P9 Exploring the Fourier Transform for Compressed Sensing Reconstructions in the MeerKAT era

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February 28, 2019

Abstract

The new MeerKAT Radio Interferometer poses an image reconstruction problem on a large scale. It measures an incomplete set of Fourier components, which have to be reconstructed by an imaging algorithm. Compressed Sensing reconstructions have the potential to improve the effective accuracy of MeerKAT, but so far have higher runtime costs compared to state-of-the-art CLEAN implementation. Both Compressed Sensing and CLEAN reconstructions use the non-uniform FFT approximation to cycle between Fourier and image space. But compared to CLEAN, Compressed Sensing algorithms need more cycles to converge, which is one of the reasons why they have higher runtime costs.

In this project, we investigate if replacing the non-uniform FFT approximation reduces the runtime costs of Compressed Sensing reconstructions. We discuss three alternatives and decided to create a new algorithm using the direct Fourier Transform. We leveraged the starlet transform and created a Compressed Sensing algorithm, which only needs to calculate the transform for non-zero basis functions. Our algorithm does not need iterative approximation algorithms of the Fourier Transform. With Coordinate Descent as the optimization algorithm, our approach lends itself to distributed image reconstruction. Although our algorithm was able to reduce the runtime costs it is still too expensive for large scale reconstructions.

Currently, there is no clear alternative to the non-uniform FFT approximation. Creating a cost effective and distributable Compressed Sensing algorithm is still an open problem.

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1 Larger runtime costs for Compressed Sensing Reconstructions

The MeerKAT instrument produces a new magnitude of data volume. An image with several million pixels gets reconstructed from billions of Visibility measurements. Although MeerKAT measures a large set of Visibilities, the measurements are still incomplete. We do not have all the information available to reconstruct an image. Essentially, this introduces ”fake” structures in the image, which a reconstruction algorithm has to remove. Additionally, the measurements are corrupted by noise.

Large scale reconstruction problem.

The inteferometer introduces two forms of corruption into the measurements:

The measurements are subjected to both noise and corruption from the instrument itself.

An image reconstruction algorithm should remove the

Noisy measurements, and the instrument itself introduces ”fake” image structures. Large scale image reconstruction problem. Several different reconstruction algorithms were developed for this problem, which can be separated into two classes: Algorithms based on CLEAN, which are cheaper to compute and Compressed Sensing based algorithms, which create higher quality reconstructions.

CLEAN uses a deconvolution

Compressed Sensing based algorithms have an image regularization.

This project searches for a way to reduce the runtime costs of Compressed Sensing based algorithms. One of the reasons is that both algorithms use the non-uniform FFT Approximation, but Compressed Sensing algorithms need more non-uniform FFT Approximation than CLEAN.

We need the non-uniform FFT to transform. In the state-of-the-art Architectures, we need several non-uniform FFT operations for a single reconstruction. It is an expensive operation,

we try to reduce the number of non-uniform FFT. One part of speeding up compressed Sensing reconstructions is reducing the number of non-uniform FFT approximations.

State-of-the-art CLEAN and Compressed Sensing algorithms use a similar Architecture, but with important differences.

1.1 The Major Cycle Architecture

Figure 1 depicts the Major Cycle Architecture used by CLEAN algorithms. First, the Visibilities get transformed into an image with the non-uniform FFT. The resulting image contains corruptions of the measurement instrument. A deconvolution algorithm, typically CLEAN, removes the corruption of the instrument with a deconvolution. The residual image, which should contain mostly noise, gets transformed back into residual Visibilities and the cycle starts over.

In the Major Cycle Architecture, we need several deconvolution attempts before it has distinguished the noise from the measurements. For MeerKAT reconstruction with CLEAN, we need approximately 4-6 non-uniform FFT cycles.

CLEAN deconvolutions are not trivial to distribute.

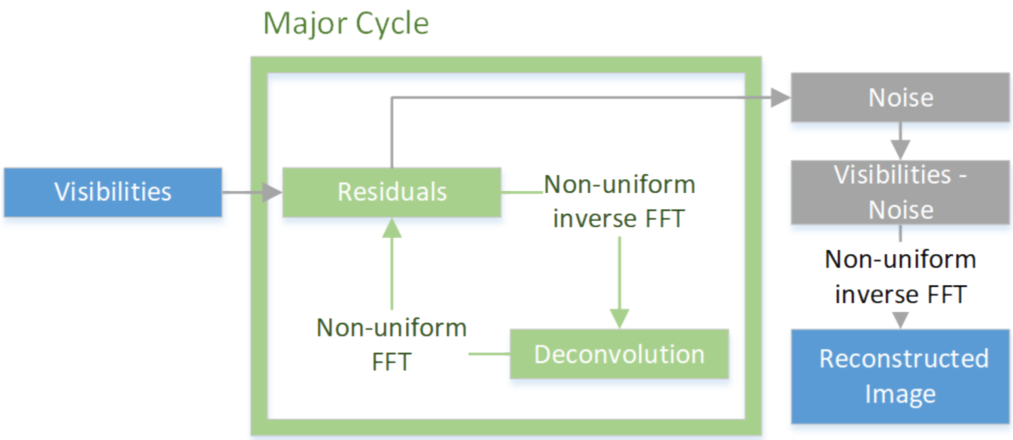

Figure 1: The Major Cycle Architecture

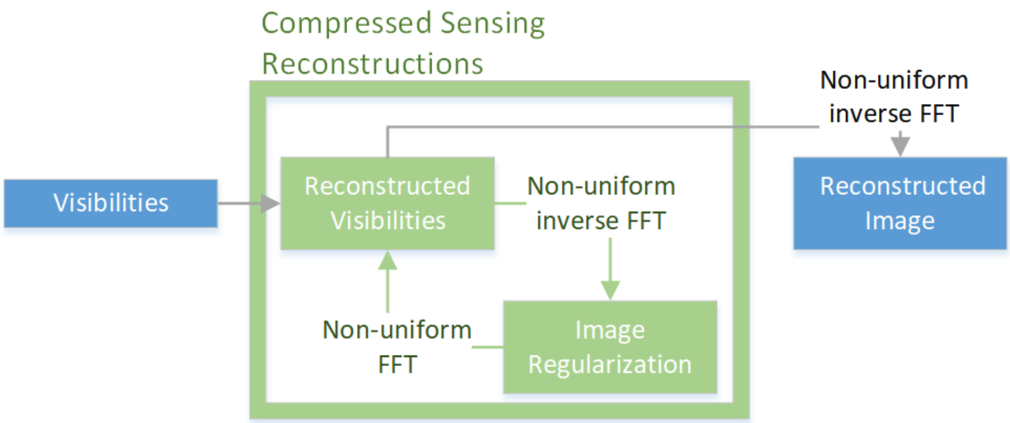

Figure 2: State-of-the-art Compressed Sensing Reconstruction Architecture

1.2 Compressed Sensing Architecture

Figure 2 depicts the architecture used by Compressed Sensing reconstructions. The Visibilities get transformed into an image with the non-uniform FFT approximation. The algorithm then modifies the image so it reduces the regularization penalty. The modified image gets transformed back to Visibilities and the algorithm then minimizes the difference between measured and reconstructed Visibilities. This is repeated until the algorithm converges to an optimum.

In this architecture, state-of-the-art Compressed Sensing algorithms need approximately 10 or more non-uniform FFT cycles to converge. It is one source for the higher runtime costs. There is one upside in this architecture: State-of-the-art algorithms managed to distribute the ”Image Regularization” operation.

1.3 Hypothesis for speeding up Compressed Sensing Algorithms

Compressed Sensing Algorithms are not bound to the Architecture presented in section 1.2. For example, we can design a Compressed Sensing based deconvolution algorithm and use the Major Cycle Architecture instead.

Our hypothesis is: We can create a Compressed Sensing based deconvolution algorithm which is both distributable and creates higher quality reconstructions than CLEAN. Because it also uses the Major Cycle architecture, we reckon that the Compressed Sensing deconvolution requires a comparable number of non-uniform FFT cycles to CLEAN. This would result in a Compressed Sensing based reconstruction algorithm with similar runtime costs to CLEAN, but higher reconstruction quality and higher potential for distributed computing.

2 Conclusion

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3 Ehrlichkeitserklärung

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Windisch, February 28, 2019

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