Dynamic Coalescence as a Mathematical Model of Leadership and Empirical Evidence of the Value of Strategic Sacrifice

Advisor: Dr. Sean Warnick **Major: Mathematics**

Project Purpose

The purpose of this project is to provide deeper insights regarding leadership behaviors that lead to high team productivity by contributing a quantitatively rigorous model of leadership and empirical evidences of the theory to the current scholarly discussion. The project will culminate with statistical evidences that the theory and its applications are veritable.

Project Importance

All great leaders have been revealed as the solution to a daunting problem facing humankind. For as long as we have recorded history we have human organizations, and, from families to cities to entire nations, every human organization of notable success in cooperation and survival has had a leader (Grint, 2011). Conversely, many CEOs have recognized the lack of qualified leadership talent as the most significant constraint on growth (Gandossy, 2008). From business administration to classroom education, leadership development faces a number of obstacles, particularly in determining how to define leadership, how to measure leadership, and what behaviors make a leader great (Northouse, 2016). This project is designed to offer a quantitatively rigorous understanding of leader-follower interactions in a team to help organizations overcome these obstacles and to provide key insights regarding the leadership behaviors that lead to high team performance.

Project Overview

A leader is often perceived as a person who assumes power or control over a group of people, regardless of how that control was acquired. Leadership, then, is the way or degree to which a leader foments the cooperation of self-interested individuals to achieve a common goal. One lens through which scientists can observe these leader-group dynamics is through team productivity, where production is the clear objective and the roles of team members are typically predetermined. Mathematically, we can examine a team as though it were an economic market where each team member is a participant in overall market production, but, established in a competitive setting, each team member is also individually incentivized to outperform his or her counterpart in the market.

In small, free markets with rational participants, competition pushes market production toward the sub-optimal Nash equilibrium. At the Nash Equilibrium, the overall producer surplus is lower than it would be if the players were fully cooperative. In 2016, Hurst and Warnick presented a two-player gradient play differential game in a Bertrand producer market with quadratic payoffs as a model of inter-firm behavior in a competitive environment and presented feedback dynamics which induce cooperation in this setting. They showed that the following price equilibrium is a stable equilibrium much closer to the perfectly collusive equilibrium, where the β dynamics represent profit-sharing as side-payments between firms.

$$\hat{p}_j^{eq} = \frac{(1-\beta_j)A_{ij}\hat{p}_i+\beta_i A_{ij}p_i}{-2A_{jj}} \text{ where } \dot{\beta}_i = -A_{ij}\hat{p}_j p_i + \frac{(1-\beta_i)(A_{ij}p_i)^2}{-2A_{ii}}$$

Just as firms can share resources in an economic market, resources like time, energy, and attention can be disbursed between members of a team. Side-payments in a team reduce the incentives of self-interested individuals to compete with their teammates, thereby inducing cooperation and boosting the team's production equilibrium. This model provides a theoretical foundation to explain team dynamics, to study the various components of the system, and to predict results of this behavior in a team setting. The practice of interpersonal investment is a hallmark of great leaders and the essence of our quantitative model; it is a behavior that I call *strategic sacrifice*.

Strategic sacrifice is a decision made by the leader of a team to invest his or her own meaningful resources to incentivize his or her followers to enter or remain in a collusive state. Given the mathematics, we anticipate that this state of collusion implies a state of optimal profit and production. Because strategic sacrifice is often exhibited through activities such as mentorship, constructive feedback, and employee

development, it should not seem far-fetched that this strategy would increase team productivity. However, this oligopoly model represents reality only to the degree to which we can realistically analyze individuals on a team as rational decision makers in an economic market.

Empirical analyses can help determine the validity of this representation. I have the opportunity to analyze a series of proprietary surveys distributed through several companies of variegated size (both in revenues and number of employees) across Asia. The original dataset contains 77 variables describing an organizations aptitude in various leadership behaviors, which I have collapsed and categorized into 6 dependent variables that capture how well a leader 1.) motivates, 2.) assesses, 3.) develops, 4.) manages, 5.) communicates with, and 6.) sacrifices for his or her team. The independent variable of interest is productivity of the organization as measured by sales per employee, and I have control variables to capture the effects of the size of the organization by number of employees, the organization's annual revenue, the office's country, and the industry of each company.

Preliminary analyses by methods of OLS and quantile regression suggest that an organization should expect more than a 30% average marginal increase in production due to the effects of strategic sacrifice (see Figure 1). This suggests that our model is a legitimate representation of reality in teams and a solid foundation on which we can move forward in our attempt to integrate this idea into modern leadership theory. It also provided unexpected insight regarding the role of effective communication within a team, and the interplay between communication and strategic sacrifice, a point of further analysis.

Variable	Coefficient	Std. Error	t	P > |t|	95% C.I.
Motivate	-.1008353	.0780251	-1.29	0.197	[-.2543048, .0526341]
Assess	.0762411	.08568	0.89	0.374	[-.092285, .2447672]
Develop	.0015116	.054557	0.03	0.978	[-.1057979, .1088211]
Manage	.0224384	.050585	0.44	0.658	[-.0770584, .1219353]
Communication	.1336869	.0672183	2.03	0.044	[.0039556, .2683823]
StrategicSacrifice	.2301569	.1257003	2.70	0.007	[.0920139, .5864999]
Number of Employees	.0338898	.0374483	0.90	0.366	[-.0397681, .1075478]
Yearly Revenue	.0186693	.0169235	1.10	0.271	[-.014618, .0519566]
Country of Data Origin	.0944666	.0474882	1.99	0.047	[.0010609, .1878723]

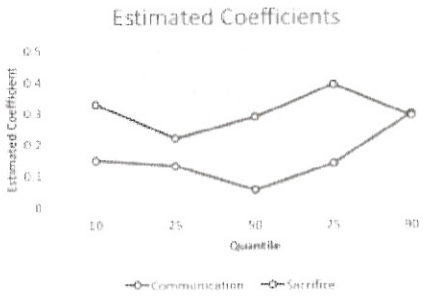

Figure 1: OLS Summary Statistics & graph of controlled quantile regression

Moving forward, I will finalize the mathematical model of the leader-team setting, extend the statistical analyses and inferences, and perform a more in-depth literature review to determine where this project fits in the conversation. Based on my studies up to this point, I anticipate that it will contribute new knowledge and a new perspective regarding servant-leadership and its importance in team productivity.

Qualifications of Thesis Committee

Dr. Warnick is a valuable expert in Control Theory, a branch of mathematics fundamental to my proposed model. I have worked in Dr. Warnick's research lab for 2 years, and I have been working on this leadership project for more than half of that time. Through the lab, we have worked closely with professional experts in the field of leadership studies. Dr. Warnick has been researching dynamic coalition formation for more than a decade, and this research is fundamental to my model of leadership.

Dr. Gibb Dyer is a leading expert in the study of team productivity, as evidenced by the publication of his recent book: "Team Building: Proven Strategies for Improving Team Performance." I have spent the last year and a half working on the mathematical foundations of my model, but my understanding was greatly restricted until I began studying with Dr. Dyer. His insights regarding both team performance and leadership theory have already been monumental in my progress toward completing this project by informing my quantitative model and helping me to understand my place in the scholarly discussion.

Project Timetable

I began working on this project in September 2016. I will finish my literature review and model by December 20, 2017. I will have completed the statistical analyses, inference, and discussion by May 1, 2018, and I will have a final draft ready for publication and defense by June 1, 2018.

He needs to defend by May 11. Please refer to the deadlines on our website.

Culminating Experience

I will submit this paper for publication in either the *Journal of Organizational Behavior* or the *Academy of Management Review*. I will also present my research at the annual student research conference held by the College of Physical and Mathematical Sciences in the spring of 2018.