24-774: Special Topics in ACSI
Laboratory 3: System Identification

Background: For Lab 2 we gave you a system model that was "close enough" to perform controller design. This is common: generate a controller based on a rough model of the system, and then use experiments to identify the system parameters for further controller design. Now we will do this identification, but simplify the process so that we are only identifying the closed loop system (unless you want to do the bonus).

Figure 1: Tumbbler hardware. The hardware has various sensors (wheel encoders, IMU, ultrasonic) that can be used to control the 4 degree of freedom robot.

Background

As in Lab 2 we will ignore the yaw angle of the robot, i.e. we will look only at straight line translation of the system. The basic system dynamics from before will help, and are included here for completeness.

The system dynamics are exactly the same of those of a Segway or hover board type system, and there are many sources online that derive these dynamics. A good resource for the overall approach is given by https://kth.diva-portal.org/smash/get/diva2:916184/FULLTEXT01.pdf. The linearized equations about the upright position are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I_{pend} + m_{pend} l^2) f}{I_{pend}(m_{cart} + m_{pend}) + m_{cart} m_{pend} l^2} & \frac{m_{pend}^2 g l^2}{I_{pend}(m_{cart} + m_{pend}) + m_{cart} m_{pend} l^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m_{pend} l f}{I_{pend}(m_{cart} + m_{pend}) + m_{cart} m_{pend} l^2} & \frac{m_{pend} g l (m_{cart} + m_{pend})}{I_{pend}(m_{cart} + m_{pend}) + m_{cart} m_{pend} l^2} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \dfrac{I_{pend} + m_{pend}l^2}{I_{pend}\left(m_{cart} + m_{pend}\right) + m_{cart}m_{pend}l^2} \\ 0 \\ \dfrac{m_{pend}l}{I_{pend}\left(m_{cart} + m_{pend}\right) + m_{cart}m_{pend}l^2} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

for the state vector $x = \begin{bmatrix} x \\ \acute{x} \\ \theta \\ \acute{\theta} \end{bmatrix}$ and input of force. To convert between motor voltage and force we can use a simple linear approximation as shown in Section 3 in the reference.

$$u = \frac{2k_T}{Rr}V,$$

where u is the force, k_T is the motor torque constant, R is the motor resistance, r is the wheel radius, and V is the motor voltage.

As seen in the reference, the parameters are typically estimated using CAD models / experimentation; for this lab we will provide you with values that are close enough to perform the design.

Parameter	Value
m_{cart}	0.493 kg
m_{pend}	0.312 kg
I_{pend}	0.00024 kg.m^2
L	0.04 m
f	0.01 N.s/m
k_T	0.11 N.m/A
R	10 ohm
r	0.0335 m

Frequency Domain System Identification

For this unstable system we have no choice but to use closed loop system identification techniques. Using the LQR controller that you developed in Lab 2, we will apply inputs to the system and measure the outputs. For starters, let's apply sinusoids of fixed frequency and study the system response.

1. Use a computer algebra package to find the transfer function of the closed loop system from voltage input to angle / position with your LQR controller applied. This should be a 2×1 transfer function matrix. Feel free to assume that your state observer is perfect, i.e. it's OK to assume $C = I$ here. Find the answer for your specific controller values and the nominal plant parameters and plot the Bode plot using Matlab.
2. Now create a set of 5 logarithmically spaced frequencies between .3 Hz and 3 Hz (the *logspace* command might help). Add a sinusoidal input on top of the control signal and measure the system outputs. Adjust the size of the sinusoid until you see a clear sinusoid in the angle and position measurements. NOTE: the angle measurement will likely show the sinusoid much more clearly – that is normal.
3. Plot the sinusoidal input signal and outputs on the same plot. Measure the magnitude and phase shift for each element of the transfer function matrix based on the graphs. Plot these measurements on top of the Bode plots from Part 1. How well do they match?
4. Now let's try to automate the computation from Part 4. Use the Goertzel algorithm (see 'doc goertzel') to compute the single point FFT's corresponding to the frequencies you measured. Plot the Goertzel measurements on the Bode plots from Part 1 and again compare the match quality. For simplicity, here is a simple snippet of code that demonstrates the Matlab usage:

5. Increase the sinusoid amplitude by a factor of 2 (if you lose the ability to balance, it's OK to pull it back a bit). Repeat Part 3 or Part 4 (just for 2 frequencies of your choice). Is the system behaving linearly in this input range?

System Identification Toolbox

Let's now apply a white noise identification and use the System ID Toolbox to get an estimate of the system model.

1. Apply an excitation to the system as above but use a random input rather than a sinusoid. This will allow us to measure the frequency response in a single measurement. Capture at least 30 seconds of data of input signal and outputs (angle and position).

2. Enter the data from above into the system identification toolbox. Estimate a state space model of the system and a transfer function model of the system. Plot the Bode plot of these systems against the measurements you made from the frequency domain tests and against the nominal plant model.

3. Compare the results from the white noise and frequency domain identification methods. Which appears to give more accurate results?

BONUS

Plant Model Update

Using the method of your choice, find a state space or transfer function model of the plant based on measurements of the close loop system. Plot the Bode plot of your identified plant against the nominal plant model.

Reporting

Compile a single PDF lab report following the guidelines below.

Your report should be organized in terms of numbered items in the lab procedure. For each numbered item in the lab procedure you must address the following items at a minimum:

1.) The details of all calculations involved in generating your results. Be sure to highlight the main results.
2.) Presentation of your results in the form of plots and tables. This should include all relevant plots and Simulink models. Do not present plots that use the black background that is the Simulink scope default.
3.) General discussion. What sense do you make of the results? What can you conclude?
4.) Answers to all the discussion questions in the lab procedure.

After completing these tasks for all numbered items in the lab procedure, complete the following sections to finish your report:

- Conclusions: What were the main results? What did you learn (if anything) by completing the lab? What suggestions do you have to make the lab better or more interesting?
- References: Compile all of your references into a single section at the end of the document. I highly recommend the use of a reference manager, e.g. Bibtex, EndNote, etc.