Applied Math HW

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1 Problem

Find the general solution to $y' + 15t^{14}y = t^{15}$. Use the variable $I = \int e^{t^{15}} dt$ where it occurs in your answer since this integral is not easily computable. Note that the arbitrary constant C would come from actually computing the integral I, so you do not need to write it.

1.1 Step 1

We will find an appropriate integrating factor $\mu(t)$. Multiply both sides of the equation by that factor, and express the left side as the derivative of a product.

$$\mu(t) = e^{\int 15t^{14} dt} \tag{1}$$
$$= e^{t^{15}} \tag{2}$$

$$y' + 15t^{14}y = t^{15} \tag{3}$$
$$\implies y'e^{t^{15}} + 15t^{14}e^{t^{15}} = t^{15}e^{t^{15}} \tag{4}$$
$$\implies \frac{d}{dt}[e^{t^{15}} y] = t^{15}e^{t^{15}}. \tag{5}$$

1.2 Step 2

We will use integration by parts to find the solution for the differential equation.

$$\implies e^{t^{15}} y = \int t^{15}e^{t^{15}} dt \tag{6}$$
$$\implies e^{t^{15}} y = \int tt^{14}e^{t^{15}} dt \tag{7}$$
$$\implies e^{t^{15}} y = \frac{t}{15}e^{t^{15}} - \frac{1}{15}\int e^{t^{15}} dt \tag{8}$$
$$\implies e^{t^{15}} y = \frac{t}{15}e^{t^{15}} - \frac{I}{15}. \tag{9}$$

So we finally have

$$y = \frac{t}{15} - \frac{I}{15} e^{-t^{15}}. \tag{10}$$