TOPIC

1. PROPERTIES OF WELL FORMED FORMULA
2. CONVERSION TO CluASAL FORM
3. INFERENCE RULES

OPERTIES OF WELL FORMED FORMU

1. Proposition logic uses a symbolic “language” to represent the logical structure, or form, of a compound proposition.

2. Has rules of syntax- grammatical rules for putting symbols together in the right way.

3. Any expression that obey the syntactic rules of propositional logic is called a WELL FORMES FORMULA , or WFF

Definition

A well formed formula is any formula that is capable of being generated by some combination of seven information rules.

- Every propositional letter of PL (e.g. A, B, C) is a WFF.
- If P is a WFF, then ⏶(P) is a WFF.
- If P and Q are WFF, then (P⏴Q) is a WFF.
- If P and Q are WFF, then (P⏵Q) is a WFF.
- If P and Q are WFF, then (P≣Q) is a WFF.
- If P and Q are WFF, then (P⊻Q) is a WFF.
- Nothing else is a wff except what can be formed by repeated application of 1-6.

Example(WFF)

- P
- (P■Q)
- ▲((P▶▲(Q)))
- ▲(▲(P◀(Q)))

Example(not WFF)

- P▲
- PQ
- ◀▲(Q))
- ▲▲P◀▲Q▶S

Three type of WFF: Atomic WFFs

Definition

An Atomic wff in PL is a well formed formula in PL consisting only of a single propositional letter.

Example:

>P

>Q

>A

>B

Complex WFFs

A complex wff in PL is a well formed formula in PL that contains at least one propositional letter and a truth- functional operator.

Example:

▲(P)

(P◀Q)

(P▬(A))

Literal Wff

A literal wff in PL is a well formed formula in PL that consist of an atomic wff P or a negated atomic wff ⏶(P).

Example:

❑P

❑⏶(P)

❑Q

❑⏶(Q)

Conversion to clausal form /normal form

Firstly :

1. **PL does not permit us to make generalized statement about classes of similar object.**
2. **These are serious limitation when reasoning about real world entity.**
3. **E.g. statement=> it should be possible to conclude that john must take the pascal course.**

ALL STUDENT IN COMPUTER SCIENCE MUST TAKE PASCAL

JOHN IS A COMPUTER SCIENCE MAJOR

SYNTAX OF FOPL

1.CONNECTIVES: These are 5 connective symbols => (not or negation ▲)

(AND or conjunction ◀)

(OR or disjunction ▶)

(implication

(equivalence or if and only if

2. QUANTIFIERS: 2 quantifier symbols are

(existential quantifier)

(universal quantifier)

3. CONSTANT: constant are fixed value terms that belong to a

4. VARIABLE: variable are terms that assume different value over a given domain.

5. FUNCTION : function symbol denote relations defined on a domain D.

6. PREDICATE: predicate symbols denote relations or functional mapping from the elements of a domains D to the value true and false.

capital letter and capitalized word such as P, Q , R, EQUAL, MARRIED are used to represent predicate.

Symbolic formation

E1: All employee earning $1400 or more per year pay taxes.
E2: Some employee are sick today.
E3: no employee earns more than the president.

Predicate and functions:

E(x) for x is an employee.
P(x) for x is president.
i(x) for income of x . (lower case denotes a function).
GE(u,v) for u is greater than or equal to v.

Using abbreviation E1, E2, E3

E1’: [illegible] &GE [illegible] 1400 [illegible]

E2’: [illegible] E(y [illegible]

E3’: [illegible] &P [illegible] GE [illegible]

E1’ E2’ E3’ (woof)

Clausal form

1. **Clause:** as the disjunction of a number of literals.

2. **Ground clause**: is one in which no variable occur in the expression.

3. **Horn clause:** is a clause with a most one positive literal.

How to convert PL to

Conjunctive normal form

$(A \lor B) \land (A \lor \neg B)$

Disjunctive normal form

$(A \land B) \lor (A \land \neg B)$

1. **Remove $\rightarrow$**
2. **Remove $\leftrightarrow$**
3. **Move negation($\neg$) inward/ apply demorgan's law**
4. **Use distributive law**

Remove $\Rightarrow$

$A \Rightarrow B$

$\neg A \vee B$

Remove $\Leftrightarrow$

$A \Leftrightarrow B$

$(A \Rightarrow B) \wedge (B \Rightarrow A)$

Convert in CNF $\rightarrow$

$(A \Rightarrow B) \Rightarrow C$

$(\neg A \vee B) \Rightarrow C$

$\neg(\neg A \vee B) \vee C$

$\neg(\neg A) \wedge \neg B \vee C$

$(A \wedge \neg B) \vee C$

$(A \vee C) \wedge (B \vee C)$

CONVERT INTO DNF

▲◀🔊A▶▲B⌧◀◀🔊S☰⌧

(▲A◀B)◀(▲S▶T)

◀🔊(▲A◀B)◀▲S)▶(◀🔊▲A◀B)◀T⌧

NFERENCE RULES

INFERENCE: Deriving conclusion from evidences
RULES OF INFERENCE: Template for constructing valid argument

Types

1. Modus ponen

P→Q
P

Q

2. MODUS TOLLEN:

$P \rightarrow Q$

$\neg Q$

$\neg P$

$[(P \rightarrow Q) \wedge P] \rightarrow Q$

3.HYPOTHETICAL SYLLOGISM

$P \rightarrow Q$

$Q \rightarrow R$

$P \rightarrow R$

$[(P \rightarrow Q) \wedge \neg Q] \rightarrow \neg P$

4. Disjunctive syllogism

P▸Q

▴P

Q

[(P▸Q)◂▴P]▤Q

5. ADDITION

P

P▸Q

P▤(P▸Q)

6. RESOLUTION

P▸Q

P▸R

Q▸R

[(P▸Q)◂(▴P▸R)]⇛(Q▸R)