_l R^l + \frac{B_l}{R^{l+1}}\ for\ 0 \leq \theta \leq \frac{\pi}{2} \tag{12}$$

So in the northern hemisphere:

$$B_l = -A_l R^{2l+1} \tag{13}$$

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l (r^l - \frac{R^{2l+1}}{r^{l+1}})P_l(cos\ \theta) \tag{14}$$

From boundary condition (iii) $r >> R$ the second term is very small and thus:

$$\sum_{l=0}^{\infty} A_l r^l P_l(cos\ \theta) = -E_0 r\ cos\ \theta + C \tag{15}$$

Since the result has $cos\ \theta$ and C, only $l = 0$, $l = 1$ matter. $A_0 = C$ and $A_1 = -E_0$

$$V(r,\theta) = C\frac{R}{r} - E_0(r - \frac{R^3}{r^2})cos\ \theta \tag{16}$$

To eliminate C, I will simply drop it and test the boundary conditions. Does (17) satisfy my boundary conditions?

$$V(r,\theta) = -E_0(r - \frac{R^3}{r^2})cos\ \theta \tag{17}$$

$$(i)\ r = R:\quad 0 \stackrel{?}{=} -E_0(R - \frac{R^3}{R^2})cos\ \theta \tag{18}$$

$$0 \stackrel{\checkmark}{=} R - R \tag{19}$$

$$(ii)\ \theta = \frac{\pi}{2}:\quad 0 \stackrel{?}{=} -E_0(r - \frac{R^3}{r^2})cos\ \frac{\pi}{2} \tag{20}$$

$$0 \stackrel{\checkmark}{=} cos\ \frac{\pi}{2} \tag{21}$$

$$(iii)\ r >> R:\quad V(r,\theta) \stackrel{\checkmark}{\rightarrow} -E_0 r\ cos\ \theta : C = 0 \tag{22}$$

The conductor will look like a dipole far away.