Problem 1: Taylor, 2-11

Problem 2, Taylor, 2-15

Problem 3, Taylor, 2-55

Problem 4, Thorton & Marion, 9-60

Problem 5, Taylor 10-6 Find the CM of a uniform hemispherical shell of inner and outer radii a and b and mass M.

(a) Comment on the limiting case when $a \to 0$.

(b) Comment on the limiting case when $a \to b$.

The integral equation for the position of the center of mass is:

$$\vec{R} = \frac{1}{M}\int \vec{r}dm$$

dm stands for a bit of mass. The bit of mass can be described as the volume mass density, $\rho(\vec{r})$, times a bit of volume, dV. $\rho(\vec{r})$ is uniform and thus will be represented by a simple ρ.

A hemisphere is most easily handled when dealing with spherical coordinates.

dV in spherical is represented by:

$$dV = r^2 sin\theta dr d\phi d\theta$$

$\vec{r}$ in spherical is simply $r\hat{r}$, however, over the ϕ and θ integrals, the direction of $\vec{r}$ changes, and thus $\vec{r}$ needs to be taken to a coordinate system that doesn't change over the integrals. $\vec{r}$ expressed in cartesian is:

$$\vec{r} = rsin\theta cos\phi\hat{x} + rsin\theta sin\phi\hat{y} + rcos\theta\hat{z}$$

$\vec{R}$ can now be expressed as a spherical volume integral:

$$\vec{R} = \frac{1}{M}\int \vec{r}dm = \frac{1}{M}\int \rho(rsin\theta cos\phi\hat{x} + rsin\theta sin\phi\hat{y} + rcos\theta\hat{z})r^2 sin\theta dr d\phi d\theta$$