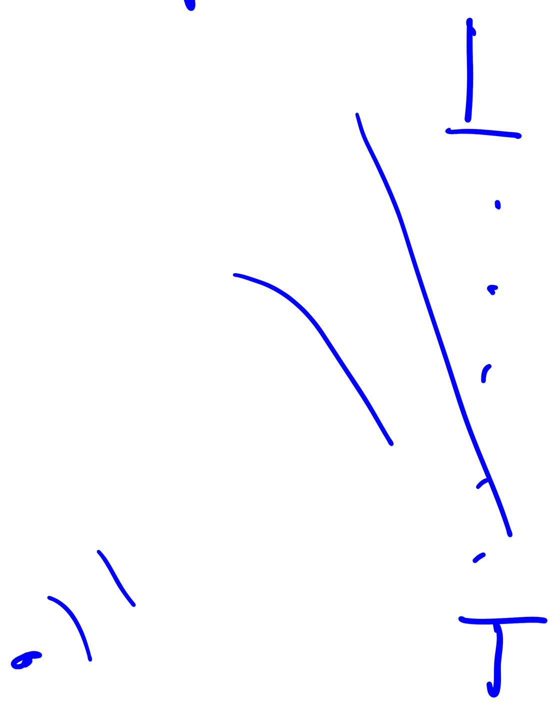
Diffraction grating

Let's consider semi-transparent object as aperture.



Without object, field at the aperture

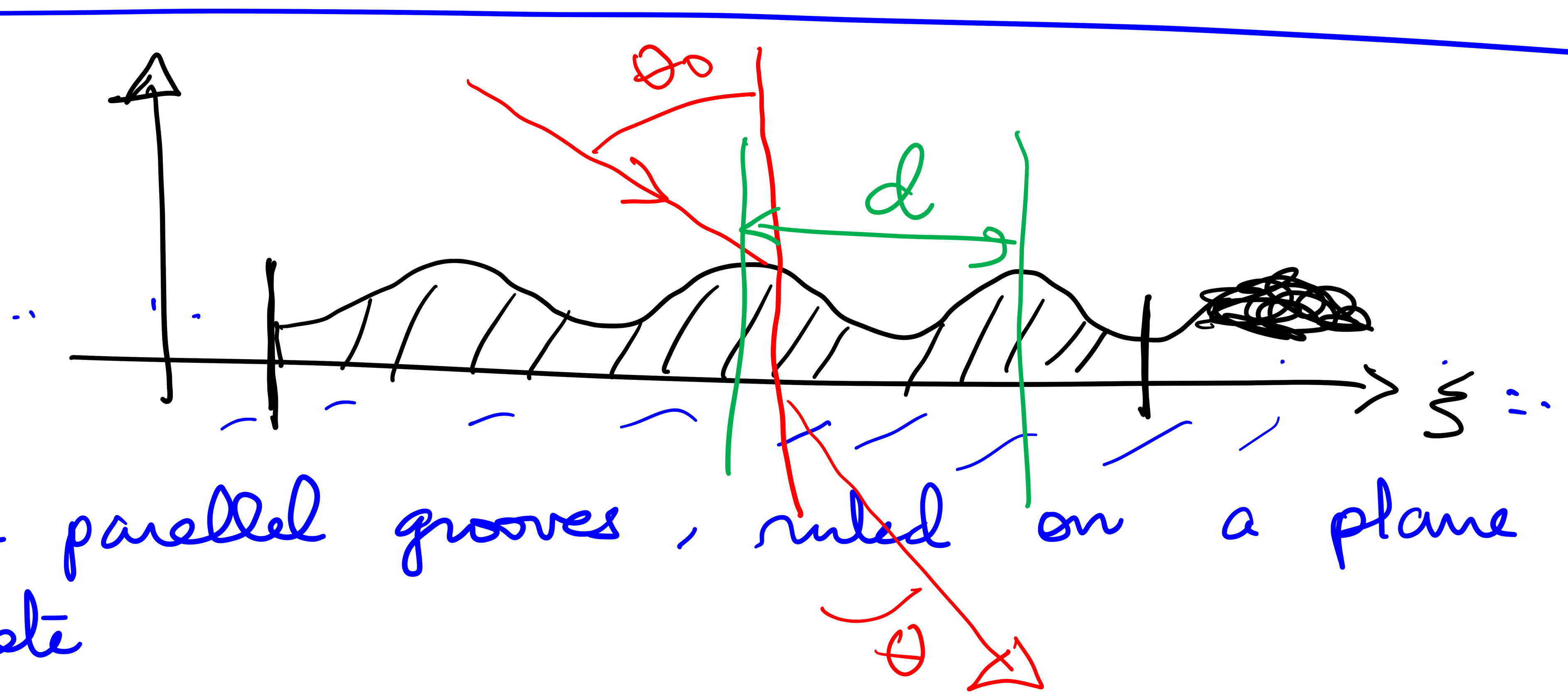
$$V_0(\xi,\eta) = A e^{ik(l_0\xi + m_0\eta)}$$

↖ plane wave

· With the object, $V(\xi,\eta)$

- Transmission function: $F(\xi,\eta) = \dfrac{V(\xi,\eta)}{V_0(\xi,\eta)}$

1D grating



N- parallel grooves, ruled on a plane glass plate

As discussed

$$l_0 = \cos(\pi/2 - \theta_0) = \sin\theta_0$$

$$l = \cos(\pi/2 - \theta) = \sin\theta$$

$$p = l - l_0 = \sin\theta - \sin\theta_0$$

$$U(P) = C \int d\xi \, e^{-ikp\xi} \cdot F(\xi) \qquad (q=0)$$

$$= C\left(\int_0^d + \int_d^{2d} + \dots \int_{(N-1)d}^{Nd} \right) d\xi \, e^{-ikp\xi} F(\xi)$$

$$= C \int_0^d \left(1 + e^{-ikpd} + e^{-2ikpd} + \dots e^{-ik(N-1)d} \right) F(\xi) \, d\xi \, e^{-ikp\xi}$$

$$= C \int_0^d \frac{F(\xi) e^{-ikp\xi}}{1 - e^{-ikpd}} \left(1 - e^{-ikNd} \right) d\xi$$

$$U(P) = U^{(0)}(P)\left[\left(\frac{1 - e^{ikNdp}}{1 - e^{-ikpd}}\right)\right]$$

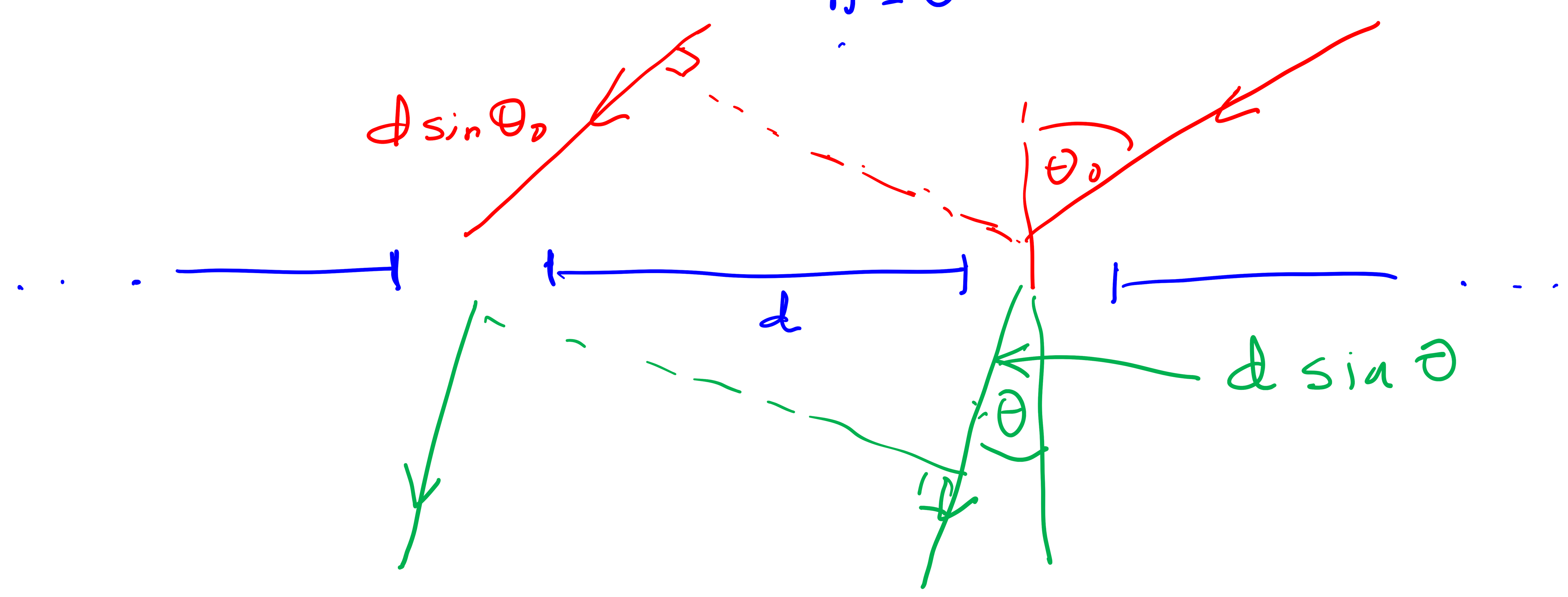
$$U^{(0)}(P) = c\int_{\text{unit cell}} e^{-ikp\xi}\, F(\xi)\, d\xi$$

$$I(P) = |U(P)|^2 = |U^{(0)}(P)|^2\left[\frac{2 - 2\cos(Nkpd)}{2 - 2\cos kpd}\right]$$

$$= |U^{(0)}(P)|^2 \left|\frac{\sin(Nkpd/2)}{\sin(kpd/2)}\right|^2$$

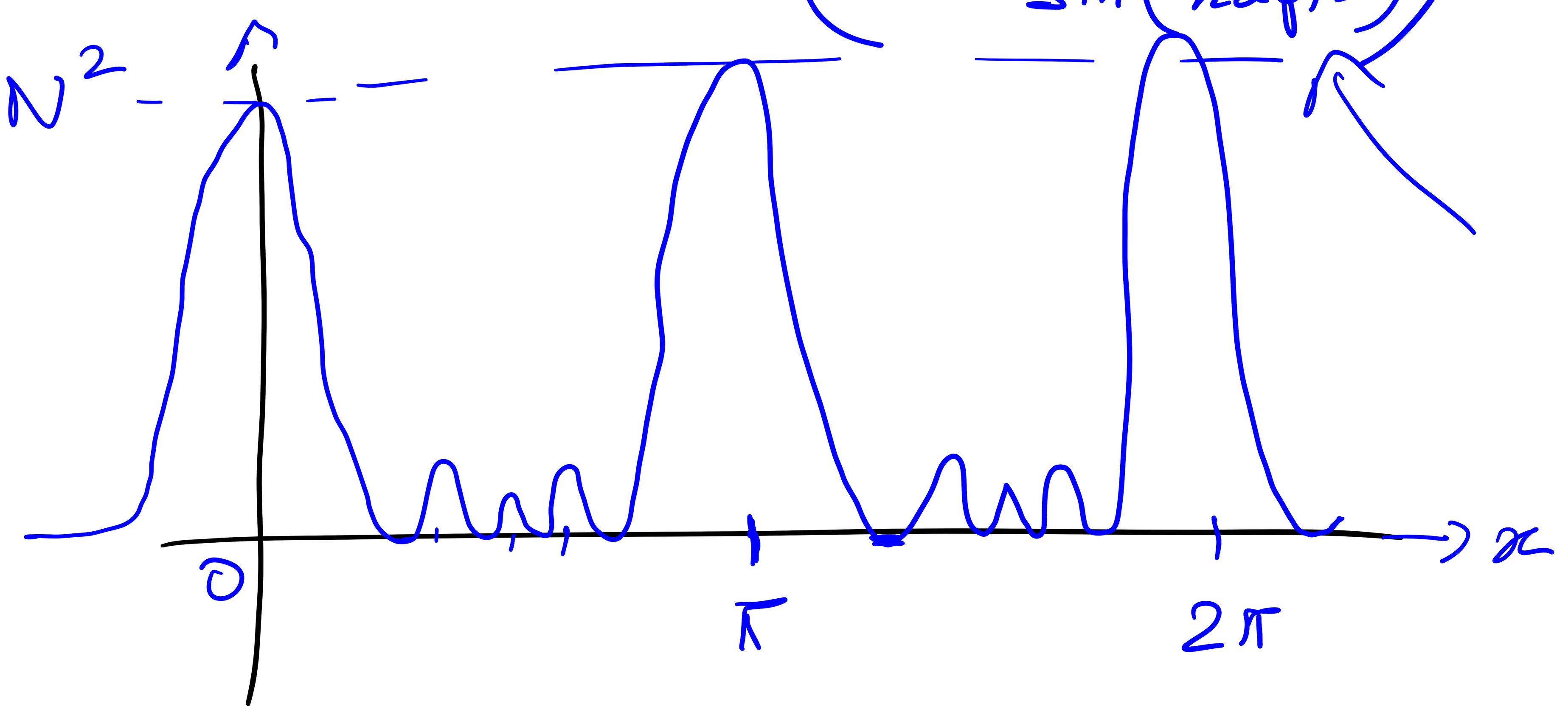
Aside

$$U(P) = U^{(0)}(P)\sum_{n=0}^{N-1} e^{-iknpd}$$



$$kd(\sin\theta - \sin\theta_0) = kdp$$

Coming back : $\left(\frac{\sin(Nkdp/2)}{\sin(kdp/2)}\right)^2 \to \left(\frac{\sin(Nx)}{\sin x}\right)^2$

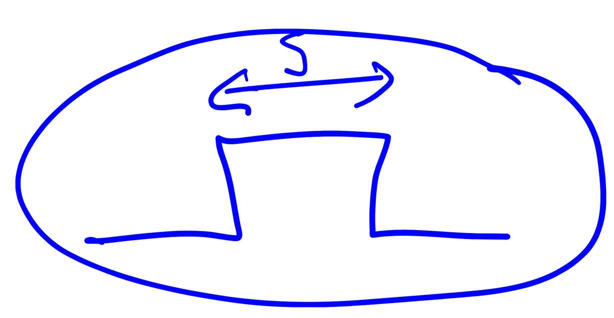


Condition for max

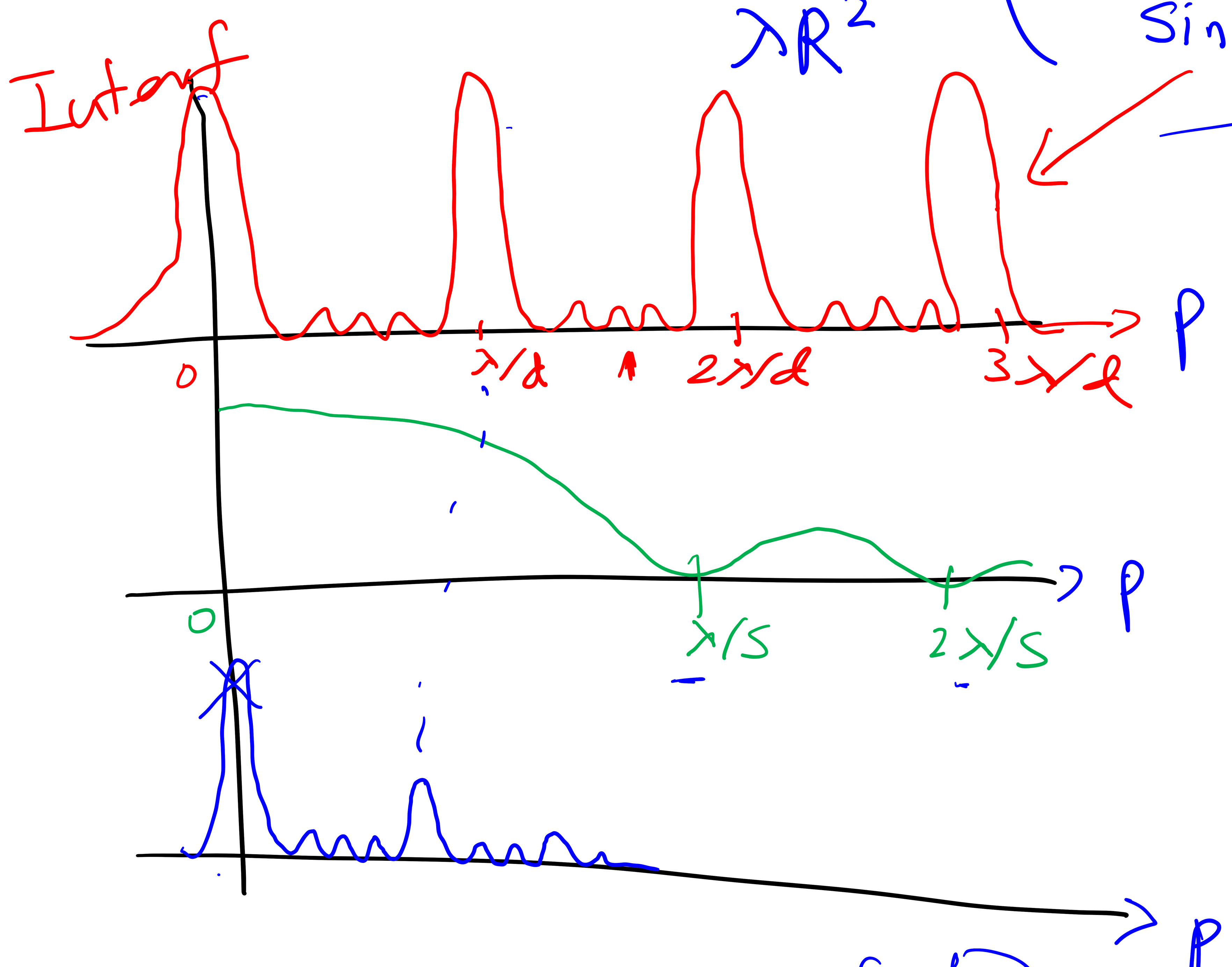
$$\frac{kdp}{2} = n\pi \Rightarrow p = \frac{n\lambda}{d}, \quad n = 0, \pm 1, \pm 2.$$

$I^{(0)}(P)$ — depends on the detailed form of the grooves

- example : rect



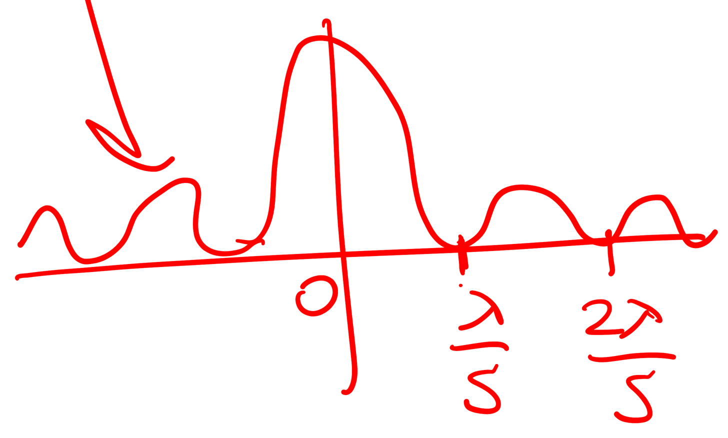
$$I(p) = \frac{P \cdot S}{\lambda R^2}\left(\frac{\sin(Nkdp/2)}{\sin(kdp/2)}\right)^2 \cdot \left(\frac{\sin(ksp/2)}{ksp/2}\right)^2$$



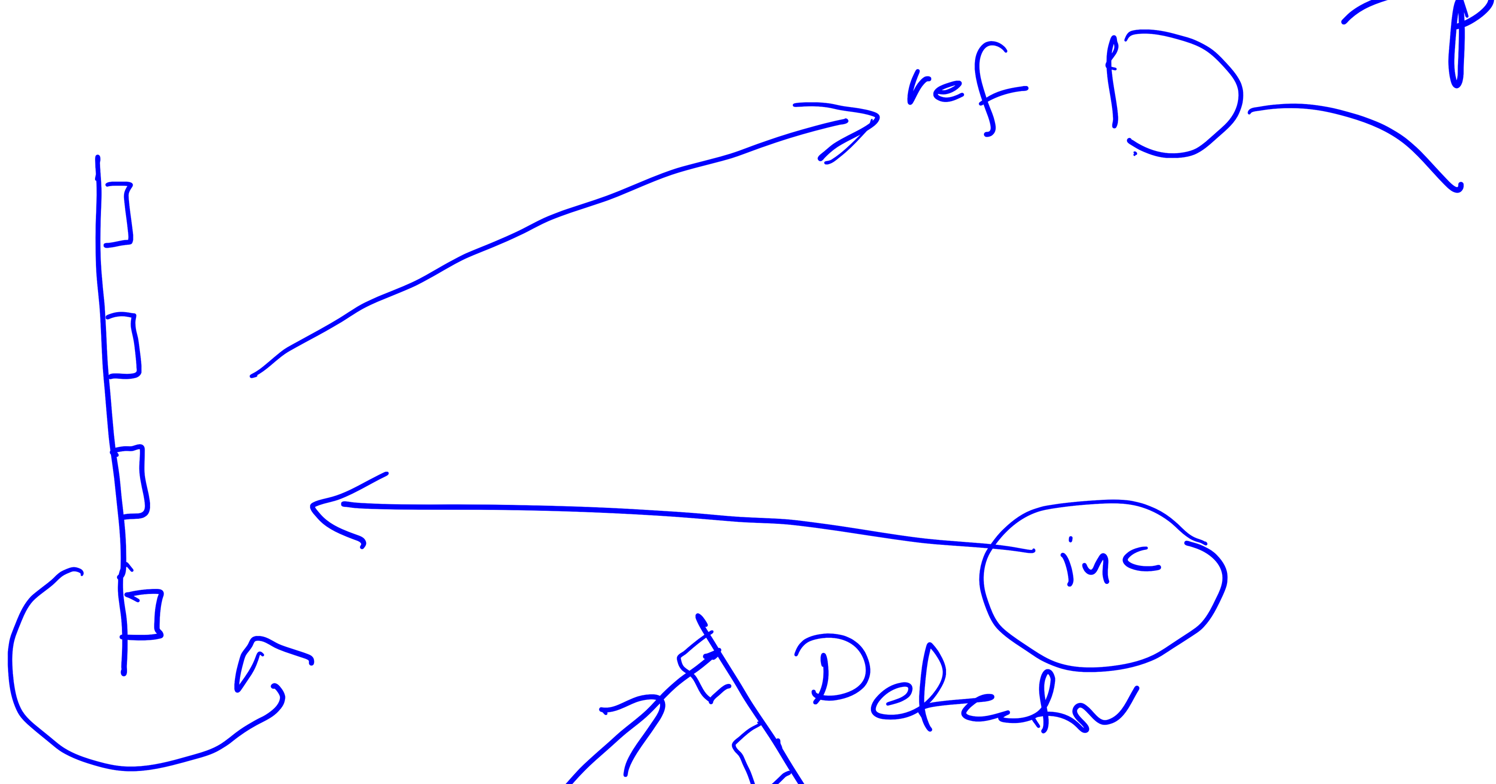
(zero)

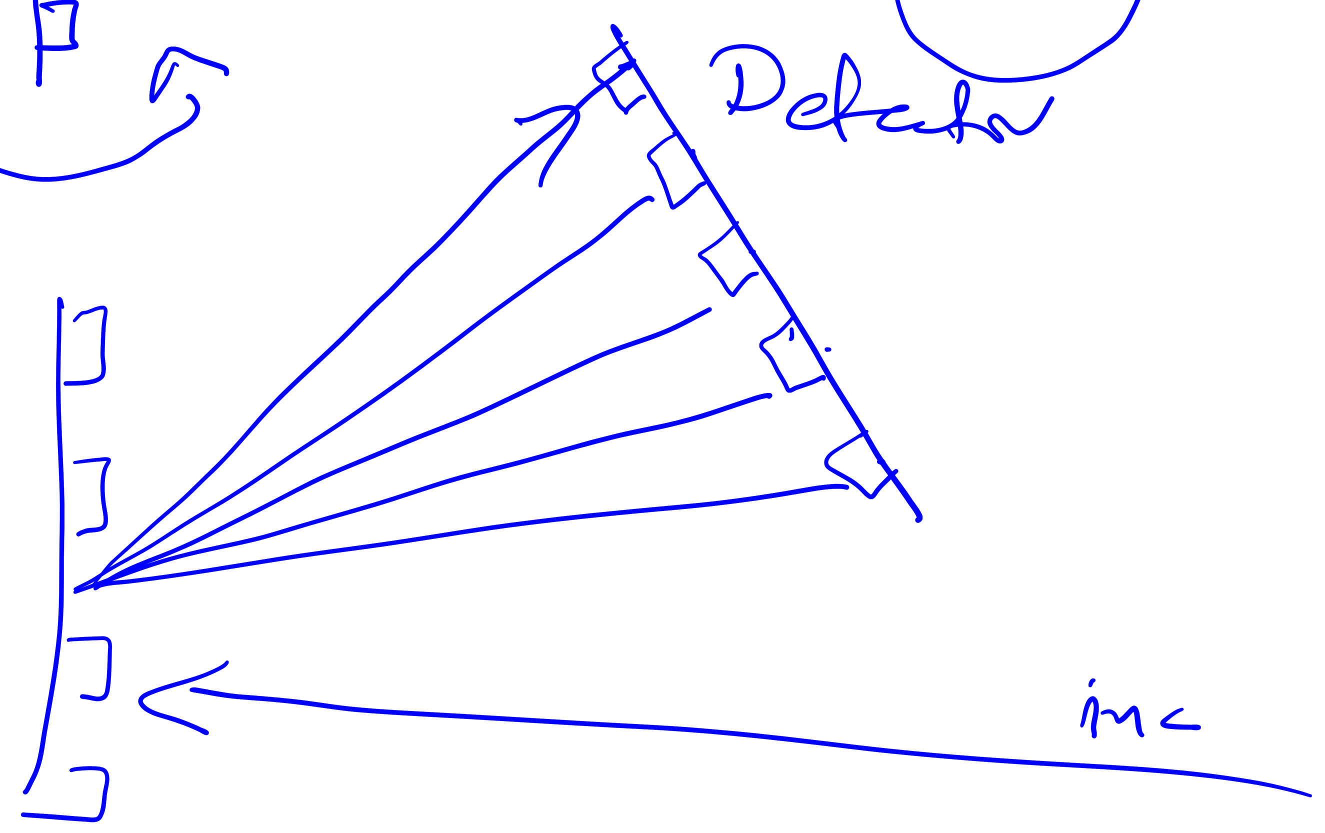
$$\frac{ksp}{2} = n\pi$$

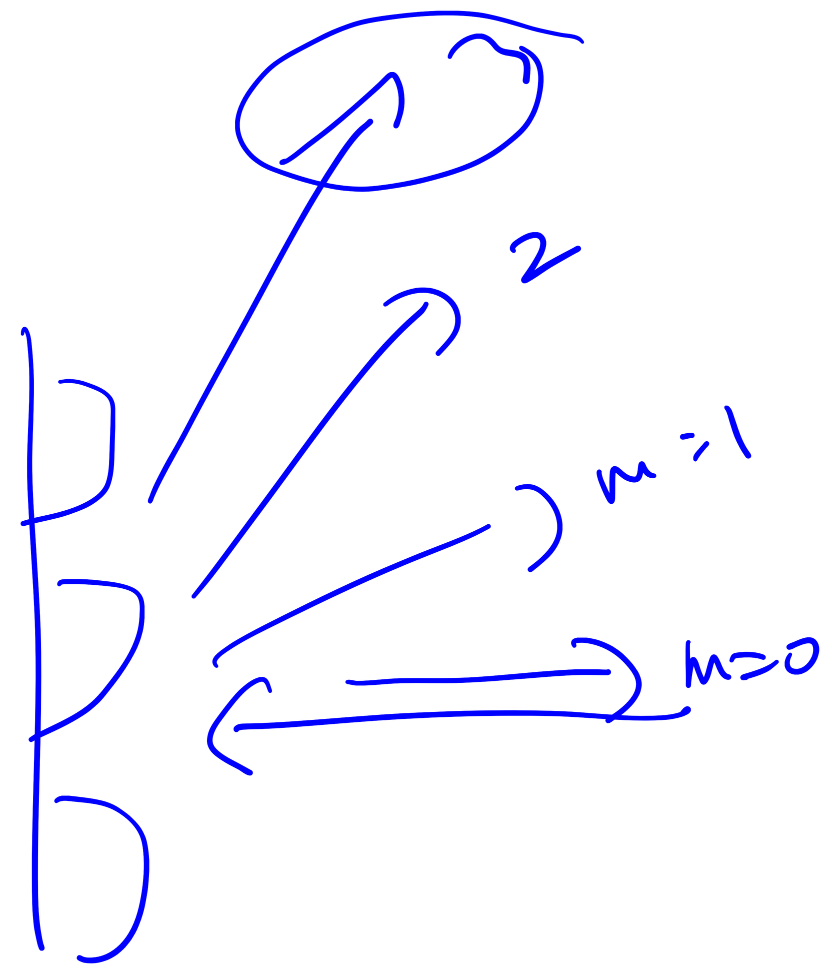
$$p = \frac{n\lambda}{s}$$



$$\frac{\lambda}{s} \gg \frac{\lambda}{d}$$







Resolving power of my grating

Separation between the primary max order "m" and the neighbouring minimum:

$$\Delta p = \frac{m\lambda}{d} - \left(\frac{m\lambda}{d} - \frac{\lambda}{Nd}\right) = \frac{\lambda}{Nd}$$

If the wavelength changes by $\Delta\lambda$,

$$\Delta p = |m| \cdot \frac{\Delta\lambda}{d}$$

$\lambda \pm \frac{1}{2}\Delta\lambda$ is just resolved when the max of one wavelength coincides with first min of the other:

$$\frac{\lambda}{Nd} = |m| \frac{\Delta\lambda}{d} \Rightarrow \boxed{\frac{\lambda}{\Delta\lambda} = |m| N}$$

Simplify: $d(\sin\theta - \sin\theta_0) = m\lambda$

$$\frac{\lambda}{\Delta\lambda} = \frac{Nd\,|\sin\theta - \sin\theta_0|}{\lambda} \leq \underbrace{\frac{2Nd}{\lambda}}$$

Example : $\underline{\Delta\lambda} = \underbrace{0.1\,A^{\circ}}$, $\underline{\lambda} = 550\ nm$, $m = 2$

$$N = \frac{\lambda}{\Delta\lambda} \cdot \underbrace{\frac{1}{|m|}} = \frac{5.5\times 10^{3}}{0.1\times 2} = \underbrace{27{,}500}$$

<u>Question</u> : Why can we not go for super-high orders ?