# PROPERTIES OF WELL FORMED FORMULA

Proposition  logic uses a symbolic “language” to represent the logical structure, or form, of a compound proposition.

Has rules of syntax-  grammatical rules for putting symbols together in the right way.

Any expression that obey the syntactic rules of propositional logic is called a WELL FORMES FORMULA , or WFF

# The Symbols of PL are:

- Basic propositions:

A1, A2, A3…….B1, B2, B3……..etc.

- Connective:

▲, ◀, ▶,

- Parentheses: ()

# WFFs of PL are defined as follows:

- **A well formed formula is any formula that is capable of being generated by some combination of seven information rules.**
- Any basic proposition is a WFF **(e.g. A, B, C)**
- If P and Q are WFFs, then so are:
  - **▲(P) is a WFF.**
  - **(P◀Q) is a WFF.**
  - **(P▶Q) is a WFF.**
  - **(P☰Q) is a WFF.**
  - **(P⬇Q) is a WFF.**
- **Nothing else is a WFF except what can be formed by repeated application of 1-6.**

# Examples of WFFs and not WFFs

- The following are the WFFs in PL
  - P
  - (P≡Q)
  - ▲((P▶▲(Q)))
  - ▲(▲(P◀(Q)))
- The following are NOT WFFs in PL
  - P▲
  - PQ
  - ◀▲(Q))
  - ▲▲P◀▲Q▶S

# Three types of WFFs

**Atomic WFFs**

An Atomic WFF in PL is a well formed formula in PL consisting only of a single propositional letter.

- The following are atomic WFFs in PL

P

Q

A

B

# Complex WFFs

A complex WFF in PL is a well formed formula in PL that contains at least one propositional letter and a truth-functional operator.

- The following are complex WFFs in PL

▲(P)

(P◄Q)

(P▬▲(A))

# Literal WFFs

A literal WFF in PL is a well formed formula in PL that consist of an atomic WFF P or a negated atomic WFF ▲(P).

- The following are Literals in PL

P

▲(P)

Q

▲(Q)

# Conversion to clausal form /normal form

1. PL does not permit us to make generalized statement about classes of similar object.
2. These are serious limitation when reasoning about real world entity.

   The word All, Some, Every, it is difficult to represent logical of these sentences with the help of PL.

Example:

ALL students are tall.

# SYNTAX OF FOPL

1. **VARIABLE:** variable are terms that assume different value over a given domain.

   Example: X, Y, Z

2. **CONNECTIVES:** These are 5 connective symbols => (not or negation ▲)

   (AND or conjunction ◀)

   (OR or disjunction ▶)

   (implication

   (equivalence or if and only if

3. **CONSTANT**: constant are fixed value terms that belong to a given domain of discourse.

   a,b,c,100 etc.

4. **FUNCTION :** function symbol denote relations defined on a domain D.

5. **PREDICATE:** predicate symbols denote relations or functional mapping from the elements of a domains D to the value true and false.

capital letter and capitalized word such as P, Q , R, EQUAL, MARRIED are used to represent predicate.

- Example:

**All boys plays football**

**sol:** Plays(boys, football) where plays is a predicate

# 6. Quantifier

**There are two quantifier symbols are**

(existential quantifier)

(universal quantifier)

**EXAMPLES:**

**ALL BOYS PLAYS FOOTBALL**

**SOL:** (X)BOYS PLAYS(BOYS, FOOTBALL)

**SOME BOYS PLAYS FOOTBALL**

**SOL:** (X)BOYS PLAYS(BOYS, FOOTBALL)

# Clausal form

1. **Clause:** as the disjunction of a number of literals.
2. **Ground clause:** is one in which no variable occur in the expression.
3. **Horn clause:** is a clause with a most one positive literal.

# How to convert PL to: CNF & DNF

- **CNF(Conjunction Normal Form)**

1. Remove
2. Remove
3. Move negation() inward/ apply demorgan's law
4. Use distributive law

# Convert in CNF

- $(A \rightarrow B) \rightarrow C$

Remove $\rightarrow$            Remove $\leftrightarrow$

$A \rightarrow B$            $A \leftrightarrow B$

$\neg A \lor B$            $(A \rightarrow B) \land (B \rightarrow A)$

- **SOL:** $(A \lor B) \rightarrow C$

$\neg(A \lor B) \lor C$

$\neg(A \land B) \lor C$

$(A \land \neg B) \lor C$

$(A \lor C) \land (B \lor C)$

# CONVERT INTO DNF(Disjunction Normal Form)

$\neg(A \wedge \neg B) \vee (S \rightarrow T)$

<u>**SOL:**</u>

$\neg(A \wedge \neg B) \vee (\neg S \wedge T)$

$(\neg A \vee B) \vee (\neg S \wedge T)$

$((\neg A \vee B) \vee \neg S) \wedge ((\neg A \vee B) \vee T)$

# INFERENCE RULES

- **INFERENCE: Deriving conclusion from evidences**
- **RULES OF INFERENCE: Template for constructing valid argument**

In predicate calculus, there are rules of inference, that can be applied to certain WFF and set of WFFs to produce new WFFs.

## 1. Modus ponen

**P**

**P → Q**

**---------------**

**Q**

## 2. MODUS TOLLEN:

▲Q

P→Q

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▲P

## 3.HYPOTHETICAL SYLLOGISM

P→Q

Q→R

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P→R

## 4. Disjunctive syllogism

$P \lor Q$

$\neg P$

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$Q$

## 5. ADDITION

$P$

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$P \lor Q$

$P \rightarrow (P \lor Q)$

# 6. RESOLUTION

$P \lor Q$

$P \lor R$

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$Q \lor R$

$[(P \lor Q) \land (\lnot P \lor R)] \to (Q \lor R)$