Fuzzy PID controller for 2D differential geometric guidance and control problem

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Abstract: A fuzzy set-point weighting proportional–integral–derivative (PID) controller is applied in the development of the flight control system for a two-dimensional differential geometric (DG) guidance and control system, whose function is to guarantee the achieved angle of attack (AOA) and track the commanded AOA efficiently. In particular, a Lyapunov stability criterion is introduced to study the relation between the stability and the output of the fuzzy inference system, and a genetic algorithm is utilised to tune the PID gains in the simulations so as to show the full potentiality of the proposed control scheme. The results demonstrate that the designed controller yields a fast-responding and stable system that is robust to parameter variations. Moreover, the DG guidance law is viable and effective in a realistic missile defence engagement.

1 Introduction

Proportional–integral–derivative (PID) controllers and their variants are extensively used in process industries because of their simple structures and robust performances [1, 2]. So far, considerable work has been published on developing methods to reduce the time spent on optimising the choice of the PID gains. Over the past two decades, the application of knowledge-based systems in process control has been growing, especially in the field of fuzzy control, in which linguistic descriptions of human expertise in control process are represented as fuzzy rules or relations. This knowledge base is used by an inference mechanism, in conjunction with some knowledge of the states of the process in order to determine control actions. Although they do not have an apparent structure of PID controllers, fuzzy logic controllers may be considered as nonlinear PID controllers whose gains can be determined online based on the system error and its time derivative [3–6].

A comparison between different fuzzy methodologies is presented in [7]. The results indicate that the main benefit in the use of fuzzy logic appears when the process nonlinearities are significant, and the fuzzy set-point weighting (FSW) technique is superior to the other fuzzy schemes, as it guarantees in general very good performance in the set-point and load disturbance step response and it requires modest implementation effort. However, the application of the FSW PID controller in missile control problems, especially in differential geometric (DG) guidance and control problems, has not been fully addressed in the open literature.

Moreover, there have not been many attempts to apply DG formulations to missile guidance problems since Adler [8] studied the three-dimensional (3D) proportional navigation (PN) guidance law in terms of the geodesic and normal curvature of the missile's path on the surface generated by the line of sight (LOS).

Chiou, Kuo and Soetanto [9–11] proposed DG guidance commands using the Frenet formulas [12]. Li *et al.* [13, 14] examined their applications in a realistic missile defense engagement. The results demonstrate that the resultant DG guidance law was shown to be a generalisation of the PN guidance law.

Ariff *et al.* [15] presented a novel DG guidance algorithm using information on the involute of the target's trajectory. White *et al.* [16] studied the application of DG formulations to a planar interception engagement. The results of their papers indicated that the DG guidance algorithm performed better than the conventional PN guidance law in most cases.

This paper differs from prior work in three main aspects. First, the FSW PID controller and feedback linearisation technique are introduced to develop the closed-loop transfer function of DG flight control system (FCS). Second, the Lyapunov stability criterion [17] is utilised to study the stability properties of the resulting FCS, as well as to determine the relation between the asymptotic stability and the output of the fuzzy inference system (FIS). Third, the performance of the derived FCS is studied and compared with other control schemes in a realistic engagement, in which case the PID gains of all the considered processes are tuned by means of the genetic algorithm [18], whose efficiency and effectiveness has been recognised in tuning of the fuzzy modules and PID controllers [6, 7, 19, 20].

2 Formulation of the missile problem

As illustrated in Fig. 1, the proposed engagement is a surface-to-air tactical missile interception scenario. Without loss of generality, the thrust $\boldsymbol{P}$, gravitational force $\boldsymbol{G}$ and atmospheric force $\boldsymbol{R}$ will be considered throughout the engagement. In the present case, the thrust acts in the direction of the axis $x_b o$ of the body frame, the gravitational force acts in the opposite direction of the axis OY_1 of the inertial frame, and in the velocity frame (throughout this paper, the word 'velocity' will only be used to designate a vector quantity: the corresponding scalar will be called the speed) the atmospheric force can be expressed simply

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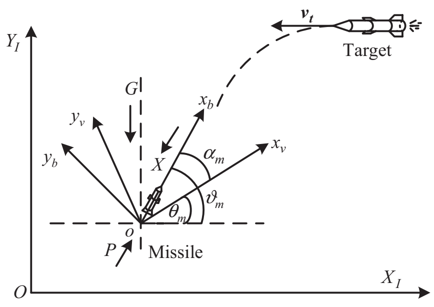

Fig. 1 *Frame definition and geometry of the engagement*

as follows [13]

$$\boldsymbol{R} = \begin{bmatrix} -X \\ Y \end{bmatrix} = \begin{bmatrix} -(C_{x0} + C_x^\alpha \alpha_{\rm m}^2) v^2 \rho(h) S/2 \\ C_y^\alpha \rho(h) v^2 S \alpha_{\rm m}^2/2 \end{bmatrix}$$

where X and Y are the atmospheric drag and lift, respectively, C_{x0}, C_x^α and C_y^α are the atmospheric coefficients, $\rho(h)$ is the air density computed as a piecewise-exponential function of the missile altitude [21], v is the free-stream speed, S is the reference area, and $\alpha_{\rm m}$ is the AOA of the missile body with respect to the inertial frame.

Therefore the motion of the missile can be formulated in the inertial frame as follows

$$\begin{aligned} m\dot{\boldsymbol{v}}_{\rm m} &= C_{\rm b}^{\rm I}\begin{bmatrix} P \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ g \end{bmatrix} + \boldsymbol{C}_{\rm v}^{\rm I}\boldsymbol{R} \\ J\dot{\omega} &= \rho(h)\boldsymbol{v}_{\rm m}^2 Sl(m_z^\alpha \alpha + m_z^\omega \omega + m_z^\delta \delta)/2 \\ \dot{\vartheta}_{\rm m} &= \omega \\ \alpha_{\rm m} &= \vartheta_{\rm m} - \theta_{\rm m} \end{aligned} \qquad (1)$$

where $(\dot{\cdot})$ is the derivative with respect to time, m is the current mass of the missile, $\boldsymbol{v}_m$ is the missile's velocity; C_b^I and C_v^I are the transformation matrices from the body frame and the velocity frame to the inertial frame, respectively, P is the magnitude of the thrust, g is the standard gravitational acceleration, $\vartheta_{\rm m}$ is the Euler angle of the missile's body frame with respect to the inertial frame, $\theta_{\rm m}$ is the rotation angle of the missile's trajectory, δ is the deflection angle commanded by the actuator system, J is the moment of inertia of the missile's body frame, ω is the angular speed of the missile's body with respect to the inertial frame, l is the mean diameter of the missile and m_z^α, m_z^ω and m_z^δ are the moment coefficients of the missile.

The corresponding equations for the above variables are

$$m = m_0 - \frac{Pt_p}{(I_{\rm s}\ g)} \qquad \theta_{\rm m} = \tan^{-1}(v_{\rm my}/v_{\rm mx})$$

where m_0 is the initial mass of the missile, $I_{\rm s}$ is the impulse, $t_{\rm p}$ is the burn time, and $v_{\rm mx}$, $v_{\rm my}$ are the components of the missile velocity in the inertial frame.

It should be noted that the above relations could also be used to describe the motion of the target, simply by changing the corresponding subscript m to t

3 Derivation of the commanded angle of attack

The 2D DG guidance curvature command in the arc length system is [9]

$$\kappa_{\rm m} = N^2 \kappa_{\rm t} \frac{\cos\eta_{\rm t}}{\cos\eta_{\rm m}} + {\rm K}\frac{r'q'}{\cos\eta_{\rm m}} \qquad (2)$$

where $(\,)'$ denotes the derivative with respect to the arc length S along the missile's trajectory, N is the target/missile speed ratio, $k_{\rm t}$ is the curvature of the target's trajectory, $\eta_{\rm t}$ and $\eta_{\rm m}$ are the lead angles of the target and missile, respectively, K is the DG guidance gain, q' is the angular rate of LOS (LOSR), and r' is the closing speed.

The DG guidance curvature command must be transformed from the arclength system to the time domain before it can be applied to a realistic engagement. This means that the derivatives of all the variables in (2) must be taken with respect to time, not the arc length, which is impossible to measure with onboard sensors.

According to classical DG theory of curvature, we have [12]

$$r' = \frac{{\rm d}r}{{\rm d}s} = \frac{{\rm d}r}{{\rm d}t}\frac{{\rm d}t}{{\rm d}s} = \frac{\dot{r}}{v_{\rm m}} \qquad (3)$$

$$q' = \frac{{\rm d}q}{{\rm d}s} = \frac{{\rm d}q}{{\rm d}t}\frac{{\rm d}t}{{\rm d}s} = \frac{\dot{q}}{v_{\rm m}} \qquad (4)$$

where v_m is the missile's speed, and $\dot{r}$, $\dot{q}$ are the closing speed and the LOSR in the time domain, respectively.

Substituting (3) and (4) into (2), in the time domain the guidance curvature command is developed in the form

$$\kappa_{\rm m}({\rm t}) = N^2 \kappa_{\rm t} \frac{\cos\eta_{\rm t}}{cos\eta_{\rm m}} + K\frac{\dot{r}\dot{q}}{v_{\rm m}^2 cos\eta_{\rm m}} \qquad (5)$$

Furthermore, we have [12]

$$\kappa_{\rm m}'' = |r_{\rm m}''|$$

where

$$r_{\rm m}'' = \frac{{\rm d}^2 r_{\rm m}}{{\rm d}s^2} = v_m \frac{{\rm d}^2 t}{{\rm d}s^2} + \left(\frac{{\rm d}t}{{\rm d}s}\right)^2 a_{\rm m} \qquad (6)$$

and

$$\frac{{\rm d}^2 t}{{\rm d}s^2} = -v_{\rm m}^{-2}\frac{{\rm d}v_{\rm m}}{{\rm d}s} = -v_{\rm m}^{-3}\frac{{\rm d}v_{\rm m}}{{\rm d}t} = -v_{\rm m}^{-3}\dot{v}_{\rm m} \qquad (7)$$

where $a_{\rm m}$ is the missile acceleration vector and, $\dot{v}_{\rm m}$ is the rate of change of the missile speed [21].

Taken together, in the time domain, the current curvature of the missile's trajectory is

$$\kappa_{\rm m} = |a_{\rm m} - \dot{v}_{\rm m} t_{\rm m}|/v_{\rm m}^2 \qquad (8)$$

Consequently, the commanded AOA $\alpha_{\rm cmd}$ can be developed by assuming that the guidance curvature command is equal to the current curvature, yielding

$$\alpha_{\rm cmd} = \{\alpha_{\rm cmd}\!:\!\{\kappa_{\rm m}(t) = \kappa_{\rm m}\}\} \qquad (9)$$

The output of the DG guidance system is designed as $\alpha_{\rm cmd}$. It must be stressed that the proposed guidance system involves feedback of angle and is different from the conventional guidance systems, which involve feedback of accelerations [21].

4 Fuzzy set-point weighting PID controller

The 'textbook' version of the PID controller is [1]

$$u(t) = K_{\rm p} e(t) + K_{\rm i}\int_0^t e(\tau)\,{\rm d}\tau + K_{\rm d}\frac{{\rm d}e(t)}{{\rm d}t} \qquad (10)$$

$$e(t) = y_{\rm sp}(t) - y(t) \qquad (11)$$

where u(t) is the control variable, $e(t)$ the system error, $K_{\rm p}$ the proportional gain, $K_{\rm i}$ the integral gain, $K_{\rm d}$ the derivative gain, and y(t) the measured output. The referenced signal $y_{sp}(t)$ is also called the set-point.

The classical tuning method of the PID controller is the Ziegler–Nichols step-response or frequency-response method, which generally results in good load disturbance attenuation but also in a large overshoot and settling time for a step response, which cannot be acceptable for a realistic FCS. An effective way to cope with this problem is to weigh the set-point for the proportional action by means of a constant $b < 1$ so that we get [1]

$$u_{spid} = K_p(by_{sp}(t) - y(t)) + K_i\int_0^t e(\tau)\,d\tau + K_d\dot{e}(t) \qquad (12)$$

The preceding relation is equivalent to

$$u_{spid} = K_p e(t) + K_i\int_0^t e(\tau)\,d\tau + K_d\dot{e}(t) + \Delta u(t) \qquad (13)$$

where

$$\Delta u(t) = K_p(b(t) - 1)y_{sp}(t) \qquad (14)$$

However, the use of set-point weighting generally leads to an increase in the rise time, since the effectiveness of the proportional action is somewhat reduced. This significant drawback can be avoided by using a FIS to determine the value of the weight $b(t)$ based on the current value of the system error $e(t)$ and its time derivative $\dot{e}(t)$. The approach proposed by Visioli [6] consists of fuzzifying the set-point weight, that is, $b(t)$, of the classical PID controller, leaving the three PID gains determined by means of the Ziegler–Nichols method to preserve a good load disturbance attenuation. In this way, we have

$$b(t) = w + f(e, \dot{e}) \qquad (15)$$

where $f(e, \dot{e})$ is the output of the FIS, and $w \le 1$ is a positive constant.

For the sake of simplicity, and from the practical point of view as well, $w = 1$ is usually selected to maintain the effectiveness of the proportional action so as to reduce the possibility of the occurrence of undershoot for a random input in the realistic engagement, in which case the plant parameters varies according to the flight altitude and Mach number. Moreover, the scale coefficients related to the two inputs of the FIS are chosen as the inverse value of the maximum allowable AOA and its time derivative so as to normalise the inputs of the FIS, and the output coefficient of the FIS is determined based on a practical tuning procedure [6].

Therefore, the proposed FSW PID controller is

$$u_{ipid} = K_p e(t) + K_i\int_0^t e(\tau)\,d\tau + K_i\dot{e}(t) + K_p f(e, \dot{e})y_{sp}(t) \quad (16)$$

Using Zadeh's logical 'AND' operator and the 'CENTROID' defuzzification method, the control surface of the designed FIS can be obtained as shown in Fig. 2. Note that the derived control surface is not flat, which means that the output solution of the designed FIS contains both linear and nonlinear terms, the latter of which has been shown to be effective for missile guidance and control systems [22]. Moreover, the proposed FSW PID controller has a structure with an extra degree of freedom (DOF) compared with the classical PID controller because the resulting signal path from y_{sp} (t) to $u(t)$ is different from that from $y(t)$ to $u(t)$. It is worth noting that the more DOF the controller has, the better performance can be guaranteed in missile guidance and control problem [23].

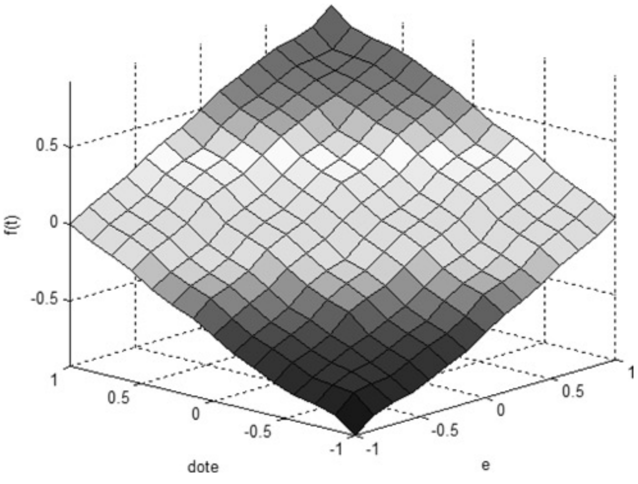

Fig. 2 *Control surface of the FSW system*

5 Flight control system

A conventional missile guidance system usually issues a guidance command in the form of the desired acceleration normal to the missile's velocity. Therefore, the function of the conventional FCS is to make the achieved normal acceleration 'track' the commanded acceleration with good fidelity. It is thus appropriate to deal with the error between the commanded and the achieved normal acceleration [21]. However, as proved previously, the output of the DG guidance system is the commanded AOA. Therefore the function of the desired FCS is how to guarantee that the achieved AOA tracks the commanded AOA as closely as possible. In this section, the FSW PID controller and the feedback linearisation technique are utilised to form the closed-loop transfer function of the DG FCS in order to meet the requirement of this angle-tracking problem.

The open-loop flow of the DG guidance and control system is indicated in Fig. 3. Notice that the proposed FCS consists of three parts. The first two parts are the designed FSW PID controller and the actuator system, while the third is the dynamics between the achieved AOA and the deflection angle, which, from (1), is of the form

$$J\ddot{\alpha}_m - B\dot{\alpha}_m - A\alpha_m = C\delta \qquad (17)$$

where $A = m_z^{\alpha}\rho(h)v_m^2 Sl/2$, $B = m_z^{\omega}\rho(h)v_m^2 Sl/2$ and $C = m_z^{\delta}\rho(h)v_m^2 Sl/2$.

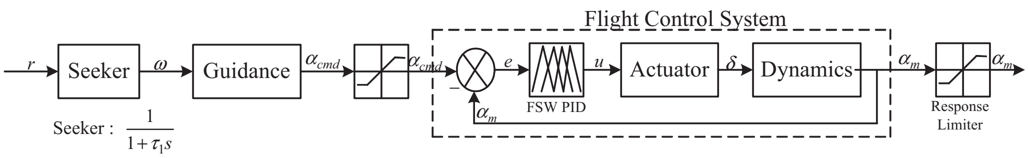

Fig. 3 *Illustration of the open-loop DG guidance and control system*

As stated previously, the system error of the desired FCS is

$$e(t) = \alpha_{\mathrm{cmd}}(t) - \alpha_{\mathrm{m}}(t) \tag{18}$$

Substituting (18) into (17), we obtain

$$J\ddot{e} - B\dot{e} - Ae = J\ddot{\alpha}_{\mathrm{cmd}} - B\dot{\alpha}_{\mathrm{cmd}} - A\alpha_{\mathrm{cmd}} - C\delta \tag{19}$$

Ignoring the dynamics of the actuator system,

$$u = \delta \tag{20}$$

Furthermore, in order to cancel the perturbation terms on the right-hand side of (19), as well as to linearise the system response, the control effort can be designed as

$$u = \frac{1}{C}[J\ddot{\alpha}_{\mathrm{cmd}} - B\dot{\alpha}_{\mathrm{cmd}} - A\alpha_{\mathrm{cmd}} + u_{\mathrm{fpid}}] \tag{21}$$

Now, according to (16), we have

$$u_{\mathrm{fpid}} = K_{\mathrm{p}}e + K_{\mathrm{i}}\int e(\tau)\,\mathrm{d}\tau + K_{\mathrm{d}}\dot{e} + K_{\mathrm{p}}f(e,\dot{e})\alpha_{\mathrm{cmd}}$$

As a result, (21) should be

$$\begin{aligned} u = \frac{1}{C}\Big\{ & J\ddot{\alpha}_{\mathrm{cmd}} + K_{\mathrm{p}}e + K_{\mathrm{i}}\int e(\tau)\,\mathrm{d}\tau \\ & + K_{\mathrm{d}}[(1 - B/K_d)\dot{\alpha}_{\mathrm{cmd}} - \dot{\alpha}_{\mathrm{m}}] + K_{\mathrm{p}}f(e,\dot{e})\alpha_{\mathrm{cmd}}\Big\} \end{aligned} \tag{22}$$

Obviously, the control law indicated by (22) consists of two parts. The first is a feedback term, $J\ddot{\alpha}_{\mathrm{cmd}}/C$, while the other is a generalization of the FSW PID controller. Moreover, it should be pointed out that $\ddot{\alpha}_{\mathrm{cmd}}$ is available from the continuity of the curvature information.

Applying (20) and (22) to (19), we have

$$\begin{aligned} & J\ddot{e} - B\dot{e} - Ae \\ & \quad = -K_{\mathrm{p}}e - K_{\mathrm{i}}\int e(\tau)\,\mathrm{d}\tau - K_{\mathrm{d}}\dot{e} - K_{\mathrm{p}}f(e,\dot{e})\alpha_{\mathrm{cmd}} \end{aligned} \tag{23}$$

Furthermore, we have

$$\begin{aligned} & J\dddot{e} + (K_{\mathrm{d}} - B)\ddot{e} + (K_{\mathrm{p}} - A)\dot{e} + K_{\mathrm{i}}e \\ & \quad = -K_{\mathrm{p}}f(e,\dot{e})\dot{\alpha}_{\mathrm{cmd}} - K_{\mathrm{p}}\dot{f}(e,\dot{e})\alpha_{\mathrm{cmd}} \end{aligned} \tag{24}$$

Hence the transfer function between the system error and the system input is

$$\frac{e(s)}{\alpha_{\mathrm{cmd}}(s)} = -\frac{K_{\mathrm{p}}f(e,\dot{e})s + K_{\mathrm{p}}\dot{f}(e,\dot{e})}{Js^3 + (K_{\mathrm{d}} - B)s^2 + (K_{\mathrm{p}} - A)s + K_{\mathrm{i}}} \tag{25}$$

Similarly, we have

$$\begin{aligned} G(s) &= \frac{\alpha_{\mathrm{m}}(s)}{\alpha_{\mathrm{cmd}}(s)} \\ &= \frac{Js^3 + (K_{\mathrm{d}} - B)s^2 + [K_{\mathrm{p}} - A + K_{\mathrm{p}}f(e,\dot{e})]s + K_{\mathrm{i}} + K_{\mathrm{p}}\dot{f}(e,\dot{e})}{Js^3 + (K_{\mathrm{d}} - B)s^2 + (K_{\mathrm{p}} - A)s + K_{\mathrm{i}}} \end{aligned} \tag{26}$$

Notice that (26) is the closed-loop transfer function of the desired FCS without inclusion of the actuator dynamics, and can be reduced to have an identical response to that derived based on the short-period approximation [21, 24] by ignoring the integral term and the effect of the FIS.

Moreover, for a skid-to-turn (STT) missile, the actuator behaves like a spring–mass damper system, whose transfer function is [21]

$$\frac{\delta(s)}{u(s)} = \frac{\omega_{\mathrm{A}}^2}{s^2 + 2\xi\omega_{\mathrm{A}}s + \omega_{\mathrm{A}}^2} \tag{27}$$

where ω_{A} is the natural frequency of the actuator dynamics and ξ is the damping ratio.

It should be noted that the derived FCS also works for a conventional acceleration-tracking FCS, that is, a PN guidance and control system, simply by replacing the system error from the angle by the acceleration or load factor.

6 Stability analysis using Lyapunov stability criterion

It is well known that Lypunov stability criteria are very powerful techniques for determining the steady-state stability of nonlinear systems. In particular, Lyapunov's first method is strictly concerned with local stability issues, where stable and unstable cases are separated on the basis of the stability or instability of linear approximations around equilibrium points. Lyapunov's second method allows consideration of asymptotic stability in the large and of global asymptotic stability as well as local stability. The advantage of the latter method lies in the fact that stability information is deduced from the sign-definiteness of the time rate of change of an arbitrarily chosen definite function of the system states as the system pursues free motion. The system trajectories need not be explicitly solved for. Thus, when the method is used successfully, the stability results are obtained rather painlessly [17]. However, the application of Lyapunov's second method to the determination of the stability condition of fuzzy FCS has not been well addressed in the open literature, which is of some concern in this section.

We now begin the study of stability by stating a preliminary definition.

Definition: Let

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2(x_1 + c) \\ \dot{x}_2 &= -x_1(x_1 + c) \end{aligned} \tag{28}$$

where c is a non-zero constant. Consider E as an equilibrium point for the system described by (28). If $V = (x_1^2 + x_2^2)/2$, we have $\dot{V} = -x_1^2 < 0$, and uniform stability of the origin of the above equations follows. However, since $\dot{x}_1 = x_2 c$ on E, solutions starting in E leave E for all $x_2 \neq 0$.

Based on the above definition and according to (24), we have

$$\dddot{e} + \frac{(K_{\mathrm{d}} - B)\ddot{e}}{J} + \frac{(K_{\mathrm{p}} - A)\dot{e}}{J} + \frac{K_{\mathrm{i}}e}{J} = -\frac{K_{\mathrm{p}}f(e,\dot{e})\dot{\alpha}_{\mathrm{cmd}}}{J} - \frac{K_{\mathrm{p}}\dot{f}(e,\dot{e})\alpha_{\mathrm{cmd}}}{J} \tag{29}$$

Then the state equations of the system are

$$\begin{aligned} x_0 &= e \\ x_1 &= \dot{x}_0 = \dot{e} \\ x_2 &= \dot{x}_1 = \ddot{e} \\ \dot{x}_2 &= -a_2 x_2 - a_1 x_1 - a_0 x_0 - b_1 f(x_0, x_1) - b_0 \dot{f}(x_0, x_1) \end{aligned} \tag{30}$$

where

$$a_0 = \frac{K_i}{J}, \quad a_1 = \frac{(K_p - A)}{J}, \quad a_2 = \frac{(K_d - B)}{J}, \quad b_1 = \frac{K_p \dot{\alpha}_{cmd}}{J}, \quad b_0 = \frac{K_p \alpha_{cmd}}{J}$$

According to the basic rules of the FIS, it must be stressed that $f(0, 0) = 0$ and $\dot{f}(0, 0) = 0$. Therefore it is clear that the equilibrium point of the system error is at the origin.

Because $K_i \ll J$, the last equation of (30) becomes

$$\dot{x}_2 = -a_2 x_2 - a_1 x_1 - b_1 f(x_0, x_1) - b_0 \dot{f}(x_0, x_1) \tag{31}$$

Then the candidate Lyapunov function is defined as

$$V = (a_1 x_1^2 + x_2^2)/2 \tag{32}$$

Obviously, V is positive-definite for $a_1 > 0$, which means that $K_p > A$.

Furthermore, we have

$$\begin{aligned} \dot{V} &= a_1 x_1 x_2 + x_2 \dot{x}_2 \\ &= -a_2 x_2^2 + [-b_1 f(x_0, x_1) - b_0 \dot{f}(x_0, x_1)] x_2 \end{aligned} \tag{33}$$

Hence

$$\dot{V} < -a_2 x_2^2 + |b_1 f(x_0, x_1) + b_0 \dot{f}(x_0, x_1)||x_2| \tag{34}$$

Therefore, in order to guarantee that $\dot{V} < 0$, we must have

$$K_d > B \quad \text{and} \quad |b_1 f(x_0, x_1) + b_0 \dot{f}(x_0, x_1)| < a_2 |x_2| \tag{35}$$

That is,

$$K_p |f(e, \dot{e})||\dot{\alpha}_{cmd}| + K_p |\dot{f}(e, \dot{e})||\alpha_{cmd}| < (K_d - B)|\ddot{e}| \tag{36}$$

Also, we have [6]

$$-1 < f(e, \dot{e}) < 1$$

Moreover, assuming that

$$|\dot{\alpha}_{cmd}|_{max} = \alpha_{dm}^{sup} \quad |\alpha_{cmd}|_{max} = \alpha_m^{sup} \tag{37}$$

we have the stability condition for this case

$$|\dot{f}(e, \dot{e})| < \frac{(K_d - B)|\ddot{e}| - K_p \alpha_{dm}^{sup}}{K_p \alpha_m^{sup}} \tag{38}$$

Consequently, the PID gains and the output of the FIS should be adjusted to satisfy the stability condition so as to guarantee that system error approaches zero in finite time, and to prevent system oscillation as well.

7 Simulation results and discussion

Previous studies on the performance of the FSW PID controller have focused mainly on its step response to a known constant plant [6, 7]. With this restriction, the statement is not practical in realistic nonlinear missile problems, where the inputs to the controller and the plant parameters are time-varying. In this section, the performance of on FCS consisting of different controllers is studied and compared in control effort terms. Moreover, a comparison between the interception performances obtained with the DG guidance law and the PN guidance law is presented in three characteristic engagements. In particular, all the included PID gains are optimised by means of a genetic algorithm [18].

The initial conditions and constants for all of the following simulations are specified as follows

initial position of missile $\boldsymbol{r}_{m0} = (0, 0)\,\mathrm{m}$
initial velocity of missile $\boldsymbol{v}_{m0} = (0, 0)\,\mathrm{m/s}$
initial position of target $\boldsymbol{r}_{t0} = (50\,000, 50\,000)\,\mathrm{m}$
initial velocity of target $\boldsymbol{v}_{t0} = (-1000, -1000)\,\mathrm{m/s}$
initial mass of missile $m_{m0} = 1000\,\mathrm{kg}$
thrust $P = 65\,000\,\mathrm{N}$
burn time 15 s
impulse 250 s
mass of target $m_t = 500\,\mathrm{kg}$
reference area of missile and target $S = 0.2\,\mathrm{m}^2$
time constant of seeker system $\tau_1 = 0.5\,\mathrm{s}$
natural frequency of actuator system $\omega_A = 100\,\mathrm{rad/s}$
damping ratio of actuator system $\xi = 0.5$
maximum permissible AOA 10°
moment of inertial $J = 100\ \mathrm{kg\ m}^2$
mean diameter of missile 2 m
initial launch angle of missile θ_{m0}=62°
simulation step 0.01 s
coefficients of moment: $m_z^{\alpha} = -0.002$, $m_z^{\omega} = 0.001$, $m_z^{\delta} = 0.003$
atmospheric coefficients: $C_{x0} = 0.0774$, $C_x^{\alpha} = 0.00084\ \mathrm{deg}^{-2}$, $C_y^{\alpha} = 0.0333\ \mathrm{deg}^{-1}$

In order to test the effectiveness of the proposed FSW PID controller, a comparison between the responses obtained with flight control systems consisting of a PID controller, a fixed value of set-point weighting (Fixed-b) PID controller and the FSW PID controller is presented in Table 1, where MD is the miss distance, defined as the closest distance between the missile and target before its divergence; Time denotes the engagement time; EN is the energy [25]. Specifically, in order to make a fair comparison, as well as to show the full potentialities of the investigated control schemes, the PID gains of all the controllers considered in this section are determined by means of an autotuning procedure based on a genetic algorithm [18], whose purpose is to search for the global optimum values for the desired gains with respect to a defined objective function. Thus it overcomes some weakness of conventional tuning methods and guarantees a better performance. In practical computations, the procedure consists of choosing a suitable bound for every gain so as to guarantee the stability of the process and the convergence of the genetic algorithm. The selected objective function subjected to

Table 1: Comparison of the different controllers

	MDm	Times	IAE	EN
Z–N	6.27	30.68	91.92	6.88
PID	4.11	30.68	55.27	4.31
Fixed-b	3.91	30.68	54.84	4.25
FSW	1.08	30.68	16.89	0.83

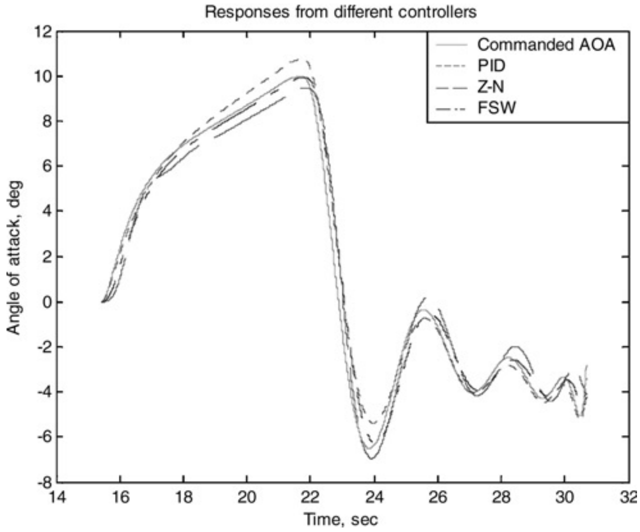

Fig. 4 *Comparison of performance of the different controllers*

minimise in these cases is the integrated absolute error (IAE), defined as

$$\mathrm{IAE} = \int_{t_0}^{t_f} (|\alpha_{\mathrm{cmd}}(t) - \alpha_{\mathrm{m}}(t)|)\,\mathrm{d}t \qquad (39)$$

Moreover, the performance of as FCS consisting of the PID controller with classical Ziegler–Nichols tuning method (Z–N) is also presented in Table 1.

Plots of responses of this angle 'track' problem are indicated in Fig. 4. Referring to Table 1, it is apparent that the proposed FSW FCS seems in general to ensure better performance than the other considered schemes, not only in the miss distance, but also in the control effort. This is because with the application of the FIS to the set-point weighting, both the rise time and the overshoot can be decreased from the classical tuning method. It is worth noting that the use of a fixed weighting parameter (Fixed-b) is almost useless if the PID gains are optimally tuned. Although the performance separation is not evident in this case where there is no saturation, it must be stressed that the FSW tuning method is more useful if saturation is significant in the process [6, 7].

Furthermore, in order to illustrate the homing performance of the DG guidance law in the presence of the designed FCS, a comparison between the interception performances obtained with the proposed DG guidance law and the benchmark guidance law, defined as the true PN (TPN) guidance law and the pure PN (PPN) guidance law [21], is performed in the proposed engagement. Three interception cases in different manoeuvre types are considered, and some of the relevant engagement characteristics are given in Table 2. Note that the threat lateral manoeuvre level increases with the case number.

The performances of the DG guidance law and the benchmark guidance law are presented in Table 3. It is apparent that the proposed DG guidance law has a similar performance to the benchmark guidance law in the first two cases. However, in the third case, which includes a stressing threat target, the performance separation becomes even more evident, and the DG guidance law performs better than other guidance schemes. Moreover, regardless of the types of targets, the DG guidance law has a longer engagement time and the control effort measured from the DG guidance law tends to stay relatively higher versus the benchmark guidance law, which indicates that the performance advantage of the DG guidance law comes at the expense of the control effort, especially in the case of intercepting a highly manoeuvring target.

Table 2: Engagement specifications

	Homing time, s	Manoeuvre, m	Manoeuver, s
Case 1	15	n/a	0
Case 2	15.2	0–30.61	1.5
Case 3	11.5	0–20	4

As illustrated in Fig. 5, unlike the conventional PN guidance law, the LOSR response produced by the DG guidance law reaches its maximum value at the beginning of the homing envelope. Subsequently it decreases gradually, and remains steady at the end. Furthermore, it should be pointed out that the DG guidance curvature command has an inverse response with the LOSR [13, 14], which indicates that the main function of the DG guidance curvature command is to null the LOSR over the engagement, thus leading to better performance.

The commanded AOA in the second case is depicted in Fig. 6. It is apparent that the DG guidance law compensates for the target's manoeuvre and engages its maximum commanded AOA at the beginning of the homing envelope in all the cases, whereas the commanded AOA produced by the benchmark guidance law tends to increase dramatically throughout the engagement, and already reaches its saturation in the final stage of the engagement. This is one indirect indicator of why the performance of the benchmark guidance law is not as good as that of the DG guidance law.

The error between the commanded and the achieved AOA of the designed FCS is presented in Fig. 7. Note that the proposed FCS converges quickly in the first two cases, and the error angle remains almost zero in the final stage of the engagement. However, the convergence of the proposed controller is slower and the magnitude of the error angle is greater in the third case, which indicates that a more sophisticated controller is expected in the case of interception of a highly manoeuvring target.

8 Conclusions

The simulation results of this paper indicate clearly that the proposed fuzzy set-point weighting PID controller outperforms conventional PID controllers in terms of two well-known overall performance indexes. Application of the designed DG guidance and control system further

Table 3: Comparison of the interception performance

	Missile distance, m			Engagement time, *t*			Guidance gain			Control energy		
	PPN	TPN	DG	PPN	TPN	DG	PPN	TPN	DG	PPN	TPN	DG
Case 1	1.14	1.05	1.16	30.65	30.65	30.66	6	6	10	1.16	1.09	1.65
Case 2	6.32	5.11	1.97	30.60	30.60	30.61	8	8	18	1.25	1.13	1.92
Case 3	22.18	18.37	2.25	26.82	26.82	26.83	16	12	18	2.30	2.17	3.76

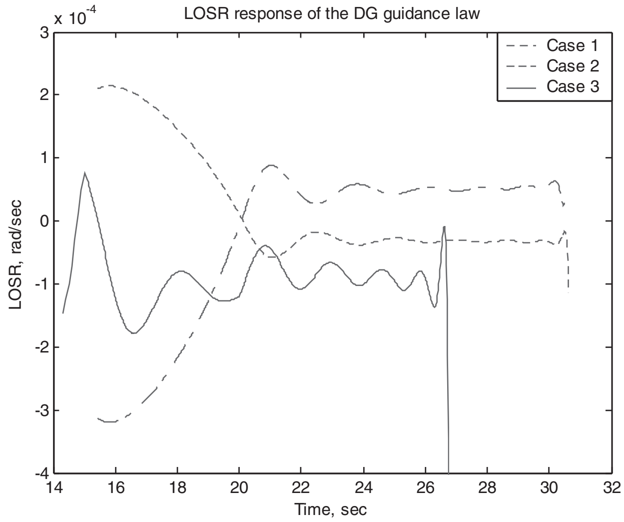

Fig. 5 *LOSR response of the DG guidance law*

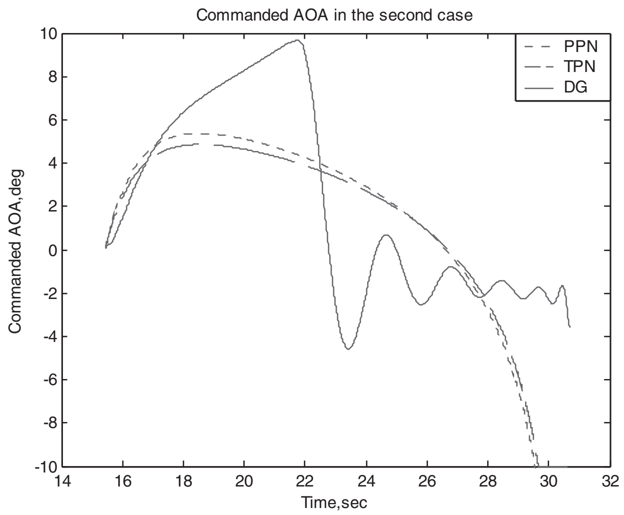

Fig. 6 *Commanded AOA in the second case*

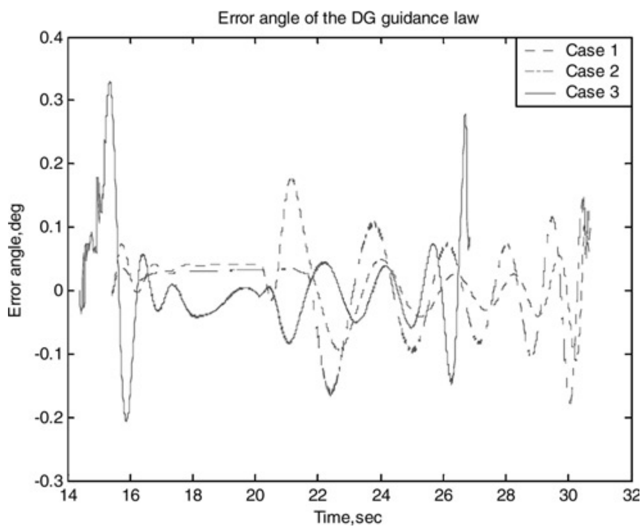

Fig. 7 *Error angle of the DG guidance and control system*

demonstrates its effectiveness and potentiality in realistic missile defense engagement. Moreover, the resulting FCS can ensure good stability, despite variations in the high-frequency dynamics of the missile actuator system.

The output of FIS usually causes oscillation and instability to the system. The results in this paper have highlighted that a Lyapunov stability criterion can be used as a constraint on the output of the FIS and the determination of the PID gains, as well as to guarantee asymptotically stability of the system without losing the effectiveness of the FIS.

The reason why the proposed FCS consumes more control effort than the classical control scheme is that the DG guidance law is more sensitive to changes in the LOS [13, 14]; thus the tendency of the commanded AOA changes according to the LOS, as illustrated in Figs. 5 and 6, which is harder to track for a PID controller than the one commanded by the PN guidance law.

Finally, the reason why the DG guidance law compensates for the target's manoeuvre earlier than the benchmark guidance law is that there are terms of the target's information in the DG guidance command, such as the curvature of the target's trajectory. Many current research projects focus on how to provide more accurate, real-time target information. The results presented in this paper demonstrate the benefits of this ongoing research for the performance of missile guidance systems.

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10 References

1 Åström, K., and Hagglund, T.: 'PID controllers: theory, design, and tuning' (ISA Press, 1995)
2 Kuo, B.C.: 'Automatic control systems' (Prentice-Hall, Englewood Cliffs, NJ, 1997)
3 Tzafestas, S.G., and Papanikolopoulos, N.P.: 'Incremental fuzzy expert PID control', *IEEE Trans. Ind. Electron.*, 1990, **37**, (5), pp. 365–371
4 He, S.Z., Tan, S., and Xu, F.L.: 'Fuzzy self-tuning of PID controllers', *Fuzzy Sets Syst.*, 1993, **56**, (1), pp. 37–46
5 Zhao, Z.Y., Tomizuka, M., and Isaka, S.: 'Fuzzy gain scheduling of PID controllers', *IEEE Trans. Syst. Man Cybern.*, 1993, **23**, (5), pp. 1392–1398
6 Visioli, A.: 'Fuzzy logic based set-point weighting for PID controllers', *IEEE Trans. Syst. Man Cybern. A*, 1999, **29**, (6), pp. 587–592
7 Visioli, A.: 'Tuning of PID controllers with fuzzy logic', *IEE Proc., Control Theory Appl.*, 2001, **148**, (1), pp. 1–8
8 Adler, F.P.: 'Missile guidance by three-dimensional proportional navigation', *J. Appl. Phys.*, 1956, **27**, (5), pp. 500–507
9 Kuo, C.Y., and Chiou, Y.C.: 'Geometric analysis of missile guidance command', *IEE Proc., Control Theory Appl.*, 2000, **147**, (2), pp. 205–211
10 Chiou, Y.C., and Kuo, C.Y.: 'Geometric approach to three-dimensional missile guidance problem', *J. Guidance Control Dyn.*, 1998, **21**, (2), pp. 335–341
11 Kuo, C.Y., Soetanto, D., and Chiou, Y.C.: 'Geometric analysis of flight control command for tactical missile guidance', *IEEE Trans. Control Syst. Tech.*, 2001, **9**, (2), pp. 234–243
12 Kreyszig, E.: 'Differential geometry' (Dover, New York, 1991)
13 Li, C.Y., Jing, W.X., Wang, H., and Qi, Z.G.: 'Application of 2D differential geometric guidance to tactical missile interception', Proc. IEEE Aerospace Conf., 2006
14 Li, C.Y., Jing, W.X., Wang, H., and Qi, Z.G.: 'Iterative solution to differential geometric guidance problem', *Aircraft Eng. Aerosp. Tech.*, 2006, **78**, (5), pp. 415–425
15 Ariff, O., Zbikowski, R., Tsourdos, A., and White, B.A.: 'Differential geometric guidance based on the involute of the target's trajectory', *J. Guidance Control Dyn.*, 2005, **28**, (5), pp. 990–996

16 White, B.A., Ariff, O., and Tsourdos, A.: 'Direct intercept guidance using differential geometric concepts'. Proc. AIAA Guidance, Navigation and Control Conf., August 2005, AIAA-2005-5969
17 Hsu, J.C., and Meyer, A.U.: 'Modern control principles and applications' (McGraw-Hill, New York, 1968)
18 Houck, C., Joines, A., and Kay, M.: 'A genetic algorithm for function optimization: a matlab implementation' (NCSU-IE TR 95-09, 1995)
19 Homaifar, A., and McCormick, E.: 'Simultaneous design of membership functions and rule sets for fuzzy controllers using genetic algorithms', *IEEE Trans. Fuzzy Syst.*, 1995, **3**, (2), pp. 129–139
20 Wang, G.P., and Kwok, D.P.: 'Auto-tuning of classical PID controllers using an advanced genetic algorithm'. Proc. Int. Conf. on Industrial Electronics, Control, Instrumentation, and Automation, 1992, pp. 1224–1229
21 Siouris, G.M.: 'Missile guidance and control systems' (Springer-Verlag, New York, 2004)
22 Meng, J.E., and Ya, L.S.: 'Hybrid fuzzy proportional-integral plus conventional derivative control of linear and nonlinear system', *IEEE Trans. Ind. Electron.*, 2001, **48**, (6), pp. 1109–1117
23 Rogers, S.: 'Missile guidance comparison'. Proc. AIAA Guidance, Navigation and Control Conf., 2004, AIAA-2004-4882
24 Friedland, B.: 'Control system design: an introduction to state space methods' (Dover, New York, 1986)
25 Oviedo, J.J.E., Boelen, T., and Overschee, P.: 'Robust advanced PID control (RaPID): PID tuning based on engineering specifications', *IEEE Control Syst. Mag.*, 2006, **26**, (1), pp. 15–19