<u>AQA A-Level Maths: Practice Paper 4</u>

Focus: Pure

Difficulty: Hard

Time: 2 hours

Marks:

Section A (multiple choice): 10 marks (15 minutes)

Section B (standard questions): 70 marks (1 hour 15 minutes)

Section C (extended question): 20 marks (30 minutes)

(Total 100 marks)

Grade Boundaries: (approximate)

A*: 80 (80%)

A: 70 (70%)

B: 60 (60%)

C: 50 (50%)

D: 40 (40%)

Main Topics Examined:

Proof, Exponentials and Logs, Differential Equations, Integration, Numerical Methods, Vectors

Advice:

1. Read the questions carefully - look out for tricks.
2. Some questions are harder than the A-level standard.
3. Apply existing knowledge to unfamiliar questions.
4. Check the fully worked solutions for any questions you missed.

Section A: Multiple choice. You are advised to spend no more than **15 minutes** in Section A.

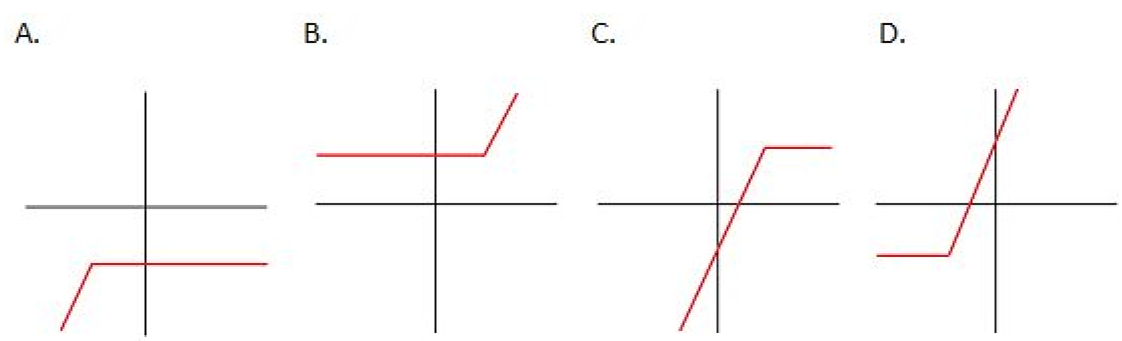
1. Complete the square of $4 + 6x - x^2$.

O $(x + 3)^2 - 13$
O $(x - 3)^2 - 5$
O $13 - (x + 3)^2$
O $13 - (x - 3)^2$ [1 mark]

2. Simplify $(x^6 + a^2x^3y) \div (x^6 - a^4y^2)$.

O $x^3 \div (a^2y)$
O $x^3 \div (x^3 - a^2y)$
O $x^2 \div (x^2 + a^2)$
O $x^2 \div (a^2 - x^3y)$ [1 mark]

3. Which of the sketches below shows the graph of $y = x - |x - 1|$?



O A
O B
O C
O D [1 mark]

4. A line L passes through the points (5, 1), (k, 4) and (-1, 9).

The value of k is

- O 2
- O 9/4
- O 11/4
- O 3 [1 mark]

5. If $y = (3x + 2)^6$, then the coefficient of x^3 in the expansion of dy/dx is

- O 1080
- O 3240
- O 9720
- O 19440 [1 mark]

6. The number of real solutions to the equation $e^{4x} + 4e^{2x} + 4 = 0$ is

- O 0
- O 1
- O 2
- O 4 [1 mark]

7. The equation of $4x^5 + x - 1 = 0$ is to be solved numerically using the iterative formula $x_{n+1} = 1 - 4x_n^5$ with initial guess x_0. The true root is $\alpha = 0.623$ to 3 decimal places.

Which of these is true?

- O For all $\{x_0 \in (0.5, 1) : x_0 \neq \alpha\}$, the iterative formula diverges.
- O The iterative formula forms a staircase when plotted on axes.
- O When $x_0 = 0.6$, the iterative formula converges to 3 d.p. in 6 iterations.
- O None of these. [1 mark]

8. The smallest positive value of θ such that

$\sin^2 2\theta + 8 \sin \theta \cos \theta \cos 2\theta - 4 \cos^2 2\theta = 0$

is

O 6.5°
O 19.8°
O 39.2°
O 50.9° [1 mark]

9. Find the area of the region bounded between the curves $y = x^2 + x$ and $y = x - x^3$.

O 1/12
O 1/6
O 1/3
O 4/3 [1 mark]

10. Which of these is true about the function $f(x) = 4x^3 + 3kx^2 - kx - 2$, defined for all real x?

O If $0 < 3k < 4$ then $f(x)$ is an increasing function for all x.

O If $|3k| = 4$, then $f(x)$ has a stationary point of inflection.

O When $f(x)$ has turning points, the difference in the y-coordinates of the maximum and minimum points of $f(x)$ is always less than 12.

O When the equation $f(x) = 0$ has three real roots, the product of these roots is independent of k. [1 mark]

Section B: Standard questions. Ensure to leave sufficient time for Section C.

11.

a. Prove that the product of four consecutive positive integers can never be equal to a perfect square. [5 marks]

b. Let p be a prime number.

i) Prove that the square root of p is irrational. [4 marks]

ii) Prove that there exists some integer $k > 0$ such that p and $p + k$ are prime. [5 marks]

iii) Explain why the result in part b.ii) proves that there are an infinite number of primes. [1 mark]

[Total for Q11: 15 marks]

12. Find the exact values of x and y in terms of a satisfying the following simultaneous equations:

$3 \log_8 (xy) = 4 \log_2 x$
$\log_2 y = a + \log_2 x$

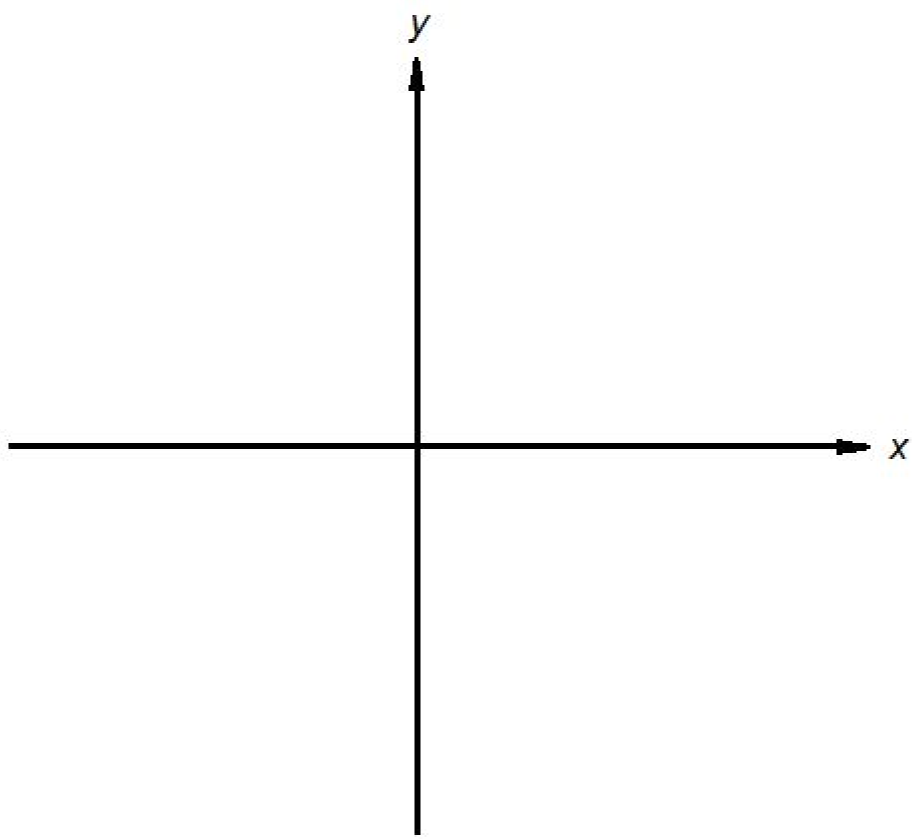
Give your answers in the form $x = p^a$ and $y = q^a$ where p and q are irrational numbers in exact form. [8 marks]

[Total for Q12: 8 marks]

13.

a. The curve C_1 is the parabola $y = x^2$. C_1 is transformed into curve C_2 by a reflection in the x-axis, followed by a translation by the vector $5a\,\mathbf{j}$, where $a > 0$ and $\mathbf{j}$ is the unit vector parallel to the y-axis. Line L is the line $y = a$.

i) Sketch, on the same axes, the graphs of C_2 and L. [2 marks]



ii) Show that the area enclosed by C_2 and L is $(32a^{3/2})/3$. [6 marks]

iii) Given that the area enclosed by C_2 and L is 108, find the exact value of a. [2 marks]

b. The horizontal line $y = k$ divides the region between C_1 and L into two equal areas.

Find the exact value of the ratio k/a. Give our answer in its simplest form.

[8 marks]

[Total for Q13: 18 marks]

14. $f(x)$ is a continuous function on the interval $[a, b]$. It is given that,

$$I = \int_a^b \frac{f(x)}{f(a+b-x)+f(x)}\,\mathrm{d}x.$$

a. By making the substitution $u = a + b - x$ and manipulating suitably, prove that

$$I = \frac{b-a}{2}.$$

[5 marks]

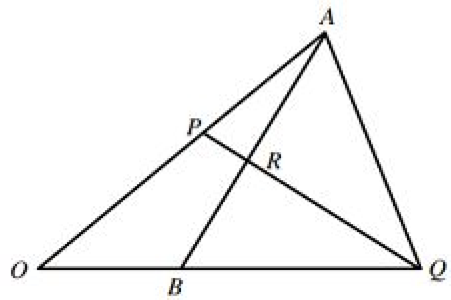
b. Hence, showing full justification, find the exact value of

$$\int_{\sqrt[3]{\ln(3)}}^{\sqrt[3]{\ln(4)}} \frac{x^2\sin(x^3)}{\sin(x^3)+\sin(\ln(12)-x^3)}\,\mathrm{d}x.$$

[3 marks]

[Total for Q14: 8 marks]

15. The figure below shows a triangle *OAQ* and lines drawn from vertices *A* and *Q*.



The point *P* lies on *OA* so that *OP* : *OA* = 3 : 5.
The point B lies on *OQ* so that *OB* : *BQ* = 1 : 2.
Vectors **a** and **b** are defined as *OA* and *OB* respectively.

a. Given that *AR* = *h AB*, where *h* is a scalar parameter with $0 < h < 1$, show that *OR* = (1 - *h*) **a** + *h* **b** [3 marks]

b. Given further that *PR* = *k PQ*, where *k* is a scalar parameter with $0 < k < 1$, find a similar expression for *OR* in terms of *k*, **a** and **b**. [3 marks]

c. Determine

i) the value of k and the value of h. [4 marks]

ii) the ratio $PR : RQ$. [1 mark]

[Total for Q15: 11 marks]

16. Show that a general solution of the differential equation $dy/dx = (2y^2 - 7y + 3) / 5$ is given by

$$y = \frac{Ae^x - 3}{2Ae^x - 1},$$

where A is an arbitrary constant. [10 marks]

Section C: Extended question.

17. For a function *f*(*x*) defined on some interval [*a*, *b*], the value of

$$I = \int_a^b f(x)\ dx$$

is approximated using the trapezium rule, with *n* intervals. The result of this approximation is *S*. The error, *E*, of this approximation is defined as the size of the difference between the approximation and the true value, i.e. $E = | S - I |$.

a. By considering a single trapezium as the sum of a rectangle and a small triangle, prove that the error in the approximation is bounded by

$$E \leq \frac{K(b-a)^3}{12n^2},$$

where *K* is the maximum value of $| f''(x) |$ in the interval $a \leq x \leq b$. You should include a sketch of the trapezium rule in use to help you. [18 marks]

b. Let

$$I = \int_0^1 x^3\ \mathrm{d}x.$$

Find the exact **difference** between the maximum error predicted by the formula in part a) and the actual error *E* when *I* is approximated with *n* = 4 trapeziums.

[2 marks]

Fully justify your answers. You are advised to use the space below to plan your answer and continue on the pages that follow.

Write your final answer to part b) in the space provided on the last question page.

More space to answer Q17.

More space to answer Q17.

More space to answer Q17.

Your answer to part b):

[Total for Q17: 20 marks]

End of Questions

Question Sources

Q11:	AQA A-Level Maths Past Paper
Q12:	MadasMaths A-Level Paper
Q14:	IIT JEE Advanced (Maths) Past Paper
Q15, 16:	MadasMaths A-Level Paper

IIT JEE is an exam in India.