Harmonic Model of Power Transformer

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Abstract: Appropriate harmonic models of all power system components including power transformers are the basis of power system harmonic analysis. Currently, several harmonic models of transformers have been proposed. This paper firstly summarizes all available models of transformers and then derives a general harmonic model for transformers. Consequentially, the paper details the computation and harmonic analysis of the magnetizing current of a single phase power transformer. Finally, based on the controlled source model of power transformers, the paper derives its practical harmonic equivalent circuit.

Keywords: Harmonic model; Power transformers; Magnetizing current;

I. Introduction

With wide application of power electronics and other harmonic producing equipment, renewed interest has been stimulated in harmonic analysis subject. presently, among all the harmonic studies, the studies on harmonic penetration and state estimation have been paid more attention. To fully understand the penetration of power system harmonics in electrical networks, appropriate harmonic models of all power system devices including power transformers are required.

Harmonic modeling of power transformer is studied in two sides: one side studies the construction of harmonic model of power transformers which is characterized by the computation and harmonic analysis of the magnetizing current; the other studies the relation between model parameters and harmonic frequencies.

In reference [1], the problem of zero sequence current generation in transmission lines connected to large converter plant was analyzed using harmonic model of power transformers. In the literature, the transformer was represented by conventional model in fundamental frequency in which the values of damper resistance and leakage reactance must be determined by short circuit tests. Five kinds of harmonic models of power transformers are utilized in reference[2], all of which neglect its inherent harmonic producing effect and represented by modified short-circuit impedance in fundamental frequency. Nonlinear models of power transformer are proposed in reference[3], in which the relation between core losses resistance and harmonic frequencies was analyzed and practical analysis methodologies were proposed. In 1996, a complete and practical harmonic model was proposed by Task Force on Harmonics Modeling and Simulation of Transmission and Distribution Committee of the IEEE Power Engineer Society[4]. Based on the computation and harmonic analysis of magnetizing current, this paper derives the general harmonic model of power transformer and its equivalent circuit.

II. Derivation of General Harmonic Model

To obtain a general harmonic model, the paper begins the derivation from the elementary electrical and magnetic equations representing the characteristics of power transformers and the electromagnetic law[5]. Since the harmonic frequencies of interest are far lower than the resonance frequency of a transformer, the effect of distributed capacitors is neglected.

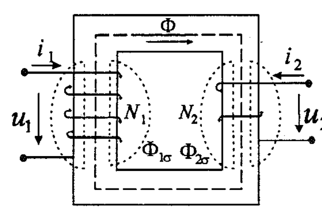

Fig. 1. Physical Model of A Single Phase Transformer

Fig.1 gives the physical model of a single phase transformer and the corresponding electrical and magnetic equations are given as follows.

$$u_1 = r_1 i_1 + L_{1\sigma}\frac{d i_1}{d t} + E_1 \qquad (1)$$

$$u_2 = r_2 i_2 + L_{2\sigma}\frac{d i_2}{d t} + E_2 \qquad (2)$$

$$\Phi = B A \qquad (3)$$

$$B = f(H) \qquad (4)$$

$$N_1 i_1 + N_2 i_2 = H l \qquad (5)$$

where

r_1 , r_2 — the resistance of primary and secondary windings.

$L_{1\sigma}$, $L_{2\sigma}$—the leakage inductance of the primary and secondary windings.

$E_1 = N_1\frac{d\Phi}{dt}$, $E_2 = N_2\frac{d\Phi}{dt}$—the induced electromagnetic forces of the primary and secondary windings respectively.

N_1 , N_2—the number of turns of the primary and secondary windings.

B , H , Φ—the intensity of magnetization ,the intensity of magnetic field and magnetic-flux in iron core of the transformer respectively.

A , l —the effective cross-section and length of the core respectively.

Dividing (5) by N_1 yields

$$i_1 + \frac{N_2}{N_1} i_2 = i_m , \qquad (6)$$

where, $i_m = \frac{Hl}{N_1}$ is the magnetizing current and generates the main magnetizing flux. Combining (3) and (4), it is clearly that the value of the magnetizing current is related to the physical parameters, the magnetizing curve and the induced electromagnetic forces.

Based on (1), (2) and (6), the general harmonic model is easily obtained, shown in Fig.2. Obviously, the electrical and magnetic circuits can be taken into account independently and the primary and secondary electrical circuits are finally combined with an ideal transformer including the magnetizing current source. For a specified harmonic frequency, both the winding resistance and leakage inductance are constant. Thus the general harmonic model of a transformer is established if the magnetizing current and

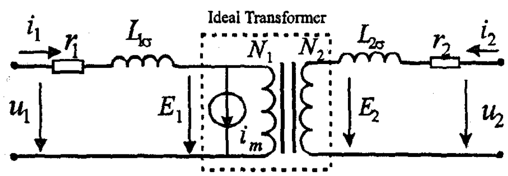

Fig.2 Complete Harmonic Model of A Transformer

its harmonics have been found.

III Characteristic Parameters of The Harmonic Model

From the analysis above, it is clear that there are three dominant characteristic parameters which are required to represent a transformer in frequency domain adequately: winding resistance, leakage inductance and magnetizing current. Currently, some studies have been carried out to measure the characteristic parameters of the harmonic model[6]. Due to the size of the experimental transformer, the inclusion derived in this literature can not be applied to general power transformers.

3.1 Winding Resistance r.

There exists much disputation on the value of resistance in harmonic frequencies. Some papers consider the value of winding resistance as constant[3,4], but a great majority of papers think that the skin effects and proximity effects should be accounted for in harmonic model. Some papers think the value of winding resistance is proportional to the harmonic order[7], or is proportional to the square root of the harmonic order[8]. To consider the influence of skin effects and proximity effects on harmonic model, this paper adopts the definition of short circuit parameters in model B of reference[2], where the resistance component is determined by the following formula:

$$r = 0.1026 k h X_{50}(J + h) \qquad (7)$$

where J is the ratio of hysterisis to eddy current losses, taken as 3 for silicon steels, and $k = 1/_{J+1}$; h is the harmonic order and X_{50} is the leakage reactance at 50Hz.

3.2 Leakage Inductance L_σ.

Since the leakage flux mainly flows along the air gap and tank of transformer, the leakage inductance is considered as constant, which is also verified by much research literature[2,3,4].

3.3 Magnetizing Current $\dot{i}_m$.

Presently, most power transformers operate in the saturated section of core materials, which makes the magnetizing current too significant to be neglected for power system harmonic analysis. In harmonic model of power transformers, the magnetizing characteristics is usually described by the magnetizing resistance representing the total core losses and the anhysteretic curve representing the saturation characteristics of core material.

3.3.1 Magnetizing Resistance R_m.

There are two kinds of losses in a transformer core: hysterisis losses and eddy current losses, which are integrated into iron core losses. Generally, the total core losses can be computed as follows:

$$P_{fe} = P_h + P_e = K_h B^s f + K_e B^2 f^2 \qquad (8)$$

where P_h and P_e are the hysterisis and eddy current losses respectively; K_h and K_e are constant, B is the maximum value of magnetic flux density; f is the frequency of the excitation; and S is the Steinmetz exponent, which typically ranges from 1.5 to 2.5, depending on the operating points of the transformer's core. If the voltage which produce the core flux is purely sinusoidal with rms magnitude V_m and frequency f, (8) may be rewritten as:

$$P_{fe} = k_h f^{1-s} V_m^s + k_e V_m^2 \qquad (9)$$

where $k_h \neq K_h$ and $k_e \neq K_e$. It is obvious that the magnetizing resistance is frequency-dependent and voltage-dependent. For the fact that the effect of hysteresis in modern transformer is much smaller than that of eddy current , and to simplify the computation, the magnetizing resistance can be considered as constant.

3.3.2 Anhysteretic Magnetization Curve

For a single phase transformer, (3) and (4) are usually jointed to be represented by Ψ–i curve which can be described by a polynomial using the Least Square Mean technique. Consequently, a set of discrete values of magnetizing current in a complete cycle can be obtained when the induced electromagnetic force is known. Thus, the magnitudes and phases of different harmonics can be analyzed by Discrete Fourier Transformation.

IV. Computation of the Magnetizing Current

Because the magnetizing current depends on the induced electromagnetic force, not on the terminal voltage directly, its computation has to proceed iteratively. For a special harmonic frequency, the horizontal branch of the harmonic model is linear, so it is convenient to compute the voltage drop produced by the magnetizing current in frequency-domain. On the other hand, due to the nonlinearity of the magnetizing characteristics, it is simpler to use time-domain techniques to compute the magnetizing current according to the anhysteretic curve. The computation steps are outlined as follows:

1) Calculate the voltage drop of the horizontal circuit using standard frequency domain analysis.

2) Convert the induced electromagnetic force phasor to the time domain and derive the representation of magnetic linkage applying the electromagnetic law.

3) Determine the active component of the magnetizing current in time domain according to the representation of induced electromagnetic force and the value of magnetizing resistance.

4) Compute the reactive component of the magnetizing current point by point in time domain based on the representation of magnetization characteristics and magnetic linkage.

5) The total magnetizing current is determined by summing two current components computed in step 3) and 4) respectively .

6)Using Discrete Fourier Transformation (DFT) to convert the total current to the frequency domain.

Generally, on the first iteration, it is assumed that the magnetizing current is set to zero and the induced electromagnetic force equals to the terminal voltage. After the initial value of the magnetizing current is determined, then the induced electromagnetic force is updated by subtracting the voltage drop across the horizontal branch from the primary terminal voltage phasor. Experience indicates that four to five iterations is usually sufficient for convergence.

V. Algorithm Verification

The test transformer is taken from reference[9] to verify the convergence of the algorithm. For lack of the magnetizing data, the current-flux curve of the test transformer is approximated by Least Square Approach based on its physical dimensions the B-H curve of a silicon steel sheet. The main operating and physical data are summarized below:

Rated apparent power: 6.25MVA

Rated Voltage: 16KV/110KV

Short circuit reactance: 22Ω

Current-flux curve: $i = 0.3588\Psi - 1.159\times10^{-4}\Psi^3 + 5.2\times10^{-8}\Psi^5$

Table 1 shows the harmonic voltage phasors of the transformer's primary terminal, which may be either the last iteration results of electric network harmonic analysis or the actual measurement of operating transformer's terminal voltage phasors. Table 2 gives the main harmonic contents of magnetizing current obtained in the 3rd and 4th iteration and the final convergent values. It has been seen that four or five iterations would give the satisfactory results within 1% error.

VI. Derivation of The Equivalent Harmonic Circuit

Although the complete harmonic model can be established after the determination of the magnetizing current, it is difficult to directly establish the nodal harmonic admittance matrix of the power transformer because the induced electromagnetic force can not be measured or evaluated. Based on the controlled voltage source model of the transformer, this paper derives its alternative harmonic equivalent circuit[10].

According to the order of harmonic current, the complete harmonic model of the transformer can be broken

Table 1. Primary terminal voltages of test transformer (kV, rad)

Harmonics	1st	3rd	5th	7th
Magnitudes	16	1.1	0.8	1
Angles	0.5	3.5	1.8	2

Table 2. Computation results of magnetizing current(A, rad)

Harmonic		3rd	4th	Convergent
1st	Mag	65.5622	65.8744	65.9546
	Ang	1.9859	1.9857	1.9856
3rd	Mag	27.0328	27.2776	27.3407
	Ang	3.1302	3.1266	3.1254
5th	Mag	9.5005	9.6701	9.7162
	Ang	-2.3006	-2.3084	-2.3117
7th	Mag	2.1681	2.2726	2.3109
	Ang	-2.2407	-2.2263	-2.2255

into a set of uncoupled harmonic model shown in Figure 3.

For a special harmonic frequency, yields

$$\dot{I}_{ph} = \dot{I}_{mh} + k\,\dot{I}_{sh} \tag{10}$$

$$\dot{U}_{ph} = \dot{E}_h + \dot{I}_{ph} Z_{ph} \tag{11}$$

$$k\,\dot{E}_h = \dot{U}_{sh} + \dot{I}_{sh} Z_{sh} \tag{12}$$

Substitution of (10) and (11) into (12), thus

$$\begin{aligned} k\dot{U}_{ph} &= \dot{U}_{sh} + \dot{I}_{sh} Z_{sh} + k\,\dot{I}_{ph} Z_{ph} \\ &= \dot{U}_{sh} + \dot{I}_{sh} Z_{sh} + k\,\dot{I}_{mh} Z_{ph} + k^2\,\dot{I}_{sh} Z_{ph} \end{aligned} \tag{13}$$

Consequently,

$$\dot{I}_{sh} = \frac{k}{k^2 Z_{ph} + Z_{sh}}\dot{U}_{ph} - \frac{1}{k^2 Z_{ph} + Z_{sh}}\dot{U}_{sh} + i_{2h} \tag{14}$$

$$\dot{I}_{ph} = \frac{k^2}{k^2 Z_{ph} + Z_{sh}}\dot{U}_{ph} - \frac{k}{k^2 Z_{ph} + Z_{sh}}\dot{U}_{sh} + i_{1h} \tag{15}$$

From equations (14) and (15), the equivalent harmonic model of transformer can be derived, shown in Figure 4, Here,

$$i_{1h} = \frac{Z_{sh}}{k^2 Z_{ph} + Z_{sh}}\dot{I}_{mh} \tag{16}$$

$$i_{2h} = -\frac{kZ_{ph}}{k^2 Z_{ph} + Z_{sh}}\dot{I}_{mh} \tag{17}$$

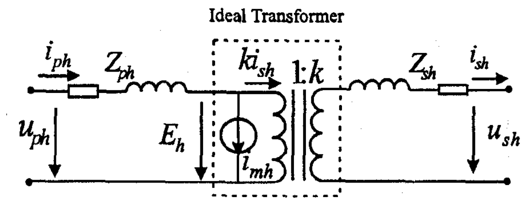

Figure 3 Uncoupled harmonic model of transformer

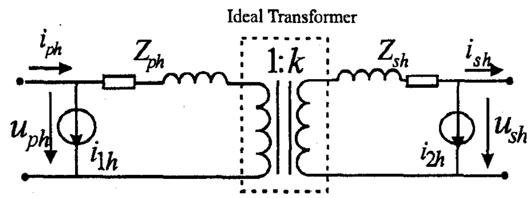

Figure 4 Equivalent harmonic model of transformer

VII. Conclusion

Appropriately modeling of all electrical elements is the basis of power system harmonic analysis. According to the characteristics of power system harmonic analysis, this paper derives the general harmonic model of power transformer and also proposed the practical computation and harmonic analysis methodologies of magnetizing current. Based on the controlled voltage source model of transformers, the paper derives the harmonic equivalent circuit suitable to power system harmonic analysis.

VIII. References

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