

16-833: Robot Localization and Mapping, Fall 2020

Homework 3 - Linear and Nonlinear SLAM Solvers

Due: Monday November 16, 11:59pm, 2020

Your homework should be submitted as a **typeset PDF file** along with a folder including **code only (no data)**. The PDF must be submitted on **Gradescope**, and code submitted on **Canvas**. If you have questions, post them on Piazza or come to office hours. Please do not post solutions or codes on Piazza. This homework must be done **individually**, and plagiarism will be taken seriously. You are free to discuss and troubleshoot with others, but the code and writeup must be your own. Note that you should list the name and Andrew ID of each student you have discussed with on the first page of your PDF file.

1. 2D Linear SLAM

In this problem you will implement your own 2D linear SLAM solver in the `linear/` folder. Everything you need to know to complete this problem was covered in class, so refer to your notes and lecture slides for guidance.

We will be using the least squares formulation of the SLAM problem, which was presented in class:

$$\begin{aligned} x^* &= \arg \min_x \sum \|h_i(x) - z_i\|_{\Sigma_i}^2 \\ &\vdots \\ &\approx \arg \min_x \|\mathbf{A}x - b\|^2 \end{aligned} \tag{1}$$

where z_i is the i -th measurement, $h_i(x)$ is the corresponding prediction function, and $\|a\|_{\Sigma}^2$ denotes the squared Mahalanobis distance: $a^T \Sigma^{-1} a$.

In this problem, the state vector is comprised of the trajectory of robot positions and the observed landmark positions. Both positions are simply (x, y) coordinates. There are two types of measurements: odometry and landmark measurements. Odometry measurements give a relative $(\Delta x, \Delta y)$ displacement from the previous position to the next position (in global frame). Landmark measurements also give a relative displacement $(\Delta x, \Delta y)$ from the robot position to the landmark (also in global frame).

(a) Write out the measurement functions and their Jacobians (for both the odometry measurements and landmark measurements). For simplicity, please parameterize these functions on only the relevant state variables rather than the entire state vector. Use h_o and h_m to denote the odometry and measurement functions, respectively, and their Jacobians are H_o and H_m . Use $\mathbf{r} = \begin{bmatrix} r_x & r_y \end{bmatrix}^T$ and $\mathbf{l} = \begin{bmatrix} l_x & l_y \end{bmatrix}^T$ to denote robot and landmark positions, respectively. Use \mathbf{r}^{t-1} to denote the previous robot pose and \mathbf{r}^t the next robot pose in an odometry measurement. (5 points)

(b) Once you have these functions, you can complete the file `create_Ab_linear.m` to generate the linear system. Simply choose a variable ordering (for convenience in constructing \mathbf{A} and b , *not for efficiency*) and fill in the correct blocks based on the measurements and Jacobians. *Add a prior on the first pose placing it at the origin.* If you

do not, your system will not be fully constrained (up to an arbitrary global transformation) and will not line up with the ground truth. Finally, complete the function `format_solution()` located in the `util/` directory. Simply format the data from your state vector into a trajectory and list of landmarks as specified in the file. The MATLAB function `reshape()` should be useful here. (5 points)

(c) Now that you have the linear system, experiment with five different ways to solve it. Complete the functions in `solve_pinv.m`, `solve_chol1.m`, `solve_qr1.m`, `solve_chol2.m`, and `solve_qr2.m`. Follow these guidelines for each type of solution:

i) `solve_pinv()` - do not use MATLAB's `pinv()` function - it will not work on sparse matrices. Do not use “\” either. Simply solve the normal equations naively by computing the pseudo-inverse yourself. (5 points)

ii) `solve_chol1()` - use MATLAB's `chol()` function and the provided `forward_sub()` and `back_sub()` functions in the `util/` folder to solve it. Return the solution as well as the computed R factor. (5 points)

iii) `solve_qr1()` - use MATLAB's `qr()` function. Look at the options listed in the documentation for sparse matrices (“`help qr`” or “`doc qr`”). Since you need to compute $Q^T b$ to solve the system, choose the option that returns such a vector as well as the R factor. Utilize the “economy size” version if it is possible to do so. Why can you or why can you not utilize this version? (10 points)

iv) `solve_chol2()` - use the same `chol()` function you used for `solve_chol1()` but with a fill-in-reducing ordering. (5 points)

v) `solve_qr2()` - use the same `qr()` function you used for `qr1()` but with a fill-in-reducing ordering. (5 points)

You can test a specific solution type by running the function `slam_2D_linear()` with the corresponding solution name (`pinv`, `chol`, `qr1`, `cho2`, or `qr2`).

(d) Once you are sure each of your decompositions is correct, compare the efficiency of these methods using the following datasets.

i) `2D_linear` dataset: Compare these methods to each other by running `slam_2D_linear()` with no input argument. The function should print out the run time for each of these algorithms and the evaluation of the resulting solution. Include the two figures that are generated - the resulting trajectory and the sparsity patterns of each decomposition. List the decompositions in order of efficiency, from *most efficient* to *least efficient*. Explain why they are in this order. (10 points)

ii) `2D_linear_loop` dataset: Change the input data file in `slam_2D_linear()` to the other listed data file, `2D_linear_loop.mat`. Compare your methods on this dataset. Include the two figures that are generated - the resulting trajectory and the sparsity patterns of each decomposition. List the decompositions in order of efficiency, from *most efficient* to *least efficient*. Is this order the same as the last dataset? Why or why not? (10 points)

2. 2D Nonlinear SLAM

Now you are going to build off of your linear SLAM algorithm to create a nonlinear SLAM algorithm in the `nonlinear/` folder. The problem set-up is exactly the same as in the linear problem, except we introduce a nonlinear measurement function that returns a bearing angle θ and range d (in robot's body frame, notice we assume that this robot always perfectly facing the x -direction of the global frame), which together describe the vector from the robot to the landmark:

$$h_m(r_x, r_y, l_x, l_y) = \begin{bmatrix} \text{atan2}(l_y - r_y, l_x - r_x) \\ \left((l_x - r_x)^2 + (l_y - r_y)^2\right)^{\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} \theta \\ d \end{bmatrix} \quad (2)$$

(a) In your nonlinear algorithm, you'll need to predict measurements based on the current state estimate. Fill in the functions `meas_odom.m` and `meas_landmark.m` with the corresponding measurement functions. The odometry

measurement function is the same linear function we used in the linear SLAM algorithm, while the landmark measurement function is the new nonlinear function introduced in Equation 2. (5 points)

(b) Derive the jacobian of the nonlinear landmark function in your writeup and implement the function `meas_landmark_jacobian.m` to calculate the jacobian at the provided linearization point. (5 points)

(c) Implement `create_Ab_nonlinear.m` to generate the linear system A and b at the current linearization point. Remember to include a *prior* on the first pose to tie it down to the origin. Also implement `error_nonlinear.m`. This function will be similar to `create_Ab_nonlinear.m` but instead of generating the linear system, it will compute the total sums of squares error evaluated at the current state estimate. This function is necessary for our implementation of the Gauss-Newton algorithm, which we have provided for you in `gauss_newton.m`. (15 points)

Take caution when computing differences between angles in both of these functions. If you take the difference between two angles that are around π , but one is positive and another is negative, you may end up with a difference that is close to $\pm 2\pi$, when the real difference should be around 0. To alleviate this, normalize angles to the interval $(-\pi, \pi]$ after taking differences.

(d) Choose your favorite solver from Section 2 (b) and implement it in the file `solve_linear_system.m`. The function `solve_linear_system()` is applied to solve the linearized system in each Gauss-Newton iteration. Now test your SLAM algorithm by running `slam_2D_nonlinear.m`. MATLAB's debugging tools (setting breakpoints and such) may prove useful if you need to debug your code. You may also want to uncomment some of the `fprintf` lines in `gauss_newton.m` to get more information about the state of the system. When implemented properly, your solution should follow the ground truth trajectory closely and pass the evaluation at the end of the run. Show the resulting trajectory and report the error of your final solution compared to that of the pure odometry trajectory. (10 points)

(e) We implemented this nonlinear SLAM algorithm in an incremental fashion above. Why can't we solve it all at once in a batch optimization? (5 points)

3. Code Submission

Do not change the structure in the `slam_solvers/` folder. Please do not upload the `data/` folder or any other data or image.