

PROBABILITY

Probability is the measure of uncertainty of various phenomenon, numerically. It can have positive value from 0 to 1.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty.

$$\text{Probability} = \frac{\text{no. of favorable outcome}}{\text{total no. of outcomes}}$$

Approach to Probability:

- Statistical approach** : Observation & data collection
- Classical approach**: Only Equal probable events
- Axiomatic approach**: For real life events. It closely relates to set theory.

2. Random Experiments:

An experiment is called random experiment if it satisfies the following two conditions:

- It has more than one possible outcome.
- It is not possible to predict the outcome in advance.

Outcomes: a possible result of a random experiment is called its outcome.

Sample space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by the symbol S. Example: In Toss of a coin, Sample space is Head, Tail.

Sample point: Each element of the sample space is called a sample point. E.g. in toss of a coin, Head is a Sample point.

3. Event:

It is the set of favorable outcome.

Any subset E of a sample space S is called an event. E.g. Event of getting odd outcome in a throw of a die

Occurrence of an event: the event E of a sample space S is said to have occurred if the outcome ω of the experiment is such that $\omega \in E$. If the outcome ω is such that $\omega \notin E$, we say that the event E has not occurred.

Types of Event

- Impossible and Sure Events
- Simple Event
- Compound Event

Impossible and Sure Events:

The empty set ϕ and the sample space S describe events. **Impossible event** is denoted by ϕ , while the whole sample space, S, is called the **Sure Event**.

E.g. in rolling a die, impossible event is that number is more than 6 & Sure event is the event of getting number less than or equal to 6.

Simple Event:

If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

In a sample space containing n distinct elements, there are exactly n simple events.

E.g. in rolling a die, Simple event could be the event of getting 4.

Compound Event:

If an event has more than one sample point, it is called a Compound event.

E.g. in rolling a die, Simple event could be the event of getting even number

Algebra of Events

- Complementary Event
- Event 'A or B'
- Event 'A and B'
- Event 'A but not B'

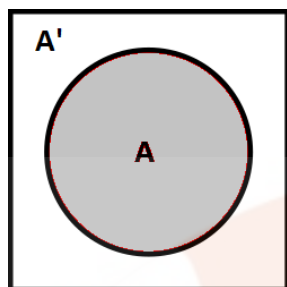
Complementary Event

Complementary event to A = 'not A'

Example: If event A = Event of getting odd number in throw of a die, that is {1, 3, 5}

Then, Complementary event to A = Event of getting even number in throw of a die, that is {2, 4, 6}

$A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A$ (Where S is the Sample Space)

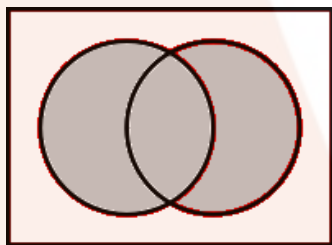


Event (A or B)

Union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both.

When the sets A and B are two events associated with a sample space, then ' $A \cup B$ ' is the event 'either A or B or both'. This event ' $A \cup B$ ' is also called ' A or B '.

Event ' A or B ' = $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$.

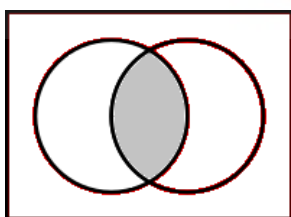


$A \cup B$

Event 'A and B'

Intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B . i.e., which belong to both ' A and B '. If A and B are two events, then the set $A \cap B$ denotes the event ' A and B '.

Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

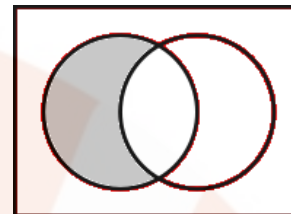


$A \cap B$

Event 'A but not B'

$A - B$ is the set of all those elements which are in A but not in B . Therefore, the set $A - B$ may denote the event ' A but not B '.

$$A - B = A \cap B'$$



$A \cap B'$

Mutually exclusive events

Events A and B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e., if they cannot occur simultaneously.

Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Then A & B are mutually exclusive events, they cannot occur simultaneously.

Simple events of a sample space are always mutually exclusive.

Exhaustive events

Lot of events that together forms sample space.

Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Even A & B together forms exhaustive events as it forms Sample Space.

4. Axiomatic Approach to Probability:

It is another way of describing probability. Here Axioms or rules are used.

Let S be sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval $[0,1]$ satisfying the following axioms

- i) For any event E , $P[E] \geq 0$
- ii) $P[S] = 1$
- iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(\phi) = 0$. Let $F = \phi$ and $E = \phi$ be two disjoint events,

$\therefore P(E \cup \phi) = P(E) + P(\phi)$ or $P(E) = P(E) + P(\phi)$ i.e., $P(\phi) = 0$

Let S be a sample space containing outcomes $\omega_1, \omega_2, \dots, \omega_n$, i.e., $S = \{\omega_1, \omega_2, \dots, \omega_n\}$ then

- i. $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
- ii. $P(\omega_1) + P(\omega_2) + \dots + P(\omega_n) = 1$
- iii. For any event E , $P(E) = \sum_{\omega_i \in E} P(\omega_i)$
- iv. $P(\phi) = 0$

Probabilities of equally likely outcomes:

Let $P(\omega_i) = p$, for all $\omega_i \in S$ where $0 \leq p \leq 1$,

then $p = \frac{1}{n}$ where n = number of elements.

Let S be a sample space and E be an event, such that $n(S) = n$ and $n(E) = m$. If each outcome is equally likely, then it follows that

$$P(E) = \frac{m}{n}$$

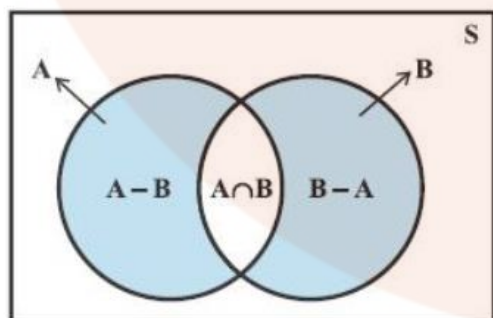
$$= \frac{\text{Number of Outcomes favourable to } E}{\text{Total possible outcomes}}$$

Probability of the event 'A or B' :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability of the event 'A and B'

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$



Probability of the event 'Not A'

$$P(A') = P(\text{not } A) = 1 - P(A)$$