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Capacitated Reliable Facility Location in Presence of Disruption: A Column Generation Approach

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Because of today's globalized threats that comes in addition to, e.g., labor disruptions or failures resulting from harsh weather conditions, there has been a renewed interest in resilient facility location. In this paper, we revisit the p-center location under disruptions with a column generation formulation in order to address the scalability limitations of the previous formulations and solution schemes. Also, we include capacity constraints as well as a more accurate estimate of the required resource taking into account that not all failures occur at the same time, and therefore some backup resource can be shared. Intensive numerical experiments complete the paper, with some comparisons with previously proposed models. Conclusions are drawn in the last section.

Key words: Reliable Facility Location, Disruptions, Column Generation, Backup Facility

1. Introduction

The logistics networks are referred to the entire chain of distribution centers and transportation of goods or services from the supplier to the final costumers or users. On time production and delivery, resilience supplier and limitation on inventory is the main concern of enterprises in modern logistics systems. Interdiction of logistics system, as a model to make sure that the system is resilience, is usually motivated within a defense or military context. The main application of interdiction or protection model is for logistics network to real world problem when there is a disruption. Disruption is the result of an event that causes an unplanned, negative deviation from the expected delivery according to organization objectives. In the context of facility location, disruption affects facilities so that some users need to be directed to other facilities. They may also affect the routes between users and facilities. Designing a resilient logistics network in presence of disruptions has gained a lot of attention in recent years. Design of reliable logistics network to avoid disruption can be accomplished by improvement of existing facilities and also set a backup facility when a facility is under disruption. We will focus on disruptions affecting facility locations.

The p-median problem (PMP) as a classical location problem search for locating p facilities (medians) to minimize the sum of the distances from each demand vertex to its nearest facility on a network (graph). The capacitated p-median problem (CPMP) considers capacities for the product or services to be given by each facility. The total demand by users cannot exceed total facilities capacities.

In this paper we examine a column generation approach to the capacitated facility location problem in presence of uncertain disruption. The identified restricted master problem (RMP) optimize the covering of 1-median clusters satisfying a set of capacity constraints, and new columns are generated solving capacitated sub-problem, which consider the restricted master dual variables and the clusters capacities. Two set of models are proposed to solve the problem. in the first model the primary and backup supplier are considered separately and the capacity of the supplier also considered separately for being primary or backup. In the second model which named shared model, the capacity of facilities shared for being primary or backup supplier.

2. Related Work

2.1. Reliable Facility Location Models subject to Disruption

Decisions about facility location are either costly or difficult to reverse. The impact of decisions will remain for a long time horizon. The parameter estimation (e.g., costs, demands, transportation times) may be inaccurate due to poor measurements. Disruption effects on the logistics networks parameters as discussed before (*e.g.*, time, cost, and availability of facility). In case of uncertainty of disruption, the parameters such as recognizing demand point or distances have been studied by authors and several models have been developed for facility location under uncertainty (Snyder (2006)). In the sequel, we classify the work made on the facility location under disruption and the solution methods related to the design of reliable facility location under disruption.

2.2. Classification of Facility Location Problems

There are various studies on the reliable facility location problems in logistics networks. Most of them considered the un-capacitated p-median problem (UFLP) (Lim et al. (2010)). Its objective is to find the optimal location based on the average distances between locations and users. The capacitated p-median problem (CPMP) (Lorena and Senne (2004)) adds capacity constraints on the facilities, but has not yet been studied in the context of disruptions. To our knowledge, facility location under uncertainty can be divided by to two main categories: *(i)* uncertainty on customer demand, and travel time(cost)between facility locations and customers, *(ii)* uncertainty on availability of facility locations.

The facility location under uncertainty for category (i) are well reviewed by Snyder (2006). In this case, problems are categorized into three main categories. *(i-a)*, when there are certainties on demand or travel time values. *(i-b)*, when there are uncertainties on demand or travel time whose values are governed by probability distributions that are known by the decision maker. *(i-c)*, when demand or travel time is uncertain, and furthermore, no information about probabilities is known. In the context of *(ii)*, authors have worked along two directions for the design of resilient facility location models: *(ii-a)* fortification of a subset of facilities subject to some number or budget constraints (e.g., Scaparra and Church (2012)) and *(ii-b)* establishing some backup facilities (e.g., Lim et al. (2010)).

2.3. Models of the Literature

We first look into the papers related to first category (category *(i)*). The methods to solve this problem are reviewed by Snyder (2006). The category *(i-a)* is the classical facility location problem. the category *(i-b)* is modeled as stochastic facility location problems (for instances, Louveaux and Peeters (1992), Chen et al. (2006)), and *(i-c)* is modeled as robust facility location problems (for instance, Averbakh (2003), Snyder (2006)).

Now we review the papers dealing with category *(ii)*, starting with those which used the concept of fortification, or equivalently interdiction(category *(ii-a)*). In this case, single and multiple disruptions are defined in facility location problem (*e.g.*, Losada et al. (2010)). Different level of disruption as partial and complete is modeled in this problem (*e.g.*, Liberatore et al. (2012)). In case of the capacitated problem the disruption can reduce the capacity of supplier (Scaparra and Church (2012))or facility can loose its capacity (Atoei et al. (2013)). Different size of facility location fortification problem is solved in literature. For instances, 50 to 150 demand nodes (Church and Scaparra (2007), Liberatore et al. (2011)) and 20 to 40 facility nodes (Losada et al. (2010), Scaparra and Church (2012)) to support demand nodes are solved by authors.

The objectives of facility location under disruption are to minimize the impact of disruption. It can be minimizing the worst-case impact of disruption or worse-case disruption scenario (e.g., Losada et al. (2012)). It can be facility protection or fortification based on the first investment (Church and Scaparra (2007)). Other objectives proposed by authors are minimizing of recovery of disrupted facilities (*e.g.*, Liberatore et al. (2014)).

The most method to solve the proposed models are Bender decomposition (Azad et al. (2013), Losada et al. (2010)), Lagrangian relaxation (Snyder and Daskin (2005)), pre processing techniques based on the valid lower and upper bound, and heuristics methods(Liberatore et al. (2011)).

We now review the papers dealing with the backup facility location (category *(ii-b)*). In this case, the authors considered for each user there is a primary facility and a backup facility or a layer of backup facilities (Lim et al. (2010)). Also there is no capacity constraint for potential facilities. Lim et al. (2010) considered hardening selected facilities. They dividing facilities as unreliable and another that is reliable. Different size of problem is considered by authors. Li et al. (2013) solved the problem with size of 150 demand nodes and 30 to 50 supplier. Lim et al. (2010) employed a data set of 263 nodes representing the largest cities in the contiguous 48 states in the United States. They solved the problem where different combination of reliable and unreliable facilities are considered (for instance, 11 unreliable facilities and 4 reliable facilities). The method, which is used in case of solving backup facility location models, is Lagrangian relaxation (Li et al. (2013), Lim et al. (2010)).

In continue we look into the summary of related problem including data sets, objectives and solution approaches in table 1.

Reference	problem feature fortification (Y/N)	backup (Y/N)	objective cost	capacity constraint	exact/ heuristics	largest size of data
Snyder and Daskin (2005)	Y		p^M	No	LR	150
Church and Scaparra (2007)	Y		p^M	No	ILP	150(dif.)
Scaparra et al. (2008)		Y	Max-covering	No	Greedy algo.	316(dif.)
Lim et al. (2010)		Y	p^M	No	LR	263
Liberatore et al. (2011)	Y		Max-covering	No	heuristics	263
Scaparra and Church (2012)		Y	Multi obj.	Yes	Tri-level	150
Losada et al. (2012)			Max-covering	No	Stochastic	150
Liberatore et al. (2012)	Y		Multi obj.	Yes	Tri-level	305(dif.)
Li et al. (2013)	Y	Y	p^M	No	LR	150
Qin et al. (2013)	Y		Protection	Yes	two-stage Sto.	rand.
Hernandez et al. (2014)			Multi obj.	No	Robust	100

Table 1 **Logistics Networks Disruption: Facility Location Problem**

3. Problem Statement

We denote by I the set of customers, J the set of potential locations for the facilities, and p the maximum number of facilities to open. Each customer $i \in I$ has demand D_i, and let Q_j be the capacity of location j. Let COST_{ij} be the transport cost of demands between facility location $j \in J$ and customer $i \in I$ (with the convention $\text{COST}_{ij} = 0$ for all $i \in I$). For a given facility location j with a set I_j of assigned customers, the transportation cost related to j can be written as follows:

$$\text{COST}_j = \sum_{i \in I_j} d_{ij} = \text{COST}_j^W + \text{COST}_j^B$$

Each customer is assigned a primary supplier and a different backup supplier. Associated with each facility location j is the failure probability $0 \leq q_j \leq 1$.

Under the assumption of a single facility failure (we have time to recover from a facility failure before a new one occurs), we need to optimize the facility capacities. The probability of a simultaneous failure of its primary and backup supplier is negligible. Consider two users i_1 and i_2, each assigned to a different facility location for their primary supplier, say j_1 for i_1, and j_2 for i_2. Assume that both i_1 and i_2 have the same backup supplier, say j_3. When checking the capacity constraint for j_3, we can only consider $\max\{d_1, d_2\}$ since i_1 and i_2 will not need to recourse to facility in j_3 at the same time.

Additional assumptions:

- The events of facility failures are independent
- For any customer, if the primary supplier fails, the backup supplier is available
- If a facility is fortified, it becomes non-failable
- If a facility fails, it becomes unavailable

4. Capacitated Reliable Facility Problem

We propose a new decomposition model in which the sets of configurations for primary and backup facility is considered as one decision variable (z_c). The capacity constraint is in pricing problem and we control both demand for primary and backup based on the selected facility capacity. Also assigning facility as primary or backup is in pricing problem(equations (10),(11))

4.1. Variables and Parameters

<u>The set of decision variables:</u>

$z_c \in \{0,1\}$. $z_c = 1$ if configuration c is selected in the optimal solution, 0 otherwise.

$x_j \in \{0,1\}$. $x_j = 1$ if facility location is selected and fortified, 0 otherwise.

$y_j \in \{0,1\}$. $y_j = 1$ if facility location j is open and is used as a primary or a backup supplier, 0 otherwise.

<u>Notations:</u>

For a given facility location j, let C_j be the set of facility location j configurations, where $c \in C_j$ is characterized by the set of users assigned to a facility in location j with

$a_i^{\mathrm{W},c} \in \{0,1\}$. $a_i^{\mathrm{W},c} = 1$ if customer i uses the facility of configuration c as a primary facility location, 0 otherwise.

$a_i^{\mathrm{B},c} \in \{0,1\}$. $a_i^{\mathrm{B},c} = 1$ if customer i uses the facility of configuration c as a backup facility location, 0 otherwise.

<u>Parameters:</u>

The setup cost s_j is a fixed cost required to implement facility fortification (the costs of contract negotiation, overhead, personnel training, etc.). The variable fortification cost varies with the amount of reliability improvement of the facility. (the cost of acquiring and installing the units of protective measures, the cost of procurement and storage of backup inventory, and the cost of hiring extra workforce, etc.). We define r_j as the cost associated with the unit reduction in the failure probability of facility j. The total available fortification budget is equal to B.

4.2. Master Problem

$$\min \quad \sum_{c\in C} \text{COST}_c z_c - PENAL(\sum_{j\in J} q_j x_j) \tag{1}$$

or by removing constraint (8)

$$\min \sum_{c\in C} \text{COST}_c z_c + PENAL_1(\sum_{j\in J} q_j \left(\sum_{c\in C_j} a_i^{\text{W},c} z_c \right)) - PENAL_2(\sum_{j\in J} q_j x_j)$$

where

$$\text{COST}_c = \sum_{i\in I} (a_i^{\text{W},c} + a_i^{\text{B},c}) d_{ij} \qquad c \in C_j.$$

Constraints are written as follows:

$$\begin{aligned}
&\sum_{c\in C_j} z_c = y_j && j \in J && (2)\\
&x_j \le y_j && j \in J && (3)\\
&\sum_{j\in J} y_j \le p && && (4)\\
&\sum_{c\in C} a_i^{\text{W},c} z_c = 1 && i \in I && (5)\\
&\sum_{c\in C_j} a_i^{\text{W},c} z_c + \sum_{c\in C} a_i^{\text{B},c} z_c \le 2 - x_j && i \in I, j \in J && (6)\\
&\sum_{c\in C} a_i^{\text{B},c} z_c \ge \sum_{c\in C_j} a_i^{\text{W},c} z_c - x_j && i \in I, j \in J && (7)\\
&\sum_{j\in J} q_j \left(\sum_{c\in C_j} a_i^{\text{W},c} z_c \right) \le \sum_{j\in J} q_j \left(\sum_{c\in C_j} a_i^{\text{B},c} z_c \right) && && \\
&\qquad - \sum_{c\in C} a_i^{\text{B},c} z_c + 1 && i \in I && (8)\\
&\sum_{j\in J} (s_j + r_j q_j) x_j \le B && && (9)\\
&y_j \in \{0,1\} && j \in J && \\
&x_j \in \{0,1\} && j \in J && \\
&z_c \in \{0,1\} && c \in C. &&
\end{aligned}$$

Constraints (2) checks whether location j is opened for a facility that will be used as a primary or a backup supplier. If $y_j^w = 0$, no facility is open in location j. If $y_j^w = 1$, one facility is opened in location j, and we make sure to select exactly one primary/backup facility configuration in location j.

Constraints (3) ensure that $x_j = 1$ if and only location j is fortified and open.

Constraint (4) sets the limit on the number of facilities.

Constraint (5) guarantee that each user i is assigned to a primary supplier.

Constraints (6) and (7) guarantee that each user i is assigned to a backup supplier, if its primary supplier is not a fortified facility. Indeed, consider (6) for the facility location $j^w(i)$ that is the primary supplier of user i. Then, $\sum\limits_{c\in C_j} a_i^{\mathrm{W},c} z_c = 1$. Consequently, if $x_{j^w(i)} = 1$, then $\sum\limits_{c\in C} a_i^{\mathrm{B},c} z_c = 0$. On the other hand, if $x_{j^w(i)} = 0$, then according to constraint (7), then if $\sum\limits_{c\in C} a_i^{\mathrm{B},c} z_c = 1$, meaning that user i needs a backup facility. Note that, due to the constraints in the pricing problem, a given facility location cannot be used both as a primary and a backup supplier.

Constraint (8) guarantee, for any user i, a selection of the primary supplier with a failure probability that is smaller than the failure of the backup supplier if the primary supplier is not fortified. Let us consider a particular user i. If its primary supplier is fortified, due to constraints (6) and (7), $\sum\limits_{c\in C} a_i^{\mathrm{B},c} z_c = 0$, and consequently $\sum\limits_{j\in J} q_j(\sum\limits_{c\in C_j} a_i^{\mathrm{B},c} z_c) = 0$ as well. This is turn implies that $\sum\limits_{j\in J} q_j(\sum\limits_{c\in C_j} a_i^{\mathrm{W},c} z_c) \leq 1$ which is always true due to constraints (5). On the other hand, if the primary supplier is not fortified, $\sum\limits_{j\in J} q_j(\sum\limits_{c\in C_j} a_i^{\mathrm{B},c} z_c) = 1$, and constraint (8) becomes:

$$\sum_{j\in J} q_j(\sum_{c\in C_j} a_i^{\mathrm{W},c} z_c) \geq \sum_{j\in J} q_j(\sum_{c\in C_j} a_i^{\mathrm{B},c} z_c)$$

Indeed, If user i is assigned to a non fortified facility $j^{\mathrm{W}}(i)$ as its primary supplier, then $\sum\limits_{c\in C} a_i^{\mathrm{B},c} z_c = 1$ and $x_{j^{\mathrm{W}}(i)} = 1$ due to constraints (6). Let $j^{\mathrm{B}}(i)$ be the location of its backup supplier. Then, $\sum\limits_{c\in C_{j^{\mathrm{B}}(i)}} a_i^{\mathrm{B},c} z_c = 1$, and consequently, $x_{j^{\mathrm{B}}(i)} = 1$. On the other hand, for $j \neq j^{\mathrm{B}}(i)$, $\sum\limits_{c\in C_{j^{\mathrm{B}}(i)}} a_i^{\mathrm{B},c} z_c = 0$, therefore the facility located in j may or may not be fortified. If user i is assigned to a fortified facility $j^{\mathrm{W}}(i)$ as its primary supplier, then $\sum\limits_{c\in C} a_i^{\mathrm{B},c} z_c = 0$ and again, facility located in j may or may not be fortified.

Constraints (9) enforce the budget constraint on the selection of fortified facilities.

4.3. Pricing Problem

We now write the pricing problem (PP_j) for potential facility location j. Let $u_j^{(2)} \lesssim 0$, $u_i^{(5)} \lesssim 0$, $u_{ij}^{(6)} \leq 0$, and $u_{ij}^{(7)} \geq 0$, be the values of dual variables associated with constraints (2), (5), (6), (7),and (8) respectively.

$$\min \overline{\text{COST}}_j^{\text{PP}^{\text{NO\_SHARE}}} = \text{COST}_j - u_j^{(2)} - \sum_{i \in I} u_i^{(5)} a_i^{\text{W}} - \sum_{i \in I} u_{ij}^{(6)} \left(a_i^{\text{B}} + a_i^{\text{W}}\right) - \sum_{j' \in J: j' \neq j} \sum_{i \in I} u_{ij'}^{(6)} a_i^{\text{B}}$$
$$- \sum_{i \in I} u_{ij}^{(7)} (a_i^{\text{B}} - a_i^{\text{W}}) - \sum_{j' \in J: j' \neq j} \sum_{i \in I} u_{ij'}^{(7)} a_i^{\text{B}}$$

where

$$\text{COST}_j = \sum_{i \in I} (a_i^{\text{W}} + a_i^{\text{B}}) d_{ij}$$

subject to:

$$\sum_{i \in I} D_i \left(a_i^{\text{W}} + a_i^{\text{B}}\right) \leq Q_j \tag{10}$$

$$a_i^{\text{W}} + a_i^{\text{B}} \leq 1 \qquad i \in I \tag{11}$$

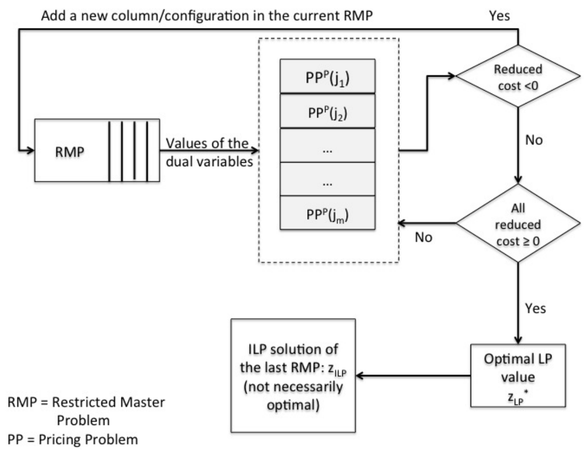
$$a_i^{\text{W}}, a_i^{\text{B}} \in \{0, 1\} \qquad i \in I. \tag{12}$$

Constraint (10) enforce the differentiated facility location capacity constraints. Note that we have an overestimation of the capacity under the assumption of a single facility location.

4.4. Solution Process

The flowchart of the column generation solution is depicted in Figure 1. Note that there are different pricing problems, and a good strategy is to solve them in a round robin order. In other words, memorize the last pricing problem that was solved, and next time a pricing problem has to be solved, solve the following one (with respect to the last pricing problem we solved) in the order in which we have stored the pricing problems.

Figure 1 Column generation

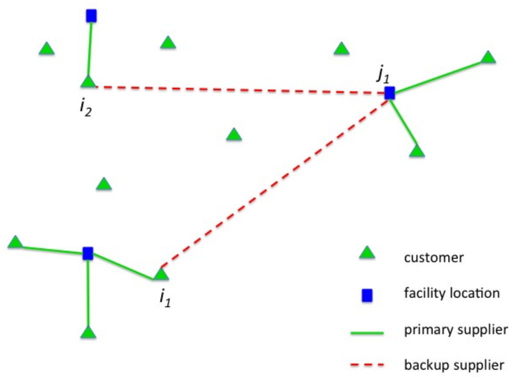


The next section is we will look into shared backup facility location. Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure of a failure before another one occurs

5. Shared Backup Facility Problem

In the previous model, capacity requirement is overestimated as it does not take into account that under the assumption of single facility failure, backup resource can be shared. Let us have a look to the example depicted in Figure 2. Therein, users i_1 and i_2 have different primary suppliers, while they share the same backup supplier (j_1). Under the assumption that at most one supplier will fail at a time, and that we have the time to fix the failure of a failure before another one occurs, backup resources of i_1 and i_2 can be shared, i.e., instead of requiring $D_1 + D_2$ with respect to facility location j_1, $\max\{D_1, D_2\}$ suffice. We will next explain how to modify the optimization

Figure 2 Sharing Backup Resources



models of Section 4 in order to take into account the sharing of the backup resources.

5.1. Master Problem without any information a_i^{W}

Capacity constraints can no more be taken care in the pricing problem. We therefore need to add the following constraints in the master problem:

$$\sum_{c \in C_j} \sum_{i \in I} D_i \, a_i^{\text{W}} z_c \leq Q_j^{\text{W}} \qquad j \in J \tag{13}$$

$$Q_j^{\text{W}} + Q_j^{\text{B}} \leq Q_j \qquad j \in J \tag{14}$$

$$Q_j^{\text{W}}, Q_j^{\text{B}}, Q_j \geq 0 \qquad j \in J. \tag{15}$$

How to compute the resource requirements for the backup:
$x_{ijj'}^{\text{B}} = 1$ if i uses j as a primary supplier and j' as a backup supplier, 0 otherwise.

$$\sum_{c \in C_j} a_i^{\text{W}} z_c + \sum_{c \in C_{j'}} a_i^{\text{B}} z_c - 1 \leq x_{ijj'} \tag{16}$$

$$x_{ijj'} \leq \sum_{c \in C_j} a_i^{\mathrm{W}} z_c \tag{17}$$

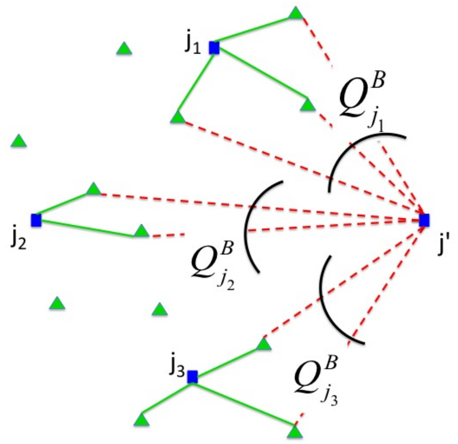
$$x_{ijj'} \leq \sum_{c \in C_{j'}} a_i^{\mathrm{B}} z_c \tag{18}$$

$$Q_j^{\mathrm{B}} = \max_{j \in J} \{Q_{jj'}\} \tag{19}$$

where

$$Q_{jj'} = \sum_{i \in I} D_i x_{ijj'}.$$

Figure 3 **Computing the shared backup resource requirements**



The Constraint 16 and 19 can be mixed and written as below:

$$\sum_{i \in I} D_i \left(\sum_{c \in C_j} a_i^{\mathrm{W}} z_c + \sum_{c \in C_{j'}} a_i^{\mathrm{B}} z_c - 1 \right) \leq \sum_{i \in I} D_i x_{ijj'}$$

and then

$$\sum_{i \in I} D_i \left(\sum_{c \in C_j} a_i^{\mathrm{W}} z_c + \sum_{c \in C_{j'}} a_i^{\mathrm{B}} z_c - 1 \right) \leq Q_{jj'}$$

Because the objective function is minimization we can ignore constraints **??** and 18. Then we can formulate the problem as below:

$$\min \sum_{c \in C} \mathrm{COST}_c z_c + PENAL_1 (\sum_{j \in J} q_j \left(\sum_{c \in C_j} a_i^{\mathrm{W},c} z_c \right)) - PENAL_2 (\sum_{j \in J} q_j x_j) \tag{20}$$

Subject to:

$$\sum_{c \in C_j} z_c = y_j \qquad j \in J \tag{21}$$

$$x_j \leq y_j \qquad j \in J \tag{22}$$

$$\sum_{j \in J} y_j \leq p \tag{23}$$

$$\sum_{c \in C} a_i^{\mathrm{W},c} z_c = 1 \qquad i \in I \tag{24}$$

$$\sum_{c \in C_j} a_i^{\mathrm{W},c} z_c + \sum_{c \in C} a_i^{\mathrm{B},c} z_c \leq 2 - x_j \qquad i \in I, j \in J \tag{25}$$

$$\sum_{c \in C} a_i^{\mathrm{B},c} z_c \geq \sum_{c \in C_j} a_i^{\mathrm{W},c} z_c - x_j \qquad i \in I, j \in J \tag{26}$$

$$\sum_{j \in J} (s_j + r_j q_j) x_j \leq B \tag{27}$$

$$\sum_{c \in C_j} \sum_{i \in I} D_i \, a_i^{\mathrm{W}} z_c \leq Q_j^{\mathrm{W}} \qquad j \in J \tag{28}$$

$$Q_j^{\mathrm{W}} + Q_j^{\mathrm{B}} \leq Q_j \qquad j \in J \tag{29}$$

$$Q_j^{\mathrm{B}} \geq Q_{jj'} \qquad j \in J \tag{30}$$

$$\sum_{i \in I} D_i \left(\sum_{c \in C_j} a_i^{\mathrm{W}} z_c + \sum_{c \in C_{j'}} a_i^{\mathrm{B}} z_c - 1 \right) \leq Q_{jj'} \tag{31}$$

$$Q_j^{\mathrm{W}}, Q_j^{\mathrm{B}}, Q_j \geq 0 \qquad j \in J. \tag{32}$$

$$Q_{jj'} \geq 0 \qquad j, j' \in J. \tag{33}$$

$$y_j \in \{0,1\} \qquad j \in J \tag{34}$$

$$x_j \in \{0,1\} \qquad j \in J \tag{35}$$

$$z_c \in \{0,1\} \qquad c \in C. \tag{36}$$

5.2. Pricing Problem

The pricing problem is modified as follows. We now write the pricing problem (PP_j) for potential facility location j. Let $u_j^{(21)} \lesseqgtr 0$, $u_i^{(24)} \lesseqgtr 0$, $u_{ij}^{(25)} \leq 0$, $u_{ij}^{(26)} \geq 0$, $u_j^{(28)} \leq 0$, and $u_j^{(31)} \leq 0$, be the values of dual variables associated with constraints (21), (24), (25), (26), (28),and (31) respectively.

$$\min \overline{\mathrm{COST}_j^{\mathrm{SHARED}}} = \overline{\mathrm{COST}_j^{\mathrm{NO\_SHARE}}} - u_j^{(28)} \sum_{i \in I} a_i^{\mathrm{W}} - \sum_{j \in J} \sum_{i \in I} u_j^{(31)} a_i^{\mathrm{W}} - \sum_{j' \in J : j' \neq j} \sum_{i \in I} u_{j'}^{(31)} a_i^{\mathrm{B}}$$

subject to:

$$a_i^{\mathrm{W}} + a_i^{\mathrm{B}} \leq 1 \qquad i \in I \tag{37}$$

$$a_i^{\mathrm{W}}, a_i^{\mathrm{B}} \in \{0,1\} \qquad i \in I. \tag{38}$$

5.3. Shared protection assuming the primary/working elements are given

Variables and Parameters

$z_c \in \{0,1\}$. $z_c = 1$ if configuration c is selected in the optimal solution, 0 otherwise.

$x_j \in \{0,1\}$. $x_j = 1$ if facility location is selected and fortified, 0 otherwise.

$y_j \in \{0,1\}$. $y_j = 1$ if facility location j is open and is used as a primary or a backup supplier, 0 otherwise.

Q_j^{B} Backup capacity for facility J

We assume here that location and user assignment to the primary facilities are known, through the coefficients a_{ij}^{W}: equal to 1 if user i is assigned to a primary facility located in j, 0 otherwise. Moreover,

$$Q_j^{\mathrm{W}} = \sum_{i \in I} a_{ij}^{\mathrm{W}} D_i.$$

Base on the previous model, we can do a precalculation: for a_{ij}^{W}=1 then y_j=1 according to connected user to j as the primary working path. Therefore, we can observe the number of y_j that is selected, then we can know the occupied facilities in p of constriant 42. For a given facility location j, let C_j be the set of facility location j configurations, where $c \in C_j$ is characterized by the set of users assigned to a facility in location j with

$a_i^{\mathrm{B},c} \in \{0,1\}$. $a_i^{\mathrm{B},c} = 1$ if customer i uses the facility of configuration c as a backup facility location, 0 otherwise.

$$\min \sum_{c \in C} \mathrm{COST}_c^B z_c - \mathrm{PENAL}_2 \times \sum_{j \in J} q_j x_j \tag{39}$$

subject to:

$$\sum_{c \in C_j} z_c = y_j \qquad j \in J \tag{40}$$

$$x_j \leq y_j \qquad j \in J \tag{41}$$

$$\sum_{j \in J} y_j \leq p \tag{42}$$

$$\sum_{c \in C} a_i^{\mathrm{B},c} z_c + x_j \geq a_{ij}^{\mathrm{W}} \qquad i \in I, j \in J \tag{43}$$

$$\sum_{c \in C} a_i^{\mathrm{B},c} z_c + x_j \leq 2 - a_{ij}^{\mathrm{W}} \qquad i \in I, j \in J \tag{44}$$

$$\sum_{j \in J} (s_j + r_j q_j) x_j \leq B \tag{45}$$

$$Q_j^{\mathrm{W}} + Q_j^{\mathrm{B}} \leq Q_j \qquad j \in J \tag{46}$$

$$\sum_{i \in I} D_i a_{ij}^{\mathrm{W}} \sum_{c \in C_{j'}} a_i^{\mathrm{B},c} z_c \leq Q_j^{\mathrm{B}} \qquad j, j' \in J : j \neq j' \tag{47}$$

$$Q_j^{\mathrm{B}} \geq 0 \qquad j \in J \tag{48}$$

$$y_j \in \{0,1\} \qquad j \in J \tag{49}$$

$$x_j \in \{0,1\} \qquad j \in J \tag{50}$$

$$z_c \in \{0,1\} \qquad c \in C \tag{51}$$

5.4. Pricing Problem

The pricing problem is modified as follows. We now write the pricing problem (PP_j) for potential facility location j. Let $u_j^{(40)} \lesseqgtr 0$, $u_{ij}^{(43)} \geq 0$, $u_{ij}^{(44)} \leq 0$, and $u_{ij}^{(47)} \leq 0$ be the values of dual variables associated with constraints (40), (43), (44) and (47) respectively.

$$\min \overline{\text{COST}_j^{\text{SHARED}}} = \text{COST}_j - u_j^{(40)} - \sum_{i \in I} u_{ij}^{(43)} a_i^{\text{B}} - \sum_{i \in I} u_{ij}^{(44)} a_i^{\text{B}} - \sum_{i \in I} (\sum_{j' \in J : j' \neq j} u_{ij'}^{(44)} a_i^{\text{B}}) - \sum_{i \in I} D_i a_{ij}^{\text{W}} \sum_{j' \in J : j' \neq j} u_{jj'}^{(47)} a_i^{\text{B}}$$

subject to:

$$a_{ij}^{\text{W}} + a_i^{\text{B}} \leq 1 \qquad i \in I \tag{52}$$

$$a_i^{\text{B}} \in \{0,1\} \qquad i \in I. \tag{53}$$

6. Numerical Results

6.1. Data Sets and Parameters

Models and algorithms proposed in the previous sections were tested on two data sets taken from Snyder and Daskin (2005), which can be found in the online appendix of Daskin (1995), with 49 and 88 users, with $m = n$, together with their demand and distance values. We assume that the transportation cost is proportional to the Euclidean distances. We generated the location capacity values as follows. Let $\overline{D} = \sum_{i \in I} D_i / p$ be the average demand per facility location (under the assumption there are p facilities and load is balanced among the facilities), where the demand values are taken from Snyder and Daskin (2005) p=5, 10, 20. Then, for each potential facility location j, we computed the Q_j capacity value as a randomly generated value in the interval $[2\overline{D}, 2.2\overline{D}]$. The fixed cost s_j based on Snyder and Daskin (2005) are drown from $U \sim [500, 1500]$ and rounded to nearest integer. Following Lim et al. (2010), the hardening cost is set as follows: $r_j = 0.2 \times s_j$. q_j the probability of failure is generated randomly by uniform distribution $U \sim [0, 0.05]$.

$\text{PENAL}_1 = 100$, $\text{PENAL}_2 = 100$.

PENAL_1 and PENAL_2 depend on the problem size and cost are changing.

The budget for fortification is calculated as below:

$$\overline{B} = \sum_{j \in J} (s_j + r_j q_j)$$

and the budget depends on percentage of opened facilities we decide to fortify (f is the percentage of facility we decide to fortify). In this case p is number of facilities to open and the budget is:

$$B = \frac{\overline{B}}{f \times p}$$

Percentage of resource saving in table 5 is shown by $RS\%$ is calculated based on the used capacity as backup divided by total demand:

$$RS\% = (1 - (Q^{\mathrm{B}}/Q^{\mathrm{W}})) \times 100$$

6.2. Generation of Initial Solutions

We provide an initial set of columns to both implemented models.

In Model DIS_FACLOC_F, each column is associated to a facility location, together with its set of assigned users, for which it is either a primary or a backup facility. First order the facility location in the increasing order of their q_j values. Taking into account the budget constraint, fortify as many locations as possible, considering the facility location in the increasing order of their fortification. For the fortified facility locations, assign as many primary users as possible taking into account the facility capacity constraints. If some users are still without a primary supplier, assign them in priority to the remaining unfortified facility location with the smallest q_j.

In Model DIS_FACLOC_FSR, the results for primary allocation (user primary assignment $a_i^{\mathrm{B},c}$) from the DIS_FACLOC_Fis used as a_{ij}^{W}. In this case the primary assignment is given and we are looking for the optimal allocation for backup facility while we do fortification. To initial the solution we rank the selected facilities for primary supplier based on their failure probability. Then we fortified facilities with highest probabilities up to budget constraint is satisfied. We assign the back up supplier for those users which supported by not fortified facilities.

6.3. Model Accuracy

We first report on the computational times and the accuracies of the solutions. Results are summarized in Table2 for DIS_FACLOC_F. The two test cases results are shown in this table. The first column is number of potential facilities available to open. p is the number of facilities which can be opened. The $z^{\star}_{\mathrm{LP}}$ is linear optimal solution for this problem. The $\tilde{z}^{\mathrm{W}}_{\mathrm{ILP}}$ is the integer optimal solution for primary user allocation and $\tilde{z}^{\mathrm{B}}_{\mathrm{ILP}}$ the integer optimal solution for backup user allocation. The gap calculated based on the formula: $\frac{(\tilde{z}^{\mathrm{W}}_{\mathrm{ILP}}+\tilde{z}^{\mathrm{B}}_{\mathrm{ILP}})-z^{\star}_{\mathrm{LP}}}{(\tilde{z}^{\mathrm{W}}_{\mathrm{ILP}}+\tilde{z}^{\mathrm{B}}_{\mathrm{ILP}})}\%$. through the columns 7 to 9 in the table 2, the number of initial,generated, and selected columns are shown. the CPU usage (time to solve) is shown in last column.

As it is shown the model in larger size problem behave better than in small size one and the gap is less But the time and the generated column is more in large size problem because there are more option to generate column and select.

The optimal solution (transportation cost) either LP or ILP is decreasing when the p increasing in both cases 49 and 88 nodes.

Nodes	p	$z'^{\star}_{\text{LP}}$	$z^{\star}_{1_{\text{LP}}}$	$z^{\star}_{2_{\text{LP}}}$	$z^{\star}_{\text{ILP}}$	$z^{\star}_{\text{LP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap (%)	# columns i	g	s	CPU (sec.)
49	5	198.6	7.8	2.1	203.8	200.1	141.6	62.2	1.8%	5	394	5	1103
49	10	173.8	5.4	2.7	178.3	174.6	140.6	37.7	2.1%	10	556	10	1589
49	20	119.0	5.8	4.6	123.2	119.3	94.4	28.7	3.2%	20	689	20	1804
88	5	630.9	27.3	4.4	643.9	636.5	489.7	154.2	1.1%	5	512	5	2537
88	10	564.8	21.6	7.6	581.7	578.8	504.9	76.8	0.5%	10	637	10	3203
88	20	529.7	19.8	11.3	537.7	532.3	492.5	45.2	1.0%	20	809	17	3872

Table 2 Computational Times and Solution Accuracies (Model dis_facloc_f)

The results for DIS_FACLOC_FSR, are summarized in Table3. in this case we are looking into backup allocation based on the primary assignment. In comparison between $\tilde{z}_{\text{ILP}}$ in Table3 and $\tilde{z}^{\text{B}}_{\text{ILP}}$in Table2 shows that by increasing p the optimal solution will decrease in both test cases and also the gap is increased in the second model (DIS_FACLOC_FSR).

Nodes	p	$z'^{\star}_{\text{LP}}$	$z^{\star}_{2_{\text{LP}}}$	$z^{\star}_{\text{LP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap (%)	# columns i	g	s	CPU (sec.)
49	5	62.2	0	62.2	62.2	0.0%	5	0	0	0
49	10	27.9	3.1	24.8	26.3	5.8%	10	72	4	394
49	20	22.8	3.7	19.1	19.9	3.7%	20	83	5	433
88	5	154.2	0	154.2	154.2	0.0%	5	0	0	0
88	10	40.7	0.4	40.3	41.3	3.5%	10	102	3	539
88	20	28.2	0.9	27.3	28.6	4.6%	20	87	2	435

Table 3 Computational Times and Solution Accuracies (Model dis_facloc_fsr)

6.4. Penalty analysis

In this section we calculate the optimal solution once without penalties and once including penalties. In the equation 54, the objective function include three elements. The first element is calculating the transportation cost (distances) based on the selected configurations. The second elements in this objective function is for favoring selecting those facilities as primary with lower failure probability. The third element is for prioritizing of fortification of those facilities with higher failure probability. The second and third elements of objective function are normalized using $PENAL_1$ and $PENAL_2$.

$$\min \sum_{c \in C} \text{COST}_c z_c + PENAL_1 (\sum_{j \in J} q_j \left(\sum_{c \in C_j} a_i^{\text{W},c} z_c \right)) - PENAL_2 (\sum_{j \in J} q_j x_j) \tag{54}$$

To analyze the effect of the penalties on objective, we move these two elements in objective function to set of constraints. To calculate upper bound for this constraints, we have two terms related to $PENAL_1$ and $PENAL_2$ and we first consider them equal zero and then we consider a value for them like # and ##. We add two constraints 55 and 56 while we remove second and third terms from objective function.

$$z_1 = \sum_{j\in J} q_j \left(\sum_{c\in C_j} a_i^{\mathrm{W},c} z_c \right) \tag{55}$$

$$z_2 = \sum_{j\in J} q_j x_j \tag{56}$$

Table 4

PENAL1	*PENAL2*	Nodes	p	weighted $z^\star_{\mathrm{LP}}$	weighted $\tilde{z}_{\mathrm{ILP}}$	non-weighted $z^\star_{\mathrm{LP}}$	non-weighted $\tilde{z}^{\mathrm{W}}_{\mathrm{ILP}}$	non-weighted $\tilde{z}^{\mathrm{B}}_{\mathrm{ILP}}$	gap(%)	gap/z(%)
0.1	1.3	49	5	204.3	208.1	200.1	141.6	62.2	1.8%	1.7%
		49	10	176.5	180.9	174.6	140.6	37.7	2.1%	2.4%
		49	20	120.2	123.1	119.3	94.4	28.7	3.2%	2.3%
0.7	6.5	49	5	218.9	221.1	201.6	141.6	62.2	1.8%	0.99%
		49	10	191.3	195.6	173.8	140.6	37.7	2.5%	2.1%
		49	20	139.0	141.6	121.7	94.4	28.7	1.2%	1.8%
7	65	49	5	295.3	309.3	258.7	196.5	76.2	5.1%	4.5%
		49	10	255.7	264.2	219.9	178.7	58.6	7.3%	3.2%
		49	20	187.9	197.8	152.3	121.8	41.4	6.6%	5.0%
7	0	49	5	360.2	371.1	241.9	182.2	69.2	3.7%	2.9%
		49	10	323.8	336.8	205.7	164.3	52.6	5.1%	3.8%
		49	20	255.4	262.3	140.4	108.6	35.4	2.5%	2.6%
0	65	49	5	132.7	143.5	226.1	161.4	73.8	3.8%	7.5%
		49	10	97.1	110.6	189.4	153.6	48.1	5.4%	12.2%
		49	20	47.7	50.4	137.4	109.5	34.3	4.5%	5.3%

Table 4 Results under different Penalty value(PENAL1, PENAL2)

6.5. Resource Sharing

We now investigate the resource saving when sharing the backup resources. Results are summarized in Table 5. In this table the number of nodes and p value is shown in first and second columns. In the columns three, the Q^{W} is the total primary capacity which is needed to support all users as primary supplier. The results in columns four to seven is related to first model(DIS_FACLOC_F) as follow: In the column four, the Q^{B} is the total backup capacity which is needed to support those users need to be supported as backup. In the column five, the $RS\%$ is the percentage of resource saving for backup support and the number of fortified facilities is shown in column six.

the total needed capacity is shown in column seven for model DIS_FACLOC_F. It is the same for model DIS_FACLOC_FSRin column eight to eleven.

We solved the experimental test cases with same amount of fortification and the same budget of fortification. The results show that in small size test cases the model DIS_FACLOC_F, and model DIS_FACLOC_FSR, are performing almost same while in case of 49 nodes and $p = 10$ the model DIS_FACLOC_FSR, perform better than model DIS_FACLOC_F, and there are more resource saving.

The results show that in large size test cases the model DIS_FACLOC_FSR, performs better than model DIS_FACLOC_F. the percentage of resource saving is increasing with increasing of p value. The resource saving is significant in case of 88 nodes and $p = 20$. the total needed backup capacity is 258.2 and the resource saving percentage is 96.86% in case of model DIS_FACLOC_FSR, which in case of model DIS_FACLOC_F, it is 1,901.1 and 76.86% mean while the number of fortified facilities is the same and it is equal 9.

			DIS_FACLOC_F				DIS_FACLOC_FSR			
Nodes	p	Q^{W}	Q^{B}	$RS\%$	#fortified	Q	Q^{B}	$RS\%$	#fortified	Q
49	5	247,051.3	96,409.7	60.98%	3	343,461.0	96,409.7	60.98%	3	343,461
49	10	247,051.3	115,399.9	53.29%	6	362,451.2	60,947.6	75.32%	6	308,026
49	20	247,051.3	30,185.8	87.78%	13	277,237.1	31,413.7	87.28%	13	278,465
88	5	8,213.9	3,143.5	61.31%	3	11,357.4	3,143.5	61.31%	3	11,357.4
88	10	8,213.9	2,238.4	72.75%	6	10,452.3	1083.4	86.81%	6	9297.3
88	20	8,213.9	1,901.1	76.86%	9	10,115.0	258.2	96.86%	9	8472.1

Table 5 Resource Saving

6.6. Scenario *B=0*

In this section we assume that there is no budget in order to fortify the facilities failure. In this case we need both primary and backup facility supplier for each user. In table 6 and table 7 we calculated the optimal solution and needed capacity for two models (DIS_FACLOC_F, and DIS_FACLOC_FSR).

Indeed by setting fortification budget equal zero we should have backup for each user and we are removing the fortification from the problem. In this case we remove the constraint ...

6.7. Different Objective Function

In this section we analyze the impact of different objective function in the model DIS_FACLOC_F. In continue we propose different objective function and we analyze the different solution based on the different objectives.

Nodes	p	$z^\star_{\text{LP}}$	$\tilde{z}^{\text{W}}_{\text{ILP}}$	$\tilde{z}^{\text{B}}_{\text{ILP}}$	gap (%)	# initial	# generated columns	# selected	CPU times (sec.)
49	5	272.2	139.4	135.706	1.0%	5	769	5	2631
49	10	235.9	113.5	126.475	1.6%	10	924	10	3371
49	20	166.6	91.2	80.920	2.9%	20	1120	20	4223
88	5	925.8	531.2	402.241	0.8%	5	972	5	3966
88	10	880.4	501.3	386.952	0.8%	10	1267	10	4839
88	20	787.1	486.1	306.289	0.7%	20	1576	20	6034

Table 6 Computational Times and Solution Accuracies (Model dis_facloc_f) when B=0

Nodes	p	Q^{W}	DIS_FACLOC_F			DIS_FACLOC_FSR		
			Q^{B}	RS%	Q	Q^{B}	RS%	Q
49	5	247,051.3	247,051.3	0%	494,102.6	112,804.5	54.3%	359,855.8
49	10	247,051.3	247,051.3	0%	494,102.6	81,461.3	67 %	328,512.6
49	20	247,051.3	247,051.3	0%	494,102.6	59,739.2	75.8%	306,790.5
88	5	8,213.9	8,213.9	0%	16,427.8	3,982.6	51.5%	12,196.5
88	10	8,213.9	8,213.9	0%	16,427.8	2,801.4	65.9%	11,015.3
88	20	8,213.9	8,213.9	0%	16,427.8	2,103.9	74.4%	10,317.8

Table 7 No Facility Fortification Resource Saving

6.7.1. Objective 1 :Minimization of transportation cost

The objective function in this section is based on the model in section 4. In this model the objective is considered based on the distance or transportation cost which we call it $Z^{\overline{Obj1}}$ in this section.

$$Z^{\overline{Obj1}} = \sum_{c \in C} \text{COST}_c z_c \tag{57}$$

Where

$$\text{COST}_{c_j} = \sum_{i \in I} (a_i^{\text{W}} + a_i^{\text{B}}) d_{ij}$$

6.7.2. Objective 2 : Minimization of resource usage

The objective function in this section is based on capacity usage which we call it $Z^{\overline{Obj2}}$.

$$Z^{\overline{Obj2}} = \sum_{j \in J} (Q_j^{\text{W}} + Q_j^{\text{B}}) \tag{58}$$

Where

$$Q_j^{\text{W}} = \sum_{c \in C_j} \quad \sum_{i \in I} D_i a_i^{\text{W},c} z_c$$
$$Q_j^{\text{B}} = \sum_{c \in C_j} \quad \sum_{i \in I} D_i a_i^{\text{B},c} z_c$$

The pricing problem is modified as follows. We now write the pricing problem (PP_j) for potential facility location j. Let $u_j^{(40)} \lesseqgtr 0$, $u_{ij}^{(43)} \geq 0$, $u_{ij}^{(44)} \leq 0$, and $u_{ij}^{(47)} \leq 0$ be the values of dual variables associated with constraints (40), (43), (44) and (47) respectively.

$$\min \overline{\text{COST}_j^{\text{SHARED}}} = \text{COST}_j - u_j^{(40)} - \sum_{i\in I} u_{ij}^{(43)} a_i^{\text{B}} - \sum_{i\in I} u_{ij}^{(44)} a_i^{\text{B}} - \sum_{i\in I} \Big(\sum_{j'\in J: j'\neq j} u_{ij'}^{(44)} a_i^{\text{B}} \Big) - \sum_{i\in I} D_i a_{ij}^{\text{W}} \sum_{j'\in J: j'\neq j} u_{jj'}^{(47)} a_i^{\text{B}}$$

subject to:

$$a_{ij}^{\text{W}} + a_i^{\text{B}} \leq 1 \qquad i \in I \tag{59}$$

$$a_i^{\text{B}} \in \{0,1\} \qquad i \in I. \tag{60}$$

Where

$$\text{COST}_j = \sum_{c\in C_j} \sum_{i\in I} D_i a_i^{\text{W},c} z_c + \sum_{c\in C_j} \sum_{i\in I} D_i a_i^{\text{B},c} z_c$$

We summarized the the different solution for 49/88 nodes and $p = 10$ in table 8

Node		$\overline{Z^{Obj1}}$	$\overline{Z^{Obj2}}$
49	$dist^{\text{W}}$ primary distance	140.6	152.3
	$dist^{\text{B}}$ backup distance	37.7	74.8
	Q^{W} primary resource	247,051.3	247,051.3
	Q^{B} backup resource	115,399.9	87,524.9
88	$dist^{\text{W}}$primary distance	504.897	537.124
	$dist^{\text{B}}$ backup distance	76.834	110.507
	Q^{W} primary resource	8,213.9	8,213.9
	Q^{B} backup resource	2,238.4	1,655.5

Table 8 Different objective function solutions for 49 and 88 nodes test case

7. Conclusion and Future Work

We have presented a new integer-linear programming model for identifying optimal fortification strategies of logistics networks in the event of disruption. We have tested this model on two different geographical data sets, using

-In this paper the fortification is binary. If we fortified the facility there is no disruption which in real word is not a case. For future work we suggest to consider partial fortification and also partial disruption with different probability.

- In this paper, in the model DIS_FACLOC_FSR, the input data for primary assignment is based on the solution of model DIS_FACLOC_Fwhich we suggest to solve the second model individually.

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