(1)(a)

P	Q	$(P \vee \sim Q) \Rightarrow Q$				
T	T	T	T	F	T	T
T	F	T	T	T	F	F
F	T	F	F	F	T	T
F	F	F	T	T	F	F

(b) THIS STATEMENT IS NOT VALID, BECAUSE IT IS NOT TRUE FOR EVERY VALUE OF P AND Q. IN THIS FORM OF LOGIC, A STATEMENT IS VALID IFF IT IS A TAUTOLOGY.

(C) THIS STATEMENT IS SATISFIABLE. IT IS NOT A CONTRADICT-ION, I.E. THERE EXIST VALUES OF P AND Q WHICH MAKE THE STATEMENT (CONDITIONALLY) TRUE.

(2)(a)

$$\exists x \in \mathbb{R} \ni p(x).$$

$p(x) :=$ "X IS NOT BEAUTIFUL."

(b) $\forall x \in \mathbb{R}, \neg(p(x))$.

(c) FOR EVERY REAL NUMBER X, IT IS NOT THE CASE THAT X IS NOT BEAUTIFUL.

(3) $\forall q \in \mathbb{Q}, \exists n \in \mathbb{Z} \ni q \in \mathbb{Z} \Rightarrow q = \frac{n}{5}$.

SUPPOSE THAT IF q IS AN INTEGER, THEN IT CAN BE EXPRESSED AS A RATIO OF INTEGERS WITH DENOMINATOR 5. SUPPOSE q IS AN INTEGER.

LET n BE SOME INTEGER, SO THAT WE HAVE

$$q = \frac{n}{5}.$$

THEN EQUIVALENTLY, WE HAVE

$$5q = n.$$

SUBSTITUTING FOR n, WE ~~SEE~~ SEE THAT

$$q = \frac{n}{5} = \frac{5q}{5}$$

SINCE THE PRODUCT OF INTEGERS IS ITSELF AN INTEGER, $5q$ IS AN INTEGER.

SO, THEREFORE, FOR ALL $q \in \mathbb{Z}$, q CAN BE DENOTED AS A FRACTION OF INTEGERS WITH DENOMINATOR 5 WHEN WE CHOOSE $5q$ AND 5 AS THOSE INTEGERS.

(4) $\forall q \in \mathbb{Q}, q = \frac{n}{5} \wedge n \in \mathbb{Z} \Rightarrow q \in \mathbb{Z}$.

SUPPOSE (BY CONTRADICTION) THAT q IS AN INTEGER. LET $n = 7$, SO THAT WE HAVE q EQUIVALENT TO $7/5$.

BUT IT IS ALSO THE CASE THAT

$$q = \frac{7}{5} = 1.4,$$

WHICH IS RATIONAL BUT NOT AN INTEGER.