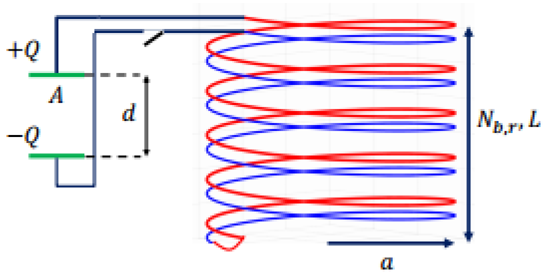
Problem 1

A parallel plate capacitor is formed by two circular plates of area A and placed at a distance d apart. It is connected with two inter-winding circular coils. The coils have the same radius a and total length l but different density of wires: n_r and n_b rounds per unit length respectively. Initially the capacitor has a charge Q, and the switch is closed at $t = 0$. (1) How is the charge changing as a function of time? (2) Calculate the magnetic energy in the coil and the electric energy in the capacitor, how does the total change with time? (3) Although there is no resistance in the circuit, people find the energy gradually getting lost, where does it go? Use Poynting vectors to help your reasoning.



At time $t = 0$ when the switch is closed, a current will begin to form. When the switch is closed, the current will flow from the positive plate to the negative plate and thus the positive current will be defined as top plate to bottom plate or according to the picture, clockwise in the wires near the capacitor. The current that forms will be:

$$I = -\dot{Q}$$

The two solenoids will resist the change of current creating an emf value equal to the negative inductance value times the change in current:

$$\varepsilon_L = -L\dot{I} = L\ddot{Q}$$

The total emf in this system has terms from both the solenoids and the capacitor:

$$\varepsilon = \varepsilon_C + \varepsilon_L = \frac{Q}{C} + L\ddot{Q}$$

The circuit doesn't include a resistor and thus the total emf must equal 0:

$$\frac{Q}{C} + L\ddot{Q} = 0$$

Rewriting:

$$\ddot{Q} = \frac{1}{LC}Q$$

Solution to this dif eq:

$$Q(t) = Q_0 e^{i\sqrt{\frac{1}{LC}}t}$$

The inductance value is determined by a combination of the two solenoids. The current that forms will be the same in both the solenoids, however, the current through the red solenoid and the current through the blue solenoid will be in opposite directions. The two solenoids can be thought of as a single solenoid with $n_r - n_b$ total number of turns. This works because any flux generated through a red loop will have equal and opposite flux generated through a blue loop. The total will depend on one loop having more turns than the other.

The inductance of a solenoid is $L = \frac{N\Phi}{I}$, where N is the total number of turns,A is the cross sectional area, and Φ is the total magnetic flux. $\Phi = \mu_0 \frac{NIA}{L}$, this makes the inductance:

$$L = \mu_0 \frac{N^2 A_L}{l}$$

The inductance of the modified solenoid is then:

$$L = \mu_0 (n_r - n_b)^2 l A_L$$

The capacitance of this capacitor is:

$$C = \frac{\epsilon_0 A_C}{d}$$

$Q(t)$ now becomes:

$$Q(t) = Qe^{i\sqrt{\frac{d}{\mu_0 \epsilon_0 (n_r - n_b)^2 l A_L A_C}}t}$$

$A_L = \pi a^2$ and $A_C = A$ thus $Q(t)$ becomes:

$$Q(t) = Qe^{i\sqrt{\frac{d}{\mu_0 \epsilon_0 (n_r - n_b)^2 l \pi a^2 A}}t}$$

Solving for the magnetic energy in the coil:

The magnetic energy E_B is:

$$E_L = \frac{1}{2}LI^2$$

The current is equal to the change of charge with respec to time or:

$$I = \dot{Q} =$$

The energy of the capacitor is:

$$E_C = \frac{1}{2}\frac{Q^2}{C}$$

Problem 2

Same configuration as Problem 1. Now if an electron escaped from the conductor at $t = 0$ with zero velocity, describe its trajectory as a cunction of t. If you can not solve for the trajectory, make sure you have the right equations. Even though we may not be able to solve for the equations, there are several categories of trajectories and extreme cases you can physically anticipate, try to list them and explain.