Syntax and Semantics of Propositional logic

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Knowledge ?

- Consists of facts, concepts and rules.
- awareness or familiarity gained by experiences of facts, data, and situations.
- Study of knowledge is called **epistemology**.

- there is one decision maker which :
- act by sensing the environment and using knowledge.
- But if the knowledge part will not present then,
- it cannot display intelligent behavior.

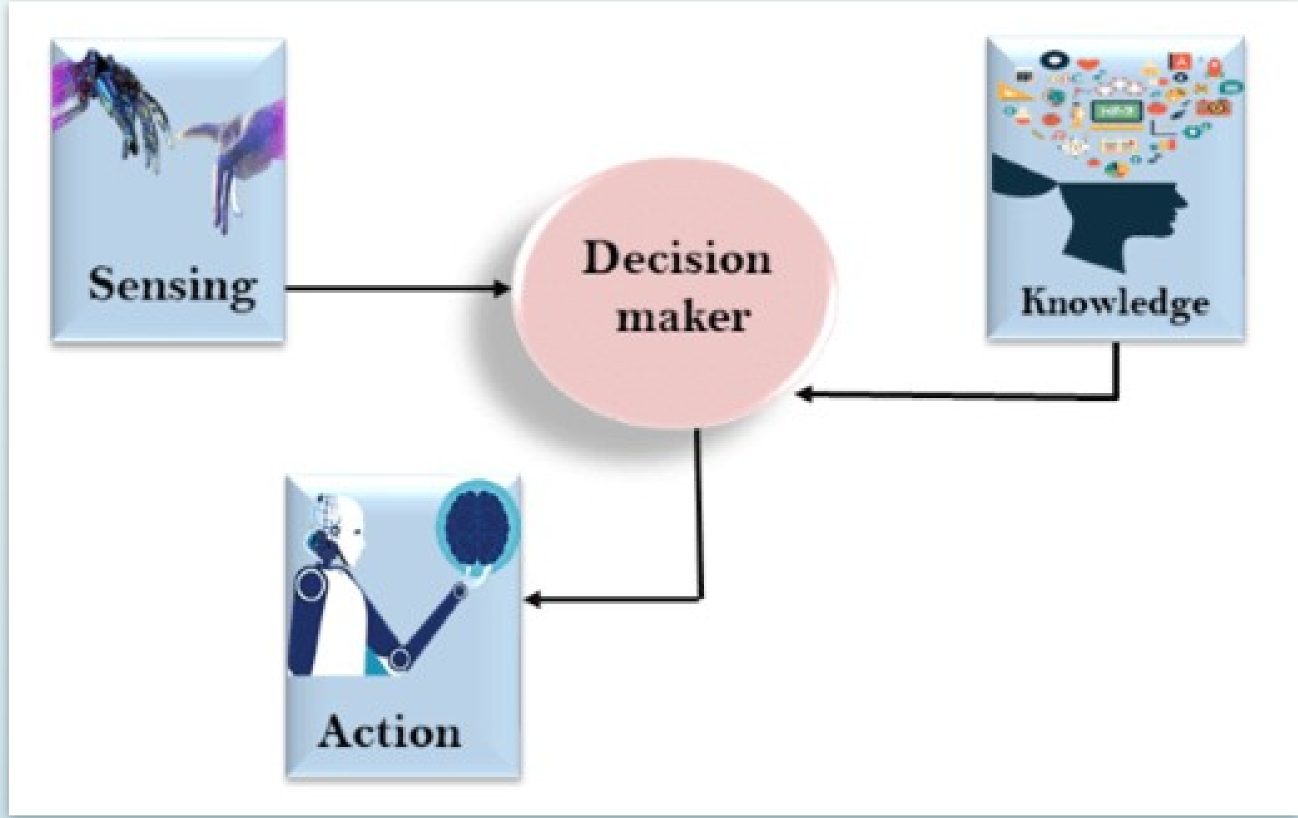
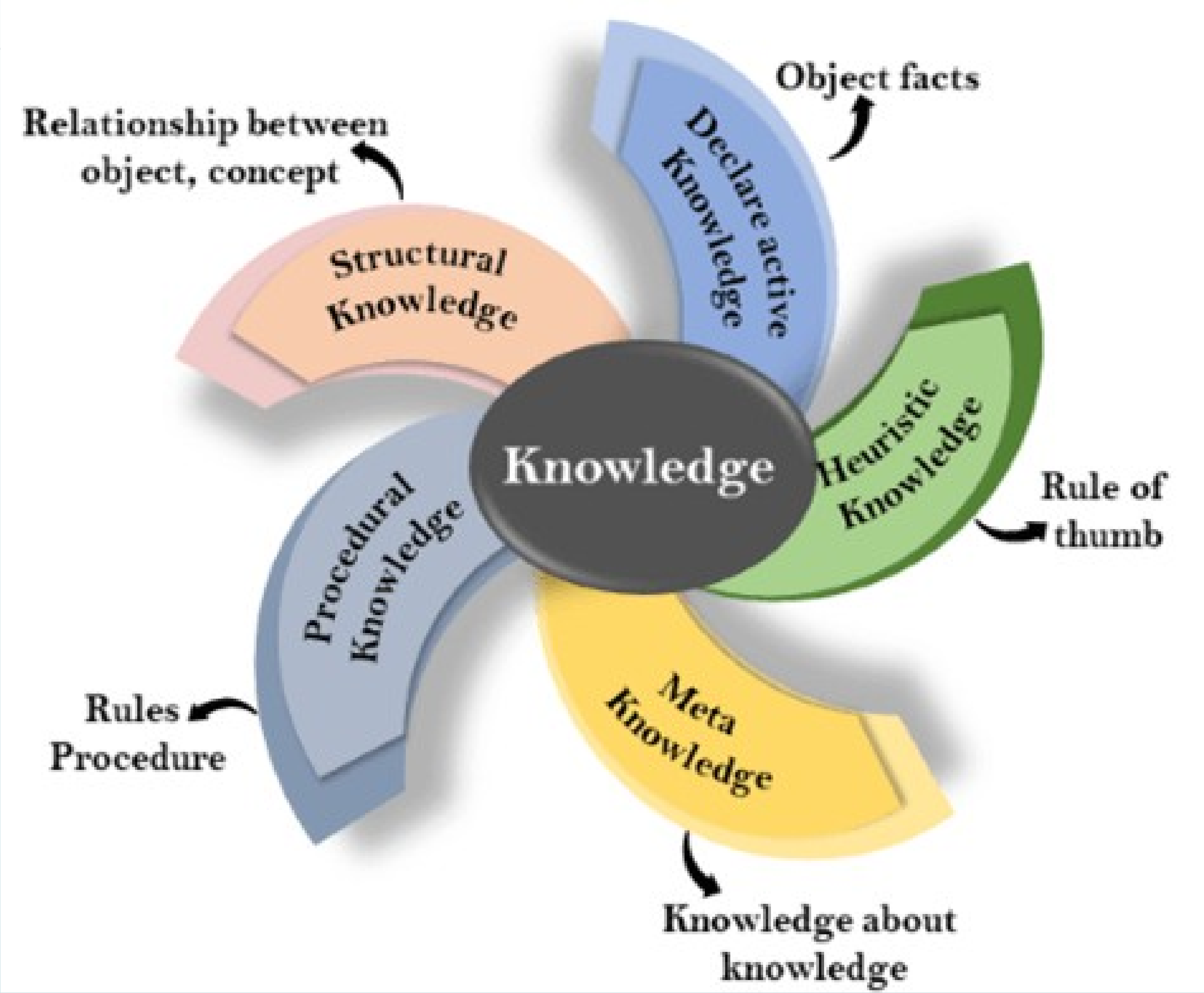


Figure1

Types of knowledge



Declarative Knowledge:

- Declarative knowledge is to know about something.
- It includes concepts, facts, and objects.

Procedural Knowledge

- It is also known as imperative knowledge.
- Procedural knowledge is a type of knowledge which is responsible for knowing how to do something
- It includes rules, strategies, procedures, agendas, etc.

Meta-knowledge:

- Knowledge about the other types of knowledge is called Meta-knowledge.

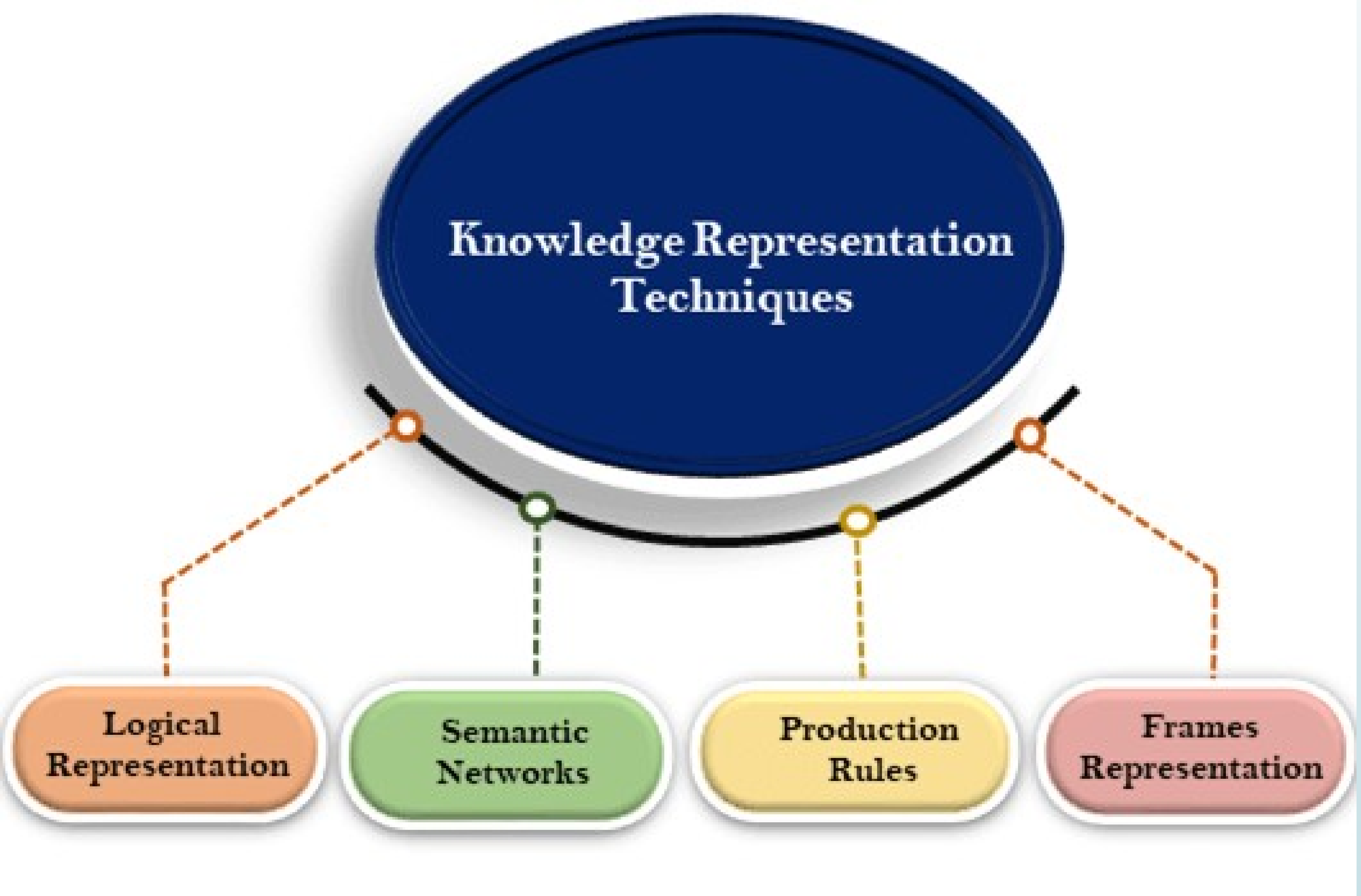
Heuristic knowledge:

- Heuristic knowledge is representing knowledge of some experts in a filed or subject.
- Heuristic knowledge is rules of thumb based on previous experiences, awareness of approaches, and which are good to work but not guaranteed.

Structural knowledge:

- Structural knowledge is basic knowledge to problem-solving.
- It describes relationships between various concepts such as kind of, part of, and grouping of something.
- It describes the relationship that exists between concepts or objects.

Techniques of knowledge representation



Logical Representation

- language with some concrete rules which deals with propositions
- means drawing a conclusion based on various conditions.
- consists of precisely defined **syntax** and **semantics**
- categorised into mainly two logics:

  Propositional Logics

  Predicate logics

Syntax and Semantics

Syntax

- rules which decide how we can construct legal sentences in the logic.
- determines which symbol we can use in knowledge representation.
- How to write those symbols.

Sementics

- Semantics are the rules by which we can interpret the sentence in the logic.
- Semantic also involves assigning a meaning to each sentence.

Propositional Logic

- A proposition is a declarative statement which is either true or false
- technique of knowledge representation in logical and mathematical form.
- Propositional logic is also called Boolean logic as it works on 0 and 1.
- Propositions can be either true or false, but it cannot be both.
- consists of an object, relations or function, and **logical connectives**.

- Examples :
- 3+3= 7(False proposition)
- 5 is a prime number.

Syntax of propositional logic

- The syntax of propositional logic defines the allowable sentences for the knowledge representation

Types of Propositions:

- There are two types of Propositions:
- **Atomic Propositions**
- **Compound propositions**

Atomic Propositions

- Atomic propositions are the simple propositions.
- It consists of a single proposition symbol.
- These are the sentences which must be either true or false.

- 2+2 is 4, it is an atomic proposition
- "The Sun is cold" is also a proposition

Compound propositions

- constructed by combining simpler or atomic propositions
- using parenthesis and logical connectives.

- "It is raining today, and street is wet."
- "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives:

- used to connect two simpler propositions
- representing a sentence logically.
- We can create compound propositions with the help of logical connectives.
- There are mainly five connectives

- **Negation**
- **Conjunction**
- **Disjunction**
- **Implication**
- **Biconditional**

Summarized table for Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Negation:

- A sentence such as ¬ P is called negation of P.
- A literal can be either Positive literal or negative literal.

For Negation:

P	¬ P
True	**False**
False	**True**

Conjunction

- A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction.

Example:

- Rohan is intelligent and hardworking.
- It can be written as,

P= Rohan is intelligent,

Q= Rohan is hardworking. → **P ∧ Q**.

For Conjunction:

P	Q	P∧Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

Disjunction

A sentence which has ∨ connective, such as **P** ∨ **Q**. is called disjunction, where P and Q are the propositions.

Example: "Ritika is a doctor or Engineer",
Here P= Ritika is Doctor.
Q= Ritika is Doctor, so we can write it as **P** ∨ **Q**.

For disjunction:

P	Q	P V Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

Implication

- A sentence such as P → Q, is called an implication.
- Implications are also known as if-then rules.
- It can be represented as

  If it is raining, then the street is wet.

  Let P= It is raining,
- Q= Street is wet,
- so it is represented as P → Q

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

Biconditional

- A sentence such as **P ⇔ Q is a Biconditional sentence,**
- **example If I am breathing, then I am alive**

P= I am breathing,

Q= I am alive, it can be represented as P ⇔ Q.

For Biconditional:

P	Q	P⇔ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Logical equivalence

- Logical equivalence is one of the features of propositional logic.
- Two propositions are said to be logically equivalent if and only if
- the columns in the truth table are identical to each other.

A	B	¬A	¬A∨ B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Rules of Inference

- we need intelligent computers which can create new logic from old logic or by evidence,
- **so generating the conclusions from evidence and facts is termed as Inference**.
- terminologies related to inference rules are:
- Implication
- Converse
- Contrapositive
- Inverse

Terminologies

- **Implication:** It is one of the logical connectives which can be represented as $P \to Q$. It is a Boolean expression.
- **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \to P$.
- **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as $\neg Q \to \neg P$.
- **Inverse:** The negation of implication is called inverse. It can be represented as $\neg P \to \neg Q$.

Truth table, we can prove that P → Q is equivalent to ¬ Q → ¬ P, and Q→ P is equivalent to ¬ P → ¬ Q.

P	Q	P → Q	Q→ P	¬ Q → ¬ P	¬ P → ¬ Q.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Types of Inference rules

1. Modus Ponens

- if P and P → Q is true, then we can infer that Q will be true.

Notation for Modus ponens: $$\frac{P \to Q, \quad P}{\therefore Q}$$

Proof by Truth table:

P	Q	$P \to Q$
0	0	0
0	1	1
1	0	0
1	1	1

2. Modus Tollens

- if P→ Q is true and ¬ **Q is true, then ¬ P** will also true

Notation for Modus Tollens: $\dfrac{P \to Q, \;\; \sim Q}{\sim P}$

Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \to Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

3. Hypothetical Syllogism

- if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation

Example:

Statement-1: If you have my home key then you can unlock my home. **P→Q**

Statement-2: If you can unlock my home then you can take my money. **Q→R**

Conclusion: If you have my home key then you can take my money. **P→R**

Proof by truth table:

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

4. Disjunctive Syllogism

- if P ∨ Q is true, and ¬P is true, then Q will be true.

Notation of Disjunctive syllogism: $\dfrac{P \lor Q,\quad \neg P}{Q}$

Proof by truth-table:

P	Q	$\neg P$	$P \lor Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

5. Addition

- it states that If P is true, then P ∨ Q will be true.

Notation of Addition: $\dfrac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P

Statement-2: I have Chocolate ice-cream.

Conclusion: I have vanilla or chocolate ice-cream. ==> (P∨Q)

Proof by Truth-Table:

P	Q	$P \vee Q$
0	**0**	**0**
1	**0**	**1**
0	**1**	**1**
1	**1**	**1**

6. Simplification

- if **P ∧ Q** is true, then **Q or P** will also be true. It can be represented as

Notation of Simplification rule: $\frac{P \wedge Q}{Q}$ Or $\frac{P \wedge Q}{P}$

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

Limitations of Propositional logic:

- We cannot represent relations like ALL, some, or none with propositional logic. Example:
  - **All the girls are intelligent.**
  - **Some apples are sweet.**
- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

First-Order Logic

- in propositional logic, we can only represent the facts, which are either true or false
- PL is not sufficient to represent the complex sentences or natural language statements
- The propositional logic has very limited expressive power.
- Consider the following sentence, which we cannot represent using PL logic.
- **"Some humans are intelligent", or**
- **"Sachin likes cricket."**

First-Order logic

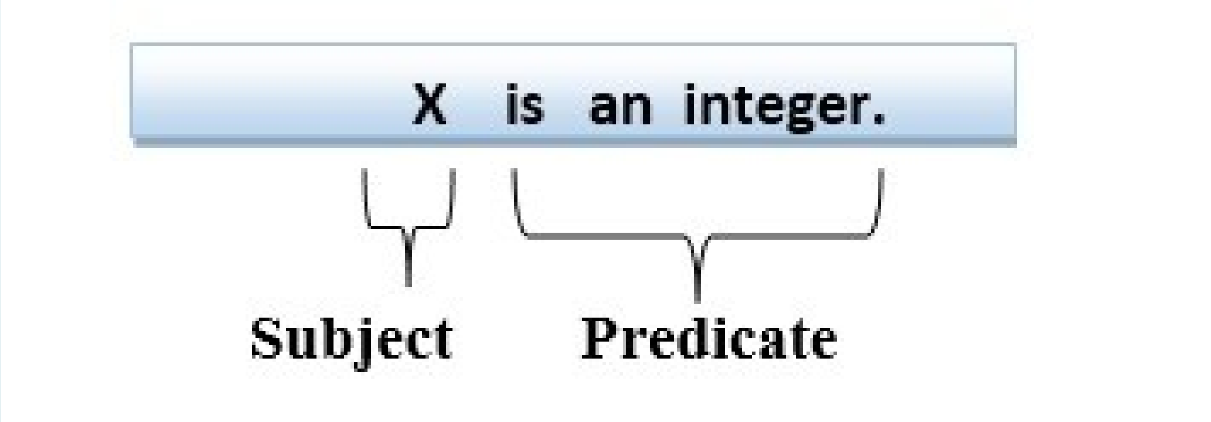
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as **Predicate logic or First-order predicate logic**.
- also assumes the following things in the world:

Objects: A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...

Relations: sister,brother

Function: Father of,, ...

- **First-order logic statements can be divided into two parts:**
- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.
- **Consider the statement: "x is an integer."**,
- the first part x is the subject of the statement and second part "is an integer," is known as a predicate.

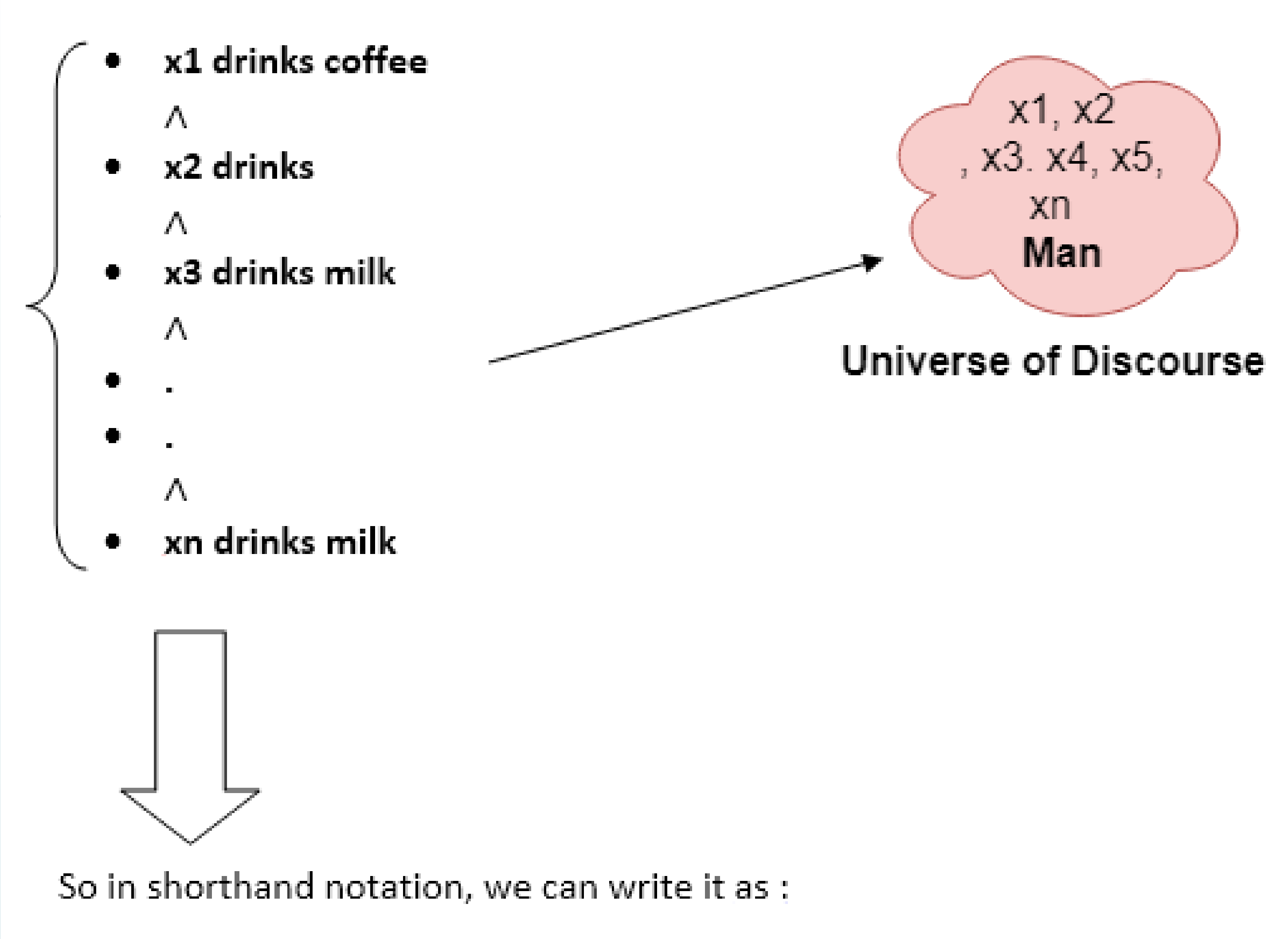


Quantifiers in First-order logic

- A quantifier is a language element which generates quantification,
- quantification specifies the quantity of specimen in the universe of discourse.
- There are two types of quantifier:
  - **Universal Quantifier, (for all, everyone, everything)**
  - **Existential quantifier, (for some, at least one).**

Universal Quantifier

- Universal quantifier is a symbol of logical representation,
- which specifies that the statement within its range is true for everything or every instance of a particular thing.
- represented by a symbol ∀

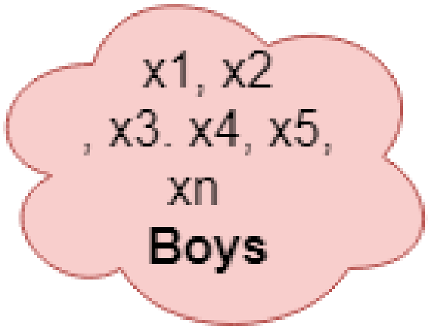


$\forall x \; man(x) \rightarrow drink(x, coffee).$

Existential Quantifier

- which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
- **There exists a 'x.'**
- **For some 'x.'**
- **For at least one 'x.'**

- x1 is intelligent
  V
- x2 is intelligentV
- x3 is intelligent
  V
- .
- .
  V
- xn is intelligent



Universe of Discourse

So in short-hand notation, we can write it as:

$\exists x$: boys(x) $\wedge$ intelligent(x)