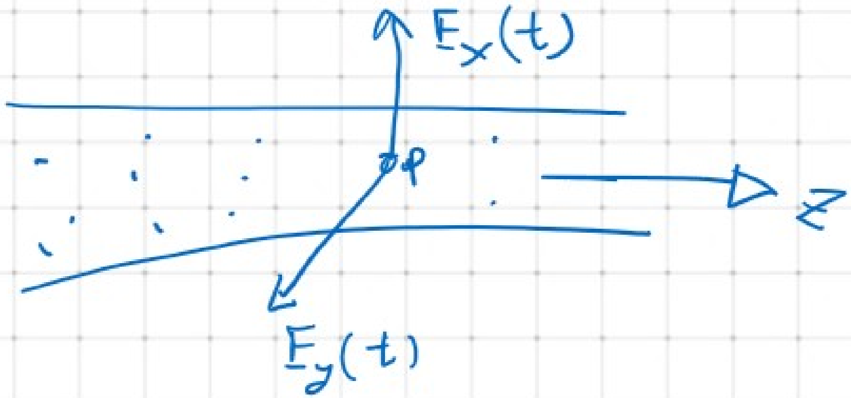
Including the vector nature of light



If the beam is quasi-monochromatic

$$E_x(t) = a_1(t)\, e^{i(\phi_1(t) - \bar{\omega} t)}$$

$$E_y(t) = a_2(t)\, e^{i(\phi_2(t) - \bar{\omega} t)}$$

Define COHERENCY MATRIX $\longrightarrow$ correlation properties of $\bar{E}$-field at a given point:

$$J \longrightarrow \begin{bmatrix} \langle E_x^*(t)\, E_x(t) \rangle & \langle E_x(t)\, E_y^*(t) \rangle \\ \langle E_y(t)\, E_x^*(t) \rangle & \langle E_y^*(t)\, E_y(t) \rangle \end{bmatrix}$$

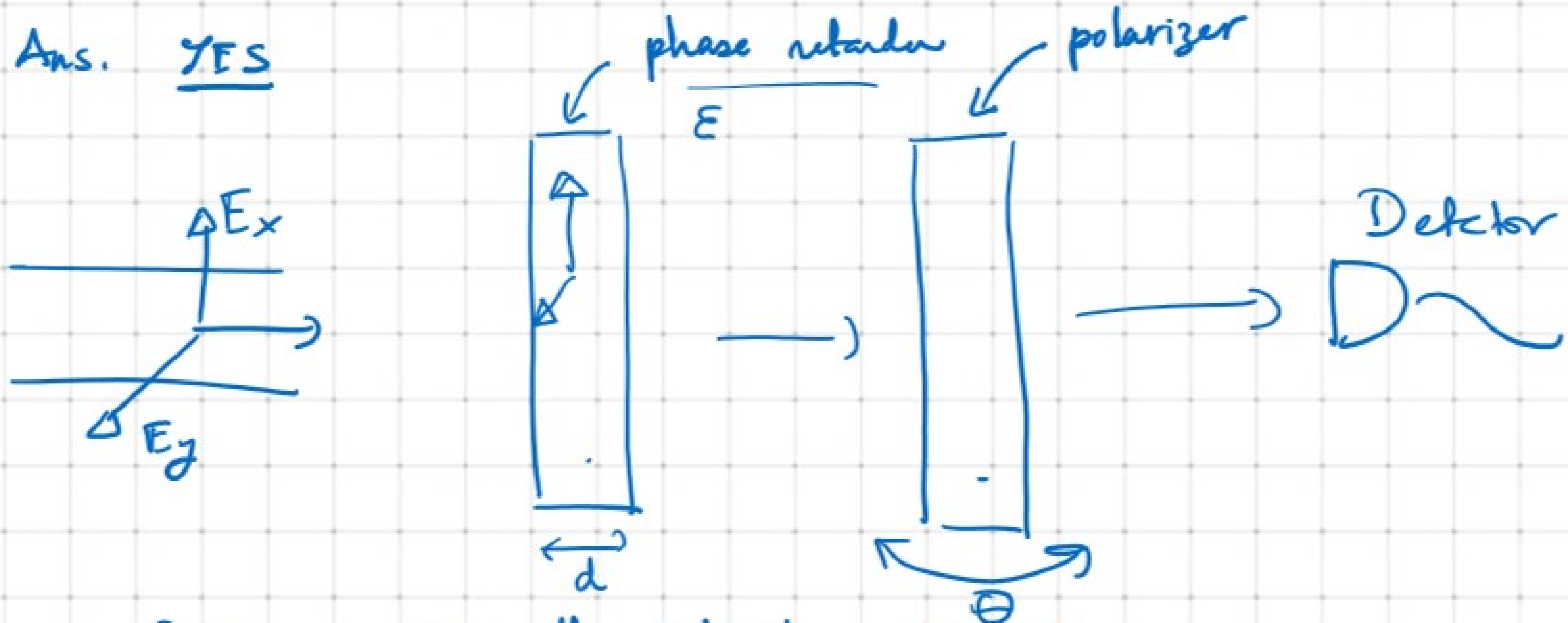
equal-time correlation MATRIX

* Diagonal elements $\longrightarrow$ average intensities in the two polarizations

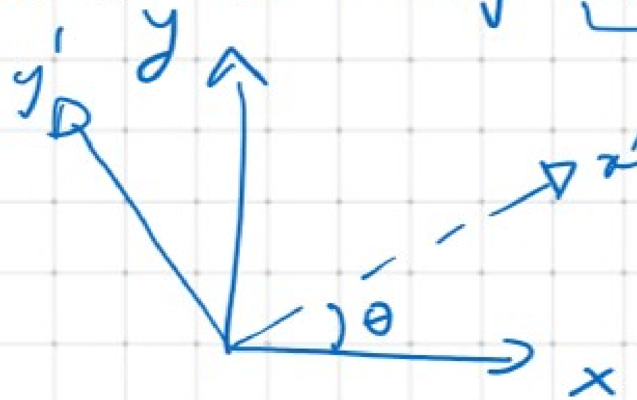
* Off-diagonal $\longrightarrow$ analogous to mutual coherence with some caveats*

* Question : Can off diagonal components of J be measured via simple experiment ?

- Ans. <u>YES</u>



Let's consider the intensity $\underbrace{I(\theta, \varepsilon)}$



Component of $\vec{E}$ along an axis at angle θ wrt x :

$$E\ (t;\ \theta, \varepsilon) = E_x \cos\theta + E_y e^{i\varepsilon} \sin\theta$$

$$I(\theta, \varepsilon) = \langle E(t;\theta,\varepsilon)\, E^*(t;\theta,\varepsilon)\rangle$$

$$= \langle \underbrace{E_x(t)\, E_x^*(t)}\rangle \cos^2\theta + \langle \underbrace{E_y(t)\, E_y^*(t)}\rangle \sin^2\theta$$

$$+ \langle \underbrace{E_x(t)\, E_y^*(t)}\rangle e^{-i\varepsilon} \cos\theta \cdot \sin\theta$$

$$+ \langle \underbrace{E_y(t)\, E_x^*(t)}\rangle e^{i\varepsilon} \cos\theta \cdot \sin\theta$$

$$= J_{xx}\cos^2\theta + J_{yy}\sin^2\theta + \underline{J_{xy}\, e^{-i\varepsilon} \cos\theta \cdot \sin\theta}$$

$$+ \underline{J_{yx}\, e^{i\varepsilon} \cos\theta \cdot \sin\theta}$$

Envelopes:

$$\hat{J} \longrightarrow \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \langle a_1^2 \rangle & \langle a_1 a_2\, e^{i(\phi_1 - \phi_2)} \rangle \\ \langle a_1 a_2\, e^{-i(\phi_1 - \phi_2)} \rangle & \langle a_2^2 \rangle \end{bmatrix}$$

- $\mathrm{Tr}(\hat{J})$ = total intensity of light
- $J_{ij} = J_{ji}^* \longrightarrow \hat{J}$ is <u>Hermitian</u>
- One can normalize J_{xy}:

$$j_{xy} = |j_{xy}|\, e^{i\beta_{xy}} = \frac{J_{xy}}{\sqrt{J_{xx}}\,\sqrt{J_{yy}}}$$

* One can normalize J_{xy}:

$$\hat{j}_{xy} = |j_{xy}| e^{i\beta_{xy}} = \frac{J_{xy}}{\sqrt{J_{xx}}\sqrt{J_{yy}}}$$

$\longrightarrow$ Cauchy Schwarz $\Rightarrow$ $|j_{xy}| \leq 1$

* $\det(\hat{J}) = J_{xx} J_{yy} - J_{xy} J_{yx} \geq 0 \longleftarrow$

* Going back to $I(\theta, \varepsilon)$

$$I(\theta, \varepsilon) = J_{xx} \cos^2\theta + J_{yy} \sin^2\theta + 2\sqrt{J_{xx}}\sqrt{J_{yy}}\, |j_{xy}| \cos(\beta_{xy} - \varepsilon) \cos\theta \sin\theta$$

$$= I^{(1)} + I^{(2)} + 2\sqrt{I^{(1)}}\sqrt{I^{(2)}}\, |j_{xy}| \cos(\beta_{xy} - \varepsilon)$$

* Finally: to measure $\hat{J}$ (One choice of measurements)

$\{\theta, \varepsilon\}$:

$\{0^\circ, 0^\circ\}$, $\{45^\circ, 0^\circ\}$, $\{90^\circ, 0^\circ\}$, $\{135^\circ, 0^\circ\}$, $\{45^\circ, 90^\circ\}$, $\{135^\circ, 90^\circ\}$.

- **Exercise:**

$$J_{xx} = I(0^\circ, 0^\circ), \quad J_{yy} = I(90^\circ, 0^\circ)$$

$$J_{xy} = \frac{1}{2}\left[I(45^\circ, 0) - I(135^\circ, 0)\right] + \frac{i}{2}\left[I(45^\circ, 90^\circ) - I(135^\circ, 90^\circ)\right]$$

$$J_{yx} = \frac{1}{2}\left[I(45^\circ, 0) - I(135^\circ, 0)\right] - \frac{i}{2}\left[I(45^\circ, 90^\circ) - I(135^\circ, 90^\circ)\right]$$

Question: How does the observed intensity change when one of the parameters $[\theta, \varepsilon]$ is kept fixed?

Case I: We keep θ fixed

$$I_{\substack{max(\varepsilon) \\ min(\varepsilon)}} = J_{xx}\cos^2\theta + J_{yy}\sin^2\theta \pm 2|J_{xy}|\cos\theta \cdot \sin\theta$$

$$\frac{I_{max(\varepsilon)} - I_{min(\varepsilon)}}{I_{max(\varepsilon)} + I_{min(\varepsilon)}} = \frac{|J_{xy}|\sin 2\theta}{J_{xx}\cos^2\theta + J_{yy}\sin^2\theta}$$

Max : $\beta_{xy} - \varepsilon = 2m\pi$

Min $\beta_{xy} - \varepsilon = (2m+1)\pi$

Case II : Keep ε- fixed

$$I(\theta, \varepsilon) = \frac{1}{2}(J_{xx} + J_{yy}) + J_{xx}\left(\cos^2\theta - \frac{1}{2}\right) + J_{yy}\left(\sin^2\theta - \frac{1}{2}\right) + |J_{xy}| \sin 2\theta \cdot \cos(\beta_{xy} - \varepsilon)$$

$$= \frac{1}{2}(J_{xx} + J_{yy}) + \frac{(J_{xx} - J_{yy})}{2} \cos 2\theta + |J_{xy}| \sin 2\theta \cos(\beta_{xy} - \varepsilon)$$

$$A \cos x + B \sin x = \sqrt{A^2 + B^2} \cos\left(x - \tan^{-1}\frac{B}{A}\right)$$

$$\frac{1}{2}(J_{xx} + J_{yy}) + R \cos(2\theta - \alpha)$$

$$R = \sqrt{\frac{(J_{xx} - J_{yy})^2}{4} + |J_{xy}|^2 \cos^2(\beta_{xy} - \varepsilon)}$$

$$\alpha = \tan^{-1}\left(\frac{2|J_{xy}| \cos(\beta_{xy} - \varepsilon)}{J_{xx} - J_{yy}}\right)$$

$$I_{\substack{max \\ min}}(\theta) = \frac{1}{2}(J_{xx} + J_{yy}) \pm R$$

$$R_{max}(\Sigma) = \frac{1}{2}\sqrt{(J_{xx} - J_{yy})^2 + 4|J_{xy}|^2}$$

$$= \frac{1}{2}\sqrt{J_{xx}^2 + J_{yy}^2 + 2J_{xx}J_{yy} - 4J_{xx}J_{yy} + 4|J_{xy}|^2}$$

$$= \frac{1}{2}(J_{xx} + J_{yy})\sqrt{1 - \frac{4\det(\hat{J})}{(J_{xx} + J_{yy})^2}}$$

$$I_{\substack{max(\theta,\Sigma) \\ min(\theta,\Sigma)}} = \frac{1}{2}(J_{xx} + J_{yy})\left[1 \pm \sqrt{1 - \frac{4\det(\hat{J})}{(Tr\,\hat{J})^2}}\right]$$

$$\frac{I_{max}(\theta,\Sigma) - I_{min}(\theta,\Sigma)}{I_{max}(\theta,\Sigma) + I_{min}(\theta,\Sigma)} = \sqrt{1 - \frac{4\det\hat{J}}{(Tr\,\hat{J})^2}}$$