Comenius University Bratislava
Faculty of mathematics, physics and informatics

Rearrangement Problem of Bicolor Arrays by Prefix Reversals

Masters thesis

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Poďakovanie: Tu môžete poďakovať školiteľovi, prípadne ďalším osobám, ktoré vám s prácou nejako pomohli, poradili, poskytli dáta a podobne.

Abstrakt

TODO

Kľúčové slová: TODO

Abstract

TODO

Keywords:

Contents

List of Figures

List of Tables

Úvod

Chapter 1

State of the art

In this chapter, we will discuss current state of art of the pancake problem. Generalizations of the problem will be reviewed as well as the formulation of the problem in its most basic form and we will provide an overview of different approaches.

1.1 First formulation of the pancake sorting problem

The first formulation of the *pancake sorting problem* is given by *Jacob E. Goodman*, in 1975, and is as follows:

> The chef in our place is sloppy, and when he prepares a stack of pancakes they come out all different sizes. Therefore, when I deliver them to a customer, on the way to the table I rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. If there are n pancakes, what is the maximum number of flips (as a function f(n) of n) that I will ever have to use to rearrange them?

The pancake sorting problem, in its both original and burnt variant, has been a known problem in computer science for almost 50 years. Following the definition, *William H.Gates and Christos H.Papadimitriou*, former being the Microsoft founder, have researched it. In their work [5], they showed upper bound of $f(n) \leq \frac{5n+5}{3}$ and also lower bound $\frac{17n}{16} \leq f(n) \iff n \pmod{16} \equiv 0$.

They also introduced the burnt pancake problem, where each pancake has a *burnt* side. Pancakes are considered to be sorted, when all of the pancakes are facing *unburnt* side up. That is, if even number of flips are performed during the sorting. They also derived bounds for the burnt pancake problem, stating

$$\frac{3n}{2} - 1 \leq g(n) \leq 2n + 3$$

Table 1.1: Concrete computed numbers for pancake sorting problem

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
f(n)	0	1	3	4	5	7	8	9	10	11	13	14	15	16	17	18	19

, where $g(n)$ is denoting the corresponding burnt pancake problem function.

Lower bounds derived by Gates were quickly disproved by *Heydari* in [2], where they argued $\frac{9n}{8} + 2 \leq f(n)$. Almost simultaneously, Heydari et. al have found the permutation φ_n, where $n \pmod{14} \equiv 0$, to be bounded by $\frac{15n}{14} \leq \varphi_n \leq \frac{8n}{7} - 1$, consequently setting the lower bound $\frac{15n}{14} \leq f(n)$.

Most recent upper bounds were given by Chitturi in [4], where they proved $f(n) \leq \frac{18}{11}n + O(1)$.

The pancake sorting problem has been proven to be a $NP-hard$ problem, as shown in [3], as recently as 2015.

In conclusion, in its basic formulation, the pancake sorting problem is currently bounded by

$$\lfloor \frac{15}{14}n \rfloor \leq f(n) \leq \frac{18}{11}n + O(1)$$

It is also important to note that the preferred representation for the pancake sorting problem has been algebraic permutations. The concrete values of $f(n)$ are known to for $n \leq 17$, however it was shown that permutations are not the correct representation for computing pancake numbers for $n \leq 13$. Other method was introduced in [6], using graph theory and computing the diameter of a pancake graph with $n!$ vertices, each one labeling a unique permutatio, witch edges that connect vertices that are transposible by one prefix reversal. Concrete numbers for $f(n)$, with $14 \leq n \leq 15$ were computed, along with $n \leq 17$, as was shown in [1].

Table 1.1 shows computed numbers for pancake sorting problem.

1.2 Introduction to two dimensional pancake problem

Two dimensional pancake sorting problem has been introduced by *Yamamura and co.*, in [7]. This generalization works on a $n \times m$ array, with two different possible flips - horizontal and vertical, more on variant can be read in 1.3. They proved that any array is reachable by prefix reversals, unless both number of columns and rows are divisible by 4. Unlike in the pancake sorting problem, where it is proven any permutation is sortable by prefix reversals.

TODO pridaj veci z tejto prace, vytvorenu teoriu a bounds

In the next chapter, we will introduce another generalizations of the pancake problem, such as two-dimensional and burnt two-dimensional variants.

1.3 Two dimensional burnt pancake problem

Latest generalization of the problem is studied in [8]. They introduce a rearrangement problem of two-dimensional bicolor arrays by prefix reversals to be a generalization of the two dimensional burnt pancake problem. Unlike the previous two dimensional pancake problem, where group theory terminology was proven to be sufficient, in this generalization, it is argued that using equivalence classes in terms of a groupoid action is a more suitable tool.

An algorithm and an estimate of minimal number of prefix reversals needed are given as well

Chapter 2

Theory behind two dimensional burnt pancake problem

Chapter 3

Three-dimensional pancake sorting problem

3.1 Comparison with two-dimensional definition

3.2 Two-dimensional studied swaps

3.3 Rubic's cube swapping

3.4 Definition of 3d pancake sorting problem

3.5 Possible applications

Chapter 4

Graph theory and pancakes flipping

Záver

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