AQA A-Level Maths: Practice Paper 1

Focus: Pure

Difficulty: Hard

Time: 2 hours

Marks:

Section A (multiple choice): 10 marks (15 minutes)

Section B (standard questions): 70 marks (1 hour 15 minutes)

Section C (extended question): 20 marks (30 minutes)

(Total 100 marks)

Grade Boundaries: (approximate)

A*: 80 (80%)

A: 70 (70%)

B: 60 (60%)

C: 50 (50%)

D: 40 (40%)

Main Topics Examined:

Algebra, Parametric Equations, Sequences and Series,
Trigonometry,

Differential Equations

Advice:

1. Read the questions carefully - look out for tricks.
2. Some questions are harder than the A-level standard.
3. Apply existing knowledge to unfamiliar questions.
4. Check the fully worked solutions for any questions you missed.

Section A: Multiple choice. You are advised to spend no more than **15 minutes** in Section A.

1. Factorise $2x^2 - xy - y^2$.

O $(x - y)^2$
O $(2x - y)^2$
O $(2x - y)(x + y)$
O $(2x + y)(x - y)$ [1 mark]

2. The coefficient of x^2 in the expansion of $(2x^2 - x + 4)(1 - x - 8x^2)$ is

O -32
O -29
O 29
O 32 [1 mark]

3. The solution(s) to the equation $|2x - 5| = 1 - 6x$, is/are

O $x = -1$ only
O $x = 3/4$ only
O $x = -1$ or $x = 3/4$
O no solutions [1 mark]

4. The lines $3x + ay = 2$ and $bx - y = a - b$ are perpendicular in the xy-plane, where both a and b are non-zero.

Which of the relationships between a and b is true?

O $ab = 3$
O $ab = -3$
O $a/b = 3$
O $a/b = -3$ [1 mark]

5. k is the smallest positive value of x which is a solution to both the equations $2 \sin x + 1 = 0$ and $2 \cos 2x = 1$.

How many values of x in the range $0 \leq x \leq k$ are solutions to at least one of these equations?

O 0
O 2
O 3
O 4 [1 mark]

6. The functions $f(x)$, $g(x)$ and $h(x)$ are related by $f'(x) = g(x + 1)$ and $g'(x) = h(x - 1)$.

Find an expression for f ''(2x) in terms of h.

O $h(2x)$
O $4\ h(2x)$
O $2\ h'(2x)$
O $h(2x + 1)$ [1 mark]

7. A vector **v** has magnitude m, where $m \neq 0$. Which of these vectors points in the opposite direction as **v** with magnitude m^2?

O $\mathbf{v} - m\mathbf{v}$
O $m\mathbf{v}$
O $-m\mathbf{v}$
O $-m^2\mathbf{v}$ [1 mark]

8. In a geometric series S, the sum to infinity of S is 32/31 times the sum of the first 5 terms of S. The common ratio of S is

O 1/2
O 1/4
O 1/32
O 5/32 [1 mark]

9. Given that m is a real constant satisfying

$$\int_0^2 x^m \mathrm{d}x = \frac{16\sqrt{2}}{7} \quad \text{and} \quad \int_0^2 x^{m+1} \mathrm{d}x = \frac{32\sqrt{2}}{9},$$

then the value of m is

O 5/2
O 7/2
O 9/2
O 11/2 [1 mark]

10. If the function $f(a)$ is defined as

$$f(a) = \int_0^1 (x^2 - a)^2 \; dx$$

then the minimum value of $f(a)$ is

O 3/20
O 4/45
O 7/13
O 1 [1 mark]

Section B: Standard questions. Ensure to leave sufficient time for Section C.

11. The straight line *L* with equation $y = t(x - 2)$, where t is a parameter, intersects the circle with equation $x^2 + y^2 = 1$ at two distinct points *A* and *B*.

The midpoint of *AB* is *M*.

a. The line *L* passes through a certain point *P* for all *t*. State the coordinates of P.

[1 mark]

b. Show that, for any real value of *t*, the coordinates of *M* are given by

$$M = \left(\frac{2t^2}{1+t^2}, -\frac{2t}{1+t^2}\right).$$

[7 marks]

c. Hence show that the locus of M as t varies is a circle, stating its radius and the coordinates of its centre. [5 marks]

[Total for Q11: 13 marks]

12. The sum to *n* terms of a series *a* satisfies

$$\sum_{k=1}^{n} a_k = \log_{10}\left(\frac{(n+1)(n+2)}{2}\right)$$

a. Find a simplified expression for a_n. [3 marks]

b. Find the exact value of

$$\sum_{k=1}^{20} a_{2k},$$

giving your answer in terms of a base-10 logarithm. [4 marks]

c. Explain why the sum to infinity of *a* diverges. [1 mark]

[Total for Q12: 8 marks]

13. You are given that

$$y = \tan\left(\frac{\alpha}{2}\right)$$

a. Show that

$$\sin(\alpha) = \frac{2y}{1+y^2}$$

[4 marks]

b. Given that

$$\cos(\alpha) = \frac{1-y^2}{1+y^2},$$

make suitable substitutions to solve the equation

$$4\tan\left(\frac{\alpha}{2}\right) + 3\cot(\alpha)\sec^2(\alpha) = 0\ , \quad 0 \leq \alpha \leq 2\pi.$$

[8 marks]

c. x is a real number such that $0 < x < 1$. Use a suitable reflection identity for $\sin(x)$ to show that

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

[3 marks]

d. Hence, verify that

$$x = \sin\left(\frac{1}{4}\pi + \frac{1}{4}\sqrt{\pi^2 - 2\sqrt{2}}\right)$$

is a solution to the equation

$$\sin^{-1}(x)\cos^{-1}(x) = \frac{\sqrt{2}}{8}$$

[4 marks]

[Total for Q13: 19 marks]

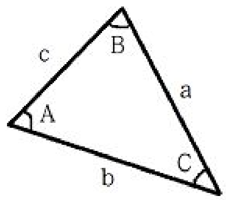
14. x and y are variables such that $x + y \geq a$ and $x - y \leq -1$ where a is a constant.

The **smallest** possible value of $x + ay$ is 7.

Find the value of a. Fully justify your answer. [9 marks]

[Total for Q14: 9 marks]

15. Triangle *ABC* has side lengths *a*, *b* and *c* opposite angles *A*, *B* and *C* respectively as shown in the diagram below:



a. The area of triangle *ABC* is *T*. By solving the cosine rule, namely

$$a^2 = b^2 + c^2 - 2bc\cos A$$

simultaneously with a suitable formula for the area of the triangle, prove that

$$T = \sqrt{s(s-a)(s-b)(s-c)}$$

where *s* is **half** the perimeter of triangle *ABC*. [10 marks]

(More space to answer Q15a on the next page.)

b. Hence, find the exact area of a triangle with sides of length 4 cm, 6 cm and 8 cm.

[1 mark]

[Total for Q15: 11 marks]

16. At time t = 0, one litre of a certain liquid chemical is added to a tank containing 20 litres of water. The chemical reacts with the water forming a gas, and as a result of this reaction, both the volumes of the water and the chemical are reduced.

At time t minutes since the chemical reaction started, the volumes of the chemical and the water used up in the reaction are (1 - v) litres and 4(1 - v) litres respectively.

The rate at which the volume of the chemical in the tank reduces, is proportional to the product of the volume of the chemical and the volume of the water, still left in the tank.

Given that 2 minutes after the reaction started, the volume of the chemical remaining is exactly 4/19 of a litre, show that

$$2^t = \frac{v+4}{5v}$$. [10 marks]

(More space to answer Q16 on the next page.)

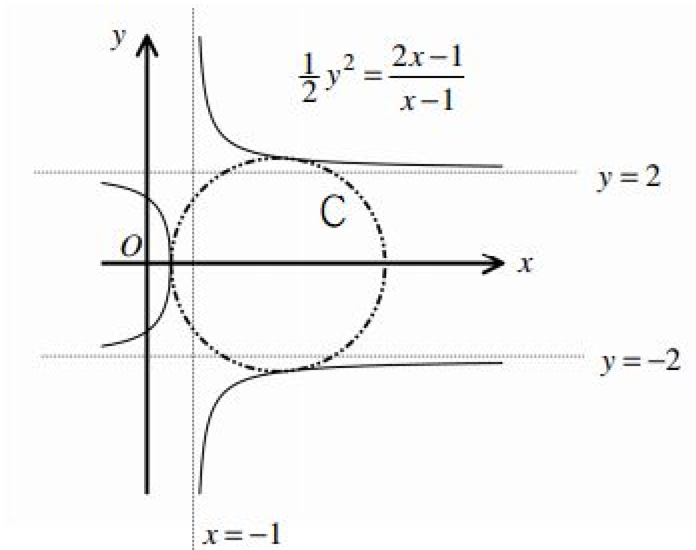
[Total for Q16: 10 marks]

Section C: Extended question.

17. The figure below shows the curve

$$\frac{1}{2}y^2 = \frac{2x-1}{x-1}$$

whose three asymptotes are shown as the dotted lines x = -1 and y = ± 2. A circle C is drawn, so that it touches all three branches of the curve, as shown.



Determine the Cartesian equation of C in its simplest form.

Fully justify your answer. You are advised to use the space below to plan your answer and continue on the pages that follow.

Write your final answer in the space provided on the last question page.

[20 marks]

More space to answer Q17.

More space to answer Q17.

More space to answer Q17.

Your answer:

[Total for Q17: 20 marks]

End of Questions

Question Sources

Q5: ENGAA Past Paper (Mathematics)
Q6: STEP I Past Paper
Q9: ENGAA Past Paper (Mathematics)
Q10: STEP I Past Paper

Q11: MadasMaths Paper
Q12: CSAT Past Paper (Math B)
Q14: Gaokao Past Paper (Math)
Q16: MadasMaths Undergraduate Paper

Q17: MadasMaths Undergraduate Paper (Challenge)

ENGAA is an entrance exam for studying Engineering at Cambridge.
STEP I in an entrance exam for studying Maths or Physics at Oxbridge or Durham.
CSAT is an exam in South Korea.
Gaokao is an exam in China.