STATISTICAL TABLES (ONLINE)

Table A: Empirical Cumulative Distribution of τ

Significance level	0.01	0.025	0.05	0.10
Sample Size T	The τ statistic: No Constant or Time Trend ($a_0 = a_2 = 0$)			
25	−2.65	−2.26	−1.95	−1.60
50	−2.62	−2.25	−1.95	−1.61
100	−2.60	−2.24	−1.95	−1.61
250	−2.58	−2.24	−1.95	−1.62
300	−2.58	−2.23	−1.95	−1.62
∞	−2.58	−2.23	−1.95	−1.62
	The τ_μ statistic: Constant but No Time Trend ($a_2 = 0$)			
25	−3.75	−3.33	−2.99	−2.62
50	−3.59	−3.22	−2.93	−2.60
100	−3.50	−3.17	−2.89	−2.59
250	−3.45	−3.14	−2.88	−2.58
500	−3.44	−3.13	−2.87	−2.57
∞	−3.42	−3.12	−2.86	−2.57
	The τ_τ statistic: Constant + Time Trend			
25	−4.38	−3.95	−3.60	−3.24
50	−4.15	−3.80	−3.50	−3.18
100	−4.05	−3.73	−3.45	−3.15
250	−3.99	−3.69	−3.43	−3.13
500	−3.97	−3.67	−3.42	−3.13
∞	−3.96	−3.67	−3.41	−3.12

Source: The table is reproduced from Fuller (1996).

Table B: Empirical Distribution of Φ

Significance level	0.10	0.05	0.025	0.01
Sample size T		Φ_1		
25	4.12	5.18	6.30	7.88
50	3.94	4.86	5.80	7.06
100	3.86	4.71	5.57	6.70
250	3.81	4.63	5.45	6.52
500	3.79	4.61	5.41	6.47
∞	3.78	4.59	5.38	6.43
		Φ_2		
25	4.67	5.68	6.75	8.21
50	4.31	5.13	5.94	7.02
100	4.16	4.88	5.59	6.50
250	4.07	4.75	5.40	6.22
500	4.05	4.71	5.35	6.15
∞	4.03	4.68	5.31	6.09
		Φ_3		
25	5.91	7.24	8.65	10.61
50	5.61	6.73	7.81	9.31
100	5.47	6.49	7.44	8.73
250	5.39	6.34	7.25	8.43
500	5.36	6.30	7.20	8.34
∞	5.34	6.25	7.16	8.27

Table C: Critical Values for the Engle–Granger Cointegration Test

T	1%	5%	10%	1%	5%	10%
	Two Variables			***Three Variables***		
50	−4.123	−3.461	−3.130	−4.592	−3.915	−3.578
100	−4.008	−3.398	−3.087	−4.441	−3.828	−3.514
200	−3.954	−3.368	−3.067	−4.368	−3.785	−3.483
500	−3.921	−3.350	−3.054	−4.326	−3.760	−3.464
	Four Variables			***Five Variables***		
50	−5.017	−4.324	−3.979	−5.416	−4.700	−4.348
100	−4.827	−4.210	−3.895	−5.184	−4.557	−4.240
200	−4.737	−4.154	−3.853	−5.070	−4.487	−4.186
500	−4.684	−4.122	−3.828	−5.003	−4.446	−4.154

The critical values are for cointegrating relations (with a constant in the cointegrating vector) estimated using the Engle–Granger methodology.
Source: Critical values are interpolated using the response surface in MacKinnon (1991).

Table D: Residual Based Cointegration Test with $I(1)$ and $I(2)$ Variables

		Intercept Only				**Linear Trend**			
		$m_2 = 1$* *prob-value		***$m_2 = 2$* *prob-value***		***$m_2 = 1$* *prob-value***		***$m_2 = 2$* *prob-value***	
m_1	***T***	**0.01**	**0.05**	**0.01**	**0.05**	**0.01**	**0.05**	**0.01**	**0.05**
0	50	−4.18	−3.51	−4.70	−4.02	−4.66	−4.01	−5.14	−4.45
	100	−4.09	−3.42	−4.51	−3.86	−4.55	−3.90	−4.93	−4.31
	250	−4.02	−3.38	−4.35	−3.80	−4.41	−3.83	−4.81	−4.20
1	50	−4.65	−3.93	−5.15	−4.40	−5.11	−4.42	−5.62	−4.89
	100	−4.51	−3.89	−4.85	−4.26	−4.85	−4.26	−5.23	−4.62
	250	−4.39	−3.80	−4.71	−4.18	−4.73	−4.19	−5.11	−4.50
2	50	−4.93	−4.30	−5.54	−4.77	−5.47	−4.74	−5.98	−5.17
	100	−4.81	−4.25	−5.29	−4.59	−5.21	−4.58	−5.59	−4.93
	250	−4.77	−4.16	−5.06	−4.49	−5.07	−4.51	−5.35	−4.80
3	50	−5.38	−4.71	−5.76	−5.08	−5.89	−5.13	−6.23	−5.48
	100	−5.20	−4.56	−5.58	−4.92	−5.52	−4.91	−5.97	−5.25
	250	−5.05	−4.48	−5.44	−4.83	−5.38	−4.78	−5.69	−5.07
4	50	−5.81	−5.09	−6.24	−5.48	−6.35	−5.47	−6.64	−5.82
	100	−5.58	−4.93	−5.88	−5.20	−5.86	−5.20	−6.09	−5.50
	250	−5.39	−4.28	−5.64	−5.07	−5.66	−5.08	−5.95	−5.34

Note: m_1 is the number of $I(1)$ variables and m_2 is the number of $I(2)$ variables on the right-hand side of the multicointegrating relationship.

Source: The critical values for the intercept only case are from Haldrup (1994) and critical values for the linear trend are from Engsted, Gonzalo and Haldrup (1997).

Table E: Empirical Distributions of the λ_{max} and λ_{trace} Statistics

	Significance level							
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
	λ_{max} and λ_{trace} statistics without any deterministic regressors							
$n-r$	λ_{max}				λ_{trace}			
1	2.86	3.84	4.93	6.51	2.86	3.84	4.93	6.51
2	9.52	11.44	13.27	15.69	10.47	12.53	14.43	16.31
3	15.59	17.89	20.02	22.99	21.63	24.31	26.64	29.75
4	21.56	23.80	26.14	28.82	36.58	39.89	42.30	45.58
5	27.62	30.04	32.51	35.17	54.44	59.46	62.91	66.52
	λ_{max} and λ_{trace} statistics with drift							
$n-r$	λ_{max}				λ_{trace}			
1	2.69	3.76	4.95	6.65	2.69	3.76	4.95	6.65
2	12.07	14.07	16.05	18.63	13.33	15.41	17.52	20.04
3	18.60	20.97	23.09	25.52	26.79	29.68	32.56	35.65
4	24.73	27.07	28.98	32.24	43.95	47.21	50.35	54.46
5	30.90	33.46	35.71	38.77	64.84	68.52	71.80	76.07
	λ_{max} and λ_{trace} statistics with a constant in the cointegrating vector							
	λ_{max}				λ_{trace}			
1	7.52	9.24	10.80	12.97	7.52	9.24	10.80	12.95
2	13.75	15.67	17.63	20.20	17.85	19.96	22.05	24.60
3	19.77	22.00	24.07	26.81	32.00	34.91	37.61	41.07
4	25.56	28.14	30.32	33.24	49.65	53.12	56.06	60.16
5	31.66	34.40	36.90	39.79	71.86	76.07	80.06	84.45

Source: Osterwald–Lenum (1992).

Table F: Critical Values for $\beta_1 = 0$ in the Error−Correction Model

k		$T^a = 50$	$T^a = 100$	$T^a = 200$	$T^a = 500$
			No Intercept or Trend ($d = 0$)		
2	1%	−3.309	−3.259	−3.235	−3.220
	5%	−2.625	−2.609	−2.602	−2.597
	10%	−2.273	−2.268	−2.266	−2.265
3	1%	−3.746	−3.683	−3.652	−3.633
	5%	−3.047	−3.026	−3.016	−3.009
	10%	−2.685	−2.680	−2.677	−2.675
4	1%	−4.088	−4.015	−3.979	−3.957
	5%	−3.370	−3.348	−3.337	−3.331
	10%	−3.000	−2.997	−2.995	−2.994
			Intercept but no Trend ($d = 1$)		
2	1%	−3.954	−3.874	−3.834	−3.811
	5%	−3.279	−3.247	−3.231	−3.221
	10%	−2.939	−2.924	−2.916	−2.911
3	1%	−4.268	−4.181	−4.138	−4.112
	5%	−3.571	−3.538	−3.522	−3.512
	10%	−3.216	−3.205	−3.199	−3.195
4	1%	−4.537	−4.446	−4.401	−4.374
	5%	−3.819	−3.789	−3.774	−3.765
	10%	−3.453	−3.447	−3.444	−3.442
			Intercept and Trend ($d = 2$)		
2	1%	−4.451	−4.350	−4.299	−4.269
	5%	−3.778	−3.733	−3.710	−3.696
	10%	−3.440	−3.416	−3.405	−3.398
3	1%	−4.712	−4.605	−4.552	−4.519
	5%	−4.014	−3.971	−3.949	−3.935
	10%	−3.662	−3.643	−3.634	−3.629
4	1%	−4.940	−4.831	−4.776	−4.743
	5%	−4.221	−4.182	−4.162	−4.150
	10%	−3.857	−3.846	−3.840	−3.837

Note: T^a is the adjusted sample size equal to $T-(2k-1)-d$ where T is the usable sample size, d is the number of deterministic regressors, and k is the number of $I(1)$ variables in the model. The critical values are calculated using equation (26) in Ericsson and MacKinnon (2002).

Table G: Critical Values for Threshold Unit Roots

Panel (a): Consistent Estimate of the Threshold Using the TAR Model

T	No Lagged Changes				One Lagged Change				Four Lagged Changes			
	90%	95%	97.5	99%	90%	95%	97.5	99%	90%	95%	97.5	99%
50	5.15	6.19	7.25	8.64	5.55	6.62	7.66	9.10	5.49	6.55	7.59	9.00
100	5.08	6.06	6.93	8.19	5.39	6.34	7.30	8.54	5.38	6.32	7.29	8.56
250	5.11	6.03	6.88	8.04	5.26	6.12	6.99	8.14	5.36	6.29	7.15	8.35

Panel (b): Consistent Estimate of the Threshold Using the M−TAR Model

	No Lagged Changes				One Lagged Change				Four Lagged Changes			
	90%	95%	97.5	99%	90%	95%	97.5	99%	90%	95%	97.5	99%
50	5.02	6.05	7.09	8.59	4.98	6.07	7.15	8.56	4.93	5.96	7.01	8.48
100	4.81	5.77	6.73	7.99	4.77	5.71	6.56	7.90	4.74	5.70	6.67	7.97
250	4.70	5.64	6.51	7.64	4.64	5.54	6.40	7.56	4.64	5.54	6.39	7.61

Panel (c): Known Threshold Value in the M−TAR Model

	No Lagged Changes				One Lagged Change				Four Lagged Changes			
	90%	95%	97.5	99%	90%	95%	97.5	99%	90%	95%	97.5	99%
50	4.21	5.19	6.15	7.55	4.12	5.11	6.05	7.25	3.82	4.73	5.65	6.84
100	4.11	5.04	5.96	7.10	4.08	4.97	5.87	7.06	3.81	4.72	5.63	6.83
250	4.08	4.97	5.83	6.91	4.05	4.93	5.78	6.83	3.69	4.71	5.63	6.78