Part A

Q1.

Normal order reduction:

$$
\begin{aligned}
&(\lambda xyz \mid xz(yyz))(\lambda x \mid x)(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda x \mid (\lambda yz \mid xz(yyz)))(\lambda x \mid x)(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda x \mid (\lambda y \mid (\lambda z \mid xz(yyz))))(\lambda x \mid x)(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda y \mid (\lambda z \mid (\lambda x \mid x)z(yyz)))(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda z \mid (\lambda x \mid x)z((\lambda x \mid xy)(\lambda x \mid xy)z))\ a \\
&\rightarrow (\lambda x \mid x)a((\lambda x \mid xy)(\lambda x \mid xy)a) \\
&\rightarrow a((\lambda x \mid xy)(\lambda x \mid xy)a) \\
&\rightarrow a(((\lambda x \mid xy)y)a) \\
&\rightarrow a(yya)
\end{aligned}
$$

Applicative order reduction:

$$
\begin{aligned}
&(\lambda xyz \mid xz(yyz))(\lambda x \mid x)(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda x \mid (\lambda yz \mid xz(yyz)))(\lambda x \mid x)(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda yz \mid (\lambda x \mid x)z(yyz))(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda yz \mid z(yyz))(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda y \mid (\lambda z \mid z(yyz)))(\lambda x \mid xy)\ a \\
&\rightarrow (\lambda z \mid z((\lambda x \mid xy)(\lambda x \mid xy)z))a \\
&\rightarrow (\lambda z \mid z(((\lambda x \mid xy)y)z))a \\
&\rightarrow (\lambda z \mid z((yy)z))a \\
&\rightarrow a(yya)
\end{aligned}
$$

Q2.

(a) Knowing XOR has the following logic

```
XOR (a, b):
  if a:
    return not(b)
  else:
    return b
```

we can combine this with the lambda expression given for NOT and OR to obtain

$$
\begin{aligned}
&x \text{ XOR } y \\
&\to (\lambda xy \mid x(\text{NOT } y)y) \\
&\to (\lambda xy \mid x(\lambda y \mid yFT)y) \\
&\to (\lambda xy \mid x(yFT)y)
\end{aligned}
$$

(b)
- For XOR TF,

$$
\begin{aligned}
&(\lambda xy \mid x(yFT)y) \\
&\to T(FFT)F \\
&\to T(T)F \\
&\to T
\end{aligned}
$$

- For XOR TT,

$$
\begin{aligned}
&(\lambda xy \mid x(yFT)y) \\
&\to T(TFT)T \\
&\to T(F)T \\
&\to F
\end{aligned}
$$

Q3.

(a) The result of evaluating the expression is 13 and the last context created during evaluation is $\{x \to 5, y \to 3\}\cup$ CT0

(i) Evaluate (lambda (x) (lambda (y) (+ (* 2 x) y))) 5
- CT1 = $\{x \to 5\}\cup$ CT0
- Then, [F1, CT1] = [lambda (y) (+ (* 2 x) y), CT1] where F1 = (lambda (y) (+ (* 2 x) y))

(ii) Evaluate outer expression with [F1, CT1]: (F1 3)
- CT2 = $\{y \to 3\}\cup$ CT1 = $\{x \to 5, y \to 3\}\cup$ CT0
- Then, [F2, CT2] = [(+ (* 2 x) y), CT2] where F2 = (+ (* 2 x) y)

(iii) Evaluate [F2, CT2]: (F2)
- F2 evaluates to (+ (* 2 x) y) with CT2 = $\{x \to 5, y \to 3\}\cup$ CT0

(iv) Finally,
- (+ (* 2 x) y) = (+ (* 2 5) 3) = 13

(b) The result of evaluating the expression is 9 and the last context created during evaluation is $\{x \to 5, y \to 3\}\cup$ CT0

(i) Evaluate (lambda (x) (+ 1 x))

- CT1 = CT0
- Then, [F1, CT1] = [(+ 1 x), CT1] where F1 = (+ 1 x)

(ii) Evaluate the second argument 8

(iii) Evaluate outer expression [F1, CT1] and 8: (F1 8)

- CT2 = $\{x \rightarrow 8\} \cup$ CT1 = $\{x \rightarrow 8\} \cup$ CT0
- Then,[F2, CT2] = [(+ 1 x), CT2] where F2 = (+ 1 x)

(iv) Evaluate [F2, CT2]: (F2)

- F2 evaluates to (+ 1 x) with CT2 = $\{x \rightarrow 8\} \cup$ CT0

(v) Finally,

- (+ 1 x) = (+ 1 8) = 9