PHY 493 HW 2

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1 a $\quad A + B \longrightarrow C_1 + C_2 + \ldots + C_n, \quad P_A = (E, \vec{p}_A), \quad P_B = (M_B, \vec{0})$

$$P_A^2 = E^2 + M_A^2$$

$$(P_A + P_B)^2 = \left(\sum_n P_{C_n} \right)^2$$

we can choose to evaluate the LHS in the given frame and the RHS in the center of momentum frame (where we also assume there is 0 kinetic energy):

$$-M_A^2 - M_B^2 + 2(-E M_B + 0) = -\left(\sum_n M_{C_n} \right)^2$$

$$\Rightarrow \boxed{E = \frac{\left(\sum_n M_{C_n} \right)^2 - M_A^2 - M_B^2}{2 M_B}}$$

b i $\quad \pi^- + p \rightarrow K^0 + \Sigma^0, \quad M_{\pi^-} = 139.6 \; MeV/c^2, \quad M_p = 938.3 \; MeV/c^2$

$$M_{K^0} = 497.6 \; MeV/c^2, \quad M_{\Sigma^0} = 1193.6 \; MeV/c^2$$

$$\boxed{E = 1045 \; MeV}$$

ii $\quad p + p \rightarrow p + p + \pi^0, \quad M_{\pi^0} = 135.0 \; MeV/c^2$

$$\boxed{E = 1218 \; MeV}$$

iii $\quad \pi^- + p \rightarrow p + \bar{p} + n, \quad M_n = 939.6 \; MeV/c^2$

$$\boxed{E = 3747 \; MeV}$$

2 a $\quad \pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \vec{p}_\mu = -\vec{p}_\nu, \quad M_\nu \approx 0$

$$+ M_\pi^2 = -M_\mu^2 + 2\left(-p_\mu \sqrt{p_\mu^2 + M_\mu^2} - p_\mu^2 \right)$$

$$\Rightarrow (M_\pi^2 - M_\mu^2)^2 + p_\mu^4 - 2 p_\mu^2 (M_\pi^2 - M_\mu^2) = 4 p_\mu^2 (p_\mu^2 + M_\mu^2)$$

$$\Rightarrow 3 p_\mu^4 + p_\mu^2 (6 M_\mu^2 - 2 M_\pi^2) - (M_\pi^2 - M_\mu^2)$$

$$\Rightarrow p_\mu^2 = \frac{1}{6} \left[-(6 M_\mu^2 - 2 M_\pi^2) \pm \sqrt{(6 M_\mu^2 - 2 M_\pi^2)^2 + 12 (M_\pi^2 - M_\mu^2)} \right], \quad \begin{array}{l} 6 M_\mu > 2 M_\pi \\ \text{so must be } + \end{array}$$

$$\Rightarrow P_\mu = \frac{1}{\sqrt{6}} \sqrt{\sqrt{(6M_\mu^2 - 2M_\pi^2)^2 + 12(M_\pi^2 - M_\mu^2)} - (6M_\mu^2 - 2M_\pi^2)}$$

$$E = \sqrt{P_\mu^2 + M_\mu^2}$$

b $M_\pi = 139.57 \text{ MeV}/c^2, \quad M_\mu = 105.66 \text{ MeV}/c^2$

$$P_{\mu^+} = 0.54472 \text{ MeV}/c$$

c   At best we can Lorentz boost in the same direction as the μ^+, and at worst we can boost in the opposite direction.

$$\gamma = \frac{\sqrt{P_\pi^2 + M_\pi^2}}{M_\pi}, \quad \beta = \frac{P_\pi}{\gamma M_\pi} \quad \left(\text{The Lorentz factor and the velocity}\right)$$

Maximum P_μ':

$$P_\mu' = \gamma(P_\mu + \beta E_\mu) = \frac{\sqrt{P_\pi^2 + M_\pi^2}}{M_\pi} P_\mu + \frac{P_\pi}{M_\pi}\sqrt{P_\mu^2 + M_\pi^2}$$

Minimum P_μ':

$$P_\mu' = \gamma(P_\mu - \beta E_\mu) = \frac{\sqrt{P_\pi^2 + M_\pi^2}}{M_\pi} P_\mu - \frac{P_\pi}{M_\pi}\sqrt{P_\pi^2 + M_\pi^2}$$

For $P_\pi = 3 \cdot 10^3 \text{ MeV}/c^2$,

$$64.54 \text{ GeV}/c^2 < P_\mu' \le 64.57 \text{ GeV}/c^2$$

3 $\Sigma^{*0} \to \pi^- + \Sigma^+, \quad \pi^0 + \Sigma^0, \quad \pi^+ + \Sigma^-$

$\quad |1,0\rangle \quad\quad |1,-1\rangle \; |1,1\rangle \quad |1,0\rangle \; |1,0\rangle \quad |1,1\rangle \; |1,-1\rangle \quad$ Isospin states $|I, I_3\rangle$

$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|1,-1\rangle|1,1\rangle - |1,-1\rangle|1,1\rangle\right) \quad$ From PDG Clebsch-Gordan table

We see that (from isospin conservation) we would expect the $\pi^- \Sigma^+$ and $\pi^+ \Sigma^-$ in equal proportion and no $\pi^0 \Sigma^0$ decays.

So for 10,000 decays, we would expect $\boxed{5,000 \; \pi^- \Sigma^+ \text{ and } 5,000 \; \pi^+ \Sigma^-}$

4 Electromagnetic interactions conserve parity, so
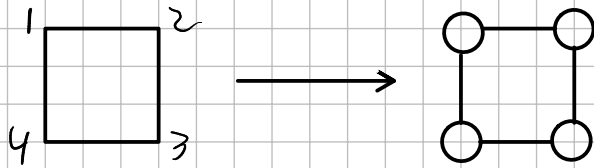$$\eta \to 3\pi^0, \quad \eta \to \pi^+ + \pi^- + \pi^0$$
suggests that η has odd parity since all pions have odd parity and $(-1)^3 = -1$. The decay
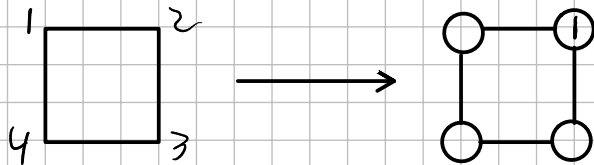$$\eta \to \pi^+ + \pi^-$$
cannot happen then, since $-1 \neq (-1)^2$.

5 **a** Consider how the vertices transform:

we place the vertices in order. There are 4 choices for 1.

Having made this choice, 2 must be adjacent to 1, and there are 2 choices for this.
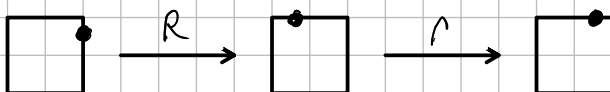

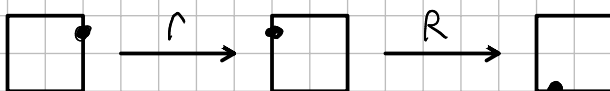There is only one choice left for 3 and then for 4, so there are
$$4 \cdot 2 \cdot 1 \cdot 1 = 8$$
symmetries.

b The group is non-abelian. Consider a point on a side along with the clockwise rotation by $90°$ R and the reflection about the vertical r:

but

In fact, $rR = R^{-1}r$