

P9 Exploring the Fourier Transform for Compressed Sensing Reconstructions in the MeerKAT era

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Abstract

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1 Introduction

Universe

Accuracy

No

Inverse Problems

1.1 Inverse Problem

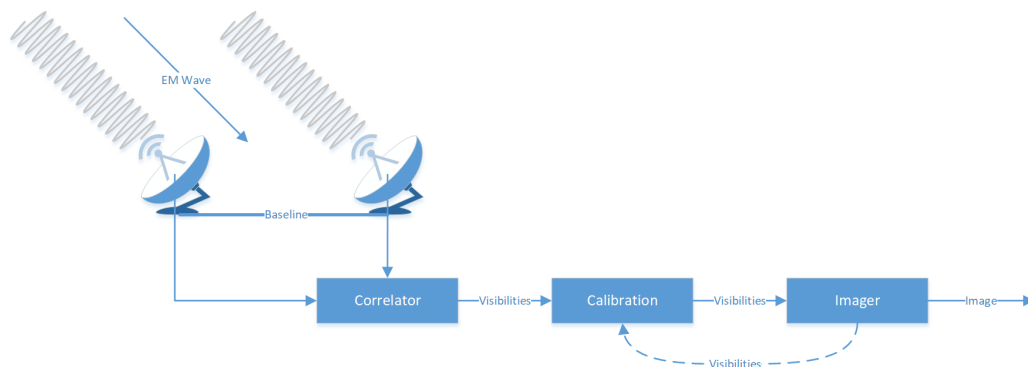


Figure 1: Interferometer System

$$V(u, v, w) = \iint \frac{I(x, y)}{\sqrt{1 - x^2 - y^2}} e^{2\pi i [ux + vy + w(\sqrt{1 - x^2 - y^2} - 1)]} dx dy \quad (1.1)$$

1.2 Image Reconstruction

2 Larger runtime costs for Compressed Sensing Reconstructions

The MeerKAT instrument produces a new magnitude of data volume. An image with several million pixels gets reconstructed from billions of Visibility measurements. Although MeerKAT measures a large set of Visibilities, the measurements are still incomplete. We do not have all the information available to reconstruct an image. Essentially, this introduces "fake" structures in the image, which a reconstruction algorithm has to remove. Additionally, the measurements are noisy.

We require an image reconstruction algorithm which removes the "fake" structures from the image, and removes the noise from the measurements. The large data volume of MeerKAT requires the algorithm to be both scalable and distributable. Over the years, several reconstruction algorithms were developed, which can be separated into two classes: Algorithms based on CLEAN, which are cheaper to compute and algorithms based on Compressed Sensing, which create higher quality reconstructions.

CLEAN based algorithms represent the reconstruction problem as a deconvolution. First, they calculate the "dirty" image, which is corrupted by noise and fake image structures. The incomplete measurements essentially convolve the image with a Point Spread Function (*PSF*). CLEAN estimates the *PSF* and searches for a deconvolved version of the dirty image. In each CLEAN iteration, it searches for the highest pixel in the dirty image, subtracts a fraction *PSF* at the location. It adds the fraction to the same pixel location of the "cleaned" image. After several iterations, the cleaned image contains the deconvolved version of the dirty image. CLEAN accounts for noise by stopping early. It stops when the highest pixel value is smaller than a certain threshold. This results in a light-weight and robust reconstruction algorithm. CLEAN is comparatively cheap to compute, but does not produce the best reconstructions and is difficult to distribute on a large scale.

Compressed Sensing based algorithms represent the reconstruction as an optimization problem. They search for the optimal image which is as close to the Visibility measurements as possible, but also has the smallest regularization penalty. The regularization encodes our prior knowledge about the image. Image structures which were likely measured by the instrument result in a low regularization penalty. Image structures which were likely introduced by noise or the measurement instrument itself result in high penalty. Compressed Sensing based algorithms explicitly handle noise and create higher quality reconstructions than CLEAN. State-of-the-art Compressed Sensing algorithms show potential for distributed computing. However, they currently do not scale on MeerKATs data volume. They require too many computing resources compared to CLEAN based algorithms.

This project searches for a way to reduce the runtime costs of Compressed Sensing based algorithms. One reason for the higher costs is due to the non-uniform FFT Cycle. State-of-the-art CLEAN and Compressed Sensing based algorithms both use the non-uniform FFT approximation in a cycle during reconstruction. The interferometer measures the Visibilities in a continuous space in a non-uniform pattern. The image is divided in a regularly spaced, discrete pixels. The non-uniform FFT creates an approximate, uniformly sampled image from the non-uniform measurements. Both, CLEAN and Compressed Sensing based algorithms use the non-uniform FFT to cycle between non-uniform Visibilities and uniform image. However, a Compressed Sensing algorithm requires more non-uniform FFT cycles for reconstruction.

CLEAN and Compressed Sensing based algorithms use the non-uniform FFT in a similar manner. However, there are slight differences in the architecture. This project hypothesises that The previous project searched for an alternative to the non-uniform FFT cycle. Although there are alternatives, there is currently no replacement which leads to lower runtime costs for Compressed Sensing. Current research is focused on reducing the number of non-uniform FFT cycles for Compressed Sensing algorithms.

CLEAN based algorithms use the Major Cycle Architecture for reconstruction. Compressed Sensing based algorithms use a similar architecture, but with slight modifications. Our hypothesis is that we may reduce the number of non-uniform FFT cycles for Compressed Sensing by using CLEAN's Major Cycle Architecture.

2.1 CLEAN: The Major Cycle Architecture

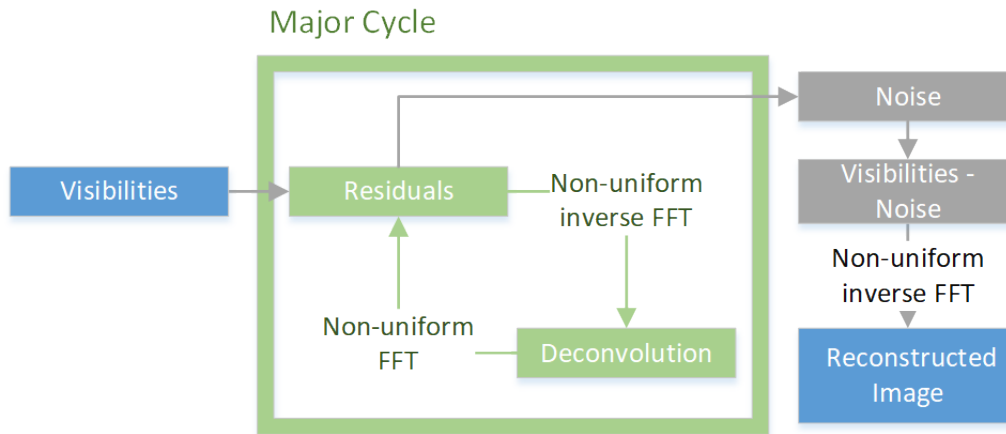


Figure 2: The Major Cycle Architecture

Figure 2 depicts the Major Cycle Architecture used by CLEAN algorithms. First, the Visibilities get transformed into an image with the non-uniform FFT. The resulting dirty image contains the corruptions of the measurement instrument and noise. A deconvolution algorithm, typically CLEAN, removes the corruption of the instrument with a deconvolution. When the deconvolution stops, it should have removed most of the observed structures from the dirty image. The rest, mostly noisy part of the dirty image gets transformed back into residual Visibilities and the cycle starts over.

In the Major Cycle Architecture, we need several deconvolution attempts before it has distinguished the noise from the measurements. Both the non-uniform FFT and the deconvolution are approximations. By using the non-uniform FFT in a cycle, it can reconstruct an image at a higher quality. For MeerKAT reconstruction with CLEAN, we need approximately 4-6 non-uniform FFT cycles for a reconstruction.

2.2 Compressed Sensing Architecture

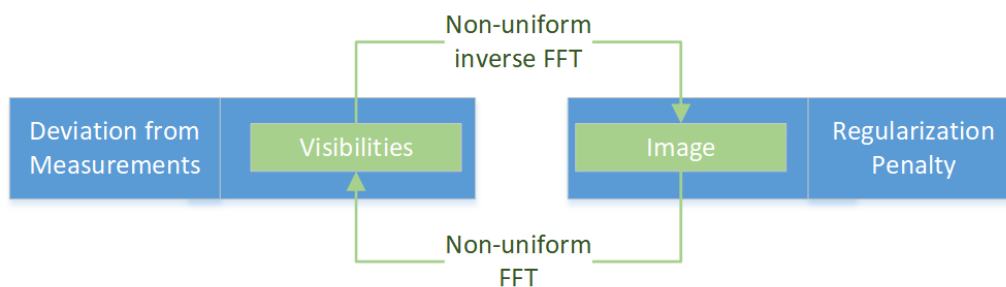


Figure 3: State-of-the-art Compressed Sensing Reconstruction Architecture

Figure 3 depicts the architecture used by Compressed Sensing reconstructions. The Visibilities get transformed into an image with the non-uniform FFT approximation. The algorithm then modifies the image so it reduces the regularization penalty. The modified image gets transformed back to Visibilities and the algorithm then minimizes the difference between measured and reconstructed Visibilities. This is repeated until the algorithm converges to an optimum.

In this architecture, state-of-the-art Compressed Sensing algorithms need approximately 10 or more non-uniform FFT cycles to converge. It is one source for the higher runtime costs. For MeerKAT reconstructions

the non-uniform FFT tends to dominate the runtime costs. A CLEAN reconstruction with the Major Cycle Architecture already spends a large part of its time in the non-uniform FFT. Compressed Sensing algorithms need even more non-uniform FFT cycle on top of the "Image Regularization" step being generally more expensive than CLEAN deconvolution. There is one upside in this architecture: State-of-the-art algorithms managed to distribute the "Image Regularization" operation.

2.3 Hypothesis for reducing costs of Compressed Sensing Algorithms

Compressed Sensing Algorithms are not bound to the Architecture presented in section 2.2. For example, we can design a Compressed Sensing based deconvolution algorithm and use the Major Cycle Architecture instead.

Our hypothesis is: We can create a Compressed Sensing based deconvolution algorithm which is both distributable and creates higher quality reconstructions than CLEAN. Because it also uses the Major Cycle architecture, we reckon that the Compressed Sensing deconvolution requires a comparable number of non-uniform FFT cycles to CLEAN. This would result in a Compressed Sensing based reconstruction algorithm with similar runtime costs to CLEAN, but higher reconstruction quality and higher potential for distributed computing.

2.4 State of the art: WSCLEAN Software Package

2.4.1 W-Stacking Major Cycle

2.4.2 Deconvolution Algorithms

CLEAN MORESANE

2.5 Distributing the Image Reconstruction

2.5.1 Distributing the Non-uniform FFT

2.5.2 Distributing the Deconvolution

3 Distributed Image Reconstruction for Radio Interferometers

In Astronomy, instruments with higher angular resolution allows us to measure ever smaller structures in the sky. For Radio frequencies, the angular resolution is bound to the antenna dish diameter, which puts practical and financial limitations on the highest possible angular resolution. Radio Interferometers get around this limitation by using several smaller antennas instead. Together, they act as a single large antenna with higher angular resolution at lower financial costs compared to single dish instruments.

Each antenna pair of an Interferometer measures a single Fourier component of the observed image. Retrieving the observed image from the measurements is essentially calculating the Fourier Transform. However, since the Interferometer only measured a limited number of Fourier components, the resulting image is "dirty". It is essentially the observed image convolved with a Point Spread Function (PSF). There are different ways of deconvolving the image. Find the most likely deconvolution of the dirty image. Deconvolution is not a unique solution. there are potentially many different deconvolved images, the reconstruction algorithm should find the most likely one.

Fastest architecture. Major Cycle 4 architecture. The Major Cycle contains Three operations, Gridding, inverse FFT and Deconvolution.

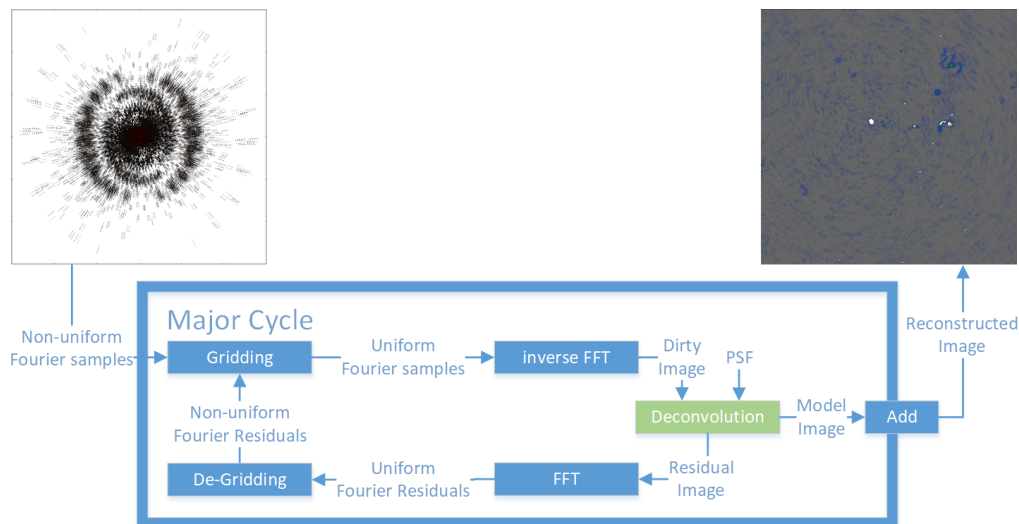


Figure 4: The Major Cycle Architecture of image reconstruction algorithms

Gridding takes the non-uniformly sampled Fourier measurements and interpolates them on a regularly spaced grid. Afterwards, we can use the FFT to calculate the inverse Fourier transform and arrive at the dirty image. A deconvolution algorithm then takes the dirty image and the PSF and puts out two images. The "model" image, which contains the deconvolved version of the dirty image, and the residual image. The inverse operations get applied to the residual image, FFT and De-gridding and the whole process gets restarted from the beginning. This is a single Major Cycle. The final image is the addition of all the major cycles Gridding, Deconvolution and PSF are only approximations in the Major Cycle framework. By repeating the process, we reduce the errors.

In this architecture, the choice of deconvolution algorithm mainly decides how good the reconstruction is. The most expensive operation in terms of runtime is Gridding. State-of-the-art Deconvolution algorithm. State of the art was mainly to run this on a single machine.

3.1 Distributed computing for large scale Reconstructions

New Interferometer produce an ever increasing number of measurements. Sooner or later, the Major Cycle needs to work on distributed computing. State-of-the-art Gridding and Deconvolution algorithms allow only for limited distribution.

Recently, Veeneboer et al[3] managed to design a Gridder on the GPU, reducing the wall-clock time of the most expensive operations by several factors. Same idea can potentially be used for distributing the Gridder on multiple machines. Applying the Theory of Compressed Sensing to deconvolution algorithms, we can create an algorithm with better reconstruction quality and suited to distributed computing.

In this project, We want to create a distributed gridding and deconvolution algorithm and explore the difficulties in distributing the image reconstruction. Compare the reconstruction quality.

4 Handling the Data Volume

The new data volume is a challenge to process for both algorithms and computing infrastructure. Push for parallel and distributed algorithms. For Radio Interferometer imaging, we require specialized algorithms. The two distinct operations, non-uniform FFT and Deconvolution, were difficult algorithms for parallel or distributed computing.

The non-uniform FFT was historically what dominated the runtime []. Performing an efficient non-uniform FFT for Radio Interferometers is an active field of research[1, 2], continually reducing the runtime costs of the operation. Recently, Veeneboer et al[3] developed a non-uniform FFT which can be fully executed on the GPU. It speeds up the most expensive operation.

In Radio Astronomy, CLEAN is the go-to deconvolution algorithm. It is light-weight and compared to the non-uniform FFT, a cheap algorithm. It is also highly iterative, which makes it difficult for effective parallel or distributed implementations. However, compressed sensing based deconvolution algorithms can be developed with distribution in mind.

4.1 Fully distributed imaging algorithm

Current imaging algorithms push towards parallel computing with GPU acceleration. But with Veeneboer et al's non-uniform FFT and a compressed sensing based deconvolution, we can go a step further and create a distributed imaging algorithm.

5 Conclusion

References

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6 Ehrlichkeitserklärung

Hiermit erkläre ich, dass ich die vorliegende schriftliche Arbeit selbstständig und nur unter Zuhilfenahme der in den Verzeichnissen oder in den Anmerkungen genannten Quellen angefertigt habe. Ich versichere zudem, diese Arbeit nicht bereits anderweitig als Leistungsnachweis verwendet zu haben. Eine Überprüfung der Arbeit auf Plagiate unter Einsatz entsprechender Software darf vorgenommen werden.

Windisch, March 14, 2019

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