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Adaptive generalised predictive temperature control for air conditioning systems

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Abstract: This study presents an adaptive stable generalised predictive control (ASGPC) for temperature control of a class of air-conditioning systems. The proposed ASGPC method is more general and practical than the approaches presented by Rossiter *et al.* and Kouvaritakis *et al.* A real-time stable generalised predictive control (GPC) control algorithm along with a recursive least-squares parameter estimator is proposed to control the rotational speed of the water chiller pump, thereby adjusting the rate of heat exchange in a refrigeration circulation system in order to achieve temperature set-point tracking. Computer simulations are conducted to verify effectiveness of the proposed control method. Through experimental results, the proposed control method is shown capable of giving satisfactory temperature set-point tracking performance and disturbance rejection capability.

1 Introduction

Owing to demands of improving daily life, people have paid much effort to construct comfortable living environments. In addition, many high-tech industries (e.g. optic-electronic, electronic, information, biotech and medical industries) have been unceasingly upgrading their products by employing appropriate production environments. Among these environments, temperature control has been considered as a key technology to achieve comfortable space for human being or qualities of these high-tech products by means of refrigeration or air-conditioning process. On the other hand, energy saving for the air-conditioning process has attracted much attention in recent years because of remarkable increase of gas price. Hence, the air-conditioning technology with precise temperature tracking and energy-saving schemes has become increasingly important in the field of heating, ventilation and air condition (HVAC).

With the advent of fast and efficient microprocessors and microcontrollers, these high-speed computational devices have been used to achieve high-performance and sophisticated control functions together with high sampling rates for various kinds of non-linear as well as linear industrial processes with input–output models [1–5]. In particular, the refrigeration and air-conditioning systems under consideration often have many properties, such as long time delay, time varying and non-linearities [6]. For example, because of highly non-linear water–steam phase transformation, the coolants, such as water and refrigerant, usually make overall refrigeration and air-conditioning system greatly non-linear. Therefore such processes cannot be described by a simple mathematical equation with constant parameters. To obtain good control performance, it is necessary to develop a kind of simplified mathematical model with variable coefficients or a class of simple non-linear system model in order to describe dynamics of these processes [7]. In this paper, the simplified linear, discrete-time mathematical model with variable coefficients will be adopted to model the small-scale air-conditioning system based on the reaction curve method [8] under open-loop test; afterwards, the well-known digital recursive least-squares estimation method with forgetting factor [9] will be on-line used to find the system parameters in real time. The control policy [for instance generalised predictive control (GPC)] is then synthesised based on the certain equivalence principle to achieve goals of precise temperature setpoint tracking and energy saving.

GPC was first presented by Clarke *et al.* [8, 10]. GPC provides the following advantages: (i) it obviates steady-state error of system and exhibits remarkable robustness with respect to model mismatch and un-modelled dynamics; (ii) many practical tuning knobs can be adjusted so as to improve control performance; and (iii) future predictions with state and control constraints are calculated [1, 10]. Although the GPC strategy has demonstrated many aforementioned merits and advantages, it still suffers from the closed-loop stability problem. To overcome the stability problem, Rossiter *et al.* and other researchers [1, 2] presented a stable generalised predictive control (SGPC) method. In these studies [1, 2], a stable feedback controller was first used to achieve guaranteed stability in the inner loop, and then a similar method of GPC was employed to attain desired control performance; however, the closed-loop characteristic equations, or the Bezout identities, are set to be unity, making infeasible in designing practical controllers for most industrial processes and machines.

As GPC control is very easy to understand and implement, this model-based predictive control method has become very popular in both industry and academia. Indeed, such a control strategy has gained many successful applications in the industry [3, 11–20]. In particular, many control methods for HVAC systems have been proposed by full use of GPC strategies, such as predictive control based on local linear fuzzy models for an industrial-scale cross-flow heat exchanger [21], multiple fuzzy model-based temperature predictive control for HVAC systems [22], GPC-PID (proportional-integral-derivative) control strategy with hierarchical structure for a cooling coil unit [23], support vector regression model predictive control for an HVAC plant [24], practical GPC with decentralised identification approach to HVAC systems [25], statistical multiple-input multiple-output (MIMO) controller performance monitoring [26] and structure design and indirect adaptive general predictive temperature control of a class of passive HVAC [27].

The objective of this paper is to develop an adaptive GPC for temperature control of a class of air-conditioning system with variable water volume (VWV) adjustment and variable-frequency motor control. The proposed control method combines the well-known recursive least-squares parameter method and the SGPC strategy given in [1, 2], in order to accomplish satisfactory tracking and guaranteed stability simultaneously. The proposed SGPC method is more general than the approach presented in [1, 2], whose Bezout identity must be equal to unity. The effectiveness and performance of the proposed method will be exemplified by conducting several computer simulations and experiments on the experimental air-conditioning system.

In comparison with the Abu-Ayyad and Dubay's method in [20], their technical merit hinges on the performance improvement of an adaptive GPC (AGPC) for controlling non-linear systems by considering that the dynamic matrix comprises only the two parameters which directly introduces the non-linear characteristics of the controlled and the manipulated variables, thereby giving a more accurate prediction of $\hat{Y}_{\mathrm{m}}$ at any closed-loop state y_{m} of the process. Unlike this approach, our proposed technique puts emphasis on improvement of the transient performance of a SGPC method by assigning appropriate pole locations in the feedback control loop, and generation of optimal predictive control signals by incorporating the feedback controller. Similar to the Abu-Ayyad and Dubay's approach [20], the proposed method also uses the same first-order plus dead time (FOPDT) model with on-line parameter estimation for controlling a class of industry non-linear plants at fixed operating points, which can be modelled as linearised system models. Hence, the proposed control method is weak in giving satisfactory transient performance for those highly non-linear processes investigated in the Abu-Ayyad and Dubay's paper [20]. In addition, when compared with the Rousseau's approach in [27], the proposed method contributes to improvement of the SGPC method presented by Rossiter *et al.* [1] and Kouvaritakis *et al.* [2], and establishment of a real-time adaptive control algorithm with application to a small-scale air-conditioning system. Hence, the main difference between the Rousseau's [27] method and the proposed method is that our proposed method first uses a feedback controller design to achieve internally asymptotically stability of the controlled plant with a desired characteristic equation, and then employs GPC to obtain the optimal control signals, whereas Rousseau's [27] approach focused on the similar indirect GPC with a distributed architecture. From theoretical view of point, our proposed method is shown to ensure internally stable, whereas the stability of the Rousseau's [27] method is not explicitly proven.

The rest of the paper is outlined as follows. In Section 2 some useful mathematical preliminaries are briefly recalled for developing the proposed control method. Section 3 rigorously describes the AGPC method, and presents its real-time control algorithm. In Section 4 experimental setup and computer simulations are conducted to examine the feasibility and effectiveness of the proposed controller, and experimental results are used for illustration of the merit and performance of the controlled air-conditioning system. Section 5 concludes the paper.

2 Mathematical preliminaries

This section will recall some useful mathematical preliminaries for the proposed predictive control method described in Section 3. In doing so, it is necessary to have the following four definitions [1].

Definition 1: Let $\mathbf{F}$ be the vector space of all polynomial $f(z)$ in z^{-1} of degree $n-1$, and choose $[1, z^{-1}, \ldots, z^{-(n-1)}]$ as its basis set. $D\{f(z)\}$ represents the degree $n-1$ of $f(z)$. Let $\mathbf{f}$ be the $\mathbf{R}^n$ column vector of the coordinates of $f(z)$ with respect to this basis set.

Definition 2: For a causal polynomial $f(z) \in \mathbf{F}$, let the convolution matrix $\mathbf{C}_f \in \mathbf{R}^{n\times n}$ be of the form $[\mathbf{C}_f]_{ij} = f_{i-j}$, where f_k is the coefficient of z^{-k} in $f(z)$

$$\mathbf{C}_f = \begin{bmatrix} C_{f11} & C_{f12} & \cdots & C_{f1n} \\ C_{f21} & C_{f22} & \cdots & C_{f2n} \\ \vdots & \vdots & \vdots & \vdots \\ C_{fn1} & C_{fn2} & \cdots & C_{fnn} \end{bmatrix}$$

$$= \begin{bmatrix} f_0 & f_{-1} & \cdots & f_{1-n} \\ f_1 & f_0 & \cdots & f_{2-n} \\ \vdots & \vdots & \vdots & \vdots \\ f_{n-1} & f_{n-2} & \cdots & f_0 \end{bmatrix} = \begin{bmatrix} f_0 & 0 & \cdots & 0 \\ f_1 & f_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ f_{n-1} & f_{n-2} & \cdots & f_0 \end{bmatrix}$$

Definition 3: Let the matrix formed out of the first μ columns of the convolution matrix $\mathbf{C}_f$ be denoted by $\boldsymbol{\Gamma}_f$, and the matrix formed out of the remainder last $n-\mu$ columns of $\mathbf{C}_f$ be $\mathbf{M}_f$. Furthermore, let $\mathbf{e}_i$ be the ith standard basis vector in $\mathbf{R}^n$; then $\boldsymbol{\Gamma}_f$ and $\mathbf{M}_f$ are expressed by

$$\boldsymbol{\Gamma}_f = \mathbf{C}_f[\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_\mu]$$

$$= \begin{bmatrix} f_0 & 0 & 0 & \cdots & 0 \\ f_1 & f_0 & 0 & \cdots & 0 \\ f_2 & f_1 & f_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ f_{n-1} & f_{n-2} & \cdots & \cdots & f_0 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & 1 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} f_0 & 0 & \cdots & 0 \\ f_1 & f_0 & \ddots & \vdots \\ f_2 & f_1 & \ddots & 0 \\ \vdots & \vdots & \ddots & f_0 \\ \vdots & \vdots & \ddots & \vdots \\ f_{n-1} & f_{n-2} & \cdots & f_{n-\mu} \end{bmatrix}$$

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$$\mathbf{M}_f = \mathbf{C}_f[\mathbf{e}_{\mu+1}, \ldots, \mathbf{e}_n]$$

$$= \begin{bmatrix} f_0 & 0 & 0 & \cdots & 0 \\ f_1 & f_0 & 0 & \cdots & 0 \\ f_2 & f_1 & f_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ f_{n-1} & f_{n-2} & \cdots & \cdots & f_0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \ddots & \vdots \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_0 & 0 & \ddots & \vdots \\ f_1 & f_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ f_{n-\mu-1} & f_{n-\mu-2} & \cdots & f_0 \end{bmatrix}$$

Definition 4: For $f(z) \in \mathbf{F}$, $\mathbf{H}_f$ represents the Hankel matrix in $\mathbf{R}^{n\times n}$ by $[\mathbf{H}_f]_{ij} = f_{i-1+j}$, where f_k denotes the coefficient of z^{-k} in $f(z)$

$$H_f = \begin{bmatrix} H_{f11} & H_{f12} & \cdots & H_{f1n} \\ H_{f21} & H_{f22} & \cdots & H_{f2n} \\ \vdots & \vdots & \vdots & \vdots \\ H_{fn1} & H_{fn2} & \cdots & H_{fnn} \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & \cdots & f_n \\ f_2 & f_3 & \cdots & f_{n+1} \\ \vdots & \vdots & \vdots & \vdots \\ f_n & f_{n+1} & \cdots & f_{2n-1} \end{bmatrix}$$

$$= \begin{bmatrix} f_1 & f_2 & \cdots & f_{n-1} & 0 \\ f_2 & \cdots & f_{n-1} & 0 & 0 \\ \vdots & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & \vdots \\ f_{n-1} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

By Definition 2, it is easy to show Lemma 1 using the fact that the multiplication of the convolution matrix is commutative.

Lemma 1: For $f(z)$, $h(z) \in \mathbf{F}$, then $\mathbf{C}_f\mathbf{C}_h = \mathbf{C}_h\mathbf{C}_f$.

3 Adaptive generalised predictive temperature control

This section aims at describing the design strategy of the AGPC for small scale air-conditioning system. This controller is synthesised as the following four basic steps. First, the recursive least square estimation (RLSE) method is utilised to obtain the system's parameters, which are then used for controller design. Second, based on the estimated parameters, a feedback controller is designed to achieve the internal stability, namely that the inner loop with the feedback controller is guaranteed asymptotically stable. Third, an output prediction method is described using Hankel and convolution matrices, and then a stable predictive controller is proposed to output the present optimal control signal based on the minimisation of the errors between the future desired trajectory and the predicted system trajectory, and the future controls. Last but not least, a real-time adaptive control algorithm is presented to control the small-scale air-conditioning system.

3.1 Mathematical model and real-time parameter estimation

Since most air-conditioning processes with time delays are operated around operating points, it is reasonable to model these processes as the first-order system models with time delay because of the fact that the open-loop step responses of the small-scale air-conditioning systems often exhibit typical reaction curves with time delay, as shown in Fig. 1. Moreover, the discrete-time mathematical models of these air-conditioning systems can be described by using a constant sampling period T by

$$(1 - az^{-1})y(k) = (b_0 + b_1z^{-1})z^{-d}u(k) \quad (1)$$

where $y(k)$ and $u(k)$ denote the system input and output in time k, respectively; z is the shifting operator such that $z^{-1}\{y(k)\} = y(k-1)$, $z^{-n}\{y(k)\} = y(k-n)$ and $z^n\{y(k)\} = y(k+n)$. The system delay d is assumed to be known, and can be obtained from its reaction curve experimentation. Notice that system (1) is asymptotically stable because of the physical phenomenon of air-conditioning systems.

The parameter estimation is devoted to using the recursive least-square (RLS) estimation method to identify the parameters of the system model (1), and the RLS estimation method with forgetting factor is expressed by

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) + K(k)(y(k) - \phi^{\mathrm{T}}(k)\hat{\theta}(k-1)) \\ K(k) &= P(k-1)\phi(k)(\lambda + \phi^{\mathrm{T}}(k)P(k-1)\phi(k))^{-1} \\ P(k) &= (I - K(k)\phi^{\mathrm{T}}(k))P(k-1)/\lambda \end{aligned} \quad (2)$$

where $\hat{\theta}(k) = [\hat{a}(k) \;\; \hat{b}_0(k) \;\; \hat{b}_1(k)]^{\mathrm{T}}$; $\phi^{\mathrm{T}}(k) = [y(k-1) \; u(k-d) \; u(k-d-1)]$; $K(k)$ is an error correct gain vector; $P(k)$ is an 3×3 symmetrical matrix; $P(0) = \rho \times I_3$ and ρ is a large and positive real number; λ is a forgetting factor; and $\lambda \neq 0 \in R^1$.

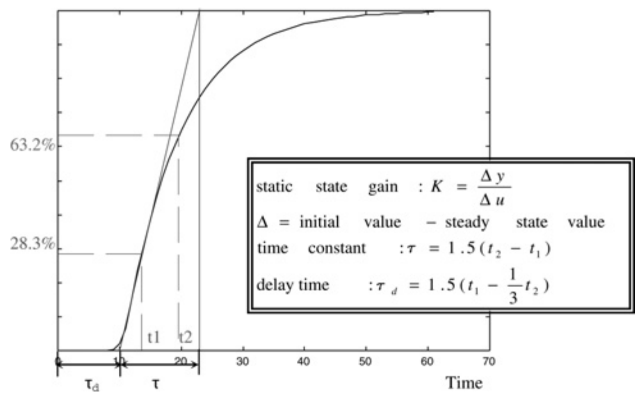

Fig. 1 *Characteristics of typical open-loop step response (the reaction curve) of air-conditioning systems*

Note that the RLS method must be used together with pseudo random binary testing signals, in order to achieve fast convergence of these estimated parameters. For details of the pseudo random binary testing signals, the reader is referred to [9].

Remark 1: On basis of the Abu-Ayyad and Dubay's work [20], the non-linear effect can be incorporated into the system parameters of the FOPDT model. From this point of view, the proposed control method adopts the similar idea to obtain the system parameters with the non-linear effects nearby fixed operating points, but the original discrete-time model (1) cannot be used to incorporate the non-linear effects for highly non-linear plant models.

3.2 Feedback loop design

On basis of the SGPC method proposed by the researchers [1, 2], this subsection will synthesise a feedback controller for system (1) with desired closed-loop characteristics. In doing so, system (1) is re-expressed by

$$A(z)y(k) = B(z)\Delta u(k) \tag{3}$$

where $\delta(z) = 1 - z^{-1}$, $A(z) = \delta(z)(1 - \hat{a}z^{-1})$, $B(z) = z^{-1}b(z)$, $b(z) = (\hat{b}_0 + \hat{b}_1 z^{-1})z^{-d+1}$ and $\Delta u(k) = \delta(z)u(k)$; $B(z)$ and $A(z)$ have no common unstable zeros. The aim of the controller design is to use the polynomial approach to finding the feedback controller with the control increment $\Delta u(k)$ based on the transfer function, $g(z) = B(z)/A(z)$, of the system model (3). As depicted in Fig. 2, the stabilising controller is parameterised as $\tilde{K}(z) = N(z)/M(z)$, where the two polynomials $N(z)$ and $M(z)$ are determined in the following equation

$$A(z)M(z) + B(z)N(z) = Q(z) \tag{4}$$

where $Q(z)$ is the desired stable characteristic polynomial of the closed-loop system, and both polynomials $N(z)$ and $M(z)$ are an admissible solution pair of (4). From Fig. 2, one obtains the following relationships

$$\frac{y(z)}{c(z)} = \frac{B(z)/[M(z)A(z)]}{1 + [B(z)N(z)]/[M(z)A(z)]} = B(z)/Q(z)$$

$$\frac{\Delta u(z)}{c(z)} = \frac{1/M(z)}{1 + [B(z)N(z)]/[M(z)A(z)]} = A(z)/Q(z)$$

where $c(z)$ represents the temperature command, or the temperature set point.

Since the two polynomials $A(z)$ and $B(z)$ are relatively co-prime and the closed-loop characteristic equation $Q(z)$ with desired poles is given, both polynomials $M(z)$ and $N(z)$ of the controller can be solved by the Sylvester equation. As a result, with the stabilising controller, the inner closed-loop system has the desired roots inside the unit circle, thus showing its asymptotical stability and desirable transient performance.

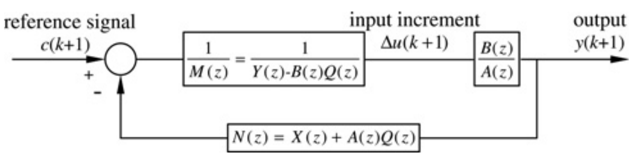

Fig. 2 *Feedback structure*

3.3 Output prediction

In order to obtain the output prediction, one rewrites the system model (3) as

$$A(z)y(k+1) = b(z)\Delta u(k) \tag{5}$$

and expresses the control signal in Fig. 2 by

$$\begin{aligned} M(z)\Delta u(k) &= c(k) - N(z)y(k) \\ &= c(k) - z^{-1}N(z)y(k+1) \end{aligned} \tag{6}$$

From Definitions 2 and 4, (4)–(6) can be rewritten in terms of Hankel matrices and convolution matrices

$$\mathbf{C}_b\mathbf{C}_{z^{-1}N} + \mathbf{C}_A\mathbf{C}_M = \mathbf{C}_Q \tag{7}$$

$$\mathbf{H}_A \underset{\leftarrow}{\mathbf{y}} + \mathbf{C}_A \underset{\rightarrow}{\mathbf{y}} = \mathbf{H}_b\Delta \underset{\leftarrow}{\mathbf{u}} + \mathbf{C}_b\Delta \underset{\rightarrow}{\mathbf{u}} \tag{8}$$

$$\mathbf{H}_M\Delta \underset{\leftarrow}{\mathbf{u}} + \mathbf{C}_M\Delta \underset{\rightarrow}{\mathbf{u}} = \underset{\rightarrow}{\mathbf{c}_0} - \mathbf{H}_{z^{-1}N} \underset{\leftarrow}{\mathbf{y}} - \mathbf{C}_{z^{-1}N} \underset{\rightarrow}{\mathbf{y}} \tag{9}$$

where $\mathbf{H}_A$, $\mathbf{H}_b$, $\mathbf{H}_M$, $\mathbf{H}_{z^{-1}N}$ are the Hankel matrices in $\mathbf{R}^{n\times n}$, respectively; $\mathbf{C}_A$, $\mathbf{C}_b$, $\mathbf{C}_M$, $\mathbf{C}_{z^{-1}N}$ and $\mathbf{C}_Q$ are, respectively, the convolution matrices in $\mathbf{R}^{n\times n}$; n is the output horizon and a positive integer. Moreover, the column vectors in (8) and (9) are

$$\underset{\rightarrow}{\mathbf{y}} = [y(k+1), \ldots, y(k+n)]^{\mathrm{T}}$$

$$\underset{\leftarrow}{\mathbf{y}} = [y(k), y(k-1), \ldots, y(k-n+1)]^{\mathrm{T}}$$

$$\Delta \underset{\rightarrow}{\mathbf{u}} = [\Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+n-1)]^{\mathrm{T}}$$

$$\Delta \underset{\leftarrow}{\mathbf{u}} = [\Delta u(k-1), \ldots, \Delta u(k-n)]^{\mathrm{T}}$$

$$\underset{\rightarrow}{\mathbf{c}_0} = [c(k), \ldots, c(k+n-1)]^{\mathrm{T}}$$

Theorem 1: Under the notation given above, one obtains

$$\underset{\rightarrow}{\mathbf{y}} = \mathbf{C}_Q^{-1}\mathbf{C}_b \underset{\rightarrow}{\mathbf{c}_0} - \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_2\Delta \underset{\leftarrow}{\mathbf{u}} \tag{10}$$

$$\Delta \underset{\rightarrow}{\mathbf{u}} = \mathbf{C}_Q^{-1}\mathbf{C}_A \underset{\rightarrow}{\mathbf{c}_0} - \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_4\Delta \underset{\leftarrow}{\mathbf{u}} \tag{11}$$

with

$$\mathbf{p}_1 = \mathbf{C}_Q^{-1}(\mathbf{C}_b\mathbf{H}_{z^{-1}N} + \mathbf{C}_M\mathbf{H}_A),$$

$$\mathbf{p}_2 = \mathbf{C}_Q^{-1}(\mathbf{C}_b\mathbf{H}_M - \mathbf{C}_M\mathbf{H}_b)$$

$$\mathbf{p}_3 = \mathbf{C}_Q^{-1}(\mathbf{C}_A\mathbf{H}_{z^{-1}N} - \mathbf{C}_{z^{-1}N}\mathbf{H}_A),$$

$$\mathbf{p}_4 = \mathbf{C}_Q^{-1}(\mathbf{C}_A\mathbf{H}_M + \mathbf{C}_{z^{-1}N}\mathbf{H}_b)$$

Proof: First, use the formula $\mathbf{C}_f\mathbf{C}_h = \mathbf{C}_h\mathbf{C}_f$ in Lemma 1 and (7), pre-multiply (8) by $\mathbf{C}_M$, and subtract the term of (9) by pre-multiplying $\mathbf{C}_b$, thereby yielding the vector of future outputs in (10). Next, pre-multiply (8) by $\mathbf{C}_{z^{-1}N}$, and add

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the term of (9) by pre-multiplying $\mathbf{C}_A$, and solve for the future control incremental vector in (11). □

3.4 Generalised predictive controller design

This subsection is devoted to developing the GPC based on the derived output prediction (10). Similar to the procedure of standard GPC design, the proposed generalised predictive controller decomposes the reference signal vector $\underset{\rightarrow}{\mathbf{c}_0}$ in $\mathbf{R}^n$ into the lead $\mathbf{R}^\mu$ vector $\underset{\rightarrow}{\mathbf{c}}$ and the followed $\mathbf{R}^{n-\mu}$ vector $\mathbf{c}^\infty$, that is

$$\underset{\rightarrow}{\mathbf{c}_0} = [\underset{\rightarrow}{\mathbf{c}}^{\mathrm{T}}, \mathbf{c}^{\infty\mathrm{T}}]^{\mathrm{T}}$$

where the former vector $\underset{\rightarrow}{\mathbf{c}}$ means the reference signal for transient responses, and this vector is to be determined. The latter vector $\mathbf{c}^\infty$ is for the steady-state response, and can be pre-determined by practical requirements. In the steady state, the output signal y is expected to equal the reference trajectory $\underset{\rightarrow}{\mathbf{r}} = [r(k+1), \ldots, r(k+n)]^{\mathrm{T}}$, and the steady-state value of the term $N(z)y$ should be equal to the reference signal c. Therefore the steady-state vector is chosen by

$$\mathbf{c}^\infty = \mathbf{C}_{\overline{N}} \underset{\rightarrow}{\mathbf{r}}$$

where $\mathbf{C}_{\overline{N}}$ denotes the matrix formed by the last $n-\mu$ rows of $\mathbf{C}_N$. For the case of step set point r, $\mathbf{C}_{\overline{N}}$ can be replaced by the last $n-\mu$ rows of the identity matrix multiplied by the DC gain of the polynomial $N(z)$. Thus, it is assumed that only the first μ future values of the reference signal can be set as arbitrary values.

Theorem 2: For a reference horizon μ, the predicted values of the output in (10) and control increment in (11) are given by

$$\underset{\rightarrow}{\mathbf{y}} = \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \mathbf{y}^\infty - \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_2 \Delta \underset{\leftarrow}{\mathbf{u}} \tag{12}$$

$$\Delta \underset{\rightarrow}{\mathbf{u}} = \boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \Delta\mathbf{u}^\infty - \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_4 \Delta \underset{\leftarrow}{\mathbf{u}} \tag{13}$$

where

$$\begin{aligned}
\mathbf{p}_1 &= \mathbf{C}_Q^{-1}(\mathbf{C}_b \mathbf{H}_{z^{-1}N} + \mathbf{C}_M \mathbf{H}_A),\\
\mathbf{p}_2 &= \mathbf{C}_Q^{-1}(\mathbf{C}_b \mathbf{H}_M - \mathbf{C}_M \mathbf{H}_b),\\
\mathbf{p}_3 &= \mathbf{C}_Q^{-1}(\mathbf{C}_A \mathbf{H}_{z^{-1}N} - \mathbf{C}_{z^{-1}N} \mathbf{H}_A)\\
\mathbf{p}_4 &= \mathbf{C}_Q^{-1}(\mathbf{C}_A \mathbf{H}_M + \mathbf{C}_{z^{-1}N} \mathbf{H}_b)\\
\mathbf{y}^\infty &= \mathbf{M}_{Q^{-1}b}\mathbf{C}^\infty, \quad \Delta\mathbf{u}^\infty = \mathbf{M}_{Q^{-1}A}\mathbf{c}^\infty
\end{aligned}$$

Proof: Given μ and based on the definitions of $\boldsymbol{\Gamma}_f$ and $\mathbf{M}_f$, the vector $\mathbf{C}_Q^{-1}\mathbf{C}_b \underset{\rightarrow}{\mathbf{c}_0}$ in (10) and the vector $\mathbf{C}_Q^{-1}\mathbf{C}_A \underset{\rightarrow}{\mathbf{c}_0}$ in (11) can be decomposed into the output prediction vector $\underset{\rightarrow}{\mathbf{c}}$ and the steady-state vector $\mathbf{c}^\infty$, that is

$$\begin{aligned}
\mathbf{C}_Q^{-1}\mathbf{C}_b \underset{\rightarrow}{\mathbf{c}_0} &\triangleq \mathbf{C}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}_0} = [\,\boldsymbol{\Gamma}_{Q^{-1}b} \quad \mathbf{M}_{Q^{-1}b}\,] \begin{bmatrix} \underset{\rightarrow}{\mathbf{c}} \\ \mathbf{c}^\infty \end{bmatrix}\\
&= \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \mathbf{M}_{Q^{-1}b}\mathbf{c}^\infty \triangleq \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \mathbf{y}^\infty
\end{aligned}$$

$$\begin{aligned}
\mathbf{C}_Q^{-1}\mathbf{C}_A \underset{\rightarrow}{\mathbf{c}_0} &\triangleq \mathbf{C}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}_0} = [\,\boldsymbol{\Gamma}_{Q^{-1}A} \quad \mathbf{M}_{Q^{-1}A}\,] \begin{bmatrix} \underset{\rightarrow}{\mathbf{c}} \\ \mathbf{c}^\infty \end{bmatrix}\\
&= \boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \mathbf{M}_{Q^{-1}A}\mathbf{c}^\infty \triangleq \boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \Delta\mathbf{u}^\infty
\end{aligned}$$

□

The optimal control of the proposed GPC is based on the minimisation of the following performance

$$J = \| \underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{y}} \|^2 + \sigma \|\Delta \underset{\rightarrow}{\mathbf{u}}\|^2 \tag{14}$$

The optimal control law satisfies the following result.

Theorem 3: The optimal signal of the proposed GPC law is expressed by

$$M(z)\Delta u(k) = c(k) - N(z)y(k) \tag{15}$$

where

$$c(k) = \mathbf{p}_{\mathbf{r}}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} + \mathbf{p}_{\mathbf{y}}^{\mathrm{T}} \underset{\leftarrow}{\mathbf{y}} + \mathbf{p}_{\mathbf{u}}^{\mathrm{T}} \Delta \underset{\leftarrow}{\mathbf{u}} \tag{16}$$

$$\begin{aligned}
\mathbf{p}_{\mathbf{r}}^{\mathrm{T}} &= \mathbf{e}_1^{\mathrm{T}}[\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} + \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A}]^{-1}\\
&\quad \times (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} - \boldsymbol{\Gamma}_{Q^{-1}b}^{T}\mathbf{M}_{Q^{-1}b}\mathbf{C}_{\overline{N}} - \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\mathbf{M}_{Q^{-1}A}\mathbf{C}_{\overline{N}})\\
\mathbf{p}_{\mathbf{y}}^{\mathrm{T}} &= \mathbf{e}_1^{\mathrm{T}}[\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} + \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A}]^{-1}\\
&\quad \times (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}\mathbf{p}_1 + \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\mathbf{p}_3)\\
\mathbf{p}_{\mathbf{u}}^{\mathrm{T}} &= \mathbf{e}_1^{\mathrm{T}}[\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} + \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A}]^{-1}\\
&\quad \times (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}\mathbf{p}_2 + \sigma\boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}\mathbf{p}_4)
\end{aligned} \tag{17}$$

Proof: Substituting the reference trajectory $\underset{\rightarrow}{\mathbf{r}}$, the output prediction vector $\underset{\rightarrow}{\mathbf{y}}$ in (12) and the future control vector $\Delta \underset{\rightarrow}{\mathbf{u}}$ in (13) into the performance cost J in (14), one finds the optimal vector $\underset{\rightarrow}{\mathbf{c}}$ satisfying $(\partial J/\partial \underset{\rightarrow}{\mathbf{c}}) = 0$. Hence

$$\begin{aligned}
J &= \| \underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{y}} \|^2 + \sigma \|\Delta \underset{\rightarrow}{\mathbf{u}}\|^2\\
&= (\underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{y}})^{\mathrm{T}}(\underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{y}}) + \sigma(\Delta \underset{\rightarrow}{\mathbf{u}})^{\mathrm{T}}(\Delta \underset{\rightarrow}{\mathbf{u}})\\
&= (\underset{\rightarrow}{\mathbf{r}} - \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} - \mathbf{y}^\infty + \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} + \mathbf{p}_2 \Delta \underset{\leftarrow}{\mathbf{u}})^{\mathrm{T}}\\
&\quad \times (\underset{\rightarrow}{\mathbf{r}} - \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} - \mathbf{y}^\infty + \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} + \mathbf{p}_2 \Delta \underset{\leftarrow}{\mathbf{u}})\\
&\quad + \sigma(\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \Delta\mathbf{u}^\infty - \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_4 \Delta \underset{\leftarrow}{\mathbf{u}})^{\mathrm{T}}\\
&\quad \times (\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \Delta\mathbf{u}^\infty - \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_4 \Delta \underset{\leftarrow}{\mathbf{u}})\\
&= \underset{\rightarrow}{\mathbf{r}}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} - \underset{\rightarrow}{\mathbf{c}}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}}(\underset{\rightarrow}{\mathbf{r}} - \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} - \mathbf{y}^\infty + \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} + \mathbf{p}_2 \Delta \underset{\leftarrow}{\mathbf{u}})\\
&\quad - \underset{\rightarrow}{\mathbf{r}}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \mathbf{y}^{\infty\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}}\\
&\quad - \underset{\leftarrow}{\mathbf{y}}^{\mathrm{T}} \mathbf{p}_1^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} - \Delta \underset{\leftarrow}{\mathbf{u}}^{\mathrm{T}} \mathbf{p}_2^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \cdots\\
&\quad + \sigma \underset{\rightarrow}{\mathbf{c}}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}}(\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \Delta\mathbf{u}^\infty - \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \mathbf{p}_4 \Delta \underset{\leftarrow}{\mathbf{u}})\\
&\quad + \sigma\Delta\mathbf{u}^{\infty\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} - \sigma \underset{\leftarrow}{\mathbf{y}}^{\mathrm{T}} \mathbf{p}_3^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}}\\
&\quad - \sigma\Delta \underset{\leftarrow}{\mathbf{u}}^{\mathrm{T}} \mathbf{p}_4^{\mathrm{T}}\boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \cdots
\end{aligned}$$

The optimal GPC control laws (15–17) are obtained from finding its minimum with respect to the optimal vector $\underset{\rightarrow}{\mathbf{c}}$, that is,

$$
\begin{aligned}
\frac{\partial J}{\partial \underset{\rightarrow}{\mathbf{c}}} = & -\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} + \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} \underset{\rightarrow}{\mathbf{c}} + \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{y}^{\infty} - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_1 \underset{\leftarrow}{\mathbf{y}} \\
& - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_2 \Delta \underset{\leftarrow}{\mathbf{u}} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A} \underset{\rightarrow}{\mathbf{c}} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \Delta \mathbf{u}^{\infty} \\
& - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_3 \underset{\leftarrow}{\mathbf{y}} - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_4 \Delta \underset{\leftarrow}{\mathbf{u}} = 0
\end{aligned}
$$

which yields

$$
\begin{aligned}
\underset{\rightarrow}{\mathbf{c}} = & [\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A}]^{-1} \\
& \times \{(\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{y}^{\infty} - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \Delta \mathbf{u}^{\infty}) \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_1 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_3) \underset{\leftarrow}{\mathbf{y}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{T} \mathbf{p}_2 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{T} \mathbf{p}_4) \Delta \underset{\leftarrow}{\mathbf{u}} \} \\
= & [\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A}]^{-1} \\
& \times \{(\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{M}_{Q^{-1}b} \mathbf{C}_{\overline{N}} \underset{\rightarrow}{\mathbf{r}} - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{M}_{Q^{-1}A} \mathbf{C}_{\overline{N}} \underset{\rightarrow}{\mathbf{r}}) \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_1 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_3) \underset{\leftarrow}{\mathbf{y}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_2 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_4) \Delta \underset{\leftarrow}{\mathbf{u}} \} \\
= & [\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A}]^{-1} \\
& \times \{(\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{M}_{Q^{-1}b} \mathbf{C}_{\overline{N}} - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{M}_{Q^{-1}A} \mathbf{C}_{\overline{N}}) \underset{\rightarrow}{\mathbf{r}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_1 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_3) \underset{\leftarrow}{\mathbf{y}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_2 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_4) \Delta \underset{\leftarrow}{\mathbf{u}} \}
\end{aligned}
$$

and

$$
\begin{aligned}
c(k) = & [1 \quad 0 \quad \cdots \quad 0][c(k) \quad c(k+1) \quad \cdots \quad c(k+\mu-1)]^{\mathrm{T}} \\
= & \mathbf{e}_1^{\mathrm{T}} \underset{\rightarrow}{\mathbf{c}} = \mathbf{e}_1^{\mathrm{T}} [\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}b} + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \boldsymbol{\Gamma}_{Q^{-1}A}]^{-1} \\
& \times \{(\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} - \boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{M}_{Q^{-1}b} \mathbf{C}_{\overline{N}} - \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{M}_{Q^{-1}A} \mathbf{C}_{\overline{N}}) \underset{\rightarrow}{\mathbf{r}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_1 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_3) \underset{\leftarrow}{\mathbf{y}} \\
& + (\boldsymbol{\Gamma}_{Q^{-1}b}^{\mathrm{T}} \mathbf{p}_2 + \sigma \boldsymbol{\Gamma}_{Q^{-1}A}^{\mathrm{T}} \mathbf{p}_4) \Delta \underset{\leftarrow}{\mathbf{u}} \} \\
= & \mathbf{P}_{\mathbf{r}}^{\mathrm{T}} \underset{\rightarrow}{\mathbf{r}} + \mathbf{P}_{\mathbf{y}}^{\mathrm{T}} \underset{\leftarrow}{\mathbf{y}} + \mathbf{P}_{\mathbf{u}}^{\mathrm{T}} \Delta \underset{\leftarrow}{\mathbf{u}}
\end{aligned}
$$

From (16), it follows that

$$
\begin{aligned}
c(k+1) &= p_r(z) r(k+n) + p_u(z) \Delta u(k) + p_y(z) y(k+1) \\
&= p_r(z) r(k+n) + z^{-1} p_u(z) \Delta u(k+1) + p_y(z) y(k+1)
\end{aligned} \tag{18}
$$

and

$$
p_r(z) = P\{\mathbf{p}_{\mathbf{r}}\}, \quad p_u(z) = P\{\mathbf{p}_{\mathbf{u}}\}, \quad p_y(z) = P\{\mathbf{p}_{\mathbf{y}}\}
$$

where the operator $P\{x\}$ maps the vector x into its corresponding polynomial [1, 2]. □

Remark 2: In this section, no dual control approach is used, whereas an indirect self-tuning scheme in [9] is adopted to synthesise the predictive controller based on the minimisation of the predictive performance index J in (14) under the assumption that the system parameters in (1) can be on-line successfully estimated by the RLS estimation method with forgetting factor. Since the closed-loop identification condition of the plant model (1) holds, the system parameters in (1) will converge to their true values. Once the RLS estimator works as designed, the predictive controller will generate the optimal control signals in (15) for the process, and the control signals will be further processed by the stabilising controller developed in Section 3.2 such that the appropriate control actions are taken to actuate the process output to achieve the desired tracking signals.

3.5 Real-time adaptive control algorithm

In this subsection, a real-time control algorithm including the RLS parameter estimator is employed to recursively update the system's parameters and then the adaptive stable generalised predictive control (ASGPC) controller's parameters based on the certainty equivalent principle. Fig. 3 depicts the system structure of the GPC law without the RLS parameter estimation method. Fig. 4 presents the structure of the proposed adaptive predictive control method. The computations at each time instant k can be described as the following steps:

Step 1: Collect and record the measured data of the plant output $y(k)$ and input $u(k)$.
Step 2: Obtain the initial model parameters a, b_0 and b_1 of the system using (1).
Step 3: Set $Q(z)$, d, n, μ, σ and λ.
Step 4: Measure the current process output $y(k)$.

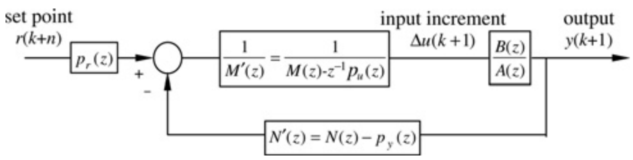

Fig. 3 *SGPC control feedback structure*

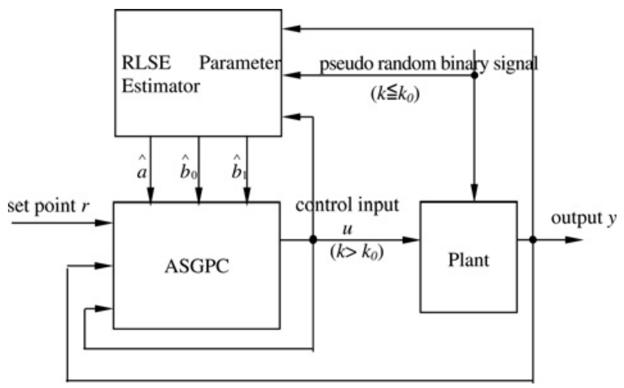

Fig. 4 *ASGPC control structure with the RLS parameter estimator*

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Step 5: Estimate the model parameters $\hat{a}$, $\hat{b}_0$, $\hat{b}_1$, using (2) and pseudo random binary signals (PRBS) when time instant k is less than k_0 where k_0 can be selected according to the transient response of the system.
Step 6: Update the model parameters using (2) when time instant k is greater than k_0.
Step 7: Compute the control signal $u(k)$ by (15–17).
Step 8: Output the control signal $u(k)$ to the controlled plant.
Step 9: Repeat Steps 4–8.

Once the estimated parameters of system (1) have reached steady-state values, it is easy to show the stability of the resulting control system; this is because the inner loop is guaranteed stable by the proposed stabilising controller. In what follows, the experimental results of the controlled process will be shown to accurately track the desired outputs and remain on set points.

4 Simulations, experimental results and discussion

This section emphasises on examining the performance of the proposed ASGPC for temperature control of an experimental air-conditioning system with VWV adjustment and variable-frequency motor control. In the sequel, particular effort is paid to first describe the experimental air-conditioning system, and then to illustrate the feasibility and applicability of the proposed control method by conducting computer simulations and experimental results.

4.1 Experiment setup

Fig. 5 describes the experimental air-conditioning system with an industrial personal computer (IPC) as a main controller, and Fig. 6 shows the block diagram of the overall control system with the ASGPC controller implemented by the IPC. In Fig. 5, the variable-frequency induction motor with its drive circuitry takes appropriate control signals, ranging from 0 to 10 Vdc, to produce the corresponding variable rotational speeds from 300 to 1800 rpm, in order to drive the pump in the water chiller machine to achieve heat exchange. In the heat exchanger, the low-temperature ice water passes through the cold store room and takes out of the heat inside the room with the air circulation driven by the fan. Hence, the temperature in the store room decreases in proportional to flow rate of the passing water chiller if the speed of the fan is kept constant; if the flow rate becomes fast, then the temperature in the store room will decrease quickly. In order to accomplish the temperature set-point tracking, the temperature sensor PT100 is employed to measure the temperature inside the cold store room via its temperature-to-resistance transducer. The transducer is of high-performance linearity and reliability, and its output voltage ranges from 0 to 5 Vdc.

The signal taken by the temperature sensor PT100 is digitised and converted into its digital form by the data acquisition card, called RT-DAC3; the digital signal is then send back to the IPC for further control purpose. The main task of the IPC controller is to periodically read the temperature signal, to compare it with the reference temperature trajectory, to execute the proposed real-time control algorithm, and, afterwards, to output the control signal to the data acquisition card RT-DAC3. The resulting control signal is transferred into a physical control signal ranging from 0 to 10 Vdc, thereby altering the speed of the variable-frequency controlled induction motor and then changing the flow rate of the chilled water. The real-time control algorithm in Section 3.5 was implemented by the IPC controller using the standard C programming language, and the data acquisition card RT-DAC3 was employed to collect all the control signals and the controlled temperatures inside the cold store room. As a result, the temperature of the cold store room was maintained at the desired temperature by the proposed control algorithm.

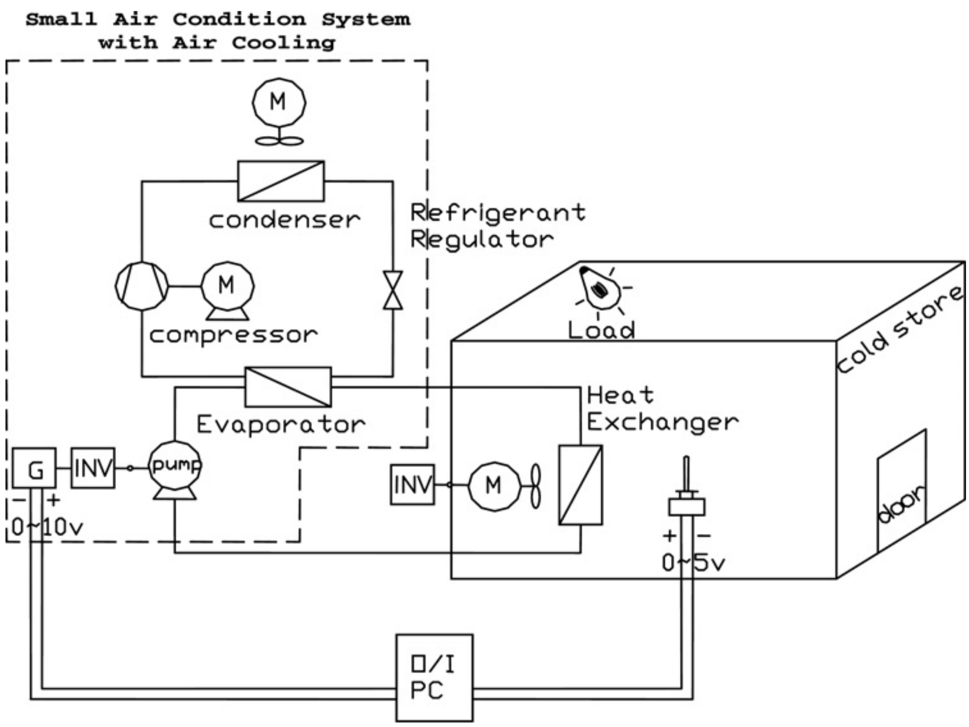

Fig. 5 *Schematic of the experimental air-conditioning temperature control system*

doi: 10.1049/iet-cta.2010.0085

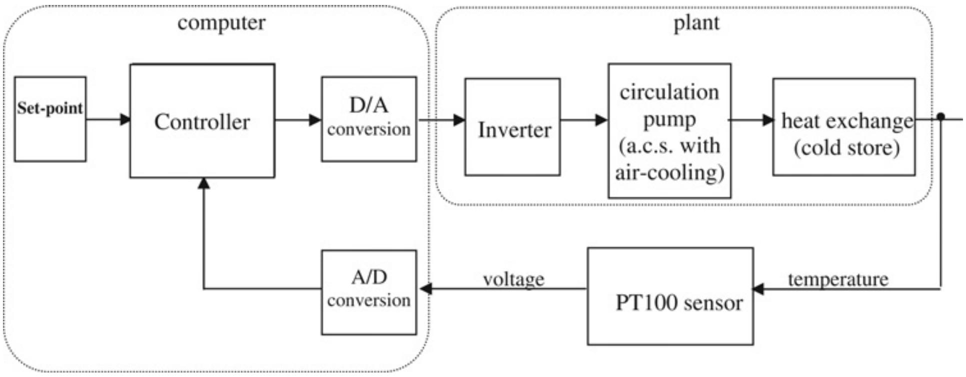

Fig. 6 *Block diagram of the proposed ASGPC control system*

4.2 Computer simulations and discussion

Before proceeding with the real-time control experiment, it is necessary to conduct computer simulations to verify the feasibility of the proposed control approach. To efficiently perform the numerical simulation, this subsection uses Matlab software to do this task. The key parameters are set as follows; the output horizon n is 3, the reference horizon μ is 1, the weight σ for control increment in the performance cost is 0.02, the forgotten factor λ of the RLSE method is 0.999, the closed-loop characteristic polynomial is set by $Q(z) = 1 - 1.5z^{-1} + 0.75z^{-2} - 0.125z^{-3}$, and the simulation time is 400 s. In order to satisfy the persistent excitation conditions for the RLS method, PRBS are adopted for the parameter estimation in the first ten samples during simulation, thus accelerating parameter convergence. In doing so, the real control signal is set as 3.75 Vdc when the PRBS is unity, and the signal is 2.71 Vdc when the PRBS is null.

To avoid too slow rotational speeds of the pump connected to the variable-frequency controlled induction motor, the control signals are limited to the range from 1.67 to 10 Vdc, thereby resulting in that the lowest rotational speed of the pump is 300 rpm (its working frequency is 10 Hz), and its highest rotation speed is 1800 rpm. Worthy of mention is that too slow speeds of the pump will cause the refrigeration capability of the system drop seriously, and even turn off the compressor of air-conditioning system. In addition, a dither disturbance is added into the simulation in order to facilitate the estimation process. Fig. 7 depicts the simulated result comparisons of the temperature tracking responses and the disturbance rejection capability with a step disturbance with an amplitude 0.05 in the 250th sampling instant between the proposed ASGPC, SGPC and PID controllers. The results reveal that, although the settling times of the proposed ASGPC are slightly slower than those of the SGPC under set point and disturbance changes, the ASGPC has less oscillatory control and output signals than the SGPC does after a constant disturbance is applied, thereby showing that the ASGPC has a better disturbance rejection ability. In addition, in comparison with the PID controller with the three-term parameters of $k_p = 0.012$, $k_i = 0.00008$ and $k_d = 24$. Notice that the three-term parameters of the PID controller are obtained as the

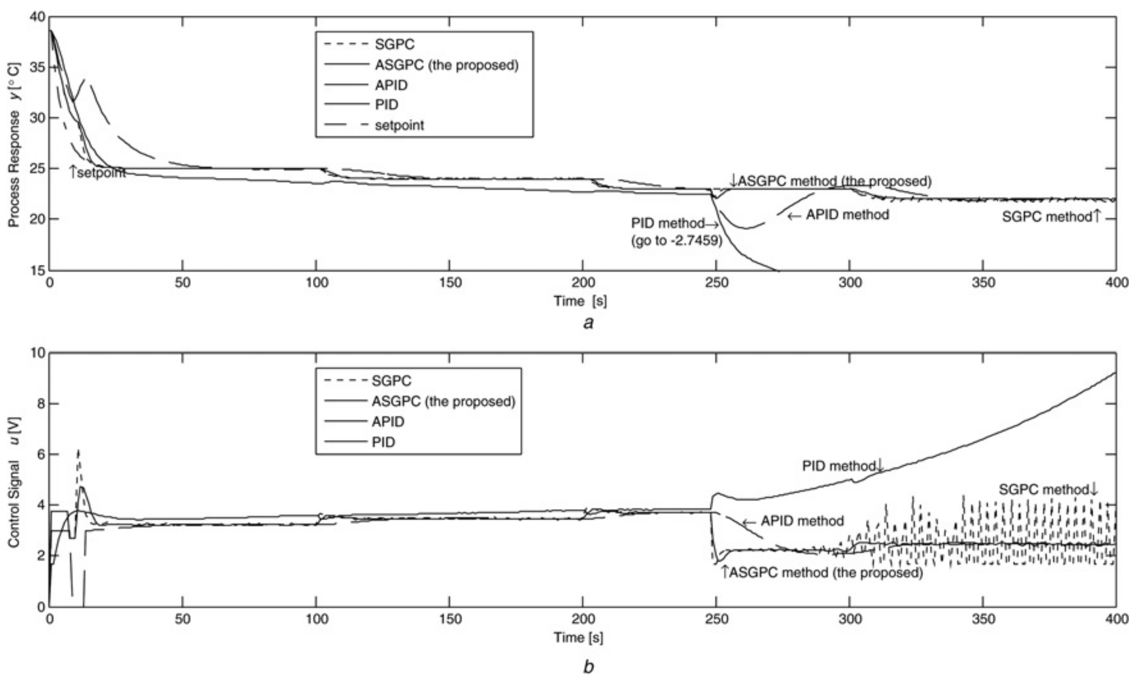

Fig. 7 *Simulated temperature control in air-conditioning system with a step disturbance with an amplitude 0.05 in the 250th sampling instant*

a Set-point tracking responses y of SGPC (dot) [1, 2], the proposed ASGPC (solid), the APID (dash) and the PID (solid) controllers
b Control signals u of SGPC (dot) [1, 2], the proposed ASGPC (solid), APID (dash) and PID (solid) controllers

doi: 10.1049/iet-cta.2010.0085

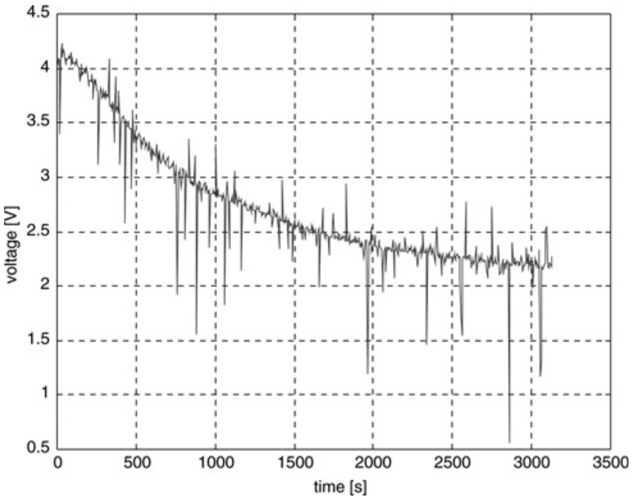

Fig. 8 *Actual open-loop step response of the experimental system*

following steps. First, get the actual open-loop step response of the experimental system in Fig. 8. Second, use the reaction curve method and the Ziegler–Nichols tuning method to obtain analog three-term control parameters, k_p, k_i and k_d. Finally, obtain the discrete controller by $D(z) = (u(z)/e(z)) = k_p + k_i(T/2)((z+1)/(z-1)) + k_d((z-1)/(Tz))$ and then conduct computer simulations to obtain the output response. The simulation results in Fig. 7 indicate that the PID control is poor in set-point tracking responses, and even unstable after the applied constant disturbance at the 250th sampling instant. Through the numerical scenarios, the proposed ASGPC has been shown to outperform the SGPC and the PID controller in tracking reference temperature trajectories and disturbance rejection.

Note that if the three-term control parameters of the used PID controller are determined via the Ziegler–Nichols tuning method, then it is true that, in comparison with the proposed method, the fixed-parameter PID control indeed has poor control performance in presence of external disturbances. Generally speaking, if a different parameter tuning method for PID control is used, then the result would be changed. This is also true for adaptive PID (APID) controllers because different adaptive rules for adjusting PID control parameters will result in different control performance. For performance comparison, the APID controller in [28], which is a fuzzy supervised generalised predictive PID controller, is used to conduct the same computer simulations shown in Fig. 7. The APID controller combines the RLSE method, a generalised predictive PID controller, and uses a fuzzy supervisor to on-line tune the three-term parameters of the PID controller. Through simulation results in Fig. 7, the proposed control method outperforms the APID method because the proposed controller has faster transient responses under set-point changes and external step disturbances.

4.3 Experimental results and discussion

The following two experiments were conducted to investigate the performance of the proposed ASGPC controller. In both experiments, the reference temperature trajectory was set to the most comfortable dry bulb temperatures from 22 to 25°C, and all the controller settings are the same to those in Section 4.2.

The first experiment was carried out to collect the input–output data of the open-loop step response by directly sending a control signal of 10 Vdc to the motor driver. Fig. 8 displays the open-loop step response where the sampling time is 30 s, and the output signals range from 1 to 5 Vdc, which are proportional to the corresponding temperatures from 0 to 50°C. From the measured data, the time delay of the system was determined and the three parameters of the overall system were recursively estimated by the RLS method where $y(k)$ denotes the difference between the given temperature and the measured temperature. Afterwards, it is expressed by the following the first-order system model with time-delay d

$$a(z)y(k) = b_p(z)z^{-d}u(k) + \xi(k) \tag{19}$$

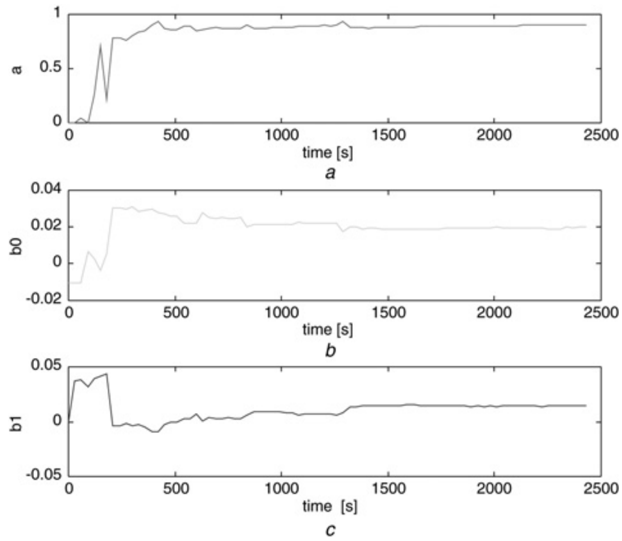

Fig. 9 *Convergent time histories of the three estimated parameters in the real-time temperature control experiment in air-conditioning system*

a Model parameter a
b Model parameter b_0
c Model parameter b_1

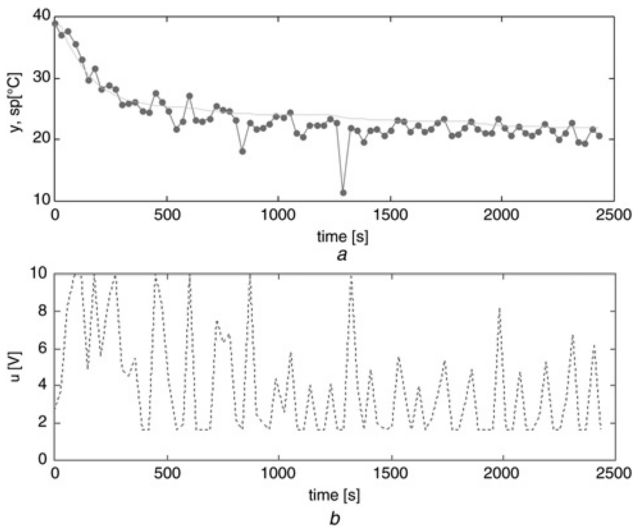

Fig. 10 *Experimental results of the real-time temperature control in air-conditioning system with the time locus*

a Set-point tracking response y (dot-dash) and set point (solid)
b Control signal u(dot)

where $a(z) = 1 - 0.8992z^{-1}$, $b_p(z) = 0.0195 + 0.0144z^{-1}$, $d = 1$, $\xi(k)$ is the discrete-time white Gaussian noise with zero mean.

The second experiment was performed to examine the performance of the proposed ASGPC control method. Figs. 9 and 10, respectively, show the behaviour of the estimated parameters and the experimental results of the controlled air-conditioning system. Through the experimental results, the proposed adaptive generalised predictive control has been shown capable of giving satisfactory temperature tracking performance for the experimental air-conditioning system.

5 Conclusions

This paper has developed a systematic design approach for adaptive stable generalised predictive temperature control of a class of air-conditioning system. The empirical model of the air-conditioning system is approximately established based on the open-loop step response method, that is, the reaction curve method in [8], and its model parameters are on-line estimated by the recursive least square method with forgetting factor. With the identified model, an improved SGPC controller has been established by letting the closed-loop characteristic equation be a desired one, in order to achieve guaranteed stability and desired control performance. The proposed SGPC method is more general and practical than the approaches presented by Rossiter *et al.* [1] and Kouvaritakis *et al.* [2]. The feasibility and effectiveness of the proposed AGPC method have been successfully exemplified by conducting several computer simulations on the air-conditioning system. After simulations, the real-time control algorithm has been implemented for temperature control of the air-conditioning system for the cold store room. Although the measurement noise and long sampling period resulted in fluctuating output responses during experimentation, the experimental results have shown that the proposed control method is capable of giving satisfactory temperature control performance for the air-conditioning system. An interesting topic for future research would be to include the FOPDT model in [20] to improve the proposed ASGPC method and to extend the proposed control method to more general HVAC systems.

6 Acknowledgments

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