PHY 493 HW 3

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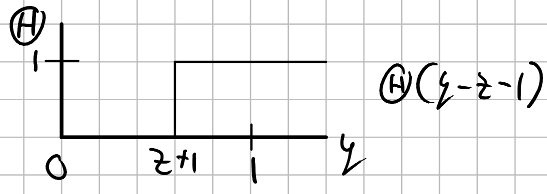
1 **a** $\tau_\mu = 2.197 \cdot 10^{-6}\,s, \quad N_0 = 10^6, \quad t = 2.2 \cdot 10^{-5}$

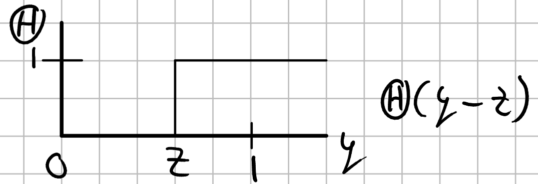
$$N = N_0 e^{-t/\tau_\mu} = 44.78 \approx 44$$

b $\tau_{\pi^-} = 2.603 \cdot 10^{-8}$

$$P(t > 1s) = e^{-(1s)/\tau_{\pi^-}} \approx e^{-10^8} \approx 0$$

2 $f(x) = \begin{cases} 1 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}, \quad g(y) = \begin{cases} 2y & \text{if } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$





$$h(z) = \int f(x)\,dx \int g(y)\,\delta(z-x-y)\,dy$$
$$= \int_0^1 f(z-y)\,g(y)\,dy$$
$$= \int_0^1 \big(\Theta(z-y) - \Theta(z-y-1)\big)\,2y\,dy$$
$$= \int_0^1 \big(\Theta(y-z-1) - \Theta(y-z)\big)\,2y\,dy$$
$$= (1-(z+1))\left[\Theta(z+1) - \Theta(z+1-1)\right] - (1-z)\left[\Theta(z) - \Theta(z-1)\right]$$
$$= -z\,\Theta(z+1) + (2z-1)\,\Theta(z) - (z-1)\,\Theta(z-1)$$

$$h(z) = \begin{cases} -z & \text{if } -1<z<0 \\ z-1 & \text{if } 0<z<1 \\ 0 & \text{if } 1<z \end{cases}$$

3 **a** $A + B \to A + B$

$$d\sigma = |M|^2 \frac{\hbar^2 S}{4\sqrt{(p_A \cdot p_B)^2 - (m_A m_B)^2}} \frac{d^3\vec{p}_{A'}}{2(2\pi)^3 E_{A'}} \frac{d^3\vec{p}_{B'}}{2(2\pi)^3 E_{B'}} (2\pi)^4 \delta^4(p_A + p_B - p_{A'} - p_{B'})$$

$$= |M|^2 \frac{\hbar^2 S}{4\sqrt{(p_A \cdot p_B)^2 - (m_A m_B)^2}} \frac{1}{4(2\pi)^2} \frac{\delta(E_A + E_B - E_{A'} - E_{B'})}{E_{B'}} \frac{d^3\vec{p}_{A'}}{E_{A'}}$$

Using spherical coordinates $d^3\vec{p}_{A'} = q^2\,dq\,d\Omega$, evaluating in the center-of-momentum frame

$$\vec{p}_A = -\vec{p}_B \equiv \vec{p}, \quad E_A + E_B = m_A + m_B, \quad E_{B'} = \sqrt{m_B^2 + (\vec{p}_A + \vec{p}_B - \vec{p}_A{}')}$$
$$E_{A'} = \sqrt{m_A^2 + q^2} \qquad\qquad = \sqrt{m_B^2 + q^2}$$

So integrating yields

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 S}{64\pi^2\sqrt{p^2-(m_A m_B)^2}} \int_0^{\infty} |M|^2 \frac{\delta(m_A+m_B-\sqrt{m_A^2+q^2}-\sqrt{m_B^2+q^2})}{\sqrt{m_A^2+q^2}\sqrt{m_B^2+q^2}} q^2 dq$$

Making the substitution

$$\xi = \sqrt{m_A^2+q^2} + \sqrt{m_B^2+q^2} \Rightarrow d\xi = \frac{q\,\xi\,dq}{\sqrt{m_A^2+q^2}\,\sqrt{m_B^2+q^2}}$$

$$\Rightarrow \xi(\xi - 2\sqrt{m_B^2+q^2}) = m_A^2 - m_B^2$$

$$\Rightarrow \xi^2 - m_A^2 + m_B^2 = 2\xi\sqrt{m_B^2+q^2}$$

$$\Rightarrow \frac{1}{4\xi^2}(\xi^2 - m_A^2 + m_B^2)^2 - m_B^2 = q^2$$

we may evaluate the integral as

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 S}{64\pi^2\sqrt{p^2-(m_A m_B)^2}} \int_0^{\infty} |M|^2 \delta(m_A+m_B-\sqrt{m_A^2+q^2}-\sqrt{m_B^2+q^2}) \frac{q}{\xi} d\xi$$