Section A: Fast.

1. If θ is acute and sec θ = $\frac{5}{3}$ then the value of cot θ is

O $\frac{3}{5}$

O $\frac{5}{3}$

O $\frac{3}{4}$

O $\frac{4}{3}$

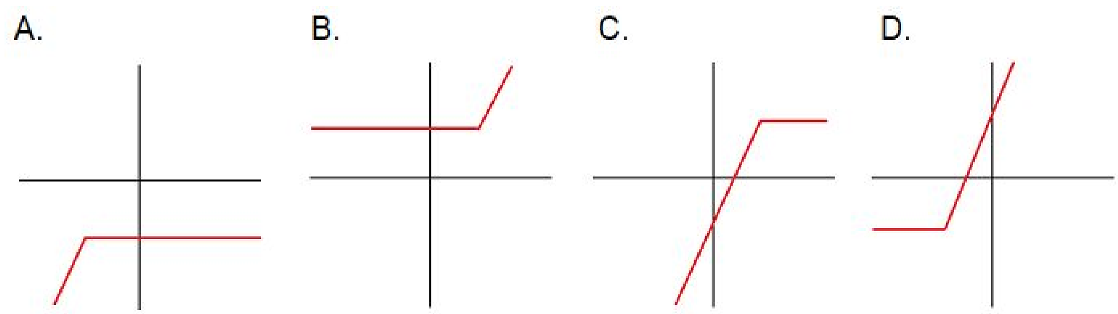
2. The correct factorisation of $3x^2 - 2xy - y^2$ is

O $(3x + y)^2$
O $(3x + y)(x - y)$
O $(3x - y)(x + y)$
O $(3x + y)^3 - (3x - y)^3$

3. The coefficient of x^2 in the expansion of $(2x^2 - x + 4)(1 - x - 8x^2)$ is

O 29
O -32
O -29
O 32

4. Which of the graphs below sketches the function $y = x - |x - 1|$?



O A
O B
O C
O D

5. *P* and Q are variables such that *P* is inversely proportional to the square of *Q*.

If Q is increased by 40%, then to the nearest percent, by what has *P* **decreased**?

O 44%
O 49%
O 51%
O 96%

6. Three variables *x*, *y* and *z* are known to be related to each other by:

x is directly proportional to the square of *z*.
y is inversely proportional to the cube of *z*.

Which of the following correctly describes the relationship between *x* and *y*?

O The square of *x* is directly proportional to the cube of *y*.
O The square of *x* is inversely proportional to the cube of *y*.
O The cube of *x* is directly proportional to the square of *y*.
O The cube of *x* is inversely proportional to the square of *y*.

7. The graph of the function $y = \sin x$ is reflected in the line $x = \pi$, then reflected in the line $y = 2$, where x is in radians.

The equation of the resulting graph is

O $y = 4 + \sin x$
O $y = 4 - \sin x$
O $y = 4 + \cos x$
O $y = 4 - \cos x$

8. Which of these graphs is strictly convex for all real x?

O $y = 1$
O $y = x$
O $y = x^2$
O $y = x^3$

9. Which of these lines passes through the point (-1, 5) and is parallel to the line $x + y = 0$?

O $y = 4 - x$
O $y = x - 4$
O $5y - x + 1 = 0$
O $5x - y + 1 = 0$

10. A line L passes through the points (5, 1), (k, 4) and (-1, 9).

The value of k is

O 2
O 9/4
O 11/4
O 3

11. The lines $3x + ay = 2$ and $bx - y = a - b$ are perpendicular.

Which of the relationships between a and b is true?

O $ab = 3$
O $ab = -3$
O $a/b = 3$
O $a/b = -3$

12. A sequence is given by $u_n = \sin(45n^o)$.

The period of this sequence is

O 4
O 8
O 45
O 360

13. Which of these integers is divisible by all primes less than 10?

O 720
O 1260
O 3240
O 4480

14. If θ is obtuse and positive, then

O cos θ is always positive
O cos θ could be either positive or negative
O cot θ is always negative
O cot θ could be either positive or negative

15. The value of $2 \times (1 + 2 + 3 + 4 + \ldots + n)$, where n is a positive integer, is given by

O $2n(n + 1)$
O $n(n + 1)$
O $n^2 - n + 1$
O $n^2 + n - 1$

16. The derivative of $\ln 3x$ with respect to x is

O $1/x$
O $1/(3x)$
O $3/x$
O $-1/(3x)$

17. A vector **v** has magnitude m, where $m \neq 0$.

Which of these vectors points in the opposite direction as **v** with magnitude m^2?

O $\mathbf{v} - m\mathbf{v}$
O $m\mathbf{v}$
O $-m\mathbf{v}$
O $-m^2\mathbf{v}$

18. Which of these is true about the graph of $xy = 4$ in the Cartesian plane?

O It is symmetrical about the x-axis.
O It is symmetrical about the y-axis.
O Both of the above.
O None of the above.

19. A correct completed square form of $4x - 1 - x^2$ is

O $(x - 2)^2 - 3$
O $3 + (2 - x)^2$
O $(2 - x)^2 - 3$
O $3 - (2 - x)^2$

20. The discriminant of $(x + a)(x - a)$ is

O $2a$
O $-2a$
O $4a^2$
O $-4a^2$

21. The complete set of solutions to $-8 < 6 - \frac{x}{2}$ is

O $x < 4$
O $x > 4$
O $x < 28$
O $x > 28$

22. The correct simplification of

$$4 + \frac{4 - x^2}{x^2 - 2x}$$

is

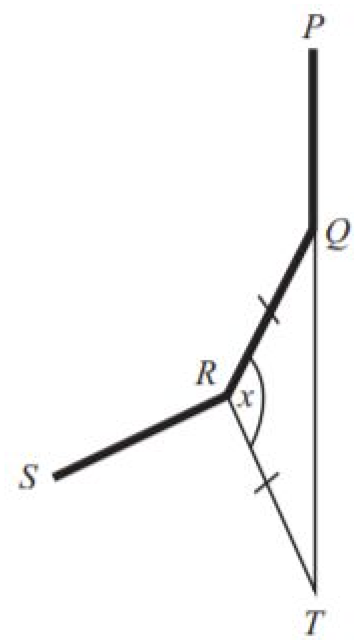
O $3 - \frac{2}{x}$

O $3 + \frac{2}{x}$

O $5 - \frac{2}{x}$

O $5 + \frac{2}{x}$

23. In the diagram below, $PQRS$ is part of a regular n-sided polygon. The side PQ is extended to T such that PQT is a straight line. The length of RQ is the same as the length of RT.



Find an equation for n in terms of x°, where x° is the size of angle $\angle QRT$.

O $n = 360 \div (180 - x^{\circ})$
O $n = 720 \div (180 - x^{\circ})$
O $n = 360 \div (360 - x^{\circ})$
O $n = 720 \div (360 - x^{\circ})$

24. $f(x) = x^3 + px^2 + qx + p^2$ where p and q are real.

Given that 2 is a root of $f(x)$ and $f(1) = -3.5$, find all possible value(s) of p.

O $p = -1 \pm \frac{\sqrt{6}}{3}$
O $p = 1$ or $p = -3$
O $p = 1$
O there are no values of p

25. A square $PQRS$ is drawn above the x-axis with the side PQ on the x-axis. P is the point (-5, 0) and Q is the point (1, 0). A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

O $x^2 + y^2 + 4x - 6y + 4 = 0$
O $x^2 + y^2 - 10x + 6y - 36 = 0$
O $x^2 + y^2 - 4x + 6y - 4 = 0$
O $x^2 + y^2 + 10x - 6y + 36 = 0$

26. The complete set of values of a for which the equation $3x^2 = (a + 2)x - 3$ has two real distinct roots is

O $-4\sqrt{2} < a < 4\sqrt{2}$
O $a < -4\sqrt{2},\ a > 4\sqrt{2}$
O $-8 < a < 4$
O $a < -8,\ a > 4$

27. The indefinite integral of $f(x) = \sin x + 3 \sec^2 x$ with respect to x is given by

O $\cos x - \sec^3 x + C$
O $3 \tan x - \sin x + C$
O $\cos x + \sec^3 x + C$
O $3 \tan x - \cos x + C$

28. Which techniques, applied one after the other, would be most suitable for evaluating

$$\int \frac{\cos x(\sin x - 1)}{\sin^2 x + 5\sin x + 6}\ \mathrm{d}x$$

O Integration by parts then by substitution
O Integration by substitution then by partial fractions
O Integration by quotient rule then by parts
O Integration by substitution then by parts

29. The only correct trigonometric identity from the options below is

- O $\cos^2 2x - \sin^2 2x \equiv \sin 4x$
- O $\tan^2 x + \sec^2 x \equiv 1$
- O $\csc^2 x - \cot^2 x \equiv 1$
- O $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 \equiv 1$

30. The sum of all positive factors of 72 is

- O 168
- O 195
- O 225
- O 232

31. If $x + \frac{1}{x} = 5$, then the value of $x^2 + \frac{1}{x^2}$ is

- O 21
- O 23
- O 25
- O 27

32. Let a and b be constants such that $ab = 1$. Let

$$P = \frac{a}{a+1} + \frac{b}{b+1} \quad \text{and} \quad Q = \frac{1}{a+1} + \frac{1}{b+1}.$$

What can be deduced about P and Q?

- O P is always greater than Q
- O P is always less than Q
- O P and Q are always equal to 1
- O P and Q are always equal but not always equal to 1

33. Points *A* and *B* have coordinates (1, 2) and (-2, 5) in the Cartesian plane with point *O* as its origin. The point *P* lies on the line segment *AB* such that the distance *AP* : *PB* is in the ratio 1 : 2.

Which of these is true?

O *P* lies on the *x*-axis
O *P* lies on the *y*-axis
O The distance *OP* is $\sqrt{5}$ units
O The distances *OA* : *OB* are in the ratio 1 : $\sqrt{3}$

34. The maximum number of inflection points of a polynomial curve of degree 4 is

O 1
O 2
O 3
O 4

35. Which one of the following expressions is undefined?

O $\tan(45^{\circ})$
O $\cot(\pi/2)$
O $\cos^{-1}(-0.5)$
O $\ln(0)$

36. A group of drivers, consisting of 200 women and 300 men, was asked if they passed their driving test at the first attempt. Altogether 167 of the group said they passed at the first attempt. Of the women, 143 said they did not pass at the first attempt.

How many of the men said they passed at the first attempt?

O 10
O 24
O 57
O 110

37. *PQRST* is a regular pentagon, and *RSU* is an equilateral triangle where point *U* lies inside the pentagon.

What is the size of the acute angle *STU*?

(All points are labelled clockwise, in order.)

O 48°
O 54°
O 60°
O 66°

38. The original price of an item is *p*. The price is **increased by** 125%. The increased price is then **decreased by** 40% to *q*.

The value of the ratio $\frac{q}{p}$ is

O $\frac{17}{20}$

O $\frac{27}{20}$

O $\frac{45}{28}$

O $\frac{33}{28}$

39. Evaluate

$$\log_2\left(\frac{5}{4}\right) + \log_2\left(\frac{6}{5}\right) + \log_2\left(\frac{7}{6}\right) + \cdots + \log_2\left(\frac{64}{63}\right).$$

O -2
O 3
O 4
O 6

40. What is the coefficient of x^3 in the expansion of $(1-2x)^5(1+2x)^5$?

O -6400
O -640
O -80
O 0

Section B: Standard.

1. By considering the graph of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 20$, find the complete set of values of the constant k for which the equation $f(x) = k$ has exactly four distinct real roots.

   O $-12 < k < 15$
   O $15 < k < 20$
   O $k > 20$
   O $7 < k < 20$

2. The curve $y = x^2$ is translated by the vector $4\mathbf{i} + 3\mathbf{j}$ and then reflected in the line $y = -1$.

   Which one of the following is an equation of the resulting curve?

   O $y = (x - 3)(5 - x)$
   O $y = (x - 2)(x - 6)$
   O $y = 8x - 20 - x^2$
   O $y = 8x - 21 - x^2$

3. Given that $7 \cos \theta - 3 \tan \theta \sin \theta = 1$, which one of the following is true?

   O $\cos \theta = -3/5$ or $-1/2$
   O $\cos \theta = -3/5$ or $1/2$
   O $\cos \theta = 3/5$ or $-1/2$
   O $\cos \theta = 3/5$ or $1/2$

4. By considering the small-angle approximation for cos x, it can be deduced that,

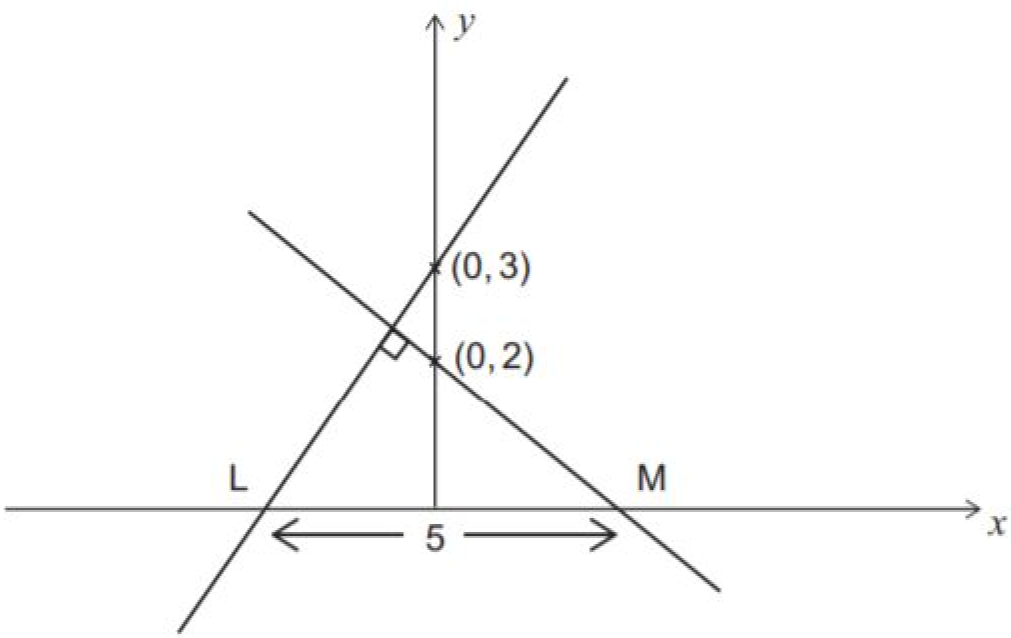
O $\cos^{-1} x \approx 1 - \frac{x^2}{2}$, for small x

O $\cos^{-1} x \approx 1 - \frac{x^2}{2}$, for x close to 1 such that $x \leq 1$

O $\cos^{-1} x \approx \sqrt{2 - 2x}$, for small x

O $\cos^{-1} x \approx \sqrt{2 - 2x}$, for x close to 1 such that $x \leq 1$

5. The straight line with equation, $y = mx + 3$ where $m > 0$, $m \neq 1$ is perpendicular to the line with equation $y = px + 2$. The lines cut the x-axis at the points L and M respectively.



The length of LM is 5 units. What is the value of $m + p$ given that $m > 1$?

O -8/3
O -5/6
O 8/3
O 5/6

6. Define the constant $K = ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)$ for real a, b and c.

By considering the value of K (taken separately) when $a = b$, when $b = c$ and when $a = c$, it can be deduced that the factorisation of K is

O $K = abc(a + b + c)(ab + bc + ac)$
O $K = (a + b)(b + c)(a + c)$
O $K = (a - b - c)(b - a - c)(c - a - b)$
O $K = -(a - b)(b - c)(c - a)(a + b + c)$

7. Given that θ is obtuse, which of the following must be true?

1 $\sin\theta \cos\theta$ is always negative
2 $\cos\theta \cot\theta$ is always positive
3 $\cos^2\theta > \sin^2\theta$

O **1** and **2** only
O **2** and **3** only
O **1** and **3** only
O **1**, **2** and **3**

8. The random variable Z is distributed with the standard normal distribution.

What is the interquartile range of Z?

O 0.1974
O 0.6745
O 0.6915
O 1.3490

9. Let $f(x) = (x-1)(x-2)(x-3)(x-5)$ for all positive real *x*.

Further, define

$$F(x) = \int_0^x f(t)\ \mathrm{d}t$$

with the same domain as *f*(*x*).

By considering the graph of $y = f(x)$, which of these is true?

- O *F*(*x*) has local maximum points at *x* = 2 and *x* = 5
- O *F*(*x*) has local minimum at *x* = 1 and *x* = 3
- O *F*(*x*) is a polynomial of degree 4
- O The coefficient of *x* in *F*(*x*) is 30

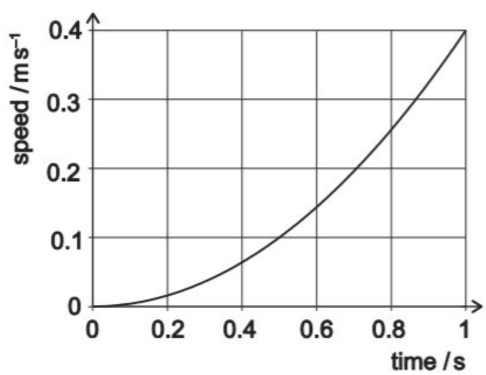
10. A ball of mass m = 0.5 kg is at rest a distance d above the flat floor of a spacecraft. Installed in the floor is an artificial gravity generator. There is no air in the spacecraft and the spacecraft is free from any influence of gravity.

The generator is switched on at time t = 0 s and produces a gravitational field g that increases linearly with time, such that the force exerted on the ball is given by $0.2t$ newtons at any time t.

Which of the following statements is true?

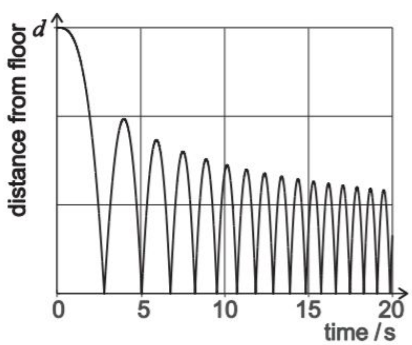
O The magnitude of the gravitational acceleration g in the spacecraft is given by $0.1t$ at any time t

O Assuming that the ball does not hit the floor within the first second of motion, the speed-time graph of the ball is



O The time taken for the ball to first hit the floor is $\left(\frac{15d}{2}\right)^{\frac{1}{3}}$ seconds.

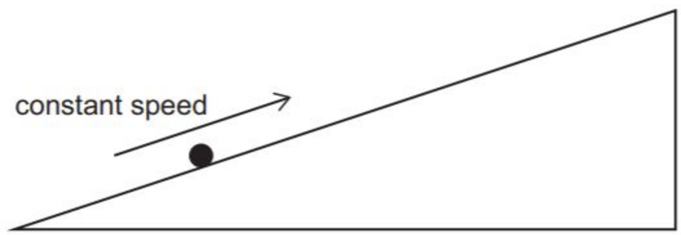
O The distance-time graph of the ball on a timescale showing the ball repeatedly bouncing on the spacecraft floor could be shown by



11. How many solutions of the equation $2\sin^3\theta = \sin\theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$?

O 2
O 3
O 4
O 5

12. The diagram represents a mass that is moving in a straight line at constant speed up a slope of constant gradient.



Which pair of statements **must** be true?

1 All the forces acting on the mass are equal in magnitude.
2 More than two forces act on the mass.
3 The force of friction on the mass is equal to the driving force.
4 The weight of the mass acts in the opposite direction to the contact force.
5 There is no air resistance acting on the mass.
6 There is no resultant force acting on the mass.

O **1** and **5**
O **2** and **6**
O **3** and **4**
O **1** and **6**

13. The two functions *f* and *g* satisfy $f'(x) = ax + g(x)$, where *a* is a constant.

Given that $\int_2^4 g(x)\,dx = 12$ and $f(4) = 18 + f(2)$, what is the value of *a*?

O 1
O 3
O 5
O 6

14. A man is cycling along a straight horizontal road at a constant speed of 9 ms^{-1}. He passes a boy who is cycling at 5 ms^{-1} in the same direction.

When the man is level with the boy, the boy begins to accelerate at a constant rate of 0.8 ms^{-2}. The boy maintains this constant acceleration and the man continues at constant speed until the boy passes the man.

What is the time interval between the two instances when the man and the boy are level?

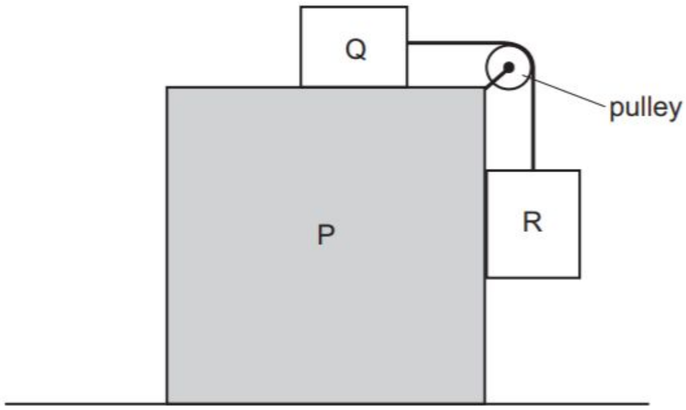
O 5 s
O 10 s
O 22.5 s
O 35 s

15. The functions *f*(*x*), *g*(*x*) and *h*(*x*) are related by $f'(x) = g(x+1)$ and $g'(x) = h(x-1)$.

Express $f''(2x)$ in terms of h.

O $h(2x)$
O $4h(2x)$
O $2h'(2x)$
O $h(2x+1)$

16. A block *P* has a smaller block *Q* resting on its top surface. *Q* is connected to a hanging block, *R*, by a light, inextensible string. The string passes over a smooth pulley which is connected to block *P*, as shown in the diagram.



The masses of blocks *P*, *Q* and *R* are m_P, m_Q and m_R respectively.

P is accelerated horizontally to the right by an external force. While this is happening, *Q* and *R* do not move relative to *P*.

What is the acceleration of *P*? (Gravitational field strength = g.)

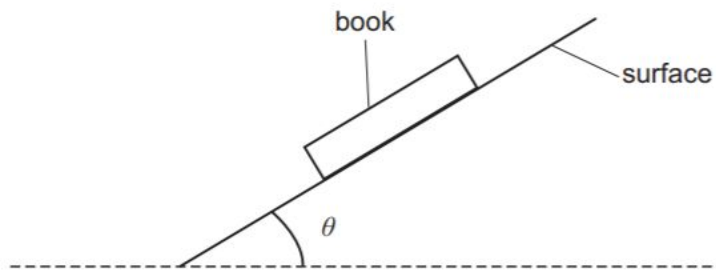
O $\frac{m_Q g}{m_R}$

O $\frac{m_R g}{m_Q}$

O $\frac{m_R g}{m_R + m_Q}$

O $\frac{(m_Q + m_R) g}{m_P + m_Q + m_R}$

17. A book of mass *m* rests on a rough surface. The surface is now tilted as shown:



When the angle of tilt θ is 20°, the book slides down the slope at a constant speed.

What is the acceleration of the book down the slope when the angle of tilt is 25°?

(Gravitational acceleration = *g*; neglect air resistance.)

O $g(\sin 20^\circ - \cos 25^\circ \tan 25^\circ)$
O $g(\cos 20^\circ - \sin 20^\circ \tan 25^\circ)$
O $g(\cos 25^\circ - \sin 20^\circ \tan 20^\circ)$
O $g(\sin 25^\circ - \cos 25^\circ \tan 20^\circ)$

18. The correct simplification of

$$\frac{1}{2+\frac{4}{3-\frac{5}{x+1}}}$$

is

O $\frac{2x-3}{10x}$

O $\frac{3x-2}{10x}$

O $\frac{16x-7}{120x}$

O $\frac{7x-16}{120x}$

19. Differentiate $f(x) = |x|$ with respect to x.

(Assume $x \neq 0.$)

O 1

O -1

O $\frac{x}{|x|}$

O $-\frac{x}{|x|}$

20. An icosahedral dice has its 20 faces numbered each with the distinct integers from 1 to 20. The dice is rolled 3 times and the **largest** number from these three rolls is recorded as the random variable X.

Which of these is true?

O $X \sim B(20, 0.05)$

O The mean of X is 10

O $P(X \leq 10) = \frac{1}{8}$

O $P(X = k) = 1 - \left(\frac{k}{20}\right)^3$ for all integers $1 \leq k \leq 20$

21. During hot days, an ice cream van sells a large number of ice cream cones containing either 1, 2 or 3 scoops of ice cream. The respective probabilities of a customer buying a 1, 2 or 3 scoop ice cream cone are 1/6, 1/2 or 1/3.

A random sample of *n* customers is examined, each customer having bought an ice cream cone from this van. The probability that more than *n* scoops of ice cream are ordered by these *n* customers is greater than 0.9999.

What is the smallest possible value of *n*?

O 5
O 6
O 7
O 8

22. Consider the following set of data, which can be considered a random sample:

5, 10, 11, 11, 14, 17, 19, 24, 30, 36, 40, 45

Which of these, if any, is/are true?

1 there are no outliers in this data
2 the data is positively skewed

O **1** only
O **2** only
O Both **1** and **2**
O Neither **1** nor **2**

23. An appropriate statistical method to test for correlation between “proportion of a country’s population living in urban areas” and “GDP per capita of a country” is

O Spearman’s rank correlation coefficient
O Pearson’s product-moment correlation coefficient
O Hypothesis test on a Normal distribution
O All of the above would be appropriate

24. A computer is used to flip a virtual fair coin 500 times. The random variable X represents the number of heads that appear.

If X is approximated as having a normal distribution, then the correct way to estimate the probability of obtaining between 100 and 400 heads, inclusive, is

O $P(100 \le X \le 400)$ under $X \sim N(250, 125)$
O $P(100 < X < 400)$ under $X \sim N(250, 0.25)$
O $P(99.5 \le X \le 400.5)$ under $X \sim N(250, 125)$
O $P(100.5 \le X \le 399.5)$ under $X \sim N(250, 125)$

25. $f(x)$ is the series approximation to $\frac{1+x}{1-x^2}$, expanded to the first three terms.

If the maximum error in this approximation is less than 1% for all $|x| < a$, then the smallest possible value of a is

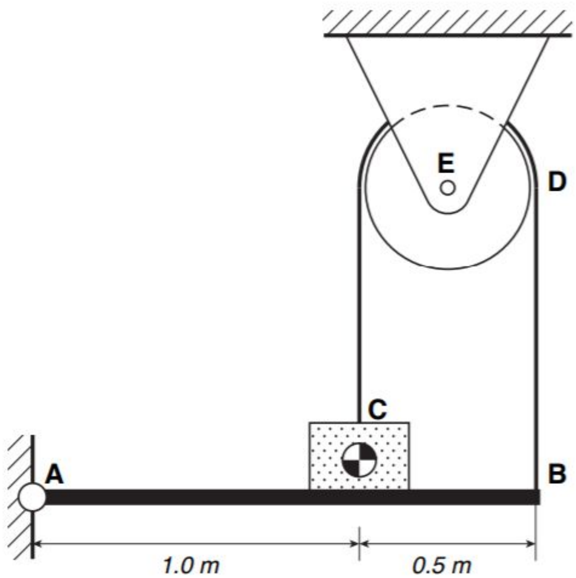
O $\frac{1}{100}$
O $\frac{1}{\sqrt[3]{100}}$
O $\frac{1}{10}$
O $\frac{1}{\sqrt[3]{10}}$

26. The function $f(x) = (x^4 + x^3)^8$ is differentiated n times with respect to x, and the resulting function is a non-zero constant.

What is the value of this constant?

O $8 \times 24!$
O $7 \times 8! \times 12!$
O $32!$
O $12 \times (8!)^4$

27. A uniform horizontal bar AB of weight 100 N is pinned to the wall at A. The string BDC passes over a smooth light pulley and is attached to the block C of weight 250 N, as shown. The pulley is free to rotate about E.

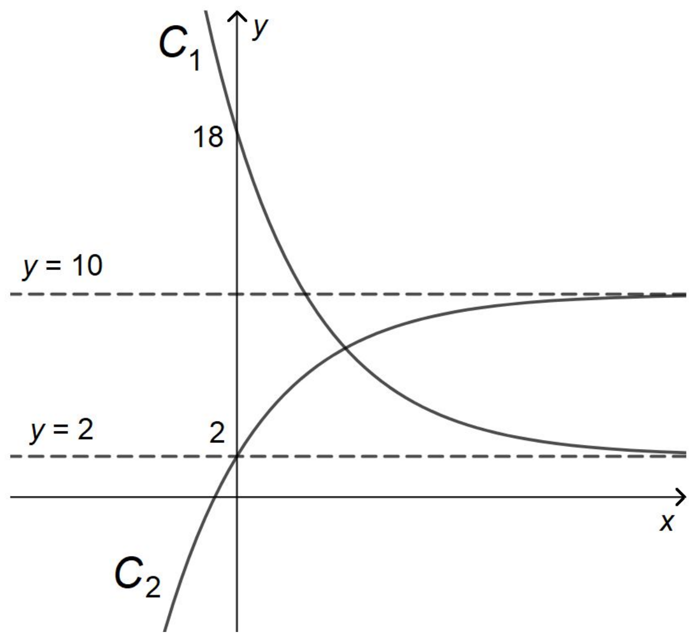


The vertical reaction force at A is R_A, and the reaction between the block C and the bar AB is R_{bar}.

What are R_A and R_{bar}?

O R_A = 90 N, R_{bar} = 120 N
O R_A = 90 N, R_{bar} = 130 N
O R_A = 100 N, R_{bar} = 120 N
O R_A = 100 N, R_{bar} = 130 N

28. The graphs of two exponential curves, C_1 and C_2, are shown, along with their respective asymptotes.



These curves are the result of a sequence of transformations. The transformations are as follows:

Starting from curve $y = e^{-x}$,

Step **1**: translation by vector $a\,\mathbf{j}$
Step **2**: stretch parallel to y-axis, scale factor b, this produces curve C_1
Step **3**: stretch parallel to y-axis, scale factor c
Step **4**: reflection in the line $y = d$, this produces curve C_2

Which of these is true?

O The equation of C_1 is $y = 2 + 18e^{-x}$
O The equation of C_2 is $y = 10 - 8e^{x}$
O $a = 2$ and $b = 6$
O $c = 1/2$ and $d = 11/2$

29. The sum of the first *n* cube numbers is given by

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2$$

for all $n \in \mathbb{N}$.

Use the above identity to factorise the following alternating sum of cubes:

$$1^3 - 2^3 + 3^3 - 4^3 + 5^3 \cdots - (2k)^3$$

for all $k \in \mathbb{N}$.

O $-k^2(k+1)(4k+1)$
O $-k^2(4k-3)^2$
O $-k^2(4k+3)^2$
O $-k^2(k-1)(4k-1)$

30. Let *a* be a non-negative integer. The value of a^3 - a + 1 is

O even, if *a* is even, and odd, if *a* is odd
O always 1 more than a multiple of 6
O never a perfect square
O never prime

31. Which of the following options could be used to prove the statement

$$\cos\theta + \sin\theta \leq \sqrt{2}$$

for all real θ ?

1 Proof by deduction
2 Proof by exhaustion
3 Proof by contradiction
4 Disproof of $\cos\theta + \sin\theta > \sqrt{2}$ by counterexample

O **1** or **3**
O **2** or **4**
O **1** or **4**
O **2** or **3**

32. Some experimental data is gathered and the natural logarithm of the dependent variable, ln *y*, is plotted against the independent variable, *t*, on the vertical and horizontal axes respectively.

The resulting graph is a straight line, passing through points (0, 2 ln 5) and (2, 4 ln 5).

Express *y* in terms of *t*.

O $y = \ln(5 + 2t)$
O $y = 5^{t+2}$
O $y = 5 + e^{2t}$
O $y = t^{2 - \ln 5}$

33. Find and simplify fully,

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln \sqrt{\frac{1-x}{1+x}}$$

O $\dfrac{1}{x^2+1}$
O $\dfrac{1}{x^2-1}$
O $\dfrac{1}{1-x^2}$
O $\sqrt{\dfrac{x}{x^2+1}}$

34. The constant term in the expansion of $\left(2x + \frac{3}{x^2}\right)^{12}$ is

O 495
O 20,736
O 10,264,320
O 479,001,600

35. A seed is planted. After one month, there is one twig containing two leaves. In the following month, the twig grows two further twigs, each new twig again containing two leaves.

If in successive months each new twig produces two further twigs with their leaves, how many leaves will be on the plant after 10 months?

(You may assume that no leaves fall off the plant, and once a new twig has produced two twigs it does not produce further twigs in subsequent months.)

O 1,023
O 1,024
O 2,046
O 2,048

36. For what range(s) of *x* is the inequality $5 - 3x < \frac{2}{x}$ satisfied?

O $\frac{2}{3} < x < 1$

O $x < \frac{2}{3}$ or $x > 1$

O $0 < x < \frac{2}{3}$ or $x > 1$

O $x < 0$ or $\frac{2}{3} < x < 1$

37. A rower wants to cross a river to a point on the opposite river bank directly opposite from where she will start. The distance to cover is 100 m, and she wants to cross in 10 seconds. The river flows uniformly at 7.5 m/s.

How fast must she row her boat, and at what angle, **relative** to the velocity of the flowing water? (Round the angle to the nearest degree.)

O speed = 12.5 ms^{-1}, angle = 37°
O speed = 12.5 ms^{-1}, angle = 127°
O speed = 6.6 ms^{-1}, angle = 37°
O speed = 6.6 ms^{-1}, angle = 127°

38. S is an inductive sequence of the Fibonacci type. The first few terms of S are 4, 5, 9, 14, 23, ...

What is the next term in the sequence?

O 28
O 34
O 37
O 39

39. A large number of people enter a maze. At each junction, each person randomly takes the left or right turns, with probabilities P(left) = $\frac{1}{3}$ and P(right) = $\frac{2}{3}$.

After three such junctions, what is the most likely combination of turns people will have taken?

O left three times
O right three times
O left twice and right once
O left once and right twice

40. Let $f(x) = \frac{2x}{\sqrt{4-x^2}}$. When the series expansion of $f(x)$ is truncated to the first 3 terms, the percentage error in calculating $\int_0^1 f(x)\, dx$, to 2 significant figures, is

O 0.14%
O 0.44%
O 1.6%
O 14%

41. A box contains three red balls, three blue balls and three green balls.

If three balls are drawn from the box at random, the probability that they are all different colour is

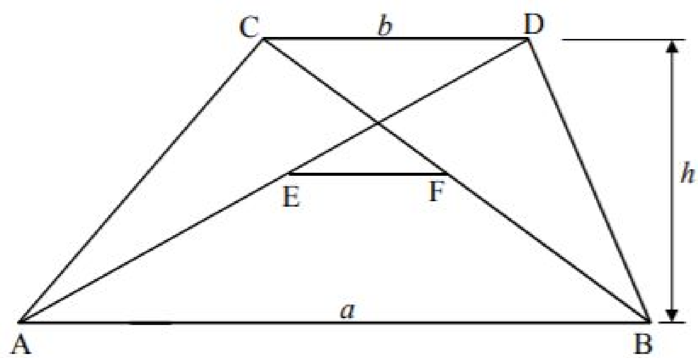
O $\frac{2}{15}$

O $\frac{2}{81}$

O $\frac{4}{27}$

O $\frac{9}{28}$

42. Trapezium ABDC is shown below, with |AB| = *a* and |CD| = *b*. The perpendicular height of the trapezium is *h*. E and F are the midpoints of the diagonals AD and BC respectively.



What is the length of line segment EF?

O $\frac{a+b}{2}$

O $\frac{a-b}{2}$

O $\frac{3abh}{2(a^2-b^2)}$

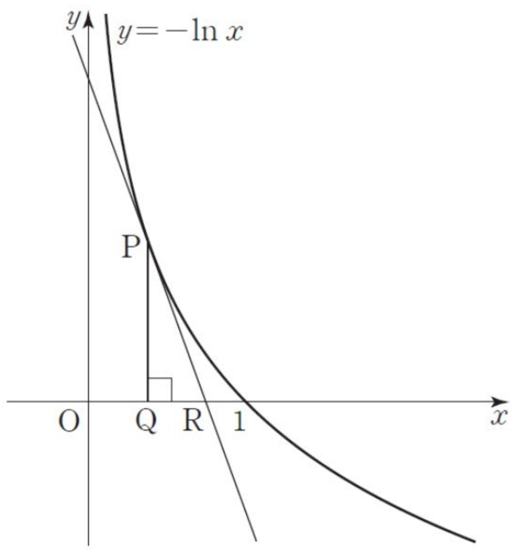
O $\frac{h(a+b)}{2(a-b)}$

43. Factorise fully $x^2 + 2xy - 3y^2 + 4x - 4y$.

- O $(x - 3y + 4)(x + y)$
- O $(x + y - 4)(x - 3y)$
- O $(x - y + 4)(x - 3y)$
- O $(x + 3y + 4)(x - y)$

Section C: Hard.

1. The point P lies on the curve $y = -\ln x$ and has coordinates $(t, -\ln t)$ where $0 < t < 1$. Let Q be the point on the x-axis with x-coordinate equal to t, and R be the point where the tangent to the curve at P intersects the x-axis.



What is the maximum area of triangle PQR as t varies?

- O $\frac{1}{e^2}$
- O $\frac{2}{e^2}$
- O $\frac{1}{e^{\sqrt{2}}}$
- O $\frac{2}{e^{\sqrt{2}}}$

2. Define the set S = $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20} \cdots\right\}$.

Which of these are correct ways to represent S in set-builder notation?

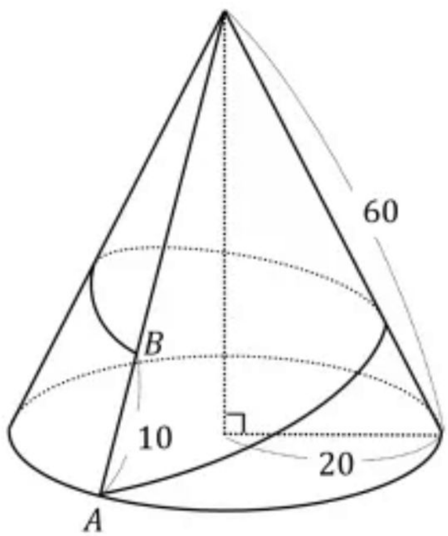
1 $$S = \left\{\frac{n}{3n-1} : n \in \mathbb{N}\right\}$$

2 $$S = \left\{\frac{p}{q} : p \in \mathbb{N},\ pq \cup \mathbb{P} = \varnothing\right\}$$

3 $$S = \{n \in \mathbb{Q} : n(3k-1) \in \mathbb{N},\ k \in \mathbb{N}\}$$

O **1** only
O **1** and **2** only
O **1** and **3** only
O **2** and **3** only

3. The diagram illustrates a right circular cone-shaped mountain, with radius 20 units and slant length 60 units. The points *A* and *B* lie on the same line of steepest ascent up the mountain at a distance of 10 units apart, as shown.



A sightseeing train is to be built travelling around the mountain, starting from point *A* and ending at point *B*. The curve shown represents the **shortest distance** track for the train, which first goes uphill from *A* and then downhill to *B*.

What is the length of the downhill portion of the track?

- O $\frac{200}{\sqrt{19}}$
- O $\frac{300}{\sqrt{30}}$
- O $\frac{300}{\sqrt{91}}$
- O $\frac{400}{\sqrt{91}}$

4. Regular n-sided polygon $A_1A_2A_3...A_n$ (labelled clockwise) shares side A_1A_2 with the regular k-sided polygon $A_1A_2B_3B_4B_5...B_k$ (labelled anticlockwise), where $n > k \geq 5$.

Which of these is always true? (Points are labelled in order.)

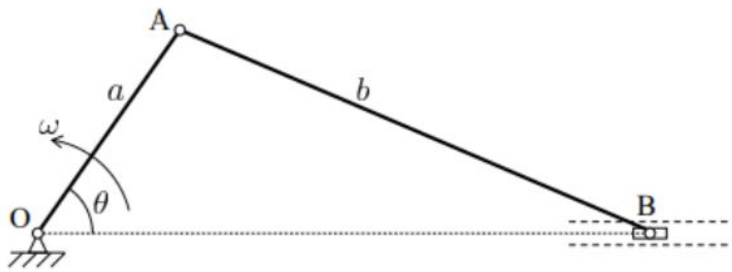
O The size of angle $B_kA_1A_n$, in degrees, is $\dfrac{180(k+n)}{kn}$.

O Triangle $A_1A_3B_3$ cannot have an obtuse angle.

O Points A_n, A_1 and B_3 cannot be collinear.

O The compound shape must have an even number of lines of symmetry.

5. A mechanical crank OA is driven by a piston AB such that it rotates at a constant rate as shown below. Point B is constrained to move horizontally and O is fixed.



At time $t = 0$, the whole system lies almost collinear on OB (i.e. $\theta \approx 0$, $\theta > 0$) and piston B is pushed inwards at a variable speed along its axis to turn the crank.

The rate of rotation, ω, is such that the angle θ (in radians) increases at a constant rate of ω radians per second.

The lengths of the inextendible rods OA and AB are a and b, where $b > a$.

At any time t, the **speed** of piston B is given by

O $$a\omega \sin \omega t + \frac{a^2\omega \sin \omega t \cos \omega t}{\sqrt{b^2 - a^2 \sin^2 \omega t}}$$

O $$-a\omega \sin \omega t - \frac{a^2\omega \sin \omega t \cos \omega t}{\sqrt{b^2 - a^2 \sin^2 \omega t}}$$

O $$a^2\omega^2 \sin^2 \omega t + \frac{a^4\omega^2 \sin^2 \omega t \cos^2 \omega t}{b^2 - a^2 \sin^2 \omega t}$$

O $$a^2\omega^2 \sin^2 \omega t - \frac{a^4\omega^2 \sin^2 \omega t \cos^2 \omega t}{a^2 - b^2 \sin^2 \omega t}$$

6. A small boat floats on the sea. It encounters waves of the form

$$y(x,t) = A\sin(kx - \omega t)$$

where y is the height of the water above its mean displacement at position x and time t, and A, k and ω are constants.

If the wavelength (peak-to-peak distance) of the water waves is 10 m, the amplitude is 0.5 m, and the waves travel at a horizontal speed of 2 ms^{-1}, then which of the following is true?

- O The value of k is $\frac{\pi}{10}$ (neglecting units)
- O The units of ω are ms^{-1}
- O The maximum vertical velocity of the boat is $\frac{\pi}{5}$ ms^{-1}
- O The value of A is equal to the maximum vertical acceleration of the boat (neglecting units)

7. A short-barrelled machine gun stands on horizontal ground. The gun fires an extremely large number of bullets, from ground level, at the same initial speed continuously for a period of one second, but does not fire outside of this interval.

During this time period, the angle of elevation of the barrel, $\alpha(t)$, decreases from its initial angle of $\frac{\pi}{4}$ to $\frac{\pi}{6}$ at the moment it fires the last bullet.

The initial speed of the bullets and the variation of the barrel elevation are such that all the bullets fired by the machine gun land on the ground at the **same time**.

Which function describes the variation of α with time *t* seconds after the first bullet was fired?

(Apply all standard projectile motion assumptions.)

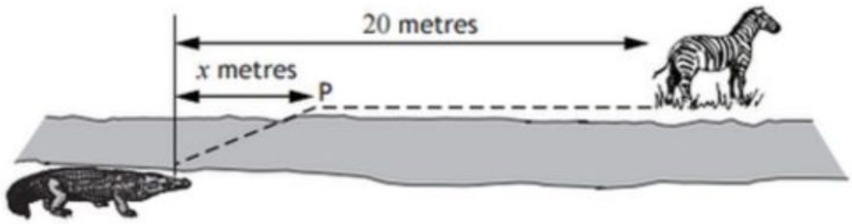
O $\alpha(t) = \sin^{-1}\left(\frac{1}{2} + \frac{1-t}{2(1+\sqrt{2})}\right)$

O $\alpha(t) = \sin^{-1}\left(\frac{1}{2} - \frac{t+1}{2(1-\sqrt{2})}\right)$

O $\alpha(t) = \cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{t+1}{2(1-\sqrt{3})}\right)$

O $\alpha(t) = \cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{1-t}{2(1+\sqrt{3})}\right)$

8. A crocodile is stalking a stationary prey located 20 metres further upstream on the opposite bank of a river.



The crocodile swims across the river a **horizontal** distance x in a straight line and then moves directly to the prey for the remainder of the distance as shown above. The speed of the crocodile is different (but constant) on land and in water.

The time taken for the crocodile to get to the prey is modelled by the equation

$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$

where x is as defined in the figure and T is measured in **tenths** of a second.

Which of these is true?

1 The shortest possible time the crocodile can reach the prey is 9.8 seconds

2 The width of the river is 3.6 metres

3 The speed of the crocodile travelling on land is 2.5 metres per second

O **1** and **2** only
O **2** and **3** only
O **1** and **3** only
O **1**, **2** and **3**

9. A parachutist jumps out of a plane at height h. They are subject to air resistance with a force of magnitude αv^2. The speed of the parachutist's motion satisfies

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \alpha v^2$$

Eventually, the parachutist approaches a terminal velocity, long before reaching the ground. The magnitude of this terminal velocity is

- O $\sqrt{\frac{mg}{\alpha}}$
- O $\frac{1}{2}\ln\frac{mg}{\alpha}$
- O $\frac{1}{2}\ln\left(1+\frac{\alpha}{mg}\right)$
- O $\sqrt{\frac{\alpha}{mg}}$

10. If *a* is a non-zero constant such that

$$\sin(x+y) = \frac{11}{a} \quad \text{and} \quad \sin(x-y) = \frac{2}{a}$$

then the value of $\frac{\tan x}{\tan y}$ is

- O $\frac{11}{9}$
- O $\frac{13}{11}$
- O $\frac{11}{7}$
- O $\frac{13}{9}$

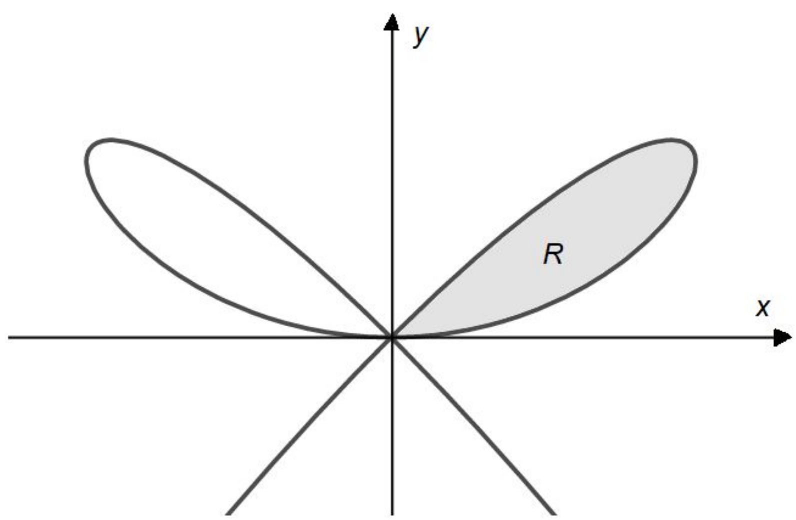
11. A circular wheel of unit radius rolls (without slipping) at a constant unit speed along smooth horizontal ground.

A fixed point P is attached to the circumference of the wheel. At time $t = 0$, the point P is located at the point of contact between the wheel and the ground.

Find the area of the region in the plane of the wheel bounded by the ground and the locus of P as the wheel turns through a single revolution.

O 2π
O 3π
O 4π
O 6π

12. The graph of $x^4 + y^3 = x^2 y$ is shown, with a closed-loop region R shaded.



By transforming the curve into a set of suitable parametric equations, or otherwise, find the area of region R.

O $\frac{1}{65}$
O $\frac{2}{65}$
O $\frac{2}{105}$
O $\frac{4}{105}$

13. You are given that

$$\frac{\mathrm{d}}{\mathrm{d}t}\tan^{-1} t = \frac{1}{t^2+1}.$$

Find and simplify fully, by any means necessary,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\frac{\sqrt[4]{\tan^{4/3} x - \tan^{2/3} x + 1}}{\sqrt{\tan^{2/3} x + 1}}\right) + \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2\tan^{2/3} x - 1}{\sqrt{3}}\right)\right]$$

for all valid *x*.

O $\sqrt[3]{\tan x}$

O $\sqrt[6]{\tan x}$

O $\sqrt[4]{\tan^{-1} x}$

O $(\tan^{-1} x)^{\tan x}$

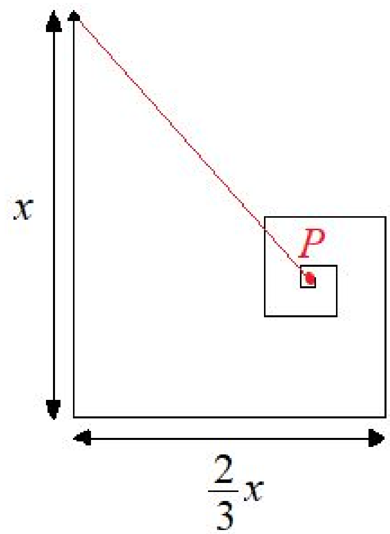
14. The number of stationary points on the curve with equation

$$3x^5 + y^{10} = 5x^3y + 18$$

is

O 4
O 6
O 22
O 30

15. An ant travels along a straight path with a distance of x meters. From then on, it turns left and covers $\frac{2}{3}$ of the straight distance it traveled before turning, and continues doing this until it eventually reaches an unknown point P.



The distance of *P* to the ant's starting position is kx. The exact value of *k* is

O $\frac{3\sqrt{7}}{7}$

O $\frac{3\sqrt{13}}{13}$

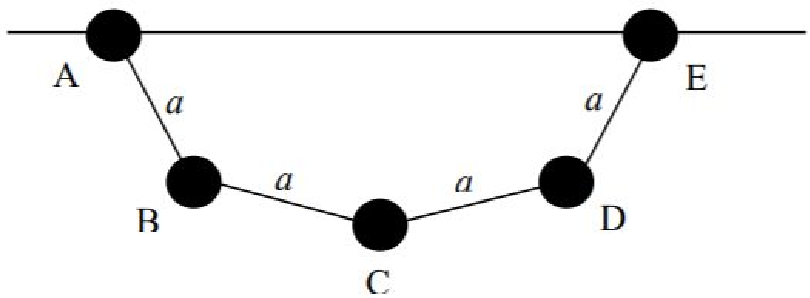
O $\frac{13\sqrt{7}}{37}$

O $\frac{7\sqrt{3}}{13}$

16. Two identical beads A and E, each of mass m, are threaded on a rough straight horizontal wire. The beads are joined by a light inextensible string of length $4a$.

Particles B, C, D, each of mass $2m$, are attached to the string so that $AB = BC = CD = DE = a$. The system hangs freely under gravity in a vertical plane as shown in the figure.

The coefficient of friction between each bead and the wire is $\frac{1}{4}$.



For the system to lie in a symmetrical shape as shown, the maximum possible distance AE is

O $\dfrac{2}{\sqrt{3}}a$

O $(2+\sqrt{3})a$

O $\dfrac{2\sqrt{15}}{3}a$

O $2a\left(\dfrac{1}{\sqrt{10}}+\dfrac{1}{\sqrt{2}}\right)$

End of Questions

Question Sources

Questions sourced from (generally in order of increasing difficulty)

- AQA / Edexcel / OCR A-level past papers
- MCAT past exams (American medical college entrance exam)
- NSAA past papers (Cambridge entrance exam - Natural Sciences)
- Brilliant.org, i-want-to-study-engineering.org, madasmaths.com
- Various international university admissions tests
  (UEE - NUS, Singapore; Gaokao - China; CSAT - South Korea; JEE - IIT, India)