ROBUST PID CONTROLLER AUTOTUNING WITH A PHASE SHAPER [1]

YangQuan Chen *, **Kevin L. Moore** *,
Blas M. Vinagre **, and **Igor Podlubny** ***

* *Center for Self-Organizing and Intelligent Systems (CSOIS), Dept. of Electrical and Computer Engineering, Utah State University, Logan, UT84322-4160, USA*
** *Dept. of Electronic and Electromechanical Engineering, University of Extremadura, 06071-Badajoz, Spain*
*** *Dept. of Informatics and Process Control, Technical University of Kosice, 042 00 Kosice, Slovak Republic*

Abstract: In our previous work (Chen et al., 2003a), a robust PID autotuning method was proposed by using the idea of "flat phase", i.e., the phase derivative w.r.t. the frequency is zero at a given frequency called the "tangent frequency" so that the closed-loop system is robust to gain variations and the step responses exhibit an iso-damping property. However, the width of the achieved phase flatness region is hard to adjust. In this paper, we propose a phase shaping idea to make the width of the phase flatness region adjustable. With a suitable phase shaper, we are able to determine the width of the flat phase region so as to make the whole design procedure of a robust PID controller much easier and the system performance can be enhanced more significantly. The plant gain and phase at the desired frequency, which are identified by several relay feedback tests in an iterative way, are used to estimate the derivatives of the amplitude and phase of the plant with respect to the frequency at the same frequency point by the well known Bode's integral relationship. Then, these derivatives are used to design the proposed robust PID controller. The phase shaper, based on the idea of FOC (Fractional Order Calculus), is actually a fractional order integrator or differentiator. In this paper, no plant model is assumed during the controller design. Only several relay tests and calculations are needed. Simulation examples illustrate the effectiveness and the simplicity of the proposed method with an iso-damping property.

Keywords: Phase shaping, robust PID tuning, iso-damping, flat phase region, relay feedback tuning, Bode's integrals, fractional order calculus.

[1] For submission to IFAC FDA04. Jan. 2004. Corresponding author: Dr YangQuan Chen. E-mail: `yqchen@ece.usu.edu` or `yqchen@ieee.org`; Tel. 01-435-7970148; Fax: 01-435-7973054. URL: `http://www.csois.usu.edu/people/yqchen`. This work is supported in part by the New Faculty Research Grant of Utah State University. This project has also been funded in part by the National Academy of Sciences under the Collaboration in Basic Science and Engineering Program/Twinning Program supported by Contract No. INT-0002341 from the National Science Foundation. The contents of this publication do not necessarily reflect the views or policies of the National Academy of Sciences or the National Science Foundation, nor does mention of trade names, commercial products or organizations imply

1. INTRODUCTION

There is a magic number α, the ratio between the integral time T_i and the derivative time T_d, in the modified Ziegler-Nichols method for PID controller design. This magic number α is chosen as a constant, i.e., $T_i = \alpha T_d$, in order to obtain a unique solution of PID control parameter setting. The control performances are heavily influenced by the choice of α as observed in (Tan et al., 1996). Recently, the role of α has drawn much attention from researchers, e.g., (Wallén et al., 2002; Panagopoulos et al., 1999; Kristiansson and Lennartsson, 1999). For the Ziegler-Nichols PID tuning method, α is generally assigned as 4 (Astrom and Hagglund, 1995). Wallén, Åström and Hägglund proposed that the tradeoff between the practical implementation and the system performance is the major reason for choosing the ratio between T_i and T_d as 4 (Wallén et al., 2002).

In our recent work (Chen et al., 2003a), a new relationship between T_i and T_d was given instead of the equation $T_i = 4T_d$ proposed in the modified Ziegler-Nichols method (Astrom and Hagglund, 1995; Hang et al., 1991). It was proposed in (Chen et al., 2003a) to add an additional condition called the "flat phase condition" that the phase Bode plot at a specified frequency w_c where the sensitivity circle tangentially touches the Nyquist curve is locally flat which implies that the system will be more robust to gain variations. In other words, if the gain increases or decreases a certain percentage, the gain margin will remain unchanged. Therefore, in this case, the step responses under various gains changing around the nominal gain will exhibit an iso-damping property, i.e., the overshoots of step responses will be almost the same. As presented in (Chen et al., 2003a), this additional condition can be expressed as $\frac{\mathrm{d}\angle G(s)}{\mathrm{d}s}|_{s=jw_c} = 0$ which can be equivalently expressed as

$$\angle \frac{\mathrm{d}G(s)}{\mathrm{d}s}|_{s=jw_c} = \angle G(s)|_{s=jw_c} \qquad (1)$$

where w_c is the frequency at the tangent point as mentioned in the above, called "tangent frequency" (Chen et al., 2003a). In (1),

$$G(s) = K(s)P(s) \qquad (2)$$

is the transfer function of the open loop system including the controller $K(s)$ and the plant $P(s)$ and the PID controller can be expressed as

$$K(s) = K_p(1 + \frac{1}{T_i s} + T_d s). \qquad (3)$$

endorsement by the National Academy of Sciences or the National Science Foundation.

PID controller designed by the "flat phase" tuning method proposed in (Chen et al., 2003a) can exhibit a good iso-damping performance for some classes of plants. There are three important constants in this new tuning method, namely, the "tangent phase" Φ_m, the "tangent frequency w_c and the "gain adjustment ratio" β which are required to design a PID controller with iso-damping property. However, the "flat phase" tuning method can not determine the width of the flat phase region. Therefore, the limited width of the flat phase makes the sensitivity circle very difficult to be be tangentially touched by the Nyquist curve on the flat phase. Consequently, it is hard to select Φ_m, w_c and β properly, if not impossible.

The main contribution of this paper is the use of a modified tuning method which gives a PID controller $K(s)$ and a phase shaper $C(s)$ both to achieve the condition (1) and to determine the width of the flat phase region. Comparing to the tuning method proposed in (Chen et al., 2003a), in the modified tuning method, the PID controller does not need to fulfil all the phase requirement by itself alone. The PID controller $K(s)$ is used to just determine the upper limit frequency of the flat phase region. After that, a phase shaper, which comes from the idea of the approximate fractional order differentiator or integrator (Manabe, 1961; Oustaloup et al., 1996; Podlubny, 1999; Raynaud and Zergaïnoh, 2000; Vinagre and Chen, 2002), is applied to achieve the lower limit frequency and also make the flat phase exactly match the phase requirement. The approximation method for the fractional order calculus operators used here is the continued fraction expansion (CFE) of the Tustin operator (Chen and Moore, 2002). Clearly, if the width of the flat phase region can be determined, it is much easier to design a robust PID controller which can ensure that the sensitivity circle tangentially touches the Nyquist curve on the local flat phase region.

The remaining parts of this paper are organized as follows. In Sec. 2, a modified flat phase tuning method is proposed. The phase shaper idea is discussed in detail in Sec. 3. In Sec. 4, the whole design procedures of the PID controller and the phase shaper are summarized. Some simulation examples are presented in Sec. 5 for illustrations. Finally, Sec. 6 concludes this paper with some remarks on further investigations.

2. A MODIFIED FLAT PHASE TUNING METHOD

As discussed in (Chen et al., 2003a), for the PID controller tuning, we concentrate on the frequency range around the "tangent frequency". If the "tangent phase" Φ_m and the "tangent frequency" w_c

are pre-specified, $\angle P(jw_c)$, $|P(jw_c)|$ and $s_p(w_c)$ can be obtained, where $\angle P(jw_c)$ is the phase and $|P(jw_c)|$ is the gain of the plant at the specific frequency w_c; $s_p(w_c)$ represents the derivative of the phase of the open loop system, which can be approximated by Bode's Integral (Karimi et al., 2002b,a) as follows:

$$s_p(w_c) = w_c \frac{\mathrm{d}\angle P(jw)}{\mathrm{d}w}|_{w_c}$$

$$\approx \angle P(jw_c) + \frac{2}{\pi}[\ln|K_g| - \ln|P(jw_c)|] \quad (4)$$

in which $|K_g| = P(0)$ is the static gain of the plant.

Furthermore, the PID controller parameters can be set as follows:

$$K_p = \frac{1}{|P(jw_c)\sqrt{1+\tan^2(\Phi_m - \angle P(jw_c))}|}, (5)$$

$$T_i = \frac{-2}{w_c[s_p(w_c) + \hat{\Phi}) + \tan^2(\hat{\Phi})s_p(w_c)]}, \quad (6)$$

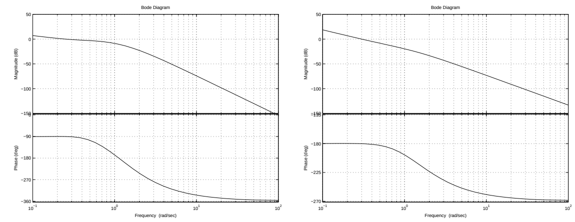
$$T_d = \frac{-T_i w_0 + 2s_p(w_0) + \sqrt{\Delta}}{2s_p(w_0)w_0^2 T_i} \quad (7)$$

where $\hat{\Phi} = \Phi_m - \angle P(jw_c)$ and $\Delta = T_i^2 w_0^2 - 8s_p(w_0)T_i w_0 - 4T_i^2 w_0^2 s_p^2(w_0)$ (Chen et al., 2003a).

In the modified tuning method, for the open loop system $G(s) = C(s)K(s)P(s)$, the PID controller $K(s)$ and the phase shaper $C(s)$ are designed separately. We use the same tuning method proposed in (Chen et al., 2003a) to design the PID controller here. In designing the PID controllers, the following guidelines should be observed:

- For the plant without integrator whose static phase equals $0°$, selecting $\Phi_m = 90°$, under the condition (1), we obtain the phase plot of $K(s)P(s)$ with a flat phase at $-90°$ for all the frequencies below w_c as shown in Fig. 1(a);
- For the plant with an integrator whose static phase equals $-90°$, selecting $\Phi_m = 0°$, we obtain the phase plot of $K(s)P(s)$ with a flat phase at $-180°$ for all the frequencies below w_c as shown in Fig. 1(b).

The above observations inform us that since we have already obtained a flat phase at $-90°$ or $-180°$, the only thing that needs to be done is just moving the flat phase to our desired phase requirement $-\pi + \Phi_m$, which means we would better have a phase compensator with a constant phase Θ ($-90° < \Theta < 90°$) which is the most important characteristics of fractional order differentiators or integrators s^α ($-1 < \alpha < 1$) (Manabe, 1961). Figure 2(a) gives the Bode plot of the fractional order integrator $s^{-0.5}$ which has a constant phase at $-45°$. Therefore, we can simply select the phase shaper as a fractional order differentiator/integrator.



(a) The flat phase region of $K(s)P(s)$ ($P(s) = \frac{1}{(s+1)^5}$) for lower frequencies

(b) The flat phase region of $K(s)P(s)$ ($P(s) = \frac{1}{s(s+1)^3}$) for lower frequencies

Fig. 1. Comparisons of the achieved flat phase regions for plants with and without an integrator

3. THE PHASE SHAPER

3.1 FOC Approximation

From the discussions in the previous section, clearly, the phase shaper comes from the idea of FOC (Fractional Order Calculus) (Oustaloup et al., 1996; Podlubny, 1999; Raynaud and Zergaïnoh, 2000; Vinagre and Chen, 2002). However, in practice, fractional order integrators or differentiators can not exactly be achieved or implemented with the ideal Bode plot shown in Fig. (2)(a) because they are infinite dimensional linear filters. A band-limit FOC implementation is important in practice, i.e., the finite-dimensional approximation of FOC should be done in a proper range of frequencies of practical interest (Chen and Moore, 2002; Oustaloup et al., 2000). Therefore, we can only design a phase compensator having a constant phase within a proper frequency range of interest.

In general, there are several approximation methods for FOC which can be divided into discretization method and frequency domain fitting method (Oustaloup et al., 2000; Chen et al., 2003b). Oustaloup proposed a continuous time frequency domain fitting method (Oustaloup et al., 2000) that can directly give the approximate s-transfer function. The existing discretization methods, e.g., (Machado, 1997; Vinagre et al., 2001), applied the direct power series expansion (PSE) of the Euler operator, continuous fractional expansion (CFE) of the Tustin operator and numerical integration based method.

3.2 Phase Shaper Realization

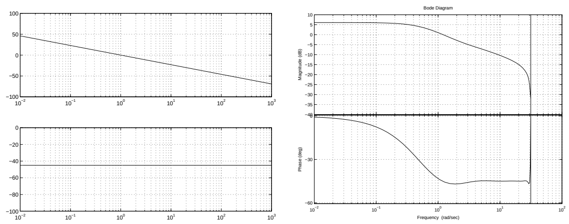
In designing a phase shaper, two factors in selecting the approximation method should be considered:

1) The phase shaper has a flat phase within the desired frequency range;
2) the phase shaper should have a lower order.

Therefore, in our study, a fourth order continued fraction expansion (CFE) of Tustin operator is employed which can give us a satisfying approximation result. The obtained discretized approximation of the fractional order integrator $s^{-0.5}$ with the discretization sampling time $T_s = 0.1s$ is give by

$$C(z) = \frac{3.578z^4 + 1.789z^3 - 2.683z^2 - 0.894z + 0.224}{16z^4 - 8z^3 - 12z^2 + 4z + 1}. \quad (8)$$

which its Bode plot shown in Fig. 2(b).



(a) The Bode plot of $s^{-0.5}$ (continuous) (b) The Bode plot of $s^{-0.5}(discretized)$

Fig. 2. Comparison of Bode plots for $s^{-0.5}$ and the discretized approximation using CFE of Tustin operator (4/4)

From Fig. 2(b).it is seen that the phase of (8) is nearly constant at $-45°$ within the frequency range between 4 rad./sec. and 30 rad./sec. The position of the constant phase area is greatly related with the discretization sampling time T_s and the width of that area shown on the Bode plot is fixed. To make the analysis more convenient, we transform the z-transfer function (8) to the s-transfer function (9) using the Tustin operator.

$$C(s) = \frac{0.025s^4 + 17.9s^3 + 1252s^2 + 1.67e004s + 3.58e004}{s^4 + 186.7s^3 + 5600s^2 + 3.2e004s + 1.78e004} \quad (9)$$

The Bode plot of (9) is shown in Fig. 3. The transfer function (9) shows us an illustrative example of a phase shaper with the property of locally constant phase Θ ($-90° < \Theta < 90°$). The position of the constant phase region is adjustable

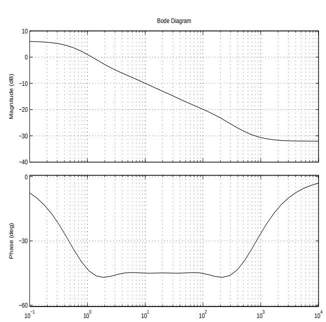


Fig. 3. Bode plot of the continuous-time fourth order approximation using CFE of the Tustin operator

by selecting different T_s. Combining the PID controller $K(s)$ which makes the system $K(s)P(s)$ have a flat phase in the lower frequency area, the phase shaper $C(s)$ can be used to ensure that the open loop system $C(s)K(s)P(s)$ has the flat phase with the expected width centered at the desired position. It is obvious that the constant phase area of $C(s)$ and the flat phase area of $K(s)P(s)$ must have an intersection and w_c for the PID controller design turns into the upper limit of the flat phase of the open loop system and the lower limit of the flat phase is determined roughly by $\frac{1}{10T_s}$ rad./sec.

4. DESIGN PROCEDURE

As discussed in Sec. 2, the PID controller and the phase shaper are designed separately. In what follows, the design procedures will be summarized

4.1 PID Controller Design

How to determine $s_p(w_c)$ was discussed in (Chen et al., 2003a) based on the experimental measurement of $\angle P(jw_c)$ and $|P(jw_c)|$. Therefore, let us summarize what are known at this point for PID controller design. We are given i) w_c; ii) $\Phi_m = 90°$ or $180°$; iii) measurement of $\angle P(jw_c)$ and $|P(jw_c)|$ (Chen et al., 2003a) and iv) an estimation of $s_p(w_c)$ (Chen et al., 2003a).

Then, using (5), (6) and (7), we can retrieve the PID parameters K_p, T_i and T_d.

4.2 Phase Shaper Design

The steps for designing phase shaper include i) selecting α, based on the phase margin requirement for the open loop system, for the fractional order integrator or differentiator s^α; ii) calculating the approximation transfer function for the fractional order integrator or differentiator; iii) selecting a proper discretization sampling time T_s to determine the position of the constant phase area of the approximation transfer function.

4.3 Gain Adjustment

Note that, among the above design procedures, only the phase requirement for the open loop system $C(s)K(s)P(s)$ is considered. However, we also need to care about the gain so that the sensitivity circle touches the flat phase region of the Nyquist curve exactly and the gain crossover frequency is settled within the flat phase. Therefore, a gain β is used to match the gain condition

$$G(jw_{gc}) = \beta C(jw_{gc})K(jw_{gc})P(jw_{gc}) = 1. \quad (10)$$

where w_{gc} is the desired gain crossover frequency of the open loop system ($\frac{1}{10T_s} < w_{gc} < w_c$). It is suggested to select w_{gc} at the midpoint of the flat phase area.

Equivalently, we use $\beta C(s)$ to update $C(s)$ so that the open loop system $C(s)K(s)P(s)$ matches both of the phase and gain requirement.

4.4 Selection of w_c and T_s

Because w_c and T_s determine the width and the position of the flat phase, it is very important to give a guidance to select w_c and T_s. Two factors influence the selections of w_c and T_s: 1) the desired gain crossover frequency w_{gc} should be within the flat phase region; 2) the flat phase area may not be so wide as well, i.e., the width is below 0.2 rad./sec. For better performance, it is suggested that w_c <0.3 rad./sec.

5. ILLUSTRATIVE SIMULATION

The modified tuning method presented above will be illustrated via some simulation examples. In the simulation, the following classes of plants, studied in (Wallén et al., 2002), will be used.

$$P_n(s) = \frac{1}{(s+1)^{(n+3)}}, n = 1, 2, 3, 4; \quad (11)$$

$$P_5(s) = \frac{1}{s(s+1)^3}; \quad (12)$$

$$P_6(s) = \frac{1}{(s+1)^4} e^{-s}; \quad (13)$$

$$P_7(s) = \frac{1}{s(s+1)^3} e^{-s}; \quad (14)$$

5.1 The General Plant $P_2(s)$

We still consider $P_2(s)$ in (11) first, which was studied in (Wallén et al., 2002; Chen et al., 2003a). For the PID controller design, because the plant $P_2(s)$ does not include any integrator, Φ_m should be set as 90° and w_c = 0.25 rad./sec. With these specifications, the PID controller $K_2(s)$ is designed as

$$K_2(s) = 1.095(1 + \frac{1}{4.892s} + 1.829s). \quad (15)$$

The specifications of the phase shaper $C_2(s)$ are set as $\alpha = -0.5$, which means that we use the fractional order integrator $s^{-0.5}$ as the original form of the phase shaper, T_s = 1 sec. and β =

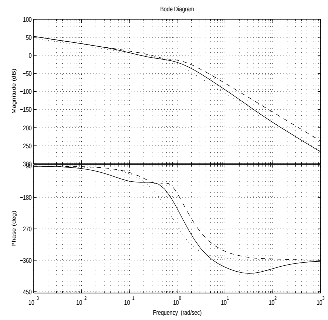


Fig. 4. Bode diagram comparison (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat phase" PID; Solid line: The proposed.)

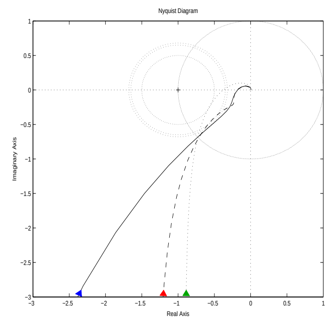


Fig. 5. Nyquist diagram comparison (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat phase" PID; Solid line: The proposed.)

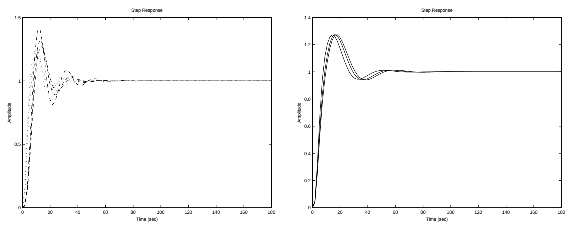
9.091. The phase shaper designed by the proposed method is

$$C_2(s) = \frac{0.0226s^4 + 1.626s^3 + 11.38s^2 + 15.18s + 3.252}{s^4 + 18.67s^3 + 56s^2 + 32s + 1.778}. (16)$$

For comparison, the corresponding PID controller designed by the modified Ziegler-Nichols method is $K_{2z} = 0.232(1 + \frac{1}{1.011s} + 0.253s)$ while the corresponding PID controller designed by the "flat phase" tuning method (Chen et al., 2003a) is $K_{2f} = 0.671(1 + \frac{1}{2.149s} + 1.657s)$.

The Bode and the Nyquist plots are compared in Figs. 4 and 5.

From Fig. 4, it is seen that the phase plot between 0.1 rad./sec. and 0.3 rad./sec. is flat. The phase margin roughly equals 45°. In Fig. 5, the Nyquist curve of the open loop system is tangential to the sensitivity circle at the flat phase. And we can also see that the flat phase is wide enough to accommodate the gain variation of the plant. The step responses of the closed-loop systems are compared in Fig. 6. Comparing the closed-loop system with the proposed modified controller to that with the modified Ziegler-Nichols controller, the overshoots of the step response from the proposed scheme remain invariant under gain variations. However, the overshoots of the modified Ziegler-Nichols controller change remarkably.



(a) Step responses of the closed-loop system with the modified Ziegler-Nichols and the "flat phase" PID controllers

(b) Step response of the closed-loop system with the "flat phase" PID controller plus a phase shaper

Fig. 6. Comparisons of step responses (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat Phase" PID; Solid line: The proposed. For all schemes, gain variations 1, 1.1, 1.3 are considered in step responses)

5.2 Plant With An Integrator $P_5(s)$

This case is omitted due to space limitation. Please refer to the combined case in Sec. 5.4.

5.3 Plant With A Time Delay $P_6(s)$

This case is omitted due to space limitation. Please refer to the combined case in Sec. 5.4.

5.4 Plant With An Integrator And A Time Delay $P_7(s)$

For the plant with an integrator and a time delay $P_7(s)$, the proposed PID controller is $K_7(s) = 0.228(1+\frac{1}{4.002s}+1.343s)$ with respect to w_c=0.25 rad./sec. and Φ_m=0°. The proposed phase shaper is

$$C_7(s) = \frac{4.528s^4 + 56.35s^3 + 112.7s^2 + 42.93s + 1.59}{0.5062s^4 + 24.3s^3 + 113.4s^2 + 100.8s + 14.4}$$

with respect to $\alpha = 0.5$, $T_s = 1.5$ sec. and $\beta = 0.022$.

The controller designed by the modified Ziegler-Nichols method is $K_{7z} = 0.266(1 + \frac{1}{10.136s} + 2.534s)$. The corresponding PID controller designed by the "flat phase" tuning method (Chen et al., 2003a) is $K_{7f} = 0.268(1 + \frac{1}{10.795s} + 2.438)$.

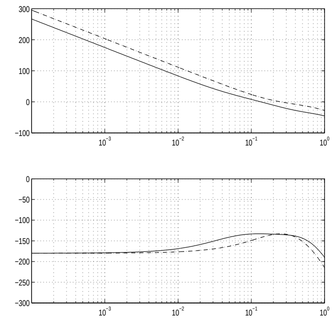


Fig. 7. System With Integrator And Delay: Bode diagram comparison (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat Phase" PID; Solid line: The proposed)

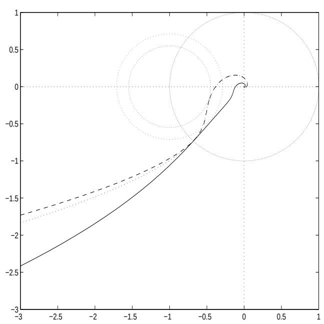
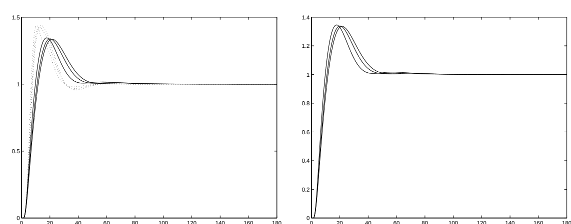


Fig. 8. System With Integrator And Delay: Nyquist diagram comparison (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat Phase" PID; Solid line: The proposed)



(a) Step Responses of system with Modified Ziegler-Nichols controller and "flat phase" PID controller

(b) Step response of system with phase shaper

Fig. 9. System With An Integrator And A Time Delay: Comparisons of step responses (Dashed line: The modified Ziegler-Nichols; Dotted line: "Flat Phase" PID; Solid line: The proposed. For all schemes, gain variations 1, 1.1, 1.3 are considered in step responses)

6. CONCLUSION

This paper presents an extension of our previous work where a robust PID autotuning method was proposed by using the idea of "flat phase", i.e., the phase derivative w.r.t. the frequency is zero at a given frequency called the "tangent frequency" so that the closed-loop system is robust to gain variations and the step responses exhibit an iso-damping property. However, the width of the achieved phase flatness region is hard to adjust. A phase shaping idea is proposed to make the width of the phase flatness region adjustable. With a suitable phase shaper, we are able to determine the width of the flat phase region so as to make the whole design procedure of a robust PID controller much easier and the system performance can be enhanced more significantly. The plant gain and phase at the desired frequency, which are identified by several relay feedback tests in an iterative way, are used to estimate the derivatives of the amplitude and phase of the plant with respect to the frequency at the same frequency point by the well known Bode's integral relationship. Then, these derivatives are used to design the proposed robust PID controller. The phase shaper, based on the idea of FOC (Fractional Order Calculus), is actually a fractional order integrator or differentiator. In this paper, no plant model is assumed during the controller design. Only several relay tests and calculations are needed. Simulation examples illustrate the effectiveness and the simplicity of the proposed method with an iso-damping property. From the illustrative simulation, it can be seen that the proposed phase shaping approach to robust PID controller tuning gives a satisfying performance for a class of plants.

Our further research efforts include 1) Testing on more types of plants; 2) Experiment on real plants 3) Exploration of nonminimum phase, open loop unstable systems.

7. ACKNOWLEDGMENTS

The authors are grateful to Professor Li-Chen Fu, Editor-in-Chief of Asian Journal of Control for providing a complimentary copy of the "Special Issue on Advances in PID Control", Asian J. of Control (vol. 4, no. 4). The simulation study was helped by C. H. Hu.

REFERENCES

Karl J. Astrom and Tore Hagglund. *PID Controllers: Theory, Design, and Tuning.* ISA - The Instrumentation, Systems, and Automation Society (2nd edition), 1995.

Y. Q. Chen, C. H. Hu, and K. L. Moore. Relay feedback tuning of robust PID controllers with iso-damping property. In *Proceedings of The 42nd IEEE Conference on Decision and Control*, Hawaii, 2003a.

Y. Q. Chen and K. L. Moore. Discretization schemes for fractional-order differentiators and integrators. *IEEE Trans. Circuits Syst. I*, 49:363–367, March 2002.

YangQuan Chen, B. M. Vinagre, and Igor Podlubny. On fractional order disturbance observers. In *Proc. of The First Symposium on Fractional Derivatives and Their Applications at The 19th Biennial Conference on Mechanical Vibration and Noise, the ASME International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (ASME DETC2003)*, pages 1–8, `DETC2003/VIB--48371`, Chicago, Illinois, 2003b.

C. C. Hang, K. J. Åström, and W. K. Ho. Refinements of the Ziegler-Nichols tuning formula. *IEE Proc. Pt. D*, 138(2):111–118, 1991.

A. Karimi, D. Garcia, and R. Longchamp. Iterative controller tuning using Bode's integrals. In *Proceedings of the 41st IEEE Conference on Decision and Control*, pages 4227–4232, Las Vegas, Nevada, 2002a.

A. Karimi, D. Garcia, and R. Longchamp. PID controller design using Bode's integrals. In *Proceedings of the American Control Conference*, pages 5007–5012, Anchorage, AK, 2002b.

B. Kristiansson and B. Lennartsson. Optimal PID controllers including roll off and Schmidt predictor structure. In *Proceedings of IFAC 14th World Congress*, volume F, pages 297–302, Beijing, P. R. China, 1999.

J. A. T. Machado. Analysis and design of fractional-order digital control systems. *J. Syst. Anal. Modeling-Simulation*, 27:107–122, 1997.

S. Manabe. The non-integer integral and its application to control systems. *ETJ of Japan*, 6(3-4):83–87, 1961.

A. Oustaloup, F. Levron, F. Nanot, and B. Mathieu. Frequency band complex non integer differentiator: Characterization and synthesis. *IEEE Trans. Circuits Syst. I*, 47:25–40, Jan. 2000.

A. Oustaloup, X. Moreau, and M. Nouillant. The crone suspension. *Control Eng. Pract.*, 4(8):1101–1108, 1996.

H. Panagopoulos, K. J. Åström, and T. Hägglund. Design of PID controllers based on constrained optimization. In *Proceedings of the American Control Conference*, San Diego, CA, 1999.

Igor Podlubny. Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers. *IEEE Trans. Automatic Control*, 44(1):208–214, 1999.

H. F. Raynaud and A. Zergaïnoh. State-space representation for fractional order controllers. *Automatica*, 36(7): 1017–1021, 2000.

K. K. Tan, T. H. Lee, and Q. G. Wang. Enhanced automatic tuning procedure for process control of PI/PID controllers. *AlChE Journal*, 42(9):2555–2562, 1996.

B. M. Vinagre, I. Petras, P. Merchant, and L. Dorcak. Two digital realization of fractional controllers: Application to temperature control of a solid. In *in Proc. Eur. Control Conf. (ECC01)*, pages 1764–1767, Porto, Portugal, 2001.

Blas M. Vinagre and YangQuan Chen. Lecture note on fractional calculus applications in automatic control and robotics. In Blas M. Vinagre and YangQuan Chen, editors, *The 41st IEEE CDC2002 Tutorial Workshop # 2*, pages 1–310. [Online] `http://mechatronics.ece.usu.edu/foc/cdc02_tw2_ln.pdf`, Las Vegas, Nevada, USA, 2002.

A. Wallén, K. J. Åström, and T. Hägglund. Loop-shaping design of PID controllers with constant t_i/t_d ratio. *Asian Journal of Control*, 4(4):403–409, 2002.