The Principle of Mathematical Induction

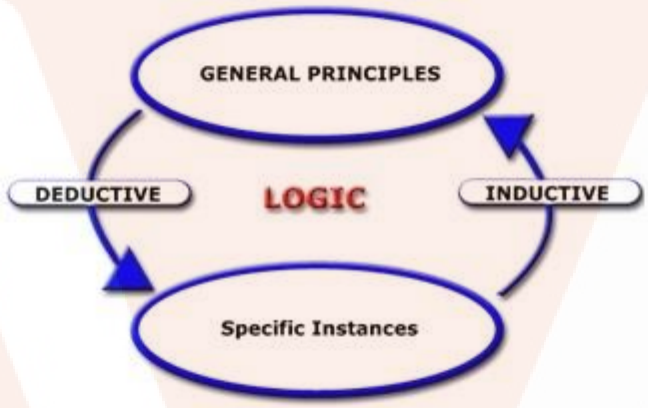
Deduction: Generalization of Specific Instance

Example : Rohit is a man & All men eat food, therefore, Rohit eats food.

Induction: Specific Instances to Generalization

Example : Rohit eats food. Vikash eats food. Rohit and Vikash are men. Then, All men eat food

Statement is true for n=1, n=k & n=k+1, then, the Statement is true for all natural numbers n.



Steps of Principle of Mathematical Induction:

Step 1: Let *P(n)* be a result or statement formulated in terms of n(given question).
Step 2: Prove that *P(1)* is true
Step 3: Assume that *P(k)* is true
Step 4: Using Step 3, prove that *P(k+1)* is true
Step 5: Thus *P(1)* is true and *P(k+1)* is true whenever *P(k)* is true.
Hence, by the Principle of Mathematical Induction, *P(n)* is true for all natural numbers n.

Example: Prove that $2^n > n$ for all positive integers n
Solution:
Step 1: Let P(n): $2^n > n$
Step 2: When n =1, $2^1 > 1$. Hence P(1) is true.
Step 3: Assume that P(k) is true for any positive integer k, i.e., $2^k > k$ ... (1)
Step 4: We shall now prove that P(k +1) is true whenever P(k) is true.
Multiplying both sides of (1) by 2, we get
$2 * 2^k > 2 * k$
i.e., $2^{k+1} > 2k$
or, $2^{k+1} > k + k$
or, $2^{k+1} > k + 1$ (since k>1)
Therefore, P(k + 1) is true when P(k) is true. Hence, by principle of mathematical induction, P(n) is true for every positive integer n.