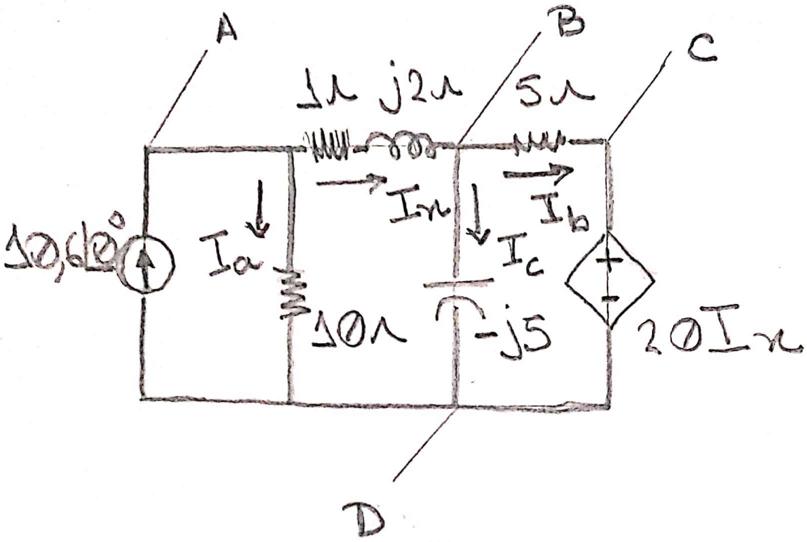


$$I_1 = 10,6\angle 0^\circ \text{ A}$$

$$G_1 = 1/10 \text{ S}$$

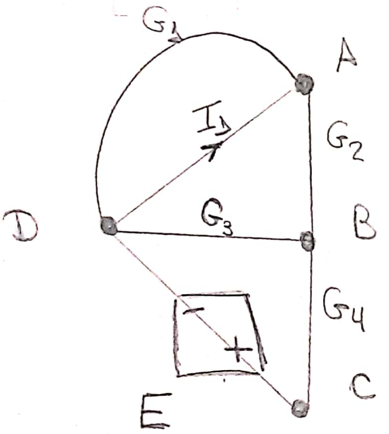
$$G_2 = \frac{1}{1+j2} \text{ S}$$

$$G_3 = \frac{1}{-j5} \text{ S}$$

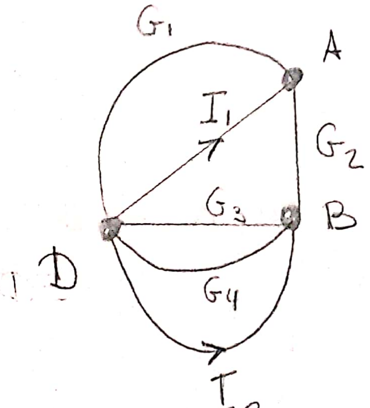
$$G_4 = \frac{1}{5} \text{ S}$$

$$\boxed{I_x = (e_A - e_B) \cdot G_2}$$

grafo de potencial



Super nó s: D ← C



$$e_C - e_D = E$$

$$\boxed{e_C - e_D = 20 I_x}$$

$$\boxed{I_2 = 20 I_x \cdot G_4}$$

$$V = Z . I$$

$$V = \frac{1}{G} . I \Rightarrow G . V = I \Rightarrow G^{-1} . G . V = G^{-1} . I$$

$$V = G^{-1} . I$$

$$\begin{bmatrix} e_A \\ e_B \\ e_S \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & (G_2+G_3+G_4) & -(G_3+G_4) \\ -G_1 & -(G_3+G_4) & (G_1+G_3+G_4) \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \\ -(I_1+I_2) \end{bmatrix}$$

$$\begin{cases} I_2 = 20 I_x G_4 \\ I_x = (e_A - e_B) . G_2 \end{cases}$$

$$I_2 = 20 . (e_A - e_B) G_2 . G_4$$

$$= \underbrace{20 G_2 G_4}_{K} e_A - \underbrace{20 G_2 G_4}_{K} e_B$$

$$I = \begin{vmatrix} I_1 \\ 20 G_2 G_4 e_A - 20 G_2 G_4 e_B \\ -I_1 + 20 G_2 G_4 e_A + 20 G_2 G_4 e_B \end{vmatrix}$$

$$I = \begin{vmatrix} I_1 \\ K e_A - K e_B \\ -I_1 - K e_A + K e_B \end{vmatrix}$$

$$\begin{bmatrix} e_A \\ e_B \\ e_D \end{bmatrix} = \underbrace{\begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2-K & (G_2+G_3+G_4)+K & -(G_3+G_4) \\ -G_1+K & -(G_3+G_4)-K & (G_1+G_3+G_4) \end{bmatrix}}^{-1} \begin{bmatrix} I_1 \\ 0 \\ I_1 \end{bmatrix}$$

$$K = 20\, G_2 G_4 = 0.8 - j1.6$$

$$G = \begin{bmatrix} 0.3-j0.4 & -0.2+j0.4 & -0.1 \\ -1+j2 & 1.2-j1.8 & -0.2-j0.2 \\ 0.7-j1.6 & -1+j1.4 & 0.3+j0.2 \end{bmatrix}$$

$$\begin{bmatrix} e_A \\ e_B \\ e_D \end{bmatrix} = \begin{bmatrix} 0.3-j0.4 & -0.2+j0.4 & -0.1 \\ -1+j2 & 1.2-j1.8 & -0.2-j0.2 \\ 0.7-j1.6 & -1+j1.4 & 0.3+j0.2 \end{bmatrix}^{-1} \begin{bmatrix} -10.6 \\ 0 \\ -10.6 \end{bmatrix}$$

$$e_A = -0,0231 + j6,40 \text{ V}$$

$$e_B = -0,4231 - j2,799 \text{ V}$$

$$e_D = -68,423 - j23,2006 \text{ V}$$

$$I_x = (e_A - e_B) . G_2 = 3,76 + j1,68 \text{ A}$$

$I_a = (e_a - e_5) \cdot G_1 = 6,84 - j1,68 \text{ A}$

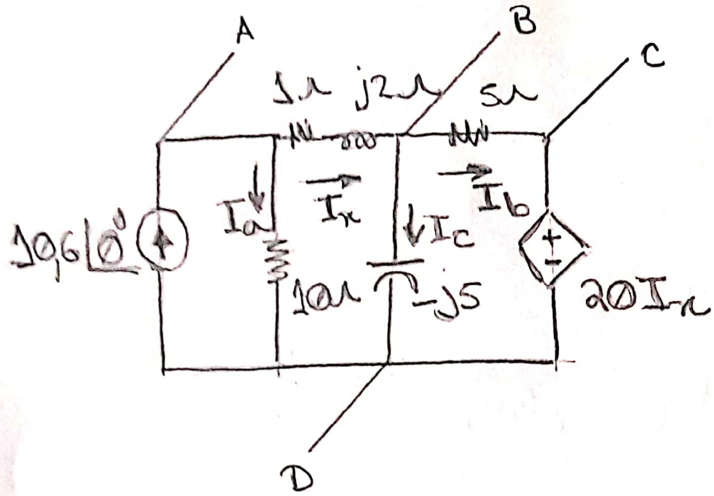
$e_C - e_D = 20 \cdot I_x$

$e_C = 20 I_x + e_D = 6,776 + j56,8 \text{ V}$

$I_b = (e_B - e_C) \cdot G_4 = -1,44 - j11,92 \text{ A}$

$I_c = (e_B - e_D) \cdot G_3 = 5,2 + j13,6 \text{ A}$

Solução 2



C ← D

$E_C - E_D = 20 I_x$

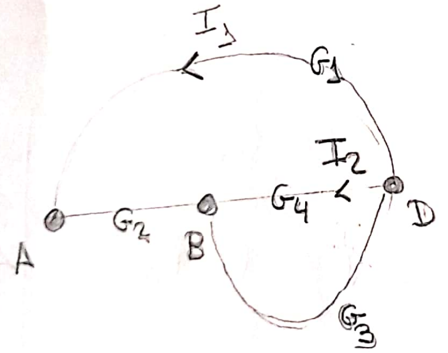
restrição

$I_x = (E_A - E_B) \cdot G_2$

$G_1 = \frac{1}{10} S \qquad G_3 = \frac{1}{-j5} S$

$G_2 = \frac{1}{1+j2} S \qquad G_4 = \frac{1}{5} S \qquad I_1 = 10,6 \angle 0°$

Grafo



$$G = \begin{vmatrix} (G_1+G_2) & -G_2 & -G_1 \\ -G_2 & (G_2+G_3+G_4) & -(G_3+G_4) \\ -G_1 & -(G_3+G_4) & G_1+G_3+G_4 \end{vmatrix}$$

$I_2 = 20 I_x \cdot G_4$

$$I = \begin{vmatrix} I_1 \\ I_2 \\ -(I_1+I_2) \end{vmatrix}$$

$$\begin{bmatrix} E_A \\ E_B \\ E_D \end{bmatrix} = \begin{bmatrix} (G_1+G_2) & -G_2 & -G_1 \\ -G_2 & (G_2+G_3+G_4) & -(G_3+G_4) \\ -G_1 & -(G_3+G_4) & (G_1+G_3+G_4) \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_1 \\ 20 I_x G_4 \\ -(I_1 + 20 I_x G_4) \end{bmatrix}$$

$$I_x = (E_A - E_B)\,G_2$$

$$I = \begin{vmatrix} I_1 \\ 20(E_A - E_B)G_2G_4 \\ -(I_1 + 20(E_A - E_B)G_2G_4) \end{vmatrix}$$

$$I = \begin{vmatrix} I_1 \\ 20G_2G_4E_A - 20G_2G_4E_B \\ -I_1 - 20G_2G_4E_A + 20G_2G_4E_B \end{vmatrix}$$

$$\begin{bmatrix} E_A \\ E_B \\ E_D \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2-20G_2G_4 & (G_2+G_3+G_4)+20G_2G_4 & -(G_3+G_4) \\ -G_1+20G_2G_4 & -(G_3+G_4)-20G_2G_4 & (G_1+G_3+G_4) \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ 0 \\ -I_1 \end{bmatrix}$$

Ommitamos E_D, ou seja $E_D = 0$

$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 \\ -(G_1+20G_2G_4) & (G_2+G_3+G_4)+20G_2G_4 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ 0 \end{bmatrix}$$

$$E_A = 68.4 - j16.8$$

$$E_B = 68.0 - j26.0$$

Como $E_D = 0$ entao $E_C = 20 I_x$

$E_C - E_D = 20 I_x$

$E_C = 20 I_x + E_D = 20 I_x$

$I_x = (E_A - E_B) . G_2$ $\Rightarrow$

$E_C = 20 (E_A - E_B) . G_2$

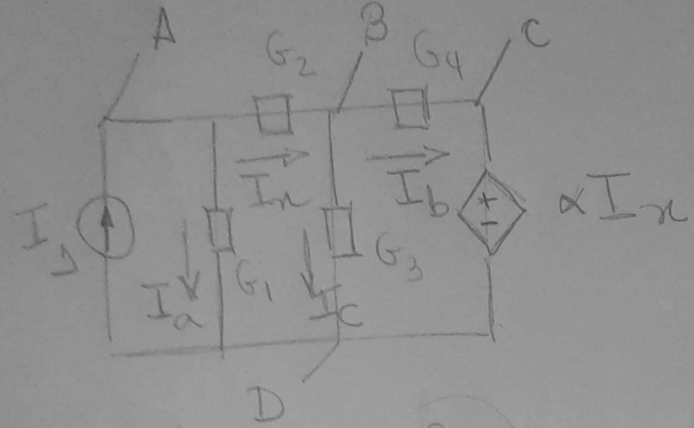
$E_C = 75.2 + j33.6$

$I_a = (E_A - E_D) . G_1 = 6.84 - j1.68$ A

$I_b = (E_B - E_C) . G_4 = -1.44 - j11.92$ A

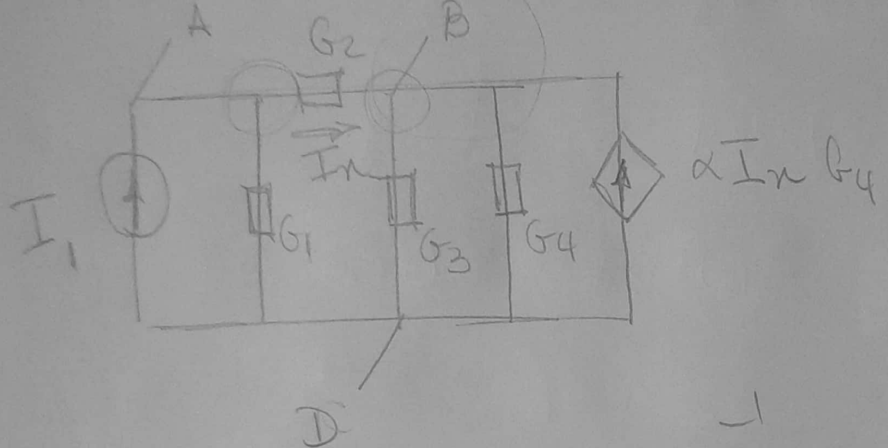
$I_c = (E_B - E_D) . G_3 = 5.2 + j13.6$ A

Solução 3



$$E_C - E_D = \alpha I_x$$

$$I_x = (E_A - E_B)\, G_2$$



$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 + G_4 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ \alpha I_x G_4 \end{bmatrix}$$

$$\alpha I_x G_4 = \alpha (E_A - E_B) G_2 G_4$$

$$= \alpha G_2 G_4 E_A - \alpha G_2 G_4 E_B$$

$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -(G_1 + \alpha G_2 G_4) & (G_2 + G_3 + G_4 + \alpha G_2 G_4) \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ \emptyset \end{bmatrix}$$

E_A ✓ $\qquad I_x = (E_A - E_B) \cdot G_2$

E_B ✓ $\qquad E_C = \alpha I_x + E_D$

$\qquad E_C = \alpha (E_A - E_B) G_2$

$E_D = \emptyset$

E_A

E_B

$E_D = \emptyset$

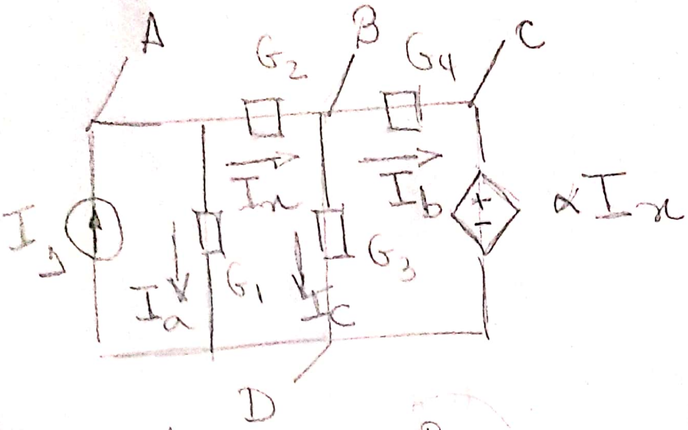
$E_C = \alpha (E_A - E_B) G_2$

$I_a = (E_A - E_D) \cdot G_1 = E_A \cdot G_1$

$I_b = (E_B - E_C) \cdot G_4$

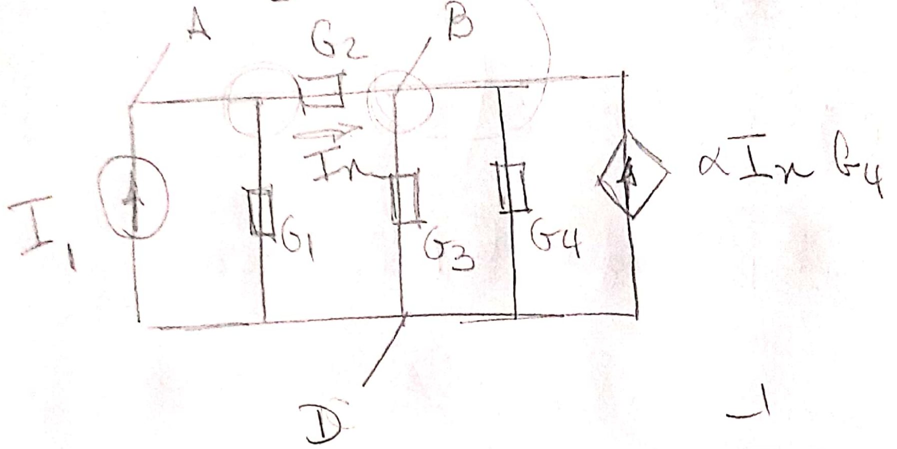
$I_c = (E_B - E_D) \cdot G_3 = E_B G_3$

Solução 3



$$E_C - E_D = \alpha I_x$$

$$I_x = (E_A - E_B)\, G_2$$



$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 + G_4 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ \alpha I_x G_4 \end{bmatrix}$$

$$\alpha I_x G_4 = \alpha (E_A - E_B) G_2 G_4$$
$$= \alpha G_2 G_4 E_A - \alpha G_2 G_4 E_B$$

$$\begin{bmatrix} E_A \\ E_B \end{bmatrix} = \begin{bmatrix} G_1 + G_2 & -G_2 \\ -(G_1 + \alpha G_2 G_4) & (G_2 + G_3 + G_4 + \alpha G_2 G_4) \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ \varnothing \end{bmatrix}$$

E_A ✓

E_B ✓

$E_D = \varnothing$

$$I_x = (E_A - E_B) \cdot G_2$$

$$E_C = \alpha I_x + E_D$$

$$E_C = \alpha (E_A - E_B) G_2$$