Extended Generalized Total Least Squares Method for the Identification of Bilinear Systems

Seokwon Han, Jinyoung Kim, and Koengmo Sung

***Abstract*— The extended generalized total least squares (e-GTLS) method is proposed as one of the bilinear system parameters. Considering that the input is noise free and that bilinear system equation is linear with respect to the output, we extend the GTLS problem. The extended GTLS problem is reduced to an unconstrained minimization problem and is then solved by the Newton–Raphson method. We compare the GTLS method and the extended GTLS method as far as the accuracy of the estimated system parameters is concerned.**

I. Introduction

There have been various studies on bilinear systems due to their simple structure, their similiarity to linear systems, and their applicability to real processes [1]–[3]. However, there is little research on the identification of bilinear systems in the presence of an additive noise. On the other hand, there are extensive studies on the identification of noisy linear systems, but most of the identification methods applied to noisy linear systems are not applicable to the identification of noisy nonlinear systems. Recently, the generalized total least squares (GTLS) method was successfully applied to the identification of noisy linear systems [4], [5]. It is possible to apply the GTLS method directly to the identification of bilinear systems through minor modifications to the covariance matrix of the errors in a row of the data matrix. However, the direct application of the GTLS problem neglects the structure of the data matrix of bilinear systems, which arises from only the output being corrupted by noise and from the fact that the bilinear system equation is linear with respect to the output.

In this correspondence, we extend the GTLS problem that considers the special structure of the data matrix of bilinear system equations. We show that the extended GTLS (e-GTLS) problem is reduced to an unconstrained minimization problem using the method of Lagrange multipliers. The e-GTLS program is then solved iteratively using the Newton–Raphson method. Through computer simulation, we compare the performances of the e-GTLS method with those of the GTLS method.

II. The Structure of the Data Matrix of Bilinear System Equations

We consider the bilinear system, which is the single input, single output (SISO), time-invariant system. The bilinear system is described by (1) and (2). It is assumed that the order of the system and the delay in the system are known.

$$x_t = \sum_{i=1}^{p} a_i x_i + \sum_{j=0}^{q} b_j u_{t-d-j} + \sum_{k_1=1}^{l} \sum_{k_2=1}^{m} c_{k_1 k_2} x_{t-k_1} u_{t-d+1-k_2} \tag{1}$$

$$y_t = x_t + e_t \tag{2}$$

where

- $\{u_t\}$ input,
- $\{x_t\}$ uncorrupted output,
- $\{y_t\}$ corrupted output,
- $\{e_t\}$ additive noise,
- d delay of the system.

We assume that the covariance matrix of the additive noise is known up to a factor of proportionality. For simplicity, we also assume that l is less than p in (1).

In general, the least square (LS) method is used for the estimation of the system parameters $\{a_i, b_j, c_{k_1 k_2}\}$ when the input u_t and the output y_t are known. However, the parameters estimated by the LS method are biased because the noise n_t is correlated with the output. Thus, the LS method fails to estimate system parameters correctly.

Let $\mathbf{h}$ be $[\mathbf{b}^T, \mathbf{a}^T, \mathbf{c}_1^T \cdots \mathbf{c}_m^T]^T$. By considering N overdetermined bilinear equations, that is, $t = 1, 2, \cdots, N$, the system equations are represented in matrix form:

$$[\mathbf{U} \quad \mathbf{Y} \quad \mathbf{Z}_1 \quad \cdots \quad \mathbf{Z}_m] \cdot \mathbf{h} \approx \mathbf{0} \tag{3}$$

where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1^r \\ \mathbf{u}_2^r \\ \vdots \\ \mathbf{u}_N^r \end{bmatrix} = [\mathbf{u}_1^c \quad \mathbf{u}_2^c \quad \cdots \quad \mathbf{u}_{q+1}^c]$, $\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1^r \\ \mathbf{y}_2^r \\ \vdots \\ \mathbf{y}_N^r \end{bmatrix} = [\mathbf{y}_1^c \quad \mathbf{y}_2^c \quad \cdots \quad \mathbf{y}_{p+1}^c]$, $\mathbf{u}_t^r = \begin{bmatrix} u_{t-d} \\ u_{t-d-1} \\ \vdots \\ u_{t-d-q} \end{bmatrix}^T$, $\mathbf{y}_t^r = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix}^T$, $\mathbf{Z}_k =$

$$\begin{bmatrix} u_{-d+2-k}y_0 & u_{-d+2-k}y_{-1} & \cdots & u_{-d+2-k}y_{1-l} \\ u_{-d+3-k}y_1 & u_{-d+3-k}y_0 & \cdots & u_{-d+3-k}y_{2-l} \\ \vdots & \vdots & \vdots & \vdots \\ u_{-d+N+1-k}y_{N-1} & u_{-d+N+1-k}y_{N-2} & \cdots & u_{-d+N+1-k}y_{N-l} \end{bmatrix},$$

$\mathbf{a} = \begin{bmatrix} -\mathbf{1} \\ \mathbf{a_1} \\ \vdots \\ \mathbf{a_p} \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -\mathbf{1} \\ \mathbf{b_1} \\ \vdots \\ \mathbf{b_q} \end{bmatrix}$, $\mathbf{c}_k = \begin{bmatrix} c_{1k} \\ c_{2k} \\ \vdots \\ c_{lk} \end{bmatrix}$, $\mathbf{U} \in R^{N\times(q+1)}$, $\mathbf{Y} \in R^{N\times(p+1)}$, $\mathbf{u}_i^r \in R^{1\times(q+1)}$, $\mathbf{u}_i^c \in R^{N\times 1}$, $\mathbf{y}_i^r \in R^{1\times(p+1)}$, $\mathbf{y}_i^c \in R^{N\times 1}$, $\mathbf{Z}_k \in R^{N\times l}$, $\mathbf{a} \in R^{(p+1)\times 1}$, $\mathbf{b} \in R^{(q+1)\times 1}$, and $\mathbf{c}_k \in R^{l\times 1}$.

Let $\mathbf{D}$ be $[\mathbf{U} \quad \mathbf{Y} \quad \mathbf{Z}_1 \quad \cdots \quad \mathbf{Z}_m]$. In the data matrix $\mathbf{D}$,$\mathbf{Y}$ and $\mathbf{Z}_k$ are subject to error due to additive noise. From the data matrix $\mathbf{D}$, it can be observed that there is a special relation between the matricies $\mathbf{Y}$ and $\mathbf{Z}_k$. Let $\mathbf{Y}$ be partitioned as $[\mathbf{Y}_1 \quad \mathbf{Y}_2 \quad \mathbf{Y}_3]$, where $\mathbf{Y}_1 = [\mathbf{y}_1^c] \in \mathrm{R}^{N\times 1}$,$\mathbf{Y}_2 = [\mathbf{y}_2^c \cdots \mathbf{y}_{l+1}^c] \in \mathrm{R}^{N\times l}$, and $\mathbf{Y}_3 = [\mathbf{y}_{l+2}^c \cdots \mathbf{y}_{p+1}^c] \in \mathrm{R}^{N\times(p-l)}$. In addition, define $\mathbf{W}_k$ as $[\mathbf{u}_k^c \quad \cdots \quad \mathbf{u}_k^c] \in R^{N\times l}$, that is, all the columns of $\mathbf{W}_k$ are the kth column vector of $\mathbf{U}$. Then, the following equation is satisfied.

$$\mathbf{Z}_k = \mathbf{W}_k \otimes \mathbf{Y}_2 \tag{4}$$

where $\mathbf{W}_k \otimes \mathbf{Y}_2$is the direct or Hadamard product. Then, (3) is represented by using the relation of (4).

$$[\mathbf{U} \quad \mathbf{Y} \quad \mathbf{W}_1 \otimes \mathbf{Y}_2 \quad \cdots \quad \mathbf{W}_m \otimes \mathbf{Y}_2] \cdot \mathbf{h} \approx \mathbf{0}. \tag{5}$$

III. Application of the GTLS Method to the Bilinear System Identification

In this section, the GTLS method is directly applied to estimation of the system parameters. Let the data matrix $\mathbf{D}$ be par-

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titioned as $[\mathbf{D}_1\, \mathbf{D}_2]$, where $\mathbf{D}_1 = [\mathbf{U}] \in R^{N\times(q+1)}$and $\mathbf{D}_2 = [\mathbf{Y}\, \mathbf{W}_1 \otimes \mathbf{Y}_2 \cdots \mathbf{W}_m \otimes \mathbf{Y}_2] \in R^{N\times(p+lm+1)}$. Only the matrix $\mathbf{D}_2$ is subject to error. Then, the GTLS problem for the bilinear system identification is described as minimization of $\|\hat{\mathbf{D}}_2\, \mathbf{R}_{\mathbf{D}_2}^{-1\backslash 2}\|_{\mathrm{F}}$ with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $\left[\mathbf{D}_1\, \mathbf{D}_2 + \hat{\mathbf{D}}_2\right] \mathbf{h} = 0$ and $a_0 = -1$, where

$\mathbf{R}_{D_2}$ covariance matrix of the errors in rows of the matrix,
$\|\cdot\|_F$ Frobenius norm,
$\hat{\mathbf{D}}_2$ error matrix of $\mathbf{D}_2$ [4].

We assume that the covariance matrix of the additive noise $\{e_t\}$ is known up to a factor of propotionality. Using this assumption and (2), we can calculate the error vector of the ith row of $\mathbf{Z}_{\mathrm{k}}$ and calculate the noise covariance matrix $\mathbf{R}_{\mathbf{D}_2}$. Substituting (2) in the the matrix and inspecting it, we observe that the error vector of the tth row $\hat{\mathbf{d}}_{2t}^{\mathrm{r}}$ is represented as follows:

$$\hat{\mathbf{d}}_{2t}^{\mathrm{r}} = [\hat{\mathbf{y}}_t^r\, \hat{\mathbf{z}}_{1t}^{\mathrm{r}} \cdots \hat{\mathbf{z}}_{mt}^r]$$

where

$$\hat{\mathbf{y}}_t^r = [e_t \cdots e_{t-p}] \text{ and}$$
$$\hat{z}_{kt}^r = [u_{-d+(t+1)-k}e_{t-1} \cdots u_{-d+(t+1)-k}e_{t-l}]$$

Let $\mathbf{R}_{ee}$ be the $(p+1)$ by $(p+1)$ covariance matrix of the additive noise. Then, $\mathbf{R}_{\mathbf{D}_2}$ is represented by (6).

$$\mathbf{R}_{\mathbf{D_2}} = E\left[(\hat{\mathbf{d}}_{2t}^{\mathrm{r}})^{\mathrm{T}}\, \hat{\mathbf{d}}_{2t}^{\mathrm{r}}\right] = \begin{bmatrix} \mathbf{\Gamma}_{\mathbf{aa}} & \mathbf{\Gamma}_{\mathbf{ac}_1} & \cdots & \mathbf{\Gamma}_{\mathbf{ac}_m} \\ \mathbf{\Gamma}_{\mathbf{c}_1\mathbf{a}} & \mathbf{\Gamma}_{\mathbf{c}_1\mathbf{c}_1} & \cdots & \mathbf{\Gamma}_{\mathbf{c}_1\mathbf{c}_m} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{\Gamma}_{\mathbf{c}_m\mathbf{a}} & \mathbf{\Gamma}_{\mathbf{c}_m\mathbf{c}_1} & \cdots & \mathbf{\Gamma}_{\mathbf{c}_m\mathbf{c}_m} \end{bmatrix} \tag{6}$$

where $\mathbf{\Gamma}_{\mathbf{aa}}(i,j) = \mathbf{R}_{ee}(i,j)$, $\mathbf{\Gamma}_{\mathbf{ac}_n}(i,j) = \mathbf{\Gamma}_{\mathbf{c}_n\mathbf{a}}(i,j) = \mathbf{R}_{ee}(i,j)m_u$, $\mathbf{\Gamma}_{\mathbf{c_{k_1}c_{k_2}}}(i,j) = \mathbf{\Gamma}_{\mathbf{c}_{k_2}\mathbf{c}_{k_1}}(i,j) = \mathbf{R}_{\mathbf{ee}}(i,j) \cdot\mathbf{R}_{uu}(\mathrm{k}_1, \mathrm{k}_2)$, m_u is the mean of the input, and $\mathbf{R}_{uu}(k_1, k_2)$ is the autocovariance matrix of the input. Using the above $\mathbf{R}_{\mathbf{D}_2}$, we can solve the GTLS problem[4].

IV. Extension of the GTLS Problem for the Bilinear System Identification

A. The Definition of the e-GTLS Problem

The GTLS method does not consider the special structure of the data matrix $\mathbf{D}$. That is, the error matrices of $\mathbf{Y}$ and $\mathbf{Z_k}$ are treated independently. Therefore, it is necessary to modify the GTLS problem so that the relation of (4) is considered. From (4), we observed that if the error matrix of $\mathbf{Y}$, i.e., $\hat{\mathbf{Y}}$, is determined, the error matrix of $\mathbf{Z_k}$, i.e., $\hat{\mathbf{Z}}_{\mathbf{k}}$, is determined by

$$\hat{\mathbf{Z}}_{\mathbf{k}} = \mathbf{W}_{\mathbf{k}} \otimes \hat{\mathbf{Y}}_2$$

where $\hat{\mathbf{Y}}_2$ is the error matrix of $\mathbf{Y}_2$. Then, the GTLS problem is extended as minimization of $\|\hat{\mathbf{Y}}\, \mathbf{R}_{ee}^{-1\backslash 2}\|_F$ with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to

$$a_0 = -1 \tag{7}$$

and

$$[\mathbf{U}\quad \mathbf{Y}+\hat{\mathbf{Y}}\quad \mathbf{W}_1 \otimes (\mathbf{Y}_2+\hat{\mathbf{Y}}_2)\quad \cdots\quad \mathbf{W}_m \otimes (\mathbf{Y}_2+\hat{\mathbf{Y}}_2)]\cdot\mathbf{h} = \mathbf{0} \tag{8}$$

where $\hat{\mathbf{Y}} = [\hat{\mathbf{Y}}_1\, \hat{\mathbf{Y}}_2\, \hat{\mathbf{Y}}_3]$, $\hat{\mathbf{Y}}_i$ is the error matrix of $\mathbf{Y}_i$, $\mathbf{R}_{ee}$ is the covariance matrix of the additive noise, and $\mathbf{R}_{ee} \in R^{(p+1)\times(q+1)}$. In the e-GTLS problem, the weighting matrix $\mathbf{R}_{ee}^{-1\backslash 2}$ is used for the decorrelation of the error vector $\hat{\mathbf{y}}_{\mathrm{i}}$. It can be explained as follows. In the ideal case, $\hat{\mathbf{y}}_i = [\, e_i\, e_{i-1} \cdots e_{i-p}\,]^T$, and $E[\hat{\mathbf{y}}_i\, \hat{\mathbf{y}}_t^T] = \mathbf{R}_{ee}$. That is, the estimated error vector is correlated. Therefore, it is necessary to whiten the error vector by $\mathbf{R}_{ee}^{-1\backslash 2}$.

B. The Solution to the e-GTLS Problem

In the e-GTLS problem, there are two constraints represented by (7) and (8). The constraint of (8) can be removed by the method of Lagrange multipliers. Define the function of

$$L(\hat{\mathbf{Y}}, \lambda) = \sum_{\mathrm{i=1}}^{\mathrm{N}} \hat{\mathbf{y}}_i^r \mathbf{R}_{ee}^{-1} \hat{\mathbf{y}}_i^{r\,\mathrm{T}} + \lambda^T\{[\mathbf{U}\, \mathbf{Y} + \hat{\mathbf{Y}}\, \mathbf{W}_1 \otimes (\mathbf{Y}_2 + \hat{\mathbf{Y}}_2) \cdots \mathbf{W}_m \otimes (\mathbf{Y}_2 + \hat{\mathbf{Y}}_2)] \cdot \mathbf{h}\} \tag{9}$$

where $\lambda = [\lambda_1\, \lambda_2 \cdots \lambda_N]^T$. Differentiating (9) with respect to λ_i and $\hat{\mathbf{y}}_i^r$, we obtain (10) and (11). Because $\mathbf{R}_{ee}^{-1}$ is the symmetric matrix

$$\frac{\partial L}{\partial \hat{\mathbf{y}}_i^r} = 2\hat{\mathbf{y}}_i^r \mathbf{R}_{\mathrm{ee}}^{-1} + \lambda_i(\mathbf{a} + \mathbf{s}_i)^{\mathrm{T}} = \mathbf{0} \tag{10}$$

$$\frac{\partial L}{\partial \lambda_i} = \mathbf{u}_i^{\mathrm{r}}\mathbf{b} + (\mathbf{y}_i^r + \hat{\mathbf{y}}_i^r)(\mathbf{a} + s_i) = 0 \tag{11}$$

where $\mathbf{s}_i = [0\quad \sum_{j=1}^m \mathbf{w}_{ji}^r \otimes \mathbf{c}_j^T\quad 0\quad \cdots\quad 0]^T$, and $\mathbf{w}_{ji}^r$ is the ith row vector of the matrix $\mathbf{W}_j$. From (10) and (11), we obtain s_i and $\hat{\mathbf{y}}_i^r$ by some calculus. They are

$$\lambda_i = \frac{2\{\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}\}}{(\mathbf{a} + \mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a} + \mathbf{s}_i)} \tag{12}$$

and

$$\hat{\mathbf{y}}_i^r = \frac{\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}}{(\mathbf{a} + \mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a} + \mathbf{s}_i)}(\mathbf{a} + \mathbf{s}_i)^T\mathbf{R}_{ee} \tag{13}$$

Substituting (13) into the e-GTLS formulation, we obtain the e-GTLS problem as minimization of

$$\sum_{i=1}^{N} \hat{\mathbf{y}}_i^r \mathbf{R}_{ee}^{-1} (\hat{\mathbf{y}}_i^r)^T = \sum_{i=1}^{N} \frac{\{\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}\}^T\, \{\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}\}}{(\mathbf{a} + \mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a} + \mathbf{s}_i)} \tag{14}$$

with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $a_0 = -1$. Defining the numerator and denominator in the above e-GTLS problem as $\mathbf{h}^{\mathrm{T}}\mathbf{N}_t\mathbf{h} = [\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}]^{\mathrm{T}}[\mathbf{y}_i^r(\mathbf{a} + \mathbf{s}_i) + \mathbf{u}_i^r\mathbf{b}]$ and $\mathbf{h}^{\mathrm{T}}\mathbf{\Delta}_t\mathbf{h} = (\mathbf{a} + \mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a} + \mathbf{s}_i)$, where

$$\mathbf{N}_t = \begin{bmatrix} \mathbf{u}_i^{r\,T} \\ \mathbf{y}_i^{r\,T} \\ (\mathbf{w}_{1i}^{\mathrm{r}} \otimes \mathbf{y}_{2i}^{\mathrm{r}})^{\mathrm{T}} \\ \vdots \\ (\mathbf{w}_{mi}^r \otimes \mathbf{y}_{2i}^{\mathrm{r}})^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_i^{r\,T} \\ \mathbf{y}_i^{r\,T} \\ (\mathbf{w}_{1i}^{\mathrm{r}} \otimes \mathbf{y}_{2i}^{\mathrm{r}})^{\mathrm{T}} \\ \vdots \\ (\mathbf{w}_{mi}^r \otimes \mathbf{y}_{2i}^{\mathrm{r}})^{\mathrm{T}} \end{bmatrix}^T$$

$$\mathbf{\Delta}_t = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \hat{\mathbf{\Gamma}}_{\mathbf{aa},t} & \hat{\mathbf{\Gamma}}_{\mathbf{ac}_1,t} & \cdots & \hat{\mathbf{\Gamma}}_{\mathbf{ac}_m,t} \\ 0 & 0 & \hat{\mathbf{\Gamma}}_{\mathbf{c}_1\mathbf{a},t} & \hat{\mathbf{\Gamma}}_{\mathbf{c}_1\mathbf{c}_1,t} & \cdots & \hat{\mathbf{\Gamma}}_{\mathbf{c}_1\mathbf{c}_m,t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \hat{\mathbf{\Gamma}}_{\mathbf{c}_m\mathbf{a},t} & \hat{\mathbf{\Gamma}}_{\mathbf{c}_m\mathbf{c}_1,t} & \cdots & \hat{\mathbf{\Gamma}}_{\mathbf{c}_m\mathbf{c}_m,t} \end{bmatrix}$$

$$\hat{\mathbf{\Gamma}}_{\mathbf{aa},t} = \mathbf{R}_{ee} \in R^{(p+1)\times(p+1)}$$

$$\hat{\mathbf{\Gamma}}_{\mathbf{ac}_n,t} = \hat{\mathbf{\Gamma}}_{\mathbf{c}_n\mathbf{a},t} = \begin{bmatrix} \mathbf{r}_{ee21}^r \otimes \mathbf{w}_{ni}^r \\ \vdots \\ \mathbf{r}_{ee2(p+1)}^r \otimes \mathbf{w}_{ni}^r \end{bmatrix} \in R^{(p+1)\times l}$$

$$\hat{\mathbf{\Gamma}}_{\mathbf{c}_{k_1}\mathbf{c}_{k_2},t} = \hat{\mathbf{\Gamma}}_{\mathbf{c}_{k_2}\mathbf{c}_{k_1},t} = \begin{bmatrix} \mathbf{r}_{ee22}^r \\ \vdots \\ \mathbf{r}_{ee2(l+1)}^r \end{bmatrix} \otimes (\mathbf{w}_{k_1t}^{r\;\mathbf{T}}\mathbf{w}_{k_1t}^r) \in R^{(l\times l)}$$

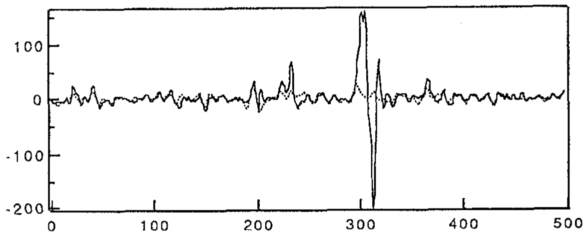

Fig. 1. utput of the bilinear system (solid line) and linear system (dotted line).

$$\mathbf{R}_{ee} = [\mathbf{R}_{ee1}\ \mathbf{R}_{ee2}\ \mathbf{R}_{ee3}] =$$
$$\begin{bmatrix} r_{ee}(1,1) & \mathbf{r}^r_{ee21} & r_{ee}(1,1+2) & \cdots & r_{ee}(1,\mathrm{p}+1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{ee}(\mathrm{p}+1,1) & \mathbf{r}^r_{ee2p} & r_{ee}(\mathrm{p}+1,1+2) & \cdots & r_{ee}(\mathrm{p}+1,\mathrm{p}+1) \end{bmatrix}$$

we get the e-GTLS problem as a minimization of $\rho(\mathbf{h})$ with respect to $\{a_i, b_j, c_{k\_1k_2}\}$ and subject to $a_0 = -1$, where

$$\rho(\mathbf{h}) = \sum_{t=1}^{N} \frac{\mathbf{h}^T \mathbf{N}_t \mathbf{h}}{\mathbf{h}^T \mathbf{\Delta}_t \mathbf{h}}. \tag{15}$$

The above e-GTLS problem can be viewed as an unconstrainted minimization problem because a_0 can be fixed to -1. The e-GTLS problem is a nonlinear optimization problem. Therefore, it is impossible to solve the e-GTLS problem in closed form. To solve the problem, we adopted the Newton–Raphson metod with the first and second derivatives of $\rho(\mathbf{h})$ with respect to $\mathbf{h}_i$ and $\mathbf{h}_l\,\mathbf{h}_m$.

V. Simulation

In this section, we examine the effectiveness of the e-GTLS method and its recursive algorithm through computer simulation. The following bilinear model is used in our simulation:

$$x_t = 1.5x_{t-1} - 0.7x_{t-2} + 0.8u_t + 0.5u_{t-1} + 0.24x_{t-1}u_t \tag{16}$$
$$y_t = x_t + \varepsilon e_t \tag{17}$$

where ε is a constant for varying SNR. The input series $\{u_t\}$ is generated from AR(1) model $u_t = 0.5u_{t-1} + \eta_t$, where $\{\eta_t\}$ is a Gaussian white noise $N(0,1)$. The additive noise $\{e_t\}$ satisfies ARMA (2,1) $e_t = 0.9e_{t-1} + 0.4e_{t-2} + \nu_t + 0.6\nu_{t-1}$, where $\{\nu_t\}$ is a gaussian white noise $N(0,1)$ as well as $\{\eta_t\}$. Fig. 1 shows the examples of the series $\{y_t\}$. In Fig. 1, the dotted line represents the time series generated by the ARMA(2,1) model, which neglects the term of $x_{t-1}u_t$ in (16). The e-GTLS and GTLS methods were used to estimaste the parameters of the bilinear system described by (11a) and (11b) with the given noise covariance matrix. The covariance matrix of the noise $\{e_t\}$ is given by the following matrix:

$$\mathbf{R}_{ee} = \begin{bmatrix} 1. & 0.738 & 0.262 \\ 0.738 & 1. & 0.738 \\ 0.262 & 0.738 & 1. \end{bmatrix}$$

where the diagonals are scaled to 1 for simplicity. One hundred Monte Carlo trials were performed at each SNR. The mean and standard deviation of the estiamated system parameters by the both method were obtained for SNR's ranging from 18 to 30 dB at 3-dB intervals. Five hundred samples were used for each trial. Fig. 2(a) and (b) show the bias and standard deviation of the estimated parameters through both methods. From the figures, it is observed that the bias and the variance of the estimated parameters via the e-GTLS method are generally smaller that those via the GTLS method, especially, the standard deviations of the AR, and are reduced from one quater to one tenth of those via the GTLS method.

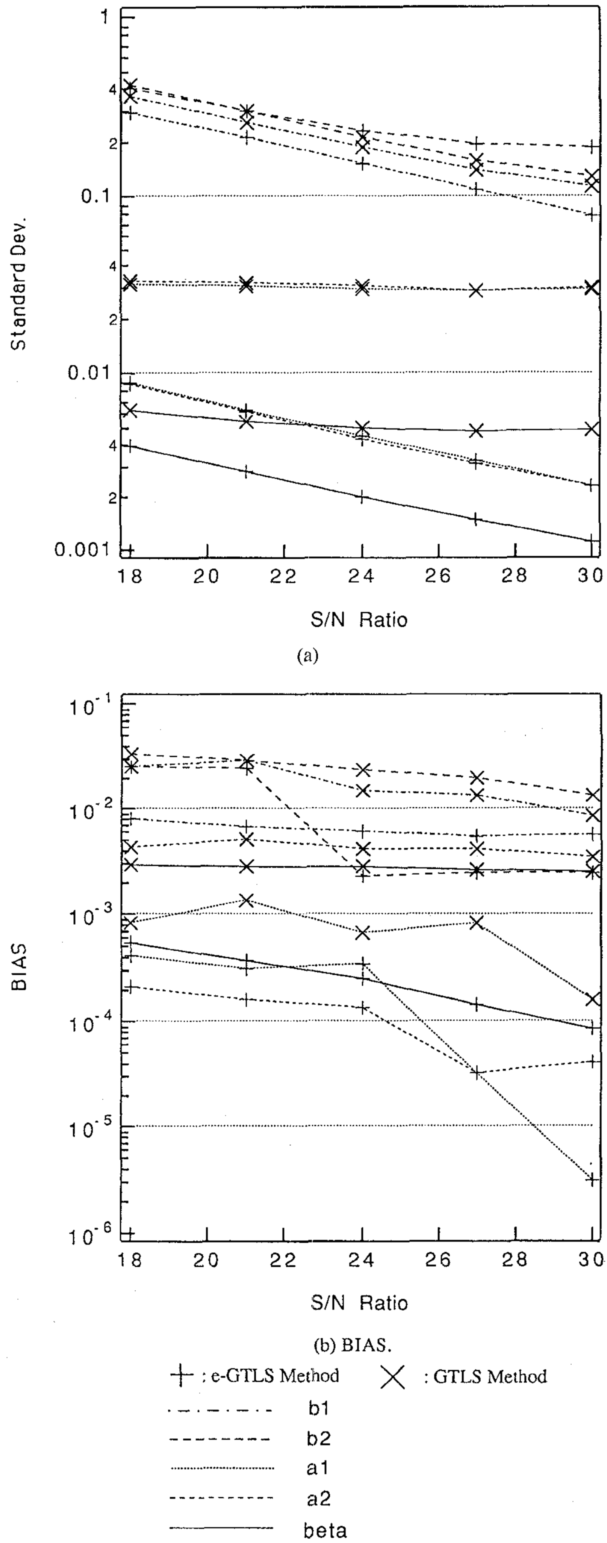

Fig. 2. Comparison of standard deviation and bias between e-GTLS method and GTLS method: (a) Standard deviation; (b) bias + : e-GTLS method × : GTLS method —·—·—·—·— : b_1 — — — — — : b_2 ·················· : a_1 — — — — — — — — : a_2 ———————— : c.

VI. Conclusion

In this correspondence, we have presented a new method to estimate the parameters of bilinear systems. Considering the structure of the data matrix of the bilinear system equations, we extended the GTLS problem and defined the e-GTLS problem. We proposed the solving method of the e-GTLS problem using the Newton–Raphson method. Through computer simulations, we showed that the e-GTLS method is an unbiased estimator of the bilinear system parameters and that the performances of the e-GTLS method are better than those of the GTLS method.

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Localizing Vapor-Emitting Sources by Moving Sensors

Boaz Porat and Arye Nehorai

Abstract—**The authors have recently explored the use of novel concentration sensors for detecting and localizing vapor-emitting sources. In the present correspondence we propose to replace stationary sensors by moving sensors, thus gaining the following two advantages. 1) A single moving sensor can accomplish the task of an array of stationary sensors by exploiting spatial and temporal diversity. 2) The sensor motion can be planned in real time to optimize localization performance.**

I. Introduction

Recently, there has been considerable progress in the development of sensors for low concentration vapors. Sensors of this kind are potentially useful in applications such as explosive detection, drug detection, sensing leakage of hazardous chemicals, pollution sensing, and environmental studies.

In a previous paper we have examined the problem of detecting and localizing vapor-emitting sources by stationary sensor arrays [1]. Based on the diffusion equation, we developed models for spatial distributions of vapors in various environments, and used them to develop measurement models. We then derived detection and estimation algorithms, analyzed their performance, and conducted some simulation studies.

This correspondence looks at the problem of vapor source localization when the sensors are permitted to move. Such a sensor can achieve spatial diversity by taking measurements at different locations at different times. In principle, a single sensor can thus perform tasks that would need an array of stationary sensors (for example, a minimum of five stationary sensors was shown in [1] to be necessary for localization in three dimensions). As a possible scenario, consider a vehicle, equipped with vapor sensors, moving in an open area where hazardous materials may be present, and whose task is to detect and localize such materials. The vehicle may be driven by a protected person or be remote controlled. Assume that the vehicle's path can be selected in real time subject to speed and maneuvering limitations and possible geometric constraints (e.g., obstacles). The goal is to use the freedom in the vehicle's path to optimize localization under a properly defined criterion. In a more general setting, we can consider several vehicles performing a coordinated search. One can liken this scenario to a search by trained dogs, since dogs use their sensing ability and freedom of motion in a similar manner. The great difference is, of course, that dogs function individually and do not share their respective information, while an automated system of the type we propose optimizes performance based on all available information.

The optimality criterion used in this correspondence for planning the sensors' motion is to reduce as much as possible the expected location estimation error after the next measurement is taken. This is accomplished by computing the gradients of the Cramér–Rao bound (CRB) on the location error with respect to the sensors' coordinates, and moving the sensors opposite to the corresponding gradient directions. Since location estimation is performed using maximum likelihood, the actual accuracy is expected to be close to the CRB, so by minimizing the CRB we can hope to minimize the actual estimation error.

Passive localization through spatial and temporal diversity has been treated by many authors in various contexts. Worthwhile noting are the fundamental works of Moura [2]–[4]; relevant to the present work are [5] and [6], which deal with trajectory optimization based on the Fisher information and the CRB, respectively.

This correspondence describes the mathematical details of the proposed localization method and illustrates it by an example.

II. Mathematical Modeling

In this section we describe the physical and measurement models. Here we consider only the simplest case—that of a point source in an infinite volume, emitting vapors at a constant rate. More general models are developed in [1].

Let $c(\vec{r}, t)$ be the diffusing substance concentration at a point $\vec{r} = (x, y, z)$ and time t, in Kg/m^3. The classical diffusion equation for a source-free volume and constant diffusivity is

$$\frac{\partial c}{\partial t} = \kappa \nabla^2 c \tag{1}$$

where κ is the *diffusivity*, in units of m^2/s.

Consider a point source at $\vec{\rho} = [\chi, \upsilon, \zeta]$ in an infinite medium, releasing a diffusing substance at a constant rate of μ Kg/s, starting

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