Compressed Sensing Image Reconstruction for CASA

Jonas Schwammberger

Today

Abstract

Solar Flares Abstract

Contents

1 Image Reconstruction for Interferometers

The angular resolution of a radio antenna is proportional to the wavelength divided by the dish diameter. To achieve high angular resolution the diameter should be as large as possible. But real world limitations like steering accuracy and price limit the diameter. Currently the largest single dish antenna has a diameter of about 500 meters[**?**].

However, Radio Interferometers, where many antennas together act as a single instrument, achieve angular resolutions that are comparable to a dish with several kilometer diameters.

In the past, interferometers like VLA, ALMA and LOFAR have made numerous discoveries.

However, Radio Interferometers achieve angular resolutions that are comparable to a single dish with a diameter of several kilometers. Instead of one antenna, Interferometers use pairs of smaller antennas spaced apart.

Baseline.

Interferometers do not observe the sky directly. Each antenna pair measure Fourier Components (Visibilities) of the sky brightness. The observed image has to be reconstructed from the measured Visibilites. Since the interferometer can only observe a limited number of Visibilities, the reconstruction is an ill-posed inverse problem. For small Field of View imaging, the CLEAN class of Algorithms[1][2][3][4] have been developed and is the de-facto standard in Radio Astronomy. It is not guaranteed to reconstruct the true image in theory. In practice it has produced remarkable results with expert tuning. New generation Interferometers like ASKAP, Pathfinder and SKA are built with wide Field of View imaging in mind. The CLEAN Algorithms have been extended for Wide Field of Views, but require even more tuning by experts.

The Theory of Compressed Sensing[5][6] has seen success in solving ill-posed inverse problems. It is flexible in its application and has produced remarkable results image de-noising[**?**], in-painting[**?**] and super-resolution[**?**]. Applying Compressed Sensing to wide Field of View imaging is an active field of research. In the last decade numerous approaches have been developed showing the potential of Compressed Sensing: Accurately modelling the effects of wide Field of View imaging, while reducing the tunable parameters and possibly super-resolved images[7]. Current research focuses on how the effects of wide Field of View can be accurately modelled while still being computationally efficient.

In this project, a proof of concept Compressed Sensing approach was developed and implemented in the Common Astronomy Software Application(CASA). []The approach is focused on small Field of View imaging and the reduction of expert intervention.

1.1 Deconvolution: Ill-posed Inverse Problem

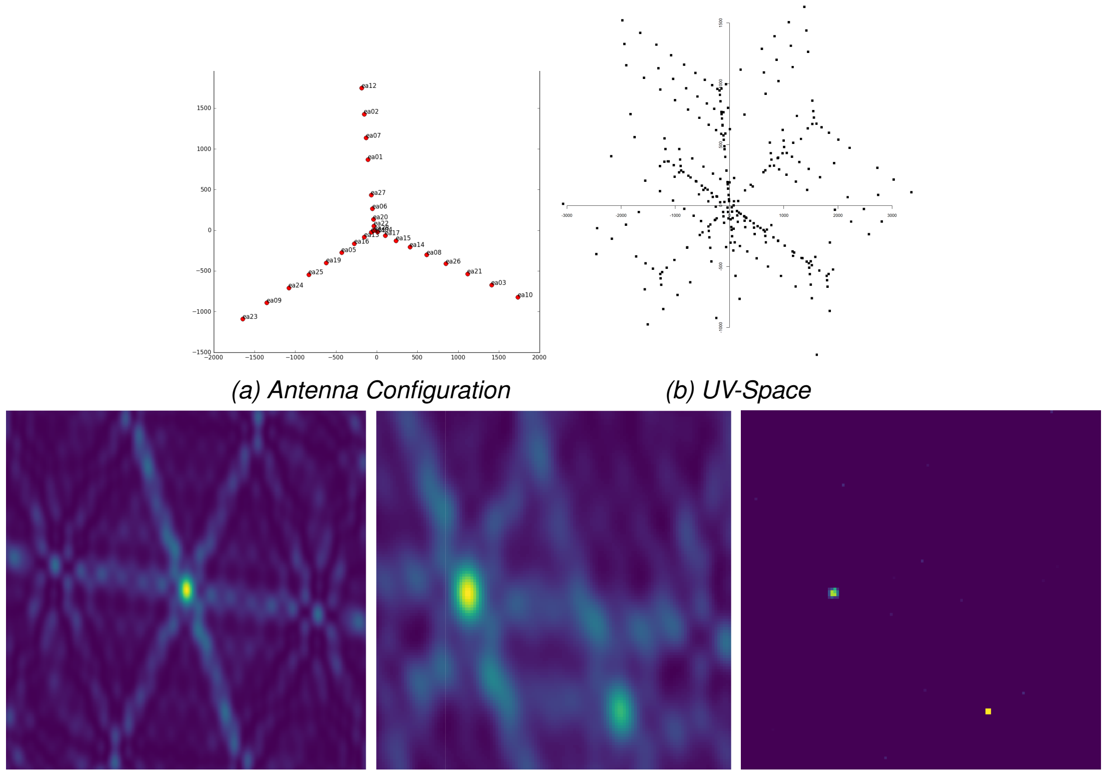
small field of view, the interferometer measures (almost) Fourier Components of the sky brightness. Inverse FFT works, it creates an image. Since but since it only measures a limited set of Fourier Components, the image is corrupted.

Try to find the true image from undersampled measurements. It is formulated as as a deconvolution problem: The corruption is modelled as a point spread function (PSF). The task is to deconvolve the dirty image, removing the corruption and restoring the original image.

$$x \star PSF + N = I_{dirty} \tag{1.1}$$

CASA produces a dirty image and a PSF.

In this project, the PSF models the instrumental effects of undersampling, antenna beam patterns. This problem



(a) Antenna Configuration (b) UV-Space

(c) PSF (d) dirty image (e) true image

Figure 1: Deconvolution Problem VLA: Retrieve the true image when only PSF and dirty image are known

1.2 Approximation with CLEAN

In each iteration of CLEAN, it searches the highest peak of the dirty image and removing a fraction of the PSF at that point. It stops until the next highest peak is below a threshold, or if the maximum number of iterations was reached. The fraction of the PSF, threshold and number of iterations are all tunable by the user. State of the art implementations expose even more parameters. The reconstruction quality depends on the chosen parameters and require extensive user input.

CLEAN does not solve the deconvolution problem (1.1) directly. Instead, it greedily minimizes the objective (1.2). It is easy to see that if CLEAN minimizes the objective to zero, it has found a solution to the original deconvolution problem in a noiseless environment.

Note that the L0 norm[1] acts as the indicator[2] function.

$$\underset{x}{minimize} \ \|I_{dirty} - x \star PSF\|_2^2 + \lambda \, \|x\|_0 \tag{1.2}$$

Since the original problem is ill-posed, the objective (1.2) may have several zero points. In practice, CLEAN is stopped before it reaches zero. The addition of noise can add spurious peaks in the dirty image. By stopping

[1] This is a common abuse of notation in Compressed Sensing literature: The "L0 norm" is not a norm.

[2] For the L0 norm to work we need to define $0^0 = 0$

early, CLEAN regularizes the objective. It assumes only a limited number of point sources exist in the image. The larger the magnitude of the peak, the more likely it is to be a real point source.

In short, CLEAN does a greedy approximation of the deconvolution problem, and assumes the resulting image consists out of a few point sources. The question remains, how close the CLEAN approximation is to the true image? If the true image consists out of a few point sources, CLEAN produces a good approximation. Extended emissions however are harder for CLEAN to reproduce. The peak of extended sources is lower than that of point sources. It is harder for CLEAN to distinguish extended sources from noise.

The CLEAN regularization scheme is not ideal for extended sources. Ideally another way of regularization would be chosen for extended emissions, but the regularization is a fixed part of the CLEAN algorithm.

1.3 The Compressed Sensing Reconstruction Framework

In the Compressed Sensing Framework, an approach is split into three separate parts:

- An objective with a data and regularization term.
- A prior function $p()$ in which transforms the image into a sparse domain.
- An optimization algorithm that is suited for the objective.

To demonstrate the flexibility of the Compressed Sensing Framework, we convert CLEAN into a Compressed Sensing approach. First we add a regularization term to (1.2) and arrive at the new objective (1.3). The objective contains the original CLEAN data term and a new regularization term. The data term forces the reconstruction to be close to the measurement, while the regularization term forces the reconstruction to be plausible. λ models the expected noise in the problem. Note that the $\left\|Px\right\|_0$ acts as an indicator function.

$$\underset{x}{minimize}\ \left\|D_{dirty} - x \star PSF\right\|_2^2 \,+\, \lambda\, p(x) \tag{1.3}$$

The Prior P transforms the image in a sparse domain. CLEAN assumes the x contains a few point sources. In Compressed Sensing terminology, it assumes x is sparse in image space. Since x is already an image, the Prior P in Compressed Sensing CLEAN is the identity matrix.

The last step is choosing a similar optimization algorithm: In every iteration, CLEAN searches the highest peak in the dirty image. Matching Pursuit is a greedy optimization algorithm. In every iteration it searches the step which minimizes (1.3) the most. This Compressed Sensing approach is similar to CLEAN, but the new objective has a unique global minimum even with the presence of noise. The tunable parameters of CLEAN are replaced by a single parameter λ.

The strength of Compressed Sensing Framework is its flexibility: The CLEAN prior works well on point sources, but is not ideal for extended emissions. In this Framework, the prior P can be replaced without changing the objective or the optimization algorithm. This has led to increased interest in Compressed Sensing for wide Field of View imaging.

The compressed sensing framework

Clean in the cs framework

The guarantees of Compressed Sensing reconstruction: Incoherent from the measurement space and sparse space is sparse.

Incoherence is easy. Interferometers measure in the Fourier space(This is an approximation for small field of view imaging. The approximation breaks apart in wide field of view). The image space is maximally incoherent from the Fourier space. Intuitively, A change in a single pixel will change all fourier components. A change in a single fourier component, changes all pixels.

maximize the information gained for each element in the sparse space.

The sparse space is here to distinguish true image from unlikely candidates. It models our prior knowledge.

Then, one can reconstruct the true image from undersampled measurements. How many measurements are needed? that depends on how sparse it is.

Taking again CLEAN as an example, if we know the image contains only one point source, we can locate it with only a few Visibilities. However if the image contains many point sources located closely together, we need more Visibilities.

The average case analysis is not trivial,

2 Inverse Problem for wide Field of View Imaging

So far the small Field of View inverse problem has been introduced where each antenna pair measures a Visibility of the sky brightness distribution. This leads to the small Field of View measurement equation (2.1). It is identical to the two dimensional Fourier Transform. In practice the Fast Fourier Transform (FFT) is used, since it scales with $n\,log(n)$ instead of n^2 pixels.

$$V(u,v) = \int\int x(l,m)e^{2\pi i(ux+vy)}dldm \tag{2.1}$$

For wide Field of View imaging, two effects break the two dimensional Fourier Transform relationship: Non-coplanar Baselines and the celestial sphere which lead to the measurement equation (2.2). Note that for small Field of View $1 - x^2 - y^2 \ll 1$, and (2.2) reduces to the 2d measurement equation (2.1).

$$V(u,v,w) = \int\int \frac{X(x,y)}{\sqrt{1-x^2-y^2}}e^{2\pi i(ux+vy+w\sqrt{1-x^2-y^2})}dxdy \tag{2.2}$$

Non-coplanar Baselines lead to a third component w for each Visibility. Figure 2 shows the the u v and w coordinate system. w is essentially the pointing direction of the instrument. The UV-Plane is the projection of the antennas on a plane perpendicular to the pointing direction. Which point in the UV-Plane get sampled and what w component it has depends on the pointing direction. If the instrument points straight up, the UV-Plane is a tangent to earth's surface, and the w term compensates for earth's surface curvature. If however the instrument points at the horizon, the projected UV-Plane gets squashed and w compensates for antennas which lie far behind the UV-Plane. In essence, w is a phase delay that corrects antenna positions in three dimensions. The wide Field of View measurement equation (2.2) would account for the w phase delay, but it breaks the the two dimensional Fourier relationship and the FFT cannot be used. The W-Projection [8] algorithm approximates the effect of the w term restores the two dimensional Fourier relationship.

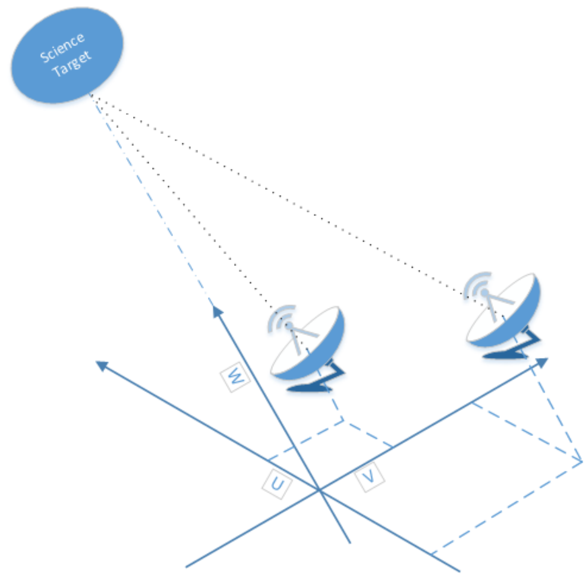


Figure 2: U V and W coordinate space

A-Projection [9]

2.1 Directionally Dependent Effects (DDE)

spread spectrum phenomenon

2.2 Calibration

2.2.1 Self-Calibration

3 Compressed Sensing Image Reconstruction

The Framework Flexibility:

Gurobi[10] was used

3.1 Sparseland Prior and Overcomplete Representations

L0, L1, L2 Priors

The Compressed Sensing CLEAN objective (1.3) uses the L0 norm for it's regularization term, which means the Objective Function is not convex. There are specialized solvers for the L0 compressed sensing. The L1 relaxation however is practically guaranteed to have the same minimum as the L0 norm and results in a convex objective function. Since Gurobi works better on the L1 relaxation it was chosen for this project.

The flexibility allows for a lot of freedom in design which can lead to different approaches for the same problem. So far, there are no 'best practices' for Astronomy: No prior, objective or optimization algorithm works strictly better than every other choice. Furthermore the choices for the individual parts influence each other. Compressed Sensing is flexible, but not every optimization algorithm works with every prior.

3.2 The Sparseland Prior and Incoherence: How Compressed Sensing works

For Compressed Sensing we need a Prior P in which our signal can be sparsely represented. It is not guaranteed that such a space exists, but for natural signals there always seem to be. This has led to the idea of the Sparseland Prior (3.1) which is at the core of Compressed Sensing: We assume for our signal x there exists a dictionary D. Each entry represents a signal part which can be present. D is potentially a large, but has a finite number entries. We assume that any x can only consist of a few signal parts of D. This means the coefficients for the signal parts in the dictionary α are all zero except for s entries for all valid x.

$$x = D\alpha \qquad x \in \mathbb{R}^n, \alpha \in \mathbb{R}^m, D \in \mathbb{R}^{n*m}, \qquad n \leq m$$
$$\|\alpha\|_0 = s \qquad s \ll n \leq m \tag{3.1}$$

In image compression this phenomenon was can already be observed: image depicting nature scenes tend to be sparse in the wavelet domain. If x in (3.1) are nature scenes, we can create a Dictionary D of wavelets. A single image x can be represented with a few wavelets, meaning the number of non-zero entries s in α is far lower than the number of pixels n. All that is left to do for compression is save the non-zero entries of α. Two effects are of note: When x is noisy, or when x is not a nature image, the resulting α is not sparse. In Compressed Sensing this fact is exploited to reconstruct the true image from under-sampled measurements.

Back to the ill-posed inverse problem of interferometry: We measure the complex Visibilities of a band-limited signal. The Nyquist-Shannon rate states that if our band limited signal has at most frequency f, our sample frequency needs to be higher than $2f$. For n pixels, this is the case when we measure all n complex Visibilites (two samples per Visibility). If we have fewer samples, the Nyquist Shannon theorem states we cannot reconstruct the true image.

But what if we know our image is a Sparseland Signal and we happen to know the dictionary? Let us assume our image consists of $n = 20 * 20$ pixels and our dictionary of $m = 1000$ wavelets. Further assume at most $s = 10$ of the wavelets are non-zero for a given image. Could one just measure the 10 non-zero components of α and reconstruct the image? If we have prior knowledge about the location of the non-zero components, we would need 10 samples to reconstruct the image. Sadly, this is not the case in general. With the first sample we have about a $1/100$ chance of measuring a non-zero component. Note that if we measure a non-zero component, we learn $1/10$ of the information about the image. If we hit a zero component, we learn practically nothing. The question is, is there a way we can maximize our information gain of the non-zero components for each sample? In fact, there is: By having the measurement space be as incoherent as possible from the dictionary space, we maximize the information gained per sample.

[How many samples are needed]

Constructing an incoherent sampling space is surprisingly easy. Random projections are likely to produce a incoherent sampling space. Since we use an interferometer, we are bound to measure in Fourier Space. Depending on the prior, the Fourier Space might be coherent with the dictionary space and we do gain the maximum amount of information. As discussed in section 2, wide Field of View imaging breaks the two dimensional Fourier relationship. McEwen et al[11] showed that the wide Field of View measurement equation can help with incoherence, and demonstrated higher image reconstruction quality on simulated data.

The Sparseland prior is the basis of which Compressed Sensing builds. The dictionary can contain any function and is not limited to wavelets. It can even contain a mixture of for example wavelets, gaussians and cosine functions. Sparseland priors lend themselves to over-complete representations, where the number of dictionary entries is multiple times higher than the number of pixels($m \gg n$).

An appropriate prior for Radio Astronomy is still under research, currently Starlets[12] and Curvelets[13] are on top of the foodchain

3.3 Choosing the Objective Function

There are three different Compressed Sensing objectives: The analysis method, where the image x is minimized directly, the synthesis method where the sparse vector α is minimized, or by in-painting the missing Visibilities V_2.

$$\begin{aligned}
analysis: &\quad \underset{x}{minimize}\ \left\|D_{dirty} - x \star PSF\right\|_2^2 \quad + \lambda \left\|Px\right\|_1 \\
synthesis: &\quad \underset{\alpha}{minimize}\ \left\|D_{dirty} - D\alpha \star PSF\right\|_2^2 + \lambda \left\|\alpha\right\|_1 \\
in-painting: &\quad \underset{V_2}{minimize}\ \left\|D_{dirty} - F^{-1}MV_2\right\|_2^2 \quad + \lambda \left\|PF^{-1}V_2\right\|_1
\end{aligned}$$

All three objective functions have the same global minimum. Retrieving x for the analysis objective is trivial, or the second and third objective x can be retrieved by $x = D\alpha$ and by $x = F^{-1}V_2$ respectively. [Empirical and theoretical studies have shown an advantage of the analysis objective over the other two [**?**]]. However, depending on the measurement space, prior and optimization algorithm, one objective may be easier to solve than others. The analysis objective is not useful when there is no transformation from x into the sparse space. This is the case for most over-complete priors: In that case, P is a $m * n$ matrix and $Px \neq \alpha$. The synthesis method just requires a transformation from the sparse space to image $D\alpha = x$. Similarly one might chose the in-painting method when the prior is a convolution in image space: Convolutions in image space are equivalent to a multiplication in Fourier Space.

wide field imaging considerations

$$\underset{x}{minimize}\ \left\|V - MF_{WFOV}x\right\|_2^2 + \lambda \left\|Px\right\|_1 \tag{3.2}$$

A-projection lofar [14]

3.4 Compressed Sensing Reconstruction Algorithms in Astronomy

3.4.1 PURIFY

Prior: Mixture of Dirac functions and Daubechies Wavelet (DB1 - DB8)

Objective: analysis

Optimizer: SDMM

Dirac is a fancy way of saying ”it is sparse in pixel space”

3.4.2 Vis-CS

Prior: dictionary of gaussians

Objective: Synthesis

Optimizer: Coordinate descent

3.4.3 SASIR

Prior: Starlets

Objective: in-painting

Optimizer: FISTA

Figure 4: Restoration Process in CASA

3.5 Implementation In CASA

CASA is a software package built for solving the deconvolution problem for instruments. Casa works in two separate cycles, the major and minor cycle. The major cycle fourier transforms the Visibilities to the image space and back. The minor cycle is the deconvolution algorithm, which tries to find the true image from a dirty image and a PSF.

The major cycle creates the dirty image and PSF, and at the end of the deconvolution algorithm, transforms the image back to the Fourier Space.

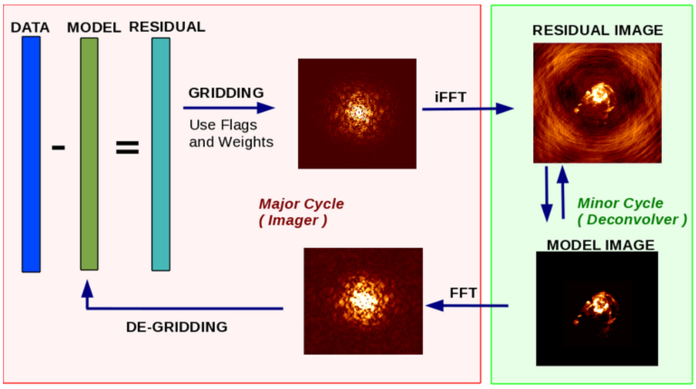


Figure 3: Casa Major Minor Cycle, source [15]

In each major cycle, the Model Visibilities get updated, it is a $\mathcal{X}^2$ approximation of the Visibilities.

Major cycle CASA is a software package built for solving the deconvolution problem for instruments like VLA and ALMA. ”Data” Column measurements(calibrated), model column contains the ”true” visibilities and the residual column only noise. The architecture is oriented after the CLEAN algorithm, it is split in a major and minor cycle.3. The first part of the major cycle produces the dirty image and the PSF. The minor cycle is where a deconvolution algorithm ”cleans” the dirty image, several iterations of CLEAN. Major cycle ends with the forward fourier transform. Chi^2 approximation of the visibilities. At the end of several major cycle, the Model column should contain an approximation of the true visibilities while the Residuals should be pure noise.

Wide field of view imaging aka A- and W- Projections are handled in the major cycle. most often a CLEAN derivate.

Dirty image, Model image and cleaned image

The idea of the dirty beam and the clean beam. The output of CASA is the model image convolved with the clean beam plus residuals. Because the model image contains many small peaks, any structure smaller than the clean beam is implausible. Convolving with a gaussian is essentially reducing the resolution. But this is not the case. CLEAN can lead to implausible model images depending on the content: If only a few point sources are visible, clean is plausibe. But for extended emissions clean produces a an area of many peaks which is not true.. With compressed sensing, the ideal prior leads to the true model image.

CASA can be extended new deconvolution algorithms, changing minor cycles. During the project it was evaluated if CASA could be modified so wide Field of View imaging can handled by the minor cycle. It was not possible. The implementation is restricted to the deconvolution in the data term. This excludes the in-painting objective function. Or that the data term minimizes on the Visibilities directly.

4 Reconstruction for VLA Observations

Compressed Sensing Reconstruction used on two different tasks:

- Center detection on sunburst data
- Reconstruction from incomplete measurements of Supernova Remnant G55

Two different problems. One is the

Structure potentially smaller than the primary beam.

4.1 Sunburst Center Detection

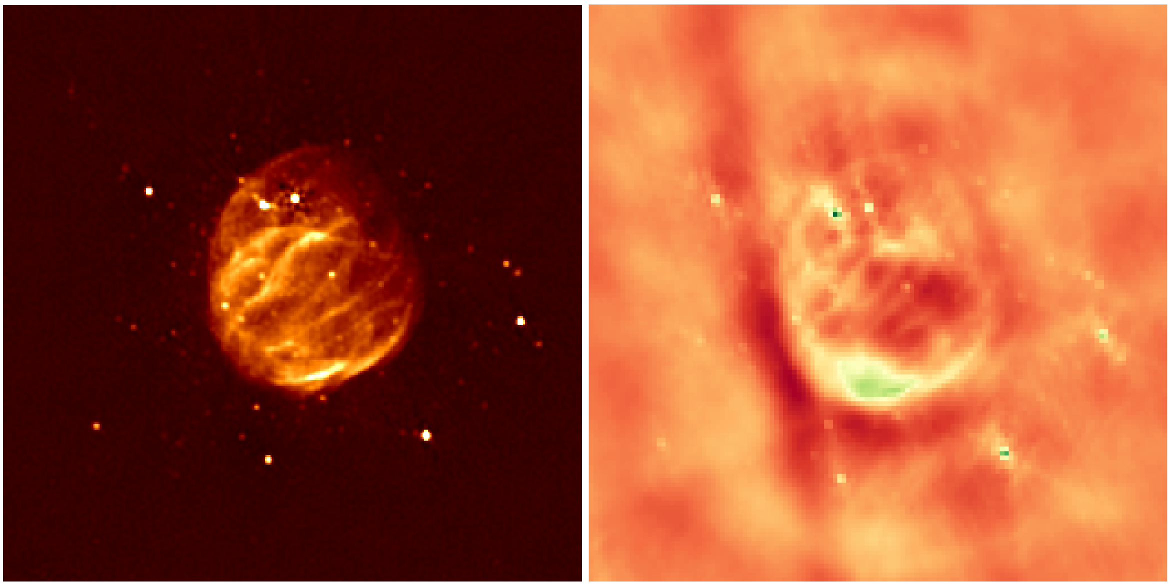
Event date.

Sub- Primary Beam. CS Objective Function

Figure: Dirty Map Peak, CLEAN, Single Peak Clean, Single Peak CS Reconstruction

Wider Variance

4.2 Reconstruction of Supernova Remnant G55



(a) Reconstruction by NRAO. Source:[16]   *(b) Dirty Image*

Figure 5: SNR G55 source observed by VLA.

Data from the supernova remnant G55 are publicly available through the CASA imaging tutorial[17]. It is a 10 second calibrated observation by VLA. 5 is the dirty image from the 10 second observation. The full dataset contains 8 hours worth of data, which is not readily available. It is unknown if reconstruction 5a was created with the full 8 hours or with another observation. The deconvolution algorithm is also unknown. For this project, the reconstructed image is assumed to show the true image of the sky.

The true image 5 has an "egg shaped" supernova remnant with a number of strong and faint point sources. Several tasks: Point Source detection and modelling of extended emissions. Point Sources inside extended emissions. The ideal Prior would

The dirty image5b is of course corrupted by a point spread function, but it is not the only effect: There is negative "trench" striking through the image as well as brighter regions around the remnant.

Small field of View imaging was employed, even though this task is more a wide field imaging problem. It makes it harder for the deconvolution algorithms.

Several Priors were used with the an analysis objective and Gurobi as the optimizer. Simple Priors

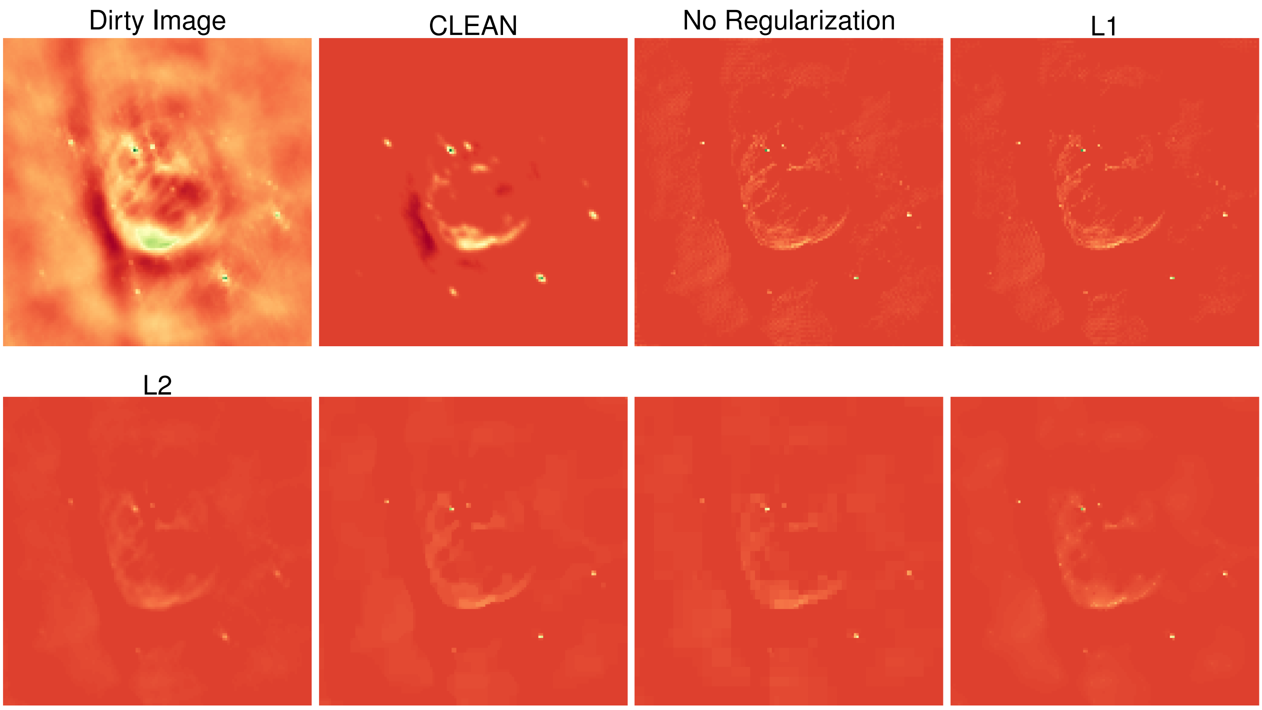
CLEAN algorithm, parameters were used from the CASA imaging tutorial.

All algorithms constrain the model image to be non-negative. Physical plausible and shown to produce better results on synthetic data[11]

Each Prior has a λ, it changes for each prior. The Miller[18] λ estimation was used and is shown in equation (4.1). For the estimation an approximation of the solution x is needed. In this project, the deconvolution was calculated without regularization and used to estimate the λ parameter for each prior.

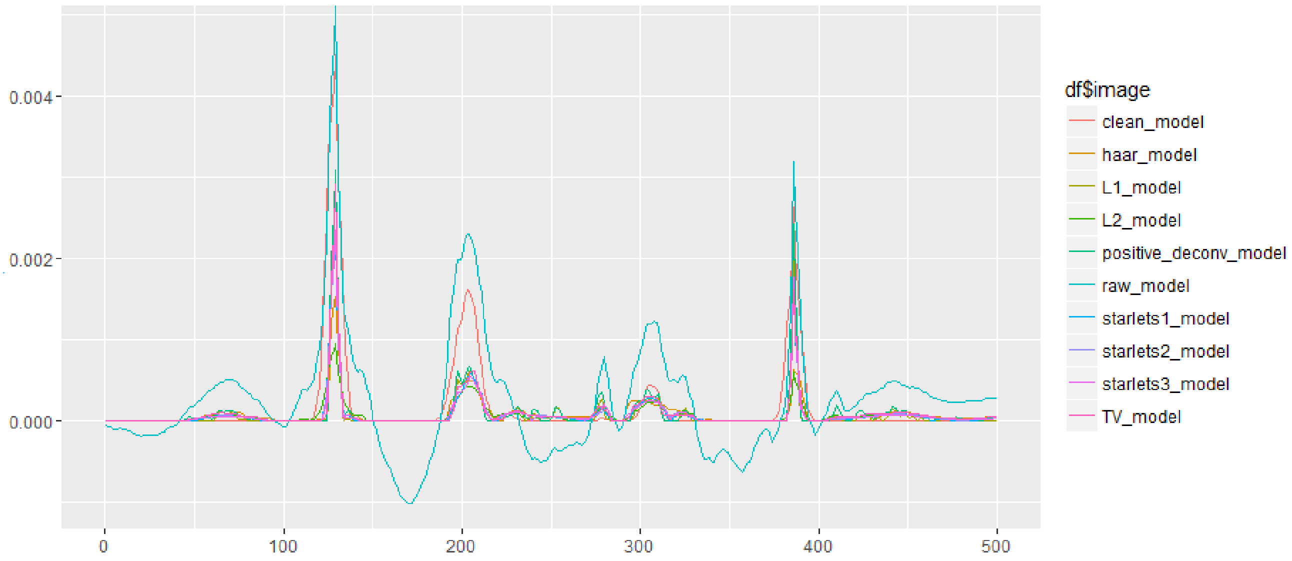
$$\lambda = e/E \tag{4.1}$$

two figures, one with all reconstructed images for all algorithms. A cut through the image showing the intensity profile.



CLEAN: Detects the brightest point sources. But only finds part of the extended emission. Limited resolution of the center. Not all point sources can be detected. CLEAN models the divet as a region of negative emission (parameters can be changed to stop this behaviour). in the profile large peaks but also wide.

No Regularization: Detects the Egghead of the Remnant. Detects point sources in the fake extended emissions. The center has more details than clean. The non-negative Constraint stops the CS algorithms.



L1: There is no visible difference between L1 and no regularization. Interaction with the miller lambda estimation. Extended emissions are not forced to be connected: the fake emissions have "holes" that are not plausible. Rocky ride in the profile.

L2: Forces the extended emissions to be more plausible, more details visible in the center. L2 also forces the point sources to be [lower and wider]. In the profile at around X shows more details in extended emissions.

L1+L2: Since L1 does a good job for point sources and L2 finds high-resolved details in extended sources, why does one not combine both? idea to combine both regularizations, get the best from both worlds. Flexibility of CS allows this prior. Tradeoff between point sources and extended. How to chose the tradeoff is not trivial, here it was assumed to be equal. In this example, all pixels are very close to zero (Maximum: 0.0076 in Dirty Image), Miller lambda estimation is dominated by the L1 term while the L2 term gets neglected. The larger the values in the dirty image, the more L2 dominates over L1. Combination is not trivial.

Total Variation: Simple prior that was used in Image denoising. Reduces the gradient over the whole image. [It tries to have as few changes in the image as possible.] The reconstructed image shows both extended emissions and point sources. It has trouble with point sources inside extended emissions. In this case it cuts off the point source and the peak in image **??** is not here.

Starlets: A more sophisticated try at combining both.

[No free lunch theorem.] The regularization decides what is noise and what is true. Search for regularization that finds the true image in every observation. CS is flexible and allows for a combination of regularization.

starlets finds a lot of smaller point sources, but they do

Problem with memory, $x \star PSF$ gets modeled as a vector matrix multiplication Px. The image x and PSF with dimensions of $128 * 128$, result in a matrix of size $128^2 * 128^2$. The memory requirement scales quadratic with the number of pixels.

5 Conclusion and Future Development

Flexible approach.

Flexibility allows multiple ways to solve the same problem. Optimal solution does not exist yet.

Limited through CASA interface. and the chosen Optimization Algorithm

small Field of View. Self-cal application

Currently infeasible for large scale, new larger instruments.

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6 Ehrlichkeitserklärung

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Windisch, July 22, 2018

Jonas Schwammberger