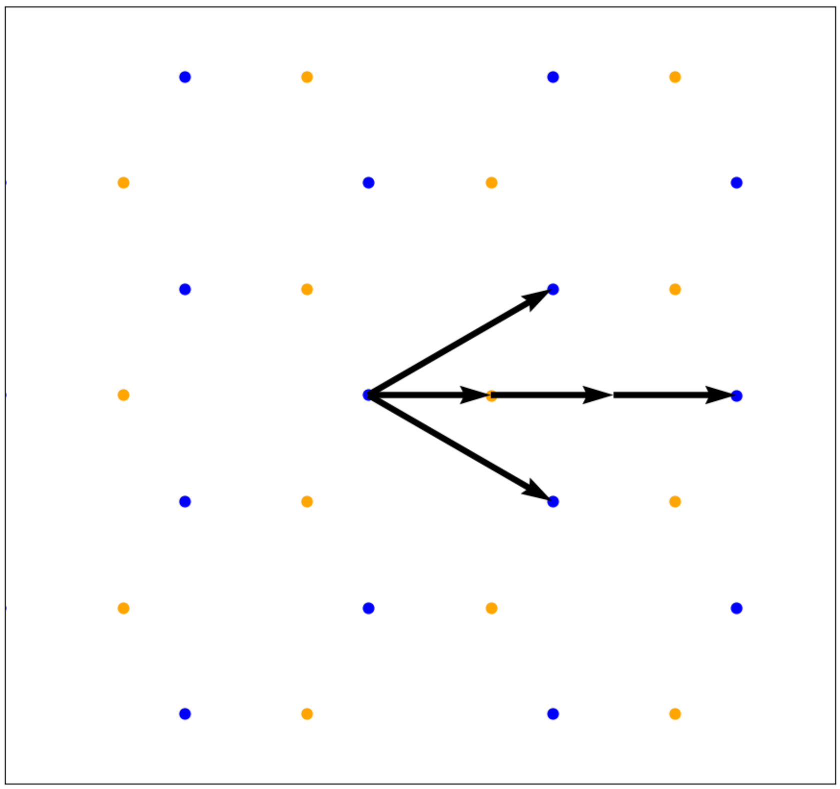
$$\vec{a}_1 = a\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}, \qquad \vec{a}_2 = a\begin{pmatrix}\frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{pmatrix}$$

$$\vec{d} = \frac{1}{3}\left(\vec{a}_1 + \vec{a}_2\right) = \frac{a}{3}\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix}$$



$$\vec{r}_B = m\vec{a}_1 + \vec{d} + 3n\vec{d} = am\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix} + \frac{a}{3}\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix} + \frac{3na}{3}\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix}$$

$$\vec{r}_B = a\left[m\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix} + \frac{1}{3}\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix} + n\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix}\right] = a\begin{pmatrix}m\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3} + n\sqrt{3}\\ m\frac{1}{2}\end{pmatrix}$$

$$|\vec{r}_B| = a\sqrt{\left(m\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3} + n\sqrt{3}\right)^2 + \left(m\frac{1}{2}\right)^2} = a\sqrt{m^2 + 3n^2 + 3mn + m + 2n + \frac{1}{3}}$$

$$\vec{r}_A = 3l\vec{d} = \frac{3la}{3}\begin{pmatrix}\sqrt{3}\\ 0\end{pmatrix} = \begin{pmatrix}la\sqrt{3}\\ 0\end{pmatrix}$$

$$|\vec{r}_A| = la\sqrt{3}$$

Condição: achar l, m e n de forma que $|\vec{r}_A| = |\vec{r}_B|$. Depois, obter posições usando os índices encontrados e gerar o ângulo entre os vetores.

$$|\vec{r}_A| = |\vec{r}_B|$$

$$la\sqrt{3} = a\sqrt{m^2 + 3n^2 + 3mn + m + 2n + \frac{1}{3}}$$

$$l = \sqrt{\frac{m^2 + 3n^2 + 3mn + m + 2n + \frac{1}{3}}{3}}$$

$$l = \sqrt{\frac{m^2}{3} + n^2 + mn + \frac{m}{3} + \frac{2n}{3} + \frac{1}{9}}$$

$$3l^2 = m^2 + 3n^2 + 3mn + m + 2n + \frac{1}{3}$$

$$m^2 + 3n^2 + 3mn + m + 2n + \frac{1}{3} - 3l^2 = 0$$