Question 1

Normal order reduction

$$
\begin{aligned}
(\lambda xyz|xz(yyz))(\lambda x|x)(\lambda x|xy)a &= (\lambda x|(\lambda y|(\lambda z|xz(yyz))))(\lambda x|x)(\lambda x|xy)a \\
&= (\lambda y|(\lambda z|(\lambda x|x)z(yyz)))(\lambda x|xy)a \\
&= (\lambda z|(\lambda x|x)z((\lambda x|xy)(\lambda x|xy)z))a \\
&= (\lambda x|x)a((\lambda x|xy)(\lambda x|xy)a) \\
&= a((\lambda x|xy)(\lambda x|xy)a) \\
&= a(((\lambda x|xy))a) \\
&= a(((y)y)a) \\
&= a(yya)
\end{aligned}
$$

Applicative order reduction

$$
\begin{aligned}
(\lambda xyz|xz(yyz))(\lambda x|x)(\lambda x|xy)a &= (\lambda yz|(\lambda x|x)z(yyz))(\lambda x|xy)a \\
&= (\lambda yz|z(yyz))(\lambda x|xy)a \\
&= (\lambda z|z((\lambda x|xy)(\lambda x|xy)z))a \\
&= (\lambda z|z(((\lambda x|xy)y)z))a \\
&= (\lambda z|z(((y)y)z))a \\
&= a(((y)y)a) \\
&= a(yya)
\end{aligned}
$$

Question 2

(a)

$$
\begin{aligned}
x \text{ OR } y &= (\lambda xy|x\text{T}y) \\
\text{NOT } x &= (\lambda x|x\text{FT})
\end{aligned}
$$

We can intuitively determine the logic behind XOR as `xor(x, y) = {x ? not y : y}`. Knowing this, we can formulate the following lambda expression:

$$
\begin{aligned}
x \text{ XOR } y &= (\lambda xy|(x(\lambda y|y\text{FT})y)) \\
&= (\lambda xy|(x(y\text{FT})y)) \\
&= (\lambda xy|x(y\text{FT})y)
\end{aligned}
$$

(b)
- $$\begin{aligned}\text{XOR T F} &= (\lambda xy|x(y\text{FT})y)\\ &= \text{T(FFT)F}\\ &= \text{TTF}\\ &= \text{T}\end{aligned}$$

- $$\begin{aligned}\text{XOR T T} &= (\lambda xy|x(y\text{FT})y)\\ &= \text{T(TFT)T}\\ &= \text{TFT}\\ &= \text{F}\end{aligned}$$

Question 3

(a) `(((lambda (x) (lambda (y) (+ (* 2 x) y))) 5) 3)`

- `(((lambda (x) (lambda (y) (+ (* 2 x) y))) 5) 3)`
- `((lambda (x) (lambda (y) (+ (* 2 x) y))) 5)`
- `(lambda (x) (lambda (y) (+ (* 2 x) y)))`
- `(lambda (y) (+ (* 2 x) y))`
  $\text{CT1} = [x \to 5] \cup \text{CT0}$
  $[e, \text{CT1}] = [$`(lambda (y) (+ (* 2 x) y))`$, \text{CT1}]$
- `(+ (* 2 x) y)`
  $\text{CT2} = [x \to 5] \cup [y \to 3] \cup \text{CT0} = [x \to 5, y \to 3]$
  $[e1, \text{CT2}] = [$`(+ (* 2 x) y)`$, \text{CT2}]$
- `(+ (* 2 x) y)`
- `(+ (* 2 5) 3)` $= 13$

Result: 13
Last context: $\{x \to 5, y \to 3\} \cup \text{CT0}$

(b) `((lambda (x y) (x y)) (lambda (x) (+ 1 x)) 8)`

- `(lambda (x) (+ 1 x))` in $[x \to$ `(lambda (x) (+ 1 x))`$, y \to 8] \cup \text{CT0 CT1} = \text{CT0}$
  $[e, \text{CT1}] = [$`(+ 1 x)`$, \text{CT1}]$

- `(lambda (x y) (x y))`
  $[e1, \{\}] =$ [`(+ (* 2 x) y)`, $\{\}] =$ [lambda (x) (+ 1 x), x $\rightarrow$ [lambda (x) (+ 1 x), $\{\}$]]
- `(+ 1 x)` with CT2 $= \{x \rightarrow 8\} \cup$ CT0
- `(+ 1 8)` $= 9$

Result: 9

Last context: $\{x \rightarrow 8\} \cup$ CT0