1. Taylor, Problem 3.10

Consider a rocket(initial mass m_0 accelerating from rest in free space). At first, as it speeds up, its momentum p increases, but as its mass m decreases p eventually begins to decrease. For what value of m is p maximum?

The total momentum of the rocket at time t is:

$$P(t) = mv$$

A short time later $t + dt$ the mass of the rocket has changed to $m + dm$ and the velocity of the rocket has changed to $v + dv$. There has also been fuel ejected, the mass of the fuels is $-dm$ and it's velocity compared to a stationary frame is now $v - v_{ex}$. The total momentum of the rocket and the ejected fuel at this new time, ignoring the double infitessimal $dmdv$ is:

$$P(t + dt) = (m + dm)(v + dv) - dm(v - v_{ex}) = mv + mdv + v_{ex}dm$$

The change in total momentum over this time period is:

$$dP = P(t + dt) - P(t) = mdv + v_{ex}dm$$

The change in total momentum is 0, because no external forces are acting apon the system. Thus:

$$mdv = -v_{ex}dm$$

Separation of variables allows us to solve for the velocity of the rocket as a function of its mass:

$$dv = -v_{ex}\frac{dm}{m}$$

If the exhaust velocity is constant, integration gives:

$$v - v_0 = v_{ex}ln(m_0/m)$$

2. Taylor, Problem 3.14

Consider a rocket subject to a linear resistive force, $f = bv$, but no other external forces. Use the equation $m\dot{v} = v_{ex}\dot{m} + F_{ext}$ (from Problem 3.11) to show that if the rocket starts from rest and ejects mass at a constant rate $k = \dot{m}$, then its speed is given by

$$v = \frac{k}{b}v_{ex}\left[1 - \left(\frac{m}{m_0}\right)^{b/k}\right]$$