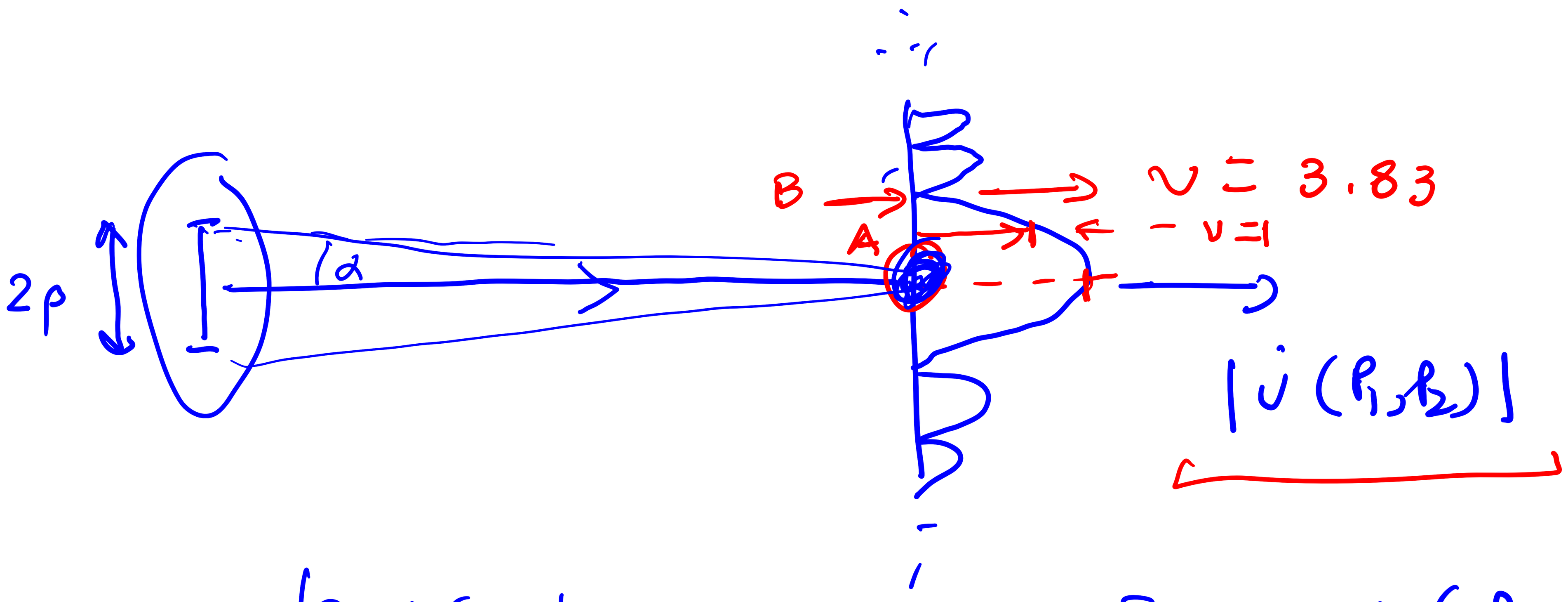


$$\left|\frac{2J_1(v)}{v}\right| = |j(P_1,P_2)|$$

$\left|\frac{2J_1(v)}{v}\right|$ decreases from 1 (at $\nu=0$) to 0.88 (at $\nu=1$)

$$\nu = \frac{\bar{k}\rho d}{R} \simeq 1 \Rightarrow d = \frac{1}{2\pi}\frac{\bar{\lambda}R}{\rho} \approx \underbrace{0.16\frac{\bar{\lambda}R}{\rho}}$$

$$\Delta A = \pi\cdot\left(\frac{0.16\bar{\lambda}R}{2\rho}\right)^2 = \pi\cdot\left(\frac{0.16\bar{\lambda}}{2\alpha}\right)^2$$

$$= 0.063\,\frac{R^2\bar{\lambda}^2}{S}$$

area of source

Example : SUNLIGHT :

$$2\alpha = 0^\circ 32' \sim 0.0093 \text{ rad}$$

Diameter 'd' of the area of coherence

$$d = 0.16\bar{\lambda}/(0.0093/2) = 34\bar{\lambda}$$

$$\bar{\lambda} = 550 \text{ nm}$$

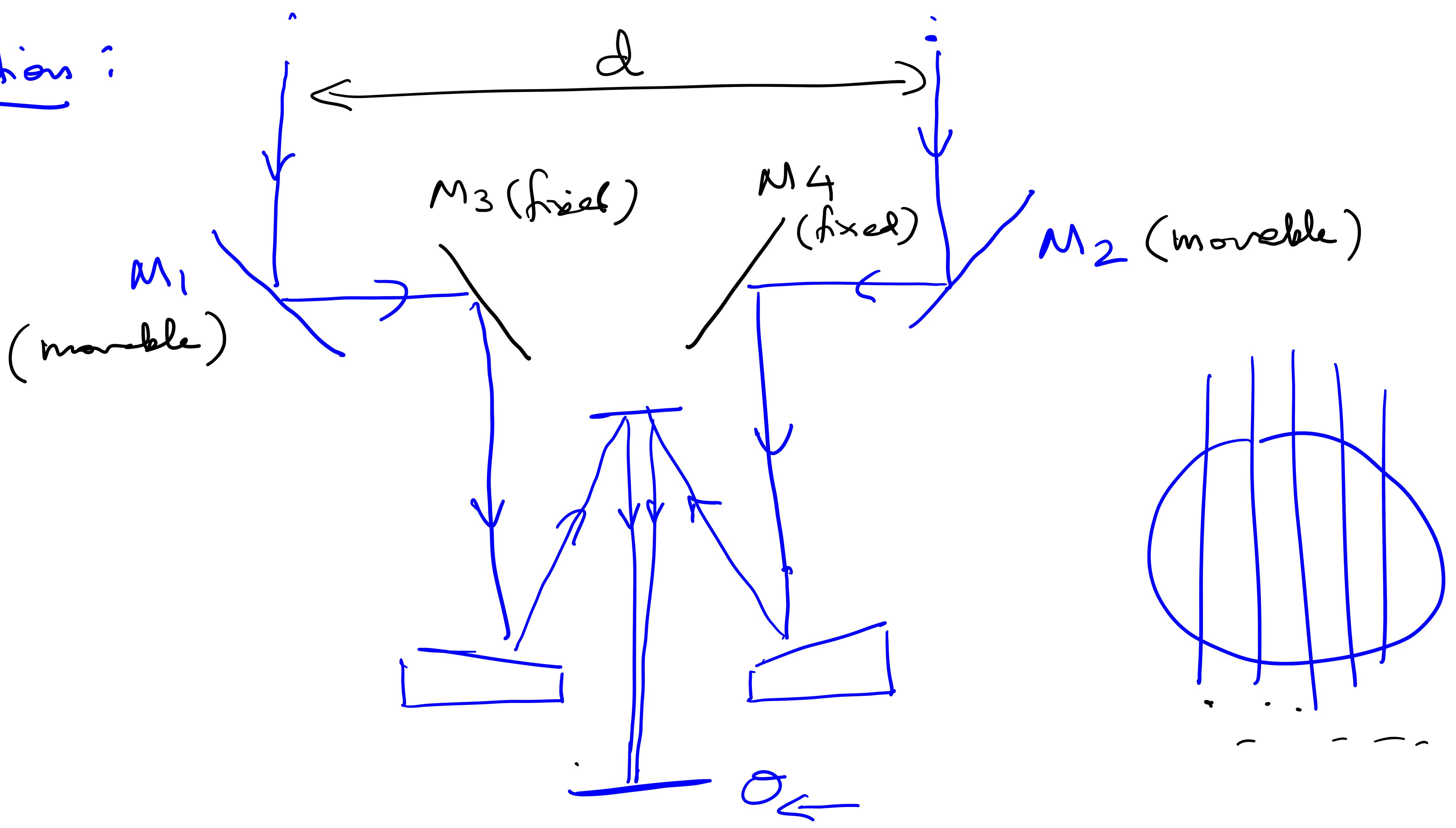
$$d = 0.019 \text{ mm} \quad \text{(small)}$$

Seminal Experiment utilises this degree of coherence

Michelson's method of measuring stellar diameters

Problem : angular diameters subtended by stars on earth are extremely small → Hard to measure even with the largest telescope.

Solution :



Method : (1) Keep changing 'd' till the fringe visibility becomes zero $\to d_0$

(2) $\alpha = 0.61 \bar{\lambda}/d_0$

fringes can be dispersed at the last stage for λ- selection

CAN you explain this ?

Answer :

- Outer mirrors M_1 and M_2 receive partially coherent light
- According to van-Cittert Zernike theorem, $|\gamma| << 1$

$$j(M_1, M_2) = \frac{\iint_\sigma i(u,v)\, e^{ik_0[(x_2-x_1)u + (y_2-y_1)v]}\, du\, dv}{\iint_\sigma i(u,v)\, du\, dv}$$

where we have re-expressed :

$$u = \xi/R, \quad v = \eta/R$$

- We already have derived equal time complex degree of coherence & Visibility

$$|j(M_1, M_2)| = \left| \frac{2J_1(k_0 \rho d/R)}{k_0 \rho d/R} \right| = \left| \frac{2J_1(k_0 \alpha d)}{k_0 \alpha d} \right|$$

- Zero of $J_1(v)$ occurs when $v = 3.83$

$$\underline{k_0}\, \alpha\, d_0 = 3.83$$

$$d_0 = 0.61\, \lambda_0/\alpha$$

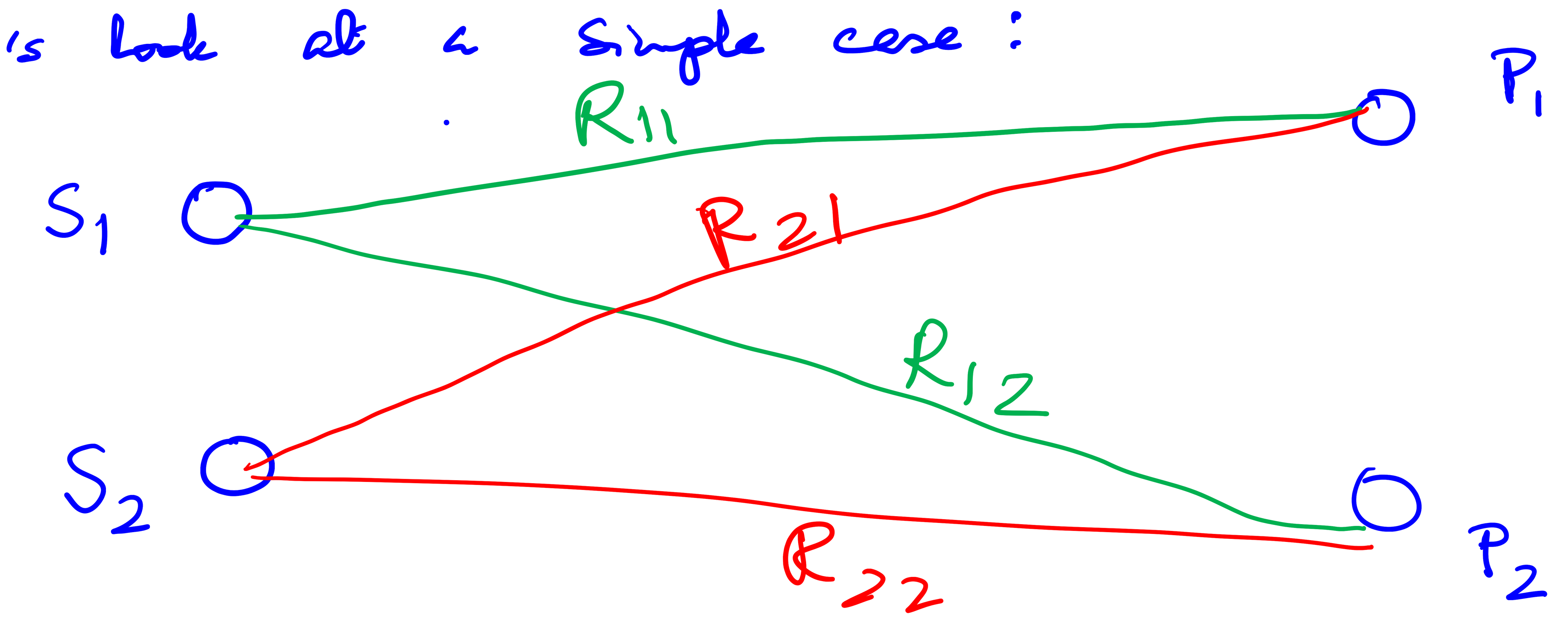
Michelson's result $\boxed{\alpha = \frac{0.61\, \lambda_0}{d_0}}$ $\square$

PARADOX

↳ we just proved that a spatially incoherent source will generate a field which is coherent over large regions of space

- Spatial coherence is generated in the process of ~~propagation~~
- HOW COME ?

Let's look at a simple case:



$V_1(P_1,t) \to$ field at P_1 due to S_1

$V_2(P_1,t) \to$ . . . P_1 . . . S_2

⋮

* $|R_{11}-R_{12}|$ small compared to $2\pi c/\Delta\omega$

  $\Rightarrow V_1(P_2,t) \simeq V_1(P_1,t)$

* $|R_{22}-R_{21}|$ small compared to coherence length — (1)

  $V_2(P_2,t) \simeq V_2(P_1,t)$

* Total field at P_1

  $V(P_1,t) = V_1(P_1,t) + V_2(P_1,t)$

* Total field at P_2

  $V(P_2,t) = V_1(P_2,t) + V_2(P_2,t)$

* $V_1(P_1,t)$ & $V_2(P_1,t)$ should be un-correlated (generated by statistically indep. sources)

* Similarly $V_2(P_2,t)$ & $V_1(P_2,t)$ are uncorrelated

* But the sums $V_1(P_1,t) + V_2(P_1,t)$

  and $V_2(P_2,t) + V_1(P_2,t)$