

Question 1

(a):

$$m \frac{dv}{dt} = -mg + c_1 v$$

If v is positive, (moving up), the drag term $c_1 v$ would be positive and thus accelerating, this cannot be a drag force. If v is negative (moving down), the drag term $c_1 v$ would also be negative and again, be causing an acceleration, and not a drag, (a) represents neither situation.

(b):

$$m \frac{dv}{dt} = -mg - c_1 v$$

If v is positive, the drag term would be negative, thus it would be resisting the direction of motion, and thus be a drag force. If v is negative, the drag term would be positive, thus also resisting the direction of motion. (b) Represents a ball that could be traveling up or down.

(c):

$$m \frac{dv}{dt} = -mg + c_2 v^2$$

No matter if the ball is moving up or down, the drag term is positive. The only time this can represent a drag force is if the ball is moving down, and the positive value would be resisting the motion. Down only.

(d):

$$m \frac{dv}{dt} = -mg - c_2 v^2$$

No matter if the ball is moving up or down, the drag term is negative. The only time this can represent a drag force is if the ball is moving up, and the negative value would be resisting the motion. Up only.

Question 2

(a): The terminal velocity of an object occurs when the sum of the forces from gravity and drag = 0:

$$\begin{aligned}0 &= mg - c_2 v^2 \\ mg &= c_2 v^2\end{aligned}$$

Solving for v gives:

$$\begin{aligned}\frac{mg}{c_2} &= v^2 \\ v_{term} &= \sqrt{\frac{mg}{c_2}}\end{aligned}$$

(b): Terminal velocity for an object subject to both drag forces is a bit more complicated. The same holds true however, that terminal velocity occurs when the sum of the gravity force and drag forces equals 0:

$$0 = mg - c_1 v - c_2 v^2$$

Defining a few terms to make this cleaner:

$$\begin{aligned}k_1 &= \frac{c_1}{m}, \quad k_2 = \frac{c_2}{m} \\ k_2 v^2 + k_1 v - g &= 0\end{aligned}$$

Quadratic:

$$\begin{aligned}a &= k_2, \quad b = k_1, \quad c = g \\ v_{term} &= \frac{-k_1 \pm \sqrt{(k_1)^2 + 4k_2 g}}{2k_2}\end{aligned}$$

Question 3

$$\left| \frac{c_2 v^2}{c_1 v} \right| = \frac{0.22v |v| D^2}{(1.55 * 10^{-4})vD} = (1.4 * 10^3) |v| D$$

Ball: $D = 10\text{cm} = 0.1\text{m}$, $m = 200\text{g} = 0.2\text{kg}$

(a): I will assume that the quadratic term dominates over the linear term when the ratio is 20 or higher. Solving for v :

$$20 = (1.4 * 10^3) |v| D$$

$$|v| = \frac{20}{(1.4 * 10^3)D} \approx 0.14 \frac{m}{s}$$

The Quadratic term dominates at velocities above $0.14 \frac{m}{s}$.

I will assume the linear term dominates when the inverse ratio is 20 or higher:

$$\frac{1}{20} = (1.4 * 10^3) |v| D$$

$$|v| = \frac{1}{20(1.4 * 10^3)D} \approx 3.6 * 10^{-4} \frac{m}{s}$$

The linear term dominates at speeds below $3.6 * 10^{-4} \frac{m}{s}$.

(b): The terminal speed occurs when the sum of the forces acting on the ball equals 0:

$$\sum F = 0 = -mg - c_1 v + c_2 v^2$$

$$v_{term} = \frac{c_1 \pm \sqrt{(c_1)^2 + 4mgc_2}}{2c_2}$$

$$v_{term} = \frac{1.55 * 10^{-4} * .1 \pm \sqrt{(1.55 * 10^{-4} * .1)^2 + 4 * .2 * 9.8 * .22 * .1^2}}{2 * .22 * .1^2}$$

$$v_{term} = \frac{1.55 * 10^{-5} \pm \sqrt{2.4025 * 10^{-10} + .017248}}{.0044}$$

$$v_{term} \approx \frac{1.55 * 10^{-5} \pm \sqrt{.017248}}{.0044}$$

$$v_{term} \approx \frac{1.55 * 10^{-5} \pm .131332}{.0044}$$

$$v_{term} \approx \frac{.13135}{.0044} = 29.9 \frac{m}{s} (plus)$$

$$v_{term} \approx \frac{-.13132}{.0044} = -29.8 \frac{m}{s} (minus)$$

The terminal velocity for the ball is approximately $29.9 \frac{m}{s}$.

(c): For the ball, the terminal velocity is in the range of speeds dominated by the quadratic drag term. If I had to choose one to keep, the quadratic drag would be more accurate, as the linear drag is negligible.