Direct State funding of Chilean universities

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1 Calculation

1.1 Yearly evaluation

1.1.1 Determination

Art. 2 of Decree with Force of Law 4 of 1980, with modifications from Art. 1 of Ministry of Education Decree 116 of 2002, indicates that 5% part of the total funding of year $n+1$ is distributed to University i according to its metrics measured in year n. They involve

- $U_{i,n}$, the number of undergraduate students ("estudiantes de pregrado");
- $M_{i,n}$, the number of majors ("carreras");
- $S_{i,n}$, the number of equivalent full-time scholars ("académicos"), i.e. professors and researchers;
- $P_{i,n}$, the number of equivalent full-time scholars with a post-graduate title such as master or a PhD;
- $G_{i,n}$, the number of research grants ("proyectos");
- $P^{\mathrm{I}}_{i,n}$, the number of Web of Science publications (WoS)[1];
- and $P^{\mathrm{S}}_{i,n}$, the number of non-WoS publications indexed by the Scientific Electronic Library Online (Scielo) Chile.

The metrics, defined in the aformentioned decrees, are ratios meant to measure an output v. staff efficiency[2]

$$x_{i,n,1} = U_{i,n}/M_{i,n}, \tag{1a}$$
$$x_{i,n,2} = U_{i,n}/S_{i,n}, \tag{1b}$$
$$x_{i,n,3} = P_{i,n}/S_{i,n}, \tag{1c}$$
$$x_{i,n,4} = G_{i,n}/S_{i,n}, \tag{1d}$$
$$x_{i,n,5} = (P^{\mathrm{I}}_{i,n} + \frac{33}{100}P^{\mathrm{S}}_{i,n})/S_{i,n} \tag{1e}$$

According to Art. 3 of Ministry of Education Decree 128 of 1991, the evaluation formula renormalises the aforementioned ratios in this way[3]:

$$\mu_{n,k} = \frac{1}{N}\sum_j x_{j,n,k} \qquad \text{(mean)} \tag{2a}$$
$$\sigma_{n,k} = \sqrt{\frac{1}{N}\Big(\sum_j x^2_{j,n,k}\Big) - N\mu^2_{n,k}} \qquad \text{(std. dev.)} \tag{2b}$$
$$\xi_{i,n,k} = \frac{x_{i,n,k} - \mu_{n,k}}{\sigma_{n,k}} \qquad \text{(reduced coeff.)} \tag{2c}$$
$$y_{i,n,k} = \exp\left[-\frac{7}{5} + \frac{\xi_{i,n,k}}{4}\right]^3 \tag{2d}$$

where N is the total number of universities. The transform in Eq. (2c) ensures that Universities are compared by how much they deviate from the mean. The exponential in Eq. (2d) is supposed to simulate a biological growth. Figure 2 displays the exponential nature of the rating.

Table 1: Coefficients used for university evaluation since 1998.

	ratio	value
c_1	students-to-majors	0.01
c_2	students-to-staff	0.14
c_3	postgrad staff-to-staff	0.24
c_4	grants-to-staff	0.25
c_5	papers-to-staff	0.35

Art. 2 of Decree with Force of Law 4 of 1980 indicates that 5% of the funding is indexed on a weighted average of the metrics $y_{i,n,k}$ (k in $1\cdots5$) (see Sect. 1.1.1). The weights c_k may vary from year to year, but have been constant since 1998 (see Table 1). University i is thus assigned a score

$$y_{i,n} = \sum_k c_k y_{i,n,k}. \tag{3a}$$

and, using the total score

$$y_n = \sum_i y_{i,n}, \tag{3b}$$

a funding share

$$f_{i,n} = \frac{y_{i,n}}{y_n} \tag{3c}$$

[1] At the time of the Decree 116 it was known as ISI

[2] While the number of publications is defined by the number of WoS plublications plus one third of Scielo one by Ministry of Education Decree 116 of 2002, the Ministry has consistently used factor 0.33 instead of 1/3 for the calculation.

[3] Although not specified by the decree, the Ministry has consistently used the population variance, not the sample variance, for the calculation.

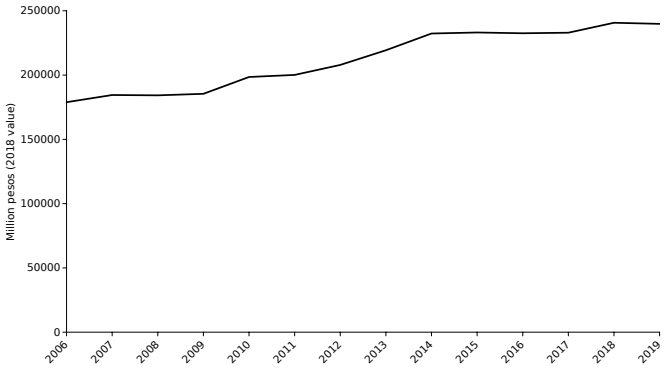

Figure 1: Evolution of total direct State funding to Chilean Universities, in 2018 pesos (inflation-corrected).

1.1.2 Marginal earnings

In this section, we focus on a yearly snapshot and drop the n index in the formulae. We examine the case where university i decides to increase one of its ratios number k by a small number of standard deviations $\Delta\xi_{i,k}$, so that the ratio $x_{i,k}$ improves by $\Delta x_{i,k} = \sigma_k \Delta\xi_{i,k}$.

For any university j, the new value of the score $y_{j,k}$ is usually modified because the mean and standard deviation are changed via $x_{i,k}$. The difference $\Delta y_{j,k}$ is given by differentiating Eq. (2d), and in turn Eqs. (2a–2c) on which it depends. The calculation, detailed in in Appendix A, yields

$$\frac{\Delta y_{j,k}}{y_{j,k}} = \frac{3}{4}\left(\frac{\xi_{j,k}}{4} - \frac{7}{5}\right)^2 \left(\delta_{ij} - \frac{1}{N} - \frac{\xi_{i,k}\xi_{j,k}}{N}\right) \Delta\xi_{i,k}, \quad (4)$$

where $\delta_{ij} = 1$ if universities i and j are the same and zero otherwise.

The meaning of Eq. (4) is the following:

1. $(\xi_{j,k}/4 - 7/5)^2$ *factor*: The relative improvement depends on the relative standing of the University in the ranking. A university lagging behind by 2 standards deviations gets a relative improvement 4,5 times higher than a university standing out by 2 standard deviations.

2. δ_{ij} *term:* University i generally benefits from an increase of its own ratio $\Delta x_{i,k}$: the δ_{ij} ($= 1$ for $i = j$) term in the equation is the only one that is not in $1/N$. However, if $|\xi_{i,k}| > \sqrt{N-1} \approx 5$ standard deviations, University could lose from improving. Nevertheless, the data of the Ministery from 2006 to 2018 can be used to show that the highest deviation any of the ratio has ever reached is 3.7.

3. University j may benefit from, or be harmed by, the improvement of University i. There are two effects at play.

   (a) $1/N$ *term*: The increase of the mean, would on its own hurt all other universities as their position relative to the mean $\xi_{j,k}$ would drop (see the $1/N$ term in the equation).

   (b) $\xi_{i,j}\xi_{j,k}/N$ *term*: However, the modification of the standard deviation works both ways. Intuitively, if a university with a high $\xi_{i,k} > 0$ ($x_{i,k} > \mu_k$) improves, it will increase the standard deviation, so that all universities deviate less from the mean: other universities with $\xi_{j,k} > 0$ will lose some of their good standing and lower tier ones with $\xi_{j,k} < 0$ will decrease their lag . Conversely, on can see that the improvement of a University with a lower rank $\xi_{j,k} < 0$, by decreasing the standard deviation of the sample when it goes closer to the mean, will help those with good standing to stand out more and harm other lower tier ones.

   For both effects combined, University j benefits if $\xi_{i,k}\xi_{j,k} < -1$ and is harmed otherwise.

To determine the additional funding fraction Δf_j, we propagate Eq. (4) into Eqs. (3a–3c):

$$\frac{\Delta f_j}{f_j} = c_k \left[\left(1 - \frac{y_j}{y}\right)\frac{\Delta y_{j,k}}{y_j} - \sum_{l\neq j} \frac{y_l}{y}\frac{\Delta y_{l,k}}{y_l}\right]. \quad (5)$$

The first term in the square brackets has the same sign as $\Delta y_{j,k}$ because, by definition, $y_j < y$. In most cases, the second term is smaller than the first one because the $\Delta y_{l,k}$ partially cancel out (some positives and some negatives) and $y_l < y$. It means that the funding received by a university that has an improved rating normally receives additional funding. It is possible, though, that a university with a very small $\Delta y_{j,k}$ (e.g. $\xi_{j,k} < -2$) will be harmed by increasing its score, because the other, larger, $\Delta y_{l,k}$ coould lead to a second term larger than the first term under these circumstances. In years 2006–2019 it has ocurred once, very marginally, in 2015, for Universidad de Talca. It would have received 1,000 CLP less had it substituted four regular professors with ones owning a postgraduate degree (improvement of $y_{\text{U. Talca},2015,3}$). It happened on that year that Universidad de Talca had the highest negative standard deviation observed for any metrics in the period 2006–2019 ($\xi \approx -2.8$).

1.2 Time evolution

1.2.1 Total funding

The total funding in year n, that we note F_n, is a slowly increasing series (see Fig. 1). In half of the years it approximately follows the consumer price index, but it has received a modest boost in other years. The average inflation-corrected increase has been 2.3% per year in period 2006–2019. This increases matches the increase in undergraduate students (+2.2% in 2006–2018), real wages (+2%?), and GDP per capita (+2.xx%?). Increase in standard of living and student population are long-term trends that I would expect to hold for at least the next decade, so we can safely assume that University funding by the State will still follow this trend.

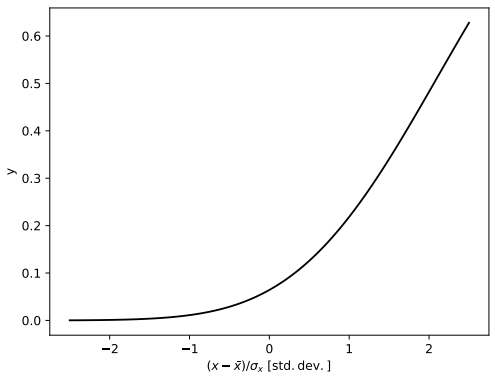

Figure 2: Transformation of the metric $x_{i,n,k}$ into $y_{i,n,k}$, before a weighted sum $\sum_k c_{i,n,k} y_{i,n,k}$ is performed to determine the rating of university i in year n.

For predictions, beyond 2019, I will therefore assume that

$$F_{n+1} = F_n(1+q) \tag{6}$$

where $q = 2\%$.

1.3 Time-evolution

Let $F_{i,n}$ be the funding received by university i at year n. Art. 2 of Decree with Force of Law 4 of 1980 indicates that 5% of the funding is indexed on metrics $y_{i,n,k}$ and 95% of the funding is related to the previous year's share of the total funding. So,

$$F_{i,n+1} = \left(\frac{19}{20} \frac{F_{i,n}}{F_n} + \frac{1}{20} \frac{y_{i,n}}{\sum_j y_{j,n}} \right) F_{n+1}. \tag{7}$$

1.4 Checks

Yearly evaluation I have checked the calculations of the 5% using open data from the Eduction Ministry for years 2006 to 2018. For each year since 2011 and 2007–2009, the percentages I derived (see Table 5 for 2018) match within numerical rounding errors (8 digits) with those of the Ministry. The subsidies I predict for each university differ by at most CLP 1,000 (USD 1.50) with the official ones due to rounding errors, as the accounting unit used in the official documents is 1,000 Chilean pesos. In 2010, the Ministry used the 2009 calculation with 2008 metrics, instead of 2009 ones, leading to large differences if the 2009 metrics given in the Ministry's spreadsheet is used. Difference are zero within rounding errors using 2008 data instead. In 2006, there is an unexplained 0.01% discrepancy between my determination and the Ministry's. The official file from the ministry with added columns showing my calculations are available from github project `https://github.com/loqueelvientoajuarez/afd`.[4]. The detail of calculations for year 2018 is given in Appendix. B. For that year, my calculations match exactly the Ministery's to the peso.

Time-evolution I have checked the recurrence formula Eq. (7) using the total amount given for each year F_n. The 95% funding is well predicted from year to year, except again for 2010, where I had to substitude 2008 funding percentages to the expected 2009 ones. Because of rounding errors cumulating from year to year, the amounts I predict for 2018 differ by up to 7,000 pesos (approx USD 10) with those of the Ministry.

Marginal earnings Marginal earnings have been determined by two methods. The first one, using differential calculus in Sect. 1.1.2, and the second one, by doing the full calculation with Eq. (7) using the new values of the coefficients. We have checked that both method agree within a few significant digits as long as the variations remain small.

2 Value of a paper

If an additional paper is published by a researcher of University i in year n, it will reflect in the 5% funding of year $n+1$. Let us call $\Delta F_{i,n+1}$ the additional earnings of the university in that year. In the subsequent years, it will reflect via the 95% (first term of the right handside of Eq. (7)) in this way:

$$\Delta F_{i,n+k} = \frac{19}{20} \Delta F_{n+k-1} F_{n+k} / F_{n+k-1},$$

so, using Eq. (6),

$$\Delta F_{i,n+k} = \Delta F_{n+k-1} \frac{19(1+q)}{20} \tag{8}$$

The additional funding obtained by the university in all years is therefore

$$\begin{aligned} \Delta F_i &= \sum_{k=1}^{+\infty} \Delta F_{i,n+k} \\ &= \sum_{k=1}^{+\infty} \frac{19(1+q)}{20} \Delta F_{i,n+1} \\ &= \frac{20}{1-19q} \Delta F_{i,n+1} \\ &\approx 32 \Delta F_{i,n+1}. \end{aligned} \tag{9}$$

The determination of $\Delta F_{i,n+1}$ is straightforward. The calculations in Eqs. (1a–7)) are done with the metrics provided by the Ministry (see Sect. 1.1.1) and for the same

[4]The original ministry file can be obtained from `http://dfi.mineduc.cl/usuarios/MECESUP/File/2018/instrumentos/AFD/AFD_2006_al_2018_MontosVariables5xc(1).xlsx` and my calculations from `https://github.com/loqueelvientoajuarez/afd/blob/master/src/tabla-afd.xlsx`

Table 2: Additional earnings in 2019 Chilean pesos for thes marginal improvement of 2018 metrics: an additional one-year contract of a postgraduate professor, an additional one-year research grant, and an additional Web of Science publication. 2019 funding is accurate to 1,000 pesos. The total funding assumed that the State funding remains constant in real terms, which it has never failed to be in the past 15 years. A research grant typically lasts 3 years and will carry the same level of funding for each year it is active. An additional tenure-track/tenured professor will bring as much funding as the years they stay hired.

universidad	postgraduate staff		research grant		WoS publication	
	2019 [CLP]	all years [CLP]	2019 [CLP]	all years [CLP]	2019 [CLP]	all years [CLP]
U.de Chile	303,000	6,060,000	952,000	19,040,000	517,000	10,340,000
P.U.Católica de Chile	330,000	6,600,000	981,000	19,620,000	531,000	10,620,000
U. de Concepción	1,098,000	21,960,000	1,104,000	22,080,000	640,000	12,800,000
U. Católica de Valparaíso	2,902,000	58,040,000	3,229,000	64,580,000	1,753,000	35,060,000
U. Téc. Federico Sta.Maria	505,000	10,100,000	2,397,000	47,940,000	1,378,000	27,560,000
U. de Santiago	316,000	6,320,000	960,000	19,200,000	418,000	8,360,000
U. Austral	1,128,000	22,560,000	1,286,000	25,720,000	798,000	15,960,000
U. Católica del Norte	302,000	6,040,000	818,000	16,360,000	964,000	19,280,000
U. de Valparaíso	657,000	13,140,000	1,014,000	20,280,000	357,000	7,140,000
U. de Antofagasta	1,285,000	25,700,000	916,000	18,320,000	1,102,000	22,040,000
U. de la Serena	227,000	4,540,000	1,008,000	20,160,000	1,376,000	27,520,000
U. de Bio Bio	4,045,000	80,900,000	1,108,000	22,160,000	669,000	13,380,000
U. de la Frontera	2,378,000	47,560,000	4,705,000	94,100,000	2,781,000	55,620,000
U. de Magallanes	2,847,000	56,940,000	3,108,000	62,160,000	3,023,000	60,460,000
U. de Talca	3,828,000	76,560,000	3,177,000	63,540,000	1,054,000	21,080,000
U. de Atacama	20,000	400,000	323,000	6,460,000	384,000	7,680,000
U. de Tarapacá	5,596,000	111,920,000	1,485,000	29,700,000	2,178,000	43,560,000
U. Arturo Prat	137,000	2,740,000	439,000	8,780,000	97,000	1,940,000
U. Metropolitana	1,960,000	39,200,000	272,000	5,440,000	89,000	1,780,000
U. de Playa Ancha	3,762,000	75,240,000	1,180,000	23,600,000	209,000	4,180,000
U.Tecnológica Metropolitana	451,000	9,020,000	519,000	10,380,000	250,000	5,000,000
U. de Los Lagos	886,000	17,720,000	723,000	14,460,000	227,000	4,540,000
U. Católica de Maule	1,918,000	38,360,000	544,000	10,880,000	343,000	6,860,000
U. Católica de Temuco	1,583,000	31,660,000	574,000	11,480,000	192,000	3,840,000
U. C.de la Sant.Concepción	712,000	14,240,000	571,000	11,420,000	325,000	6,500,000
U. de O'Higgins	22,842,000	456,840,000	12,518,000	250,360,000	8,501,000	170,020,000
U. de Aysén	81,972,000	1,639,440,000	64,904,000	1,298,080,000	9,284,000	185,680,000

Table 3: University earnings in 2018 Chilean pesos for an additional WoS paper in 2017, assuming that the total State funding increases 2% per year in real terms.

university	first year	all years
U. de Chile	493 000	15 903 000
P. U. Católica de Chile	497 000	16 032 000
U. de Concepción	612 000	19 742 000
U. Católica de Valparaíso	1 658 000	53 483 000
U. Téc. Fedérico Sta. María	1 328 000	42 839 000
U. de Santiago	407 000	13 129 000
U. Austral	673 000	21 710 000
U. Católica del Norte	930 000	30 000 000
U. de Valparaíso	456 000	14 710 000
U. de Antofagasta	1 172 000	37 806 000
U. de la Serena	969 000	31 258 000
U. de Bio Bio	600 000	19 355 000
U. de la Frontera	2 582 000	83 290 000
U. de Magallanes	1 107 000	35 710 000
U. de Talca	1 674 000	54 000 000
U. de Atacama	395 000	12 742 000
U. de Tarapacá	2 273 000	73 323 000
U. Arturo Prat	133 000	4 290 000
U. Metropolitana	152 000	4 903 000
U. de Playa Ancha	176 000	5 613 000
U. Tecnológica Metropolitana	334 000	10 744 000
U. de Los Lagos	266 000	8 581 000
U. Católica de Maule	317 000	10 226 000
U. Católica de Temuco	290 000	9 355 000
U. C. de la Sant. Concepción	213 000	6 871 000
U. de O'Higgins	8 565 000	276 290 000
U. de Aysén	7 096 000	228 903 000

ones with an additional publication. The difference in funding is $\Delta F_{i,n+1}$.

Table 3 gives the 2018 funding a University would have received, had an additional 2017 paper been published. I have made the hypothesis, that no other Traditional University has co-authored the paper, in which case the amount may vary.

My figures are much larger than those derived by Ramírez and Alfaro [2012]. The reasons are that (a) only consider the first five years after the paper is published while the half-life of the 95% dampening is 14 years, meaning that they underestimate the total revenue obtained with a paper by a factor of ≈ 4; (b) include an additional dampening of 8% per year that they do not justify and is not based on any kind of calculation by the Ministry, meaning that they underestimate the additional funding by a factor of ≈ 2.5 to 4;[5]; (c) use 2010 data, meaning that the monetary incentive is larger than the 2018 one by a factor of ≈ 2; and (d) seem to use different values for the coefficients than those retroactively published in 201. Their Fig. 4 doesn't match the corrected coefficients we derive for 2009, 2010, or 2011. Actually, both our data and Ministry's figures for years 2006 to 2017 show a systematic discrepancy between U. de Chile and P. U. Católica de Chile of the order of 25-35% in weighted sum of corrected coefficients and share of the 5%, while their Figure gives about 10%.

A Derivation of equation

The variation in $\Delta y_{j,k}$ is linked to $\Delta\xi_{i,k}$ via the derivative:

$$\Delta y_{j,k} \approx \frac{\partial\, y_{j,k}}{\partial x_{i,k}}\,\Delta x_{i,k}, \tag{10}$$

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\,\frac{\partial\,\xi_{i,k}}{\partial x_{i,k}}\,\sigma_k \Delta\xi_{i,k}, \tag{11}$$

so, substituting Eq. (2c), for the second factor

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\left[\frac{\partial\, x_{j,k}}{\partial x_{i,k}} - \frac{\partial\,\mu_k}{\partial x_{i,k}} - \frac{x_{j,k}-\mu_k}{\sigma_k}\frac{\partial\,\sigma_k}{\partial x_{i,k}}\right]\Delta\xi_{i,k} \tag{12}$$

and, backsubstituting Eq. (2c),

$$\approx \frac{\mathrm{d}\, y_{j,k}}{\mathrm{d}\xi_{i,k}}\left[\frac{\partial\, x_{j,k}}{\partial x_{i,k}} - \frac{\partial\,\mu_k}{\partial x_{i,k}} - \xi_{j,k}\frac{\partial\,\sigma_k}{\partial x_{i,k}}\right]\Delta\xi_{i,k}. \tag{13}$$

The first factor is the derivative of the function in the right handside of Eq. (2d). It is:

$$\frac{\mathrm{d}\, y_{i,k}}{\mathrm{d}\xi_{i,k}} = \frac{3}{4}\left[-\frac{7}{5} + \frac{\xi_{i,k}}{4}\right] y_{i,k}. \tag{14}$$

In the second factor, the first term is one if $i = j$, $x_{j,k}$ and $x_{i,k}$ being then the same variable, and zero otherwise.

[5] Quite the contrary, the constant increase of the total funding (consumer price index +2%) calls for an amplification of 2%

The second term is the variation of the mean when one of the term varies, it is therefore $1/N$ the variation of the individual term. So,

$$\frac{\partial x_{j,k}}{\partial x_{i,k}} = \delta_{ij}, \tag{15}$$

$$\frac{\partial \mu_k}{\partial x_{i,k}} = \frac{1}{N}\sum_j \frac{\partial x_{j,k}}{\partial x_{i,k}} = \frac{1}{N}\sum_j \delta_{ij} = \frac{1}{N}. \tag{16}$$

The last term requires some more calculation. We use Eq. (2b):

$$\frac{\partial \sigma_k}{\partial x_{i,k}} = \frac{\partial}{\partial x_{i,k}}\sqrt{\frac{1}{N}\Big(\sum_j x_{j,n,k}^2\Big) - \mu_{n,k}^2}, \tag{17}$$

$$= \frac{1}{2\sigma_k}\frac{\partial}{\partial x_{i,k}}\left[\frac{1}{N}\Big(\sum_j x_{j,k}^2\Big) - \mu_k^2\right], \tag{18}$$

$$= \frac{1}{2\sigma_k}\left[\frac{2}{N}\sum_k x_{j,k}\frac{\partial x_{j,k}}{\partial x_{i,k}} - 2\mu_k\frac{\partial \mu_k}{\partial x_{i,k}}\right], \tag{19}$$

so, using Eq. (15) and Eq. (16),

$$= \frac{1}{2\sigma_k}\left[\frac{2\xi_{i,k}}{N} - \frac{2\mu_k}{N}\right] \tag{20}$$

and, finally, with Eq. (2c),

$$= \frac{\xi_{i,k}}{N}. \tag{21}$$

B Direct state funding in 2018

Table 4 and 5 show the metrics used by the Ministry in 2018 and the calculation details for x_k and y_k.

Bibliography

References

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Table 4: Metrics used for the Direct State funding (aporte fiscal directo) of main Chilean Universities in 2018. U, the number of undergrad students; M, the number of majors; S, the number of (equivalent) full-time professors and researchers ("académico"); P, the number of (equivalent) full-time staff with post-graduate title; G, the number of research grants; P^{I}, the number of ISI publications; and P^{S}, the number of non-ISI publications indexed by the Scientif Electronic Library Online Chile.

University	U	M	S	P	G	P^{I}	P^{S}
U. de Chile	30480	77	2236.64	1499.84	855.5	2305	279
P. U. Católica de Chile	26767	76	2232.60	1508.94	763.0	2171	237
U. de Concepción	24666	90	1432.16	1129.67	388.0	1050	121
U. Católica de Valparaíso	14121	52	633.04	518.95	209.0	545	69
U. Téc. Fedérico Sta. María	15105	77	677.03	405.92	169.0	522	6
U. de Santiago	18645	68	1122.57	695.14	210.0	565	58
U. Austral	13218	60	911.62	628.02	184.0	534	66
U. Católica del Norte	10407	52	590.90	362.66	63.0	328	34
U. de Valparaíso	14737	60	873.13	557.72	120.0	409	42
U. de Antofagasta	6369	56	399.75	256.79	39.0	207	11
U. de la Serena	7084	41	370.42	209.56	28.0	165	14
U. de Bio Bio	11028	62	498.67	426.73	66.0	198	26
U. de la Frontera	9346	48	423.96	300.01	160.0	450	40
U. de Magallanes	2962	27	268.08	129.13	27.0	106	15
U. de Talca	9342	41	465.00	427.80	124.0	312	43
U. de Atacama	6359	71	317.73	138.05	5.0	78	3
U. de Tarapacá	8525	63	358.23	305.34	36.0	248	32
U. Arturo Prat	4326	39	441.08	227.30	16.0	52	18
U. Metropolitana	4548	24	325.96	212.83	3.0	33	8
U. de Playa Ancha	7747	52	421.98	309.35	30.0	65	18
U. Tecnológica Metropolitana	7970	36	297.30	175.73	13.0	61	5
U. de Los Lagos	4150	43	430.32	254.29	36.0	97	11
U. Católica de Maule	6955	28	405.88	281.93	22.0	95	24
U. Católica de Temuco	8404	57	492.29	340.62	42.0	125	26
U. C.de la Sant.Concepción	8844	31	497.69	285.65	24.0	107	11
U. de O'Higgins	0	0	34.86	29.03	4.0	14	0
U. de Aysén	0	0	15.35	11.28	1.0	3	0

Table 5: Calculation details for the 5% direct State funding (aporte fiscal directo) of main Chilean Universities in 2018.

University	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5	(%)	CLP
U. de Chile	396	0.561	13.6	0.033	0.671	0.062	0.382	0.512	1.072	0.475	10.43	1 220 349 000
P. U. Católica de Chile	352	0.421	12.0	0.021	0.676	0.066	0.342	0.406	1.007	0.411	8.76	1 025 067 000
U. de Concepción	274	0.204	17.2	0.081	0.789	0.217	0.271	0.241	0.761	0.198	6.39	748 010 000
U. Católica de Valparaíso	272	0.199	22.3	0.215	0.820	0.278	0.330	0.377	0.897	0.307	9.88	1 155 812 000
U. Téc. Fedérico Sta. María	196	0.072	22.3	0.216	0.600	0.023	0.250	0.200	0.774	0.207	5.25	614 754 000
U. de Santiago	274	0.205	16.6	0.071	0.619	0.030	0.187	0.105	0.520	0.072	2.32	271 561 000
U. Austral	220	0.103	14.5	0.042	0.689	0.078	0.202	0.124	0.610	0.109	3.09	362 052 000
U. Católica del Norte	200	0.077	17.6	0.088	0.614	0.028	0.107	0.036	0.574	0.092	2.03	237 356 000
U. de Valparaíso	246	0.145	16.9	0.075	0.639	0.040	0.137	0.056	0.484	0.060	1.87	218 799 000
U. de Antofagasta	114	0.016	15.9	0.060	0.642	0.042	0.098	0.032	0.527	0.074	1.73	202 772 000
U. de la Serena	173	0.049	19.1	0.121	0.566	0.013	0.076	0.022	0.458	0.052	1.49	173 877 000
U. de Bio Bio	178	0.054	22.1	0.209	0.856	0.359	0.132	0.052	0.414	0.041	4.75	555 201 000
U. de la Frontera	195	0.071	22.0	0.206	0.708	0.097	0.377	0.499	1.093	0.496	11.52	1 348 115 000
U. de Magallanes	110	0.015	11.0	0.016	0.482	0.003	0.101	0.033	0.414	0.041	0.84	97 927 000
U. de Talca	228	0.115	20.1	0.146	0.920	0.518	0.267	0.233	0.701	0.159	8.52	997 062 000
U. de Atacama	90	0.009	20.0	0.144	0.434	0.001	0.016	0.008	0.249	0.015	0.95	110 755 000
U. de Tarapacá	135	0.025	23.8	0.271	0.852	0.351	0.100	0.033	0.722	0.172	6.31	738 384 000
U. Arturo Prat	111	0.015	9.8	0.010	0.515	0.005	0.036	0.011	0.131	0.006	0.26	30 451 000
U. Metropolitana	190	0.065	14.0	0.036	0.653	0.049	0.009	0.007	0.109	0.005	0.70	81 804 000
U. de Playa Ancha	149	0.032	18.4	0.104	0.733	0.128	0.071	0.021	0.168	0.008	1.78	208 595 000
U. Tecnológica Metropolitana	221	0.105	26.8	0.400	0.591	0.020	0.044	0.013	0.211	0.011	2.38	278 301 000
U. de Los Lagos	97	0.011	9.6	0.010	0.591	0.020	0.084	0.025	0.234	0.013	0.56	66 056 000
U. Católica de Maule	248	0.151	17.1	0.080	0.695	0.083	0.054	0.016	0.254	0.015	1.39	162 712 000
U. Católica de Temuco	147	0.031	17.1	0.078	0.692	0.081	0.085	0.026	0.271	0.017	1.43	167 667 000
U. C.de la Sant.Concepción	285	0.231	17.8	0.092	0.574	0.015	0.048	0.014	0.222	0.012	0.89	104 634 000
U. de O'Higgins	0	0.001	0.0	0.000	0.833	0.307	0.115	0.041	0.402	0.038	3.17	370 823 000
U. de Aysén	0	0.001	0.0	0.000	0.735	0.130	0.065	0.019	0.195	0.010	1.29	150 972 000

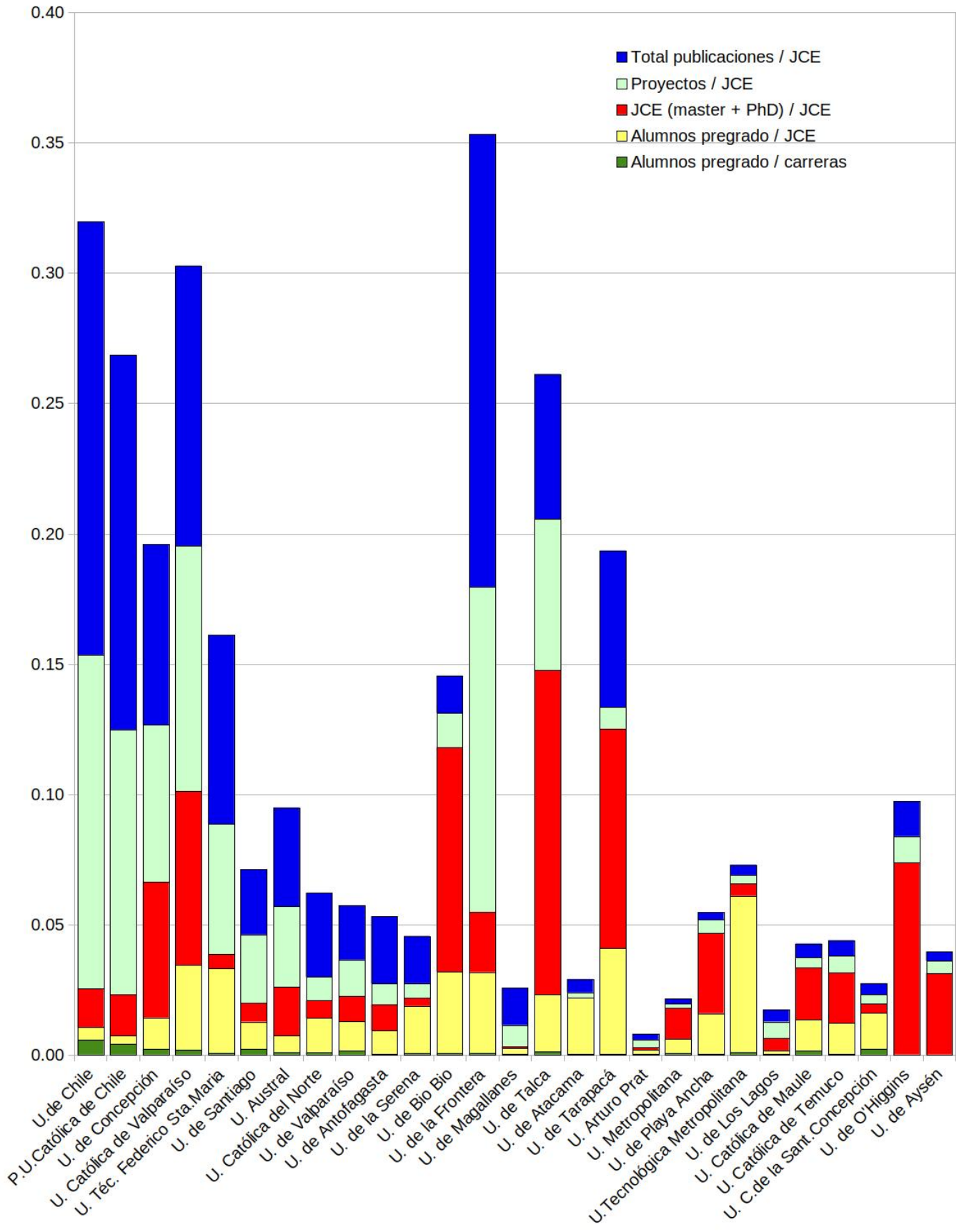

Figure 3: Graphical representation of the contributions of each AFD coefficient ($c_k y_k$) to each University's score.