

AQA A-Level Maths: Practice Paper 2

Focus: Statistics

Difficulty: Hard

Time: 1 hour 30 minutes

Marks:

Section A (multiple choice): 10 marks (15 minutes)

Section B (standard questions): 70 marks (1 hour 15 minutes)

(Total 80 marks)

Grade Boundaries: (approximate)

A*: 64 (80%)

A: 56 (70%)

B: 48 (60%)

C: 40 (50%)

D: 32 (40%)

Main Topics Examined:

Histograms, Summary Statistics, Conditional Probability,

Tree Diagrams, Binomial Distribution, Normal Hypothesis Testing

No questions are asked about a specific Large Data Set.

Section A: Multiple choice. You are advised to spend no more than **15 minutes** in Section A.

1. The interquartile range of the dataset below

4.3, 4.7, 4.8, 4.9, 5.1, 5.4, 5.5

is

- ☐ 0.5
- ☐ 0.6
- ☐ 0.7
- ☐ 0.8

[1 mark]

2. To understand the academic performance of 1,000 students, the systematic sampling method is adopted to choose 40 samples.

What should the sampling interval be?

- ☐ 24
- ☐ 25
- ☐ 40
- ☐ 50

[1 mark]

3. The Oakwood Jogging Club consists of 7 men and 6 women who go for a 5 mile run every Thursday. It is decided that a team of 8 runners would be picked at random out of the 13 runners, to represent the club at a larger meeting.

The probability that this team has more women than men is

- ☐ $\frac{6}{13}$
- ☐ $\frac{2}{11}$
- ☐ $\frac{7}{39}$
- ☐ $\frac{56}{101}$

[1 mark]

4. Which of these is **not** a requirement for the validity of the binomial model for a discrete random variable?

- ☐ Each trial must have equally likely outcomes.
- ☐ Each trial has only two mutually exclusive outcomes.
- ☐ Each trial is independent.
- ☐ Each trial has the same probability of success. [1 mark]

5. The probability that Anna wakes up before her alarm rings is 0.4.

The mean and variance of the number of times that Anna wakes up before her alarm rings, in the next 7 mornings, are

- ☐ Mean = 2.8, Variance = 1.68
- ☐ Mean = 2.8, Variance = 1.30
- ☐ Mean = 3.5, Variance = 2.8
- ☐ Mean = 3.5, Variance = 1.68 [1 mark]

6. C and D are events. The correct simplification of the event

$(C' \cap D') \cup (C \cap D') \cup (C \cap D)$ is

- ☐ $(C' \cap D)$
- ☐ $(C' \cap D)'$
- ☐ $(C \cap D')'$
- ☐ $(C \cap D)$ [1 mark]

7. A test statistic has distribution $B(25, p)$.

If the hypotheses for this test statistic are

$$H_0 : p = 0.35, \quad H_1 : p \neq 0.35$$

at a significance level of 5%, then the critical region for this test is

- ☐ $\{0, 1, 2, 3, 4\} \cup \{14, 15, 16, \dots, 24, 25\}$
- ☐ $\{0, 1, 2, 3\} \cup \{15, 16, 17, \dots, 24, 25\}$
- ☐ $\{5, 6, 7, \dots, 12, 13\}$
- ☐ $\{4, 5, 6, \dots, 13, 14\}$

[1 mark]

8. The mean age of the twenty members of a running club is exactly 28. The mean age increases by exactly 2 years when two new members join.

What is the mean age of the two new members?

- ☐ 20 years
- ☐ 30 years
- ☐ 40 years
- ☐ 50 years

[1 mark]

9. Edna wishes to investigate the energy intake from eating out at restaurants for the households in her village. She wants a sample of 100 households. She has a list of all 2065 households in the village. Ralph suggests this selection method:

“Number the households 0000 to 2064. Obtain 100 different four-digit random numbers between 0000 and 2064 and select the corresponding households for inclusion in the investigation.”

- a. What is the population for this investigation?

- ☐ Edna and Ralph
- ☐ The 2065 households in the village
- ☐ The energy intake for the village from eating out
- ☐ The 100 households selected

[1 mark]

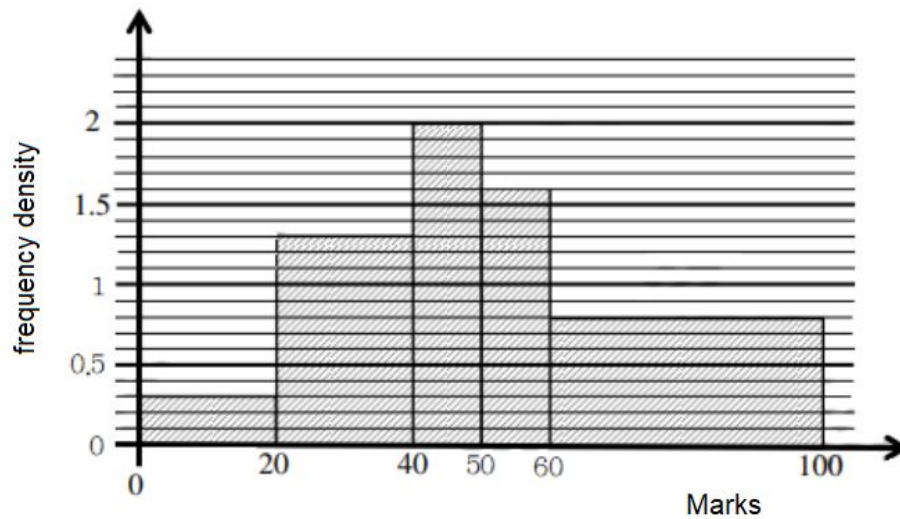
- b. What is the sampling method suggested by Ralph?

- ☐ Quota
- ☐ Systematic
- ☐ Stratified
- ☐ Simple random

[1 mark]

Section B: Standard questions. Ensure you leave enough time for Section C.

10. The histogram below shows the distribution of the marks of 250 students who recently sat the same exam.



- a. The grade boundaries for an A in this exam was 60-72 marks, and for a B was 52-59, both inclusive.

Estimate how many students achieved **either** grade A **or** B in this exam.

[4 marks]

- b. Use the histogram to estimate the median mark.

[2 marks]

c. Calculate estimates for the

i) mean

ii) standard deviation

of the marks of these students.

[4 marks]

[Total for Q10: 10 marks]

11. The masses, x kg, of 40 dancers were measured and the results were summarised as shown below.

$$\sum (x_n - 50) = 140 \quad \text{and} \quad \sum (x_n - 50)^2 = 4490.$$

The mean and standard deviation of x for this sample is μ and σ respectively.

- a. Find the values of μ and σ . [6 marks]

- b. State the value of

$$\sum (x_n - \mu)$$

[1 mark]

[Total for Q11: 7 marks]

12. The mean and standard deviation of the test marks of 40 pupils in a Mathematics class are 65 and 18, respectively.

The mean and standard deviation of the test marks of the 24 boys in the class are 72 and 20, respectively.

Find the mean and standard deviation of the test marks of the 16 girls in the class. [9 marks]

[Total for Q12: 9 marks]

13. A sample of 200 families were asked about the ethnicity of their recent orders for takeaway food. In this sample, the only responses were either Chinese (C), Indian (I), both Chinese and Indian, or neither Chinese nor Indian.

It was found that 136 families responded neither Chinese nor Indian.

Given that $P(C) = 0.23$ and $P(C \mid I) = 0.4$, find the probability that a family regularly orders from both Chinese and Indian takeaways. [7 marks]

[Total for Q13: 7 marks]

14. Newborn babies are tested for a mild illness which affects 1 in 500 babies. The result of a test is either positive or negative. A positive test implies the baby has the illness. However, the test is not perfect:

- for babies with the illness, the probability of a positive result is 0.99
- for babies without the illness, the probability of a negative result is 0.95

a. Draw a probability tree diagram to illustrate this situation. [2 marks]

b. Find the probability that

i) the result is positive [2 marks]

ii) the test gives the correct diagnosis [2 marks]

- c. Given that the result of a test is positive, show that the conditional probability that the baby has the illness is 0.038 (2 s.f.) and make one comment about the quality of the test based on your answer. [4 marks]

- d. It is required to raise the probability found in part c) by improving the testing process. To achieve this, it is intended to increase the probability of negative results for babies who do not have the illness from its current value of 0.95 to p .

Find the value of p that would raise the probability in part c) to 0.5.

[5 marks]

[Total for Q14: 15 marks]

15.

- a. X is a discrete random variable with a binomial distribution. The mean of X is 4.245 and the standard deviation of X is 1.745.

Find $P(2 \leq X \leq 6)$.

[6 marks]

- b. A different discrete random variable Y satisfies $Y \sim B(n, p)$, and that $P(Y = 2) = P(Y = 3)$.

Show that the mean of Y is $3 - p$.

[6 marks]

[Total for Q15: 12 marks]

16. A group of 1200 soldiers completed an assault course, early in the morning. A random sample of 60 soldiers were selected from the group of 1200. The time taken by each of these 60 soldiers to complete the assault course, x minutes, was recorded and:

- the sum of all x is 1350
- the sum of all x^2 is 30685

a. Find unbiased estimates for the mean and variance of the time taken by the 1200 soldiers who completed the course early in the morning. [2 marks]

- b. A group of 1500 soldiers completed the same assault course, late in the afternoon. A random sample of 60 soldiers were selected from the group of 1500. The time taken by each of these 60 soldiers to complete the assault course, y minutes, was recorded and it was found that y had a sample mean of 24.1 and a sample variance of 5.48.

Test, at the 5% significance level, whether or not the mean time of the 1500 soldiers which completed the assault course in the afternoon is different to that of the 1200 soldiers which completed the same course in the morning.

In your answer, you should:

- state your hypotheses clearly,
- define random variables,
- show clearly how you derive the relevant z-score value, and
- conclude your test in context.

[8 marks]

(More space for Q16b)

[Total for Q16: 10 marks]

End of Questions

Question Sources

- Q2: Gaokao Past Paper (Math)
Q3, 5, 7: MadasMaths Paper
Q8: ENGAA Past Paper (Mathematics)
Q9: AQA A-Level Maths Past Paper
- Q10, 11, 12: MadasMaths Paper
Q13: AQA A-Level Maths Past Paper
Q14: MEI Past Paper
Q15, 16: MadasMaths Paper

ENGAA is an entrance exam for studying Engineering at Cambridge.
Gaokao is an exam in China.