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A SWITCHING ADAPTIVE VSS/FUZZY CONTROL

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Abstract

In the field of adaptive fuzzy control, there has been a severe deficiency by assuming the premise variables will usually stay within the universe of discourse in the derivation of stability of the adaptive control system. To overcome this deficiency, we develop a switching adaptive control scheme using only essential qualitative information of the plant to attain asymptotical stability of the adaptive control system for a typical first-order nonlinear system without imposing the mentioned severe assumption. The switching adaptive control system consists of an adaptive VSS controller for coarse control, an adaptive fuzzy controller for fine control, and a hysteresis switching mechanism. An adaptive VSS control scheme is proposed to force the state to enter the universe of discourse in finite time. While the premise variable is within the universe of discourse, an adaptive fuzzy control is proposed to learn the capability to stabilize the plant. At the boundary of the universe of discourse, a hysteresis switching scheme between the two controllers will be proposed. We show that after finite times of switching, the premise variables of the fuzzy system will remain within the universe of discourse and stability of the closed-loop system can be attained by applying Lyapunov direct method.

Keywords:

Adaptive fuzzy control, Adaptive VSS control , Hysteresis switching mechanism

1. Introduction

Recently, fuzzy adaptive control has become a popular method for nonlinear adaptive control. However, in most of the related literature [1]-[14], it is assumed that the trajectories of the premise variables are located in the universe of discourse of the fuzzy systems so that the universal approximation property can be invoked in the analysis of system behavior to guarantee system stability. This assumption is infeasible in an adaptive control system since the system must pass through a learning period in which the system behavior can not be known in advance. To attack the above mentioned problem, the so called semi-global stability is induced in the field of adaptive fuzzy control. Typically with the semi-global stability as done in [15], the universe of discourse depends on unknown system parameters and the initial states. One deficiency is that we can not explicitly define the universe of discourse in advance so as to ensure the premise variables will remain in the corresponding universes of discourse.

In this study, to attain global stability in stead of semi-global stability, we propose a fuzzy adaptive control algorithm based on switch control theory. When the premise variables are outside the compact universe of discourse, an adaptive sliding mode control algorithm is used to force the trajectories of the premise variables to enter that compact set. On the other hand, when the premise variables are confined in the universe of discourse, a fuzzy adaptive control algorithm is used to attain global stability. However, during the learning period of the adaptive fuzzy controller, switching between the above two control algorithms may happen at the boundary of the universe of discourse. In this study, we shall propose a hysterisis switching adaptive control algorithm and study the stability issue of the closed-loop system.

The remainder of this paper is organized as follows. Problem formulation and the definition of the hysteresis switching adaptive control are discussed in Section 2. The adaptive VSS controller is proposed and analyzed in Section 3. Then, the considered adaptive fuzzy control is presented in Section 4. Analysis of the switching behaviour of the control system is made in Section 5. Finally, conclusions and discussions are given in Section 6.

Notations and Definitions

For a vector $x = [\ x_1\ \ x_2\ \ \cdots\ \ x_n\ ]^T$, the associated swap operation is defined as swap(x)=$[\ x_n\ \ x_{n-1}\ \ \cdots\ \ x_1\ ]^T$. For a vector x, we write $x \geq 0$ if every entry of x is greater than or equal to zero.

2. Problem formulation of the hysteresis switching adaptive control

We consider the plant

$$\dot{x} = f(x) + u \tag{1}$$

where $f(x)$ is a scalar nonlinear function of the scalar variable

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x and $u \in R^1$ is the system input. For the nonlinear function $f(x)$, we make the following assumptions.

Assumption 1 *$f(x)$ is a continuous function for $x \in (-\infty, \infty)$ and admits its maximum $f_{\max}$ on the compact (closed and bounded) connected set Ω_x with*

$$f_{\max} = \max_{x \in \Omega_x} |f(x)| \tag{2}$$

where Ω_x is the universe of discourse of the fuzzy system to be defined later and $f_{\max}$ is an unknown positive number.

Assumption 2 *The function $f(x)$ satisfies*

$$\left| \frac{df(x)}{dx} \right| \leq \kappa_f \tag{3}$$

for $x \in \Omega_x$ where κ_f is an unknown positive number.

Assumption 3 *For $x \notin \Omega_x$, there is an upper bound $\psi(x)$ of $f(x)$ satisfying*

$$|f(x)| \leq c_1^* |x| + c_2^* |x|^2 = \psi(x) \text{ for } x \notin \Omega_x \tag{4}$$

where c_1^* and c_2^* are unknown positive parameters.

Assumption 4 *We assume that for $x \in \Omega_x$*

$$\begin{cases} xf(x) > 0, & \text{if } x \neq 0 \\ f(x) = 0, & \text{if } x = 0 \end{cases} \tag{5}$$

and $f(x)$ is a convex function. Also to simplify system analysis, we shall assume $f(x)$ is an odd function, i.e.,

$$f(-x) = -f(x) \tag{6}$$

In this section, we shall consider the case that the structure of the nonlinear function $f(x)$ is unknown and a fuzzy approximator $F(x|\theta)$ will be used to approximate $f(x)$ in the universe of discourse $\Omega_x = [-1, 1]$ where x is the only premise variable. Basically, when the state trajectory $x(t)$ is outside the universe of discourse Ω_x, by utilizing the structure information of $f(x)$ given in (4) in Assumption 3, we shall develop an adaptive VSS control $u_{VSS}(t)$ to force the state trajectory entering Ω_x. On the other hand, if the state trajectory $x(t)$ is staying within Ω_x, an adaptive fuzzy control $u_{fuzzy}(t)$ will be applied to further ensure that the system will be ultimately asymptotically stable. Since switching between these two control laws with infinite frequency at the boundary of the region Ω_x may happen, we shall use a hysteresis switching control as described in the following to avoid this problem. Let h, with $0 < h < 1$, be the hysteresis size and define the hysteresis zone Ω_h as $\Omega_h = \{x | 1 - h \leq |x| \leq 1\}$. The hysteresis switching control structure, as shown in Fig. 1, is described as follows. At $t = 0$, the control structure is defined as

$$u(0) = \begin{cases} u_{VSS}(0), & \text{if } |x(t)| > 1 - h \\ u_{fuzzy}(0), & \text{if } |x(t)| \leq 1 - h \end{cases} \tag{7}$$

For $t > 0$, while $x(t)$ is outside the hysteresis zone Ω_h, the control input $u(t)$ is defined as

$$u(t) = \begin{cases} u_{VSS}(t), & \text{if } |x(t)| > 1 \text{ (i.e. } x(t) \notin \Omega_x) \\ u_{fuzzy}(t), & \text{if } |x(t)| < 1 - h \end{cases} \tag{8}$$

and on the contrary, while $x(t)$ is inside the hysteresis zone Ω_h, $u(t)$ is defined as

$$u(t) = \begin{cases} u_{VSS}(t), & \text{if } u(t_-) = u_{VSS}(t_-) \\ u_{fuzzy}(t), & \text{if } u(t_-) = u_{fuzzy}(t_-) \end{cases} \tag{9}$$

We note that while applying the adaptive VSS control law $u_{VSS}(t)$, the tuning parameters in the adaptive fuzzy control will be kept invariant. On the other hand, while applying the adaptive fuzzy control law $u_{fuzzy}(t)$, the tuning parameters in the adaptive VSS control will be frozen.

The problem to be attacked is formulated as follows. For the plant in (1) under **Assumption** 1- **Assumption** 4, we shall construct an adaptive VSS control, an adaptive fuzzy control, and a switching mechanism between the previous two controllers so that the tuning parameters in the two adaptive controllers are bounded and $x(t) \to 0$ as $t \to \infty$.

3. Design and analysis of the adaptive VSS control

In this section, an adaptive VSS control will be proposed and the system behavior will be analyzed. Recall that the system function $f(x)$ has an upper bound $\psi(x)$ with the structural information indicated in (4) for $x \notin \Omega_x$. Here, we shall develop an adaptive VSS control $u_{VSS}(t)$ to force the state trajectory entering Ω_x when the state trajectory is outside the region Ω_x. To attain this goal, we shall construct estimates $\hat{c}_1$ and $\hat{c}_2$ of c_1^* and c_2^*, respectively, so that the following inequality

$$|f(x)| \leq \hat{c}_1 |x| + \hat{c}_2 |x|^2 \text{ for } x \notin \Omega_x$$

can be attained. The tuning laws of $\hat{c}_1$ and $\hat{c}_2$ are given as

$$\dot{\hat{c}}_1 = \Gamma_1 |x|^2, \; \hat{c}_1(0) = 0 \tag{10}$$

$$\dot{\hat{c}}_2 = \Gamma_1 |x|^3, \; \hat{c}_2(0) = 0 \tag{11}$$

Based on the estimates $\hat{c}_1$ and $\hat{c}_2$, the proposed adaptive VSS control law will be defined as

$$u_{VSS} = -(\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r |x|)\text{sign}(x) \tag{12}$$

where r is a given positive constant.

Lemma 1 *Consider the adaptive VSS control system defined by (1), (12), (10), and (11). The trajectories of $x(t)$, $\hat{c}_1(t)$, and $\hat{c}_2(t)$ are bounded over the time interval (t_0, ∞) where t_0 is an arbitrary initial time, and $x(t)$ converges to the origin. Moreover, there is a finite time t_1 such that $x(t_1) = 1 - h$ if*

$x(t_0) > 1-h$ or $x(t_1) = -(1-h)$ if $x(t_0) < -(1-h)$ where t_1 is a time instant with $t_1 \leq t_0 + T_1$ and

$$T_1 = \frac{V_a(t_0)}{r(1-h)^2} \tag{13}$$

where $\tilde{c}_1 = \hat{c}_1 - c_1^*, \tilde{c}_2 = \hat{c}_2 - c_2^*$, and $V_a = \frac{1}{2}x^2 + \frac{1}{2}\Gamma_1^{-1}\tilde{c}_1^2 + \frac{1}{2}\Gamma_1^{-1}\tilde{c}_2^2$.

4. Design of the adaptive fuzzy control

The rule base of the T-S fuzzy system $F(x|\theta)$ is defined as: for $1 \leq l \leq L$,

$$\text{Rule } l: \text{If } x \text{ is } F_l, \text{then } y = \theta_l.$$

where F_l is the fuzzy set with membership function $\mu_{F_l}(x)$ and θ_l is the value specified in the antecedent part of the $l-$th rule. The number L, which is the total number of rules, will be chosen as an odd number. A typical case is shown in Fig. 2 so that the set of IF-THEN rules is complete, consistent, and continuous [16]. Based on the above rule base, the T-S fuzzy system, consisting of the singleton fuzzyifier, the product inference engine, and the center average defuzzifier [16], can be expressed as

$$F(x,\theta) = \xi^T(x)\theta \tag{14}$$

where

$$\begin{aligned} \theta &= [\theta_1, ..., \theta_L]^T, \\ \xi_l(x) &= \frac{\mu_{F_l}(x)}{\sum_i^L \mu_{F_i}(x)}, \\ \xi(x) &= [\xi_1(x), ..., \xi_L(x)]^T \end{aligned} \tag{15}$$

From Fig. 2, we can observe that

$$\sum_{i=1}^{L} \mu_{F_i}(x) = 1,$$

for any $x \in \Omega_x$ and

$$\xi(x) = \left[\mu_{F_1}(x), ..., \mu_{F_L}(x)\right]^T \tag{16}$$

From the triangular membership function shown in Fig. 2, we have, for any $x \in \Omega_x$,

$$\|\xi(x)\|^2 = \mu_{F_i}^2(x) + \mu_{F_{i+1}}^2(x) \tag{17}$$

for $1 \leq i \leq L-1$ and

$$\mu_{F_i}(x) + \mu_{F_{i+1}}(x) = 1 \tag{18}$$

From (17) and (18), it is obvious that

$$\frac{1}{2} \leq \|\xi(x)\|^2 \leq 1 \tag{19}$$

Note that since $\Omega_x = [-1, 1]$ is symmetric with respect to the origin, the rule base will be chosen to symmetric in the sense that

$$\xi(-x) = \bar{\xi}(x) \tag{20}$$

where $\bar{\xi}(x) = \text{swap}(\xi(x))$.

Now let A_i, for $1 \leq i \leq L$, be the support of the membership function $\mu_{F_i}(x)$, i.e.,

$$A_i = \left\{x \in \Omega_x \left|\mu_{F_i}(x) > 0\right.\right\}$$

Denote d_i as the center of the membership function $\mu_{F_i}(x)$ for $1 \leq i \leq L$ and γ_i as the point such that $\mu_{F_i}(\gamma_i) = \mu_{F_{i+1}}(\gamma_i)$ for $1 \leq i \leq L-1$. For the convenience of further analysis, now partition the universe of discourse Ω_x as $\Omega_x = \cup_{i=1}^{2L-2}\Omega_{x,i}$ where

$$\begin{aligned} \Omega_{x,2i-1} &= [d_i, \gamma_i), \text{ for } 1 \leq i \leq \frac{L-1}{2} \\ \Omega_{x,2i} &= [\gamma_i, d_{i+1}), \text{ for } 1 \leq i \leq \frac{L-1}{2} - 1 \\ \Omega_{x,2i-1} &= (d_i, \gamma_i], \text{ for } \frac{L+3}{2} \leq i \leq L-1 \\ \Omega_{x,2i} &= (\gamma_i, d_{i+1}], \text{ for } \frac{L+1}{2} \leq i \leq L-1 \end{aligned}$$

and

$$\Omega_{x,L-1} = [\gamma_{\frac{L-1}{2}}, d_{\frac{L+1}{2}}), \Omega_{x,L} = [d_{\frac{L+1}{2}}, \gamma_{\frac{L+1}{2}}]$$

We make a final note that the fuzzy system $F(x,\theta)$ in (14) admits a linear approximator structure with respect to the parameter vector θ and

$$F(x,\theta_1) - F(x,\theta_2) = \xi^T(x)(\theta_1 - \theta_2) \tag{21}$$

For $x \in \Omega_x$, we can approximate the system function $f(x)$ by the fuzzy system $F(x,\theta) = \xi^T(x)\theta$ so that

$$\min_\theta \|f(x) - F(x,\theta)\|_\infty = W$$

for some $W > 0$ due to the universal approximation property of the constructed fuzzy system [16] and the infinite norm is defined as

$$\|f(x)\|_\infty = \sup_{x\in\Omega_x} |f(x)|$$

Let's denote a best fitted parameter θ^* as

$$\theta^* \in \arg\min_\theta \|f(x) - F(x,\theta)\|_\infty$$

For $x \in \Omega_x$, we then have

$$|f(x) - F(x,\theta^*)| \leq W \tag{22}$$

Finally, with respect to the membership functions shown in Fig. 2, the hysteresis size h defined in (8) will be chosen such that

$$0 < h \leq \frac{2}{2(L-1)} - \varepsilon_h \tag{23}$$

where ε_h is a small positive constant.

Since the adaptive VSS controller also has the ability to ensure that $x(t)$ is bounded, we shall use the following simplified adaptive fuzzy control law

$$\dot{\hat{\theta}} = \Gamma_2 \xi x \quad (24)$$

$$u(t) = -\hat{\theta}^T \xi(x) \quad (25)$$

5. Analysis of the hysteresis switching adaptive control

Based on the adaptive VSS control and the adaptive fuzzy control, we shall study the proposed hysteresis switching adaptive control defined as in (8) and (9). The adaptive VSS control law is defined as

$$\begin{cases} \dot{\hat{c}}_1 = \Gamma_1 |x|^2,\ c_1(0) = 0, \\ \dot{\hat{c}}_2 = \Gamma_1 |x|^3,\ c_2(0) = 0, \\ \dot{\hat{\theta}} = 0, \\ u_{VSS} = -(\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r|x|)\mathrm{sign}(x) \end{cases} \quad (26)$$

which results in a closed-loop dynamics defined as

$$\begin{cases} \dot{x} = f(x) - (\hat{c}_1 |x| + \hat{c}_2 |x|^2 + r|x|)\mathrm{sign}(x), \\ \dot{\hat{c}}_1 = \Gamma_1 |x|^2,\ c_1(0) = 0, \\ \dot{\hat{c}}_2 = \Gamma_1 |x|^3,\ c_2(0) = 0 \end{cases} \quad (27)$$

On the other hand, by letting $\Gamma_2 = I$ in (24), the adaptive fuzzy control law is defined as

$$\begin{cases} \dot{\hat{c}}_1 = 0 \\ \dot{\hat{c}}_2 = 0 \\ \dot{\hat{\theta}} = \xi x,\ \hat{\theta}(0) = 0 \\ u_{fuzzy} = -\hat{\theta}^T \xi(x) \end{cases} \quad (28)$$

which results in a closed-loop dynamics defined as

$$\begin{cases} \dot{x} = f(x) - \hat{\theta}^T \xi(x) \\ \dot{\hat{\theta}} = \xi x \end{cases} \quad (29)$$

Note that the online fuzzy approximator is given by

$$F(x|\hat{\theta}) = \hat{\theta}^T \xi(x)$$

According to the tuning law of $\hat{\theta}$ defined in (28) and the definition of the vector $\xi(x)$ in (16), some further properties of $\hat{\theta}$ can be discovered.

Lemma 2 *Due to the structure of the fuzzy system and the tuning law of $\hat{\theta}(t)$ defined in (28), we have the following results. (i) If $A_i \subset [0,1]$, then $\hat{\theta}_i(t) \geq 0$ and $\hat{\theta}_i(t)$ is a monotone increasing function of time. On the other hand, if $A_i \subset [-1,0]$, then $\hat{\theta}_i(t) \leq 0$ and $\hat{\theta}_i(t)$ is a monotone decreasing function of time. (ii) If $A_i \subset [0,1]$ and there is time t_0 such that $x(t_0) \in A_i$, then $\hat{\theta}_i(t) > 0$ for $t \geq t_0$. Similarly, if $A_i \subset [-1,0]$ and there is time t_0 such that $x(t_0) \in A_i$, then $\hat{\theta}_i(t) < 0$ for $t \geq t_0$. (iii) If $x(t) \in [1-h,1]$, then $\hat{\theta}^T(t)\xi(x(t)) \geq 0$. On the other hand, if $x(t) \in [-1,-(1-h)]$, then $\hat{\theta}^T(t)\xi(x(t)) \leq 0$.*

Lemma 3 *The response $x(t)$ of the hysteresis switching adaptive control defined as in (8), (9), (26), and (28) is symmetric in the sense that if $\{x(t),\hat{\theta}(t)\}$ and $\{y(t),\check{\theta}(t)\}$ are the system responses corresponding to the initial states $x(0)$ and $-x(0)$, respectively, then $y(t) = -x(t)$ and $\check{\theta}(t) = -\bar{\theta}(t)$ where $\bar{\theta}(t) = swap(\hat{\theta}(t))$.*

Due to the symmetry of the responses of the switching control system as described in Lemma 3, we shall assume $x(0) > 0$ in the analysis of the dynamics of the switching control system. If $x(0) > 1-h$, then the adaptive VSS control law in (26) will ensure that there is a finite time t_1 such that $x(t_1) = 1-h$ and $x(t) > 1-h$ for $t \in [0,t_1)$. At $t = t_1$, the adaptive fuzzy control law in (28) will then be applied.

5.1 Analysis of switching behavior

In this section, we shall focus on discussing switching behavior of the switching control law at the boundaries of the hysteresis zone $\Omega_h = [1-h,1] \cup [-1,-(1-h)]$. For further analysis, we shall need some definitions.

Definition 1 *We say that* continuous switching of N times at the positive boundary $x = 1-h$ *happens at $t = t_i$ for $1 \leq i \leq N$ with $t_i < t_{i+1}$ if there are finite time instants t_{N+1} and $\bar{t}_i$ with $t_i < \bar{t}_i < t_{i+1}$ for $1 \leq i \leq N$ such that (i) the adaptive VSS control is applied in (t_0,t_1) for some $t_0 < t_1$, (ii) the adaptive fuzzy control law is used for $t \in [t_i,\bar{t}_i]$ with $x(t) \in [1-h,1]$ for $1 \leq i \leq N$, (iii) the adaptive VSS control law is applied within the interval $(\bar{t}_i,t_{i+1})$ for $1 \leq i \leq N$, and (iv) the adaptive fuzzy control law is used after $t = t_{N+1}$ such that there is no time instant $\bar{t}_{N+1}$ such that $\{x(t)\,|t_{N+1} \leq t \leq \bar{t}_{N+1}\} \subset [1-h,1]$, $x(\bar{t}_{N+1}) = 1$, and the adaptive VSS control law is applied after $t = \bar{t}_{N+1}$.*

Switching at the negative boundary $x = -(1-h)$ is defined similarly.

Definition 2 *We say that a switch at the positive boundary $x = 1-h$ (or at the negative boundary $x = -(1-h)$) happens N times at $t = t_i$ for $1 \leq i \leq N$ with $t_i < t_{i+1}$ if a switching at the positive boundary $x = 1-h$ (or at the negative boundary $x = -(1-h)$) happens at $t = t_i$ for $1 \leq i \leq N$. Similarly, as $N \to \infty$, we shall say a switch at the positive boundary $x = 1-h$ (or at the negative boundary $x = -(1-h)$) happens infinite times at $t = t_i$ for $1 \leq i < \infty$ with $t_i < t_{i+1}$ or a switch at the positive boundary $x = 1-h$*

(or at the negative boundary $x = -(1-h)$) happens infinite times since $t = t_1$.

Definition 3 *We say that there is no switching happened at $t = t_1$ at the boundary $x = 1-h$ (or at the boundary $x = -(1-h)$) if (i) the adaptive VSS control is applied in (t_0, t_1) for some $t_0 < t_1$, (ii) the adaptive fuzzy control law is applied after $t = t_1$, and (iii) neither continuous switching of finite times nor continuous switching of infinite times happens at the boundary $x = 1-h$ (or at the boundary $x = -(1-h)$) since $t = t_1$.*

Since the system response is symmetric as stated in Lemma 3, we shall focus on analyzing the switching property at the boundary $x = 1-h$.

Lemma 4 *Assume that there is no switching happened at the boundary $x = 1-h$ at $t = t_1$. Then it is impossible that $\{x(t)\,|t \geq t_1\} \subset [1-h, 1]$.*

In the following, we shall first investigate the learning capability of the adaptive tuning law when a switching at the boundary happens.

Lemma 5 *Suppose that a switching at the boundary $x = 1-h$ happens at $t = t_1$ and the adaptive fuzzy control is applied during the interval $[t_1, \bar{t}_1]$. Let t_2 be the next time when the adaptive fuzzy control law is applied. Then, the time difference $\bar{t}_1 - t_1$ can be estimated as*

$$\bar{t}_1 - t_1 \geq \frac{h}{f_{\max}} \tag{30}$$

Moreover, we have

$$F(x(t_2), \hat{\theta}(t_2)) \geq F(x(t_1), \hat{\theta}(t_1)) + \Delta_F \tag{31}$$

where

$$\Delta_F = \frac{1}{2}\frac{h(1-h)}{f_{\max}}$$

Similarly, if a switching at the boundary $x = -(1-h)$ happens at $t = t_1$, then we have

$$F(x(t_2), \hat{\theta}(t_2)) \leq F(x(t_1), \hat{\theta}(t_1)) - \Delta_F \tag{32}$$

Lemma 6 *It is impossible that a continuous switching of infinite times at the positive boundary $x = 1-h$ happens since a finite time $t = t_1$.*

Lemma 7 *It is impossible that a switching at the positive boundary $x = 1-h$ (or at the negative boundary $x = -(1-h)$) happens infinite times since some finite time $t = t_1$.*

Lemma 8 *Under the specified switching mechanism in (7)-(9) including the adaptive VSS control law in (26) and the adaptive fuzzy control law in (28), we have the following results.*

(i) there is a finite time t_{f_0} such that $x(t) \in \Omega_x$ and the adaptive fuzzy control is used for $t \geq t_{f_0}$ and

(ii) the parameters $\hat{c}_1(t)$ and $\hat{c}_2(t)$ in the adaptive VSS control are bounded for $t \in [0, \infty)$.

5.2 Convergence analysis

Following from Lemma 8, the adaptive fuzzy control is applied for $t \geq t_{f_0}$ where t_{f_0} is a finite time and $x(t) \in \Omega_x$ for $t \geq t_{f_0}$. Recall that $\{\Omega_{x,i}\}_{i=1}^{2L-2}$ is a partition of the universe of discourse Ω_x. Particularly, let $\Omega_{x,0} = \Omega_{x,L-1} \cup \Omega_{x,L} = \left[\gamma_{\frac{L-1}{2}}, \gamma_{\frac{L+1}{2}}\right]$ and thus $0 \in \Omega_{x,0}$. Now define sets S_i for $1 \leq i \leq 2L-2$ such that

$$S_i = \{t\,|x(t) \in \Omega_{x,i},\ t \geq t_{f_0}\}$$

and denote the time length $\sigma(S_i)$ as the Borel measure of the set S_i.

Lemma 9 *The the time length $\sigma(S_i)$ is finite for $1 \leq i \leq L-3$ and $L+2 \leq i \leq 2L-2$. Therefore there is a finite time t_f with $t_f \geq t_{f_0}$ such that $x(t) \in \Omega_{x,0} = \cup_{i=L-2}^{L+1}\Omega_{x,i}$ and the adaptive control is used for $t \geq t_f$.*

Theorem 1 *Assume that $f(x)$ is a convex function in $[0,1]$ and a concave function in $[-1,0]$. Then $\lim_{t\to\infty} x(t) = 0$ and $\hat{\theta}(t)$ is bounded over $[0,\infty)$.*

6. Conclusions and Discussions

In the field of adaptive fuzzy control, there has been a severe deficiency by assuming the premise variables will usually stay within the universe of discourse in derivation of stability of the adaptive control system. To overcome this deficiency, we have developed a switching adaptive control scheme to attain asymptotical stability of the adaptive control system for a typical first-order nonlinear system without imposing the mentioned severe assumption.

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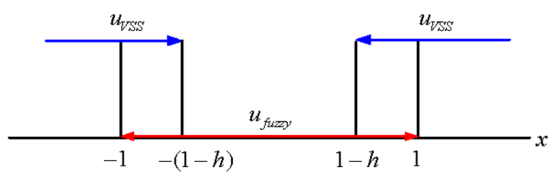

Figure 1. Illustration of the hysteresis switching control.

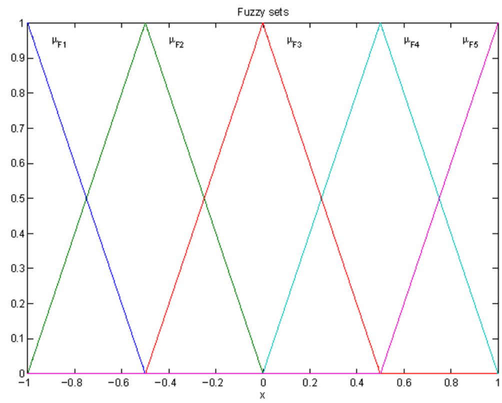

Figure 2. A typical case of the fuzzy sets in the rule base.

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