PHY 493 HW 2

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$\boxed{1}$ $\boxed{a}$ $A + B \longrightarrow C_1 + C_2 + \ldots + C_n$, $P_A = (E, \vec{p}_A)$, $P_B = (m_B, \vec{0})$

$$p_A^2 = E^2 + m_A^2$$

$$(P_A + P_B)^2 = \left(\sum_n P_{C_n}\right)^2$$

we can choose to evaluate the LHS in the given frame and the RHS in the center of momentum frame (where we also assume there is 0 kinetic energy):

$$-m_A^2 - m_B^2 + 2(-E m_B + 0) = -\left(\sum_n m_{C_n}\right)^2$$

$$\Rightarrow \boxed{E = \frac{\left(\sum_n m_{C_n}\right)^2 - m_A^2 - m_B^2}{2 m_B}}$$

$\boxed{b}$ $\boxed{i}$ $\pi^- + p \rightarrow K^0 + \Sigma^0$, $m_{\pi^-} = 139.6\ \mathrm{MeV}/c^2$, $m_p = 938.3\ \mathrm{MeV}/c^2$
$m_{K^0} = 497.6\ \mathrm{MeV}/c^2$, $m_{\Sigma^0} = 1193.6\ \mathrm{MeV}/c^2$

$$\boxed{E = 1045\ \mathrm{MeV}}$$

$\boxed{ii}$ $p + p \rightarrow p + p + \pi^0$, $m_{\pi^0} = 135.0\ \mathrm{MeV}/c^2$

$$\boxed{E = 1218\ \mathrm{MeV}}$$

$\boxed{iii}$ $\pi^- + p \rightarrow p + \bar{p} + n$, $m_n = 939.6\ \mathrm{MeV}/c^2$

$$\boxed{E = 3747\ \mathrm{MeV}}$$

$\boxed{2}$ $\boxed{a}$ $\pi^+ \rightarrow \mu^+ + \nu_\mu$, $\vec{p}_\mu = -\vec{p}_\nu$, $m_\nu \approx 0$

$$-m_\pi^2 = -m_\mu^2 + 2\left(-p_\mu \sqrt{p_\mu^2 + m_\mu^2} - p_\mu^2\right)$$

$$\Rightarrow (m_\pi^2 - m_\mu^2)^2 + p_\mu^4 - 2p_\mu^2(m_\pi^2 - m_\mu^2) = 4p_\mu^2(p_\mu^2 + m_\mu^2)$$

$$\Rightarrow 3p_\mu^4 + p_\mu^2(6m_\mu^2 - 2m_\pi^2) - (m_\pi^2 - m_\mu^2)$$

$$\Rightarrow p_\mu^2 = \frac{1}{6}\left[-(6m_\mu^2 - 2m_\pi^2) \pm \sqrt{(6m_\mu^2 - 2m_\pi^2)^2 + 12(m_\pi^2 - m_\mu^2)}\right],$$ $6m_\mu > 2m_\pi$, so must be +

$$\Rightarrow p_\mu = \frac{1}{\sqrt{6}}\sqrt{\sqrt{(6m_\mu^2-2m_\pi^2)^2+12(m_\pi^2-m_\mu^2)} - (6m_\mu^2-2m_\pi^2)}$$
$$E = \sqrt{p_\mu^2+m_\mu^2}$$

b $m_\pi = 139.57\ MeV/c^2$, $m_\mu = 105.66\ MeV/c^2$

$$p_{\mu^+} = 0.54472\ MeV/c$$

c At best we can Lorentz boost in the same direction as the μ^+, and at worst we can boost in the opposite direction.

$$\gamma = \frac{\sqrt{p_\pi^2+m_\pi^2}}{m_\pi},\quad \beta = \frac{p_\pi}{\gamma m_\pi}$$ (The Lorentz factor and the velocity)

Maximum p_μ':

$$p_\mu' = \gamma(p_\mu + \beta E_\mu) = \frac{\sqrt{p_\pi^2+m_\pi^2}}{m_\pi}p_\mu + \frac{p_\pi}{m_\pi}\sqrt{p_\pi^2+m_\pi^2}$$

minimum p_μ':

$$p_\mu' = \gamma(p_\mu - \beta E_\mu) = \frac{\sqrt{p_\pi^2+m_\pi^2}}{m_\pi}p_\mu - \frac{p_\pi}{m_\pi}\sqrt{p_\pi^2+m_\pi^2}$$

For $p_\pi = 3\cdot 10^3\ MeV/c^2$,

$$64.54\ GeV/c^2 < p_\mu' \leq 64.57\ GeV/c^2$$

3 $\Sigma^{*0} \rightarrow \pi^- + \Sigma^+,\quad \pi^0 + \Sigma^0,\quad \pi^+ + \Sigma^-$

$|1,0\rangle \quad |1,-1\rangle\ |1,1\rangle \quad |1,0\rangle\ |1,0\rangle \quad |1,1\rangle\ |1,-1\rangle$ Isospin states $|I, I_3\rangle$

$|1,0\rangle = \frac{1}{\sqrt{2}}(|1,-1\rangle|1,1\rangle - |1,-1\rangle|1,1\rangle)$ From PDG Clebsch-Gordan table

we see that (from isospin conservation) we would expect the $\pi^-\Sigma^+$ and $\pi^+\Sigma^-$ in equal proportion and no $\pi^0\Sigma^0$ decays.

So for 10,000 decays, we would expect 5,000 $\pi^-\Sigma^+$ and 5,000 $\pi^+\Sigma^-$

$\boxed{4}$ Electromagnetic interactions conserve parity, so

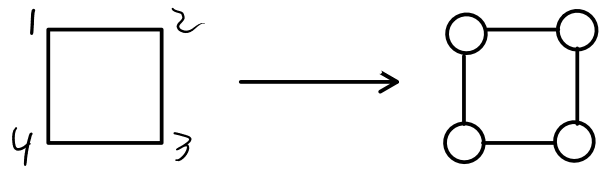
$$\eta \to 3\pi^0, \quad \eta \to \pi^+ + \pi^- + \pi^0$$

suggests that η has odd parity since all pions have odd parity and $(-1)^3 = -1$. The decay

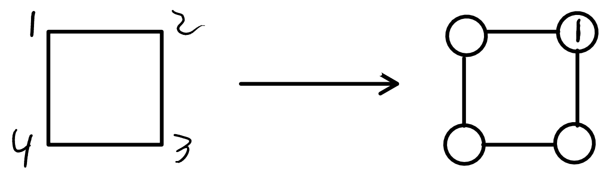
$$\eta \to \pi^+ + \pi^-$$

cannot happen then, since $-1 \neq (-1)^2$.

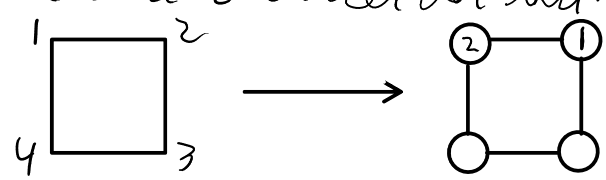
$\boxed{5}$ $\boxed{a}$ Consider how the vertices transform:



we place the vertices in order. There are 4 choices for 1.



Having made this choice, 2 must be adjacent to 1, and there are 2 choices for this.

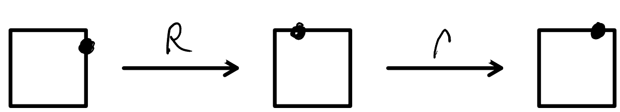


There is only one choice left for 3 and then for 4, so there are

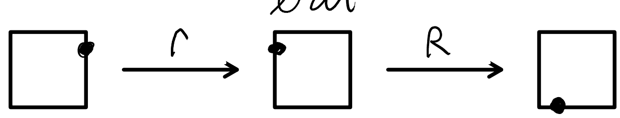
$$4 \cdot 2 \cdot 1 \cdot 1 = 8$$

symmetries.

$\boxed{b}$ The group is non-abelian. Consider a point on a side along with the clockwise rotation by $90°$ R and the reflection about the vertical r:



but



In fact, $rR = R^{-1}r$