

Q.2: McIntyre 2.17

A spin-1 particle is in the state

$$|\Psi\rangle = \frac{1}{\sqrt{30}} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix}$$

a) What are the possible results of a measurement of the spin component S_z , and with what probabilities would they occur? Calculate the expectation value of the spin component S_z .

b) Calculate the expectation value of the spin component S_x . *Suggestion:* Use matrix mechanics to calculate the expectation value.

The state of the system is:

$$|\Psi\rangle = \frac{1}{\sqrt{30}}(1|1\rangle + 2|0\rangle + 5i|-1\rangle)$$

The possible measurements of S_z for the spin-1 system are \hbar , $0\hbar$, $-\hbar$, for $|1\rangle$, $|0\rangle$, $|-1\rangle$ respectively.

The probability of measuring \hbar is:

$$\begin{aligned} \mathcal{P}_1 &= |\langle 1 | \Psi \rangle|^2 \\ &= \left| \langle 1 | \left[\frac{1}{\sqrt{30}} |1\rangle + \frac{2}{\sqrt{30}} |0\rangle + \frac{5i}{\sqrt{30}} |-1\rangle \right] \right|^2 \\ &= \left| \frac{1}{\sqrt{30}} \langle 1 | 1 \rangle + \frac{2}{\sqrt{30}} \langle 1 | 0 \rangle + \frac{5i}{\sqrt{30}} \langle 1 | -1 \rangle \right|^2 \\ &= \left| \frac{1}{\sqrt{30}} \right|^2 = \frac{1}{30} \end{aligned}$$

The probability of measuring $0\hbar$ is:

$$\begin{aligned} \mathcal{P}_0 &= |\langle 0 | \Psi \rangle|^2 \\ &= \left| \langle 0 | \left[\frac{1}{\sqrt{30}} |1\rangle + \frac{2}{\sqrt{30}} |0\rangle + \frac{5i}{\sqrt{30}} |-1\rangle \right] \right|^2 \\ &= \left| \frac{2}{\sqrt{30}} \right|^2 = \frac{4}{30} \end{aligned}$$

The probablility of measuring $-\hbar$ is:

$$\begin{aligned}\mathcal{P}_{-1} &= |\langle -1 | \Psi \rangle|^2 \\ &= \left| \langle -1 | \left[\frac{1}{\sqrt{30}} |1\rangle + \frac{2}{\sqrt{30}} |0\rangle + \frac{5i}{\sqrt{30}} |-1\rangle \right] \right|^2 \\ &= \left| \frac{5i}{\sqrt{30}} \right|^2 = \frac{25}{30}\end{aligned}$$

The expectation value of S_z is:

$$\begin{aligned}\langle S_z \rangle &= \langle \Psi | S_z | \Psi \rangle \\ &= \hbar(\mathcal{P}_1) + 0\hbar(\mathcal{P}_0) - \hbar(\mathcal{P}_{-1}) \\ &= \frac{\hbar}{30} - \frac{25\hbar}{30} = -\frac{24}{30}\hbar = -\frac{4}{5}\hbar\end{aligned}$$

The expectation value of the spin component S_x will be found using the suggested method of matrices. The matrix representation of the S_x operator is:

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

And the expectation value os S_x is:

$$\begin{aligned}\langle S_x \rangle &= \langle \Psi | S_x | \Psi \rangle \\ &= \frac{1}{\sqrt{30}} \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{30}} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \\ &= \frac{\sqrt{2}}{60} \hbar \begin{pmatrix} 1 & 2 & -5i \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \\ &= \frac{\sqrt{2}}{60} \hbar \begin{pmatrix} 2 & 1 - 5i & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5i \end{pmatrix} \\ &= \frac{\sqrt{2}}{60} \hbar (2 + (2 - 10i) + 10i) \\ \langle S_x \rangle &= \frac{\sqrt{2}}{15} \hbar\end{aligned}$$

Q.3 McIntyre 5.11

A particle is in the ground state of an infinite square well. The potential wall at $x = L$ suddenly moves to $x = 3L$ such that the well is now three times its original size. Find the probabilities that the particle is measured to have the ground state energy or the first excited state energy of the new well.

Initially, the particle is in the ground state of the initial configuration and its energy eigenstate wave function is:

$$|\Psi\rangle = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

After the boundary shift, the new allowable states are:

$$|\phi_n\rangle = \sqrt{\frac{2}{3L}} \sin \frac{n\pi x}{3L}$$