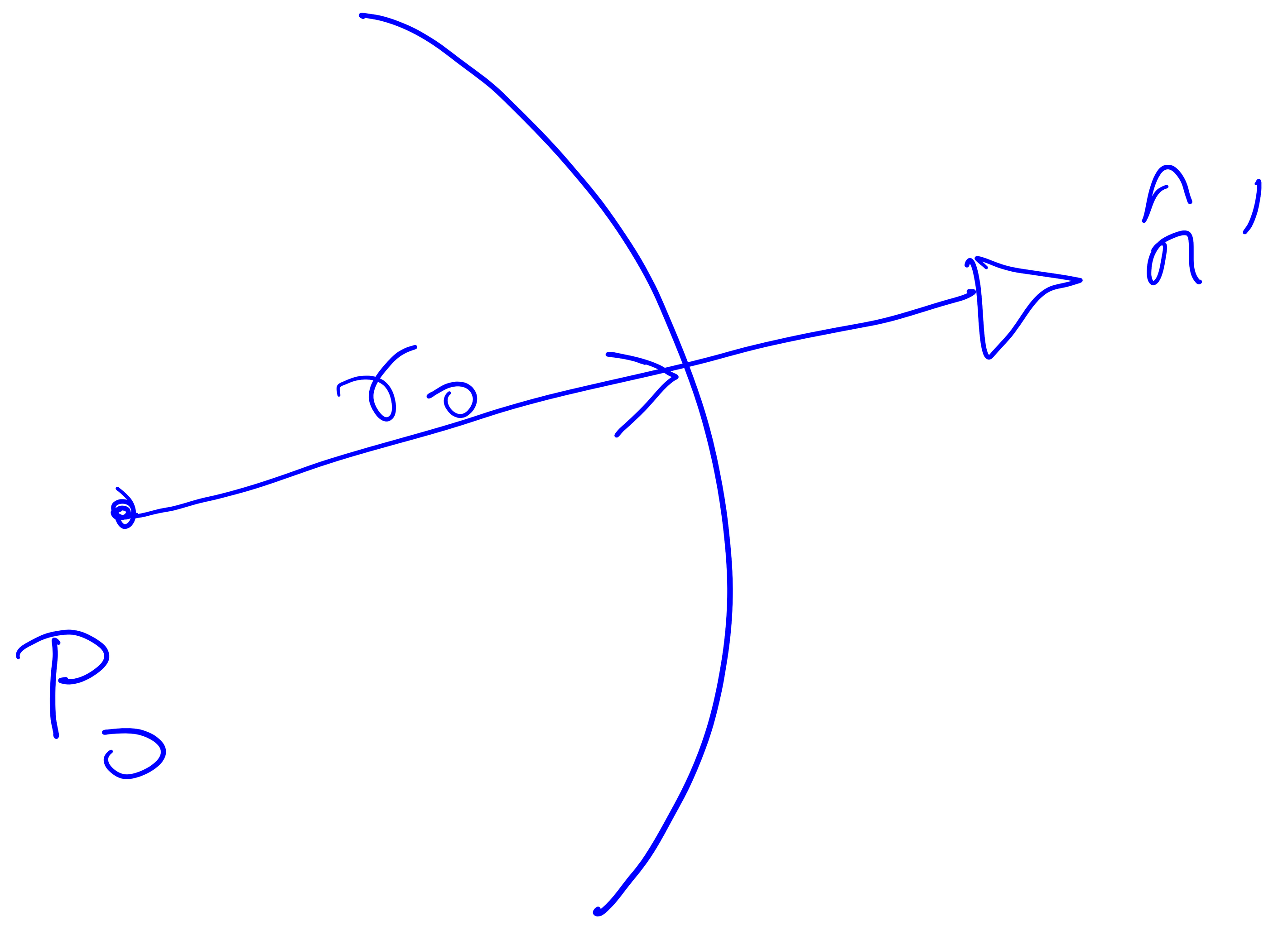


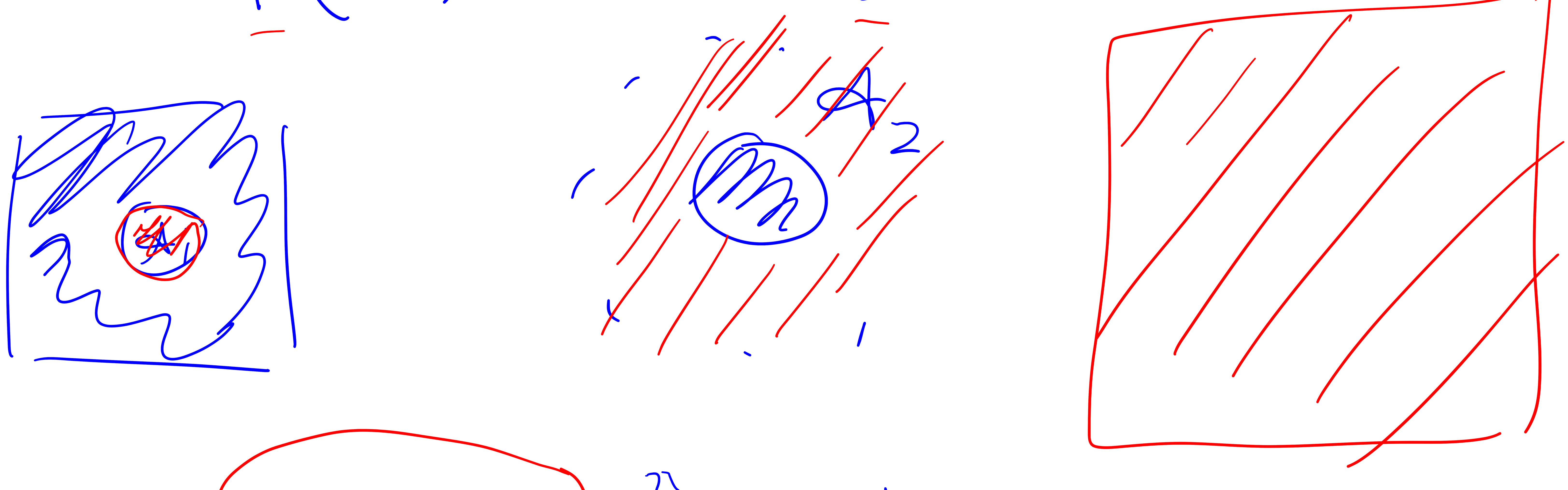
$$(n', r_0) = 0$$

$$(1 + \cos\chi)$$

$$\iint_{A_1} + \iint_{A_2} = \iint$$

entire wavefront

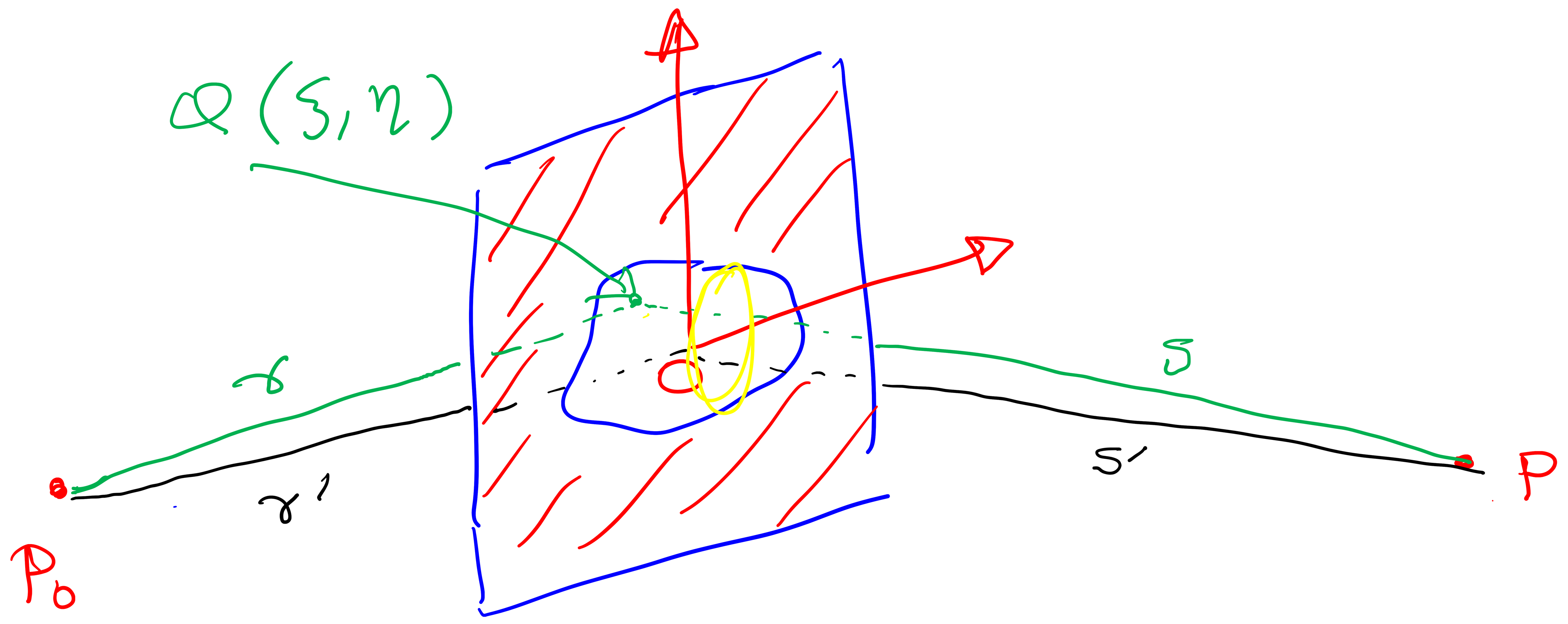
$U_1(P)$ $U_2(P)$ empty screen $U(P)$



Diffraction: "Fraunhofer" & "Fresnel"

Diffraction integral

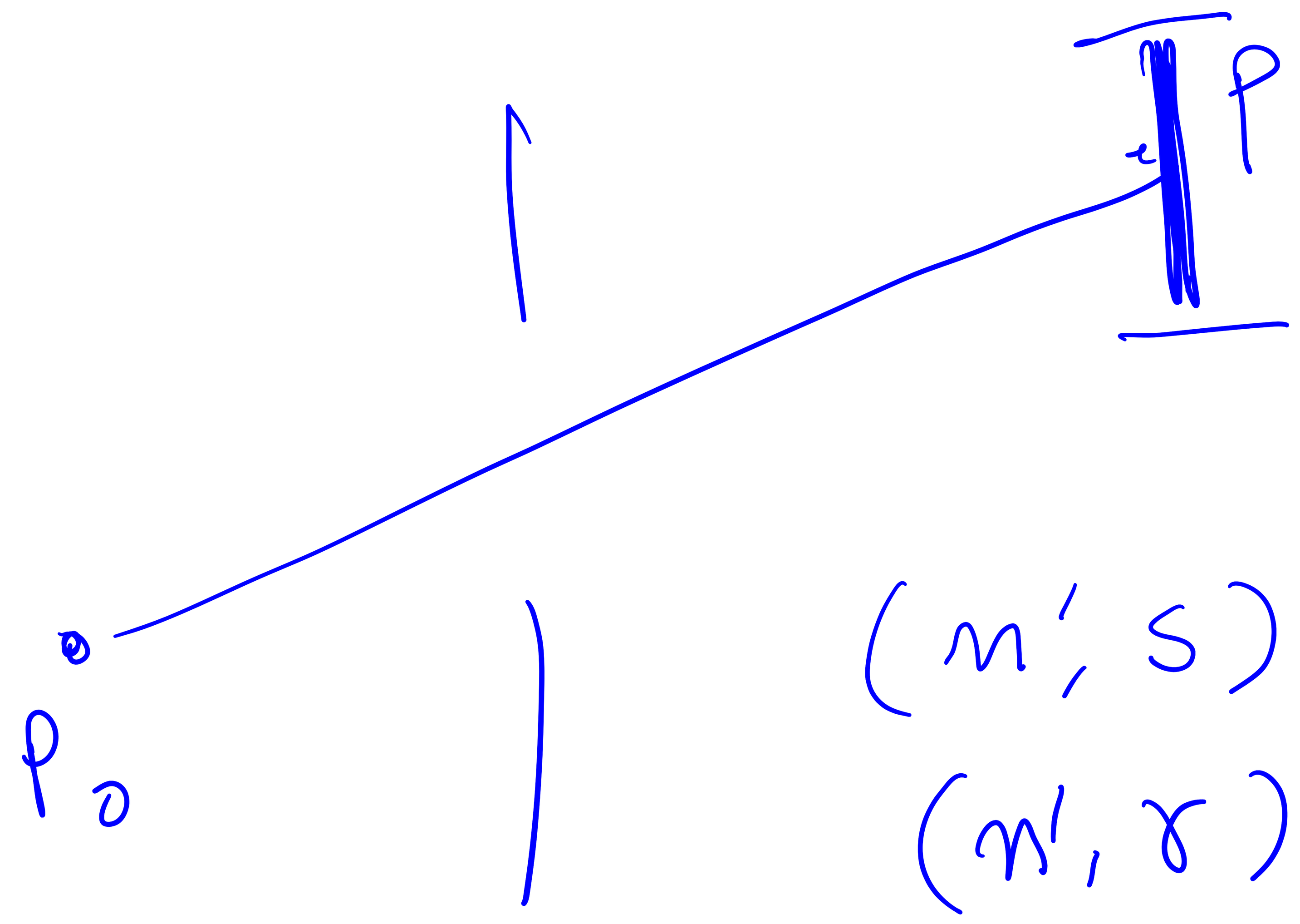
$$U(P) = -\frac{iA}{2\lambda}\iint_A dS \, \frac{e^{ik(r+s)}}{rs}\left[\cos(n', r) - \cos(n', s)\right]$$

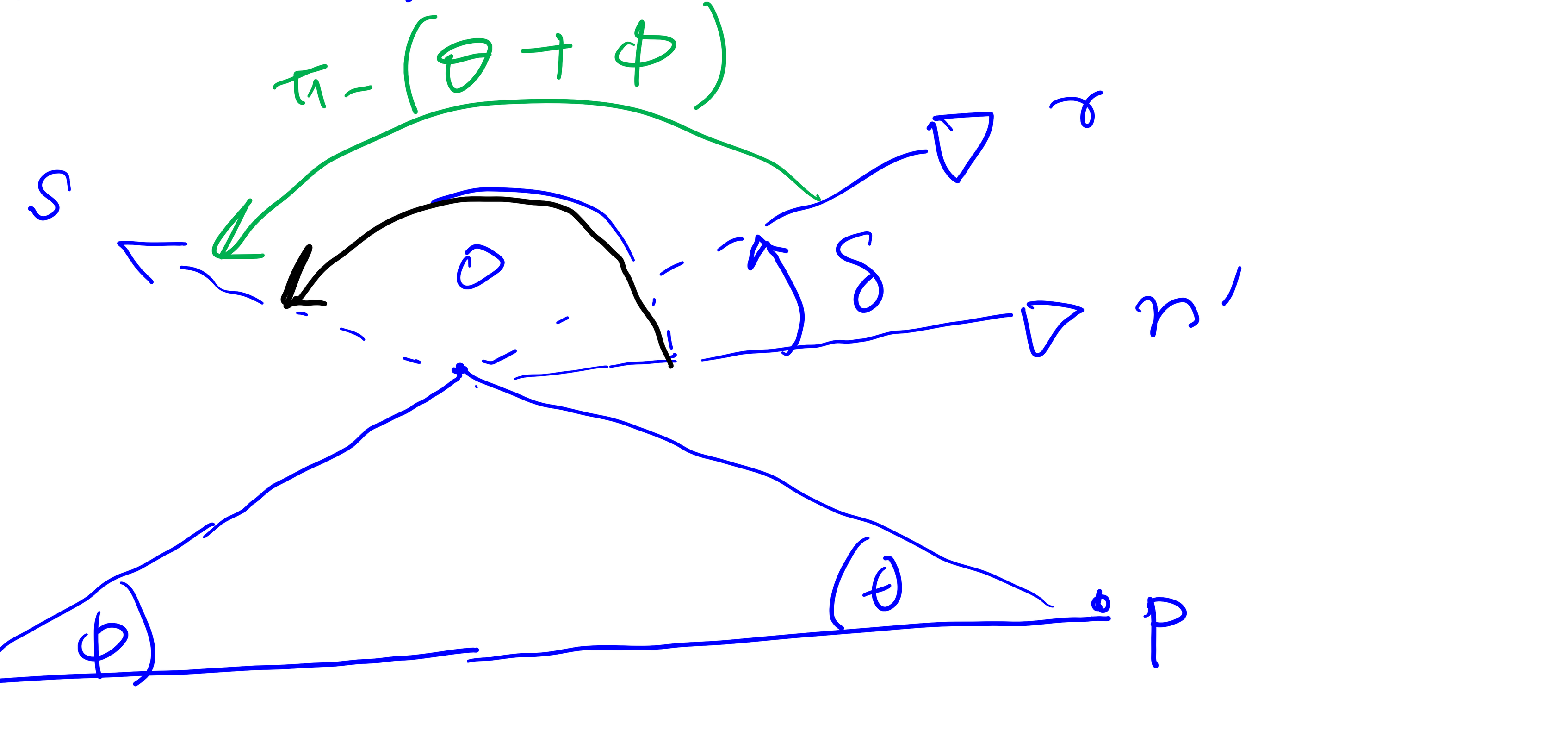


Assumptions : (1) OP_0 , OP $\gg$ W aperture ✓

$\Rightarrow$ inclination factor const over the A

(2)





$$(n', s) = \delta + \pi - \underbrace{(\theta+\phi)}$$

$$(n', r) = \delta \qquad \phi, \theta \sim 0$$

$$\cos(n', r) - \cos(n', s) = 2\cos\delta \quad ✓$$

$$r\,s \longrightarrow r'\,s'$$

$$U(P) \approx \frac{-iA}{\lambda}\,\frac{\cos\delta}{r's'} \iint_{A} e^{ik(r+s)}\,dS \qquad — (2)$$

$$P_0 \to (x_0, y_0, z_0)\ ; \quad P \to (x, y, z)$$

$$Q = (\xi, \eta, 0)$$

$$\left.\begin{aligned} |QP_0|^2 &= r^2 = (x_0-\xi)^2 + (y_0-\eta)^2 + z_0^2 \\ |QP|^2 &= s^2 = (x-\xi)^2 + (y-\eta)^2 + z^2 \end{aligned}\right\}$$

$$r'^2 = x_0^2 + y_0^2 + z_0^2$$

$$s'^2 = x^2 + y^2 + z^2$$

$$r^2 = r'^2 - 2(x_0\xi + y_0\eta) + \xi^2 + \eta^2$$

$$r = r'\left[1 - \frac{2(x_0\xi + y_0\eta)}{r'^2} + \frac{\xi^2+\eta^2}{r'^2}\right]^{1/2}$$

$$= r'\left[1 - \frac{x_0\xi + y_0\eta}{r'^2} + \frac{\xi^2+\eta^2}{2r'^2} - \frac{\frac{4}{8}(x_0\xi + y_0\eta)^2}{r'^4} + \dots\right]$$

$$r = r' - \frac{x_0\xi + y_0\eta}{r'} + \frac{\xi^2+\eta^2}{2r'} - \frac{(x_0\xi + y_0\eta)^2}{2r'^3} + \dots$$

$$s = s' - \frac{x\xi + y\eta}{s'} + \frac{\xi^2+\eta^2}{2s'} - \frac{(x\xi + y\eta)^2}{2s'^3}$$

Substituting in Kirchhoff.

$$U(P) = -\frac{iA}{\lambda}\frac{\cos\delta}{r's'}e^{ik(r'+s')}\iint_A e^{ikf(\xi,\eta)}\,d\xi\,d\eta$$

$$f(\xi,\eta) = -\frac{x_0\xi + y_0\eta}{r'} - \frac{x\xi + y\eta}{s'} + \frac{1}{2}\left(\frac{1}{r'} + \frac{1}{s'}\right)(\xi^2+\eta^2) - \frac{(x_0\xi + y_0\eta)^2}{2r'^3} - \frac{(x\xi + y\eta)^2}{2s'^3} + \dots$$

$$\vec{P_0 O} = \left(\frac{0-x_0}{r'}, \frac{0-y_0}{r'}\right) = (l_0, m_0)$$

$$\vec{OP} = \left(\frac{x-0}{s'}, \frac{y-0}{s'}\right) = (l, m)$$

$$f(\xi,\eta) = (l_0 - l)\xi + (m_0 - m)\eta$$

$$+ \frac{1}{2}\left(\frac{1}{r'}+\frac{1}{s'}\right)(\xi^2+\eta^2)$$

$$- \frac{(l_0\xi + m_0\eta)^2}{2r'} - \frac{(l\xi + m\eta)^2}{2s'}$$

FRAUNHOFER : Quadratic & higher order can be neglected in f

FRESNEL : Need to keep quadratic terms

$$\frac{1}{2}k\left|\left(\frac{1}{r'}+\frac{1}{s'}\right)(\xi^2+\eta^2) - \frac{(l_0\xi + m_0\eta)^2}{r'} - \frac{(l\xi+m\eta)^2}{s'}\right|$$

$$\ll 2\pi$$

"Cauchy Schwarz"

$$(l_0\xi + m_0\eta)^2 \leq (l_0^2+m_0^2)(\xi^2+\eta^2)$$

$$|\quad| \leq \frac{1}{2}k\left|\left(\frac{1}{r'}+\frac{1}{s'}\right) + \frac{l_0^2+m_0^2}{r'} + \frac{l^2+m^2}{s'}\right| \cdot (\xi^2+\eta^2)$$

$$\leq \frac{1}{2}k \cdot 2\left(\frac{1}{r'}+\frac{1}{s'}\right)(\xi^2+\eta^2)$$

$$\ll 2\pi$$

$$\left(\frac{1}{|r'|}+\frac{1}{|s'|}\right)(\xi^2+\eta^2) \ll \lambda$$

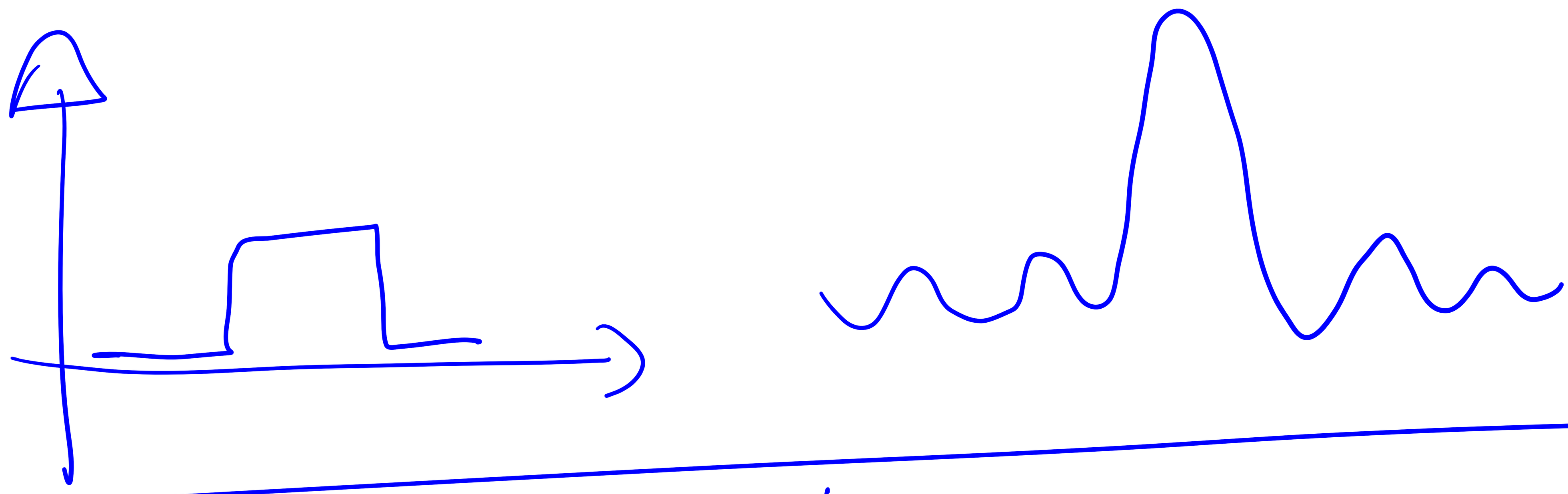
$$|s'| \gg \underbrace{(\xi^2+\eta^2)_{max}/\lambda}$$

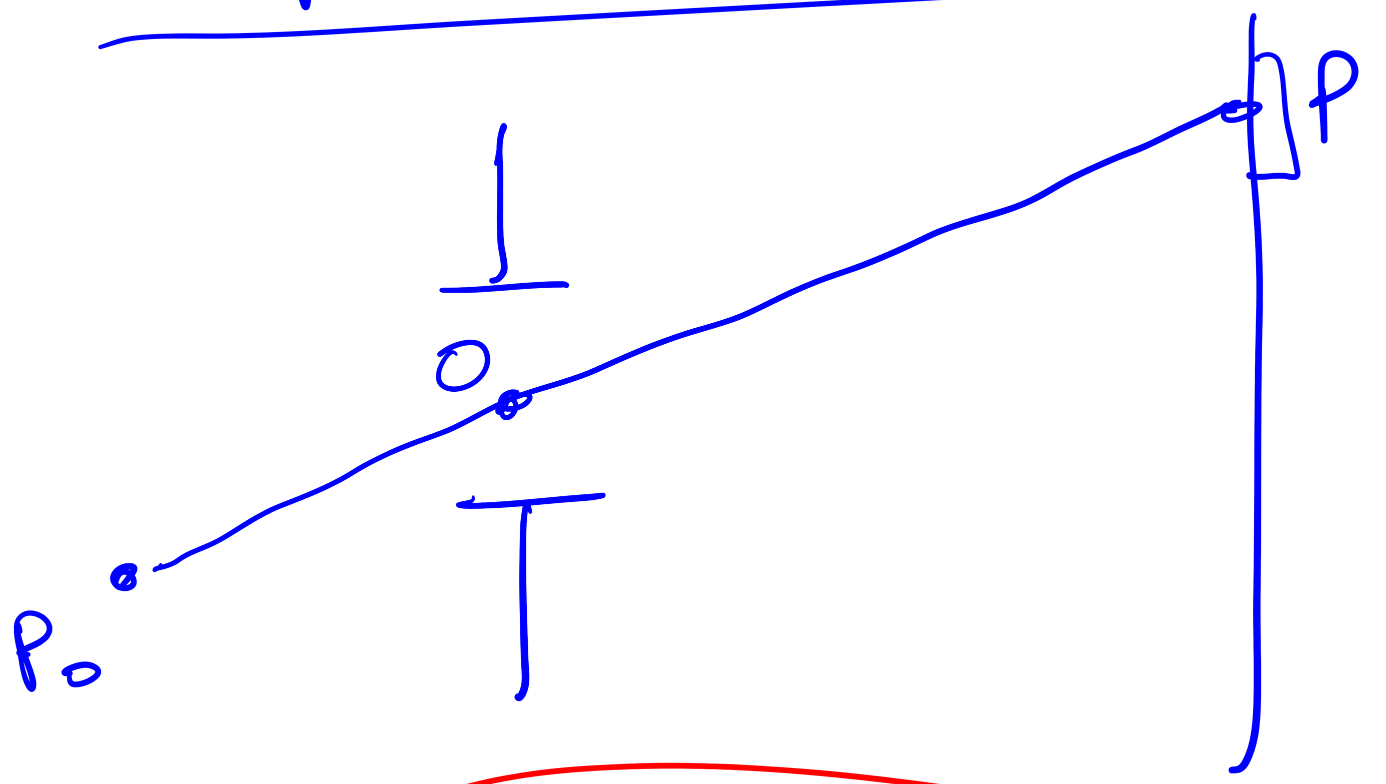
Find Fraunhofer:

$$U(P) = \underbrace{C}\cdot \iint_{\mathcal{A}} e^{-ik(p\xi+q\eta)}\, d\xi\, d\eta$$

$p = l - l_0$

$q = m - m_0$





$|OP| \sim R$

$dA = R^2\, dp\, dq$

$\mathcal{P}$ = ~~total~~ power incident on $\mathcal{A}$

$$\iint |U(p,q)|^2 R^2\, dp\, dq = \mathcal{P}$$

Aside

$$U(p,q) = \iint G(\xi,\eta)\, e^{-\frac{2\pi}{\lambda}(p\xi+q\eta)}\, d\xi\, d\eta$$

pupil function

$$G(\xi,\eta) = \begin{cases} C(\xi) & \text{in aperture} \\ 0 & \text{outside} \end{cases}$$

Parseval identity

$$\delta(x-x_0) = \int e^{i2\pi p(x-x_0)}\, dp$$

$$\iint |G(\xi,\eta)|^2\, d\xi\, d\eta = \frac{1}{\lambda^2}\iint |U(p,q)|^2\, dp\, dq$$

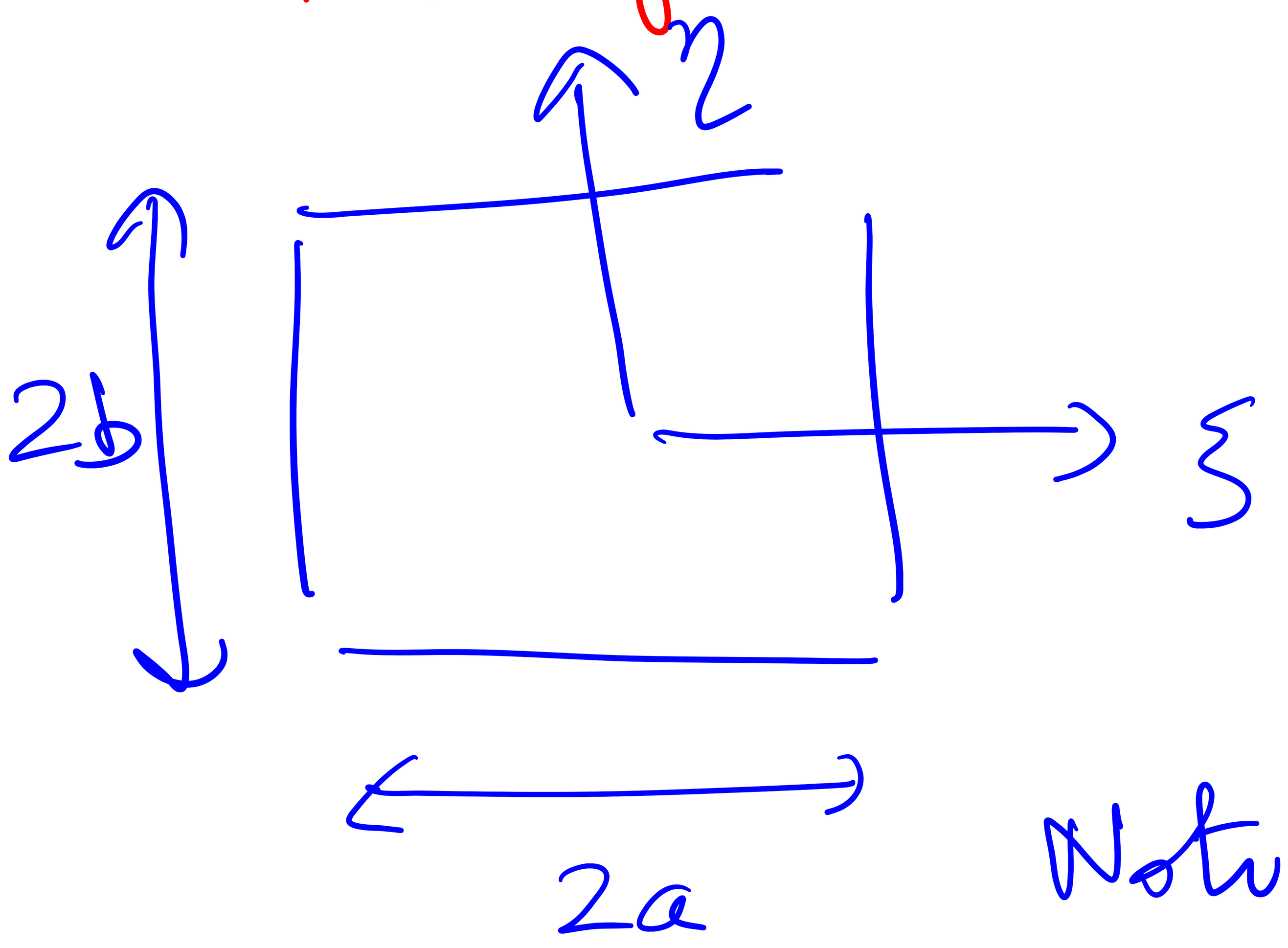
$$\Rightarrow |C|^2 D = \frac{1}{\lambda^2} \frac{P}{R^2}$$

$$\boxed{C = \sqrt{\frac{P}{D}} \frac{1}{\lambda R}}$$

$$\boxed{U(p,q) = \sqrt{\frac{P}{D}} \frac{1}{\lambda R} \iint_A e^{-ik(p\xi + q\eta)} d\xi\, d\eta}$$

$\square$

Examples: Rectangular aperture



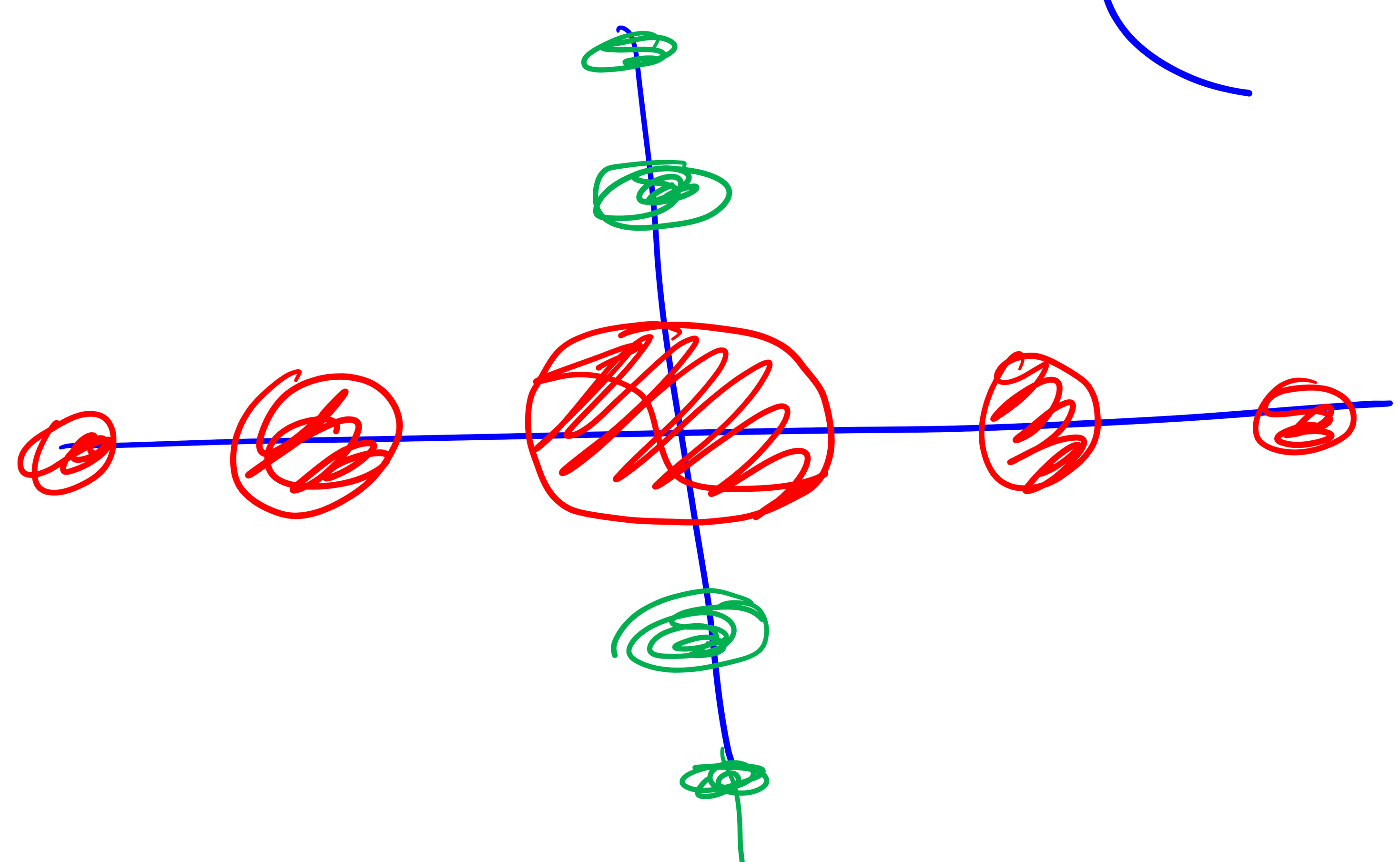
$$U(p) = C \int_{-a}^{a} \int_{-b}^{b} e^{-ik(p\xi + q\eta)} d\xi\, d\eta$$

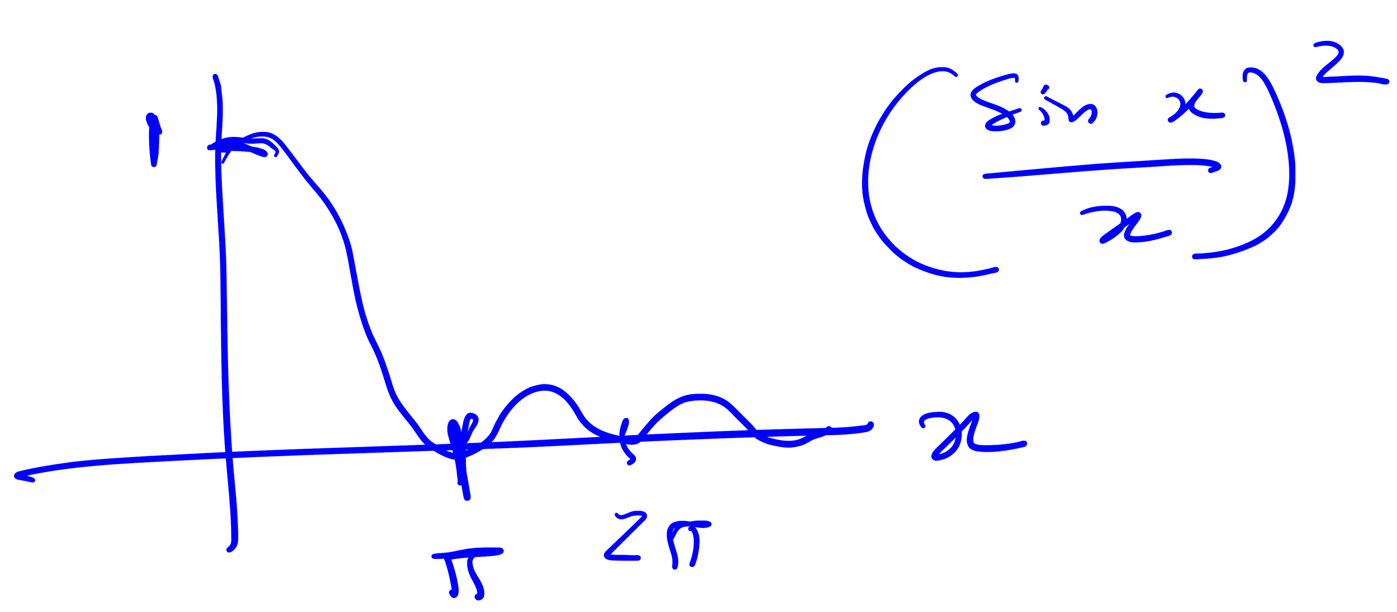
Note: $\displaystyle\int_{-a}^{a} e^{-ikp\xi} d\xi = \frac{1}{-ikp}\left(e^{-ikpa} - e^{+ikpa}\right)$

$$= 2\frac{\sin(kpa)}{kp}$$

$$U(p) = C \cdot 4ab \cdot \frac{\sin(kpa)}{kpa} \cdot \frac{\sin(kqb)}{kqp}$$

$$I(p) = I_0 \left(\frac{\sin(kpa)}{kpa}\right)^2 \left(\frac{\sin(kqb)}{kqb}\right)^2$$





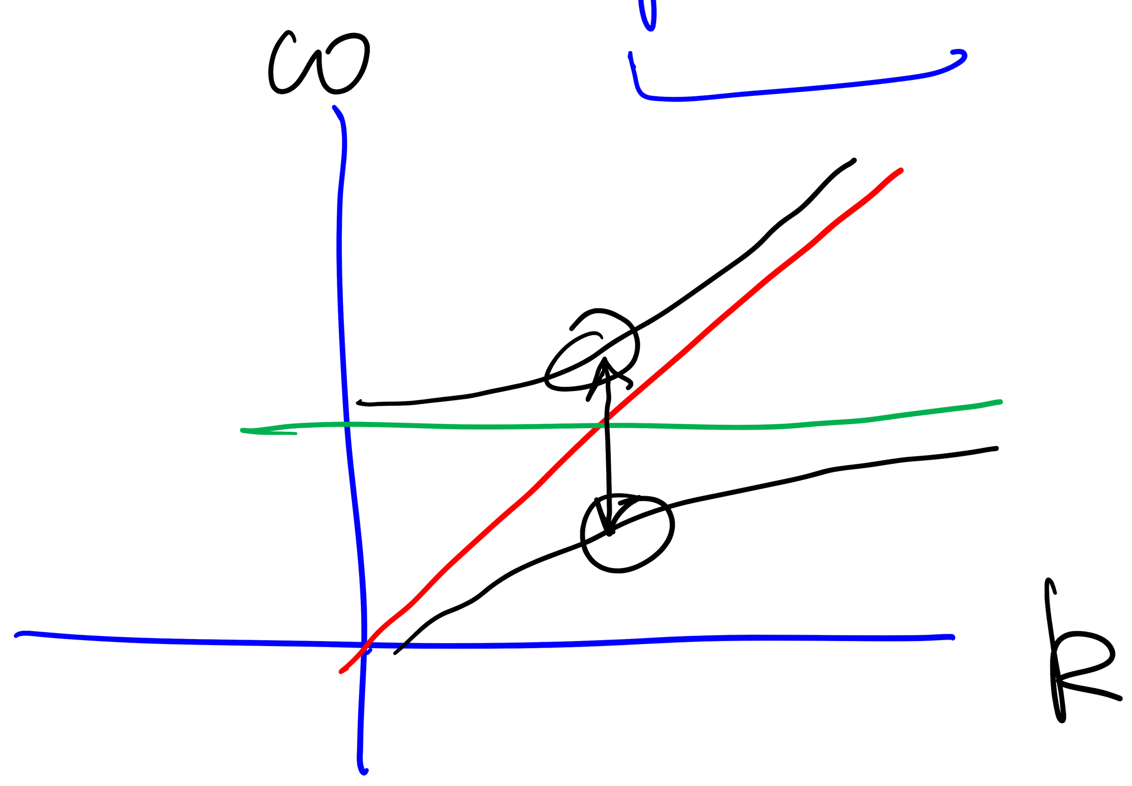
① Circular aperture (Hecht)

② YDSE (slits have non-negligible widths)

$$\omega^2 = c^2 k^2 + \omega_0^2$$

Coupled mode

photon + plasma



$\omega = ck$ ✓