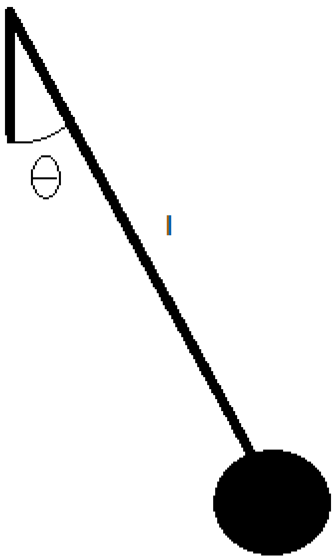

Question 1, Thornton & Marion, 7-24

Consider a simple plane pendulum consisting of a mass m attached to a string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate.

$$\frac{\partial l}{\partial t} = -\alpha = constant$$

The suspension point remains fixed. Compute the Lagrangian and Hamiltonian functions. Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system.

The position of the pendulum at any point in time is:

$$\vec{r} = lsin(\theta)\hat{x} - lcos(\theta)\hat{y}$$

The length l and the angle θ are both functions of time. Taking the time derivative of r gives:

$$\dot{r} = [\dot{l}sin(\theta) + lcos(\theta)\dot{\theta}]\hat{x} - [\dot{l}cos(\theta) - lsin(\theta)\dot{\theta}]\hat{y}$$

The velocity will be used in the kinetic energy, v^2:

$$\begin{aligned}\dot{r}^2 =& \dot{l}^2 sin^2(\theta) + l^2cos^2(\theta)\dot{\theta}^2 + 2l\dot{l}\dot{\theta}sin(\theta)cos(\theta)\\ &+ \dot{l}^2 cos^2\theta + l^2sin^2(\theta)\dot{\theta}^2 - 2l\dot{l}\dot{\theta}sin(\theta)cos(\theta)\\ \dot{r}^2 =& \dot{l}^2 + l^2\dot{\theta}^2\end{aligned}$$

The kinetic energy T is:

$$T = \frac{1}{2}m(\dot{l}^2 + l^2\dot{\theta}^2)$$

The potential energy of this system is mgh, where h is a time dependent function of l and θ:

$$h = -lcos(\theta)$$

The potential energy U is:

$$U = -mglcos(\theta)$$

The Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2}m(\dot{l}^2 + l^2\dot{\theta}^2) + mglcos(\theta)$$

$\mathcal{H}$