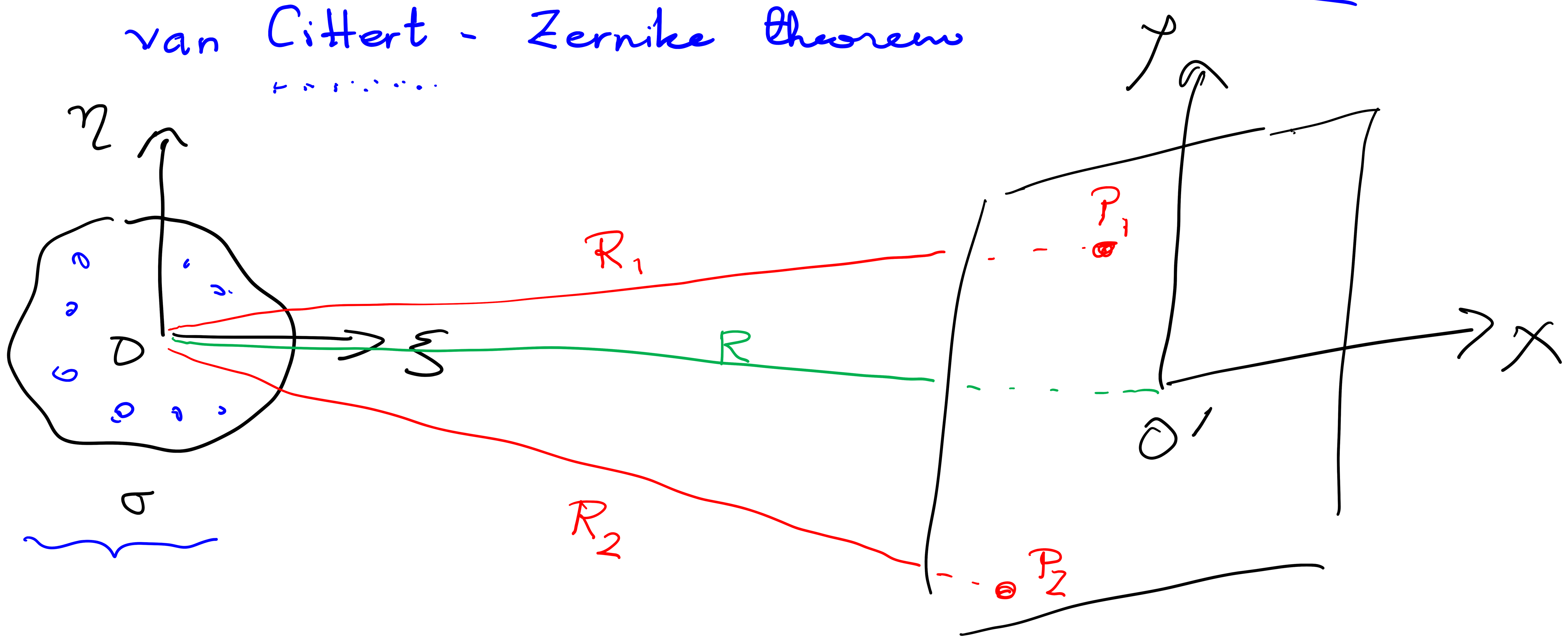
Degree of coherence for an extended source

van Cittert - Zernike theorem



If the source is made of many different elements

$$V_1(t) = \sum_m V_{m1}(t), \quad V_2(t) = \sum_m V_{m2}(t)$$

"Mutual" intensity

$$\boxed{J(P_1, P_2)} = \langle V_1(t)\, V_2^*(t) \rangle$$

$$= \sum_m \langle V_{m1}(t)\, V_{m2}^*(t) \rangle + \cancel{\sum_{m \neq n} \langle V_{m1}(t)\, V_{n2}^*(t) \rangle}$$

Vibrations from diff elements on 'σ' are statistically independ-ent & they have mean $= 0$

$$\langle V_{m1}(t)\, V_{n2}^*(t) \rangle = \langle V_{m1}(t) \rangle \langle V_{n2}^*(t) \rangle = 0 \qquad m \neq n \qquad [1]$$

If we consider a single source element $d\sigma_m$:

$$V_{m1}(t) = A_m\left(t - \frac{R_{m1}}{v}\right) \frac{e^{-2\pi i \bar{\nu}(t - R_{m1}/v)}}{R_{m1}}$$

$$V_{m2}(t) = A_m\left(t - \frac{R_{m2}}{c}\right) \frac{e^{-2\pi i \bar{\nu}(t - R_{m2}/v)}}{R_{m2}}$$

$$\langle V_{m1}(t)\, V_{m2}^*(t) \rangle$$

$$= \langle A_m(t-t_1) A_m^*(t-t_2) \rangle \frac{e^{2\pi i \bar{\nu}(R_{m1}-R_{m2})/v}}{R_{m1} R_{m2}}$$

stationarity

$$\langle A_m(t) A_m^*\left(t - \underbrace{\frac{R_{m2}-R_{m1}}{v}}\right)\rangle$$

$$\underline{|R_{m2}-R_{m1}| << \frac{1}{\Delta\nu}}$$

$$J(P_1,P_2) = \sum_m \underbrace{\langle A_m(t) A_m^*(t) \rangle}_{I(S)\,dS} \frac{e^{i2\pi\bar{\nu}(R_{m1}-R_{m2})/v}}{R_{m1}\cdot R_{m2}}$$

continuum limit

$$\boxed{J(P_1,P_2) = \int_\sigma I(S) \frac{e^{i\bar{k}(R_1-R_2)}}{R_1 R_2} dS}$$ $\square$

VAN-CITTERT ZERNIKE TH.

Complex degree of coherence

$$j(P_1,P_2) = \frac{1}{\sqrt{I(P_1)}\sqrt{I(P_2)}} \int_\sigma dS \frac{I(S)\, e^{i\bar{k}(R_1-R_2)}}{R_1 R_2}$$

$$I(P_1) = J(P_1,P_1) = \int_\sigma dS \frac{I(S)}{R_1^2}$$

$$I(P_2) = J(P_2,P_2) = \int_\sigma dS \frac{I(S)}{R_2^2}$$

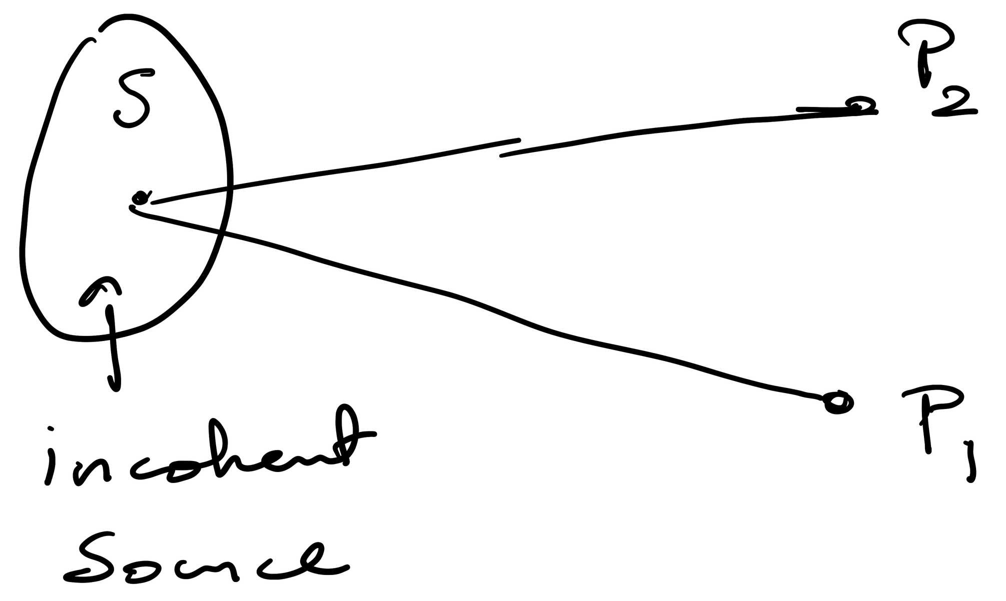
STATEMENT: Equal time complex degree of coherence $j(P_1,P_2)$ for an extended incoherent quasi-monochromatic source is equal to normalized complex amplitude of a diffraction pattern

↳ this diffraction pattern is obtained by replacing the source by a diffracting aperture of same shape and size and filling with with a spherical wave converging at P_2.

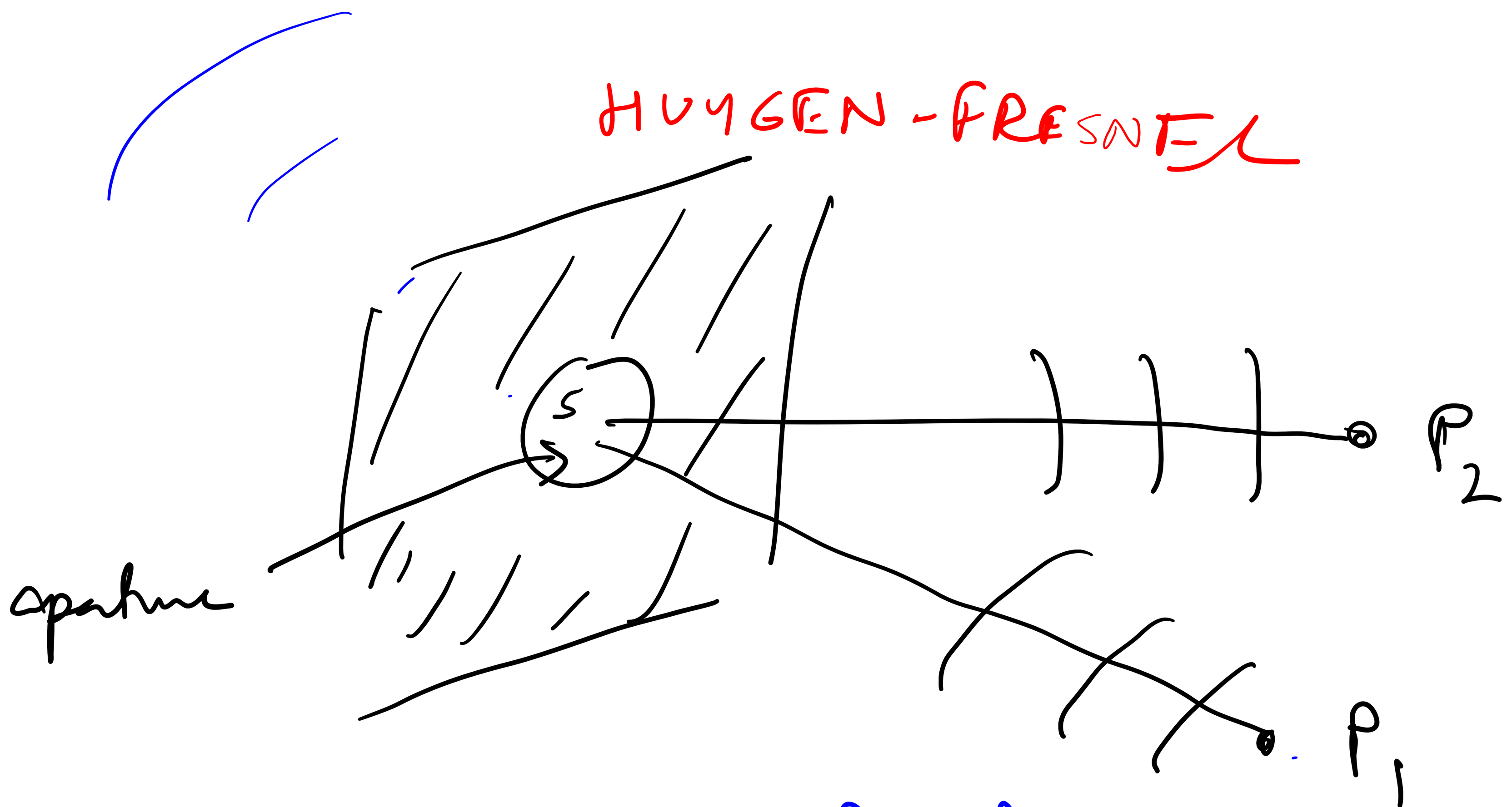
Amplitude distribution of this wavefront is proportional to the intensity distribution on the original source

ANALOGY ~

VAN-CITTERT ZERNIKE



HUYGEN-FRESNEL



Equal time degree of coherence of the field at P_1 & P_2 :

$$j(P_1, P_2) = \frac{1}{N} \int_\sigma \underbrace{I(S)}\, \frac{e^{i\bar{k}(R_2 - R_1)}}{R_1 R_2}\, dS$$

Normalized complex ampl. $U(P_2)$ due to diffraction of monochromatic spherical wave converging to P_1 :

$$U(P_2) = \frac{1}{N'} \int_A a(S) \underbrace{\frac{e^{-ikR_1}}{R_1}}_{\text{incident converging}} \cdot \underbrace{\frac{e^{ikR_2}}{R_2}}_{\text{diverging spherical wave}} dS$$

FRAUNHOFER LIMIT OF VAN-CITTERT Zernike

$$R_1 = (x_1 - \xi)^2 + (y_1 - \eta)^2 + R^2$$

$$R_1 \simeq R + \frac{1}{2R}\left[(x_1 - \xi)^2 + (y_1 - \eta)^2\right]$$

$$R_2 \simeq \text{similar}$$

$$R_2 - R_1 = \underbrace{\frac{x_1^2 + y_1^2 - (x_2^2 + y_2^2)}{2R}} - \frac{\left[(x_1 - x_2)\xi + (y_1 - y_2)\eta\right]}{R}$$

Let $\frac{x_1 - x_2}{R} = p$, $\frac{y_1 - y_2}{R} = q$

$$\psi = \bar{k}\left(\frac{x_1^2 + y_1^2 - x_2^2 - y_2^2}{2R}\right) = \bar{k}\,(OP_1 - OP_2)$$

Substituting in VCZ :

$$j_{12} = \frac{e^{i\psi} \iint_\sigma I(\xi, \eta)\, e^{-i\bar{k}(p\xi + q\eta)}\, d\xi\, d\eta}{\iint_\sigma I(\xi, \eta)\, d\xi\, d\eta}$$

Example : Uniform circular source (radius = ρ)

$$\tilde{I}(f,g) = i_0 \iint_{\xi^2+\eta^2<\rho^2} e^{-i(f\xi+g\eta)}\, d\xi\, d\eta$$

Exercise from Hecht

$$\tilde{I}(\bar{k}p, \bar{k}q) = \pi\rho^2 \left[\frac{2J_1\left(\bar{k}\rho\sqrt{p^2+q^2}\right)}{\bar{k}\rho\sqrt{p^2+q^2}}\right]$$

$$j_{12} = e^{i\psi}\left[\frac{2J_1(v)}{v}\right]$$

$$v = \bar{k}\rho\sqrt{p^2+q^2} = \frac{2\pi}{\lambda}\,\frac{\rho}{R}\sqrt{(X_1-X_2)^2+(Y_1-Y_2)^2}$$