Bang-Bang, PID and Full State Feedback Controller Evaluation

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***Abstract*— The objective of this study was to evaluate the performance of various controllers, including Bang-Bang, PID, and Full State Feedback controllers, in controlling a step and sine wave response system. The study aimed to determine the impact of different controller parameters on the system's response and to compare the performance of the different controllers. The results showed that full state feedback and PID control had the best response characteristics and the benefits and drawbacks of each depending on the most critical response characteristic.**

I. Introduction

THE goal of this lab is to implement and compare the effectiveness of various closed-loop control schemes, used to control the position of the motor platen system. Bang – bang, PID and full state feedback controllers were implemented, and their signal response characteristics were evaluated. This lab is also intended to provide intuition for the programming and tuning of these controllers.

The controllers were implemented using the class LabVIEW VI. The bang-bang controller was evaluated at a range of control efforts in response to a square and sine wave input signal, the output was expected to be less jerky at lower control efforts. The PID controller was first evaluated as just a proportional controller using various proportional gain constants, the controller response was again evaluated in response to square and sine wave inputs. The PID was then tuned for the fastest rise time at a set max overshoot in response to a square wave input.

Bang-bang controllers are some of the most basic control methods. The plant effort is set at a constant value and the direction of the plant effort is set by the sign of the system error. PID is the most used control method. PID controllers sum the error, integral of error and the derivative of the error, each scaled by its own gain constant to achieve a desired system response. The system response can be tuned by changing the gain values. A full state feedback controller is a modern control method that leverages a state space model of the system to achieve desired response characteristics. Ackermann's formula can then be used to pick the poles of the system to match desired output characteristics.

II. Procedure

LabVIEW Setup

The class LabVIEW VI was modified with the addition of a rotation error calculation, and a toggleable controller selector. Toggleable sine and square wave signal generators were used to generate the desired angle output of the system. The system rotation angle was controlled so the system error was calculated as the difference between desired and actual platen position. The integral and derivative values for the PID and full state feedback controllers were calculated using MathToolHigherOrder.vi. The controllers output a 10-bit signed integer value to control the motor. The class VI recorded data for time, desired platen angle, actual platen angle in degrees, error, and output command signal and platen angular velocity in degrees per second.

Hardware Setup

The hardware setup was identical to Lab1, the system was controlled by an encoded DC motor connected to a 9V motor controller which was controlled by a SADI DAQ. The motor torque was controlled by a 10-bit PWM signal which was output by the LabVIEW VI [1].

Controller Setup

Bang-bang control checked the sign of the system error, $e(t)$, to toggle a positive or negative constant motor effort, $u(t)$, with the only hyperparameter being the percent of maximum effort applied, c (1).

$$u(t) = \begin{Bmatrix} +c & e(t) > 0 \\ -c & e(t) < 0 \end{Bmatrix} \tag{1}$$

PID control calculates the control effort based on the magnitude, integral and derivative of the error, with each value scaled by a hyperparameter constant k_p, k_i, k_d (2).

$$u(t) = k_p e(t) + k_i \int e(t)dt + k_d \dot{e}(t) \tag{2}$$

The full state feedback controller states $x_1(t), x_2(t)$, were the error and derivative of error (3). The constants k_1 and k_2

were calculated using Ackermann's pole placement formula [2] and using a 2nd order desired response equation with a 2% settling time of 0.3 seconds and 5% overshoot.

$$u(t) = -[k_1 \quad k_2]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} \tag{3}$$

A system transfer function from Lab 1 was used to solve Ackermann's formula for optimal pole placement (4). The values of

$$\mathrm{H}(s) = \frac{1.83117}{0.029003s + 1} \tag{4}$$

Experimental Design

The signal generators were set to produce waves at 0.25Hz frequency and an amplitude of 113 degrees. The bang-bang controller was evaluated at effort levels of 25%, 50% and 100% for both the sine and square wave inputs separately, while allowing for at least four complete periods at each level.

The PID controller was first evaluated a with the integral and derivative gains set to 0, and only varying the proportional gain. Thus, the P-controller was evaluated the same as the bang-bang controller for both sine and square wave inputs separately, at proportional gain levels of 0.75, 2 and 5. The PID controller was then tuned for the fastest possible rise time with less than 10% overshoot. The tuned PID controller was then tested in response to the square wave input.

The full state feedback controller was implemented by setting the PID controller k_i to 0 and setting k_p and k_d equal to k_1 and k_2 respectively. The calculated gain values were $k_1 = 8.7676$ and $k_2 = 0.0803$. This controller was then tested using a square wave input.

Data Processing and Controller Evaluation

The collected data for the 14 total test conditions was processed in Python. One period of square or sine wave input was isolated for each test condition and the mean of the absolute value, standard deviation and root mean square of the error and command signal were calculated. The error was values aim to give insight into the effectiveness of the controllers at achieving the desired state, and the command signal into the amount of effort it took the system to achieve that performance.

For the square wave test conditions, the overshoot, 90% rise time and 2% settling time of the systems were also calculated.

III. Results

The error based statistical evaluations in Table I show the error performance of each controller in response to sine and square wave inputs respectively. Lower error metrics indicate that the system followed the desired system state more closely. Similarly, Table II shows the statistical evaluation of the motor command value which was output from the controllers. Higher command values indicate that it took more resources to achieve the system output effectively indicating the cost of each controller.

Rows showing the control effort, c, correspond with the bang-bang controller at the shown effort level. Rows with the controller k_p correspond with the P-controller with the integral and derivative gains set to 0 and the proportional gain set to the indicated value. PID and full state controllers were only evaluated for square wave inputs, so sine inputs were not applicable.

For 2nd order performance metrics, Table III, settling time values for the bang-bang controller was not applicable as the output was continuously oscillating even when a steady input was given, thus the system never settled. None of the P-controller tests overshot their set point so their overshoot values were 0. The P-controller with $k_p = 0.75$ never reached 90%, of its desired angle so the rise time and settling time metrics were not applicable. With $k_p = 2$, the system reached 90% of the desired output, but not 98% so the rise time metric was applicable, but the settling time metric was not.

TABLE I
ERROR PERFORMANCE METRICS OF CONTROLLERS

Controller	Mean (abs)		Standard Deviation		RMS	
	Square	Sine	Square	Sine	Square	Sine
c = 0.25	59.10	2.770	95.55	3.324	95.56	3.324
c = 0.5	35.80	4.337	71.57	5.068	71.67	5.069
c = 1.0	54.05	38.03	73.73	43.02	74.17	43.03
k_p = 0.75	101.2	70.64	111.2	78.46	111.2	8.60
k_p = 2	55.07	45.73	82.44	50.69	82.46	50.69
k_p= 5	23.63	22.02	57.58	24.44	57.59	24.44
PID	20.58	NA	56.13	NA	56.13	NA
Full state	19.39	NA	55.48	NA	55.49	NA

All units = degrees. NA = not applicable.

TABLE II
COMMAND PERFORMANCE METRICS OF CONTROLLERS

Controller	Mean (abs)		Standard Deviation		RMS	
	Square	Sine	Square	Sine	Square	Sine
c = 0.25	255.8	255.8	255.7	255.7	255.8	255.8
c = 0.5	511.5	511.5	511.4	511.4	511.5	511.5
c = 1.0	1023	1023	1022	1022	1023	1023
k_p = 0.75	75.59	52.74	83.04	58.58	83.06	58.68
k_p = 2	108.8	90.40	163.1	100.2	163.2	100.2
k_p= 5	114.8	107.0	253.6	118.7	253.6	118.7
PID	199.1	NA	548.2	NA	548.2	NA
Full state	160.9	NA	468.6	NA	468.6	NA

All units = PWM command units 0-1023. NA = not applicable.

TABLE III
2ND ORDER PERFORMANCE METRICS OF CONTROLLERS

Controller	Overshoot (deg)	90% Rise Time (sec)	Settling Time (sec)
c = 0.25	5.098	0.9124	NA
c = 0.5	13.22	0.4565	NA
c = 1.0	69.68	0.3495	NA
k_p = 0.75	0	NA	NA
k_p = 2	0	1.131	NA
k_p= 5	0	0.4484	0.6798
PID	2.227	0.2992	0.3303
Full state	0.9116	0.3088	0.3503

Units: deg = degrees, sec = seconds.

IV. Discussion

The goal of this lab was to implement and evaluate the effectiveness of bang-bang, P, PID, and full state feedback controllers.

Comparing the bang-bang and P-controller, the bang-bang controller showed the best mean error at 50% effort while the P-controller showed improving error with increasing k_p values. The bang-bang controllers showed high error at low control efforts due to the long response time, it also showed high mean error at high control efforts due to large oscillations when the system should have been at a steady state. The bang-bang controller effort, shown in Table II, was equivalent to the input effort as the system was always active at that effort level, the P-controller, while also showing increased mean command output, had significantly lower effort across all gain values.

With increasing effort level, the bang-bang controller showed greater overshoot, however, rise time was also greatly improved.

The P-controllers showed improved error metrics and response time at higher values of k_p at the cost of increase command values. The system did not become unstable at any gain values.

The PID and full state feedback controllers closely matched low error metrics at relatively high command performance compared to the P-controllers. The PID controller had about 1.3 degrees more overshoot, but slightly better rise and settling times than the full state feedback controller. The full state controller did however exhibit 20% better command performance than the PID controller. This indicates that the full state controller can achieve similar, or better performance metrics as a well-tuned PID controller, while requiring less energy to achieve it. The square wave response of the full state feedback controller shown in Figure 1 demonstrates the rapid response of the system with minimal overshoot.

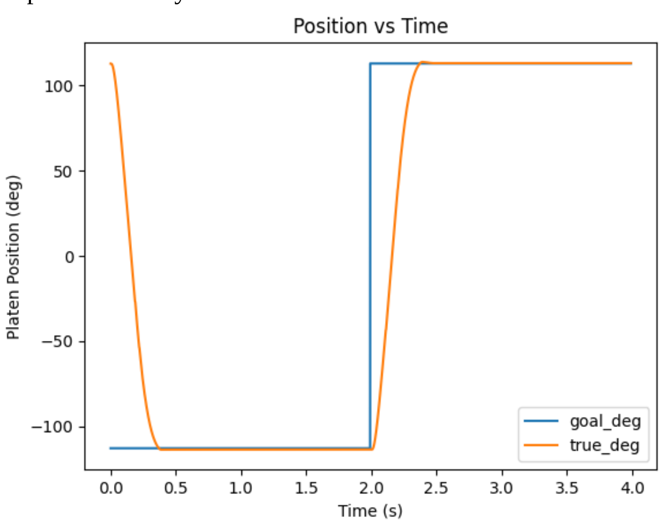

Fig. 1. Response of the full state feedback controller to a square wave input of 0.25Hz and an amplitude of 113 degrees. The system shows a rapid response with minimal overshoot.

V. Conclusion

The results show that full state feedback and PID controllers generally yield the best results for controlling the class motor platen system for desired platen angle. For future test, it would be beneficial to include a deadband for the bang-bang controller to better assess its settling time.

References

[1] Out of the Box ER. (2024, Jan 12). Pendulum v2.1 Assembly. [Online].
[2] EML 4314C Class Notes Spring 2024.