

The precession and nutation of a spinning top is an analysis of a rotating rigid body system.

Inertia Tensor

Products of Inertia:

$$\begin{aligned} L_x &= I_{xz}\omega \quad \text{and} \quad L_y = I_{yz}\omega \\ \text{where : } I_{xz} &= - \sum m_\alpha x_\alpha z_\alpha, \\ I_{yz} &= - \sum m_\alpha y_\alpha z_\alpha, \\ I_{zz} &= \sum m_\alpha (x_\alpha^2 + y_\alpha^2) \end{aligned}$$

Inertia Tensor:

$$\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

The spinning top has axial symmetry along its core, (upright z -axis), this property makes the off diagonal components of the inertia tensor all equal to 0.

The diagonals:

For this example, the top will be portrayed as a uniform density cone.

The I_{zz} product of inertia using cylindrical coordinates is:

$$\begin{aligned} \varrho &= M/V = \frac{3M}{\pi R^2 h} \\ r &= \frac{Rz}{h} \end{aligned}$$

$$\begin{aligned} I_{zz} &= \varrho \int_V dV \rho^2 = \varrho \int_0^h dz \int_0^{2\pi} d\phi \int_0^r \rho d\rho \rho^2 \\ &= \end{aligned}$$