PHY 493 HW 6

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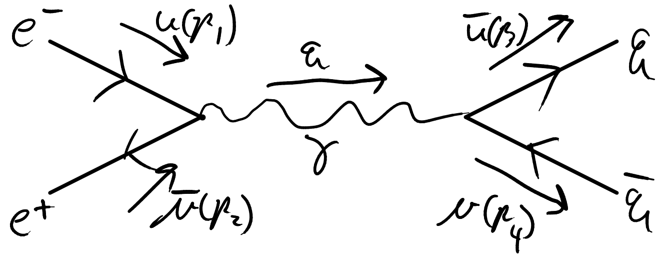
1 **a** $e^+ + e^- \to q + \bar{q}$, $E \gg m_e, m_q$, $Q_q = e_q/e$



Note: I did this out in full because I didn't quite get it in HW 5.

b $-i\mathcal{M}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$

$$= \int \bar{v}(p_2)\left[i g_e \gamma^\mu (2\pi)^4 \delta^4(p_1+p_2-q)\right] u(p_1) \frac{-i g_{\mu\nu}}{q^2}$$

$$\bar{u}(p_3) c_i^\dagger \left[i Q_q g_e \gamma^\nu (2\pi)^4 \delta^4(q-p_3-p_4)\right] c_i v(p_4) \frac{d^4 q}{(2\pi)^4}$$

$$= \frac{i Q_q g_e^2}{(p_2+p_3)^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right] c_i^\dagger c_i (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

$$\Rightarrow \mathcal{M} = \frac{Q_q g_e^2}{(p_2+p_3)^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right] c_i^\dagger c_i$$

$$\Rightarrow \langle|\mathcal{M}|^2\rangle = \underbrace{\frac{1}{4}}_{\text{avg. spin in}} \Big(\underbrace{\sum_{i,j} c_i^\dagger c_j}_{\text{color out}}\Big) \frac{Q_q^2 g_e^4}{(p_2+p_3)^4} tr\left(\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu(\not{p}_2 - m_e)\right)$$

$$tr\left(\gamma_\mu(\not{p}_4 - m_q)\gamma_\nu(\not{p}_3 + m_q)\right)$$

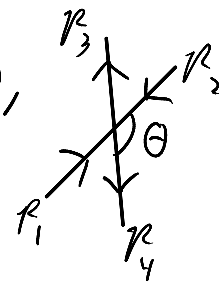
$$= \left(\frac{3}{4}\right)\frac{(4\pi e_q e)^2}{(p_2+p_3)^4} \boxed{tr\left(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2 - \gamma^\mu m_e \gamma^\nu m_e\right)} = A$$

$$\boxed{tr\left(\gamma_\mu \not{p}_4 \gamma_\nu \not{p}_3 - \gamma_\mu m_q \gamma_\nu m_q\right)}. = B$$

In the center-of-momentum frame,

$p_1 = (E, \vec{p}_e)$, $p_2 = (E, -\vec{p}_e)$, $p_3 = (E, \vec{p}_q)$, $p_4 = (E, -\vec{p}_q)$,

so then



$$A = tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2) - tr(\gamma^\mu m_e \gamma^\nu m_e)$$

$$= p_{1\alpha} p_{2\beta}\, tr(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta) - m_e^2\, tr(\gamma^\mu\gamma^\nu)$$

$$= 4 p_{1\alpha} p_{2\beta}\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} + g^{\alpha\nu}g^{\mu\beta}\right) - 4 m_e^2 g^{\mu\nu}$$

$$= 4\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(m_e^2 + p_1 \cdot p_2)\right)$$

$$\approx 4\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1 \cdot p_2)\right), \text{ since } p_1 \cdot p_2 = E^2 + |\vec{p}_e|^2 \gg m_e^2.$$

Similarly,

$$B = 4(p_{4\mu}p_{3\nu} + p_{4\nu}p_{3\mu} - g_{\mu\nu}(p_3 \cdot p_4)).$$

Then

$$\langle |M|^2 \rangle = \frac{12(4\pi e_q e)^2}{(p_1+p_2)^4}\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1\cdot p_2)\right]\left[p_{4\mu}p_{3\nu} + p_{4\nu}p_{3\mu} - g_{\mu\nu}(p_4\cdot p_3)\right]$$

$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)\right]$$

$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[(E^2 + |\vec{p}_e||\vec{p}_q|\cos\theta)^2 + (E^2 - |\vec{p}_e||\vec{p}_q|\cos\theta)^2\right]$$

$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[2E^4 + 2|\vec{p}_e|^2|\vec{p}_q|^2\cos^2\theta\right]$$

$$= \frac{48(4\pi e_q e)^2}{(p_1+p_2)^4}E^4\left[1+\cos^2\theta\right]$$

Note

$$(p_1+p_2)^2 = 2m_e^2 + 2(E^2 + |\vec{p}_e|^2) = 2m_e + 2(E^2 + E^2 - m_e^2) = 4E^2,$$

$$|\vec{p}_e|^2|\vec{p}_q|^2 = (E^2 - m_e^2)(E^2 - m_q^2) \simeq E^4.$$

We finally have

$$\boxed{\langle |M|^2 \rangle} = \frac{48}{4^2}(4\pi e_q e)^2(1+\cos^2\theta) = \boxed{48\pi^2 e_q^2 e^2(1+\cos^2\theta)}.$$

The cross section is then

$$\frac{d\sigma_q}{d\Omega} = \frac{1}{(8\pi)^2}\frac{\langle |M|^2\rangle}{(2E)^2}\frac{|\vec{p}_q|}{|\vec{p}_e|} \simeq \frac{1}{(8\pi)^2}\frac{\langle |M|^2\rangle}{(2E)^2}\frac{E}{E} = \frac{3e_q^2e^2}{16E^2}(1+\cos^2\theta),$$

$$\Rightarrow \sigma_q = \frac{3e_q^2e^2}{16E^2}2\pi\int_0^\pi (1+\cos^2\theta)(\sin\theta)\,d\theta = \frac{3\pi e_q^2 e^2}{8E^2}\left(2+\frac{2}{3}\right)$$

$$\boxed{\sigma_q = \frac{\pi e_q^2 e^2}{E^2}}$$

$\boxed{c}$ The relevant quark flavors with $m_q < 30$ GeV are u, d, c, s, b (all but t). So, summing over these flavors gives a total cross section of

$$\boxed{\sum_q \sigma_q} = \frac{\pi e^4}{E^2}\sum_q \frac{e_q^2}{e^2} = \frac{\pi\alpha^2}{E^2}\left[2\left(\frac{2}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2\right] = \boxed{\frac{11\pi\alpha^2}{9E^2}}.$$

Comparing to the $e^+e^- \to \mu^+\mu^-$ cross section σ_μ, we find the $e^+e^- \to q\bar{q}$ reaction almost 4 times more likely.

$$\boxed{\sum_q \sigma_q/\sigma_\mu} = \frac{11\pi\alpha^2}{9E^2}\Big/\frac{\pi\alpha^2}{3E^2} = \frac{11}{3} \simeq \boxed{3.67}$$