Physics 230 Notes

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Chapter 1

Electric Charge and Electric Field

1.1 Electric Charge

Electromagnetism (EM) affects only charged particles, mainly electrons and protons. All particles have charges that are integer multiples of the elementary charge e such that the charge is given by

$$q = ne, \tag{1.1}$$

where q represents charge (C), n represents an integer, and $e = 1.6022 \times 10^{-19}$ C represents the elementary charge.

Electric charge is conserved. This means that the total charge of any isolated system with no charge moving in or out stays the same – charge is never created or destroyed.

Coulomb's Law states that charges of the same sign repel and charges of the opposite sign attract. Furthermore, the force F produced by charges can be calculated via

$$F = k\frac{q_1 q_2}{r^2}, \tag{1.2}$$

where $k = 9.0 \times 10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}$, q_1 represents the first charge, q_2 represents the second charge, and r represents the separation distance. Note that k can also be representented as

$$k = \frac{1}{4\pi\epsilon_0}. \tag{1.3}$$

The **principle of superposition of forces** is the vector sum of the individual forces:

$$F = k\frac{q_1 q_2}{(r_{12})^2}r_{12} + k\frac{q_1 q_2}{(r_{13})^2}r_{13} = q_1\vec{E}. \tag{1.4}$$

1.2 Electric Field and Electric Forces

The **electric field of a point charge** at $\vec{r}$ is given by

$$\vec{E}(\vec{r}) = \frac{kq}{r^3}\vec{r} = \frac{\vec{F}}{q}, \tag{1.5}$$

where q denotes the charge of the point source (C), r denotes the radial distance of the point charge from the origin, and $\vec{r}$ represents the displacement vector of the point from the origin. We can also calculate the magnitude of the eletric field via

$$E = k\frac{|q|}{r^2}. \tag{1.6}$$

The **electric field of a group of charges** is the superposition of all the electric forces from all the charges. This can be approximated with volume charge density (ρ) via

$$\vec{E}(\vec{r_o}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r})(\vec{r_0}-\vec{r})}{|\vec{r_0}-\vec{r}|^3}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z, \tag{1.7}$$

where $\epsilon_0 = 8.854 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m}^2}$, $\rho(\vec{r})$ is the surface charge density, and $\vec{r}$ is the radius.

For **a conductor without current flow**, the charge all resides on the surface. Using the surface charge density $\sigma(\vec{r})$, we can approximate the electric field via

$$\vec{E}(\vec{r_o}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r})(\vec{r_0}-\vec{r})}{|\vec{r_0}-\vec{r}|^3}\,\mathrm{d}A. \tag{1.8}$$

If one has a **thin wire** where all the charge resides, with linear charge density $\lambda(\vec{r}) = \frac{\mathrm{d}q}{\mathrm{d}l}$, where $\mathrm{d}l$ is the element of length along the wire, then

$$\vec{E}(\vec{r_o}) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\vec{r})(\vec{r_0}-\vec{r})}{|\vec{r_0}-\vec{r}|^3}\,\mathrm{d}l. \tag{1.9}$$

Example. For a ring of total charge Q on a circle of radius a, the circumference is $2\pi a$ and therefore the linear charge density is $\lambda = \frac{Q}{2\pi a}$. The electric field is therefore

$$\begin{aligned}
\vec{E}(\vec{r_0}) &= \frac{1}{4\pi\epsilon_0}\int \frac{\vec{r_0}-\vec{r}}{|\vec{r_0}-\vec{r}|^3}\lambda(\vec{r})\,\mathrm{d}l = E_x\hat{i} && (1.10)\\
&= \frac{1}{4\pi\epsilon}\frac{Qx}{(x^2+a^2)^{3/2}}\hat{i} && (1.11)\\
&= \frac{Q\hat{i}}{4\pi\epsilon_0 x^2} \quad \text{(For } x >> a\text{)}. && (1.12)
\end{aligned}$$

1.3 Electric Field Lines

Electric field lines have tangent vectors parallel to the electric field and begin only on positive charges and end only on negative charges, though they can also go to infinity in either direction.

1.4 Electric Dipoles

An **electric dipole** is a pair of point charges of equal magnitude and opposite sign (a positive charge q and a negative charge $-q$ separated by a distance d.

An **electric dipole moment** is the product of the positive charge q and the displacement d it is separated from the negative charge $-q$, given by

$$\vec{p} = q\vec{d}. \tag{1.13}$$

Using the volume charge density formula $\rho(\vec{r}) = \rho(x\vec{i} + y\vec{j} + z\vec{k}) = \rho(x, y, z)$, we can approximate the electric dipole of a huge number of elementary charges (total charge is neutral) via

$$\vec{P} = \int \rho(\vec{r})\vec{r}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z. \tag{1.14}$$

The total force on an electric dipole is just the net force from an external electric field (the electric field from all the other charges that are not part of the electric dipole). This is true for the total torque.

In a uniform electric field $\vec{E}$, the net force on a electric dipole is 0. However, if the electric dipole monent vector $\vec{p}$ is not parallel to $\vec{E}$, then a torque is exerted on the dipole that changes its angular momentum $\vec{L}$ by $\frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \vec{\tau}$. This vector torque is calculated via

$$\vec{\tau} = \vec{P} \times \vec{E}, \tag{1.15}$$

where $\vec{P}$ is the electric dipole moment, $\vec{E}$ is the electric field, and the direction of τ is perpendicular to both $\vec{P}$ and $\vec{E}$. The magnitude of torque can also be found via

$$\tau = pE\sin\phi, \tag{1.16}$$

where p is the magnitude of the electric dipole moment $\vec{p}$, E is the magnitude of the electric field $\vec{E}$, and ϕ is the angle between $\vec{p}$ and $\vec{E}$.

To calculate the potential energy of a dipole, we use

$$U = -\vec{p} \cdot \vec{E}. \tag{1.17}$$

where $\vec{p}$ is the electric dipole moment and $\vec{E}$ is the electric field.

We can approximate the field of an electric dipole at $r >> d$ with binomial expansion using

$$\vec{E}(\vec{r}) \approx \frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{4\pi\epsilon_0 r^3}, \tag{1.18}$$

where $\vec{p}$ is the electric dipole moment, $\hat{r}$ is the unit vector in the direction or $\vec{r}$, and $\epsilon_0 = 8.854 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2}$ (we probably don't need to know this formula for the exam tbh).

1.5 Gauss's Law

Gauss's law states that the outward flux of the electric field through a closed surface is equal to the total charge inside the surface divided by the electric constant, ϵ_0:

$$\Phi_E \equiv \oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q_{encl}}{\epsilon_0}. \tag{1.19}$$

Let us examine what this equation means.

We define the **mass flux** through a surface via

$$\Phi_M = \frac{\mathrm{d}M}{\mathrm{d}t}, \tag{1.20}$$

where M is the mass and t is time. The surface that we pick to evaluate the flow through is sometimes called a *Gaussian surface*. We also always consider a *stationary* situation, where things do not change with time at a given location. Flux is essentially anything measurable that can be transported (ex. mass, momentum, energy, heat, etc.). The **mass flux** through a surface can be calculated with a surface integral via

$$\Phi_M = \int \rho_m \vec{v} \cdot \mathrm{d}\vec{A}, \tag{1.21}$$

where ρ_m is the mass density and $\vec{V}$ is the velocity. For a tilted surface, the mass flux can be calculated as

$$\Phi_M = \rho_M \vec{v} \cdot \vec{A}. \tag{1.22}$$

When no mass is created or destroyed, we have that

$$\Phi_M = \frac{-\,\mathrm{d}M_{encl}}{\mathrm{d}t}. \tag{1.23}$$

Note that for any *closed surface*, the mass flux is always zero if the flow is stationary and if there is no creation of air, i.e.,

$$\Phi_M = 0. \tag{1.24}$$

This is analogous to Gauss's law for a closed surface with no charges inside, i.e.,

$$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0} = 0. \tag{1.25}$$

Mass creation is when:

$$\Phi_M + \frac{\mathrm{d}M}{\mathrm{d}t} > 0, \tag{1.26}$$

and mass destruction is when

$$\Phi_M + \frac{\mathrm{d}M}{\mathrm{d}t} < 0. \tag{1.27}$$

Conservation of charge states that

$$\Phi_Q + \frac{\mathrm{d}Q}{\mathrm{d}t} = 0. \tag{1.28}$$

Second Law of thermodynamics states that

$$\Phi_S + \frac{\mathrm{d}S}{\mathrm{d}t} \geq 0. \tag{1.29}$$

Gauss's law and spherical symmetry. Using coulomb's law for one charge

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^2} \tag{1.30}$$

For more than one charge, you need to do superposition

Every region totally inside a conductor is neutral. Excess charge all lies on the surface (static). An interior surface of a conductor, surrounding a cavity, only has charge if there is non-zero charge inside the cavity; the two charges cancel

Spherical symmetry

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Electric Potential

Chapter 3

Electric Potential

Chapter 4

Capacitance and Dielectrics

Chapter 5

Current, Resistance, and Electromotive Force

Chapter 6

Direct-Current Circuits

Now we shall look in more detail at electrical circuits, which have a network of conductors and other devices connected together. Generally, the conductors can be idealized as thin wires, each of which carries a constant current in a stationary situation, and junctions where all the incoming currents get divided up into all the outgoing currents, with no charge of either sign building up at the junction. This was given by **Kirchhoff's junction rule** or **current law** or **first law: Sum of currents into a junction is zero**.

Since in a stationary situation the electric force is a **conservative force**, with a **potential** (potential energy per charge, measured in the SI unit of $1\,\mathrm{V} = 1\,\mathrm{J/C}$) that is defined up to one overall additive constant (so that the potential differences or voltages between two points are uniquely defined), we get **Kirchhoff's loop rule** or **voltage law** or **second law: the algebraic sum of potential differences around any closed loop is zero** (the algebraic sum means keeping track of signs. If we have a potential increase from a to b, so $V_b - V_a = V^{ba} = -V^{ab} > 0$, the potential difference is positive; if $V_b < V_a$, $V_b - V_a = V^{ba} = -V^{ab} < 0$, this is negative).

Now let us use Kirchhoff's rules to get the effective resistance R_{eff} of N resistors, of resistances R_i with i running from 1 to N, that is, $R_1, R_2, \ldots, R_N$, when the resistors are connected in **series** and in **parallel**. In series, we have

$$R_{\text{eff}} = R_{\text{eq}} = \sum R_i = R_1 + R_2 + \cdots + R_N. \tag{6.1}$$

In parallel, we have

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}. \tag{6.2}$$

A wire of cross section A can be considered as a parallel arrangement of wires of cross sections adding up to A, which fits $\frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma}{L} A$, adding up the A's.

Chapter 7

Magnetic Field and Magnetic Forces

Chapter 8

Sources of Magnetic Field

Chapter 9

Electromagnetic Induction

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Inductance

Chapter 11

Electromagnetic Waves