PHY 493 HW 3

Nicholas Todaro
2/19/19

1 a $\tau_\mu = 2.197 \cdot 10^{-6}\,s$, $N_0 = 10^6$, $t = 2.2 \cdot 10^{-5}$

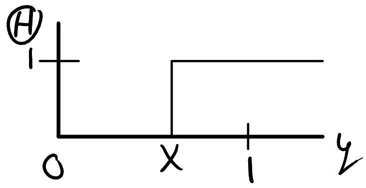
$$N = N_0 e^{-t/\tau_\mu} = 44.78 \approx \boxed{44}$$

b $\tau_{\pi^-} = 2.603 \cdot 10^{-8}$

$$P(t > 1s) = e^{-(1s)/\tau_{\pi^-}} \approx e^{-10^8} \approx 0$$

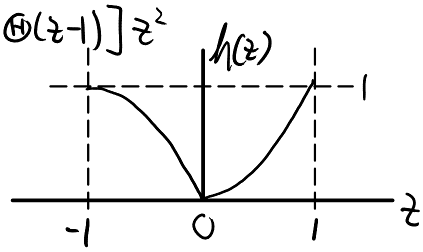
2 $f(x) = \begin{cases} 1 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$, $g(y) = \begin{cases} 2y & \text{if } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$, Θ is the Heaviside step function

$$\begin{aligned} h(z) &= \int f(x)\,dx \int g(y)\, \delta(z-x-y)\,dy \\ &= \int_0^1 f(z-y)\, g(y)\,dy \\ &= \int_0^1 \big(\Theta(z-y) - \Theta(z-y-1)\big)\, 2y\,dy \\ &= \int_0^1 \big(\Theta(y-z-1) - \Theta(y-z)\big)\, 2y\,dy \\ &= \Theta(-z-1)\int_0^1 2y\,dy + [\Theta(z+1) - \Theta(z+1-1)]\int_{z+1}^1 2y\,dy - \Theta(-z)\int_0^1 2y\,dy \\ &\quad - [\Theta(z) - \Theta(z-1)]\int_z^1 2y\,dy \\ &= [\Theta(-z-1) - \Theta(-z)] + [\Theta(z+1) - \Theta(z)](1-(z+1)^2) + [\Theta(z) - \Theta(z-1)](z^2-1) \\ &= [\Theta(z) - \Theta(z-1)] + [\Theta(z+1) - \Theta(z)](1-(z+1)^2) + [\Theta(z) - \Theta(z-1)](z^2-1) \\ &= [\Theta(z+1) - \Theta(z)](-z^2-2z) + [\Theta(z) - \Theta(z-1)]\,z^2 \end{aligned}$$



$\Theta(y-x)$

$$\boxed{h(z) = \begin{cases} 2|z| - z^2 & \text{if } -1 < z \le 0 \\ z^2 & \text{if } 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}}$$



3 a $A + B \to A + B$, $p_A + \begin{pmatrix} m_B \\ 0 \end{pmatrix} = p_{A'} + \begin{pmatrix} m_B \\ 0 \end{pmatrix}$ if $E_A \ll m_B$

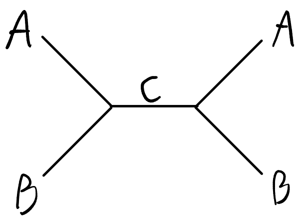
From Griffiths Eq. 6.4.2

$$\frac{d\sigma}{d\Omega} = \frac{S|\mathcal{M}|^2}{(8\pi)^2(E_A+E_B)^2}\frac{|\vec{p}_f|}{|\vec{p}_i|}$$

where $\vec{p}_i = \vec{p}_A$ and $\vec{p}_f = \vec{p}_{A'} = \vec{p}_A$ since we approximate $\vec{p}_B = \vec{p}_{B'} = 0$. Since the incoming pair is distinct as well as the outgoing, $S=1$. So

$$\frac{d\sigma}{d\Omega} = \frac{1|\mathcal{M}|^2}{(8\pi)^2(E_A+m_B)^2} \approx \frac{|\mathcal{M}|^2}{(8\pi m_B)^2}$$

b



We do ~~not~~ have an s-channel diagram because that would require AAC and BBC vertices

c

$$-i\mathcal{M}(2\pi)^4\delta^4(p_A+p_B-p_{A'}-p_{B'})$$

$$=\int(2\pi)^4\delta^4(p_A+p_B-p_C)(-ig)\frac{i}{q^2-m_C^2}(-ig)(2\pi)^4\delta^4(p_C-p_{A'}-p_{B'})\frac{d^4q}{(2\pi)^4}$$

$$=\frac{-ig}{u-m_C^2}(2\pi)^4\delta^4(p_A+p_B-p_{A'}-p_{B'})$$

$$\Rightarrow \boxed{\mathcal{M}=\frac{g}{u-m_C^2}}$$

d

$$\frac{d\sigma}{d\Omega}=\frac{g^2}{(8\pi m_B)^2(u-m_C^2)^2}$$

$$u-m_C^2=(p_A+p_B)^2-m_C^2=m_A^2+m_B^2-2E_Am_B-m_C^2$$

Since $E_A \ll m_B \Rightarrow E_Am_B \ll m_B^2$ and $m_A, m_C \ll m_B$,

$$u-m_C^2 \approx m_B^2$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\Omega}=\frac{g^2}{64\pi^2 m_B^4}}$$

e Since there is no angular dependence, the total crosssection is

$$\boxed{\sigma=4\pi\frac{d\sigma}{d\Omega}=\frac{g^2}{16\pi m_B^2}}$$