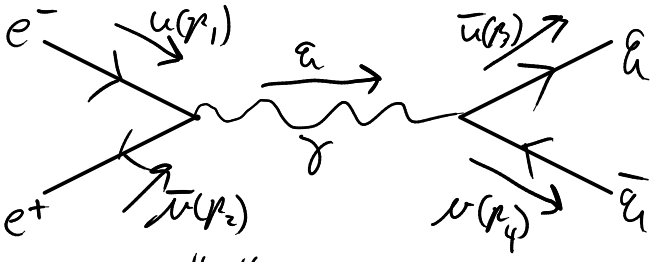
PHY 493 HW 6

Nicholas Lodoreb

1 **a** $e^+ + e^- \to q + \bar{q}$, $E \gg m_e, m_q$, $Q_q = e_q/e$



Note: I did this out in full because I didn't quite get it in HW 5.

b $-i\mathcal{M}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$

$$= \int \bar{v}(p_2)\left[ie_e\gamma^\mu(2\pi)^4\delta^4(p_1+p_2-q)\right]u(p_1)\frac{-iq_{\mu\nu}}{q^2}$$
$$\bar{u}(p_3)C_i^\dagger\left[iQ_q e_e\gamma^\nu(2\pi)^4\delta^4(q-p_3-p_4)\right]C_i v(p_4)\frac{d^4q}{(2\pi)^4}$$

$$= \frac{iQ_q e_e^2}{(p_2+p_3)^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right]C_i^\dagger C_i(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$\Rightarrow \mathcal{M} = \frac{Q_q e_e^2}{(p_2+p_3)^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right]C_i^\dagger C_i$$

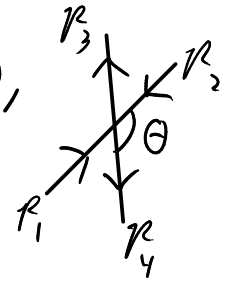
$$\Rightarrow \langle|\mathcal{M}|^2\rangle = \frac{1}{4}\left(\sum_{i,i} C_i^\dagger C_i\right)\frac{Q_q^2 e_e^4}{(p_2+p_3)^4}\,\mathrm{tr}\left(\gamma^\mu(\not{p}_1+m_e)\gamma^\nu(\not{p}_2-m_e)\right)$$
$$\mathrm{tr}\left(\gamma_\mu(\not{p}_4-m_q)\gamma_\nu(\not{p}_3+m_q)\right)$$

(avg. spin in; color out)

$$= \left(\frac{3}{4}\right)\frac{(4\pi Q_q e)^2}{(p_2+p_3)^4}\boxed{\mathrm{tr}\left(\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2 - \gamma^\mu m_e\gamma^\nu m_e\right) = A}$$
$$\boxed{\mathrm{tr}\left(\gamma_\mu\not{p}_4\gamma_\nu\not{p}_3 - \gamma_\mu m_q\gamma_\nu m_q\right) = B}$$

In the center-of-momentum frame,

$p_1 = (E, \vec{p}_e)$, $p_2 = (E, -\vec{p}_e)$, $p_3 = (E, \vec{p}_q)$, $p_4 = (E, -\vec{p}_q)$,



so then

$$A = \mathrm{tr}\left(\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2\right) - \mathrm{tr}\left(\gamma^\mu m_e\gamma^\nu m_e\right)$$
$$= p_{1\alpha}p_{2\beta}\,\mathrm{tr}\left(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right) - m_e^2\,\mathrm{tr}\left(\gamma^\mu\gamma^\nu\right)$$
$$= 4p_{1\alpha}p_{2\beta}\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} + g^{\alpha\nu}g^{\mu\beta}\right) - 4m_e^2 g^{\mu\nu}$$

$$= 4\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(m_e^2 + p_1\cdot p_2)\right)$$
$$\approx 4\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1\cdot p_2)\right), \text{ since } p_1\cdot p_2 = E^2 + |\vec{p}_e|^2 \gg m_e^2.$$

Similarly,

$$B = 4\left(p_{4\mu}p_{3\nu} + p_{4\nu}p_{3\mu} - g_{\mu\nu}(p_3\cdot p_4)\right).$$

Then

$$\langle |M|^2\rangle = \frac{12(4\pi e_q e)^2}{(p_1+p_2)^4}\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1\cdot p_2)\right]\left[p_{4\mu}p_{3\nu} + p_{4\nu}p_{3\mu} - g_{\mu\nu}(p_4\cdot p_3)\right]$$
$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[(p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4)\right]$$
$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[\left(E^2 + |\vec{p}_e||\vec{p}_q|\cos\theta\right)^2 + \left(E^2 - |\vec{p}_e||\vec{p}_q|\cos\theta\right)^2\right]$$
$$= \frac{24(4\pi e_q e)^2}{(p_1+p_2)^4}\left[2E^4 + 2|\vec{p}_e|^2|\vec{p}_q|^2\cos^2\theta\right]$$
$$= \frac{48(4\pi e_q e)^2}{(p_1+p_2)^4}E^4\left[1+\cos^2\theta\right]$$

Note

$$(p_1+p_2)^2 = 2m_e^2 + 2\left(E^2 + |\vec{p}_e|^2\right) = 2m_e + 2(E^2 + E^2 - m_e^2)$$
$$= 4E^2,$$

$$|\vec{p}_e|^2|\vec{p}_q|^2 = (E^2 - m_e^2)(E^2 - m_q^2) \approx E^4.$$

we finally have

$$\langle |M|^2\rangle = \frac{48}{4^2}(4\pi e_q e)^2(1+\cos^2\theta) = 48\pi^2 e_q^2 e^2(1+\cos^2\theta).$$

The cross section is then

$$\frac{d\sigma_q}{d\Omega} = \frac{1}{(8\pi)^2}\frac{\langle |M|^2\rangle}{(2E)^2}\frac{|\vec{p}_q|}{|\vec{p}_e|} \approx \frac{1}{(8\pi)^2}\frac{\langle |M|^2\rangle}{(2E)^2}\frac{E}{E} = \frac{3e_q^2e^2}{16E^2}(1+\cos^2\theta),$$

$$\Rightarrow \sigma_q = \frac{3e_q^2e^2}{16E^2}2\pi\int_0^\pi (1+\cos^2\theta)(\sin\theta)\,d\theta = \frac{3\pi e_q^2e^2}{8E^2}\left(2+\frac{2}{3}\right)$$

$$\sigma_q = \frac{\pi e_q^2 e^2}{E^2}$$

[c] The relevant quark flavors with $m_q < 30$ GeV are u, d, c, s, b (all but t). So, summing over these flavors gives a total cross section of

$$\sum_q \sigma_q = \frac{\pi e^4}{E^2} \sum_q \frac{e_q^2}{e^2} = \frac{\pi\alpha^2}{E^2}\left[2\left(\frac{2}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2\right] = \frac{11\pi\alpha^2}{9E^2}.$$

Comparing to the $e^+e^- \to \mu^+\mu^-$ cross section σ_μ, we find the $e^+e^- \to q\bar{q}$ reaction almost 4 times more likely:

$$\sum_q \sigma_q/\sigma_\mu = \frac{11\pi\alpha^2}{9E^2} \Big/ \frac{\pi\alpha^2}{3E^2} = \frac{11}{3} \approx 3.67.$$