P7 Compressed Sensing Image Reconstruction for CASA

Jonas Schwammberger

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Abstract

abstract plcaceholder

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1 Large Scale Image Reconstruction for the MeerKAT Telescope

Image reconstruction on the large scale

Large scale image reconstruction applied to data from the MeerKAT Telescope.

$$V(u,v) = \int \int x(x,y) e^{2\pi i (ux+vy)} \, dx \, dy \tag{1.1}$$

sampling pattern is defined by the instrument, in MeerKAT's case the sampling Visibilities are:

1. Not Uniform: Pockets of dense samples.
2. Incomplete: Information for the Image Reconstruction was not measured by the instrument

1.1 The Major Cycle Architecture

Algorithms in Radio Astronomy employ the major cycle architecture. The figure shows the major cycle framework. In a major cycle consists of two parts: The non-uniform FFT and an optimization algorithm.

The non-uniform FFT is responsible for approximating a regularly spaced image from the measurements, and for approximating the measurements corresponding to an image. Non-uniform FFT's are fast approximation algorithms, but by being approximation algorithms, they introduce errors.

A optimization algorithm uses the image and removes the effect of incomplete samples. In the past, CLEAN algorithms were used to remove the effects. For the future, algorithms based on the theory of compressed sensing show promises in image quality.

A full major cycle consists of the following operations: First, it approximates the regularly spaced the regularly spaced image from the measurements. Then the optimization algorithm removes the effects of Incomplete measurements and returns the corresponding image. The major cycle then approximates the measurements corresponding to the image with the non-uniform FFT. The residual measurements are used in the next Major Cycle. In each cycle, two errors get simultaneously reduced:

1. The Error introduced by the non-uniform FFT.
2. The Error introduced by the incomplete measurements.

After several major cycles the algorithm converges on a regularly spaced image which has a small error from non-uniform samples, and a small error from incomplete measurements.

For the optimization algorithm, the CLEAN class of algorithms get used. But algorithms based on the theory of compressed sensing have been shown to produce superior images.

1.2 Compressed Sensing Reconstructions

However, compressed Sensing algorithms come with the drawback of requiring more major cycles.

Current Compressed Sensing reconstructions reduce the number of major cycles. However, the question is if Compressed Sensing can use a different architecture, and scale better to problems of the size of MeerKAT.

There are ways to get rid of the major cycle, but overall the complexity could not be reduced.

2 Challenges for imaging MeerKAT data

terabytes of measurements.

Large image size 32k squared

Wide field of view imaging

Calibration

Full polarization imaging

Wide band imaging

2.1 Wide Field of View Imaging

So far the small Field of View inverse problem has been introduced where each antenna pair measures a Visibility of the sky brightness distribution. This leads to the small Field of View measurement equation (2.1). It is identical to the two dimensional Fourier Transform. In practice the Fast Fourier Transform (FFT) is used, since it scales with $n\,log(n)$ instead of n^2 pixels.

$$V(u,v) = \int\int x(l,m)e^{2\pi i(ux+vy)}dldm \tag{2.1}$$

For wide Field of View imaging, two effects break the two dimensional Fourier Transform relationship: Non-coplanar Baselines and the celestial sphere which lead to the measurement equation (2.2). Note that for small Field of View $1-x^2-y^2 \ll 1$, and (2.2) reduces to the 2d measurement equation (2.1).

$$V(u,v,w) = \int\int \frac{X(x,y)}{\sqrt{1-x^2-y^2}}e^{2\pi i(ux+vy+w\sqrt{1-x^2-y^2})}dxdy \tag{2.2}$$

Non-coplanar Baselines lead to a third component w for each Visibility. Figure 1 shows the the u v and w coordinate system. w is essentially the pointing direction of the instrument. The UV-Plane is the projection of the antennas on a plane perpendicular to the pointing direction. Which point in the UV-Plane get sampled and what w component it has depends on the pointing direction. If the instrument points straight up, the UV-Plane is a tangent to earth's surface, and the w term compensates for earth's surface curvature. If however the instrument points at the horizon, the projected UV-Plane gets squashed and w compensates for antennas which lie far behind the UV-Plane. In essence, w is a phase delay that corrects antenna positions in three dimensions. The wide Field of View measurement equation (2.2) would account for the w phase delay, but it breaks the the two dimensional Fourier relationship and the FFT cannot be used. The W-Projection [?] algorithm approximates the effect of the w term restores the two dimensional Fourier relationship.

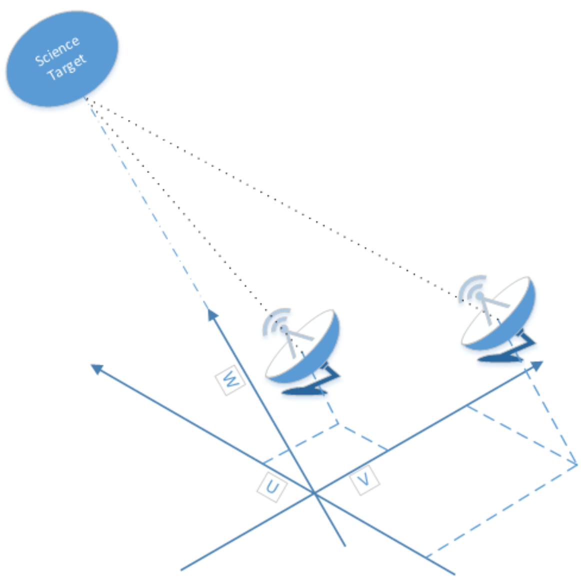

Figure 1: U V and W coordinate space

2.2 State of the Art: Distributing the W-Term with WSCLEAN

Tackling the problem State of the Art:Distributing the W-Term with WSCLEAN

Spherical Harmonics

FFT

uv smooth

Coordinate Descent

A-Projection [**?**]

3 Eliminating the Major Cycle

There are some ways of potentially eliminating the major cycle.

3.1 Optimized non-uniform FFT

$$\underset{x}{minimize}\, y - Ax \tag{3.1}$$

3.2 Spherical Harmonics

fourier expands into spherical harmonics

Fast spherical harmonics transform exists, but is not faster out of the box

solving everything on the sphere, ”spherical haar wavelets”

3.3 UV-Smooth

3.4 Coordinate Descent

$x = \Psi\alpha$

4 Coordinate Descent

4.1 The Starlet Regularization

4.2 Active set Heuristic with Starlets

4.3 Results on Simulated Data

4.3.1 Point Sources

4.3.2 Gaussian and Point Sources

4.4 Estimate on MeerKAT size data

5 Conclusion

References

[1] National Radio Astronomy Observations. tclean overview, 2016.

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6 Ehrlichkeitserklärung

Hiermit erkläre ich, dass ich die vorliegende schriftliche Arbeit selbstständig und nur unter Zuhilfenahme der in den Verzeichnissen oder in den Anmerkungen genannten Quellen angefertigt habe. Ich versichere zudem, diese Arbeit nicht bereits anderweitig als Leistungsnachweis verwendet zu haben. Eine Überprüfung der Arbeit auf Plagiate unter Einsatz entsprechender Software darf vorgenommen werden.
Windisch, December 17, 2018

Jonas Schwammberger