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Nonlinear System Identification Based on Genetic Algorithm and Grey Function*

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Abstract **- The paper presents a method for the identification of nonlinear system parameters by using an improved Genetic Algorithm and Grey Function. The paper firstly outlines several commonly used nonlinear identification methods such as RLS, RIV and COR and also their drawbacks. Then, a method based on the Genetic Algorithm and Grey Function is proposed and given in detail in the paper. Finally, a simulation experiment to TV set production data of an electronic factory was carried out. The simulations show that the method can gain good results and is also simple and effective.**

Index Terms - Genetic Algorithm, Nonlinear system, Grey function

I. INTRODUCTION

In economics, people often use production function to analyse the quantitative relationship between investment and productivity. Among these production functions, Cobb-Douglas production function is one of the most popular and its mathematic model is shown as:

$$y = a x_1^{b1} x_2^{b2} \qquad (1)$$

in the equation, y is productivity, x_1 and x_2 represent production investment , a is conversion factor, $b1$ and $b2$ are production factors. Three model parameters a , $b1$, $b2$ are needed to be identified in the model identification. Several identification methods have been reported to identify the model parameters, such as Recursive Least Squares (RLS), Recursive method of instrumental variable (RIV) and Correlative Function method (COR), etc. RLS is usually considered that it is simple to use and costs little computation but it gives large estimation error when the system is influenced by colourful noises. Although COR can acquire more accurate parameter estimation but the estimation error is still not satisfying. RIV can get accurate parameter but it costs large amount of complex computation. Most of methods used in the system identification are based on the assumption that the searching space of identified system model is continuous and differentiable. If the searching space is undifferentiable or parameter span is nonlinear, traditional recurrent methods can not gain the global optimization [2-9]. However, Genetic Algorithm (hereafter referred to as GA) does not need to assume that the searching space is differentiable or continuous. GA employs random, yet directly, search for locating the globally optimal solution and can search discrete, noisy and multimodal space [10-13]. In this paper, GM(0,h) combined with an improved GA was applied to identify Cobb-Douglas production function model, which can be described as the following equation (1), and also build a Grey-Cobb-Douglas production function model.

II. RESEARCH METHODOLOGY

A. Grey system theory and method

Grey system theory is mainly applied to problems raised in the modeling, prediction, decision and control of those systems such as uncertain system model, incomplete behavior information, and uncertain system operation. Grey system modeling is the basis and its most characteristic modeling is to time series GM（Grey Model）. It utilizes system information to make abstract concept to be quantitative, and then optimize modeling. Thus, Grey Model can show its advantage under the condition that probability characteristics or subjection characteristics can not be found. Social, economic and biological systems are clearly layered, random in dynamic changes, uncertainty of index data and ambiguous in structure relationship. For example，technical methods, human factors and natural environment changes may cause data deviations, loss and even untrue, which is considered as grey[1].

The core of Grey system theory and method is Grey Dynamic Model and its characteristics are to build functions and grey derivative equations. Grey Dynamic Model is built on the basis of Grey Model concept and is a modeling method taking derivative fitting as its core. The methodology of Grey System modeling is to directly convert time series into derivative functions and then acquire a Grey Dynamic Model derived from abstract system. A GM(n,h) model built on grey system is the time continuous function model of a derivation equation. In the model, n is the order of derivative equation and h is the number of variables.

B. Basic GM model

A GM(n,h) model is a derivative equation which has n order and h variables. According to different n and h, GM models can be classified in three kinds:

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(1) Predication model

A GM(n,1) model only has one variable and is a predication model. Its requirement to data is time series to be able to reflect effect of predicted objects. The value of n is usually less than 3 and n=1 in most cases because the computation is more complex and computation accuracy may not be good if n is bigger. Thus, when n=1, the model is expressed as GM(1,1)， and named as single series one order linear dynamic model.

(2) State model

It is expressed as GM(1,h) and reflects how h-1 variables affect variation ratio of the derivative of dependant variable. So, it is named as one order linear equation of h series because of h>1.

(3) Static model

GM(0,h) model does not include derivative term, so it is considered as a static model. Its mathematic equation is shown as:

$$X_1^{(1)}(t) = b_1 X_2^{(1)}(t) + b_2 X_3^{(1)}(t) + \ldots + b_{h-1} X_h^{(1)}(t) + a$$

Its coefficient vectors can be solved by using least square. It is similar with multi-variable linear regression equation but only need to appropriate transformation of sequence, as shown,

$$X_i^{(0)}(t) \rightarrow X_i^{(1)}(t)$$

coefficient vector is expressed as $\hat{a}$ ：

$$\hat{a} = [b_1, b_{2,} \ldots, b_{h-1}, a]^T$$

Thus：

$$\hat{a} = [B^T B]^{-1} B^T Y_n$$

$$B = \begin{bmatrix} X_2^{(1)}(1) & X_3^{(1)}(1) & \ldots X_h^{(1)}(1) & 1 \\ X_2^{(1)}(2) & X_3^{(1)}(2) & \ldots X_h^{(1)}(2) & 1 \\ \ldots\ldots & & & \\ X_2^{(1)}(n) & X_3^{(1)}(n) & \ldots X_h^{(1)}(n) & 1 \end{bmatrix}$$

$$Y_n = [X_1^{(1)}(1), X_1^{(1)}(2), \ldots., X_1^{(1)}(n)]^T$$

GM(0,h) model can be gained if coefficients are solved and put into the equation.

C. Computation of GM(0,h) model：

GM(0,h) model is similar with multi variable linear regression equation but there are still some differences. Linear modeling is on the basis of original data, but GM(0,h) modeling is on the basis of added sum sequence of original data.

1). do one added generation operation (1-AGO) of h data sequence, $X_i^{(1)}(k) = \sum_{j=1}^{k} X_i^{(0)}(j)$, $(k = 1,2,\ldots,n)$;

2). According equations above, construct B and Y_n;

3). Solve $\hat{a}$ by using least square method, but Genetic Algorithm is used to solve $\hat{a}$;

4). Roots are put into equations and get GM(0,h) model.

5). Verify the model.

D. Genetic Algorithm (GA)

GA is a robust search and optimization technique, which simulates heritage mechanism in the nature and natural evolutionary. When it is applied to a practical problem, parameters of the problem need be encoded into strings. Several strings form an individual and many individuals form a population. Reproduction, Crossover and Mutation techniques are applied to evolve and locate global optimum of the parameters in the population. In addition, a fitness function needs to be formed and control parameters of GA such as crossover probability P_c , mutation probability P_m and population N will be selected. Given parameters to be identified $a_1, a_2, \cdots, a_n$ are encoded into strings as following,

a_1	a_2	$\cdots$	a_n

The length of each parameter is set according to actual requirement of research objects. For example, the length of each parameter is L, the whole length of the string is K=L×N.
Fitness function gives a fitness value for each individual in the population and its selection is related with the researched problem. The variance of fitness function usually is required to correspond to the variance of objective function and then the individual with the best fitness will be the best solution.
The working parameters of GA are usually set as: crossover probability $P_c \in [0.25, 1.0]$, mutation probability $P_m \in [0.005, 0.05]$, the size of population $N \in [30, 100]$.

E. Adaptive Genetic Algorithm (AGA)

An Adaptive Genetic Algorithm was applied to do the parameter identification of a bilinear system. AGA takes advantage of adaptive crossover probability and mutation probability [3]-[6],[9]. The structure of AGA is shown as follows.

1) Adaptive crossover probability p_c and adaptive mutation probability p_m can be expressed as the following formula:

$$P_c = \begin{cases} k_1 (f_{\max} - f')/(f_{\max} - \bar{f}) & ,f' \geq \bar{f} \\ k_2 & ,f' < \bar{f} \end{cases}$$

$$P_m = \begin{cases} k_3 (f_{\max} - f)/(f_{\max} - \bar{f}) & ,f \geq \bar{f} \\ k_4 & ,f < \bar{f} \end{cases}$$

constants $k_1, k_2, k_3, k_4 \leq 1.0$; $f_{\max}$ is a maximum fitness value in each population; $\bar{f}$ is an average fitness value in each population. f' is the bigger fitness value within the selected two strings to do a crossover operation. f is the maximum fitness value in the selected strings to do a mutation operation.

2) Crossover in two positions.

3) Crossover positions are selected in unequal probabilities [14].

4) Elitist Strategy

5) The convergence criterion is that the fitness value of each individual equals to the average fitness value.

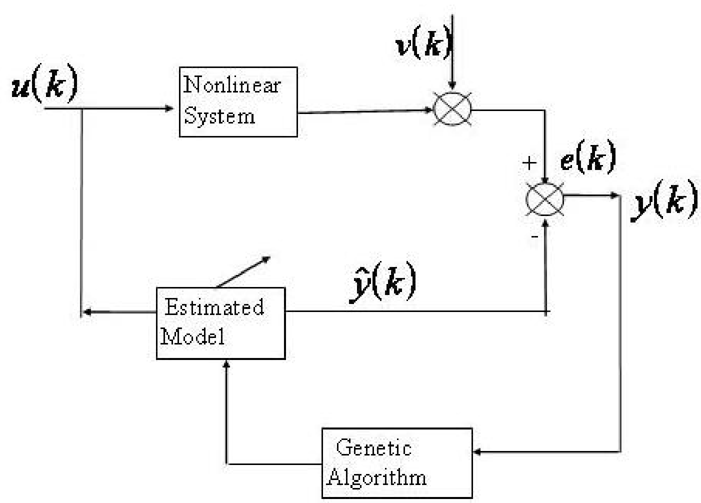

Fig. 1 a diagram of nonlinear system identification by using GA

F. Parameter identification of a Grey-Cobb-Douglas model by using AGA

Fig. 1 shows that GA is applied to identify the parameters of a Grey-Cobb-Douglas model. The procedure of the identification algorithm is as follows:

1) Set the encoding method and decide the binary encoding length of each parameter length and the length of the formed individual.

2) Set the working parameter of GA. Randomly produce N binary strings and initialize population.

3) Decode binary strings to parameters of the bilinear system.

4) Calculate the fitness of each group of parameters and evaluate each individual

A parameter identification problem actually is a minimization problem and the objective function J needs to be minimized. Hence, the objective function needs to be converted to a fitness function when GA is applied. Define

$$f(k) = \begin{cases} C_{\max} - J(k) & J(k) < C_{\max} \\ 0 & Other \end{cases},$$

Where $J(k) = \sum_{j=0}^{s} e^2 (k-j)$, the sum of squared errors of system outputs.

5) Perform reproduction, crossover and mutation to generate the next population.

6) Repeat the 3), 4) ,5) steps until the convergence is reached.

7) Parameter value represented by the individual which has the best fitness in the population is the result of identified system parameter..

In this paper, a Grey-Cobb-Douglas model is built by using GM(0,h) modeling method. It is assumed that $y^0(k)$ and $x_i^0(k)$ are time sequences of productivity and investment (k =1,2,... n ; i =1,2,... h), n is sampling number , h is investment factor. 1-AGO is applied to $y^0(k)$ and $x_i^0(k)$, thus:

$$y^1(k) = \sum_{j=1}^{k} y^0(j)$$

$$x_i^1(k) = \sum_{j=1}^{k} x_i^0(j) \qquad (2)$$

So, Cobb-Douglas production function based on the data of $y^1(k)$ and $x_i^1(k)$ is shown as:

$$y^1(k) = a\, x_1^{1b1}\, x_2^{1b2} \qquad (3)$$

The equation (3) is defined as Grey-Cobb-Douglas model. The following work is to identify parameters a , $b1$, $b2$ of the model by using AGA. Then the three parameters are put into equation (3) and Grey-Cobb-Douglas can be acquired. Through the following equation:

$$y^0(k) = y^1(k) - y^1(k-1) \qquad (4)$$

Model value of Cobb-Douglas production function can be acquired.

III. SIMULATIONS

TV set production data of an electronic factory are used for case study. According to the algorithm proposed in the paper, working parameters of GA are population $N = 80$; adaptive crossover possibility $k_1 = k_2 = 0.76$ and mutation possibility, $k_3 = k_4 = 0.005$; Crossover at multiple different positions and inverse operation is also applied; the length of chromosome is 20; selection strategy is R=E; All system parameters to be identified are within the span [0,3.5]. The system was designed to be sampled every three genetic operations. The convergence was reached after 852 genetic operations. The simulation result is shown in Table 1 and the sum of squared parameter errors is 0.003101.

$$y^1(k) = 3.200394\, x_1^{1(0.908805)}\, x_2^{1(0.125568)} \tag{5}$$

a =3.200394

$b1$ =0.908805

$b2$ =0.125568

To verify the model accuracy, x_1^1 and x_2^1 are put into formula (5) correspondingly. Then, match values can be solved by using formula (4) and shown as follows：

Table 1

No.	Year	Real value	Model value	Deviation	Relative error (%)
1	1966	17.501	18.465	-0.964	-0.511
2	1967	20.510	20.325	0.184	0.899
3	1968	23.934	22.137	1.796	7.506
4	1969	25.614	25.425	1.188	4.465
5	1970	26.894	26.035	0.858	3.192
6	1971	29.707	28.990	0.716	2.410
7	1972	31.731	32.278	-0.547	-1.724
8	1973	33.455	34.349	-0.894	-2.674
9	1974	37.193	38.696	-1.503	-4.042
10	1975	40.384	41.900	-1.516	-3.755
11	1976	43.942	43.980	-0.038	-0.087
12	1977	46.460	46.602	0.857	1.806
13	1978	47.866	48.378	-0.512	-1.071
14	1979	50.311	50.992	-0.661	-1.314
15	1980	53.283	52.758	0.524	0.984

In the table, match values are compared with real values. It is found that the sum-up of relative errors in 15 years is 34%， the biggest relative error is 7.506%, average relative error is 2.27%. These results show that the method proposed in the paper is effective.

IV. CONCLUSION

The paper describes a method for parameter identification of nonlinear system by using a method based on improved Genetic Algorithm and Grey function. Simulations to TV set production data of an electronic factory have been carried out to investigate the effectiveness of proposed method. It is shown that good results could be obtained. Comparing with RLS and COR, the method is simpler and more effective.

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