AQA AS Further Maths: Practice Paper 1

Focus: Pure

Difficulty: Hard

Time: 2 hours 30 minutes

Marks: (Recommended Timings)

Section A (multiple choice): 10 marks (15 minutes)

Section B (standard questions): 70 marks (1 hour 30 minutes)

Section C (extended question): 20 marks (45 minutes)

(Total 100 marks)

Grade Boundaries: (approximate)

A: 70 (70%)

B: 60 (60%)

C: 50 (50%)

D: 40 (40%)

E: 30 (30%)

Main Topics Examined:

Hyperbolic Functions, Proof by Induction, Graphs of Rational Functions, Complex Numbers, Polynomials, Further Vectors

Advice:

1. The questions in each section are in no particular order.
2. Questions are at A-level standard difficulty, with many above.
3. You may wish to take a short break between Sections B and C.
4. Plan your answers to Section C carefully before beginning.
5. Check the fully worked solutions for any questions you missed.

Section A: Multiple choice. You are advised to spend no more than **15 minutes** in Section A.

1. A student attempts to use proof by induction to show that $n^2 - n$ is odd for all $n \in \mathbb{R}$. They argue as follows:

> Assume true for $n = k$, where k is a positive integer.
>
> For $k + 1$, we have $(k + 1)^2 - (k + 1) = k^2 + 2k + 1 - k - 1$
> $= k^2 + k$
> $= k^2 - k + 2k$
> which must be odd, since $k^2 - k$ is assumed to be odd and $2k$ is even.
>
> Therefore, true for $n = k \Rightarrow$ true for $n = k + 1$.
> Hence by induction, $n^2 - n$ is odd for all positive integers n.

What mistake has the student made in this argument?

O It was assumed that $k^2 - k$ is odd before it was proven
O The general form for an odd integer, i.e. $n = 2k + 1$, was not used
O The argument is incomplete since there is no established base case
O There is an algebraic error in the inductive step

[1 mark]

2. The polynomial $P(x)$ is given by $P(x) = x^2 + 5x + n$, where n is an integer.

The smallest value of n such that $P(x)$ has complex roots is

O 6
O 25/4
O 7
O 25

[1 mark]

3. The circle C is defined as the plot of all z satisfying $\left|z-\sqrt{3}-i\right|=1$ on an Argand diagram.

Which of the following is true?

- O $|z|$ takes its minimum value on C when $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$.
- O $\arg z$ takes its maximum value on C when $z=\frac{\sqrt{3}}{2}-\frac{3}{2}i$.
- O For all z on C, $\frac{3}{2}\le|z|\le\frac{7}{2}$.
- O For all z on C, $-\frac{\pi}{6}\le\arg z\le\frac{\pi}{3}$. [1 mark]

4. The matrices **A** and **B** are defined in terms of a real parameter t by

$$\mathbf{A}=\begin{pmatrix}1 & 2 & 1\\ 2 & t & 4\\ 3 & 2 & -1\end{pmatrix} \text{ and } \mathbf{B}=\begin{pmatrix}15 & -4 & -1\\ -2t & 4 & 2\\ 17 & -4 & -3\end{pmatrix}$$

Which of these is true?

- O The determinant of **A** is proportional to t.
- O When t = -1 and t = 4, **AB** = 3(**A** + **B**).
- O When t = 7, **AB** = 4**I**.
- O For all t, det(**AB**) > det(**A**) det(**B**). [1 mark]

5. The quadratic equation $z^2 + iz - 1 = 0$ has complex roots ω and σ.

Which of these is true?

- O $\omega^* = \sigma$
- O $\omega + \sigma = i$
- O $\omega\sigma = -1$
- O $\omega^2 + \sigma^2 = 2 - i$ [1 mark]

6. The roots of the equation $x^4 - px^3 + qx^2 - pqx + 1 = 0$ are α, β, γ and δ.

What is the value of $(\alpha + \beta + \gamma)(\alpha + \beta + \delta)(\alpha + \gamma + \delta)(\beta + \gamma + \delta)$?

O -1
O 1
O *pq*
O p^2q^2 [1 mark]

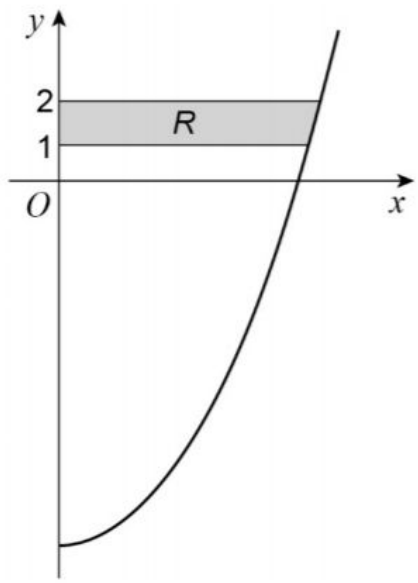
7. The coefficient of x^3 in the Maclaurin series expansion of ln(1 - *x*) is

O 1/3
O -1/3
O 1/6
O -1/6 [1 mark]

8. Which of these is **not** an asymptote to the curve $y = \frac{1}{x^2 - 4}$?

O $x = 0$
O $y = 0$
O $x = 2$
O $x = -2$ [1 mark]

9. The diagram shows the curve $y = 18x^2 - 9$ for $x \geq 0$.



A solid is formed when the region R is rotated through 360° about the y-axis.

The volume of this solid is $k\pi$, where the value of k is

O $\frac{7}{6}$

O $\frac{7}{12}$

O 33

O $\frac{33}{4}$ [1 mark]

10. The coordinates of the points A and B are $(3, -2, 1)$ and $(5, 3, 0)$ respectively.

The line l has equation $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} - 3\mathbf{k})$.

The acute angle between l and the line AB, to 3 significant figures, is

O 16.8°
O 33.6°
O 56.4°
O 73.2° [1 mark]

Section B: Standard questions. Ensure to leave sufficient time for Section C.

11. Matthew is finding a formula for the inverse function arsinh *x*. He writes his steps as follows:

$$\text{Let } y = \sinh x$$

$$y = \frac{1}{2}(e^x - e^{-x})$$

$$2y = e^x - e^{-x}$$

$$0 = e^x - 2y - e^{-x}$$

$$0 = (e^x)^2 - 2ye^x - 1$$

$$0 = (e^x - y)^2 - y^2 - 1$$

$$y^2 + 1 = (e^x - y)^2$$

$$\pm\sqrt{y^2+1} = e^x - y$$

$$y \pm \sqrt{y^2+1} = e^x$$

To find the inverse function, swap x and y: $x \pm \sqrt{x^2+1} = e^y$

$$\ln\left(x \pm \sqrt{x^2+1}\right) = y$$

$$\text{arsinh}\, x = \ln\left(x \pm \sqrt{x^2+1}\right)$$

a. Identify, and explain, the error in Matthew's proof. [2 marks]

b. Solve ln $(x + \sqrt{x^2 + 1}) = 3$. [1 mark]

c. Derive a similar formula for arcosh *x*. Fully justify your answer. [6 marks]

[Total for Q11: 9 marks]

12.

a. Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[4 marks]

b. **Hence** show that

$$\sum_{r=1}^{2n} r(r-1)(r+1) = n(n+1)(2n-1)(2n+1)$$

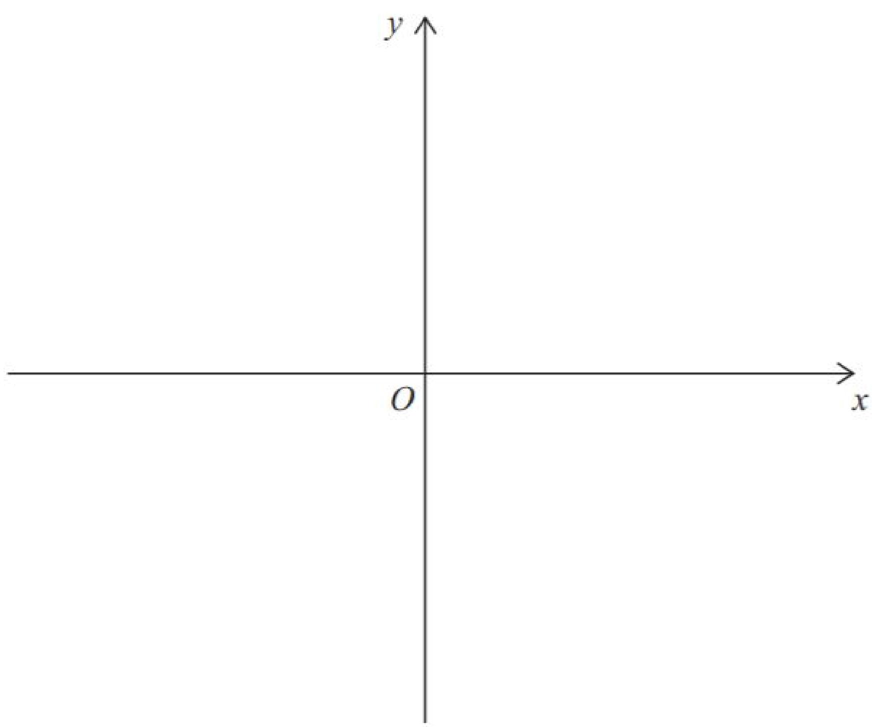
[4 marks]

[Total for Q12: 8 marks]

13. The graph of the rational function $y = f(x)$ intersects the x-axis exactly once at $(-3, 0)$. The graph has exactly two asymptotes, $y = 2$ and $x = 1$.

a. Find $f(x)$. [2 marks]

b. Sketch the graph of the function. [3 marks]



c. Find the range of values of x for which $f(x) \leq 5$. [4 marks]

[Total for Q13: 9 marks]

14. There is a unique complex number w that satisfies both

$$|w - 3| = 2 \quad \text{and} \quad \arg(w + 1) = \alpha$$

where α is a constant such that $0 < \alpha < \pi$.

a. i) Find the value of α. [2 marks]

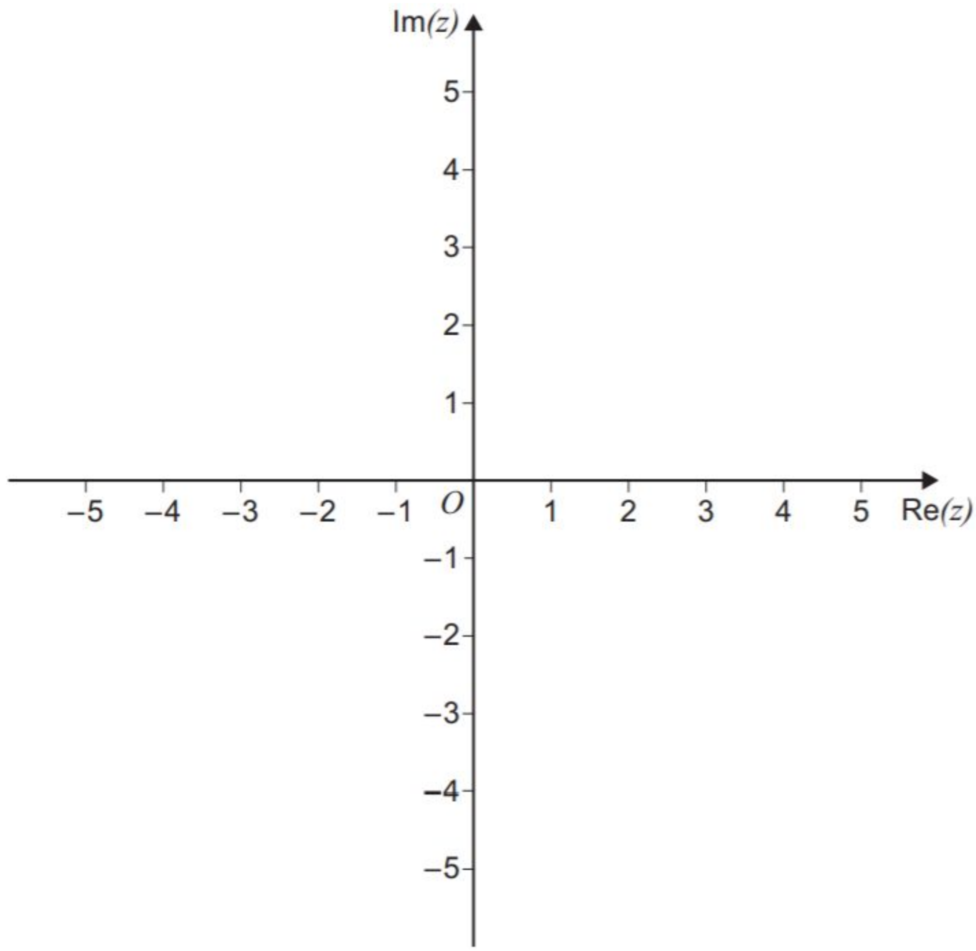
ii) Express w in the form $r(\cos\theta + i\sin\theta)$.

Give r in exact form and θ to two significant figures. [4 marks]

b. Consider the set

$$P = \{z \in \mathbb{C} : 3 \leq |z - 1 - i| \leq 4\}.$$

i) On the Argand diagram below, sketch the locus of $z \in P$. [3 marks]



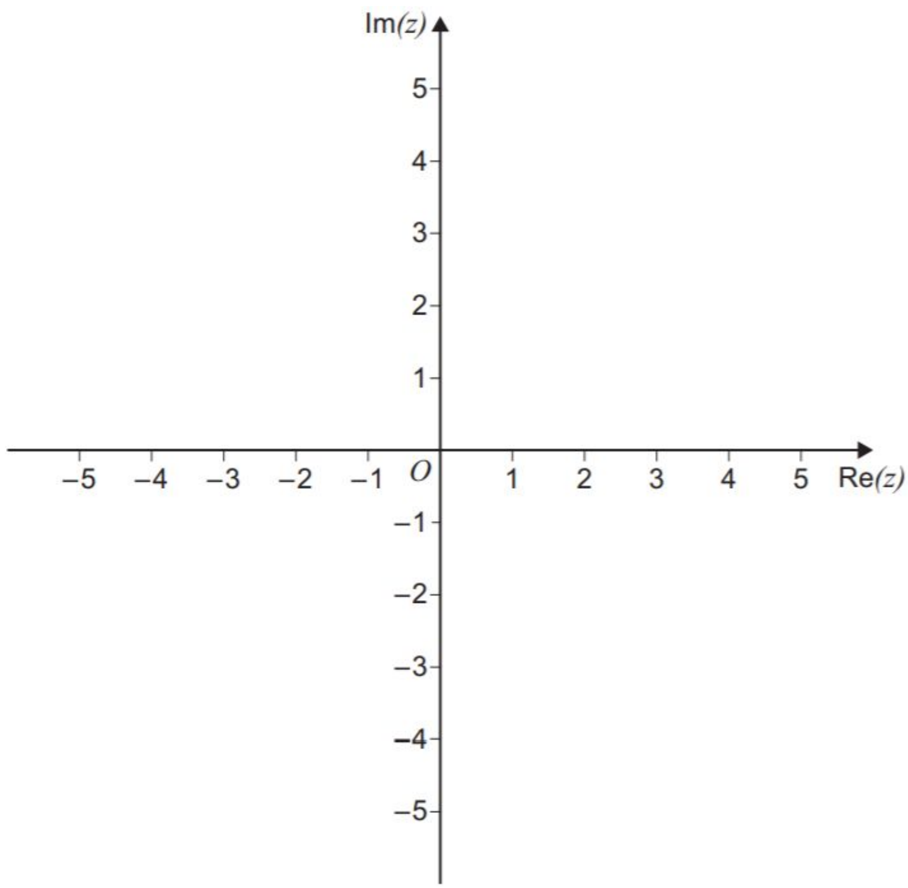
ii) Consider the set S defined as

$$S = P \cap \{z \in \mathbb{C} : 0 \leq \arg(z - 1 - i) \leq \frac{\pi}{3}\}.$$

Find the maximum value of Re(z) + Im(z) for all $z \in S$.

Give your answer in exact form. The Argand diagram has been repeated below to help you.

[3 marks]



[Total for Q14: 12 marks]

15. α, β and γ are the real roots of the cubic equation $x^3 + mx^2 + nx + 2 = 0$.

By considering $(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2$, prove that $m^2 \geq 3n$.

Fully justify your answer. [5 marks]

[Total for Q15: 5 marks]

16. This question is about modelling using vectors.

a. A theme park has two zip wires. Sarah models the two zip wires as straight lines using coordinates in metres.

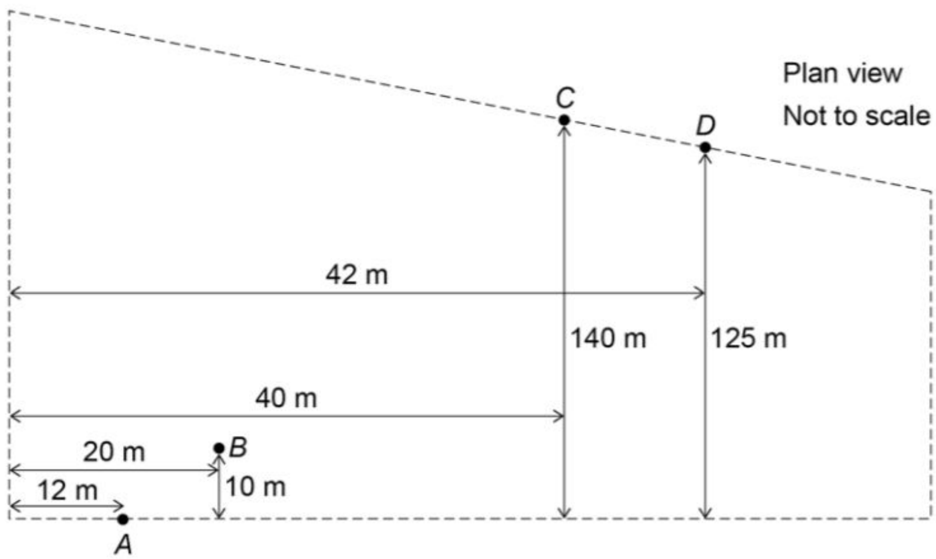
The ends of one wire are located at (0, 0, 0) and (0, 100, 20). The ends of the other wire are located at (10, 0, 20) and (10, 100, 5).

i) Use Sarah's model to find the shortest distance between the zip wires. [7 marks]

ii) State one way in which Sarah's model could be refined. [1 mark]

b. A lighting engineer is setting up part of a display inside a large building. The diagram shows a plan view of the area in which he is working. He has two lights, which project narrow beams of light.

One is set up at a point 3 metres above the point *A* and the beam from this light hits the wall 23 metres above the point *D*. The other is set up 1 metre above the point *B* and the beam from this light hits the wall 29 metres above the point *C*.



i) By creating a suitable model, show that the beams of light intersect.

[6 marks]

ii) Find the acute angle between the two beams of light. [3 marks]

iii) State one way in which the model you created in part b.i) could be refined. [1 mark]

[Total for Q16: 18 marks]

17. Find all solutions to the equation $\sinh^2 2x - \cosh 2x = 1$.

Give your answers in the form $x = \frac{1}{2} \ln(a \pm \sqrt{b})$ where *a* and *b* are integers.

Fully justify your answer. [9 marks]

[Total for Q17: 9 marks]

Section C: Extended question. Aim to spend no more than 45 minutes on this question.

18. In the Cartesian plane, circle C has equation $x^2 + y^2 - 8x = 0$ and hyperbola H has equation $4x^2 - 9y^2 = 36$.

A line L of positive gradient is tangent to H at point A and tangent to C at point B.

Find, in any order:

- an equation for L, in its simplest form
- the coordinates of the intersections of C and H, in exact form
- the area of the smaller region bounded by C and H, to 3 significant figures

Fully justify your answers. You are advised to plan your answer first below and continue on the pages that follow.

[20 marks]

Write your final answers on the last page in the spaces provided.

[More space for Q18]

[More space for Q18]

[More space for Q18]

[More space for Q18]

Equation for *L*:

Coordinates of intersections of *C* and *H*:

Specified area bounded by *C* and *H*:

[Total for Q18: 20 marks]

End of Questions

Question Sources

Q1,2,6,8-10:	AQA AS Further Maths Past Paper
Q11-16:	AQA AS Further Maths Past Paper
Q17:	MEI Past Paper
Q18:	IIT JEE

MEI is a maths-specific exam board.
IIT JEE is an exam in India.