Physics Solutions (Problem Solving)

Section A: Multiple Choice

1. **Answer:** Lights after a delay with a final brightness the same as X

   Working: The induced emf in the coil will oppose the cell emf and cause a delay in the current to lamp Y.

2. **Answer:** 2 : 1

   Working: Let T_1 and T_2 be tension in steel and copper wire respectively.
   Resolving forces:
   Horizontal: $T_1 \cos 120 = -T_2 \cos 30 \rightarrow 1/2\ T_1 = \sqrt{3}/2\ T_2 \rightarrow T_1 = \sqrt{3}\ T_2$
   $E = (FL)/(Ax) \rightarrow x = (FL)/(AE)$
   $\rightarrow$ ratio copper / steel = $((1*\sqrt{3})/(0.5*1)) / ((\sqrt{3}*1)/(0.5*2))$
   $= 2$

3. **Answer:** Elastic potential energy stored per unit volume of material

   Working: Consider a graph of stress vs strain when Hooke's law is obeyed.
   Graph is triangular → area = ½ * stress * strain
   Using stress = F/A and strain = x/L,
   Area = ½ (Fx)/(AL)
   Since elastic potential energy = ½ Fx and volume = AL,
   Area = elastic potential energy / volume
   → represents EPE stored per unit volume

4. **Answer:** 241

   Working: Conservation of momentum:
   $(mu/240) = (m + m/240)v$
   $\rightarrow (m/240) * u = (241m/240) * v$
   $\rightarrow u/v = (241/240) / (1/240) = 241$

5. **Answer:** 17000 N to the right

Working: Conservation of mass: rate of flow in = rate of flow out

Rate in: 5 * 0.6 = 3 m^3/s

Rate out: 3 = V * 0.25 → V = 12 m/s

Force = rate of change of momentum

= mass flow rate * change in speed

F = (3 * 800) * (12 – 5) = 16800 = 17000 N (2 s.f.)

Direction is to the right since the oil increased in speed

6. **Answer:** **A**

Working: Consider the temperature increasing linearly from 0 ºC to 100 ºC. Resistance initially decreases rapidly then levels off.

So from V = IR → *I* = V/R, if R decreases rapidly then *I* will *increase* rapidly. So the scale spacings need to be further apart to correct for the small change in temperature. At higher temperatures, increasing the temp further has little effect on R and hence I, so the scale needs to be closer together to correct for the small movement of the ammeter needle. This is A.

(Not D since the orientation is the wrong way around.)

7. **Answer:** 1350 Ω

Working: When in the light, V across resistor = 4 so V across LDR = 0.5 V.

When in the dark, V across resistor = 2 so V across LDR = 2.5 V.

Current in light: V = IR → I = V/R = 4/1200 = 0.00333 A

→ Resistance of LDR in light = V/I = 0.5/0.00333 = 150 Ω

Current in dark: I = 2/1200 = 0.001667 A

→ Resistance of LDR in dark = 2.5/0.001667 = 1500 Ω

Difference in resistance is 1500 - 150 = 1350 Ω

8. **Answer:** The complete diffraction pattern shifts in the *y*-direction because the path difference required for a maxima to appear has changed.

Working: Time period of light in air: λ/c = 600 nm / c = T
Time to travel 300 nm in vacuum = 300 nm / c = T/2
Time to travel 300 nm in material = 300 nm / (c/2) = T
Therefore, after passing through the material, the light passing through one slit will be half a cycle out of phase with the other. Before, a maximum forms if path difference is $n\lambda$ or $2n\pi$ phase difference, but now since the initial phase difference is π, only π, 3π, 5π etc more phase difference is required. So the maxima form at different path differences and hence at different positions (shifted in y-direction).

9. **Answer:** **2** and **3** only

Working: Processes I and III: V and T appear to be inversely proportional. Charles' law states that for constant pressure, V and T should be directly proportional which is not what we see in the graph.
Process II takes place at constant temperature, so there can be no change in internal energy. Since the volume increased, the gas must have done work on the surroundings but by conservation of energy this work must have come from absorbing heat energy.
Process IV also takes place at constant temperature, but this time volume has decreased so work was done *on* the gas. Again to maintain constant internal energy this additional work must have been released as heat energy.

10. **Answer:** $1/2\ M\omega_0^2(R - r)^2$

Working: KE of ring = ½ mv^2 where v is linear speed of the centre of mass of the ring. $v = \omega r$ where r is radius of centre of mass.
Radius of circular path of centre = radius of ring – radius of cone
= R – r
$\rightarrow v = \omega(R - r)$
$\rightarrow KE = \frac{1}{2} M\ \omega^2(R - r)^2$

11.

a. Time between adjacent peaks = 0.34 - 0.31 = 0.03 = T [1 mark]
$\rightarrow$ f = 1/T = 1/0.03 = 33.3 Hz [1 mark]
$c = f\lambda \rightarrow \lambda = c/f = (3 * 10^8)/33.3 = 9.0 \times 10^6$ m [1 mark]
(Accept any answer between 7.5×10^7 and 1.0×10^8 m.)

b. i) Maximum strain = 1.2×10^{-21} [1 mark]
Maximum extension = $4000 * 1.2 \times 10^{-21} = 4.8 \times 10^{-18}$ m [1 mark]
Diameter of a proton is approximately 10^{-15} m
$\rightarrow$ ratio = 0.0048 times the diameter of a proton [1 mark]
$\rightarrow$ No, this does not apply (not approximately 0.001)

ii) half wavelength path difference means waves in antiphase [1 mark]
so destructive interference takes place [1 mark]
this results in zero amplitude, (so no signal detected) [1 mark]
a change in length will result in a change in path/phase difference, so signal detected [1 mark]

iii) if initially the path difference is zero there will be a maximum signal [1 mark]
a change from max amplitude would represent a much smaller percentage (therefore less sensitive) [1 mark]

c. i) the gravitational force is a very weak/the weakest force *or* gravitational waves have very large wavelength so very low energy [1 mark]
so instruments require a very high resolution to detect [1 mark]

ii) photons have momentum (by de Broglie equation)
or
gravitational fields distort spacetime [1 mark]

12.

a. SHM eqn: $a = -\omega^2 x$, Newton's 2nd law: $ma = -kx \rightarrow a = -kx/m$

$\rightarrow -kx/m = -\omega^2 x \rightarrow \omega^2 = k/m$ [1 mark]

Effective spring constant = 1/(1/22 + 1/22) = 11 [1 mark]

$\rightarrow \omega = \sqrt{(11/0.12)} = 9.574$ [1 mark]

$2\pi f = 9.574 \rightarrow f = 1.5$ Hz [1 mark]

b. i) As magnet A moves, its coil experiences a change of magnetic flux [1 mark]

The change in magnetic flux linkage induces an emf in the coil (by Faraday's law) [1 mark]

The induced emf causes a current to flow in both coils [1 mark]

The current in the second coil causes a force to act on magnet B, driving magnet B into oscillation [1 mark]

Because both mass-spring systems have the same period/frequency [1 mark]

Resonance occurs and so magnet B oscillates with increasing amplitude [1 mark]

The amplitude of the oscillations of magnet A will start to decrease to conserve total kinetic energy of the magnets [1 mark]

ii) Maximum kinetic energy of each magnet is proportional to the square of its amplitude *or* $E_k(max) = 1/2\ mv^2_{max} = 1/2\ m\omega^2 A^2$ [1 mark],

so sum of the amplitudes squared must be constant [1 mark]

increase in amplitude of B cannot be more than the initial amplitude of A (since magnet A will still be oscillating) [1 mark]

iii) The phase difference of the two magnets' oscillations will change by 180^o [1 mark]

13.

a. Consider velocities in each direction of a single particle. Let the angle of elevation of the particle be θ from the ground.

	s	u	v	a	t
horizontal	r	v cos θ	v cos θ	0	t
vertical	0	v sin θ	-v sin θ	-g	t

In vertical direction: v = u + at → -v sin θ = v sin θ - gt [1 mark] → 2v sin θ = gt

$$\sin\theta = \frac{gt}{2v} \to \sin^2\theta = \frac{g^2t^2}{4v^2}$$ [2 marks]

In horizontal direction: s = ut + ½ at^2 → r = vt cos θ [1 mark]

$$\cos\theta = \frac{r}{vt} \to \cos^2\theta = \frac{r^2}{v^2t^2}$$ [1 mark]

Using the identity $\sin^2\theta + \cos^2\theta = 1$ (from right triangle),

$$\frac{g^2t^2}{4v^2} + \frac{r^2}{v^2t^2} = 1$$ [1 mark]

$$g^2t^4 + 4r^2 = 4v^2t^2 \to g^2t^4 - 4v^2t^2 + 4r^2 = 0$$ [1 mark]

Using the quadratic formula to solve for t^2,

$$t^2 = \frac{4v^2 \pm \sqrt{(4v^2)^2 - 4(g^2)(4r^2)}}{2g^2}$$ [1 mark]

$$t^2 = \frac{4v^2 \pm \sqrt{16(v^4 - g^2r^2)}}{2g^2}$$ [1 mark]

$$2g^2t^2 = 4v^2 \pm 4\sqrt{v^4 - g^2r^2}$$

$$\frac{1}{2}g^2t^2 = v^2 \pm \sqrt{v^4 - g^2r^2}$$ [1 mark]

b. Different initial angle of elevation [1 mark] the path with a higher angle has a longer time of flight than the path with smaller angle [1 mark]

14.

a. i) Momentum (of the objects) is conserved [1 mark]
Kinetic energy (of the objects) is not conserved / total kinetic energy after is less than before [1 mark]

ii) To conclude a one to one interaction, must have observed that increasing the intensity of the incident X-rays has no effect on the change in wavelength/recoil speed of electrons [1 mark] since this implies more photons can affect a single electron [1 mark]

To conclude the interaction is not with the nucleus, must have observed no difference in results when graphite was changed with another material [1 mark] since this would change the atoms/number of protons in the nucleus [1 mark]

b. i) Conservation of momentum:
Horizontal direction: $p_i = p \cos φ + p_f \cos θ$ [1 mark]
$\rightarrow \cos φ = (p_i - p_f \cos θ)/p$
Vertical direction: $0 = p_f \sin θ - p \sin φ$ [1 mark]
$\rightarrow \sin φ = (p_f \sin θ)/p$
Squaring both sides and adding,
$\sin^2 φ + \cos^2 φ = 1 \rightarrow (p_i - p_f \cos θ)^2/p^2 + (p_f \sin θ)^2/p^2 = 1$ [1 mark]
$p_i^2 + p_f^2 \cos^2 θ - 2p_ip_f \cos θ + p_f^2 \sin^2 θ = p^2$
Again using $\sin^2 φ + \cos^2 φ = 1$,
$p_i^2 + p_f^2 - 2p_ip_f \cos θ = p^2$ [1 mark]

(Alternative method: use vector triangle of momentum vectors then use cosine rule)

ii) Relativistic energy of electron = rest mass energy + kinetic energy
$= m_ec^2 + K$ [1 mark]
Using the relativistic equation, let $E = m_ec^2 + K$:
$\rightarrow (m_ec^2 + K)^2 = p^2c^2 + m_e^2c^4$ [1 mark]
$\rightarrow K^2 + m_e^2c^4 + 2Km_ec^2 = p^2c^2 + m_e^2c^4$
$\rightarrow K^2 + 2Km_ec^2 = p^2c^2$
$\rightarrow K^2/c^2 + 2 K m_e = p^2$ [1 mark]

iii) From conservation of energy, $E_i = E_f + K \rightarrow K = E_i - E_f$. Replacing K with this in the energy equation,

$\rightarrow (E_i - E_f)^2/c^2 + 2 (E_i - E_f) m_e = p^2$

$\rightarrow (E_i^2 + E_f^2 - 2 E_i E_f) / c^2 + 2 m_eE_i - 2 m_eE_f = p^2$ [1 mark]

Since $E = mc^2$ and for a photon, $p = mc \rightarrow E = pc$,

$\rightarrow (p_i^2c^2 + p_f^2c^2 - 2 p_ic\, p_fc) / c^2 + 2 m_ep_ic - 2 m_ep_fc = p^2$

$\rightarrow p_i^2 + p_f^2 - 2 p_ip_f + 2 m_e p_ic - 2 m_e p_fc = p^2$ [1 mark]

Replacing p^2 with the second equation,

$\rightarrow p_i^2 + p_f^2 - 2 p_ip_f + 2 m_e p_ic - 2 m_e p_fc = p_i^2 + p_f^2 - 2p_ip_f \cos \theta$

$\rightarrow 2 p_ip_f - 2 m_e p_ic + 2 m_e p_fc = 2p_ip_f \cos \theta$

$\rightarrow 2 p_ip_f (1 - \cos \theta) = 2 m_e p_ic - 2 m_e p_fc$ [1 mark]

Using De Broglie's equation, let λ_i and λ_f be initial and final wavelengths:

$\lambda = h/p \rightarrow p = h/\lambda$,

$\rightarrow 2 h/\lambda_i * h/\lambda_f (1 - \cos \theta) = 2 m_e * h/\lambda_i * c - 2 m_e * h/\lambda_f * c$ [1 mark]

$\rightarrow 2 h(1 - \cos \theta)/(\lambda_i * \lambda_f) = 2 (m_ec/\lambda_i - m_ec/\lambda_f)$

$\rightarrow 2 h(1 - \cos \theta)/(\lambda_i * \lambda_f) = 2 (m_ec\lambda_f - m_ec\lambda_i) / (\lambda_i * \lambda_f)$

$\rightarrow h(1 - \cos \theta) = m_ec(\lambda_f - \lambda_i)$ [1 mark]

Since $\lambda_f - \lambda_i$ is change in wavelength, let $\Delta\lambda = \lambda_f - \lambda_i$:

$h(1 - \cos \theta) = m_e c \Delta\lambda$

$\rightarrow \Delta\lambda = (h / (m_e c)) * (1 - \cos \theta)$. [1 mark]

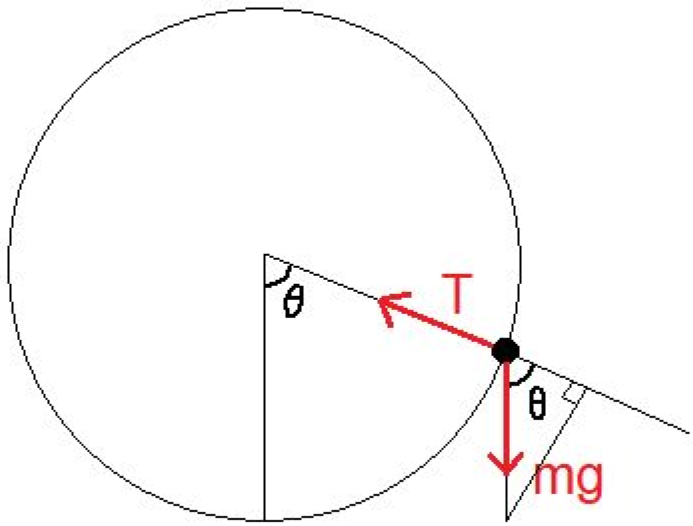
Since $-1 \leq \cos \theta \leq 1$, maximum value of $\Delta\lambda$ is

$\Delta\lambda = 2 h /(m_e c) = 2 * (6.63 \times 10^{-34}) / (9.11 \times 10^{-31} * 3 \times 10^8)$

$= 4.85 \times 10^{-12}$ m [1 mark]

15.

a. i) Free body diagram:



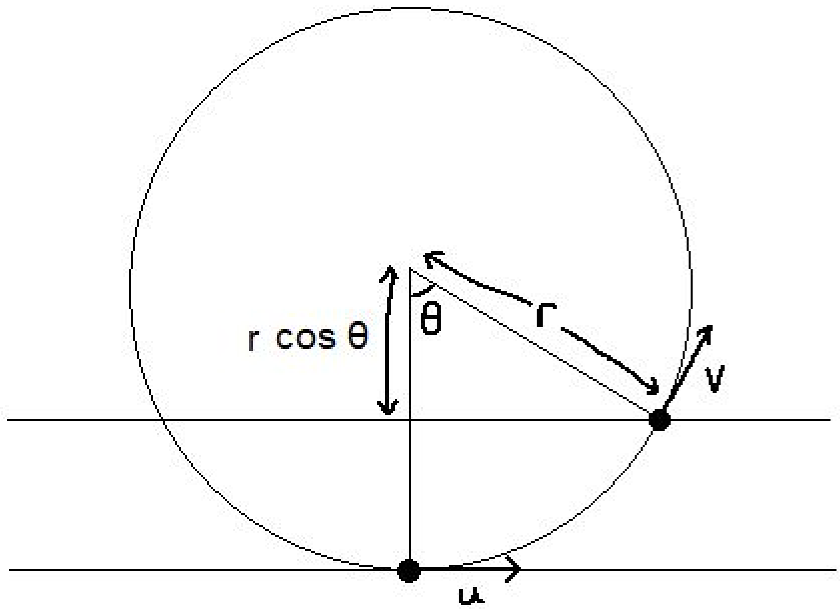
[1 mark]

Component of weight acting away from centre of circle = mg cos θ
[1 mark]
Tension is provided by centripetal force:
T - mg cos θ = ma
T - mg cos θ = mv^2/r [1 mark]

ii) For angles such that cos θ is positive *or* in the lower half of the circle [1 mark], the weight force acts against / has a component in the opposite direction of the tension [1 mark]

iii) v is **not** constant [1 mark] since: (any one of the following)
the weight acts on (with a component in the direction of) the velocity (which is not cancelled out by the tension since centripetal force has no tangential component)
or
kinetic energy must vary since GPE varies with the height (by conservation of energy)
[1 mark]

iv) Consider the ball moving through an angle θ from the bottom:



Energy before = energy after

KE before = KE after + GPE after (defining GPE at bottom = 0)

Using trig, height from P to centre level = r cos θ

Gain in height = r - r cos θ [1 mark]

$1/2\ mu^2 = 1/2\ mv^2 + mg(r - r\cos\theta)$ [1 mark]

$1/2\ mu^2 = 1/2\ mv^2 + mgr(1 - \cos\theta)$

$u^2 = v^2 + 2gr(1 - \cos\theta)$ [1 mark]

b. i) The ball is most likely to fall off at the top since this is where its kinetic energy is a minimum.

The force equation at the top is:

$N + mg = mv^2/r$ where N is normal contact force [1 mark]

On the point of falling off, N = 0, so $mg = mv^2/r$

$\rightarrow v^2 = gr$ [1 mark]

Using energy equation, $u^2 = v^2 + 2gr$ [1 mark]

$\rightarrow u^2 = gr + 2gr = 3gr$

$\rightarrow u = \sqrt{(3gr)}$ [1 mark]

ii) In scenario 1, the centripetal force is provided by the contact force which can only be exerted towards the centre of the circle, but in scenario 2 the contact force can be exerted in either direction. [1 mark]
This does not affect the validity of the model, assuming the object remains in circular motion. [1 mark]

iii) Friction acts tangentially against the velocity at all times, with magnitude proportional to mg cos θ since cos θ > 1 for all angles on the hemisphere *or* since contact force is always positive. [1 mark].

So $a = v^2/r$ decreases and since since mg cos θ - N = ma, the contact force N must increase. [1 mark]

So it takes longer for N to fall to zero where the ball would lose contact. [1 mark]