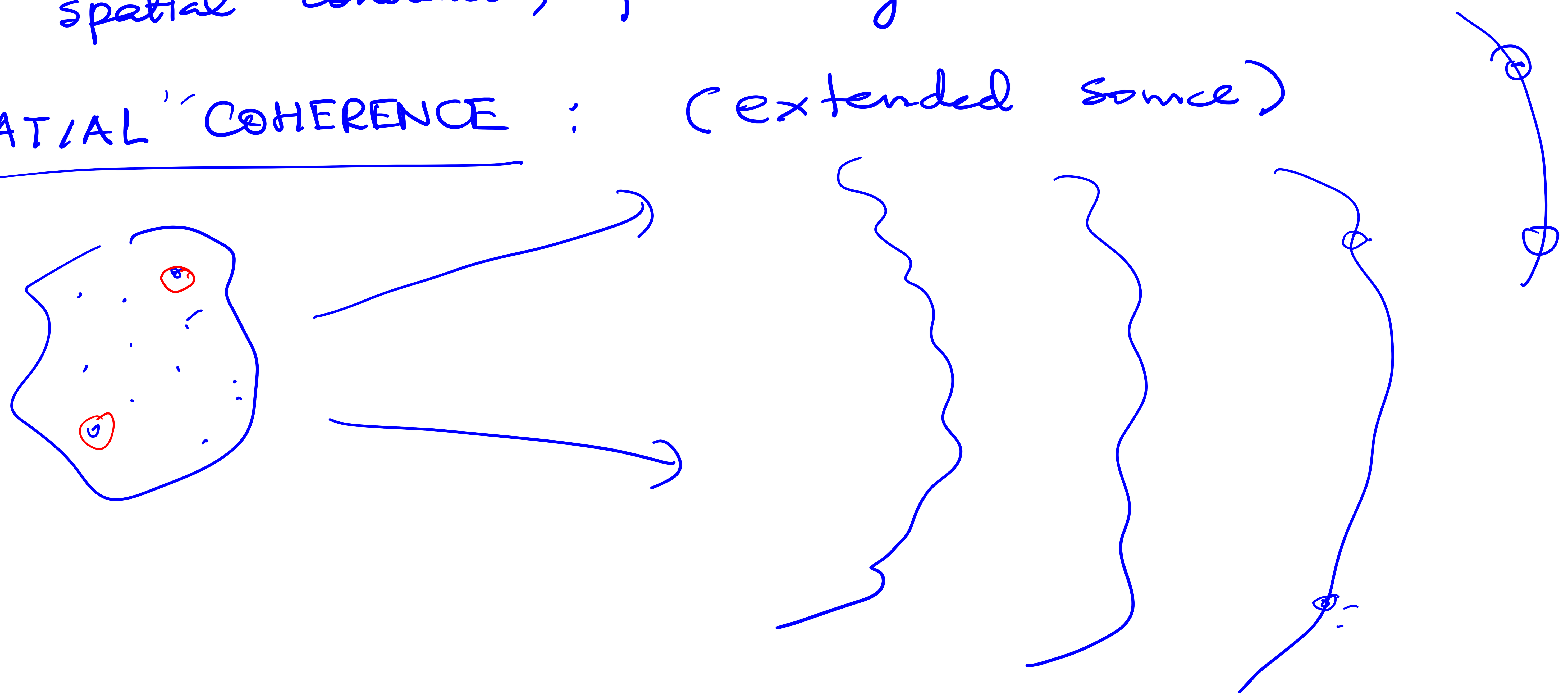
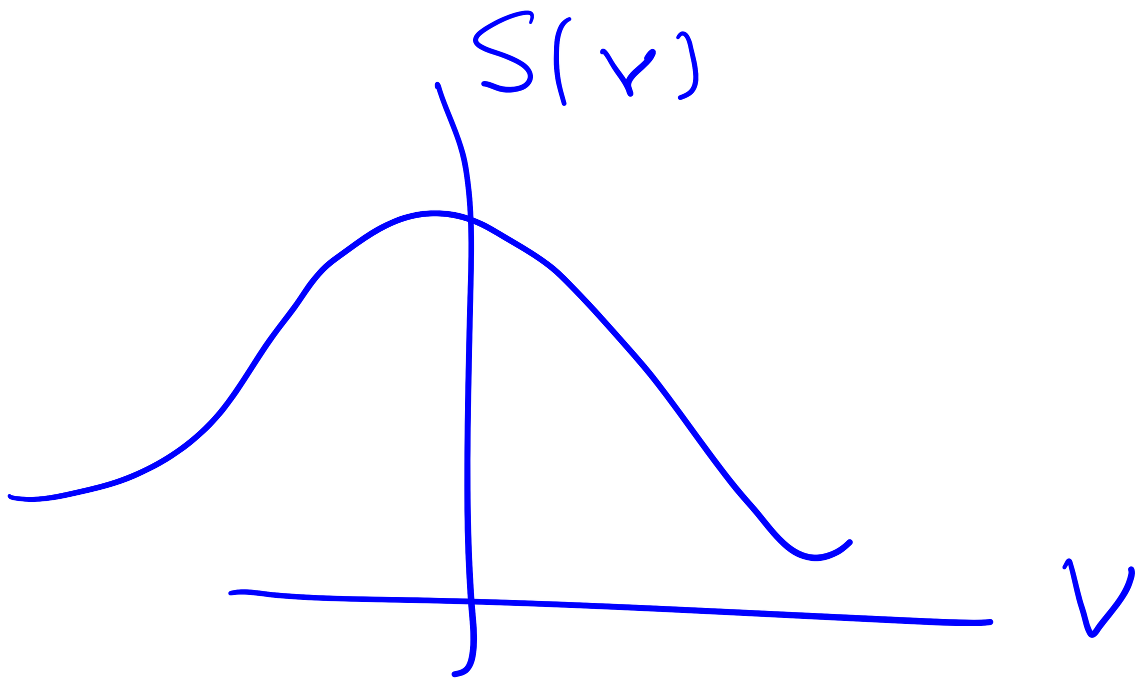
COHERENCE

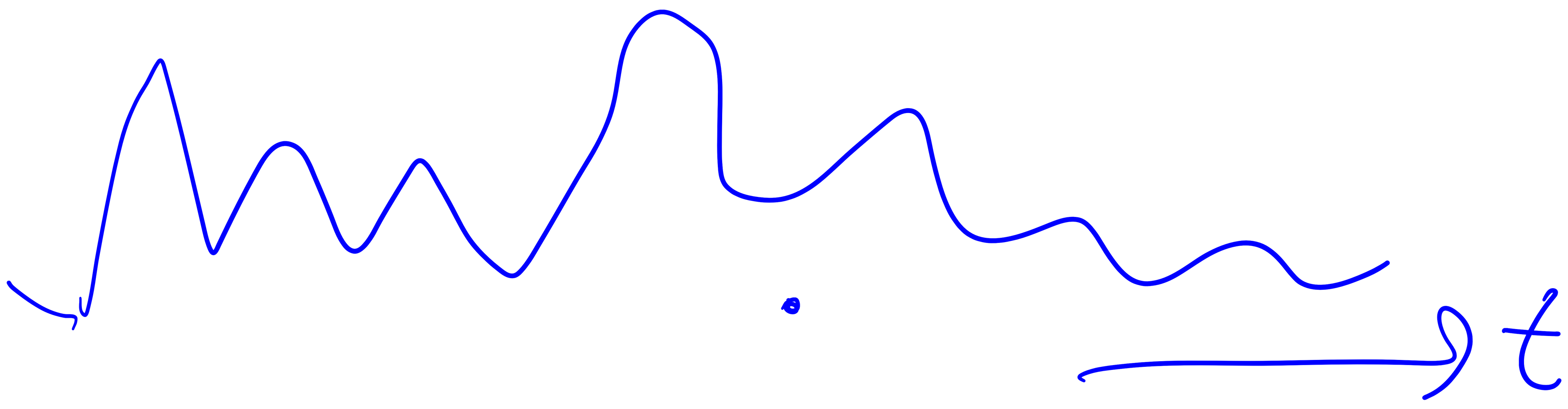
In high school, you have learnt about "temporal and spatial coherence", qualitatively.

"SPATIAL" COHERENCE : (extended source)



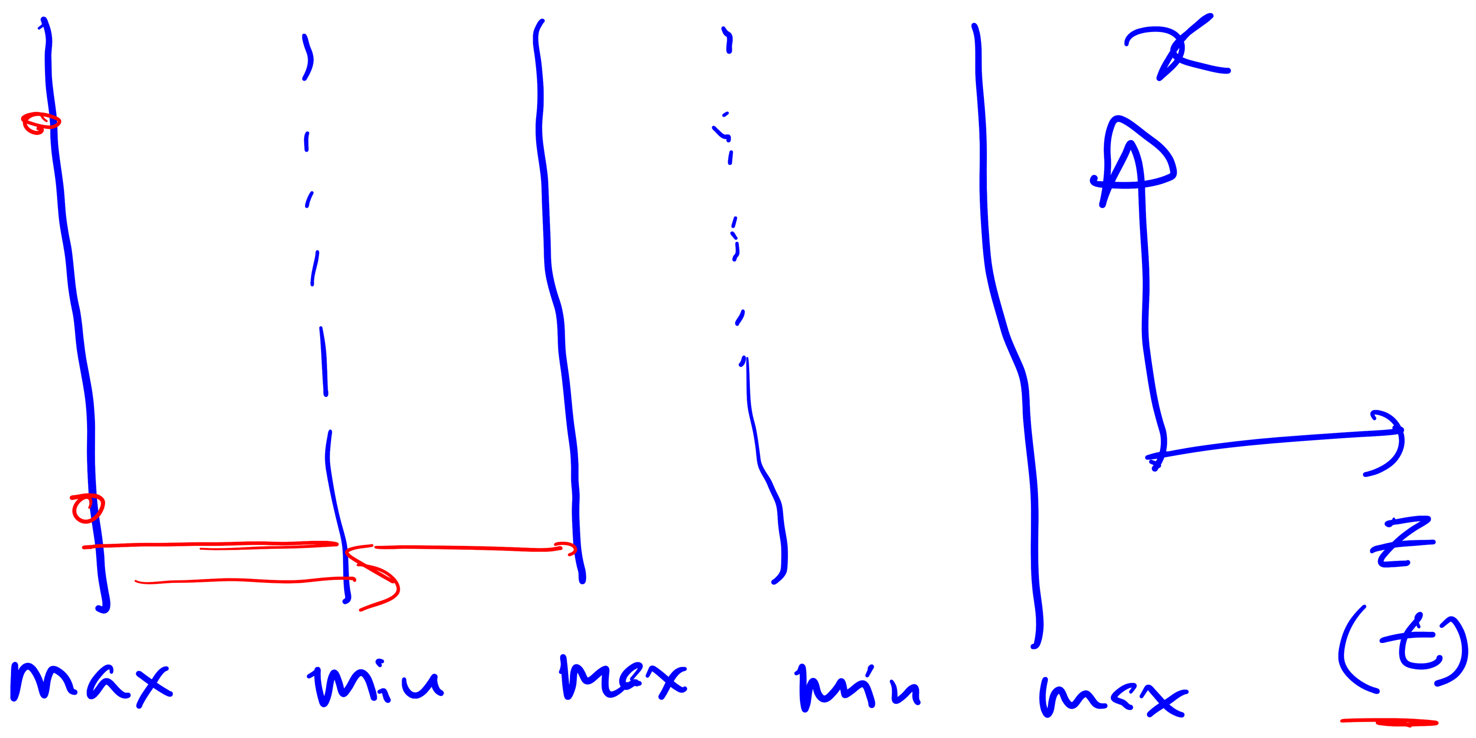
TEMPORAL COHERENCE (non-monochromatic)



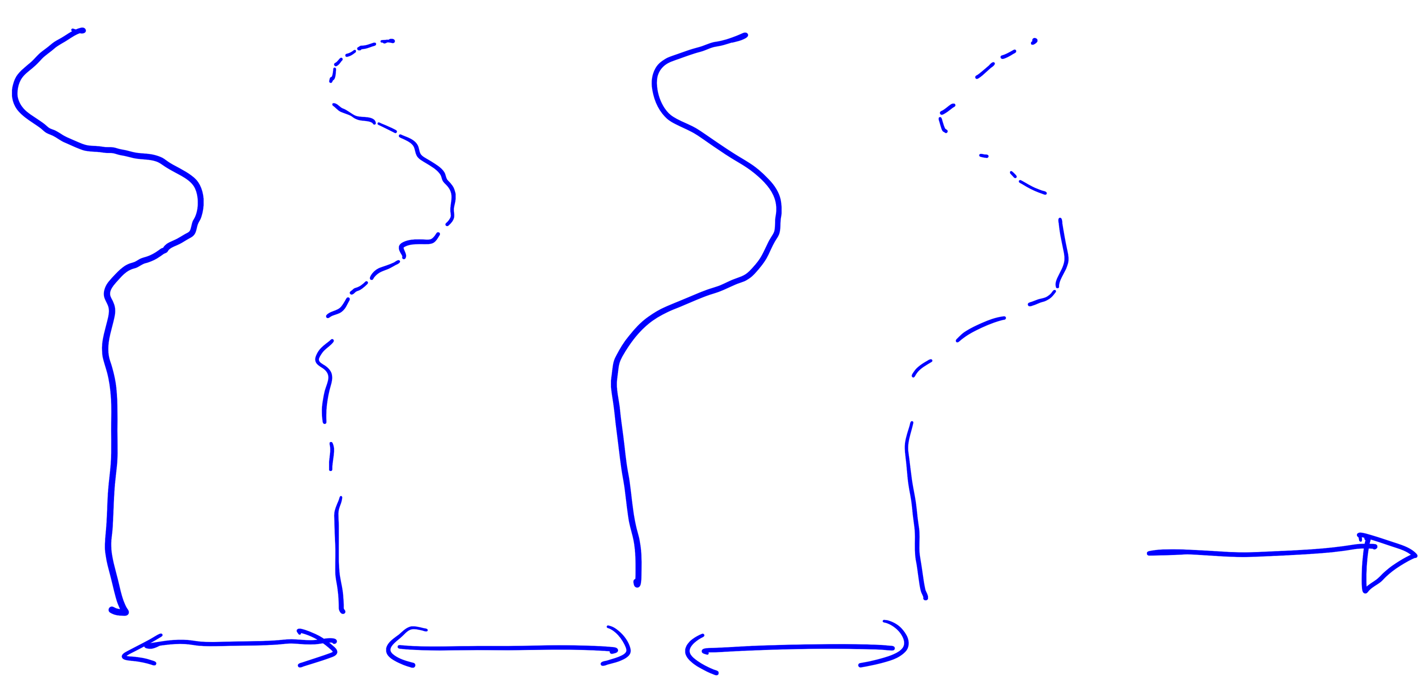


Examples

(1) Spatially & temporally coherent $\bar{k}$

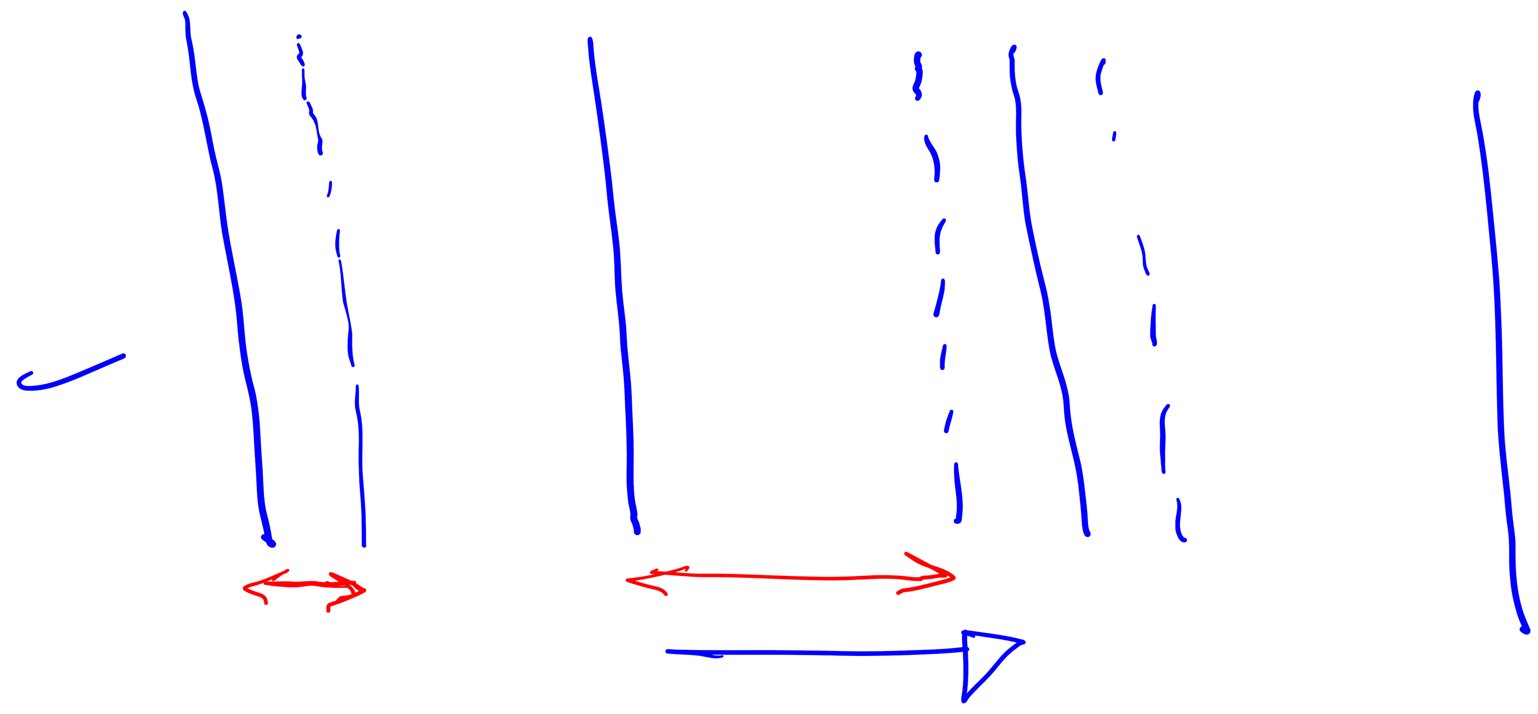


(2) Temporal coherence
Spatial incoherence

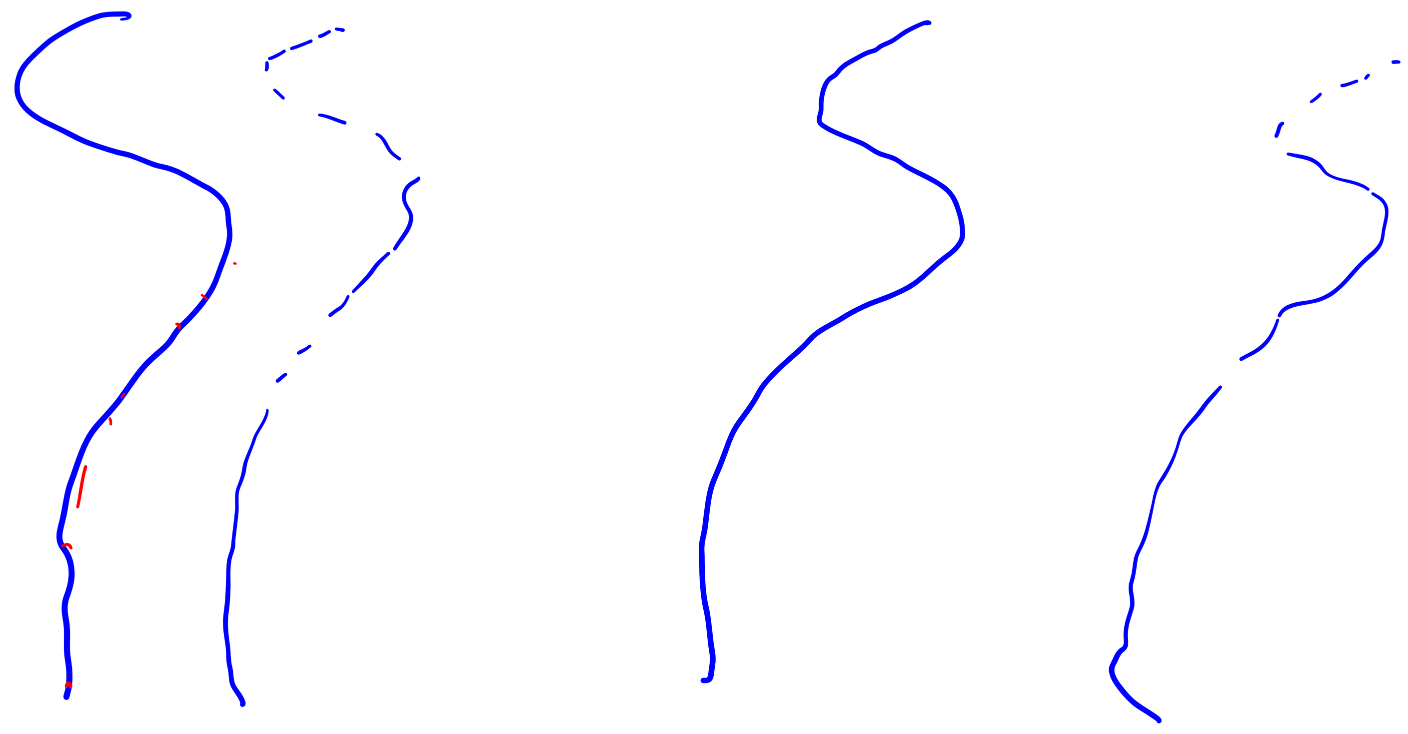


(3) Spatial coherence

temporally incoherence



(4) Spatial & temporal incoherence



Question: How to describe partially coherent light?

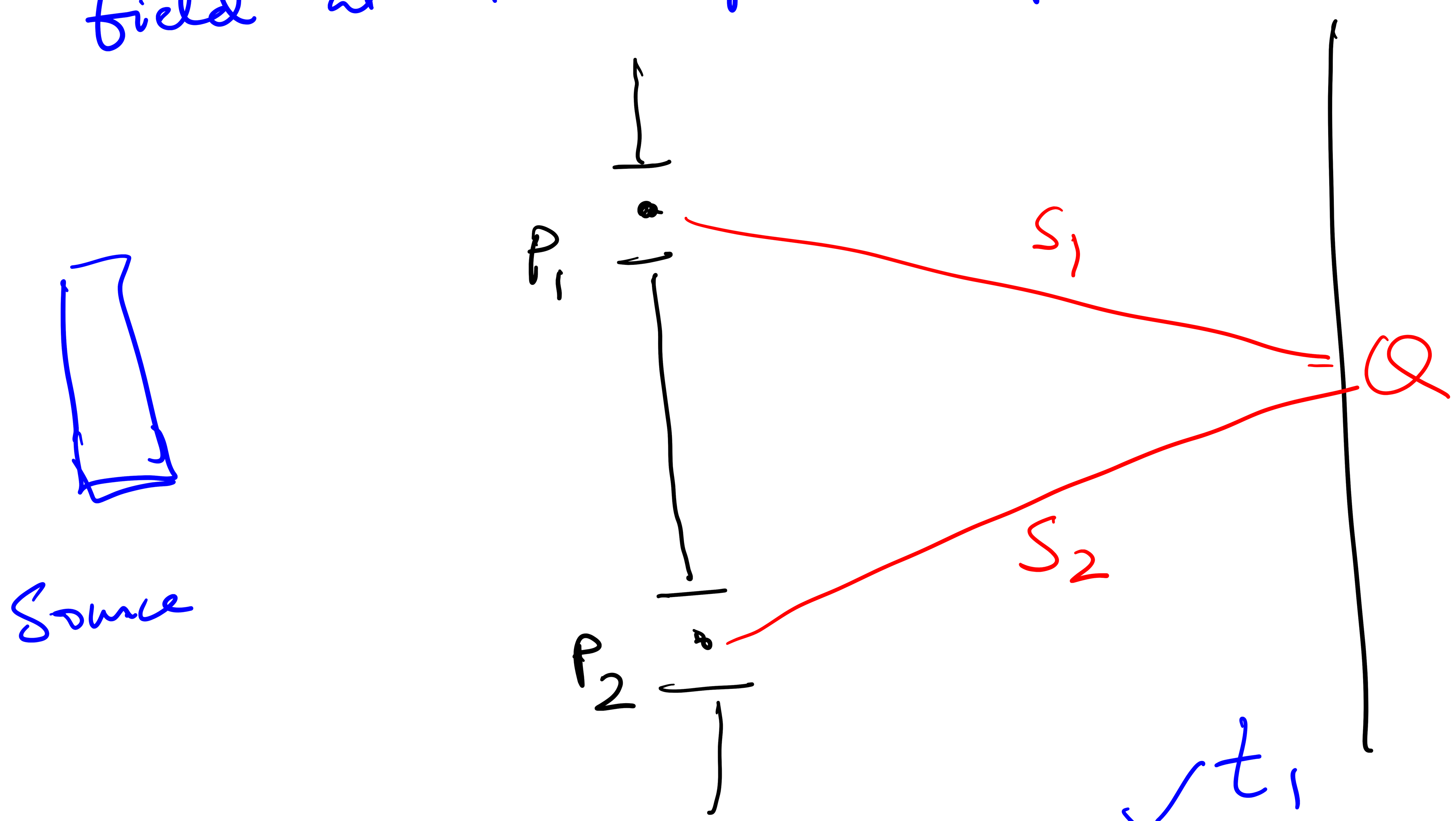
Correlations: Extended polychromatic source

Real field $V^{(r)}(P,t) \longrightarrow V(P,t)$

- Detector records time averaged intensities

$$I(P) = 2\langle V^{(r)2}(P,t)\rangle = \langle V(P,t)\, V^*(P,t)\rangle$$

- Suppose we want to look at the properties of the field at two spatial points P_1, P_2



$$V(Q,t) = K_1 V\Big(P_1, \overbrace{t - \frac{S_1}{c}}^{t_1}\Big) + K_2 V\Big(P_2, \underbrace{t - \frac{S_2}{c}}_{t_2}\Big)$$

(K_i's geometrical factors like $1/S_i$)

$$I(Q) = |K_1|^2 \underbrace{\langle V_1(t-t_1)\, V_1^*(t-t_1)\rangle} + |K_2|^2 \langle V_2(t-t_2)\, V_2^*(t-t_2)\rangle$$

$+ \ldots$

$$+ \underbrace{K_1 K_2^*}\ \underbrace{\langle V_1(t-t_1)\, V_2^*(t-t_2)\rangle} + \underbrace{K_1^* K_2}\ \langle V_1^*(t-t_1) \cdot V_2(t-t_2)\rangle$$

- "Stationarity" of the field
  ↳ can shift the time origin in temporal averages

$$I(Q) = |K_1|^2 I_1 + |K_2|^2 I_2 + 2|K_1 K_2|\,\mathrm{Re}\left(\langle V_1(t+\tau)\, V_2^*(t)\rangle\right)$$

$$\tau = \frac{s_2 - s_1}{c}$$

$$= \underbrace{|K_1|^2 I_1}_{2} + \underbrace{|K_2|^2}\ \underbrace{I_2} + 2|K_1 K_2|\, \Gamma_{12}^{(r)}\left(\frac{s_2 - s_1}{c}\right)$$

- Mutual coherence function

$$\Gamma_{12}(\tau) = \langle V_1(t+\tau)\, V_2^*(t)\rangle_t$$

- Self-coherence function (if P_1 & P_2 coincide)

$$\Gamma_{11}(\tau) = \langle V_1(t+\tau)\, V_1^*(t)\rangle_t$$

- If $\tau = 0$

$$\Gamma_{11}(0) = I_1, \qquad \Gamma_{22}(0) = I_2$$

- Normalized mutual coherence function

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)}\sqrt{\Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1}\cdot\sqrt{I_2}}$$

Def. "Complex degree of coherence"

$$I(Q) = \underbrace{I^{(1)}(Q)} + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)\, I^{(2)}(Q)}\ \underbrace{\gamma_{12}^{(r)}\left(\frac{S_2 - S_1}{c}\right)} \ \square$$

Interference Law for stationary fields

Wavefront division $\longrightarrow$ $\gamma_{12}(\tau)$

Amplitude division $\longrightarrow$ $\gamma_{11}(\tau)$

How to measure $\gamma_{12}^{(r)}$

Problem : Given a source, given two points P_1 & P_2, $\gamma_{12}^{(r)}(\tau)$

Solution : (1) . . . - - 2 pinholes on an opaque screen

(2) $I(Q)$ on a screen YDSE

$P_2Q - P_1Q = c\tau$

(3) $I^{(1)}(Q)$, $I^{(2)}(Q)$

(4) $$\gamma_{12}^{(r)}(\tau) = \frac{I(Q) - I^{(1)}(Q) - I^{(2)}(Q)}{2\sqrt{I^{(1)}(Q)}\sqrt{I^{(2)}(Q)}}$$

What values can γ_{12} take?

$$\lim_{T\to\infty} \frac{1}{2T}\left|\underbrace{\int_{-T}^{T} \underbrace{V(P_1, t+\tau)}\, V^*(P_2, t)}\right|^2 \leq$$

$$|\gamma_{12}(\tau)| \leq 1$$

But where is partial coherence?

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)|\, e^{i(\alpha_{12}(\tau) - 2\pi \overset{\text{mean freq}}{\bar{\nu}} \tau)}$$

$$\alpha_{12}(\tau) = 2\pi\bar{\nu}\tau + \arg(\gamma_{12}(\tau))$$

$$I(Q) = I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)}\sqrt{I^{(2)}(Q)}\,|\gamma_{12}(\tau)| \cos(\alpha_{12}(\tau) - \underbrace{2\pi\bar{\nu}\tau}_{\delta})$$

If $|\gamma_{12}(\tau)| = 1$: $I(Q)$ of monochromatic $\bar{\nu}$

COHERENT

$|\gamma_{12}(\tau)| = 0$: No interference $\Rightarrow$ INCOHERENT

$|\gamma_{12}(\tau)| < 1$: PARTIALLY COHERENT

$$I(Q) = |\gamma_{12}(\tau)| \left[I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)}\sqrt{I^{(2)}(Q)} \cdot \cos(\alpha_{12}(\tau) - \delta) \right]$$
$$+ (1 - |\gamma_{12}(\tau)|) \left[I^{(1)}(Q) + I^{(2)}(Q) \right]$$

Coherent superposition of beams of intensities $\underline{|\gamma_{12}(\tau)|\, I^{(i)}(Q)}$ & phase diff $\alpha_{12}(\tau) - \delta$

incoherent superposition of beam of intensity $\underline{(1 - |\gamma_{12}(\tau)|)}\, I^{(i)}(Q)$

$i = 1, 2$

$$\frac{I_{coh}}{I_{incoh}} = \frac{|\gamma_{12}(\tau)|}{1 - |\gamma_{12}(\tau)|}$$

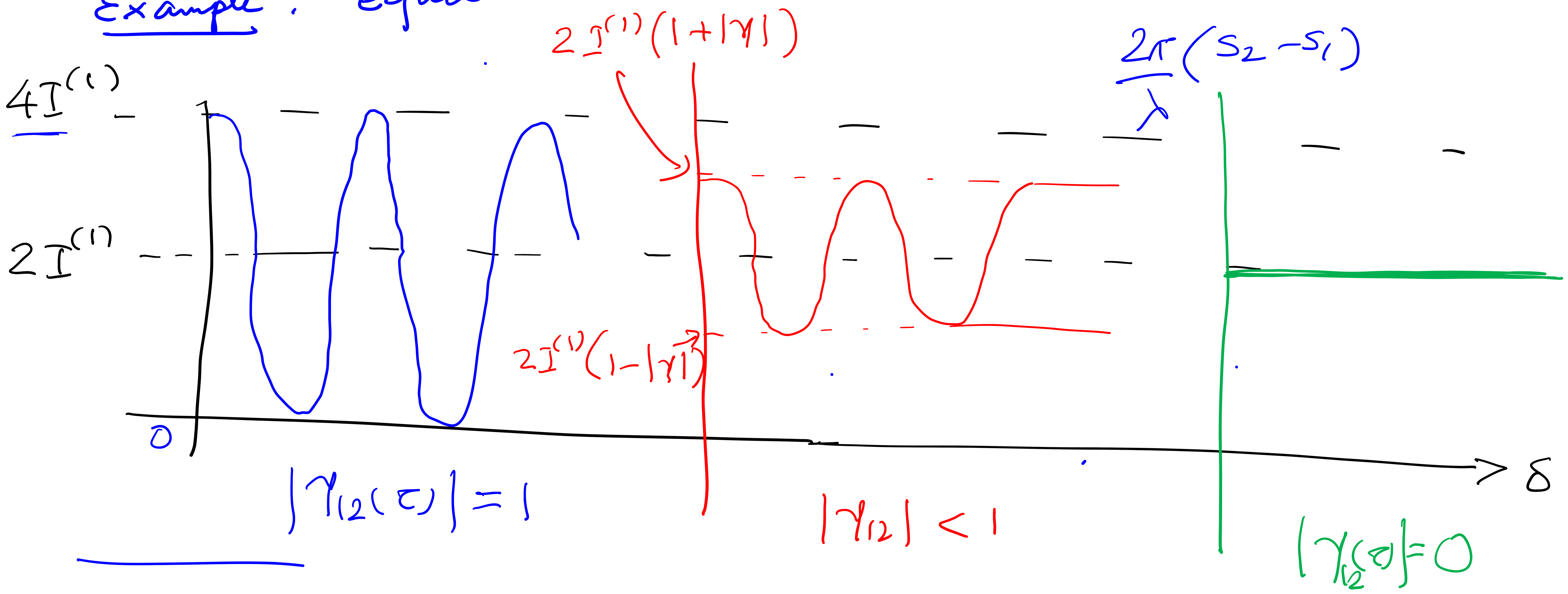
$$\frac{I_{coh}}{I_{tot}} = \underbrace{|\gamma_{12}(\tau)|}$$

* Relation between degree of coherence & interference pattern

$$I(Q) = \underline{I^{(1)}}(Q) + \underline{I^{(2)}}(Q) + 2\sqrt{I^{(1)}(Q)}\cdot\sqrt{I^{(2)}(Q)}\,|\gamma_{12}(\tau)|\cos(\alpha_{12}(\tau) - \delta)$$

$$\delta = \frac{2\pi}{\lambda}(s_2 - s_1)$$

Example: equal intensities



Fringe visibility $V(Q) = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$$I_{\substack{max\\min}} = I^{(1)}(Q) + I^{(2)}(Q) \pm 2\sqrt{I^{(1)}(Q)\, I^{(2)}(Q)}\,|\gamma_{12}(\tau)|$$

$$\underbrace{V(Q)} = \frac{2\sqrt{I^{(1)}(Q)\, I^{(2)}(Q)}}{I^{(1)}(Q) + I^{(2)}(Q)} \underbrace{|\gamma_{12}(\tau)|}$$

$$\gamma_{12} = |\gamma_{12}|\, e^{i\alpha_{12}(\tau)}$$