Arithmetic and Geometry

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Lecture 1: What is a number?

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The most important mathematical object are the natural numbers. These have obviously existed for hundreds of thousand years. Without getting too sophisticated, the starting point of mathematics is an empty page.

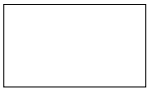

Figure 1: A Box Of Nothing

Now we introduce something: | represents a single entity we call one. The next idea is adding | to itself: || which we call two, ||| three, |||| four, ||||| five, and so on.

Definition 1. A natural number is a string of ones.

The natural numbers form a sequence, they are naturally ordered. Every natural number can be associated by the next one called the successor:

$$s(1) = ||, \quad s(2) = |||, \quad s(3) = ||||$$

Another important concept is the notion of relative size, which one is bigger/smaller?

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Compare the two natural numbers by pairing the ones in each sequence in both strings. If there are some leftovers, then that string of ones is the larger number.

Definition 2. Notion of equality: represent a number n or m where $n = m \Leftrightarrow$ every ones in n can be paired up with ones in m.

Definition 3. Notion of inequality: represent a number n or m where $n < m \Leftrightarrow n$ comes before m in the sequence of natural numbers.

Lecture 2: Arithmetic with numbers

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Definition 4. A natural number is a string of 1's:

$$1, 11, 111, 1111, 11111, 111111, 1111111, \ldots.$$

Now let's go back in time 1000000 years and imagine a person counting buffaloes and antilopes. The figure represents buffaloes (squares) and antilopes (triangles).

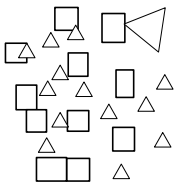


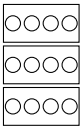
Figure 2: Buffaloes and antilopes

Number of squares: 11111111111
Number of triangles: 11111111111
Total: 111111111111111111111111111

Definition 5. The sum of numbers n and m is the combination of the strings of 1's. It is written n+m.

Laws of addition

- n + m = m + n (commmunative) $\quad 1111 + 111 = 111 + 1111$
- (k + n) + m = k + (n + m) (associative) $\quad (11 + 111) + 1111 = 11 + (111 + 1111)$
- n + 1 = s (n) (succesor)



1111 cicles in a box
111 boxes
Total = $111 \times 1111 = 111111111111$ O's.

Definition 6 (Product). The product of numbers n and m is the string formed by a copy of m for every 1 in n. It is written $n \times m$.

Laws of multiplication

- $n \times m = m \times n$ (commmunative)
- $(k \times n) \times m = k \times (n \times m)$ (associative)
- $n \times 1 = n$ (identity)

Distributive laws

- $k \times (n + m) = (k \times n) + (k \times m) = k \times n + k \times m$
- $(k + n) \times m = k \times m + n \times m$