System Identification Based on Particle Swarm Optimization Algorithm

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Abstract

Regarding the problem of conducting system identification with sample data, a new identification method that typical mathematical models constitute a system model through intercombination is studied. The basic theory of this method is: transform the problem of system structure identification into a problem of combinatorial optimization, and then use the particle swarm optimization (PSO) algorithm to realize both structure and parameter identification of the system at the same time. In order to further enhance the identification capability of the PSO algorithm, an improved PSO algorithm is used for system identification. The identification algorithm given in this paper has been proved to be reasonable and effective by results of analog simulation and of actual system identification.

1. Introduction

The system identification [1] is to determine a model that is equivalent to the identified system from a group of given models based on the input and output data. The purpose of system identification is to establish a mathematical model and use the results of system identification to resolve practical problems. System identification using sample data has been a major study them in various fields. Despite being relatively mature and well-developed, conventional system identifica-tions still have many limitations[2]. Although such shortcomings of conventional system identification methods may be solved to a certain extent by some new methods for system identification, such as the system identifications based on neural network and based on genetic algorithm, these new methods typically fail to properly handle system identification when the structure and parameters are completely unknown. With regard to this problem, document [3] offers a new method for identifying system model based on typical models and sample data, thus the problem of system identification is converted into a problem of combinatorial optimization. Then we use the genetic algorithm to solve this problem. The realization process of the genetic algorithm involves bit string coding, which has both binary numbers and real numbers. This brings difficulties for the design of genetic operations such as crossover, mutation and selection, thus influencing the efficiency of the algorithm. The PSO algorithm is an evolutionary computing technique [4] put forward by Kennedy, Eberhard et.al. in 1995. The PSO algorithm is based on the simulation of predation by birds and the thought of swam intelligence. Compared to conventional optimization methods, it has excellent optimized performances and characteristics, and has been extensively used in such fields as the optimization of complication function, neural network training, combinatorial optimization and pattern recognition. The capability of the PSO algorithm to solve the problem of combinatorial optimization and parameter optimization provides an important way for system identification.

This paper puts forward the opinion that typical mathematical models constitute a system model through intercombination and then the PSO algorithm can be used to realize the structure and parameter identification of the system. A particle swarm algorithm with stronger global search capability, which will change the inertia weight dynamically, is used to improve the accuracy of identification. The simulation experiment has verified the rationality and effectiveness of methods in this paper.

2. Problem description for system identification

The system identification [3] discussed in this paper is to select from multiple sub-models a combination of sub-models which can fit the sample data in an optimal way, and determine the parameters of this model.

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For static systems with multiple input and single output, provided that y is an observable output variable of the system, and it may be influenced by m input variables $x_1, x_2, \cdots, x_m$, n groups of sample data obtained from the system can be described as follows:

$$(y_i, x_{1i}, x_{2i}, \cdots, x_{ji}, \cdots, x_{mi})$$

Where: x_{ji} represents the sample data No. j of the group No.i, y_i is the output value of the group No.i. $j = 1, 2, \cdots, m$; $i = 1, 2, \cdots, n$

The purpose of system identification is to , through the sample data, find out variables that influence the system output and how these variables influence the system.

Definition 1 Provided that the model obtained from sample data is the sample data model and that this model is superposed by all possible sub-models, while the sub-model is comprised of meta-models (typical mathematical model).

Definition 2 Provided that the single variable x_i exerts influence on system output in the form of $f(x_i)$, then $f(x_i)$ is called single-variable meta-model.

Provided that N_1 is the number of single-variable meta-model, considering that the input variables can influence the output variable in the form of various models, then the sample data model can be described as:

$$y = p_0 + \sum_{k=1}^{N_1} \sum_{i=1}^{m} f_k(x_i)$$

Where, p_0 is the constant term.

Definition 3 Provided that there are two variables x_i , x_j influencing system output in the form of $f(x_i, x_j), x_i \neq x_j$, where $f(x_i, x_j)$ can not be decomposed into the form of $f(x_i, x_j)$, then $f(x_i, x_j)$ can be called double-variable meta-model.

Provided that N_2 is the number of double-variable meta-model and considering various possible forms of combination of all input variables, the sample data model can be described as:

$$y = p_0 + \sum_{k=1}^{N_1} \sum_{i=1}^{m} f_k(x_i) + \sum_{k=1}^{N_2} \sum_{i=1}^{m} \sum_{\substack{j=1 \\ i \neq j}}^{m} f_k(x_i, x_j)$$

The general form of data sample model can be described as:

$$y = p_0 + \sum_{k=1}^{N} M_k(x, p_{k1}, p_{k2}, \cdots, p_{km_k})$$

Where p_0 is the constant term, N is the number of sub-models, $M_k(x, p_{k1}, p_{k2}, \cdots, p_{km_k})$ is the sub-model No.k comprised of meta-models and variables, where x can represent either single variable or multiple-variable, $k = 1,2,\cdots,N$; m_k is the number of parameters in the model M_k , p_{kj} is parameter No.j in $M_k(x, p_{k1}, p_{k2}, \cdots, p_{km_k})$, $j = 1,2,\cdots,m_k$.

The sample data model consists of sub-models, which are comprised of meta-models and independent variables, so the selection of meta-model plays a decisive role in system identification. During the simulation research, the following typical meta-models are selected in this paper:

(1) Linear model $y = bx$

(2) Exponential function model $y = ae^{bx}$

(3) Negative exponential function model $y = ae^{\frac{b}{x}}$

(4) Power function model $y = ax^b$

(5) Logarithmic function model $y = a\ln(b + x)$

(6) Gompertz model $y = ka^{b^x}$

(7) Periodic function model $y = a\sin(b + cx)$

(8) Hyperbola function model $y = \frac{a}{b + x}$

(9) S pattern model $y = \frac{1}{a + be^{-cx}}$

(10) Logistic model $y = \frac{1}{a + bc^x}$

(11) CobbDouglas production function

$$y = a x_1^{b_1} x_2^{b_2} \cdots x_k^{b_k}$$

3. System identification based on the PSO

3.1. Basic particle swarm optimization (BPSO) algorithm

The PSO algorithm is an algorithm of global random optimization. In PSO algorithm, each solution of the optimization problem corresponds to a bird in the search space, or called “particle”. All particles have a fitness determined by the optimized function. In the mean time, each particle has their own speed to determine the direction of flying and distance [5]. PSO is first initialized into a swarm of random particles (random solutions), and by following current optimal particle, all particles then search in the solution space until the optimal solution is found. During each iteration process, particle updates itself by tracing two “extremums”: first is the optimal solution that the particle has searched out so far, and it is called individual extremum p; second is the optimal solution that the whole particle swarm has searched out so far, and it is called global extremum g. When these two optimal values are found, each particle updates its own

velocity and position [4] according to the following formulas:

$$v_{ij}(t+1) = v_{ij}(t) + c_1 r_1 (p_{ij} - x_{ij}(t)) + c_2 r_2 (g_j - x_{ij}(t)) \quad (1)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (2)$$

Where $\mathbf{v_{ij}(t)}$ represents component of dimension No.j of velocity vector of particle i at time t, $\mathbf{x_{ij}(t)}$ represents component of dimension No.j of the position vector of particle i at time t, p_{ij} represents component of dimension No.j of the historically optimal position vector of particle i at time t, g_j represents component of dimension No.j of the historically optimal position vector of particle swarm at time t, $\mathbf{r_1}$,$\mathbf{r_2}$ are two random numbers between 0 and 1, and c_1 , c_2 represent group cognitive coefficient.

3.2. **Improved particle swarm optimization (IPSO) algorithm**

In practical application, the BASO algorithm is liable to occur problems such as premature convergence and poor capability of local optimization. In order to overcome the problem of premature convergence and balance the global and local search capability of the algorithm, researchers have made a series of improvement for the BPSO algorithm in succession [6-8]. Reference document [6] introduces the inertia weight w for the velocity term in formula (1), i.e.

$$v_{ij}(t+1) = w v_{ij}(t) + c_1 r_1 (p_{ij} - x_{ij}(t)) + c_2 r_2 (g_j - x_{ij}(t)) \quad (3)$$

Ren et.al. [8] put forward the conception of "change rate of focusing distance", and expressed the inertia weight w in formula (3) as the function of the change rate of focusing distance, enabling the algorithm to be of dynamic self-adaptivity. The change rate of focusing distance of the particle currently is defined as:

$$k = \frac{MaxDist - MeanDist}{MaxDist} \quad (4)$$

Where:

$$MeanDist = \frac{(\sum_{i=1}^{N} \sqrt{\sum_{j=1}^{D} (g_j - p_{ij})^2})}{N},$$

$$MaxDist = \max_{i=1,2,\cdots,N} (\sqrt{\sum_{j=1}^{D} (g_j - p_{ij})^2})$$

Where N is the number of particles and D is the dimension of each particle. A self-adaptive function in which non-linear inertia weight is decreasing can be given by utilizing the change rate of focusing distance.

$$w = \begin{cases} (\alpha_1 + \frac{|r|}{2.0}) |\ln k|, & |k| > 1 \\ \alpha_1 \alpha_2 + \frac{|r|}{2.0}, & 0.05 \le |k| \le 1 \\ (\alpha_2 + \frac{|r|}{2.0}) \frac{1}{|\ln k|}, & |k| < 0.05 \end{cases} \quad (5)$$

Where, $\alpha_1 = 0.3$, $\alpha_2 = 0.2$, r is a random number within the interval of [0,1]. In this strategy of selection, w value is randomly selected to make w self-adaptively adjusted according to the change rate of focusing distance. A new improved particle swarm optimization (IPSO) algorithm will be obtained by combining formula (3), (4), (5) and (2). In this paper, this IPSO algorithm, which changes inertia weight dynamically, will be used to solve the problem of system identification.

3.3. Particle coding rule

The methods for system identification discussed in this paper can be divided into the problems of sub-model selection (i.e. the system structure identification) and model parameter identification. Selecting from multiple sub-models a combination of sub-models which can fit the sample data in an optimal way belongs to one kind of combinatorial optimization problem. There is no conventional method for parameter identification for the structure of a selected group of mathematical models is unknown. The problem of parameter identification belongs to the problem of optimization. To successfully use PSO algorithm to solve the problem of system identification, the key lies in how to conduct particle coding and establish a proper fitness function[5]. The method of system identification is, essentially, the problem of how to select sub-models and determine parameters for them. Therefore, Author thinks that particle coding can be conducted from these two aspects. The sub-model is either selected or not, so coding of binary system can be adopted and parameters of the model can be mapped as particle coding in the form of real numbers. As a consequence, the following particle coding rules can be formed in this paper.

$$[p_0, K_1, p_{11}, p_{12}, \cdots, p_{1m_1}, \cdots, K_k, p_{k1}, p_{k2}, \cdots, p_{km_k}, \cdots, K_N, p_{N1}, p_{N2}, \cdots, p_{Nm_N}]$$

Where K_k is the selection gene of the sub-model No.K, $K_k = 1$ signifies that the sub-model is selected while $K_k = 0$ signifies that it is not selected.

3.4. **Design of fitness function**

The purpose of system identification is to offer a model which can fit the sample data very well, which means making the calculated system output

y_{mk} approach the actual system output y_k as much as possible. The closer those two values are, the better the fitting effect will be. Therefore, the following criterion function [1] can be taken as the fitness function in this paper.

$$f = \sqrt{\sum_{k=1}^{n} (y_{mk} - y_k)^2} \qquad (6)$$

Where, y_{mk} represents the model output results of the sample data of group No.k

3.5. Procedures of system identification

With the backing of the preparation and introduction above, the main procedures of the algorithm for system identification based on PSO are as follows:

Step 1: Initialization, randomly generate initial solutions (particles) x_i of N and initial velocity v_i of N, where K_k is randomly taken as 0 or 1.

Assess the fitness of each particle, initialize p_i with its own position of each particle and initialize g with the particle of the minimum fitness in the swarm.

Step 2: Update the velocity of each particle in each dimension according to formula (1) or (3) and limit it within the maximum velocity $v_{\max}$; then update the position of each particle in each dimension according to formula (2), where conduct mutation over K_k by reversing $0 \leftrightarrow 1$.

Step 3: Calculate fitness of the new position of each particle according to formula (6).

Step 4: For each particle, if the fitness of current position $\mathbf{x_i}$ is better than the fitness of the historically optimal position p_i, replace p_i with $\mathbf{x_i}$.

Step 5: For all particles, if the fitness of currently optimal position is better than the fitness of the historically optimal position g, replace g with $\mathbf{x_i}$.

Step 6: if the stopping rule of algorithm is met (e.g., specified iteration steps are achieved or the fitness value exceeds certain threshold), the algorithm stops, and output optimization result, or else return to step 2.

4. Results of simulation experiment

Simulation experiment is conducted to verify the performance and efficient of the algorithm for system identification proposed in this paper. With Pentium(R) D CPU 2.66GHz and a memory of 1024MB as the experiment platform, all codes are realized by programming in Matlab7.6.0(R2008a). Number of particles N=15, parameters $\mathbf{c_1 = c_2 = 2.0}$, and maximum truncated generations are 1000.

In order to facilitate comparison, system model provided by reference document [3] is used:

$$y = 25 + \frac{1}{0.02 + 10e^{-0.25x}} + 5\sin(3 + 0.5x) + 2\sin(1.5 + x) \qquad (7)$$

30 groups of sample data is generated according to formula (7). Assume the model structure is unknown beforehand, we can select all meta-models provided in this paper to conduct system identification. After using BPSO and IPSO algorithms to carry out multiple times of simulation, we can select better results, respectively shown as follows:

$$y = 25.0316 + \frac{1}{0.0195 + 10.3712\, e^{-0.2512x}} + 5.0035\sin(2.9641 + 0.5006x) + 2.0183\sin(1.5286 + 1.0023x)$$

$$y = 25.0018 + \frac{1}{0.0204 + 10.0803e^{-0.2497x}} + 4.9986\sin(3.0041 + 0.4973x) + 2.0008\sin(1.4985 + 1.0004x)$$

Table 1 Mean square error of various methods

methods	mean square error	maximum deviations
GA[3]	0.1646	0.8782
BPSO	0.0684	0.5837
IPSO	0.0053	0.0793

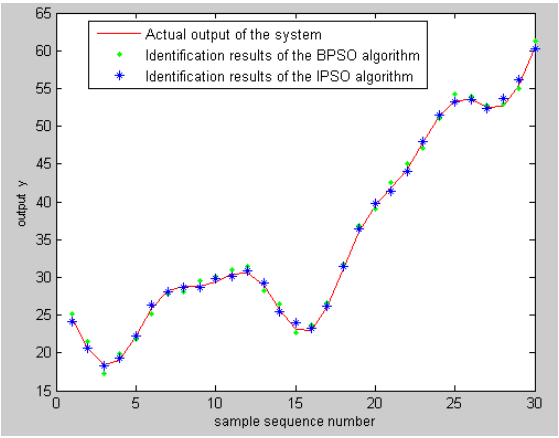

Figure 1. Results of system identification

Mean square error and maximum deviations of various methods are shown in table 1, and fitting results of algorithms in this paper are shown in figure 1. Simulation results have shown that the PSO algorithm can identify the system structure and acquire ideal model parameters, and results obtained by the IPSO algorithm is better than those by the BPSO algorithm.

On the basis of 30 groups of sample data generated according to formula (7), random disturbance quantity

will be generated within the range of [-10%, 10%] with probability of 10%, which shall be superposed to the sample data. The better identification results respectively are:

$$y = 25.3962 + \frac{1}{0.0257 + 12.5726e^{-0.2372x}} + 5.0863\sin(3.486 + 0.5628x) + 2.3053\sin(1.4795 + 1.0753x)$$

$$y = 25.0723 + \frac{1}{0.0201 + 10.0985e^{-0.240157x}} + 4.9426\sin(2.9253 + 0.4953x) + 1.9263\sin(1.5006 + 1.0002x)$$

Table 2 Mean square error of various methods with radom disturbance

methods	mean square error	Maximum deviations
GA[3]	1.0515	5.6653
BPSO	0.8135	2.6823
IPSO	0.0426	1.0257

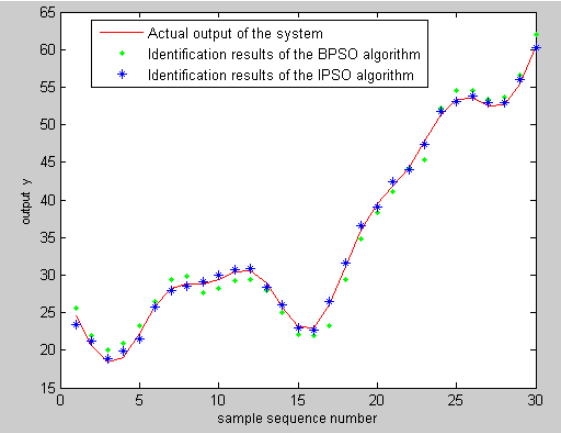

Figure 2. Results of system identification with radom disturbance

Mean square error and maximum deviations of various methods are shown in table 2, and fitting results of algorithms are shown in figure 2. It can be seen from figure 2 that the appearance of disturbance has certain influence on the solving capability of the algorithm and model results for identification. However, the IPSO algorithm proposed in this paper still can ideally identify the system structure and model parameters, and there is no big difference between fitting effect of model obtained by this algorithm and that under no disturbance. Compared to the results obtained in reference document [3], the estimated parameter values obtained by methods in this paper is closer to the actual value, and the identification accuracy is obviously improved. In addition, methods in this paper do not have the trouble brought by the genetic manipulation such as crossover and mutation of the genetic algorithm, and are easier to realize.

5. Conclusion

With regard to the problem of conducting system identification with sample data, this paper assumes that the system model is comprised of system input variables in the form of typical mathematical models through intercombination. Therefore, the problem of system structure identification is transformed into a combinatorial optimization problem of selecting from multiple typical mathematical models a group of models which can fit the sample data in an optimal way. The IPSO algorithm can be used to realize the structure and parameter identification of the system at the same time. The inertia weight of the IPSO algorithm given in this paper can adjust self-adaptively based on the change rate of focusing distance, so the global and local search abilities can be adjusted more flexibly. Therefore, identification results of the IPSO algorithm are obviously better than those of BPSO algorithm and genetic algorithm.

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