<u>Physics Solutions (Multiple Choice)</u>

Section A: Fast

1. **Answer:** 2.0 ms^{-1}

   Working: All GPE of the bob at the top is converted to KE at the bottom.
   $mgh = ½ mv^2 \rightarrow v^2 = 2gh = 2(9.81)(1.3 - 1.1) = 3.92$
   $\rightarrow v = 1.98 = 2.0$ ms^{-1} (2 s.f.)

2. **Answer:** micro, milli, centi, kilo

   Working: micro < milli < centi < kilo: $10^{-6} < 10^{-3} < 10^{-2} < 10^{3}$

3. **Answer:** kg m^{-1} s^{-2}

   Working: Pressure = Force / Area = (Mass * Acceleration) / Area
   [P] = (kg ms^{-2}) / m^2 = kg m^{-1} s^{-2}

4. **Answer:** decreasing and downwards

   Working: Accelerating downwards → Resultant force is downwards
   Magnitude of acceleration decreases from 9.81 (*g*) to 0 at terminal velocity → decreasing

5. **Answer:** 3.7 N

   Working: From vector (right-angled) triangle, use Pythagoras:
   $4.5^2 + F^2 = 5.8^2 \rightarrow F = 3.66 = 3.7$ N (2 s.f.)

6. **Answer:** Kinetic energy of molecules and temperature of substance

Working: During a change of state, temperature is constant, kinetic energy is constant and potential energy becomes less negative (as intermolecular forces are overcome, freeing the particles) so internal energy overall increases.

7. **Answer:** 0.9 %

Working: Volume of cylinder = area of cross section x height
$= (\pi d^2/4) * L$
V is proportional to d^2 and to L
% uncertainty in d is doubled and add % in L
% in V = 2*(0.01/5) + (0.1/20) = 0.009 = 0.9 %

8. **Answer:** 430 rad s^{-1}

Working: $x = A \cos(\omega t) \rightarrow v = -A\omega \sin(\omega t)$
Maximum value of $\sin(\omega t)$ is $1 \rightarrow v_{max} = -A\omega$
Since given speed we can take magnitude (ignore minus sign)
$v_{max} = A\omega \rightarrow \omega = v_{max} / A = 24 / 0.056 = 429 = 430$ rad s^{-1} (2 s.f.)

9. **Answer:** 0.063 rad s^{-2}

Working: Using $v = \omega r \rightarrow \omega = v/r$,
Initial angular speed, $\omega_0 = u/r = 9/20 = 0.45$ rad s^{-1}
Final angular speed, $\omega = v/r = 14/20 = 0.7$ rad s^{-1}
Rate of change = $\Delta\omega/\Delta t = (0.7 - 0.45) / 4 = 0.0625 = 0.063$ (2 s.f.)
Units: angular speed / time = rad s^{-1} / s = rad s^{-2}

10. **Answer:** *x* + *y* + *z*

Working: Just as gravitational forces act on the centre of mass, electric forces can be thought of as acting on the centre of charge. Since the spheres are uniformly charged, this is at their centres.

11. **Answer:** 2.5×10^{18}

Working: Kirchhoff's first law: 0.06 = 0.02 + current at Y
Current at Y = 0.04 → Charge = Q = It = 0.04(10) = 0.4 C
Charge per electron = e = 1.6×10^{-19} C
Total electrons = 0.4 / (1.6×10^{-19}) = 2.5×10^{18}

12. **Answer:** 3.0 m

Working: Waves arrive constructively → 0 / 2π / 4π / 6π etc phase difference
Path difference = 20 - 16 = 4 m
Any wavelength whose multiple includes 4 m will have a multiple of 2π phase difference.
1, 2 and 4 all divide into 4 but 3 does not.

13. **Answer:** straight current-carrying wire, with current going into plane of paper

Working: Cannot be a permanent magnet, these produce symmetric magnetic fields so would point the compasses in opposite directions. Also cannot be a solenoid since this produces the same shape of magnetic field as a permanent magnet.
Using right hand grip rule: if current goes into the page then a magnetic field is produced clockwise which agrees with the orientation of the compasses.

14. **Answer:** **A** and **B** only

Working: Electromagnetic waves slow down when refracted into a different medium of higher refractive index (optical density), so **C** is false.

15. **Answer:** $f_1 = f_2$

Working: Since for an electromagnetic wave, E = hf (i.e. energy is solely dependent on frequency), by conservation of energy, frequency cannot change.

16. **Answer:** Conservation of energy

Working: Kirchhoff's second law: the sum of potential differences around a closed loop is zero. Charges gain/lose energy when accelerated through a potential difference (E = QV), so the sum of these energy changes overall must be zero, which is the direct statement of conservation of energy.
(Kirchhoff's first law (current) is conservation of charge.)

17. **Answer:** Bone has a larger X-ray attenuation coefficient than muscle

Working: Impedance matching gel → difference in acoustic impedances is small → most ultrasound is transmitted → little is reflected
Bone contains calcium, which is a heavier atom than other elements found in the body (muscle is proteins: carbon, oxygen, nitrogen, hydrogen etc) → more absorption
Z = ρc
Piezoelectric effect: mechanical (elastic potential) energy becomes electrical energy and vice versa

18. **Answer:** $E = \frac{kL}{A}$

Working: Young modulus, E = stress / strain
Stress = F/A and strain = x/L
→ E = (F/A) / (x/L) = (FL)/(Ax)
But from Hooke's law, F = kx → k = F/x
→ E = kL/A

19. **Answer:** 0.40 V

Working: Faraday's law: e.m.f. = - rate of change of flux linkage
Straight line graph → rate of change is gradient
e.m.f. = - ($-8 * 10^{-4}$ - $8 * 10^{-4}$) / ($4 * 10^{-3}$) (notice units on axes)
= 0.4 V

20. **Answer:** **D**

Working: Total binding energy = (binding energy per nucleon) * (nucleon number)
So we need the nucleus furthest up and to the right to maximise both these quantities → D.

21. **Answer:** **A**, **B** and **C**

Working: E = hf = QV → V is proportional to f → inversely proportional to λ.

22. **Answer:** The photoelectric current produced in the circuit increases with the intensity of the incident light

Working: Work function: (minimum) energy required to remove one electron from the **highest** energy level of a surface atom.
Stopping potential: p.d. that stops the photoelectric current when the material is connected to the positive terminal of a source.
Higher intensity → more photons → more electrons (since one-to-one interaction) → more charge flow → more current
Energy of photons > energy of electrons (since some is 'consumed' by the work function) → frequency of photons > frequency of electrons (as waves) → λ of photons < λ of electrons (as waves)

23. **Answer:** 7.5 rad s^{-1}

Working: 1 revolution = 2π radians. 72 revolutions per minute = 72(2π)
= 144π radians per minute = 144π / 60 = 7.5 radians per second.

24. **Answer:** momentum, angular speed, magnetic flux linkage

Working: Vectors: momentum, force, flux density, field strength
Scalars: flux linkage, current, Young modulus, half-life, capacitance, all constants
(Current is a scalar because although it has a direction within a wire, it does not obey vector addition laws i.e. the direction of two wires at a junction does not affect the current out.
Angular speed is known as a *pseudovector*: it has a mathematically assigned direction that is almost always ignored in basic calculations, always considered a scalar at A-level)

25. **Answer:** **A**

Working: From the potential divider equation, as R increases, the V across it also increases. But the output p.d. cannot exceed the p.d. provided by the cell (Kirchhoff's second law) so the graph must increase up to a limit.

26. **Answer:** 1.5×10^{-17} C

Working: $E = Q/(4\pi\varepsilon_0 r^2) \rightarrow Q = 4\pi\varepsilon_0 r^2 E$
$Q = 4\pi(8.854 * 10^{-12})(2 * 10^{-8})^2(3.3 * 10^8) = 1.47 \times 10^{-17}$ C

27. **Answer:** 0.20 mm s^{-1}

Working: I = Anev and R = ρL/A
Same metal material → n and ρ are same in both
Connected in parallel → V is the same in both → IR is the same in both (Ohm's law)
Combining, IR = AnevR = AnevρL/A = nevρL = constant
Removing constants: vL = constant
(0.6)(L) = v(3L) → v = 0.2

28. **Answer:** $t = 0$: maximum; $t = 10$ s: 37% of the current at $t = 0$

Working: Discharging → quantities (current, charge, p.d. across) start from a maximum and decay to zero exponentially (using $x = x_0 \exp(-t/RC)$ from $dx/dt = -kx$)
After one time constant, quantity has reduced to 1/e to its original value = 37%.

29. **Answer:** $0.60 = 1.50\ e^{-0.10\,t}$

Working: Consider p.d. across capacitor (charging)
$V = V_0\ (1 - \exp(-t/10))$: Given V is across the resistor so p.d. across capacitor is $1.5 - 0.6 = 1.5(1 - \exp(-0.1t))$
$\rightarrow 0.90 = 1.5 - 1.5 \exp(-0.1t)$
$\rightarrow 0.60 = 1.5 \exp(-0.1t)$ (1st answer)
Or consider p.d. across resistor:
$0.60 = 1.5 - (1.5 - 1.5 \exp(-0.1t))$
$0.60 = 1.5 \exp(-0.1t)$ (same answer)

30. **Answer:** 2.1 nm

Working: $hf = \Phi + KE_{max} \rightarrow hc/\lambda = \Phi + p^2/2m$
$p = h/\lambda_{DeBr} \rightarrow hc/\lambda = \Phi + (h^2/2m\lambda^2_{DeBr})$
$(6.63 * 10^{-34} * 3 * 10^{8}) / (470 * 10^{-9}) = (2.3 * 1.6 * 10^{-19})$
$+ (6.63 * 10^{-34})^2 / (2 * 9.11 * 10^{-31} * \lambda^2_{DeBr})$
$\rightarrow \lambda_{DeBr} = 2.1 * 10^{-9} = 2.1$ nm

31. **Answer:** 5.6 ms^{-2}

Working: stopping distance = thinking distance + braking distance
$38 = (18 * 0.5) + s \rightarrow s = 29$ m
s = 29, u = 18, v = 0, a = ?, t = ?
$v^2 - u^2 = 2as \rightarrow a = (v^2 - u^2)/(2s) = (0^2 - 18^2)/(2*29) = -5.586\ ms^{-2}$
Magnitude of deceleration = $|a| = 5.586 = 5.6\ ms^{-2}$ (2 s.f.)

32. **Answer:** ρ decreases as T increases since the additional energy releases more electrons into the conduction band.

Working: 1st option is for metals.
3rd option assumes material properties do not change with temp.
4th option is for superconductors.

33. **Answer:** 3 kg

Working: $E_k = \frac{1}{2} mv^2$ and $p = mv \rightarrow E_k = p^2/(2m) \rightarrow m = p^2/(2E_k)$
$m = 30^2/(2*150) = 3$ kg

34. **Answer:** 2.11×10^{-9} m^2

Working: Length of wire = circumference of one turn * number of turns
$L = \pi(8 * 10^{-3}) * 50 = 1.257$ m
$R = \rho L/A \rightarrow A = \rho L/R = (4.2 * 10^{-7})(1.257)/250 = 2.11 * 10^{-9}$ m^2

35. **Answer:** 0.5 A

Working: Connected in series → currents must be same in both.
Total voltages must add to 6 V.
Only points on graph are at (2 V, 0.5 A) and (4 V, 0.5 A)
→ current = 0.5 A

36. **Answer:** a little below 6 V

Working: LDR resistance increases significantly in total darkness → almost all p.d. across it → almost 6 V

37. **Answer:** 0.30 m

Working: speed = distance / time → distance = speed * time
= $(3 * 10^8) * 10^{-9} = 0.3$ m

38. **Answer:** polarisation

Working: Sound waves are longitudinal so their vibrations are restricted to the same line as their direction of propogation (energy transfer). So they can not be plane polarised (requires oscillation in a plane)

39. **Answer:** 0.16 I

Working: Intensity is proportional to amplitude squared.
180^o out of phase → destructive interference → amplitudes subtract
Amplitude = a - 0.6a = 0.4a
Intensity = $(0.4a)^2 = 0.16a^2 = 0.16$ I

40. **Answer:** using laser light with a higher frequency

Working: For single-slit diffraction, d sin θ = nλ
First order → n = 1 → d sin θ = λ
Moving laser has no effect
Moving screen further makes maxima further away
Grating with more lines → d goes down → sin θ goes up
→ maxima moves away
Higher frequency → shorter wavelength → sin θ goes down
→ maxima comes closer

41. **Answer:** Compton effect

Working: Simple scattering: no change
Compton effect: energy lost → lower frequency → longer λ
Pair production: no change
Photoelectric effect: X-ray is absorbed

42. **Answer:** 360 Hz

Working: Stationary waves for one end open, one end closed satisfy
$L = \lambda/4, 3\lambda/4, 5\lambda/4$ etc. Since $v = f\lambda \rightarrow \lambda = v/f$
$L = v/4f, 3v/4f, 5v/4f$ etc.
If the first (fundamental) harmonic is 120 Hz, the next harmonics must have 3, 5, 7 etc times this value.
The only option is 360 (3*120).

43. **Answer:** **B**

Working: The Sun is a main-sequence star (B).

44. **Answer:** 10^{-23} kg ms^{-1}

Working: Wavelength of X-rays is approximately 10^{-10} m.
$\lambda = h/p \rightarrow p = h/\lambda = (6.63 * 10^{-34})/(10^{-10}) = 6.63 * 10^{-24}$
This rounds up to 10^{-23}.

45. **Answer:** **B**

Working: Similar to Sun → same absorption lines but Doppler shifted by some proportional (not constant) amount.
A is shifted by a constant, C and D are different.

46. **Answer:** positron

Working: Beta-plus decay is $u \rightarrow d + e^+ +$ neutrino
Only positron is emitted and feels the electromagnetic force (neutrino is only affected by weak nuclear force and gravity)

47. **Answer:** All the particles move with equal kinetic energy but in random directions.

Working: From the Maxwell-Boltzmann distribution model, particles have random velocities → random speed → random kinetic energy

48. **Answer:** **A**

Working: Consider the temperature increasing linearly from 0 ºC to 100 ºC. Resistance initially decreases rapidly then levels off.
So from V = IR → *I* = V/R, if R decreases rapidly then *I* will *increase* rapidly. So the scale spacings need to be further apart to correct for the small change in temperature. At higher temperatures, increasing the temp further has little effect on R and hence I, so the scale needs to be closer together to correct for the small movement of the ammeter needle. This is A. (Not D: oriented wrong way around.)

49. **Answer:** Brightness of B_1 decreases; brightness of B_2 increases

Working: Brightness (radiant power) is proportional to $P = IV = I^2R$
Let initial (closed switch) voltages and currents be V_{B1}, I_{B1}, V_{B2}, I_{B2}.
Switch closed: $V_{B1} = 2 * V_{B2}$ and $I_{B1} = 2 * I_{B2} \rightarrow P_{B1} = 4 * P_{B2}$.
When the switch opens, B_1 and B_2 become in series, so their currents must be equal. Since they also have equal resistance (identical bulbs), their p.d.s must also be equal. So now $P_{B1} = P_{B2}$.
B_1 went from 80% (4:1) power consumption to 50% → decreased.
B_2 went from 20% power consumption to 50% (1:1) → increased.

50. **Answer:** $\dfrac{p_0}{10\rho g}$

Working: Hydrostatic pressure due to a column of liquid = $\Delta p = h\rho g$.
If the pressure at the top is now $9p_0/10$ and at the bottom it is p_0, the difference is due to the hydrostatic pressure:
$\rightarrow p_0 - 9p_0/10 = L\rho g \rightarrow (1/10)\, p_0 = L\rho g \rightarrow L = (1/10)\,(p_0 / \rho g)$.

51. **Answer:** 10 up quarks, 11 down quarks

Working: $^{7}_{3}Li$: 3 protons, 4 neutrons
Proton: 2 up quarks, 1 down quark
Neutron: 1 up quark, 2 down quarks
Total:
= (3 * 2) + (4 * 1) = 10 up quarks, and
= (4 * 2) + (3 * 1) = 11 down quarks

52. **Answer:** All of these materials

Working: Area of roof = 50 * 100 = 5000 m^2
Mass of roof = 5000 * 100 = 500000 kg
Weight of roof = Mg = 4.9 MN
Area of wall tops = 2 * (50 * 0.1 + 100 * 0.1) = 30 m^2
Stress on walls = 4.9 * 10^{-6} / 30 = 163000 N m^{-2}
= 0.16 N mm^{-2}
This is less than all materials given so can support all.

53. **Answer:** slightly greater than 0^o

Working: Since c > 1 and 1.5 > 1, each successive plate has higher refractive index than the last, so there will not be any total internal reflection. So the angle of refraction will always be less (refracted closer to the normal each time) than the angle of incidence.
Repeating this process many times, the angle of refraction will decrease further and further, closer to 0^o (almost parallel to normal).

54. **Answer:** Magnitude: *BI*, direction: downwards

Working: Field due to P, Q and R rotate anti-clockwise due to right-hand grip rule. Since *r* and *I* (and therefore |**B**| from each wire) are the same, P and R cancel out so only Q has an effect (down).

55. **Answer:** 1.5 kg

Working: Applying Newton's second law,
Weight - Air resistance = ma
mg - 12 = 2m → 8m = 12 → m = 1.5 kg

56. **Answer:** 3.0 W

Working: Let resistance of each resistor be *R*.
Total resistance of circuit = *R*/2 + *R* = 3*R*/2.
Power = I^2R
Fraction of current passing through X = 1/2
(identical resistances → current splits in half at junction)
Fraction of resistance of X = R/(3R/2) = ⅔
Fraction of power = $(1/2)^2$ * (2/3) = 1/6
Power in X = 1/6 * 18 = 3

57. **Answer:** None of them

Working: 1: False - the epicentre must lie on the perpendicular bisector line of A and B, not necessarily at the midpoint.
2: False - Z could be anywhere
3: False - Z must be 240 km further from the epicentre than A and B, but this does not guarantee it is within this range of A and/or B.

58. **Answer:** $1 - \frac{k \cos\theta}{\sin\theta}$

Working: We want (final KE) / (initial PE). By conservation of energy,
= (initial PE - work done by friction) / (initial PE)
= 1 - (work done by friction) / (initial PE)
= 1 - (kRL) / (mg L sin θ)
= R = mg cos θ → = 1 - (kmg L cos θ) / (mg L sin θ)
Cancel out mgL → = 1 - (k cos θ) / (sin θ)

(The constraint of *k* < tan θ ensures that friction is not strong enough hold the block in place and prevent it from sliding down.)

59. **Answer:** 14 N

Working: Acceleration of whole system = constant
$= F/m = 30/(3 + 4 + 6 + 2) = 30/15 = 2\ ms^{-2}$.
Applying Newton's second law to Z alone, (T_{YZ}: tension in rod YZ)
$30 - T_{YZ} = 2 * 2 \rightarrow T_{YZ} = 26$ N
Applying Newton's second law to Y alone, (T_{XY}: tension in rod XY)
$26 - T_{XY} = 6 * 2 \rightarrow T_{XY} = 14$

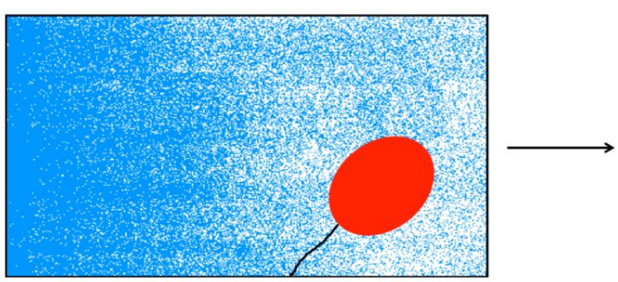
60. **Answer:** **D**

Working: Y moves at speed 2v in direction opposite to X
→ initial velocity = -2v
Conservation of momentum:
mv + (2m)(-2v) = -mv + final momentum of Y
Final momentum of Y = -3mv + mv = -2mv
Final velocity of Y = (-2mv)/(2m) = -v

(Could also have visualised by intuition that Y would not change direction so must be D)

61. **Answer:** The balloon sways forwards.

Working: When the truck accelerates, a force acts on the truck (say, to the right). This force does **not** act on the air in the back of the truck, which therefore remains at rest (Newton's third law; inertia). So the air moves to to the left making that side more dense. The balloon (helium) now floats in a density (and hence pressure) differential and so sways to the right, 'floating' above the dense air.

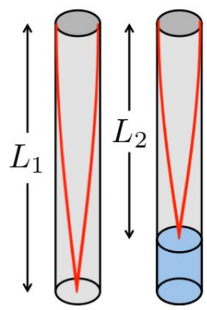


62. **Answer:** The electron degeneracy pressure being constant despite variation in stellar mass

Working: Chandrasekhar limit = 1.44 * solar mass
Below this mass, the pressure at the core is sufficiently repelled by the electrons, but above, the electron degeneracy pressure is overcome, collapsing the star rapidly (hence unstable) and resulting in a supernova.

63. **Answer:** 5 cm

Working: Frequencies in ratio 3 : 4 → wavelengths in ratio 4 : 3 (since $v = f\lambda$).



L_1 = 20 cm → L_2 = 15 cm → depth of water = 5 cm.

64. **Answer:** **(B)**

Working: When falling: $K = 1/2\ mv^2$
Since when falling, $v \propto t$ (from $v = u + at$ with $u = 0$) so
$K \propto t^2$ (a parabolic curve going upwards - either **B** or **C**)
During collision, the speed drops to zero as the ball is compressed and returns to its original size so energy also drops sharply.

65. **Answer:** ratio: 1.0×10^{-8} P: microwave Q: X-ray

Working: $c = f\lambda \rightarrow f \propto 1/\lambda$. Approximate wavelengths by magnitude in m:
gamma = 10^{-12}, X-ray = 10^{-10}, UV = 10^{-8}, Vis = 10^{-7}, IR = 10^{-5}, Micro = 10^{-2}, Radio = 10^{0}. The only correct pair differing by 10^8 are microwave and X-ray.

66. **Answer:** 330 W

Working: $V_P / V_S = n_P / n_S = I_S / I_P \rightarrow I_S = 4 * (240 / 4800) = 0.2$ A
$P = I^2R = 0.2^2 * 1500 = 330$ W

67. **Answer:** l = 0.80 m, d = 4 cm, θ = 40 °C

Working: rate of conduction (W) ∝ area, ∝ 1/distance, ∝ temp gradient
→ for lowest rate of conduction, d should be small, l should be large, and θ should be closer to 90 °C

68. **Answer:** 8.0 N

Working: Kinetic energy in terms of momentum: $E_k = p^2/(2m) \rightarrow p = \sqrt{(2mE_k)}$
$\Delta p = \sqrt{(2 * 4 * 200)} - \sqrt{(2 * 4 * 32)} = 24$ Ns
impulse of force: $J = \Delta p = Ft \rightarrow F = 24/3 = 8$ N.

69. **Answer:** potential difference across the ends

Working: $I = nevA \rightarrow v = \dfrac{I}{Ane}$.
Since I = V/R, and R will be constant for a given solution of same dimensions (based on resistivity, a material property), then v ∝ V.
v ∝ 1/n → as concentration of ions increases, n (number density) increases so v decreases.
v ∝ 1/e → as charge per ion increases, v decreases.
(Length of the container does not affect it.)

70. **Answer:** 17 km

Working: Instead of using mgh for the GPE we must instead use E = -GMm/r.
initial KE + initial GPE = final GPE
$0.5 * 0.005 * 240^2 - 6.67 * 10^{-11} * 7.4 * 10^{22} * 0.005 / (1.7 * 10^6)$
$= - 6.67 * 10^{-11} * 7.4 * 10^{-22} * 0.005 / (h + 1.7 * 10^6)$
Solving for h, we get h = 17032 m = 17 km.

71. **Answer:** compliance

Working: Ultimate tensile strength: maximum stress before material failure
Depends on material properties → bulk
Compliance = 1/Stiffness = 1/k → not a material property (k varies with dimension by k = λ/L where λ is modulus of elasticity and λ = EA (E: Young modulus, A: cross sectional area)
Young modulus: bulk property since independent of dimension (material property)
Yield stress: maximum stress before plastic deformation
Depends on material properties → bulk

72. **Answer:** 820 kPa

Working: 120 lbs / 1 in^2 = (120 * 0.454) = 54.48 kg / in^2
= 54.48 / $(0.0254)^2$ = 84444 kg m^{-2}
But pressure is measured in Newtons per metre squared and this is mass per metre squared, so multiply by *g*:
84444 * 9.81 = 828000 N m^{-2} = 830 kPa.

73. **Answer:** magnitude of the gravitational field strength

Working: $V = -GM/r$ → gradient = $dV/dr = GM/r^2$
Since field strength, $E = V/r$ and $E = -GM/r^2$, we have
gradient = $-E$ → gradient = $|E|$

74. **Answer:**

3	58

Working: Neutrons in fissile uranium nucleus = 235 + 1 = 236
Remaining neutrons = 236 - 88 - 145 = 3
Protons in ^{145}La = 92 - 35 = 57
Beta decay converts a neutron into a proton, so
protons in decay product = 57 + 1 = 58

75. **Answer:** $x - 3$

Working: x - 3 protons are balanced by x - 3 electrons to make a neutral atom overall. (The Q^{2+} ion would have x - 5 electrons.)

76. **Answer:** distance z

Working: Using conservation of energy,
→ initial KE + initial GPE = final KE + final GPE
→ final KE = initial KE - (final GPE - initial GPE)
→ final KE = initial KE - change in GPE
→ $1/2\ mv^2 = ½\ mu^2 + mgz$ [since $x - y = z \rightarrow y - x = -z$]
Multiplying through by 2/m,
→ $v^2 = u^2 + 2gz$
Since u and g are known, if we knew z we could calculate v.

77. **Answer:** The variation of the wavelength (y-axis) of waves with a speed of $0.2\ ms^{-1}$ with their frequency (x-axis)

Working: Gradient = 20/100 = 0.2
1st case: $F = ma \rightarrow a = (1/m)F \rightarrow$ gradient = 1/5 = 0.2
2nd case: $V = IR \rightarrow I = (1/R)V \rightarrow$ gradient = 1/5 = 0.2
3rd case: $E_k = ½\ mv^2 \rightarrow E_k = (m/2)v^2 \rightarrow$ gradient = 0.4/2 = 0.2
4th case: $v = f\lambda \rightarrow \lambda = v/f \rightarrow$ gradient is not constant
(reciprocal graph)

78. **Answer:** 13.4 billion years

Working: 1 megaparsec = $3.09 * 10^{19}$ km
→ $H_0 = (73\ km\ s^{-1}\ Mpc^{-1}) / (3.09 * 10^{19}\ km\ Mpc^{-1})$
→ $H_0 = 2.362 * 10^{-18}\ s^{-1}$
→ $t \approx H_0^{-1} = 4.233 * 10^{17}$ s
Converting to billions of years,
→ $t = 4.233 * 10^{17} / (60 * 60 * 24 * 365 * 10^9)$
→ t = 13.4… billion years.

79. **Answer:** 11

Working: Diffraction slit separation = 1/(300 * 10^3) = 3.333 * 10^{-6} m^{-1}
Using the formula $d \sin \theta = n\lambda$, with θ at the maximum possible diffraction angle of 90º (the edges of the screen), we get
3.333 * 10^{-6} * sin(90) = n(630 * 10^{-9})
→ n = 5.29
So the highest order spot is 5 (rounding down)
These spots appear on both sides → 10 spots due to diffraction
There is also the zero-order spot i.e. at θ = 0 for light that goes straight through without any diffraction.
→ total 11 spots.

80. **Answer:** 0%

Working: Isothermal → constant temperature → constant internal energy
From r.m.s. E_k = 3/2 kT, we find
1/2 mc^2 = constant → c^2 = constant
→ no change.

Section B: Standard

1. **Answer:** 1500 kg m^{-3}

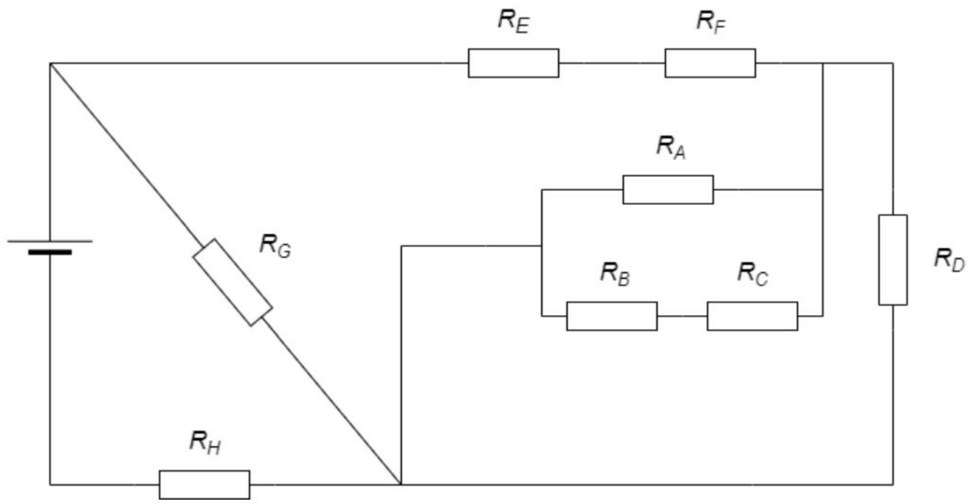
Working: Liquids are at equilibrium → no resultant force → pressure exerted by both sides is equal
Hydrostatic pressure = hρg → hρ = constant
(0.55)ρ = (0.80)(1000) → ρ = 1454 = 1500 (2 s.f.)

2. **Answer:** Elastic potential energy stored per unit volume of material

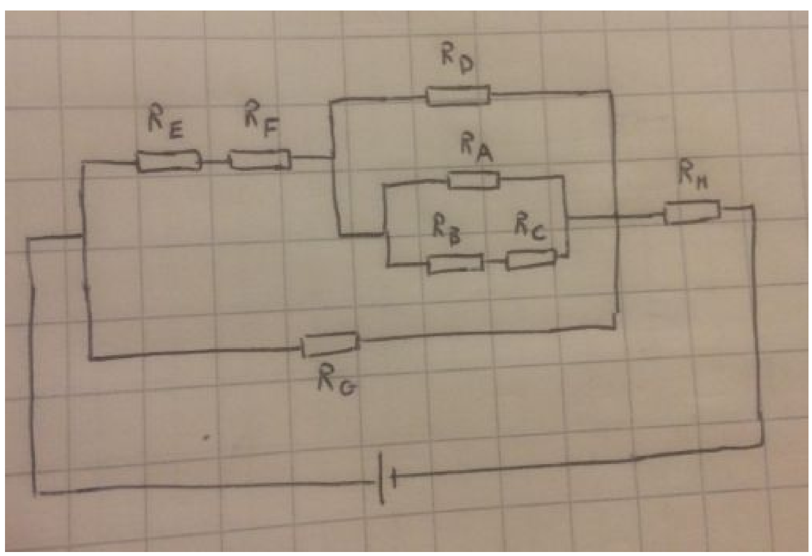
Working: Consider a graph of stress vs strain when Hooke's law is obeyed.
Graph is triangular → area = ½ * stress * strain
Using stress = F/A and strain = x/L,
Area = ½ (Fx)/(AL)
Since elastic potential energy = ½ Fx and volume = AL,
Area = elastic potential energy / volume
→ represents EPE stored per unit volume

3. **Answer:** 65.2 Ω

Working: For reference, label the resistors $R_A \dots R_H$:



It will be helpful to redraw the circuit replacing the diagonal with only vertical and horizontal lines, so the series and parallel branches become more obvious:



Add R_E and R_F (series) = 16 + 47 = 63
Add R_A, R_B and R_C (series then parallel)
= $(1/(123 + 21) + 1/54)^{-1}$ = 39.27
That branch is in parallel with R_D: = $(1/39.27 + 1/71)^{-1}$ = 25.29
Add series then parallel: $(1/106 + 1/(63 + 25.29))^{-1}$ = 48.17
Finally, add series: 48.17 + 17 = 65.17 = 65.2 Ω.

4. **Answer:** 32 m

Working: Transverse wave → particle oscillates vertically (perpendicular to direction of propagation)
Distance in one cycle = up, down, down, up → 4 * amplitude
= 4*0.04 = 0.16 m
Time period = 1/10 = 0.1 s → cycles = 20/0.1 = 200
Total distance = 0.16 * 200 = 32 m

5. **Answer:** Momentum is transferred through the wall to the Earth.

Working: Speeds are different → kinetic energy has changed → inelastic.
When a particle collides with a plane surface, the impulsive (contact) force acts normal to the plane and any other force (friction) acts parallel.
Check components of velocity parallel to the plane:
Before: 7 cos 40 = 5.36, after: 5 cos 26 = 4.49
Velocity parallel to plane has decreased → friction has had an effect → surface is not smooth.
→ resultant of contact and friction is not normal to wall.
→ momentum must have been transferred to conserve.

6. **Answer:** $v = k \sqrt{d}$, where k has units of $m^{1/2}s^{-1}$

Working: $v = k\, d^n$, taking logs on both sides:
$\ln v = \ln(k\, d^n)$, using identities:
$\ln v = \ln k + n \ln d$
$y \quad = c \quad + m\, x$
→ gradient = n = 0.5
→ $v = k\, d^{0.5} = k\sqrt{d}$
Units of k: $k = v/\sqrt{d}$, $[k] = ms^{-1}/m^{1/2} = m^{1/2}s^{-1}$.

7. **Answer:** $(3.81 \pm 0.25) \times 10^{26}$ W

Working: Luminosity is $L = 4\pi r^2 \sigma T^4$

Mean value: $L = 4\pi(1.39/2 \times 10^9)^2(5.67 \times 10^{-8})(5770)^4$

$= 3.81 \times 10^{26}$

Uncertainties:

r^2 : 2 * (0.04/1.39) = 0.05755

σ (due to significant figures): 0.005/5.67 = 0.00088

T^4 : 4 * (9/5770) = 0.00624

Total percentage (fractional) uncertainty

= 0.05755 + 0.00088 + 0.00624 = 0.06467 = 6.467 %

Range = 0.06467 * $(3.81 \times 10^{26}) = 2.5 \times 10^{25} = 0.25 \times 10^{26}$

8. **Answer:** Diffraction effects could be a problem because the distance between the transmitting antennas is comparable to the wavelength.

Working: Diffraction occurs noticeably when the gap between two sources is close to the wavelength.

$\lambda = c/f = (3 * 10^8) / (600 * 10^6) = 0.5$ m.

Distance between maxima along line at receiver = $\lambda d/a$

(from $\lambda = ax/d$) = 0.5 * 1000 / 1 = 500 m.

Since $\lambda = 0.5 \sim 1$, diffraction effects could be significant.

Since 500 m >> λ, the distances between maxima are not close.

(1st option is false: frequency has no effect on diffraction)

(2nd option is false: wavelength is close to gap size)

(3rd option is false: distance between maxima is large)

9. **Answer:** About 12% of the electricity generated is lost in the distribution network.

Working: 1st option is false: ‘efficiency’ is with reference to the amount of energy from the fuel source which is converted into useful electrical energy – this does not refer to the amount of electrical energy which is generated or consumed.
2nd option is false: again ‘efficiency’ is with reference to how much of the consumed energy is converted into useful other forms – this is not relevant to the difference between generation and consumption.
4th option seems unlikely as all electronics are tested and electricity consumption is managed centrally.
By process of elimination, 3rd option is correct.

10. **Answer:** **1**, **2** and **3**

Working: Torque = force * distance between coupled forces
The force is provided by the magnetic field:
$F = BIL$ so torque $\tau = BILd$
where d = distance AD and L = distance AB
So all options increase one of these variables.

11. **Answer:** None of them

Working: 1. False: since oscillations are vertical and mass is non-negligible there will be variations in GPE which contribute to total energy.
2: False: since at $x = 0$ (equilibrium), there is no resultant force, so weight = tension $\rightarrow mg = kx \rightarrow x = mg/k$ which is not zero.
3: False: definition of SHM is acceleration opposite/proportional to displacement not velocity.

12. **Answer:** 1.9 s

Working: $s = 49$, $u = ?$, $v = 26$, $a = 0.8$, $t = ?$
$v^2 = u^2 + 2as \rightarrow u = \sqrt{(v^2 - 2as)} = 24.446\ ms^{-1}$
$v = u + at \rightarrow t = (v - u)/a = (26 - 24.446)/0.8 = 1.94\ s = 1.9\ s$ (2 s.f.)
(*Alternative*: rearrange $s = vt - ½\ at^2$)

13. **Answer:** 0.40 m to the right of the pivot

Working: Let x = displacement of centre of gravity from pivot, with positive values meaning to the right. Taking moments about the pivot:
(0.80 * 60) = (1.20 * 35) + (x * 15)
(g has been cancelled out: these are mass-moments)
48 = 42 + 15x → x = 6/15 = 0.40 m
Positive → to the right of pivot

14. **Answer:** 6400 J

Working: Energy before = energy after
KE before + GPE before = KE after + GPE after + energy lost
$(½ * 200 * 5^2) + (200 * 8 * 10) = (½ * 200 * 9^2) + (200 * 10 * 2) + E$
E = 18500 - 12100 = 6400 J (2 s.f.)

15. **Answer:** 4/5 *R*

Working: Same material → same resistivity, also same length.
Using $R = \rho L/A$,
Resistor Y: $R = \rho L/A = \rho L/(\pi d^2/4) = 4\rho L/(\pi d^2)$
Resistor X: d is half → d^2 is quarter: $R = \rho L/(\pi d^2)$
Total resistance (parallel) = $(4\rho L/(\pi d^2))(\rho L/(\pi d^2))/(5\rho L/(\pi d^2))$
= (4R)(R)/(5R) = (4/5)R

16. **Answer:** 70 Bq

Working: 24 hours later: X has undergone (24/4.8) = 5 half lives and Y has undergone (24/8) = 3 half lives.
New activity of X = 320 * $(1/2)^5$ = 320/32 = 10 Bq
New activity of Y = 480 * $(1/2)^3$ = 480/8 = 60 Bq
Total activity = 10 + 60 = 70 Bq

17. **Answer:** The filament consumes less electrical power towards the end of the life of the bulb

Working: Towards the end of the life filament will become thinner. Resistance will increase and so consumed power will be less ($P = V^2/R$), so it will emit less light. Temperature distribution will be non uniform (evaporation of tungsten and hence change in internal energy varies). At the position where temperature is maximum, the filament will break. Black body radiation curve will become flat so the filament consumes less electrical power towards the end of the life of the bulb.

18. **Answer:** **1**, **2** and **3**

Working: 1. True: work done to overcome resistive forces/additional energy used for KE of mass will not be accounted for so the motor is actually providing more power than measured.
2. True: only other quantities are time and height, these are measured with metre ruler (resolution 1 mm, 0.1 %) and timer (resolution 0.01 s, expect < 1 % since time to lift likely > 1 s)
3: P_{input} = 1/0.15 P_{output} = 20/3 P_{output}
P_{output} = GPE gained / time taken = mgh / t = (m/t) gh, h = 1.0 m
= gradient * g → P_{input} = 20/3 * gradient * g = (g/0.15) * gradient
→ derived formula is correct but does not take into account KE
→ only valid when KE ~ 0 → only valid at low speeds → 3 is true.

19. **Answer:** kg m^3 A^{-2} s^{-3}

Working: Resistivity: $R = \rho L/A \rightarrow \rho = AR/L$
$[\rho] = \Omega$ m
$\Omega = V / A = J\ C^{-1}\ A^{-1}$
$J = kg\ m^2\ s^{-2}$ (from kinetic energy equation), C = A s (by definition)
$\rightarrow \Omega = kg\ m^2\ s^{-2}\ A^{-1}\ s^{-1}\ A^{-1} = kg\ m^2\ A^{-2}\ s^{-3}$
$\rightarrow [\rho] = kg\ m^3\ A^{-2}\ s^{-3}$

20. **Answer:** 270 K

Working: The first 5 g evaporates completely
→ energy needed to heat and boil 5 g liquid = energy released
$\rightarrow (5 * c * (80 - 30)) + (5 * L) = M_{cup} * C_{cup} * (110 - 80)$
$\rightarrow 250c + 5L = 30\ M_{cup}C_{cup}$ (Eqn. 1)
The next 80 g cools the cup to 50 ^oC:
→ energy needed to warm 80 g liquid = energy released
$\rightarrow 80 * c * (50 - 30) = M_{cup} * C_{cup} * (80 - 50)$
$\rightarrow 1600c = 30\ M_{cup}C_{cup}$ (Eqn. 2)
Divide Eqn 1 by Eqn 2:
$\rightarrow (250c + 5L) / 1600c = 1$
$\rightarrow 250/1600 + (L/c) / 320 = 1$
$\rightarrow L/c = 320 * (1 - 250/1600) = 270$
Units: $(J\ g^{-1}) / (J\ g^{-1}\ K^{-1}) = K$

21. **Answer:** The angular frequency of the rotating plate is about 16 rad s^{-1}.

Working: Time between pulses = time period = 0.4 s
Angular frequency = $2\pi/T = 2\pi/0.4 = 15.7$ rad s^{-1} ~ 16 rad s^{-1}
When in the light, V across resistor = 4 so V across LDR = 0.5 V.
When in the dark, V across resistor = 2 so V across LDR = 2.5 V.
Current in light: $V = IR \rightarrow I = V/R = 4/1200 = 0.00333$ A
→ Resistance of LDR in light = V/I = 0.5/0.00333 = 150 Ω
Current in dark: I = 2/1200 = 0.001667 A
→ Resistance of LDR in dark = 2.5/0.001667 = 1500 Ω
Difference in resistance is 1500 - 150 = 1350 Ω, not 450 Ω.

22. **Answer:** The complete diffraction pattern shifts in the *y*-direction because the path difference required for a maxima to appear has changed.

Working: Time period of light in air: λ/c = 600 nm / c = T
Time to travel 300 nm in vacuum = 300 nm / c = T/2
Time to travel 300 nm in material = 300 nm / (c/2) = T
Therefore, after passing through the material, the light passing through one slit will be half a cycle out of phase with the other.
Before, a maximum forms if path difference is $n\lambda$ or $2n\pi$ phase difference, but now since the initial phase difference is π, only π, 3π, 5π etc more phase difference is required. So the maxima form at different path differences and hence at different positions (shifted in y-direction).

23. **Answer:** the ball rises and falls through equal distances between impacts.

Working: The (signed: below axis = negative) area under the velocity-time graph represents displacement.
First option is false: acceleration is always -g except at the instant of collision where it is a very large positive value.
Second option is true but does not explain the result.
Third option is false since speed from the graph clearly decreases with each collision.

24. **Answer:** $2\rho hg$

Working: Increase in pressure of flask = hydrostatic pressure of liquid
If the RHS of the liquid moves up by distance h, then the LHS must also move down by h. So the difference in height between the levels is now 2h.
Then $p = \rho$ * height * $g = 2\rho hg$.

25. **Answer:** **A**

Working: (B) is wrong: on the graph, the maximum value of Y is equal to/slightly greater than the initial value of X. This cannot be true since one nuclei of X forms one nuclei of Y and since there is still some X remaining the amount of Y must be less than its initial.
(C) is wrong: here the amount of Y appears not to decrease at all but it should since Y is also radioactive.
(D) is wrong: here the half life of Y appears to be less than/the same as that of X.
By process of elimination, the answer is A.

26. **Answer:** $3T$ years

Working: After T years → 12.5% of the total becomes 25%
After $2T$ years → 25% becomes 50%
After $3T$ years → 50% becomes 100%
After this the plant output becomes less than the city's needs.

27. **Answer:** +*q*: executes SHM; -*q*: continues moving in direction of its displacement

Working: The field lines run in the *x*-direction, opposing each other and falling to zero in the centre line. Let *d* be the half-distance between the charge lines. Then the resultant electric field acting on the charge at displacement *x* is

$$E = \frac{\lambda}{2\pi\varepsilon_0(d+x)} - \frac{\lambda}{2\pi\varepsilon_0(d-x)}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{2x}{x^2 - d^2}$$

Assume *x* << *d* (small displacement). Then the denominator approaches $-d^2$ and so we get

$$E = -\frac{\lambda}{\pi\varepsilon_0 d^2} \cdot x$$

Since F = qE, and E is proportional to -x, we find force (and hence acceleration) is proportional to negative displacement when q > 0 and proportional to positive displacement when q < 0.
So +ve q satisfies SHM condition while force for -ve q continues Increasing.

28. **Answer:** The voltmeter displays +5 V as soon as the switch is pressed, and displays -5 V after a long time

Working: Effective circuit resistance (combining in parallel)
= R = $(1/50 + 1/25)^{-1}$ = 50/3 kΩ = 16.667 kΩ
Effective circuit capacitance (combining in parallel)
= C = 40 + 20 = 60 μF
Time constant = RC = 1 s
At t = 0, capacitors act as open switch and voltmeter shows -5 V.
At t = ∞, capacitors acts as open switch and no current flows through voltmeter (very high resistance of voltmeter) so it's +5V.
After a long time no current flows since both capacitor
and voltmeter do not allow current to flow.

29. **Answer:** the graph of $v(x)$

Working: Derive a formula for current (valid when flux is changing, else zero)

$$\varepsilon = V = IR \to I = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{\Delta BA}{\Delta t} = -\frac{BL}{R}\frac{\Delta x}{\Delta t} = -\frac{BLv}{R}$$

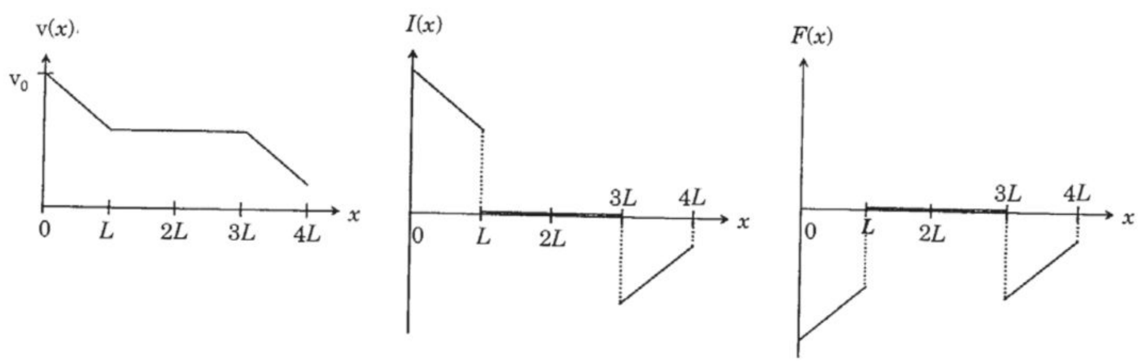
(the validity of this formula needs to be considered carefully - it is a simplification of applying calculus techniques. It can be thought of intuitively however by considering the change in area of the square exposed to the magnetic field.)

For 0 < x < L, the magnetic flux is changing from zero to a positive value so there is a positive rate of change. So by Lenz's law, a force will be exerted against the motion (negative), decreasing *v* and hence decreasing *I* (by the derived formula). All graphs match.

For L < x < 3L, the magnetic flux density is constant so the flux linkage is constant, and so no change and hence no emf and no force (constant velocity). All graphs match this.

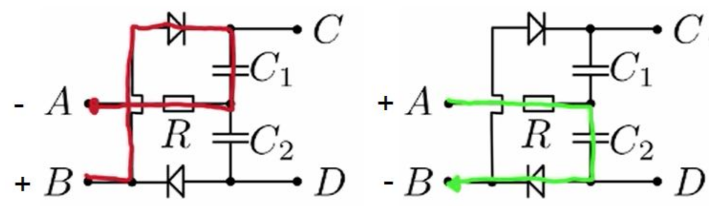
For 3L < x < 4L, the magnetic flux changes from a positive value to zero so there is a negative rate of change. A force will be exerted to the left (negative) so velocity will decrease again. This time however, the rate of change of area (and hence flux linkage) means the current is induced in the opposite direction with decreasing magnitude. I(x) and F(x) are both different here.

Correct graphs:



30. **Answer:** The two capacitors alternate between charging and retaining their charge, and the charging cycles are **180° out of phase** with each other.

Working: Consider the first oscillation of the a.c. current.
A is negative terminal → A held at 0 V → B oscillates between -340 V and 340 V. Suppose it starts at 340 V. Then current flows from B to A through the red path (shown below, left) through the diode (forward-bias) and charges capacitor C_1 while C_2 remains unaffected.



when V(B) > 0   when V(B) < 0

Then consider when the voltage drops below zero, to -340 V. Now A is at a higher potential than B so current flows the other way - the opposing diodes ensuring this time C_2 is charged by the green path (shown above, right).
So the capacitors take turns getting charged while the other retains its current level of charge.

After charging is complete, the p.d. across C_1 and C_2 are each 340 V and fixed, so CD is a d.c. source with e.m.f. 680 V.

31. **Answers:** a. The balls will bounce back to the bottom plate, carrying the opposite charge (same magnitude, opposite sign) as they went up with.

b. proportional to $V_0^{\ 2}$

Working: After hitting the top plate, the balls will get negatively charged and will now get attracted to the bottom plate which is positively charged. The motion of the balls will be periodic but **not SHM** since forces in electric fields are (inversely) proportional to x^2 not x.

If Q is charge on balls, then Q ∝ V_0 (since the balls act as spherical shell capacitors and Q = VC).
Let *t* be the time to rise to the top. From SUVAT and **E**-field eqns,

$$h = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{QV_0}{mh}\right)t^2$$

Since *h* is constant, we find

$$t \propto \frac{1}{V_0^2}$$

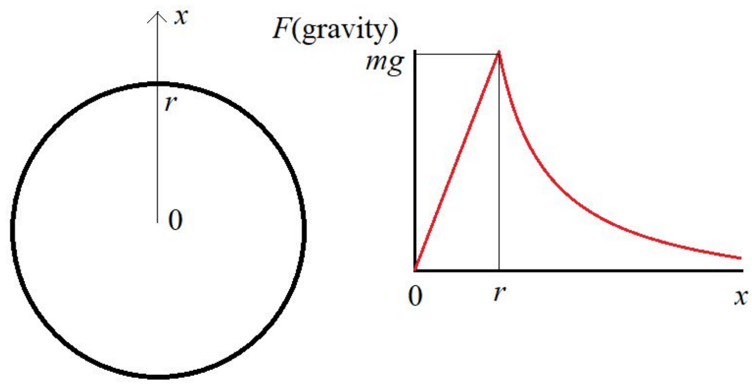
and since I = Q/t, we get I ∝ V_0^2.

32. **Answer:** The ratio of $\frac{d_{fog}}{d_{clear}}$ decreases with an increase in density of fog.

Working: Attenuation coefficient is dependent on how much the fog absorbs light (i.e. how fast the intensity drops off). So it is independent of the brightness of the surroundings.
Since intensity is usually inverse square, but with the fog it further decays exponentially making the overall relationship decay even faster, i.e. $I(x) = I_0 * \exp(-\lambda x) * 1/x^2$.
When the fog gets thicker, the light gets harder to see (since higher attenuation), so d_{fog} decreases.

33. **Answer:** at *B*, the surface of the plant

Working: Gravity decreases (inverse square) outside the planet, but at a point inside, the portion of the mass closer to the centre only affects the gravity. Since as *r* decreases, *V* (and hence *m*) decreases with r^3, and so the force decreases with $F = GMm/r^2 \propto r^3 / r^2 \propto r$:



34. **Answer:** $C = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$

Working: Q = VC → C = Q/V

V = potential difference between the two spheres

ΔV = potential at inner surface - potential at outer surface

$$\Delta V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

Alternative method: the shell capacitor is essentially two isolated spherical capacitors in series, with $C = 4\pi\varepsilon_0 a$ and $C = 4\pi\varepsilon_0 b$

Now combining capacitances in series,

$$C_{total} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{16\pi^2\varepsilon_0^2 ab}{4\pi\varepsilon_0(a+b)} = 4\pi\varepsilon_0\left[\frac{ab}{a+b}\right] = 4\pi\varepsilon_0\left[\frac{1}{a} + \frac{1}{b}\right]^{-1}$$

36. **Answer:** 53^o

Working: Angle of refraction: $\sin(45) = 1.33 \sin \theta_r \rightarrow \theta_r = 32^o$
Using simple trigonometry, $\tan \theta = 3 / (1 + 2 \tan 32) = 1.33$
$\rightarrow \theta = 53^o$.

37. **Answer:** 4 %

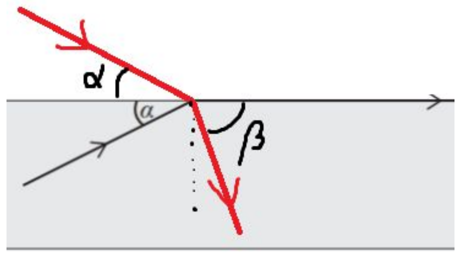
Working: Number of small lines per main scale division
= Distance between main lines / Resolution
$= 0.05 \times 10^{-2} / 10^{-5} = 50$.
$\rightarrow$ extension = (45 - 20) = 25 divisions = $25 * 10^{-5} = 2.5 * 10^{-4}$ m
$\rightarrow$ error in extension = $\Delta x/x = (10^{-5}) / (2.5 * 10^{-4}) = 0.04 = 4\%$

38. **Answer:** If the wires are made of different materials, then the different thermal expansions of the wires due to ambient temperature variation during the experiment will introduce an error.

Working: The materials should be the same so that any variation due to temperature is accounted for by moving the micrometer along, eliminating it.
If length is shorter then percentage error increases.
After adding/removing a load, we should wait a little time so that the wire has chance to fully extend before taking its measurement.
The purpose of the test weight is to ensure that both wires are taut, but it is not needed that there be no tension. Usually there will be a reasonable amount of tension initially which is taken as the initial length.

39. **Answer:** $\cos\beta = \cos^2\alpha$

Working: Diagram:



Let the refractive index of liquid be *n*.
Black (lower incident) ray is at the critical angle: $1/\sin(90 - \alpha) = n$
$\rightarrow n = 1/\cos\alpha$
Using Snell's law for the red (upper incident) ray,
$\sin(90 - \alpha) = n \sin(90 - \beta)$
$\rightarrow \cos\alpha = n \cos\beta$
$\rightarrow \cos\alpha = 1/\cos\alpha * \cos\beta$
$\rightarrow \cos\beta = \cos^2\alpha$

40. **Answer:** $\Delta T_X < \Delta T_Y < \Delta T_Z$

Working: Temperature rise → energy transferred to water in fixed time
→ power dissipated as heat by resistors
$P = V^2/R$ (since V is constant, $\Delta T \propto 1/R$ (roughly)).
Let resistance of one resistor be R.
Equivalent resistance of X = 2R
Equivalent resistance of Y = $R + (1/R + 1/R)^{-1} = R + R/2 = 1.5\ R$
Equivalent resistance of Z = $(1/(2R) + 1/R)^{-1} = 2R^2 / 3R = 0.67\ R$
So resistances are Z > Y > X
→ temperature changes are X < Y < Z (reverse order).

41. **Answer:** 0.01 W

Working: $E = ½\ kx^2 \rightarrow x_{max} = \sqrt{(2 * 0.25 / 50)} = 0.1$ m
$E = ½\ Tx \rightarrow T_{max} = 2 * 0.25 / 0.1 = 5$ N
→ time stretching = 5 / 0.2 = 25 s
Power = energy / time = 0.25 / 25 = 0.01 W

42. **Answer:** **1** and **2** only

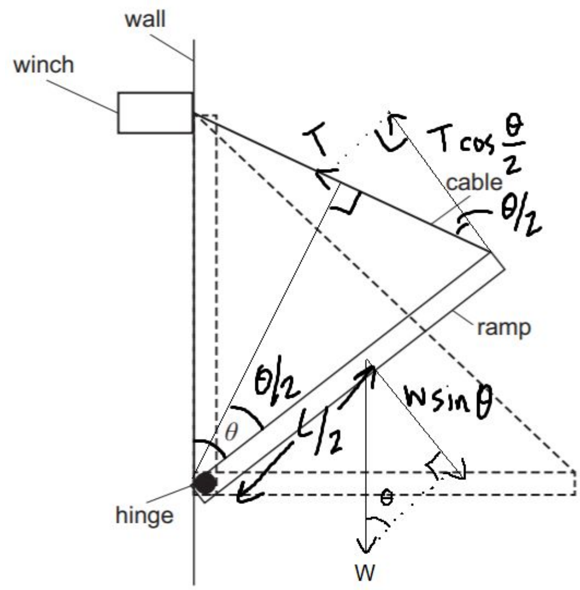
Working: Since P has lower resistance than R, the potential at the top terminal of the voltmeter is higher than at the bottom.
1: if resistance x is added in parallel with P, this would further decrease the resistance of that branch so the p.d. would increase.
2: if in series with Q then resistance of top branch is $4x$ while bottom is $3x$. So more current flows in bottom branch, increasing the p.d. drop across R and hence further increasing the p.d. across the voltmeter.
3: if in parallel with R then resistance of that branch decreases, bringing the p.d. drops across it and P closer, decreasing p.d. across voltmeter.

43. **Answer:** **1** and **3** only

Working: We cannot neglect air resistance since the graph is not a straight line (if no air resistance then gradient = $-g$ = constant).
1: Highest point → speed = 0 → drag force = 0 → only force acting is gravity → acceleration = g = free fall. All other times have some speed so will have some drag decreasing the acceleration.
2: Since the areas above and below the curve must be equal (because distance up = distance down), the time must be longer in the falling portion to counter the decrease in speed.
3: on the way up, initial KE = final GPE + air resistance losses
→ final GPE < initial KE
on the way down, initial GPE = final KE + air resistance losses
→ initial GPE > final KE

44. **Answer:** 70 kN

Working: Let weight of bridge be W, length of ramp be L,
tension in cable be T. Resolving forces perpendicular to the ramp,



Now taking moments about the hinge,
L/2 * (W sin θ) = L * (T cos θ/2)
→ T = (W sin θ) / (2 cos θ/2)
Now let θ = 2*x* → T = (W sin 2x) / (2 cos x)
Using the given formula,
T = (2W sin x cos x) / (2 cos x)
T = W sin x
Since θ varies from 0 to 90°, x varies from 0° to 45°.
Since sin θ is increasing in this interval, the maximum value is
when x = 45 → T = W sin 45° = W/√2.
Now, W = 10 * 1000 * 10 = 100 000 N
→ T = 100000 / √2 = 70710 N → 70 kN

45. **Answer:**

X to Y	between 45^o and 60^o

Working: Let the refractive index of X be n_X and that of Y be n_Y.
$n_X = 1/\sin(45^o) = \sqrt{2} = 1.414\ldots$
$n_Y = 1/\sin(60^o) = 2\sqrt{3}/3 = 1.155\ldots$
So X is more optically dense than Y.
Critical angle is defined for light going from high optical density into low optical density, so direction of incidence is X into Y.
Critical angle when X and Y form a boundary,
$\sin \theta_C = n_Y / n_X = (2\sqrt{3}) / (3\sqrt{2}) = \sqrt{6} / 3 = 0.816$
$\rightarrow \theta_C = 54.7^o \rightarrow$ between 45^o and 60^o.

46. **Answer:**

0.40	$N\ m^{-2}\ s^2$

Working: Terminal velocity = asymptotic gradient = (480 - 200) / (12 - 6.4)
= 50 ms^{-1}.
Drag force = $F = k(50)^2 = 2500k$
At terminal velocity, drag = weight
$\rightarrow 2500k = 100 * 10$
$\rightarrow k = 0.40$
Units = $[F] / [v]^2 = N / (ms^{-1})^2 = N\ m^{-2}\ s^2$

47. **Answer:** 20 000 years

Working: Ratio of living to dead:
$(10^3 / 10^{15}) / (1 / 10^{13}) = 10$
Half lives passed = $\log_2(10) = \ln(10) / \ln(2) = 3.322$
Time passed = 3.322 * 6000 = 19932 = 20000 years

48. **Answer:** Lights after a delay with a final brightness the same as X

Working: The induced emf in the coil will oppose the cell emf and cause a delay in the current to lamp Y.

49. **Answer:** Two springs in parallel combine their force constants in the same way two resistors in series combine their resistances.

Working: Springs and capacitances combine similarly (they both store energy), while resistors combine in the opposite way (it dissipates energy). All are of the form $x_T = x_1 + x_2$ or $1/x_T = 1/x_1 + 1/x_2$.

50. **Answer:** **1** and **3** only

Working: **1**: GPE lost by spring = mgh, where m = mass of spring,
h = decrease in height of **centre of mass** of spring
If full spring extends by *x*, then centre of mass moves down by ½ *x* (since half-way point down the spring)
Now EPE stored in spring = ½ Tx = ½ mgx = mgh = GPE.
2: Equilibrium position is when no force acts.
Since at length 1.27 cm, the tension and self-weight are balanced, adding any other mass would change the equilibrium position to something longer than this.
3: mg = kx → mg = k(0.0007) → k/m = g/0.0007 = 14041.28…
= 14000 s^{-2}

Section C: Hard

1. **Answer:** $\dfrac{18\rho dL^2}{5M}$

Working: In general, $R = \rho L/A$

Let A be the area of one wire.

The seven wires are in parallel so the total resistance is

$= (1/R_{Cu} + 1/R_{Al} + 1/R_{Al} + 1/R_{Al} + 1/R_{Al} + 1/R_{Al} + 1/R_{Al})^{-1}$

$= (A/(2\rho L) + 6A/(3\rho L))^{-1}$

$= (A/(2\rho L) + 2A/(\rho L))^{-1}$

$= ((A\rho L + 4A\rho L) / (2\rho^2L^2))^{-1}$

$= (5A / 2\rho L)^{-1}$

$= 2\rho L / (5A)$

Considering the mass of the cable,

M = mass of copper + mass of aluminium

$M = 3d * AL + d * 6AL$

$M = 9dAL$

Rearrange → $A = M / (9dL)$

Subbing back into resistance equation,

→ $R = 2\rho L / (5 * (M / (9dL)))$

$= 2\rho L / (5M / 9dL)$

$= 18\rho dL^2 / 5M$

2. **Answer:** 7^o: violet (dimmest), 12^o: red (brightest)

Working: Colours with wavelength closest to diffraction spacing are diffracted through a larger angle.
Diffraction spacing = 1/(300 * 1000) = 3.3×10^{-6} m = 3300 nm
So longer wavelengths of light (red) are closest to this (800 nm)
So red is diffracted more while violet is diffracted less
On the graph, red has higher intensity than violet.

3. **Answer:** $\frac{\sqrt{2}}{2}$

Working: Conservation of energy: $\frac{1}{2} kx^2 + \frac{1}{2} mv^2$ = constant
$\rightarrow kx^2 + mv^2$ = constant
When equal: $kx^2 = mv^2$. Using $x = A \sin(\omega t)$ and $v = A\omega \cos(\omega t)$,
$k A^2 \sin^2 \omega t = m A^2 \omega^2 \cos^2 \omega t$
$k A^2 \sin^2 \omega t = m A^2 \omega^2 (1 - \sin^2 \omega t)$
$(k A^2 + m A^2 \omega^2) \sin^2 \omega t = m A^2 \omega^2 \rightarrow \sin \omega t = \sqrt{(m \omega^2 / (k + m \omega^2))}$
We need to express k in terms of $m \omega^2$.
$F = -kx \rightarrow ma = -kx \rightarrow a = -(k/m) x$
From the definition of SHM, we can see that $\omega^2 = k/m \rightarrow k = m\omega^2$
$\rightarrow \sin \omega t = \sqrt{(m \omega^2 / (m \omega^2 + m \omega^2))} = \sqrt{(1/2)} = \sqrt{2} / 2$
$\rightarrow x = (\sqrt{2} / 2) A \rightarrow p = \sqrt{2} / 2$

4. **Answer:** **2** and **3** only

Working: Processes I and III: V and T appear to be inversely proportional. Charles' law states that for constant pressure, V and T should be directly proportional which is not what we see in the graph.
Process II takes place at constant temperature, so there can be no change in internal energy. Since the volume increased, the gas must have done work on the surroundings but by conservation of energy this work must have come from absorbing heat energy.
Process IV also takes place at constant temperature, but this time volume has decreased so work was done *on* the gas. Again to maintain constant internal energy this additional work must have been released as heat energy.

5. **Answer:** The amplitude of the maximum net e.m.f. induced due to both the

loops is equal to the amplitude of maximum e.m.f. induced in the smaller loop alone.

Working: Magnetic flux linkage = BA cos θ where θ is the angle between the normal vector to the plane of the loop and the magnetic field.
Since θ = ωt, Φ = BA cos ωt (where A is area in general)
emf = - dΦ/dt = BAω sin ωt
emf induced in smaller loop = BAω sin ωt
emf induced in larger loop = 2 BAω sin ωt
Notice that the emf induced in the two loops have opposite polarity (direction) because of the given orientation of the wires.
So net emf = | larger - smaller | = BAω sin ωt
→ 4th option is true (they are equal)
→ 1st and 3rd options are false (area component with flux linkage is A cos ωt but emf is proportional to sin ωt)
When loop is perpendicular to paper → parallel to magnetic field
→ minimum (zero) flux linkage → maximum rate of change
→ 2nd option is false.

6. **Answer:** $2I_1 + 4I_3 = 13$

Working: Using KVL in left loop (clockwise as positive):
$5 - 2\ I_1 - I_2 = 0 \rightarrow 2\ I_1 + I_2 = 5$
Using KVL in right loop (anticlockwise as positive):
$8 - 4\ I_3 - I_2 = 0 \rightarrow I_2 + 4\ I_3 = 8$
Using KVL in whole outside loop (anticlockwise as positive)
$8 - 4\ I_3 + 2\ I_1 - 5 = 0 \rightarrow 4\ I_3 - 2\ I_1 = 3$
(5 is negative since the emf opposes the 8 V cell's emf)
(pd across 2 Ω resistor is positive since current flows clockwise through it so the double negative becomes positive.)
Only equation that does not appear is $2I_1 + 4I_3 = 13$

7. **Answer:** $\frac{1}{2}M\omega_0^{\ 2}(R-r)^2$

Working: KE of ring = ½ mv^2 where v is linear speed of the centre of mass of the ring. v = ωr where r is radius of centre of mass.
Radius of circular path of centre = radius of ring – radius of cone = R – r
→ $v = \omega(R - r) \rightarrow v^2 = \omega^2(R - r)^2$
→ $KE = \frac{1}{2} M \omega^2(R - r)^2$

8. **Answer:** 1.50

Working: Using units of μF, V and μC for convenience,
Charge stored initially on C_3 = Q = VC = 8 μC
So 3 μC have been lost to C_1 and C_2.
Capacitance of dielectric capacitor = $\varepsilon_r A/d$ but $C_2 = 1 = A/d$
→ Capacitance of $C_1 = \varepsilon_r$ μF.
Effective capacitance of left branch = $(1/1 + 1/\varepsilon_r)^{-1} = \varepsilon_r / (1 + \varepsilon_r)$
Consider the ratio of capacitances in the left and right branches.
Left: Q = 3, Right: Q = 5
→ Capacitance of left = 3/5 * Capacitance of right
→ $\varepsilon_r / (1 + \varepsilon_r) = 3/5 * 1 \rightarrow 5\varepsilon_r = 3(1 + \varepsilon_r) \rightarrow 2\varepsilon_r = 3 \rightarrow \varepsilon_r = 3/2 = 1.50$

9. **Answer:** 5.0 litres

Working: Initial activity = 2.4×10^5 Bq.
Half-lives passed = 11.5 hrs / (8 * 24) = 0.06
Activity after 11.5 hrs = $2.4 \times 10^5 \times (\frac{1}{2})^{0.06} = 2.302 \times 10^5$ Bq
Considering concentration ratios, $115 / (2.302 \times 10^5) = 2.5 / V$
→ V = 5004 ml = 5.0 litres

10. **Answer:** 17000 N to the right

Working: Conservation of mass: rate of flow in = rate of flow out

Rate in: 5 * 0.6 = 3 m^3/s

Rate out: 3 = V * 0.25 → V = 12 m/s

Force = rate of change of momentum

= mass flow rate * change in speed

F = (3 * 800) * (12 – 5) = 16800 = 17000 N (2 s.f.)

Direction is to the right since the oil increased in speed

11. **Answer:** **1** and **2** only

Working: Rearranging the equation, [μ_0] = 1 / [c^2 ε_0]

Units of ε_0 are F m^{-1} or C^2 N^{-1} m^{-2}

= 1 / ((m $s^{-1})^2$ * (C^2 N^{-1} m^{-2}))

= m^{-2} s^2 C^{-2} N m^2

= N C^{-2} s^2

= N A^{-2}

Check (2): Wb = T m^2 = N A^{-1} m^{-1} m^2 = N A^{-1} m

→ (2) is N A^{-1} m A^{-1} m^{-1} = N A^{-2} (matches (1))

Check (3): kg m s^{-1} A^{-1} = N s A^{-1} (does not match)

12. **Answer:** $ma + 4v + k(L - 0.1) = 0$

Working: F = ma. Be careful with signs: using convention + is right, - is left,

Forces acting are:

Tension in spring: F = -kx (negative sign since tension is opposite to displacement)

Extension = Length of spring - Natural length

x = L - 0.1 → F = -k(L - 0.1),

Viscous/resistive drag force due to dashpot: |F| = 4V.

Since the system is rigid (moves as one), V = speed of mass = v.

When v is positive (moving to the right), the drag force will oppose the motion so force will be negative → F = -4v.

Combining, F = ma = -k(L - 0.1) - 4v

→ ma + 4v + k(L - 0.1) = 0.

13. **Answer:** **1** and **2** only

Working: Magnet B's amplitude increases due to resonance of force induced on magnet (by Faraday's law).

By conservation of energy, the kinetic energy (and hence speed and amplitude) of magnet A initially must become shared between both magnets. Since magnet A will still be oscillating, the energy of B must be less and hence its amplitude will also be less.

e.m.f. = - rate of change of flux linkage = -d(BA)/dt. Assuming the magnet's field lines point straight down, the magnetic flux density (field strength) will be a function of the magnet's position, i.e. the spring's extension and so its rate of change will be a function of its velocity. Since velocity oscillates, e.m.f will also oscillate and will decrease since it is induced in the opposite direction (by magnet B; Lenz's law) as oscillations continue. The forces acting on each magnet have the same frequency (identical springs) and so there will be a constant phase difference. Let x be displacement of A from equilibrium: using the approximation, B $\propto$ x → e.m.f. $\propto$ v → e.m.f. is maximum when A is at equilibrium position → force on B is maximum → acceleration of B is maximum → displacement of B is maximum → 90^o out of phase.

14. **Answer:** 2.1 kW

Working: In 3 hours, the device transfers (E = Pt)
= 3000 * 3 * 60 * 60 = 32.4 MJ of energy to the water circulation.
The energy required to heat the water from 10 to 30 ^oC (E = mcΔT):
= 120 * 4200 * 20 = 10080 J = 10.08 MJ
So the cooler must remove (32.4 - 10.08) = 22.32 MJ:
Power = 22.32 * 10^6 / (3 * 60 * 60)
= 2067 = 2100 W = 2.1 kW (2 s.f.)

15. **Answer:** $a_2 - a_1 = a_1 - a_3$

Working: Consider the motion relative to the moving pulley.

For any two objects A and B (x, v, a = position, velocity, accel.)

$x_A = x_B + x_{A \text{ relative to } B} \rightarrow v_A = v_B + v_{A \text{ relative to } B} \rightarrow a_A = a_B + a_{A \text{ relative to } B}$

Applying this relative acceleration equation to each hanging block (where A is each of the blocks and B is the hanging pulley),

Hanging block mass M: $a_2 = a_1 + a_{\text{block M relative to pulley}}$

Hanging block mass 2M: $a_3 = a_1 + a_{\text{block 2M relative to pulley}}$

Since the length of string between the hanging masses is constant, the difference of their positions relative to the position of the pulley is constant, so their rate of change (velocities) is zero and hence their accelerations is also zero.

Adding the equations together,

$\rightarrow a_2 + a_3 = 2a_1 + (a_{\text{block M relative to pulley}} + a_{\text{block 2M relative to pulley}})$

$\rightarrow a_2 + a_3 = 2a_1 + (0)$

$\rightarrow a_2 - a_1 = a_1 - a_3$ (subtract a_1 from both sides)

16. **Answer:** 3

Working: Let the point at distance *x* be *P*.

Distance from S_1 to P and from S_2 to P = $\sqrt{(x^2 + d^2)}$.

Time for ray from $S_1 = \sqrt{(x^2 + d^2)} / c$

Time for ray from $S_2 = \sqrt{(x^2 + d^2)} / (\frac{3}{4} c)$ [since $n = 4/3$]

So 'distance' of the second ray if it were travelling in air in the same timeframe would be $4/3 * \sqrt{(x^2 + d^2)}$. [we use this distance since we will combine the distances as if they were 'number of cycles']

For a maximum, the path difference must be an integer multiple of the wavelength:

$\rightarrow 4/3 * \sqrt{(x^2 + d^2)} - \sqrt{(x^2 + d^2)} = m\lambda$

$\rightarrow \sqrt{(x^2 + d^2)} = 3m\lambda$

$\rightarrow x^2 + d^2 = 9m^2\lambda^2$

$\rightarrow x^2 = 9m^2\lambda^2 - d^2$

$\rightarrow p^2 = 9 \rightarrow p = 3.$

17. **Answer:**

$\Delta h = \dfrac{p}{\rho g}$	$T^2 = \dfrac{4\pi^2 h}{g}$	friction with the sides of the tube

Working: Hydrostatic pressure formula: $\Delta p = \rho g \Delta h$

Pressure at A = pressure of gas + atmospheric pressure = $p + p_{atm}$

Pressure at B = atmospheric pressure = p_{atm}

$\rightarrow (p + p_{atm}) - p_{atm} = \rho g \Delta h \rightarrow p = \rho g \Delta h$

$\rightarrow \Delta h = \dfrac{p}{\rho g}$.

Let x be the distance of the water level from equilibrium i.e. $2x = \Delta h$.

Force acting on water: $F = pA = \rho A g\, \Delta h = 2\rho A g\, x$

When x is positive (say, downwards), the force will be upwards, so the sign is negative $\rightarrow F = -2\rho A g\, x$

Now, let the acceleration be a and the mass of the water be m.

$$a = \frac{F}{m} = -\frac{2\rho A g}{\rho V} \cdot x$$

Using the given approximation $V/A = 2h$ (this represents the approximation that the length of the water tube equals twice the height, i.e. the radius of the curved portion of the tube is small compared to the height), we get

$$a = -\frac{g}{h} \cdot x$$

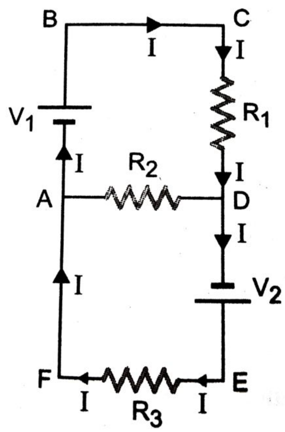
This matches the SHM equation form, $a = -\omega^2 x$, with $\omega^2 = g/h$. Then,

$$\omega = \sqrt{\frac{g}{h}} \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}} \rightarrow T^2 = \frac{4\pi^2 h}{g}$$

Damping is caused by energy dissipation from kinetic energy to other forms (i.e. heat). Since the amplitude of oscillation is small, the liquid will be moving quite slowly ($v_{max} = \omega x$) and so turbulent dissipation will be very small (negligible). However, if the liquid is dense, friction with the sides of the tube will remove energy over time, bringing the liquid to rest.

18. **Answer:** $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

Working: Let $I_2 = 0$. Assume the current will flow clockwise (if not, I will be negative).



Using KVL in the top loop,

$V_1 = IR_1 \rightarrow I = \frac{V_1}{R_1}$ *(Eqn 1)*

Using KVL in the whole circuit loop,

$V_1 + V_2 - IR_1 - IR_3 = 0 \rightarrow V_1 + V_2 = I(R_1 + R_3) \rightarrow I = \frac{V_1+V_2}{R_1+R_3}$ *(Eqn 2)*

Equating *1* and *2*,

$$\frac{V_1}{R_1} = \frac{V_1 + V_2}{R_1 + R_3} \rightarrow V_1 R_3 = V_2 R_1$$

Now check the given cases.

Let $V_1 = V_2 \rightarrow R_1 = R_3$ means no current (first two options)

Let $V_1 = 2V_2 \rightarrow R_1 = 2R_3$ means no current

Let $2V_1 = V_2 \rightarrow 2R_1 = R_3$ means no current (last option)

The third option does not match, so it must produce a current.

(R_2 can be anything).

19. **Answer:** $R\left(\frac{h\rho g+P}{x\rho g+P}\right)^{\frac{1}{3}}$

Working: Consider the density of the gas in the bubble.

$\rho_{gas} = m/V \rightarrow \rho_{gas} \propto 1/V$ (since mass of gas, m, remains constant.)

Since temperature is constant (isothermal process), as pressure is also inversely proportional to volume (Boyle's law) so

$p_{bubble} \propto \rho_{gas} \propto 1/V$.

Since $V \propto r^3$ (radius of bubble), we now have $p_{bubble} \propto 1/r^3$.

At any depth,

pressure exerted by bubble = hydrostatic pressure + atm pressure

At the bottom: $k/R^3 = h\rho g + P$

At a depth *x*: $k/r^3 = x\rho g + P$

Dividing the first equation by the second,

$(k/R^3) / (k/r^3) = [h\rho g + P] / [x\rho g + P]$

$\rightarrow r^3 / R^3 = [h\rho g + P] / [x\rho g + P]$

$\rightarrow r^3 = R^3 [h\rho g + P] / [x\rho g + P]$

$\rightarrow r = R ((h\rho g + P) / (x\rho g + P))^{1/3}$.