Handbook of

PI and PID Controller Tuning Rules

2nd Edition

Aidan O'Dwyer

Imperial College Press

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HANDBOOK OF PI AND PID CONTROLLER TUNING RULES (2nd Edition)

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Dedication

Once again, this book is dedicated with love to Angela, Catherine and Fiona and to my parents, Sean and Lillian.

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Preface

Proportional Integral (PI) and Proportional Integral Derivative (PID) controllers have been at the heart of control engineering practice for seven decades. However, in spite of this, the PID controller has not received much attention from the academic research community until the past fifteen years, when work by K.J. Åström, T. Hägglund and F.G. Shinskey, among others, has sparked a revival of interest in the use of this "workhorse" of controller implementation.

There is strong evidence that PI and PID controllers remain poorly understood and, in particular, poorly tuned in many applications. It is clear that the many controller tuning rules proposed in the literature are not having an impact on industrial practice. One reason is that the tuning rules are not very accessible, being scattered throughout the control literature; in addition, the notation used is not unified. The purpose of this book is to bring together and summarise, using a unified notation, tuning rules for PI and PID controllers. The author restricts the work to tuning rules that may be applied to the control of processes with time delays (dead times); in practice, this is not a significant restriction, as most process models have a time delay term.

It is the author's belief that this book will be useful to control and instrument engineering practitioners and will be a useful reference for students and educators in universities and technical colleges.

I would like to thank the School of Control Systems and Electrical Engineering, Dublin Institute of Technology, for providing the facilities needed to complete the book. Finally, I am deeply grateful to my mother, Lillian and my father, Sean, for their inspiration and support over many

years and to my family Angela, Catherine and Fiona, for their love and understanding.

Aidan O'Dwyer

Contents

Chapter 1

Introduction

1.1 Preliminary Remarks

The ability of proportional integral (PI) and proportional integral derivative (PID) controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. Koivo and Tanttu (1991), for example, suggest that there are perhaps 5-10% of control loops that cannot be controlled by single input, single output (SISO) PI or PID controllers; in particular, these controllers perform well for processes with benign dynamics and modest performance requirements (Hwang, 1993; Åström and Hägglund, 1995). It has been stated that 98% of control loops in the pulp and paper industries are controlled by SISO PI controllers (Bialkowski, 1996) and that, in process control applications, more than 95% of the controllers are of PID type (Åström and Hägglund, 1995). The PI or PID controller implementation has been recommended for the control of processes of low to medium order, with small time delays, when parameter setting must be done using tuning rules and when controller synthesis is performed either once or more often (Isermann, 1989).

However, despite decades of development work, surveys indicating the state of the art of control industrial practice report sobering results. For example, Ender (1993) states that, in his testing of thousands of control loops in hundreds of plants, it has been found that more than 30% of installed controllers are operating in manual mode and 65% of loops operating in automatic mode produce less variance in manual than in automatic (i.e. the automatic controllers are poorly tuned). The situation does not appear to have improved in recent years as Van Overschee and De Moor (2000) report that 80% of PID controllers are badly tuned; 30% of PID controllers operate in manual with another 30% of the controlled loops increasing the short term variability of the process to be controlled (typically due to too strong integral action). The authors state that 25% of all PID controller loops use default factory settings, implying that they have not been tuned at all.

These and other surveys (well summarised by Yu, 1999, pages 1-2) show that the determination of PI and PID controller tuning parameters is a vexing problem in many applications. The most direct way to set up controller parameters is the use of tuning rules; obviously, the wealth of information on this topic available in the literature has been poorly communicated to the industrial community. One reason is that this information is scattered in a variety of media, including journal papers, conference papers, websites and books over a period of seventy years. The author has recorded 408 separate sources of tuning rules since the first such rule was published by Callender *et al.* (1935/6). In a striking statistic, 293 sources of tuning rules have been recorded since 1992, reflecting the upsurge of interest in the use of the PID controller recently.

The purpose of this book is to bring together, in summary form, the tuning rules for PI and PID controllers that have been developed to compensate SISO processes with time delay. Tuning rules for the variations that have been proposed in the 'ideal' PI and PID controller structure are included. Considerable variations in the ideal PID controller structure, in particular, are encountered; these variations are explored in more detail in Chapter 2.

1.2 Structure of the Book

Tuning rules are set out in the book in tabular form. This form allows the rules to be represented compactly. The tables have four or five columns, according to whether the controller considered is of PI or PID form, respectively. The first column in all cases details the author of the rule and other pertinent information. The final column in all cases is labelled "Comment"; this facilitates the inclusion of information about the tuning rule that may be useful in its application. The remaining columns detail the formulae for the controller parameters.

Chapter 2 explores the range of PI and PID controller structures proposed in the literature. It is often forgotton that different manufacturers implement different versions of the PID controller algorithm (in particular); therefore, controller tuning rules that work well in one PID architecture may work poorly on another. This chapter also details the process models used to define the controller tuning rules.

Chapters 3 and 4 of the book detail, in tabular form, tuning rules for setting up, respectively, PI controllers and PID controllers (and their variations), for a wide variety of process models. P controller and I controller tuning rules are also defined (as subsets of PI controller tuning rules), and PD controller tuning rules are defined (as a subset of PID controller tuning rules), for processes whose model includes an integrator. One hundred and eighty-three such tables are provided altogether. To allow the reader to access data readily, the author has arranged that each table start on its own page of the book; each table is preceded by the controller used, together with a block diagram showing the unity feedback closed loop arrangement of the controller and process model.

In Chapter 5 of the book, analytical calculations of the gain and phase margins of a large sample of PI and PID controller tuning rules are determined, when the process is modelled in first order lag plus time delay (FOLPD) form, at a range of ratios of time delay to time constant of the process model. Results are given in graphical form.

An important feature of the book is the unified notation that is used for the tuning rules; a glossary of the symbols used is provided in Appendix 1. Appendix 2 outlines the range of methods that are used to determine process model parameters; this information is presented in summary form, as this topic could provide data for a book in itself. However, sufficient information, together with references, is provided for the interested reader.

Finally, a comprehensive reference list is provided. In particular, the author would like to recommend the contributions by McMillan (1994), Åstrom and Hagglund (1995), Shinskey (1994), (1996), Tan *et al.* (1999a), Yu (1999), Lelic and Gajic (2000) and Ang *et al.* (2005) to the interested reader, which treat comprehensively the wider perspective of PID controller design and application.

Chapter 2

Controller Architecture

2.1 Introduction

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$U(s) = G_c(s)E(s) \tag{2.1}$$

with

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2.2}$$

and with K_c = proportional gain, T_i = integral time constant and T_d = derivative time constant. If $T_i = \infty$ and $T_d = 0$ (i.e. P control), then it is clear that the closed loop measured value, y, will always be less than the desired value, r (for processes without an integrator term, as a positive error is necessary to keep the measured value constant, and less than the desired value). The introduction of integral action facilitates the achievement of equality between the measured value and the desired value, as a constant error produces an increasing controller output. The introduction of derivative action means that changes in the desired value may be anticipated, and thus an appropriate correction may be added prior to the actual change. Thus, in simplified terms, the PID controller allows contributions from present controller inputs, past controller inputs and future controller inputs.

Many variations of the PI and, in particular, the PID controller structure have been proposed. As Tan *et al.* (1999a) suggest, one important reason for the non-standard structures is due to the transition of the controllers from pneumatic implementation through electronic implementation to the present microprocessor implementation. The variations in the controller structures are detailed below.

2.2 PI Controller Structures

Tuning rules have been detailed for seven PI controller structures:

1. Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$ (2.3)

2. Ideal controller in series with a first order lag:
$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1}{1 + T_f s} \qquad (2.4)$$

3. Ideal controller in series with a second order filter (also labelled the 'generalised PID' controller (Lee and Shi, 2002)):
$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2}\right) \qquad (2.5)$$

4. Controller with set-point weighting:
$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s) \qquad (2.6)$$

5. Controller with proportional term acting on the output 1:
$$U(s) = \frac{K_c}{T_i s}E(s) - K_c Y(s) \qquad (2.7)$$

6. Controller with proportional term acting on the output 2:
$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_1 Y(s) \qquad (2.8)$$

7. Controller with a double integral term:

$$G_c(s) = K_c\left(1 + \frac{1}{T_{i1}s} + \frac{1}{T_{i2}s^2}\right) \tag{2.9}$$

2.3 PID Controller Structures

Forty-six such structures have been considered. The labelling of these structures has not been consistent in the literature; for example, the first PID controller structure considered (labelled the ideal controller below) has also been labelled the 'non-interacting' controller (McMillan, 1994), the 'ISA' algorithm (Gerry and Hansen, 1987) or the 'parallel non-interacting' controller (Visioli, 2001). The controller structures have been divided into four types, as described below.

2.3.1 *Ideal PID controller structure and its variations*

1. Ideal controller: $$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2.10}$$

   A variation of the controller is labelled the 'parallel' controller structure (McMillan, 1994). This variation has also been labelled the 'ideal parallel', 'noninteracting', 'independent' or 'gain independent' algorithm:

   $$G_c(s) = K_c + \frac{1}{T_i s} + T_d s \tag{2.11}$$

   The controller structure is used in the following products:
   (a) Allen Bradley PLC5 product (McMillan, 1994)
   (b) Bailey FC19 PID algorithm (EZYtune, 2003)
   (c) Fanuc Series 90-30 and 90-70 independent form PID algorithm (EZYtune, 2003)
   (d) Intellution FIX products (McMillan, 1994)
   (e) Honeywell TDC3000 Process Manager Type A, non-interactive mode product (ISMC, 1999)
   (f) Leeds and Northrup Electromax 5 product (Åström and Hägglund, 1988)

(g) Yokogawa Field Control Station (FCS) PID algorithm (EZYtune, 2003).

2. Ideal controller in series with a first order lag:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1} \tag{2.12}$$

3. Ideal controller in series with a first order filter:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{b_{f1}s + 1}{a_{f1}s + 1} \tag{2.13}$$

4. Ideal controller in series with a second order filter:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2} \tag{2.14}$$

5. Ideal controller with weighted proportional term:

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right) \tag{2.15}$$

6. Ideal controller with first order filter and setpoint weighting 1:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\left[R(s)\frac{1 + 0.4T_r s}{1 + sT_r} - Y(s)\right] \tag{2.16}$$

7. Ideal controller with first order filter and setpoint weighting 2:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\left[R(s)\frac{1 + 0.4T_r s}{1 + sT_r} - Y(s)\right] - K_0 Y(s) \tag{2.17}$$

8. Ideal controller with first order filter and dynamics on the controlled variable:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\frac{Ts + (1 + K)}{Ts + 1}E(s) - \frac{K}{Ts + 1}Y(s) \tag{2.18}$$

9. Controller with filtered derivative:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right) \tag{2.19}$$

This structure is used in the following products:

(a) Bailey Net 90 PID error input product with N = 10 (McMillan, 1994) and FC156 Independent Form PID algorithm (EZYtune, 2003)

(b) Concept PIDP1 and PID1 PID algorithms (EZYtune, 2003)

(c) Fischer and Porter DCU 3200 CON PID algorithm with N = 8 (EZYtune, 2003)

(d) Foxboro EXACT I/A series PIDA product (in which it is an option labelled ideal PID) (Foxboro, 1994).

(e) Hartmann and Braun Freelance 2000 PID algorithm (EZYtune, 2003)

(f) Modicon 984 product with $2 \le N \le 30$ (McMillan, 1994; EZYtune, 2003)

(g) Siemens Teleperm/PSC7 ContC/PCS7 CTRL PID products with N = 10 (ISMC, 1999) and the S7 FB41 CONT_C PID product (EZYtune, 2003).

10. Controller with filtered derivative in series with a second order filter:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2}\right) \tag{2.20}$$

11. Controller with filtered derivative with setpoint weighting 1:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s}{1 + a_{f1}s}R(s) - Y(s)\right) \tag{2.21}$$

12. Controller with filtered derivative with setpoint weighting 2:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2}R(s) - Y(s)\right) \quad (2.22)$$

13. Controller with filtered derivative with setpoint weighting 3:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2 + b_{f3}s^3}{1 + a_{f1}s + a_{f2}s^2 + a_{f3}s^3}R(s) - Y(s)\right) \quad (2.23)$$

14. Controller with filtered derivative with setpoint weighting 4:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2 + b_{f3}s^3 + b_{f4}s^4}{1 + a_{f1}s + a_{f2}s^2 + a_{f3}s^3 + a_{f4}s^4}R(s) - Y(s)\right) \quad (2.24)$$

15. Controller with filtered derivative and dynamics on the controlled variable:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right)E(s) - K_0 Y(s) \quad (2.25)$$

16. Ideal controller with setpoint weighting 1:

$$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{K_c}{T_i s}\left(F_i R(s) - Y(s)\right) + K_c T_d s\left(F_d R(s) - Y(s)\right) \quad (2.26)$$

17. Ideal controller with setpoint weighting 2:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\left[\frac{1}{T_i T_d s^2 + T_i s + 1}R(s) - Y(s)\right] \quad (2.27)$$

18. Blending controller:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{s} \tag{2.28}$$

This structure is used in the Yokogawa EFCS/EFCD Field Control Station product (Chen *et al.*, 2001).

2.3.2 *Classical PID controller structure and its variations*

19. Classical controller 1: This controller is also labelled the 'cascade' controller (Witt and Waggoner, 1990), the 'interacting' or 'series' controller (Poulin and Pomerleau, 1996), the 'interactive' controller (Tsang and Rad, 1995), the 'rate-before-reset' controller (Smith and Corripio, 1997), the 'analog' controller (St. Clair, 2000) or the 'commercial' controller (Luyben, 2001).

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1 + sT_d}{1 + s\frac{T_d}{N}} \tag{2.29}$$

The structure is used in the following products:

(a) Honeywell TDC Basic/Extended/Multifunction Types A and B products with $N = 8$ (McMillan, 1994)
(b) Toshiba TOSDIC 200 product with $3.33 \le N \le 10$ (McMillan, 1994)
(c) Foxboro EXACT Model 761 product with $N = 10$ (McMillan, 1994)
(d) Honeywell UDC6000 product with $N = 8$ (Åström and Hägglund, 1995)
(e) Honeywell TDC3000 Process Manager product – Type A, interactive mode with $N = 10$ (ISMC, 1999)
(f) Honeywell TDC3000 Universal, Multifunction and Advanced Multifunction products with $N = 8$ (ISMC, 1999)
(g) Foxboro EXACT I/A Series PIDA product (in which it is an option labelled series PID) (Foxboro, 1994).

20. Classical controller 2: $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + NT_d s}{1 + T_d s}\right)$ (2.30)

21. Series controller (Classical controller 3): This controller is also labelled the 'interacting' controller, the 'analog algorithm' (McMillan, 1994) or the 'dependent' controller (EZYtune, 2003).

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + sT_d) \tag{2.31}$$

The structure is used in the following products:
(a) Turnbull TCS6000 series product (McMillan, 1994)
(b) Alfa-Laval Automation ECA400 product (Åström and Hägglund, 1995)
(c) Foxboro EXACT 760/761 product (Åström and Hägglund, 1995).

22. Classical controller 4: This controller is also labelled the 'interacting' controller (Fertik, 1975).

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(1 + \frac{T_d s}{1 + s\frac{T_d}{N}}\right) \tag{2.32}$$

The structure is used in the following products:
(a) Bailey FC156 Classical Form PID product (EZYtune, 2003)
(b) Fischer and Porter DCI 4000 PID algorithm (EZYtune, 2003)

2.3.3 *Non-interacting PID controller structure and its variations*

23. Non-interacting controller 1: This controller is also labelled the 'reset-feedback' controller (Huang *et al.*, 1996).

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s)\right) \tag{2.33}$$

This structure is used in the following products:

(a) Bailey Fisher and Porter 53SL6000 and 53MC5000 products with N = 0 (ISMC, 1999).
(b) Moore Model 352 Single-Loop Controller product (Wade, 1994).

24. Non-interacting controller based on the parallel controller structure:
 24a. Non-interacting controller 2a:

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\dfrac{sT_d}{N}}Y(s) \tag{2.34}$$

 24b. Non-interacting controller 2b:

$$U(s) = \left(K_c+\frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\dfrac{sT_d}{N}}Y(s) \tag{2.35}$$

25. Non-interacting controller based on the two degree of freedom structure 1: This controller is also labelled the 'm-PID' controller (Huang *et al.*, 2000) the 'ISA-PID' controller (Leva and Colombo, 2001) and the 'P-I-PD (only P is DOF) incomplete 2DOF algorithm' by Mizutani and Hiroi (1991).

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\dfrac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\dfrac{T_d}{N}s}\right)R(s) \tag{2.36}$$

This structure is used in the following products:
(a) Bailey Net 90 PID PV and SP product (McMillan, 1994)
(b) Yokogawa SLPC products with $\alpha=-1$, $\beta=-1$, N=10 (McMillan, 1994)
(c) Omron E5CK digital controller with $\beta=1$ and N=3 (ISMC, 1999).

26. Non-interacting controller based on the two degree of freedom structure 2:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s} - \frac{\chi}{1 + T_i s}\right)R(s) \tag{2.37}$$

27. Non-interacting controller based on the two degree of freedom structure 3:

$$U(s) = K_c[(b-1) + (c-1)T_d s]R(s) + K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\left[R(s) - \frac{1}{(1 + sT_f)^2}Y(s)\right] \tag{2.38}$$

28. PD-I-PD (only PD is DOF) incomplete 2DOF algorithm (Mizutani and Hiroi, 1991):

$$U(s) = K_c\left[\alpha + \frac{1}{T_i s}\left(\frac{\chi T_d s}{1 + \frac{T_d}{N}s} - \frac{1}{1 + T_i s}\right)\right]R(s) - K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)Y(s) \tag{2.39}$$

However, no tuning rules are available for this controller structure.

29. PI-PID (only PI is DOF) incomplete 2DOF algorithm (Mizutani and Hiroi, 1991):

$$U(s) = K_c\left[\alpha + \frac{1}{T_i s} - \frac{\beta}{T_i s(1 + T_i s)}\right]R(s) - K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)Y(s) \tag{2.40}$$

However, no tuning rules are available for this controller structure.

30. Super (or complete) 2DOF PID algorithm (Mizutani and Hiroi, 1991):

$$U(s) = K_c\left[\alpha + \frac{1}{T_i s}\left(1 - \frac{\beta}{1+T_i s} + \frac{\chi T_d s}{1+\frac{T_d}{N}s}\right)\right]R(s)$$
$$- K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+\frac{T_d}{N}s}\right)Y(s) \quad (2.41)$$

Mizutani and Hiroi (1991) state that $0 \le \alpha \le 1$, $\beta \ge 1$ and $\chi \ge 1$, and suggest starting values of $\alpha = 0.4$, $\beta = 1.35$ and $\chi = 1.25$. However, no tuning rules are available for this controller structure.

31. Non-interacting controller 3:

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+\frac{sT_d}{N}}Y(s) \quad (2.42)$$

This structure is used in the following products:
(a) Allen Bradley SLC5/02, SLC5/03, SLC5/04, PLC5 and Logix5550 products (EZYtune, 2003).
(b) Modcomp product with N = 10 (McMillan, 1994).

32. Non-interacting controller 4: This controller is also labelled the 'PI+D' controller (Chen, 1996).

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s) \quad (2.43)$$

This structure is used in the following products:
(a) ABB 53SL6000 Product (ABB, 2001).
(b) Genesis product (McMillan, 1994)
(c) Honeywell TDC3000 Process Manager Type B, non-interactive mode product (ISMC, 1999)
(d) Square D PIDR PID product (EZYtune, 2003)

33. Non-interacting controller 5:

$$U(s) = K_c\left(b + \frac{1}{T_i s}\right)E(s) - (c + T_d s)Y(s) \tag{2.44}$$

34. Non-interacting controller 6 (I-PD controller):

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c(1 + T_d s)Y(s) \tag{2.45}$$

This structure is used in the Toshiba AdTune TOSDIC 211D8 product (Shigemasa *et al.*, 1987) and the Honeywell TDC3000 Process Manager Type C non-interactive mode product (ISMC, 1999).

35. Non-interacting controller 7:

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c\left(1 + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)Y(s) \tag{2.46}$$

36. Non-interacting controller 8:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)E(s) - K_i(1 + T_{di} s)Y(s) \tag{2.47}$$

37. Non-interacting controller 9:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)E(s) - \frac{T_d}{N}sY(s) \tag{2.48}$$

38. Non-interacting controller 10:

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_f\left(\frac{T_d}{T_m}s + 1\right)Y(s) \tag{2.49}$$

39. Non-interacting controller 11:

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + T_d s)E(s) - K_i(1 + T_{di}s)Y(s) \tag{2.50}$$

40. Non-interacting controller 12:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{1 + T_d s}E(s) - K_1 Y(s) \tag{2.51}$$

41. Non-interacting controller 13:

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1 + \frac{T_d}{N}s + \frac{T_d}{2N}s^2}Y(s)\right) \tag{2.52}$$

This structure is used in the Foxboro IA PID product, with $10 \le N \le 50$ (EZYtune, 2003); however, no tuning rules are available for this controller structure.

2.3.4 *Other PID controller structures*

42. Industrial controller (Kaya and Scheib, 1988):

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(R(s) - \frac{1 + T_d s}{1 + \frac{T_d s}{N}}Y(s)\right) \tag{2.53}$$

This structure is used in the following products:

(a) Fisher-Rosemount Provox product with N = 8 (ISMC, 1999; McMillan, 1994)
(b) Foxboro Model 761 product with N=10 (McMillan, 1994)
(c) Fischer-Porter Micro DCI product with N = 0 (McMillan, 1994)
(d) Moore Products Type 352 controller with $1 \le N \le 30$ (McMillan, 1994)
(e) SATT Instruments EAC400 product with N=8.33 (McMillan, 1994)
(f) Taylor Mod 30 ESPO product with N=16.7 (McMillan, 1994)

(g) Honeywell TDC3000 Process Manager Type B, interactive mode product with N = 10 (ISMC, 1999)

43. Alternative controller 1: $G_c(s) = K_c\left(\dfrac{1+T_is}{1+\dfrac{T_ds}{N}}\right)\left(\dfrac{1+T_ds}{1+\dfrac{T_ds}{N}}\right)$ (2.54)

44. Alternative controller 2:

$$G_c(s) = K_c\left(\frac{1+T_is}{1+\dfrac{T_d}{N}s}\right)\left(\frac{1+0.5\tau_m s+0.0833{\tau_m}^2 s^2}{1+\dfrac{T_d}{N}s}\right) \quad (2.55)$$

45. Alternative controller 3:

$$U(s) = \frac{K_c}{T_is}R(s) - K_c\left(1+\frac{1}{T_is}\right)\left(\frac{1+sT_d}{1+\dfrac{sT_d}{N}}\right)Y(s) \quad (2.56)$$

This structure is used in the following products:

(a) Honeywell TDC3000 Process Manager Type C, interactive mode product with N = 10 (ISMC, 1999)

(b) Honeywell TDC3000 Universal, Multifunction and Advanced Multifunction products with N = 8 (ISMC, 1999)

However, no tuning rules are available for this controller structure.

46. Alternative controller 4:

$$U(s) = K_c\left(\frac{1+sT_d}{1+\dfrac{sT_d}{N}}\right)E(s) + \frac{R(s)}{K_m} \quad (2.57)$$

2.3.5 *Comments on the PID controller structures*

In some cases, one controller structure may be transformed into another; clearly, the ideal and parallel controller structures (equations (2.10) and (2.11)) are very closely related. It is shown by McMillan (1994), among others, that the parameters of the ideal PID controller may be worked out from the parameters of the series PID controller, and vice versa. The ideal PID controller is given in Equation (2.58) and the series PID controller is given in Equation (2.59).

$$G_{cp}(s) = K_{cp}\left(1+\frac{1}{T_{ip}s}+T_{dp}s\right) \tag{2.58}$$

$$G_{cs}(s) = K_{cs}\left(1+\frac{1}{T_{is}s}\right)(1+sT_{ds}) \tag{2.59}$$

Then, it may be shown that

$$K_{cp} = \left(1+\frac{T_{ds}}{T_{is}}\right)K_{cs},\ T_{ip} = (T_{is}+T_{ds}),\ T_{dp} = \left(\frac{T_{is}}{T_{is}+T_{ds}}\right)T_{ds}.$$

Similarly, it may be shown that, provided $T_{ip} > 4T_{dp}$,

$$K_{cs} = 0.5K_{cp}\left(1+\sqrt{1-4\frac{T_{dp}}{T_{ip}}}\right),\ T_{is} = 0.5T_{ip}\left(1+\sqrt{1-4\frac{T_{dp}}{T_{ip}}}\right),$$

$$T_{ds} = 0.5T_{dp}\left(1-\sqrt{1-4\frac{T_{dp}}{T_{ip}}}\right).$$

Åström and Hägglund (1996) point out that the ideal controller admits complex zeroes and is thus a more flexible controller structure than the series controller, which has real zeroes; however, in the frequency domain, the series controller has the interesting interpretation that the zeroes of the closed loop transfer function are the inverse values of T_{is} and T_{ds}. O'Dwyer (2001b) developed a comprehensive set of tuning rules for the series PID controller, based on the ideal PID controller; as these are not strictly original tuning rules, only representative examples are included in the relevant tables.

In a similar manner, the parameters of an ideal controller in series with a first order filter may be determined from the parameters of the classical controller 1 (Witt and Waggoner, 1990) and the parameters of the non-interacting controller 10 structure may be determined from the parameters of the ideal PID controller (Kaya *et al.*, 2003).

2.4 Process Modelling

Processes with time delay may be modelled in a variety of ways. The modelling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from the tuning rules. The modelling strategy used in association with each tuning rule, as described in the original papers, is indicated in the tables (see Chapters 3 and 4). These modelling strategies are outlined in Appendix 2. The following models are defined:

1. First order lag plus time delay (FOLPD) model ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$)
2. FOLPD model with a positive zero ($G_m(s) = \dfrac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{1 + sT_{m1}}$)
3. FOLPD model with a negative zero ($G_m(s) = \dfrac{K_m (1 + sT_{m3}) e^{-s\tau_m}}{1 + sT_{m1}}$)
4. Non-model specific
5. Integral plus time delay (IPD) model ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s}$)
6. First order lag plus integral plus time delay (FOLIPD) model

$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)})$$

7. Second order system plus time delay (SOSPD) model

$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1} \text{ or } G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)})$$

8. Integral squared plus time delay (I^2PD) model ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s^2}$)
9. Second order system (repeated pole) plus integral plus time delay (SOSIPD) model ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1 + sT_m)^2}$)

10. SOSPD model with a positive zero
$$(G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1})$$
11. SOSPD model with a negative zero
$$(G_m(s) = \frac{K_m(1 + sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1})$$
12. Third order system plus time delay model
$$(G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3)e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3)} \text{ or}$$
$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})})$$
13. Fifth order system plus delay model
$$(G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5)e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5)})$$
14. Unstable FOLPD model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$)
15. Unstable FOLPD model with a positive zero
$$(G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_m s - 1})$$
16. Unstable SOSPD model (one unstable pole)
$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})})$$
17. Unstable SOSPD model with a positive zero
$$(G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}{}^2 s^2 - 2\xi_m T_{m1} s + 1})$$
18. Unstable SOSPD model (two unstable poles)
$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(T_{m2}s - 1)})$$
19. Delay model ($G_m(s) = K_m e^{-s\tau_m}$)

20. General model with a repeated pole ($G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$)

21. General model with integrator

$$\left(G_m(s) = \frac{K_m}{s} \frac{\prod_i \left(T_{m_1i}s+1\right) \prod_i \left(T_{m_2i}{}^2 s^2 + 2\xi_{m_ni} T_{m_2i} s + 1\right)}{\prod_i \left(T_{m_3i}s+1\right) \prod_i \left(T_{m_4i}{}^2 s^2 + 2\xi_{m_di} T_{m_4i} s + 1\right)} e^{-s\tau_m} \right)$$

22. General stable non-oscillating model with a time delay

Tables 1 and 2 show the number of tuning rules defined for each PI or PID controller structure and each process model. The following data is key to the process model type in Tables 1 and 2:

Model 1: Stable FOLPD model
Model 2: Non-model specific
Model 3: IPD model
Model 4: FOLIPD model
Model 5: SOSPD model
Model 6: Other models

Table 1: PI controller structure and tuning rules – a summary

Process model Controller Equation	*1*	*2*	*3*	*4*	*5*	*6*	*Total*
(2.3)	172	42	46	10	32	62	**364**
(2.4)	3	0	2	0	0	2	**7**
(2.5)	2	0	0	0	0	0	**2**
(2.6)	7	2	4	3	3	13	**32**
(2.7)	1	0	9	14	0	7	**31**
(2.8)	2	0	1	1	0	2	**6**
(2.9)	0	0	1	0	0	0	**1**
Total	**187**	**44**	**63**	**28**	**35**	**86**	**443**

Table 2: PID controller structure and tuning rules – a summary

Process model Controller Equation	*1*	*2*	*3*	*4*	*5*	*6*	*Total*
	Ideal structure + variations						
(2.10,2.11)	100	51	22	12	55	36	**276**
(2.12)	7	2	1	5	13	9	**37**
(2.13)	0	0	0	0	1	0	**1**
(2.14)	2	2	0	0	1	0	**5**
(2.15)	2	2	0	1	0	0	**5**
(2.16)	1	0	0	0	0	0	**1**
(2.17)	0	0	1	0	0	0	**1**
(2.18)	1	0	0	0	0	0	**1**
(2.19)	16	11	2	2	3	5	**39**
(2.20)	0	0	0	0	1	0	**1**
(2.21)	0	0	0	1	0	0	**1**
(2.22)	0	0	0	0	0	1	**1**
(2.23)	0	0	0	2	0	0	**2**
(2.24)	0	0	0	0	0	2	**2**
(2.25)	0	0	1	0	0	0	**1**
(2.26)	0	1	0	1	1	9	**12**
(2.27)	0	0	0	0	2	0	**2**
(2.28)	1	0	0	0	0	0	**1**
Subtotal	***130***	***69***	***27***	***24***	***77***	***62***	***389***
	Classical structure + variations						
(2.29)	54	5	8	5	20	9	**101**
(2.30)	1	0	2	1	2	0	**6**
(2.31)	6	5	0	1	7	6	**25**
(2.32)	2	1	1	1	0	2	**7**
Subtotal	***63***	***11***	***11***	***8***	***29***	***17***	***139***
	Non-interacting structure + variations						
(2.33)	1	0	0	0	5	2	**8**
(2.34)	5	0	0	0	0	0	**5**
(2.35)	6	0	0	0	0	0	**6**
(2.36)	12	1	5	6	7	13	**44**
(2.37)	4	0	0	0	0	0	**4**
(2.38)	2	0	1	0	0	0	**3**
(2.42)	0	0	0	0	0	4	**4**
(2.43)	10	3	4	2	3	0	**22**
(2.44)	2	0	0	0	1	0	**3**
(2.45)	10	0	8	11	1	0	**30**
(2.46)	1	0	0	0	0	1	**2**
(2.47)	1	0	1	2	0	4	**8**
(2.48)	0	1	0	0	0	0	**1**

Process model Controller Equation	*1*	*2*	*3*	*4*	*5*	*6*	*Total*
	Non-interacting structure + variations (continued)						
(2.49)	0	0	1	0	0	2	**3**
(2.50)	0	0	0	1	0	0	**1**
(2.51)	1	0	1	1	0	1	**4**
Subtotal	***55***	***5***	***21***	***23***	***17***	***27***	***148***
	Other controller structures						
(2.53)	8	0	0	1	0	1	**10**
(2.54)	0	0	0	2	0	0	**2**
(2.55)	0	0	0	1	0	0	**1**
(2.57)	0	0	0	0	2	0	**2**
Subtotal	***8***	***0***	***0***	***4***	***2***	***1***	***15***
Total	**256**	**85**	**59**	**59**	**125**	**107**	**691**

Table 1 shows that 82% of tuning rules have been defined for the ideal PI controller structure, with 42% of tuning rules based on a FOLPD process model.

The range of PID controller variations has lead to a less homogenous situation than for the PI controller; Table 2 shows that 40% of tuning rules have been defined for the ideal PID controller structure, with 37% of tuning rules based on a FOLPD process model. Many of the controller structures are used in a variety of manufacturers' products (as outlined in Section 2.3); clearly a range of tuning rules are available for use with these products.

2.5 Organisation of the Tuning Rules

The tuning rules are organised in tabular form in Chapters 3 and 4. Within each table, the tuning rules are classified further; the main subdivisions made are as follows:

(i) Tuning rules based on a measured step response (also called process reaction curve methods).
(ii) Tuning rules based on minimising an appropriate performance criterion, either for optimum regulator or optimum servo action.
(iii) Tuning rules that give a specified closed loop response (direct synthesis tuning rules). Such rules may be defined by specifying the

desired poles of the closed loop response, for instance, though more generally, the desired closed loop transfer function may be specified. The definition may be expanded to cover techniques that allow the achievement of a specified gain margin and/or phase margin.

(iv) Robust tuning rules, with an explicit robust stability and robust performance criterion built in to the design process.

(v) Tuning rules based on recording appropriate parameters at the ultimate frequency (also called ultimate cycling methods).

(vi) Other tuning rules, such as tuning rules that depend on the proportional gain required to achieve a quarter decay ratio or to achieve magnitude and frequency information at a particular phase lag.

Some tuning rules could be considered to belong to more than one subdivision, so the subdivisions cannot be considered to be mutually exclusive; nevertheless, they provide a convenient way to classify the rules. In the tables, all symbols used are defined in Appendix 1.

Chapter 3

Tuning Rules for PI Controllers

3.1 FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

3.1.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

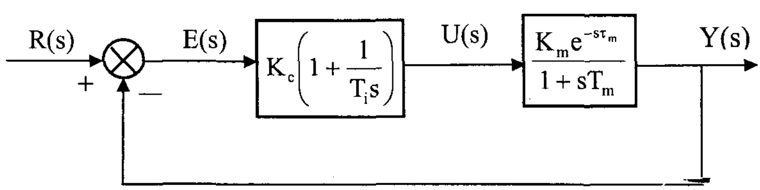

Table 3: PI controller tuning rules - FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
	Process reaction		
Callender *et al.* (1935/6). *Model: Method 1*	[1] $0.568/K_m\tau_m$	$3.64\tau_m$	$\frac{\tau_m}{T_m} = 0.3$
	[2] $0.690/K_m\tau_m$	$2.45\tau_m$	

[1] Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

[2] Decay ratio = 0.043; Period of decaying oscillation = $6.28\tau_m$

Rule	K_c	T_i	Comment
Ziegler and Nichols (1942). *Model: Method 2*	$\dfrac{0.9T_m}{K_m\tau_m}$	$3.33\tau_m$	Quarter decay ratio. $\dfrac{\tau_m}{T_m} \le 1$
Hazebroek and Van der Waerden (1950). *Model: Method 2*	$\dfrac{x_1 T_m}{K_m\tau_m}$	$x_2\tau_m$	

Coefficient values

$\dfrac{\tau_m}{T_m}$	x_1	x_2	$\dfrac{\tau_m}{T_m}$	x_1	x_2	$\dfrac{\tau_m}{T_m}$	x_1	x_2
0.2	0.68	7.14	1.1	0.90	1.49	2.0	1.20	1.00
0.3	0.70	4.76	1.2	0.93	1.41	2.2	1.28	0.95
0.4	0.72	3.70	1.3	0.96	1.32	2.4	1.36	0.91
0.5	0.74	3.03	1.4	0.99	1.25	2.6	1.45	0.88
0.6	0.76	2.50	1.5	1.02	1.19	2.8	1.53	0.85
0.7	0.79	2.17	1.6	1.06	1.14	3.0	1.62	0.83
0.8	0.81	1.92	1.7	1.09	1.10	3.2	1.71	0.81
0.9	0.84	1.75	1.8	1.13	1.06	3.4	1.81	0.80
1.0	0.87	1.61	1.9	1.17	1.03			

Rule	K_c	T_i	Comment
	$x_1 = 0.5\dfrac{\tau_m}{T_m} + 0.1$	$x_2 = \dfrac{\tau_m}{1.6\tau_m - 1.2T_m}$	$\dfrac{\tau_m}{T_m} > 3.5$
Oppelt (1951). *Model: Method 2*	[3] $K_c^{(1)}$	$1.66\tau_m$	$\tau_m >> T_m$
		$3.32\tau_m$	$\tau_m << T_m$
Moros (1999). *Model: Method 1*	$0.8T_m/K_m\tau_m$	$3\tau_m$	attributed to Oppelt
	$0.91T_m/K_m\tau_m$	$3.3\tau_m$	attributed to Rosenberg
Chien *et al.* (1952) – regulator. *Model: Method 2*; $0.1 < \dfrac{\tau_m}{T_m} < 1.0$	$\dfrac{0.6T_m}{K_m\tau_m}$	$4\tau_m$	0% overshoot
	$\dfrac{0.7T_m}{K_m\tau_m}$	$2.33\tau_m$	20% overshoot
Chien *et al.* (1952) – servo. *Model: Method 2*; $0.1 < \dfrac{\tau_m}{T_m} < 1.0$	$\dfrac{0.35T_m}{K_m\tau_m}$	$1.17T_m$	0% overshoot
	$\dfrac{0.6T_m}{K_m\tau_m}$	T_m	20% overshoot

---

[3] $K_c^{(1)} \le \dfrac{1}{K_m}\left[1.5708\dfrac{T_m}{\tau_m} + 1\right]$; recommended $K_c^{(1)} = \dfrac{1}{K_m}\left[0.77\dfrac{T_m}{\tau_m} - 1\right]$.

Rule	K_c	T_i	Comment
Reswick (1956). *Model: Method 2*; K_c, T_i *deduced from graphs*. $\frac{\tau_m}{T_m}=6.25$	$0.20/K_m$	$0.20\tau_m$	0% overshoot – regulator
	$0.30/K_m$	$0.22\tau_m$	20% overshoot – regulator
	$0.15/K_m$	$0.16\tau_m$	0% overshoot – servo
	$0.20/K_m$	$0.14\tau_m$	20% overshoot – servo
Cohen and Coon (1953). *Model: Method 2*	$\frac{1}{K_m}\left(0.9\frac{T_m}{\tau_m}+0.083\right)$	$T_i^{(2)}$ [4]	Quarter decay ratio; $0<\frac{\tau_m}{T_m}\le 1.0$
Two constraints method – Wolfe (1951). *Model: Method 4*	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2\tau_m$	Decay ratio is as small as possible; minimum error integral (regulator mode).
Two constraints criterion – Murrill (1967) – *page 356. Model: Method 5*	$\frac{0.928}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.946}$	$\frac{T_m}{1.078}\left(\frac{\tau_m}{T_m}\right)^{0.583}$	Quarter decay ratio; minimum error integral (servo mode). $0.1\le\frac{\tau_m}{T_m}\le 1.0$
Heck (1969). *Model: Method 2*	$\frac{0.8T_m}{K_m\tau_m}$	$3\tau_m$	
	0	$x_2 K_m \tau_m$	Integral controller

Coefficient values (Wolfe)

$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2
0.2	4.4	3.23	1.0	0.78	1.28
0.5	1.8	2.27	5.0	0.30	0.53

Representative coefficient values – from graph – “Störung” (Heck)

$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2
0.11	2.94	0.20	2.50	0.40	2.17	0.71	1.96
0.13	2.86	0.25	2.38	0.50	2.13		
0.14	2.78	0.29	2.35	0.56	2.08		
0.17	2.63	0.33	2.30	0.63	2.00		

---

[4] $$T_i^{(2)} = T_m\left(\frac{3.33\frac{\tau_m}{T_m}+0.31\left(\frac{\tau_m}{T_m}\right)^2}{1+2.22\frac{\tau_m}{T_m}}\right).$$

Rule	K_c	T_i	Comment
Heck (1969)-continued.	Representative coefficient values – from graph – "Führung"		

$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2
0.11	3.92	0.20	3.33	0.40	2.63	0.71	2.27
0.13	3.85	0.25	3.08	0.50	2.47		
0.14	3.64	0.29	2.94	0.56	2.41		
0.17	3.51	0.33	2.82	0.63	2.35		

Rule	K_c	T_i	Comment
Fertik and Sharpe (1979).	$\frac{0.56}{K_m}$	$0.65T_m$	*Model: Method 3*
Sakai *et al.* (1989). *Model: Method 2*	$\frac{1.2408K_m}{T_m\tau_m}$	$0.5\tau_m$	
Borresen and Grindal (1990). *Model: Method 2*	$\frac{1.0T_m}{K_m\tau_m}$	$3\tau_m$	Also described by ABB (2001)
Klein *et al.* (1992). *Model: Method 1*	$\frac{0.28T_m}{K_m(\tau_m+0.1T_m)}$	$0.53T_m$	
McMillan (1994) – *page 25. Model: Method 5*	$\frac{K_m}{3}$	τ_m	Time delay dominant processes
St. Clair (1997) – *page 22. Model: Method 5*	$\frac{0.333T_m}{K_m\tau_m}$	T_m	$\frac{\tau_m}{T_m} \ge 0.33$
Hay (1998) – *pages 197-198. Model: Method 1; Coefficients of* K_c, T_i *deduced from graphs*	x_1/K_m	$x_2\tau_m$	

Coefficient values – servo tuning

τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
0.1	6.0	9.5	0.1	4.0	10.0
0.125	4.5	7.5	0.125	3.2	8.0
0.167	3.0	5.4	0.167	2.2	6.0
0.25	2.0	3.4	0.25	1.3	4.0
0.5	0.8	1.7	0.5	0.5	2.0

Coefficient values – regulator tuning

τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
0.1	9.5	3.2	0.1	6.8	2.8
0.125	7.0	3.2	0.125	5.6	2.8
0.167	5.0	3.0	0.167	4.3	2.6
0.25	3.4	2.8	0.25	2.8	2.3
0.5	1.8	2.0	0.5	1.2	1.6

Rule	K_c	T_i	Comment
Shinskey (2000), (2001). *Model: Method 2*	$\frac{0.667T_m}{K_m\tau_m}$	$3.78\tau_m$	$\frac{\tau_m}{T_m} = 6$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td>Lipták (2001).
Model: Method 2</td><td colspan="2">$\frac{0.95T_m}{K_m\tau_m}$</td><td colspan="2">$4\tau_m$</td><td colspan="2"></td></tr>
<tr><td></td><td colspan="6">**Minimum performance index: regulator tuning**</td></tr>
<tr><td>Minimum IAE – Murrill (1967) – *pages 358-363.*</td><td colspan="2">$\frac{0.984}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.986}$</td><td colspan="2">$\frac{T_m}{0.608}\left(\frac{\tau_m}{T_m}\right)^{0.707}$</td><td colspan="2">*Model: Method 5*;
$0.1 \le \frac{\tau_m}{T_m} \le 1.0$</td></tr>
<tr><td rowspan="7">Minimum IAE – Frank and Lenz (1969).
Model: Method 1</td><td colspan="2">$\frac{1}{K_m}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$</td><td colspan="2">$\frac{\tau_m}{x_2}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Representative coefficient values – deduced from a graph</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.2</td><td>0.37</td><td>0.77</td><td>1.0</td><td>0.46</td><td>1.05</td></tr>
<tr><td>0.4</td><td>0.42</td><td>0.85</td><td>1.2</td><td>0.46</td><td>1.11</td></tr>
<tr><td>0.6</td><td>0.44</td><td>0.91</td><td>1.4</td><td>0.46</td><td>1.17</td></tr>
<tr><td>0.8</td><td>0.45</td><td>0.98</td><td></td><td></td><td></td></tr>
<tr><td>Minimum IAE – Pemberton (1972a), Smith and Corripio (1997) – *page 345.*</td><td colspan="2">$\frac{T_m}{K_m\tau_m}$</td><td colspan="2">T_m</td><td colspan="2">*Model: Method 1*;
$0.1 \le \frac{\tau_m}{T_m} \le 0.5$</td></tr>
<tr><td rowspan="2">Minimum IAE – Yu (1988).
Model: Method 1.
Load disturbance model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$.</td><td colspan="2">[5] $K_c^{(3)}$</td><td colspan="2">$T_i^{(3)}$</td><td colspan="2" rowspan="2">$\frac{\tau_m}{T_m} \le 0.35$</td></tr>
<tr><td colspan="2">[6] $K_c^{(4)}$</td><td colspan="2">$T_i^{(4)}$</td></tr>
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[5] $K_c^{(3)} = \frac{0.685}{K_m}\left(\frac{T_L}{T_m}\right)^{0.214-0.346\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.256}$,

$$T_i^{(3)} = \frac{T_m}{0.214}\left(\frac{T_L}{T_m}\right)^{1.977\frac{\tau_m}{T_m}-0.55}\left(\frac{\tau_m}{T_m}\right)^{1.123}, \ \frac{T_L}{T_m} \le 2.641\frac{\tau_m}{T_m} + 0.16 .$$

[6] $K_c^{(4)} = \frac{0.874}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.099+0.159\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.041}$,

$$T_i^{(4)} = \frac{T_m}{0.415}\left(\frac{T_L}{T_m}\right)^{-4.515\frac{\tau_m}{T_m}+0.067}\left(\frac{\tau_m}{T_m}\right)^{0.876}, \ 2.641\frac{\tau_m}{T_m} + 0.16 \le \frac{T_L}{T_m} \le 1 .$$

Rule	K_c	T_i	Comment
Minimum IAE – Yu (1988) – continued.	[7] $K_c^{(5)}$	$T_i^{(5)}$	
	[8] $K_c^{(6)}$	$T_i^{(6)}$	$\tau_m/T_m > 0.35$
Minimum IAE – Shinskey (1988) – *page 123.* *Model: Method 1*	$1.00T_m/K_m\tau_m$	$3.0\tau_m$	$\tau_m/T_m = 0.2$
	$1.04T_m/K_m\tau_m$	$2.25\tau_m$	$\tau_m/T_m = 0.5$
	$1.08T_m/K_m\tau_m$	$1.45\tau_m$	$\tau_m/T_m = 1$
	$1.39T_m/K_m\tau_m$	τ_m	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1996) – *page 38.* *Model: Method 1*	$0.95T_m/K_m\tau_m$	$3.4\tau_m$	$\tau_m/T_m = 0.1$
	$0.95T_m/K_m\tau_m$	$2.9\tau_m$	$\tau_m/T_m = 0.2$
Minimum IAE – Marlin (1995) – *pages 301-307.* *Model: Method 1.* K_c, T_i *deduced from graphs; robust to ±25% process model parameter changes.*	$1.4/K_m$	$0.24\tau_m$	$\tau_m/T_m = 0.11$
	$1.8/K_m$	$0.52\tau_m$	$\tau_m/T_m = 0.25$
	$1.4/K_m$	$0.75\tau_m$	$\tau_m/T_m = 0.43$
	$1.0/K_m$	$0.68\tau_m$	$\tau_m/T_m = 0.67$
	$0.8/K_m$	$0.71\tau_m$	$\tau_m/T_m = 1.0$
	$0.55/K_m$	$0.60\tau_m$	$\tau_m/T_m = 1.50$
	$0.45/K_m$	$0.54\tau_m$	$\tau_m/T_m = 2.33$
	$0.35/K_m$	$0.49\tau_m$	$\tau_m/T_m = 4.0$
Minimum IAE – Edgar *et al.* (1997) – *pages 8-14,8-15.* *Model: Method 1*	$0.4/K_m$	$0.5\tau_m$	Time delay dominant
	$0.94T_m/K_m\tau_m$	$4\tau_m$	Time constant dominant
Minimum IAE – Edgar *et al.* (1997) – *page 8-15.*	$0.67T_m/K_m\tau_m$	$3.5\tau_m$	*Model: Method 2*

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[7] $$K_c^{(5)} = \frac{0.871}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.015+0.384\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.055},$$

$$T_i^{(5)} = \frac{T_m}{0.444}\left(\frac{T_L}{T_m}\right)^{-0.217\frac{\tau_m}{T_m}-0.213}\left(\frac{\tau_m}{T_m}\right)^{0.867}, \quad 1 < \frac{T_L}{T_m} \le 3.$$

[8] $$K_c^{(6)} = \frac{0.513}{K_m}\left(\frac{T_L}{T_m}\right)^{0.218\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.451}, \quad T_i^{(6)} = \frac{T_m}{0.670}\left(\frac{T_L}{T_m}\right)^{-0.003\frac{\tau_m}{T_m}-0.084}\left(\frac{\tau_m}{T_m}\right)^{0.56}.$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang *et al.* (1996). *Model: Method 1*	[9] $K_c^{(7)}$	$T_i^{(7)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$
Minimum IAE – Shinskey (1988) – *page 148. Model: Method 1*	$0.58K_u$	$0.81T_u$	$\tau_m/T_m = 0.2$
	$0.54K_u$	$0.66T_u$	$\tau_m/T_m = 0.5$
	$0.48K_u$	$0.47T_u$	$\tau_m/T_m = 1$
	$0.46K_u$	$0.37T_u$	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1994) – *page 167.*	$\dfrac{K_u}{3.05 - 0.35\frac{T_u}{\tau_m}}$	[10] $T_i^{(8)}$	*Model: Method 1*
Minimum IAE – Shinskey (1996) – *page 121.*	$0.55K_u$	$0.78T_u$	*Model: Method 1*; $\tau_m/T_m = 0.2$
Minimum ISE – Hazebroek and Van der Waerden (1950). *Model: Method 2*	[11] $K_c^{(9)}$	$1.43T_m$	$\tau_m/T_m < 0.2$

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[9] $$K_c^{(7)} = \frac{1}{K_m}\left[6.4884 + 4.6198\frac{\tau_m}{T_m} + 0.8196\left(\frac{\tau_m}{T_m}\right)^{-0.9077} - 5.2132\left(\frac{\tau_m}{T_m}\right)^{-0.063}\right] + \frac{1}{K_m}\left[-7.2712\left(\frac{\tau_m}{T_m}\right)^{0.5961} - 0.7241e^{\frac{\tau_m}{T_m}}\right],$$

$$T_i^{(7)} = T_m\left[0.0064 + 3.9574\frac{\tau_m}{T_m} - 6.4789\left(\frac{\tau_m}{T_m}\right)^2 + 9.4348\left(\frac{\tau_m}{T_m}\right)^3 - 10.7619\left(\frac{\tau_m}{T_m}\right)^4\right] + T_m\left[7.5146\left(\frac{\tau_m}{T_m}\right)^5 - 2.2236\left(\frac{\tau_m}{T_m}\right)^6\right].$$

[10] $$T_i^{(8)} = T_u\left(0.87 - 0.855\frac{T_u}{\tau_m} + 0.172\left[\frac{T_u}{\tau_m}\right]^2\right).$$

[11] $$K_c^{(9)} = \frac{T_m}{K_m\tau_m}\left(0.74 + 0.3\frac{\tau_m}{T_m}\right).$$

Rule	K_c	T_i	Comment
Hazebroek and Van der Waerden (1950) – continued.	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$	

Coefficient values

$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2
0.2	0.80	7.14	0.7	0.96	2.44	2.0	1.46	1.18
0.3	0.83	5.00	1.0	1.07	1.85	3.0	1.89	0.95
0.5	0.89	3.23	1.5	1.26	1.41	5.0	2.75	0.81

Rule	K_c	T_i	Comment
Minimum ISE – Haalman (1965). *Model: Method 1*	$\frac{0.67 T_m}{K_m \tau_m}$	T_m	$M_{max} = 1.9$; $A_m = 2.36$; $\phi_m = 50^0$
Minimum ISE – Murrill (1967) – *pages 358-363. Model: Method 5*	$\frac{1.305}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.959}$	$\frac{T_m}{0.492}\left(\frac{\tau_m}{T_m}\right)^{0.739}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Minimum ISE – Frank and Lenz (1969). *Model: Method 1*	$\frac{1}{K_m}\left(x_1 + x_2 \frac{T_m}{\tau_m}\right)$	$\frac{\tau_m}{x_2}\left(x_1 + x_2 \frac{T_m}{\tau_m}\right)$	

Representative coefficient values – deduced from a graph

τ_m / T_m	x_1	x_2	τ_m / T_m	x_1	x_2
0.2	0.53	0.82	1.0	0.63	1.12
0.4	0.58	0.88	1.2	0.63	1.17
0.6	0.61	0.97	1.4	0.62	1.20
0.8	0.62	1.05			

Rule	K_c	T_i	Comment
Minimum ISE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.279}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.945}$	$\frac{T_m}{0.535}\left(\frac{\tau_m}{T_m}\right)^{0.586}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.346}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.675}$	$\frac{T_m}{0.552}\left(\frac{\tau_m}{T_m}\right)^{0.438}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Yu (1988). *Model: Method 1.*	[12] $K_c^{(10)}$	$T_i^{(10)}$	$\frac{\tau_m}{T_m} \le 0.35$

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[12] Load disturbance model $= \frac{K_L e^{-s\tau_L}}{1+sT_L}$. $K_c^{(10)} = \frac{0.921}{K_m}\left(\frac{T_L}{T_m}\right)^{0.181-0.205\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.214}$,

$$T_i^{(10)} = \frac{T_m}{0.430}\left(\frac{T_L}{T_m}\right)^{0.954\frac{\tau_m}{T_m}-0.49}\left(\frac{\tau_m}{T_m}\right)^{0.639}, \quad \frac{T_L}{T_m} \le 2.310\frac{\tau_m}{T_m} + 0.077 .$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="3">Minimum ISE – Yu (1988) – continued.</td><td colspan="2">[13] $K_c^{(11)}$</td><td colspan="2">$T_i^{(11)}$</td><td colspan="2" rowspan="2">$\frac{\tau_m}{T_m} \le 0.35$</td></tr>
<tr><td colspan="2">[14] $K_c^{(12)}$</td><td colspan="2">$T_i^{(12)}$</td></tr>
<tr><td colspan="2">[15] $K_c^{(13)}$</td><td colspan="2">$T_i^{(13)}$</td><td colspan="2">$\frac{\tau_m}{T_m} > 0.35$</td></tr>
<tr><td>Minimum ITAE - Murrill (1967) – pages 358-363. Model: Method 5</td><td colspan="2">$\frac{0.859}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.977}$</td><td colspan="2">$\frac{T_m}{0.674}\left(\frac{\tau_m}{T_m}\right)^{0.680}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 1.0$</td></tr>
<tr><td rowspan="7">Minimum ITAE – Frank and Lenz (1969). Model: Method 1</td><td colspan="2">$\frac{1}{K_m}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$</td><td colspan="2">$\frac{\tau_m}{x_2}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Representative coefficient values – deduced from a graph</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.2</td><td>0.32</td><td>0.77</td><td>1.0</td><td>0.38</td><td>1.00</td></tr>
<tr><td>0.4</td><td>0.35</td><td>0.80</td><td>1.2</td><td>0.39</td><td>1.05</td></tr>
<tr><td>0.6</td><td>0.37</td><td>0.86</td><td>1.4</td><td>0.39</td><td>1.10</td></tr>
<tr><td>0.8</td><td>0.38</td><td>0.93</td><td></td><td></td><td></td></tr>
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[13] $K_c^{(11)} = \frac{1.157}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.045+0.344\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.014}$,

$$T_i^{(11)} = \frac{T_m}{0.359}\left(\frac{T_L}{T_m}\right)^{-2.532\frac{\tau_m}{T_m}-0.292}\left(\frac{\tau_m}{T_m}\right)^{0.899}, \quad 2.310\frac{\tau_m}{T_m} + 0.077 \le \frac{T_L}{T_m} \le 1 .$$

[14] $K_c^{(12)} = \frac{1.07}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.065+0.234\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.047}$,

$$T_i^{(12)} = \frac{T_m}{0.347}\left(\frac{T_L}{T_m}\right)^{-1.112\frac{\tau_m}{T_m}-0.094}\left(\frac{\tau_m}{T_m}\right)^{0.898}, \quad 1 < \frac{T_L}{T_m} \le 3 .$$

[15] $K_c^{(13)} = \frac{1.289}{K_m}\left(\frac{T_L}{T_m}\right)^{0.04+0.067\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-0.889}, \quad T_i^{(13)} = \frac{T_m}{0.596}\left(\frac{T_L}{T_m}\right)^{0.372\frac{\tau_m}{T_m}-0.44}\left(\frac{\tau_m}{T_m}\right)^{0.46}$.

Rule	K_c		T_i		Comment	
Minimum ITAE – Yu (1988). *Model: Method 1.* Load disturbance model $= \frac{K_L e^{-s\tau_L}}{1+sT_L}$.	$K_c^{(14)}$ [16]		$T_i^{(14)}$		$\frac{\tau_m}{T_m} \le 0.35$	
	$K_c^{(15)}$ [17]		$T_i^{(15)}$			
	$K_c^{(16)}$ [18]		$T_i^{(16)}$			
	$K_c^{(17)}$ [19]		$T_i^{(17)}$		$\frac{\tau_m}{T_m} > 0.35$	
Minimum ITSE – Frank and Lenz (1969). *Model: Method 1*	$\frac{1}{K_m}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$		$\frac{\tau_m}{x_2}\left(x_1 + x_2\frac{T_m}{\tau_m}\right)$			
	Representative coefficient values – deduced from a graph					
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
	0.2	0.46	0.84	1.0	0.55	1.09
	0.4	0.50	0.89	1.2	0.54	1.15
	0.6	0.53	0.97	1.4	0.54	1.20
	0.8	0.55	1.04			

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[16] $$K_c^{(14)} = \frac{0.598}{K_m}\left(\frac{T_L}{T_m}\right)^{0.272-0.254\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.341},$$

$$T_i^{(14)} = \frac{T_m}{0.805}\left(\frac{T_L}{T_m}\right)^{0.304\frac{\tau_m}{T_m}-0.112}\left(\frac{\tau_m}{T_m}\right)^{0.196}, \quad \frac{T_L}{T_m} \le 2.385\frac{\tau_m}{T_m}+0.112.$$

[17] $$K_c^{(15)} = \frac{0.735}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.011-1.945\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.055},$$

$$T_i^{(15)} = \frac{T_m}{0.425}\left(\frac{T_L}{T_m}\right)^{-5.809\frac{\tau_m}{T_m}+0.241}\left(\frac{\tau_m}{T_m}\right)^{0.901}, \quad 2.385\frac{\tau_m}{T_m}+0.112 \le \frac{T_L}{T_m} \le 1.$$

[18] $$K_c^{(16)} = \frac{0.787}{K_m}\left(\frac{T_L}{T_m}\right)^{0.084+0.154\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.042},$$

$$T_i^{(16)} = \frac{T_m}{0.431}\left(\frac{T_L}{T_m}\right)^{-0.148\frac{\tau_m}{T_m}-0.365}\left(\frac{\tau_m}{T_m}\right)^{0.901}, \quad 1 < \frac{T_L}{T_m} \le 3.$$

[19] $$K_c^{(17)} = \frac{0.878}{K_m}\left(\frac{T_L}{T_m}\right)^{0.172-0.057\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-0.909},$$

$$T_i^{(17)} = \frac{T_m}{0.794}\left(\frac{T_L}{T_m}\right)^{0.228\frac{\tau_m}{T_m}-0.257}\left(\frac{\tau_m}{T_m}\right)^{0.489}.$$

Rule	K_c	T_i	Comment
Minimum ISTSE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.015}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.957}$	$\frac{T_m}{0.667}\left(\frac{\tau_m}{T_m}\right)^{0.552}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.065}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.673}$	$\frac{T_m}{0.687}\left(\frac{\tau_m}{T_m}\right)^{0.427}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	[20] $K_c^{(18)}$	$T_i^{(18)}$	*Model: Method 1*; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	[21] $K_c^{(19)}$	$T_i^{(19)}$	*Model: Method 31*; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.021}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.953}$	$\frac{T_m}{0.629}\left(\frac{\tau_m}{T_m}\right)^{0.546}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.076}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.648}$	$\frac{T_m}{0.650}\left(\frac{\tau_m}{T_m}\right)^{0.442}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Nearly minimum IAE, ISE, ITAE – Hwang (1995).	[22] $K_c^{(20)}$	$K_c^{(20)}/\mu_2 K_u \omega_u$	*Model: Method 35*; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$

[20] $K_c^{(18)} = \left(\frac{1.892K_mK_u + 0.244}{3.249K_mK_u + 2.097}\right)K_u$, $T_i^{(18)} = \left(\frac{0.706K_mK_u - 0.227}{0.7229K_mK_u + 1.2736}\right)T_u$.

[21] $K_c^{(19)} = \left(\frac{4.126K_m\hat{K}_u - 2.610}{5.848K_m\hat{K}_u - 1.06}\right)\hat{K}_u$, $T_i^{(19)} = \left(\frac{5.352K_m\hat{K}_u - 2.926}{5.539K_m\hat{K}_u + 5.036}\right)\hat{T}_u$.

[22] $K_c^{(20)} = (1-\mu_1)K_u$, $\mu_1 = \frac{1.14\left(1 - 0.482\omega_u\tau_m + 0.068\omega_u^2\tau_m^2\right)}{K_uK_m/1 + K_uK_m}$,

$\mu_2 = \frac{0.0694\left(-1 + 2.1\omega_u\tau_m - 0.367\omega_u^2\tau_m^2\right)}{K_uK_m/1 + K_uK_m}$, Decay ratio = 0.15.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td>Nearly minimum IAE and ITAE – Hwang and Fang (1995). Model: Method 25</td><td colspan="2">[23] $K_c^{(21)}$</td><td colspan="2">$T_i^{(21)}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.12</td></tr>
<tr><td>Gerry (2003) – minimum error: step load change. Model: Method 1</td><td colspan="2">$\frac{0.3}{K_m}$</td><td colspan="2">$0.42\tau_m$</td><td colspan="2">$\frac{\tau_m}{T_m} \ge 5$</td></tr>
<tr><td rowspan="11">Minimum IAE or ISE – ECOSSE Team (1996a). Model: Method 1.

Coefficients of K_c, T_i deduced from graphs</td><td colspan="2">x_1/K_m</td><td colspan="2">$x_2(\tau_m + T_m)$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.11</td><td>1.1</td><td>0.23</td><td>0.11</td><td>5.8</td><td>0.5</td></tr>
<tr><td>0.25</td><td>1.8</td><td>0.23</td><td>0.25</td><td>3.1</td><td>0.6</td></tr>
<tr><td>0.43</td><td>1.1</td><td>0.72</td><td>0.43</td><td>2.1</td><td>0.7</td></tr>
<tr><td>0.67</td><td>1.0</td><td>0.72</td><td>0.67</td><td>1.7</td><td>0.8</td></tr>
<tr><td>1.0</td><td>0.8</td><td>0.70</td><td>1.0</td><td>0.91</td><td>0.9</td></tr>
<tr><td>1.50</td><td>0.59</td><td>0.67</td><td></td><td></td><td></td></tr>
<tr><td>2.33</td><td>0.42</td><td>0.60</td><td></td><td></td><td></td></tr>
<tr><td>4.0</td><td>0.32</td><td>0.53</td><td></td><td></td><td></td></tr>
<tr><td rowspan="8">Minimum IE – Devanathan (1991). Model: Method 1.

Coefficients of K_c, T_i deduced from graphs</td><td colspan="2" rowspan="8">[24] $K_c^{(22)}$</td><td colspan="2">$x_2 T_u$</td><td colspan="2"></td></tr>
<tr><td colspan="4">Coefficient values</td></tr>
<tr><td>τ_m/T_m</td><td>x_2</td><td>τ_m/T_m</td><td>x_2</td></tr>
<tr><td>0.2</td><td>0.35</td><td>1.2</td><td>0.19</td></tr>
<tr><td>0.4</td><td>0.29</td><td>1.4</td><td>0.18</td></tr>
<tr><td>0.6</td><td>0.26</td><td>1.6</td><td>0.18</td></tr>
<tr><td>0.8</td><td>0.23</td><td>1.8</td><td>0.17</td></tr>
<tr><td>1.0</td><td>0.21</td><td></td><td></td></tr>
</table>

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[23] $K_c^{(21)} = \left(0.515 - 0.0521\frac{\tau_m}{T_m} + 0.0254\left[\frac{\tau_m}{T_m}\right]^2\right)K_u$,

$$T_i^{(21)} = \frac{(K_c^{(21)}/K_u\omega_u)}{\left(0.0877 + 0.0918\frac{\tau_m}{T_m} - 0.0141\left[\frac{\tau_m}{T_m}\right]^2\right)}.$$

[24] $K_c^{(22)} = \dfrac{\xi\cos\left[\tan^{-1}\left(1.57D_R - \dfrac{0.16}{D_R}\right)\right]}{K_m\cos\left[\tan^{-1}\left(\dfrac{6.28T_m}{T_u}\right)\right]}$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td></td><td colspan="6">Minimum performance index: servo tuning</td></tr>
<tr><td rowspan="6">Minimum IAE – Gallier and Otto (1968).
Model: Method 47</td><td colspan="2">x_1/K_m</td><td colspan="2">$x_2(\tau_m + T_m)$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Representative coefficient values – deduced from graphs</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.053</td><td>12</td><td>0.95</td><td>0.67</td><td>1.15</td><td>0.75</td></tr>
<tr><td>0.11</td><td>6.4</td><td>0.91</td><td>1.50</td><td>0.65</td><td>0.66</td></tr>
<tr><td>0.25</td><td>2.9</td><td>0.84</td><td>4.0</td><td>0.45</td><td>0.55</td></tr>
<tr><td>Minimum IAE – Rovira *et al.* (1969).
Model: Method 5</td><td colspan="2">$\frac{0.758}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.861}$</td><td colspan="2">$\frac{T_m}{1.020 - 0.323\frac{\tau_m}{T_m}}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 1.0$</td></tr>
<tr><td>Minimum IAE - Bain and Martin (1983).
Model: Method 1</td><td colspan="2">$\frac{1}{K_m}\left[0.3 + 0.3\frac{T_m}{\tau_m}\right]$</td><td colspan="2">$T_m + 0.4\tau_m$</td><td colspan="2">$\frac{\tau_m}{T_m} > 0.2$</td></tr>
<tr><td rowspan="2">Minimum IAE – Marlin (1995) – *pages 301-307.*
Model: Method 1</td><td colspan="2">$\frac{1.4}{K_m}$</td><td colspan="2">$0.72\tau_m$</td><td colspan="2">$\frac{\tau_m}{T_m} = 0.11$</td></tr>
<tr><td colspan="6">K_c, T_i deduced from graphs; robust to ±25% process model parameter changes</td></tr>
<tr><td>Minimum IAE – Huang *et al.* (1996).
Model: Method 1</td><td colspan="2">[25] $K_c^{(23)}$</td><td colspan="2">$T_i^{(23)}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td>Minimum IAE – Smith and Corripio (1997) – *page 345.*
Model: Method 1</td><td colspan="2">$\frac{0.6T_m}{K_m\tau_m}$</td><td colspan="2">T_m</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 1.5$</td></tr>
<tr><td>Minimum IAE – Huang and Jeng (2002).
Model: Method 1</td><td colspan="2">$\frac{0.59T_m}{K_m\tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">Minimum IAE = $2.1038\tau_m$;
$\tau_m/T_m < 0.2$</td></tr>
</table>

[25] $$K_c^{(23)} = \frac{1}{K_m}\left[-13.0454 - 9.0916\frac{\tau_m}{T_m} + 0.3053\left(\frac{\tau_m}{T_m}\right)^{-1.0169} + 1.1075\left(\frac{\tau_m}{T_m}\right)^{3.5959}\right] + \frac{1}{K_m}\left[-2.2927\left(\frac{\tau_m}{T_m}\right)^{3.6843} + 4.8259e^{\frac{\tau_m}{T_m}}\right],$$

$$T_i^{(23)} = T_m\left[0.9771 - 0.2492\frac{\tau_m}{T_m} + 3.4651\left(\frac{\tau_m}{T_m}\right)^2 - 7.4538\left(\frac{\tau_m}{T_m}\right)^3 + 8.2567\left(\frac{\tau_m}{T_m}\right)^4\right] + T_m\left[-4.7536\left(\frac{\tau_m}{T_m}\right)^5 + 1.1496\left(\frac{\tau_m}{T_m}\right)^6\right].$$

Rule	K_c	T_i	Comment
Minimum IAE – Tavakoli and Fleming (2003). *Model: Method 1*	26 $K_c^{(24)}$	$T_i^{(24)}$	$0.1 \le \frac{\tau_m}{T_m} \le 10$
	Minimum $A_m = 6dB$; minimum $\phi_m = 60^0$		
Small IAE – Hwang (1995). *Model: Method 35*	27 $K_c^{(25)}$	$K_c^{(25)}/\mu_2 K_u \omega_u$	Decay ratio = 0.1, $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{0.980}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.892}$	$\frac{T_m}{0.690 - 0.155\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.072}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.560}$	$\frac{T_m}{0.648 - 0.114\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Khan and Lehman (1996). *Model: Method 1*	28 $K_c^{(26)}$	$T_i^{(26)}$	$0.01 \le \frac{\tau_m}{T_m} \le 0.2$

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26 $K_c^{(24)} = \frac{1}{K_m}\left[0.4849\frac{T_m}{\tau_m} + 0.3047\right]$, $T_i^{(24)} = T_m\left[0.4262\frac{\tau_m}{T_m} + 0.9581\right]$.

27 $K_c^{(25)} = (1-\mu_1)K_u$, $\mu_1 = \frac{1.07(1 - 0.466\omega_u\tau_m + 0.0667\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m}$,

$$\mu_2 = \frac{0.0328(-1 + 2.21\omega_u\tau_m - 0.338\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m}, r = 0.5;$$

$$\mu_1 = \frac{1.11(1 - 0.467\omega_u\tau_m + 0.0657\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m},$$

$$\mu_2 = \frac{0.0477(-1 + 2.07\omega_u\tau_m - 0.333\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m}, r = 0.75;$$

$$\mu_1 = \frac{1.14(1 - 0.466\omega_u\tau_m + 0.0647\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m},$$

$$\mu_2 = \frac{0.0609(-1 + 1.97\omega_u\tau_m - 0.323\omega_u^2\tau_m^2)}{K_u K_m/1 + K_u K_m}, r = 1.0;$$

r = parameter related to the position of the dominant real pole.

28 $K_c^{(26)} = \left(\frac{0.7388}{\tau_m} + \frac{0.3185}{T_m}\right)\frac{T_m}{K_m}$, $T_i^{(26)} = \tau_m\left[\frac{0.7388T_m + 0.3185\tau_m}{-0.0003082T_m + 0.5291\tau_m}\right]$.

Rule	K_c	T_i	Comment
Khan and Lehman (1996) – continued.	$^{29}K_c^{(27)}$	$T_i^{(27)}$	$0.2 \le \frac{\tau_m}{T_m} \le 20$
Minimum ITAE – Rovira *et al.* (1969). *Model: Method 5*	$\frac{0.586}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.916}$	$\frac{T_m}{1.030 - 0.165\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Approximately minimum ITAE – Smith (2002).	$\frac{0.586}{K_m}\frac{T_m}{\tau_m}$	T_m	*Model: Method 1*
Minimum ISTSE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{0.712}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.968 - 0.247\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{0.786}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.559}$	$\frac{T_m}{0.883 - 0.158\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	$0.361K_u$	$^{30}T_i^{(28)}$	*Model: Method 1*; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
	$^{31}K_c^{(29)}$	$T_i^{(29)}$	*Model: Method 31*; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{0.569}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.951}$	$\frac{T_m}{1.023 - 0.179\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{0.628}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.583}$	$\frac{T_m}{1.007 - 0.167\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$

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[29] $K_c^{(27)} = \left(\frac{0.808}{\tau_m} + \frac{0.511}{T_m} - \frac{0.255}{\sqrt{T_m\tau_m}}\right)\frac{T_m}{K_m}$,

$$T_i^{(27)} = \tau_m\left[\frac{0.808T_m + 0.511\tau_m - 0.255\sqrt{T_m\tau_m}}{0.095T_m + 0.846\tau_m - 0.381\sqrt{T_m\tau_m}}\right].$$

[30] $T_i^{(28)} = 0.083(1.935K_mK_u + 1)T_u$.

[31] $K_c^{(29)} = \left(\frac{1.506K_m\hat{K}_u - 0.177}{3.341K_m\hat{K}_u + 0.606}\right)\hat{K}_u$, $T_i^{(29)} = 0.055\left(3.616K_m\hat{K}_u + 1\right)\hat{T}_u$.

Rule	K_c	T_i	Comment
Nearly minimum IAE and ITAE – Hwang and Fang (1995). *Model: Method 25*	[32] $K_c^{(30)}$	$T_i^{(30)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.03
	Minimum performance index: other tuning		
Simultaneous servo/regulator tuning – Hwang and Fang (1995). *Model: Method 25*	[33] $K_c^{(31)}$	$T_i^{(31)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.05

Rule	K_c		T_i		Comment	
[34] Minimum performance index – Wilton (1999). *Model: Method 1.* *Coefficients of* K_c *estimated from graphs*	x_1/K_m		T_m			
	Coefficient values					
	τ_m/T_m	x_1	b	τ_m/T_m	x_1	b
	0.1	750.0	0.0001	0.2	3.3675	0.008
	0.1	17.487	0.04	0.2	2.2946	0.04
	0.1	4.8990	0.2	0.2	1.7146	0.2
	0.1	2.1213	1	0.2	1.1313	1
				0.2	0.8764	5
	0.4	1.6837	0.008	0.5	150.01	0.0001
	0.4	1.5297	0.04	0.5	7.1386	0.04
	0.4	1.2247	0.2	0.5	2.6944	0.2
	0.4	0.9192	1	0.5	1.1314	1
	0.4	0.7668	5			

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[32] $$K_c^{(30)} = \left(0.438 - 0.110\frac{\tau_m}{T_m} + 0.0376\left[\frac{\tau_m}{T_m}\right]^2\right)K_u,$$

$$T_i^{(30)} = \frac{(K_c^{(30)}/K_u\omega_u)}{\left(0.0388 + 0.108\frac{\tau_m}{T_m} - 0.0154\left[\frac{\tau_m}{T_m}\right]^2\right)}.$$

[33] $$K_c^{(31)} = \left(0.46 - 0.0835\frac{\tau_m}{T_m} + 0.0305\left[\frac{\tau_m}{T_m}\right]^2\right)K_u,$$

$$T_i^{(31)} = \frac{(K_c^{(31)}/K_u\omega_u)}{\left(0.0644 + 0.0759\frac{\tau_m}{T_m} - 0.0111\left[\frac{\tau_m}{T_m}\right]^2\right)}.$$

[34] Performance index = $\int_0^\infty \left[e^2(t) + b^2K_m^2\left(y(t) - y_\infty\right)^2\right]dt$.

Rule	K_c		T_i		Comment	
Wilton (1999) - continued	Coefficient values - continued					
	τ_m/T_m	x_1	b	τ_m/T_m	x_1	b
	0.6	1.1225	0.008	0.8	0.8980	0.008
	0.6	1.1218	0.04	0.8	0.9178	0.04
	0.6	0.9798	0.2	0.8	0.7348	0.2
	0.6	0.7778	1	0.8	0.7071	1
	0.6	0.6572	5	0.8	0.6025	5
	1.0	72.004	0.0001	1.0	3.6713	0.04
	1.0	1.6657	0.2	1.0	0.8485	1
Minimum ITAE – ABB (2001). *Model: Method 7*	$\frac{0.8591}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.977}$		$1.4837T_m\left(\frac{\tau_m}{T_m}\right)^{0.68}$		$\frac{\tau_m}{T_m}<0.5$	
	Direct synthesis: time domain criteria					
Bryant *et al.* (1973). *Model: Method 1*	$\frac{0.37T_m}{K_m\tau_m}$		T_m		Critically damped dominant pole	
	$\frac{0.40T_m}{K_m\tau_m}$		T_m		Damping ratio (dominant pole) = 0.6	
	0		$x_2K_m\tau_m$			
	Coefficient values – deduced from graph					
	τ_m/T_m	x_2	τ_m/T_m	x_2		
	0.33	12.5	1.0	6.7	Critically damped dominant pole	
	0.40	11.1	2.0	4.2		
	0.50	11.1	10.0	3.0		
	0.67	9.1				
	0.33	7.1	1.0	3.2	Damping ratio (dominant pole) = 0.6	
	0.40	6.3	2.0	2.1		
	0.50	5.6	10.0	1.6		
	0.67	4.3				
Chiu *et al.* (1973). *Model: Method 1*	$\frac{\lambda T_m}{K_m(1+\lambda\tau_m)}$		T_m		λ variable; suggested values: 0.2, 0.4, 0.6, 1.0.	
	$\lambda=1.0$, OS<10%, *Model: Method 10* (Poulin and Pomerleau, 2004).					
Mollenkamp *et al.* (1973). *Model: Method 1*	$\frac{T_m}{K_m(T_{CL}+\tau_m)}$		T_m		Appropriate for "small τ_m"	

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="10">Keviczky and Csáki (1973); OS values in first row; τ_m/T_m values in first column; $K_m = 1$.</td><td colspan="2">0</td><td colspan="2">$x_2\tau_m$</td><td colspan="2">Model: Method 1</td></tr>
<tr><td colspan="6">x_2 coefficient values (estimated from graph)</td></tr>
<tr><td>τ_m/T_m OS</td><td>0%</td><td>5%</td><td>10%</td><td>15%</td><td>20%</td></tr>
<tr><td>0.1</td><td>30</td><td>22</td><td>16</td><td>12</td><td>10</td></tr>
<tr><td>0.2</td><td>18</td><td>11</td><td>9</td><td>7.5</td><td>6</td></tr>
<tr><td>0.5</td><td>8</td><td>6</td><td>5</td><td>4</td><td>3.5</td></tr>
<tr><td>1.0</td><td>5</td><td>4</td><td>3.5</td><td>3</td><td>2.5</td></tr>
<tr><td>2.0</td><td>4</td><td>3</td><td>2.5</td><td>2.2</td><td>2</td></tr>
<tr><td>5.0</td><td>3</td><td>2.4</td><td>2</td><td>1.8</td><td>1.6</td></tr>
<tr><td>10.0</td><td>2.5</td><td>2.1</td><td>1.8</td><td>1.7</td><td>1.5</td></tr>
<tr><td rowspan="2">5% overshoot – servo – Smith et al. (1975) – deduced from graph Model: Method 1</td><td colspan="2">$\frac{0.52T_m}{K_m\tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">$0.04 \le \frac{\tau_m}{T_m} \le 1.4$</td></tr>
<tr><td colspan="6">Also given by Bain and Martin (1983).</td></tr>
<tr><td>1% overshoot – servo – Smith et al. (1975) – deduced from graph</td><td colspan="2">$\frac{0.44T_m}{K_m\tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">Model: Method 1; $0.04 \le \frac{\tau_m}{T_m} \le 1.4$</td></tr>
<tr><td rowspan="2">Bain and Martin (1983). Model: Method 1</td><td colspan="2">$\frac{0.43T_m}{K_m\tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">1% overshoot – servo $\frac{\tau_m}{T_m} > 0.2$</td></tr>
<tr><td colspan="2">$\frac{1}{K_m}$</td><td colspan="2">T_m</td><td colspan="2">“Very conservative tuning for small delay”</td></tr>
<tr><td>Suyama (1992). Model: Method 1</td><td colspan="2">$\frac{0.5T_m}{K_m\tau_m}$</td><td colspan="2">T_m</td><td colspan="2">OS = 10%</td></tr>
<tr><td rowspan="2">5% overshoot – servo – Smith and Corripio (1997) – page 346. Model: Method 1</td><td colspan="2">$\frac{0.5T_m}{K_m\tau_m}$</td><td colspan="2">T_m</td><td colspan="2"></td></tr>
<tr><td colspan="6">$\phi_m = 60^0$; $0.25 \le \frac{\tau_m}{T_m} \le 1$ (Voda and Landau (1995))</td></tr>
<tr><td rowspan="7">Vítečková (1999), Vítečková et al. (2000a) (Model: Method 9 – Vítečková et al. (2000b)).</td><td colspan="2">$\frac{x_1T_m}{K_m\tau_m}$</td><td colspan="2">T_m</td><td colspan="2">Closed loop response overshoot (OS) shown</td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td></tr>
<tr><td>0.368</td><td>0</td><td>0.696</td><td>20</td><td>0.906</td><td>40</td></tr>
<tr><td>0.514</td><td>5</td><td>0.748</td><td>25</td><td>0.957</td><td>45</td></tr>
<tr><td>0.581</td><td>10</td><td>0.801</td><td>30</td><td>1.008</td><td>50</td></tr>
<tr><td>0.641</td><td>15</td><td>0.853</td><td>35</td><td></td><td></td></tr>
</table>

Rule	K_c	T_i	Comment
Mann *et al.* (2001). *Model: Method 31 or Method 33*; Closed loop response overshoot (step input) < 5%	$\frac{0.51T_m}{K_m\tau_m}$	T_m	$0<\frac{\tau_m}{T_m}<2$
	[35] $K_c^{(32)}$	$T_i^{(32)}$	
	$x_1=0.4,\ x_2=3.68$		$1<\tau_m/T_m<2$
	$x_1=0.8,\ x_2=3.28$		$2<\tau_m/T_m<4$
	$x_1=1.2,\ x_2=2.88$		$4<\tau_m/T_m$
Vítečková and Víteček (2002) *Model: Method 1*	0	[36] $T_i^{(33)}$	
	[37] $K_c^{(34)}$	$T_i^{(34)}$	
	For $0.05<\frac{\tau_m}{T_m}<0.8$, $T_i=T_m$; overshoot is under 2% (Vítečková and Víteček (2003))		
Pinnella *et al.* (1986). *Model: Method 20*	0	[38] $K_I^{(35)}$	Critically damped servo response

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[35] $$K_c^{(32)}=\frac{1}{K_m}\left[0.56+1.02\frac{T_m}{\tau_m}-\sqrt{\frac{T_m}{\tau_m}\left[1.0404\frac{T_m}{\tau_m}-x_1\right]+x_2}\right],$$

$$T_i^{(32)}=\left(\frac{\tau_m}{T_m}-1\right)\left[\frac{0.56+1.02\frac{T_m}{\tau_m}-\sqrt{\frac{T_m}{\tau_m}\left(1.0404\frac{T_m}{\tau_m}-x_1\right)+x_2}}{0.04-1.02\frac{T_m}{\tau_m}+\sqrt{\frac{T_m}{\tau_m}\left(1.0404\frac{T_m}{\tau_m}-x_1\right)+x_2}}\right]T_m.$$

[36]

$$T_i^{(33)}=-\frac{K_m}{\left[\sqrt{\frac{1}{\tau_m^2}+\frac{0.25}{T_m^2}}-\frac{1}{\tau_m}-\frac{0.5}{T_m}\right]\left[\sqrt{\left(\frac{T_m}{\tau_m}\right)^2+0.25}-\frac{T_m}{\tau_m}+0.5\right]e^{\sqrt{1+0.25\left(\frac{\tau_m}{T_m}\right)^2}-\frac{0.5\tau_m}{T_m}-1}}$$

[37] $$K_c^{(34)}=-\frac{1}{K_m}\left[\tau_mT_mx_1^2+(2T_m+\tau_m)x_1+1\right]e^{\tau_mx_1},$$

$$x_1=-\frac{2}{\tau_m}-\frac{0.5}{T_m}+\sqrt{\frac{2}{\tau_m^2}+\frac{0.25}{T_m^2}},\ T_i^{(34)}=-\frac{\tau_mT_mx_1^2+(2T_m+\tau_m)x_1+1}{x_1^2(\tau_mT_mx_1+T_m+\tau_m)}.$$

[38] $$K_I^{(35)}=\frac{1}{4K_m\tau_m^2}\left|2-\frac{\tau_m}{T_m}+\left(\frac{\tau_m}{T_m}\right)^2\right|\left|T_m\left[2-\frac{\tau_m}{T_m}+\left(\frac{\tau_m}{T_m}\right)^2+1\right]\right|e^{0.5\left[2-\frac{\tau_m}{T_m}+\left(\frac{\tau_m}{T_m}\right)^2\right]}.$$

Rule	K_c	T_i	Comment
Schneider (1988), Bekker *et al.* (1991), Khan and Lehman (1996). *Model: Method 1*	$0.368\frac{T_m}{K_m\tau_m}$	T_m	$\xi = 1$
	$0.403\frac{T_m}{K_m\tau_m}$	T_m	$\xi = 0.6$
	$1.571\frac{T_m}{K_m\tau_m}$	T_m	$\xi = 0.0$
Regulator – Gorecki *et al.* (1989). *Model: Method 1*	[39] $K_c^{(36)}$	$T_i^{(36)}$	Pole is real and has maximum attainable multiplicity
Hang *et al.* (1991). *Model: Method 1*	[40] $K_c^{(37)}$	$T_i^{(37)}$	$0.16 \le \frac{\tau_m}{T_m} < 0.96$
	Servo response: 10% overshoot, 3% undershoot		
McAnany (1993). *Model: Method 24*	[41] $K_c^{(38)}$	$T_i^{(38)}$	$T_{CL} = 1.67T_m$
Sklaroff (1992), Åström and Hägglund (1995) – *pages 250-251.*	$\frac{3}{K_m\left(1+\frac{3\tau_m}{T_m}\right)}$	T_m	*Model: Method 28*; Honeywell UDC 6000 controller

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[39] $$K_c^{(36)} = \frac{2}{K_m}\frac{T_m}{\tau_m}\left[\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}-1\right]e^{\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}-2-\frac{\tau_m}{2T_m}},$$

$$T_i^{(36)} = \tau_m\frac{1+\left(\frac{\tau_m}{2T_m}\right)^2}{3+\left(\frac{\tau_m}{2T_m}\right)+\left(\frac{\tau_m}{T_m}\right)^2+\left(\frac{\tau_m}{2T_m}\right)^3-\left[2+\left(\frac{\tau_m}{2T_m}\right)^2\right]\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}}.$$

[40] $$K_c^{(37)} = \frac{5}{6}\left(\frac{12+2\left[\dfrac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]}{15+14\left[\dfrac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]}\right)K_u,\ T_i^{(37)} = 0.2\left(\frac{4}{15}\left[\frac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]+1\right)T_u.$$

[41] $$K_c^{(38)} = \frac{\left(1.44T_m+0.72T_m\tau_m-0.43\tau_m-2.14\right)}{K_m\left(0.72\tau_m+0.36\tau_m^2+2\right)},\ T_i^{(38)} = \frac{K_m\left(5.56+2\tau_m+\tau_m^2\right)}{4T_m+1.28\tau_m-2.4}.$$

Rule	K_c	T_i	Comment
Fruehauf *et al.* (1993). *Model: Method 2*	$\frac{0.5556T_m}{\tau_m K_m}$	$5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{0.5T_m}{\tau_m K_m}$	T_m	$\frac{\tau_m}{T_m} \ge 0.33$
Kamimura *et al.* (1994). *Model: Method 1*	[42] $K_c^{(39)}$	$T_i^{(39)}$	Servo response: 10% overshoot
	[43] $K_c^{(40)}$	$T_i^{(40)}$	Servo response: 0% overshoot
	[44] $K_c^{(41)}$	$T_i^{(41)}$	"Quick" servo response: "negligible" overshoot
Zhang (1994). *Model: Method 1*	$\frac{T_m}{K_m \tau_m}$	T_m	"Good" servo response; τ_m is "small"
Davydov *et al.* (1995). *Model: Method 6*	[45] $K_c^{(42)}$	$T_i^{(42)}$	$\xi = 0.9$; $0.2 \le \tau_m/T_m \le 1$.
Kuhn (1995). *Model: Method 14*	$\frac{0.5}{K_m}$	$0.5(T_m + \tau_m)$	"Normale Einstellung"
	$\frac{1}{K_m}$	$0.7(T_m + \tau_m)$	"Schnelle Einstellung"

[42] $K_c^{(39)} = \frac{0.5(T_m + \tau_m)}{K_m[0.5\tau_m(2T_m + \tau_m) + 0.10]}$, $T_i^{(39)} = T_m + \tau_m - \frac{0.5\tau_m(2T_m + \tau_m) + 0.10}{T_m + \tau_m}$.

[43] $K_c^{(40)} = \frac{0.375(T_m + \tau_m)}{K_m[0.5\tau_m(2T_m + \tau_m) + 0.0781]}$,

$$T_i^{(40)} = T_m + \tau_m - \frac{0.5\tau_m(2T_m + \tau_m) + 0.0781}{T_m + \tau_m}.$$

[44] $K_c^{(41)} = \frac{0.425(T_m + \tau_m)}{K_m[0.5\tau_m(2T_m + \tau_m) + 0.083]}$,

$$T_i^{(41)} = T_m + \tau_m - \frac{0.5\tau_m(2T_m + \tau_m) + 0.083}{T_m + \tau_m}.$$

[45] $K_c^{(42)} = \frac{1}{K_m\left(1.905\frac{\tau_m}{T_m} + 0.826\right)}$, $T_i^{(42)} = \left(0.153\frac{\tau_m}{T_m} + 0.362\right)T_m$.

Rule	K_c	T_i	Comment
Khan and Lehman (1996). *Model: Method 1*	[46] $K_c^{(43)}$	$T_i^{(43)}$	$0.01 \le \frac{\tau_m}{T_m} \le 0.2$
	[47] $K_c^{(44)}$	$T_i^{(44)}$	$0.2 \le \frac{\tau_m}{T_m} \le 20$
Abbas (1997). *Model: Method 1*	[48] $K_c^{(45)}$	$T_m + 0.5\tau_m$	$0 \le V \le 0.2$; $0.1 \le \frac{\tau_m}{T_m} \le 5.0$
Valentine and Chidambaram (1997a).	$\frac{1}{K_m}\left[0.43 - 0.97\frac{\tau_m}{T_m}\right]$	$0.413 + 0.8\frac{\tau_m}{T_m}$	*Model: Method 1.* $\xi = 0.707$ [49]
Bi *et al.* (1999). *Model: Method 13*	$\frac{0.5064T_m}{K_m\tau_m}$	T_m	
Bi *et al.* (2000). *Model: Method 13*	$\frac{0.5064T_m}{K_m\tau_m}$	$\frac{T_m K_m}{0.5064}$	
Ettaleb and Roche (2000). *Model: Method 29*	$\frac{1}{K_m}$.	$T_m + \tau_m$	$T_{CL} = T_m + \tau_m$

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[46] $$K_c^{(43)} = \frac{T_m}{K_m}\left(\frac{0.3852}{\tau_m} + \frac{0.723}{T_m} - 0.404\frac{\tau_m}{T_m^2}\right),$$

$$T_i^{(43)} = \tau_m \frac{-0.404\tau_m^2 + 0.723T_m\tau_m - 0.3852T_m^2}{-0.525\tau_m^2 + 0.4104T_m\tau_m - 0.00024T_m^2}.$$

[47] $$K_c^{(44)} = \frac{T_m}{K_m}\left(\frac{0.404}{\tau_m} + \frac{0.256}{T_m} - \frac{0.1275}{\sqrt{\tau_m T_m}}\right),$$

$$T_i^{(44)} = \tau_m \frac{0.404T_m + 0.256\tau_m - 0.1275\sqrt{T_m\tau_m}}{0.0808T_m + 0.719\tau_m - 0.324\sqrt{T_m\tau_m}}.$$

[48] $$K_c^{(45)} = \frac{0.148 + 0.186\left(\frac{\tau_m}{T_m}\right)^{-1.045}}{K_m(0.497 - 0.464V^{0.590})}.$$

[49] $\pm 1\%$ $T_s = 2.5T_m$, $0.1 \le \frac{\tau_m}{T_m} \le 0.4$; $\pm 1\%$ $T_s = 5T_m$, $0.5 \le \frac{\tau_m}{T_m} \le 0.9$.

Rule	K_c	T_i	Comment
Chen and Seborg (2002).	[50] $K_c^{(46)}$	$T_i^{(46)}$	*Model: Method 1*
Skogestad (2003). *Model: Method 45*	$\frac{0.5T_m}{K_m\tau_m}$	$\min(T_m, 8\tau_m)$	
Gorez (2003). *Model: Method 1*	[51] $K_c^{(47)}$	T_m	$0 \le \frac{\tau_m}{\tau_m + T_m} \le \tau_m$
	[52] $K_c^{(48)}$	$(1-v)\tau_m + T_m$	$\tau_m \le \frac{\tau_m}{\tau_m + T_m} \le 1$

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[50] $K_c^{(46)} = \frac{1}{K_m}\frac{T_m\tau_m + 2T_mT_{CL} - T_{CL}^2}{(T_{CL} + \tau_m)^2}$, $T_i^{(46)} = \frac{T_m\tau_m + 2T_mT_{CL} - T_{CL}^2}{T_m + \tau_m}$,

$$0 < T_{CL} < T_m + \sqrt{T_m^2 + T_m\tau_m}\,,\quad \left(\frac{y}{d}\right)_{desired} = \frac{T_i^{(46)}se^{-\tau_m s}}{K_c(T_{CL}s+1)^2}.$$

[51] $K_c^{(47)} = \frac{T_m}{\chi K_m \tau_m}$ with

$$\chi = \frac{1}{M_s(M_s-1)} + \frac{1.5M_s^2 - 2}{0.32M_s^2(M_s-1)}\left(\frac{\tau_m}{\tau_m+T_m}\right)^2\left(3 - \frac{5\tau_m}{(\tau_m+T_m)\sqrt{M_s}}\right) - \frac{0.5}{M_s(M_s-1)}\left(1 - \frac{2.5\tau_m}{(\tau_m+T_m)\sqrt{M_s}}\right)^{2(M_s^2-1)}.$$

[52] $K_c^{(48)} = \frac{T_m}{\chi K_m \tau_m}$ with $v = 1 - 0.5\left(\frac{\frac{\tau_m}{\tau_m+T_m} - 0.4\sqrt{M_s}}{1-0.4\sqrt{M_s}}\right)^2$,

$$\chi = \frac{0.1M_s}{M_s - 1}\left[7.5 - \left(\frac{\frac{\tau_m}{\tau_m+T_m} - 0.4\sqrt{M_s}}{1-0.4\sqrt{M_s}}\right)^2\right].$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th>Comment</th></tr>
<tr><td>Sree et al. (2004), Chidambaram et al. (2005).
Model: Method 1</td><td colspan="2">$\frac{0.9179}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.8915}$</td><td colspan="2">[53] $T_i^{(49)}$</td><td>$0.1 \le \frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td>Haeri (2005).
Model: Method 1</td><td colspan="2">[54] $K_c^{(50)}$</td><td colspan="2">$0.96T_m + 0.46\tau_m$</td><td>$0 \le \frac{\tau_m}{T_m} \le 4$</td></tr>
<tr><td></td><td colspan="5">Direct synthesis: frequency domain criteria</td></tr>
<tr><td rowspan="8">Hougen (1979) – page 335.
Model: Method 1; Coefficients of K_c estimated from graphs.</td><td colspan="2">x_1/K_m</td><td colspan="2">T_m</td><td rowspan="8">Maximise crossover frequency</td></tr>
<tr><td colspan="4">Coefficient values</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>τ_m/T_m</td><td>x_1</td></tr>
<tr><td>0.1</td><td>7.0</td><td>0.6</td><td>1.1</td></tr>
<tr><td>0.2</td><td>3.5</td><td>0.7</td><td>1.0</td></tr>
<tr><td>0.3</td><td>2.2</td><td>0.8</td><td>0.8</td></tr>
<tr><td>0.4</td><td>1.8</td><td>0.9</td><td>0.75</td></tr>
<tr><td>0.5</td><td>1.4</td><td>1.0</td><td>0.7</td></tr>
<tr><td>Regulator – Gorecki et al. (1989).
Model: Method 1</td><td colspan="2">[55] $K_c^{(51)}$</td><td colspan="2">$T_i^{(51)}$</td><td>Low frequency part of magnitude Bode diagram is flat</td></tr>
</table>

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[53] $$T_i^{(49)} = T_m\left[10.59\left(\frac{\tau_m}{T_m}\right)^2 - 2.3588\frac{\tau_m}{T_m} + 0.8985\right],\ 0.1 \le \frac{\tau_m}{T_m} \le 0.4\ ;$$

$$T_i^{(49)} = T_m\left[0.7719\left(\frac{\tau_m}{T_m}\right)^4 - 3.6608\left(\frac{\tau_m}{T_m}\right)^3 + 6.5791\left(\frac{\tau_m}{T_m}\right)^2 - 5.1652\frac{\tau_m}{T_m} + 2.8059\right],$$

$$0.4 \le \frac{\tau_m}{T_m} \le 1.5\ .$$

[54] $$K_c^{(50)} = \frac{1}{K_m}\left[0.407 + \frac{6.282}{1 + 14.956\left(\frac{\tau_m}{T_m}\right)^{1.195}}\right].$$

[55] $$K_c^{(51)} = \frac{1}{K_m}\frac{1 + 3(T_m/\tau_m) + 6(T_m/\tau_m)^2 + 6(T_m/\tau_m)^3}{4\left[1 + 3(T_m/\tau_m) + 3(T_m/\tau_m)^2\right]},$$

$$T_i^{(51)} = \tau_m\frac{1 + 3(T_m/\tau_m) + 6(T_m/\tau_m)^2 + 6(T_m/\tau_m)^3}{3\left[1 + 2(T_m/\tau_m) + 2(T_m/\tau_m)^2\right]}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="11">Somani et al. (1992).
Model: Method 1;
Suggested A_m :
$1.75 \le A_m \le 3.0$</td><td colspan="2">$x_1/K_m A_m$ [56]</td><td colspan="2">$x_2\tau_m$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.0375</td><td>36.599</td><td>3.57</td><td>1.869</td><td>1.476</td><td>2.38</td></tr>
<tr><td>0.1912</td><td>7.754</td><td>3.33</td><td>2.392</td><td>1.333</td><td>2.27</td></tr>
<tr><td>0.3693</td><td>4.361</td><td>3.13</td><td>3.067</td><td>1.226</td><td>2.17</td></tr>
<tr><td>0.5764</td><td>3.055</td><td>2.94</td><td>3.968</td><td>1.146</td><td>2.08</td></tr>
<tr><td>0.8183</td><td>2.369</td><td>2.78</td><td>5.263</td><td>1.086</td><td>2.00</td></tr>
<tr><td>1.105</td><td>1.950</td><td>2.63</td><td>7.246</td><td>1.042</td><td>1.92</td></tr>
<tr><td>1.449</td><td>1.672</td><td>2.50</td><td>10.870</td><td>1.011</td><td>1.85</td></tr>
<tr><td colspan="2">[57] $K_c^{(52)}$</td><td colspan="2">$T_i^{(52)}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td>Hang et al. (1993a), (1993b) – pages 61-65.</td><td colspan="2">$\frac{\omega_p T_m}{K_m A_m}$</td><td colspan="2">[58] $T_i^{(53)}$</td><td colspan="2">Model: Method 31 or Method 33</td></tr>
</table>

[56] For $\frac{\tau_m}{T_m} < 1$, $K_c \approx \frac{0.9806}{K_m A_m}\sqrt{\frac{4}{\left(\sqrt{1.886 + 4\frac{\tau_m}{T_m}} - 1.373\right)^2} + 1}$ or

$$K_c \approx \frac{0.9806}{K_m A_m}\sqrt{1 + 1.886\left(\frac{T_m}{\tau_m}\right)^2}\ ;\ \text{for } \frac{\tau_m}{T_m} > 1,\ K_c \approx \frac{0.9806}{K_m A_m}\sqrt{1 + 8.669\left(\frac{T_m}{\tau_m}\right)^2}\ .$$

[57] $K_c^{(52)} = \frac{1}{K_m A_m}\left[0.80 + 1.33\frac{T_m}{\tau_m}\right]$, $T_i^{(52)} = \frac{5T_m}{\sqrt{1.04\left[0.80 + 1.33\frac{T_m}{\tau_m}\right]^2 - 1}}$.

[58] $T_i^{(53)} = \frac{1}{2\omega_p - \frac{4\omega_p^2\tau_m}{\pi} + \frac{1}{T_m}}$; $\frac{\tau_m}{T_m} \le 0.3$, Yang and Clarke (1996).

<table>
<tr><th>Rule</th><th colspan="8">K_c</th><th colspan="8">T_i</th><th colspan="8">Comment</th></tr>
<tr><td rowspan="5">O'Dwyer (2001a). Model: Method 1; $\frac{\tau_m}{T_m} > 0.3$, Yang and Clarke (1996).</td><td colspan="8">$\frac{x_1 T_m}{K_m \tau_m}$</td><td colspan="8">$T_m$</td><td colspan="8">$A_m = \pi/2x_1$; $\phi_m = \pi/2 - x_1$ [59]</td></tr>
<tr><td colspan="24">Representative results</td></tr>
<tr><td colspan="4">x_1</td><td colspan="4">A_m</td><td colspan="4">ϕ_m</td><td colspan="4">x_1</td><td colspan="4">A_m</td><td colspan="4">ϕ_m</td></tr>
<tr><td colspan="4">1.048</td><td colspan="4">1.5</td><td colspan="4">30^0</td><td colspan="4">0.524</td><td colspan="4">3</td><td colspan="4">60^0</td></tr>
<tr><td colspan="4">0.7854</td><td colspan="4">2</td><td colspan="4">45^0</td><td colspan="4">0.393</td><td colspan="4">4</td><td colspan="4">67.5^0</td></tr>
<tr><td>O'Dwyer (2001a). Model: Method 1</td><td colspan="8">$\frac{x_1}{\tau_m} \frac{T_m K_u T_u}{\sqrt{T_u^2 + 4\pi^2 T_m^2}}$</td><td colspan="8">$T_m$</td><td colspan="8">$A_m = \pi/2x_1$; $\phi_m = \pi/2 - x_1$</td></tr>
<tr><td rowspan="8">Chen et al. (1999b). Model: Method 1</td><td colspan="8">$\frac{x_1 T_m}{\tau_m K_m}$</td><td colspan="8">$T_m$</td><td colspan="8"></td></tr>
<tr><td colspan="24">Coefficient values</td></tr>
<tr><td colspan="3">x_1</td><td colspan="3">A_m</td><td colspan="3">ϕ_m</td><td colspan="3">M_s</td><td colspan="3">x_1</td><td colspan="3">A_m</td><td colspan="3">ϕ_m</td><td colspan="3">M_s</td></tr>
<tr><td colspan="3">0.50</td><td colspan="3">3.14</td><td colspan="3">61.4^0</td><td colspan="3">1.00</td><td colspan="3">0.72</td><td colspan="3">2.18</td><td colspan="3">48.7^0</td><td colspan="3">1.30</td></tr>
<tr><td colspan="3">0.61</td><td colspan="3">2.58</td><td colspan="3">55.0^0</td><td colspan="3">1.10</td><td colspan="3">0.76</td><td colspan="3">2.07</td><td colspan="3">46.5^0</td><td colspan="3">1.40</td></tr>
<tr><td colspan="3">0.67</td><td colspan="3">2.34</td><td colspan="3">51.6^0</td><td colspan="3">1.20</td><td colspan="3">0.80</td><td colspan="3">1.96</td><td colspan="3">44.1^0</td><td colspan="3">1.50</td></tr>
<tr><td colspan="3">0.70</td><td colspan="3">2.24</td><td colspan="3">50.0^0</td><td colspan="3">1.26</td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td></tr>
<tr><td colspan="24"></td></tr>
<tr><td>Gain margin and maximum closed loop magnitude – Chen et al. (1999a).</td><td colspan="8">$\frac{r_1 \omega_r T_m}{K_m}$</td><td colspan="8">$\frac{1}{\omega_r - \frac{4}{\pi}\tau_m \omega_r^2 + \frac{1}{T_m}}$</td><td colspan="8">Model: Method 1 [60]</td></tr>
<tr><td rowspan="2">Symmetrical optimum principle – Voda and Landau (1995). Model: Method 1</td><td colspan="8">$\frac{1}{3.5\lvert G_p(j\omega_{135^0})\rvert}$</td><td colspan="8">$\frac{4.6}{\omega_{135^0}}$</td><td colspan="8">$\frac{\tau_m}{T_m} \le 0.1$</td></tr>
<tr><td colspan="8">$\frac{1}{2.828\lvert G_p(j\omega_{135^0})\rvert}$</td><td colspan="8">$\frac{4}{\omega_{135^0}}$</td><td colspan="8">$0.1 < \frac{\tau_m}{T_m} \le 0.15$</td></tr>
</table>

[59] Given A_m, ISE is minimised when $\phi_m = 68.8884 - 34.3534A_m + 9.1606\frac{\tau_m}{T_m}$ for servo tuning (Ho *et al.* (1999)); $2 \le A_m \le 5$; $0.1 \le \tau_m/T_m \le 1$;
Given A_m, ISE is minimised when $\phi_m = 45.9848A_m^{0.2677}(\tau_m/T_m)^{0.2755}$ for regulator tuning (Ho *et al.* (1999)); $2 \le A_m \le 5$; $0.1 \le \tau_m/T_m \le 1$.

[60] $\omega_r = \frac{\pi}{4\tau_m}\frac{2r_1 A_m - 1}{(r_1 A_m)^2 - 1}$

Rule	K_c	T_i	Comment
Voda and Landau (1995) – continued.	[61] $K_c^{(54)}$	$T_i^{(54)}$	$0.15 < \frac{\tau_m}{T_m} \le 1$
Friman and Waller (1997). *Model: Method 1*	$\frac{0.1751}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{1}{\omega_{135^0}}$	$\frac{\tau_m}{T_m} > 2$. $A_m > 3$
Schaedel (1997), Henry and Schaedel (2005) - Optimal servo response.	[62] $\frac{0.5T_i^{(55)}}{K_m\left(T_1 - T_i^{(55)}\right)}$	$T_i^{(55)}$	"Normal" design – Butterworth filter. *Model: Method 21*
	[63] $\frac{0.375T_i^{(56)}}{K_m\left(T_1 - T_i^{(56)}\right)}$	$T_i^{(56)}$	"Sharp" design – Tschebyscheff filter (0.5dB). *Model: Method 21*
	$\frac{0.375T_i^{(57)}}{K_m\left(T_1 - T_i^{(57)}\right)}$	[63] $T_i^{(57)}$	Minimum ITAE (Henry and Schaedel, 2005). *Model: Method 22*
Henry and Schaedel (2005). *Model: Method 22*; Optimal regulator response; $\frac{\tau_m^a}{T_m} < 0.2$	[64] $K_c^{(58)}$	$T_i^{(58)}$	"Normal" design – Butterworth filter

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[61] $K_c^{(54)} = \frac{1}{4.6\lvert G_p(j\omega_{135^0})\rvert - 0.6K_m}$, $T_i^{(54)} = \frac{1.15\lvert G_p(j\omega_{135^0})\rvert + 0.75K_m}{\omega_{135^0}\left[2.3\lvert G_p(j\omega_{135^0})\rvert - 0.3K_m\right]}$.

[62] $T_i^{(55)} = \sqrt{T_1^2 - 2T_2^2}$,

$$T_1 = \frac{T_m}{1 + 3.45\frac{\tau_m^a}{T_m}} + \tau_m,\ T_2^2 = 1.25T_m\tau_m^a + \frac{T_m}{1 + 3.45\frac{\tau_m^a}{T_m}}\tau_m + 0.5\tau_m^2,\ 0 < \frac{\tau_m^a}{T_m} \le 0.104;$$

$$T_1 = \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + \tau_m,\ T_2^2 = T_m\tau_m^a + \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + 0.5\tau_m^2,\ \frac{\tau_m^a}{T_m} > 0.104.$$

[63] $T_i^{(56)} = T_1 - \frac{T_2^2}{T_1}$, $T_i^{(57)} = -0.64T_1 + 1.64T_1\sqrt{1 - 1.2\left(\frac{T_2}{T_1}\right)^2}$.

[64] $K_c^{(58)} = \frac{1}{K_m}\left(0.5\left[\frac{T_i^{(58)}}{T_2}\right]^2 - 1\right)$, $T_i^{(58)} = 4\frac{T_2^2}{T_1}\left(1 - 2\frac{T_2^2}{T_1^2}\right)$.

Rule	K_c	T_i	Comment
Henry and Schaedel (2005) – continued.	65 $K_c^{(59)}$	$T_i^{(59)}$	"Sharp" design – Tschebyscheff filter (0.1 dB)
	66 $K_c^{(60)}$	$T_i^{(60)}$	Minimum ITAE
Leva (2001). *Model: Method 1* Minimum $\phi_m = 50^0$	67 $K_c^{(61)}$	T_m	$\omega_c = \frac{20}{\tau_m + 5T_m}$
	68 $K_c^{(62)}$		$\omega_c = \frac{20}{\tau_m + 5T_m}$
Potočnik *et al.* (2001). *Model: Method 39*	69 $K_c^{(63)}$	$T_i^{(63)}$	$\frac{\tau_m}{T_m} \le 0.1$
	70 $K_c^{(64)}$	$T_i^{(64)}$	$0.1 < \frac{\tau_m}{T_m} < 0.15$

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65 $K_c^{(59)} = \frac{1}{K_m}\left(0.7\left[\frac{T_i^{(59)}}{T_2}\right]^2 - 1\right)$, $T_i^{(59)} = 3.11\frac{T_2^{\,2}}{T_1}\left(1 - 1.43\frac{T_2^{\,2}}{T_1^{\,2}}\right)$.

66 $K_c^{(60)} = \frac{1}{K_m}\left(0.69\left[\frac{T_i^{(59)}}{T_2}\right]^2 - 1\right)$, $T_i^{(60)} = 3.86\frac{T_2^{\,2}}{T_1}\left(1 - 1.46\frac{T_2^{\,2}}{T_1^{\,2}}\right)$.

67 $K_c^{(61)} = \min\left[\frac{0.6981T_m}{K_m\tau_m}, \frac{20T_m}{K_m(\tau_m + 5T_m)}\right]$.

68 $K_c^{(62)} = \min\left[\frac{0.6981T_m}{K_m\tau_m}, \frac{50T_m}{K_m(\tau_m + 5T_m)}\right]$.

69 $K_c^{(63)} = \frac{0.5}{\left|G_p\left(j\omega_{135^0}\right)\right|\left[1.14 + 13.24\frac{\left|G_p(j\omega_{135^0})\right|}{K_m}\right]}$,

$$T_i^{(63)} = \frac{4}{\omega_{135^0}}\left[1 + 2.48\frac{\left|G_p\left(j\omega_{135^0}\right)\right|}{K_m} + 17.40\frac{\left|G_p\left(j\omega_{135^0}\right)\right|^2}{K_m^{\,2}}\right].$$

70 $K_c^{(64)} = \frac{K_m - 2\left|G_p\left(j\omega_{135^0}\right)\right|}{2K_m\left[K_m - 2\left|G_p(j\omega_{135^0})\right|\right]}$,

$$T_i^{(64)} = \frac{\left[K_m + \sqrt{2}\left|G_p\left(j\omega_{135^0}\right)\right|\right]\left[2\left|G_p\left(j\omega_{135^0}\right)\right| - K_m\right]}{\omega_{135^0}\left|G_p\left(j\omega_{135^0}\right)\right|\left[\left|G_p(j\omega_{135^0})\right| - K_m\right]}.$$

Rule	K_c	T_i	Comment
Potočnik *et al.* (2001) – continued.	[71] $K_c^{(65)}$	$T_i^{(65)}$	$0.1 < \frac{\tau_m}{T_m} < 1$
Cluett and Wang (1997), Wang and Cluett (2000) – *page 177.* *Model: Method 1*; $0.01 < \frac{\tau_m}{T_m} \le 10$	[72] $K_c^{(66)}$	$T_i^{(66)}$	$T_{CL} = 4\tau_m$. $A_m \in [7.50, 8.28]$, $\phi_m \in [78.5^0, 79.7^0]$
	[73] $K_c^{(67)}$	$T_i^{(67)}$	$T_{CL} = 2\tau_m$. $A_m \in [4.35, 5.02]$, $\phi_m \in [70.7^0, 74.0^0]$
	[74] $K_c^{(68)}$	$T_i^{(68)}$	$T_{CL} = 1.33\tau_m$. $A_m \in [3.29, 3.89]$, $\phi_m \in [64.4^0, 70.6^0]$
	[75] $K_c^{(69)}$	$T_i^{(69)}$	$T_{CL} = \tau_m$. $A_m \in [2.74, 3.30]$, $\phi_m \in [57.8^0, 68.5^0]$
	[76] $K_c^{(70)}$	$T_i^{(70)}$	$T_{CL} = 0.8\tau_m$. $A_m \in [2.39, 2.93]$, $\phi_m \in [49.6^0, 67.1^0]$

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[71] $K_c^{(65)} = \frac{2K_m}{\omega_{135^0}}\left[0.75 + 1.15\frac{\left|G_p(j\omega_{135^0})\right|}{K_m}\right]$, $T_i^{(65)} = \frac{1}{\omega_{135^0}}\left[0.75 + 1.15\frac{\left|G_p(j\omega_{135^0})\right|}{K_m}\right]$.

[72] $K_c^{(66)} = \frac{0.019952\tau_m + 0.20042T_m}{K_m\tau_m}$, $T_i^{(66)} = \frac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425.10^{-5}T_m}\tau_m$.

[73] $K_c^{(67)} = \frac{0.055548\tau_m + 0.33639T_m}{K_m\tau_m}$, $T_i^{(67)} = \frac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032.10^{-4}T_m}\tau_m$.

[74] $K_c^{(68)} = \frac{0.092654\tau_m + 0.43620T_m}{K_m\tau_m}$, $T_i^{(68)} = \frac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550.10^{-3}T_m}\tau_m$.

[75] $K_c^{(69)} = \frac{0.12786\tau_m + 0.51235T_m}{K_m\tau_m}$, $T_i^{(69)} = \frac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988.10^{-2}T_m}\tau_m$.

[76] $K_c^{(70)} = \frac{0.16051\tau_m + 0.57109T_m}{K_m\tau_m}$, $T_i^{(70)} = \frac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355.10^{-2}T_m}\tau_m$.

Rule	K_c	T_i	Comment
Cluett and Wang (1997), Wang and Cluett (2000) – continued.	[77] $K_c^{(71)}$	$T_i^{(71)}$	$T_{CL} = 0.67\tau_m$. $A_m \in [2.11, 2.68]$, $\phi_m \in [38.1^0, 66.3^0]$
Modulus optimum principle – Cox *et al.* (1997). *Model: Method 16*	[78] $K_c^{(72)}$	$T_i^{(72)}$	$\frac{\tau_m}{T_m} \le 1$
	$\frac{0.5T_m}{K_m \tau_m}$	T_m	$\frac{\tau_m}{T_m} > 1$
Smith (1998). *Model: Method 1*	$\frac{0.35}{K_m}$	$0.42\tau_m$	6 dB gain margin – dominant delay process
Wang *et al.* (2000a). *Model: Method 31 or 32*	[79] $K_c^{(73)}$	T_m	$1.3 \le M_s \le 2$

[77] $K_c^{(71)} = \frac{0.19067\tau_m + 0.61593T_m}{K_m \tau_m}$, $T_i^{(71)} = \frac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m}\tau_m$.

[78] $K_c^{(72)} = \frac{0.5}{K_m}\left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2\tau_m + T_m\tau_m^2 + 0.667\tau_m^3}\right)$,

$$T_i^{(72)} = \left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2 + T_m\tau_m + 0.5\tau_m^2}\right).$$

[79] $K_c^{(73)} = \frac{1}{\tau_m}\left[1.451 - \frac{1.508}{M_s}\right]\frac{T_m}{K_m}$.

Rule	K_c	T_i	Comment
Wang and Shao (2000a). *Model: Method 1*	[80] $K_c^{(74)}$	$T_i^{(74)}$	$\lambda \in [1.5, 2.5]$
Chen and Yang (2000). *Model: Method 8*	$\frac{0.7T_m}{K_m \tau_m}$	T_m	$M_s = 1.26$; $A_m = 2.24$; $\phi_m = 50^0$
Hägglund and Åström (2002). *Model: Method 5*; $M_s = 1.4$	$\frac{0.25T_m}{K_m \tau_m}$	$0.8T_m$	$0.2 \le \frac{\tau_m}{T_m} < 1$
	$\frac{0.1T_m}{K_m \tau_m} + \frac{0.15}{K_m}$	$0.3\tau_m + 0.5T_m$	$\frac{\tau_m}{T_m} > 1$
	$\frac{0.15}{K_m}$	$0.3\tau_m$	$\frac{\tau_m}{T_m} \to \infty$
Hägglund and Åström (2002). *Model: Method 1*	[81] $K_c^{(75)}$	$T_i^{(75)}$	$M_s = 1.4$; $\frac{\tau_m}{T_m} \ge 0.5$

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[80] $$K_c^{(74)} = \frac{1}{\lambda f_1(\omega_{90^0})}\left[f_2(\omega_{90^0}) - \frac{1}{\omega_{90^0}}\right],$$

$$f_1(\omega_{90^0}) = \frac{K_m}{\left(1+\omega_{90^0}^2 T_m^2\right)^{1.5}}\left[(T_m + \{1+\omega_{90^0}^2 T_m^2\}\tau_m)\sin(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)\right]$$
$$- \frac{K_m}{\left(1+\omega_{90^0}^2 T_m^2\right)^{1.5}}\left[\omega_{90^0}T_m^2\cos(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)\right],$$

$$f_2(\omega_{90^0}) = -\frac{1}{1+\omega_{90^0}^2 T_m^2}\left[(T_m + \{1+\omega_{90^0}^2 T_m^2\}\tau_m)\cot(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)\right]$$
$$- \frac{\omega_{90^0}T_m^2}{1+\omega_{90^0}^2 T_m^2};$$

$$T_i^{(74)} = \frac{\omega_{90^0}\left[T_m + (1+\omega_{90^0}^2 T_m^2)\tau_m\right]\cos\theta + (1+2\omega_{90^0}^2 T_m^2)\sin\theta}{-\omega_{90^0}^3 T_m^2\cos\theta + \omega_{90^0}^2[T_m + (1+\omega_{90^0}^2 T_m^2)\tau_m]\sin\theta},$$
$$\theta = -\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m.$$

[81] $$K_c^{(75)} = \frac{1}{K_m}\left(0.14 + 0.28\frac{T_m}{\tau_m}\right),\ T_i^{(75)} = 0.33\tau_m + \frac{6.8\tau_m T_m}{10\tau_m + T_m}.$$

Rule	K_c	T_i	Comment
Leva *et al.* (2003). *Model: Method 1*	[82] $K_c^{(76)}$	T_m	minimum ϕ_m ; maximum ω_c
	[83] $K_c^{(77)}$	T_m	minimum $\phi_m = 50^0$
Mataušek and Kvaščev (2003). *Model: Method 1*	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	$x_1 = 0.5\pi - \phi_m$ [84]
Kristiansson (2003). *Model: Method 11*	[85] $K_c^{(78)}$	$\frac{1.25}{T_m}$	$1.6 \le M_s \le 1.9$
Leva *et al.* (1994). *Model: Method 43*	[86] $K_c^{(79)}$	$T_i^{(79)}$	

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[82] $K_c^{(76)} = \text{minimum}\left[\frac{T_m(0.5\pi - \phi_m)}{K_m \tau_m}, \frac{5T_m}{K_m T_{CL}}\right]$.

[83] $K_c^{(77)} = \text{minimum}\left[\frac{0.698T_m}{K_m \tau_m}, \frac{5aT_m}{K_m(\tau_m + 5T_m)}\right]$, $T_{CL} = \frac{\tau_m + 5T_m}{a}$, $a \in [4,10]$.

[84] 'Acceptable' values of x_1: $0.5 \le x_1 \le 3$. 'Recommended' values of x_1: $0.32 \le x_1 \le 0.54$ ($59^0 \le \phi_m \le 72^0$). Choose $x_1 = 0.32$ when large variations in process parameters are expected; choose $x_1 = 0.54$ to give $6\% \le OS < 13\%$ and $1.4 < M_s \le 2$.

[85] $$K_c^{(78)} = \frac{\omega_{150^0}}{K_m}\left(0.2 + \frac{0.075}{\frac{K_{150^0}}{K_m} + 0.05}\right).$$

[86] $$K_c^{(79)} = \frac{\omega_{cn} T_i^{(79)}}{K_m}\sqrt{\frac{1 + \omega_{cn}^2 T_m^2}{1 + \omega_{cn}^2 [T_i^{(79)}]^2}}, \quad T_i^{(79)} = \frac{\tan\left[\phi_m - \frac{\pi}{2} + \tau_m \omega_{cn} + \tan^{-1}(\omega_{cn} T_m)\right]}{\omega_{cn}}$$

$$\text{with } \omega_{cn} = \frac{2.82 - \phi_m - \tan^{-1}\left[\frac{T_m}{\tau_m}\left(\frac{\pi}{2} - \phi_m\right)\right]}{\tau_m}.$$

Rule	K_c	T_i	Comment
Tan *et al.* (1996). *Model: Method 30*	[87] $K_c^{(80)}$	$T_i^{(80)}$	
Xu *et al.* (2004). *Model: Method 31*	$\frac{T_m}{\lambda K_m \tau_m}$	T_m	$\lambda \in [1.5, 2.5]$
Huang *et al.* (2005). *Model: Method 42*	$\frac{0.50T_m + 0.22\tau_m}{K_m \tau_m}$	$0.9T_m + 0.4\tau_m$	
	Robust		
Rivera *et al.* (1986). *Model: Method 1*	$\frac{T_m}{\lambda K_m}$	T_m	$\lambda \ge 1.7\tau_m$, $\lambda > 0.1T_m$.
	$\lambda \ge 1.7\tau_m$, $\lambda > 0.2T_m$ (Luyben (2001)); $\lambda = 2\tau_m$ (minimum ISE - de Oliveira *et al.*, (1995))		
	$\frac{2T_m + \tau_m}{2\lambda K_m}$	$T_m + 0.5\tau_m$	$\lambda \ge 1.7\tau_m$, $\lambda > 0.1T_m$.
Chien (1988). *Model: Method 1*	$\frac{T_m}{K_m(\tau_m + \lambda)}$	T_m	$\lambda = T_m$ (Chien (1988)) [88]
Brambilla *et al.* (1990). *Model: Method 1*	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	
	No model uncertainty: $\lambda = \tau_m$, $0.1 \le \tau_m/T_m \le 1$; $\lambda = [1 - 0.5\log_{10}(\tau_m/T_m)]\tau_m$, $1 < \tau_m/T_m \le 10$.		

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[87] $K_c^{(80)} = \frac{\beta T_i^{(80)}\omega_\phi\sqrt{1 + (\beta T_m \omega_\phi)^2}}{A_m\sqrt{1 + (\beta T_i^{(80)}\omega_\phi)^2}}$, $T_i^{(80)} = \frac{1}{\beta\omega_\phi \tan\left[-\tan^{-1}\beta T_m\omega_\phi - \beta\tau_m\omega_\phi - \phi\right]}$,

$\omega_\phi < \omega_u$. $\beta = 0.8, \frac{\tau_m}{T_m} < 0.5$; $\beta = 0.5, \frac{\tau_m}{T_m} > 0.5$.

[88] $\lambda = 2\tau_m$ (Bialkowski (1996)). $\lambda > T_m + \tau_m$ (Thomasson (1997)). $\lambda \in [0.45\tau_m, 0.8\tau_m]$ (Huang *et al.* (1998)). $\lambda = 2\tau_m$ (aggressive, less robust tuning) (Gerry, (1999)); $\lambda = 2(T_m + \tau_m)$ (more robust tuning) (Gerry, (1999)); $\lambda = a.\max(T_m, \tau_m)$: $a > 3$ … slow design, $2 \le a \le 3$ … normal design, $1 \le a < 2$ …fast design, $a < 1$ … faster design (Andersson (2000) – *page 11*). *Model: Method 10.*

$\lambda \in [0.1T_m, 0.5T_m], \frac{\tau_m}{T_m} \le 0.25$; $\lambda = 1.5(\tau_m + T_m), 0.25 < \frac{\tau_m}{T_m} \le 0.75$;

$\lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 0.75$ (Leva (2001)). $\lambda \in [T_m, \tau_m]$ (Smith (2002)).

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th>Comment</th></tr>
<tr><td>Pulkkinen et al. (1993).</td><td colspan="3">$\frac{0.7T_m}{K_m\tau_m}$</td><td colspan="3">$\lambda(\tau_m+T_m)$</td><td>Model: Method 1</td></tr>
<tr><td rowspan="11">Ogawa (1995).
Model: Method 1;
Coefficients of K_c, T_i deduced from graphs</td><td colspan="3">x_1/K_m</td><td colspan="3">x_2T_m</td><td rowspan="3"></td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>$\frac{\tau_m}{T_m}$</td><td>x_1</td><td>x_2</td><td>$\frac{\tau_m}{T_m}$</td><td>x_1</td><td>x_2</td></tr>
<tr><td>0.5</td><td>0.9</td><td>1.3</td><td>2.0</td><td>0.45</td><td>2.0</td><td rowspan="2">20% uncertainty in process parameters</td></tr>
<tr><td>1.0</td><td>0.6</td><td>1.6</td><td>10.0</td><td>0.4</td><td>7.0</td></tr>
<tr><td>0.5</td><td>0.7</td><td>1.3</td><td>2.0</td><td>0.4</td><td>2.2</td><td rowspan="2">33% uncertainty in process parameters</td></tr>
<tr><td>1.0</td><td>0.47</td><td>1.7</td><td>10.0</td><td>0.35</td><td>7.5</td></tr>
<tr><td>0.5</td><td>0.47</td><td>1.3</td><td>2.0</td><td>0.32</td><td>2.4</td><td rowspan="2">50% uncertainty in process parameters</td></tr>
<tr><td>1.0</td><td>0.36</td><td>1.8</td><td>10.0</td><td>0.3</td><td>8.5</td></tr>
<tr><td>0.5</td><td>0.4</td><td>1.3</td><td>2.0</td><td>0.3</td><td>2.4</td><td rowspan="2">60% uncertainty in process parameters</td></tr>
<tr><td>1.0</td><td>0.33</td><td>1.8</td><td>10.0</td><td>0.29</td><td>9.0</td></tr>
<tr><td>Thomasson (1997).
Model: Method 1</td><td colspan="3">$\frac{\tau_m}{2K_m(\tau_m+\lambda)}$</td><td colspan="3">$0.5\tau_m$</td><td>$\tau_m \gg T_m$;
$\lambda = T_{CL}$</td></tr>
<tr><td rowspan="2">Chen et al. (1997).
Model: Method 26</td><td colspan="3" rowspan="2">$\frac{0.5T_m}{K_m \max(\tau_m, 0.5)}$</td><td colspan="3">minimum $(T_m, 6\tau_m)$</td><td>$\tau_m > 0.5$</td></tr>
<tr><td colspan="3">3</td><td>$\tau_m < 0.5$</td></tr>
<tr><td>Lee et al. (1998).
Model: Method 1</td><td colspan="3">$\frac{T_m+\frac{\tau_m^{\,2}}{2(\lambda+\tau_m)}}{K_m(\lambda+\tau_m)}$</td><td colspan="3">$T_m+\frac{\tau_m^{\,2}}{2(\lambda+\tau_m)}$</td><td>Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s+1)}$</td></tr>
<tr><td>Isaksson and Graebe (1999).
Model: Method 1</td><td colspan="3">$\frac{T_m+0.25\tau_m}{K_m\lambda}$</td><td colspan="3">$T_m+0.25\tau_m$</td><td>$T_m > \tau_m$; λ not specified</td></tr>
<tr><td>Chun et al. (1999).
Model: Method 34</td><td colspan="3">$\frac{T_m(\tau_m+2\lambda)-\lambda^2}{K_m(\tau_m+\lambda)^2}$</td><td colspan="3">$\frac{T_m(\tau_m+2\lambda)-\lambda^2}{\tau_m+T_m}$</td><td>Representative λ :
$\lambda = 0.4T_m$</td></tr>
<tr><td>Zhong and Li (2002).
Model: Method 1</td><td colspan="3">$\frac{T_m}{K_m(\alpha+\tau_m)}$</td><td colspan="3">$T_m$</td><td>$\alpha \in [\Delta\tau_m, 1.4\Delta\tau_m]$;
$\Delta\tau_m$ = time delay uncertainty.</td></tr>
<tr><td>Smith (2002).
Model: Method 1</td><td colspan="3">$\frac{1}{K_m}\frac{T_m}{\tau_m}$</td><td colspan="3">$T_m$</td><td></td></tr>
</table>

Rule	K_c	T_i	Comment
	Ultimate cycle		
McMillan (1984). *Model: Method 2 or Method 44*	$K_c^{(81)}$ [89]	$T_i^{(81)}$	Tuning rules developed from K_u, T_u
Boe and Chang (1988). *Model: Method 1*	$0.54K_u$	$1.935T_m\left(\frac{\tau_m}{T_m}\right)^{0.654}$	Quarter decay ratio
Geng and Geary (1993). *Model: Method 1*	$\frac{0.71T_m}{K_m\tau_m}$	$3.33\tau_m$	$\frac{\tau_m}{T_m} < 0.167$
Perić *et al.* (1997). *Model: Method 40*	$\frac{\hat{K}_u \tan^2\phi_m}{\sqrt{1+\tan^2\phi_m}}$	$\frac{0.1592\hat{T}_u}{\tan\phi_m}$	
Matsuba *et al.* (1998). *Model: Method 1*	$\frac{0.99}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9}$	$2.63\tau_m\left(\frac{\tau_m}{T_m}\right)^{0.097}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Huang *et al.* (2005). *Model: Method 42*	$K_c^{(82)}$ [90]	$T_i^{(82)}$	

[89] $K_c^{(81)} = \frac{1.881}{K_m}\frac{T_m}{\tau_m}\left\{\frac{1}{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}}\right\}$, $T_i^{(81)} = 1.67\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$.

[90] $$K_c^{(82)} = \frac{0.55}{K_m}\left[\frac{0.9\sqrt{\left(\hat{K}_u\right)^2-1}}{\pi-\tan^{-1}\left[\sqrt{\left(\hat{K}_u\right)^2-1}\right]}+0.4\right],$$

$$T_i^{(82)} = 0.159\hat{T}_u\left[0.9\sqrt{\left(\hat{K}_u\right)^2-1}+0.4\left\{\pi-\tan^{-1}\left[\sqrt{\left(\hat{K}_u\right)^2-1}\right]\right\}\right].$$

3.1.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1}{T_f s + 1}$$

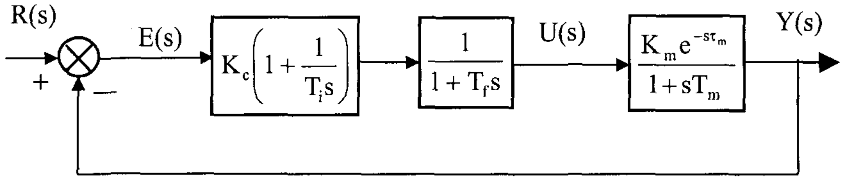

Table 4: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Lim *et al.* (1985). *Model: Method 1*	$\frac{T_m}{K_m(2\xi T_{CL2} + \tau_m)}$	T_m	$T_f = \frac{T_{CL2}^2}{2\xi T_{CL2} + \tau_m}$ [1]
	Robust		
Rivera and Jun (2000). *Model: Method 1*	$\frac{T_m}{K_m(2\tau_m + \lambda)}$	T_m	$T_f = \frac{\tau_m \lambda}{2\tau_m + \lambda}$; λ not specified

[1] Desired closed loop transfer function $= \frac{e^{-s\tau_m}}{T_{CL2}^2 s^2 + 2\xi T_{CL2} s + 1}$.

3.1.3 *Ideal controller in series with a second order filter*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+b_{f1}s+b_{f2}s^2}{1+a_{f1}s+a_{f2}s^2}\right)$$

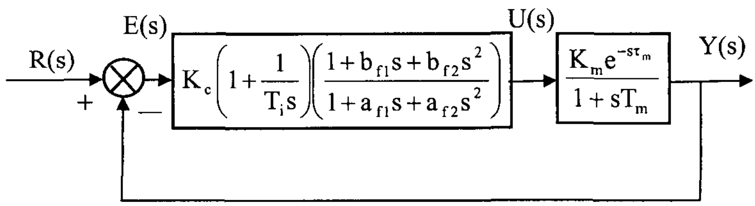

Table 5: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c		T_i		Comment	
	Direct synthesis: time domain criteria					
Tsang *et al.* (1993). *Model: Method 15*	$\frac{x_1 T_m}{K_m \tau_m}$		T_m		[2]	
	Coefficient values					
	x_1	ξ	x_1	ξ	x_1	ξ
	1.851	0.0	1.028	0.4	0.695	0.8
	1.552	0.1	0.925	0.5	0.622	0.9
	1.329	0.2	0.841	0.6	0.553	1.0
	1.160	0.3	0.768	0.7		
	Robust					
Lee and Shi (2002). *Model: Method 1*	[3] $K_c^{(83)}$		$\frac{T_m}{\gamma}$		$1<\gamma<\frac{2T_m}{\tau_m}$	

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[2] $b_{f1} = 0.5\tau_m$, $b_{f2} = 0.0833\tau_m^2$, $a_{f1} = 0.2\tau_m$, $a_{f2} = 0.01\tau_m^2$

[3] $K_c^{(83)} = \dfrac{\omega_{bw}T_m}{K_m(\omega_{bw}\gamma\tau_m+1)\left[1+\dfrac{2\omega_{bw}}{\omega_g}\tan\left(\dfrac{\omega_g\tau_m}{2}\right)\right]}$; $b_{f1} = T_m + 0.5\tau_m$; $b_{f2} = 0.5\tau_m T_m$;

$a_{f1} = \dfrac{\tau_m + 2\omega_{bw}T_m\tau_m + 2T_m}{2(\omega_{bw}\gamma\tau_m+1)}$; $a_{f2} = \dfrac{T_m\tau_m}{2(\omega_{bw}\gamma\tau_m+1)}$; suggested $\omega_g = \dfrac{0.6}{\tau_m}$.

3.1.4 *Controller with set-point weighting*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

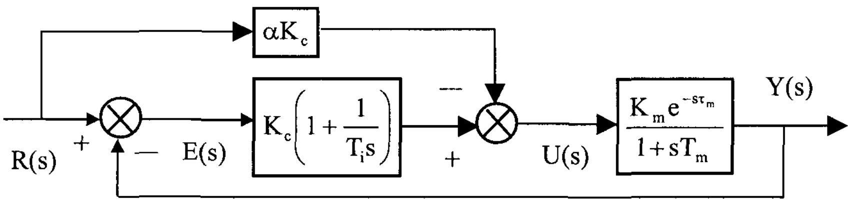

Table 6: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	Comment
	Minimum performance index: other tuning		
Servo/regulator optimisation - Taguchi and Araki (2000). *Model: Method 1*	[4] $K_c^{(84)}$	$T_i^{(84)}$	$\tau_m/T_m \le 1.0$. Overshoot (servo step) $\le 20\%$
	$\alpha = 0.6830 - 0.4242\frac{\tau_m}{T_m} + 0.06568\left(\frac{\tau_m}{T_m}\right)^2$		
	Direct synthesis: time domain criteria		
Chidambaram (2000a). *Model: Method 1*	$\frac{T_m T_i^{(85)}}{T_{CL}{}^2 + T_i^{(85)}\tau_m}$	[5] $T_i^{(85)}$	Choose T_{CL}, ξ

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[4] $$K_c^{(84)} = \frac{1}{K_m}\left(0.1098 + \frac{0.7382}{\frac{\tau_m}{T_m} - 0.002434}\right),$$
$$T_i^{(84)} = T_m\left(0.06216 + 3.171\frac{\tau_m}{T_m} - 3.058\left[\frac{\tau_m}{T_m}\right]^2 + 1.205\left[\frac{\tau_m}{T_m}\right]^3\right).$$

[5] $$T_i^{(85)} = \frac{T_m\tau_m + 2T_{CL}\xi - T_{CL}{}^2}{T_m + \tau_m},\ \alpha = \frac{T_i^{(85)} - \xi T_{CL}}{T_i^{(85)}}\ \text{or}\ \alpha = \frac{K_c K_m[T_i^{(85)} + \tau_m] - T_i^{(85)}}{2K_c K_m T_i^{(85)}}.$$

Rule	K_c	T_i	Comment
Srinivas and Chidambaram (2001).	[6] $\frac{0.9T_m}{K_m\tau_m}$	$3.33\tau_m$	*Model: Method 1*
	Direct synthesis: frequency domain criteria		
Åström and Hägglund (1995) - dominant pole design – *page 204-208. Model: Method 5 or 14*	$\frac{0.29e^{-2.7\tau+3.7\tau^2}T_m}{K_m\tau_m}$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$0.14 \le \frac{\tau_m}{T_m} \le 5.5$; $M_s = 1.4$
	$\alpha = 1 - 0.81e^{0.73\tau+1.9\tau^2}$		
	$\frac{0.78e^{-4.1\tau+5.7\tau^2}T_m}{K_m\tau_m}$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$0.14 \le \frac{\tau_m}{T_m} \le 5.5$; $M_s = 2.0$
	$\alpha = 1 - 0.44e^{0.78\tau-0.45\tau^2}$		
Åström and Hägglund (1995) - modified Ziegler-Nichols – *page 208.*	$\frac{0.4T_m}{K_m\tau_m}$	$0.7T_m$	$0.1 \le \frac{\tau_m}{T_m} \le 2$
	$\alpha = 0.5$; *Model: Method 5 or 14*		
Hägglund and Åström (2002). *Model: Method 5*; $M_s = 1.4$; $0 < \alpha < 0.5$	$\frac{0.35T_m}{K_m\tau_m} - \frac{0.6}{K_m}$	$7\tau_m$	$\frac{\tau_m}{T_m} < 0.11$
	$\frac{0.35T_m}{K_m\tau_m} - \frac{0.6}{K_m}$	$0.8T_m$	$0.11 < \frac{\tau_m}{T_m} < 0.17$
	$\frac{0.25T_m}{K_m\tau_m}$	$0.8T_m$	$0.17 < \frac{\tau_m}{T_m} < 0.2$
Hägglund and Åström (2002). *Model: Method 1*	[7] $K_c^{(86)}$	$T_i^{(86)}$	$\frac{\tau_m}{T_m} < 0.5$
	$M_s = 1.4; 0 < \alpha < 0.5$		

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[6] Sample K_c, T_i taken by the authors (corresponding to the process reaction curve tuning rules of Zeigler and Nichols (1942)); $\alpha = \frac{\tau_m}{T_i} - \frac{1}{K_c K_m} = 0.30 - 1.11\frac{\tau_m}{T_m}$ for these tuning rules.

[7] $K_c^{(86)} = \frac{1}{K_m}\left(0.14 + 0.28\frac{T_m}{\tau_m}\right)$, $T_i^{(86)} = 0.33\tau_m + \frac{6.8\tau_m T_m}{10\tau_m + T_m}$.

3.1.5 *Controller with proportional term acting on the output 1*

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c Y(s)$$

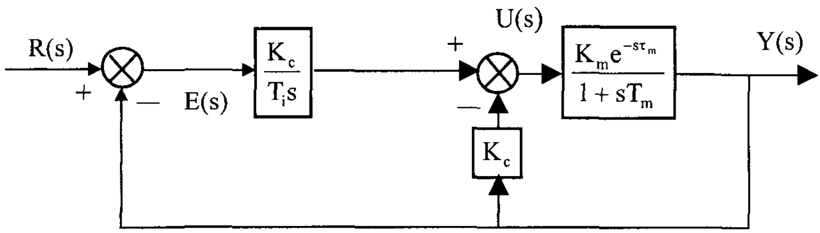

Table 7: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c			T_i			Comment		
	Minimum performance index: other tuning								
Minimum ITAE - Setiawan *et al.* (2000). *Model: Method 1*	$\frac{x_1 T_m}{K_m \tau_m}$			$x_2 \tau_m$					
	Coefficient values								
	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2
	0.5	0.554	1.805	3	0.900	0.547	7	1.740	0.413
	0.75	0.570	1.373	4	1.090	0.480	8	1.930	0.398
	1.0	0.594	1.117	5	1.290	0.445	9	2.120	0.385
	1.5	0.645	0.822	6	1.520	0.428	10	2.410	0.389
	2.0	0.720	0.679						
	Direct synthesis: time domain criteria								
Chien *et al.* (1999). *Model: Method 1*	[8] $K_c^{(87)}$			$T_i^{(87)}$			Underdamped system response - $\xi = 0.707$. $\tau_m > 0.2T_m$		

[8] $K_c^{(87)} = \frac{-T_{CL2}^2 + 1.414T_{CL2}T_m + \tau_m T_m}{K_m\left(T_{CL2}^2 + 1.414T_{CL2}\tau_m + \tau_m^2\right)}$, $T_i^{(87)} = \frac{-T_{CL2}^2 + 1.414T_{CL2}T_m + T_m\tau_m}{T_m + \tau_m}$.

3.1.6 *Controller with proportional term acting on the output 2*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_1 Y(s)$$

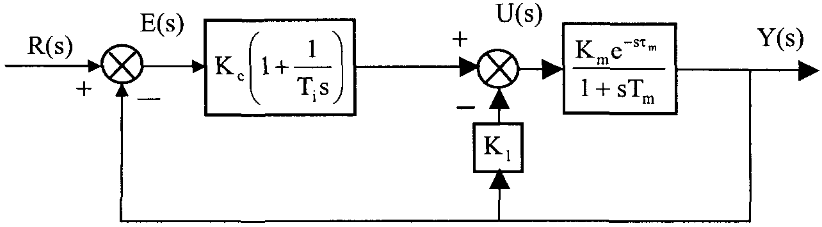

Table 8: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	Comment
	Robust		
Lee and Edgar (2002). *Model: Method 1*	[9] $K_c^{(88)}$	$T_i^{(88)}$	$K_1 = 0.25K_u$

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[9] $K_c^{(88)} = \frac{1+0.25K_u K_m}{K_m(\lambda+\tau_m)}\left[\frac{T_m - 0.25K_u K_m \tau_m}{1+0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)}\right]$,

$$T_i^{(88)} = \frac{T_m - 0.25K_u K_m \tau_m}{1+0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)}.$$

3.2 FOLPD Model with a Positive Zero $G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{1 + sT_{m1}}$

3.2.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

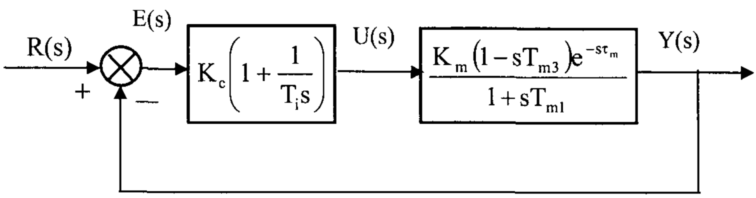

Table 9: PI controller tuning rules - FOLPD model with a positive zero $G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{1 + sT_{m1}}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2003a). *Model: Method 1*	[1] $\frac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(89)}$	

[1] γ = value of the inverse jump of the closed loop system; $\gamma < \frac{K_m T_{m3}}{T_{m1}}$.

$$T_i^{(89)} = \frac{\frac{\gamma T_{m1}}{T_{m3}}\left[T_{m3}(1 - x_1) - 0.5\tau_m(1 + x_1)\right]}{\frac{\gamma T_{m1}}{T_{m3}}(1 - x_1) - x_1};$$

$$x_1 \in \left[0.95\frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}, 0.98\frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}\right],\ 0.1 \le \frac{T_{m3}}{T_{m1}} \le 0.3;$$

$$x_1 \in \left[0.1\frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}, 0.3\frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}\right],\ \frac{T_{m3}}{T_{m1}} > 1.$$

3.2.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1}{1+T_f s}$$

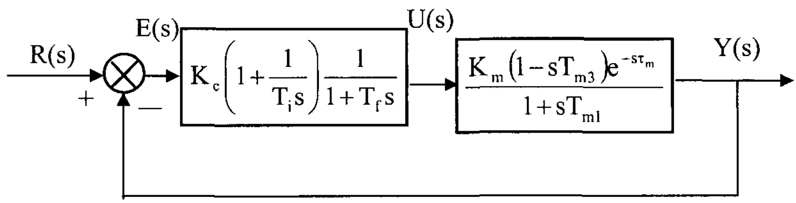

Table 10: PI controller tuning rules - FOLPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{1+sT_{m1}}$$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2003a). *Model: Method 1*	$\frac{T_{m1}}{K_m(2T_{m3}+\lambda+\tau_m)}$	T_{m1}	$T_f = \frac{T_{m3}(\lambda-\tau_m)}{2T_{m3}+\lambda+\tau_m}$

3.3 FOLPD Model with a Negative Zero $G_m(s) = \dfrac{K_m(1+sT_{m3})e^{-s\tau_m}}{1+sT_{m1}}$

3.3.1 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1}{1+T_f s}$$

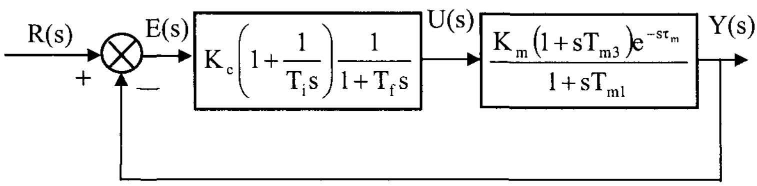

Table 11: PI controller tuning rules – FOLPD model with a negative zero $G_m(s) = \dfrac{K_m(1+sT_{m3})e^{-s\tau_m}}{1+sT_{m1}}$

Rule	K_c	T_i	Comment
	Robust		
Chang *et al.* (1997). *Model: Method 2*	$\dfrac{T_{m1}}{K_m(T_{CL}+\tau_m)}$	T_{m1}	$T_f = T_{m3}$

3.4 Non-Model Specific

3.4.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

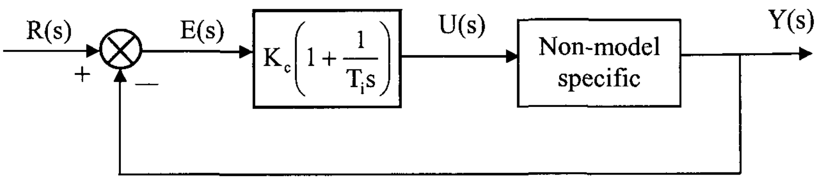

Table 12: PI controller tuning rules – non-model specific

Rule	K_c	T_i	Comment
	Ultimate cycle		
Ziegler and Nichols (1942).	$0.45K_u$	$0.83T_u$	Quarter decay ratio
Hwang and Chang (1987).	$0.45K_u$	$\frac{1}{p_1}\left(\frac{5.22}{T_u} - \frac{5.22}{T_1}\right)T_u$	
	p_1, T_1 = decay rate, period measured under proportional control when $K_c = 0.5K_u$		
Parr (1989) – *page 191.*	$0.5K_u$	$0.43T_u$	
Parr (1989) – *page 192.*	$0.33K_u$	$2T_u$	
Hang *et al.* (1993b) – *page 59.*	$0.25K_u$	$0.25T_u$	dominant time delay process
Pessen (1994).	$0.25K_u$	$0.167T_u$	dominant time delay process
McMillan (1994) – *page 90.*	$0.3571K_u$	T_u	
Åström and Hägglund (1995) – *pages 141-142.*	$0.4698K_u$	$0.4373T_u$	$A_m = 2$, $\phi_m = 20^0$
	$0.1988K_u$	$0.0882T_u$	$A_m = 2.44$, $\phi_m = 61^0$
	$0.2015K_u$	$0.1537T_u$	$A_m = 3.45$, $\phi_m = 46^0$

Rule	K_c	T_i	Comment
Calcev and Gorez (1995).	$0.3536K_u$	$0.1592T_u$	$\phi_m = 45^0$, small τ_m $\phi_m = 15^0$, large τ_m
Edgar *et al.* (1997) – *page 8-15.*	$0.59K_u$	$0.81T_u$	Minimum IAE - regulator
ABB (2001).	$0.5K_u$	$0.8T_u$	
Robbins (2002).	$0.3K_u$	[1] $T_i^{(90)}$	Minimum IAE – servo
	$0.3K_u$	[1] $T_i^{(91)}$	Good response – regulator
	Direct synthesis		
Wang *et al.* (1995b).	[2] $K_c^{(92)}$	$T_i^{(92)}$	
Vrančić *et al.* (1996), Vrančić (1996) – *page 121.*	$\dfrac{0.5A_3}{(A_1A_2 - K_m A_3)}$	$\dfrac{A_3}{A_2}$	$A_m \ge 2, \phi_m \ge 60^0$ (Vrančić *et al.* (1996))
Vrančić (1996) – *page 140.*	$0.45K_u$	$\dfrac{A_3}{A_2}$	Modified Ziegler-Nichols method
Friman and Waller (1997).	$\dfrac{0.4830}{\lvert G_p(j\omega_{150^0})\rvert}$	$\dfrac{3.7321}{\omega_{150^0}}$	$A_m > 2$, $\phi_m > 15^0$

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[1] $T_i^{(90)} = (0.24 + 0.111K_uK_m)T_u$; $T_i^{(91)} = (0.27 + 0.054K_uK_m)T_u$.

[2] For a stable process, K_m is assumed known; for an integrating process, K_m and τ_m are assumed known.

$$K_c^{(92)} = \frac{1}{\omega_{CL}} \mathrm{Im}\left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right],$$

$$T_i^{(92)} = \frac{1}{\omega_{CL}} \frac{\mathrm{Im}\left[\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right]}{\mathrm{Rl}\left[\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right]}.$$

Rule	K_c	T_i	Comment
Kristiansson and Lennartson (2000).	$\frac{1.18K_uK_m - 1.72}{K_uK_m^2}$	[3] $T_i^{(93)}$	$0.1 \le K_uK_m \le 0.5$
	$\frac{0.50K_uK_m - 0.36}{K_uK_m^2}$	[3] $T_i^{(94)}$	$K_uK_m > 0.5$
	$\frac{20K_{135^0}K_m - 160}{K_{135^0}K_m^2}$	[4] $T_i^{(95)}$	$K_uK_m < 0.1$; $K_{135^0}K_m \le 0.1$
	$\frac{5.4K_{135^0}K_m - 13.6}{K_{135^0}K_m^2}$	[4] $T_i^{(96)}$	$K_uK_m < 0.1$; $K_{135^0}K_m > 0.1$
Kristiansson and Lennartson (2002).	$\frac{0.33K_uK_m - 0.15}{K_m(0.18K_uK_m + 1)}$	$\frac{K_uK_m}{\omega_u(0.18K_uK_m + 1)}$	$K_uK_m \le 10$
	[5] $K_c^{(97)}$	$T_i^{(97)}$	$K_uK_m > 10$
Kristiansson (2003).	[6] $K_c^{(98)}$	$T_i^{(98)}$	$1.6 \le M_s \le 1.9$
	Other tuning rules		
Aikman (1950).	$0.81K_{37\%}$	$1.29T_{37\%}$	37% decay ratio
	$[0.63K_{33\%}, 0.97K_{33\%}]$	$[1.1T_{33\%}, 1.3T_{33\%}]$	33% decay ratio
Rutherford (1950).	$0.90K_{37\%}$	$T_{37\%}$	37% decay ratio
MacLellan (1950).	$1.19K_{37\%}$	$0.92T_{37\%}$	37% decay ratio
	$1.02K_{37\%}$	$0.92T_{37\%}$	37% decay ratio
Young (1955).	$K_{37\%}$	$< 1.1T_u$	37% decay ratio

[3] $T_i^{(93)} = \frac{1.18K_uK_m - 1.72}{(0.33K_uK_m - 0.17)\omega_u}$, $T_i^{(94)} = \frac{0.50K_uK_m - 0.36}{(0.33K_uK_m - 0.17)\omega_u}$.

[4] $T_i^{(95)} = \frac{20K_{135^0}K_m - 160}{(0.315K_{135^0}K_m - 0.175)\omega_u}$, $T_i^{(96)} = \frac{5.4K_{135^0}K_m - 13.6}{(1.32K_{135^0}K_m - 3.2)\omega_u}$.

[5] $K_c^{(97)} = K_{135^0}\frac{0.09K_{135^0}K_m + 0.25}{0.09K_{135^0}K_m + 1.2}$, $T_i^{(97)} = \frac{K_{135^0}K_m}{\omega_{135^0}(0.09K_{135^0}K_m + 1.2)}$.

[6] $K_c^{(98)} = \frac{\omega_{150^0}}{K_m}\left(0.2 + \frac{0.075}{\frac{K_{150^0}}{K_m} + 0.05}\right)$, $T_i^{(98)} = \omega_{150^0}\left[0.06 + 1.6\frac{K_{150^0}}{K_m} - 0.06\left(\frac{K_{150^0}}{K_m}\right)^2\right]$.

Rule	K_c	T_i	Comment
Chesmond (1982) – *page 400.*	$\frac{K_{x\%}}{0.5+0.361\ln x}$	$\frac{T_{x\%}}{1.2\sqrt{1+0.025\ln^2 x}}$	x = 100(decay ratio)
	$0.999K_{25\%}$	$0.814T_{25\%}$	Example: x = 25 i.e. quarter decay ratio
Parr (1989) – *page 191.*	$0.667K_{25\%}$	$T_{25\%}$	Quarter decay ratio
McMillan (1994) – *pages 42-43.*	$0.42K_{25\%}$	$T_{25\%}$	'Fast' tuning
	$0.33K_{25\%}$	$T_{25\%}$	'Slow' tuning
Wade (1994) – *page 114.*	$0.7515K_{25\%}$	$0.7470T_{25\%}$	
ECOSSE team (1996b).	$0.65K_{50\%}$	$0.7917T_{50\%}$	
Hay (1998) – *page 189.*	$K_{25\%}$	$T_{25\%}$	
Bateson (2002) – *page 616.*	$K_{25\%}$	T_u	'Modified' ultimate cycle method
Åström (1982).	$\frac{\sin\phi_m}{\lvert G_p(j\omega_{90^0})\rvert}$	$\frac{\tan\phi_m}{\omega_{90^0}}$	
Leva (1993).	[7] $K_c^{(99)}$	$T_i^{(99)}$	$\tan(\phi_m-\phi_\omega-0.5\pi)>0$
Åström and Hägglund (1995) – *page 248.*	$\frac{0.5}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{4}{\omega_{135^0}}$	Alfa-Laval Automation ECA400 controller
	$\frac{0.25}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{1.6}{\omega_{135^0}}$	Alfa-Laval Automation ECA400 controller – large delay
Hägglund and Åström (1991).	$0.25\hat{K}_u$	$0.2546\hat{T}_u$	*Model: Method 2*
Cox *et al.* (1994).	$\frac{0.2hT_u\sin\phi_m}{A_p}$	$0.16T_u\tan\phi_m$	
Jones *et al.* (1997). *Model: Method 2*	$0.5787\frac{h}{A_p}$	$a\hat{T}_u$	$a\in[0.7958,1.5915]$

[7] $K_c^{(99)}=\frac{\tan(\phi_m-\phi_\omega-0.5\pi)}{\lvert G_p(j\omega)\rvert\sqrt{1+\tan^2(\phi_m-\phi_\omega-0.5\pi)}}$, $T_i^{(99)}=\frac{\tan(\phi_m-\phi_\omega-0.5\pi)}{\omega}$.

Rule	K_c	T_i	Comment
NI Labview (2001). *Model: Method 2*	$0.4\hat{K}_u$	$0.8\hat{T}_u$	Quarter decay ratio
	$0.18\hat{K}_u$	$0.8\hat{T}_u$	'Some' overshoot
	$0.13\hat{K}_u$	$0.8\hat{T}_u$	'Little' overshoot
Tang *et al.* (2002). *Model: Method 4*	$\dfrac{2.5465 h \sin\phi_m}{A_p x_1 \hat{\omega}_{90^0} A_m}$	$\dfrac{\tan\phi_m}{\hat{\omega}_{90^0}}$	$0.81 \le x_1 \le 1$; $x_1 = 1$, 'small' τ_m; $x_1 = 0.91$, 'large' τ_m

3.4.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

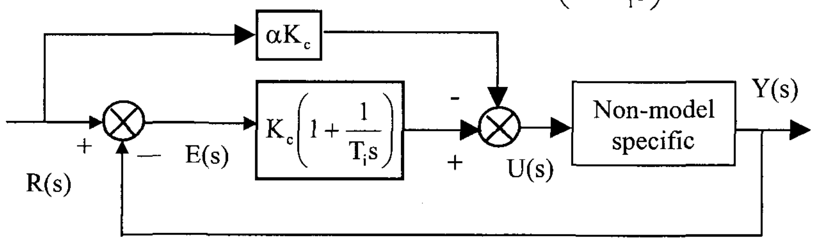

Table 13: PI controller tuning rules – non-model specific

Rule	K_c	T_i	Comment
	Direct synthesis		
Åström and Hägglund (1995) – *page 215.*	$0.053K_u e^{2.9\kappa-2.6\kappa^2}$	$0.90T_u e^{-4.4\kappa+2.7\kappa^2}$	$M_s = 1.4$
	$\alpha = 1 - 1.1e^{-0.0061\kappa+1.8\kappa^2}$; $0 < K_m K_u < \infty$.		
	$0.13K_u e^{1.9\kappa-1.3\kappa^2}$	$0.90T_u e^{-4.4\kappa+2.7\kappa^2}$	$M_s = 2.0$
	$\alpha = 1 - 0.48e^{0.40\kappa-0.17\kappa^2}$; $0 < K_m K_u < \infty$.		
Vrančić (1996) – *page 136-137.*	[8] $K_c^{(100)}$	$T_i^{(100)}$	$\alpha = [0.2, 0.5]$ - good servo & regulator action (Vrančić (1996), *page 139*)

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[8] $K_c^{(100)} = \dfrac{A_1A_2 - K_mA_3 - \chi}{(1-b^2)(K_m^2A_3 + A_1^3 - 2K_mA_1A_2)}$, $K_mA_3 - A_1A_2 < 0$;

$K_c^{(100)} = \dfrac{A_1A_2 - K_mA_3 + \chi}{(1-b^2)(K_m^2A_3 + A_1^3 - 2K_mA_1A_2)}$, $K_mA_3 - A_1A_2 > 0$; $b = 1 - \alpha$;

$\chi = \sqrt{(K_mA_3 - A_1A_2)^2 - (1-b^2)A_3(K_m^2A_3 + A_1^3 - 2K_mA_1A_2)}$;

$T_i^{(100)} = \dfrac{A_1}{K_m + \dfrac{1}{2K_c^{(100)}} + \dfrac{K_c^{(100)}K_m^2}{2}(1-b^2)}$.

3.5 IPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

3.5.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

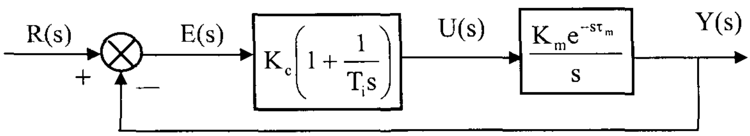

Table 14: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
	Process reaction		
Ziegler and Nichols (1942). *Model: Method 2*	$\frac{0.9}{K_m \tau_m}$	$3.33\tau_m$	Quarter decay ratio
Two constraints method – Wolfe (1951). *Model: Method 2*	$\frac{0.6}{K_m \tau_m}$	$2.78\tau_m$	Decay ratio = 0.4
	$\frac{0.87}{K_m \tau_m}$	$4.35\tau_m$	Decay ratio is as small as possible
	Minimum error integral (regulator mode).		
Coon (1956), (1964). *Model: Method 1*	$\frac{1.0}{K_m \tau_m}$	0	Quarter decay ratio; K_c, T_i *deduced from graphs*
Åström and Hägglund (1995) – *page 13. Model: Method 1*	$\frac{0.63}{K_m \tau_m}$	$3.2\tau_m$	Ultimate cycle Ziegler-Nichols equivalent
Hay (1998) – *page 188.*	$\frac{0.42}{K_m \tau_m}$	$5.8\tau_m$	*Model: Method 3*
Hay (1998) – *page 199. Model: Method 1* K_c *and* T_i *deduced from graphs*	7.5	$4.5K_m \tau_m$	$K_m \tau_m = 0.1$
	3.5		$K_m \tau_m = 0.2$
	2.5		$K_m \tau_m = 0.3$
	2.2		$K_m \tau_m = 0.4$

Rule	K_c	T_i	Comment
Hay (1998) – continued.	2.0		$K_m\tau_m = 0.5$
	1.7		$K_m\tau_m = 0.6$
Bunzemeier (1998). *Model: Method 4* $G_p = \dfrac{K_p}{s(1+sT_p)^n}$	$\dfrac{x_1}{K_m\tau_m}$	$x_2\tau_m$	0% overshoot (servo)
	$\dfrac{x_3}{K_m\tau_m}$		10% overshoot (servo)

Coefficient values

x_1	x_2	x_3	n	x_1	x_2	x_3	n
0.18	1.350	0.23	0	0.20	1.477	0.25	6
0.24	3.050	0.40	1	0.19	1.463	0.24	7
0.22	1.910	0.30	2	0.19	1.453	0.24	8
0.21	1.690	0.28	3	0.18	1.385	0.24	9
0.20	1.584	0.26	4	0.18	1.378	0.24	10
0.20	1.557	0.26	5				

Rule	K_c	T_i	Comment
	Minimum performance index: regulator tuning		
Minimum IAE – Shinskey (1988) – *page 123.*	$\dfrac{0.9524}{K_m\tau_m}$	$4\tau_m$	*Model: Method 1*
Minimum IAE – Shinskey (1994) – *page 74.*	$\dfrac{0.9259}{K_m\tau_m}$	$4\tau_m$	*Model: Method 1*
Minimum IAE – Shinskey (1988) – *page 148. Model: Method 1*	$0.61K_u$	T_u	Also specified by Shinskey (1996), *page 121.*
Minimum ISE – Hazebroek and Van der Waerden (1950).	$\dfrac{1.5}{K_m\tau_m}$	$5.56\tau_m$	*Model: Method 2*
Minimum ISE – Haalman (1965). *Model: Method 1*	$\dfrac{0.6667}{K_m\tau_m}$	0	$M_s = 1.9$, $A_m = 2.36$, $\phi_m = 50^0$
Minimum ITAE – Poulin and Pomerleau (1996). *Model: Method 1*	$\dfrac{0.5264}{K_m\tau_m}$	$4.5804\tau_m$	Process output step load disturbance
	$\dfrac{0.5327}{K_m\tau_m}$	$3.8853\tau_m$	Process input step load disturbance
	Minimum performance index: other tuning		
Skogestad (2001). *Model: Method 1*	$\dfrac{0.28}{K_m\tau_m}$	$7\tau_m$	$M_{max} = 1.4$

Rule	K_c	T_i	Comment
Skogestad (2003). *Model: Method 1*	$\frac{0.404}{K_m \tau_m}$	$7\tau_m$	$M_{max} = 1.7$
Skogestad (2001). *Model: Method 1*	$\frac{0.49}{K_m \tau_m}$	$3.77\tau_m$	$M_{max} = 2.0$
	Direct synthesis: time domain criteria		
Tyreus and Luyben (1992). *Model: Method 1 or 9*	$\frac{0.487}{K_m \tau_m}$	$8.75\tau_m$	Maximum closed loop log modulus $= 2dB$; $T_{CL} = \tau_m \sqrt{10}$
	$0.31K_u$	$2.2T_u$	
Fruehauf *et al.* (1993).	$\frac{0.5}{K_m \tau_m}$	$5\tau_m$	*Model: Method 2*
Rotach (1995). *Model: Method 5*	$\frac{0.75}{K_m \tau_m}$	$2.41\tau_m$	Damping factor for oscillations to a disturbance input = 0.75.
Wang and Cluett (1997). *Model: Method 1*	[1] $K_c^{(101)}$	$T_i^{(101)}$	$\xi = 0.707$; $T_{CL} \in [\tau_m, 16\tau_m]$
	[2] $K_c^{(102)}$	$T_i^{(102)}$	$\xi = 1$; $T_{CL} \in [\tau_m, 16\tau_m]$
Cluett and Wang (1997). *Model: Method 1*	$0.9588/K_m\tau_m$	$3.0425\tau_m$	$T_{CL} = \tau_m$
	$0.6232/K_m\tau_m$	$5.2586\tau_m$	$T_{CL} = 2\tau_m$
	$0.4668/K_m\tau_m$	$7.2291\tau_m$	$T_{CL} = 3\tau_m$
	$0.3752/K_m\tau_m$	$9.1925\tau_m$	$T_{CL} = 4\tau_m$
	$0.3144/K_m\tau_m$	$11.1637\tau_m$	$T_{CL} = 5\tau_m$
	$0.2709/K_m\tau_m$	$13.1416\tau_m$	$T_{CL} = 6\tau_m$
Poulin and Pomerleau (1999). *Model: Method 1*	$0.34K_u$ or $\frac{2.13}{K_m T_u}$	$1.04T_u$	$M_{max} = 5$ dB

[1] $K_c^{(101)} = \frac{1}{K_m \tau_m (0.7138T_{CL} + 0.3904)}$, $T_i^{(101)} = (1.4020T_{CL} + 1.2076)\tau_m$.

[2] $K_c^{(102)} = \frac{1}{K_m \tau_m (0.5080T_{CL} + 0.6208)}$, $T_i^{(102)} = (1.9885T_{CL} + 1.2235)\tau_m$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="7">Vítečková (1999), Vítečková *et al.* (2000a). *Model: Method 1*</td><td colspan="2">$x_1/K_m\tau_m$</td><td colspan="2">0</td><td colspan="2"></td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>x_1</td><td>OS</td><td>x_1</td><td>OS</td><td>x_1</td><td>OS</td></tr>
<tr><td>0.368</td><td>0%</td><td>0.696</td><td>20%</td><td>0.906</td><td>40%</td></tr>
<tr><td>0.514</td><td>5%</td><td>0.748</td><td>25%</td><td>0.957</td><td>45%</td></tr>
<tr><td>0.581</td><td>10%</td><td>0.801</td><td>30%</td><td>1.008</td><td>50%</td></tr>
<tr><td>0.641</td><td>15%</td><td>0.853</td><td>35%</td><td></td><td></td></tr>
<tr><td>Chidambaram and Sree (2003). *Model: Method 1*</td><td colspan="2">$\frac{1.1111}{K_m\tau_m}$</td><td colspan="2">$4.5\tau_m$</td><td colspan="2"></td></tr>
<tr><td rowspan="2">Huba and Žáková (2003). *Model: Method 1*</td><td colspan="2">$\frac{0.23}{K_m\tau_m}$</td><td colspan="2">$2.914\tau_m$</td><td colspan="2"></td></tr>
<tr><td colspan="2">$\frac{0.281}{K_m\tau_m}$</td><td colspan="2">$3.555\tau_m$</td><td colspan="2"></td></tr>
<tr><td rowspan="2">Skogestad (2003), (2004b). *Model: Method 1*</td><td colspan="2">$\frac{1}{K_m(T_{CL}+\tau_m)}$</td><td colspan="2">$4\xi^2(T_{CL}+\tau_m)$</td><td colspan="2">Suggested $\xi = 0.7$ or 1</td></tr>
<tr><td colspan="2">$\frac{0.5}{K_m\tau_m}$</td><td colspan="2">$8\tau_m$</td><td colspan="2">'good' robustness - $T_{CL} = \tau_m$, $\xi = 1$</td></tr>
<tr><td></td><td colspan="6">**Direct synthesis: frequency domain criteria**</td></tr>
<tr><td>Hougen (1979) – *page 333.* *Model: Method 1*</td><td colspan="2">[3] $K_c^{(103)}$</td><td colspan="2">0</td><td colspan="2">Maximise crossover frequency</td></tr>
<tr><td>Chidambaram (1994), Srividya and Chidambaram (1997).</td><td colspan="2">$\frac{0.67075}{K_m\tau_m}$</td><td colspan="2">$3.6547\tau_m$</td><td colspan="2">*Model: Method 6*; $A_m = 2$</td></tr>
<tr><td rowspan="6">Gain and phase margin – Kookos *et al.* (1999). *Model: Method 1*</td><td colspan="2">$\frac{\omega_p}{A_m K_m}$</td><td colspan="2">$\frac{1}{\omega_p(0.5\pi-\omega_p\tau_m)}$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Representative results</td></tr>
<tr><td colspan="2">$0.942/K_m\tau_m$</td><td colspan="2">$4.510\tau_m$</td><td colspan="2">$A_m = 1.5$; $\phi_m = 22.5^0$</td></tr>
<tr><td colspan="2">$0.698/K_m\tau_m$</td><td colspan="2">$4.098\tau_m$</td><td colspan="2">$A_m = 2$; $\phi_m = 30^0$</td></tr>
<tr><td colspan="2">$0.491/K_m\tau_m$</td><td colspan="2">$6.942\tau_m$</td><td colspan="2">$A_m = 3$; $\phi_m = 45^0$</td></tr>
<tr><td colspan="2">$0.384/K_m\tau_m$</td><td colspan="2">$18.710\tau_m$</td><td colspan="2">$A_m = 4$; $\phi_m = 60^0$</td></tr>
</table>

[3] Values recorded deduced from a graph:

$K_m\tau_m$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$K_c^{(103)}$	7.0	3.5	2.2	1.8	1.4	1.1	1.0	0.8	0.75	0.7

Rule	K_c	T_i	Comment
Chen *et al.* (1999a). *Model: Method 10*	$\dfrac{\left[\phi_m + \frac{\pi}{2}(A_m - 1)\right]}{K_m \tau_m (A_m^2 - 1)}$	[4] $T_i^{(104)}$	$\phi_m \ge \frac{\pi}{2}\left(1 - \frac{1.0607}{A_m}\right)$
	[5] $K_c^{(105)}$	$T_i^{(105)}$	
Cheng and Yu (2000). *Model: Method 1*	$0.5236/K_m \tau_m$	$8\tau_m$	$A_m = 2.83$; $\phi_m = 46.1^0$
O'Dwyer (2001a). *Model: Method 1*	$\dfrac{x_1}{K_m \tau_m}$	$x_2 \tau_m$	[6]

Representative coefficient values (O'Dwyer (2001a)):

x_1	x_2	A_m	ϕ_m	x_1	x_2	A_m	ϕ_m
0.558	1.4	1.5	46.2^0	0.357	4.3	4.0	60.0^0
0.484	1.55	2.0	45.5^0	0.305	12.15	5.0	75.0^0
0.458	3.35	3.0	59.9^0				

[4] $$T_i^{(104)} = \frac{\pi}{4}\left[\frac{A_m^2 - 1}{\phi_m + 0.5\pi(A_m - 1)}\right]\frac{\tau_m}{\left[0.5\pi - \phi_m - \dfrac{\phi_m + 0.5\pi(A_m - 1)}{A_m^2 - 1}\right]}.$$

[5] $$K_c^{(105)} = \frac{r_1\pi(2r_1A_m - 1)}{4K_m\tau_m(r_1^2A_m^2 - 1)},\quad T_i^{(105)} = \frac{4\tau_m(r_1^2A_m^2 - 1)^2}{\pi r_1^2 A_m (r_1A_m - 2)(2r_1A_m - 1)}$$

[6] $$A_m = \frac{\dfrac{x_2}{4x_1}\left[\dfrac{\pi}{2} + \sqrt{\dfrac{\pi^2}{4} - \dfrac{\pi}{x_2}}\right]^2}{\sqrt{1 + \dfrac{x_2^2}{4}\left[\dfrac{\pi}{2} + \sqrt{\dfrac{\pi^2}{4} - \dfrac{\pi}{x_2}}\right]^2}},\quad x_2 > 1.273;$$

$$\phi_m = \tan^{-1}\left[\sqrt{0.5x_1^2x_2^2 + 0.5x_1x_2\sqrt{x_1^2x_2^2 + 4}}\right] - \sqrt{0.5x_1^2 + 0.5\frac{x_1}{x_2}\sqrt{x_1^2x_2^2 + 4}}.$$

Rule	K_c	T_i	Comment
O'Dwyer (2001a). *Model: Method 1*	aK_u	bT_u	[7]
	Robust		
Chien (1988). *Model: Method 1*	$\frac{1}{K_m}\left(\frac{2\lambda+\tau_m}{[\lambda+\tau_m]^2}\right)$	$2\lambda+\tau_m$	[8]
Ogawa (1995). *Model: Method 1; Coefficients of* K_c *and* T_i *deduced from graphs*	$x_1/K_m\,\tau_m$	$x_2\tau_m$	
	x_1	x_2	
	0.45	11	20% uncertainty in process parameters
	0.39	12	30% uncertainty in process parameters
	0.34	13	40% uncertainty in process parameters
	0.30	14	50% uncertainty in process parameters
	0.27	15	60% uncertainty in process parameters
Alvarez-Ramirez *et al.* (1998). *Model: Method 1*	$\frac{1}{K_m}\left(\frac{1}{T_{CL}}+\frac{1}{T_m}\right)$	$T_m+\tau_m$	$T_{CL}>\tau_m$
Zhong and Li (2002). *Model: Method 2*	$\frac{2\alpha+\tau_m}{K_m(\alpha+\tau_m)^2}$	$2\alpha+\tau_m$	α not specified

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[7] $$A_m=\frac{\frac{2b}{\pi a}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{4b}}\right]^2}{\sqrt{1+\frac{1}{a^2\pi^2}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{4b}}\right]^2}},$$

$$\phi_m=\tan^{-1}\left[2\pi^2a^2b^2+2\pi ab\sqrt{4\pi^2a^2b^2+4}\right]^{0.5}-\sqrt{0.125\pi^2a^2+\frac{\pi a}{16b}\sqrt{4\pi^2a^2b^2+4}}$$

[8] $\lambda=[1/K_m,\tau_m]$ (Chien and Fruehauf (1990)); $\lambda=3\tau_m$ (Bialkowski (1996)); $\lambda>\tau_m+T_m$ (Thomasson (1997)); $\lambda=[1.5\tau_m,4.5\tau_m]$ (Zhang *et al.* (1999)). From a graph, $\lambda=1.5\tau_m$ ....Overshoot = 58%, Settling time = $6\tau_m$

$\lambda=2.5\tau_m$ ....Overshoot = 35%, Settling time = $11\tau_m$

$\lambda=3.5\tau_m$ ....Overshoot = 26%, Settling time = $16\tau_m$

$\lambda=4.5\tau_m$ ....Overshoot = 22%, Settling time = $20\tau_m$.

$\lambda=e_{max}/100K_m$; e_{max} = maximum output error after a step load disturbance (Andersson (2000) – *page 12*);

$\left(\frac{y}{d}\right)_{desired}=\frac{T_i s e^{-\tau_m s}}{K_c(\lambda s+1)^2}$ (Chen and Seborg (2002)); $\lambda=\tau_m$ (Smith (2002)).

Rule	K_c	T_i	Comment
Smith (2002). *Model: Method 1*	$\frac{1}{K_m \tau_m}$	τ_m	
Skogestad (2004a). *Model: Method 1*	[9] $K_c^{(106)}$	$\frac{4}{K_c^{(106)} K_m}$	
	Other methods		
Penner (1988). *Model: Method 1*	$\frac{0.58}{K_m \tau_m}$	$10\tau_m$	Maximum closed loop gain = 1.26
	$\frac{0.8}{K_m \tau_m}$	$5.9\tau_m$	Maximum closed loop gain = 2.0

[9] $K_c^{(106)} \geq \frac{|\Delta d_0|}{|\Delta y_{max}|}$,

$|\Delta d_0|$ = maximum magnitude of a sinusoidal disturbance over all frequencies, $|\Delta y_{max}|$ = maximum controlled variable change, corresponding to the sinusoidal disturbance.

3.5.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1}{1+T_f s}$$

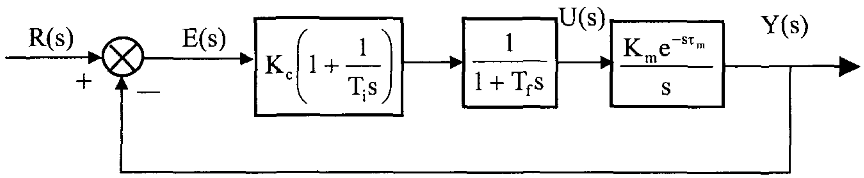

Table 15: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
	Robust		
H_∞ optimal - Tan *et al.* (1998b). *Model: Method 1*	$\frac{0.463\lambda+0.277}{K_m\tau_m}$	$\frac{\tau_m}{0.238\lambda+0.123}$	$\lambda = 0.5$; $T_f = \frac{\tau_m}{5.750\lambda+0.590}$
Rivera and Jun (2000). *Model: Method 1*	[1] $K_c^{(107)}$	$2(\tau_m+\lambda)$	λ not specified

[1] $K_c^{(107)} = \frac{2(\tau_m+\lambda)}{K_m(2\tau_m^2+4\tau_m\lambda+\lambda^2)}$, $T_f = \frac{\tau_m\lambda^2}{2\tau_m^2+4\tau_m\lambda+\lambda^2}$.

3.5.3 *Controller with set-point weighting*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

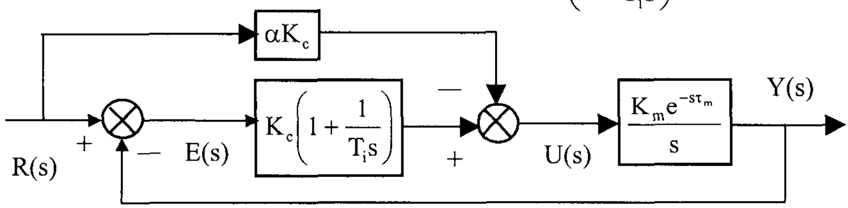

Table 16: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="2">T_i</th><th colspan="3">Comment</th></tr>
<tr><td></td><td colspan="8">Minimum performance index: servo/regulator tuning</td></tr>
<tr><td>Taguchi and Araki (2000).
Model: Method 1</td><td colspan="3">$\frac{0.7662}{K_m \tau_m}$</td><td colspan="2">$4.091\tau_m$</td><td colspan="3">$\alpha = 0.6810$, $\tau_m/T_m \le 1.0$. Overshoot (servo step) $\le 20\%$</td></tr>
<tr><td rowspan="9">Minimum ITAE - Pecharromán (2000).
Model: Method 10</td><td colspan="3">$x_1 K_u$</td><td colspan="2">$x_2 T_u$</td><td colspan="3">$K_m = 1$</td></tr>
<tr><td colspan="8">Coefficient values</td></tr>
<tr><td>x_1</td><td>x_2</td><td>α</td><td>ϕ_c</td><td>x_1</td><td>x_2</td><td>α</td><td>ϕ_c</td></tr>
<tr><td>0.049</td><td>2.826</td><td>0.506</td><td>-164^0</td><td>0.218</td><td>1.279</td><td>0.564</td><td>-135^0</td></tr>
<tr><td>0.066</td><td>2.402</td><td>0.512</td><td>-160^0</td><td>0.250</td><td>1.216</td><td>0.573</td><td>-130^0</td></tr>
<tr><td>0.099</td><td>1.962</td><td>0.522</td><td>-155^0</td><td>0.286</td><td>1.127</td><td>0.578</td><td>-125^0</td></tr>
<tr><td>0.129</td><td>1.716</td><td>0.532</td><td>-150^0</td><td>0.330</td><td>1.114</td><td>0.579</td><td>-120^0</td></tr>
<tr><td>0.159</td><td>1.506</td><td>0.544</td><td>-145^0</td><td>0.351</td><td>1.093</td><td>0.577</td><td>-118^0</td></tr>
<tr><td>0.189</td><td>1.392</td><td>0.555</td><td>-140^0</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="8">Direct synthesis</td></tr>
<tr><td rowspan="3">Chidambaram (2000b).
Model: Method 1</td><td colspan="3">$0.64/K_m\tau_m$</td><td colspan="2">$3.333\tau_m$</td><td colspan="3">$\alpha = 0.65$</td></tr>
<tr><td colspan="3">$0.487/K_m\tau_m$</td><td colspan="2">$8.75\tau_m$</td><td colspan="3">$\alpha = 0.557$</td></tr>
<tr><td colspan="8">In general, $\alpha = 0.6$ (on average) or $\alpha = 0.5\left[1 - \frac{\tau_m}{T_i}\right]$</td></tr>
<tr><td>Hägglund and Åström (2002).
Model: Method 2</td><td colspan="3">$\frac{0.35}{K_m\tau_m}$</td><td colspan="2">$7\tau_m$</td><td colspan="3">$M_s = 1.4$; $0.3 < \alpha < 0.5$</td></tr>
</table>

3.5.4 *Controller with proportional term acting on the output 1*

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

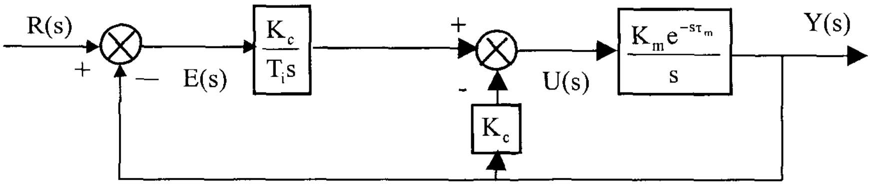

Table 17: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
	Minimum performance index: regulator tuning		
Minimum ISE – Arvanitis *et al.* (2003a).	$\frac{0.6330}{K_m \tau_m}$	$2.3800\tau_m$	*Model: Method 1*
Arvanitis *et al.* (2003a). *Model: Method 1*	[2] $\frac{0.6114}{K_m \tau_m}$	$2.7442\tau_m$	
	[3] $\frac{0.6146}{K_m \tau_m}$	$2.6816\tau_m$	
	Minimum performance index: servo tuning		
Minimum ISE – Arvanitis *et al.* (2003a).	$\frac{0.6270}{K_m \tau_m}$	$2.4688\tau_m$	*Model: Method 1*
Arvanitis *et al.* (2003a). *Model: Method 1*	[2] $\frac{0.6064}{K_m \tau_m}$	$2.8489\tau_m$	
	[3] $\frac{0.6130}{K_m \tau_m}$	$2.7126\tau_m$	

---

[2] Minimum $\int_0^\infty \left[e^2(t) + K_m{}^2 u^2(t)\right] dt$.

[3] Minimum $\int_0^\infty \left[e^2(t) + K_m{}^2 \left(\frac{du}{dt}\right)^2\right] dt$.

Rule	K_c	T_i	Comment
	Direct synthesis		
Chien *et al.* (1999). *Model: Method 2*	[4] $K_c^{(108)}$	$1.414T_{CL2} + \tau_m$	Underdamped system response - $\xi = 0.707$.
Arvanitis *et al.* (2003a). *Model: Method 1*	$\dfrac{4}{(8-\beta)K_m\tau_m}$	$\dfrac{4\tau_m}{\beta}$	$\beta = \dfrac{4}{4\xi^2 + 1}$
	Robust		
Arvanitis *et al.* (2003a). *Model: Method 1*	[5] $K_c^{(109)}$	$\dfrac{4\pi^2\tau_m}{\alpha(\pi^2 - \alpha)}$	

---

[4] $K_c^{(108)} = \dfrac{1.414T_{CL2} + \tau_m}{K_m\left(T_{CL2}^{\;2} + 1.414T_{CL2}\tau_m + \tau_m^{\;2}\right)}$.

[5] $K_c^{(109)} = \dfrac{4\pi^2}{\left[(8-\alpha)\pi^2 + \alpha^2\right]K_m\tau_m}$ with $0 < \alpha < \pi^2$;

recommended $\alpha = \dfrac{\pi^2}{2}\left(1 - \sqrt{1 - \dfrac{16}{\pi^2\left(4\xi^2 + 1\right)}}\right)$, $\xi > 0.3941$.

3.5.5 *Controller with proportional term acting on the output 2*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_1 Y(s)$$

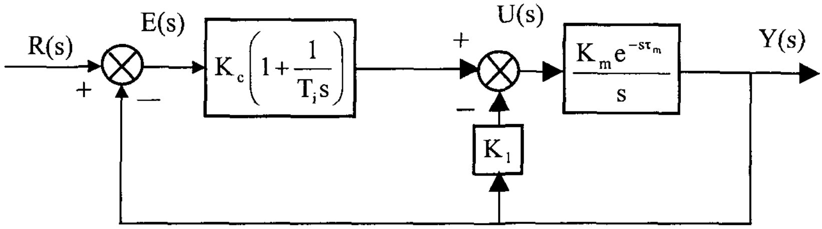

Table 18: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
	Robust		
Lee and Edgar (2002). *Model: Method 1*	[6] $K_c^{(110)}$	$T_i^{(110)}$	$K_1 = 0.25K_u$

[6] $K_c^{(110)} = \frac{0.25K_u}{(\lambda + \tau_m)}\left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}\right]$, $T_i^{(110)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$.

3.5.6 *Controller with a double integral term*

$$G_c(s) = K_c\left(1 + \frac{1}{T_{i1}s} + \frac{1}{T_{i2}s^2}\right)$$

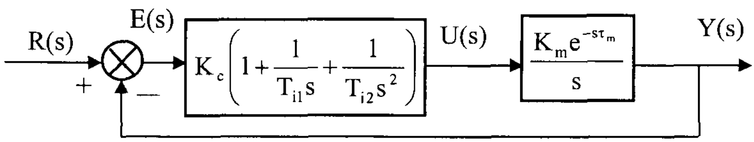

Table 19: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
	Ultimate cycle		
Belanger and Luyben (1997). *Model: Method 2*	$0.33K_u$	$T_{i1} = 2.26T_u$; $T_{i2} = 20.5T_u^{\ 2}$	Maximum closed loop log modulus = +2 dB.

3.6 FOLIPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

3.6.1 *Ideal controller* $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)$

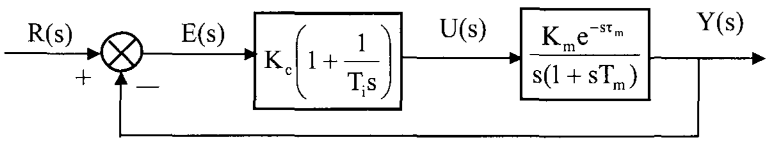

Table 20: PI controller tuning rules – FOLIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c		T_i		Comment	
	Process reaction					
Coon (1956), (1964). *Model: Method 1*; *Coefficients of* K_c *deduced from graphs*	$\dfrac{x_1}{K_m(\tau_m+T_m)}$		0		Quarter decay ratio criterion	
	Coefficient values					
	τ_m/T_m	x_1	τ_m/T_m	x_1	τ_m/T_m	x_1
	0.020	5.0	0.25	2.2	4.0	1.1
	0.053	4.0	0.43	1.7		
	0.11	3.0	1.0	1.3		
	Minimum performance index: regulator tuning					
Minimum IAE – Shinskey (1994) – *page 75.*	$\dfrac{0.556}{K_m(\tau_m+T_m)}$		$3.7(\tau_m+T_m)$		*Model: Method 1*	
Minimum IAE – Shinskey (1994) – *page 158*	$\dfrac{0.952}{K_m(T_m+\tau_m)}$		$4(T_m+\tau_m)$		*Model: Method 1*	

Rule	K_c	T_i	Comment
Minimum ITAE – Poulin and Pomerleau (1996). *Model: Method 1*	[1] $K_c^{(111)}$	$x_1(\tau_m + T_m)$	K_c *and* T_i *coefficients deduced from graph*

	Coefficient values					
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
Process output step load disturbance	0.2	5.0728	0.5231	1.2	4.7565	0.5250
	0.4	4.9688	0.5237	1.4	4.7293	0.5252
	0.6	4.8983	0.5241	1.6	4.7107	0.5254
	0.8	4.8218	0.5245	1.8	4.6837	0.5256
	1.0	4.7839	0.5249	2.0	4.6669	0.5257
Process input step load disturbance	0.2	3.9465	0.5320	1.2	4.0337	0.5312
	0.4	3.9981	0.5315	1.4	4.0278	0.5312
	0.6	4.0397	0.5311	1.6	4.0278	0.5312
	0.8	4.0397	0.5311	1.8	4.0218	0.5313
	1.0	4.0397	0.5311	2.0	4.0099	0.5314

	Direct synthesis		
Hougen (1979) – *page 338. Model: Method 1*	[2] $\frac{1.26x_1}{K_m\tau_m}$	0	Maximise crossover frequency
Poulin and Pomerleau (1999). *Model: Method 1*	$0.34K_u$ or $\frac{2.13}{K_mT_u}$	$1.04T_u$	$M_{max} = 5$ dB

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[1] $K_c^{(111)} = \frac{x_2}{K_m(\tau_m + T_m)}\sqrt{\frac{T_m^2}{x_1(\tau_m + T_m)^2} + 1}$.

[2] x_1 values recorded deduced from a graph:

τ_m/T_m	1	2	3	4	5	6	7	8	9	10
x_1	0.31	0.19	0.14	0.11	0.09	0.078	0.068	0.06	0.053	0.05

Rule	K_c	T_i	Comment
Huba and Žáková (2003). *Model: Method 1*	$^3 K_c^{(112)}$	0	
	Representative tuning rules; tuning rules for other τ_m/T_m values may be obtained from a plot.		
	$0.193/K_m\tau_m$	$3.717\tau_m$	$\tau_m/T_m = 0.5$
	$0.207/K_m\tau_m$	$3.386\tau_m$	$\tau_m/T_m = 1$
	$0.219/K_m\tau_m$	$3.127\tau_m$	$\tau_m/T_m = 2$
	Ultimate cycle		
McMillan (1984). *Model: Method 1*	$^4 K_c^{(113)}$	$T_i^{(113)}$	Tuning rules developed from K_u, T_u
Perić *et al.* (1997). *Model: Method 8*	$\dfrac{\hat{K}_u \tan^2\phi_m}{\sqrt{1+\tan^2\phi_m}}$	$\dfrac{0.1592\hat{T}_u}{\tan\phi_m}$	

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[3] $K_c^{(112)} = \dfrac{-2T_m + \sqrt{\tau_m^2 + 4T_m^2}}{K_m\tau_m^2 e^{\frac{2T_m+\tau_m-\sqrt{\tau_m^2+4T_m^2}}{2T_m}}}$.

[4] $K_c^{(113)} = \dfrac{1.477}{K_m}\dfrac{T_m}{\tau_m^2}\left\{\dfrac{1}{1+\left(\dfrac{T_m}{\tau_m}\right)^{0.65}}\right\}^2$, $T_i^{(113)} = 3.33\tau_m\left\{1+\left(\dfrac{T_m}{\tau_m}\right)^{0.65}\right\}$.

3.6.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

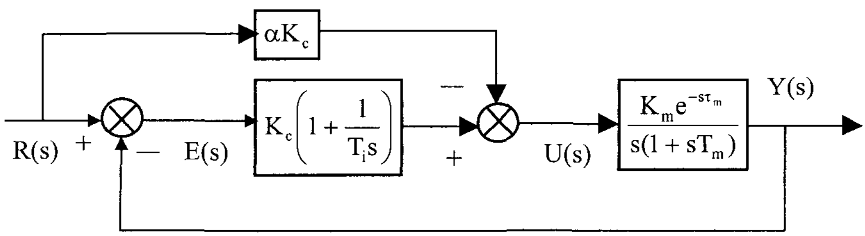

Table 21: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
	Minimum performance index: servo/regulator tuning		
Taguchi and Araki (2000). *Model: Method 1*	$K_c^{(114)}$ [5]	$T_i^{(114)}$	$\tau_m/T_m \le 1.0$. Overshoot (servo step) $\le 20\%$
Minimum ITAE - Pecharromán (2000). *Model: Method 4* $K_m = 1$; $T_m = 1$ $0.05 < \tau_m < 0.8$	$x_1 K_u$	$x_2 T_u$	

Coefficient values (Minimum ITAE - Pecharromán (2000))

x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
0.049	2.826	0.506	-164^0	0.218	1.279	0.564	-135^0
0.066	2.402	0.512	-160^0	0.250	1.216	0.573	-130^0
0.099	1.962	0.522	-155^0	0.286	1.127	0.578	-125^0
0.129	1.716	0.532	-150^0	0.330	1.114	0.579	-120^0
0.159	1.506	0.544	-145^0	0.351	1.093	0.577	-118^0
0.189	1.392	0.555	-140^0				

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[5] $K_c^{(114)} = \frac{1}{K_m}\left(0.1787 + \frac{0.2839}{\frac{\tau_m}{T_m} + 0.001723}\right)$,

$$T_i^{(114)} = 4.296 + 3.794\frac{\tau_m}{T_m} + 0.2591\left(\frac{\tau_m}{T_m}\right)^2, \quad \alpha = 0.6551 + 0.01877\frac{\tau_m}{T_m}.$$

Rule	K_c	T_i	Comment
	Direct synthesis		
Åström and Hägglund (1995) – *pages 210-212.* *Model: Method 3* $0.14 \le \frac{\tau_m}{T_m} \le 5.5$	$\frac{0.41e^{-0.23\tau+0.019\tau^2}}{K_m(T_m+\tau_m)}$	$5.7\tau_m e^{1.7\tau-0.69\tau^2}$	$M_s = 1.4$
	$\alpha = 1-0.33e^{2.5\tau-1.9\tau^2}$		
	$\frac{0.81e^{-1.1\tau+0.76\tau^2}}{K_m(T_m+\tau_m)}$	$3.4\tau_m e^{0.28\tau-0.0089\tau^2}$	$M_s = 2.0$
	$\alpha = 1-0.78e^{-1.9\tau+1.2\tau^2}$		

3.6.3 *Controller with proportional term acting on the output 1*

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

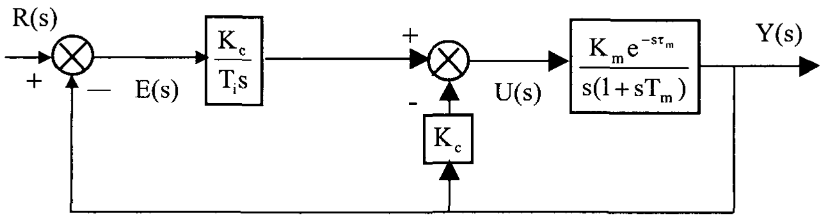

Table 22: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
	Minimum performance index: regulator tuning		
Arvanitis *et al.* (2003b). *Model: Method 1*	$\frac{4}{(8-\beta)K_m\tau_m}$	$\frac{4\tau_m}{\beta}$	
	Minimum ISE [6]		

[6] $\beta = 2.6302 - 2.1248\left(\frac{\tau_m}{T_m}\right) + 2.5776\left(\frac{\tau_m}{T_m}\right)^2 - 1.8511\left(\frac{\tau_m}{T_m}\right)^3 + 0.82557\left(\frac{\tau_m}{T_m}\right)^4$
$- 0.23641\left(\frac{\tau_m}{T_m}\right)^5 + 0.044128\left(\frac{\tau_m}{T_m}\right)^6 - 0.0053335\left(\frac{\tau_m}{T_m}\right)^7 + 0.00040201\left(\frac{\tau_m}{T_m}\right)^8$
$- 1.7164.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 + 3.1685.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \; 0 < \frac{\tau_m}{T_m} < 4.1;$

$\beta = 1.7110073 - 0.003554\left(\frac{\tau_m}{T_m} - 4.1\right), \; \frac{\tau_m}{T_m} \ge 4.1.$

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1* (continued)	Minimum performance index [7], [8]		
	Minimum performance index [9], [10]		

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[7] Performance index $= \int_0^\infty \left[e^2(t) + K_m{}^2 u^2(t)\right] dt$.

[8] $\beta = 2.1054 - 0.76462\left(\frac{\tau_m}{T_m}\right) + 0.79445\left(\frac{\tau_m}{T_m}\right)^2 - 0.51826\left(\frac{\tau_m}{T_m}\right)^3 + 0.21771\left(\frac{\tau_m}{T_m}\right)^4$

$$- 0.06\left(\frac{\tau_m}{T_m}\right)^5 + 0.010927\left(\frac{\tau_m}{T_m}\right)^6 - 0.0013006\left(\frac{\tau_m}{T_m}\right)^7 + 9.717.10^{-5}\left(\frac{\tau_m}{T_m}\right)^8$$

$$- 4.132.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9 + 7.6235.10^{-8}\left(\frac{\tau_m}{T_m}\right)^{10}, \; 0 < \frac{\tau_m}{T_m} < 4.6;$$

$$\beta = 1.69667833406 - 0.00148768\left(\frac{\tau_m}{T_m} - 4.6\right), \; \frac{\tau_m}{T_m} \ge 4.6 .$$

[9] Performance index $= \int_0^\infty \left[e^2(t) + K_m{}^2\left(\frac{du}{dt}\right)^2\right] dt$

[10] $\beta = 1.6505623 - 0.0034985\left(\frac{\tau_m}{T_m} - 8\right), \; \frac{\tau_m}{T_m} \ge 2;$

$$\beta = 2.1806 - 0.60273\left(\frac{\tau_m}{T_m}\right) + 0.3875\left(\frac{\tau_m}{T_m}\right)^2 - 0.14955\left(\frac{\tau_m}{T_m}\right)^3 + 0.034535\left(\frac{\tau_m}{T_m}\right)^4$$

$$- 0.0042243\left(\frac{\tau_m}{T_m}\right)^5 + 8.2395.10^{-5}\left(\frac{\tau_m}{T_m}\right)^6 + 5.0912.10^{-5}\left(\frac{\tau_m}{T_m}\right)^7 - 7.283.10^{-6}\left(\frac{\tau_m}{T_m}\right)^8$$

$$+ 4.2603.10^{-7}\left(\frac{\tau_m}{T_m}\right)^9 - 9.5824.10^{-9}\left(\frac{\tau_m}{T_m}\right)^{10}, \; \frac{\tau_m}{T_m} \text{ range not defined.}$$

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1*	[11] $K_c^{(115)}$	$T_i^{(115)}$	
	Minimum ISE [12]		
	Minimum performance index [13], [14]		

[11] $$K_c^{(115)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m\left[2(\tau_m + T_m)(12.5664(\tau_m + T_m) - 4) - (1.5\tau_m^2 + 3\tau_m T_m + 2T_m^2)a\right]}$$

with $a = [0.5708 + \sqrt{\dfrac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi$; $T_i^{(115)} = \dfrac{12.5664(\tau_m + T_m) - 4}{a}$.

[12] $$\chi = 0.99315 + 1.9536\left(\frac{\tau_m}{T_m}\right) - 2.5157\left(\frac{\tau_m}{T_m}\right)^2 + 1.9163\left(\frac{\tau_m}{T_m}\right)^3 - 0.8893\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.26167\left(\frac{\tau_m}{T_m}\right)^5 - 0.049809\left(\frac{\tau_m}{T_m}\right)^6 + 0.0061098\left(\frac{\tau_m}{T_m}\right)^7 - 0.00046593\left(\frac{\tau_m}{T_m}\right)^8$$
$$+ 2.0083.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.7368.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 6.0;$$
$$\chi = 1.96705757 + 0.01308189\left(\frac{\tau_m}{T_m} - 6\right), \quad \frac{\tau_m}{T_m} \ge 6.0 .$$

[13] Performance index = $\int_0^\infty \left[e^2(t) + K_m^2 u^2(t)\right]dt$

[14] $$\chi = 0.73114 + 2.3461\left(\frac{\tau_m}{T_m}\right) - 2.8509\left(\frac{\tau_m}{T_m}\right)^2 + 2.0965\left(\frac{\tau_m}{T_m}\right)^3 - 0.95252\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.27639\left(\frac{\tau_m}{T_m}\right)^5 - 0.052089\left(\frac{\tau_m}{T_m}\right)^6 + 0.0063413\left(\frac{\tau_m}{T_m}\right)^7 - 0.00048064\left(\frac{\tau_m}{T_m}\right)^8$$
$$+ 2.0611.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.8178.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5;$$
$$\chi = 1.916423116 + 0.018175439\left(\frac{\tau_m}{T_m} - 4.5\right), \quad \frac{\tau_m}{T_m} \ge 4.5 .$$

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1* (continued)	Minimum performance index [15], [16]		
	Minimum performance index: servo tuning		
Arvanitis *et al.* (2003b). *Model: Method 1*	$\dfrac{4}{(8-\beta)K_m\tau_m}$	$\dfrac{4\tau_m}{\beta}$	
	Minimum ISE [17]		

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[15] Performance index = $\displaystyle\int_0^{\infty}\left[e^2(t)+K_m{}^2\left(\frac{du}{dt}\right)^2\right]dt$

[16] $$\chi = 0.60896 + 3.0593\left(\frac{\tau_m}{T_m}\right) - 3.9339\left(\frac{\tau_m}{T_m}\right)^2 + 2.933\left(\frac{\tau_m}{T_m}\right)^3 - 1.3353\left(\frac{\tau_m}{T_m}\right)^4$$
$$+0.38696\left(\frac{\tau_m}{T_m}\right)^5 - 0.072771\left(\frac{\tau_m}{T_m}\right)^6 + 0.0088395\left(\frac{\tau_m}{T_m}\right)^7 - 0.00066864\left(\frac{\tau_m}{T_m}\right)^8$$
$$+2.8623.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 5.2941.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0<\frac{\tau_m}{T_m}<4.5;$$
$$\chi = 1.9283751 + 0.016846129\left(\frac{\tau_m}{T_m} - 4.5\right),\ \frac{\tau_m}{T_m}\ge 4.5.$$

[17] $$\beta = 1.997 - 0.82781\left(\frac{\tau_m}{T_m}\right) + 1.0002\left(\frac{\tau_m}{T_m}\right)^2 - 0.72106\left(\frac{\tau_m}{T_m}\right)^3 + 0.32269\left(\frac{\tau_m}{T_m}\right)^4$$
$$-0.092452\left(\frac{\tau_m}{T_m}\right)^5 + 0.017215\left(\frac{\tau_m}{T_m}\right)^6 - 0.0020706\left(\frac{\tau_m}{T_m}\right)^7 + 0.00015505\left(\frac{\tau_m}{T_m}\right)^8$$
$$-6.5691.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9 + 1.2025.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0<\frac{\tau_m}{T_m}<5.5;$$
$$\beta = 1.6281745 - 0.001171\left(\frac{\tau_m}{T_m} - 5.5\right),\ \frac{\tau_m}{T_m}\ge 5.5.$$

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1* (continued)	Minimum performance index [18,19]		
	Minimum performance index [20, 21]		

[18] Performance index = $\int_0^{\infty}\left[e^2(t) + K_m{}^2 u^2(t)\right]dt$

[19] $\beta = 1.5693 + 0.1066\left(\frac{\tau_m}{T_m}\right) - 0.10101\left(\frac{\tau_m}{T_m}\right)^2 + 0.05217\left(\frac{\tau_m}{T_m}\right)^3 - 0.016147\left(\frac{\tau_m}{T_m}\right)^4$

$+ 0.0030412\left(\frac{\tau_m}{T_m}\right)^5 - 0.00032187\left(\frac{\tau_m}{T_m}\right)^6 + 1.2181.10^{-5}\left(\frac{\tau_m}{T_m}\right)^7 + 9.9698.10^{-7}\left(\frac{\tau_m}{T_m}\right)^8$

$- 1.2274.10^{-7}\left(\frac{\tau_m}{T_m}\right)^9 + 3.7365.10^{-9}\left(\frac{\tau_m}{T_m}\right)^{10}, \; 0 < \frac{\tau_m}{T_m} < 4.6$.

[20] Performance index = $\int_0^{\infty}\left[e^2(t) + K_m{}^2\left(\frac{du}{dt}\right)^2\right]dt$

[21] $\beta = 0.780651417 + 4.7464131\left(\frac{\tau_m}{T_m} - 0.1\right), \; 0 < \frac{\tau_m}{T_m} < 0.3$;

$\beta = 1.729934 - 0.051535\left(\frac{\tau_m}{T_m} - 0.3\right), \; 0.3 \le \frac{\tau_m}{T_m} < 2$.

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1*	[22] $K_c^{(116)}$	$T_i^{(116)}$	
	Minimum ISE [23]		
	Minimum performance index [24, 25]		

[22] $$K_c^{(116)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m\left[2(\tau_m + T_m)(12.5664(\tau_m + T_m) - 4) - (1.5\tau_m^2 + 3\tau_m T_m + 2T_m^2)a\right]}$$

with $a = [0.5708 + \sqrt{\dfrac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi$, $T_i^{(116)} = \dfrac{12.5664(\tau_m + T_m) - 4}{a}$.

[23] $$\chi = 0.79001 + 2.114\left(\frac{\tau_m}{T_m}\right) - 2.5601\left(\frac{\tau_m}{T_m}\right)^2 + 1.8971\left(\frac{\tau_m}{T_m}\right)^3 - 0.86871\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.25371\left(\frac{\tau_m}{T_m}\right)^5 - 0.048068\left(\frac{\tau_m}{T_m}\right)^6 + 0.0058771\left(\frac{\tau_m}{T_m}\right)^7 - 0.00044707\left(\frac{\tau_m}{T_m}\right)^8$$
$$+ 1.9231.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.5718.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0 < \frac{\tau_m}{T_m} < 4.5;$$
$$\chi = 1.903177576 + 0.0181584\left(\frac{\tau_m}{T_m} - 4.5\right),\ \frac{\tau_m}{T_m} \ge 4.5.$$

[24] Performance index $= \displaystyle\int_0^\infty \left[e^2(t) + K_m^2 u^2(t)\right]dt$.

[25] $$\chi = 0.48124 + 2.5549\left(\frac{\tau_m}{T_m}\right) - 2.896\left(\frac{\tau_m}{T_m}\right)^2 + 2.0495\left(\frac{\tau_m}{T_m}\right)^3 - 0.91115\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.26094\left(\frac{\tau_m}{T_m}\right)^5 - 0.048775\left(\frac{\tau_m}{T_m}\right)^6 + 0.0059064\left(\frac{\tau_m}{T_m}\right)^7 - 0.00044618\left(\frac{\tau_m}{T_m}\right)^8$$
$$+ 1.9095.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.5335.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0 < \frac{\tau_m}{T_m} < 4.0;$$
$$\chi = 1.87054524 + 0.021802\left(\frac{\tau_m}{T_m} - 4\right),\ \frac{\tau_m}{T_m} \ge 4.0.$$

Rule	K_c	T_i	Comment
Arvanitis *et al.* (2003b). *Model: Method 1* (continued)	Minimum performance index [26], [27]		
	Direct synthesis		
Arvanitis *et al.* (2003b). *Model: Method 1*	$\frac{4}{(8-\beta)K_m\tau_m}$	$\frac{4\tau_m}{\beta}$	$\beta=\frac{4}{4\xi^2+1}$
	[28] $K^{(117)}$	$(1+4\xi^2)(\tau_m+T_m)$	

[26] Performance index $=\int_0^\infty\left[e^2(t)+K_m{}^2\left(\frac{du}{dt}\right)^2\right]dt$.

[27] $\chi=0.31373+3.7796\left(\frac{\tau_m}{T_m}\right)-5.0294\left(\frac{\tau_m}{T_m}\right)^2+3.8589\left(\frac{\tau_m}{T_m}\right)^3-1.7956\left(\frac{\tau_m}{T_m}\right)^4$

$+0.52906\left(\frac{\tau_m}{T_m}\right)^5-0.10079\left(\frac{\tau_m}{T_m}\right)^6+0.012369\left(\frac{\tau_m}{T_m}\right)^7-0.00094357\left(\frac{\tau_m}{T_m}\right)^8$

$+4.0678.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9-7.5694.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}$, $0<\frac{\tau_m}{T_m}<3.5$;

$\chi=1.86144723+0.021989\left(\frac{\tau_m}{T_m}-3.5\right)$, $\frac{\tau_m}{T_m}\geq 3.5$.

[28] $K_c{}^{(117)}=\frac{(1+4\xi^2)(\tau_m+T_m)}{K_m\left[\tau_m{}^2(0.5+8\xi^2)+\tau_m T_m(1+16\xi^2)+8\xi^2 T_m{}^2\right]}$.

3.6.4 *Controller with proportional term acting on the output 2*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_1 Y(s)$$

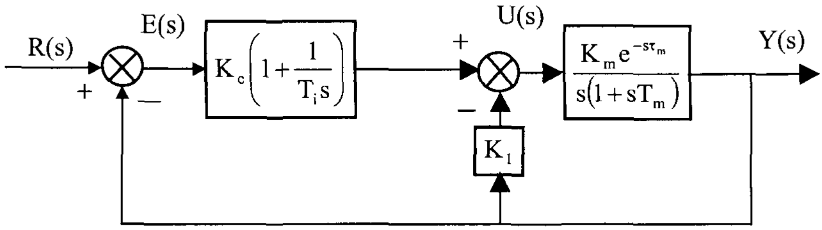

Table 23: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
	Robust		
Lee and Edgar (2002). *Model: Method 1*	[29] $K_c^{(118)}$	$T_i^{(118)}$	$K_1 = 0.25K_u$

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[29] $K_c^{(118)} = \frac{0.25K_u}{(\lambda+\tau_m)}\left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^{\ 2}}{2(\lambda+\tau_m)}\right]$, $T_i^{(118)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^{\ 2}}{2(\lambda+\tau_m)}$.

3.7 SOSPD model $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ or $\dfrac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$

3.7.1 *Ideal controller* $G_c(s) = K_c\left(1 + \dfrac{1}{T_i s}\right)$

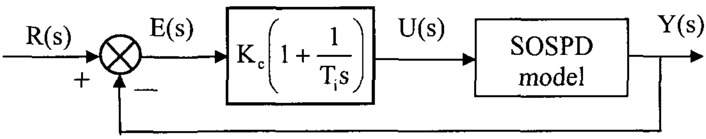

Table 24: PI controller tuning rules – SOSPD model $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ or $\dfrac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$

Rule	K_c	T_i	Comment
	Minimum performance index: regulator tuning		
Minimum IAE – Shinskey (1994) – *page 158.*	[1] $K_c^{(119)}$	$T_i^{(119)}$	*Model: Method 1*
Minimum IAE – Shinskey (1996) – *page 48. Model: Method 1* $\dfrac{\tau_m}{T_{m1}} = 0.2$	$\dfrac{0.77T_{m1}}{K_m\tau_m}$	$2.83(\tau_m + T_{m2})$	$\dfrac{T_{m2}}{T_{m1}} = 0.1$
	$\dfrac{0.70T_{m1}}{K_m\tau_m}$	$2.65(\tau_m + T_{m2})$	$\dfrac{T_{m2}}{T_{m1}} = 0.2$
	$\dfrac{0.80T_{m1}}{K_m\tau_m}$	$2.29(\tau_m + T_{m2})$	$\dfrac{T_{m2}}{T_{m1}} = 0.5$
	$\dfrac{0.80T_{m1}}{K_m\tau_m}$	$1.67(\tau_m + T_{m2})$	$\dfrac{T_{m2}}{T_{m1}} = 1.0$

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[1] $K_c^{(119)} = \dfrac{100T_{m1}}{K_m(\tau_m + T_{m2})\left(50 + 55\left[1 - e^{-\frac{T_{m1}}{\tau_m + T_{m2}}}\right]\right)}$,

$$T_i^{(119)} = \tau_m\left(0.5 + 3.5\left[1 - e^{-\frac{3T_{m1}}{(\tau_m + T_{m2})}}\right]\right).$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang *et al.* (1996). *Model: Method 1*	[2] $K_c^{(120)}$	$T_i^{(120)}$	$0 < \frac{T_{m2}}{T_{m1}} \le 1$; $0.1 \le \frac{\tau_m}{T_{m1}} \le 1$

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[2] Note: equations continued into the footnote on page 104.

$$K_c^{(120)} = \frac{1}{K_m}\left[6.4884 + 4.6198\frac{\tau_m}{T_{m1}} - 3.491\frac{T_{m2}}{T_{m1}} - 25.3143\frac{\tau_m T_{m2}}{T_{m1}^2}\right]$$

$$+\frac{1}{K_m}\left[0.8196\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9077} - 5.2132\left(\frac{\tau_m}{T_{m1}}\right)^{-0.063} - 7.2712\left(\frac{\tau_m}{T_{m1}}\right)^{0.5961}\right]$$

$$+\frac{1}{K_m}\left[-18.0448\left(\frac{T_{m2}}{T_{m1}}\right)^{0.7204} + 5.3263\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0049} + 13.9108\left(\frac{T_{m2}}{T_{m1}}\right)^{1.005}\right]$$

$$+\frac{1}{K_m}\left[0.4937\frac{T_{m2}}{\tau_m} + 19.1783\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{0.8529} + 12.2494\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{0.5613}\right]$$

$$+\frac{1}{K_m}\left[8.4355\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.557} - 17.6781\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.1818}\right]$$

$$+\frac{1}{K_m}\left[-0.7241e^{\frac{\tau_m}{T_{m1}}} - 2.2525e^{\frac{T_{m2}}{T_{m1}}} + 5.4959e^{\frac{\tau_m T_{m2}}{T_{m1}^2}}\right],$$

$$T_i^{(120)} = T_{m1}\left[0.0064 + 3.9574\frac{\tau_m}{T_{m1}} + 4.4087\frac{T_{m2}}{T_{m1}} - 6.4789\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-12.8702\frac{\tau_m T_{m2}}{T_{m1}^2} - 1.5083\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 9.4348\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[17.0736\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 15.9816\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.909\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-10.7619\left(\frac{\tau_m}{T_{m1}}\right)^4 - 10.864\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 22.3194\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-6.6602\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 6.8122\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 7.5146\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang *et al.* (1996). *Model: Method 1*	[3] $K_c^{(121)}$	$T_i^{(121)}$	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

$$+T_{m1}\left[2.8724\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 + 11.4666\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 11.1207\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-1.2174\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 4.3675\left(\frac{T_{m2}}{T_{m1}}\right)^5 - 2.2236\left(\frac{\tau_m}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[-0.112\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 + 1.0308\left(\frac{T_{m2}}{T_{m1}}\right)^6 - 1.9136\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-3.4994\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.5777\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 1.1408\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right].$$

[3] Note: equations continued into the footnote on page 105.

$$K_c^{(121)} = \frac{1}{K_m}\left[-10.4183 - 20.9497\frac{\tau_m}{T_{m1}} - 5.5175\xi_m - 26.5149\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+\frac{1}{K_m}\left[42.7745\left(\frac{\tau_m}{T_{m1}}\right)^{1.4439} + 10.5069\left(\frac{\tau_m}{T_{m1}}\right)^{0.1456} + 15.4103\left(\frac{\tau_m}{T_{m1}}\right)^{0.3157}\right]$$

$$+\frac{1}{K_m}\left[34.3236\xi_m^{3.7057} - 17.8860\xi_m^{4.5359} - 54.0584\xi_m^{1.9593}\right]$$

$$+\frac{1}{K_m}\left[22.4263\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.0541} + 2.7497\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{4.7426}\right]$$

$$+\frac{1}{K_m}\left[50.2197\xi_m^{1.8288}\left(\frac{\tau_m}{T_{m1}}\right) - 17.1968\frac{\tau_m}{T_{m1}}\xi_m^{2.7227} + 1.0293\xi_m\frac{T_{m1}}{\tau_m}\right]$$

$$+\frac{1}{K_m}\left[-16.7667e^{\frac{\tau_m}{T_{m1}}} + 14.5737e^{\xi_m} - 7.3025e^{\xi_m\frac{\tau_m}{T_{m1}}}\right],$$

$$T_i^{(121)} = T_{m1}\left[11.447 + 4.5128\frac{\tau_m}{T_{m1}} - 75.2486\xi_m - 110.807\left(\frac{\tau_m}{T_{m1}}\right)^2 - 12.282\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+T_{m1}\left[345.3228\xi_m^2 + 191.9539\left(\frac{\tau_m}{T_{m1}}\right)^3 + 359.3345\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

Rule	K_c	T_i	Comment
Minimum IAE – Shinskey (1996) – *page 121.* *Model: Method 1*	$0.48K_u$	$0.83T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$
Minimum ISE – McAvoy and Johnson (1967). *Model: Method 1; Coefficients of* K_c *and* T_i *deduced from graphs*	x_1/K_m	$x_2\tau_m$	

Coefficient values

ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2
1	0.5	0.8	1.82	4	0.5	4.3	3.45	7	0.5	7.8	3.85
1	4.0	5.7	12.5	4	4.0	27.1	6.67	7	4.0	51.2	5.88
1	10.0	13.6	25.0								

$$+T_{m1}\left[-158.7611\frac{\tau_m}{T_{m1}}\xi_m^{\,2} - 770.2897\xi_m^{\,3} - 153.633\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[-412.5409\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 - 414.7786\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 485.0976\frac{\tau_m}{T_{m1}}\xi_m^{\,3}\right]$$

$$+T_{m1}\left[864.5195\xi_m^{\,4} + 55.4366\left(\frac{\tau_m}{T_{m1}}\right)^5 + 222.2685\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[275.116\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 205.2493\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\,3} - 479.5627\left(\frac{\tau_m}{T_{m1}}\right)\xi_m^{\,4}\right]$$

$$+T_{m1}\left[-473.1346\xi_m^{\,5} - 6.547\left(\frac{\tau_m}{T_{m1}}\right)^6 - 43.2822\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5 + 99.8717\xi_m^{\,6}\right]$$

$$+T_{m1}\left[-73.5666\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m^{\,2} - 56.4418\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m^{\,3} - 37.497\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\,4}\right]$$

$$+T_{m1}\left[160.7714\frac{\tau_m}{T_{m1}}\xi_m^{\,5}\right].$$

Rule	K_c				T_i				Comment			
Minimum ITAE – Lopez *et al.* (1969). *Model: Method 7; Coefficients of* K_c *and* T_i *deduced from graphs*	x_1/K_m				$x_2 T_m$							
	Coefficient values											
	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2
	0.5	0.1	3.0	2.86	1	0.1	7.0	2.00	4	0.1	40.0	0.83
	0.5	1.0	0.2	0.83	1	1.0	0.95	2.22	4	1.0	6.0	3.33
	0.5	10.0	0.3	4.0	1	10.0	0.35	5.00	4	10.0	0.75	10.0
Minimum ITAE – Chao *et al.* (1989). *Model: Method 1*	[4] $K_c^{(122)}$				$T_i^{(122)}$							
	$0.7 \le \xi_m \le 3$; $0.05 \le \frac{\tau_m}{2\xi_m T_{m1}} \le 0.5$											
Minimum ITAE – Hassan (1993). *Model: Method 7*	[5] $K_c^{(123)}$				$T_i^{(123)}$				$0.5 \le \xi_m \le 2$, $0.1 \le \frac{\tau_m}{T_{m1}} \le 4$			

---

[4] $$K_c^{(122)} = \frac{0.09578 + 0.6206\xi_m - 0.1069\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.108+0.2558\xi_m-0.0579\xi_m^2},$$

$$T_i^{(122)} = 2\xi_m T_{m1}\left(1.31 - 0.0914\xi_m + 0.07464\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.6441+0.7626\xi_m-0.1193\xi_m^2}.$$

[5] Formulae correspond to graphs of Lopez *et al.* (1969). $K_c^{(123)}$ is obtained as follows:

$$\log\left[K_m K_c^{(123)}\right] = -0.0099458 + 1.9743700\xi_m - 3.8984310\frac{\tau_m}{T_{m1}} - 1.1225270\xi_m^2$$
$$+ 2.3493280\left(\frac{\tau_m}{T_{m1}}\right)^2 + 1.9126490\xi_m\frac{\tau_m}{T_{m1}} + 0.0754568\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2$$
$$- 0.5594610\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.7930846\xi_m^2\frac{\tau_m}{T_{m1}} + 0.2412770\xi_m^3$$
$$- 0.3567954\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0024730\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 0.0478593\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2$$
$$- 0.1354322\xi_m^3\frac{\tau_m}{T_{m1}} - 1.1756470\xi_m^3\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.0096974\xi_m^3\left(\frac{\tau_m}{T_{m1}}\right)^3;$$

$T_i^{(123)}$ is obtained as follows (equation continued into the footnote on page 107):

$$\log\left[\frac{T_i^{(123)}}{T_{m1}}\right] = 0.9321658 - 0.7200651\xi_m - 3.4227010\frac{\tau_m}{T_{m1}} + 0.0432084\xi_m^2$$

Rule	K_c	T_i	Comment
Minimum ITSE – Chao *et al.* (1989). *Model: Method 1*	[6] $K_c^{(124)}$	$T_i^{(124)}$	
	$0.7 \le \xi_m \le 3$; $0.05 \le \frac{\tau_m}{2\xi_m T_{m1}} \le 0.5$		
Nearly minimum IAE, ISE, ITAE – Hwang (1995). *Model: Method 10*	[7] $K_c^{(125)}$	$T_i^{(125)}$	Decay ratio = 0.2

$$+1.4965440\left(\frac{\tau_m}{T_{m1}}\right)^2+5.3636520\xi_m\frac{\tau_m}{T_{m1}}-0.0848431\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2$$

$$-1.6011510\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2-2.1777800\xi_m^2\frac{\tau_m}{T_{m1}}+0.0404586\xi_m^3$$

$$-0.1769621\left(\frac{\tau_m}{T_{m1}}\right)^3+0.0912008\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3+0.1501474\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2$$

$$+0.3033341\xi_m^3\frac{\tau_m}{T_{m1}}+0.1753407\xi_m^3\left(\frac{\tau_m}{T_{m1}}\right)^2-0.0622614\xi_m^3\left(\frac{\tau_m}{T_{m1}}\right)^3.$$

[6] $$K_c^{(124)}=\frac{0.3366+0.6355\xi_m-0.1066\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.0167+0.1743\xi_m-0.03946\xi_m^2},$$

$$T_i^{(124)}=2\xi_m T_{m1}\left(1.9255-0.3607\xi_m+0.1299\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.5848+0.7294\xi_m-0.1136\xi_m^2}.$$

[7] $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$, $0.6\le\xi_m\le 4.2$. Equations continued into the footnote on page 108.

$$K_c^{(125)}=\left(1-\frac{0.622\left[1-0.435\omega_H\tau_m+0.052(\omega_H\tau_m)^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,$$

$$T_i^{(125)}=\frac{K_c^{(125)}(1+K_H K_m)}{0.0697\omega_H K_m\left(1+0.752\omega_H\tau_m-0.145\omega_H^2\tau_m^2\right)},\ \varepsilon<2.4;$$

$$K_c^{(125)}=\left(1-\frac{0.724\left[1-0.469\omega_H\tau_m+0.0609(\omega_H\tau_m)^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,$$

$$T_i^{(125)}=\frac{K_c^{(125)}(1+K_H K_m)}{0.0405\omega_H K_m\left(1+1.93\omega_H\tau_m-0.363\omega_H^2\tau_m^2\right)},\ 2.4\le\varepsilon<3;$$

$$K_c^{(125)}=\left(1-\frac{1.26(0.506)^{\omega_H\tau_m}\left[1-1.07/\varepsilon+0.616/\varepsilon^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,$$

Rule	K_c		T_i		Comment	
	Minimum performance index: servo tuning					
Minimum IAE – Gallier and Otto (1968). *Model: Method 19.*	x_1/K_m		$x_2\left(T_{m1}+T_{m2}+\tau_m\right)$		$T_{m1}=T_{m2}$	
	Representative coefficient values – deduced from graphs					
	$\dfrac{\tau_m}{2T_{m1}}$	x_1	x_2	$\dfrac{\tau_m}{2T_{m1}}$	x_1	x_2
	0.053	3.6	1.35	0.67	0.76	0.72
	0.11	2.3	1.17	1.50	0.53	0.60
	0.25	1.35	0.93	4.0	0.43	0.51
Minimum IAE - Huang *et al.* (1996). *Model: Method 1*	[8] $K_c^{(126)}$		$T_i^{(126)}$		$0<\dfrac{T_{m2}}{T_{m1}}\le 1$; $0.1\le\dfrac{\tau_m}{T_{m1}}\le 1$	

$$T_i^{(125)}=\frac{K_c^{(125)}\left(1+K_HK_m\right)}{0.0661\omega_HK_m\left(1+0.824\ln\left[\omega_H\tau_m\right]\right)\left(1+1.71/\varepsilon-1.17/\varepsilon^2\right)},\ 3\le\varepsilon<20;$$

$$K_c^{(125)}=\left(1-\frac{1.09\left[1-0.497\omega_H\tau_m+0.0724\left(\omega_H\tau_m\right)^2\right]}{K_HK_m/\left(1+K_HK_m\right)}\right)K_H,$$

$$T_i^{(125)}=\frac{K_c^{(125)}\left(1+K_HK_m\right)}{0.054\omega_HK_m\left(-1+2.54\omega_H\tau_m-0.457\omega_H^{\,2}\tau_m^{\,2}\right)},\ \varepsilon>20.$$

[8] Note: equations continued into the footnote on page 109.

$$K_c^{(126)}=\frac{1}{K_m}\left[-13.0454-9.0916\frac{\tau_m}{T_{m1}}+2.6647\frac{T_{m2}}{T_{m1}}+9.162\frac{\tau_mT_{m2}}{T_{m1}^{\,2}}\right]$$

$$+\frac{1}{K_m}\left[0.3053\left(\frac{\tau_m}{T_{m1}}\right)^{-1.0169}+1.1075\left(\frac{\tau_m}{T_{m1}}\right)^{3.5959}-2.2927\left(\frac{\tau_m}{T_{m1}}\right)^{3.6843}\right]$$

$$+\frac{1}{K_m}\left[-31.0306\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8476}-13.0155\left(\frac{T_{m2}}{T_{m1}}\right)^{2.6083}+9.6899\left(\frac{T_{m2}}{T_{m1}}\right)^{2.9049}\right]$$

$$+\frac{1}{K_m}\left[-0.6418\frac{T_{m2}}{\tau_m}+18.9643\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.2016}-39.7340\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{1.3293}\right]$$

$$+\frac{1}{K_m}\left[28.155\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.801}-2.0067\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{3.956}\right]$$

$$+\frac{1}{K_m}\left[4.8259e^{\frac{\tau_m}{T_{m1}}}+2.1137e^{\frac{T_{m2}}{T_{m1}}}+8.4511e^{\frac{\tau_mT_{m2}}{T_{m1}^{\,2}}}\right],$$

Rule	K_c	T_i	Comment
Minimum IAE - Huang *et al.* (1996). *Model: Method 1*	[9] $K_c^{(127)}$	$T_i^{(127)}$	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

$$T_i^{(126)} = T_{m1}\left[0.9771 - 0.2492\frac{\tau_m}{T_{m1}} + 0.8753\frac{T_{m2}}{T_{m1}} + 3.4651\left(\frac{\tau_m}{T_{m1}}\right)^2 - 3.8516\frac{\tau_m T_{m2}}{T_{m1}^2}\right]$$

$$+ T_{m1}\left[7.5106\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 7.4538\left(\frac{\tau_m}{T_{m1}}\right)^3 + 11.6768\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+ T_{m1}\left[-10.9909\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 16.1461\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 8.2567\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+ T_{m1}\left[-18.1011\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 6.2208\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 21.9893\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+ T_{m1}\left[15.8538\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 4.7536\left(\frac{\tau_m}{T_{m1}}\right)^5 + 14.5405\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+ T_{m1}\left[-2.2691\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 - 8.387\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+ T_{m1}\left[-16.651\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 7.1990\left(\frac{T_{m2}}{T_{m1}}\right)^5 + 1.1496\left(\frac{\tau_m}{T_{m1}}\right)^6\right]$$

$$+ T_{m1}\left[-4.728\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 + 1.1395\left(\frac{T_{m2}}{T_{m1}}\right)^6 + 0.6385\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+ T_{m1}\left[1.0885\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 3.1615\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 4.5398\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right].$$

[9] Note: equations continued into the footnote on page 110.

$$K_c^{(127)} = \frac{1}{K_m}\left[-10.95 - 1.8845\frac{\tau_m}{T_{m1}} - 3.4123\xi_m + 4.5954\xi_m\frac{\tau_m}{T_{m1}} - 1.7002\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+ \frac{1}{K_m}\left[-2.1324\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 - 14.4149\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right) - 0.7683\xi_m^3\right]$$

$$+\frac{1}{K_m}\left[7.5142\left(\frac{\tau_m}{T_{m1}}\right)^{0.421}+3.7291\left(\frac{\tau_m}{T_{m1}}\right)^{0.1984}+5.3444\left(\frac{\tau_m}{T_{m1}}\right)^{1.8033}\right]$$

$$+\frac{1}{K_m}\left[-0.0819\xi_m{}^{19.5419}-3.603\xi_m{}^{1.0749}+7.1163\xi_m{}^{1.1006}\right]$$

$$+\frac{1}{K_m}\left[3.206\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.6753}-7.8480\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.1642}+11.3222\xi_m{}^{1.9948}\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+\frac{1}{K_m}\left[2.4239e^{\frac{\tau_m}{T_{m1}}}+3.4137e^{\xi_m}+1.0251e^{\frac{\xi_m\tau_m}{T_{m1}}}-0.5593\xi_m\frac{T_{m1}}{\tau_m}\right],$$

$$T_i^{(127)}=T_{m1}\left[2.4866-23.3234\frac{\tau_m}{T_{m1}}+5.3662\xi_m+65.6053\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[29.0062\xi_m\frac{\tau_m}{T_{m1}}-24.1648\xi_m{}^2-83.6796\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-135.9699\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2+43.1477\frac{\tau_m}{T_{m1}}\xi_m{}^2+51.9749\xi_m{}^3\right]$$

$$+T_{m1}\left[86.0228\left(\frac{\tau_m}{T_{m1}}\right)^4+70.4553\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3+153.4877\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-125.0112\frac{\tau_m}{T_{m1}}\xi_m{}^3-68.5893\xi_m{}^4-62.7517\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[27.6178\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-152.7422\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3+20.8705\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3\right]$$

$$+T_{m1}\left[54.0012\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4+58.7376\xi_m{}^5+13.1193\left(\frac{\tau_m}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[20.2645\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5-23.2064\xi_m{}^6-61.6742\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2\right]$$

$$+T_{m1}\left[136.2439\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3-95.4092\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4+20.4168\frac{\tau_m}{T_{m1}}\xi_m{}^5\right].$$

Rule	K_c	T_i	Comment
Minimum ISE – Keviczky and Császi (1973). *Model: Method 1* *Representative* K_c, T_i *coefficients estimated from graph*; $K_m = 1$	x_1	$x_2\tau_m$	

Coefficient values

ξ	0.1		0.2		0.5		1.0		2.0	
τ_m/T_{m1}	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2	x_1	x_2
0.1	0.26	1.5	0.55	2	2.2	3.5	6	6	14	9
0.2	0.21	1.3	0.4	1.5	1.5	2.8	3	4.5	9	8
0.5	0.15	1.2	0.25	1.3	0.8	2	2	3.5	5	7
1.0	0.12	1.1	0.2	1.2	0.65	1.8	1.2	3.3	3	6.5
2.0	0.1	1.0	0.16	1.2	0.4	1.9	0.9	3.5	1.8	7
5.0	0.1	1.2	0.16	1.4	0.4	3.0	0.65	5.0	1.0	8
10.0	0.15	1.6	0.2	2.0	0.45	5.5	0.6	7.5	0.7	10

Other coefficient values

$\dfrac{\tau_m}{T_{m1}}$	x_1	x_2	ξ	$\dfrac{\tau_m}{T_{m1}}$	x_1	x_2	ξ
0.2	30	15	5.0	2.0	4	15	5.0
0.5	13	14	5.0	2.0	8	30	10.0
0.5	30	30	10.0	5.0	1.9	17	5.0
1.0	7	14	5.0	5.0	3.4	32	10.0
1.0	14	30	10.0	10.0	1.1	19	5.0
				10.0	2.0	34	10.0

Rule	K_c	T_i	Comment
Minimum ITAE – Chao *et al.* (1989). *Model: Method 1*	[10] $K_c^{(128)}$	$T_i^{(128)}$	
	$0.7 \le \xi_m \le 3$; $0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$		
Minimum ITSE – Chao *et al.* (1989). *Model: Method 1*	[11] $K_c^{(129)}$	$T_i^{(129)}$	
	$0.7 \le \xi_m \le 3$; $0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$		

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[10] $$K_c^{(128)} = \frac{0.2763 + 0.3532\xi_m - 0.06955\xi_m^{\ 2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.3772-0.2414\xi_m+0.03342\xi_m^{\ 2}},$$

$$T_i^{(128)} = 2\xi_m T_{m1}\left(0.9953 + 0.07857\xi_m + 0.005317\xi_m^{\ 2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.3276+0.3226\xi_m-0.05929\xi_m^{\ 2}}.$$

[11] $$K_c^{(129)} = \frac{0.3970 + 0.4376\xi_m - 0.08168\xi_m^{\ 2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.5629-0.1187\xi_m+0.01328\xi_m^{\ 2}},$$

$$T_i^{(129)} = 2\xi_m T_{m1}\left(1.3414 - 0.1487\xi_m + 0.07685\xi_m^{\ 2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.4287+0.1981\xi_m-0.01978\xi_m^{\ 2}}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="10">Nearly minimum IAE, ISE, ITAE – Hwang (1995).
Model: Method 10</td><td colspan="2">[12] $K_c^{(130)}$</td><td colspan="2">$T_i^{(130)}$</td><td colspan="2"></td></tr>
<tr><td colspan="7">For $\xi_m \le 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$:</td></tr>
<tr><td colspan="7">Coefficient values</td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td><td>x_5</td><td>x_6</td><td>x_7</td></tr>
<tr><td>0.822</td><td>-0.549</td><td>0.122</td><td>0.0142</td><td>6.96</td><td>-1.77</td><td>0.786</td></tr>
<tr><td>x_8</td><td>x_9</td><td>x_{10}</td><td>x_{11}</td><td>x_{12}</td><td>x_{13}</td><td>x_{14}</td></tr>
<tr><td>-0.441</td><td>0.0569</td><td>0.0172</td><td>4.62</td><td>-0.823</td><td>1.28</td><td>0.542</td></tr>
<tr><td>x_{15}</td><td>x_{16}</td><td>x_{17}</td><td>x_{18}</td><td>x_{19}</td><td>x_{20}</td><td>x_{21}</td></tr>
<tr><td>-0.986</td><td>0.558</td><td>0.0476</td><td>0.996</td><td>2.13</td><td>-1.13</td><td>1.14</td></tr>
<tr><td>x_{22}
-0.466</td><td>x_{23}
0.0647</td><td>x_{24}
0.0609</td><td>x_{25}
1.97</td><td>x_{26}
-0.323</td><td></td><td></td></tr>
</table>

[12] Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$.

$$K_c^{(130)} = \left(1 - \frac{x_1\left[1 + x_2\omega_H\tau_m + x_3(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H ,$$

$$T_i^{(130)} = \frac{K_c^{(130)}(1 + K_H K_m)}{x_4\omega_H K_m\left(1 + x_5\omega_H\tau_m + x_6\omega_H^2\tau_m^2\right)},\ \varepsilon < 2.4 ;$$

$$K_c^{(130)} = \left(1 - \frac{x_7\left[1 + x_8\omega_H\tau_m + x_9(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H ,$$

$$T_i^{(130)} = \frac{K_c^{(130)}(1 + K_H K_m)}{x_{10}\omega_H K_m\left(1 + x_{11}\omega_H\tau_m + x_{12}\omega_H^2\tau_m^2\right)},\ 2.4 \le \varepsilon < 3 ;$$

$$K_c^{(130)} = \left(1 - \frac{x_{13}x_{14}^{\omega_H\tau_m}\left[1 + x_{15}/s + x_{16}/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H ,$$

$$T_i^{(130)} = \frac{K_c^{(130)}(1 + K_H K_m)}{x_{17}\omega_H K_m\left(1 + x_{18}\ln[\omega_H\tau_m]\right)\left(1 + x_{19}/s + x_{20}/s^2\right)},\ 3 \le \varepsilon < 20 ;$$

$$K_c^{(130)} = \left(1 - \frac{x_{21}\left[1 + x_{22}\omega_H\tau_m + x_{23}(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H ,$$

$$T_i^{(130)} = \frac{K_c^{(130)}(1 + K_H K_m)}{x_{24}\omega_H K_m\left(-1 + x_{25}\omega_H\tau_m + x_{26}\omega_H^2\tau_m^2\right)},\ \varepsilon > 20 .$$

Rule	K_c	T_i	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) - continued	For $\xi_m > 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$:		

Coefficient values

x_1	x_2	x_3	x_4	x_5	x_6	x_7
0.794	-0.541	0.126	0.0078	8.38	-1.97	0.738
x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
-0.415	0.0575	0.0124	4.05	-0.63	1.15	0.564
x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}
-0.959	0.773	0.0335	0.947	1.9	-1.07	1.07
x_{22}	x_{23}	x_{24}	x_{25}	x_{26}		
-0.466	0.0667	0.0328	2.21	-0.338		

For $\xi_m > 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$ and

$\xi_m \le 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$:

Coefficient values

x_1	x_2	x_3	x_4	x_5	x_6	x_7
0.789	-0.527	0.11	0.009	9.7	-2.4	0.76
x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
-0.426	0.0551	0.0153	4.37	-0.743	1.22	0.55
x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}
-0.978	0.659	0.0421	0.969	2.02	-1.11	1.11
x_{22}	x_{23}	x_{24}	x_{25}	x_{26}		
-0.467	0.0657	0.0477	2.07	-0.333		

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th>Comment</th></tr>
<tr><td></td><td colspan="5">Direct synthesis</td></tr>
<tr><td rowspan="12">Bryant et al. (1973).
Model: Method 1</td><td colspan="2">0</td><td colspan="2">13 $x_2 K_m T_{m1}$</td><td>$T_{m1} > T_{m2}$</td></tr>
<tr><td colspan="2">$\frac{x_1}{K_m \tau_m}$</td><td colspan="2">T_{m1}</td><td>$T_{m1} > T_{m2}$</td></tr>
<tr><td colspan="5">Coefficient values – deduced from graph</td></tr>
<tr><td>τ_m/T_{m1}</td><td>x_1</td><td>τ_m/T_{m1}</td><td>x_1</td><td></td></tr>
<tr><td>0.33</td><td>0.08</td><td>1.0</td><td>0.15</td><td rowspan="4">Critically damped dominant pole</td></tr>
<tr><td>0.40</td><td>0.09</td><td>2.0</td><td>0.24</td></tr>
<tr><td>0.50</td><td>0.09</td><td>10.0</td><td>0.33</td></tr>
<tr><td>0.67</td><td>0.11</td><td></td><td></td></tr>
<tr><td>0.33</td><td>0.14</td><td>1.0</td><td>0.31</td><td rowspan="4">Damping ratio (dominant pole) = 0.6</td></tr>
<tr><td>0.40</td><td>0.16</td><td>2.0</td><td>0.47</td></tr>
<tr><td>0.50</td><td>0.18</td><td>10.0</td><td>0.64</td></tr>
<tr><td>0.67</td><td>0.23</td><td></td><td></td></tr>
</table>

[13] Representative x_2 values, summarized as follows, are deduced from graphs:

x_2 value	τ_m/T_{m1}						
ξ_m	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.4	-	-	5.81	7.46	9.17	10.9	12.7
0.6	-	5.13	6.94	8.40	10.0	11.8	14.1
0.8	-	5.99	7.30	9.17	11.9	15.4	20.0
0.9	-	5.81	7.30	10.0	14.1	19.6	28.6
1.0	7.94	9.01	11.5	13.9	16.4	18.9	-
1.1	8.93	9.90	12.3	14.5	16.9	19.6	-
1.2	9.80	10.8	13.2	15.4	17.9	20.4	-
1.5	12.2	13.3	15.4	18.2	20.0	22.7	-
1.9	15.4	16.7	18.9	21.3	23.8	25.6	-

Rule	K_c	T_i	Comment
Bryant *et al.* (1973) - continued	[14] x_1/K_m	$x_2 K_m T_{m1}$	$T_{m1} > T_{m2}$
Hougen (1979) – *pages 345-346. Model: Method 1*	[15] $K_c^{(131)}$	$T_i^{(131)}$	Maximise crossover frequency

---

[14] Representative x_1 values, summarised as follows, are deduced from graphs:

x_1 value	τ_m/T_{m1}					
ξ_m	1.0	2.0	3.0	4.0	5.0	6.0
0.4	-	0.006	0.076	0.103	0.115	0.122
0.6	-	0.039	0.093	0.114	0.122	0.120
0.8	0.055	0.110	0.127	0.114	0.093	0.075
0.9	0.139	0.147	0.122	0.093	0.070	0.052

Representative x_2 values, summarised as follows, are deduced from graphs:

x_2 value	τ_m/T_{m1}					
ξ_m	1.0	2.0	3.0	4.0	5.0	6.0
0.4	-	5.81	7.46	9.17	10.9	12.7
0.6	5.13	6.94	8.40	10.0	11.8	14.1
0.8	5.99	7.30	9.17	11.9	15.4	20.0
0.9	5.81	7.30	10.0	14.1	19.6	28.6

[15] $$K_c^{(131)} = \frac{x_1}{K_m}\left[1 + \frac{1}{T_{m1}T_{m2}}\left(\frac{0.7T_{m2}(T_{m1}+T_{m2}) + T_{m1}^2}{T_{m1}+0.7T_{m2}}\right)^2\right],$$

$$T_i^{(131)} = (T_{m1} + 0.7T_{m2})\left(1 + 0.1\frac{\tau_m}{T_{m1}}\right);$$

x_1 values are obtained as follows:

x_1 value	τ_m/T_{m1}				
T_{m2}/T_{m1}	0.1	0.2	0.3	0.4	0.5
0.1	0.31	0.19	0.15	0.11	0.09
0.3	0.6	0.42	0.35	0.28	0.23
0.5	0.65	0.5	0.45	0.46	0.32
1.0	0.7	0.6	0.5	0.45	0.38
	τ_m/T_{m1}				
T_{m2}/T_{m1}	0.6	0.7	0.8	0.9	1.0
0.1	0.085	0.075	0.07	0.065	0.06
0.3	0.2	0.18	0.16	0.155	0.15
0.5	0.29	0.26	0.24	0.23	0.22
1.0	0.34	0.32	0.31	0.29	0.28

Rule	K_c	T_i	Comment
Hougen (1988). *Model: Method 1*	[16] $K_c^{(132)}$	$T_i^{(132)}$	Five criteria are fulfilled
Somani *et al.* (1992). [17]	$x_1/K_m A_m$	$x_2 \tau_m$	
	Coefficient values – see page 117		

[16] Equations for $K_c^{(132)}$ deduced from graph;

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.73\log_{10}\left(\frac{T_{m2}}{\tau_m}\right)+0.65\right]}, \ \frac{T_{m2}}{T_{m1}} = 0.1 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.61\log_{10}\left(\frac{T_{m2}}{\tau_m}\right)+0.29\right]}, \ \frac{T_{m2}}{T_{m1}} = 0.2 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.48\log_{10}\left(\frac{T_{m2}}{\tau_m}\right)+0.05\right]}, \ \frac{T_{m2}}{T_{m1}} = 0.5 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.41\log_{10}\left(\frac{T_{m2}}{\tau_m}\right)-0.06\right]}, \ \frac{T_{m2}}{T_{m1}} = 1 . \ T_i^{(132)} = T_{m1} + 0.7T_{m2} + 0.12\tau_m .$$

[17] For $\frac{25T_{m1}T_{m2}}{T_i} << 1$,

$$K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m}\right)^2 \left[2.944 - \tan^{-1}\left(\frac{T_{m1}}{\tau_m} + \frac{T_{m2}}{\tau_m}\right)\right]^2} \sqrt{1 + \left(\frac{T_{m2}}{\tau_m}\right)^2 \left[2.944 - \tan^{-1}\left(\frac{T_{m1}}{\tau_m} + \frac{T_{m2}}{\tau_m}\right)\right]^2} \text{ or}$$

$$K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m}\right)^2 \left[2.944 - \frac{T_{m1}}{\tau_m} - \frac{T_{m2}}{\tau_m}\right]^2} \sqrt{1 + \left(\frac{T_{m2}}{\tau_m}\right)^2 \left[2.944 - \frac{T_{m1}}{\tau_m} - \frac{T_{m2}}{\tau_m}\right]^2} ;$$

For $\frac{25T_{m1}T_{m2}}{T_i} >> 1$, $K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m}\right)^2 x_1^2} \sqrt{1 + \left(\frac{T_{m2}}{\tau_m}\right)^2 x_1^2}$, with

$$x_1 = 0.981 + \left[0.5\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right) + 0.945 + \sqrt{0.25\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right)^2 + 0.945\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right)}\right]^{0.333}$$

$$+ \left[0.5\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right) + 0.945 - \sqrt{0.25\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right)^2 + 0.945\left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}}\right)}\right]^{0.333} .$$

Rule	K_c			T_i			Comment	
	Coefficient values							
Somani *et al.* (1992) – continued. *Model: Method 1*; Suggested A_m: $1.75 \le A_m \le 3.0$	$\frac{\tau_m}{T_{m2}}$	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	$\frac{\tau_m}{T_{m2}}$	$\frac{\tau_m}{T_{m1}}$	x_1	x_2
	0.1	0.147	11.707	12.5	1.00	0.082	12.389	6.25
	0.1	0.440	10.091	8.33	1.00	0.437	3.462	5.00
	0.1	0.954	10.315	6.25	1.00	1.016	2.370	4.17
	0.1	1.953	11.070	5.00	1.00	2.075	2.087	3.57
	0.1	4.587	12.207	4.17	1.67	0.169	6.871	5.00
	0.1	13.89	13.337	3.70	1.67	0.581	2.773	4.17
	0.125	0.121	11.318	12.5	1.67	1.242	1.930	3.57
	0.125	0.404	8.613	8.33	1.67	2.445	1.624	3.13
	0.125	0.897	8.510	6.25	1.67	4.651	1.957	3.13
	0.125	1.842	8.997	5.00	2.5	0.156	7.607	4.55
	0.125	4.219	9.840	4.17	2.5	0.338	4.015	4.17
	0.125	11.628	10.707	3.70	2.5	0.559	2.797	3.85
	0.25	0.064	14.494	11.11	2.5	1.182	1.848	3.33
	0.25	0.256	6.504	8.33	5.0	0.075	16.187	4.17
	0.25	0.667	5.136	6.25	5.0	0.238	5.623	3.85
	0.25	1.406	4.961	5.00	5.0	0.433	3.448	3.57
	0.25	3.021	5.174	4.17	5.0	0.667	2.52	3.33
	0.25	6.452	5.502	3.70	5.0	0.951	2.014	3.13
	0.5	0.062	14.918	8.33	10.0	0.073	17.650	3.85
	0.5	0.376	4.353	6.25	10.0	0.235	5.996	3.57
	0.5	0.909	3.274	5.00	10.0	0.424	4.524	3.33
	0.5	1.880	3.025	4.17	10.0	0.649	2.641	3.13
	0.5	3.401	3.038	3.70	10.0	0.917	2.093	2.94
Tan *et al.* (1996). *Model: Method 2*	[18] $K_c^{(133)}$			$T_i^{(133)}$			$T_{m1} = T_{m2}$	
Poulin *et al.* (1996). *Model: Method 1*	$\frac{T_{m1}}{K_m(T_{m1}+\tau_m)}$			T_{m1}			$T_{m1} > 5T_{m2}$; Minimum $\phi_m = 58^0$	

[18] $$K_c^{(133)} = \frac{\beta T_i^{(133)}\omega_\phi\sqrt{1+(\beta T_{m1}\omega_\phi)^2}}{A_m\sqrt{1+(\beta T_i^{(133)}\omega_\phi)^2}}, \beta = 0.8, \frac{\tau_m}{T_{m1}} < 0.5;\ \beta = 0.5, \frac{\tau_m}{T_{m1}} > 0.5;\ \omega_\phi < \omega_u;$$

$$T_i^{(133)} = \frac{1}{\beta\omega_\phi \tan\left[-2\tan^{-1}\beta T_m\omega_\phi - \beta\tau_m\omega_\phi - \phi_m\right]}.$$

Rule	K_c	T_i	Comment
Schaedel (1997). *Model: Method 1*	[19] $K_c^{(134)}$	$T_i^{(134)}$	"Normal" design
	[20] $K_c^{(135)}$	$T_i^{(135)}$	"Sharp" design
Pomerleau and Poulin (2004). *Model: Method 5*	$\frac{T_{m1}}{K_m(T_{m1}+\tau_m)}$	$1.5T_{m1}$	$T_{m1} = T_{m2}$; OS<10%
Leva *et al.* (2003). *Model: Method 1*	$\frac{T_{m1}}{2K_m(T_{m2}+\tau_m)}$	$4(T_{m2}+\tau_m)$	$T_{m1} >> T_{m2} + \tau_m$; Symmetrical optimum principle
	Robust		
Brambilla *et al.* (1990). *Model: Method 1* *λ values obtained from graph*	$\frac{T_{m1}+T_{m2}+0.5\tau_m}{K_m\tau_m(2\lambda+1)}$	$T_{m1}+T_{m2}+0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_{m1}+T_{m2}} \le 10$

Coefficient values

$\frac{\tau_m}{T_{m1}+T_{m2}}$	λ	$\frac{\tau_m}{T_{m1}+T_{m2}}$	λ
0.1	3.0	2.0	0.4
0.2	1.8	5.0	0.2
0.5	1.0	10.0	0.2
1.0	0.6		

[19] $K_c^{(134)} = \frac{0.5T_i^{(134)}}{K_m\left(2\xi_m T_{m1} + \tau_m - T_i^{(134)}\right)}$,

$$T_i^{(134)} = \sqrt{(2\xi_m T_{m1} + \tau_m)^2 - 2\left(T_{m1}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m^2\right)}.$$

[20] $K_c^{(135)} = \frac{0.375}{K_m}\frac{4\xi_m^2 T_{m1}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m^2 - T_{m1}^2}{T_{m1}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m^2}$,

$$T_i^{(135)} = \frac{4\xi_m^2 T_{m1}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m^2 - T_{m1}^2}{2\xi_m T_{m1} + \tau_m}.$$

Rule	K_c	T_i	Comment
Kristiansson (2003). *Model: Method 1*	[21] $K_c^{(136)}$	$2\xi_m T_{m1}$	
	[22] $K_c^{(137)}$		Suggested $M_{max} = 1.7$, $\beta = 10$
Panda *et al.* (2004). *Model: Method 1*	[23] $K_c^{(138)}$	$T_i^{(138)}$	[24]
	[25] $K_c^{(139)}$	$T_i^{(139)}$	

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[21] $K_c^{(136)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left[\alpha - \sqrt{\alpha^2 - 1 + \frac{1}{M_{max}^2}}\right]$, $\alpha = 1 + \frac{T_{m2}}{\tau_m}\left[1 - \sqrt{1 - \frac{1}{M_{max}^2}}\right]$.

[22] $K_c^{(137)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left[1 + 0.1913\frac{T_{m2}}{\tau_m} - \sqrt{0.346 + 0.3826\frac{T_{m2}}{\tau_m} + 0.0366\frac{T_{m2}^2}{\tau_m^2}}\right]$.

[23] $K_c^{(138)} = \frac{8\xi_m^2 T_{m1} + 2\xi_m \tau_m + T_{m1}}{2\lambda K_m}$, $T_i^{(138)} = \frac{4\xi_m^2 T_{m1} + \xi_m \tau_m + 0.5 T_{m1}}{2\xi_m}$.

[24] $\lambda \ge \text{maximum}\left[0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9T_{m2}/T_{m1})}, 1.7\tau_m\right]$.

[25] $K_c^{(139)} = \frac{0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}}}{\lambda K_m} + \frac{0.5\tau_m}{\lambda K_m}$,

$T_i^{(139)} = 0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + 0.5\tau_m$.

3.7.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

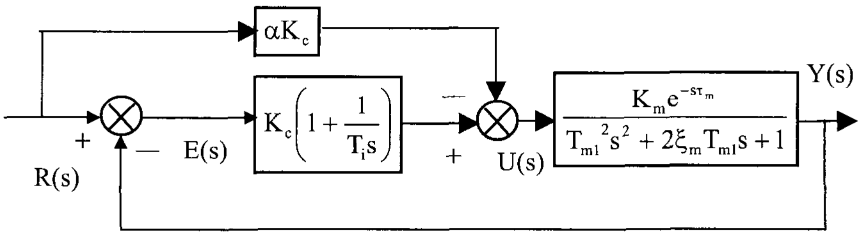

Table 25: PI controller tuning rules – SOSPD model $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	Comment
	Minimum performance index: servo/regulator tuning		
Taguchi and Araki (2000). *Model: Method 1*	[1] $K_c^{(140)}$	$T_i^{(140)}$	$\xi_m = 1$; $\tau_m/T_m \le 1.0$; Overshoot (servo step) $\le 20\%$

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[1] $K_c^{(140)} = \dfrac{1}{K_m}\left(0.3717 + \dfrac{0.5316}{\dfrac{\tau_m}{T_m} + 0.0003414}\right)$,

$$T_i^{(140)} = T_m\left(2.069 - 0.3692\frac{\tau_m}{T_m} + 1.081\left[\frac{\tau_m}{T_m}\right]^2 - 0.5524\left[\frac{\tau_m}{T_m}\right]^3\right),$$

$$\alpha = 0.6438 - 0.5056\frac{\tau_m}{T_m} + 0.3087\left(\frac{\tau_m}{T_m}\right)^2 - 0.1201\left(\frac{\tau_m}{T_m}\right)^3.$$

Rule	K_c		T_i		Comment			
Taguchi and Araki (2000) - continued	[2]$K_c^{(141)}$		$T_i^{(141)}$		$\xi_m = 0.5$			
Minimum ITAE - Pecharromán (2000), Pecharromán and Pagola (2000). *Model: Method 11* $K_m = 1$; $T_{m1} = 1$; $\xi_m = 1$	$x_1 K_u$		$x_2 T_u$					
	Coefficient values							
	x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
	0.147	1.150	0.411	-146^0	0.280	0.512	0.281	-73^0
	0.170	1.013	0.401	-140^0	0.291	0.503	0.270	-63^0
	0.1713	1.0059	0.4002	-139.7^0	0.297	0.483	0.260	-52^0
	0.195	0.880	0.386	-133^0	0.303	0.462	0.246	-41^0
	0.210	0.720	0.342	-125^0	0.307	0.431	0.229	-30^0
	0.234	0.672	0.345	-115^0	0.317	0.386	0.171	-19^0
	0.249	0.610	0.323	-105^0	0.324	0.302	0.004	-10^0
	0.262	0.568	0.308	-94^0	0.320	0.223	-0.204	-6^0
	0.274	0.545	0.291	-84^0				

---

[2] $K_c^{(141)} = \dfrac{1}{K_m}\left(0.1000 + \dfrac{0.05627}{[\frac{\tau_m}{T_m} + 0.06041]^2}\right)$,

$$T_i^{(141)} = T_m\left(4.340 - 16.39\frac{\tau_m}{T_m} + 30.04\left[\frac{\tau_m}{T_m}\right]^2 - 25.85\left[\frac{\tau_m}{T_m}\right]^3 + 8.567\left[\frac{\tau_m}{T_m}\right]^4\right),$$

$$\alpha = 0.6178 - 0.4439\frac{\tau_m}{T_m} - 7.575\left(\frac{\tau_m}{T_m}\right)^2 + 9.317\left(\frac{\tau_m}{T_m}\right)^3 - 3.182\left(\frac{\tau_m}{T_m}\right)^4.$$

3.8 SOSIPD Model – Repeated Pole $\dfrac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

3.8.1 *Controller with set-point weighting*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

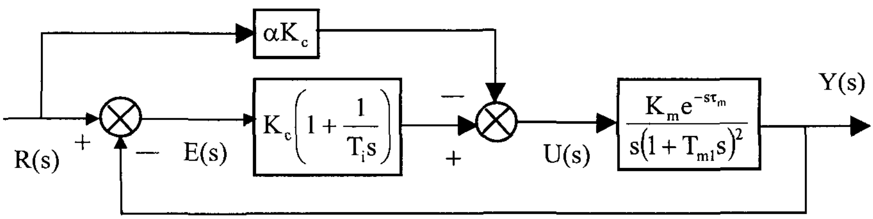

Table 26: PI controller tuning rules – SOSIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

Rule	K_c		T_i		Comment			
	Minimum performance index: servo/regulator tuning							
Taguchi and Araki (2000). *Model: Method 1*	[4] $K_c^{(142)}$		$8.549+4.029\dfrac{\tau_m}{T_{m1}}$		$\tau_m/T_{m1} \le 1.0$; Overshoot (servo step) $\le 20\%$			
Minimum ITAE - Pecharromán (2000). *Model: Method 2*	$x_1 K_u$		$x_2 T_u$					
	Coefficient values							
	x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
$K_m = 1$; $T_{m1} = 1$	0.049	2.826	0.506	-164^0	0.218	1.279	0.564	-135^0
$0.1 < \tau_m < 10$	0.066	2.402	0.512	-160^0	0.250	1.216	0.573	-130^0
	0.099	1.962	0.522	-155^0	0.286	1.127	0.578	-125^0
	0.129	1.716	0.532	-150^0	0.330	1.114	0.579	-120^0
	0.159	1.506	0.544	-145^0	0.351	1.093	0.577	-118^0
	0.189	1.392	0.555	-140^0				

[4] $K_c^{(142)} = \dfrac{1}{K_m}\left(0.07368 + \dfrac{0.3840}{\dfrac{\tau_m}{T_{m1}} + 0.7640}\right)$, $\alpha = 0.6691 + 0.006606\dfrac{\tau_m}{T_{m1}}$.

3.9 SOSPD Model with a Positive Zero $G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

3.9.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

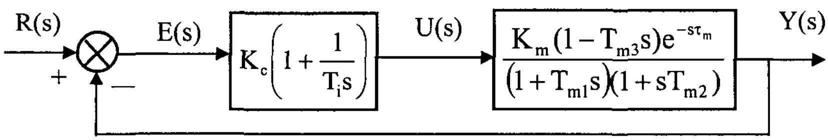

Table 27: PI controller tuning rules – SOSPD model with a positive zero $G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Pomerleau and Poulin (2004). *Model: Method 3*	$\frac{1}{K_m}\frac{T_{m1}}{T_{m1} + T_{m3} + \tau_m}$	$1.5T_{m1}$	OS<10%; $T_{m1} = T_{m2}$

Rule	K_c		T_i		Comment
	Direct synthesis: frequency domain criteria				
Khodabakhshian and Golbon (2004). *Model: Method 1*	[5] $K_c^{(143)}$		$T_i^{(143)}$		Recommended $M_{max} = 0\text{dB}$; $T_{m1} > T_{m2}$
	Ultimate cycle				
Luyben (2000). *Model: Method 1*	$\dfrac{K_u}{x_1 K_m}$		[6] $\dfrac{T_u}{T^1}$		$T_{m1} = T_{m2}$
T_{m3}/T_{m1} values	0.2	0.4	0.8	1.6	
x_1 values	3.9	3.3	2.8	3.0	$\tau_m/T_{m3} = 0.2$
	3.8	3.4	3.0	2.9	$\tau_m/T_{m3} = 0.4$
	4.1	3.6	3.2	3.1	$\tau_m/T_{m3} = 0.6$
	4.3	3.8	3.4	3.2	$\tau_m/T_{m3} = 0.8$
	4.8	4.3	3.7	3.3	$\tau_m/T_{m3} = 1.0$
	5.2	4.6	3.9	3.4	$\tau_m/T_{m3} = 1.2$
	5.5	5.0	4.4	3.7	$\tau_m/T_{m3} = 1.4$
	6.0	5.4	4.7	3.8	$\tau_m/T_{m3} = 1.6$

[5] $K_c^{(143)} = \dfrac{T_i^{(143)}}{K_m}\sqrt{\dfrac{(T_{m1}T_{m2})^2\omega^6 + (T_{m1}^2 + T_{m2}^2)\omega^4 + \omega^2}{(T_{m1}T_{m3})^2\omega^4 + (T_{m1}^2 + T_{m3}^2)\omega^2 + 1}}$, with ω given by solving

$$\cos^{-1}\left(\frac{1 - 10^{-0.1M_{max}}}{2}\right) = 0.5\pi + \tan^{-1}\left(T_i^{(143)}\omega\right) - \tan^{-1}(T_{m3}\omega) - \tan^{-1}(T_{m2}\omega) - \tan^{-1}(T_{m1}\omega) - \tau_m\omega\,;$$

$$T_i^{(143)} = \frac{1}{T_{m1}}\left(1 + 0.175\frac{\tau_m}{T_{m1}} + 0.3\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 0.2\frac{T_{m2}}{T_{m1}}\right),\ \frac{\tau_m}{T_{m1}} < 2\,;$$

$$T_i^{(143)} = \frac{1}{T_{m1}}\left(0.65 + 0.35\frac{\tau_m}{T_{m1}} + 0.3\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 0.2\frac{T_{m2}}{T_{m1}}\right),\ \frac{\tau_m}{T_{m1}} > 2\,.$$

[6] $T^1 = \left(0.5 + 1.56\dfrac{T_{m3}}{T_{m1}}\right) + \left(3.44 - 1.56\dfrac{T_{m3}}{T_{m1}}\right)\dfrac{\tau_m}{T_{m3}}$.

3.10 SOSPD Model (repeated pole) with a Negative Zero

$$G_m(s) = \frac{K_m(1+T_{m3}s)e^{-s\tau_m}}{(1+sT_{m1})^2}$$

3.10.1 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1}{1+T_f s}$$

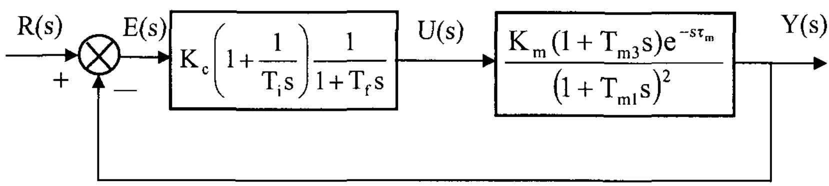

Table 28: PI controller tuning rules – SOSPD model with a negative zero $G_m(s) = \frac{K_m(1+T_{m3}s)e^{-s\tau_m}}{(1+sT_{m1})^2}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Pomerleau and Poulin (2004). *Model: Method 3*	$\frac{1}{K_m}\frac{T_{m1}}{T_{m1}+\tau_m}$	$1.5T_{m1}$	$T_f = T_{m3}$; OS<10%

3.11 Third Order System plus Time Delay Model

$$G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^3 + a_3 s^3} e^{-s\tau_m} \text{ or}$$

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

3.11.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

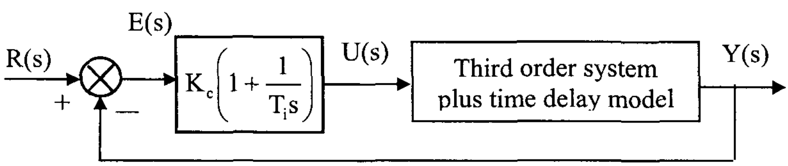

Table 29: PI controller tuning rules – Third order system plus time delay model $G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^3 + a_3 s^3} e^{-s\tau_m}$ or $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$.

Rule	K_c	T_i	Comment
	Direct synthesis		
Hougen (1979) - *pages 350-351.* *Model: Method 1*	$\frac{0.7}{K_m}\left(\frac{T_{m1}}{\tau_m}\right)^{0.333}$	$T_i^{(144)}$	$\frac{\tau_m}{T_{m1}} > 0.04$; $T_{m1} \ge T_{m2} \ge T_{m3}$
	[1] $K_c^{(144)}$		$\frac{\tau_m}{T_{m1}} \le 0.04$; $T_{m1} \ge T_{m2} \ge T_{m3}$

---

[1] $K_c^{(144)} = \frac{1}{2K_m}\left[0.7\left(\frac{T_{m1}}{\tau_m}\right)^{0.333} + 0.8\frac{T_{m1} + T_{m2} + T_{m3}}{(T_{m1}T_{m2}T_{m3})^{0.333}}\right]$,

$T_i^{(144)} = 1.5\tau_m^{0.08}\sqrt{T_{m1}(T_{m2} + T_{m3})}$.

Rule	K_c	T_i	Comment
Vrančić *et al.* (1996), Vrančić (1996) – *page 121. Model: Method 1*	$\frac{0.5A_3}{(A_1A_2 - K_mA_3)}$ [2]	$\frac{A_3}{A_2}$	$A_m \ge 2, \phi_m \ge 60^0$ (Vrančić *et al.* (1996))
Vrančić *et al.* (2004a). *Model: Method 1*	$\frac{0.5A_3}{(A_1A_2 - K_mA_3)}$	[3] $T_i^{(145)}$	
	0	$\frac{A_1A_3}{A_1A_2 - K_mA_3}$	
Vrančić *et al.* (2004b).	[4] $K_c^{(146)}$	$T_i^{(146)}$	*Model: Method 1*

[2] $A_1 = K_m(a_1 - b_1 + \tau_m)$, $A_2 = K_m(b_2 - a_2 + A_1a_1 - b_1\tau_m + 0.5\tau_m^2)$,

$$A_3 = K_m(a_3 - b_3 + A_2a_1 - A_1a_2 + b_2\tau_m - 0.5b_1\tau_m^2 + 0.167\tau_m^3).$$

[3] $$T_i^{(145)} = \frac{A_1A_3(A_1A_2 - K_mA_3))}{(A_1A_2 - K_mA_3)^2 + K_mA_3(A_1A_2 - K_mA_3) + 0.25K_m^2A_3^2}.$$

[4] $$K_c^{(146)} = \frac{A_1A_2 - A_3K_m - [\mathrm{sign}(A_1A_2 - A_3K_m)]A_1\sqrt{A_2^2 - A_1A_3}}{A_3K_m^2 - 2A_1A_2K_m + A_1^3},$$

$$T_i^{(146)} = 2A_1\frac{A_1A_2 - A_3K_m - [\mathrm{sign}(A_1A_2 - A_3K_m)]A_1\sqrt{A_2^2 - A_1A_3}}{(A_3K_m^2 - 2A_1A_2K_m + A_1^3)(1 + K_c^{(146)}K_m)^2}.$$

3.11.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

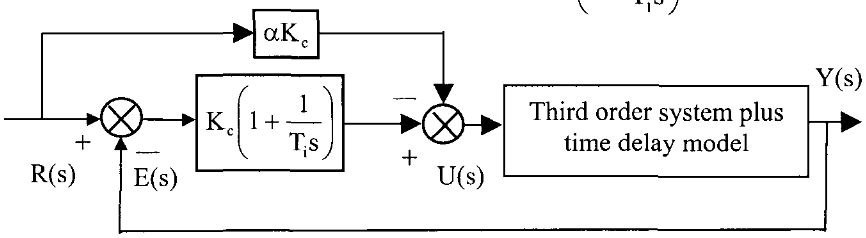

Table 30: PI controller tuning rules – Third order system plus time delay model $G_m(s) = K_m \dfrac{1+b_1 s+b_2 s^2+b_3 s^3}{1+a_1 s+a_2 s^3+a_3 s^3} e^{-s\tau_m}$ or $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$.

Rule	K_c	T_i	Comment
	Minimum performance index: servo/regulator tuning		
Taguchi and Araki (2000). *Model: Method 1*	[5] $K_c^{(147)}$	$T_i^{(147)}$	$\frac{\tau_m}{T_m} \le 1.0$
	Overshoot (servo step) $\le 20\%$; $T_{m1} = T_{m2} = T_{m3}$		

---

[5] $K_c^{(147)} = \dfrac{1}{K_m}\left(0.2713 + \dfrac{0.7399}{\dfrac{\tau_m}{T_m} + 0.5009}\right)$, $\alpha = 0.4908 - 0.2648\dfrac{\tau_m}{T_m} + 0.05159\left(\dfrac{\tau_m}{T_m}\right)^2$,

$$T_i^{(147)} = T_m\left(2.759 - 0.003899\frac{\tau_m}{T_m} + 0.1354\left[\frac{\tau_m}{T_m}\right]^2\right).$$

Rule	K_c	T_i	Comment
	Direct synthesis		
Vrančić (1996) – *page 136-137. Model: Method 1*	[6] $K_c^{(148)}$	$T_i^{(148)}$	$b = [0.5, 0.8]$ - good servo and regulator response (*page 139*); $b = 1-\alpha$

[6] $$K_c^{(148)} = \frac{A_1A_2 - K_mA_3 - \chi}{\left(1-b^2\right)\left(K_m^2A_3 + A_1^3 - 2K_mA_1A_2\right)},\ K_mA_3 - A_1A_2 < 0;$$

$$\chi = \sqrt{\left(K_mA_3 - A_1A_2\right)^2 - \left(1-b^2\right)A_3\left(K_m^2A_3 + A_1^3 - 2K_mA_1A_2\right)};$$

$$K_c^{(148)} = \frac{A_1A_2 - K_mA_3 + \chi}{\left(1-b^2\right)\left(K_m^2A_3 + A_1^3 - 2K_mA_1A_2\right)},\ K_mA_3 - A_1A_2 > 0.$$

$$T_i^{(148)} = \frac{A_1}{K_m + \dfrac{1}{2K_c^{(148)}} + \dfrac{K_c^{(148)}K_m^2}{2}\left(1-b^2\right)} \text{ with}$$

$$A_1 = K_m\left(a_1 - b_1 + \tau_m\right),\ A_2 = K_m\left(b_2 - a_2 + A_1a_1 - b_1\tau_m + 0.5\tau_m^2\right),$$

$$A_3 = K_m\left(a_3 - b_3 + A_2a_1 - A_1a_2 + b_2\tau_m - 0.5b_1\tau_m^2 + 0.167\tau_m^3\right).$$

3.12 Unstable FOLPD Model $G_m(s)=\dfrac{K_m e^{-s\tau_m}}{T_m s-1}$

3.12.1 *Ideal controller* $G_c(s)=K_c\left(1+\dfrac{1}{T_i s}\right)$

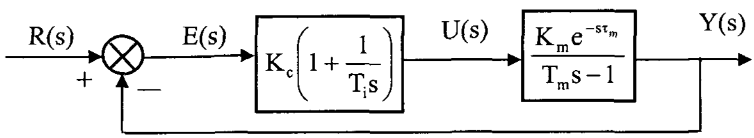

Table 31: PI controller tuning rules – unstable FOLPD model $G_m(s)=\dfrac{K_m e^{-s\tau_m}}{T_m s-1}$

Rule	K_c	T_i	Comment
	Minimum performance index: servo tuning		
Minimum ITSE - Majhi and Atherton (2000). *Model: Method 1*	[1] $K_c^{(149)}$	$T_i^{(149)}$	$0<\frac{\tau_m}{T_m}<0.693$
Minimum ITSE - Majhi and Atherton (2000). *Model: Method 3*	[2] $K_c^{(150)}$	$T_i^{(150)}$	$0<\frac{\tau_m}{T_m}<0.693$; $K=-A_p/K_m h$

[1] $K_c^{(149)}=\dfrac{1}{K_m}\left(0.889+\dfrac{e^{-\tau_m/T_m}-0.064}{e^{\tau_m/T_m}-0.990}\right)$, $T_i^{(149)}=\dfrac{2.6316T_m\left(e^{\tau_m/T_m}-0.966\right)}{\left(e^{-\tau_m/T_m}-0.377\right)}$.

[2] $K_c^{(150)}=\dfrac{1}{K_m}\left(0.889+\dfrac{1}{K(1+K)}\right)$, $T_i^{(150)}=\dfrac{T_m K(1+K)}{0.5(1-0.6382K)\tanh^{-1}K}$.

Rule	K_c	T_i	Comment
	Direct synthesis		
De Paor and O'Malley (1989). *Model: Method 1*	[3] $K_c^{(151)}$	$T_i^{(151)}$	$A_m = 2$; $\frac{\tau_m}{T_m} < 1$
Venkatashankar and Chidambaram (1994). *Model: Method 1*	[4] $K_c^{(152)}$	$25(T_m - \tau_m)$	$\frac{\tau_m}{T_m} < 0.67$
Chidambaram (1995b). *Model: Method 1*	$\frac{1}{K_m}\left(1 + 0.26\frac{T_m}{\tau_m}\right)$	$25T_m - 27\tau_m$	$\frac{\tau_m}{T_m} < 0.6$
Chidambaram (1997).	$\frac{1.678}{K_m}\ln\left(\frac{T_m}{\tau_m}\right)$	$0.4015T_m e^{5.8\tau_m/T_m}$	*Model: Method 1*
Valentine and Chidambaram (1998). *Model: Method 1*	$\frac{2.695}{K_m}e^{-1.277\tau_m/T_m}$	$0.4015T_m e^{5.8\tau_m/T_m}$	[5] $\xi = 0.333$; $\frac{\tau_m}{T_m} < 0.7$

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[3] $$K_c^{(151)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)}\sin\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right],$$

$$T_i^{(151)} = \frac{T_m}{\left[\sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)}\right]\tan(0.5\phi)},\ \phi = \tan^{-1}\sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)} - \sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}.$$

[4] $$K_c^{(152)} = \frac{1}{K_m}\sqrt{\left(0.98\sqrt{1 + \frac{0.04T_m^2}{(T_m - \tau_m)^2}}\right)\left(\frac{25}{\tau_m}\right)\beta(T_m - \tau_m)\sqrt{\frac{1 + \frac{\beta^2 T_m^2}{\tau_m^2}}{1 + \beta^2\frac{625}{\tau_m^{\ 2}}(T_m - \tau_m)^2}}},$$

$$\beta = 1.373,\ \frac{\tau_m}{T_m} < 0.25;\ \beta = 0.953,\ 0.25 \le \frac{\tau_m}{T_m} < 0.67.$$

[5] $\pm 1\%$ $T_s = 7.5T_m$, $\tau_m/T_m = 0.1$; $\pm 1\%$ $T_s = 8.75T_m$, $\tau_m/T_m = 0.2$; $\pm 1\%$ $T_s = 11.25T_m$, $\tau_m/T_m = 0.3$; $\pm 1\%$ $T_s = 15.0T_m$, $\tau_m/T_m = 0.4$; $\pm 1\%$ $T_s = 17.5T_m$, $\tau_m/T_m = 0.5$; $\pm 1\%$ $T_s = 20.0T_m$, $\tau_m/T_m = 0.6$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="11">Ho and Xu (1998).
Model: Method 1</td><td colspan="2">$\frac{\omega_p T_m}{A_m K_m}$</td><td colspan="2">[6] $T_i^{(153)}$</td><td colspan="2">$\frac{\tau_m}{T_m} < 0.62$</td></tr>
<tr><td colspan="6">Representative results</td></tr>
<tr><td colspan="2">$\frac{0.5775T_m}{K_m \tau_m}$</td><td colspan="2">$\frac{\tau_m T_m}{0.3212T_m - \tau_m}$</td><td colspan="2">Good servo response. $A_m = 2.3$, $\phi_m = 25^0$</td></tr>
<tr><td colspan="2">$\frac{0.6918T_m}{K_m \tau_m}$</td><td colspan="2">$\frac{\tau_m T_m}{0.3359T_m - \tau_m}$</td><td colspan="2">Good regulator response. $A_m = 1.9$, $\phi_m = 22.5^0$</td></tr>
<tr><td colspan="2">$x_1 T_m / K_m \tau_m$</td><td colspan="2">$-T_m$</td><td colspan="2"></td></tr>
<tr><td colspan="6">Other representative results: coefficient values</td></tr>
<tr><td>x_1</td><td>A_m</td><td>ϕ_m</td><td>x_1</td><td>A_m</td><td>ϕ_m</td></tr>
<tr><td>1.0472</td><td>1.5</td><td>30^0</td><td>0.5236</td><td>3</td><td>60^0</td></tr>
<tr><td>0.7854</td><td>2</td><td>45^0</td><td>0.3927</td><td>4</td><td>67.5^0</td></tr>
<tr><td rowspan="2">Luyben (1998).
Model: Method 1</td><td colspan="2">$0.31K_u$</td><td colspan="2">$2.2T_u$</td><td colspan="2">$T_{CL} = 3.16\tau_m$</td></tr>
<tr><td colspan="6">Maximum closed loop log modulus = 2 dB</td></tr>
<tr><td>Gaikward and Chidambaram (2000).
Model: Method 1</td><td colspan="2">[7] $K_c^{(154)}$</td><td colspan="2">$T_i^{(154)}$</td><td colspan="2">$0.1 \le \frac{\tau_m}{T_m} \le 0.6$</td></tr>
<tr><td>Chidambaram (2000c).
Model: Method 1</td><td colspan="2">[8] $K_c^{(155)}$</td><td colspan="2">$T_i^{(155)}$</td><td colspan="2">'small' $\frac{\tau_m}{T_m}$</td></tr>
</table>

[6] $T_i^{(153)} = \dfrac{1}{1.57\omega_p - \omega_p^2 \tau_m - \dfrac{1}{T_m}}$.

[7] $K_c^{(154)} = \dfrac{1}{K_m}\left[0.2619 + 0.7266\dfrac{T_m}{\tau_m}\right]$, $T_i^{(154)} = T_m\left[0.193 + 4.19\dfrac{\tau_m}{T_m} + 8.214\left(\dfrac{\tau_m}{T_m}\right)^2\right]$.

[8] $K_c^{(155)} = \dfrac{T_m T_i^{(155)}}{0.04T_S^2\xi^2 + T_i^{(155)}\tau_m}$, $T_i^{(155)} = \dfrac{0.04T_S^2\xi^2 + T_m\tau_m + 0.4T_S\xi^2}{T_m - \tau_m}$.

Rule	K_c	T_i	Comment
[9]Chandrashekar *et al.* (2002). *Model: Method 1*	$\frac{49.535}{K_m} e^{-21.644\tau_m/T_m}$	$0.1523 T_m e^{7.9425\tau_m/T_m}$	$\frac{\tau_m}{T_m} < 0.1$
	$\frac{0.8668}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.8288}$		$0.1 \le \frac{\tau_m}{T_m} \le 0.7$
	Alternative tuning rules		
Coefficients of K_c and T_i recorded in tables; T_{CL2} as in footnote	$6.0000/K_m$	$0.6000T_m$	$\tau_m/T_m = 0.1$
	$3.2222/K_m$	$1.4500T_m$	$\tau_m/T_m = 0.2$
	$2.2963/K_m$	$2.6571T_m$	$\tau_m/T_m = 0.3$
	$1.8333/K_m$	$4.4000T_m$	$\tau_m/T_m = 0.4$
	$1.5556/K_m$	$7.0000T_m$	$\tau_m/T_m = 0.5$
	$1.3265/K_m$	$14.6250T_m$	$\tau_m/T_m = 0.6$
	$1.1875/K_m$	$31.0330T_m$	$\tau_m/T_m = 0.7$
Sree *et al.* (2004). *Model: Method 1*	$\frac{0.8624}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.9744}$	[10] $T_i^{(156)}$	$0.01 \le \frac{\tau_m}{T_m} \le 0.6$
	Robust		
Rotstein and Lewin (1991). *Model: Method 1*	$\frac{T_m \lambda\left(\frac{\lambda}{T_m}+2\right)}{\lambda^2 K_m}$	$\lambda\left(\frac{\lambda}{T_m}+2\right)$	λ obtained graphically – sample values below
	K_m uncertainty = 50%	$\tau_m/T_m = 0.2$	$\lambda \in [0.6T_m, 1.9T_m]$

---

[9] Desired closed loop transfer function $= \frac{(\eta s+1)e^{-s\tau_m}}{(T_{CL2}s+1)^2}$;

$T_{CL2} = \left(0.025 + 1.75\frac{\tau_m}{T_m}\right)T_m$, $\frac{\tau_m}{T_m} \le 0.1$; $T_{CL2} = 2\tau_m$, $0.1 \le \frac{\tau_m}{T_m} \le 0.5$;

$T_{CL2} = 2\tau_m + 5\tau_m\left(\frac{\tau_m}{T_m} - 0.5\right)$, $0.5 \le \frac{\tau_m}{T_m} \le 0.7$.

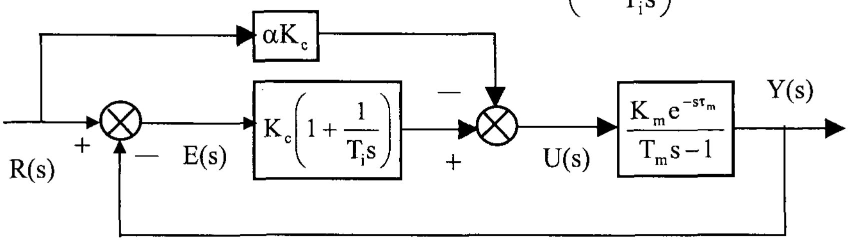
[10] $T_i^{(156)} = T_m\left[143.34\left(\frac{\tau_m}{T_m}\right)^3 - 73.912\left(\frac{\tau_m}{T_m}\right)^2 + 19.039\frac{\tau_m}{T_m} - 0.2276\right]$.

Rule	K_c	T_i	Comment
Rotstein and Lewin (1991) - continued	K_m uncertainty = 30%	$\tau_m/T_m = 0.2$	$\lambda \in [0.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.4$	$\lambda \in [1.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.6$	$\lambda \in [3.9T_m, 4.1T_m]$
Tan *et al.* (1999c). *Model: Method 1*	[11] $K_c^{(157)}$	$T_i^{(157)}$	$\frac{\tau_m}{T_m} < 0.5$

[11] $K_c^{(157)} = \dfrac{1}{K_m\left(0.1486\dfrac{\tau_m}{T_m} + 0.6564\right)}$, $T_i^{(157)} = \dfrac{T_m\left(12.7708\dfrac{\tau_m}{T_m} + 3.8019\right)}{0.1486\dfrac{\tau_m}{T_m} + 0.6564}$.

3.12.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$



<u>Table 32</u>: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
	Direct synthesis		
Chidambaram (2000c). *Model: Method 1*	$K_c^{(158)}$ [12]	$T_i^{(158)}$	'small' τ_m/T_m
	$\frac{2.695}{K_m}e^{-1.277\tau_m/T_m}$ [13]	$0.4015T_m e^{5.8\tau_m/T_m}$	'Dominant pole placement' method; $\xi = 0.333$
Jhunjhunwala and Chidambaram (2001). *Model: Method 1*	$K_c^{(159)}$ [14]	$T_i^{(159)} = 0.324T_m e^{\frac{6.461\tau_m}{T_m}}$	$\frac{\tau_m}{T_m} \le 0.2$
	$0.876\left(\frac{\tau_m}{T_m}\right)^{-0.898}$		$0.2 \le \frac{\tau_m}{T_m} \le 0.6$

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[12] $K_c^{(158)} = \frac{T_m T_i^{(158)}}{0.04{T_S}^2\xi^2 + T_i^{(158)}\tau_m}$, $T_i^{(158)} = \frac{0.04{T_S}^2\xi^2 + T_m\tau_m + 0.4T_S\xi^2}{T_m - \tau_m}$,

$$\alpha = 1 - \frac{0.2T_S\xi^2(T_m - \tau_m)}{0.04{T_S}^2\xi^2 + T_m\tau_m + 0.4T_S\xi^2}.$$

[13] $\alpha = 0.733 + 0.6\frac{\tau_m}{T_m} - 0.36\left(\frac{\tau_m}{T_m}\right)^2$; $0.1 \le \frac{\tau_m}{T_m} \le 0.6$.

[14] $K_c^{(159)} = \frac{1}{K_m}\left[10.7572 - 35.1\frac{\tau_m}{T_m}\right]$, $\alpha = \frac{K_c^{(159)}K_m(2 - T_m) + \left(K_c^{(159)}K_m - 1\right)T_i^{(159)}}{2K_c^{(159)}K_m}$.

Rule	K_c	T_i		Comment
Chandrashekar *et al.* (2002)[15]. *Model: Method 1*	$\frac{49.535}{K_m}e^{-21.644\tau_m/T_m}$	$0.1523T_m e^{7.9425\tau_m/T_m}$		$\frac{\tau_m}{T_m} < 0.1$
	$\frac{0.8668}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.8288}$			$0.1 \le \frac{\tau_m}{T_m} \le 0.7$
	Alternative tuning rules			
	K_c	T_i	α	
Coefficients of K_c *and* T_i *recorded in tables;* T_{CL2} *as in footnote*	$6.0000/K_m$	$0.6000T_m$	0.6667	$\tau_m/T_m = 0.1$
	$3.2222/K_m$	$1.4500T_m$	0.7246	$\tau_m/T_m = 0.2$
	$2.2963/K_m$	$2.6571T_m$	0.7742	$\tau_m/T_m = 0.3$
	$1.8333/K_m$	$4.4000T_m$	0.8182	$\tau_m/T_m = 0.4$
	$1.5556/K_m$	$7.0000T_m$	0.8571	$\tau_m/T_m = 0.5$
	$1.3265/K_m$	$14.6250T_m$	0.8974	$\tau_m/T_m = 0.6$
	$1.1875/K_m$	$31.0330T_m$	0.9323	$\tau_m/T_m = 0.7$
Hägglund and Åström (2002). *Model: Method 1*; $M_s = 1.4$; $0.3 < \alpha < 0.5$	$0.28/K_mT_m$	$8\tau_m$		$\frac{\tau_m}{T_m} = 0.02$
		$7.8\tau_m$		
	$0.2/K_mT_m$	$11\tau_m$		$\frac{\tau_m}{T_m} = 0.05$
	$0.28/K_mT_m$	$10\tau_m$		
	$0.13/K_mT_m$	$7\tau_m$		$\frac{\tau_m}{T_m} = 0.10$
	$0.28/K_mT_m$	$19\tau_m$		
	$0.28/K_mT_m$	$135\tau_m$		$\tau_m/T_m = 0.15$

---

[15] Desired closed loop transfer function $= \dfrac{(\eta s+1)e^{-s\tau_m}}{(T_{CL2}s+1)^2}$;

$$T_{CL2} = \left(0.025 + 1.75\frac{\tau_m}{T_m}\right)T_m,\ \frac{\tau_m}{T_m} \le 0.1;\ T_{CL2} = 2\tau_m,\ 0.1 \le \frac{\tau_m}{T_m} \le 0.5;$$

$$T_{CL2} = 2\tau_m + 5\tau_m\left(\frac{\tau_m}{T_m} - 0.5\right),\ 0.5 \le \frac{\tau_m}{T_m} \le 0.7.$$

$$\alpha = 0.505 + 3.426\frac{\tau_m}{T_m} - 10.57\left(\frac{\tau_m}{T_m}\right)^2,\ \frac{\tau_m}{T_m} \le 0.2;$$

$$\alpha = 0.713 + 0.232\frac{\tau_m}{T_m},\ 0.2 \le \frac{\tau_m}{T_m} \le 0.7.$$

3.12.3 *Controller with proportional term acting on the output 1*

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c Y(s)$$

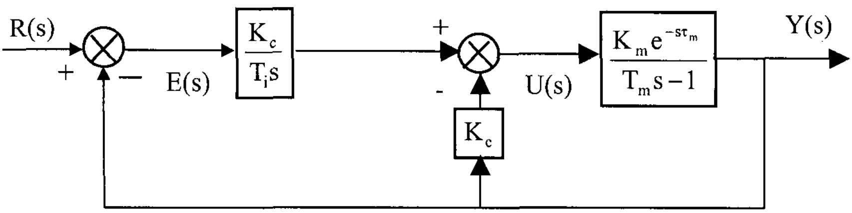

Table 33: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
	Minimum performance index		
Minimum ISE – Paraskevopoulos *et al.* (2004). *Model: Method 1*	[1] $K_c^{(160)}$	$\frac{1.916\tau_m + \frac{4.78\tau_m^2}{T_m}}{0.88 - \frac{\tau_m}{T_m}}$	$0.05 \le \frac{\tau_m}{T_m} \le 0.85$; regulator tuning
		$\frac{2.725\tau_m + \frac{1.71\tau_m^2}{T_m}}{0.88 - \frac{\tau_m}{T_m}}$	$0.05 \le \frac{\tau_m}{T_m} \le 0.85$; servo tuning

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[1] $K_c^{(160)} = \sqrt{K_c^{(min)} K_c^{(max)}}$

with $K_c^{(min)} = \frac{\omega_{min} T_i}{K_m}\sqrt{\frac{1 + \omega_{min}^2 T_m^2}{1 + \omega_{min}^2 T_i^2}}$, $K_c^{(max)} = \frac{\omega_{max} T_i}{K_m}\sqrt{\frac{1 + \omega_{max}^2 T_m^2}{1 + \omega_{max}^2 T_i^2}}$ and

$$\omega_{min} = \frac{1}{T_m}\left[\frac{T_i}{T_m} - \frac{\tau_m}{T_m + T_i}\right],\ \omega_{max} = \frac{\frac{\pi^2}{2\tau_m}\left[\frac{T_i}{T_i + \tau_m} - \frac{\tau_m}{T_m}\right]}{\frac{\tau_m}{T_m} + \pi\left[\frac{T_i}{T_i + \tau_m} - \frac{\tau_m}{T_m}\right]},\ T_i\omega_{max}^2 >> 1.$$

Rule	K_c	T_i	Comment
	Minimum performance index: other tuning		
[2] Paraskevopoulos *et al.* (2004). *Model: Method 1*	[1] $K_c^{(160)}$ (page 137)	[3] $T_i^{(160)}$	$0.01 \le \frac{\tau_m}{T_m} \le 0.3$
		[3] $T_i^{(161)}$	$0.3 \le \frac{\tau_m}{T_m} \le 0.85$
Minimise $\int_0^\infty \left[e^2(t) + \left(\frac{du}{dt} \right)^2 \right] dt$ - Paraskevopoulos *et al.* (2004).	[1] $K_c^{(160)}$ (page 137)	$\dfrac{2.58\tau_m + \dfrac{3.60\tau_m^{\ 3}}{T_m^{\ 2}}}{0.88 - \dfrac{\tau_m}{T_m}}$	$0.03 \le \frac{\tau_m}{T_m} \le 0.85$; *Model: Method 1*
	Direct synthesis		
Paraskevopoulos *et al.* (2004). *Model: Method 1*	[1] $K_c^{(160)}$ (page 137)	$> T_i^{(min)}$ [4]	$0.02 < \frac{\tau_m}{T_m} < 0.7$
		$> \dfrac{1.35\tau_m}{\left(1 - \dfrac{\tau_m}{T_m}\right)^2}$	$0.1 < \frac{\tau_m}{T_m} < 0.9$
		$x_1\tau_m + x_2\sqrt{\tau_m T_m}$	$\frac{\tau_m}{T_m} < 0.3$

Coefficient values

x_1	x_2	ξ	x_1	x_2	ξ
0.7269	2.5362	0.5	6.975	5.7053	1.5
1.7016	3.0585	0.75	12.452	8.4358	2
3.0696	3.7509	1			

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[2] Minimise performance index = $\int_0^\infty \left[e^2(t) + (u(t) - u_\infty)^2 \right] dt$

[3] $$T_i^{(160)} = T_m \left[0.747 + 3.66 \frac{\tau_m}{T_m} + 18.64 \left(\frac{\tau_m}{T_m} \right)^3 \right]; \quad T_i^{(161)} = T_m \left[\frac{4.52 \dfrac{\tau_m}{T_m} + 0.476 \left(\dfrac{\tau_m}{T_m} \right)^3}{0.88 - \dfrac{\tau_m}{T_m}} \right].$$

[4] $$T_i^{(min)} = T_m [1.3146 \frac{\tau_m}{T_m} - 2.3805 \left(\frac{\tau_m}{T_m} \right)^2 + 57.622 \left(\frac{\tau_m}{T_m} \right)^3 - 158.99 \left(\frac{\tau_m}{T_m} \right)^4 + 173.41 \left(\frac{\tau_m}{T_m} \right)^5].$$

Rule	K_c	T_i	Comment
Paraskevopoulos *et al.* (2004) – continued.	[1] $K_c^{(160)}$ (page 137)	$x_1\tau_m + x_2 T_m + x_3 {\tau_m}^6 {T_m}^{-5}$	$0.2 < \frac{\tau_m}{T_m} < 0.7$

Coefficient values

x_1	x_2	x_3	ξ	x_1	x_2	x_3	ξ
4.59	-0.19	8.28	0.5	12.41	1.89	51.95	1.5
6.16	0.028	9.90	0.75	19.91	3.44	95	2
8.21	0.37	16.88	1				

Rule	K_c	T_i	Comment
Paraskevopoulos *et al.* (2004). *Model: Method 1*	[1] $K_c^{(160)}$ (page 137)	[5] $T_i^{(162)}$	$\frac{\tau_m}{T_m} < 0.2$

Coefficient values

x_1	x_2	T_{CL2}	x_1	x_2	T_{CL2}
0.409	0.2817	0.5	2.353	2.67	1.5
0.717	0.6542	0.75	4.07	4.76	2
1.142	1.177	1	8.88	10.73	3

Rule	K_c	T_i	Comment
	[1] $K_c^{(160)}$ (page 137)	$T_m\left[x_1 + x_3\left(\frac{\tau_m}{T_m}\right)^3\right]$	$0.2 < \frac{\tau_m}{T_m} < 0.7$

Coefficient values

x_1	x_2	T_{CL2}	x_1	x_2	T_{CL2}
1.518	10.54	0.5	13	49.66	1.5
3.34	16.89	0.75	23.08	82.87	2
5.81	29.17	1	51.94	176.8	3

Robust

Rule	K_c	T_i	Comment
Zhang and Xu (2002). *Model: Method 1*	[6] $K_c^{(163)}$	$T_i^{(163)}$	$\tau_m < T_{m1}$; for $\frac{\tau_m}{T_m} \le 0.5$, $\lambda = 2 - 5\tau_m$

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[5] $$T_i^{(162)} = T_m\left[x_1 - \frac{x_2}{\frac{\tau_m}{T_m} - \sqrt{\frac{\tau_m}{T_m}}}\right]$$

[6] $$K_c^{(163)} = \frac{\lambda^2 + 2\lambda T_m + T_m\tau_m}{K_m(\lambda + \tau_m)^2},\ T_i^{(163)} = \frac{\lambda^2 + 2\lambda T_m + T_m\tau_m}{T_m - \tau_m}.$$

3.12.4 *Controller with proportional term acting on the output 2*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_1 Y(s)$$

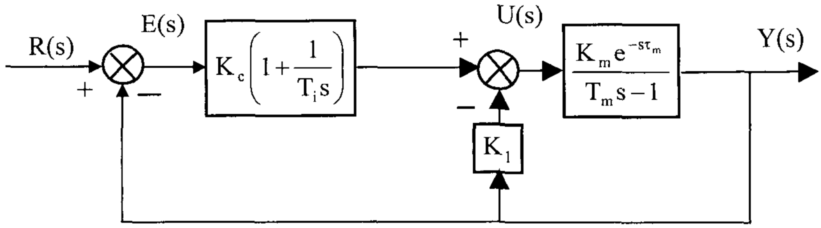

Table 34: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
	Robust		
Chidambaram (1997). *Model: Method 1*	$\frac{T_m - \tau_m}{\lambda K_m}$	[7] $T_i^{(164)}$	$\lambda > 1.7$; $\frac{\tau_m}{T_m} < 1$
Lee and Edgar (2002). *Model: Method 1*	[8] $K_c^{(165)}$	$T_i^{(165)}$	

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[7] $$T_i^{(164)} = \frac{T_m - \left(\frac{T_m}{\tau_m}\right)^{0.5}\tau_m}{\left(\frac{T_m}{\tau_m}\right)^{0.5} - 1} + 0.5\tau_m,\ K_1 = \frac{1}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.5}.$$

[8] $$K_c^{(165)} = \frac{K_1^{(165)}K_m - 1}{K_m(\lambda + \tau_m)}\left[\frac{T_m - K_1^{(165)}K_m\tau_m}{K_1^{(165)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)}\right],$$

$$T_i^{(165)} = \frac{T_m - K_1^{(165)}K_m\tau_m}{K_1^{(165)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)},$$

$$K_1^{(165)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)}\sin\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right].$$

3.13 Unstable FOLPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}s - 1}$$

3.13.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

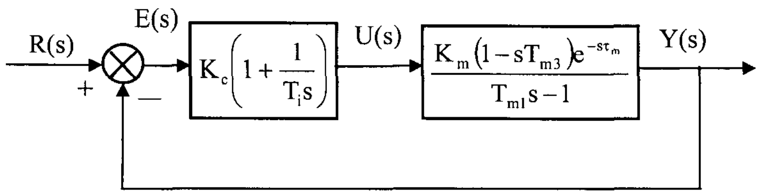

Table 35: PI controller tuning rules – unstable FOLPD model with a positive zero - $G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}s - 1}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2003c). *Model: Method 1*	[1] $\frac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(166)}$	

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[1] γ = the negative of the value of the inverse jump of the closed loop system; $\gamma \ge \frac{T_{m3}}{T_{m1}}$.

$$T_i^{(166)} = \frac{\frac{\gamma T_{m1}}{T_{m3}}\left[T_{m3}(1 - x_1) - 0.5\tau_m(1 + x_1)\right]}{\frac{\gamma T_{m1}}{T_{m3}}(1 - x_1) + x_1}, \quad x_1 > \frac{\gamma T_{m1}}{\gamma T_{m1} - T_{m3}}.$$

3.13.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1}{1+T_f s}$$

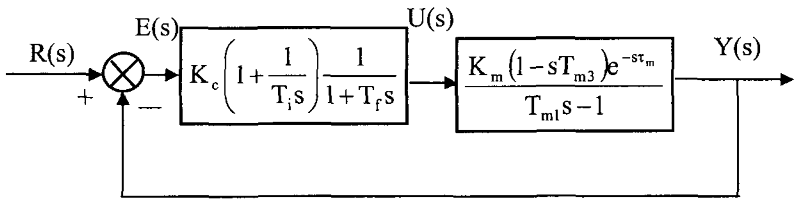


Table 36: PI controller tuning rules – unstable FOLPD model with a positive zero - $G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}s-1}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2003b). *Model: Method 1*	[2] $K_c^{(167)}$	$T_i^{(167)}$	$\frac{\tau_m}{T_{m1}} - \frac{\tau_m}{T_{m3}} < 0.62$

[2] Desired closed loop transfer function = $\frac{(1-sT_{m3})(1+T_i^{(167)}s)e^{-s\tau_m}}{(1+T_{CL2}^{(1)}s)(1+T_{CL2}^{(2)}s)}$.

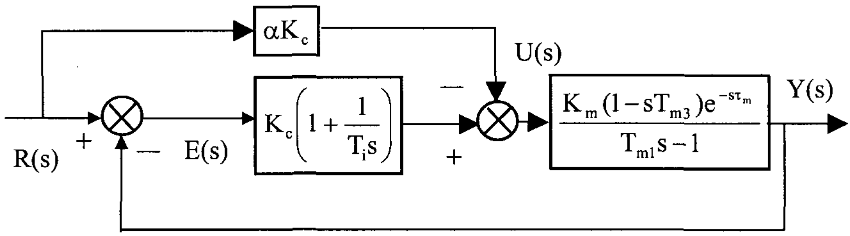
$$K_c^{(167)} = -\frac{T_i^{(167)}}{K_m\left(T_{CL2}^{(1)}+T_{CL2}^{(2)}+T_{m3}+\tau_m-T_i^{(167)}\right)},$$

$$T_i^{(167)} = \frac{T_{m1}\left(T_{CL2}^{(1)}T_{CL2}^{(2)}-T_{m3}\tau_m+\left[T_{CL2}^{(1)}+T_{CL2}^{(2)}+T_{m3}+\tau_m\right]T_{m1}\right)}{T_{m1}^{\;2}+T_{m3}\tau_m-(\tau_m+T_{m3})T_{m1}},$$

$$T_f = \frac{T_{m3}\tau_m T_i^{(167)}}{T_{m1}\left(T_{CL2}^{(1)}+T_{CL2}^{(2)}+T_{m3}+\tau_m-T_i^{(167)}\right)}.$$

3.13.3 *Controller with set-point weighting*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$



<u>Table 37</u>: PI controller tuning rules – unstable FOLPD model with a positive zero - $G_m(s) = \dfrac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}s - 1}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2003b). *Model: Method 1*	[3] $K_c^{(168)}$	$T_i^{(168)}$	$\dfrac{\tau_m}{T_{m1}} - \dfrac{\tau_m}{T_{m3}} < 0.62$

[3] Desired closed loop transfer function = $\dfrac{(1 - sT_{m3})(1 + T_i^{(168)}s)e^{-s\tau_m}}{(1 + T_{CL2}^{(1)}s)(1 + T_{CL2}^{(2)}s)}$.

$$K_c^{(168)} = -\frac{T_i^{(168)}}{K_m\left(T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(168)}\right)},$$

$$T_i^{(168)} = \frac{T_{m1}\left(T_{CL2}^{(1)}T_{CL2}^{(2)} - T_{m3}\tau_m + \left[T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m\right]T_{m1}\right)}{T_{m1}^{\,2} + T_{m3}\tau_m - (\tau_m + T_{m3})T_{m1}},$$

$$T_f = \frac{T_{m3}\tau_m T_i^{(168)}}{T_{m1}\left(T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(168)}\right)},\quad \alpha = 1 - 0.5\left(\frac{T_{m3} + T_{CL2}^{(1)} + T_{CL2}^{(2)}}{T_i^{(168)}}\right).$$

Rule	K_c	T_i	Comment
Sree and Chidambaram (2003c). *Model: Method 1*	[4] $\dfrac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(169)}$	

[4] γ = the negative of the value of the inverse jump of the closed loop system;

$$\gamma \geq \frac{T_{m3}}{T_{m1}}.\ T_i^{(169)} = \frac{\frac{\gamma T_{m1}}{T_{m3}}\left[T_{m3}(1-x_1) - 0.5\tau_m(1+x_1)\right]}{\frac{\gamma T_{m1}}{T_{m3}}(1-x_1)+x_1},\ x_1 > \frac{\gamma T_{m1}}{\gamma T_{m1} - T_{m3}};$$

$$\alpha = 1 - 0.5\left(\frac{T_{m3} + T_{CL2}^{(1)} + T_{CL2}^{(2)}}{T_i^{(169)}}\right).$$

Desired closed loop transfer function = $\dfrac{(1 - sT_{m3})\left(1 + T_i^{(169)}s\right)e^{-s\tau_m}}{\left(1 + T_{CL2}^{(1)}s\right)\left(1 + T_{CL2}^{(2)}s\right)}$

(Sree and Chidambaram (2003b)).

3.14 Unstable SOSPD Model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

3.14.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

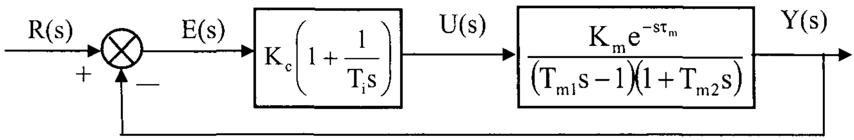


Table 38: PI controller tuning rules – unstable SOSPD model (one unstable pole) $G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$

Rule	K_c		T_i		Comment	
	Minimum performance index: regulator tuning					
Minimum ITAE – Poulin and Pomerleau (1996). *Model: Method 1*	$K_c^{(170)}$ [1]		$\frac{4T_{m1}(\tau_m + T_{m2})}{x_1 T_{m1} - 4(\tau_m + T_{m2})}$		*Coefficients of* K_c, T_i *deduced from graph*	
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
Process output step load disturbance	0.05	0.9479	2.3546	0.30	1.6163	2.6612
	0.10	1.0799	2.4111	0.35	1.7650	2.7368
	0.15	1.2013	2.4646	0.40	1.9139	2.8161
	0.20	1.3485	2.5318	0.45	2.0658	2.9004
	0.25	1.4905	2.5992	0.50	2.2080	2.9826
Process input step load disturbance						
	0.05	1.1075	2.4230	0.25	1.6943	2.7007
	0.10	1.2013	2.4646	0.35	1.8161	2.7637
	0.15	1.3132	2.5154	0.40	1.9658	2.8445
	0.20	1.4384	2.5742	0.45	2.1022	2.9210
	0.25	1.5698	2.6381	0.50	2.2379	3.0003

---

[1] $K_c^{(170)} = \dfrac{x_2 T_{m1}\sqrt{1 + \dfrac{x_1 T_{m2}^2}{4(\tau_m + T_{m2})^2}}}{K_m\left(x_1 T_{m1} - 4[\tau_m + T_{m2}]\right)}$.

Rule	K_c	T_i	Comment
	Direct synthesis		
Poulin *et al.* (1996). *Model: Method 1*	$-\frac{\omega T_{m1}}{\tau_m}$	[2] $T_i^{(171)}$	minimum $\phi_m = 25^0$
	$-\frac{3.53}{K_m}$	T_{m1}	$T_{m2} + \tau_m < 0.16T_{m1}$; $\phi_m \in [25^0, 90^0]$
	Ultimate cycle		
McMillan (1984). *Model: Method 1*	[3] $K_c^{(172)}$	$T_i^{(172)}$	Tuning rules developed from K_u, T_u

[2] $T_i^{(171)} = \frac{4T_{m1}(\tau_m + T_{m2})}{1.3T_{m1} - 4(\tau_m + T_{m2})}$, $0.16T_{m1} \le T_{m2} + \tau_m \le 0.3T_{m1}$; ω = frequency where phase of $G_m(j\omega)G_c(j\omega)$ is maximum.

[3] $$K_c^{(172)} = \frac{1.477}{K_m}\frac{T_{m1}T_{m2}}{\tau_m^{\,2}}\left\{\frac{1}{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}}\right\}^2,$$

$$T_i^{(172)} = 3.33\tau_m\left\{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}\right\}.$$

3.15 Unstable SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}{}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

3.15.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

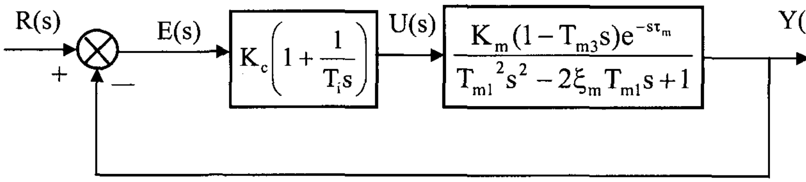


Table 39: PI controller tuning rules – unstable SOSPD model with a positive zero $G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}{}^2 s^2 - 2\xi_m T_{m1}s + 1}$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2004). *Model: Method 1*	[4] $K_c{}^{(173)}$	$T_i{}^{(173)}$	

---

[4] Desired closed loop transfer function $= \frac{(1 - sT_{m3})(1 + T_i{}^{(173)}s)e^{-s\tau_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}$.

$$K_c{}^{(173)} = \frac{T_i{}^{(173)}}{K_m\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m - T_i{}^{(173)}\right)}$$

$$T_i{}^{(173)} = -\frac{T_{m1}{}^2\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m\right) - T_{CL}^{(1)}T_{CL}^{(2)}x_1}{T_{m3}\tau_m - T_{m1}{}^2}, \text{ with}$$

$$x_1 = \frac{T_{m1}\left(T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m\right)\left(-2\xi_m T_{m3}\tau_m + T_{m1}(T_{m3} + \tau_m)\right) - x_2}{\left(-2\xi_m T_{m1} + T_{m3} + \tau_m\right)T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}{}^2 + (T_{m3}\tau_m - T_{m1}{}^2)\left(T_{CL}^{(1)} + T_{CL}^{(2)} + 2\xi_m T_{m1}\right)},$$

and with $x_2 = \left(T_{m3}\tau_m - T_{m1}{}^2\right)\left(T_{CL}^{(1)}T_{CL}^{(2)} - \tau_m T_{m3}\right)$; $-\tau_m\left(\frac{2\xi_m}{T_{m1}} + \frac{1}{T_{m3}}\right) < 0.62$.

3.15.2 *Controller with set-point weighting*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$

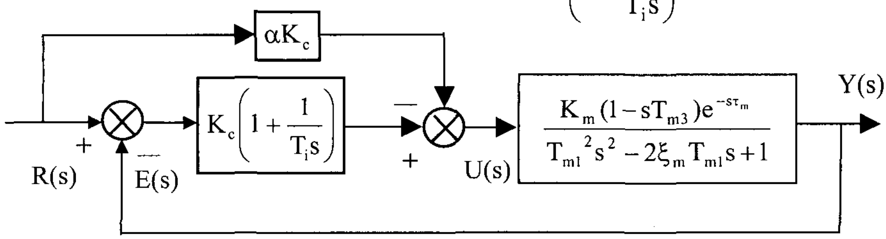


Table 40: PI controller tuning rules — unstable SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	Comment
	Direct synthesis: time domain criteria		
Sree and Chidambaram (2004). *Model: Method 1*	[5] $K_c^{(174)}$	$T_i^{(174)}$	

---

[5] Desired closed loop transfer function $= \dfrac{(1 - sT_{m3})(1 + T_i^{(174)}s)e^{-s\tau_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}$.

$$K_c^{(174)} = \frac{T_i^{(174)}}{K_m\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m - T_i^{(174)}\right)},\ -\tau_m\left(\frac{2\xi_m}{T_{m1}} + \frac{1}{T_{m3}}\right) < 0.62\ ;$$

$$T_i^{(174)} = -\frac{T_{m1}^2\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m\right) - T_{CL}^{(1)}T_{CL}^{(2)}x_1}{T_{m3}\tau_m - T_{m1}^2}, \text{ with}$$

$$x_1 = \frac{T_{m1}\left(T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m\right)\left(-2\xi_m T_{m3}\tau_m + T_{m1}(T_{m3} + \tau_m)\right) - x_2}{\left(-2\xi_m T_{m1} + T_{m3} + \tau_m\right)T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}^2 + (T_{m3}\tau_m - T_{m1}^2)\left(T_{CL}^{(1)} + T_{CL}^{(2)} + 2\xi_m T_{m1}\right)},$$

and with $x_2 = \left(T_{m3}\tau_m - T_{m1}^2\right)\left(T_{CL}^{(1)}T_{CL}^{(2)} - \tau_m T_{m3}\right)$, $\alpha = 1 - \dfrac{T_{m3} + 0.5\left(T_{CL}^{(1)} + T_{CL}^{(2)}\right)}{T_i^{(174)}}$.

3.16 Delay Model $G_m(s) = K_m e^{-s\tau_m}$

3.16.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

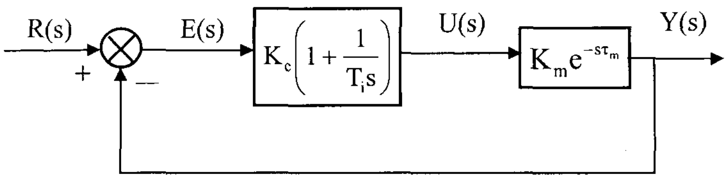


Table 41: PI controller tuning rules – $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	Comment
	Process reaction		
Callender *et al.* (1935/6). *Model: Method 1*	[1] $0.568/K_m\tau_m$	$3.64\tau_m$	*Representative results deduced from graphs*
	[2] $0.65/K_m\tau_m$	$2.6\tau_m$	
	[3] $0.79/K_m\tau_m$	$3.95\tau_m$	
	[4] $0.95/K_m\tau_m$	$3.3\tau_m$	
Two constraints method – Wolfe (1951). *Model: Method 1*	$\frac{0.2}{K_m\tau_m}$	$0.3\tau_m$	Decay ratio is as small as possible
	Minimum error integral (regulator mode)		
	Minimum performance index: regulator tuning		
Haalman (1965) – minimum ISE. *Model: Method 1*	0	$1.5\tau_m$	
	$M_s = 1.9$; $A_m = 2.36$; $\phi_m = 50^0$		
Gerry (1998) – minimum error: step load change.	$\frac{0.3}{K_m}$	$0.42\tau_m$	*Model: Method 1*
Gerry and Hansen (1987) – minimum IAE.	$\frac{0.345}{K_m}$	$0.455\tau_m$	*Model: Method 1*

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[1] Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

[2] Decay ratio = 0.1; Period of decaying oscillation = $8\tau_m$

[3] Decay ratio = 0.12; Period of decaying oscillation = $6.28\tau_m$

[4] Decay ratio = 0.33; Period of decaying oscillation = $5.56\tau_m$

Rule	K_c	T_i	Comment
Shinskey (1988) – minimum IAE - *page 123.*	$\frac{0.43}{K_m}$	$0.5\tau_m$	*Model: Method 1*
Shinskey (1994) – minimum IAE. *Model: Method 1*	$0.4/K_m$	$0.5\tau_m$	*page 67*
	0	$1.6K_m\tau_m$	*page 63*
Shinskey (1988) – minimum IAE – *page 148.*	$\frac{0.4255}{K_m}$	$0.25T_u$	*Model: Method 1*
Shinskey (1996) – minimum IAE – *page 167.*	$0.4K_u$	$0.25T_u$	*Model: Method 1*
	Minimum performance index: servo tuning		
Minimum IAE – Huang and Jeng (2002). *Model: Method 1*	$\frac{0.36}{K_m}$	$0.47\tau_m$	Minimum IAE = $1.377\tau_m$. $A_m = 2$; $\phi_m = 60^0$.
Minimum ISE - Keviczky and Csáki (1973) *Model: Method 1*	0	$1.33\tau_m$	Minimum ISE = $1.53\tau_m$; $K_m = 1$
	0.5	$0.635\tau_m$	$K_m = 1$
	Minimum performance index: other tuning		
Fertik (1975). *Model: Method 1*	$0.35/K_m$	$0.4\tau_m$	K_c, T_i *values deduced from graph*
Åström and Hägglund (2000). *Model: Method 1*	$\frac{0.25}{K_m}$	$0.35\tau_m$	Maximise K_c/T_i subject to $M_{max} = 2$
Skogestad (2001). *Model: Method 1*	$0.16/K_m$	$0.340\tau_m$	$M_{max} = 1.4$
Skogestad (2003). *Model: Method 1*	$0.200/K_m$	$0.318\tau_m$	$M_{max} = 1.6$
Skogestad (2001). *Model: Method 1*	$0.26/K_m$	$0.306\tau_m$	$M_{max} = 2.0$
Åström and Hägglund (2004). *Model: Method 1*	x_1/K_m	$x_2\tau_m$	

Coefficient values (Åström and Hägglund (2004))

x_1	x_2	M_{max}	x_1	x_2	M_{max}
0.057	0.400	1.1	0.211	0.342	1.6
0.103	0.389	1.2	0.227	0.334	1.7
0.139	0.376	1.3	0.241	0.326	1.8
0.168	0.363	1.4	0.254	0.320	1.9
0.191	0.352	1.5	0.264	0.314	2.0

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="2">Comment</th></tr>
<tr><td></td><td colspan="8">Direct synthesis</td></tr>
<tr><td rowspan="2">Van der Grinten (1963).
Model: Method 1</td><td colspan="3">$0.5/K_m$</td><td colspan="3">$0.5\tau_m$</td><td colspan="2">Step disturbance</td></tr>
<tr><td colspan="3">[5] $K_c^{(175)}$</td><td colspan="3">$T_i^{(175)}$</td><td colspan="2">Stochastic disturbance</td></tr>
<tr><td rowspan="4">Buckley (1964).
Model: Method 1</td><td colspan="3">0</td><td colspan="3">$1.45K_m\tau_m$</td><td colspan="2">$M_{max} = 2dB$</td></tr>
<tr><td colspan="3">$0.075/K_m$</td><td colspan="3">$0.1\tau_m$</td><td colspan="2" rowspan="3">Representative results;
$M_{max} = 2dB$</td></tr>
<tr><td colspan="3">$0.53/K_m$</td><td colspan="3">τ_m</td></tr>
<tr><td colspan="3">$0.55/K_m$</td><td colspan="3">$1.59\tau_m$</td></tr>
<tr><td rowspan="2">Bryant *et al.* (1973).
Model: Method 1</td><td colspan="3">0</td><td colspan="3">$2.70K_m\tau_m$</td><td colspan="2">Critically damped dominant pole</td></tr>
<tr><td colspan="3">0</td><td colspan="3">$2.50K_m\tau_m$</td><td colspan="2">Damping factor (dominant pole) = 0.6</td></tr>
<tr><td rowspan="8">Keviczky and Csáki (1973).
Model: Method 1; Representative K_c, T_i *values estimated from graphs*; $K_m = 1$</td><td colspan="3">0</td><td colspan="3">$x_2\tau_m$</td><td colspan="2"></td></tr>
<tr><td colspan="8">Coefficient values</td></tr>
<tr><td>x_2</td><td>OS</td><td>x_2</td><td>OS</td><td>x_2</td><td>OS</td><td>x_2</td><td>OS</td></tr>
<tr><td>0.64</td><td>105%</td><td>1.00</td><td>50%</td><td>1.5</td><td>17%</td><td>2</td><td>4%</td></tr>
<tr><td colspan="3">x_1</td><td colspan="3">$2\tau_m$</td><td colspan="2"></td></tr>
<tr><td colspan="8">Coefficient values</td></tr>
<tr><td>x_1</td><td>OS</td><td>x_1</td><td>OS</td><td>x_1</td><td>OS</td><td>x_1</td><td>OS</td></tr>
<tr><td>0.68</td><td>5%</td><td>0.72</td><td>10%</td><td>0.75</td><td>15%</td><td>0.78</td><td>20%</td></tr>
<tr><td rowspan="3">Kamimura *et al.* (1994).
Model: Method 1</td><td colspan="3">0</td><td colspan="3">$2K_m\tau_m$</td><td colspan="2">OS = 10%</td></tr>
<tr><td colspan="3">0</td><td colspan="3">$2.6667K_m\tau_m$</td><td colspan="2">OS = 0%</td></tr>
<tr><td colspan="3">0</td><td colspan="3">$2.3529K_m\tau_m$</td><td colspan="2">"Quick" servo response; "negligible" overshoot</td></tr>
<tr><td>Hansen (2000).
Model: Method 1</td><td colspan="3">$0.2/K_m$</td><td colspan="3">$0.3\tau_m$</td><td colspan="2"></td></tr>
<tr><td></td><td colspan="8">Robust</td></tr>
<tr><td>Bequette (2003) – *page 300.*</td><td colspan="3">$\dfrac{\tau_m}{K_m(2\lambda + \tau_m)}$</td><td colspan="3">$0.5\tau_m$</td><td colspan="2">*Model: Method 1*</td></tr>
<tr><td>Skogestad (2003).
Model: Method 1</td><td colspan="3">$0.294/K_m$</td><td colspan="3">$0.5\tau_m$</td><td colspan="2">IMC based rule</td></tr>
<tr><td></td><td colspan="8">Other tuning</td></tr>
<tr><td>Skogestad (2003).</td><td colspan="3">$0.25/K_m$</td><td colspan="3">$0.333\tau_m$</td><td colspan="2">*Model: Method 1*</td></tr>
</table>

[5] $K_c^{(175)} = \dfrac{e^{-\omega_d\tau_m}}{K_m}\left(1 - 0.5e^{-\omega_d\tau_m}\right)$, $T_i^{(175)} = \dfrac{\tau_m}{e^{-\omega_d\tau_m}}\left(1 - 0.5e^{-\omega_d\tau_m}\right)$.

3.17 General Model with a Repeated Pole

3.17.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$

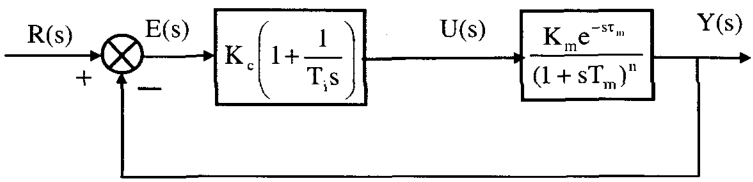

Table 42: PI controller tuning rules – general model with a repeated pole

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$$

Rule	K_c	T_i	Comment
	Direct synthesis: frequency domain criteria		
Schaedel (1997). *Model: Method 2*	$\frac{0.5}{K_m\left[n^{0.5}-1\right]}$	$\frac{\tau_m + T_{ar}}{\sqrt{n}}$	'Normal' design
	$\frac{0.375}{K_m}\left[\frac{n+1}{n-1}\right]$	$\frac{(n+1)(\tau_m + T_{ar})}{2n}$	'Sharp' design

3.18 General Model with Integrator

$$G_m(s)=\frac{K_m}{s}\frac{\prod_i\left(T_{m_1i}s+1\right)\prod_i\left(T_{m_2i}{}^2s^2+2\xi_{m_ni}T_{m_2i}s+1\right)}{\prod_i\left(T_{m_3i}s+1\right)\prod_i\left(T_{m_4i}{}^2s^2+2\xi_{m_di}T_{m_4i}s+1\right)}e^{-s\tau_m}$$

3.18.1 *Ideal controller* $G_c(s)=K_c\left(1+\frac{1}{T_is}\right)$

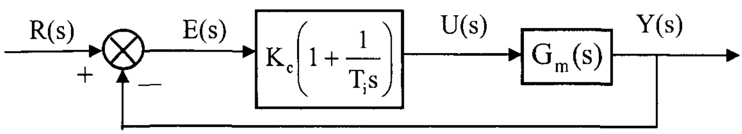

Table 43: PI controller tuning rules – general model with integrator

$$G_m(s)=\frac{K_m}{s}\frac{\prod_i\left(T_{m_1i}s+1\right)\prod_i\left(T_{m_2i}{}^2s^2+2\xi_{m_ni}T_{m_2i}s+1\right)}{\prod_i\left(T_{m_3i}s+1\right)\prod_i\left(T_{m_4i}{}^2s^2+2\xi_{m_di}T_{m_4i}s+1\right)}e^{-s\tau_m}$$

Rule	K_c	T_i	Comment
	Direct synthesis		
Loron (1997). *Model: Method 1*	[6] $K_c^{(176)}$	$\alpha_1 T_Q$	Symmetrical optimum method
	$K_c^{(177)}$		Non-symmetrical optimum method

---

[6] $K_c^{(176)}=\frac{1}{K_m\sqrt{\alpha_1 T_Q}}$, $\alpha_1=\left(\frac{1+\sin\phi_m}{\cos\phi_m}\right)^2$

$$T_Q=\left[\sum_i T_{m_3i}+2\sum_i \xi_{m_di}T_{m_4i}\right]-\left[\sum_i T_{m_1i}+2\sum_i \xi_{m_ni}T_{m_2i}\right]+\tau_m .$$

$K_c^{(177)}=\frac{\lambda}{K_m\sqrt{\alpha_2 T_Q}}$, $\lambda=\sqrt{\frac{M_c{}^2}{M_c{}^2-1}}$, M_c = desired closed loop resonant peak,

$\alpha_2=\left(\frac{1+\sin\Delta\phi}{\cos\Delta\phi}\right)^2$, $\Delta\phi=\cos^{-1}(1/\lambda)$.

Chapter 4

Tuning Rules for PID Controllers

4.1 FOLPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$

4.1.1 *Ideal controller* $G_c(s) = K_c\left(1 + \dfrac{1}{T_i s} + T_d s\right)$

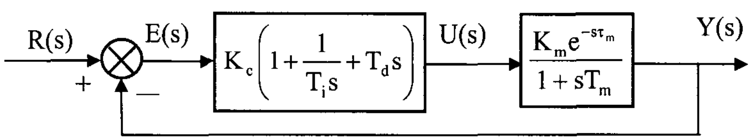

Table 44: PID controller tuning rules – FOLPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Callender *et al.* (1935/6). *Model: Method 1*	[1] $\dfrac{1.066}{K_m \tau_m}$	$1.418\tau_m$	$0.353\tau_m$ or $0.47\tau_m$	$\dfrac{\tau_m}{T_m} = 0.3$
Ziegler and Nichols (1942). *Model: Method 2*	$\dfrac{aT_m}{K_m \tau_m}$, $a \in [1.2, 2]$	$2\tau_m$	$0.5\tau_m$	Quarter decay ratio

[1] Decay ratio = 0.043; Period of decaying oscillation = $6.28\tau_m$

Rule	K_c	T_i	T_d	Comment
Chien *et al.* (1952) – regulator. *Model: Method 2*	$\frac{0.95T_m}{K_m\tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
	$\frac{1.2T_m}{K_m\tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
Chien *et al.* (1952) – servo. *Model: Method 2*	$\frac{0.6T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	0% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
	$\frac{0.95T_m}{K_m\tau_m}$	$1.36T_m$	$0.47\tau_m$	20% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
Cohen and Coon (1953). *Model: Method 2*	[1] $K_c^{(178)}$	$T_i^{(178)}$	$\frac{0.37\tau_m}{1+0.19\frac{\tau_m}{T_m}}$	Quarter decay ratio. $0<\frac{\tau_m}{T_m}\le 1$
Three constraints method – Murrill (1967) - *page 356. Model: Method 5*	$\frac{1.370}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.950}$	$\frac{T_m}{1.351}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	[2] $T_d^{(179)}$	$\frac{K_c K_m T_d}{T_m}=0.5$; $0.1\le\frac{\tau_m}{T_m}\le 1$
	Quarter decay ratio; minimum integral error (servo mode).			
Åström and Hägglund (1988) - *pages 120-126.*	$\frac{3}{K_m}$	$T_{90\%}$	$0.5\tau_m$	*Model: Method 23*
	$T_{90\%}$ = 90% closed loop step response time (Leeds and Northrup Electromax V controller).			
Parr (1989) – *page 194.*	$\frac{1.25T_m}{K_m\tau_m}$	$2.5\tau_m$	$0.4\tau_m$	*Model: Method 2*
Borresen and Grindal (1990).	$\frac{T_m}{K_m\tau_m}$	$3\tau_m$	$0.5\tau_m$	*Model: Method 2*

[1] $$K_c^{(178)}=\frac{1}{K_m}\left(1.35\frac{T_m}{\tau_m}+0.25\right),\ T_i^{(178)}=T_m\left(\frac{2.5\frac{\tau_m}{T_m}+0.46\left(\frac{\tau_m}{T_m}\right)^2}{1+0.61\frac{\tau_m}{T_m}}\right).$$

[2] $$T_d^{(179)}=0.365T_m\left(\frac{\tau_m}{T_m}\right)^{0.950}.$$

Rule	K_c		T_i		T_d		Comment	
Sain and Özgen (1992).	$K_c^{(180)}$ [3]		$T_i^{(180)}$		$T_d^{(180)}$		*Model: Method 24*	
Connell (1996) – *page 214. Model: Method 2*	$\frac{1.6T_m}{K_m\tau_m}$		$1.6667\tau_m$		$0.4\tau_m$		Approximate quarter decay ratio	
Hay (1998) – *pages 197,198. Model: Method 1*	x_1/K_m		$x_2\tau_m$		$x_3\tau_m$			
Coefficients of K_c, T_i, T_d *deduced from graphs*	Coefficient values - servo tuning							
	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.125	6.5	9.0	0.47	0.125	5.0	9.8	0.34
	0.167	5.0	6.5	0.45	0.167	3.7	7.0	0.32
	0.25	3.5	4.2	0.40	0.25	2.5	4.8	0.30
	0.5	1.8	2.2	0.35	0.5	1.1	2.5	0.27
	Coefficient values - regulator tuning							
	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.125	9.8	2.0	0.52	0.1	10.0	2.2	0.35
	0.167	7.2	1.8	0.50	0.125	8.0	2.2	0.35
	0.25	4.5	1.6	0.45	0.167	6.0	2	0.34
	0.5	2.2	1.4	0.35	0.25	4.0	1.8	0.32
					0.5	2.0	1.4	0.28
Moros (1999). *Model: Method 1*	$\frac{1.2T_m}{K_m\tau_m}$		$2\tau_m$		$0.42\tau_m$		attributed to Oppelt	
	$\frac{1.2T_m}{K_m\tau_m}$		$2\tau_m$		$0.44\tau_m$		attributed to Rosenberg	
Lipták (2001).	$\frac{0.85T_m}{K_m\tau_m}$		$1.6\tau_m$		$0.6\tau_m$		*Model: Method 2*	
ControlSoft Inc. (2005).	$\frac{2}{K_m}$		$\tau_m + T_m$		$T_d^{(181)}$ [4]		*Model: Method 10*	

[3] $K_c^{(180)} = \frac{1}{K_m}\left(0.6939\frac{T_m}{\tau_m} + 0.1814\right)$, $T_i^{(180)} = \frac{0.8647T_m + 0.226\tau_m}{\frac{T_m}{\tau_m} + 0.8647}$,

$$T_d^{(180)} = \frac{0.0565T_m}{0.8647\frac{T_m}{\tau_m} + 0.226}.$$

[4] $T_d^{(181)} = \max\left[\frac{\tau_m}{3}, \frac{T_m}{6}\right]$ (slow loop); $T_d^{(181)} = \min\left[\frac{\tau_m}{3}, \frac{T_m}{6}\right]$ (fast loop).

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE – Murrill (1967) – *pages 358-363.*	$\frac{1.435}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{\tau_m}{T_m}\right)^{0.749}$	$T_d^{(182)}$ [5]	*Model: Method 5*; $0.1 \le \frac{\tau_m}{T_m} \le 1$
Minimum IAE – Marlin (1995) – *pages 301-307.* *Model: Method 8*	$1.3/K_m$	$0.7\tau_m$	$0.01\tau_m$	$\tau_m/T_m = 0.43$
	$1.0/K_m$	$0.68\tau_m$	$0.05\tau_m$	$\tau_m/T_m = 0.67$
	$0.85/K_m$	$0.65\tau_m$	$0.08\tau_m$	$\tau_m/T_m = 1.0$
	$0.65/K_m$	$0.65\tau_m$	$0.10\tau_m$	$\tau_m/T_m = 1.5$
	$0.45/K_m$	$0.55\tau_m$	$0.14\tau_m$	$\tau_m/T_m = 2.33$
	$0.4/K_m$	$0.50\tau_m$	$0.21\tau_m$	$\tau_m/T_m = 4.0$
	Coefficients of K_c, T_i, T_d *estimated from graphs; robust to* ±25% *process model parameter changes*			
Minimum IAE – Peng and Wu (2000). *Model: Method 17*	$6.558K_m$	$0.925T_m$	$0.061T_m$	$\tau_m/T_m = 0.1$
	$2.311K_m$	$0.677T_m$	$0.189T_m$	$\tau_m/T_m = 0.2$
	$1.479K_m$	$0.633T_m$	$0.233T_m$	$\tau_m/T_m = 0.3$
	$1.117K_m$	$0.647T_m$	$0.250T_m$	$\tau_m/T_m = 0.4$
	$0.947K_m$	$0.705T_m$	$0.268T_m$	$\tau_m/T_m = 0.5$
	$0.825K_m$	$0.752T_m$	$0.289T_m$	$\tau_m/T_m = 0.6$
	$0.731K_m$	$0.792T_m$	$0.312T_m$	$\tau_m/T_m = 0.7$
	$0.695K_m$	$0.871T_m$	$0.316T_m$	$\tau_m/T_m = 0.8$
	$0.630K_m$	$0.895T_m$	$0.326T_m$	$\tau_m/T_m = 0.9$
	$0.651K_m$	$1.030T_m$	$0.306T_m$	$\tau_m/T_m = 1.0$
Minimum IAE – Pessen (1994). *Model: Method 1*	$0.7K_u$	$0.4T_u$	$0.149T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Modified minimum IAE – Cheng and Hung (1985).	$\frac{3}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{\tau_m}{T_m}\right)^{0.749}$	$T_d^{(183)}$ [6]	*Model: Method 12*

[5] $T_d^{(182)} = 0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$.

[6] $T_d^{(183)} = 0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$.

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Murrill (1967) – *pages 358-363.*	$\frac{1.495}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.945}$	$\frac{T_m}{1.101}\left(\frac{\tau_m}{T_m}\right)^{0.771}$	$0.56T_m\left(\frac{\tau_m}{T_m}\right)^{1.006}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$; *Model: Method 5*
Minimum ISE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.473}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{1.115}\left(\frac{\tau_m}{T_m}\right)^{0.753}$	[7] $T_d^{(184)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.524}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.735}$	$\frac{T_m}{1.130}\left(\frac{\tau_m}{T_m}\right)^{0.641}$	[7] $T_d^{(185)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Ho *et al.* (1998). *Model: Method 1*	[8] $K_c^{(186)}$	$T_i^{(186)}$	$T_d^{(186)}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Minimum ITAE – Murrill (1967) – *pages 358-363.*	$\frac{1.357}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.947}$	$\frac{T_m}{0.842}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	[9] $T_d^{(187)}$	*Model: Method 5* $0.1 \le \frac{\tau_m}{T_m} \le 1$
Minimum ISTSE - Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.468}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{0.942}\left(\frac{\tau_m}{T_m}\right)^{0.725}$	[10] $T_d^{(188)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.515}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.730}$	$\frac{T_m}{0.957}\left(\frac{\tau_m}{T_m}\right)^{0.598}$	[10] $T_d^{(189)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$

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[7] $T_d^{(184)} = 0.550T_m\left(\frac{\tau_m}{T_m}\right)^{0.948}$, $T_d^{(185)} = 0.552T_m\left(\frac{\tau_m}{T_m}\right)^{0.851}$

[8] $K_c^{(186)} = \frac{1.0722}{K_m}\frac{\phi_m^{-0.116}}{A_m^{0.8432}}\left(\frac{\tau_m}{T_m}\right)^{-0.908}$, $T_i^{(186)} = \frac{1.2497T_m\phi_m^{1.0082}}{A_m^{0.2099}}\left(\frac{\tau_m}{T_m}\right)^{0.3678}$,

$$T_d^{(186)} = \frac{0.4763T_m\phi_m^{-0.328}}{A_m^{0.0961}}\left(\frac{\tau_m}{T_m}\right)^{1.0317};$$

Given A_m, ISE is minimised when $\phi_m = 46.5489A_m^{0.2035}(\tau_m/T_m)^{0.3693}$ (Ho *et al.* (1999)); $2 \le A_m \le 5$; $0.1 \le \tau_m/T_m \le 1.0$.

[9] $T_d^{(187)} = 0.381T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}$.

[10] $T_d^{(188)} = 0.443T_m\left(\frac{\tau_m}{T_m}\right)^{0.939}$, $T_d^{(189)} = 0.444T_m\left(\frac{\tau_m}{T_m}\right)^{0.847}$.

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993).	[11] $K_c^{(190)}$	$T_i^{(190)}$	$0.144T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 1*
	[12] $K_c^{(191)}$	$T_i^{(191)}$	$0.15\hat{T}_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 31*
Minimum ISTES – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.531}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.960}$	$\frac{T_m}{0.971}\left(\frac{\tau_m}{T_m}\right)^{0.746}$	[15] $T_d^{(192)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.592}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.705}$	$\frac{T_m}{0.957}\left(\frac{\tau_m}{T_m}\right)^{0.597}$	[15] $T_d^{(193)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum error – step load change – Gerry (1998).	$\frac{0.3}{K_m}$	$0.5\tau_m$	T_m	$\frac{\tau_m}{T_m} > 5$; *Model: Method 1*
Nearly minimum IAE, ISE, ITAE – Hwang (1995).	[16] $K_c^{(194)}$	$T_i^{(194)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\varepsilon_1 < 2.4$
	Model: Method 35; Decay ratio = 0.15; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$			

[11] $K_c^{(190)} = \frac{4.434K_mK_u - 0.966}{5.12K_mK_u + 1.734}K_u$, $T_i^{(190)} = \frac{1.751K_mK_u - 0.612}{3.776K_mK_u + 1.388}T_u$.

[12] $K_c^{(191)} = \frac{6.068K_m\hat{K}_u - 4.273}{5.758K_m\hat{K}_u - 1.058}\hat{K}_u$, $T_i^{(191)} = \frac{1.1622K_m\hat{K}_u - 0.748}{2.516K_m\hat{K}_u - 0.505}\hat{T}_u$.

[15] $T_d^{(192)} = 0.413T_m\left(\frac{\tau_m}{T_m}\right)^{0.933}$, $T_d^{(193)} = 0.414T_m\left(\frac{\tau_m}{T_m}\right)^{0.850}$.

[16] $K_c^{(194)} = \left(K_{H1} - \frac{0.674\left[1 - 0.447\omega_{H1}\tau_m + 0.0607\omega_{H1}^2\tau_m^2\right]}{K_m/(1 + K_{H1}K_m)}\right)$,

$$T_i^{(194)} = \frac{K_c^{(194)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0607(1 + 1.05\omega_{H1}\tau_m - 0.233\omega_{H1}^2\tau_m^2)}.$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) – continued.	[17] $K_c^{(195)}$	$T_i^{(195)}$		$2.4 \le \varepsilon_1 < 3$
	[18] $K_c^{(196)}$	$T_i^{(196)}$		$3 \le \varepsilon_1 < 20$
	[19] $K_c^{(197)}$	$T_i^{(197)}$		$\varepsilon_1 \ge 20$
Nearly minimum IAE, ITAE – Hwang and Fang (1995). *Model: Method 25*	[20] $K_c^{(198)}$	$T_i^{(198)}$	$T_d^{(198)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.12

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[17] $$K_c^{(195)} = \left(K_{H1} - \frac{0.778\left[1 - 0.467\omega_{H1}\tau_m + 0.0609\omega_{H1}^2\tau_m^2\right]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(195)} = \frac{K_c^{(195)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0309\left(1 + 2.84\omega_{H1}\tau_m - 0.532\omega_{H1}^2\tau_m^2\right)}.$$

[18] $$K_c^{(196)} = \left(K_{H1} - \frac{1.31(0.519)^{\omega_{H1}\tau_m}\left[1 - 1.03/\varepsilon_1 + 0.514/\varepsilon_1^2\right]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(196)} = \frac{K_c^{(196)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0603(1 + 0.929\ln[\omega_{H1}\tau_m])\left(1 + 2.01/\varepsilon_1 - 1.2/\varepsilon_1^2\right)}.$$

[19] $$K_c^{(197)} = \left(K_{H1} - \frac{1.14\left[1 - 0.482\omega_{H1}\tau_m + 0.068\omega_{H1}^2\tau_m^2\right]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(197)} = \frac{K_c^{(197)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0694\left(-1 + 2.1\omega_{H1}\tau_m - 0.367\omega_{H1}^2\tau_m^2\right)}.$$

[20] $$K_c^{(198)} = \left[0.802 - 0.154\frac{\tau_m}{T_m} + 0.0460\left(\frac{\tau_m}{T_m}\right)^2\right]K_u,$$

$$T_i^{(198)} = \frac{K_c^{(198)}}{K_u\omega_u\left[0.190 + 0.0532\frac{\tau_m}{T_m} - 0.00509\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$T_d^{(198)} = \left[0.421 + 0.00915\frac{\tau_m}{T_m} - 0.00152\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c^{(198)}}.$$

Rule	K_c	T_i	T_d	Comment
O'Connor and Denn (1972). *Model: Method 1*; $K_m = T_m$; $\frac{\tau_m}{T_m} < 0.2$	Minimum $0.5\int_0^\infty \left[a^2 y(t) + \left(\frac{du(t)}{dt}\right)^2\right]dt$			
	[21] $\frac{a\tau_m^2 + 2x_1}{(2+x_1)\tau_m}$	$\frac{a\tau_m^2 + 2x_1}{2\tau_m a}$	$\frac{x_1\tau_m}{\tau_m^2 a + 2x_1}$	Recommended $a \in \left[\frac{0.6}{\tau_m^2}, \frac{1}{\tau_m^2}\right]$
Syrcos and Kookos (2005). *Model: Method 1*	[22] $K_c^{(199)}$	$T_i^{(199)}$	$T_d^{(199)}$	$2 \le \frac{\tau_m}{T_m} \le 5$
	Minimum performance index – constraints are placed on the process input and output signals			
Minimum IE – Devanathan (1991). *Model: Method 1*; *Coefficients of* K_c, T_i *provided are deduced from graphs*	[23] $K_c^{(200)}$	$0.32T_u$	$0.08T_u$	
		x_2T_u	x_3T_u	
	Coefficient values			

τ_m/T_m	x_2	x_3	τ_m/T_m	x_2	x_3
0.2	0.31	0.08	1.2	0.26	0.07
0.4	0.29	0.07	1.4	0.26	0.07
0.6	0.29	0.07	1.6	0.24	0.06
0.8	0.29	0.07	1.8	0.24	0.06
1.0	0.28	0.07			

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[21] $$x_1 = \frac{a\tau_m^2 - 2\frac{\tau_m}{T_m} + 2\sqrt{\left(\frac{\tau_m}{T_m}\right)^2 + 2\tau_m^2 a}}{2 + \frac{\tau_m}{T_m}}.$$

[22] $$K_c^{(199)} = \frac{1}{K_m}\left[0.31 + 0.6\frac{T_m}{\tau_m}\right],\ T_i^{(199)} = T_m\left[0.777 + 0.45\frac{\tau_m}{T_m}\right],$$
$$T_d^{(199)} = T_m\left[0.44 - 0.56\left(\frac{T_m}{\tau_m}\right)^{2.2}\right].$$

[23] $$K_c^{(200)} = \frac{\xi\cos\left[\tan^{-1}\left(1.57D_R - \frac{0.16}{D_R}\right)\right]}{K_m\cos\left[\tan^{-1}\left(\frac{6.28T_m}{T_u}\right)\right]}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th colspan="2">Comment</th></tr>
<tr><td></td><td colspan="8">Minimum performance index: servo tuning</td></tr>
<tr><td rowspan="6">Minimum IAE – Gallier and Otto (1968). Model: Method 1</td><td colspan="2">x_1/K_m</td><td colspan="2">$x_2(T_m+\tau_m)$</td><td colspan="2">$x_3(T_m+\tau_m)$</td><td colspan="2"></td></tr>
<tr><td colspan="8">Representative coefficient values – deduced from graphs</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>x_3</td><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>x_3</td></tr>
<tr><td>0.053</td><td>12.3</td><td>0.95</td><td>0.0215</td><td>0.67</td><td>1.45</td><td>0.75</td><td>0.155</td></tr>
<tr><td>0.11</td><td>6.8</td><td>0.93</td><td>0.041</td><td>1.50</td><td>0.85</td><td>0.63</td><td>0.21</td></tr>
<tr><td>0.25</td><td>3.4</td><td>0.88</td><td>0.083</td><td>4.0</td><td>0.57</td><td>0.53</td><td>0.19</td></tr>
<tr><td>Minimum IAE – Rovira et al. (1969).</td><td colspan="2">$\frac{1.086}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.869}$</td><td colspan="2">$\frac{T_m}{0.740-0.13\frac{\tau_m}{T_m}}$</td><td colspan="2">[24] $T_d^{(201)}$</td><td colspan="2">Model: Method 5 $0.1<\frac{\tau_m}{T_m}\le 1$</td></tr>
<tr><td rowspan="8">Minimum IAE – Marlin (1995) – pages 301-307. Model: Method 1</td><td colspan="2">$1.3/K_m$</td><td colspan="2">$0.93\tau_m$</td><td colspan="2">$0.02\tau_m$</td><td colspan="2">$\tau_m/T_m=0.25$</td></tr>
<tr><td colspan="2">$1.0/K_m$</td><td colspan="2">$0.80\tau_m$</td><td colspan="2">$0.05\tau_m$</td><td colspan="2">$\tau_m/T_m=0.43$</td></tr>
<tr><td colspan="2">$0.8/K_m$</td><td colspan="2">$0.71\tau_m$</td><td colspan="2">$0.06\tau_m$</td><td colspan="2">$\tau_m/T_m=0.67$</td></tr>
<tr><td colspan="2">$0.7/K_m$</td><td colspan="2">$0.73\tau_m$</td><td colspan="2">$0.09\tau_m$</td><td colspan="2">$\tau_m/T_m=1.0$</td></tr>
<tr><td colspan="2">$0.55/K_m$</td><td colspan="2">$0.64\tau_m$</td><td colspan="2">$0.12\tau_m$</td><td colspan="2">$\tau_m/T_m=1.5$</td></tr>
<tr><td colspan="2">$0.48/K_m$</td><td colspan="2">$0.56\tau_m$</td><td colspan="2">$0.15\tau_m$</td><td colspan="2">$\tau_m/T_m=2.33$</td></tr>
<tr><td colspan="2">$0.4/K_m$</td><td colspan="2">$0.52\tau_m$</td><td colspan="2">$0.21\tau_m$</td><td colspan="2">$\tau_m/T_m=4.0$</td></tr>
<tr><td colspan="8">Coefficients of K_c, T_i, T_d estimated from graphs; robust to $\pm 25\%$ process model parameter changes.</td></tr>
<tr><td>Minimum IAE – Sadeghi and Tych (2003).</td><td colspan="2">[25] $K_c^{(202)}$</td><td colspan="2">$T_i^{(202)}$</td><td colspan="2">$\frac{1.45933\tau_m}{\frac{\tau_m}{T_m}+3.27873}$</td><td colspan="2">Model: Method 1 $0.1\le\frac{\tau_m}{T_m}\le 1.0$</td></tr>
<tr><td rowspan="2">Wang et al. (1995a). Model: Method 1 $0.05<\frac{\tau_m}{T_m}<6$</td><td colspan="2">[26] $K_c^{(203)}$</td><td colspan="2" rowspan="2">$T_m+0.5\tau_m$</td><td colspan="2" rowspan="2">$\frac{0.5T_m\tau_m}{T_m+0.5\tau_m}$</td><td colspan="2">Minimum IAE</td></tr>
<tr><td colspan="2">[26] $K_c^{(204)}$</td><td colspan="2">Minimum ISE</td></tr>
</table>

[24] $T_d^{(201)}=0.348T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}$.

[25] $K_c^{(202)}=\frac{1}{K_m}\left(0.26266+0.82714\frac{T_m}{\tau_m}\right)$, $T_i^{(202)}=T_m\left(0.28743\frac{\tau_m}{T_m}+1.35955\right)$.

[26] $K_c^{(203)}=\frac{\left(0.7645+\frac{0.6032}{\tau_m/T_m}\right)(T_m+0.5\tau_m)}{K_m(T_m+\tau_m)}$, $K_c^{(204)}=\frac{\left(0.9155+\frac{0.7524}{\tau_m/T_m}\right)(T_m+0.5\tau_m)}{K_m(T_m+\tau_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.048}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	[27] $T_i^{(205)}$	$T_d^{(205)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.154}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.567}$	[28] $T_i^{(206)}$	$T_d^{(206)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Ho *et al.* (1998). *Model: Method 1*	[29] $K_c^{(207)}$	$T_i^{(207)}$	$T_d^{(207)}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \le \frac{\tau_m}{T_m} \le 1.0$.
Minimum ISE – Sadeghi and Tych (2003).	[30] $K_c^{(208)}$	$T_i^{(208)}$	$\frac{1.93576\tau_m}{\frac{\tau_m}{T_m} + 3.83528}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$; *Model: Method 1*
Minimum ITAE – Rovira *et al.* (1969).	$\frac{0.965}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.85}$	[31] $T_i^{(209)}$	$T_d^{(209)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$; *Model: Method 5*

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[27] $$T_i^{(205)} = \frac{T_m}{1.195 - 0.368\frac{\tau_m}{T_m}},\ T_d^{(205)} = 0.489T_m\left(\frac{\tau_m}{T_m}\right)^{0.888}.$$

[28] $$T_i^{(206)} = \frac{T_m}{1.047 - 0.220\frac{\tau_m}{T_m}},\ T_d^{(206)} = 0.490T_m\left(\frac{\tau_m}{T_m}\right)^{0.708}.$$

[29] $$K_c^{(207)} = \frac{1.8578}{K_m}\frac{\phi_m^{0.0821}}{A_m^{0.9087}}\left(\frac{\tau_m}{T_m}\right)^{-0.9471},\ T_d^{(207)} = \frac{0.4899T_m\phi_m^{0.1457}}{A_m^{0.0845}}\left(\frac{\tau_m}{T_m}\right)^{1.0264},$$

$$T_i^{(207)} = \frac{0.0211T_m[1 + 0.3289A_m + 6.4572\phi_m + 25.1914(\tau_m/T_m)]}{1 + 0.0625A_m - 0.8079\phi_m + 0.347(\tau_m/T_m)};$$

Given A_m, with $2 \le A_m \le 5$ and $0.1 \le \tau_m/T_m \le 1.0$, ISE is minimised when $\phi_m = 29.7985 + \frac{62.1189}{A_m} + \frac{40.3182\tau_m}{T_m} - \frac{76.2833\tau_m}{A_m T_m}$ (Ho *et al.* (1999)).

[30] $$K_c^{(208)} = \frac{1}{K_m}\left(0.40455 + 0.96441\frac{T_m}{\tau_m}\right),\ T_i^{(208)} = T_m\left(0.43770\frac{\tau_m}{T_m} + 1.39588\right).$$

[31] $$T_i^{(209)} = \frac{T_m}{0.796 - 0.1465\frac{\tau_m}{T_m}},\ T_d^{(209)} = 0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Wang *et al.* (1995a). *Model: Method 1*	[32] $K_c^{(210)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ITAE – Sadeghi and Tych (2003).	[33] $K_c^{(211)}$	$T_i^{(211)}$	$\frac{1.55237\tau_m}{\frac{\tau_m}{T_m} + 4.00000}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$; *Model: Method 1*
Modified minimum ITAE – Cheng and Hung (1985).	$\frac{1.2}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.855}$	[34] $T_i^{(212)}$	$T_d^{(212)}$	$\xi = 0.707$; *Model: Method 12*
Modified minimum ITAE – Smith (2003).	$\frac{0.965}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.855}$	$1.26T_m$	$0.308\tau_m$	*Model: Method 1*
Minimum ISTSE – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.042}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	[35] $T_i^{(213)}$	$T_d^{(213)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.142}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.579}$	[36] $T_i^{(214)}$	$T_d^{(214)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$

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[32] $$K_c^{(210)} = \frac{\left(0.7303 + \frac{0.5307}{\tau_m/T_m}\right)(T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}.$$

[33] $$K_c^{(211)} = \frac{1}{K_m}\left(0.20360 + 0.80451\frac{T_m}{\tau_m}\right),\ T_i^{(211)} = T_m\left(0.29349\frac{\tau_m}{T_m} + 1.29110\right).$$

[34] $$T_i^{(212)} = \frac{T_m}{0.796 - 0.147\frac{\tau_m}{T_m}},\ T_d^{(212)} = 0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}.$$

[35] $$T_i^{(213)} = \frac{T_m}{0.987 - 0.238\frac{\tau_m}{T_m}},\ T_d^{(213)} = 0.385T_m\left(\frac{\tau_m}{T_m}\right)^{0.906}.$$

[36] $$T_i^{(214)} = \frac{T_m}{0.919 - 0.172\frac{\tau_m}{T_m}},\ T_d^{(214)} = 0.384T_m\left(\frac{\tau_m}{T_m}\right)^{0.839}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993).	$0.509K_u$	[37] $T_i^{(215)}$	$0.125T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 1*
	$0.604\hat{K}_u$	[37] $T_i^{(216)}$	$0.130\hat{T}_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 31*
Minimum ISTES – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{0.968}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.904}$	[38] $T_i^{(217)}$	$T_d^{(217)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.061}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.583}$	[39] $T_i^{(218)}$	$T_d^{(218)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Nearly minimum IAE, ISE, ITAE – Hwang (1995). *Model: Method 35*	[40] $K_c^{(219)}$	$T_i^{(219)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\varepsilon_1 < 2.4$
	[41] $K_c^{(220)}$	$T_i^{(220)}$		$2.4 \le \varepsilon_1 < 3$

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[37] $T_i^{(215)} = 0.051(3.302K_mK_u + 1)T_u$, $T_i^{(216)} = 0.04\left(4.972K_m\hat{K}_u + 1\right)\hat{T}_u$.

[38] $T_i^{(217)} = \dfrac{T_m}{0.977 - 0.253\dfrac{\tau_m}{T_m}}$, $T_d^{(217)} = 0.316T_m\left(\dfrac{\tau_m}{T_m}\right)^{0.892}$.

[39] $T_i^{(218)} = \dfrac{T_m}{0.892 - 0.165\dfrac{\tau_m}{T_m}}$, $T_d^{(218)} = 0.315T_m\left(\dfrac{\tau_m}{T_m}\right)^{0.832}$.

[40] $K_c^{(219)} = \left(K_{H1} - \dfrac{0.822\left[1 - 0.549\omega_{H1}\tau_m + 0.112\omega_{H1}^2\tau_m^2\right]}{K_m/(1 + K_{H1}K_m)}\right)$,

$$T_i^{(219)} = \frac{K_c^{(219)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0142\left(1 + 6.96\omega_{H1}\tau_m - 1.77\omega_{H1}^2\tau_m^2\right)} .$$

[41] $K_c^{(220)} = \left(K_{H1} - \dfrac{0.786\left[1 - 0.441\omega_{H1}\tau_m + 0.0569\omega_{H1}^2\tau_m^2\right]}{K_m/(1 + K_{H1}K_m)}\right)$,

$$T_i^{(220)} = \frac{K_c^{(220)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0172\left(1 + 4.62\omega_{H1}\tau_m - 0.823\omega_{H1}^2\tau_m^2\right)} .$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) - continued	42 $K_c^{(221)}$	$T_i^{(221)}$		$3 \le \varepsilon_1 < 20$
	43 $K_c^{(222)}$	$T_i^{(222)}$		$\varepsilon_1 \ge 20$
	Decay ratio = 0.1; $0.1 \le \frac{\tau_m}{T_m} \le 2.0$			
Nearly minimum IAE, ITAE – Hwang and Fang (1995).	44 $K_c^{(223)}$	$T_i^{(223)}$	$T_d^{(223)}$	*Model: Method 25*
	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.03			

42 $$K_c^{(221)} = \left(K_{H1} - \frac{1.28(0.542)^{\omega_{H1}\tau_m}\left[1 - 0.986/\varepsilon_1 + 0.558/\varepsilon_1^{\ 2}\right]}{K_m/(1 + K_{H1}K_m)}\right),$$

$$T_i^{(221)} = \frac{K_c^{(221)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0476(1 + 0.996\ln[\omega_{H1}\tau_m])(1 + 2.13/\varepsilon_1 - 1.13/\varepsilon_1^{\ 2})}.$$

43 $$K_c^{(222)} = \left(K_{H1} - \frac{1.14\left[1 - 0.466\omega_{H1}\tau_m + 0.0647\omega_{H1}^{\ 2}\tau_m^{\ 2}\right]}{K_m/(1 + K_{H1}K_m)}\right),$$

$$T_i^{(222)} = \frac{K_c^{(222)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0609(-1 + 1.97\omega_{H1}\tau_m - 0.323\omega_{H1}^{\ 2}\tau_m^{\ 2})}.$$

44 $$K_c^{(223)} = \left[0.537 - 0.0165\frac{\tau_m}{T_m} + 0.00173\left(\frac{\tau_m}{T_m}\right)^2\right]K_u,$$

$$T_i^{(223)} = \frac{K_c^{(223)}}{K_u\omega_u\left[0.0503 + 0.163\frac{\tau_m}{T_m} - 0.0389\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$T_d^{(223)} = \left[0.350 - 0.0344\frac{\tau_m}{T_m} + 0.00644\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c^{(223)}}.$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: other tuning			
Simultaneous Servo/regulator tuning – nearly minimum IAE, ITAE – Hwang and Fang (1995).	[45] $K_c^{(224)}$	$T_i^{(224)}$	$T_d^{(224)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.05; *Model: Method 25*
Servo or regulator tuning – minimum IAE – Huang and Jeng (2003).	$\dfrac{0.36 + 0.76\frac{T_m}{\tau_m}}{K_m}$	$0.47\tau_m + T_m$	$\dfrac{0.47 T_m \tau_m}{0.47\tau_m + T_m}$	*Model: Method 37*; $\frac{\tau_m}{T_m} < 0.33$
	[46] $K_c^{(225)}$	$T_i^{(225)}$	$T_d^{(225)}$	$\frac{\tau_m}{T_m} \ge 0.33$
Åström and Hägglund (2004). *Model: Method 1*	$\dfrac{x_1\tau_m + x_2 T_m}{K_m \tau_m}$	$\dfrac{x_3\tau_m + x_4 T_m}{\tau_m + \alpha_5 T_m}\tau_m$	$\dfrac{x_6 T_m \tau_m}{\tau_m + x_7 T_m}$	

Coefficient values (Åström and Hägglund (2004)):

x_1	x_2	x_3	x_4	x_5	x_6	x_7	Comment
0.057	0.139	0.400	0.923	0.012	1.59	4.59	$M_{max} = 1.1$
0.103	0.261	0.389	0.930	0.040	1.62	4.44	$M_{max} = 1.2$
0.139	0.367	0.376	0.900	0.074	1.66	4.39	$M_{max} = 1.3$
0.168	0.460	0.363	0.871	0.111	1.70	4.37	$M_{max} = 1.4$
0.191	0.543	0.352	0.844	0.146	1.74	4.35	$M_{max} = 1.5$

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[45] $$K_c^{(224)} = \left[0.713 - 0.176\frac{\tau_m}{T_m} + 0.0513\left(\frac{\tau_m}{T_m}\right)^2\right]K_u,$$

$$T_i^{(224)} = \frac{K_c^{(224)}}{K_u\omega_u\left[0.149 + 0.0556\frac{\tau_m}{T_m} - 0.00566\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$T_d^{(224)} = \left[0.371 - 0.0274\frac{\tau_m}{T_m} + 0.00557\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c^{(224)}}.$$

[46] $$K_c^{(225)} = \frac{0.8194 T_n + 0.2773\tau_m}{K_m \tau_m^{0.9738} T_m^{0.0262}},\ T_i^{(225)} = 1.0297 T_m + 0.3484\tau_m,$$

$$T_d^{(225)} = \left(\frac{0.4575 T_m + 0.0302\tau_m}{1.0297 T_m + 0.3484\tau_m}\right)\tau_m.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td rowspan="7">Åström and Hägglund (2004) - continued.</td><td colspan="7">Coefficient values</td><td rowspan="2"></td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td><td>x_5</td><td>x_6</td><td>x_7</td></tr>
<tr><td>0.211</td><td>0.616</td><td>0.342</td><td>0.820</td><td>0.179</td><td>1.78</td><td>4.34</td><td>$M_{max} = 1.6$</td></tr>
<tr><td>0.227</td><td>0.681</td><td>0.334</td><td>0.799</td><td>0.209</td><td>1.81</td><td>4.33</td><td>$M_{max} = 1.7$</td></tr>
<tr><td>0.241</td><td>0.740</td><td>0.326</td><td>0.781</td><td>0.238</td><td>1.84</td><td>4.32</td><td>$M_{max} = 1.8$</td></tr>
<tr><td>0.254</td><td>0.793</td><td>0.320</td><td>0.764</td><td>0.264</td><td>1.87</td><td>4.31</td><td>$M_{max} = 1.9$</td></tr>
<tr><td>0.264</td><td>0.841</td><td>0.314</td><td>0.751</td><td>0.288</td><td>1.89</td><td>4.30</td><td>$M_{max} = 2.0$</td></tr>
<tr><td></td><td colspan="7">Direct synthesis: time domain criteria</td><td></td></tr>
<tr><td rowspan="2">Van der Grinten (1963).
Model: Method 1</td><td colspan="2">$\frac{1}{K_m}\left(0.5+\frac{T_m}{\tau_m}\right)$</td><td colspan="2">$T_m + 0.5\tau_m$</td><td colspan="2">$\frac{T_m \tau_m}{\tau_m + 2T_m}$</td><td>Step disturbance</td></tr>
<tr><td colspan="2">[47] $K_c^{(226)}$</td><td colspan="2">$T_i^{(226)}$</td><td colspan="2">$T_d^{(226)}$</td><td>Stochastic disturbance</td></tr>
<tr><td>Regulator – Gorecki et al. (1989).
Model: Method 1</td><td colspan="2">[48] $K_c^{(227)}$</td><td colspan="2">$T_i^{(227)}$</td><td colspan="2">$T_d^{(227)}$</td><td>Pole is real and has maximum attainable multiplicity; $\tau_m/T_m < 2$</td></tr>
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[47] $$K_c^{(226)} = \frac{e^{-\omega_d \tau_m}}{K_m}\left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m}e^{-\omega_d \tau_m}\right),$$

$$T_i^{(226)} = \frac{\tau_m}{e^{-\omega_d \tau_m}}\left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m}e^{-\omega_d \tau_m}\right),\ T_d^{(226)} = \frac{\left(1 - 0.5e^{-\omega_d \tau_m}\right)T_m}{1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m}e^{-\omega_d \tau_m}}.$$

[48] $$K_c^{(227)} = \frac{2}{K_m}\frac{T_m}{\tau_m}\left[6 + \frac{\tau_m}{2T_m}x_1 - 9 - \left(\frac{\tau_m}{2T_m}\right) - \left(\frac{\tau_m}{2T_m}\right)^2\right]e^{x_1 - 3 - \frac{\tau_m}{2T_m}},$$

$$T_i^{(227)} = \tau_m \frac{\left[6 + \frac{\tau_m}{2T_m}\right]x_1 - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m}\right)^2}{\left[21 + 3\frac{\tau_m}{T_m} + \left(\frac{\tau_m}{2T_m}\right)^2\right]x_1 - 36 - 4.5\frac{\tau_m}{T_m} - 6\left(\frac{\tau_m}{2T_m}\right)^2 - \left(\frac{\tau_m}{2T_m}\right)^3},$$

$$T_d^{(227)} = 0.5\tau_m \frac{x_1 - 1}{\left[6 + \frac{\tau_m}{2T_m}\right]x_1 - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m}\right)^2},\ x_1 = \sqrt{3 + \left(\frac{\tau_m}{2T_m}\right)^2}.$$

Rule	K_c	T_i	T_d	Comment
Saito *et al.* (1990). *Model: Method 18*	$\frac{0.215\tau_m + T_m}{1.37K_m\tau_m}$	$0.315\tau_m + T_m$	$\frac{0.315T_m\tau_m}{0.315\tau_m + T_m}$	$\frac{\tau_m}{T_m} < 1$
			[49] $T_d^{(228)}$	$1 \le \frac{\tau_m}{T_m} < 10$
Suyama (1992). *Model: Method 1*	[50] $K_c^{(229)}$	$T_m + 0.309\tau_m$	$T_d^{(229)}$	OS=10%
Fruehauf *et al.* (1993). *Model: Method 2*	$\frac{0.56T_m}{\tau_m K_m}$	$5\tau_m$	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{0.5T_m}{\tau_m K_m}$	T_m	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} \ge 0.33$
Juang and Wang (1995). *Model: Method 1*	[51] $K_c^{(230)}$	$T_i^{(230)}$	$T_d^{(230)}$	$T_{CL} = \alpha T_m$, $0 < \alpha < 1$
Abbas (1997) *Model: Method 1*	[52] $K_c^{(231)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0 \le V \le 0.2$; $0.1 \le \frac{\tau_m}{T_m} \le 5.0$
Camacho *et al.* (1997).	$\frac{1}{K_m}\frac{T_m + \tau_m}{T_m\tau_m}$	$\frac{4T_m\tau_m}{T_m + \tau_m}$	$\frac{T_m\tau_m}{T_m + \tau_m}$	*Model: Method 1*

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[49] $T_d^{(228)} = \dfrac{0.315\tau_m T_m + 0.003\tau_m^2}{0.315\tau_m + T_m}$.

[50] $K_c^{(229)} = \dfrac{1}{K_m}\left[0.7236\dfrac{T_m}{\tau_m} + 0.2236\right]$, $T_d^{(229)} = \dfrac{2.236T_m\tau_m}{7.236T_m + 2.236\tau_m}$.

[51] $$K_c^{(230)} = \frac{\alpha + \frac{\tau_m}{T_m} + 0.5\left(\frac{\tau_m}{T_m}\right)^2}{K_m\left(\alpha + \frac{\tau_m}{T_m}\right)^2},\ T_i^{(230)} = T_m\frac{\alpha + \frac{\tau_m}{T_m} + 0.5\left(\frac{\tau_m}{T_m}\right)^2}{\left(\alpha + \frac{\tau_m}{T_m}\right)},$$

$$T_d^{(230)} = \frac{0.5\left(\frac{\tau_m}{T_m}\right)^2 T_m\left(\alpha + \frac{\tau_m}{T_m} - 0.5\alpha\frac{\tau_m}{T_m}\right)}{\left(\alpha + \frac{\tau_m}{T_m}\right)\left(\alpha + \frac{\tau_m}{T_m} + 0.5\left[\frac{\tau_m}{T_m}\right]^2\right)}.$$

[52] $$K_c^{(231)} = \frac{0.177 + 0.348\left(\frac{\tau_m}{T_m}\right)^{-1.002}}{K_m(0.531 - 0.359V^{0.713})}.$$

Rule	K_c	T_i	T_d	Comment
Cluett and Wang (1997), Wang and Cluett (2000). *Model: Method 1*; $0.01 \le \frac{\tau_m}{T_m} \le 10$	53 $K_c^{(232)}$	$T_i^{(232)}$	$T_d^{(232)}$	
	$T_{CL} = 4\tau_m$, $A_m \in [7.97, 8.56]$, $\phi_m \in [79.67^0, 79.70^0]$			
	54 $K_c^{(233)}$	$T_i^{(233)}$	$T_d^{(233)}$	
	$T_{CL} = 2\tau_m$, $A_m \in [4.84, 5.31]$, $\phi_m \in [73.9^0, 74.14^0]$			
	55 $K_c^{(234)}$	$T_i^{(234)}$	$T_d^{(234)}$	
	$T_{CL} = 1.33\tau_m$, $A_m \in [3.81, 4.16]$, $\phi_m \in [69.84^0, 70.70^0]$			
	56 $K_c^{(235)}$	$T_i^{(235)}$	$T_d^{(235)}$	
	$T_{CL} = \tau_m$, $A_m \in [3.30, 3.56]$, $\phi_m \in [65.86^0, 68.50^0]$			
	57 $K_c^{(236)}$	$T_i^{(236)}$	$T_d^{(236)}$	
	$T_{CL} = 0.8\tau_m$, $A_m \in [3.00, 3.19]$, $\phi_m \in [60.60^0, 67.21^0]$			

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53 $K_c^{(232)} = \frac{0.019952\tau_m + 0.20042T_m}{K_m\tau_m}$, $T_i^{(232)} = \frac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425.10^{-5}T_m}\tau_m$,

$$T_d^{(232)} = \frac{-0.0069905\tau_m + 0.029480T_m}{0.029773\tau_m + 0.29907T_m}\tau_m.$$

54 $K_c^{(233)} = \frac{0.055548\tau_m + 0.33639T_m}{K_m\tau_m}$, $T_i^{(233)} = \frac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032.10^{-4}T_m}\tau_m$,

$$T_d^{(233)} = \frac{-0.016651\tau_m + 0.093641T_m}{0.093905\tau_m + 0.56867T_m}\tau_m.$$

55 $K_c^{(234)} = \frac{0.092654\tau_m + 0.43620T_m}{K_m\tau_m}$, $T_i^{(234)} = \frac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550.10^{-3}T_m}\tau_m$,

$$T_d^{(234)} = \frac{-0.024442\tau_m + 0.17669T_m}{0.17150\tau_m + 0.80740T_m}\tau_m.$$

56 $K_c^{(235)} = \frac{0.12786\tau_m + 0.51235T_m}{K_m\tau_m}$, $T_i^{(235)} = \frac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988.10^{-2}T_m}\tau_m$,

$$T_d^{(235)} = \frac{-0.030407\tau_m + 0.27480T_m}{0.25285\tau_m + 1.0132T_m}\tau_m.$$

57 $K_c^{(236)} = \frac{0.16051\tau_m + 0.57109T_m}{K_m\tau_m}$, $T_i^{(236)} = \frac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355.10^{-2}T_m}\tau_m$,

$$T_d^{(236)} = \frac{-0.035204\tau_m + 0.38823T_m}{0.33303\tau_m + 1.1849T_m}\tau_m.$$

Rule	K_c	T_i	T_d	Comment
Cluett and Wang (1997), Wang and Cluett (2000) – continued.	[58] $K_c^{(237)}$	$T_i^{(237)}$	$T_d^{(237)}$	
	$T_{CL} = 0.67\tau_m$, $A_m \in [2.80, 2.95]$, $\phi_m \in [52.35^0, 66.45^0]$			
Valentine and Chidambaram (1997a).	[59] $K_c^{(238)}$	$T_i^{(238)}$	$T_d^{(238)}$	$\xi = 0.707$; *Model: Method 1*
Morilla *et al.* (2000). *Model: Method 28*	[60] $K_c^{(239)}$	$T_i^{(239)}$	$\alpha T_i^{(239)}$	$\alpha = 0.1$; $\delta_0 = [0.2, 0.5]$

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[58] $K_c^{(237)} = \dfrac{0.19067\tau_m + 0.61593T_m}{K_m\tau_m}$, $T_i^{(237)} = \dfrac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m}\tau_m$,

$$T_d^{(237)} = \frac{-0.039589\tau_m + 0.51941T_m}{0.40950\tau_m + 1.3228T_m}\tau_m .$$

[59] $K_c^{(238)} = \dfrac{1}{K_m}\left[0.746 - 1.242\ln\left(\dfrac{\tau_m}{T_m}\right)\right]$,

$$T_i^{(238)} = T_m\left[0.3106 + 1.372\frac{\tau_m}{T_m} - 0.545\left(\frac{\tau_m}{T_m}\right)^2\right], T_d^{(238)} = \frac{T_m^{\ 2}}{16.015T_m - 11.702\tau_m} .$$

$\pm 1\%$ $T_S = 2.5T_m$, $0.1 \le \dfrac{\tau_m}{T_m} \le 0.5$; $\pm 1\%$ $T_S = 3.0T_m$, $\dfrac{\tau_m}{T_m} = 0.6$; $\pm 1\%$ $T_S = 3.75T_m$, $\dfrac{\tau_m}{T_m} = 0.7$; $\pm 1\%$ $T_S = 4.3T_m$, $\dfrac{\tau_m}{T_m} = 0.8$; $\pm 1\%$ $T_S = 5.0T_m$, $\dfrac{\tau_m}{T_m} = 0.9$.

[60] $K_c^{(239)} = \dfrac{T_i^{(239)}\omega_{n0}}{K_m[2\delta_0 + \omega_{n0}(\tau_m - T_i^{(239)})]}$, $T_i^{(239)} = \dfrac{\tau_m}{1 + \sqrt{1 - 4\alpha}}$,

$$\omega_{n0} = \frac{-\delta_0 T_m + \sqrt{\delta_0^{\ 2}T_m^{\ 2} + T_m\left(\tau_m - T_i^{(239)}\right) - \alpha[T_i^{(239)}]^2}}{T_m\left(\tau_m - T_i^{(239)}\right) - \alpha[T_i^{(239)}]^2}$$

with $\delta_0 = \dfrac{1}{\sqrt{1 + \left(\dfrac{2\pi}{\log_e[b/a]}\right)^2}}$, $\dfrac{b}{a}$ = desired closed loop response decay ratio.

Rule	K_c	T_i	T_d	Comment
Mann *et al.* (2001). *Model: Method 31 or Method 33*	[61] $K_c^{(240)}$	$T_i^{(240)}$	$T_d^{(240)}$	OS < 5%; $0 < \tau_m/T_m < 1$
	[62] $K_c^{(241)}$	$T_i^{(241)}$	$T_d^{(241)}$	OS < 5%; $1 \le \tau_m/T_m < 2$
Chen and Seborg (2002).	[63] $K_c^{(242)}$	$T_i^{(242)}$	$T_d^{(242)}$	*Model: Method 1*

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[61] $$K_c^{(240)} = \frac{\left[0.770 + 0.245\left(\frac{\tau_m}{T_m}\right)^{0.854}\right]T_m}{K_m \tau_m},\quad T_i^{(240)} = \left[1.262 + 0.147\left(\frac{\tau_m}{T_m}\right)^{0.854}\right]T_m,$$

$$T_d^{(240)} = \frac{0.262 + 0.147\left(\frac{\tau_m}{T_m}\right)^{0.854}}{0.770 + 0.245\left(\frac{\tau_m}{T_m}\right)^{0.854}}\tau_m.$$

[62] $$K_c^{(241)} = \frac{\left[0.603 + 0.275\left(\frac{\tau_m}{T_m}\right)^{-2.4}\right]T_m}{K_m \tau_m},\quad T_i^{(241)} = \left[1.1618 + 0.165\left(\frac{\tau_m}{T_m}\right)^{-2.4}\right]T_m,$$

$$T_d^{(241)} = \frac{0.1618 + 0.165\left(\frac{\tau_m}{T_m}\right)^{-2.4}}{0.603 + 0.275\left(\frac{\tau_m}{T_m}\right)^{-2.4}}\tau_m.$$

[63] $$K_c^{(242)} = \frac{1}{K_m}\frac{\left(2T_m\tau_m + 0.5\tau_m^2\right)\left(3T_{CL} + 0.5\tau_m\right) - 2T_{CL}^3 - 3T_{CL}^2\tau_m}{2T_{CL}^3 + 3T_{CL}^2\tau_m + 0.5\tau_m^2\left(3T_{CL} + 0.5\tau_m\right)},$$

$$T_i^{(242)} = \frac{\left(2T_m\tau_m + 0.5\tau_m^2\right)\left(3T_{CL} + 0.5\tau_m\right) - 2T_{CL}^3 - 3T_{CL}^2\tau_m}{\tau_m\left(2T_m + \tau_m\right)},$$

$$T_d^{(242)} = \frac{3T_{CL}^2\tau_m T_m + 0.5T_m\tau_m^2\left(3T_{CL} + 0.5\tau_m\right) - 2\left(T_m + \tau_m\right)T_{CL}^3}{\left(2T_m\tau_m + 0.5\tau_m^2\right)\left(3T_{CL} + 0.5\tau_m\right) - 2T_{CL}^3 - 3T_{CL}^2\tau_m};$$

$$\left(\frac{y}{d}\right)_{desired} = \frac{T_i^{(242)}s\left(1 + 0.5\tau_m s\right)e^{-s\tau_m}}{K_c^{(242)}\left(T_{CL}s + 1\right)^3}.$$

Rule	K_c	T_i	T_d	Comment
Vítečková and Víteček (2002). *Model: Method 1*	[64] $K_c^{(243)}$	$T_i^{(243)}$	$T_d^{(243)}$	
	For $0.05 < \frac{\tau_m}{T_m} < 1.6$, $T_i = 1.2T_m$; overshoot is under 2% (Vítečková and Víteček (2003)).			
Gorez (2003). *Model: Method 1*	$\frac{(1-v)\tau_m + T_m}{\chi K_m \tau_m}$	$(1-v)\tau_m + T_m$	$\frac{(1-v)\tau_m T_m}{(1-v)\tau_m + T_m}$	[65]
Sree *et al.* (2004). *Model: Method 1*	$\frac{1}{K_m}\left(\frac{T_m}{\tau_m} + 0.5\right)$	$T_m + 0.5\tau_m$	[66] $T_d^{(244)}$	
	$\frac{1.377}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8422}$	[67] $T_i^{(245)}$	$T_d^{(245)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$

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[64] $K_c^{(243)} = \frac{e^{\tau_m x_1}}{K_m}\left[\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2)x_1^2 + \tau_m x_1 - 1\right]$,

$$x_1 = -\frac{3}{\tau_m} - \frac{0.5}{T_m} + \sqrt{\frac{3}{\tau_m^2} + \frac{0.25}{T_m^2}} \; ;$$

$$T_i^{(243)} = -2\left[\frac{\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2)x_1^2 + \tau_m x_1 - 1}{x_1^3(\tau_m^2 T_m x_1 + 2\tau_m T_m + \tau_m^2)}\right],$$

$$T_d^{(243)} = -0.5\left[\frac{\tau_m^2 T_m x_1^2 + (4\tau_m T_m + \tau_m^2)x_1 + 2\tau_m + 2T_m}{\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2)x_1^2 + \tau_m x_1 - 1}\right].$$

[65] For $0 \le \frac{\tau_m}{\tau_m + T_m} \le \tau_m$, $v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$,

$$\chi = \frac{1}{2M_s(M_s - 1)} + \frac{(1.3M_s^2 - 1)}{2M_s(M_s - 1)} \frac{1.56\tau_m^2(0.5\tau_m + 3T_m)}{(\tau_m + T_m)^3};$$

For $\tau_m \le \frac{\tau_m}{\tau_m + T_m} \le 1$, $v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$, $\chi = \frac{0.65M_s}{M_s - 1}$.

[66] $T_d^{(244)} = \frac{0.5\tau_m(T_m + 0.1667\tau_m)}{T_m + 0.5\tau_m}$.

[67] $T_i^{(245)} = 1.085T_m\left(\frac{\tau_m}{T_m}\right)^{0.4777}$, $T_d^{(245)} = T_m\left[0.3899\frac{\tau_m}{T_m} + 0.0195\right]$.

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="3">T_d</th><th colspan="3">Comment</th></tr>
<tr><td></td><td colspan="12">Direct synthesis: frequency domain criteria</td></tr>
<tr><td rowspan="2">Regulator – Gorecki et al. (1989).</td><td colspan="3">[68] $K_c^{(246)}$</td><td colspan="3">$T_i^{(246)}$</td><td colspan="3">$T_d^{(246)}$</td><td colspan="3">Model: Method 1</td></tr>
<tr><td colspan="12">Low frequency part of magnitude Bode diagram is flat.</td></tr>
<tr><td rowspan="10">Somani et al. (1992). Model: Method 1; Suggested A_m : $1.75 \le A_m \le 3.0$</td><td colspan="3">[69] $K_c^{(247)}$</td><td colspan="3">$x_2\tau_m$</td><td colspan="3">$x_3\tau_m$</td><td colspan="3"></td></tr>
<tr><td colspan="12">Coefficient values</td></tr>
<tr><td colspan="2">τ_m/T_m</td><td colspan="2">x_2</td><td colspan="2">x_3</td><td colspan="2">τ_m/T_m</td><td colspan="2">x_2</td><td colspan="2">x_3</td></tr>
<tr><td colspan="2">0.0375</td><td colspan="2">3.57</td><td colspan="2">0.71</td><td colspan="2">1.869</td><td colspan="2">2.38</td><td colspan="2">0.48</td></tr>
<tr><td colspan="2">0.1912</td><td colspan="2">3.33</td><td colspan="2">0.67</td><td colspan="2">2.392</td><td colspan="2">2.27</td><td colspan="2">0.45</td></tr>
<tr><td colspan="2">0.3693</td><td colspan="2">3.13</td><td colspan="2">0.63</td><td colspan="2">3.067</td><td colspan="2">2.17</td><td colspan="2">0.43</td></tr>
<tr><td colspan="2">0.5764</td><td colspan="2">2.94</td><td colspan="2">0.59</td><td colspan="2">3.968</td><td colspan="2">2.08</td><td colspan="2">0.42</td></tr>
<tr><td colspan="2">0.8183</td><td colspan="2">2.78</td><td colspan="2">0.56</td><td colspan="2">5.263</td><td colspan="2">2.00</td><td colspan="2">0.40</td></tr>
<tr><td colspan="2">1.105</td><td colspan="2">2.63</td><td colspan="2">0.53</td><td colspan="2">7.246</td><td colspan="2">1.92</td><td colspan="2">0.38</td></tr>
<tr><td colspan="2">1.449</td><td colspan="2">2.50</td><td colspan="2">0.50</td><td colspan="2">10.870</td><td colspan="2">1.85</td><td colspan="2">0.37</td></tr>
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[68] $$K_c^{(246)} = \frac{1}{K_m} \frac{1}{2\dfrac{\tau_m}{T_i^{(246)}}\left(\dfrac{T_m}{\tau_m}+1\right)-2},$$

$$T_i^{(246)} = \tau_m \frac{7 + 42\dfrac{T_m}{\tau_m} + 135\left(\dfrac{T_m}{\tau_m}\right)^2 + 240\left(\dfrac{T_m}{\tau_m}\right)^3 + 180\left(\dfrac{T_m}{\tau_m}\right)^4}{15\left[2\dfrac{T_m}{\tau_m}+1\right]\left[1 + 3\dfrac{T_m}{\tau_m} + 6\left(\dfrac{T_m}{\tau_m}\right)^2\right]},$$

$$T_d^{(246)} = \tau_m \frac{1 + 7\dfrac{T_m}{\tau_m} + 27\left(\dfrac{T_m}{\tau_m}\right)^2 + 60\left(\dfrac{T_m}{\tau_m}\right)^3 + 60\left(\dfrac{T_m}{\tau_m}\right)^4}{7 + 42\dfrac{T_m}{\tau_m} + 135\left(\dfrac{T_m}{\tau_m}\right)^2 + 240\left(\dfrac{T_m}{\tau_m}\right)^3 + 180\left(\dfrac{T_m}{\tau_m}\right)^4}.$$

[69] For $\dfrac{\tau_m}{T_m} < 1$, $K_c^{(247)} \approx \dfrac{0.7809}{K_m A_m}\sqrt{\dfrac{4}{\left(\sqrt{0.803 + 4\dfrac{\tau_m}{T_m}} + 0.896\right)^2} + 1}$ or

$$K_c^{(247)} \approx \frac{0.7809}{K_m A_m}\sqrt{1 + 0.803\left(\frac{T_m}{\tau_m}\right)^2}\ ;\ K_c^{(247)} \approx \frac{0.7809}{K_m A_m}\sqrt{1 + 0.455\left(\frac{T_m}{\tau_m}\right)^2}\ ,\ \frac{\tau_m}{T_m} > 1.$$

Rule	K_c		T_i		T_d		Comment	
Chidambaram (1995a). *Model: Method 1*	[70] $K_c^{(248)}$		$T_i^{(248)}$		$T_d^{(248)}$			
	x_1/K_mA_m		$x_2\tau_m$		$x_3\tau_m$		Alternative	
	Coefficient values							
	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.062	29.770	2.34	0.37	0.389	5.104	2.19	0.35
	0.083	22.100	2.33	0.37	0.543	3.781	2.14	0.34
	0.150	12.470	2.29	0.37	1.03	2.262	2.00	0.32
	0.196	9.670	2.27	0.36	2.92	1.209	1.72	0.28
	0.290	6.686	2.23	0.36	3.69	1.105	1.67	0.27
	0.339	5.784	2.21	0.35				
Chidambaram (1995a). *Model: Method 1*	$\frac{1.20T_m}{K_m\tau_m}$		$2.4\tau_m$		$0.38\tau_m$		$A_m = 1.5$	
	$\frac{1.03T_m}{K_m\tau_m}$		$2.4\tau_m$		$0.38\tau_m$		$A_m = 1.75$	

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[70] $$K_c^{(248)} \approx \frac{1}{K_mA_m}\left[\frac{3.42}{\left(\sqrt{4.45+4\frac{\tau_m}{T_m}}-2.11\right)^2}+0.85\right] \text{ or } \frac{0.85}{K_mA_m}\sqrt{1+4.45\left(\frac{T_m}{\tau_m}\right)^2};$$

$$T_i^{(248)} \approx \frac{5T_m}{\sqrt{1.17\left[\frac{3.42}{\left(\sqrt{4.45+4\frac{\tau_m}{T_m}}-2.11\right)^2}+0.85\right]^2-1}} \text{ or } \frac{5T_m}{\sqrt{4.45\left(\frac{T_m}{\tau_m}\right)^2-0.15}};$$

$$T_d^{(248)} \approx \frac{0.8T_m}{\sqrt{1.17\left[\frac{3.42}{\left(\sqrt{4.45+4\frac{\tau_m}{T_m}}-2.11\right)^2}+0.85\right]^2-1}} \text{ or } \frac{0.8T_m}{\sqrt{4.45\left(\frac{T_m}{\tau_m}\right)^2-0.15}}.$$

Rule	K_c	T_i	T_d	Comment
Chidambaram (1995a) – continued.	$\frac{0.90T_m}{K_m\tau_m}$	$2.4\tau_m$	$0.38\tau_m$	$A_m = 2.0$
Branica *et al.* (2002). *Model: Method 1*; $M_s = 1.8$	[71] $K_c^{(249)}$	$T_i^{(249)}$	$T_d^{(249)}$	$0.1 \le \frac{\tau_m}{T_m} \le 0.5$
	[72] $K_c^{(250)}$	$T_i^{(250)}$	$T_d^{(250)}$	$0.5 < \frac{\tau_m}{T_m} \le 1.1$
	[73] $K_c^{(251)}$	$T_i^{(251)}$	$T_d^{(251)}$	$1.1 < \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTSE – Zhuang and Atherton (1993). *Model: Method 1*	[74] $mK_u\cos(\phi_m)$	$T_i^{(252)}$	$\frac{T_i^{(252)}}{\alpha}$	$A_m = 2$, $\phi_m = 60^0$

[71] $K_c^{(249)} = \frac{1}{K_m}\left(0.021 + 0.788\frac{T_m}{\tau_m}\right)$, $T_i^{(249)} = 1.534\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.326}$,

$$T_d^{(249)} = 0.233\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.055}.$$

[72] $K_c^{(250)} = \frac{1}{K_m}\left(0.292 + 0.671\frac{T_m}{\tau_m}\right)$, $T_i^{(250)} = 1.239\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.554}$,

$$T_d^{(250)} = 0.230\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.069}.$$

[73] $K_c^{(251)} = \frac{1}{K_m}\left(0.338 + 0.623\frac{T_m}{\tau_m}\right)$, $T_i^{(251)} = 1.228\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.480}$,

$$T_d^{(251)} = 0.231\tau_m\left(\frac{\tau_m}{T_m}\right)^{-0.120}.$$

[74] $m = 0.614(1 - 0.233e^{-0.347K_mK_u})$, $\phi_m = 33.8^0\left(1 - 0.97e^{-0.45K_mK_u}\right)$, $0.1 \le \frac{\tau_m}{T_m} \le 2.0$,

$$\alpha = 0.413(3.302K_mK_u + 1),\ T_i^{(252)} = \alpha\frac{\tan(\phi_m) + \sqrt{\frac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}.$$

Rule	K_c			T_i			T_d			Comment		
Minimum ISTSE – Zhuang and Atherton (1993) – continued.	[75] $mK_u \cos(\phi_m)$			$T_i^{(252)}$ (page 176)			$\frac{T_i^{(252)}}{\alpha}$			$A_m = 2$, $\phi_m = 60^0$		
Li *et al.* (1994). *Model: Method 1*	[76] $K_c^{(253)}$			$4T_d^{(253)}$			$T_d^{(253)}$					
	ϕ_m	A_m	η	ϕ_m	A_m	η	ϕ_m	A_m	η	ϕ_m	A_m	η
	15^0	2	2.8	30^0	1.67	3.8	45^0	1.25	4.6	60^0	1.11	5.4
	20^0	1.67	3.2	35^0	1.43	4.0	50^0	1.25	4.9	65^0	1.11	5.5
	25^0	1.67	3.5	40^0	1.43	4.2	55^0	1.25	5.2			
	$\frac{4h}{\pi A_p A_m}$			$0.3183T$			$0.0796T$			simplified algorithm		
Tan *et al.* (1996). *Model: Method 30*	$\frac{K_u}{A_m}\cos\phi_m$			$\alpha T_d^{(254)}$			[77] $T_d^{(254)}$			α chosen arbitrarily		
	$\frac{K_\phi}{A_m}$			[78] $T_i^{(255)}$			$T_d^{(255)}$			Arbitrary A_m, ϕ_m at ω_ϕ		

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[75] *Model: Method 31*; $m = 0.613(1 - 0.262e^{-0.44K_m \hat{K}_u})$, $\phi_m = 33.2^0\left(1 - 1.38e^{-0.68K_m \hat{K}_u}\right)$,

$\alpha = 1.687K_m \hat{K}_u$, $0.1 \le \frac{\tau_m}{T_m} \le 2.0$.

[76] $$K_c^{(253)} = \frac{4h\cos\phi_m}{\pi A_m\left[\sqrt{A_p^2 - b^2} + \frac{6\pi b}{T}T_d^{(253)}\right]},\ T_d^{(253)} = \frac{T}{4\pi}\left[\eta + \sqrt{\eta^2 + \frac{4}{\eta}}\right],$$

$$\eta = \frac{\left[\tan\phi_m - \left(b\big/\sqrt{A_p^2 - b^2}\right)\right]}{\left[1 + \left(b\big/\sqrt{A_p^2 - b^2}\right)\right]}$$ with $\pm b$ = deadband of relay, T = limit cycle period.

[77] $$T_d^{(254)} = T_u\frac{\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}}{4\pi}$$; recommended $A_m = 2$, $\phi_m = 45^0$.

[78] $$T_i^{(255)} = \frac{r_2K_\phi\left(\omega_u^2 - \omega_\phi^2\right)}{\omega_u\omega_\phi^2\sqrt{K_u^2 - r_2^2K_\phi^2}},\ T_d^{(255)} = \frac{\omega_u\sqrt{K_u^2 - r_2^2K_\phi^2}}{r_2K_\phi\left(\omega_u^2 - \omega_\phi^2\right)},\ r_2 = 0.1 + 0.9\frac{K_u}{K_\phi}.$$

Rule	K_c	T_i	T_d	Comment
Friman and Waller (1997). *Model: Method 1*; $A_m > 2$	$0.25K_u$	$1.1879T_u$	$0.2970T_u$	$\tau_m/T_m < 0.25$, $\phi_m > 60^0$
	$\frac{0.4830}{\left\|G_p(j\omega_{150^0})\right\|}$	$\frac{2.6064}{\omega_{150^0}}$	$\frac{0.6516}{\omega_{150^0}}$	$0.25 \le \frac{\tau_m}{T_m} \le 2.0$ $\phi_m > 45^0$
	Robust			
Rivera *et al.* (1986). *Model: Method 1*	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + 0.5\tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.1T_m$, $\lambda \ge 0.8\tau_m$.
	$0.5\tau_m \le \lambda \le 3\tau_m$ (Zou *et al.* (1997))			
Brambilla *et al.* (1990). *Model: Method 1*	[79] $\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0.1 \le \frac{\tau_m}{T_m} \le 10$
	No model uncertainty - $\lambda \approx 0.35\tau_m$			
Lee *et al.* (1998) *Model: Method 1*	$\frac{T_i^{(256)}}{K_m(\lambda + \tau_m)}$	[80] $T_i^{(256)}$	$T_d^{(256)}$	$T_{CL} = \lambda$
	[81] $K_c^{(257)}$	$T_i^{(257)}$	$T_d^{(257)}$	
Gerry (1999). *Model: Method 10*	$\frac{T_m}{K_m(\lambda + 0.5\tau_m)}$	T_m	$0.5\tau_m$	
	$\lambda = 2\tau_m$ (aggressive, less robust tuning); $\lambda = 2(T_m + \tau_m)$ (more robust tuning)			

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[79] $\lambda \in [0.1T_m, 0.5T_m], \frac{\tau_m}{T_m} \le 0.25$;

$$\lambda = 1.5(\tau_m + T_m), 0.25 < \frac{\tau_m}{T_m} \le 0.75 \,;\, \lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 0.75 \text{ (Leva (2001)).}$$

[80] $T_i^{(256)} = T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$, $T_d^{(256)} = \frac{\tau_m^2}{2(\lambda + \tau_m)}\left(1 - \frac{\tau_m}{3T_i^{(256)}}\right)$.

[81] $K_c^{(257)} = \frac{T_i^{(257)}}{K_m(2T_{CL} + \tau_m - \alpha)}$, $T_i^{(257)} = T_m + \alpha - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha}$,

$$T_d^{(257)} = \frac{T_m\alpha - \frac{0.167\tau_m^3 - 0.5\alpha\tau_m^2}{2T_{CL} + \tau_m - \alpha}}{T_i^{(257)}} - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$\alpha = T_m - T_m\left(1 - \frac{T_{CL}}{T_m}\right)^2 e^{-\frac{\tau_m}{T_m}}.$$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
McMillan (1984). *Model: Method 2 or Method 44*	[82] $K_c^{(258)}$	$T_i^{(258)}$	$T_d^{(258)}$	Tuning rules developed from K_u, T_u
Geng and Geary (1993). *Model: Method 1*	$\frac{0.94T_m}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.167$
Wojsznis and Blevins (1995).	$\frac{T_m + 0.5\tau_m}{aK_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$1.3 \le a \le 1.6$; *Model: Method 36*
Perić *et al.* (1997). *Model: Method 40*	$\hat{K}_u \cos\phi_m$	$\alpha T_d^{(259)}$	[83] $T_d^{(259)}$	Recommended $\alpha = 4$
Matsuba *et al.* (1998). *Model: Method 1*	[84] $K_c^{(260)}$	$T_i^{(260)}$	$T_d^{(260)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Wojsznis *et al.* (1999). *Model: Method 1*	$\frac{K_u \cos\phi_m}{A_m}$	[85] $T_i^{(261)}$	$\alpha T_i^{(261)}$	

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[82] $$K_c^{(258)} = \frac{1.415}{K_m}\frac{T_m}{\tau_m}\left\{\frac{1}{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}}\right\},\ T_i^{(258)} = \tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\},$$
$$T_d^{(258)} = 0.25\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}.$$

[83] $$T_d^{(259)} = \left(\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}\right)\frac{\hat{T}_u}{4\pi}.$$

[84] $$K_c^{(260)} = \left[\frac{1.31}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9}, \frac{2.19}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9}\right],\ T_i^{(260)} = 1.59\tau_m\left(\frac{T_m}{\tau_m}\right)^{0.097},$$
$$T_d^{(260)} = 0.40\tau_m\left(\frac{T_m}{\tau_m}\right)^{0.097}.$$

[85] $$T_i^{(261)} = \left(\tan\phi_m + \sqrt{4\alpha + \tan^2\phi_m}\right)\frac{T_u}{4\pi\alpha}.$$

Rule	K_c	T_i	T_d	Comment
Wojsznis *et al.* (1999) – continued.	$0.5K_u \cos\phi_m$	[86] $T_i^{(262)}$	$0.15T_i^{(262)}$	Default values
	$0.38K_u$	$1.2T_u$	$0.18T_u$	$\phi_m = 45^0$; $\tau_m/T_m < 0.2$
	$0.27K_u$	$0.87T_u$	$0.13T_u$	$A_m = 3$, $\phi_m = 33^0$; $\tau_m/T_m \approx 0.25$
Wojsznis *et al.* (1999). *Model: Method 36*	[87] $K_c^{(263)}$	$T_i^{(263)}$	$0.125T_i^{(263)}$	$a_1 \in [0.3, 0.4]$; $a_2 \in [0.25, 0.4]$; $0.2 \le \dfrac{\tau_m}{T_m} \le 0.7$

[86] $T_i^{(262)} = 0.53T_u\left(\tan\phi_m + \sqrt{0.6 + \tan^2\phi_m}\right)$.

[87] $K_c^{(263)} = K_u\left(a_2 + 0.25\left[1.0 - a_1 - \dfrac{0.6}{1 + e^{-\left(\frac{T_u}{\tau_m} - 7.0\right)}}\right]\right)$, $T_i^{(263)} = T_u\left(a_1 + \dfrac{0.6}{1 + e^{-\left(\frac{T_u}{\tau_m} - 7.0\right)}}\right)$.

4.1.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

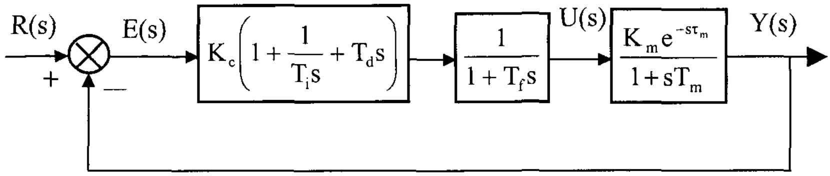

Table 45: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Schaedel (1997). *Model: Method 21*	[1] $K_c^{(264)}$	$\frac{T_1^2 - T_2^2}{T_1 - T_d^{(264)}}$	$T_d^{(264)}$	$T_f = T_d^{(264)}/N$ $5 \le N \le 20$
	Robust			
Horn *et al.* (1996). *Model: Method 1*	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$T_f = \frac{\lambda \tau_m}{2(\lambda + \tau_m)}$, $\lambda \in [\tau_m, T_m]$
	$\lambda = \max(0.25\tau_m, 0.2T_m)$ (Luyben (2001)); $\lambda > 0.25\tau_m$ (Bequette, (2003)).			

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[1] $K_c^{(264)} = \dfrac{0.375 T_i^{(264)}}{K_m\left(T_1 + \dfrac{T_d^{(264)}}{N} - T_i^{(264)}\right)}$, $T_d^{(264)} = \dfrac{T_2^2}{T_1} - \dfrac{T_3^3}{T_2^2}$;

$$T_1 = \frac{T_m}{1 + 3.45\dfrac{\tau_m^a}{T_m}} + \tau_m,\ T_2^2 = 1.25 T_m \tau_m^a + \frac{T_m}{1 + 3.45\dfrac{\tau_m^a}{T_m}}\tau_m + 0.5\tau_m^2,\ T_3 = 0,$$

$$0 < \frac{\tau_m^a}{T_m} \le 0.104;$$

$$T_1 = \frac{0.7 T_m^2}{T_m - 0.7\tau_m^a} + \tau_m,\ T_2^2 = T_m \tau_m^a + \frac{0.7 T_m^2}{T_m - 0.7\tau_m^a} + 0.5\tau_m^2,$$

$$T_3^3 = \frac{0.952 T_m^2(\tau_m^a - 0.1)}{T_m - 0.7\tau_m^a} + T_m \tau_m \tau_m^a + \frac{0.35 T_m^2 \tau_m^2}{T_m - 0.7\tau_m^a} + 0.167\tau_m^3,\ \frac{\tau_m^a}{T_m} > 0.104.$$

Rule	K_c	T_i	T_d	Comment
H_∞ optimal – Tan *et al.* (1998b). *Model: Method 1*	$K_c^{(265)}$ [2]	$T_m + 0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m + 2T_m}$	
	$\lambda = 2$: 'fast' response; $\lambda = 1$: 'robust' tuning; $\lambda = 1.5$: recommended			
Lee and Edgar (2002). *Model: Method 1*	$K_c^{(266)}$ [3]	$T_i^{(266)}$	$T_d^{(266)}$	$T_f = T_d^{(266)}$
Huang *et al.* (2002). *Model: Method 1*	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m}{2T_m + \tau_m}$	$T_f = \frac{\lambda\tau_m}{2(\lambda + \tau_m)}$
	$\lambda = 0.65\tau_m$ - minimum ISE; $\lambda = 0.5\tau_m$ - overshoot to a step input $\le 5\%$, $A_m = 3$. In general, $A_m = 2(\lambda/\tau_m + 1)$.			
Zhang *et al.* (2002).	$K_c^{(267)}$ [4]	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	*Model: Method 1*

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[2] $K_c^{(265)} = \frac{0.265\lambda + 0.307}{K_m}\left(\frac{T_m}{\tau_m} + 0.5\right)$, $T_f = \frac{\tau_m}{5.314\lambda + 0.951}$.

[3] $K_c^{(266)} = \frac{1}{K_m(\lambda + \tau_m)}\left[T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m\sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}}\right]$,

$$T_i^{(266)} = T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m\sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}},$$

$$T_d^{(266)} = \tau_m\sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}}.$$

[4] $K_c^{(267)} = \frac{1}{K_m}\left(\frac{T_m + 0.5\tau_m}{2\lambda + 0.5\tau_m}\right)$, $T_f = \frac{\lambda^2}{2\lambda + 0.5\tau_m}$ or $T_f = \frac{0.1T_m\tau_m}{\tau_m + 2T_m}$.

4.1.3 *Ideal controller in series with a second order filter*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

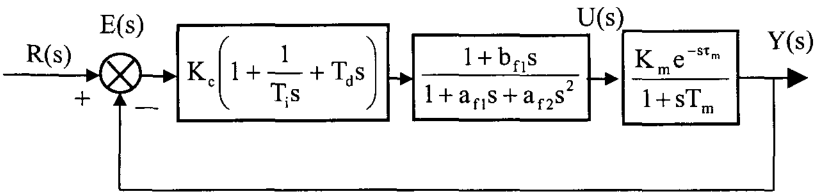

Table 46: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Kristiansson (2003). *Model: Method 11*	[5] $K_c^{(268)}$	$T_i^{(268)}$	$T_d^{(268)}$	$0.05 \le \frac{\tau_m}{T_m} \le 2$; $1.6 \le M_s \le 1.9$

[5] Equations continued into the footnote on page 184;

$$K_c^{(268)} = \frac{2\left[1.1\frac{T_m}{\tau_m} - 0.1\right]\left[0.06 + 0.68\frac{\tau_m}{T_m} - 0.12\left(\frac{\tau_m}{T_m}\right)^2\right]}{K_m\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$T_i^{(268)} = \frac{2T_m\left[0.06 + 0.68\frac{\tau_m}{T_m} - 0.12\left(\frac{\tau_m}{T_m}\right)^2\right]}{\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003) – continued.	[6] $K_c^{(269)}$	$T_i^{(269)}$	$T_d^{(269)}$	$0.7 < \frac{\tau_m}{T_m} < 2$; $1.6 \le M_s \le 1.9$
	Robust			
Horn *et al.* (1996). *Model: Method 1*	[7] $K_c^{(270)}$	$T_m + 0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m + 2T_m}$	$\lambda > \tau_m$, $\lambda < T_m$.

$$T_d^{(268)} = \frac{0.5T_m\left[0.06 + 0.68\frac{\tau_m}{T_m} - 0.12\left(\frac{\tau_m}{T_m}\right)^2\right]}{\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},\ b_{1f} = 0,\ a_{1f} = 0.8T_f,\ a_{2f} = T_f^2 \text{ with}$$

$$T_f = \frac{T_m\left[0.06 + 0.68\frac{\tau_m}{T_m} - 0.12\left(\frac{\tau_m}{T_m}\right)^2\right]^2\left[1.1\frac{T_m}{\tau_m} - 0.1\right]}{\min\left\{5 + 2\frac{T_m}{\tau_m}, 25\right\}}.$$

[6] $$K_c^{(269)} = \frac{0.6667(T_m + \tau_m)\left[1.1\frac{T_m}{\tau_m} - 0.1\right]}{K_m\tau_m\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$T_i^{(269)} = \frac{0.6667(T_m + \tau_m)}{\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},\ T_d^{(269)} = \frac{0.1667(T_m + \tau_m)}{\left[0.68 + 0.84\frac{\tau_m}{T_m} - 0.25\left(\frac{\tau_m}{T_m}\right)^2\right]},$$

$$b_{1f} = 0,\ a_{1f} = 0.8T_f,\ a_{2f} = T_f^2 \text{ with } T_f = \frac{(T_m + \tau_m)^2\left[1.1\frac{T_m}{\tau_m} - 0.1\right]}{9T_m\left(5 + 2\frac{T_m}{\tau_m}\right)}.$$

[7] $$K_c^{(270)} = \frac{2T_m + \tau_m}{2(2\lambda + \tau_m - b_{f1})K_m},\ b_{f1} = \frac{\lambda^2\tau_m + 2T_m\tau_m(\tau_m - \lambda)}{T_m(\tau_m + 2\lambda)} + \frac{2\lambda(2T_m - \lambda)}{(\tau_m + 2\lambda)},$$

$$a_{f1} = \frac{2\lambda\tau_m + 2\lambda^2 + b_{f1}\tau_m}{2(2\lambda + \tau_m - b_{f1})},\ a_{f2} = \frac{\lambda^2\tau_m}{2(2\lambda + \tau_m - b_{f1})}.$$

4.1.4 *Ideal controller with weighted proportional term*

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$$

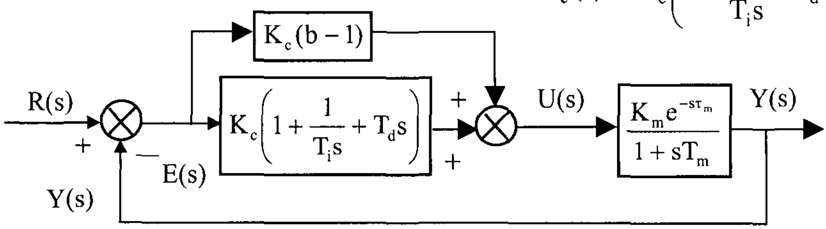

Table 47: PID controller tuning rules – FOLPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Åström and Hägglund (1995) – *pages 208-210.* *Model: Method 5 or Method 14*	$\dfrac{3.8e^{-8.4\tau+7.3\tau^2}T_m}{K_m\tau_m}$	$5.2\tau_m e^{-2.5\tau-1.4\tau^2}$ or $0.46T_m e^{2.8\tau-2.1\tau^2}$	$0.89\tau_m e^{-0.37\tau-4.1\tau^2}$ or $0.077T_m e^{5.0\tau-4.8\tau^2}$	$M_s = 1.4$; $0.14 \le \dfrac{\tau_m}{T_m} \le 5.5$
	$b = 0.40e^{0.18\tau+2.8\tau^2}$			
	$\dfrac{8.4e^{-9.6\tau+9.8\tau^2}T_m}{K_m\tau_m}$	$3.2\tau_m e^{-1.5\tau-0.93\tau^2}$ or $0.28T_m e^{3.8\tau-1.6\tau^2}$	$0.86\tau_m e^{-1.9\tau-0.44\tau^2}$ or $0.076T_m e^{3.4\tau-1.1\tau^2}$	$M_s = 2.0$; $0.14 \le \dfrac{\tau_m}{T_m} \le 5.5$
	$b = 0.22e^{0.65\tau+0.051\tau^2}$			
Gorez (2003). *Model: Method 1*	$\dfrac{(1-v)\tau_m + T_m}{\chi K_m \tau_m}$	$(1-v)\tau_m + T_m$	$\dfrac{(1-v)\tau_m T_m}{(1-v)\tau_m + T_m}$	[8]
	$b = \dfrac{(\chi-1)\tau_m - T_m}{(1-v)\tau_m + T_m} + 1$			

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[8] For $0 \le \dfrac{\tau_m}{\tau_m + T_m} \le \tau_m$, $v = \dfrac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$

$$\chi = \frac{1}{2M_s(M_s-1)} + \frac{(1.3M_s^2-1)}{2M_s(M_s-1)}\frac{1.56\tau_m^2(0.5\tau_m + 3T_m)}{(\tau_m+T_m)^3};$$

For $\tau_m \le \dfrac{\tau_m}{\tau_m + T_m} \le 1$, $v = \dfrac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$, $\chi = \dfrac{0.65M_s}{M_s - 1}$.

4.1.5 *Ideal controller with first order filter and set-point weighting 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\left[R(s)\frac{1 + 0.4T_r s}{1 + sT_r} - Y(s)\right]$$

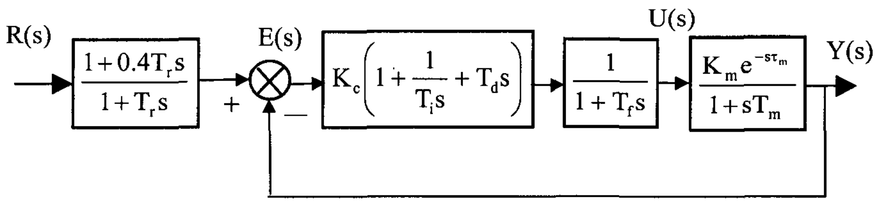

Table 48: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Normey-Rico *et al.* (2000). *Model: Method 1*	$\frac{0.375(\tau_m + 2T_m)}{K_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$T_f = 0.13\tau_m$ $T_r = 0.5\tau_m$

4.1.6 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right)$

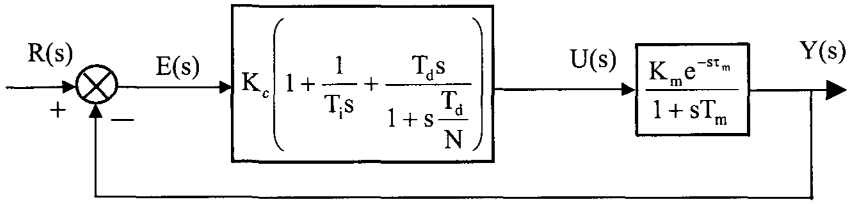

Table 49: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Tavakoli and Tavakoli (2003). *Model: Method 1*; $0.1 \le \frac{\tau_m}{T_m} \le 2$; $N = 10$	[1] $K_c^{(271)}$	$T_i^{(271)}$	$0.0111T_m$	Minimum IAE
	[2] $K_c^{(272)}$	$T_i^{(272)}$		Minimum ISE
	$\frac{1}{K_m}\left(\frac{0.8}{\frac{\tau_m}{T_m} + 0.1}\right)$	$\tau_m\left(0.3 + \frac{T_m}{\tau_m}\right)$	$\tau_m\left(\frac{0.06}{\frac{\tau_m}{T_m} + 0.04}\right)$	Minimum ITAE

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[1] $K_c^{(271)} = \frac{1}{K_m}\left(\frac{1}{\frac{\tau_m}{T_m} + 0.2}\right)$, $T_i^{(271)} = \tau_m\left(\frac{0.3\frac{\tau_m}{T_m} + 1.2}{\frac{\tau_m}{T_m} + 0.08}\right)$ $\tau_m\left(\frac{0.06}{\frac{\tau_m}{T_m} + 0.04}\right)$.

[2] $K_c^{(272)} = \frac{1}{K_m}\left(\frac{0.3\frac{\tau_m}{T_m} + 0.75}{\frac{\tau_m}{T_m} + 0.05}\right)$, $T_i^{(272)} = 2.4\tau_m\left(\frac{1}{\frac{\tau_m}{T_m} + 0.4}\right)$.

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Davydov *et al.* (1995). *Model: Method 6*	[3] $K_c^{(273)}$	$T_i^{(273)}$	$T_d^{(273)}$	$\xi = 0.9$; $0.2 \le \tau_m / T_m \le 1$ $N = K_m$
	[4] $K_c^{(274)}$	$T_i^{(274)}$	$T_d^{(274)}$	
Kuhn (1995). *Model: Method 14*; N = 5	$1/K_m$	$0.66(\tau_m + T_m)$	$0.167(\tau_m + T_m)$	"Normale Einstellung"
	$2/K_m$	$0.8(\tau_m + T_m)$	$0.194(\tau_m + T_m)$	"Schnelle Einstellung"

[3] $K_c^{(273)} = \dfrac{1}{K_m\left(1.552\dfrac{\tau_m}{T_m} + 0.078\right)}$, $T_i^{(273)} = \left(0.186\dfrac{\tau_m}{T_m} + 0.532\right)T_m$,

$$T_d^{(273)} = 0.25\left(0.186\frac{\tau_m}{T_m} + 0.532\right)T_m .$$

[4] $K_c^{(274)} = \dfrac{1}{K_m\left(1.209\dfrac{\tau_m}{T_m} + 0.103\right)}$, $T_i^{(274)} = \left(0.382\dfrac{\tau_m}{T_m} + 0.338\right)T_m$,

$$T_d^{(274)} = 0.4\left(0.382\frac{\tau_m}{T_m} + 0.338\right)T_m .$$

Rule	K_c	T_i	T_d	Comment
Schaedel (1997)	[5] $K_c^{(275)}$	$T_i^{(275)}$	$T_d^{(275)}$	*Model: Method 21*
Goodwin *et al.* (2001) – *page 426.*	[6] $K_c^{(276)}$	$T_i^{(276)}$	$T_d^{(276)}$	*Model: Method 1*

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[5] $K_c^{(275)} = \dfrac{0.375T_i^{(275)}}{K_m\left(T_1 + \dfrac{T_d^{(275)}}{N} - T_i^{(275)}\right)}$, $T_i^{(275)} = \dfrac{T_1^{\,2} - T_2^{\,2}}{T_1 - T_d^{(275)}}$, $T_d^{(275)} = \dfrac{T_2^{\,2}}{T_1} - \dfrac{T_3^{\,3}}{T_2^{\,2}}$;

$$T_1 = \frac{T_m}{1 + 3.45\dfrac{\tau_m^a}{T_m}} + \tau_m \,,\; T_2^{\,2} = 1.25T_m\tau_m^a + \frac{T_m}{1 + 3.45\dfrac{\tau_m^a}{T_m}}\tau_m + 0.5\tau_m^{\,2} \,,\; T_3 = 0 \,,$$

$$0 < \frac{\tau_m^{\ a}}{T_m} \le 0.104\,;$$

$$T_1 = \frac{0.7T_m^{\,2}}{T_m - 0.7\tau_m^a} + \tau_m \,,\; T_2^{\,2} = T_m\tau_m^a + \frac{0.7T_m^{\,2}}{T_m - 0.7\tau_m^a} + 0.5\tau_m^{\,2} \,,$$

$$T_3^{\,3} = \frac{0.952T_m^{\,2}\left(\tau_m^a - 0.1\right)}{T_m - 0.7\tau_m^a} + T_m\tau_m\tau_m^a + \frac{0.35T_m^{\,2}\tau_m^{\,2}}{T_m - 0.7\tau_m^a} + 0.167\tau_m^{\,3} \,,\; \frac{\tau_m^a}{T_m} > 0.104 \,.$$

$$N = \frac{T_d^{(275)}}{-\left(\dfrac{a + \tau_m - T_i^{(275)}}{2}\right) + \sqrt{\dfrac{\left(a + \tau_m - T_i^{(275)}\right)^2}{4} + 0.375\dfrac{T_i^{(275)}T_d^{(275)}}{K_v}}} \,,$$

$0.05 \le a \le 0.2$, K_v = prescribed overshoot in the manipulated variable for a step response in the command variable.

[6] $K_c^{(276)} = \dfrac{(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) - 4T_{CL2}^{\,2}}{K_m\left(16\xi^2 T_{CL2}^{\,2} + 8\xi T_{CL2}\tau_m + \tau_m^{\,2}\right)}$,

$$T_i^{(276)} = \frac{(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) - 4T_{CL2}^{\,2}}{2\left(16\xi^2 T_{CL2}^{\,2} + 8\xi T_{CL2}\tau_m + \tau_m^{\,2}\right)}\left(4\xi T_{CL2} + \tau_m\right),$$

$$T_d^{(276)} = \left(16\xi^2 T_{CL2}^{\,2} + 8\xi T_{CL2}\tau_m + \tau_m^{\,2}\right) \frac{\left[(4\xi T_{CL2} + \tau_m)^2\tau_m T_m - 2T_{CL2}^{\,2}(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) + 8T_{CL2}^{\,4}\right]}{(4\xi T_{CL2} + \tau_m)^3\left[(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) - 4T_{CL2}^{\,2}\right]},$$

$$N = \frac{4\xi T_{CL2} + \tau_m}{2T_{CL2}^{\,2}}T_d^{(276)} \text{ with } G_{CL}(s) = \frac{1}{T_{CL2}^{\,2}s^2 + 2\xi T_{CL2}s + 1}.$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + 0.5\tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda = [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
Morari and Zafiriou (1989). *Model: Method 1*	[7] $K_c^{(277)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.25\tau_m$, $\lambda > 0.2T_m$
Gong *et al.* (1996). *Model: Method 1*	[8] $K_c^{(278)}$	$T_m + 0.3866\tau_m$	$\frac{0.3866T_m\tau_m}{T_m + 0.3866\tau_m}$	$N = [3,10]$
Maffezzoni and Rocco (1997).	[9] $K_c^{(279)}$	$T_i^{(279)}$	$T_d^{(279)}$	*Model: Method 12*
Kasahara *et al.* (1999).	[10] $K_c^{(280)}$	$T_i^{(280)}$	$T_d^{(280)}$	Robust to 20% change in plant parameters
	Model: Method 1; N = 10; $0 < \frac{\tau_m}{T_m} < 1$			

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[7] $K_c^{(277)} = \frac{1}{K_m}\left(\frac{T_m + 0.5\tau_m}{\lambda + \tau_m}\right)$, $N = \frac{2T_m(\lambda + \tau_m)}{\lambda(2T_m + \tau_m)}$.

[8] $K_c^{(278)} = \frac{T_m + 0.3866\tau_m}{K_m(\lambda + 1.0009\tau_m)}$, $\lambda = \frac{(0.1388 + 0.1247N)T_m + 0.0482N\tau_m}{0.3866(N-1)T_m + 0.1495N\tau_m}\tau_m$.

[9] $K_c^{(279)} = \frac{2T_m(\tau_m + x_1) + \tau_m^2}{2K_m(\tau_m + x_1)^2}$, $T_i^{(279)} = T_m + \frac{\tau_m^2}{2(\tau_m + x_1)}$,

$$T_d^{(279)} = \frac{\tau_m^2[2T_m(\tau_m + x_1) - \tau_m x_1]}{2(\tau_m + x_1)[2T_m(\tau_m + x_1) + \tau_m^2]}, \quad N = \frac{2T_m(\tau_m + x_1) - \tau_m x_1}{x_1[2T_m(\tau_m + x_1) + \tau_m^2]}$$

$x_1 = 0.25\tau_m$ for uncertainty on τ_m only; $x_1 > 0.1T_m, \frac{\tau_m}{T_m} < 0.4$ for uncertainty on the minimum phase dynamics within the control band.

[10] $K_c^{(280)} = 0.6K_u\left[0.718 - 0.057\frac{\tau_m}{T_m}\right]$, $T_i^{(280)} = 0.5T_u\left[0.296 + 0.838\frac{\tau_m}{T_m}\right]$,

$$T_d^{(280)} = 0.125T_u\left[0.635 - 1.091\frac{\tau_m}{T_m}\right].$$

Rule	K_c	T_i	T_d	Comment
Kasahara *et al.* (1999) – continued.	[11] $K_c^{(281)}$	$T_i^{(281)}$	$T_d^{(281)}$	Robust to 30% change in plant parameters
	[12] $K_c^{(282)}$	$T_i^{(282)}$	$T_d^{(282)}$	Robust to 40% change in plant parameters
	[13] $K_c^{(283)}$	$T_i^{(283)}$	$T_d^{(283)}$	Robust to 50% change in plant parameters
Leva and Colombo (2000).	[14] $K_c^{(284)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$T_d^{(284)}$	λ not specified; *Model: Method 1*

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[11] $K_c^{(281)} = 0.6K_u\left[0.690 - 0.076\frac{\tau_m}{T_m}\right]$, $T_i^{(281)} = 0.5T_u\left[0.274 + 0.788\frac{\tau_m}{T_m}\right]$,

$$T_d^{(281)} = 0.125T_u\left[0.526 - 0.057\frac{\tau_m}{T_m}\right].$$

[12] $K_c^{(282)} = 0.6K_u\left[0.331 + 0.149\frac{\tau_m}{T_m}\right]$, $T_i^{(282)} = 0.5T_u\left[0.041 + 0.857\frac{\tau_m}{T_m}\right]$,

$$T_d^{(282)} = 0.125T_u\left[0.495 - 0.064\frac{\tau_m}{T_m}\right].$$

[13] $K_c^{(283)} = 0.6K_u\left[0.195 + 0.266\frac{\tau_m}{T_m}\right]$, $T_i^{(283)} = 0.5T_u\left[-0.013 + 0.731\frac{\tau_m}{T_m}\right]$,

$$T_d^{(283)} = 0.125T_u\left[0.489 - 0.065\frac{\tau_m}{T_m}\right].$$

[14] $K_c^{(284)} = \frac{1}{K_m}\frac{2(\lambda + \tau_m)T_m + \tau_m^2}{2(\lambda + \tau_m)^2}$, $T_d^{(284)} = \frac{T_m(\lambda + \tau_m)}{\left[2(\lambda + \tau_m)T_m + \tau_m^2\right]}$,

$$N = \frac{2T_m(\lambda + \tau_m)^2}{\lambda\left[2(\lambda + \tau_m)T_m + \tau_m^2\right]}.$$

Rule	K_c	T_i	T_d	Comment
Leva (2001). *Model: Method 1*	[15] $K_c^{(285)}$	$T_m + \frac{\tau_m^2}{2(\lambda+\tau_m)}$	$T_d^{(285)}$	
Leva *et al.* (2003).	[16] $K_c^{(286)}$	$T_m + \frac{\tau_m^2}{2(\lambda+\tau_m)}$	$T_d^{(286)}$	*Model: Method 1*
Zhang *et al.* (2002).	[17] $K_c^{(287)}$	$T_i^{(287)}$	$T_d^{(287)}$	*Model: Method 1*

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[15] $K_c^{(285)} = \frac{1}{K_m}\frac{2(\lambda+\tau_m)T_m+\tau_m^{\ 2}}{2(\lambda+\tau_m)^2}$, $N = \frac{2T_m(\lambda+\tau_m)^2}{\lambda\left[2(\lambda+\tau_m)T_m+\tau_m^{\ 2}\right]}-1$,

$T_d^{(285)} = \tau_m^{\ 2}\frac{2\lambda T_m + (2T_m-\lambda)\tau_m}{2(\lambda+\tau_m)\left[2T_m(\lambda+\tau_m)+\tau_m^{\ 2}\right]}$. $\lambda \in [0.1T_m, 0.5T_m]$, $\frac{\tau_m}{T_m} \le 0.25$;

$\lambda = 1.5(\tau_m + T_m)$, $0.25 < \frac{\tau_m}{T_m} \le 0.75$; $\lambda = 3(\tau_m + T_m)$, $\frac{\tau_m}{T_m} > 0.75$.

[16] $K_c^{(286)} = \frac{1}{K_m}\frac{2(\lambda+\tau_m)T_m+\tau_m^{\ 2}}{2(\lambda+\tau_m)^2}$, $T_d^{(286)} = \frac{\tau_m T_m(\lambda+\tau_m)}{\left[2(\lambda+\tau_m)T_m+\tau_m^{\ 2}\right]}$,

$N = \frac{2T_m(\lambda+\tau_m)^2}{\lambda\left[2(\lambda+\tau_m)T_m+\tau_m^{\ 2}\right]}$. $\lambda \in [0.1T_m, 0.5T_m]$, $\frac{\tau_m}{T_m} \le 0.33$;

$\lambda = 1.5(\tau_m + T_m)$, $0.33 < \frac{\tau_m}{T_m} \le 3$; $\lambda = 3(\tau_m + T_m)$, $\frac{\tau_m}{T_m} > 3$.

[17] $K_c^{(287)} = \frac{0.5\tau_m + T_m - \frac{T_d^{(287)}}{N}}{K_m(2\lambda + 0.5\tau_m)}$, $T_i^{(287)} = 0.5\tau_m + T_m - \frac{T_d^{(287)}}{N}$,

$T_d^{(287)} = \frac{\tau_m T_m}{2T_i^{(287)}} - \frac{T_d^{(287)}}{N}$, $\frac{T_d^{(287)}}{N} = \frac{\lambda^2}{2\lambda + 0.5\tau_m}$ or N = 10.

4.1.7 *Blending controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{s}$

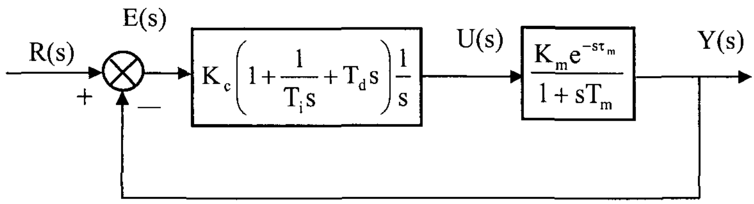

Table 50: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Chen *et al.* (2001). *Model: Method 1*	[18] $K_c^{(288)}$	$T_i^{(288)}$	$T_d^{(288)}$	
	Representative λ values			
	$\lambda = 9.56\tau_m$, $M_{max} = 1.20$; $\lambda = 4.68\tau_m$, $M_{max} = 1.26$; $\lambda = 2.19\tau_m$, $M_{max} = 1.50$; $\lambda = 1.40\tau_m$, $M_{max} = 2.00$; $\lambda = 1.11\tau_m$, $M_{max} = 2.50$; $\lambda = 0.95\tau_m$, $M_{max} = 3.00$.			

[18] $K_c^{(288)} = \frac{1}{K_m}\frac{T_m + \tau_m + 2\lambda}{(\lambda + \tau_m)^2}$, $T_i^{(288)} = T_m + \tau_m + 2\lambda$, $T_d^{(288)} = \frac{T_m(2\lambda + \tau_m)}{T_m + \tau_m + 2\lambda}$.

4.1.8 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

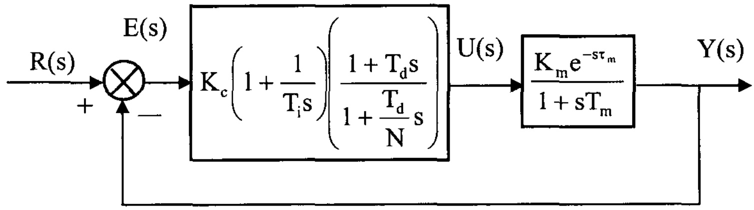

Table 51: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Witt and Waggoner (1990). *Model: Method 2*	$\frac{x_1 T_m}{K_m \tau_m}$, $x_1 \in [0.6,1.0]$	τ_m	τ_m	Equivalent to Ziegler and Nichols (1942); $N \in [10,20]$
Witt and Waggoner (1990). *Model: Method 2*	[1] $K_c^{(289)}$	$T_i^{(289)}$	$T_d^{(289)}$	Preferred tuning; $N \in [10,20]$

[1] Tuning equivalent to Cohen and Coon (1953).

$$K_c^{(289)} = \frac{1.350\frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}}{2K_m},$$

$$T_i^{(289)} = \frac{T_m}{1.350\frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}},$$

$$T_d^{(289)} = \frac{T_m}{1.350\frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}}.$$

Rule	K_c	T_i	T_d	Comment
Witt and Waggoner (1990) (continued).	[2] $K_c^{(290)}$	$T_i^{(290)}$	$T_d^{(290)}$	Alternate tuning; $N \in [10,20]$
St. Clair (1997) – *page 21. Model: Method 2*	$\frac{T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'aggressive' tuning; $N \ge 1$
	$\frac{0.5T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'conservative' tuning; $N \ge 1$
Harrold (1999). *Model: Method 10*; N not specified	$1/K_m$	T_m	$\le 0.25T_m$	$\tau_m/T_m \le 0.25$
	$0.5/K_m$			$\tau_m/T_m \approx 0.5$
	$0.25/K_m$			$\tau_m/T_m \ge 1$
Shinskey (2000). *Model: Method 2*	$\frac{0.889T_m}{K_m \tau_m}$	$1.75\tau_m$	$0.70\tau_m$	$\frac{\tau_m}{T_m} = 0.167$; N not specified
	Minimum performance index: regulator tuning			
Minimum IAE – Huang and Chao (1982).	$\frac{0.817}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.982}$	[3] $T_i^{(291)}$	$T_d^{(291)}$	N = 8; *Model: Method 1*

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[2] $$K_c^{(290)} = \frac{1.350\frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}}{2K_m},$$

$$T_i^{(290)} = \frac{T_m}{1.350\frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}},$$

$$T_d^{(290)} = \frac{T_m}{1.350\frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m}\sqrt{0.7425 + 0.0150\frac{\tau_m}{T_m} + 0.0625\left(\frac{\tau_m}{T_m}\right)^2}}.$$

[3] $$T_i^{(291)} = 0.903T_m\left(\frac{\tau_m}{T_m}\right)^{0.780}, \quad T_d^{(291)} = 0.602T_m\left(\frac{\tau_m}{T_m}\right)^{0.954}.$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Kaya and Scheib (1988). *Model: Method 5*	[4] $K_c^{(292)}$	$T_i^{(292)}$	$T_d^{(292)}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum IAE – Witt and Waggoner (1990). *Model: Method 2*	[5] $K_c^{(293)}$	$T_i^{(293)}$	$T_d^{(293)}$	Preferred tuning
	[6] $K_c^{(294)}$	$T_i^{(294)}$	$T_d^{(294)}$	Alternate tuning
	$0.1 < \frac{\tau_m}{T_m} \le 0.258$; $N \in [10,20]$			
Minimum IAE - Shinskey (1988) – *page 143. Model: Method 1*	$0.95T_m/K_m\tau_m$	$1.43\tau_m$	$0.52\tau_m$	$\tau_m/T_m = 0.2$
	$0.95T_m/K_m\tau_m$	$1.17\tau_m$	$0.48\tau_m$	$\tau_m/T_m = 0.5$
	$1.14T_m/K_m\tau_m$	$1.03\tau_m$	$0.40\tau_m$	$\tau_m/T_m = 1$
	$1.39T_m/K_m\tau_m$	$0.77\tau_m$	$0.35\tau_m$	$\tau_m/T_m = 2$

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[4] $K_c^{(292)} = \frac{0.98089}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.76167}$, $T_i^{(292)} = \frac{T_m}{0.91032}\left(\frac{\tau_m}{T_m}\right)^{1.05211}$,

$$T_d^{(292)} = 0.59974T_m\left(\frac{\tau_m}{T_m}\right)^{0.89819}.$$

[5] $K_c^{(293)} = \frac{0.718}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.921}\left[1+\sqrt{1-1.693\left(\frac{\tau_m}{T_m}\right)^{-1.886}}\right]$,

$$T_i^{(293)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1-\sqrt{1-1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}},\ T_d^{(293)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1+\sqrt{1+1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}}.$$

[6] $K_c^{(294)} = \frac{0.718}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.921}\left[1-\sqrt{1-1.693\left(\frac{\tau_m}{T_m}\right)^{-1.886}}\right]$,

$$T_i^{(294)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1-\sqrt{1+1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}},\ T_d^{(294)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1+\sqrt{1-1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Shinskey (1996) – *page 119.*	$0.96T_m/K_m\tau_m$	$1.43\tau_m$	$0.52\tau_m$	*Model: Method 1* $\tau_m/T_m = 0.2$
Minimum IAE – Edgar *et al.* (1997) – *pages 8-14, 8-15.*	$0.94T_m/K_m\tau_m$	$1.5\tau_m$	$0.55\tau_m$	*Model: Method 1* Time constant dominant; N = 10
Minimum IAE – Edgar *et al.* (1997) – *page 8-15.*	$0.88T_m/K_m\tau_m$	$1.8\tau_m$	$0.70\tau_m$	*Model: Method 2;* N = 10
Minimum IAE – Smith and Corripio (1997) – *page 345.*	$\dfrac{T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	*Model: Method 1*; $0.1 \le \dfrac{\tau_m}{T_m} \le 1.5$
Minimum IAE – Shinskey (1988) – *page 148. Model: Method 1*	$0.56K_u$	$0.39T_u$	$0.14T_u$	$\tau_m/T_m = 0.2$
	$0.49K_u$	$0.34T_u$	$0.14T_u$	$\tau_m/T_m = 0.5$
	$0.51K_u$	$0.33T_u$	$0.13T_u$	$\tau_m/T_m = 1$
	$0.46K_u$	$0.28T_u$	$0.13T_u$	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1994) – *page 167.*	$\dfrac{K_u\tau_m}{3\tau_m - 0.32T_u}$	[7] $T_i^{(295)}$	$0.14T_u$	*Model: Method 1*; N not specified
Minimum ISE – Huang and Chao (1982).	$\dfrac{1.101}{K_m}\left(\dfrac{T_m}{\tau_m}\right)^{0.881}$	[8] $T_i^{(296)}$	$T_d^{(296)}$	*Model: Method 1*; N = 8
Minimum ISE – Kaya and Scheib (1988). *Model: Method 5*	[9] $K_c^{(297)}$	$T_i^{(297)}$	$T_i^{(297)}$	$0 < \dfrac{\tau_m}{T_m} \le 1$; N=10

[7] $T_i^{(295)} = T_u\left(0.15\dfrac{T_u}{\tau_m} - 0.05\right)$.

[8] $T_i^{(296)} = 1.134T_m\left(\dfrac{\tau_m}{T_m}\right)^{0.883}$, $T_d^{(296)} = 0.563T_m\left(\dfrac{\tau_m}{T_m}\right)^{0.881}$.

[9] $K_c^{(297)} = \dfrac{1.11907}{K_m}\left(\dfrac{T_m}{\tau_m}\right)^{0.89711}$, $T_i^{(297)} = \dfrac{T_m}{0.7987}\left(\dfrac{\tau_m}{T_m}\right)^{0.9548}$,

$$T_d^{(297)} = 0.54766T_m\left(\frac{\tau_m}{T_m}\right)^{0.87798}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Huang and Chao (1982).	$\frac{0.745}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.036}$	[10] $T_i^{(298)}$	$T_d^{(298)}$	*Model: Method 1*; N = 8
Minimum ITAE – Kaya and Scheib (1988).	[11] $K_c^{(299)}$	$T_i^{(299)}$	$T_d^{(299)}$	$0 < \frac{\tau_m}{T_m} \le 1$
	N=10; *Model: Method 5*			
Minimum ITAE – Witt and Waggoner (1990).	[12] $K_c^{(300)}$	$T_i^{(300)}$	$T_d^{(300)}$	Preferred tuning
	[13] $K_c^{(301)}$	$T_i^{(301)}$	$T_d^{(301)}$	Alternate tuning
	Model: Method 2; $N \in [10,20]$			

[10] $T_i^{(298)} = 0.771T_m\left(\frac{\tau_m}{T_m}\right)^{0.595}$, $T_d^{(298)} = 0.597T_m\left(\frac{\tau_m}{T_m}\right)^{1.006}$.

[11] $K_c^{(299)} = \frac{0.77902}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.06401}$, $T_i^{(299)} = \frac{T_m}{1.14311}\left(\frac{\tau_m}{T_m}\right)^{0.70949}$,

$$T_d^{(299)} = 0.57137T_m\left(\frac{\tau_m}{T_m}\right)^{1.03826}.$$

[12] $K_c^{(300)} = \frac{0.679}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.947}\left[1+\sqrt{1-1.283\left(\frac{\tau_m}{T_m}\right)^{-1.733}}\right]$, $0.1 < \frac{\tau_m}{T_m} < 0.379$;

$$T_i^{(300)} = \frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1-\sqrt{1-1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}},\ T_d^{(300)} = \frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1+\sqrt{1+1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}}.$$

[13] $K_c^{(301)} = \frac{0.679}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.947}\left[1-\sqrt{1-1.283\left(\frac{\tau_m}{T_m}\right)^{-1.733}}\right]$,

$$T_i^{(301)} = \frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1-\sqrt{1+1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}},\ T_d^{(301)} = \frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1+\sqrt{1-1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Ou and Chen (1995).	$K_c^{(302)} = \frac{1.357}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.947}$	$T_i^{(302)} = \frac{T_m}{0.842}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	[14] $T_d^{(302)}$	*Model: Method 46*
Minimum ITSE – Huang and Chao (1982).	$\frac{0.994}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.907}$	[15] $T_i^{(303)}$	$T_d^{(303)}$	*Model: Method 1;* N = 8
	Minimum performance index: servo tuning			
Minimum IAE – Huang and Chao (1982).	$\frac{0.673}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-1.00}$	[16] $T_i^{(304)}$	$T_d^{(304)}$	*Model: Method 1;* N = 8
Minimum IAE – Kaya and Scheib (1988). *Model: Method 5*	$\frac{0.65}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.04432}$	[17] $T_i^{(305)}$	$T_d^{(305)}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10

[14] $T_d^{(302)} = 0.381 T_m \left(\frac{\tau_m}{T_m}\right)^{0.995}$,

$$N = \pm \frac{0.75 K_m K_c^{(302)} T_d^{(302)}}{T_i^{(302)} + K_m K_c^{(302)}\left(T_d^{(302)} + T_i^{(302)} - \tau_m - T_m\right)},\ N > 0 .$$

[15] $T_i^{(303)} = 1.032 T_m \left(\frac{\tau_m}{T_m}\right)^{0.869}$, $T_d^{(303)} = 0.570 T_m \left(\frac{\tau_m}{T_m}\right)^{0.884}$.

[16] $T_i^{(304)} = \frac{T_m}{0.148\left(\frac{\tau_m}{T_m}\right) + 0.979}$, $T_d^{(304)} = 0.502 T_m \left(\frac{\tau_m}{T_m}\right)^{1.032}$.

[17] $T_i^{(305)} = \frac{T_m}{0.9895 + 0.09539\frac{\tau_m}{T_m}}$, $T_d^{(305)} = 0.50814 T_m \left(\frac{\tau_m}{T_m}\right)^{1.08433}$

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Witt and Waggoner (1990). *Model: Method 2*	[18] $K_c^{(306)}$	$T_i^{(306)}$	$T_d^{(306)}$	Preferred tuning; $0.1 \le \frac{\tau_m}{T_m} \le 1$; $N \in [10,20]$
	[19] $K_c^{(307)}$	$T_i^{(307)}$	$T_d^{(307)}$	Alternate tuning
Minimum IAE – Smith and Corripio (1997) – *page 345. Model: Method 1*	$\frac{0.83T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_m} \le 1.5$; N not specified

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[18] $$K_c^{(306)} = \frac{1.086}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.869}\left[1+\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.914}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}\right],$$

$$T_i^{(306)} = \frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1-\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}},$$

$$T_d^{(306)} = \frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1+\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}}.$$

[19] $$K_c^{(307)} = \frac{1.086}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.869}\left[1-\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.914}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}\right],$$

$$T_i^{(307)} = \frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1+\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}},$$

$$T_d^{(307)} = \frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1-\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Huang and Chao (1982).	$\frac{0.787}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.964}$	[20] $T_i^{(308)}$	$T_d^{(308)}$	*Model: Method 1*; $N = 8$
Minimum ISE – Kaya and Scheib (1988). *Model: Method 5*	[21] $K_c^{(309)}$	$T_i^{(309)}$	$T_d^{(309)}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ITAE – Huang and Chao (1982).	$\frac{0.684}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.995}$	[22] $T_i^{(310)}$	$T_d^{(310)}$	*Model: Method 1*; $N = 8$
Minimum ITAE - Kaya and Scheib (1988). *Model: Method 5*	[23] $K_c^{(311)}$	$T_i^{(311)}$	$T_d^{(311)}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10

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[20] $T_i^{(308)} = 1.678T_m\left(\frac{\tau_m}{T_m}\right)^{-1.028}$, $T_d^{(308)} = 0.594T_m\left(\frac{\tau_m}{T_m}\right)^{0.971}$.

[21] $K_c^{(309)} = \frac{0.71959}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.03092}$, $T_i^{(309)} = \frac{T_m}{1.12666 - 0.18145\frac{\tau_m}{T_m}}$,

$T_d^{(309)} = 0.54568T_m\left(\frac{\tau_m}{T_m}\right)^{0.86411}$.

[22] $T_i^{(310)} = \frac{T_m}{0.114\left(\frac{\tau_m}{T_m}\right) + 0.986}$, $T_d^{(310)} = 0.491T_m\left(\frac{\tau_m}{T_m}\right)^{1.049}$.

[23] $K_c^{(311)} = \frac{1.12762}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.80368}$, $T_i^{(311)} = \frac{T_m}{0.99783 + 0.02860\frac{\tau_m}{T_m}}$,

$T_d^{(311)} = 0.42844T_m\left(\frac{\tau_m}{T_m}\right)^{1.0081}$.

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Witt and Waggoner (1990). *Model: Method 2*	[24] $K_c^{(312)}$	$T_i^{(312)}$	$T_d^{(312)}$	Preferred tuning; $0.1 \le \frac{\tau_m}{T_m} \le 1$; $N \in [10,20]$
	[25] $K_c^{(313)}$	$T_i^{(313)}$	$T_d^{(313)}$	Alternate tuning

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[24] $$K_c^{(312)} = \frac{0.965}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.85}\left[1+\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.929}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}\right],$$

$$T_i^{(312)} = \frac{0.616T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}}{1-\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}},$$

$$T_d^{(312)} = \frac{0.616T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}}{1+\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}}.$$

[25] $$K_c^{(313)} = \frac{0.965}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.85}\left[1-\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.929}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}\right],$$

$$T_i^{(313)} = \frac{0.616T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}}{1+\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}},$$

$$T_d^{(313)} = \frac{0.616T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}}{1-\sqrt{1-1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796-0.1465\frac{\tau_m}{T_m}\right\}}}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th colspan="2">Comment</th></tr>
<tr><td>Minimum ITAE – Ou and Chen (1995).</td><td colspan="2">$K_c^{(314)} = \frac{0.965}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.85}$</td><td colspan="2">$T_i^{(314)}$ [26]</td><td colspan="2">$T_d^{(314)}$</td><td colspan="2">Model: Method 46</td></tr>
<tr><td>Minimum ITSE – Huang and Chao (1982).</td><td colspan="2">$\frac{0.718}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.00}$</td><td colspan="2">$T_i^{(315)}$ [27]</td><td colspan="2">$T_d^{(315)}$</td><td colspan="2">Model: Method 1; N = 8</td></tr>
<tr><td></td><td colspan="8">Direct synthesis</td></tr>
<tr><td rowspan="6">Tsang et al. (1993). Model: Method 15</td><td colspan="2">$\frac{x_1 T_m}{K_m \tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">$0.25\tau_m$</td><td colspan="2">N = 2.5</td></tr>
<tr><td colspan="8">Coefficient values</td></tr>
<tr><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td></tr>
<tr><td>1.6818</td><td>0.0</td><td>0.9916</td><td>0.3</td><td>0.6693</td><td>0.6</td><td>0.4957</td><td>0.9</td></tr>
<tr><td>1.3829</td><td>0.1</td><td>0.8594</td><td>0.4</td><td>0.6000</td><td>0.7</td><td>0.4569</td><td>1.0</td></tr>
<tr><td>1.1610</td><td>0.2</td><td>0.7542</td><td>0.5</td><td>0.5429</td><td>0.8</td><td></td><td></td></tr>
<tr><td>Tsang and Rad (1995). Model: Method 15</td><td colspan="2">$\frac{0.809T_m}{K_m \tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">$0.5\tau_m$</td><td colspan="2">Overshoot = 16%; N=5</td></tr>
<tr><td>Smith and Corripio (1997) – page 346. Model: Method 1</td><td colspan="2">$\frac{0.5T_m}{K_m \tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">$0.5\tau_m$</td><td colspan="2">Servo – 5% overshoot; N not specified</td></tr>
</table>

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[26] $$T_i^{(314)} = \frac{T_m}{0.796 - 0.1465\frac{\tau_m}{T_m}},\quad T_d^{(314)} = 0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929},$$

$$N = \pm\frac{0.75K_m K_c^{(314)} T_d^{(314)}}{T_i^{(314)} + K_m K_c^{(314)}\left(T_d^{(314)} + T_i^{(314)} - \tau_m - T_m\right)},\quad N > 0.$$

[27] $$T_i^{(315)} = \frac{T_m}{0.063\left(\frac{\tau_m}{T_m}\right) + 1.061},\quad T_d^{(315)} = 0.596T_m\left(\frac{\tau_m}{T_m}\right)^{1.028}.$$

Rule	K_c	T_i	T_d	Comment
Schaedel (1997) *Model: Method 21*	$^{28}K_c^{(316)}$	$T_1 - \frac{T_2^2}{T_1}$	$\sqrt{T_1^2 - 2T_2^2}$	N not defined
O'Dwyer (2001a). *Model: Method 1*	$\frac{x_1 T_m}{K_m \tau_m}$	$\frac{T_m}{N}$	T_m	$A_m = \pi/2x_1N$; $\phi_m = 0.5\pi - x_1N$
	Representative result			
	$\frac{0.079T_m}{K_m \tau_m}$	$0.1T_m$	T_m	$A_m = 2.0$; $\phi_m = 45^0$
Wang *et al.* (2002). *Model: Method 1*	$\frac{1}{K_m\left(1+\frac{\tau_m}{2T_m}\right)}$	$\frac{T_m}{K_m\left(1+\frac{\tau_m}{2T_m}\right)}$	$0.5\tau_m$	Labeled IMC; N = 10
	$\frac{0.65}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.04432}$	$^{29}T_i^{(317)}$	$T_d^{(317)}$	Labeled IAE setpoint; N = 10
	$^{30}K_c^{(318)}$	$T_i^{(318)}$	$T_d^{(318)}$	Labeled IAE load; N = 10

$$^{28}K_c^{(316)} = \frac{0.375\left(T_1 - \frac{T_2^2}{T_1}\right)}{K_m\left(\frac{\sqrt{T_1^2 - 2T_2^2}}{N} + \frac{T_2^2}{T_1} - \sqrt{T_1^2 - 2T_2^2}\right)},$$

$$T_1 = \frac{T_m}{1 + 3.45\frac{\tau_m^a}{T_m}} + \tau_m,\ T_2^2 = 1.25T_m\tau_m^a + \frac{T_m}{1 + 3.45\frac{\tau_m^a}{T_m}}\tau_m + 0.5\tau_m^2,\ 0 < \frac{\tau_m^a}{T_m} \le 0.104;$$

$$T_1 = \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + \tau_m,\ T_2^2 = T_m\tau_m^a + \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + 0.5\tau_m^2,\ \frac{\tau_m^a}{T_m} > 0.104.$$

$$^{29}T_i^{(317)} = \frac{T_m}{0.9895 + 0.09539\frac{\tau_m}{T_m}},\ T_d^{(317)} = 0.50814\left(\frac{\tau_m}{T_m}\right)^{1.08433}.$$

$$^{30}K_c^{(318)} = \frac{0.98089}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.76167},\ T_i^{(318)} = 1.09851T_m\left(\frac{\tau_m}{T_m}\right)^{1.05211},$$

$$T_d^{(318)} = 0.59974T_m\left(\frac{\tau_m}{T_m}\right)^{0.89819}.$$

Rule	K_c	T_i	T_d	Comment
Wang *et al.* (2002) – continued.	[31] $K_c^{(319)}$	$T_i^{(319)}$	$T_d^{(319)}$	Labeled ISE setpoint; N = 10
	[32] $K_c^{(320)}$	$T_i^{(320)}$	$T_d^{(320)}$	Labeled ISE load; N = 10
	[33] $K_c^{(321)}$	$T_i^{(321)}$	$T_d^{(321)}$	Labeled ITAE setpoint; N = 10
	[34] $K_c^{(322)}$	$T_i^{(322)}$	$T_d^{(322)}$	Labeled ITAE load; N = 10
Huang *et al.* (2005). *Model: Method 42*	$\frac{0.65T_m}{K_m\tau_m}$	T_m	$0.4\tau_m$	N=20 $A_m = 2.7$, $\phi_m = 65^0$
	The above is a representative, default, tuning rule; other such rules may be deduced from a graph provided by the authors for other A_m, ϕ_m values.			

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[31] $K_c^{(319)} = \frac{0.71959}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.03092}$, $T_i^{(319)} = \frac{T_m}{1.12666 - 0.18145\frac{\tau_m}{T_m}}$,

$$T_d^{(319)} = 0.54568\left(\frac{\tau_m}{T_m}\right)^{0.86411}.$$

[32] $K_c^{(320)} = \frac{1.11907}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.89711}$, $T_i^{(320)} = 1.2520T_m\left(\frac{\tau_m}{T_m}\right)^{0.95480}$,

$$T_d^{(320)} = 0.54766T_m\left(\frac{\tau_m}{T_m}\right)^{0.87798}.$$

[33] $K_c^{(321)} = \frac{1.12762}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.80368}$, $T_i^{(321)} = \frac{T_m}{0.9978 + 0.02860\frac{\tau_m}{T_m}}$,

$$T_d^{(321)} = 0.42844\left(\frac{\tau_m}{T_m}\right)^{1.0081}.$$

[34] $K_c^{(322)} = \frac{0.77902}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.06401}$, $T_i^{(322)} = 0.87481T_m\left(\frac{\tau_m}{T_m}\right)^{0.70949}$,

$$T_d^{(322)} = 0.57137T_m\left(\frac{\tau_m}{T_m}\right)^{1.03826}.$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Harris and Tyreus (1987). *Model: Method 1*	$\frac{T_m}{K_m(\tau_m+\lambda)}$	T_m	$0.5\tau_m$	$N=(\tau_m+\lambda)/\lambda$; $\lambda \ge \text{maximum}\,[0.45\tau_m, 0.1T_m]$
Chien (1988). *Model: Method 1*	$\frac{T_m}{K_m(\lambda+0.5\tau_m)}$	T_m	$0.5\tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
	$\frac{0.5\tau_m}{K_m(\lambda+0.5\tau_m)}$	$0.5\tau_m$	T_m	
Zhang *et al.* (1996). *Model: Method 38*	35 $K_c^{(323)}$	$0.5\tau_m$	T_m	$\lambda \in [0.2\tau_m, 1.2\tau_m]$
	36 $K_c^{(324)}$	T_m	$0.5\tau_m$	
Zhang *et al.* (2002).	37 $K_c^{(325)}$	T_m	$0.5\tau_m$	*Model: Method 1*
Shi and Lee (2002). *Model: Method 1*	38 $K_c^{(326)}$	T_m	$\frac{1}{\omega_g}\tan\left(\frac{\omega_g\tau_m}{2}\right)$	Suggested $\omega_g=\frac{0.6}{\tau_m}$
Lee and Shi (2002). *Model: Method 1*	39 $K_c^{(327)}$	T_m	$0.5\tau_m$	Suggested $\omega_g=\frac{0.6}{\tau_m}$
		40 $T_i^{(328)}$		

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35 $K_c^{(323)}=\frac{0.5\tau_m}{K_m(0.5\tau_m+2\lambda)}$, $N=\frac{T_m(2\lambda+0.5\tau_m)}{\lambda^2}$.

36 $K_c^{(324)}=\frac{T_m}{K_m(0.5\tau_m+2\lambda)}$, $N=\frac{0.5\tau_m(2\lambda+0.5\tau_m)}{\lambda^2}$.

37 $K_c^{(325)}=\frac{T_m}{K_m(2\lambda+0.5\tau_m)}$, N = 10 or $N=\frac{\lambda^2}{0.5\tau_m(2\lambda+0.5\tau_m)}$.

38 $K_c^{(326)}=\frac{\omega_{bw}T_m}{K_m\left[1+\frac{2\omega_{bw}}{\omega_g}\tan\left(\frac{\omega_g\tau_m}{2}\right)\right]}$, $N=1+\frac{2\omega_{bw}}{\omega_g}\tan\left(\frac{\omega_g\tau_m}{2}\right)$.

39 $K_c^{(327)}=\frac{\omega_{bw}T_m}{K_m\left[1+\frac{2\omega_{bw}}{\omega_g}\tan\left(\frac{\omega_g\tau_m}{2}\right)\right]}$, $N=1+\frac{2\omega_{bw}}{\omega_g}\tan\left(\frac{\omega_g\tau_m}{2}\right)$.

40 $T_i^{(328)}=\frac{T_m}{\gamma}$, $1<\gamma<1+\frac{T_m}{\tau_m}+\frac{1}{\omega_{bw}\tau_m}$.

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Huang *et al.* (2005). *Model: Method 42*	[41] $K_c^{(329)}$	$T_i^{(329)}$	$T_d^{(329)}$	N=20 $A_m = 2.7$, $\phi_m = 65^0$
	The above is a representative, default, tuning rule; other such rules may be deduced from a graph provided by the authors for other A_m, ϕ_m values.			

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[41] $K_c^{(329)} = \frac{0.65}{K_m}\left[\frac{\sqrt{\left(\hat{K}_u\right)^2 - 1}}{\pi - \tan^{-1}\left[\sqrt{\left(\hat{K}_u\right)^2 - 1}\right]}\right]$, $T_i^{(329)} = 0.159\hat{T}_u\sqrt{\left(\hat{K}_u\right)^2 - 1}$,

$T_d^{(329)} = 0.064\hat{T}_u\left[\pi - \tan^{-1}\sqrt{\left(\hat{K}_u\right)^2 - 1}\right]$.

4.1.9 *Classical controller 2* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+NT_d s}{1+T_d s}\right)$

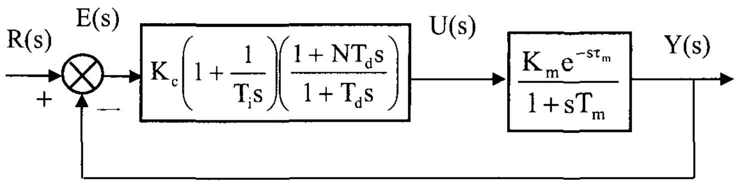

Table 52: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Hougen (1979) – *page 335. Model: Method 1*	[1] $K_c^{(330)}$	T_m	$0.45\frac{\tau_m}{N}$	Maximise crossover frequency; $N \in [10,30]$

[1] Values deduced from graph.
$K_c^{(330)} = 9.0/K_m, \tau_m/T_m = 0.1$; $K_c^{(330)} = 4.2/K_m, \tau_m/T_m = 0.2$;
$K_c^{(330)} = 2.8/K_m, \tau_m/T_m = 0.3$; $K_c^{(330)} = 2.1/K_m, \tau_m/T_m = 0.4$;
$K_c^{(330)} = 1.7/K_m, \tau_m/T_m = 0.5$; $K_c^{(330)} = 1.45/K_m, \tau_m/T_m = 0.6$;
$K_c^{(330)} = 1.2/K_m, \tau_m/T_m = 0.7$; $K_c^{(330)} = 1.1/K_m, \tau_m/T_m = 0.8$;
$K_c^{(330)} = 0.95/K_m, \tau_m/T_m = 0.9$; $K_c^{(330)} = 0.85/K_m, \tau_m/T_m = 1.0$.

4.1.10 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$$

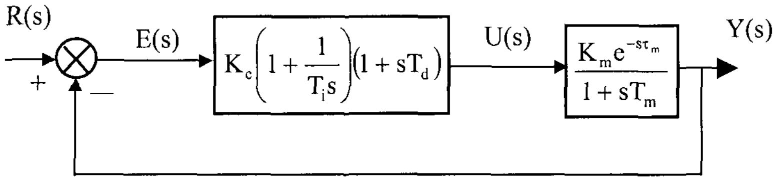

Table 53: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Kraus (1986). *Model: Method 19*	$\frac{0.833T_m}{K_m\tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller pre-tuning
Tan *et al.* (1999a) – *page 25.*	$\frac{0.6T_m}{K_m\tau_m}$	τ_m	τ_m	*Model: Method 2*
O'Dwyer (2001b). *Chien et al. (1952) equivalent – regulator*	$\frac{x_1 T_m}{K_m\tau_m}$	$x_2\tau_m$	$x_3\tau_m$	*Model: Method 2*
	Representative results			
	x_1	x_2	x_3	
	0.7236	1.8353	0.5447	0% overshoot; $0.1 < \tau_m/T_m < 1$
	0.84	1.4	0.6	20% overshoot; $0.1 < \tau_m/T_m < 1$
O'Dwyer (2001b). *Model: Method 2*	Representative results - *Chien et al. (1952) equivalent – servo*			
	[2] $K_c^{(331)}$	$T_i^{(331)}$	$T_d^{(331)}$	0% overshoot; $\tau_m/T_m < 0.5$

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[2] $K_c^{(331)} = \frac{0.3T_m}{K_m\tau_m}\left[1-\sqrt{1-\frac{2\tau_m}{T_m}}\right]$, $T_i^{(331)} = \frac{T_m}{2}\left[1+\sqrt{1-\frac{2\tau_m}{T_m}}\right]$,

$$T_d^{(331)} = \frac{T_m}{2}\left[1-\sqrt{1-\frac{2\tau_m}{T_m}}\right]$$

Rule	K_c		T_i		T_d		Comment	
O'Dwyer (2001b) – continued.	[3] $K_c^{(332)}$		$T_i^{(332)}$		$T_d^{(332)}$		20% overshoot; $\tau_m/T_m < 0.7234$	
	Direct synthesis							
Tsang *et al.* (1993). *Model: Method 15*	$\frac{x_1 T_m}{K_m \tau_m}$		T_m		$0.25\tau_m$			
	x_1	ξ	x_1	ξ	x_1	ξ	x_1	ξ
	1.8194	0.0	1.0894	0.3	0.7482	0.6	0.5709	0.9
	1.5039	0.1	0.9492	0.4	0.6756	0.7	0.5413	1.0
	1.2690	0.2	0.8378	0.5	0.6170	0.8		
	Ultimate cycle							
Pessen (1994). *Model: Method 1*	$0.35K_u$		$0.25T_u$		$0.25T_u$		$0.1 \le \frac{\tau_m}{T_m} \le 1$	

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[3] $K_c^{(332)} = \frac{0.475T_m}{K_m \tau_m}\left[1+\sqrt{1-1.3824\frac{\tau_m}{T_m}}\right]$, $T_i^{(332)} = 0.68T_m\left[1+\sqrt{1-1.3824\frac{\tau_m}{T_m}}\right]$,

$$T_d^{(332)} = 0.68T_m\left[1-\sqrt{1-1.3824\frac{\tau_m}{T_m}}\right]$$

4.1.11 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$

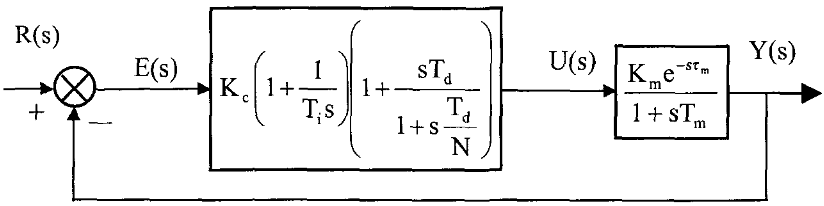

Table 54: PID controller tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Hang *et al.* (1993b) – *page 76.* *Model: Method 2*	$\frac{0.83T_m}{K_m \tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller 'pretune'; N not specified
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_m}{K_m(\lambda+0.5\tau_m)}$	T_m	$0.5\tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
	$\frac{0.5\tau_m}{K_m(\lambda+0.5\tau_m)}$	$0.5\tau_m$	T_m	

4.1.12 *Non-interacting controller 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s)\right)$$

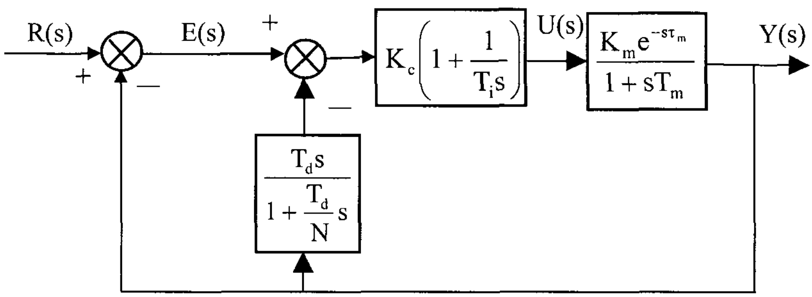

Table 55: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000). *Model: Method 1*	$\frac{0.8(T_m + 0.4\tau_m)}{K_m \tau_m}$	$T_m + 0.4\tau_m$	$\frac{0.4 T_m \tau_m}{T_m + 0.4\tau_m}$	$N = \min[20, 20T_d]$; regulator tuning
	$\frac{0.6(T_m + 0.4\tau_m)}{K_m \tau_m}$			$N = \min[20, 20T_d]$; servo tuning

4.1.13 *Non-interacting controller 2a*

$$U(s)=K_c\left(1+\frac{1}{T_i s}\right)E(s)-\frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$$

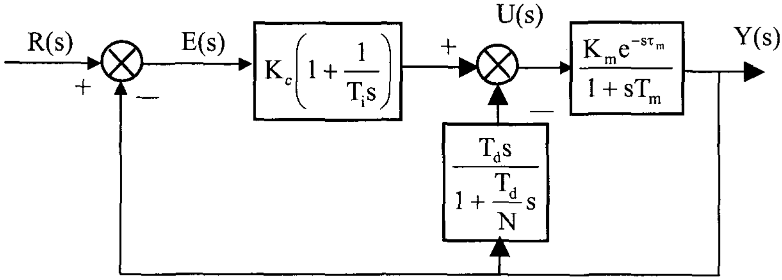

Table 56: PID controller tuning rules – FOLPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Minimum ISE - Zhuang and Atherton (1993). *Model: Method 1*	$\frac{1.260}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.887}$	$T_i^{(333)}$ [4]	$T_d^{(333)}$	$0.1\le\frac{\tau_m}{T_m}\le1.0$
	$\frac{1.295}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.619}$	$T_i^{(334)}$ [5]	$T_d^{(334)}$	$1.1\le\frac{\tau_m}{T_m}\le2.0$
Minimum ISTSE - Zhuang and Atherton (1993).	$\frac{1.053}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.930}$	$T_i^{(335)}$ [6]	$T_d^{(335)}$	$0.1\le\frac{\tau_m}{T_m}\le1.0$; *Model: Method 1*

[4] $T_i^{(333)}=\frac{T_m}{0.701-0.147\frac{\tau_m}{T_m}}$, $T_d^{(333)}=0.375T_m\left(\frac{\tau_m}{T_m}\right)^{0.886}$, N=10.

[5] $T_i^{(334)}=\frac{T_m}{0.661-0.110\frac{\tau_m}{T_m}}$, $T_d^{(334)}=0.378T_m\left(\frac{\tau_m}{T_m}\right)^{0.756}$, N=10.

[6] $T_i^{(335)}=\frac{T_m}{0.736-0.126\frac{\tau_m}{T_m}}$, $T_d^{(335)}=0.349T_m\left(\frac{\tau_m}{T_m}\right)^{0.907}$, N=10.

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993) (continued)	$\frac{1.120}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.625}$	[7] $T_i^{(336)}$	$T_d^{(336)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$; N=10
Minimum ISTES – Zhuang and Atherton (1993). *Model: Method 1*	$\frac{0.942}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.933}$	[8] $T_i^{(337)}$	$T_d^{(337)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.001}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.624}$	[9] $T_i^{(338)}$	$T_d^{(338)}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	[10] $K_c^{(339)}$	$T_i^{(339)}$	$0.112T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 1*
	[11] $K_c^{(340)}$	$0.271\hat{K}_u \hat{T}_u K_m$	$0.116\hat{T}_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; *Model: Method 31*

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[7] $$T_i^{(336)} = \frac{T_m}{0.720 - 0.114\frac{\tau_m}{T_m}},\ T_d^{(336)} = 0.350T_m\left(\frac{\tau_m}{T_m}\right)^{0.811}.$$

[8] $$T_i^{(337)} = \frac{T_m}{0.770 - 0.130\frac{\tau_m}{T_m}},\ T_d^{(337)} = 0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.897},\ N=10.$$

[9] $$T_i^{(338)} = \frac{T_m}{0.754 - 0.116\frac{\tau_m}{T_m}},\ T_d^{(338)} = 0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.813},\ N=10.$$

[10] $$K_c^{(339)} = \frac{4.437K_mK_u - 1.587}{8.024K_mK_u - 1.435}K_u,\ T_i^{(339)} = 0.037(5.89K_mK_u + 1)T_u,\ N=10.$$

[11] $$K_c^{(340)} = \frac{2.354K_m\hat{K}_u - 0.696}{3.363K_m\hat{K}_u + 0.517}\hat{K}_u,\ N=10.$$

4.1.14 *Non-interacting controller 2b*

$$U(s) = \left(K_c + \frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1 + \frac{sT_d}{N}} Y(s)$$

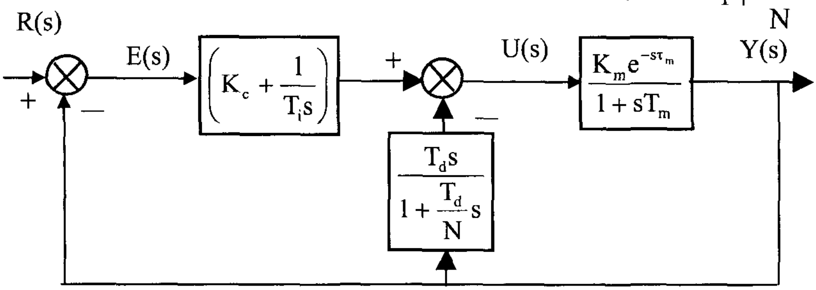

Table 57: PID controller tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Kaya and Scheib (1988).	[12] $K_c^{(341)}$	$T_i^{(341)}$	$T_d^{(341)}$	*Model: Method 5*
Minimum ISE - Kaya and Scheib (1988).	[13] $K_c^{(342)}$	$T_i^{(342)}$	$T_d^{(342)}$	

[12] $K_c^{(341)} = \frac{1.31509}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8826}$, $T_i^{(341)} = \frac{T_m}{1.2587}\left(\frac{\tau_m}{T_m}\right)^{1.3756}$,

$T_d^{(341)} = 0.5655 T_m \left(\frac{\tau_m}{T_m}\right)^{0.4576}$, $0 < \frac{\tau_m}{T_m} \le 1$; N=10.

[13] $K_c^{(342)} = \frac{1.3466}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9308}$, $T_i^{(342)} = \frac{T_m}{1.6585}\left(\frac{\tau_m}{T_m}\right)^{1.25738}$,

$T_d^{(342)} = 0.79715 T_m \left(\frac{\tau_m}{T_m}\right)^{0.41941}$, $0 < \frac{\tau_m}{T_m} \le 1$; N=10.

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Kaya and Scheib (1988).	[14] $K_c^{(343)}$	$T_i^{(343)}$	$T_d^{(343)}$	*Model: Method 5*
	Minimum performance index: servo tuning			
Minimum IAE - Kaya and Scheib (1988).	[15] $K_c^{(344)}$	$T_i^{(344)}$	$T_d^{(344)}$	*Model: Method 5*
Minimum ISE - Kaya and Scheib (1988).	[16] $K_c^{(345)}$	$T_i^{(345)}$	$T_d^{(345)}$	
Minimum ITAE - Kaya and Scheib (1988).	[17] $K_c^{(346)}$	$T_i^{(346)}$	$T_d^{(346)}$	

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[14] $K_c^{(343)} = \frac{1.3176}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7937}$, $T_i^{(343)} = \frac{T_m}{1.12499}\left(\frac{\tau_m}{T_m}\right)^{1.42603}$,

$$T_d^{(343)} = 0.49547 T_m \left(\frac{\tau_m}{T_m}\right)^{0.41932}, \quad 0 < \frac{\tau_m}{T_m} \le 1;\ N=10.$$

[15] $K_c^{(344)} = \frac{1.13031}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.81314}$, $T_i^{(344)} = \frac{T_m}{5.7527 - 5.7241\frac{\tau_m}{T_m}}$,

$$T_d^{(344)} = 0.32175 T_m \left(\frac{\tau_m}{T_m}\right)^{0.17707}, \quad 0 < \frac{\tau_m}{T_m} \le 1;\ N=10.$$

[16] $K_c^{(345)} = \frac{1.26239}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8388}$, $T_i^{(345)} = \frac{T_m}{6.0356 - 6.0191\frac{\tau_m}{T_m}}$,

$$T_d^{(345)} = 0.47617 T_m \left(\frac{\tau_m}{T_m}\right)^{0.24572}, \quad 0 < \frac{\tau_m}{T_m} \le 1;\ N=10.$$

[17] $K_c^{(346)} = \frac{0.98384}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.49851}$, $T_i^{(346)} = \frac{T_m}{2.71348 - 2.29778\frac{\tau_m}{T_m}}$,

$$T_d^{(346)} = 0.21443 T_m \left(\frac{\tau_m}{T_m}\right)^{0.16768}, \quad 0 < \frac{\tau_m}{T_m} \le 1;\ N=10.$$

4.1.15 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$

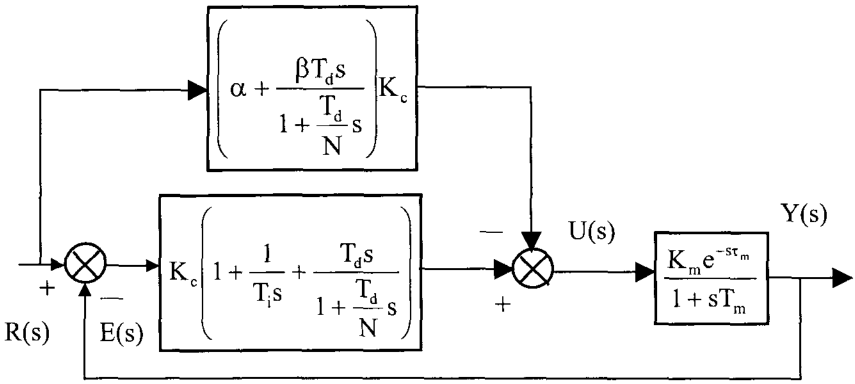

Table 58: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Hiroi and Terauchi (1986) – regulator. *Model: Method 2*; $\alpha = 0.6$; $\beta = 1$	$\frac{0.95T_m}{K_m \tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: Servo/regulator tuning			
Taguchi and Araki (2000).	[1] $K_c^{(347)}$	$T_i^{(347)}$	$T_d^{(347)}$	Overshoot (servo step) $\le 20\%$
	Minimum performance index: other tuning			
Shen (2002). *Model: Method 5*; N=0, $\beta = 0$, $M_s \le 2$	Minimum $\int_0^\infty \lvert r(t) - y(t)\rvert dt + \int_0^\infty \lvert d(t) - y(t)\rvert dt$			
	[2] $K_c^{(348)}$	$T_i^{(348)}$	$T_d^{(348)}$	$0 < \frac{\tau_m}{T_m} < \infty$

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[1] $K_c^{(347)} = \frac{1}{K_m}\left(0.1415 + \frac{1.224}{\frac{\tau_m}{T_m} - 0.001582}\right)$,

$T_i^{(347)} = T_m\left(0.01353 + 2.200\frac{\tau_m}{T_m} - 1.452\left[\frac{\tau_m}{T_m}\right]^2 + 0.4824\left[\frac{\tau_m}{T_m}\right]^3\right)$,

$T_d^{(347)} = T_m\left(0.0002783 + 0.4119\frac{\tau_m}{T_m} - 0.04943\left[\frac{\tau_m}{T_m}\right]^2\right)$,

$\alpha = 0.6656 - 0.2786\frac{\tau_m}{T_m} + 0.03966\left[\frac{\tau_m}{T_m}\right]^2$, N fixed (but not specified);

$\beta = 0.6816 - 0.2054\frac{\tau_m}{T_m} + 0.03936\left[\frac{\tau_m}{T_m}\right]^2$, $\frac{\tau_m}{T_m} \le 1.0$. *Model: Method 1.*

[2] $K_c^{(348)} = \frac{T_m}{K_m\tau_m}\exp\left[2.94 - 11.63\left(\frac{\tau_m}{\tau_m + T_m}\right) + 11.15\left(\frac{\tau_m}{\tau_m + T_m}\right)^2\right]$,

$T_i^{(348)} = \tau_m \exp\left[1.88 - 3.63\left(\frac{\tau_m}{\tau_m + T_m}\right) + 0.86\left(\frac{\tau_m}{\tau_m + T_m}\right)^2\right]$,

$T_d^{(348)} = \tau_m \exp\left[-0.25 - 0.06\left(\frac{\tau_m}{\tau_m + T_m}\right) - 1.99\left(\frac{\tau_m}{\tau_m + T_m}\right)^2\right]$,

$\alpha = 1 - \exp\left[-0.22 - 0.90\left(\frac{\tau_m}{\tau_m + T_m}\right) + 1.45\left(\frac{\tau_m}{\tau_m + T_m}\right)^2\right]$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th>T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td></td><td colspan="6">Direct synthesis</td></tr>
<tr><td>Kasahara et al. (1997).
Model: Method 1</td><td colspan="2">[3] $K_c^{(349)}$</td><td>$T_i^{(349)}$</td><td colspan="2">$T_d^{(349)}$</td><td>Closed loop transfer function is 4th order; $N = 0$</td></tr>
<tr><td rowspan="2">Huang et al. (2000).
Model: Method 1</td><td colspan="2">$\frac{\lambda(T_m + 0.4\tau_m)}{K_m\tau_m}$</td><td>$T_m + 0.4\tau_m$</td><td colspan="2">$\frac{0.4T_m\tau_m}{T_m + 0.4\tau_m}$</td><td>For $\lambda = 0.80$, $A_m = 2.13$</td></tr>
<tr><td colspan="6">$\alpha = \frac{T_m}{T_m + \alpha\tau_m} - 1$, $\beta = 0$, $N = \min[20, 20T_d]$.</td></tr>
<tr><td>Srinivas and Chidambaram (2001).</td><td colspan="2">[4] $K_c^{(350)}$</td><td>$T_i^{(350)}$</td><td colspan="2">$T_d^{(350)}$</td><td>Model: Method 1; $N = \infty$</td></tr>
<tr><td rowspan="6">Kim et al. (2004).
Model: Method 1</td><td colspan="2">x_1/K_m</td><td>x_2T_m</td><td colspan="2">x_3T_m</td><td></td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>τ_m/T_m</td><td>x_1</td><td>x_2</td><td>x_3</td><td>α</td><td>β</td></tr>
<tr><td>0.1</td><td>12.5</td><td>0.22</td><td>0.04</td><td>0.68</td><td>0.75</td></tr>
<tr><td>0.2</td><td>6.1</td><td>0.41</td><td>0.08</td><td>0.63</td><td>0.70</td></tr>
<tr><td>0.3</td><td>4.1</td><td>0.57</td><td>0.11</td><td>0.62</td><td>0.70</td></tr>
</table>

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[3] $K_c^{(349)} = \frac{1}{K_m}\left[\frac{0.2\tau_m^3 + 1.2\tau_m^2T_m + 5.4\tau_mT_m^2 + 9.6T_m^3}{\tau_m(\tau_m + 3T_m)^2}\right]$,

$$T_i^{(349)} = 1.667\tau_m\frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.389\tau_m^2\frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(349)} = \frac{T_m^2\tau_m(\tau_m + 3T_m)}{1.2(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2},$$

$$\alpha = 1 - 0.723\tau_m\left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m}\right)\left[1.667\tau_m\left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m}\right) - 1.389\frac{\tau_m^2(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4}\right]^{-1},$$

$$\beta = 1 - 0.261\tau_m^2\left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m}\right)^2\frac{1}{T_i^{(349)}T_d^{(349)}}.$$

[4] Sample $K_c^{(350)}$, $T_i^{(350)}$, $T_d^{(350)}$ taken by the authors correspond to the process reaction tuning rules of Zeigler and Nichols (1942); $\alpha = \frac{\tau_m}{T_i^{(350)}} - \frac{1}{K_c^{(350)}K_m} = 0.5 - 0.83\frac{\tau_m}{T_m}$ and $\beta = \frac{1}{T_d^{(350)}}\left[\tau_m - \frac{T_m}{K_c^{(350)}K_m} - \frac{\tau_m^2}{2T_i^{(350)}}\right] = -0.16$ for this tuning rule.

Rule	K_c	T_i	T_d	Comment	
Kim *et al.* (2004) - continued	Coefficient values				
	τ_m/T_m	x_1	x_2	x_3	α	β
	0.4	3.1	0.71	0.15	0.59	0.69
	0.5	2.5	0.83	0.18	0.58	0.69
	0.6	2.11	0.94	0.21	0.56	0.69
	0.7	1.82	1.05	0.24	0.54	0.69
	0.8	1.61	1.13	0.28	0.51	0.68
	0.9	1.44	1.22	0.31	0.48	0.69
	1.0	1.33	1.26	0.34	0.49	0.66
	OS < 20% *(servo step); settling time* ≤ *that of corresponding rule of* Chien *et al.* (1952); a cost function is minimized.					
	Robust					
Leva and Colombo (2004).	[5] $K_c^{(351)}$	$T_i^{(351)}$	$T_d^{(351)}$	*Model: Method 41*	
	Ultimate cycle					
Hang and Åström (1988a).	[6] $0.6K_u$	$0.5T_u$	$0.125T_u$	*Model: Method 2*	
	[7] $0.6K_u$	$T_i^{(352)}$	$0.125T_u$		

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[5] $K_c^{(351)} = \dfrac{1}{K_m(\tau_m+\lambda)}\left[T_m + \dfrac{\tau_m^2}{2(\tau_m+\lambda)}\right]$, $T_i^{(351)} = T_m + \dfrac{\tau_m^2}{2(\tau_m+\lambda)}$,

$T_d^{(351)} = \dfrac{\tau_m T_m(\tau_m+\lambda)}{2T_m(\tau_m+\lambda)+\tau_m^2} - \dfrac{\lambda\tau_m}{2(\tau_m+\lambda)}$, $N = \dfrac{2T_m(\tau_m+\lambda)^2}{\lambda\left[2T_m(\tau_m+\lambda)+\tau_m^2\right]} - 1$.

α not specified; $\beta = 1$. $\lambda \ge \dfrac{3\Delta\tau_m}{\pi}$, $\Delta\tau_m$ = maximum variation in the time delay, with only variation in the time delay considered (Leva and Colombo (2004)); λ chosen so that nominal cut-off frequency = $\hat{\omega}_u$ (Leva, 2005).

[6] $\alpha = 1 - 2(x-0.1) - \dfrac{1.66\tau_m}{T_m}$, $\dfrac{\tau_m}{T_m} < 0.3$; $\alpha = 1 - 2x - \dfrac{\tau_m}{T_m}$, $0.3 \le \dfrac{\tau_m}{T_m} < 0.6$;

x = % overshoot. N=10, $\beta = 1$.

[7] $T_i^{(352)} = 0.5\left(1.5 - 0.83\dfrac{\tau_m}{T_m}\right)T_u$. $\alpha = \dfrac{\tau_m}{T_m} - 0.6$, $0.6 \le \dfrac{\tau_m}{T_m} < 0.8$;

$\alpha = 0.2$, $0.8 \le \dfrac{\tau_m}{T_m} < 1.0$.

Rule	K_c	T_i	T_d	Comment
Hang and Åström (1988a) - continued	$0.6K_u$	$0.335T_u$	$0.125T_u$	$\frac{\tau_m}{T_m} > 1.0$; $\alpha = 0.2$
Hang *et al.* (1991) - servo.	[8] $0.6K_u$	$0.5T_u$	$0.125T_u$	*Model: Method 2*; $\beta = 1$
	[9] $0.6K_u$	$0.222K'T_u$	$0.125T_u$	
Hang and Cao (1996). *Model: Method 31*	$0.6K_u$	[10] $T_i^{(353)}$	$T_d^{(353)}$	$0.1 \le \frac{\tau_m}{T_m} < 0.5$; N=10; $\beta = 1$

[8] $0.16 \le \frac{\tau_m}{T_m} < 0.57$; N = 10. $\alpha = 1 - \frac{15 - K'}{15 + K'}$, 10% overshoot;

$\alpha = 1 - \frac{36}{27 + 5K'}$, 20% overshoot with $K' = 2\left(\frac{11[\tau_m/T_m] + 13}{37[\tau_m/T_m] - 4}\right)$.

[9] $0.57 \le \frac{\tau_m}{T_m} < 0.96$; N =10. $\alpha = 1 - \frac{8}{17}\left(\frac{4}{9}K' + 1\right)$, 20% overshoot, 10% undershoot.

[10] $T_i^{(353)} = \left(0.53 - 0.22\frac{\tau_m}{T_m}\right)T_u$, $T_d^{(353)} = \left(0.53 - 0.22\frac{\tau_m}{T_m}\right)\frac{T_u}{4}$. α takes on values of 1.0, 0.2 and $0.40(\tau_m/T_m)^2 - 0.05(\tau_m/T_m) + 0.58$ at different points on the open loop process step response.

4.1.16 *Non-interacting controller based on the two degree of freedom structure 2*

$$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}-\frac{\chi}{1+T_i s}\right)R(s)$$

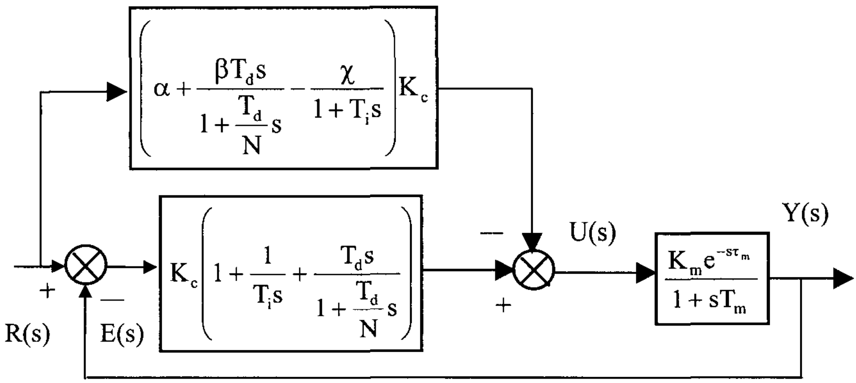

Table 59: PID tuning rules – FOLPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Hiroi and Terauchi (1986) - regulator. *Model: Method 2*	$\frac{0.95T_m}{K_m\tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
	$\frac{1.2T_m}{K_m\tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1<\frac{\tau_m}{T_m}<1$
	$\alpha=0.6$; $\beta=1$ or $\beta=-1.48$; $\chi=0.15$			

4.1.17 *Non-interacting controller based on the two degree of freedom structure 3*

$$U(s) = K_c\left[(b-1)+(c-1)T_d s\right]R(s) + K_c\left(1+\frac{1}{T_i s}+T_d s\right)\left[R(s)-\frac{1}{(1+sT_f)^2}Y(s)\right]$$

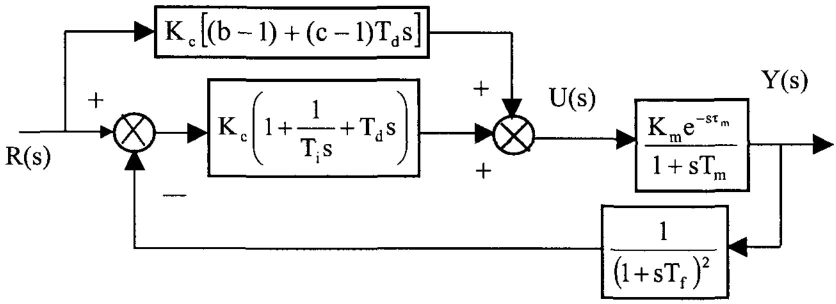

Table 60: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: other tuning			
Åström and Hägglund (2004).	[11] $K_c^{(354)}$	$T_i^{(354)}$	$\frac{0.5\tau_m T_m}{0.3\tau_m + T_m}$	*Model: Method 1*
	No precisely defined performance specification			
Åström and Hägglund (2004).	[12] $K_c^{(355)}$	$T_i^{(355)}$	$T_d^{(355)}$	*Model: Method 45*
	No precisely defined performance specification			

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[11] $K_c^{(354)} = \frac{1}{K_m}\left[0.2 + 0.45\frac{T_m}{\tau_m}\right]$, $T_i^{(354)} = \frac{0.4\tau_m + 0.8T_m}{\tau_m + 0.1T_m}\tau_m$; $b = 0$, $\frac{\tau_m}{T_m} \le 1$; $b = 1$, $\frac{\tau_m}{T_m} > 1$; $c = 0$; $T_f = 0.1\tau_m$.

[12] $K_c^{(355)} = \frac{1}{K_m}\left[0.2 + 0.45\frac{T_m + 0.5T_f}{\tau_m + 0.5T_f}\right]$, $b = 1$, $c = 0$. $T_f = \frac{T_d^{(355)}}{N}$, $N \in [4,10]$;

$$T_i^{(355)} = \frac{0.4\tau_m + 0.8T_m + 0.6T_f}{\tau_m + 0.1T_m + 0.55T_f}(\tau_m + 0.5T_f),$$

$$T_d^{(355)} = \frac{0.5(\tau_m + 0.5T_f)(T_m + 0.5T_f)}{0.3\tau_m + T_m + 0.65T_f}; \quad \frac{\tau_m}{T_m} > 1 .$$

4.1.18 *Non-interacting controller 4*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$$

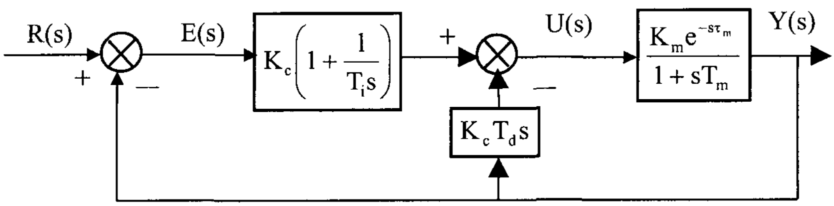

Table 61: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
VanDoren (1998).	$\frac{1.5T_m}{K_m\tau_m}$	$2.5\tau_m$	$0.4\tau_m$	*Model: Method 2*
ABB (2001). *Model: Method 2*	$\frac{1.25T_m}{K_m\tau_m}$	$2\tau_m$	$0.5\tau_m$	
	Minimum performance index: regulator tuning			
Minimum IAE - Shinskey (1996) – *page 117.*	$1.32T_m/K_m\tau_m$	$1.80\tau_m$	$0.44\tau_m$	$\tau_m/T_m = 0.1$; *Model: Method 1*
Minimum IAE – Shinskey (1988) – *page 143. Model: Method 1*	$1.32T_m/K_m\tau_m$	$1.77\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.2$
	$1.35T_m/K_m\tau_m$	$1.43\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.5$
	$1.49T_m/K_m\tau_m$	$1.17\tau_m$	$0.37\tau_m$	$\tau_m/T_m = 1$
	$1.82T_m/K_m\tau_m$	$0.92\tau_m$	$0.32\tau_m$	$\tau_m/T_m = 2$
Minimum IAE – Edgar *et al.* (1997) – *pages 8-14, 8-15.*	$1.30T_m/K_m\tau_m$	$1.8\tau_m$	$0.45\tau_m$	*Model: Method 1*
	$1.33T_m/K_m\tau_m$	$2.1\tau_m$	$0.63\tau_m$	*Model: Method 2*
	Minimum performance index: other tuning			
Minimum IAE – Shinskey (1988) – *page 148. Model: Method 1*	$0.77K_u$	$0.48T_u$	$0.11T_u$	$\tau_m/T_m = 0.2$
	$0.70K_u$	$0.42T_u$	$0.12T_u$	$\tau_m/T_m = 0.5$
	$0.66K_u$	$0.38T_u$	$0.12T_u$	$\tau_m/T_m = 1$
	$0.60K_u$	$0.34T_u$	$0.12T_u$	$\tau_m/T_m = 2$

Rule	K_c	T_i	T_d	Comment
Shinskey (1994) – *page 167.*	[1] $K_c^{(356)}$	$0.125\dfrac{T_u^{\ 2}}{\tau_m}$	$0.12T_u$	Minimum IAE; *Model: Method 1*
ABB (2001).	[2] $K_c^{(357)}$	$T_i^{(357)}$	$T_d^{(357)}$	*Model: Method 7*
	Direct synthesis			
Haeri (2002)	[3] $K_c^{(358)}$	$T_i^{(358)}$	$T_d^{(357)}$	*Model: Method 1*

[1] $K_c^{(356)} = \dfrac{K_u}{3.73 - 0.69\dfrac{T_u}{\tau_m}}$, $\dfrac{T_u}{\tau_m} < 2.7$; $K_c^{(356)} = \dfrac{K_u}{2.62 - 0.35\dfrac{T_u}{\tau_m}}$, $\dfrac{T_u}{\tau_m} \ge 2.7$.

[2] $K_c^{(357)} = \dfrac{1.3570}{K_m}\left(\dfrac{T_m}{\tau_m}\right)^{0.947}$, $T_i^{(357)} = 1.1760T_m\left(\dfrac{\tau_m}{T_m}\right)^{0.738}$,

$$T_d^{(357)} = 0.3810T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}, \ \frac{\tau_m}{T_m} \ge 0.5 .$$

[3] $$K_c^{(358)} = \frac{1}{K_m}\left[\frac{6.84}{1 + 2.33\left(\dfrac{\tau_m}{T_m}\right)^{0.7} + 7.82\left(\dfrac{\tau_m}{T_m}\right)^{3.5}} + 0.64\right],$$

$$T_i^{(358)} = \tau_m\left[0.95 + 2.58\frac{\tau_m}{T_m} + 3.57\left(\frac{\tau_m}{T_m}\right)^2\right], \ 0 < \frac{\tau_m}{T_m} \le 0.29 ,$$

$$T_i^{(358)} = \tau_m\left[2.15 - 0.76\frac{\tau_m}{T_m} + 0.33\left(\frac{\tau_m}{T_m}\right)^2\right], \ 0.29 < \frac{\tau_m}{T_m} < 4 ;$$

$$T_d^{(358)} = \tau_m\left[0.29\left(\frac{\tau_m}{T_m + \tau_m}\right) + 3.94\left(\frac{\tau_m}{T_m + \tau_m}\right)^2 - 4.65\left(\frac{\tau_m}{T_m + \tau_m}\right)^{2.8}\right], 0 < \frac{\tau_m}{T_m} \le 1.79 ;$$

$$T_d^{(358)} = \tau_m\left[0.87\left(\frac{\tau_m}{T_m}\right) - 0.49\left(\frac{\tau_m}{T_m}\right)^2 + 0.09\left(\frac{\tau_m}{T_m}\right)^{2.8}\right], \ 1.79 < \frac{\tau_m}{T_m} < 4 .$$

4.1.19 *Non-interacting controller 5*

$$U(s) = K_c\left(b + \frac{1}{T_i s}\right)E(s) - \left(c + T_d s\right)Y(s)$$

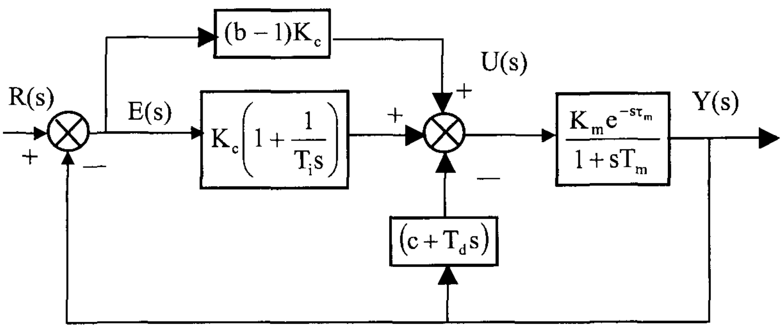

Table 62: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Chidambaram (2000a). *Model: Method 1*	[4] $K_c^{(359)}$	$T_i^{(359)}$	$T_d^{(359)}$	$c = K_c^{(359)}$
	[5] $\frac{1.2\tau_m}{K_m T_m}$	$0.5\tau_m$	$\frac{0.15\tau_m^2}{K_m T_m}$	$c = \frac{1.2\tau_m}{K_m T_m}$

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[4] $K_c^{(359)} = \frac{1}{K_m}\left[4.114 - 6.575\frac{\tau_m}{T_m} + 3.342\left(\frac{\tau_m}{T_m}\right)^2\right]$,

$T_i^{(359)} = T_m\left[0.311 + 1.372\frac{\tau_m}{T_m} - 0.545\left(\frac{\tau_m}{T_m}\right)^2\right]$, $T_d^{(359)} = \frac{K_c^{(359)} T_i^{(359)}}{\left[16.016 - 11.7\left(\frac{\tau_m}{T_m}\right)\right]}$,

$b = 0.793 - 3.149\frac{\tau_m}{T_m} + 8.405\left(\frac{\tau_m}{T_m}\right)^2 - 8.431\left(\frac{\tau_m}{T_m}\right)^3$, $\frac{\tau_m}{T_m} \le 0.5$,

$b = 0.0584 + 0.44\frac{\tau_m}{T_m}$, $0.5 \le \frac{\tau_m}{T_m} \le 0.9$.

[5] $b = 0.3238 + 0.4332\frac{\tau_m}{T_m}$, $\frac{\tau_m}{T_m} \le 0.8$; $b = 0.8507$, $\frac{\tau_m}{T_m} \approx 0.9$.

4.1.20 *Non-interacting controller 6 (I-PD controller)*

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c(1 + T_d s)Y(s)$$

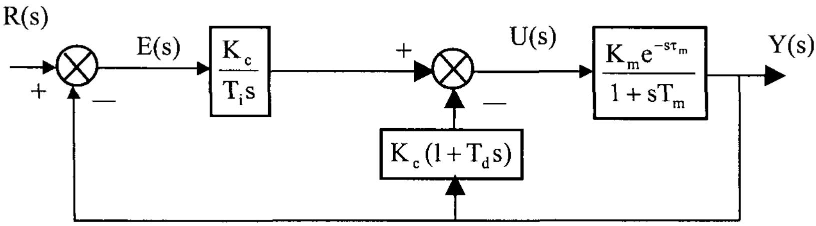

Table 63: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Suyama (1993).	[6] $K_c^{(360)}$	$T_i^{(360)}$	$T_d^{(360)}$	*Model: Method 1*
	Closed loop transfer function is 4th order			
Shigemasa *et al.* (1987), Suyama (1993).	[7] $K_c^{(361)}$	$T_i^{(361)}$	$T_d^{(361)}$	*Model: Method 1*
	Closed loop transfer function is 4th order Butterworth			

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[6] $$K_c^{(360)} = \frac{1}{K_m}\left[\frac{0.2\tau_m^{\ 3} + 1.2\tau_m^{\ 2}T_m + 5.4\tau_m T_m^{\ 2} + 9.6T_m^{\ 3}}{\tau_m(\tau_m + 3T_m)^2}\right],$$

$$T_i^{(360)} = 1.667\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.389\tau_m^{\ 2}\frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(360)} = \frac{T_m^{\ 2}\tau_m(\tau_m + 3T_m)}{1.2(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

[7] $$K_c^{(361)} = \frac{1.3319(\tau_m + 2T_m)^3}{K_m\tau_m(\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(361)} = 2.2532\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.692\tau_m^{\ 2}\frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(361)} = \tau_m(\tau_m + 3T_m)\frac{1.5095(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.3319(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

<table>
<tr><th>Rule</th><th>K_c</th><th>T_i</th><th>T_d</th><th>Comment</th></tr>
<tr><td rowspan="6">Shigemasa et al. (1987), Suyama (1993) (continued). Model: Method 1</td><td>[8] $K_c^{(362)}$</td><td>$T_i^{(362)}$</td><td>$T_d^{(362)}$</td><td rowspan="6">Toshiba AdTune TOSDIC 211D8 product</td></tr>
<tr><td colspan="3">Closed loop transfer function is 4th order ITAE</td></tr>
<tr><td>[9] $K_c^{(363)}$</td><td>$T_i^{(363)}$</td><td>$T_d^{(363)}$</td></tr>
<tr><td colspan="3">Closed loop transfer function is 4th order Bessel</td></tr>
<tr><td>[10] $K_c^{(364)}$</td><td>$T_i^{(364)}$</td><td>$T_d^{(364)}$</td></tr>
<tr><td colspan="3">Closed loop transfer function is 4th order Binomial</td></tr>
</table>

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[8] $$K_c^{(362)} = \frac{2.6242(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(362)} = 1.8898\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 0.720\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(362)} = \tau_m(\tau_m + 3T_m)\frac{2.3130(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{2.6242(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

[9] $$K_c^{(363)} = \frac{0.9450(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(363)} = 3.3333\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 3.527\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(363)} = \tau_m(\tau_m + 3T_m)\frac{1.3444(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.9450(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

[10] $$K_c^{(364)} = \frac{0.5622(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(364)} = 5.3337\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 9.488\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(364)} = \tau_m(\tau_m + 3T_m)\frac{1.1244(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{0.5622(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

Rule	K_c	T_i	T_d	Comment
Chien *et al.* (1999).	[11] $K_c^{(365)}$	$T_i^{(365)}$	$T_d^{(365)}$	*Model: Method 1*
Kasahara *et al.* (2001). *Model: Method 1*	[12] $K_c^{(366)}$	$T_i^{(366)}$	$T_d^{(366)}$	OS = 0%
	[13] $K_c^{(367)}$	$T_i^{(367)}$	$T_d^{(367)}$	OS = 10%
Ozawa *et al.* (2003).	[14] $K_c^{(368)}$	$T_i^{(368)}$	$T_d^{(368)}$	*Model: Method 1* $OS \in [0\%,10\%]$
	Minimum performance index: servo tuning			
Minimum ISE – Argelaguet *et al.* (2000).	$\dfrac{2T_m + \tau_m}{2K_m \tau_m}$	$T_m + 0.5\tau_m$	$\dfrac{T_m \tau_m}{2T_m + \tau_m}$	*Model: Method 1*

[11] Underdamped system response - $\xi = 0.707$. $\tau_m > 0.2T_m$.

$$K_c^{(365)} = \frac{1.414T_{CL2}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL2}^2}{K_m\left(T_{CL2}^2 + 0.707T_{CL2}\tau_m + 0.25\tau_m^2\right)},$$

$$T_i^{(365)} = \frac{1.414T_{CL2}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL2}^2}{T_m + 0.5\tau_m},$$

$$T_d^{(365)} = \frac{0.707T_m T_{CL2}\tau_m + 0.25T_m\tau_m^2 - 0.5\tau_m T_{CL2}^2}{T_m\tau_m + 0.25\tau_m^2 + 1.414T_{CL2}T_m - T_{CL2}^2}.$$

[12] $$K_c^{(366)} = \frac{0.281(\tau_m + 2T_m)^3}{K_m\tau_m(\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(366)} = 5.33\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 18.98\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(366)} = \tau_m(\tau_m + 3T_m)\frac{0.562(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{0.281(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

[13] $$K_c^{(367)} = \frac{1.066T_m - 0.467\tau_m}{K_m\tau_m},\quad T_i^{(367)} = \frac{4.695\tau_m(1.066T_m - 0.467\tau_m)}{\tau_m + 2T_m},$$

$$T_d^{(367)} = \tau_m\left(\frac{0.331T_m - 0.334\tau_m}{1.066T_m - 0.467\tau_m}\right).$$

[14] $$K_c^{(368)} = \frac{1.6193(\tau_m + 2T_m)^3}{K_m\tau_m(\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(368)} = 2.7051\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.6706\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(368)} = \tau_m(\tau_m + 3T_m)\frac{1.7793(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.6193(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.$$

4.1.21 *Non-interacting controller 7*

$$U(s)=\frac{K_c}{T_i s}E(s)-K_c\left(1+\frac{T_d s}{1+\frac{T_d}{N}s}\right)Y(s)$$

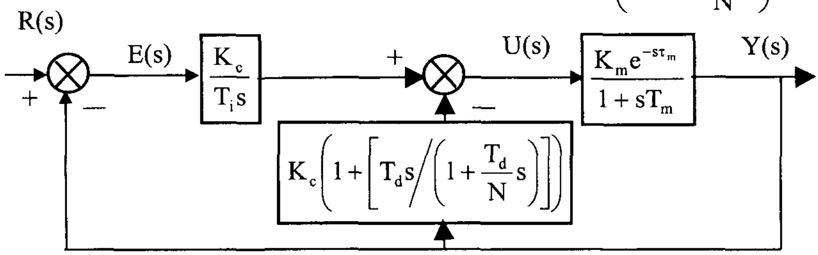

Table 64: PID controller tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Ogawa and Katayama (2001).	[15] $K_c^{(369)}$	$T_i^{(369)}$	$T_d^{(369)}$	N=10
	Minimum ISE - servo. *Model: Method 1*			

[15] Desired closed loop response $=\frac{e^{-s\tau_{CL}}}{(1+sT_{CL})^2}$. $K_c^{(369)}=\frac{1}{K_m}\left(\frac{\frac{\tau_{CL}}{T_m}-2\frac{T_{CL}}{T_m}+4}{\frac{\tau_{CL}}{T_m}+2\frac{T_{CL}}{T_m}}\right)$,

$$T_i^{(369)}=T_m\left(\frac{\left(\frac{\tau_{CL}}{T_m}+2\frac{T_{CL}}{T_m}\right)\left(\frac{\tau_{CL}}{T_m}-2\frac{T_{CL}}{T_m}+4\right)}{2\frac{\tau_{CL}}{T_m}+4}\right),$$

$$T_d^{(369)}=T_m\left(\frac{\frac{\tau_{CL}}{T_m}\left(\frac{\tau_{CL}}{T_m}+4\frac{T_{CL}}{T_m}-2\left(\frac{\tau_{CL}}{T_m}\right)^2\right)}{\left(\frac{\tau_{CL}}{T_m}+2\frac{T_{CL}}{T_m}\right)\left(\frac{\tau_{CL}}{T_m}-2\frac{T_{CL}}{T_m}+4\right)}\right);\ \frac{T_{CL}}{T_m}\in[0.01,1.00];$$

$$\frac{T_{CL}}{T_m}=-0.1902\left(\frac{\tau_{CL}}{T_m}\right)^2+0.6974\left(\frac{\tau_{CL}}{T_m}\right)+0.007393,\ \frac{\tau_{CL}}{T_m}\in[0.05,1.00].$$

4.1.22 *Non-interacting controller 11*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)E(s) - K_i(1+T_{di}s)Y(s)$$

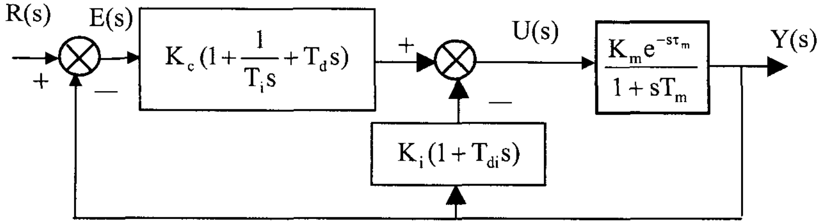

Table 65: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[16] $K_c^{(370)}$	$T_i^{(370)}$	$T_d^{(370)}$	$K_i = 0.25K_u$, $T_{di} = 0$

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[16] $$K_c^{(370)} = \frac{1+0.25K_uK_m}{K_m(\lambda+\tau_m)}\left[\frac{T_m - 0.25K_uK_m\tau_m}{1+0.25K_uK_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)}\right],$$

$$T_i^{(370)} = \frac{T_m - 0.25K_uK_m\tau_m}{1+0.25K_uK_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)},$$

$$T_d^{(370)} = \frac{1+0.25K_uK_m}{K_m(\lambda+\tau_m)}\{\frac{0.25K_uK_m\tau_m^2}{2(1+0.25K_uK_m)} - \frac{\tau_m^3}{6(\lambda+\tau_m)} + \frac{\tau_m^4}{4(\lambda+\tau_m)^2} + \frac{\tau_m^2(T_m - 0.25K_uK_m\tau_m)}{2(1+0.25K_uK_m)(\lambda+\tau_m)}.$$

4.1.23 *Non-interacting controller 12*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_d s+1}E(s) - K_1 Y(s)$$

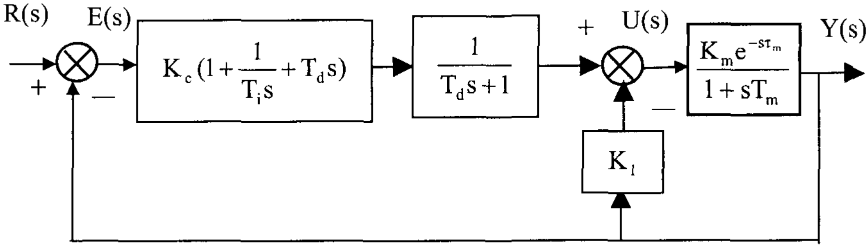

Table 66: PID tuning rules – FOLPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[17] $K_c^{(371)}$	$T_i^{(371)}$	$T_d^{(371)}$	$K_1 = 0.25K_u$

---

[17] $K_c^{(371)} = \frac{1+0.25K_u K_m}{K_m(\lambda+\tau_m)}\{\frac{T_m - 0.25K_u K_m \tau_m}{1+0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)} +$

$$\tau_m\left[\frac{0.25K_u K_m}{2(1+0.25K_u K_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1+0.25K_u K_m)(\lambda+\tau_m)}\right]^{0.5}\},$$

$T_i^{(371)} = \frac{T_m - 0.25K_u K_m \tau_m}{1+0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)} +$

$$\tau_m\left[\frac{0.25K_u K_m}{2(1+0.25K_u K_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1+0.25K_u K_m)(\lambda+\tau_m)}\right]^{0.5},$$

$T_d^{(371)} =$

$$\tau_m\left[\frac{0.25K_u K_m}{2(1+0.25K_u K_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1+0.25K_u K_m)(\lambda+\tau_m)}\right]^{0.5}.$$

4.1.24 *Industrial controller* $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$

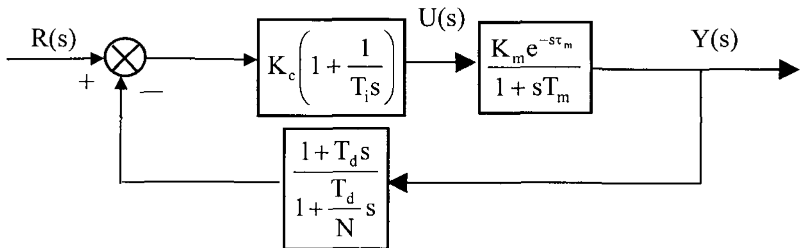

Table 67: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="1">T_i</th><th colspan="1">T_d</th><th colspan="2">Comment</th></tr>
<tr><td></td><td colspan="6">Minimum performance index: regulator tuning</td></tr>
<tr><td rowspan="3">Kaya and Scheib (1988).
Model: Method 5</td><td colspan="2">$\frac{x_1}{K_m}\left(\frac{T_m}{\tau_m}\right)^{x_2}$</td><td>$\frac{T_m}{x_3}\left(\frac{\tau_m}{T_m}\right)^{x_4}$</td><td>$x_5 T_m\left(\frac{\tau_m}{T_m}\right)^{x_6}$</td><td colspan="2">$0 < \frac{\tau_m}{T_m} \le 1$;
N=10</td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td><td>x_5</td><td>x_6</td></tr>
<tr><td>Minimum IAE</td><td>0.91</td><td>0.7938</td><td>1.01495</td><td>1.00403</td><td>0.5414</td><td>0.7848</td></tr>
<tr><td>Minimum ISE</td><td>1.1147</td><td>0.8992</td><td>0.9324</td><td>0.8753</td><td>0.56508</td><td>0.91107</td></tr>
<tr><td>Minimum ITAE</td><td>0.7058</td><td>0.8872</td><td>1.03326</td><td>0.99138</td><td>0.60006</td><td>0.971</td></tr>
<tr><td></td><td colspan="6">Minimum performance index: servo tuning</td></tr>
<tr><td rowspan="3">Kaya and Scheib (1988).
Model: Method 5</td><td colspan="2">$\frac{x_1}{K_m}\left(\frac{T_m}{\tau_m}\right)^{x_2}$</td><td>$\frac{T_m}{x_3 - x_4\frac{\tau_m}{T_m}}$</td><td>$x_5 T_m\left(\frac{\tau_m}{T_m}\right)^{x_6}$</td><td colspan="2">$0 < \frac{\tau_m}{T_m} \le 1$;
N=10</td></tr>
<tr><td colspan="6">Coefficient values</td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td><td>x_5</td><td>x_6</td></tr>
<tr><td>Minimum IAE</td><td>0.81699</td><td>1.004</td><td>1.09112</td><td>0.22387</td><td>0.44278</td><td>0.97186</td></tr>
<tr><td>Minimum ISE</td><td>1.1427</td><td>0.9365</td><td>0.99223</td><td>0.35269</td><td>0.35308</td><td>0.78088</td></tr>
<tr><td>Minimum ITAE</td><td>0.8326</td><td>0.7607</td><td>1.00268</td><td>0.00854</td><td>0.44243</td><td>1.11499</td></tr>
</table>

Rule	K_c	T_i	T_d	Comment
	Robust			
Harris and Tyreus (1987). *Model: Method 1*	$\frac{T_m}{K_m(0.5\tau_m+\lambda)}$	T_m	$0.5\tau_m$	$\lambda \ge$ max imum $[0.8\tau_m, T_m]$; N not specified
Ho *et al.* (2001). *Model: Method 1*	[18] $K_c^{(372)}$	T_m	$0.5\tau_m$	N not specified
	$\frac{1.551T_m}{K_m A_m \tau_m}$	T_m	$0.5\tau_m$	

[18] $K_c^{(372)} = \frac{T_m}{K_m(0.5\tau_m + T_{CL})}$, $T_{CL} = -0.5\tau_m + \sqrt{0.25\tau_m^{\ 2} + \omega_g^{\ -2}}$,

ω_g = frequency at which the Nyquist curve has a magnitude of unity.

4.2 Non-Model Specific

4.2.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

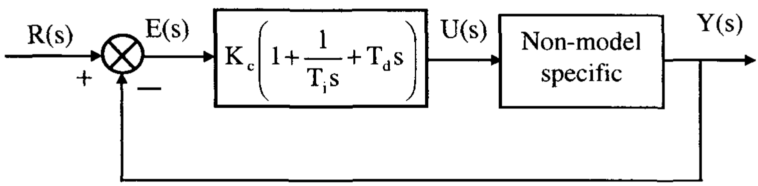

Table 68: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Ziegler and Nichols (1942).	$[0.6K_u, K_u]$	$0.5T_u$	$0.125T_u$	Quarter decay ratio
Farrington (1950).	$[0.33K_u, 0.5K_u]$	T_u	$[0.1T_u, 0.25T_u]$	
McAvoy and Johnson (1967).	$0.54K_u$	T_u	$0.2T_u$	
Atkinson and Davey (1968).	$0.25K_u$	$0.75T_u$	$0.25T_u$	
	20% overshoot – servo response			
Carr (1986); Pettit and Carr (1987).	K_u	$0.5T_u$	$0.125T_u$	Underdamped
	$0.6667K_u$	T_u	$0.167T_u$	Critically damped
	$0.5K_u$	$1.5T_u$	$0.167T_u$	Overdamped
Parr (1989) – *pages 190,191,193.*	$0.5K_u$	T_u	$0.2T_u$	$\xi \approx 0.45$
	$0.5K_u$	T_u	$0.25T_u$	
	$0.5K_u$	$0.34T_u$	$0.08T_u$	
Tinham (1989).	$0.4444K_u$	$0.6T_u$	$0.19T_u$	Less than quarter decay ratio response
Blickley (1990).	$0.5K_u$	T_u	$[0.125T_u, 0.167T_u]$	Quarter decay ratio

Rule	K_c	T_i	T_d	Comment
Corripio (1990) – *page 27.*	$0.75K_u$	$0.63T_u$	$0.1T_u$	Quarter decay ratio
Åström (1982).	$K_u \cos\phi_m$	$T_i^{(373)}$ [1]	$\alpha T_i^{(373)}$	
	Representative results			
	$0.87K_u$	$0.55T_u$	$0.14T_u$	$\phi_m = 30^0$
	$0.71K_u$	$0.77T_u$	$0.19T_u$	$\phi_m = 45^0$
	$0.50K_u$	$1.19T_u$	$0.30T_u$	$\phi_m = 60^0$
Åström and Hägglund (1984).	$\dfrac{K_u}{A_m}$	arbitrary	$\dfrac{T_u}{4\pi^2 T_i}$	Specify A_m
	$K_u \cos\phi_m$	$\alpha T_d^{(374)}$	$T_d^{(374)}$ [2]	Specify ϕ_m
Hang and Åström (1988b).	$K_u \sin\phi_m$	$\dfrac{T_u(1-\cos\phi_m)}{\pi \sin\phi_m}$	$\dfrac{T_u(1-\cos\phi_m)}{4\pi \sin\phi_m}$	
Åström and Hägglund (1988) – *page 60.*	$0.35K_u$	$0.77T_u$	$0.19T_u$	$A_m \ge 2$, $\phi_m \ge 45^0$
Åström and Hägglund (1995) – *page 141-142.*	$0.4698K_u$	$0.4546T_u$	$0.1136T_u$	$A_m = 2$, $\phi_m = 20^0$
	$0.1988K_u$	$1.2308T_u$	$0.3077T_u$	$A_m = 2.44$, $\phi_m = 61^0$
	$0.2015K_u$	$0.7878T_u$	$0.1970T_u$	$A_m = 3.45$, $\phi_m = 46^0$
De Paor (1993).	$0.906K_u$	$0.5T_u$	$0.125T_u$	$\phi_m = 25^0$
	$0.866K_u$	$0.5T_u$	$0.125T_u$	$\phi_m = 30^0$
McMillan (1994) – *page 90.*	$0.5K_u$	$0.5T_u$	$0.125T_u$	
Calcev and Gorez (1995).	$0.3536K_u$	$0.1592T_u$	$0.0398T_u$	
	$\phi_m = 45^0$, small τ_m; $\phi_m = 15^0$, large τ_m			

[1] $T_i^{(373)} = \left(\tan\phi_m + \sqrt{4\alpha + \tan^2\phi_m}\right)\dfrac{T_u}{4\alpha\pi}$. $\alpha = 0.25$ (Åström, 1982);
$\alpha = 0.5$ (Seki *et al.*, 2000); $0.1 \le \alpha \le 0.3$ (Seki *et al.*, 2000).

[2] $T_d^{(374)} = \left(\tan\phi_m + \sqrt{1 + \tan^2\phi_m}\right)\dfrac{T_u}{\pi}$.

Rule	K_c	T_i	T_d	Comment
ABB Commander 300/310 (1996).	$0.625K_u$	$0.5T_u$	$0.083T_u$	'P+D' tuning rule
Luo *et al.* (1996).	$0.48K_u$	$0.5T_u$	$0.125T_u$	
Karaboga and Kalinli (1996).	$[0.32K_u, 0.6K_u]$	$[0.213T_u, 1.406T_u]$	$[0.133T_u, 0.469T_u]$	
Luyben and Luyben (1997) – *page 97.*	$0.46K_u$	$2.20T_u$	$0.16T_u$	Based on work by Tyreus and Luyben (1992)
Tan *et al.* (1999a) – *page 26.*	$0.4K_u$	$0.5T_u$	$0.125T_u$	
Tan *et al.* (1999a) – *page 27.*	$0.5K_u$	T_u	$0.125T_u$	
	Tighter damping than quarter decay ratio tuning			
Wojsznis *et al.* (1999).	$0.4K_u$	$0.333T_u$	$0.083T_u$	
Yu (1999) – *page 11.*	$0.33K_u$	$0.5T_u$	$0.125T_u$	Some overshoot
	$0.2K_u$	$0.5T_u$	$0.125T_u$	No overshoot
Tan *et al.* (2001).	$K_c^{(375)}$	[3] $T_i^{(375)}$	$0.25T_i^{(375)}$	
Chau (2002) – *page 115.*	$0.33K_u$	$0.5T_u$	$0.333T_u$	"Just a bit of" overshoot
	$0.2K_u$	$0.5T_u$	$0.333T_u$	No overshoot
Robbins (2002).	$0.45K_u$	[4] $T_i^{(376)}$	$0.25T_u$	Minimum IAE – step change in setpoint
	$0.45K_u$	[4] $T_i^{(377)}$	$0.25T_u$	"Good" response – step change in load
Smith (2003).	$0.75K_u$	$0.625T_u$	$0.1T_u$	
	Other rules			
Rutherford (1950).	$1.25K_{37\%}$	$T_{37\%}$	$0.125T_{37\%}$	37% decay ratio
	$1.12K_{37\%}$	$T_{37\%}$	$0.25T_{37\%}$	

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[3] $K_c^{(375)} = \dfrac{T_i^{(375)}\omega_u \left|G_c'(j\omega_u)\right|}{0.25[T_i^{(375)}]^2\omega_u^2 + 1}$, $T_i^{(375)} = 0.3183T_u \tan\dfrac{0.5\pi + \angle G_c'(j\omega_u)}{2}$,

$G_c'(j\omega_u)$ = desired frequency response of controller $G_c(j\omega)$ at ω_u.

[4] $T_i^{(376)} = (0.24 + 0.111K_m K_u)T_u$, $T_i^{(377)} = (0.27 + 0.054K_m K_u)T_u$

Rule	K_c	T_i	T_d	Comment
MacLellan (1950).	$1.77K_{37\%}$	$1.29T_{37\%}$	$0.16T_{37\%}$	37% decay ratio
	$1.67K_{37\%}$	$1.29T_{37\%}$	$0.16T_{37\%}$	
	$1.49K_{37\%}$	$0.98T_{37\%}$	$0.24T_{37\%}$	
Harriott (1964) – *pages 179-180.*	$K_{25\%}$	$0.167T_{25\%}$	$0.667T_{25\%}$	Quarter decay ratio
Lipták (1970) – *pages 846-847.*	$K_{25\%}$	$0.667T_{25\%}$	$0.167T_{25\%}$	Quarter decay ratio
Chesmond (1982) – *page 400.*	[5] $K_c^{(378)}$	$T_i^{(378)}$	$0.25T_i^{(378)}$	x = 100(decay ratio)
	$0.999K_{25\%}$	$0.488T_{25\%}$	$0.122T_{25\%}$	Example: x = 25 i.e. quarter decay ratio
Parr (1989) – *page 191.*	$K_{25\%}$	$0.67T_{25\%}$	$0.17T_{25\%}$	Quarter decay ratio
McMillan (1994) – *page 43*	$0.83K_{25\%}$	$0.5T_{25\%}$	$0.1T_{25\%}$	'Fast' tuning
	$0.67K_{25\%}$	$0.5T_{25\%}$	$0.1T_{25\%}$	'Slow' tuning
Wade (1994) – *page 114.*	$[1.002K_{25\%}, 1.67K_{25\%}]$	$0.45T_{25\%}$	$0.1125T_{25\%}$	
Hay (1998) – *page 189.*	$K_{25\%}$	$0.667T_{25\%}$	$0.167T_{25\%}$	
ECOSSE team (1996b).	$0.8497K_{50\%}$	$0.475T_{50\%}$	$0.1188T_{50\%}$	
Bateson (2002) – *page 616.*	$K_{25\%}$	$0.6667T_u$	$0.1667T_u$	'modified' ultimate cycle
Rotach (1994).	[6] $K_c^{(379)}$	$T_i^{(379)}$	$-0.5\dfrac{d\phi_{\omega_{M_{max}}}}{d\omega_{M_{max}}}$	

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[5] $K_c^{(378)} = \dfrac{K_{x\%}}{0.5 + 0.361\ln x}$, $T_i^{(378)} = \dfrac{T_{x\%}}{2\sqrt{1 + 0.025\ln^2 x}}$.

[6] $K_c^{(379)} = \dfrac{M_{max}}{\sqrt{{M_{max}}^2 - 1}}\left|G_p(j\omega_{M_{max}})\right|$, $T_i^{(379)} = -\dfrac{2}{{\omega_{M_{max}}}^2\left[\dfrac{d\phi_{\omega_{M_{max}}}}{d\omega_{M_{max}}}\right]}$.

Rule	K_c	T_i	T_d	Comment
Rotach (1994) – continued.	$^{7}K_c^{(380)}$	$T_i^{(380)}$	$T_d^{(380)}$	
García and Castelo (2000).	$^{8}K_c^{(381)}$	$T_i^{(381)}$	$0.25T_i^{(381)}$	$\omega_1 < \omega_u$
Lloyd (1994). *Model: Method 3*	$0.4\hat{K}_u$	$0.3333\hat{T}_u$	$0.083\hat{T}_u$	Self-regulating processes
	$0.4\hat{K}_u$	$\hat{T}_u$	$0.159\hat{T}_u$	Non self-regulating processes
Jones *et al.* (1997).	$0.7490\dfrac{h}{A_p}$	$a\hat{T}_u$,	$0.125\hat{T}_u$	*Model: Method 2*
	$a \in [0.2526, 0.5053]$			

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[7] $K_c^{(380)} = \dfrac{M_{max}}{\sqrt{M_{max}^2 - 1}}\left|G_p(j\omega_r)\right|$,

$$T_i^{(380)} = -\frac{2}{\omega_r\left[\omega_r \dfrac{d\phi_{\omega_r}}{d\omega_r}\left\{1+\tan^2\left(\pi - \sin^{-1}\dfrac{1}{M_{max}} + \phi_{\omega_r}\right)\right\} - \tan\left(\pi - \sin^{-1}\dfrac{1}{M_{max}} + \phi_{\omega_r}\right)\right]},$$

$$T_d^{(380)} = -\frac{0.5}{\omega_r}[\omega_r \frac{d\phi_{\omega_r}}{d\omega_r}\left\{1 + \tan^2\left(\pi - \sin^{-1}\frac{1}{M_{max}} + \phi_{\omega_r}\right)\right\} + \tan\left(\pi - \sin^{-1}\frac{1}{M_{max}} + \phi_{\omega_r}\right)].$$

[8] $K_c^{(381)} = \dfrac{\cos\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right)}{\left|G_p(j\omega_1)\right|}$, $T_i^{(381)} = \dfrac{2[\sin\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right) + 1]}{\omega_1 \cos\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right)}$.

Rule	K_c	T_i	T_d	Comment
Shin *et al.* (1997).	[9] $K_c^{(382)}$	$T_i^{(382)}$	$\alpha T_i^{(382)}$	*Model: Method 2*
NI Labview (2001). *Model: Method 2*	$0.6\hat{K}_u$	$0.5\hat{T}_u$	$0.12\hat{T}_u$	Quarter decay ratio
	$0.25\hat{K}_u$	$0.5\hat{T}_u$	$0.12\hat{T}_u$	'Some' overshoot
	$0.15\hat{K}_u$	$0.5\hat{T}_u$	$0.12\hat{T}_u$	'Little' overshoot
Tang *et al.* (2002). *Model: Method 4*	$\dfrac{2.5465h\sin\phi_m}{A_p x_1 \hat{\omega}_{90^0} A_m}$	$\delta T_d^{(383)}$, $\delta \in [1.5,4]$	[10] $T_d^{(383)}$	$0.81 \le x_1 \le 1$
	$x_1 = 1$, 'small' τ_m; $x_1 = 0.91$, 'large' τ_m			
Dutton *et al.* (1997).	[11] $0.5K_u^e$	T_u^e	$0.25T_u^e$	*pages 576-8.*

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[9] $$K_c^{(382)} = \frac{\rho}{x_1\left(\alpha\omega_u T_i^{(382)} - \dfrac{1}{\omega_u T_i^{(382)}}\right) - x_2\left(\alpha\hat{\omega}_u T_i^{(382)} - \dfrac{1}{\hat{\omega}_u T_i^{(382)}}\right) - y_2 - \rho x_2},$$

$$T_i^{(382)} = \frac{(x_2 - x_1) + \sqrt{(x_1 - x_2)^2 + 4\left(\alpha\rho x_1\omega_u + \alpha y_2\hat{\omega}_u\right)\left(\dfrac{\rho x_1}{\omega_u} + \dfrac{y_2}{\hat{\omega}_u}\right)}}{2\left(\alpha\rho x_1\omega_u + \alpha y_2\hat{\omega}_u\right)},$$

$$x_1 = -\frac{1}{K_u},\ x_2 = \frac{1}{\hat{K}_u}\cos\left(\angle G_p(j\hat{\omega}_u)\right),\ \rho = \frac{\left(\omega_u - \hat{\omega}_u\right)}{\omega_u}\frac{\sqrt{1-\xi^2}}{\xi},$$

$y_2 = \dfrac{1}{\hat{K}_u}\sin\left(\angle G_p(j\hat{\omega}_u)\right)$. Typical α : 0.1; typical ξ : [0.3,0.7].

[10] $$T_d^{(383)} = \frac{-\cot\phi_m + \sqrt{\cot^2\phi_m + 4/\delta}}{2\hat{\omega}_{90^0}}.$$

[11] $K_u^e = \dfrac{K_c^*}{\sqrt{1 - \dfrac{(8-r)^2}{55}}}$, $T_u^e = \dfrac{6.283}{\omega_d}\left[1 - \dfrac{(8-r)^{3.5}}{1110}\right]^{0.286}$ with r = ratio of the height of the first peak to the height of the first trough of the closed loop response, when controller gain K_c^* is applied; ω_d = damped natural frequency.

4.2.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

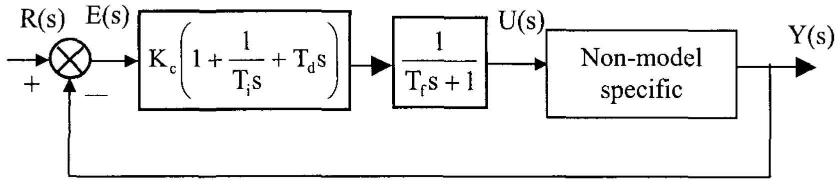

Table 69: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Other tuning: minimum performance index			
Kristiansson (2003).	[12] $K_c^{(384)}$	$T_i^{(384)}$	$T_d^{(384)}$	
	"Almost" optimal tuning rule; $K_{150^0} \geq 0.03$; $1.6 \leq M_s \leq 1.9$			

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[12] $$K_c^{(384)} = \frac{0.60}{K_m\left[\frac{K_{150^0}}{K_m} + 0.07\right]\left[0.32 + 1.6\frac{K_{150^0}}{K_m} - 0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$T_i^{(384)} = \frac{1.5}{\omega_{150^0}\left[0.32 + 1.6\frac{K_{150^0}}{K_m} - 0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$T_d^{(384)} = \frac{0.6667}{\omega_{150^0}\left[0.32 + 1.6\frac{K_{150^0}}{K_m} - 0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$T_f^{(384)} = \frac{0.4}{\omega_{150^0}\left[\frac{K_{150^0}}{K_m} + 0.07\right]\left[0.32 + 1.6\frac{K_{150^0}}{K_m} - 0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]^2 \min\left\{6 + \frac{K_m}{K_{150^0}}, 25\right\}} \text{ or}$$

$$T_f^{(384)} = \frac{1}{\omega_{150^0}\left[20\frac{K_{150^0}}{K_m} + 0.5\right]\left[0.32 + 1.6\frac{K_{150^0}}{K_m} - 0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]}.$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Wang *et al* (1995b).	[13] $K_c^{(385)}$	$T_i^{(385)}$	$T_d^{(385)}$	

[13] For a stable process, K_m is assumed known; for an integrating process, K_m and τ_m are assumed known. $K_c^{(385)} = \frac{1}{\omega_{CL}} \mathrm{Im}\left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right]$,

$$T_i^{(385)} = \frac{1}{\omega_{CL}} \frac{\mathrm{Im}\left[\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right]}{\mathrm{Rl}\left[\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})\{1 - G_{CL}(j\omega_{CL})\}}\right]}$$

$$+\frac{1}{3}\left[\mathrm{Rl}\left\{\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})[1 - G_{CL}(j\omega_{CL})]}\right\} - \mathrm{Rl}\left\{\frac{j2\omega_{CL} G_{CL}(j2\omega_{CL})}{G_p(j2\omega_{CL})[1 - G_{CL}(j2\omega_{CL})]}\right\}\right],$$

$$T_d^{(385)} = \frac{1}{3}\left[\frac{\mathrm{Rl}\left\{\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})[1 - G_{CL}(j\omega_{CL})]}\right\} - \mathrm{Rl}\left\{\dfrac{j2\omega_{CL} G_{CL}(j2\omega_{CL})}{G_p(j2\omega_{CL})[1 - G_{CL}(j2\omega_{CL})]}\right\}}{\mathrm{Im}\left\{\dfrac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})[1 - G_{CL}(j\omega_{CL})]}\right\}}\right].$$

4.2.3 *Ideal controller in series with a second order filter*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

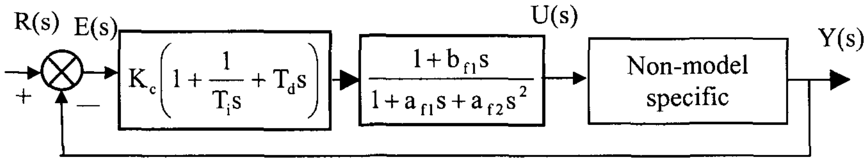

Table 70: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: other tuning			
Kristiansson *et al.* (2000).	[1] $K_c^{(386)}$	$T_i^{(386)}$	$T_d^{(386)}$	$M_s \approx 1.7$
	"Almost" optimal tuning rule			

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[1] $$K_c^{(386)} = \frac{1.6\left[1.6\dfrac{\left|G_p(j\omega_u)\right|^2}{K_m^{\,2}} - 2.3\dfrac{\left|G_p(j\omega_u)\right|}{K_m} + 1.1\right]}{K_m\left[0.37 + \dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]},\ b_{1f} = 0,\ a_{1f} = T_f,\ a_{2f} = T_f^{\,2},$$

$$T_i^{(386)} = \frac{1.6}{\omega_u\left[0.37 + \dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]},\quad T_d^{(386)} = \frac{0.625}{\omega_u\left[0.37 + \dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]},$$

$$T_f = \frac{\left[1.6\dfrac{\left|G_p(j\omega_u)\right|^2}{K_m^{\,2}} - 2.3\dfrac{\left|G_p(j\omega_u)\right|}{K_m} + 1.1\right]}{\omega_u\left[0.37 + \dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]^2\left[13 - 20\dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]},\quad \frac{\left|G_p(j\omega_u)\right|}{K_m} \le 0.5 \text{ or}$$

$$T_f = \frac{\left[1.6\dfrac{\left|G_p(j\omega_u)\right|^2}{K_m^{\,2}} - 2.3\dfrac{\left|G_p(j\omega_u)\right|}{K_m} + 1.1\right]}{3\omega_u\left[0.37 + \dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]^2\left[13 - 20\dfrac{\left|G_p(j\omega_u)\right|}{K_m}\right]},\quad \frac{\left|G_p(j\omega_u)\right|}{K_m} > 0.5 .$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003).	[2] $K_c^{(387)}$	$T_i^{(387)}$	$T_d^{(387)}$	"Almost" optimal tuning rule; $K_{150^0} \ge 0.03$; $1.6 \le M_s \le 1.9$

[2] $$K_c^{(387)} = \frac{0.60}{K_m\left[\frac{K_{150^0}}{K_m}+0.07\right]\left[0.32+1.6\frac{K_{150^0}}{K_m}-0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$T_i^{(387)} = \frac{1.5}{\omega_{150^0}\left[0.32+1.6\frac{K_{150^0}}{K_m}-0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$T_d^{(387)} = \frac{0.6667}{\omega_{150^0}\left[0.32+1.6\frac{K_{150^0}}{K_m}-0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]}, \; b_{1f} = 0,$$

$$a_{1f} = \frac{0.8}{\omega_{150^0}\left[20\frac{K_{150^0}}{K_m}+0.5\right]\left[0.32+1.6\frac{K_{150^0}}{K_m}-0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]},$$

$$a_{2f} = \frac{1}{\omega_{150^0}^{\;2}\left[20\frac{K_{150^0}}{K_m}+0.5\right]^2\left[0.32+1.6\frac{K_{150^0}}{K_m}-0.8\left(\frac{K_{150^0}}{K_m}\right)^2\right]^2}.$$

4.2.4 *Ideal controller with weighted proportional term*

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$$

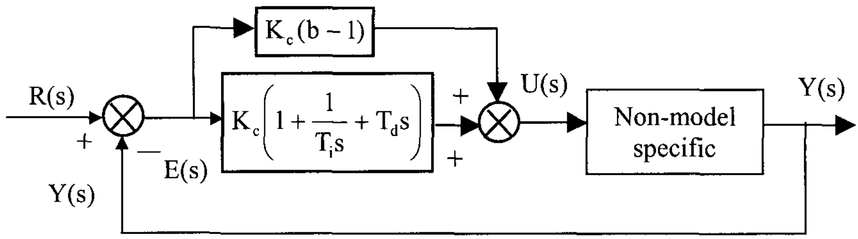

Table 71: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Åström and Hägglund (1995) – *page 217.*	[3] $K_c^{(388)}$	$T_i^{(388)}$	$T_d^{(388)}$	$0 < K_m K_u < \infty$ $M_{max} = 1.4$
	[4] $K_c^{(389)}$	$T_i^{(389)}$	$T_d^{(389)}$	$0 < K_m K_u < \infty$ $M_{max} = 2.0$
Streeter *et al.* (2003).	[5] $K_c^{(390)}$	$T_i^{(390)}$	$T_d^{(390)}$	$0 < K_m K_u < \infty$ $M_{max} = 2.0$

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[3] $K_c^{(388)} = \left(0.33e^{-0.31\kappa-\kappa^2}\right)K_u$, $T_i^{(388)} = \left(0.76e^{-1.6\kappa-0.36\kappa^2}\right)T_u$, $\left(0.33e^{-0.31\kappa-\kappa^2}\right)K_u$

$T_d^{(388)} = \left(0.17e^{-0.46\kappa-2.1\kappa^2}\right)T_u$, $b = 0.58e^{-1.3\kappa+3.5\kappa^2}$.

[4] $K_c^{(389)} = \left(0.72e^{-1.6\kappa+1.2\kappa^2}\right)K_u$, $T_i^{(389)} = \left(0.59e^{-1.3\kappa+0.38\kappa^2}\right)T_u$,

$T_d^{(389)} = \left(0.15e^{-1.4\kappa+0.56\kappa^2}\right)T_u$, $b = 0.25e^{0.56\kappa-0.12\kappa^2}$.

[5] $K_c^{(390)} = \left(0.72e^{-1.6\kappa+1.2\kappa^2}\right)K_u - 0.0012340T_u - 6.1173.10^{-6}$,

$$T_i^{(390)} = \frac{\left(0.59e^{-1.3\kappa+0.38\kappa^2}\right)\left[\left(0.72e^{-1.6\kappa+1.2\kappa^2}\right)K_u - 0.0012340T_u - 6.1173.10^{-6}\right]T_u}{\left[0.72e^{-1.6\kappa+1.2\kappa^2} - 0.040430e^{-1.3\kappa+0.38\kappa^2}\right]K_u},$$

$$T_d^{(390)} = \frac{0.108e^{-3\kappa+1.76\kappa^2}K_u T_u - 0.0026640\left(e^{T_u}\right)^{\log\left(1.6342^{\log K_u}\right)}}{0.72K_u e^{-1.6\kappa+1.2\kappa^2} - 0.0012340T_u - 6.1173.10^{-6}},$$

$$b = 0.25e^{0.56\kappa-0.12\kappa^2} + \frac{K_u}{e^{K_u}}.$$

4.2.5 *Controller with filtered derivative* $G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$

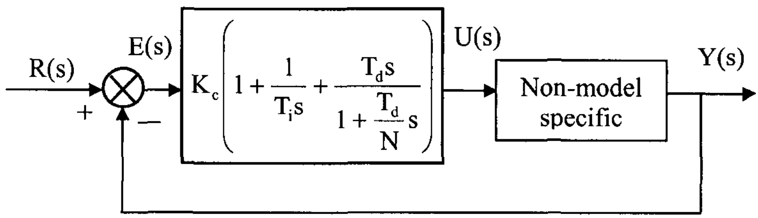

Table 72: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Leva (1993). N not specified	[6] $K_c^{(391)}$	$T_i^{(391)}$	$6.283/\beta\omega$	$\beta = 10$
	[7] $K_c^{(392)}$	$\alpha T_d^{(392)}$	$T_d^{(392)}$	$6 \le \alpha \le 10$
Yi and De Moor (1994).	[8] $K_c^{(393)}$	$T_i^{(393)}$	$T_d^{(393)}$	N not defined

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[6] $$K_c^{(391)} = \frac{\omega T_i^{(391)}}{\left|G_p(j\omega)\right|\sqrt{\left(1-\omega T_i^{(391)}\frac{2\pi}{\beta}\right)^2+\omega^2[T_i^{(391)}]^2}},$$

$$T_i^{(391)} = \frac{\tan(\phi_m - \phi_\omega - 0.5\pi)}{\omega\sqrt{1+\frac{2\pi}{\beta}\tan(\phi_m - \phi_\omega - 0.5\pi)}},\ \phi_\omega > \phi_m - \pi.$$

[7] $$K_c^{(392)} = \frac{\omega T_i^{(392)}}{\left|G_p(j\omega)\right|\sqrt{\left(1-\omega^2 T_i^{(392)} T_d^{(392)}\right)^2+\omega^2[T_i^{(392)}]^2}},$$

$$T_d^{(392)} = \frac{-\alpha\omega+\sqrt{\alpha^2\omega^2+4\alpha\omega^2\tan^2(\phi_m-\phi_\omega-0.5\pi)}}{2\alpha\omega^2\tan(\phi_m-\phi_\omega-0.5\pi)},\ \phi_\omega < \phi_m - \pi.$$

[8] $$K_c^{(393)} = \frac{0.5\cos\left[225^0 - \angle G_p(j\omega_\phi)\right]}{\left|G_p(j\omega_\phi)\right|},\ T_i^{(393)} = 4T_d^{(393)},$$

$$T_d^{(393)} = \frac{\tan\left[225^0 - \angle G_p(j\omega_\phi)\right]+\sqrt{4+\tan^2\left[225^0 - \angle G_p(j\omega_\phi)\right]}}{2\omega_\phi}.$$

Rule	K_c	T_i	T_d	Comment
Vrančić (1996) – *page 130.*	$\frac{T_i^{(394)}}{2\left(A_1 - T_i^{(394)}\right)}$	[9] $T_i^{(394)}$	$T_d^{(394)}$	$N < 10$
Vrančić (1996) – *pages 120-121.*	[10] $K_c^{(395)}$	$T_i^{(395)}$	$\frac{A_3A_4 - A_2A_5}{A_3^2 - A_1A_5}$	$N \ge 10$
Vrančić (1996) - *page 134.*	[11] $K_c^{(396)}$	$T_i^{(396)}$	$\chi T_i^{(396)}$	$N = 10$; $\chi = [0.2, 0.25]$
Vrančić *et al.* (1999).	[12] $K_c^{(397)}$	$T_i^{(397)}$	$T_d^{(397)}$	$8 \le N \le 20$

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[9] $T_i^{(394)} = \dfrac{A_3}{A_2 - T_d^{(394)}A_1 - \dfrac{[T_d^{(394)}]^2}{N}}$,

$$T_d^{(394)} = \frac{-(A_3^2 - A_5A_1) + \sqrt{\left(A_3^2 - A_5A_1\right)^2 - \frac{4}{N}(A_3A_2 - A_5)(A_5A_2 - A_4A_3)}}{\frac{2}{N}(A_3A_2 - A_5)}.$$

[10] $K_c^{(395)} = \dfrac{0.5A_3\left(A_3^2 - A_1A_5\right)}{\left(A_3^2 - A_1A_5\right)\left(A_1A_2 - A_3K_m\right) - \left(A_3A_4 - A_2A_5\right)A_1^2}$,

$$T_i^{(395)} = \frac{A_3\left(A_3^2 - A_1A_5\right)}{\left(A_3^2 - A_1A_5\right)A_2 - \left(A_3A_4 - A_2A_5\right)A_1}.$$

[11] $K_c^{(396)} = \dfrac{0.5T_i^{(396)}}{A_1 - K_mT_i^{(396)}}$, $T_i^{(396)} = \dfrac{A_2 - \sqrt{A_2^2 - 4\chi A_1A_3}}{2\chi A_1}$.

[12] $K_c^{(397)} = \dfrac{A_3}{2\left(A_1A_2 - A_0A_3 - T_d^{(397)}A_1^2 - \dfrac{[T_d^{(397)}]^2}{N}A_0A_1\right)}$,

$T_i^{(397)} = \dfrac{A_3}{A_2 - T_d^{(397)}A_1 - \dfrac{[T_d^{(397)}]^2}{N}K_m}$, $T_d^{(397)}$ is obtained by solving:

$$\frac{A_0A_3}{N^3}\left[T_d^{(397)}\right]^4 + \frac{A_1A_3}{N^2}\left[T_d^{(397)}\right]^3 - \frac{A_0A_5 - A_2A_3}{N}\left[T_d^{(397)}\right]^2 + \left(A_3^2 - A_1A_5\right)\left[T_d^{(397)}\right] + \left(A_2A_5 - A_3A_4\right) = 0.$$

Rule	K_c	T_i	T_d	Comment
Lennartson and Kristiansson (1997).	[13] $K_c^{(398)}$	$T_i^{(398)}$	$0.4T_i^{(398)}$	$K_uK_m \ge 1.67$
Kristiansson and Lennartson (1998a).	[14] $K_c^{(399)}$	$T_i^{(399)}$	$0.4T_i^{(399)}$	
		$T_i^{(400)}$	$0.4T_i^{(400)}$	
	[15] $K_c^{(401)}$	$T_i^{(401)}$	$0.4T_i^{(401)}$	$N = 2.5$
Kristiansson and Lennartson (1998b).	[16] $K_c^{(402)}$	$T_i^{(402)}$	$0.333T_i^{(402)}$	

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[13] $$K_c^{(398)} = K_uK_m \frac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]},$$

$$T_i^{(398)} = \frac{K_c^{(398)}}{-0.053\omega_u^3 + 0.47\omega_u^2 - 0.14\omega_u + 0.11},$$

$$N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right].$$

[14] $$K_c^{(399)} = K_uK_m \frac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + x_1\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]};$$

$$T_i^{(399)} = \frac{K_c^{(399)}}{-0.525\omega_u^3 + 0.473\omega_u^2 - 0.143\omega_u + 0.113},\ \frac{\omega_u}{K_uK_m} \le 0.4;$$

$$T_i^{(400)} = \frac{K_c^{(400)}}{-0.185\omega_u^3 + 1.052\omega_u^2 - 0.854\omega_u + 0.309},\ \frac{\omega_u}{K_uK_m} \ge 0.4;$$

$$N = \frac{x_1}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right];\ x_1 = 3,\ K_uK_m > 10;\ x_1 = 2.5,\ K_uK_m < 10.$$

[15] $$K_c^{(401)} = \frac{7.71}{K_m^2K_u^2} - \frac{9.14}{K_mK_u} + 3.14,\ K_uK_m > 1.67,\ \frac{\omega_u}{K_uK_m} > 0.45;$$

$$T_i^{(401)} = \frac{K_c^{(401)}}{-0.63\omega_u^3 + 0.39\omega_u^2 + 0.15\omega_u + 0.0082}.$$

[16] $$K_c^{(402)} = \frac{12K_u^3K_m^2 - 35K_u^2K_m + 30K_u}{K_m^3K_u^3 + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]},$$

$$T_i^{(402)} = \frac{K_c^{(402)}K_m^3K_u^2}{\omega_u\left[0.95K_m^2K_u^2 - 2K_mK_u + 1.4\right]},\ N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right].$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2000).	[17] $K_c^{(403)}$	$T_i^{(403)}$	$T_d^{(403)}$	$0.1 \le K_u K_m \le 0.5$
	[18] $K_c^{(404)}$	$T_i^{(404)}$	$T_d^{(404)}$	$K_u K_m > 0.5$

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[17] $$K_c^{(403)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)^2}{K_m^2 K_u(-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1\right],$$

$$T_i^{(403)} = \frac{K_m K_u(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u(-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1\right],$$

$$T_d^{(403)} = \frac{K_m K_u(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u(-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{\dfrac{(-20 + 13K_m K_u)^2(1 + 0.37K_m K_u)^2}{(1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6)^2} - 1}{\dfrac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1}\right], \quad N = \frac{T_d^{(403)}}{T_f^{(403)}},$$

$$T_f^{(403)} = \frac{K_m K_u(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u(-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2}.$$

[18] $$K_c^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)^2}{3\omega_u K_m^2 K_u^2(1 + 0.37K_m K_u)^2} \left[\frac{4.8K_m K_u(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1\right],$$

$$T_i^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{3\omega_u(1 + 0.37K_m K_u)^2} \left[\frac{4.8K_m K_u(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1\right],$$

$$T_d^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{3\omega_u(1 + 0.37K_m K_u)^2} \left[\frac{\dfrac{3K_m^2 K_u^2(1 + 0.37K_m K_u)^2}{(1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6)^2} - 1}{\dfrac{4.8K_m K_u(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1}\right],$$

$$N = \frac{T_d^{(404)}}{T_f^{(404)}}, \quad T_f^{(404)} = \frac{1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6}{3\omega_u(1 + 0.37K_m K_u)^2}.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2000) – continued.	[19] $K_c^{(405)}$	$T_i^{(405)}$	$T_d^{(405)}$	$K_u K_m < 0.1$
Kristiansson and Lennartson (2002).	[20] $K_c^{(406)}$	$T_i^{(406)}$	$T_d^{(406)}$	$K_u K_m \le 10$

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[19] $$K_c^{(405)} = \frac{(-6+3.7K_{135^0}K_m)^2}{13K_m(1.8+0.3K_mK_{135^0})^2}\left[\frac{20.8(1.8+0.3K_mK_{135^0})}{(-6+3.7K_mK_{135^0})}-1\right],$$

$$T_i^{(405)} = \frac{(-6+3.7K_{135^0}K_m)K_{135^0}K_m}{13\omega_{135^0}(1.8+0.3K_mK_{135^0})^2}\left[\frac{20.8(1.8+0.3K_mK_{135^0})}{(-6+3.7K_mK_{135^0})}-1\right],$$

$$T_d^{(405)} = \frac{(-6+3.7K_mK_{135^0})K_mK_{135^0}}{13\omega_{135^0}(1.8+0.3K_mK_{135^0})^2}\left[\frac{\dfrac{169(1.8+0.3K_mK_{135^0})^2}{(-6+3.7K_mK_{135^0})^2}}{\dfrac{20.8(1.8+0.3K_mK_{135^0})}{(-6+3.7K_mK_{135^0})}-1}-1\right],$$

$$N = \frac{T_d^{(405)}}{T_f^{(405)}},\ T_f^{(405)} = \frac{(-6+3.7K_mK_{135^0})K_mK_{135^0}}{13\omega_{135^0}(1.8+0.3K_mK_{135^0})^2}.$$

[20] $$K_c^{(406)} = \frac{K_u}{K_m}\frac{\left[1.5(K_mK_u+4)(0.4K_mK_u+0.75)-K_mK_ux_1\right]x_1}{(0.4K_mK_u+0.75)^2(4+K_mK_u)},$$

$$T_i^{(406)} = \frac{K_mK_u}{\omega_u}\frac{\left[1.5(K_mK_u+4)(0.4K_mK_u+0.75)-K_mK_ux_1\right]}{(0.4K_mK_u+0.75)^2(4+K_mK_u)},$$

$$T_d^{(406)} = \frac{K_mK_u}{\omega_u}\frac{(4+K_mK_u)}{\left[1.5(4+K_mK_u)(0.4K_mK_u+0.75)-K_mK_ux_1\right]\left(1+N^{-1}\right)},$$

$$N = \frac{T_d^{(406)}}{T_f^{(406)}},\ T_f^{(406)} = \frac{K_m^2K_u^2}{\omega_u}\frac{x_1}{(0.4K_mK_u+0.75)^2(4+K_mK_u)},$$

$$x_1 = 0.13+0.16K_mK_u-0.007K_m^2K_u^2.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2002) – continued.	[21] $K_c^{(407)}$	$T_i^{(407)}$	$T_d^{(407)}$	$K_u K_m > 10$

[21] $$K_c^{(407)} = \frac{K_{135^0}}{K_m} \frac{\left[1.5(0.44K_m K_{135^0} + 1.4)x_2 - K_m K_{135^0} x_3\right] x_3}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$T_i^{(407)} = \frac{K_m K_{135^0}}{\omega_{135^0}} \frac{\left[1.5(0.44K_m K_{135^0} + 1.4)x_2 - K_m K_u x_3\right]}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$T_d^{(407)} = \frac{K_m K_{135^0}}{\omega_{135^0}} \frac{x_2}{\left[1.5x_2(0.44K_m K_{135^0} + 1.4) - K_m K_{135^0} x_3\right]\left(1 + N^{-1}\right)},$$

$$N = \frac{T_d^{(407)}}{T_f^{(407)}}, \; T_f^{(407)} = \frac{K_m{}^2 K_{135^0}{}^2}{\omega_{135^0}} \frac{x_3}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$x_2 = \min(14 + 0.5K_m K_{135^0}, 25), \; x_3 = 1.35 + 0.35K_m K_{135^0} - 0.006K_m{}^2 K_{135^0}{}^2 .$$

4.2.6 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

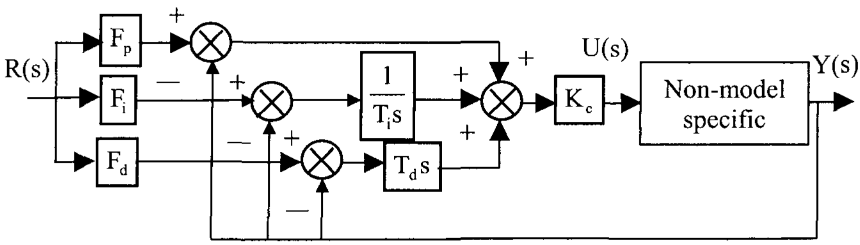

Table 73: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Mantz and Tacconi (1989).	$0.6K_u$; $F_p = 0.17$	$0.5T_u$; $F_i = 1$	$0.125T_u$; $F_d = 0.654$	Quarter decay ratio

4.2.7 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1+sT_d}{1+s\frac{T_d}{N}}$

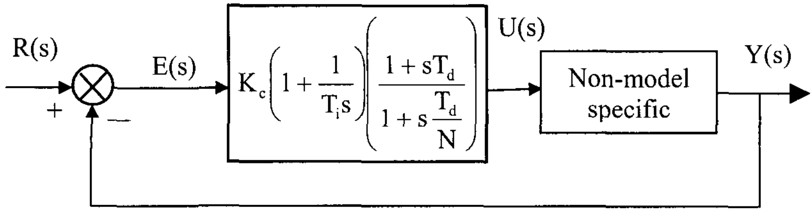

Table 74: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Edgar *et al.* (1997) –*page 8-15.*	$0.56K_u$	$0.39T_u$	$0.14T_u$	$N = 10$
	Ultimate cycle			
Kinney (1983).	$0.25K_u$	$0.5T_u$	$0.12T_u$	N not defined
Corripio (1990) – *page 27.*	$0.6K_u$	$0.5T_u$	$0.125T_u$	$10 \le N \le 20$
St. Clair (1997) – *page 17.* $N \ge 1$	$0.5K_u$	$1.2T_u$	$0.125T_u$	'aggressive' tuning
	$0.25K_u$	$1.2T_u$	$0.125T_u$	'conservative' tuning
Harrold (1999).	$0.67K_u$	$0.5T_u$	$0.125T_u$	N not defined

4.2.8 *Series controller (classical controller 3)*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + sT_d)$$

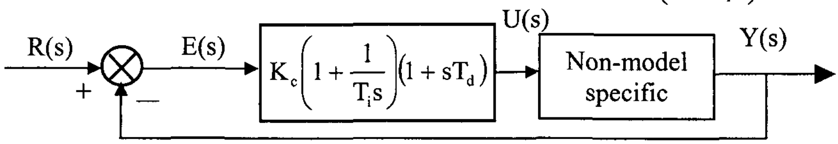

Table 75: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Pessen (1953).	$0.25K_u$	$0.33T_u$	$0.5T_u$	'optimum' servo response
	$0.2K_u$	$0.25T_u$	$0.5T_u$	
	'optimum' regulator response - step changes			
Pessen (1954).	$0.33K_u$	$0.5T_u$	$0.33T_u$	'Some'overshoot
Tan *et al.* (1999a) – *page 26.*	$0.3K_u$	$0.15T_u$	$0.25T_u$	Ziegler-Nichols ultimate cycle equivalent
O'Dwyer (2001b) – *representative results - Åström and Hägglund (1995) equivalent – page 142.*	$0.2349K_u$	$0.2273T_u$	$0.2273T_u$	$A_m = 2$, $\phi_m = 20^0$
	$0.0944K_u$	$0.6154T_u$	$0.6154T_u$	$A_m = 2.44$, $\phi_m = 61^0$
	$0.1008K_u$	$0.3939T_u$	$0.3939T_u$	$A_m = 3.45$, $\phi_m = 46^0$
Tan *et al.* (2001).	[1] $K_c^{(408)}$	$T_i^{(408)}$	$0.25T_i^{(408)}$	

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[1] $K_c^{(408)} = \dfrac{T_i^{(408)}\omega_u\left|G_c'(j\omega_u)\right|}{\sqrt{[T_i^{(408)}]^2\omega_u^2 + 1}\sqrt{0.0625[T_i^{(408)}]^2\omega_u^2 + 1}}$,

$$T_i^{(408)} = \frac{\sqrt{6.25\omega_u^2 + 4\omega_u^2\tan^2\left[\dfrac{2\angle G_c'(j\omega_u) + \pi}{2}\right]} - 2.5\omega_u}{\omega_u^2\tan\left[\dfrac{2\angle G_c'(j\omega_u) + \pi}{2}\right]},$$

$G_c'(j\omega_u)$ = desired controller frequency response at $\omega = \omega_u$.

4.2.9 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$

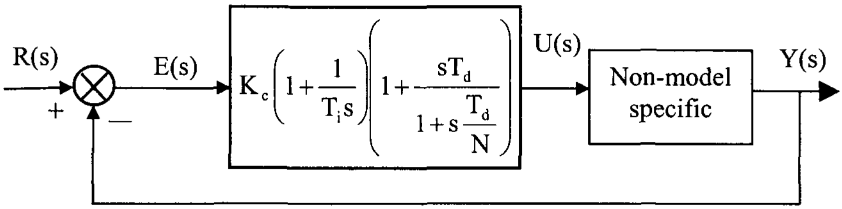

Table 76: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Hang *et al.* (1993b) - *page 58.*	$0.35K_u$	$1.13T_u$	$0.20T_u$	N not specified

4.2.10 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$

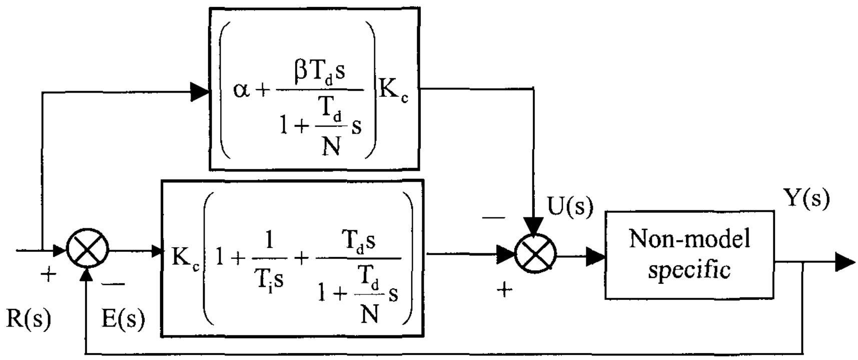

Table 77: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: other tuning			
Shen (2002).	[2] $K_c^{(409)}$	$T_i^{(409)}$	$T_d^{(409)}$	$\hat{K}_u > 1/K_m$

[2] Minimise $\int_0^\infty |r(t) - y(t)|dt + \int_0^\infty |d(t) - y(t)|dt$; N=0, $\beta = 0$, $M_s \le 2$;

$$K_c^{(409)} = \hat{K}_u \exp\left[0.17 - 2.62\Big/\left(K_m \hat{K}_u\right) + 1.79\Big/\left(K_m{}^2 \hat{K}_u^2\right)\right],$$

$$T_i^{(409)} = \hat{T}_u \exp\left[-0.02 - 2.62\Big/\left(K_m \hat{K}_u\right) + 1.34\Big/\left(K_m{}^2 \hat{K}_u^2\right)\right],$$

$$T_d^{(409)} = \hat{T}_u \exp\left[-1.70 - 0.59\Big/\left(K_m \hat{K}_u\right) - 0.25\Big/\left(K_m{}^2 \hat{K}_u^2\right)\right],$$

$$\alpha = 1 - \exp\left[-0.30 - 0.48\Big/\left(K_m \hat{K}_u\right) + 0.93\Big/\left(K_m{}^2 \hat{K}_u^2\right)\right].$$

4.2.11 *Non-interacting controller 4*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$$

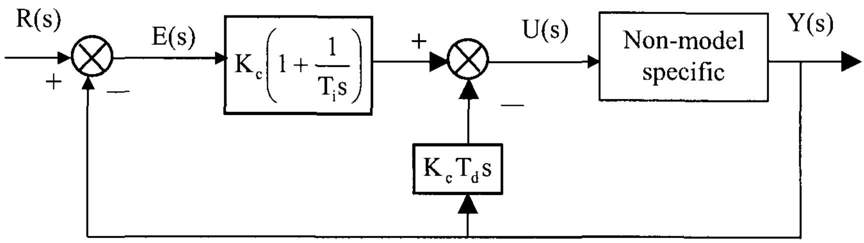

Table 78: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Edgar *et al.* (1997) – *page 8-15.*	$0.77K_u$	$0.48T_u$	$0.11T_u$	
	Ultimate cycle			
VanDoren (1998).	$0.75K_u$	$0.625T_u$	$0.1T_u$	

4.2.12 *Non-interacting controller 9*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)E(s) - \frac{T_d}{N}sY(s)$$

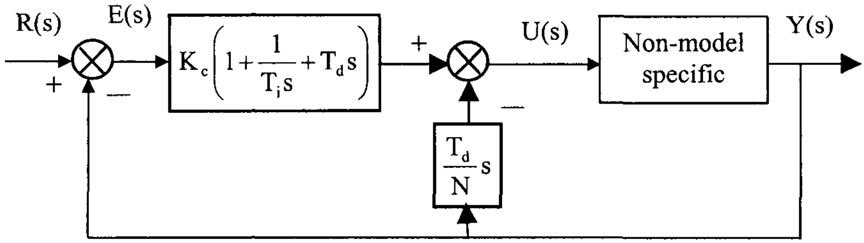

Table 79: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
	Other tuning			
Bateson (2002) - *pages 629-637.* *Model: Method 1*	[3] $K_c^{(410)}$	$T_i^{(410)}$	$\frac{2}{\omega_{180^0}}$	N = 10

[3] $K_c^{(410)} = 10^{\min\left[\left\{-\left|G_p\left(j\omega_{140^0}\right)\right|\right\} \text{ or } \left\{-\left|G_p\left(j\omega_{180^0}\right)\right|-6\right\}\right]}$, $\left|G_p\left(j\omega_{140^0}\right)\right|$ and $\left|G_p\left(j\omega_{180^0}\right)\right|$ are given in dB; $T_i^{(410)} = \min\left[\omega_{82^0}, 0.2\omega_{170^0}\right]$.

4.3 IPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

4.3.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

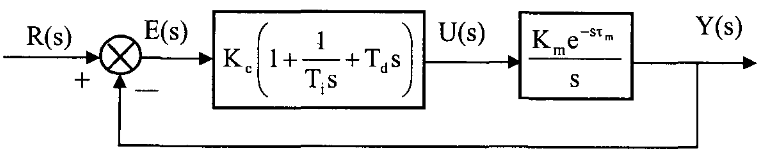

Table 80: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Ford (1953). *Model: Method 3*	$\frac{1.48}{K_m\tau_m}$	$2\tau_m$	$0.37\tau_m$	Decay ratio 2.7:1
Åström and Hägglund (1995) – *page 139.*	$\frac{0.94}{K_m\tau_m}$	$2\tau_m$	$0.5\tau_m$	*Model: Method 1*
	Ultimate cycle Ziegler-Nichols equivalent			
Hay (1998) – *page 188.*	$\frac{0.4}{K_m\tau_m}$	$3.2\tau_m$	$0.8\tau_m$	*Model: Method 3*
Hay (1998) – *page 199. Model: Method 1;* K_c, T_d *deduced from graphs*	10.0	$3.2K_m\tau_m^2$	$0.55\tau_m$	$K_m\tau_m = 0.1$
	4.0		$0.30\tau_m$	$K_m\tau_m = 0.2$
	2.5		$0.25\tau_m$	$K_m\tau_m = 0.3$
	2.0		$0.25\tau_m$	$K_m\tau_m = 0.4$
	1.8		$0.25\tau_m$	$K_m\tau_m = 0.5, 0.6$
	Minimum performance index: regulator tuning			
Visioli (2001). *Model: Method 1*	$x_1/K_m\tau_m$	$x_2\tau_m$	$x_3\tau_m$	
	Coefficient values			
	1.37	1.49	0.59	Minimum ISE
	1.36	1.66	0.53	Minimum ITSE
	1.34	1.83	0.49	Minimum ISTSE

Rule	K_c		T_i		T_d		Comment	
	Minimum performance index: servo tuning							
Visioli (2001). *Model: Method 1*	$x_1/K_m\tau_m$		0		$x_3\tau_m$			
	Coefficient values							
	1.03				0.49		Minimum ISE	
	0.96				0.45		Minimum ITSE	
	0.90				0.45		Minimum ISTSE	
	Minimum performance index: other tuning							
Åström and Hägglund (2004). *Model: Method 1*	$x_1/K_m\tau_m$		$x_2\tau_m$		$x_3\tau_m$			
	Coefficient values							
	x_1	x_2	x_3	M_{max}	x_1	x_2	x_3	M_{max}
	0.139	76.9	0.346	1.1	0.616	4.58	0.410	1.6
	0.261	23.3	0.365	1.2	0.681	3.82	0.418	1.7
	0.367	12.2	0.378	1.3	0.740	3.28	0.426	1.8
	0.460	7.85	0.389	1.4	0.793	2.89	0.434	1.9
	0.543	5.78	0.400	1.5	0.841	2.61	0.440	2.0
	Direct synthesis							
Leonard (1994). *Model: Method 1*	$\frac{0.74}{K_m\tau_m}$		$12.2\tau_m$		$0.41\tau_m$		OS (step input) < 10%; Minimum IAE (disturbance ramp).	
	$0.47K_u$		$3.05T_u$		$0.10T_u$			
Wang and Cluett (1997).	[1] $K_c^{(411)}$		$T_i^{(411)}$		$T_d^{(411)}$		*Model: Method 1*	
	[2] $K_c^{(412)}$		$T_i^{(412)}$		$T_d^{(412)}$			
	K_u, T_u *deduced from graph*							
Cluett and Wang (1997). *Model: Method 1*	$x_1/K_m\tau_m$		$x_2\tau_m$		$x_3\tau_m$		$T_{CL}=x_4\tau_m$	
	Coefficient values							
	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
	0.9588	3.0425	0.3912	1	0.3752	9.1925	0.1702	4
	0.6232	5.2586	0.2632	2	0.3144	11.1637	0.1453	5
	0.4668	7.2291	0.2058	3	0.2709	13.1416	0.1269	6

[1] $K_c^{(411)} = \frac{1}{K_m\tau_m(0.7138T_{CL}+0.3904)}$, $T_i^{(411)} = (1.4020T_{CL}+1.2076)\tau_m$,

$$T_d^{(411)} = \frac{\tau_m}{1.4167T_{CL}+1.6999};\ T_{CL}\in[\tau_m, 16\tau_m],\ \xi = 0.707.$$

[2] $K_c^{(412)} = \frac{1}{K_m\tau_m(0.5080T_{CL}+0.6208)}$, $T_i^{(412)} = (1.9885T_{CL}+1.2235)\tau_m$,

$$T_d^{(412)} = \frac{\tau_m}{1.0043T_{CL}+1.8194};\ T_{CL}\in[\tau_m, 16\tau_m],\ \xi = 1.$$

Rule	K_c	T_i	T_d	Comment
Rotach (1995). *Model: Method 5*	$\frac{1.21}{K_m \tau_m}$	$1.60\tau_m$	$0.48\tau_m$	
	Damping factor for oscillations to a disturbance input = 0.75.			
Chen *et al.* (1999a).	$\frac{1.661}{A_m K_m \tau_m}$	0	[3] $T_d^{(413)}$ [4] $T_d^{(414)}$	*Model: Method 1*
Chidambaram and Sree (2003).	$\frac{1.2346}{K_m \tau_m}$	$4.5\tau_m$	$0.45\tau_m$	*Model: Method 1*
Sree and Chidambaram (2005b).	$\frac{0.896}{K_m \tau_m}$	$2.5\tau_m$	$0.55\tau_m$	*Model: Method 1*
Chen and Seborg (2002).	[5] $K_c^{(415)}$	$T_i^{(415)}$	$T_d^{(415)}$	*Model: Method 1*
	Robust			
Zou *et al.* (1997), Zou and Brigham (1998). *Model: Method 7 or Method 8*	$\frac{2}{K_m(\lambda + 0.5\tau_m)}$	$2\lambda + \tau_m$	$\frac{\lambda + 0.25\tau_m}{2\lambda + \tau_m}\tau_m$	$0.5\tau_m \le \lambda \le 3\tau_m$

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[3] $T_d^{(413)} = 1.273\tau_m\left[1 - 0.6002A_m(1.571 - \phi_m)\right]$.

[4] $T_d^{(414)} = \tau_m\left(1.2732 - \frac{r_1 A_m}{1.6661}\right)$.

[5] $\left(\frac{y}{d}\right)_{\text{desired}} = \frac{T_i^{(415)} s(1 + 0.5\tau_m s)e^{-s\tau_m}}{K_c^{(415)}(T_{CL}s + 1)^3}$, $K_c^{(415)} = \frac{\tau_m}{K_m}\frac{(3T_{CL} + 0.5\tau_m)}{(T_{CL} + 0.5\tau_m)^3}$,

$$T_i^{(415)} = 3T_{CL} + 0.5\tau_m,\ T_d^{(415)} = \frac{1.5T_{CL}^2\tau_m + 0.75T_{CL}\tau_m^2 + 0.125\tau_m^3 - T_{CL}^3}{\tau_m(3T_{CL} + 0.5\tau_m)}.$$

4.3.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

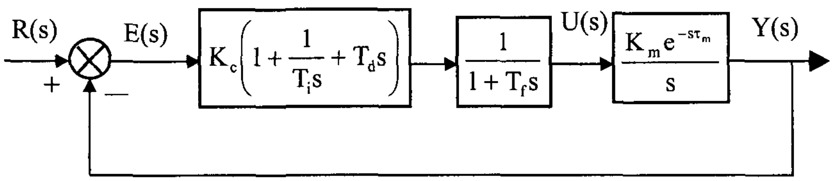

Table 81: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Rice and Cooper (2002). *Model: Method 1*	[6] $K_c^{(416)}$	$2\lambda + 1.5\tau_m$	$\frac{0.5\tau_m^2 + \lambda\tau_m}{2\lambda + 1.5\tau_m}$	Suggested $\lambda = 3.162\tau_m$

[6] $K_c^{(416)} = \frac{2\lambda + 1.5\tau_m}{K_m\left(\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2\right)}$, $T_f = \frac{0.5\lambda^2\tau_m}{\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2}$.

4.3.3 *Ideal controller with first order filter and set-point weighting 2*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}\left[R(s)\frac{1+0.4T_r s}{1+sT_r} - Y(s)\right] - K_0 Y(s)$$

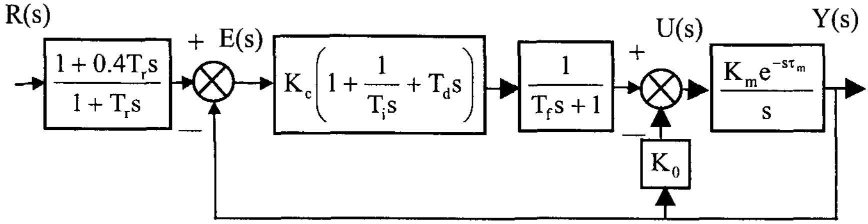

Table 82: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Normey-Rico *et al.* (2000). *Model: Method 1*	$\frac{0.563}{K_m \tau_m}$	$1.5\tau_m$	$0.667\tau_m$	$T_f = 0.13\tau_m$ $K_0 = \frac{1}{2K_m \tau_m}$ $T_r = 0.75\tau_m$

4.3.4 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{sT_d}{N}}\right)$

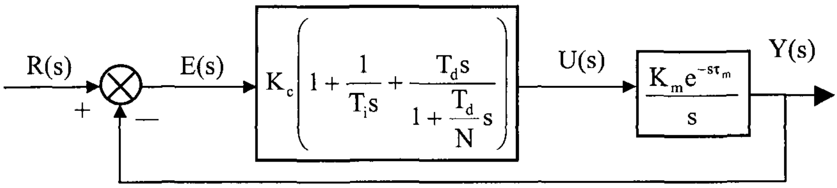

Table 83: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Kristiansson and Lennartson (2002).	[7] $K_c^{(417)}$	$T_i^{(417)}$	$T_d^{(417)}$	*Model: Method 1*
	Robust			
Chien (1988). *Model: Method 1*	[8] $K_c^{(418)}$	$2\lambda + \tau_m$	$\frac{\tau_m(\lambda + 0.25\tau_m)}{2\lambda + \tau_m}$	N=10

---

[7] $K_c^{(417)} = \frac{2.5\omega_u}{K_m}\left(-0.08 + \frac{0.055}{K_1} + \frac{0.059}{K_1^2}\right)$
$$\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right),$$

$$T_i^{(417)} = \frac{2.5}{\omega_u}\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right),$$

$$T_d^{(417)} = \frac{2.5}{\omega_u}\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right)^{-1}\left(\frac{1}{N} + 1\right)^{-1},$$

$$N = \frac{T_d^{(417)}}{T_f^{(417)}},\ T_f^{(417)} = \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.16\omega_u(10 + 1/K_1)},\ K_1 \in [0.5,1].$$

[8] $K_c^{(418)} = \frac{2\lambda + \tau_m}{K_m(0.5\lambda + \tau_m)^2}$; $\lambda \in [1/K_m, \tau_m]$ (Chien and Fruehauf (1990)).

4.3.5 *Controller with filtered derivative and dynamics on the controlled variable*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{sT_d}{N}}\right)E(s) - K_0 Y(s)$$

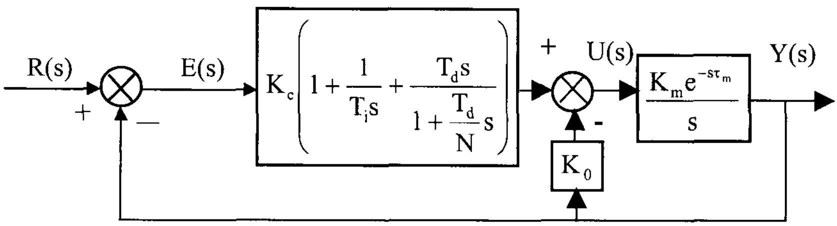

Table 84: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis – time domain criteria			
Normey-Rico *et al.* (2001). *Model: Method 1*	[9] $K_c^{(419)}$	$(1.5-\gamma)\tau_m$	$T_d^{(419)}$	$K_0 = \frac{1}{2K_m\tau_m}$
	$\frac{0.1716}{K_m\tau_m}$	τ_m	$0.5\tau_m$	$N = 1$; $K_0 = \frac{1}{2K_m\tau_m}$
	Recommended $\gamma = 0.5$; non-oscillatory response			

[9] $K_c^{(419)} = \frac{1.5-\gamma}{K_m\tau_m}\left[4\gamma + 1 - 4\sqrt{\gamma^2 + 0.5\gamma}\right]$, $T_d^{(419)} = \frac{\tau_m}{1.5-\gamma} - \gamma\tau_m$, $N = \frac{1-\gamma(1.5-\gamma)}{(1.5-\gamma)\gamma}$.

4.3.6 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

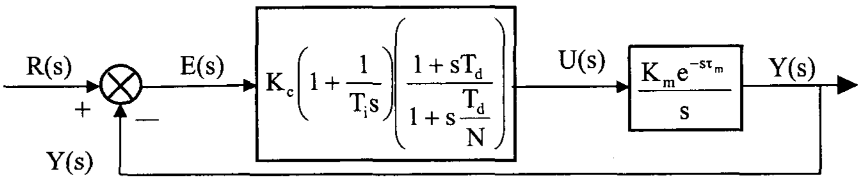

Table 85: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="3">T_d</th><th colspan="3">Comment</th></tr>
<tr><td></td><td colspan="12">Process reaction</td></tr>
<tr><td rowspan="14">Bunzemeier (1998).
Model: Method 4;
$G_p = \frac{K_p}{s(1+sT_p)^n}$</td><td colspan="3">$\frac{x_1}{K_m\tau_m}$</td><td colspan="3" rowspan="2">$x_2\tau_m$</td><td colspan="3" rowspan="2">$x_3\tau_m$</td><td colspan="3">0% overshoot (servo)</td></tr>
<tr><td colspan="3">$\frac{x_4}{K_m\tau_m}$</td><td colspan="3">10% overshoot (servo)</td></tr>
<tr><td colspan="12">Coefficient values</td></tr>
<tr><td colspan="2">x_1</td><td colspan="2">x_2</td><td colspan="2">x_3</td><td colspan="2">x_4</td><td colspan="2">N</td><td colspan="2">n</td></tr>
<tr><td colspan="2">0.21</td><td colspan="2">0.730</td><td colspan="2">0.280</td><td colspan="2">0.24</td><td colspan="2">5.60</td><td colspan="2">0</td></tr>
<tr><td colspan="2">0.68</td><td colspan="2">1.570</td><td colspan="2">0.796</td><td colspan="2">0.94</td><td colspan="2">5.99</td><td colspan="2">2</td></tr>
<tr><td colspan="2">0.47</td><td colspan="2">1.200</td><td colspan="2">0.669</td><td colspan="2">0.62</td><td colspan="2">6.03</td><td colspan="2">3</td></tr>
<tr><td colspan="2">0.35</td><td colspan="2">0.981</td><td colspan="2">0.587</td><td colspan="2">0.46</td><td colspan="2">5.99</td><td colspan="2">4</td></tr>
<tr><td colspan="2">0.30</td><td colspan="2">0.866</td><td colspan="2">0.530</td><td colspan="2">0.38</td><td colspan="2">6.02</td><td colspan="2">5</td></tr>
<tr><td colspan="2">0.27</td><td colspan="2">0.828</td><td colspan="2">0.487</td><td colspan="2">0.34</td><td colspan="2">6.01</td><td colspan="2">6</td></tr>
<tr><td colspan="2">0.25</td><td colspan="2">0.801</td><td colspan="2">0.452</td><td colspan="2">0.32</td><td colspan="2">6.03</td><td colspan="2">7</td></tr>
<tr><td colspan="2">0.23</td><td colspan="2">0.782</td><td colspan="2">0.424</td><td colspan="2">0.30</td><td colspan="2">5.97</td><td colspan="2">8</td></tr>
<tr><td colspan="2">0.22</td><td colspan="2">0.767</td><td colspan="2">0.401</td><td colspan="2">0.29</td><td colspan="2">5.99</td><td colspan="2">9</td></tr>
<tr><td colspan="2">0.22</td><td colspan="2">0.756</td><td colspan="2">0.381</td><td colspan="2">0.27</td><td colspan="2">5.95</td><td colspan="2">10</td></tr>
<tr><td></td><td colspan="12">Minimum performance index: regulator tuning</td></tr>
<tr><td>Minimum IAE – Shinskey (1988) – *page 143*.</td><td colspan="3">$\frac{0.93}{K_m\tau_m}$</td><td colspan="3">$1.57\tau_m$</td><td colspan="3">$0.58\tau_m$</td><td colspan="3">*Model: Method 2*; N not specified</td></tr>
<tr><td rowspan="2">Minimum IAE – Shinskey (1994) – *page 74*.
Model: Method 1</td><td colspan="3">$\frac{0.93}{K_m\tau_m}$</td><td colspan="3">$1.60\tau_m$</td><td colspan="3">$0.58\tau_m$</td><td colspan="3">N=10</td></tr>
<tr><td colspan="3">$\frac{0.93}{K_m\tau_m}$</td><td colspan="3">$1.48\tau_m$</td><td colspan="3">$0.63\tau_m$</td><td colspan="3">N=20</td></tr>
</table>

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Shinskey (1996) – *page 117.*	$\frac{0.93}{K_m \tau_m}$	$1.57\tau_m$	$0.56\tau_m$	*Model: Method 2*; N not specified
Minimum IAE – Shinskey (1996) – *page 121.*	$0.56K_u$	$0.39T_u$	$0.15T_u$	*Model: Method 1*; N not specified
	Robust			
Chien (1988). *Model: Method 1*	[10] $K_c^{(420)}$	$0.5\tau_m$	$2\lambda + 0.5\tau_m$	$\lambda \in [1/K_m, \tau_m]$ (Chien and Fruehauf (1990)); N=10
	[10] $K_c^{(421)}$	$2\lambda + 0.5\tau_m$	$0.5\tau_m$	
Rice and Cooper (2002). *Model: Method 1*	[11] $K_c^{(422)}$	$2\lambda + \tau_m$	$0.5\tau_m$	Suggested $\lambda = 3.162\tau_m$
	Ultimate cycle			
Luyben (1996). *Model: Method 9*	$0.46K_u$	$2.2T_u$	$0.16T_u$	Maximum closed loop log modulus of +2dB ; N=10
Belanger and Luyben (1997). *Model: Method 1*	$3.11K_u$	$2.2T_u$	$3.64T_u$	N=0.1

[10] $K_c^{(420)} = \frac{1}{K_m}\left(\frac{0.5\tau_m}{[\lambda + 0.5\tau_m]^2}\right)$, $K_c^{(421)} = \frac{1}{K_m}\left(\frac{2\lambda + 0.5\tau_m}{[\lambda + 0.5\tau_m]^2}\right)$.

[11] $K_c^{(422)} = \frac{2\lambda + \tau_m}{K_m\left(\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^{\,2}\right)}$, $N = \frac{\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^{\,2}}{\lambda^2}$.

4.3.7 *Classical controller 2* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+NT_d s}{1+T_d s}\right)$

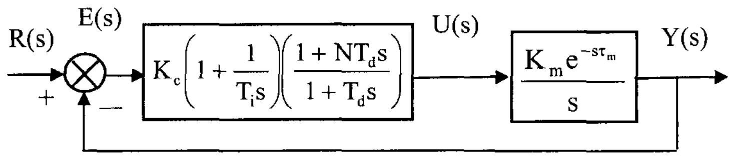

Table 86: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis – frequency domain criteria			
Hougen (1979) – *page 333-334. Model: Method 1*	[12] $K_c^{(423)}$	∞	$0.45\frac{\tau_m}{N}$	Maximise crossover frequency $N \in [10,30]$
Hougen (1988). *Model: Method 1*	[13] $K_c^{(424)}$	∞	$10^{[\log_{10}\tau_m + 0.65]}$	Five criteria are fulfilled; N = 10.

[12] Values deduced from graph:

$K_m\tau_m$	0.1	0.2	0.3	0.4	0.5
$K_c^{(423)}$	9	4.2	2.8	2.1	1.7
$K_m\tau_m$	0.6	0.7	0.8	0.9	1.0
$K_c^{(423)}$	1.45	1.2	1.1	0.95	0.85

[13] Equations deduced from graph; $K_c^{(424)} \approx \frac{1}{K_m'} 10^{-\left[\log_{10}\left(\frac{\tau_m}{T_m'}\right)\right]}$, $K_m = \frac{K_m'}{T_m'}$.

4.3.8 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$

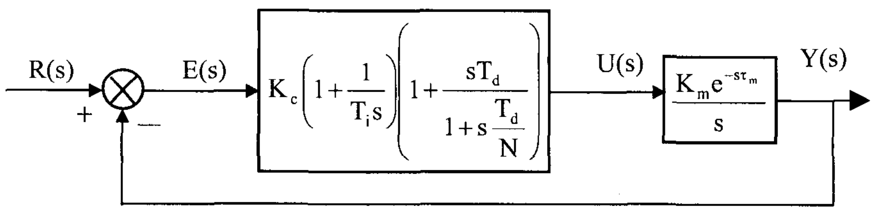

Table 87: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	[1] $K_c^{(425)}$	$2\lambda + 0.5\tau_m$	$0.5\tau_m$	$\lambda \in [1/K_m, \tau_m]$ (Chien and Fruehauf (1990)); N=10
	[2] $K_c^{(426)}$	$0.5\tau_m$	$2\lambda + 0.5\tau_m$	

[1] $K_c^{(425)} = \frac{1}{K_m}\left(\frac{2\lambda + 0.5\tau_m}{[\lambda + 0.5\tau_m]^2}\right)$

[2] $K_c^{(426)} = \frac{1}{K_m}\left(\frac{0.5\tau_m}{[\lambda + 0.5\tau_m]^2}\right)$

4.3.9 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$

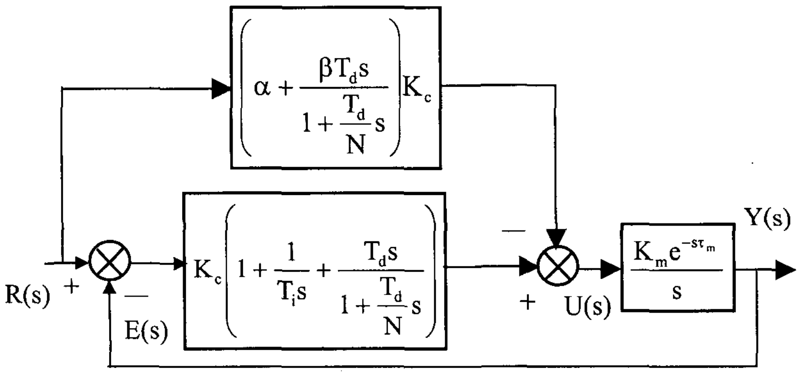

Table 88: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment	
	Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000). *Model: Method 10*	x_1K_u	x_2T_u	x_3T_u		
	Coefficient values				
	x_1	x_2	x_3	α	ϕ_c
	1.672	0.366	0.136	0.601	-164^0
	1.236	0.427	0.149	0.607	-160^0
	0.994	0.486	0.155	0.610	-155^0
	0.842	0.538	0.154	0.616	-150^0
	0.752	0.567	0.157	0.605	-145^0
	0.679	0.610	0.149	0.610	-140^0
	0.635	0.637	0.142	0.612	-135^0
	0.590	0.669	0.133	0.610	-130^0
	0.551	0.690	0.114	0.616	-125^0
	0.520	0.776	0.087	0.609	-120^0
	0.509	0.810	0.068	0.611	-118^0

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000). *Model: Method 1*	$\frac{1}{K_m}\left(\frac{1.253}{\tau_m}\right)$	$2.388\tau_m$	$0.4137\tau_m$	$\frac{\tau_m}{T_m} \le 1.0$
	$\alpha = 0.6642$, $\beta = 0.6797$, Overshoot (servo step) $\le 20\%$; N fixed but not specified			
	Direct synthesis			
Hansen (2000). *Model: Method 1*	$0.938K_m\tau_m$	$2.7\tau_m$	$0.313\tau_m$	$N = 0$; $\beta = 1$; $\alpha = 0.833$
Chidambaram (2000b). *Model: Method 1*	$\frac{0.9425}{K_m\tau_m}$	$2\tau_m$	$0.5\tau_m$	$N = \infty$; $\beta = 1$; $\alpha = 0.707$ or $\alpha = 0.65$
Chidambaram and Sree (2003). *Model: Method 1*	$\frac{1.2346}{K_m\tau_m}$	$4.5\tau_m$	$0.45\tau_m$	$N = 0$; $\beta = 0$; $\alpha = 0.6$

4.3.10 *Non-interacting controller based on the two degree of freedom structure 3*

$$U(s) = K_c\left[(b-1)+(c-1)T_d s\right]R(s) + K_c\left(1+\frac{1}{T_i s}+T_d s\right)\left[R(s)-\frac{1}{(1+sT_f)^2}Y(s)\right]$$

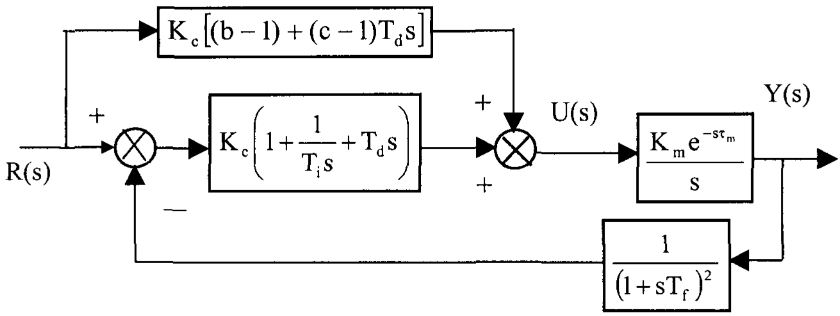

Table 89: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: other tuning			
Åström and Hägglund (2004). *Model: Method 1*	$\frac{0.45}{K_m}$	$8\tau_m$	$0.5\tau_m$	No precisely defined performance specification
	$b = 0$, $\frac{\tau_m}{T_m} \le 1$; $b = 1$, $\frac{\tau_m}{T_m} > 1$. $c = 0$. $T_f = 0.1\tau_m$			

4.3.11 *Non-interacting controller 4*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$$

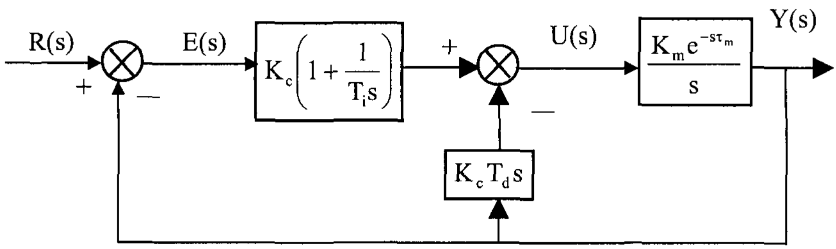

Table 90: PID controller tuning rules – IPD model $\frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE – Shinskey (1988) – *page 143. Model: Method 2*	$\frac{1.28}{K_m \tau_m}$	$1.90\tau_m$	$0.48\tau_m$	Also defined by Shinskey (1996), *page 117.*
Minimum IAE - Shinskey (1994) – *page 74.*	$\frac{1.28}{K_m \tau_m}$	$1.90\tau_m$	$0.46\tau_m$	*Model: Method 1*
Minimum IAE – Shinskey (1988) – *page 148.*	$0.82K_u$	$0.48T_u$	$0.12T_u$	*Model: Method 1*
Minimum IAE – Shinskey (1996) – *page 121.*	$0.77K_u$	$0.48T_u$	$0.12T_u$	*Model: Method 1*

4.3.12 *Non-interacting controller 6 (I-PD controller)*

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c(1 + T_d s)Y(s)$$

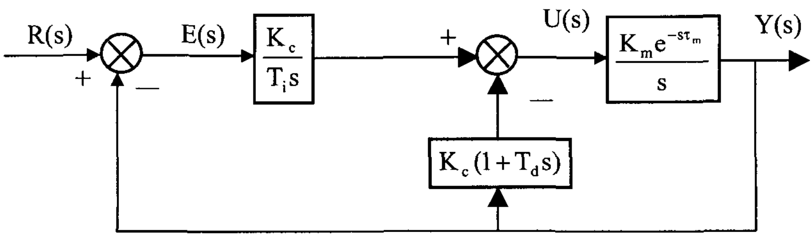

Table 91: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum ISE– Arvanitis *et al.* (2003a).	$\frac{1.4394}{K_m \tau_m}$	$2.4569\tau_m$	$0.3982\tau_m$	*Model: Method 1*
Arvanitis *et al.* (2003a).	$\frac{1.2986}{K_m \tau_m}$	$3.2616\tau_m$	$0.4234\tau_m$	*Model: Method 1*
	Minimise performance index $\int_0^\infty \left[e^2(t) + K_m{}^2 u^2(t)\right]dt$			
Arvanitis *et al.* (2003a).	$\frac{1.1259}{K_m \tau_m}$	$6.7092\tau_m$	$0.4627\tau_m$	*Model: Method 1*
	Minimise performance index $\int_0^\infty \left[e^2(t) + K_m{}^2 \left(\frac{du}{dt}\right)^2\right]dt$			
	Minimum performance index: servo tuning			
Minimum ISE– Arvanitis *et al.* (2003a).	$\frac{1.5521}{K_m \tau_m}$	$2.1084\tau_m$	$0.3814\tau_m$	*Model: Method 1*
Arvanitis *et al.* (2003a).	$\frac{1.4057}{K_m \tau_m}$	$2.5986\tau_m$	$0.4038\tau_m$	*Model: Method 1*
	Minimise performance index $\int_0^\infty \left[e^2(t) + K_m{}^2 u^2(t)\right]dt$			

Rule	K_c	T_i	T_d	Comment
Arvanitis *et al.* (2003a).	$\frac{1.3332}{K_m \tau_m}$	$3.0010\tau_m$	$0.4167\tau_m$	*Model: Method 1*
	Minimise performance index $\int_0^{\infty}\left[e^2(t) + K_m^{\,2}\left(\frac{du}{dt}\right)^2\right]dt$			
	Direct synthesis			
Chien *et al.* (1999). *Model: Method 2*	[3] $K_c^{(427)}$	$1.414T_{CL2} + \tau_m$	$T_d^{(427)}$	Underdamped system response - $\xi = 0.707$
Arvanitis *et al.* (2003a).	$\frac{16(2\xi^2+1)}{(32\xi^2+4)K_m\tau_m}$	$(2\xi^2+1)\tau_m$	$\frac{16\xi^2+4}{16(2\xi^2+1)}\tau_m$	*Model: Method 1*

[3] $K_c^{(427)} = \frac{1.414T_{CL2} + \tau_m}{K_m\left(T_{CL2}^{\,2} + 0.707T_{CL2}\tau_m + 0.25\tau_m^{\,2}\right)}$, $T_d^{(427)} = \frac{0.25\tau_m^{\,2} + 0.707T_{CL2}\tau_m}{1.414T_{CL2} + \tau_m}$.

4.3.13 *Non-interacting controller 8*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)E(s) - K_i\left(1+T_{di}s\right)Y(s)$$

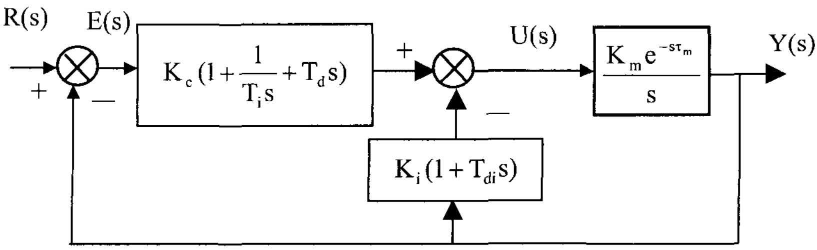

Table 92: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[4] $K_c^{(428)}$	$T_i^{(428)}$	$T_d^{(428)}$	$K_i = 0.25K_u$, $T_{di} = 0$

[4] $K_c^{(428)} = \frac{0.25K_u}{(\lambda+\tau_m)}\left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda+\tau_m)}\right]$, $T_i^{(428)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda+\tau_m)}$

$T_d^{(428)} = \frac{0.25K_u}{(\lambda+\tau_m)}\{0.25\tau_m^2 - \frac{\tau_m^3}{6(\lambda+\tau_m)} + \frac{\tau_m^4}{4(\lambda+\tau_m)^2} + \frac{\tau_m^2(1-0.25K_u K_m \tau_m)}{0.5K_u K_m(\lambda+\tau_m)}\}$.

4.3.14 *Non-interacting controller 10*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_f\left(\frac{T_d}{T_m}s+1\right)Y(s)$$

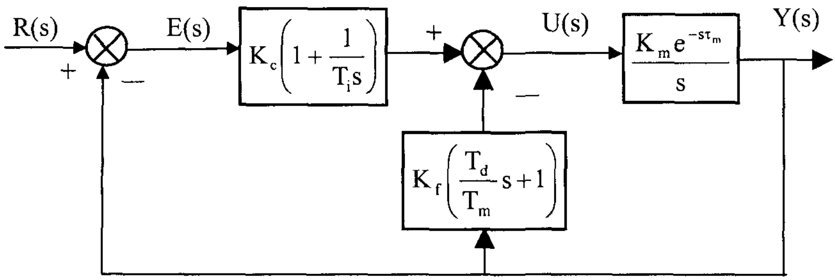

Table 93: PID tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Kaya and Atherton (1999), Kaya (2003).	$0.313\hat{K}_u$	$0.733\hat{T}_u$	[5] $T_d^{(429)}$	*Model: Method 9*

---

[5] $$T_d^{(429)} = \frac{0.753\left[\left(\frac{K_m \hat{K}_u}{\tau_m \hat{T}_u}\right)^{0.333} - \frac{1}{\tau_m}\right]\frac{\tau_m T_m}{K_m}}{\frac{0.567}{K_m}\left(\frac{K_m \hat{K}_u}{\tau_m \hat{T}_u}\right)^{0.667} - 0.313\hat{K}_u}.$$

4.3.15 *Non-interacting controller 12*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_d s + 1}E(s) - K_1 Y(s)$$

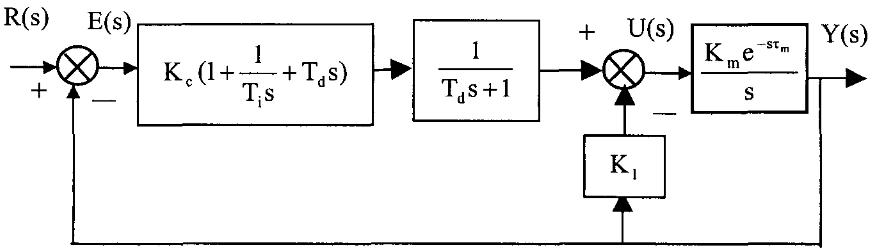

Table 94: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[6] $K_c^{(430)}$	$T_i^{(430)}$	$T_d^{(430)}$	$K_1 = 0.25K_u$

---

[6] $$K_c^{(430)} = \frac{0.25K_u}{(\lambda+\tau_m)}\left\{\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda+\tau_m)} + \tau_m\left[0.5 - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(1-0.25K_u K_m \tau_m)}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5}\right\},$$

$$T_i^{(430)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda+\tau_m)} + \tau_m\left[0.5 - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(1-0.25K_u K_m \tau_m)}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5},$$

$$T_d^{(430)} = \tau_m\left[0.5 - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(1-0.25K_u K_m \tau_m)}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5}.$$

4.4 FOLIPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

4.4.1 *Ideal controller* $G_c(s) = K_c\left(1 + \dfrac{1}{T_i s} + T_d s\right)$

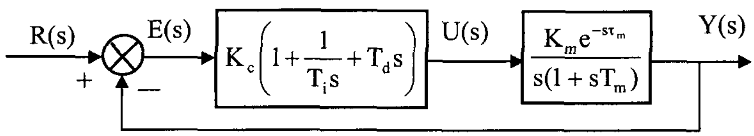

Table 95: PID controller tuning rules – FOLIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum ISE – Haalman (1965).	$\dfrac{2}{3K_m\tau_m}$	0	T_m	*Model: Method 1*
	$A_m = 2.36$; $\phi_m = 50^0$; $M_s = 1.9$.			
	Direct synthesis			
Van der Grinten (1963). *Model: Method 1*	$\dfrac{1}{K_m\tau_m}$	0	$T_m + 0.5\tau_m$	Step disturbance
	$\dfrac{e^{-2\omega_d\tau_m}}{K_m\tau_m}$	0	[1] $T_d^{(431)}$	Stochastic disturbance
Tachibana (1984).	$\dfrac{\lambda}{K_m(1+\lambda\tau_m)}$	0	T_m	*Model: Method 5*

[1] $T_d^{(431)} = \left(1 - 0.5e^{-\omega_d\tau_m} + \dfrac{T_m}{\tau_m}e^{-\omega_d\tau_m}\right)\tau_m e^{-\omega_d\tau_m}$.

<table>
<tr><th>Rule</th><th colspan="6">K_c</th><th colspan="6">T_i</th><th colspan="6">T_d</th><th colspan="6">Comment</th></tr>
<tr><td rowspan="2">Chen et al. (1999a).</td><td colspan="6">$\frac{\omega_p T_m}{K_m A_m T_d^{(432)}}$</td><td colspan="6">0</td><td colspan="6">[2] $T_d^{(432)}$</td><td colspan="6">Model: Method 1</td></tr>
<tr><td colspan="6">$\frac{r_1 \omega_r T_m}{K_m T_d^{(433)}}$</td><td colspan="6">0</td><td colspan="6">[2] $T_d^{(433)}$</td><td colspan="6">Model: Method 1</td></tr>
<tr><td rowspan="6">Vítečková (1999), Vítečková et al. (2000a). Model: Method 1</td><td colspan="6">$x_1/K_m\tau_m$</td><td colspan="6">0</td><td colspan="6">T_m</td><td colspan="6"></td></tr>
<tr><td colspan="24">Coefficient values</td></tr>
<tr><td colspan="3">x_1</td><td colspan="3">OS</td><td colspan="3">x_1</td><td colspan="3">OS</td><td colspan="3">x_1</td><td colspan="3">OS</td><td colspan="3">x_1</td><td colspan="3">OS</td></tr>
<tr><td colspan="3">0.368</td><td colspan="3">0%</td><td colspan="3">0.641</td><td colspan="3">15%</td><td colspan="3">0.801</td><td colspan="3">30%</td><td colspan="3">0.957</td><td colspan="3">45%</td></tr>
<tr><td colspan="3">0.514</td><td colspan="3">5%</td><td colspan="3">0.696</td><td colspan="3">20%</td><td colspan="3">0.853</td><td colspan="3">35%</td><td colspan="3">1.008</td><td colspan="3">50%</td></tr>
<tr><td colspan="3">0.581</td><td colspan="3">10%</td><td colspan="3">0.748</td><td colspan="3">25%</td><td colspan="3">0.906</td><td colspan="3">40%</td><td colspan="3"></td><td colspan="3"></td></tr>
<tr><td rowspan="4">O'Dwyer (2001a). Model: Method 1</td><td colspan="6">$\frac{x_1 T_m}{K_m \tau_m}$</td><td colspan="6">0</td><td colspan="6">T_m</td><td colspan="6">$A_m = 0.5/x_1$; $\phi_m = 0.5\pi - x_1$</td></tr>
<tr><td colspan="24">Representative coefficient values</td></tr>
<tr><td colspan="4">x_1</td><td colspan="4">A_m</td><td colspan="4">ϕ_m</td><td colspan="4">x_1</td><td colspan="4">A_m</td><td colspan="4">ϕ_m</td></tr>
<tr><td colspan="4">0.785</td><td colspan="4">2</td><td colspan="4">45^0</td><td colspan="4">0.524</td><td colspan="4">3</td><td colspan="4">60^0</td></tr>
<tr><td>Chen and Seborg (2002).</td><td colspan="6">[3] $K_c^{(434)}$</td><td colspan="6">$3T_{CL} + \tau_m$</td><td colspan="6">$T_d^{(434)}$</td><td colspan="6">Model: Method 1</td></tr>
<tr><td></td><td colspan="24">Robust</td></tr>
<tr><td>Rivera and Jun (2000). Model: Method 1</td><td colspan="6">$\frac{\tau_m + T_m + 2\lambda}{K_m(\tau_m + \lambda)^2}$</td><td colspan="6">$\tau_m + T_m + 2\lambda$</td><td colspan="6">$\frac{T_m(\tau_m + 2\lambda)}{\tau_m + T_m + 2\lambda}$</td><td colspan="6">$\lambda$ not specified</td></tr>
</table>

[2] $$T_d^{(432)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_m}}, \quad T_d^{(433)} = \frac{1}{\omega_r - \frac{4\omega_r^2 \tau_m}{\pi} + \frac{1}{T_m}},$$
$$\omega_r = \frac{0.25\pi(2r_1 A_m - 1)}{(r_1^2 A_m^2 - 1)\tau_m}.$$

[3] $$\left(\frac{y}{d}\right)_{desired} = \frac{(3T_{CL} + \tau_m)s e^{-s\tau_m}}{K_c^{(434)}(T_{CL}s + 1)^3}, \quad K_c^{(434)} = \frac{(3T_{CL} + \tau_m)(T_m + \tau_m)}{(T_{CL} + \tau_m)^3},$$
$$T_d^{(434)} = \frac{3T_{CL}^2 T_m + 3T_{CL} T_m \tau_m - T_{CL}^3 + T_m \tau_m^2}{(3T_{CL} + \tau_m)(T_m + \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
McMillan (1984). *Model: Method 1*	[4] $K_c^{(435)}$	$T_i^{(435)}$	$0.25T_i^{(435)}$	Tuning rules developed from K_u, T_u
Perić *et al.* (1997). *Model: Method 8*	$\hat{K}_u \cos\phi_m$	$\alpha T_d^{(436)}$	[5] $T_d^{(436)}$	Recommended $\alpha = 4$

---

[4] $K_c^{(435)} = \frac{1.111}{K_m}\frac{T_m}{\tau_m^2}\left\{\frac{1}{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}}\right\}^2$, $T_i^{(435)} = 2\tau_m\left\{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}\right\}$.

[5] $T_d^{(436)} = \left(\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}\right)\frac{\hat{T}_u}{4\pi}$.

4.4.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{1+T_f s}$$

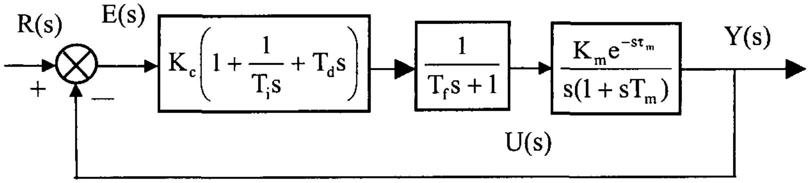

Table 96: PID controller tuning rules – FOLIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Robust			
H_∞ optimal – Tan *et al.* (1998b).	[6] $K_c^{(437)}$	$T_i^{(437)}$	$T_d^{(437)}$	*Model: Method 1*
	$\lambda = 0.5$ (performance), $\lambda = 0.1$ (robustness), $\lambda = 0.25$ (acceptable)			
Zhang *et al.* (1999). *Model: Method 1*	[7] $K_c^{(438)}$	$3\lambda + T_m + \tau_m$	$\dfrac{(3\lambda+\tau_m)T_m}{3\lambda+\tau_m+T_m}$	$1.5\tau_m \le \lambda \le 4.5\tau_m$
	$\lambda = 1.5\tau_m$, OS = 58%, $T_S = 6\tau_m$; $\lambda = 2.5\tau_m$, OS = 35%, $T_S = 11\tau_m$; $\lambda = 3.5\tau_m$, OS = 26%, $T_S = 16\tau_m$; $\lambda = 4.5\tau_m$, OS = 22%, $T_S = 20\tau_m$.			
Tan *et al.* (1998a). *Model: Method 1*; λ not specified	[8] $K_c^{(439)}$	$T_m + 8.1633\tau_m$	$\dfrac{T_m\tau_m}{0.1225T_m+\tau_m}$	$T_f = 0.5549\tau_m$
	[8] $K_c^{(440)}$	$T_m + 5.3677\tau_m$	$\dfrac{T_m\tau_m}{0.1863T_m+\tau_m}$	$T_f = 0.4482\tau_m$

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[6] $K_c^{(437)} = \dfrac{(0.463\lambda+0.277)([0.238\lambda+0.123]T_m+\tau_m)}{K_m\tau_m^2}$, $T_f = \dfrac{\tau_m}{5.750\lambda+0.590}$,

$T_i^{(437)} = T_m + \dfrac{\tau_m}{0.238\lambda+0.123}$, $T_d^{(437)} = \dfrac{T_m\tau_m}{(0.238\lambda+0.123)T_m+\tau_m}$.

[7] $K_c^{(438)} = \dfrac{3\lambda+T_m+\tau_m}{K_m(3\lambda^2+3\lambda\tau_m+\tau_m^2)}$, $T_f = \dfrac{\lambda^3}{3\lambda^2+3\lambda\tau_m+\tau_m^2}$.

[8] $K_c^{(439)} = \dfrac{0.0337T_m}{K_m\tau_m^2}\left(1+\dfrac{\tau_m}{0.1225T_m}\right)$, $K_c^{(440)} = \dfrac{0.0754T_m}{K_m\tau_m^2}\left(1+\dfrac{\tau_m}{0.1863T_m}\right)$.

Rule	K_c	T_i	T_d	Comment
Tan *et al.* (1998a) – continued.	$^9 K_c^{(441)}$	$T_m + 3.9635\tau_m$	$\frac{T_m \tau_m}{0.2523T_m + \tau_m}$	$T_f = 0.2863\tau_m$
Rivera and Jun (2000). *Model: Method 1*	$^{10} K_c^{(442)}$	$2(\tau_m + \lambda) + T_m$	$\frac{2T_m(\tau_m + \lambda)}{2(\tau_m + \lambda) + T_m}$	λ not specified

9 $K_c^{(441)} = \frac{0.1344T_m}{K_m \tau_m^2}\left(1 + \frac{\tau_m}{0.2523T_m}\right)$.

10 $K_c^{(442)} = \frac{2(\tau_m + \lambda) + T_m}{K_m\left(2\tau_m^2 + 4\tau_m\lambda + \lambda^2\right)}$, $T_f = \frac{\tau_m \lambda^2}{2\tau_m^2 + 4\tau_m\lambda + \lambda^2}$.

4.4.3 *Ideal controller with weighted proportional term*

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$$

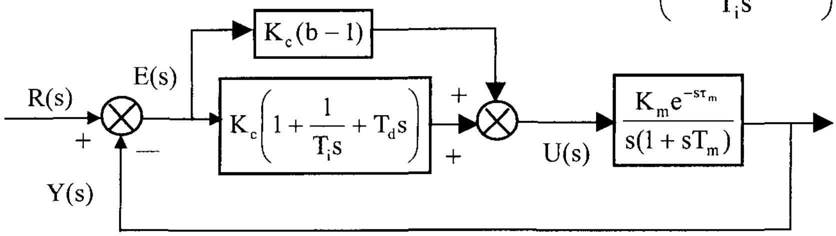

Table 97: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Åström and Hägglund (1995) – *pages 212-213. Model: Method 1 or Method 3*	$\frac{5.6e^{-8.8\tau+6.8\tau^2}}{K_m(T_m+\tau_m)}$	$1.1\tau_m e^{6.7\tau-4.4\tau^2}$	$1.7\tau_m e^{-6.4\tau+2.0\tau^2}$	$M_{max} = 1.4$
	$b = 0.12e^{6.9\tau-6.6\tau^2}$			
	$\frac{8.6e^{-7.1\tau+5.4\tau^2}}{K_m(T_m+\tau_m)}$	$1.0\tau_m e^{3.3\tau-2.3\tau^2}$	[11] $T_d^{(443)}$	$M_{max} = 2.0$

[11] $T_d^{(443)} = 0.38\tau_m e^{0.056\tau-0.60\tau^2}$.

4.4.4 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right)$

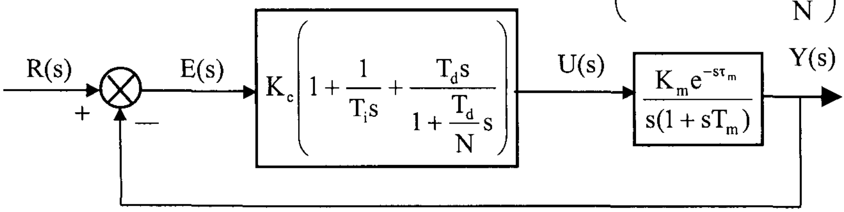

Table 98: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Kristiansson and Lennartson (2002).	[12] $K_c^{(444)}$	$T_i^{(444)}$	$T_d^{(444)}$	*Model: Method 1*; $K_1 \in [0.5,1]$
	Robust			
Chien (1988). *Model: Method 1*	$\frac{2\lambda + \tau_m + T_m}{K_m(\lambda + \tau_m)^2}$	$2\lambda + T_m + \tau_m$	$\frac{T_m(2\lambda + \tau_m)}{2\lambda + T_m + \tau_m}$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); $N = 10$

---

[12] $K_c^{(444)} = \frac{2.5\omega_u}{K_m}\left(-0.08 + \frac{0.055}{K_1} + \frac{0.059}{K_1^2}\right)\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right),$

$$T_i^{(444)} = \frac{2.5}{\omega_u}\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right),$$

$$T_d^{(444)} = \frac{2.5}{\omega_u}\left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)}\right)^{-1}\left(\frac{1}{N} + 1\right)^{-1},$$

$$N = \frac{T_d^{(444)}}{T_f^{(444)}},\ T_f^{(444)} = \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.16\omega_u(10 + 1/K_1)}.$$

4.4.5 *Controller with filtered derivative with set-point weighting 1*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s}{1 + a_{f1}s}R(s) - Y(s)\right)$$

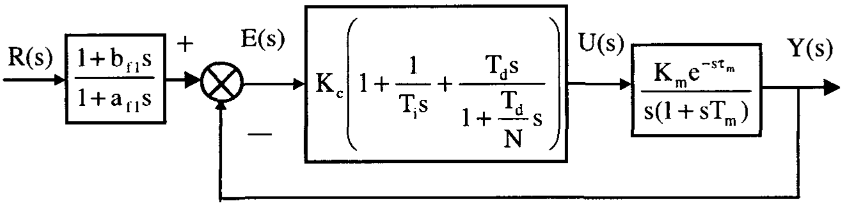

Table 99: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th>T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td></td><td colspan="6">Robust</td></tr>
<tr><td rowspan="7">Liu et al. (2003).
Model: Method 1;
recommended
$\lambda = \tau_m$</td><td colspan="2">[13] $K_c^{(445)}$</td><td>$T_i^{(445)}$</td><td colspan="2">$T_d^{(445)}$</td><td></td></tr>
<tr><td colspan="6">Representative λ values deduced from a graph</td></tr>
<tr><td>λ</td><td>OS</td><td>Rise time</td><td>λ</td><td>OS</td><td>Rise time</td></tr>
<tr><td>$0.3\tau_m$</td><td>$\approx 57\%$</td><td>$\approx 3\tau_m$</td><td>$2\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 8\tau_m$</td></tr>
<tr><td>$0.5\tau_m$</td><td>$\approx 33\%$</td><td>$\approx 3.5\tau_m$</td><td>$2.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 10.5\tau_m$</td></tr>
<tr><td>τ_m</td><td>$\approx 7\%$</td><td>$\approx 5\tau_m$</td><td>$3\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 12.5\tau_m$</td></tr>
<tr><td>$1.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 8.5\tau_m$</td><td>$3.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 14\tau_m$</td></tr>
</table>

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[13] $K_c^{(445)} = \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{K_m(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)^2}$,

$$T_i^{(445)} = \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)},$$

$$T_d^{(445)} = \frac{T_m(3\lambda + \tau_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3} - \frac{\lambda^3}{3\lambda^2 + 3\lambda\tau_m + T_m^2},$$

$$N = \frac{T_m(3\lambda + \tau_m)}{\lambda^3[(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3]} - 1,\ b_{f1} = \lambda,\ a_{f1} = 3\lambda + \tau_m.$$

4.4.6 *Controller with filtered derivative with set-point weighting 3*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\dfrac{T_d}{N}s}\right)\left(\frac{1+b_{f1}s+b_{f2}s^2+b_{f3}s^3}{1+a_{f1}s+a_{f2}s^2+a_{f3}s^3}R(s)-Y(s)\right)$$

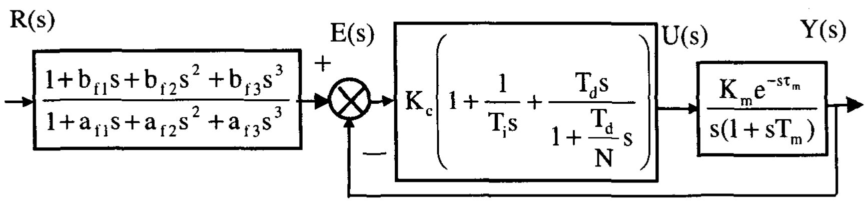

Table 100: PID controller tuning rules – FOLIPD model $G_m(s)=\dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td></td><td colspan="7">Robust</td></tr>
<tr><td rowspan="2">Liu et al. (2003).
Model: Method 1</td><td colspan="2">[1] $K_c^{(446)}$</td><td colspan="2">$T_i^{(446)}$</td><td colspan="2">$T_d^{(446)}$</td><td></td></tr>
<tr><td colspan="7">$a_{f1}=7\lambda+\tau_m$; $a_{f2}=\lambda(16\lambda+4\tau_m)$, $a_{f3}=4\lambda^2(3\lambda+\tau_m)$</td></tr>
<tr><td rowspan="5">Representative λ values (deduced from a graph)</td><td>λ</td><td>OS</td><td colspan="2">Rise time</td><td>λ</td><td>OS</td><td>Rise time</td></tr>
<tr><td>$0.3\tau_m$</td><td>$\approx 43\%$</td><td colspan="2">$\approx 3.5\tau_m$</td><td>$2\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 16.5\tau_m$</td></tr>
<tr><td>$0.5\tau_m$</td><td>$\approx 15\%$</td><td colspan="2">$\approx 4.5\tau_m$</td><td>$2.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 20\tau_m$</td></tr>
<tr><td>τ_m</td><td>$\approx 0\%$</td><td colspan="2">$\approx 8\tau_m$</td><td>$3\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 24\tau_m$</td></tr>
<tr><td>$1.5\tau_m$</td><td>$\approx 0\%$</td><td colspan="2">$\approx 12.5\tau_m$</td><td>$3.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 28\tau_m$</td></tr>
</table>

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[1] $K_c^{(446)}=\dfrac{(3\lambda+\tau_m+T_m)(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})-\lambda^3}{K_m(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})^2}$,

$T_i^{(446)}=\dfrac{(3\lambda+\tau_m+T_m)(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})-\lambda^3}{(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})}$, $b_{f1}=3\lambda$, $b_{f2}=3\lambda^2$, $b_{f3}=\lambda^3$,

$T_d^{(446)}=\dfrac{T_m(3\lambda+\tau_m)(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})}{(3\lambda+\tau_m+T_m)(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})-\lambda^3}-\dfrac{\lambda^3}{3\lambda^2+3\lambda\tau_m+T_m^{\,2}}$,

$N=\dfrac{T_m(3\lambda+\tau_m)}{\lambda^3[(3\lambda+\tau_m+T_m)(3\lambda^2+3\lambda\tau_m+\tau_m^{\,2})-\lambda^3]}-1$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="1">T_i</th><th colspan="1">T_d</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="7">Liu *et al.* (2003) – continued.</td><td colspan="6">$a_{f1} = 4\lambda + \tau_m$; $a_{f2} = \lambda(3.25\lambda + \tau_m)$, $a_{f3} = 0.25\lambda^2(3\lambda + \tau_m)$</td></tr>
<tr><td colspan="6">Representative λ values (deduced from a graph)</td></tr>
<tr><td>λ</td><td>OS</td><td>Rise time</td><td>λ</td><td>OS</td><td>Rise time</td></tr>
<tr><td>$0.3\tau_m$</td><td>$\approx 61\%$</td><td>$\approx 3\tau_m$</td><td>$2\tau_m$</td><td>$\approx 4\%$</td><td>$\approx 6.5\tau_m$</td></tr>
<tr><td>$0.5\tau_m$</td><td>$\approx 40\%$</td><td>$\approx 3\tau_m$</td><td>$2.5\tau_m$</td><td>$\approx 4\%$</td><td>$\approx 8\tau_m$</td></tr>
<tr><td>τ_m</td><td>$\approx 15\%$</td><td>$\approx 3.5\tau_m$</td><td>$3\tau_m$</td><td>$\approx 1\%$</td><td>$\approx 10\tau_m$</td></tr>
<tr><td>$1.5\tau_m$</td><td>$\approx 7\%$</td><td>$\approx 5\tau_m$</td><td>$3.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 12\tau_m$</td></tr>
</table>

4.4.7 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

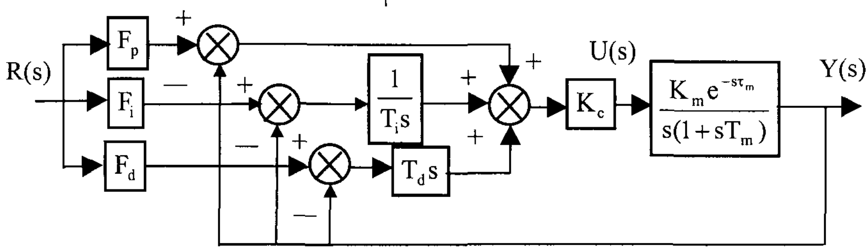

Table 101: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_me^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Oubrahim and Leonard (1998). *Model: Method 1*	$0.6K_u$, $F_p = 0.1$	$0.5T_u$, $F_i = 1$	$0.125T_u$, $F_d = 0.01$	$0.05 < \frac{\tau_m}{T_m} < 0.8$; 20% overshoot

4.4.8 *Classical controller 1* $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)\left(\dfrac{1+T_d s}{1+\dfrac{T_d s}{N}}\right)$

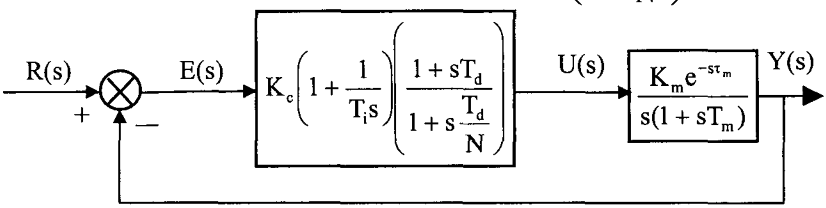

Table 102: PID controller tuning rules – FOLIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE – Shinskey (1994) – *page 75.*	$\dfrac{0.78}{K_m(\tau_m + T_m)}$	$1.38(\tau_m + T_m)$	$0.66(\tau_m + T_m)$	$\tau_m = T_m$; N=10. *Model: Method 1*
Minimum IAE – Shinskey (1994) – *pages 158-159. Model: Method 1*; N not specified	[2] $K_c^{(447)}$	$T_i^{(447)}$	$0.56\tau_m + 0.75T_m$	$\dfrac{\tau_m}{T_m} < 2$
	[3] $K_c^{(448)}$			$\dfrac{\tau_m}{T_m} \ge 2$

[2] $K_c^{(447)} = \dfrac{100}{108 K_m \tau_m\left(1.22 - 0.03\dfrac{T_m}{\tau_m}\right)}$, $T_i^{(447)} = 1.57\tau_m\left(1 + 1.2\left[1 - e^{-\frac{T_m}{\tau_m}}\right]\right)$.

[3] $K_c^{(448)} = \dfrac{100}{108 K_m \tau_m\left(1 + 0.4\dfrac{T_m}{\tau_m}\right)}$.

Rule	K_c	T_i	T_d	Comment		
Minimum ITAE – Poulin and Pomerleau (1996), (1997). *Model: Method 1*	[4] $K_c^{(449)}$	$x_2(\tau_m + T_m)$	T_m	$0 \le \frac{\tau_m}{(T_m/N)} \le 2$ $3.33 \le N \le 10$		
	Coefficient values (deduced from graphs)					
	$\frac{\tau_m}{T_m/N}$	x_1	x_2	$\frac{\tau_m}{T_m/N}$	x_1	x_2
Process output step load disturbance	0.2	5.0728	0.5231	1.2	4.7565	0.5250
	0.4	4.9688	0.5237	1.4	4.7293	0.5252
	0.6	4.8983	0.5241	1.6	4.7107	0.5254
	0.8	4.8218	0.5245	1.8	4.6837	0.5256
	1.0	4.7839	0.5249	2.0	4.6669	0.5257
Process input step load disturbance	0.2	3.9465	0.5320	1.2	4.0337	0.5312
	0.4	3.9981	0.5315	1.4	4.0278	0.5312
	0.6	4.0397	0.5311	1.6	4.0278	0.5312
	0.8	4.0397	0.5311	1.8	4.0218	0.5313
	1.0	4.0397	0.5311	2.0	4.0099	0.5314
	Direct synthesis					
Poulin *et al.* (1996). *Model: Method 1*	[5] $K_c^{(450)}$	$21(0.2T_m + \tau_m)$	T_m	$N = 5.$ $M_{max} = 1$ dB		
	Robust					
Chien (1988). *Model: Method 1*	$\frac{T_m}{K_m(\lambda + \tau_m)^2}$	T_m	$2\lambda + \tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10		
	$\frac{2\lambda + \tau_m}{K_m(\lambda + \tau_m)^2}$	$2\lambda + \tau_m$	T_m			

[4] $K_c^{(449)} = \frac{x_2}{K_m(\tau_m + T_m)}\sqrt{\frac{T_m^{\,2}}{x_1(\tau_m + T_m)^2} + 1}$.

[5] $K_c^{(450)} = \frac{0.5455}{K_m(\tau_m + 0.2T_m)}$.

4.4.9 *Classical controller 2* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+NT_d s}{1+T_d s}\right)$

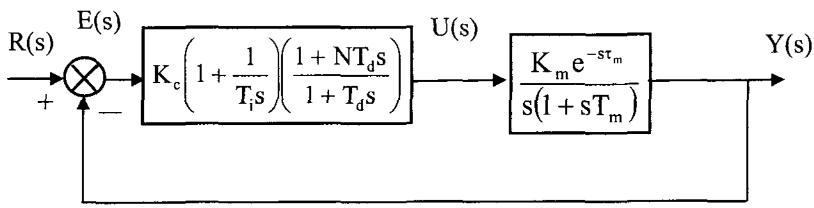

Table 103: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Hougen (1979) – *page 338. Model: Method 1*	[6] $\frac{0.106x_1}{K_m \tau_m}$	∞	$0.135{T_m}^{0.7}{\tau_m}^{0.3}$	Maximise crossover frequency; $N \in [10,30]$

---

[6] x_1 values deduced from graph:

τ_m/T_m	1	2	3	4	5	6
x_1	6	3	2.1	1.8	1.6	1.3
τ_m/T_m	7	8	9	10	11	12
x_1	1.1	1.0	0.9	0.8	0.7	0.7

4.4.10 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$$

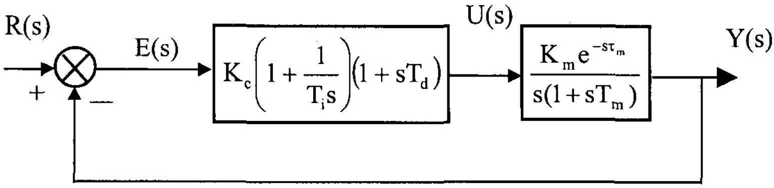

Table 104: PID tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Skogestad (2003). *Model: Method 1*	$\frac{0.5}{K_m \tau_m}$	$8\tau_m$	T_m	

4.4.11 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$

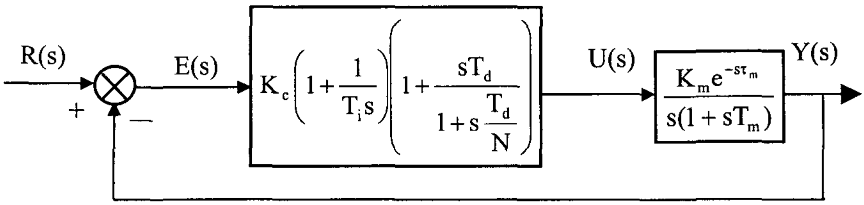

Table 105: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	$\frac{2\lambda+\tau_m}{K_m(\lambda+\tau_m)^2}$	$2\lambda+\tau_m$	T_m	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
	$\frac{T_m}{K_m(\lambda+\tau_m)^2}$	T_m	$2\lambda+\tau_m$	

4.4.12 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+(T_d/N)s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+(T_d/N)s}\right)R(s)$$

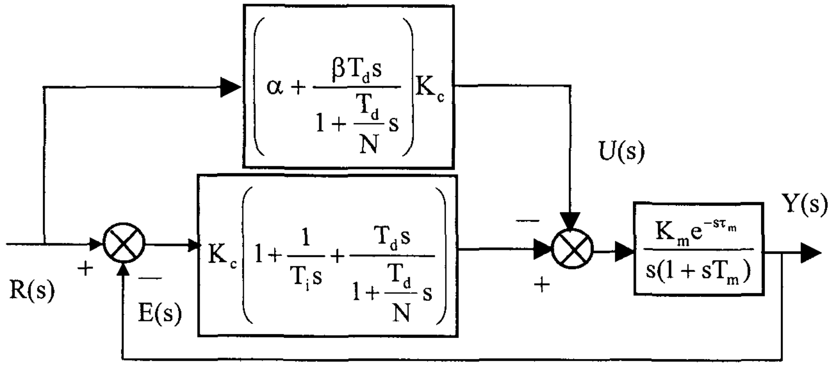

Table 106: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment	
	Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000). *Model: Method 4*; $K_m = 1$; $T_m = 1$; $\beta = 1$, $N = 10$; $0.05 < \tau_m < 0.8$	$x_1 K_u$	$x_2 T_u$	$x_3 T_u$		
	Coefficient values				
	x_1	x_2	x_3	α	ϕ_c
	1.672	0.366	0.136	0.601	-164^0
	1.236	0.427	0.149	0.607	-160^0
	0.994	0.486	0.155	0.610	-155^0
	0.842	0.538	0.154	0.616	-150^0
	0.752	0.567	0.157	0.605	-145^0
	0.679	0.610	0.149	0.610	-140^0
	0.635	0.637	0.142	0.612	-135^0
	0.590	0.669	0.133	0.610	-130^0
	0.551	0.690	0.114	0.616	-125^0
	0.520	0.776	0.087	0.609	-120^0
	0.509	0.810	0.068	0.611	-118^0

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000). *Model: Method 1*	[1] $K_c^{(451)}$	$T_i^{(451)}$	$T_d^{(451)}$	$\frac{\tau_m}{T_m} \le 1.0$
	Direct synthesis			
Wang and Cai (2001). *Model: Method 1*	[2] $K_c^{(452)}$	$T_i^{(452)}$	$T_d^{(452)}$	N=0; $\beta = 0$.
	$\frac{0.6068}{K_m \tau_m}$	$5.9784\tau_m$	[3] $T_d^{(453)}$	
Wang and Cai (2002).	$\frac{0.6189}{K_m \tau_m}$	$5.9112\tau_m$	[4] $T_d^{(454)}$	*Model: Method 1*
Xu and Shao (2003a).	[5] $K_c^{(455)}$	$2(T_m + 1.5\tau_m)$	$\frac{(T_m + 0.5\tau_m)^2}{(T_m + 1.5\tau_m)}$	*Model: Method 1*
Xu and Shao (2003c).	$\frac{1.5\tau_m + 0.5T_m}{2K_m \tau_m^2}$	$T_m + \tau_m$	$\frac{1.5T_m \tau_m}{1.5\tau_m + 0.5T_m}$	[6] *Model: Method 1*

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[1] $K_c^{(451)} = \frac{1}{K_m}\left(0.7608 + \frac{0.5184}{[\frac{\tau_m}{T_m} + 0.01308]^2}\right)$,

$T_i^{(451)} = T_m\left(0.03330 + 3.997\frac{\tau_m}{T_m} - 0.5517\left[\frac{\tau_m}{T_m}\right]^2\right)$,

$T_d^{(451)} = T_m\left(0.03432 + 2.058\frac{\tau_m}{T_m} - 1.774\left[\frac{\tau_m}{T_m}\right]^2 + 0.6878\left[\frac{\tau_m}{T_m}\right]^3\right)$,

$\alpha = 0.6647$, $\beta = 0.8653 - 0.1277(\tau_m/T_m) + 0.03330(\tau_m/T_m)^2$,

N fixed but not specified; OS $\le 20\%$.

[2] $K_c^{(452)} = \frac{1}{K_m \tau_m}\left[1.3608 - \frac{1.2064}{M_s}\right]$, $T_i^{(452)} = \frac{\tau_m(1.3608M_s - 1.2064)}{0.29M_s - 0.3016}$,

$T_d^{(452)} = \frac{(T_m + 0.1\tau_m)(1.450M_s - 1.508)}{1.3608M_s - 1.2064}$, $\alpha = \frac{0.2M_s}{1.3608M_s - 1.2064}$, $1.3 \le M_s \le 2$.

[3] $T_d^{(453)} = 0.8363(T_m + 0.1\tau_m)$, $\alpha = 0.3296$, $M_s = 1.6$ ($A_m > 2.66$, $\phi_m > 36.4^0$).

[4] $T_d^{(454)} = 0.8460(T_m + 0.1\tau_m)$, N=0, $\beta = 0$, $\alpha = 0.3232$, $A_m = 3$, $\phi_m = 60^0$.

[5] $K_c^{(455)} = \frac{0.5(T_m + 1.5\tau_m)}{K_m \tau_m (T_m + \tau_m)}$, $\alpha = \frac{\tau_m}{T_m + 1.5\tau_m}$, N=$\infty$, $\beta = 0$, $A_m > 2$, $\phi_m > 60^0$.

[6] $\alpha = \frac{\tau_m}{1.5T_m + 0.5\tau_m}$, $\beta = 0.6667$, N=∞, $A_m > 2$, $\phi_m > 60^0$.

4.4.13 *Non-interacting controller 4*

$$U(s)=K_c\left(1+\frac{1}{T_i s}\right)E(s)-K_cT_dsY(s)$$

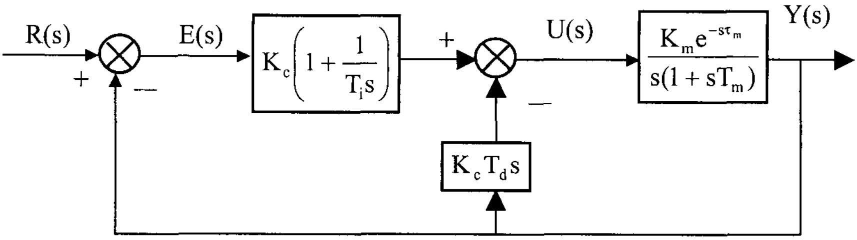

Table 107: PID controller tuning rules – FOLIPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Shinskey (1994) – *page 75.*	$\frac{1.16}{K_m(\tau_m+T_m)}$	$1.38(\tau_m+T_m)$	$0.55(\tau_m+T_m)$	*Model: Method 1*
Minimum IAE - Shinskey (1994) – *page 159.*	[7] $K_c^{(456)}$	$T_i^{(456)}$	$0.48\tau_m+0.7T_m$	*Model: Method 1*

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[7] $K_c^{(456)}=\frac{1.28}{K_m\tau_m\left(1+0.24\frac{T_m}{\tau_m}-0.14\left[\frac{T_m}{\tau_m}\right]^2\right)}$, $T_i^{(456)}=1.9\tau_m\left(1+0.75[1-e^{-\frac{T_m}{\tau_m}}]\right)$.

4.4.14 *Non-interacting controller 6 (I-PD controller)*

$$U(s) = \frac{K_c}{T_i s}E(s) - K_c(1 + T_d s)Y(s)$$

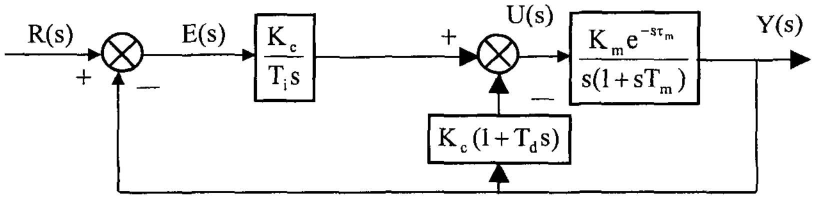

Table 108: PID tuning rules –FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Arvanitis *et al.* (2003b). Minimum ISE [8]	$\frac{16}{(16-3\beta)K_m\tau_m}$	$\frac{4}{\beta}\tau_m$	$\frac{(8-\beta)}{16}\tau_m$	*Model: Method 1*
	Minimum $\int_0^\infty \left[e^2(t) + K_m{}^2u^2(t)\right]dt$ [9]			

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[8] $\beta = 1.5841 + 0.3101\left(\frac{\tau_m}{T_m}\right) + 0.38841\left(\frac{\tau_m}{T_m}\right)^2 - 0.4622\left(\frac{\tau_m}{T_m}\right)^3 + 0.21918\left(\frac{\tau_m}{T_m}\right)^4$
$- 0.060691\left(\frac{\tau_m}{T_m}\right)^5 + 0.010771\left(\frac{\tau_m}{T_m}\right)^6 - 0.0012466\left(\frac{\tau_m}{T_m}\right)^7 + 9.1201.10^{-5}\left(\frac{\tau_m}{T_m}\right)^8$
$- 3.8302.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9 + 7.0316.10^{-8}\left(\frac{\tau_m}{T_m}\right)^{10}$.

[9] $\beta = 0.066485 + 2.0757\left(\frac{\tau_m}{T_m}\right) + 1.0972\left(\frac{\tau_m}{T_m}\right)^2 - 2.7957\left(\frac{\tau_m}{T_m}\right)^3 + 1.9019\left(\frac{\tau_m}{T_m}\right)^4$
$- 0.68418\left(\frac{\tau_m}{T_m}\right)^5 + 0.14763\left(\frac{\tau_m}{T_m}\right)^6 - 0.019733\left(\frac{\tau_m}{T_m}\right)^7 + 0.0016016\left(\frac{\tau_m}{T_m}\right)^8$
$- 7.2368.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 + 1.397.10^{-6}\left(\frac{\tau_m}{T_m}\right)^{10}$, $0 < \frac{\tau_m}{T_m} < 4.0$;

$\beta = 2.128638 - 0.00560812\left(\frac{\tau_m}{T_m} - 4.0\right)$, $\frac{\tau_m}{T_m} \geq 4.0$.

Rule	K_c	T_i	T_d	Comment
Arvanitis *et al.* (2003b). *Model: Method 1*	[10] $K_c^{(457)}$	$T_i^{(457)}$	$T_d^{(457)}$	
	Minimum ISE [11]			
	Minimum performance index [12],[13]			

[10] $$K_c^{(457)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m \tau_m [25.1328(\tau_m + T_m) - 8 - (\tau_m + 2T_m)a]}$$

$$\text{with } a = [0.5708 + \sqrt{\frac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi,$$

$$T_i^{(457)} = \frac{12.5664(\tau_m + T_m) - 4}{a}, \; T_d^{(457)} = T_m - T_m^{\,2}\frac{a}{12.5664(\tau_m + T_m) - 4}.$$

[11] $$\chi = 1.178 + 4.5663\left(\frac{\tau_m}{T_m}\right) - 7.0371\left(\frac{\tau_m}{T_m}\right)^2 + 5.5784\left(\frac{\tau_m}{T_m}\right)^3 - 2.6103\left(\frac{\tau_m}{T_m}\right)^4$$

$$+ 0.7679\left(\frac{\tau_m}{T_m}\right)^5 - 0.1458\left(\frac{\tau_m}{T_m}\right)^6 + 0.017833\left(\frac{\tau_m}{T_m}\right)^7 - 0.0013561\left(\frac{\tau_m}{T_m}\right)^8$$

$$+ 5.83.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 1.0823.10^{-6}\left(\frac{\tau_m}{T_m}\right)^{10}, \; 0 < \frac{\tau_m}{T_m} < 3.5,$$

$$\chi = 2.2036 - 0.0033077\left(\frac{\tau_m}{T_m} - 6.5\right), \; \frac{\tau_m}{T_m} \geq 3.5.$$

[12] Performance index = $\int_0^\infty [e^2(t) + K_m^{\,2}u^2(t)]dt$.

[13] $$\chi = -0.1185 + 4.781\left(\frac{\tau_m}{T_m}\right) - 3.9977\left(\frac{\tau_m}{T_m}\right)^2 + 1.733\left(\frac{\tau_m}{T_m}\right)^3 - 0.42245\left(\frac{\tau_m}{T_m}\right)^4$$

$$+ 0.057708\left(\frac{\tau_m}{T_m}\right)^5 - 0.0038421\left(\frac{\tau_m}{T_m}\right)^6 + 2.6386.10^{-5}\left(\frac{\tau_m}{T_m}\right)^7 + 1.078.10^{-5}\left(\frac{\tau_m}{T_m}\right)^8$$

$$- 4.2486.10^{-7}\left(\frac{\tau_m}{T_m}\right)^9.$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Arvanitis *et al.* (2003b).	$\dfrac{16}{(16-3\beta)K_m\tau_m}$	$\dfrac{4}{\beta}\tau_m$	$\dfrac{(8-\beta)}{16}\tau_m$	*Model: Method 1*
	Minimum ISE [14]			
	Minimum performance index [15], [16]			

[14] $$\beta = 1.2171 + 0.7339\left(\frac{\tau_m}{T_m}\right) + 0.083449\left(\frac{\tau_m}{T_m}\right)^2 - 0.14905\left(\frac{\tau_m}{T_m}\right)^3 + 0.030257\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.0045567\left(\frac{\tau_m}{T_m}\right)^5 - 0.0029943\left(\frac{\tau_m}{T_m}\right)^6 + 0.00056821\left(\frac{\tau_m}{T_m}\right)^7 - 5.4957.10^{-5}\left(\frac{\tau_m}{T_m}\right)^8$$
$$+ 2.7505.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9 - 5.6642.10^{-8}\left(\frac{\tau_m}{T_m}\right)^{10}.$$

[15] Performance index = $\int_0^\infty \left[e^2(t) + K_m{}^2 u^2(t)\right]dt$

[16] $$\beta = 0.083709 + 2.2875\left(\frac{\tau_m}{T_m}\right) - 0.18739\left(\frac{\tau_m}{T_m}\right)^2 - 1.0059\left(\frac{\tau_m}{T_m}\right)^3 + 0.79718\left(\frac{\tau_m}{T_m}\right)^4$$
$$- 0.3036\left(\frac{\tau_m}{T_m}\right)^5 + 0.067859\left(\frac{\tau_m}{T_m}\right)^6 - 0.0093104\left(\frac{\tau_m}{T_m}\right)^7 + 0.00077161\left(\frac{\tau_m}{T_m}\right)^8$$
$$- 3.5473.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 + 6.9487.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0 < \frac{\tau_m}{T_m} < 4.5$$
$$\beta = 2.444547 - 0.0146274714\left(\frac{\tau_m}{T_m} - 4.5\right),\ \frac{\tau_m}{T_m} \ge 4.5.$$

Rule	K_c	T_i	T_d	Comment
Arvanitis *et al.* (2003b). *Model: Method 1*	[17] $K_c^{(458)}$	$T_i^{(458)}$	$T_d^{(458)}$	
	Minimum ISE [18]			
	Minimum performance index [19],[20]			

---

[17] $$K_c^{(458)}=\frac{12.5664(\tau_m+T_m)-4}{K_m\tau_m[25.1328(\tau_m+T_m)-8-(\tau_m+2T_m)a]}$$
$$\text{with } a=[0.5708+\sqrt{\frac{3.4674\tau_m+3.1416T_m-1}{\tau_m}}]\chi$$
$$T_i^{(458)}=\frac{12.5664(\tau_m+T_m)-4}{a},\ T_d^{(458)}=T_m-T_m^{\ 2}\frac{a}{12.5664(\tau_m+T_m)-4}.$$

[18] $$\chi=1.1571+5.0775\left(\frac{\tau_m}{T_m}\right)-8.2021\left(\frac{\tau_m}{T_m}\right)^2+6.6353\left(\frac{\tau_m}{T_m}\right)^3-3.1355\left(\frac{\tau_m}{T_m}\right)^4$$
$$+0.92683\left(\frac{\tau_m}{T_m}\right)^5-0.17635\left(\frac{\tau_m}{T_m}\right)^6+0.02158\left(\frac{\tau_m}{T_m}\right)^7-0.0016402\left(\frac{\tau_m}{T_m}\right)^8$$
$$+7.0439.10^{-5}\left(\frac{\tau_m}{T_m}\right)^9-1.3056.10^{-6}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0<\frac{\tau_m}{T_m}<4.5$$
$$\chi=2.124307-0.004844182\left(\frac{\tau_m}{T_m}-4.5\right),\ \frac{\tau_m}{T_m}\geq 4.5\,.$$

[19] Performance index $=\int_0^\infty\left[e^2(t)+K_m^{\ 2}u^2(t)\right]dt$.

[20] $$\chi=-0.70861+7.4212\left(\frac{\tau_m}{T_m}\right)-8.1214\left(\frac{\tau_m}{T_m}\right)^2+4.8168\left(\frac{\tau_m}{T_m}\right)^3-1.7217\left(\frac{\tau_m}{T_m}\right)^4$$
$$+0.38998\left(\frac{\tau_m}{T_m}\right)^5-0.056975\left(\frac{\tau_m}{T_m}\right)^6+0.0053141\left(\frac{\tau_m}{T_m}\right)^7-0.00030233\left(\frac{\tau_m}{T_m}\right)^8$$
$$+9.3969.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9-1.1861.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10},\ 0<\frac{\tau_m}{T_m}<4.5$$
$$\chi=2.1069751341-0.0017098\left(\frac{\tau_m}{T_m}-4.5\right),\ \frac{\tau_m}{T_m}\geq 4.5\,.$$

Rule	K_c	T_i	T_d	Comment
Arvanitis *et al.* (2003b) (continued).	Minimum performance index [21,22]			
	Direct synthesis			
Arvanitis *et al.* (2003b). *Model: Method 1*	[23] $K_c^{(459)}$	$(2\xi^2+1)\tau_m$	$\frac{16\xi^2+4}{16(2\xi^2+1)}\tau_m$	
	[24] $K_c^{(460)}$	$T_i^{(460)}$	$T_d^{(460)}$	

---

[21] Performance index $= \int_0^\infty \left[e^2(t) + K_m^{\,2}\left(\frac{du}{dt}\right)^2 \right] dt$.

[22] $$\chi = 0.05486 + 0.81662\left(\frac{\tau_m}{T_m}\right) - 1.4989\left(\frac{\tau_m}{T_m}\right)^2 + 1.1955\left(\frac{\tau_m}{T_m}\right)^3 - 0.48464\left(\frac{\tau_m}{T_m}\right)^4$$
$$+ 0.11011\left(\frac{\tau_m}{T_m}\right)^5 - 0.013386\left(\frac{\tau_m}{T_m}\right)^6 + 0.00059665\left(\frac{\tau_m}{T_m}\right)^7 + 3.8633.10^{-5}\left(\frac{\tau_m}{T_m}\right)^8$$
$$- 5.307.10^{-6}\left(\frac{\tau_m}{T_m}\right)^9 + 1.6437.10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 6.0;$$
$$\chi = 2.1111 - 0.001025\left(\frac{\tau_m}{T_m} - 6\right), \quad \frac{\tau_m}{T_m} \ge 6.$$

[23] $$K_c^{(459)} = \frac{16(2\xi^2+1)}{(32\xi^2+4)K_m\tau_m}.$$

[24] $$K_c^{(460)} = \frac{\tau_m + T_m + 4\tau_m\xi^2}{K_m\tau_m^{\,2}(1+8\xi^2)}, \quad T_i^{(460)} = \tau_m + T_m + 4\tau_m\xi^2, \quad T_d^{(460)} = \frac{\tau_m T_m (1+4\xi^2)}{\tau_m + T_m + 4\tau_m\xi^2}.$$

4.4.15 *Non-interacting controller 8*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)E(s) - K_i(1 + T_{di}s)Y(s)$$

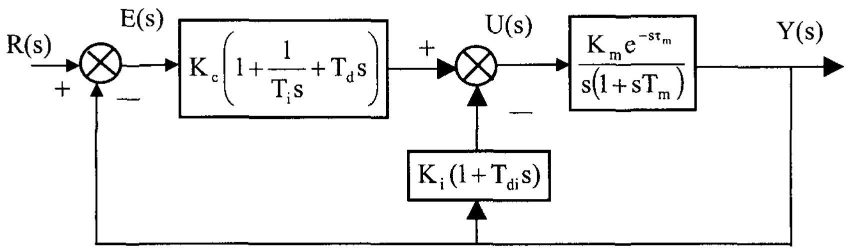

Table 109: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Wang *et al.* (2001b). *Model: Method 7*	$\frac{0.4189}{K_m \tau_m}$	$4\tau_m$	$1.25(T_m + 0.1\tau_m)$	$K_i = \frac{0.2}{K_m \tau_m}$
	$T_{di} = 0$; $A_m = 3$; $\phi_m = 60^0$			
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[1] $K_c^{(461)}$	$T_i^{(461)}$	$T_d^{(461)}$	$K_i = 0.25K_u$, $T_{di} = 0$

---

[1] $K_c^{(461)} = \frac{0.25K_u}{(\lambda + \tau_m)}\left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}\right]$, $T_i^{(461)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$,

$$T_d^{(461)} = \frac{0.25K_u}{(\lambda + \tau_m)}\left\{\frac{4T_m}{K_u K_m} + 0.5\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{\tau_m^2(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m(\lambda + \tau_m)}\right\}.$$

4.4.16 *Non-interacting controller 11*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + T_d s)E(s) - K_i(1 + T_{di}s)Y(s)$$

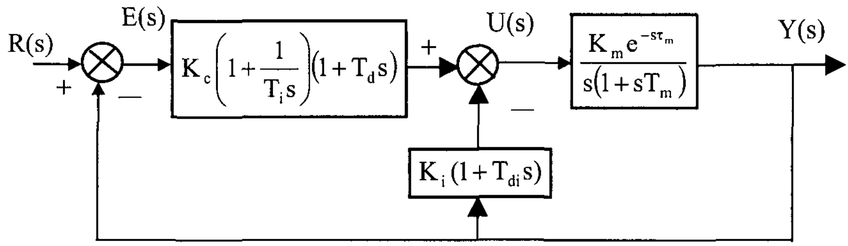

Table 110: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Majhi and Mahanta (2001). *Model: Method 6*	$\frac{0.5236}{K_m \tau_m}$	$2\tau_m e^{\tau_m s}$	T_m	$T_{di} = T_m$; $K_i = \frac{0.5}{K_m \tau_m}$; $A_m = 3$; $\phi_m = 60^0$

4.4.17 *Non-interacting controller 12*

$$U(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_d s+1}E(s)-K_1 Y(s)$$

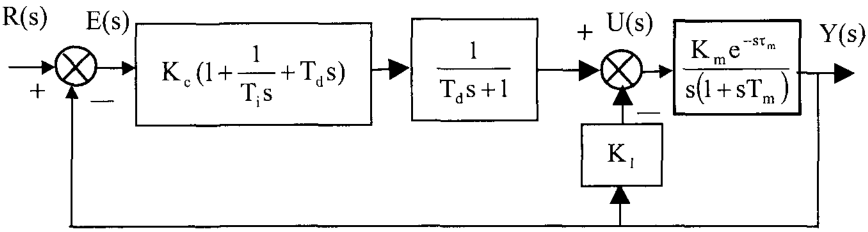

Table 111: PID controller tuning rules – FOLIPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[2] $K_c^{(462)}$	$T_i^{(462)}$	$T_d^{(462)}$	$K_1=0.25K_u$

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[2] $K_c^{(462)}=\frac{0.25K_u}{\lambda+\tau_m}\{\frac{4}{K_u K_m}-\tau_m+\frac{\tau_m^2}{2(\lambda+\tau_m)}+$

$$\left[\frac{4T_m}{K_u K_m}+0.5\tau_m^2-\frac{\tau_m^3}{6(\lambda+\tau_m)}+\frac{\tau_m^4}{4(\lambda+\tau_m)^2}+\frac{(1-0.25K_u K_m\tau_m)\tau_m^2}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5}\},$$

$$T_i^{(462)}=\frac{4}{K_u K_m}-\tau_m+\frac{\tau_m^2}{2(\lambda+\tau_m)}+$$

$$\left[\frac{4T_m}{K_u K_m}+0.5\tau_m^2-\frac{\tau_m^3}{6(\lambda+\tau_m)}+\frac{\tau_m^4}{4(\lambda+\tau_m)^2}+\frac{(1-0.25K_u K_m\tau_m)\tau_m^2}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5},$$

$$T_d^{(462)}=\left[\frac{4T_m}{K_u K_m}+0.5\tau_m^2-\frac{\tau_m^3}{6(\lambda+\tau_m)}+\frac{\tau_m^4}{4(\lambda+\tau_m)^2}+\frac{(1-0.25K_u K_m\tau_m)\tau_m^2}{0.5K_u K_m(\lambda+\tau_m)}\right]^{0.5}.$$

4.4.18 *Industrial controller* $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$

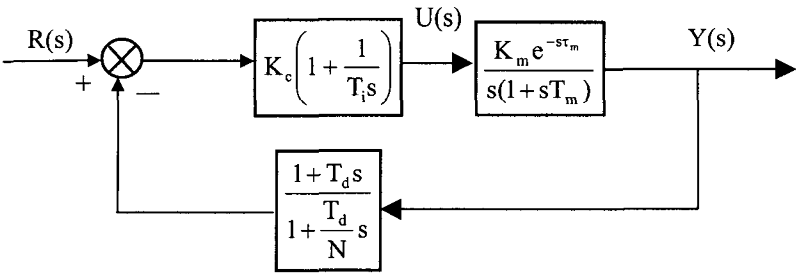
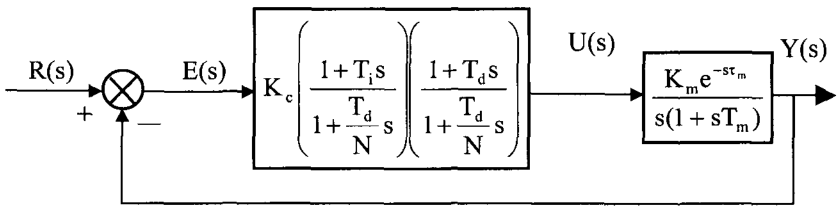


Table 112: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Skogestad (2004b). *Model: Method 1*	$\frac{1}{K_m(T_{CL}+\tau_m)}$	$4\xi^2(T_{CL}+\tau_m)$	T_m	'good' robustness - $T_{CL}=\tau_m$; ξ=0.7 or 1
	$N \in [5,10]$; N = 2 if measurement noise is a serious problem			

4.4.19 *Alternative controller 1* $G_c(s) = K_c\left(\dfrac{1+T_i s}{1+\dfrac{T_d s}{N}}\right)\left(\dfrac{1+T_d s}{1+\dfrac{T_d s}{N}}\right)$



<u>Table 113</u>: PID controller tuning rules – FOLIPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

<table>
<tr><td>Rule</td><td colspan="2">K_c</td><td colspan="2">T_i</td><td colspan="2">T_d</td><td colspan="2">Comment</td></tr>
<tr><td></td><td colspan="8">Direct synthesis</td></tr>
<tr><td rowspan="6">Tsang et al. (1993).
Model: Method 2</td><td colspan="2">$\dfrac{x_1}{K_m \tau_m}$</td><td colspan="2">T_m</td><td colspan="2">$0.25\tau_m$</td><td colspan="2">N = 2.5</td></tr>
<tr><td colspan="8">Coefficient values</td></tr>
<tr><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td><td>x_1</td><td>ξ</td></tr>
<tr><td>1.6818</td><td>0.0</td><td>0.9916</td><td>0.3</td><td>0.6693</td><td>0.6</td><td>0.4957</td><td>0.9</td></tr>
<tr><td>1.3829</td><td>0.1</td><td>0.8594</td><td>0.4</td><td>0.6000</td><td>0.7</td><td>0.4569</td><td>1.0</td></tr>
<tr><td>1.1610</td><td>0.2</td><td>0.7542</td><td>0.5</td><td>0.5429</td><td>0.8</td><td></td><td></td></tr>
<tr><td>Tsang and Rad (1995).
Model: Method 2</td><td colspan="2">$\dfrac{0.809}{K_m \tau_m}$</td><td colspan="2">$T_m$</td><td colspan="2">$0.5\tau_m$</td><td colspan="2">Maximum overshoot = 16%; N = 8.33</td></tr>
</table>

4.4.20 *Alternative controller 2*

$$G_c(s) = K_c\left(\frac{1+T_i s}{1+\frac{T_d s}{N}}\right)\left(\frac{1+0.5\tau_m s+0.0833\tau_m{}^2 s^2}{1+\frac{T_d s}{N}}\right)$$

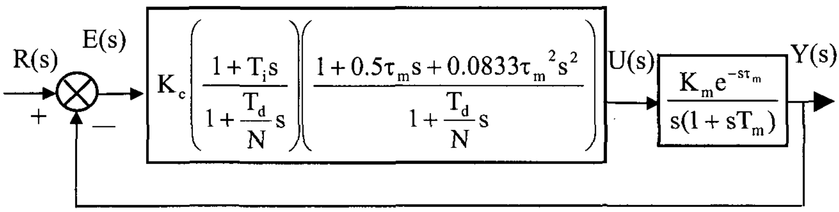


Table 114: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c		T_i		T_d		Comment	
	Direct synthesis							
Tsang *et al.* (1993). *Model: Method 2*	$\frac{x_1}{K_m\tau_m}$		T_m		$0.25\tau_m$		N = 2.5	
	Coefficient values							
	x_1	ξ	x_1	ξ	x_1	ξ	x_1	ξ
	1.8512	0.0	1.1595	0.3	0.8411	0.6	0.6219	0.9
	1.5520	0.1	1.0280	0.4	0.7680	0.7	0.5527	1.0
	1.3293	0.2	0.9246	0.5	0.6953	0.8		

4.5 SOSPD Model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

4.5.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

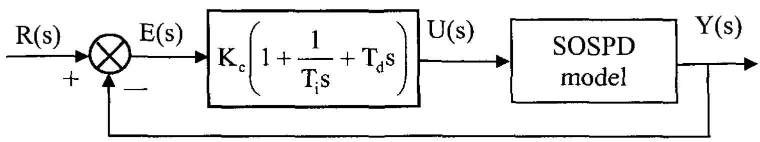


Table 115: PID tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Auslander *et al.* (1975).	[1] $K_c^{(463)}$	$T_i^{(463)}$	$0.25T_i^{(463)}$	*Model: Method 3*

[1] Note: equations continued into the footnote on page 310.

$$K_c^{(463)} = \frac{1.2}{K_m(T_{m1} - T_{m2})}\left\{\left[\tau_m + \frac{T_{m1}T_{m2}}{T_{m1} - T_{m2}}\ln\frac{T_{m1}}{T_{m2}}\right]\left[\left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}} - \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}\right]\right.$$

$$\left. - \frac{\frac{1}{(T_{m1} - T_{m2})^2}\left[\left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}} - \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}\right]^2}{1 + \frac{T_{m2}}{T_{m1} - T_{m2}}\left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}} - \frac{T_{m1}}{T_{m1} - T_{m2}}\left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}}}\right\}^{-1},$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE – Wills (1962b) - *deduced from graph. Model: Method 1*	$\frac{5.8}{K_m}$	$0.36T_u$	$0.21T_u$	Representative tuning values; $T_{m2} = \tau_m = 0.1T_{m1}$
	$\frac{5}{K_m}$	$0.29T_u$	$0.29T_u$	
Minimum IAE– Shinskey (1988) – *page 151. Model: Method 2*	$0.62K_u$	$0.38T_u$	$0.15T_u$	$\frac{T_{m2}}{T_{m2}+\tau_m} = 0.25$
	$0.68K_u$	$0.33T_u$	$0.19T_u$	$\frac{T_{m2}}{T_{m2}+\tau_m} = 0.5$
	$0.79K_u$	$0.26T_u$	$0.21T_u$	$\frac{T_{m2}}{T_{m2}+\tau_m} = 0.75$
Minimum ITAE – Lopez *et al.* (1969). *Model: Method 17*	x_1/K_m	x_2T_{m1}	x_3T_{m1}	

Representative coefficient values (from graphs)

x_1	x_2	x_3	ξ_m	τ_m/T_{m1}
25	0.5	0.25	0.5	0.1
0.7	1.3	1.2	0.5	1
0.35	5	1.0	0.5	10
25	0.5	0.2	1.0	0.1
1.8	1.7	0.7	1.0	1
9.0	2	0.45	4.0	1

$T_i^{(463)} =$

$$\frac{2T_{m1}T_{m2}}{T_{m1}-T_{m2}}\ln\left(\frac{T_{m1}}{T_{m2}}\right)+2\tau_m-\frac{\dfrac{2}{(T_{m1}-T_{m2})}\left[\left(\dfrac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}}-\left(\dfrac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}\right]}{1+\dfrac{T_{m2}}{T_{m1}-T_{m2}}\left(\dfrac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}-\dfrac{T_{m1}}{T_{m1}-T_{m2}}\left(\dfrac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Bohl and McAvoy (1976b).	[2] $K_c^{(464)}$	$T_i^{(464)}$	$T_d^{(464)}$	*Model: Method 1*

[2] $\frac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9$; $\frac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5$

$$K_c^{(464)} = \frac{10.9507}{K_m}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{-1.2096+0.1760\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.1044+0.1806\ln\left(10\frac{T_{m2}}{T_{m1}}\right)-0.2071\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right],$$

$$T_i^{(464)} = 0.2979T_{m1}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{0.7750-0.1026\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.1701+0.0092\ln\left(10\frac{T_{m2}}{T_{m1}}\right)+0.0081\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]$$

$$T_d^{(464)} = 0.1075T_{m1}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{0.6025-0.0624\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.4531-0.0479\ln\left(10\frac{T_{m2}}{T_{m1}}\right)+0.0128\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Hassan (1993).	[3] $K_c^{(465)}$	$T_i^{(465)}$	$T_d^{(465)}$	*Model: Method 7*

---

[3] $0.5 \le \xi_m \le 2$; $0.1 \le \frac{\tau_m}{T_{m1}} \le 4$. $K_c^{(465)}$ is obtained as follows:

$$\log\left[K_m K_c^{(465)}\right] = 1.9763370 - 0.6436825\xi_m - 5.1887660\frac{\tau_m}{T_{m1}} + 0.4375558\xi_m^{\ 2} + 2.9005550\left(\frac{\tau_m}{T_{m1}}\right)^2 + 3.1468010\xi_m\frac{\tau_m}{T_{m1}} - 0.1697221\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.8161808\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 - 1.2048220\xi_m^{\ 2}\frac{\tau_m}{T_{m1}} - 0.0810373\xi_m^{\ 3} - 0.4444091\left(\frac{\tau_m}{T_{m1}}\right)^3 + 0.0319431\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 0.1054399\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.1652807\xi_m^{\ 3}\frac{\tau_m}{T_{m1}} + 0.1175991\xi_m^{\ 3}\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.0375245\xi_m^{\ 3}\left(\frac{\tau_m}{T_{m1}}\right)^3 ;$$

$T_i^{(465)}$ is obtained as follows:

$$\log\left[\frac{T_i^{(465)}}{T_{m1}}\right] = -0.7865873 + 0.6796885\xi_m + 2.1891540\frac{\tau_m}{T_{m1}} - 0.3471095\xi_m^{\ 2} - 1.9003610\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.7007801\xi_m\frac{\tau_m}{T_{m1}} + 0.3077857\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.8566974\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.2535062\xi_m^{\ 2}\frac{\tau_m}{T_{m1}} + 0.0412943\xi_m^{\ 3} + 0.3484161\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.1626562\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0661899\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.2247806\xi_m^{\ 3}\frac{\tau_m}{T_{m1}} - 0.2470783\xi_m^{\ 3}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.0493011\xi_m^{\ 3}\left(\frac{\tau_m}{T_{m1}}\right)^3 ,$$

$T_d^{(465)}$ is obtained as follows (continued into footnote on page 313):

$$\log\left[\frac{T_d^{(465)}}{T_{m1}}\right] = -0.6726798 - 0.2072477\xi_m + 2.6826330\frac{\tau_m}{T_{m1}} + 0.0807474\xi_m^{\ 2} - 1.7707830\left(\frac{\tau_m}{T_{m1}}\right)^2 - 1.6685140\xi_m\frac{\tau_m}{T_{m1}} + 0.0845958\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Sung *et al.* (1996). *Model: Method 9*	[4] $K_c^{(466)}$	$T_i^{(466)}$	$T_d^{(466)}$	$0.05 < \frac{\tau_m}{T_{m1}} \le 2$

$$+0.7159307\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.5631447\xi_m^{\,2}\frac{\tau_m}{T_{m1}} - 0.0225269\xi_m^{\,3}$$

$$+0.2821874\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0616288\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0626626\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right)^2$$

$$-0.0372784\xi_m^{\,3}\frac{\tau_m}{T_{m1}} - 0.0948097\xi_m^{\,3}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.0272541\xi_m^{\,3}\left(\frac{\tau_m}{T_{m1}}\right)^3 .$$

[4] Formulae continued into footnote on page 314.

$$K_c^{(466)} = \frac{1}{K_m}\left[-0.67 + 0.297\left(\frac{\tau_m}{T_{m1}}\right)^{-2.001} + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} < 0.9 \text{ or}$$

$$K_c^{(466)} = \frac{1}{K_m}\left[-0.365 + 0.260\left(\frac{\tau_m}{T_{m1}} - 1.4\right)^2 + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} \ge 0.9;$$

$$T_i^{(466)} = T_{m1}\left[2.212\left(\frac{\tau_m}{T_{m1}}\right)^{0.520} - 0.3\right],\ \frac{\tau_m}{T_{m1}} < 0.4 \text{ or}$$

$$T_i^{(466)} = T_{m1}\left\{-0.975 + 0.910\left(\frac{\tau_m}{T_{m1}} - 1.845\right)^2 + \alpha\right\},\ \frac{\tau_m}{T_{m1}} \ge 0.4 \text{ with}$$

$$\alpha = \left[1 - e^{-\frac{\xi_m}{0.15 + 0.33\frac{\tau_m}{T_{m1}}}}\right]\left[5.25 - 0.88\left(\frac{\tau_m}{T_{m1}} - 2.8\right)^2\right];$$

$$T_d^{(466)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\xi_m}{-0.15 + 0.939\left(\frac{\tau_m}{T_{m1}}\right)^{-1.121}}}\right]\left[1.45 + 0.969\left(\frac{T_{m1}}{\tau_m}\right)^{1.171}\right] - 1.9 + 1.576\left(\frac{\tau_m}{T_{m1}}\right)^{-0.530}} .$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995). *Model: Method 10*	[5] $K_c^{(467)}$	$T_i^{(467)}$	$T_d^{(467)}$	$\varepsilon_2 < 2.4$
	[6] $K_c^{(468)}$	$T_i^{(468)}$		$2.4 \le \varepsilon_2 < 3$
	[7] $K_c^{(469)}$	$T_i^{(469)}$		$3 \le \varepsilon_2 < 20$

[5] $$K_c^{(467)} = \left(K_{H2} - \frac{0.622\left[1 - 0.435\omega_{H2}\tau_m + 0.052\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(467)} = \frac{K_c^{(467)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0697\left(1 + 0.752\omega_{H2}\tau_m - 0.145\omega_{H2}^2\tau_m^2\right)},$$

$$T_d^{(467)} = \frac{1.45(1 + K_u K_m)}{K_m^2\omega_u}\left(1 - \frac{1.16}{\varepsilon_0}\right)\left(1 - 0.612\omega_u\tau_m + 0.103\omega_u^2\tau_m^2\right).$$

[6] $$K_c^{(468)} = \left(K_{H2} - \frac{0.724\left[1 - 0.469\omega_{H2}\tau_m + 0.0609\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(468)} = \frac{K_c^{(468)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0405\left(1 + 1.93\omega_{H2}\tau_m - 0.363\omega_{H2}^2\tau_m^2\right)}.$$

[7] $$K_c^{(469)} = \left(K_{H2} - \frac{1.26(0.506)^{\omega_{H2}\tau_m}\left[1 - 1.07/\varepsilon_2 + 0.616/\varepsilon_2^2\right]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(469)} = \frac{K_c^{(469)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0661\left(1 + 0.824\ln[\omega_{H2}\tau_m]\right)\left(1 + 1.71/\varepsilon_2 - 1.17/\varepsilon_2^2\right)}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th colspan="2">Comment</th></tr>
<tr><td rowspan="2">Hwang (1995) - continued</td><td colspan="2">[8] $K_c^{(470)}$</td><td colspan="2">$T_i^{(470)}$</td><td colspan="2">$T_d^{(467)}$ (page 314)</td><td colspan="2">$\varepsilon_2 \ge 20$</td></tr>
<tr><td colspan="8">Decay ratio = 0.2; $0.2 \le \tau_m/T_{m1} \le 2.0$ and $0.6 \le \xi_m \le 4.2$</td></tr>
<tr><td></td><td colspan="8">Minimum performance index: servo tuning</td></tr>
<tr><td rowspan="2">Minimum IAE – Wills (1962b) - deduced from graph. Model: Method 1</td><td colspan="2">$6.5/K_m$</td><td colspan="2">$1.45T_u$</td><td colspan="2">$0.14T_u$</td><td colspan="2" rowspan="2">Representative tuning values; $T_{m2} = \tau_m = 0.1T_{m1}$</td></tr>
<tr><td colspan="2">$7/K_m$</td><td colspan="2">$0.95T_u$</td><td colspan="2">$0.22T_u$</td></tr>
<tr><td rowspan="7">Minimum IAE – Gallier and Otto (1968). Model: Method 1</td><td colspan="2">x_1/K_m</td><td colspan="2">[9] $T_i^{(471)}$</td><td colspan="2">$T_d^{(471)}$</td><td colspan="2">$T_{m1} = T_{m2}$</td></tr>
<tr><td colspan="8">Representative coefficient values – deduced from graphs</td></tr>
<tr><td>$\frac{\tau_m}{2T_{m1}}$</td><td>x_1</td><td>x_2</td><td>x_3</td><td>$\frac{\tau_m}{2T_{m1}}$</td><td>x_1</td><td>x_2</td><td>x_3</td></tr>
<tr><td>0.053</td><td>6.7</td><td>0.89</td><td>0.24</td><td>0.67</td><td>1.05</td><td>0.64</td><td>0.24</td></tr>
<tr><td>0.11</td><td>4.15</td><td>0.84</td><td>0.24</td><td>1.50</td><td>0.70</td><td>0.55</td><td>0.26</td></tr>
<tr><td>0.25</td><td>2.35</td><td>0.77</td><td>0.24</td><td>4.0</td><td>0.52</td><td>0.50</td><td>0.20</td></tr>
<tr><td colspan="8"></td></tr>
<tr><td>Minimum ITAE - Wills (1962a) – deduced from graph.</td><td colspan="2">$\frac{18}{K_m}$</td><td colspan="2">$\approx T_u$</td><td colspan="2">$\approx 0.2T_u$</td><td colspan="2">$T_{m1} = T_{m2}$; $\tau_m = 0.1T_{m1}$; Model: Method 1</td></tr>
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[8] $K_c^{(470)} = \left(K_{H2} - \frac{1.09\left[1 - 0.497\omega_{H2}\tau_m + 0.0724\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(470)} = \frac{K_c^{(470)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.054\left(-1 + 2.54\omega_{H2}\tau_m - 0.457\omega_{H2}^2\tau_m^2\right)}.$$

[9] $T_i^{(471)} = x_2(T_{m1} + T_{m2} + \tau_m)$, $T_d^{(471)} = x_3(T_{m1} + T_{m2} + \tau_m)$.

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Sung *et al.* (1996). *Model: Method 9*	[10] $K_c^{(472)}$	$T_i^{(472)}$	$T_d^{(472)}$	$0.05 < \frac{\tau_m}{T_{m1}} \le 2$
Nearly minimum IAE, ISE, ITAE – Hwang (1995). *Model: Method 10*	[11] $K_c^{(473)}$	$T_i^{(473)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [12]

[10] $K_c^{(472)} = \frac{1}{K_m}\left[-0.04 + \left[0.333 + 0.949\left(\frac{\tau_m}{T_{m1}}\right)^{-0.983}\right]\xi_m\right]$, $\xi_m \le 0.9$ or

$$K_c^{(472)} = \frac{1}{K_m}\left[-0.544 + 0.308\frac{\tau_m}{T_{m1}} + 1.408\left(\frac{\tau_m}{T_{m1}}\right)^{-0.832}\xi_m\right], \ \xi_m > 0.9 .$$

$T_i^{(472)} = T_{m1}\left[2.055 + 0.072\left(\tau_m/T_{m1}\right)\right]\xi_m$, $\tau_m/T_{m1} \le 1$ or

$$T_i^{(472)} = T_{m1}\left[1.768 + 0.329\left(\tau_m/T_{m1}\right)\right]\xi_m, \ \tau_m/T_{m1} > 1 .$$

$$T_d^{(472)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\left(\frac{\tau_m}{T_{m1}}\right)^{1.060}\xi_m}{0.870}}\right]\left[0.55 + 1.683\left(\frac{T_{m1}}{\tau_m}\right)^{1.090}\right]} .$$

[11] $K_c^{(473)} = \left(K_{H2} - \frac{0.822\left[1 - 0.549\omega_{H2}\tau_m + 0.112\omega_{H2}^2\tau_m^2\right]}{K_m/\left(1 + K_{H2}K_m\right)}\right)$,

$$T_i^{(473)} = \frac{K_c^{(473)}\left(1 + K_{H2}K_m\right)}{\omega_{H2}K_m 0.0142\left(1 + 6.96\omega_{H2}\tau_m - 1.77\omega_{H2}^2\tau_m^2\right)} .$$

[12] $\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$.

Rule	K_c	T_i	T_d	Comment
Hwang (1995) – continued.	[13] $K_c^{(474)}$	$T_i^{(474)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [14]
	[15] $K_c^{(475)}$	$T_i^{(475)}$		
	[16] $K_c^{(476)}$	$T_i^{(476)}$		
Hwang (1995) – continued.	[17] $K_c^{(477)}$	$T_i^{(477)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [18]

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[13] $K_c^{(474)} = \left(K_{H2} - \frac{0.786\left[1 - 0.441\omega_{H2}\tau_m + 0.0569\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(474)} = \frac{K_c^{(474)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0172\left(1 + 4.62\omega_{H2}\tau_m - 0.823\omega_{H2}^2\tau_m^2\right)}.$$

[14] $\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$.

[15] $K_c^{(475)} = \left(K_{H2} - \frac{1.28(0.542)^{\omega_{H2}\tau_m}\left[1 - 0.986/\varepsilon_2 + 0.558/\varepsilon_2^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(475)} = \frac{K_c^{(475)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0476(1 + 0.996\ln[\omega_{H2}\tau_m])(1 + 2.13/\varepsilon_2 - 1.13/\varepsilon_2^2)}.$$

[16] $K_c^{(476)} = \left(K_{H2} - \frac{1.14\left[1 - 0.466\omega_{H2}\tau_m + 0.0647\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(476)} = \frac{K_c^{(476)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0609\left(-1 + 1.97\omega_{H2}\tau_m - 0.323\omega_{H2}^2\tau_m^2\right)}.$$

[17] $K_c^{(477)} = \left(K_{H2} - \frac{0.794\left[1 - 0.541\omega_{H2}\tau_m + 0.126\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(477)} = \frac{K_c^{(477)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0078\left(1 + 8.38\omega_{H2}\tau_m - 1.97\omega_{H2}^2\tau_m^2\right)},$$

[18] $\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$.

Rule	K_c	T_i	T_d	Comment
Hwang (1995) – continued.	[19] $K_c^{(478)}$	$T_i^{(478)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [20]
	[21] $K_c^{(479)}$	$T_i^{(479)}$		
	[22] $K_c^{(480)}$	$T_i^{(480)}$		
Hwang (1995) – continued.	[23] $K_c^{(481)}$	$T_i^{(481)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [24]

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[19] $K_c^{(478)} = \left(K_{H2} - \frac{0.738\left[1 - 0.415\omega_{H2}\tau_m + 0.0575\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(478)} = \frac{K_c^{(478)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0124(1 + 4.05\omega_{H2}\tau_m - 0.63\omega_{H2}^2\tau_m^2)}.$$

[20] $\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$.

[21] $K_c^{(479)} = \left(K_{H2} - \frac{1.15(0.564)^{\omega_{H2}\tau_m}\left[1 - 0.959/\varepsilon_2 + 0.773/\varepsilon_2^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(479)} = \frac{K_c^{(479)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0335(1 + 0.947\ln[\omega_{H2}\tau_m])(1 + 1.9/\varepsilon_2 - 1.07/\varepsilon_2^2)}.$$

[22] $K_c^{(480)} = \left(K_{H2} - \frac{1.07\left[1 - 0.466\omega_{H2}\tau_m + 0.0667\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(480)} = \frac{K_c^{(480)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0328(-1 + 2.21\omega_{H2}\tau_m - 0.338\omega_{H2}^2\tau_m^2)}.$$

[23] $K_c^{(481)} = \left(K_{H2} - \frac{0.789\left[1 - 0.527\omega_{H2}\tau_m + 0.11\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right)$,

$$T_i^{(481)} = \frac{K_c^{(481)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.009(1 + 9.7\omega_{H2}\tau_m - 2.4\omega_{H2}^2\tau_m^2)}.$$

[24] $\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$, $\xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td rowspan="3">Hwang (1995) – continued.</td><td colspan="2">[25] $K_c^{(482)}$</td><td colspan="2">$T_i^{(482)}$</td><td colspan="2" rowspan="3">$\frac{0.471K_u}{K_m\omega_u}$</td><td rowspan="3">Decay ratio = 0.1; $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$; $0.6 \le \xi_m \le 4.2$ [26]</td></tr>
<tr><td colspan="2">[27] $K_c^{(483)}$</td><td colspan="2">$T_i^{(483)}$</td></tr>
<tr><td colspan="2">[28] $K_c^{(484)}$</td><td colspan="2">$T_i^{(484)}$</td></tr>
<tr><td></td><td colspan="7">Minimum performance index: other tuning</td></tr>
<tr><td rowspan="10">Wilton (1999).
Model: Method 1</td><td colspan="7">Minimum $\int_0^\infty \left[e^2(t) + x_2^2 K_m^2 (y(t) - y_\infty)^2\right]dt$; $T_{m1} > T_{m2}$</td></tr>
<tr><td colspan="2">$\frac{x_1}{K_m}$</td><td colspan="2">$T_{m1} + T_{m2}$</td><td colspan="2">$\frac{T_{m2}}{1 + T_{m2}/T_{m1}}$</td><td rowspan="3"></td></tr>
<tr><td colspan="6">Coefficient values (obtained from graphs)</td></tr>
<tr><td colspan="2">$T_{m2}/T_{m1} = 0.25$</td><td colspan="2">$T_{m2}/T_{m1} = 0.5$</td><td colspan="2">$T_{m2}/T_{m1} = 0.75$</td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_1</td><td>x_2</td><td>x_1</td><td>x_2</td><td></td></tr>
<tr><td>1250.1</td><td>0.0001</td><td></td><td></td><td></td><td></td><td rowspan="4">$\tau_m/T_{m1} = 0.1$</td></tr>
<tr><td>22.946</td><td>0.04</td><td>20.396</td><td>0.04</td><td>22.946</td><td>0.04</td></tr>
<tr><td>4.8990</td><td>0.2</td><td>4.8990</td><td>0.2</td><td>4.8990</td><td>0.2</td></tr>
<tr><td>2.1213</td><td>1</td><td>2.1213</td><td>1</td><td>2.1213</td><td>1</td></tr>
</table>

[25] $$K_c^{(482)} = \left(K_{H2} - \frac{0.76\left[1 - 0.426\omega_{H2}\tau_m + 0.0551\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right),$$
$$T_i^{(482)} = \frac{K_c^{(482)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0153\left(1 + 4.37\omega_{H2}\tau_m - 0.743\omega_{H2}^2\tau_m^2\right)}.$$

[26] $$\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2, \xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2.$$

[27] $$K_c^{(483)} = \left(K_{H2} - \frac{1.22(0.55)^{\omega_{H2}\tau_m}\left[1 - 0.978/\varepsilon_2 + 0.659/\varepsilon_2^2\right]}{K_m/(1 + K_{H2}K_m)}\right),$$
$$T_i^{(483)} = \frac{K_c^{(483)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0421(1 + 0.969\ln[\omega_{H2}\tau_m])\left(1 + 2.02/\varepsilon_2 - 1.11/\varepsilon_2^2\right)}.$$

[28] $$K_c^{(484)} = \left(K_{H2} - \frac{1.11\left[1 - 0.467\omega_{H2}\tau_m + 0.0657\omega_{H2}^2\tau_m^2\right]}{K_m/(1 + K_{H2}K_m)}\right),$$
$$T_i^{(484)} = \frac{K_c^{(484)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0477\left(-1 + 2.07\omega_{H2}\tau_m - 0.333\omega_{H2}^2\tau_m^2\right)}.$$

Rule	K_c		T_i		T_d		Comment
Wilton (1999) – continued.	$\frac{x_1}{K_m}$		$T_{m1}+T_{m2}$		$\frac{T_{m2}}{1+T_{m2}/T_{m1}}$		
	Coefficient values (obtained from graphs)						
	$T_{m2}/T_{m1}=0.25$		$T_{m2}/T_{m1}=0.5$		$T_{m2}/T_{m1}=0.75$		
	x_1	x_2	x_1	x_2	x_1	x_2	
	180.01	0.0001	230.01	0.0001	300.01	0.0001	$\tau_m/T_{m1}=0.5$
	8.1584	0.04	9.1782	0.04	10.708	0.04	
	2.9394	0.2	3.1843	0.2	3.4293	0.2	
	1.2728	1	1.3435	1	1.4142	1	
	90.004	0.0001	100.00	0.0001	112.01	0.0001	$\tau_m/T_{m1}=1$
	4.5891	0.04	4.6911	0.04	5.354	0.04	
	2.0331	0.2	2.0821	0.2	2.1311	0.2	
	0.8768	1	1.0324	1	1.0748	1	
Keane *et al.* (2000). *Model: Method 1*	29 $K_c^{(485)}$		$T_i^{(485)}$		$T_d^{(485)}$		
	Minimum ITAE servo plus ITAE regulator, with minimum M_{max} and with good noise attenuation; $T_{m1}=T_{m2}$						
Huang and Jeng (2003). *Model: Method 13*	30 $K_c^{(486)}$		$2\xi_m T_{m1}$		$T_d^{(486)}$		$\xi_m \le 1.1$

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29 $K_c^{(485)}=\frac{(\ln K_u)[T_u+T_{m1}(1+\ln K_u)]}{T_u K_m}$, $T_d^{(485)}=\frac{T_u T_{m1}}{T_u+T_{m1}(1+\ln K_u)}$,

$$T_i^{(485)}=\frac{(\ln K_u)[T_u+T_{m1}(1+\ln K_u)]}{(\ln K_u)(1+\ln K_u)+\left[\ln K_u+\ln\left(1+e^{T_m-K_u}\right)\right]\left[\ln(K_u+1.33419\tau_m)-1\right]}.$$

30 $K_c^{(486)}=\frac{1.2858T_m\xi_m}{K_m\tau_m}\left(\frac{T_m}{\tau_m}\right)^{-0.0544}$,

$$T_d^{(486)}=\frac{[(-0.0349\xi_m+1.0064)T_m+(0.4196\xi_m-0.1100)\tau_m]^2}{2T_m\xi_m}.$$

Rule	K_c	T_i	T_d	Comment
Huang and Jeng (2003) – continued.	[31] $K_c^{(487)}$	$T_i^{(487)}$	$T_d^{(487)}$	$\xi_m > 1.1$
	Servo or regulator tuning – Minimum IAE			
	Direct synthesis: Frequency domain criteria			
Hang *et al.* (1993a). *Model: Method 8*; $\frac{\tau_m}{T_{m1}} > 0.3$ (Yang and Clarke (1996)).	$\frac{\pi T_{m1}}{A_m K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$T_{m1} = T_{m2}$
	Sample results			
	$\frac{1.5708T_{m1}}{K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$A_m = 2.0$, $\phi_m = 45^\circ$
	$\frac{1.0472T_{m1}}{K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$A_m = 3.0$, $\phi_m = 60^\circ$
Ho *et al.* (1994). *Model: Method 1*	[32] $K_c^{(488)}$	$T_i^{(488)}$	$T_d^{(488)}$	$\tau_m / T_m < 2\xi_m$
Ho *et al.* (1995a). *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	[33] $T_i^{(489)}$	T_{m2}	$T_{m1} > T_{m2}$.

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[31] $$K_c^{(487)} = \frac{0.5890}{K_m \tau_m}\left(\frac{T_{m2}}{\tau_m}\right)^{-0.0030}\left(0.0052\frac{T_{m2}^2}{\tau_m} + 0.8980T_{m2} + 0.4877\tau_m + T_{m1}\right),$$

$$T_i^{(487)} = 0.0052\frac{T_{m2}^2}{\tau_m} + 0.8980T_{m2} + 0.4877\tau_m + T_{m1},$$

$$T_d^{(487)} = T_{m1}\left(\frac{0.0052\frac{T_{m2}^2}{\tau_m} + 0.8980T_{m2} + 0.4877\tau_m}{0.0052\frac{T_{m2}^2}{\tau_m} + 0.8980T_{m2} + 0.4877\tau_m + T_{m1}}\right).$$

[32] $$K_c^{(488)} = \frac{2\omega_p T_{m1}^2}{\pi A_m}\left(\frac{\pi\xi_m}{\omega_p T_{m1}} + \pi - 2\omega_p \tau_m\right),\ T_i^{(488)} = \frac{2}{\pi}T_{m1}^2\left(\frac{\pi\xi_m}{T_{m1}\omega_p} + \pi - 2\omega_p\tau_m\right),$$

$$T_d^{(488)} = \frac{\pi T_{m1}}{2\left(\frac{\pi\xi_m}{\omega_p} + \pi T_{m1} - 2T_{m1}\omega_p\tau_m\right)}.$$

[33] $$T_i^{(489)} = \frac{1}{2\omega_p - \frac{4\omega_p^2\tau_m}{\pi} + \frac{1}{T_{m1}}}.$$

Rule	K_c	T_i	T_d	Comment
Ho *et al.* (1995a) – continued.	Sample results			
	$\frac{0.7854T_{m1}}{K_m\tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$, $\phi_m = 45^\circ$
	$\frac{0.5236T_{m1}}{K_m\tau_m}$	T_{m1}	T_{m2}	$A_m = 3.0$, $\phi_m = 60^\circ$
Ho *et al.* (1997). *Model: Method 1*	[34] $K_c^{(490)}$	$T_i^{(490)}$	$T_d^{(490)}$	[35]
Leva *et al.* (1994). *Model: Method 15*	[36] $K_c^{(491)}$	$T_i^{(491)}$	$0.25T_i^{(491)}$	$\phi_m = 70^0$ (at least).

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[34] $$K_c^{(490)} = \frac{2}{\pi A_m K_m}\left(\pi\xi_m + \pi - 2\frac{\tau_m}{T_{m1}}\right),\ T_i^{(490)} = \frac{2}{\pi}T_{m1}\left(\pi\xi_m + \pi - 2\tau_m\right),$$

$$T_d^{(490)} = \frac{\pi T_{m1}^2}{2\left(\pi\xi_m T_{m1} + \pi T_{m1} - 2\tau_m\right)};\ \frac{\tau_m}{T_m} \le 1,\ 2\xi_m \ge \frac{\tau_m}{T_m}.$$

[35] $$\phi_m < \frac{\left(\pi + \sqrt{\pi^2 + \frac{8\pi\tau_m\xi_m}{T_{m1}}}\right)\left(A_m^2 - 1\right) - 2\pi A_m\left(A_m - 1\right)}{4A_m}.$$

[36] $$K_c^{(491)} = \frac{\omega_{cn}T_i^{(491)}}{K_m}\sqrt{\frac{\left(1 + \omega_{cn}^2 T_{m1}^2\right)^2 + 4\xi_m^2\omega_{cn}^2 T_{m1}^2}{\left(1 - T_i^{(491)}T_d^{(491)}\omega_{cn}^2\right)^2 + [T_i^{(491)}]^2\omega_{cn}^2}},$$

$$\omega_{cn} = \frac{1}{\tau_m}\left[4.07 - \phi_m + \tan^{-1}\left\{\frac{2\xi_m\tau_m T_{m1}\left(0.5\pi - \phi_m\right)}{\left(0.5\pi - \phi_m\right)^2 T_{m1}^2 - \tau_m^2}\right\}\right];$$

$$T_i^{(491)} = \frac{2}{\omega_{cn}}\tan\left[0.5\left\{\omega_{cn}\tau_m + \phi_m - 0.5\pi - \tan^{-1}\left(\frac{2\xi_m\omega_{cn}T_{m1}}{\omega_{cn}^2 T_{m1}^2 - 1}\right)\right\}\right],$$

$$\xi_m \le 1 \text{ or } \xi_m > 1 \text{ (with } 0.5\pi - \phi_m > \frac{3\tau_m}{T_{m1}}\left[\xi_m + \sqrt{\xi_m^2 - 1}\right]\text{)}.$$

Rule	K_c	T_i	T_d	Comment
Leva *et al.* (1994) – continued.	[37] $K_c^{(492)}$	$T_i^{(492)}$	$T_d^{(492)}$	
Wang *et al.* (1999).	$\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\frac{0.5T_{m1}}{\xi_m}$	*Model: Method 12.* [38]
Wang and Shao (1999). *Model: Method 12*	$x_1 \frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	
	Some coefficient values			
	x_1	Comment	x_1	Comment
	1.571	$A_m = 2$, $\phi_m = 45^0$	0.785	$A_m = 4$, $\phi_m = 67.5^0$
	1.047	$A_m = 3$, $\phi_m = 60^0$	0.628	$A_m = 5$, $\phi_m = 72^0$

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[37] $$K_c^{(492)} = \frac{\omega_{cn} T_{m1}^2}{K_m T_d^{(492)}} \sqrt{\frac{\omega_{cn}^2 + \frac{1}{T_{m1}^2}\left(2\xi_m\sqrt{\xi_m^2 - 1} - 1\right)}{\omega_{cn}^2 + z^2}},$$

$$\omega_{cn} = \frac{1}{\tau_m}\left[4.07 - \phi_m + \tan^{-1}\left\{\frac{2\xi_m \tau_m T_{m1}(0.5\pi - \phi_m)}{(0.5\pi - \phi_m)^2 T_{m1}^2 - \tau_m^2}\right\}\right],$$

$$z = \frac{\omega_{cn}}{\tan\left[\phi_m - 0.5\pi + \omega_{cn}\tau_m + \tan^{-1}\left\{\frac{\omega_{cn} T_{m1}}{\left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}\right\}\right]},$$

$$T_i^{(492)} = \frac{T_{m1} z + \left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}{z\left(\xi_m + \sqrt{\xi_m^2 - 1}\right)},\ T_d^{(492)} = \frac{T_{m1}}{T_{m1} z + \left(\xi_m + \sqrt{\xi_m^2 - 1}\right)},\ \xi_m > 1$$

$$\text{with } \frac{3\tau_m}{T_{m1}}\left[\xi_m - \sqrt{\xi_m^2 - 1}\right] < 0.5\pi - \phi_m \le \frac{3\tau_m}{T_{m1}}\left[\xi_m + \sqrt{\xi_m^2 - 1}\right].$$

[38] $\xi_m > 0.7071$

or $0.05 < \frac{0.7071\tau_m}{T_{m1}\sqrt{2\xi_m^2 - 1}} < 0.15$, $\frac{0.7071\tau_m}{T_{m1}\sqrt{2\xi_m^2 - 1}} > 1, \xi_m \ge 1$

or $0.05 < \frac{\xi_m \tau_m}{T_{m1}} < 0.15$, $\frac{\xi_m \tau_m}{T_{m1}} > 1, \xi_m < 1$.

Rule	K_c	T_i	T_d	Comment
Wang and Shao (2000b). *Model: Method 12*	$K_c^{(493)}$ [39]	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Suggested $M_{max} = 1.6$
Chen *et al.* (1999b). *Model: Method 1*	$\frac{x_1 \xi_m T_{m1}}{\tau_m K_m}$	$2\xi_m \tau_m$	$\frac{\tau_m}{2\xi_m}$	
	x_1	Comment		
	1.00	$A_m = 3.14$, $\phi_m = 61.4^0$, $M_s = 1.00$		
	1.22	$A_m = 2.58$, $\phi_m = 55.0^0$, $M_s = 1.10$		
	1.34	$A_m = 2.34$, $\phi_m = 51.6^0$, $M_s = 1.20$		
	1.40	$A_m = 2.24$, $\phi_m = 50.0^0$, $M_s = 1.26$		
	1.44	$A_m = 2.18$, $\phi_m = 48.7^0$, $M_s = 1.30$		
	1.52	$A_m = 2.07$, $\phi_m = 46.5^0$, $M_s = 1.40$		
	1.60	$A_m = 1.96$, $\phi_m = 44.1^0$, $M_s = 1.50$		
Leva *et al.* (2003). *Model: Method 1*	$K_c^{(494)}$ [40]	$16(T_{m2} + \tau_m)$	$4(T_{m2} + \tau_m)$	
	$K_c^{(495)}$ [41]	$5T_{m2} + 4\tau_m$	$T_d^{(495)}$	

[39] $K_c^{(493)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left(1.451 - \frac{1.508}{M_{max}}\right)$.

[40] $K_c^{(494)} = \frac{T_{m1} T_{m2}}{8K_m (T_{m2} + \tau_m)^2}$, $T_{m1} \geq 4(T_{m2} + \tau_m)$.

[41] $K_c^{(495)} = \frac{T_{m1}(5T_{m2} + 4\tau_m)}{8K_m (T_{m2} + \tau_m)^2}$, $T_d^{(495)} = \frac{4T_{m2}(T_{m2} + \tau_m)}{5T_{m2} + 4\tau_m}$, $T_{m1} \geq 8(T_{m2} + \tau_m)$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="3">T_i</th><th colspan="2">T_d</th><th colspan="2">Comment</th></tr>
<tr><td></td><td colspan="9">Direct synthesis: time domain criteria</td></tr>
<tr><td rowspan="10">Wills (1962a).
Model: Method 1
Servo response;
$\xi = 1$.
$T_{m1} = T_{m2}$;
$\tau_m = 0.1T_{m1}$</td><td colspan="2">x_1/K_m</td><td colspan="3">x_2T_u</td><td colspan="2">x_3T_u</td><td colspan="2"></td></tr>
<tr><td colspan="9">Representative coefficient values – deduced from graphs</td></tr>
<tr><td>x_1</td><td>x_2</td><td>x_3</td><td>x_1</td><td>x_2</td><td>x_3</td><td>x_1</td><td>x_2</td><td>x_3</td></tr>
<tr><td>2.5</td><td>2.9</td><td>0.035</td><td>1.6</td><td>1.4</td><td>0.143</td><td>0.2</td><td>0.48</td><td>0.143</td></tr>
<tr><td>2.75</td><td>2.9</td><td>0.073</td><td>16</td><td>1.4</td><td>0.28</td><td>8</td><td>0.48</td><td>0.36</td></tr>
<tr><td>3.2</td><td>2.9</td><td>0.143</td><td>10</td><td>1.4</td><td>0.72</td><td>0.09</td><td>0.28</td><td>0.073</td></tr>
<tr><td>20</td><td>2.9</td><td>0.285</td><td>0.42</td><td>0.70</td><td>0.073</td><td>0.1</td><td>0.28</td><td>0.143</td></tr>
<tr><td>12</td><td>2.9</td><td>0.57</td><td>0.4</td><td>0.70</td><td>0.143</td><td>0.11</td><td>0.28</td><td>0.28</td></tr>
<tr><td>1.4</td><td>1.4</td><td>0.035</td><td>10</td><td>0.70</td><td>0.36</td><td></td><td></td><td></td></tr>
<tr><td>1.5</td><td>1.4</td><td>0.073</td><td>2</td><td>0.70</td><td>0.70</td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Van der Grinten (1963).
Model: Method 1</td><td colspan="2">[42] $K_c^{(496)}$</td><td colspan="3">$T_{m1} + T_{m2} + 0.5\tau_m$</td><td colspan="2">$T_d^{(496)}$</td><td colspan="2">Step disturbance</td></tr>
<tr><td colspan="2">[43] $K_c^{(497)}$</td><td colspan="3">$T_i^{(497)}$</td><td colspan="2">$T_d^{(497)}$</td><td colspan="2">Stochastic disturbance</td></tr>
<tr><td>Pemberton (1972b).</td><td colspan="2">$\frac{2(T_{m1} + T_{m2})}{3K_m\tau_m}$</td><td colspan="3">$T_{m1} + T_{m2}$</td><td colspan="2">$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$</td><td colspan="2">*Model: Method 1*</td></tr>
<tr><td>Pemberton (1972a), (1972b).
Model: Method 1</td><td colspan="2">$\frac{(T_{m1} + T_{m2})}{K_m\tau_m}$</td><td colspan="3">$T_{m1} + T_{m2}$</td><td colspan="2">$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$</td><td colspan="2">$0.1 \le \frac{T_{m1}}{T_{m2}} \le 1.0$
$0.2 \le \frac{\tau_m}{T_{m2}} \le 1.0$</td></tr>
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[42] $$K_c^{(496)} = \frac{1}{K_m}\left(0.5 + \frac{T_{m1} + T_{m2}}{\tau_m}\right),\ T_d^{(496)} = \frac{(T_{m1} + T_{m2})\tau_m + 2T_{m1}T_{m2}}{\tau_m + 2(T_{m1} + T_{m2})}.$$

[43] $$K_c^{(497)} = \frac{e^{-\omega_d\tau_m}}{K_m}\left(1 - 0.5e^{-\omega_d\tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m}e^{-\omega_d\tau_m}\right),$$

$$T_i^{(497)} = \frac{\tau_m}{e^{-\omega_d\tau_m}}\left(1 - 0.5e^{-\omega_d\tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m}e^{-\omega_d\tau_m}\right),$$

$$T_d^{(497)} = \frac{\left(1 - 0.5e^{-\omega_d\tau_m} + \frac{T_{m1}T_{m2}e^{-\omega_d\tau_m}}{(T_{m1} + T_{m2})\tau_m}\right)(T_{m1} + T_{m2})}{1 - 0.5e^{-\omega_d\tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m}e^{-\omega_d\tau_m}}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td>Pemberton (1972b).
Model: Method 1</td><td colspan="2">$\frac{2(T_{m1}+T_{m2})}{3K_m\tau_m}$</td><td colspan="2">$T_{m1}+T_{m2}$</td><td colspan="2">$\frac{T_{m1}+T_{m2}}{4}$</td><td>$0.1 \le \frac{T_{m1}}{T_{m2}} \le 1.0$
$0.2 \le \frac{\tau_m}{T_{m2}} \le 1.0$</td></tr>
<tr><td>Chiu *et al.* (1973).
Model: Method 1</td><td colspan="2">$\frac{\lambda T_{m1}+T_{m2}}{K_m(1+\lambda\tau_m)}$</td><td colspan="2">$T_{m1}+T_{m2}$</td><td colspan="2">$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$</td><td>$\lambda$ variable; suggested values are 0.2, 0.4, 0.6 and 1.0.</td></tr>
<tr><td>Mollenkamp *et al.* (1973).
Model: Method 1</td><td colspan="2">$\frac{2\xi_m T_{m1}}{K_m(T_{CL}+\tau_m)}$</td><td colspan="2">$2\xi_m T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m}$</td><td>Appropriate for "small τ_m"</td></tr>
<tr><td rowspan="2">Smith *et al.* (1975).
Model: Method 1</td><td colspan="2">$\frac{\lambda T_{m1}}{K_m(1+\lambda\tau_m)}$</td><td colspan="2">$T_{m1}$</td><td colspan="2">$T_{m2}$</td><td>$T_{m1} > T_{m2}$</td></tr>
<tr><td colspan="7">λ = pole of specified FOLPD closed loop response.</td></tr>
<tr><td>Suyama (1992).
Model: Method 1</td><td colspan="2">$\frac{T_{m1}+T_{m2}}{2K_m\tau_m}$</td><td colspan="2">$T_{m1}+T_{m2}$</td><td colspan="2">$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$</td><td>OS=10%</td></tr>
<tr><td>Wang and Clements (1995).</td><td colspan="2">$\frac{2\lambda\xi_m T_{m1}}{K_m(1+\lambda\tau_m)}$</td><td colspan="2">$2\xi_m T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m}$</td><td>*Model: Method 16*; Underdamped response</td></tr>
<tr><td>Gorez and Klàn (2000).
Model: Method 1</td><td colspan="2">[44] $K_c^{(498)}$</td><td colspan="2">$2\xi_m T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m}$</td><td>Non-dominant time delay</td></tr>
<tr><td rowspan="6">Vítečková (1999), Vítečková *et al.* (2000a).
Model: Method 1</td><td colspan="2">$\frac{x_1(T_{m1}+T_{m2})}{K_m\tau_m}$</td><td colspan="2">$T_{m1}+T_{m2}$</td><td colspan="2">$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$</td><td rowspan="6">Closed loop overshoot (OS) defined

Overdamped process; $T_{m1} > T_{m2}$</td></tr>
<tr><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td></tr>
<tr><td>0.368</td><td>0</td><td>0.696</td><td>20</td><td>0.906</td><td>40</td></tr>
<tr><td>0.514</td><td>5</td><td>0.748</td><td>25</td><td>0.957</td><td>45</td></tr>
<tr><td>0.581</td><td>10</td><td>0.801</td><td>30</td><td>1.008</td><td>50</td></tr>
<tr><td>0.641</td><td>15</td><td>0.853</td><td>35</td><td></td><td></td></tr>
</table>

[44] $K_c^{(498)} = \frac{2\xi_m T_{m1}}{K_m(2\xi_m T_{m1} + \tau_m)}$.

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td rowspan="6">Vítečková (1999), Vítečková et al. (2000a) – continued.</td><td colspan="2">$\frac{x_1 \xi_m T_{m1}}{K_m \tau_m}$</td><td colspan="2">$2\xi_m T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m}$</td><td rowspan="6">Underdamped process; $0.5 < \xi_m \le 1$</td></tr>
<tr><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td><td>x_1</td><td>OS (%)</td></tr>
<tr><td>0.736</td><td>0</td><td>1.392</td><td>20</td><td>1.812</td><td>40</td></tr>
<tr><td>1.028</td><td>5</td><td>1.496</td><td>25</td><td>1.914</td><td>45</td></tr>
<tr><td>1.162</td><td>10</td><td>1.602</td><td>30</td><td>2.016</td><td>50</td></tr>
<tr><td>1.282</td><td>15</td><td>1.706</td><td>35</td><td></td><td></td></tr>
<tr><td rowspan="2">Skogestad (2004b). Model: Method 1</td><td colspan="2">$\frac{2\xi_m T_{m1}}{K_m (T_{CL} + \tau_m)}$</td><td colspan="2">$2\xi_m T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m}$</td><td>Recommended $T_{CL} = \tau_m$</td></tr>
<tr><td colspan="7">$A_m = 3.14$, $\phi_m = 61.4^0$, $M_{max} = 1.59$</td></tr>
<tr><td>Vítečková et al. (2000b). Model: Method 4</td><td colspan="2">$\frac{0.74 T_{m1}}{K_m \tau_m}$</td><td colspan="2">$2T_{m1}$</td><td colspan="2">$0.5T_{m1}$</td><td>$T_{m1} = T_{m2}$</td></tr>
<tr><td>Arvanitis et al. (2000).</td><td colspan="2">[45] $K_c^{(499)}$</td><td colspan="2">$2\xi_m^{des} T_{m1}$</td><td colspan="2">$\frac{T_{m1}}{2\xi_m^{des}}$</td><td>Model: Method 1</td></tr>
<tr><td>Bi et al. (2000). Model: Method 8</td><td colspan="2">$\frac{1.0128\xi_m T_{m1}}{K_m \tau_m}$</td><td colspan="2">$1.9747 K_m \tau_m$</td><td colspan="2">$\frac{0.5064 T_{m1}^2}{K_m \tau_m}$</td><td></td></tr>
<tr><td>Chen and Seborg (2002).</td><td colspan="2">[46] $K_c^{(500)}$</td><td colspan="2">$T_i^{(500)}$</td><td colspan="2">$T_d^{(500)}$</td><td>Model: Method 1</td></tr>
</table>

[45] $K_c^{(499)} = \frac{2\xi_m^{des} T_{m1}}{K_m \tau_m \left(\sqrt{2}\xi_m^{des} + 1\right)}$.

[46] $K_c^{(500)} = \frac{1}{K_m} \frac{\left[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}\right](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m}{(T_{CL} + \tau_m)^3}$,

$$T_i^{(500)} = \frac{\left[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}\right](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m}{T_{m1}T_{m2} + (T_{m1} + T_{m2} + \tau_m)\tau_m},$$

$$T_d^{(500)} = \frac{3T_{CL}^2 T_{m1}T_{m2} + T_{m1}T_{m2}\tau_m (3T_{CL} + \tau_m) - (T_{m1} + T_{m2} + \tau_m)T_{CL}^3}{\left[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}\right](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m},$$

$$\left(\frac{y}{d}\right)_{desired} = \frac{T_i^{(500)} s e^{-s\tau_m}}{K_c^{(500)} (T_{CL}s + 1)^3}.$$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th>T_d</th><th>Comment</th></tr>
<tr><td>Chen and Seborg (2002) – continued.</td><td colspan="2">[47] $K_c^{(501)}$</td><td colspan="2">$T_i^{(501)}$</td><td>$T_d^{(501)}$</td><td></td></tr>
<tr><td></td><td colspan="6">Robust</td></tr>
<tr><td rowspan="6">Brambilla et al. (1990). Model: Method 1</td><td colspan="2">[48] $K_c^{(502)}$</td><td colspan="2">$T_i^{(502)}$</td><td>$T_d^{(502)}$</td><td></td></tr>
<tr><td colspan="6">Representative coefficient values (deduced from graph)</td></tr>
<tr><td>$\dfrac{\tau_m}{T_{m1}+T_{m2}}$</td><td>$\lambda$</td><td>$\dfrac{\tau_m}{T_{m1}+T_{m2}}$</td><td>$\lambda$</td><td>$\dfrac{\tau_m}{T_{m1}+T_{m2}}$</td><td>$\lambda$</td></tr>
<tr><td>0.1</td><td>0.50</td><td>1.0</td><td>0.29</td><td>10.0</td><td>0.14</td></tr>
<tr><td>0.2</td><td>0.47</td><td>2.0</td><td>0.22</td><td></td><td></td></tr>
<tr><td>0.5</td><td>0.39</td><td>5.0</td><td>0.16</td><td></td><td></td></tr>
<tr><td>Lee et al. (1998). Model: Method 1</td><td colspan="2">$\dfrac{T_i^{(503)}}{K_m(2\lambda+\tau_m)}$</td><td colspan="2">[49] $T_i^{(503)}$</td><td>$T_d^{(503)}$</td><td></td></tr>
</table>

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[47] $$K_c^{(501)} = \frac{1}{K_m}\frac{(2\xi_m T_{m1}\tau_m + T_{m1}^2)(3T_{CL}+\tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m}{(T_{CL}+\tau_m)^3},$$

$$T_i^{(501)} = \frac{(2\xi_m T_{m1}\tau_m + T_{m1}^2)(3T_{CL}+\tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m}{T_{m1}^2 + (2\xi_m T_{m1} + \tau_m)\tau_m},$$

$$T_d^{(501)} = \frac{3T_{CL}^2 T_{m1}^2 + T_{m1}^2\tau_m(3T_{CL}+\tau_m) - (2T_{m1}\xi_m + \tau_m)T_{CL}^3}{(2\xi_m T_{m1}\tau_m + T_{m1}^2)(3T_{CL}+\tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m},$$

$$\left(\frac{y}{d}\right)_{desired} = \frac{T_i^{(501)} s e^{-s\tau_m}}{K_c^{(501)}(T_{CL}s+1)^3}.$$

[48] $$K_c^{(502)} = \frac{T_{m1}+T_{m2}+0.5\tau_m}{K_m\tau_m(2\lambda+1)},\quad T_i^{(502)} = T_{m1}+T_{m2}+0.5\tau_m,$$

$$T_d^{(502)} = \frac{T_{m1}T_{m2} + 0.5(T_{m1}+T_{m2})\tau_m}{T_{m1}+T_{m2}+0.5\tau_m},\quad 0.1 \le \frac{\tau_m}{T_{m1}+T_{m2}} \le 10.$$

[49] $$T_i^{(503)} = 2\xi_m T_{m1} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda+\tau_m)},\ T_d^{(503)} = T_i^{(503)} - 2\xi_m T_{m1} + \frac{T_{m1}^2}{T_i^{(503)}} -$$

$$\frac{\tau_m^3}{6T_i^{(503)}(2\lambda+\tau_m)};\ \text{desired closed loop response} = \frac{e^{-\tau_m s}}{(\lambda s+1)^2}.$$

Rule	K_c	T_i	T_d	Comment
Lee *et al.* (1998). *Model: Method 1*	$\frac{T_i^{(504)}}{K_m(2\lambda+\tau_m)}$	[50] $T_i^{(504)}$	$T_d^{(504)}$	
	$\lambda = \text{maximum}[0.25\tau_m, 0.2T_{m1}]$ (Panda *et al.*, 2004)			
	$\frac{T_i^{(505)}}{K_m(\lambda+\tau_m)}$	[51] $T_i^{(505)}$	$T_d^{(505)}$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s+1)}$
	$\frac{T_i^{(506)}}{K_m(\lambda+\tau_m)}$	[52] $T_i^{(506)}$	$T_d^{(506)}$	
Huang *et al.* (1998), Rivera and Jun (2000). *Model: Method 1*	$\frac{2\xi_m T_{m1}}{K_m(\tau_m+\lambda)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	
	λ specified graphically for different OS and T_R values (Huang *et al.* (1998))			
Marchetti and Scali (2000).	[53] $K_c^{(507)}$	$T_i^{(507)}$	$T_d^{(507)}$	*Model: Method 1*

[50] $T_i^{(504)} = T_{m1} + T_{m2} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda+\tau_m)}$, $T_d^{(504)} = T_i^{(504)} - T_{m1} - T_{m2} + \frac{T_{m1}T_{m2}}{T_i^{(504)}} - \frac{\tau_m^3}{6T_i^{(504)}(2\lambda+\tau_m)}$; desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s+1)^2}$.

[51] $T_i^{(505)} = 2\xi_m T_{m1} + \frac{\tau_m^2}{2(\lambda+\tau_m)}$, $T_d^{(505)} = T_i^{(505)} - 2\xi_m T_{m1}\left(\frac{T_{m1}^2 - \frac{\tau_m^3}{6(\lambda+\tau_m)}}{T_i^{(505)}}\right)$.

[52] $T_i^{(506)} = T_{m1} + T_{m2} + \frac{\tau_m^2}{2(\lambda+\tau_m)}$,

$$T_d^{(506)} = T_i^{(506)} - (T_{m1} + T_{m2})\left(\frac{T_{m1}T_{m2} - \frac{\tau_m^3}{6(\lambda+\tau_m)}}{T_i^{(506)}}\right).$$

[53] $K_c^{(507)} = \frac{2\xi_m T_{m1} + 0.5\tau_m}{K_m(\lambda+\tau_m)}$, $T_i^{(507)} = 2\xi_m T_{m1} + 0.5\tau_m$, $T_d^{(507)} = \frac{T_{m1}(T_{m1} + \xi_m\tau_m)}{2\xi_m T_{m1} + 0.5\tau_m}$.

Rule	K_c	T_i	T_d	Comment
Smith (2003). *Model: Method 1*	$\dfrac{T_{m1}}{K_m(\lambda+\tau_m)}$	T_{m1}	T_{m2}	$\lambda \in [\tau_m, T_{m2}]$, $T_{m1} \ge T_{m2}$
Kristiansson (2003).	[54] $K_c^{(508)}$	$2\xi_m T_{m1}$	$\dfrac{T_{m1}}{2\xi_m}$	*Model: Method 1*
	Ultimate cycle			
Wills (1962a). *Model: Method 1* Quarter decay ratio tuning. $T_{m1} = T_{m2}$; $\tau_m = 0.1T_{m1}$	x_1/K_m	$x_2 T_u$	$x_3 T_u$	

Coefficient values (deduced from graph)

x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
10	2.9	0.036	28	1.4	0.143	18	0.48	0.143
8	1.4	0.036	6	0.67	0.068	0.9	0.28	0.143
14	2.9	0.068	2	0.48	0.068	18	0.67	0.24
33	2.9	0.143	0.6	0.28	0.068	18	0.48	0.24
18	1.4	0.068	30	0.67	0.143	13	0.28	0.28

Rule	K_c	T_i	T_d	Comment
Bohl and McAvoy (1976b).	[55] $K_c^{(509)}$	$T_i^{(509)}$	$T_d^{(509)}$	*Model: Method 1*
Landau and Voda (1992). *Model: Method 1*	[56] $K_c^{(510)}$	$\dfrac{4+\beta}{\beta}\dfrac{1}{\omega_{135^0}}$	$\dfrac{4}{4+\beta}\dfrac{1}{\omega_{135^0}}$	$1 \le \beta \le 2$
	$3K_u$	$0.51T_u$	$0.13T_u$	$\omega_u \tau_m \le 0.16$

[54] $K_c^{(508)} = \dfrac{2\xi_m T_{m1}}{K_m \tau_m}\left[1 - \dfrac{1}{M_{max}}\right]$. Recommended values of M_{max} : 1.4,1.7,2.0.

[55] $$K_c^{(509)} = \frac{2.2689}{K_m}\left[\left(\frac{\tau_m}{T_u}\right)^{2.0302+0.9553\ln\left(\frac{\tau_m}{T_u}\right)}\right]\left[\left(K_u K_m\right)^{1.3135+0.1809\ln(K_u K_m)+0.5219\ln\left(\frac{\tau_m}{T_u}\right)}\right],$$

$$T_i^{(509)} = 0.2693T_u\left[\left(\frac{\tau_m}{T_u}\right)^{-0.0517-0.0643\ln\left(\frac{\tau_m}{T_u}\right)}\right]\left[\left(K_u K_m\right)^{0.5914-0.0832\ln(K_u K_m)+0.0687\ln\left(\frac{\tau_m}{T_u}\right)}\right],$$

$$T_d^{(509)} = 0.0068T_u\left[\left(\frac{\tau_m}{T_u}\right)^{-4.2613-1.5855\ln\left(\frac{\tau_m}{T_u}\right)}\right]\left[\left(K_u K_m\right)^{-1.1436-0.2658\ln(K_u K_m)-1.1100\ln\left(\frac{\tau_m}{T_u}\right)}\right].$$

[56] $K_c^{(510)} = \dfrac{4+\beta}{4}\dfrac{\beta}{2\sqrt{2}\left|G_p(j\omega_{135^0})\right|}$.

Rule	K_c	T_i	T_d	Comment
Landau and Voda (1992) – continued.	$1.9K_u$	$0.64T_u$	$0.16T_u$	$0.16 < \omega_u \tau_m \le 0.2$

4.5.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

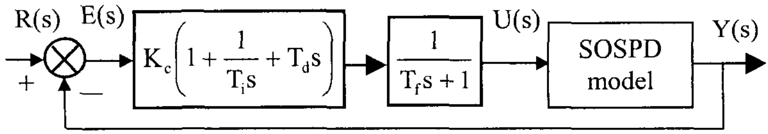

Table 116: PID controller tuning rules – SOSPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Lim *et al.* (1985).	[1] $K_c^{(511)}$	$T_{m1} + T_{m2}$	$\dfrac{T_{m1}T_{m2}}{T_{m1} + T_{m1}}$	*Model: Method 1*
Arvanitis *et al.* (2000). *Model: Method 1*	[2] $K_c^{(512)}$	$2\xi_m^{des} T_{m1}$	$\dfrac{T_{m1}}{2\xi_m^{des}}$	$T_{CL2} > \dfrac{\tau_m}{\sqrt{2}}$
Panda *et al.* (2004). *Model: Method 1*	$\dfrac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\dfrac{T_{m1}}{2\xi_m}$	$T_f = \dfrac{T_{m1}}{2N\xi_m}$, N not defined
	Direct synthesis: frequency domain criteria			
Huang *et al.* (2005). *Model: Method 14*	$\dfrac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\dfrac{T_{m1}}{2\xi_m}$	$T_f = \dfrac{0.025T_{m1}}{\xi_m}$
	$A_m \approx 3$, $\phi_m \approx 60^0$			
	Robust			
Hang *et al.* (1993a). *Model: Method 8*	$\dfrac{2T_{m1} + \tau_m}{2(\lambda + \tau_m)K_m}$	$T_{m1} + 0.5\tau_m$	$\dfrac{T_{m1}\tau_m}{2T_{m1} + \tau_m}$	$\lambda > 0.25\tau_m$. $T_f = \dfrac{\lambda\tau_m}{2(\lambda + \tau_m)}$
	Model has a repeated pole (T_{m1}).			

[1] $K_c^{(511)} = \dfrac{T_{m1} + T_{m2}}{K_m(2\xi T_{CL2} + \tau_m)}$, $T_f = \dfrac{T_{CL2}^2}{2\xi T_{CL2} + \tau_m}$.

[2] $K_c^{(512)} = \dfrac{2\xi_m^{des} T_{m1}}{K_m(2\xi_m^{des} T_{CL2} + \tau_m)}$, $T_f = \dfrac{2(2T_{CL2}^2 - \tau_m^2)}{2\xi_m^{des} T_{CL2} + \tau_m}$.

Rule	K_c	T_i	T_d	Comment
Rivera and Jun (2000). *Model: Method 1*; λ not specified	$\frac{2\xi_m T_{m1}}{\lambda}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \tau_m$
	$\frac{2\xi_m T_{m1}}{K_m(2\tau_m + \lambda)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \frac{\tau_m \lambda}{2\tau_m + \lambda}$
Lee and Edgar (2002). *Model: Method 1*	[3] $K_c^{(513)}$	$T_i^{(513)}$	$T_d^{(513)}$	
Zhang *et al.* (2002). *Model: Method 1* 5% overshoot: $\lambda = 0.5\tau_m$	$\frac{T_{m1} + T_{m2}}{K_m(\tau_m + 2\lambda)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	$T_f = \frac{\lambda^2}{2\lambda + \tau_m}$; H_∞ method
	[4] $K_c^{(514)}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	$T_{m1} + T_{m2}$	$T_f = \frac{\lambda \tau_m}{\lambda + 2\tau_m}$; H_2 method
	$\frac{T_i^{(515)}}{K_m(2\lambda + \tau_m)}$	[5] $T_i^{(515)}$	$T_d^{(515)}$	$T_f = 0$; Maclaurin method

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[3] $$K_c^{(513)} = \frac{1}{K_m(2\lambda + \tau_m)}\left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} + \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}\right)\frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}}\right),$$

$$T_i^{(513)} = 2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} + \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}\right)\frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}},$$

$$T_d^{(513)} = \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}\right)\frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}}.$$

[4] $$K_c^{(514)} = \frac{T_{m1} T_{m2}}{K_m(T_{m1} + T_{m2})(\lambda + 2\tau_m)}.$$

[5] $$T_i^{(515)} = T_{m1} + T_{m2} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}, \quad T_d^{(515)} = \frac{T_{m1} T_{m2} - \frac{\tau_m^3}{12\lambda + 6\tau_m}}{T_i^{(515)}} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003). *Model: Method 1*	[6] $K_c^{(516)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	
	[7] $K_c^{(517)}$			
Panda *et al.* (2004). *Model: Method 1*	[8] $K_c^{(518)}$	$T_i^{(518)}$	$T_d^{(518)}$	
	[9] $K_c^{(519)}$	$T_i^{(519)}$	$T_d^{(519)}$	

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[6] $$K_c^{(516)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left[\alpha - \sqrt{\alpha^2 - 1 + \frac{1}{M_{max}^2}}\right],\ \alpha = 1 + \frac{T_f}{\tau_m}\left[1 - \sqrt{1 - \frac{1}{M_{max}^2}}\right], T_f = \frac{T_{m1}}{\beta}.$$

[7] $$K_c^{(517)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left[1 + 0.019\frac{T_{m1}}{\tau_m} - \sqrt{0.346 + 0.038\frac{T_{m1}}{\tau_m} + 0.00036\frac{T_{m1}^2}{\tau_m^2}}\right];$$

Suggested $M_{max} = 1.7$, $\beta = 10$.

[8] $$K_c^{(518)} = \frac{2\xi_m^2 T_{m1} + 0.25T_{m1} + 0.5\xi_m \tau_m}{K_m(\lambda + 2\xi_m T_{m1})},$$

$$T_i^{(518)} = \frac{4\xi_m^2 T_{m1} + 0.5T_{m1} + \xi_m \tau_m}{2\xi_m},\ T_d^{(518)} = \frac{(T_{m1} + 2\xi_m \tau_m)T_{m1}\xi_m}{4\xi_m^2 T_{m1} + \xi_m \tau_m + 0.5T_{m1}},$$

$$T_f = \frac{\lambda(T_{m1} + 2\xi_m \tau_m)}{4\xi_m \lambda + 4\xi_m \tau_m + 2T_{m1}},\ \lambda = \text{maximum}\left[0.25\left(\frac{T_{m1}}{2\xi_m} + \tau_m\right), 0.4\xi_m T_{m1}\right].$$

[9] $$K_c^{(519)} = \frac{1}{K_m}\frac{0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172T_{m1}e^{-(6.9T_{m2}/T_{m1})} + \frac{0.558T_{m2}T_{m1}}{T_{m1} + 1.208T_{m2}} + 0.5\tau_m}{\lambda + 0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172T_{m1}e^{-(6.9T_{m2}/T_{m1})}},$$

$$T_i^{(519)} = 0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172T_{m1}e^{-(6.9T_{m2}/T_{m1})} + \frac{0.558T_{m2}T_{m1}}{T_{m1} + 1.208T_{m2}} + 0.5\tau_m,$$

$$T_d^{(519)} = \frac{\left[0.828 + 0.812\frac{T_{m2}}{T_{m1}} + 0.172T_{m1}e^{-(6.9T_{m2}/T_{m1})}\right]\left[\frac{1.116T_{m2}T_{m1}}{T_{m1} + 1.208T_{m2}} + \tau_m\right]}{1.656 + 1.624\frac{T_{m2}}{T_{m1}} + 0.344T_{m1}e^{-(6.9T_{m2}/T_{m1})} + \frac{1.116T_{m2}T_{m1}}{T_{m1} + 1.208T_{m2}} + \tau_m},$$

$$T_f = \frac{\lambda[1.116T_{m1}T_{m2} + \tau_m(T_{m1} + 1.208T_{m2})]}{2(\lambda + \tau_m)(T_{m1} + 1.208T_{m2}) + 2.232T_{m2}T_{m1}},$$

$$\lambda = \text{maximum}\left[\frac{0.279T_{m2}T_{m1}}{T_{m1} + 1.208T_{m2}} + 0.25\tau_m, 0.1656 + 0.1624\frac{T_{m2}}{T_{m1}} + 0.0344T_{m1}e^{-(6.9T_{m2}/T_{m1})}\right]$$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Huang *et al.* (2005). *Model: Method 14*	[10] $K_c^{(520)}$	$T_i^{(520)}$	$T_d^{(520)}$	$T_f = \frac{T_d^{(520)}}{20}$
	$A_m \approx 3, \phi_m \approx 60^0$			

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[10] $K_c^{(520)} = 0.080 \frac{\hat{K}_u \hat{T}_u}{K_m \tau_m} \sin\left(\frac{6.283\tau_m}{\hat{T}_u}\right)$, $T_i^{(520)} = 0.159 \hat{K}_u \hat{T}_u \sin\left(\frac{6.283\tau_m}{\hat{T}_u}\right)$,

$$T_d^{(520)} = 0.159 \frac{\hat{T}_u}{\hat{K}_u}\left[\frac{1 + \hat{K}_u \cos\left(\frac{6.283\tau_m}{\hat{T}_u}\right)}{\sin\left(\frac{6.283\tau_m}{\hat{T}_u}\right)}\right].$$

4.5.3 *Ideal controller in series with a first order filter*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{b_{f1}s + 1}{a_{f1}s + 1}$$

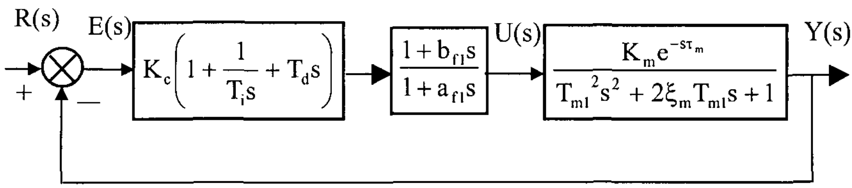

Table 117: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Jahanmiri and Fallahi (1997). *Model: Method 6*	$\frac{2\xi_m T_{m1}}{K_m(\tau_m + \lambda)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$b_{f1} = 0.5\tau_m$, $a_{f1} = \frac{\lambda\tau_m}{2(\tau_m + \lambda)}$
	$\lambda = 0.25\tau_m + 0.1\xi_m T_{m1}$			

4.5.4 *Ideal controller in series with a second order filter*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

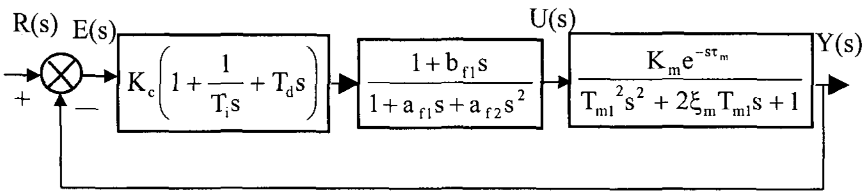

Table 118: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Kristiansson (2003). *Model: Method 1*	$K_c^{(521)}$ [11]	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$M_{max} = 1.7$, $\beta = 10$.
	$b_{1f} = 0$, $a_{1f} = 0.08T_{m1}$, $a_{2f} = 0.01T_{m1}^2$			

[11] $K_c^{(521)} = \frac{2\xi_m T_{m1}}{K_m \tau_m}\left[1 + 0.019\frac{T_{m1}}{\tau_m} - \sqrt{0.346 + 0.038\frac{T_{m1}}{\tau_m} + 0.00036\frac{T_{m1}^2}{\tau_m^2}}\right]$.

4.5.5 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{sT_d}{N}}\right)$

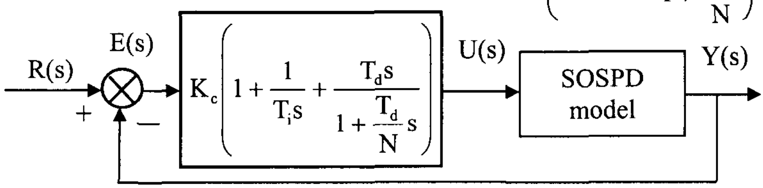

Table 119: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Hang *et al.* (1994). *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	[12] $T_i^{(522)}$	T_{m2}	$N = 20$. $T_{m1} > T_{m2}$
	Sample result			
	$\frac{0.7854 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$, $\phi_m = 45^\circ$
	Robust			
Hang *et al.* (1994). *Model: Method 1*	$\frac{T_{m1}}{K_m(T_{CL} + \tau_m)}$	T_{m1}	T_{m2}	$N = 20$. $T_{m1} > T_{m2}$
Zhong and Li (2002).	[13] $K_c^{(523)}$	$T_i^{(523)}$	$T_d^{(523)}$	*Model: Method 1*

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[12] $T_i^{(522)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_{m1}}}$.

[13] $K_c^{(523)} = \frac{2\xi_m T_{m1}(2\alpha + \tau_m) - \alpha^2}{K_m(2\alpha + \tau_m)^2}$, $T_i^{(523)} = \frac{2\alpha + \tau_m}{2\xi_m T_{m1}(2\alpha + \tau_m) - \alpha^2}$,

$$T_d^{(523)} = T_i^{(523)}\left[T_{m1}^2 - \frac{2\xi_m T_{m1}\alpha^2(2\alpha + \tau_m) - \alpha^4}{(2\alpha + \tau_m)^2}\right], N = \frac{2\alpha + \tau_m}{\alpha^2} T_d^{(523)}.$$

4.5.6 Controller with filtered derivative in series with a second order filter

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)\left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2}\right)$$

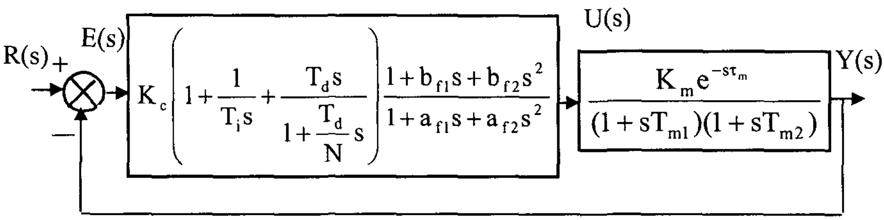

Table 120: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Shi and Lee (2004). *Model: Method 1*	[14] $K_c^{(524)}$	$T_i^{(524)}$	$T_d^{(524)}$	

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[14] $K_c^{(524)} = \frac{\omega_{-6dB}}{K_m(2 + \omega_{-6dB}\tau_m)}\left(T_{m1} + T_{m2} - \frac{0.5}{\omega_{-6dB}}\right)$, $b_{f1} = \frac{1 + \omega_{-6dB}\tau_m}{2\omega_{-6dB}}$,

$b_{f2} = \frac{\tau_m}{4\omega_{-6dB}}$, $a_{f1} = \frac{1}{2\omega_{-6dB}}$, $a_{f2} = \frac{\tau_m}{2\omega_{-6dB}(1 + \omega_{-6dB}\tau_m)}$,

$T_i^{(524)} = T_{m1} + T_{m2} - \frac{0.5}{\omega_{-6dB}}$, $T_d^{(524)} = \frac{T_{m1}T_{m2}\omega_{-6dB}^2 - 0.5[(T_{m1} + T_{m2})\omega_{-6dB} - 0.5]}{\omega_{-6dB}[(T_{m1} + T_{m2})\omega_{-6dB} - 0.5]}$,

$N = \frac{0.5[(T_{m1} + T_{m2})\omega_{-6dB} - 0.5]}{T_{m1}T_{m2}\omega_{-6dB}^2 - 0.5[(T_{m1} + T_{m2})\omega_{-6dB} - 0.5]}$.

4.5.7 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

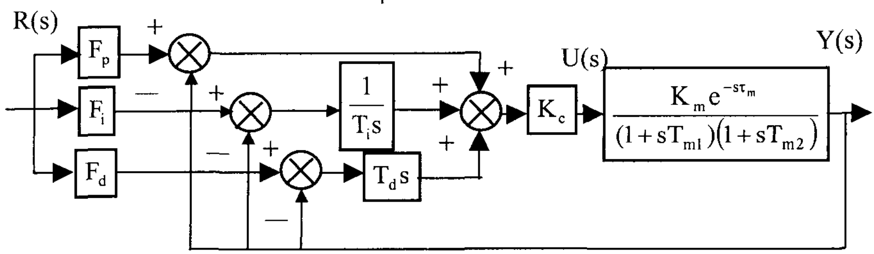

Table 121: PID tuning rules – SOSPD model $G_m(s) = \frac{K_me^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
Oubrahim and Leonard (1998). *Model: Method 1*; $T_{m1} = T_{m2}$; $0.1 < \frac{\tau_m}{T_{m1}} < 3$	[1] $0.6K_u$	$0.5T_u$ $F_i = 1$	$0.125T_u$	10% overshoot
	[2] $0.6K_u$	$0.5T_u$ $F_i = 1$	$0.125T_u$	20% overshoot

[1] $F_p = 1.3\frac{16 - K_mK_u}{17 + K_mK_u}$, $F_d = 1.69\left(\frac{16 - K_mK_u}{17 + K_mK_u}\right)^2$.

[2] $F_p = \frac{38}{29 + 3.5K_mK_u}$, $F_d = \frac{1.717}{(1 + 0.121K_mK_u)^2}$.

4.5.8 *Ideal controller with set-point weighting 2*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\left[\frac{1}{T_i T_d s^2 + T_i s + 1}R(s) - Y(s)\right]$$

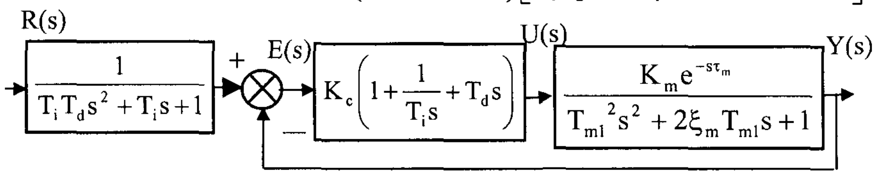

Table 122: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Arvanitis *et al.* (2000).	[3] $K_c^{(525)}$	$T_i^{(525)}$	$T_d^{(525)}$	*Model: Method 1*
	[4] $K_c^{(526)}$	$T_i^{(526)}$	$T_d^{(526)}$	

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[3] $K_c^{(525)} = \frac{\chi T_{m1}}{K_m \tau_m}\left(2\xi_m + \frac{T_{m1}}{\tau_m}\right)$, $0 < \chi < 1$, with suggested $\chi = 0.4$;

$$T_i^{(525)} = \frac{\tau_m}{a^2}\left[a + 2\xi^2\left(\frac{1}{\chi}-1\right) + 2\xi\sqrt{\left(\frac{1}{\chi}-1\right)\left(a + \xi^2\left(\frac{1}{\chi}-1\right)\right)}\right], \text{ with}$$

$$1 + \frac{\tau_m^2}{\chi T_{m1}(2\xi_m \tau_m + T_{m1})} + \xi^2\left(\frac{1}{\chi}-1\right) \ge 0 \text{ and with } a = 1 + \frac{\tau_m^2}{\chi T_{m1}(2\xi_m \tau_m + T_{m1})};$$

$$T_d^{(525)} = \frac{1}{\chi}\frac{T_{m1}\tau_m}{2\xi_m \tau_m + T_{m1}}.$$

[4] $K_c^{(526)} = \frac{1}{K_m \tau_m}\frac{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}{T_{CL2}^2 + \tau_m(2\xi T_{CL2} + \tau_m)}$,

$$T_i^{(526)} = \frac{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}{\tau_m^2 + \tau_m^2 T_{m1}(2\xi_m \tau_m + T_{m1})},$$

$$T_d^{(526)} = \frac{T_{m1}^2\left[T_{CL2}^2 + \tau_m(2\xi T_{CL2} + \tau_m)\right]}{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}.$$

4.5.9 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

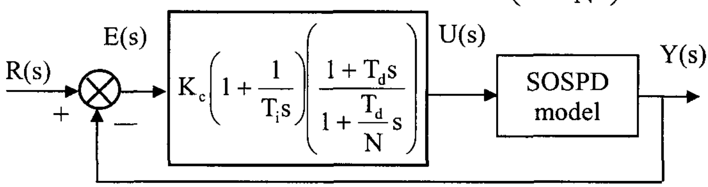

Table 123: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Shinskey (1988) – *page 149. Model: Method 1*; N not specified	$\frac{0.80T_{m1}}{K_m\tau_m}$	$1.5(T_{m2}+\tau_m)$	$0.60(T_{m2}+\tau_m)$	$\frac{T_{m2}}{T_{m2}+\tau_m}=0.25$
	$\frac{0.77T_{m1}}{K_m\tau_m}$	$1.2(T_{m2}+\tau_m)$	$0.70(T_{m2}+\tau_m)$	$\frac{T_{m2}}{T_{m2}+\tau_m}=0.5$
	$\frac{0.83T_{m1}}{K_m\tau_m}$	$0.75(T_{m2}+\tau_m)$	$0.60(T_{m2}+\tau_m)$	$\frac{T_{m2}}{T_{m2}+\tau_m}=0.75$
Minimum IAE – Chao *et al.* (1989). *Model: Method 1*	[5] $K_c^{(527)}$	$T_i^{(527)}$	$T_d^{(527)}$	$N=8$
	$0.8 \le \xi_m \le 3;\ 0.05 \le \frac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			

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[5] $$K_c^{(527)} = \frac{-0.01133+0.5017\xi_m-0.07813\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.890+0.7902\xi_m-0.1554\xi_m^2},$$

$$T_i^{(527)} = 2\xi_m T_{m1}\left(1.235-0.4126\xi_m+0.09873\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.03793+0.3975\xi_m-0.05354\xi_m^2},$$

$$T_d^{(527)} = 2\xi_m T_{m1}\left(1.214-0.6250\xi_m+0.1358\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.5696-0.8484\xi_m+0.0505\xi_m^2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Shinskey (1994) – *page 159. Model: Method 1* N not specified	[6] $K_c^{(528)}$	$T_i^{(528)}$	$T_d^{(528)}$	$\frac{T_{m2}}{\tau_m} \le 3$
	$\frac{2.5T_{m1}}{K_m\tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\frac{T_{m2}}{\tau_m} > 3$
Minimum IAE - Shinskey (1996) – *page 121. Model: Method 1* N not specified	$0.59K_u$	$0.36T_u$	$0.26T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$
Minimum IAE Shinskey (1996) – *page 119. Model: Method 1* $\frac{\tau_m}{T_{m1}} = 0.2$; N not specified	$\frac{0.85T_{m1}}{K_m\tau_m}$	$1.98\tau_m$	$0.86\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{0.87T_{m1}}{K_m\tau_m}$	$2.30\tau_m$	$1.65\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{1.00T_{m1}}{K_m\tau_m}$	$2.50\tau_m$	$2.00\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{1.25T_{m1}}{K_m\tau_m}$	$2.75\tau_m$	$2.75\tau_m$	$\frac{T_{m2}}{T_{m1}} = 1.0$
Minimum ISE – McAvoy and Johnson (1967).	$\frac{x_1}{K_m}$	$x_2\tau_m$	$x_3\tau_m$	*Model: Method 1*

Representative coefficient values (deduced from graph)

Rule	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	x_3	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	x_3
N = 20	1	0.5	0.7	0.97	0.75	4	0.5	3.0	1.16	0.85
	1	4.0	7.6	3.33	2.03	4	4.0	22.7	1.89	1.28
	1	10.0	34.3	5.00	2.7	7	0.5	5.4	1.19	0.85
						7	4.0	40.0	1.64	1.14
N = 10	1	0.5	0.9	1.10	0.64	4	0.5	3.2	1.33	0.78
	1	4.0	8.0	4.00	1.83	4	4.0	23.9	2.17	1.17
	1	10.0	33.5	6.25	2.43	7	0.5	5.9	1.39	0.78
						7	4.0	42.9	1.89	1.04

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[6] $$K_c^{(528)} = \frac{100}{\left(48 + 57\left[1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right]\right)\frac{K_m\tau_m}{T_{m1}}\left(1 + 0.34\frac{T_{m2}}{\tau_m} - 0.2\left[\frac{T_{m2}}{\tau_m}\right]^2\right)},$$

$$T_i^{(528)} = \tau_m\left(1.5 - e^{-\frac{T_{m1}}{1.5\tau_m}}\right)\left(1 + 0.9\left[1 - e^{-\frac{T_{m2}}{\tau_m}}\right]\right), T_d^{(528)} = 0.56\tau_m\left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right) + 0.6T_{m2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Chao *et al.* (1989). *Model: Method 1*	[7] $K_c^{(529)}$	$T_i^{(529)}$	$T_d^{(529)}$	$N = 8$
	$0.8 \le \xi_m \le 3;\ 0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			
Minimum ITAE – Chao *et al.* (1989). *Model: Method 1*	[8] $K_c^{(530)}$	$T_i^{(530)}$	$T_d^{(530)}$	$0.8 \le \xi_m \le 3$

[7] $$K_c^{(529)} = \frac{0.2538 + 0.3875\xi_m - 0.04283\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.652+0.5416\xi_m-0.09544\xi_m^2},$$

$$T_i^{(529)} = 2\xi_m T_{m1}\left(1.8283 - 0.8083\xi_m + 0.1852\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.1363+0.2342\xi_m-0.01445\xi_m^2},$$

$$T_d^{(529)} = 2\xi_m T_{m1}\left(1.077 - 0.4952\xi_m + 0.1062\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.4772+0.0442\xi_m+0.01694\xi_m^2}.$$

[8] $$K_c^{(530)} = \frac{-0.1033 + 0.5347\xi_m - 0.07995\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.798+0.7191\xi_m-0.1416\xi_m^2},$$

$$T_i^{(530)} = 2\xi_m T_{m1}\left(0.9096 - 0.1299\xi_m + 0.04179\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.1277+0.5220\xi_m-0.08629\xi_m^2},$$

$$T_d^{(530)} = 2\xi_m T_{m1}\left(1.2226 - 0.6456\xi_m + 0.1373\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.4780-0.03653\xi_m+0.04523\xi_m^2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Chao *et al.* (1989) – continued.	$K_c^{(531)}$ [9]	$T_i^{(531)}$	$T_d^{(531)}$	$0.3 < \xi_m < 0.8$
	$N = 8;\ 0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			
Minimum ITSE – Chao *et al.* (1989). *Model: Method 1*	$K_c^{(532)}$ [10]	$T_i^{(532)}$	$T_d^{(532)}$	$0.8 \le \xi_m \le 3$

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[9] $$K_c^{(531)} = \frac{1.3292 - 2.8527\xi_m + 2.0365\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.6159 - 0.06616\xi_m + 0.5351\xi_m^2},$$

$$T_i^{(531)} = 2\xi_m T_{m1}\exp(x_1),\ x_1 = \left(3.3368 - 7.7919\xi_m + 4.3556\xi_m^2\right) + \left(1.4094 - 4.4581\xi_m + 3.4857\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(0.1160 - 0.8142\xi_m + 0.7402\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2,$$

$$T_d^{(531)} = 2\xi_m T_{m1}\exp(x_2),\ x_2 = \left(2.3054 - 6.4328\xi_m + 3.9743\xi_m^2\right) + \left(1.8243 - 5.6458\xi_m + 4.6238\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(0.2866 - 1.4727\xi_m + 1.2893\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

[10] $$K_c^{(532)} = \frac{0.1034 + 0.4857\xi_m - 0.0709\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.713 + 0.6238\xi_m - 0.1178\xi_m^2},$$

$$T_i^{(532)} = 2\xi_m T_{m1}\left(1.5144 - 0.6060\xi_m + 0.1414\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.1008 + 0.2625\xi_m - 0.02203\xi_m^2},$$

$$T_d^{(532)} = 2\xi_m T_{m1}\left(1.0783 - 0.4886\xi_m + 0.1038\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{0.4316 + 0.08163\xi_m + 0.009173\xi_m^2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITSE – Chao *et al.* (1989) – continued.	[11] $K_c^{(533)}$	$T_i^{(533)}$	$T_d^{(533)}$	$0.3 < \xi_m < 0.8$
	$N = 8;\ 0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			
	Minimum performance index: servo tuning			
Minimum IAE – Chao *et al.* (1989). *Model: Method 1*	[12] $K_c^{(534)}$	$T_i^{(534)}$	$T_d^{(534)}$	$N = 8$
	$0.8 \le \xi_m \le 3;\ 0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			

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[11] $$K_c^{(533)} = \frac{1.8342 - 3.8984\xi_m + 2.7315\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.5641+0.1628\xi_m+0.09337\xi_m^2},$$

$$T_i^{(533)} = 2\xi_m T_{m1}\exp(x_1),\ x_1 = \left(3.2026 - 7.4812\xi_m + 4.3686\xi_m^2\right) + \left(0.9475 - 3.6318\xi_m + 2.9969\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(-0.06927 - 0.3875\xi_m + 0.4220\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2;$$

$$T_d^{(533)} = 2\xi_m T_{m1}\exp(x_2),\ x_2 = \left(1.7393 - 3.7971\xi_m + 1.7608\xi_m^2\right) + \left(1.0983 - 1.5794\xi_m + 1.0744\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(0.01553 - 0.01414\xi_m - 0.009083\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

[12] $$K_c^{(534)} = \frac{-0.1779 + 0.6763\xi_m - 0.1264\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.9941+0.1669\xi_m-0.04381\xi_m^2},$$

$$T_i^{(534)} = 2\xi_m T_{m1}\left(0.2857 + 0.4671\xi_m - 0.08353\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_1},$$

$$x_1 = 0.1665 - 0.3324\xi_m + 0.1776\xi_m^2 - 0.02897\xi_m^3;$$

$$T_d^{(534)} = 2\xi_m T_{m1}\exp(x_2),\ x_2 = \left(-0.8776 + 0.4353\xi_m - 0.1237\xi_m^2\right) + \left(-2.0552 + 2.786\xi_m - 0.5687\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(-0.5008 + 0.6232\xi_m - 0.1430\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Chao *et al.* (1989). *Model: Method 1*	[13] $K_c^{(535)}$	$T_i^{(535)}$	$T_d^{(535)}$	$N = 8$
	$0.8 \le \xi_m \le 3$; $0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			
Minimum ITAE – Chao *et al.* (1989). *Model: Method 1*	[14] $K_c^{(536)}$	$T_i^{(536)}$	$T_d^{(536)}$	$0.8 \le \xi_m \le 3$

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[13] $$K_c^{(535)} = \frac{0.1446 + 0.4302\xi_m - 0.07501\xi_m^{\ 2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.214+0.3123\xi_m-0.06889\xi_m^{\ 2}};$$

$$T_i^{(535)} = 2\xi_m T_{m1}\exp(x_1),\ x_1 = \left(-0.1256 + 0.1434\xi_m - 0.03277\xi_m^{\ 2}\right) + \left(-2.2502 + 3.7015\xi_m - 1.7699\xi_m^{\ 2} + 0.2680\xi_m^{\ 3}\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(-0.4823 + 0.9479\xi_m - 0.4913\xi_m^{\ 2} + 0.07792\xi_m^{\ 3}\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2;$$

$$T_d^{(535)} = 2\xi_m T_{m1}\left(0.9105 - 0.3874\xi_m + 0.08662\xi_m^{\ 2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_2},$$

$$x_1 = 0.0779 + 0.2855\xi_m - 0.01985\xi_m^{\ 2}.$$

[14] $$K_c^{(536)} = \frac{-0.2641 + 0.7736\xi_m - 0.1486\xi_m^{\ 2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.7225-0.04344\xi_m-0.001247\xi_m^{\ 2}};$$

$$T_i^{(536)} = 2\xi_m T_{m1}\left(0.22257 + 0.7452\xi_m - 0.1451\xi_m^{\ 2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_1},$$

$$x_1 = 0.03138 - 0.04430\xi_m + 0.01120\xi_m^{\ 2};$$

$$T_d^{(536)} = 2\xi_m T_{m1}\exp(x_2),\ x_2 = \left(0.05515 - 0.7031\xi_m + 0.1433\xi_m^{\ 2}\right) + \left(-1.2256 + 1.8544\xi_m - 0.3366\xi_m^{\ 2}\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(-0.2315 + 0.3402\xi_m - 0.06757\xi_m^{\ 2}\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Chao *et al.* (1989). *Model: Method 1*	15 $K_c^{(537)}$	$T_i^{(537)}$	$T_d^{(537)}$	$0.3<\xi_m<0.8$
	$N=8;\ 0.05\le\frac{\tau_m}{2\xi_m T_{m1}}\le 0.5$			
Minimum ITSE – Chao *et al.* (1989). *Model: Method 1*	16 $K_c^{(538)}$	$T_i^{(538)}$	$T_d^{(538)}$	$0.8\le\xi_m\le 3$

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15 $$K_c^{(537)}=\frac{1.1295-2.8584\xi_m+2.1176\xi_m^{\,2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.9904-2.5311\xi_m+2.8008\xi_m^{\,2}};$$

$$T_i^{(537)}=2\xi_m T_{m1}\left(8.6286-20.70\xi_m+13.203\xi_m^{\,2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_1},$$

$$x_1=0.7962-2.4809\xi_m+1.7611\xi_m^{\,2};$$

$$T_d^{(537)}=2\xi_m T_{m1}\exp(x_2),\ x_2=\left(3.9285-12.874\xi_m+8.6434\xi_m^{\,2}\right)$$
$$+\left(3.2228-12.034\xi_m+9.2547\xi_m^{\,2}\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)$$
$$+\left(0.6366-3.0208\xi_m+2.4603\xi_m^{\,2}\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

16 $$K_c^{(538)}=\frac{0.0820+0.4471\xi_m-0.07797\xi_m^{\,2}}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-0.9246+0.07955\xi_m-0.02436\xi_m^{\,2}};$$

$$T_i^{(538)}=2\xi_m T_{m1}\left(0.5930+0.1457\xi_m-0.02062\xi_m^{\,2}\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_1},$$

$$x_1=-0.2201+0.1576\xi_m-0.03202\xi_m^{\,2};$$

$$T_d^{(538)}=2\xi_m T_{m1}\exp(x_2),\ x_2=\left(-0.6534-0.1770\xi_m-0.0240\xi_m^{\,2}\right)$$
$$+\left(-1.0018+1.5914\xi_m-0.2831\xi_m^{\,2}\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)$$
$$+\left(-0.2291+0.3064\xi_m-0.0634\xi_m^{\,2}\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITSE – Chao *et al.* (1989) – continued.	[17] $K_c^{(539)}$	$T_i^{(539)}$	$T_d^{(539)}$	$0.3 < \xi_m < 0.8$
	$N = 8;\ 0.05 \le \dfrac{\tau_m}{2\xi_m T_{m1}} \le 0.5$			
	Direct synthesis			
Smith *et al.* (1975). *Model: Method 1*	$\dfrac{\lambda T_{m1}}{K_m(\lambda\tau_m + 1)}$	T_{m1}	T_{m2}	$\lambda = \dfrac{1}{T_{CL}}$; N not specified
Smith *et al.* (1975). *Model: Method 1*	$\dfrac{x_1 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$\rho = \dfrac{\tau_m}{T_{m1} + T_{m2}}$; $N = 10$
Sklaroff (1992).	[18] $K_c^{(540)}$	$T_{m1} + T_{m2}$	$\dfrac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	
	Also given by Åström and Hägglund (1995) – *page 250-251*. *Model: Method 7*. N = 8 – Honeywell UDC6000 controller.			

Coefficient values (deduced from graph) – Smith *et al.* (1975), *Model: Method 1*

ξ_m	ρ	x_1	ξ_m	ρ	x_1	ξ_m	ρ	x_1
6	≥ 0.02	0.51	3	≥ 0.04	0.50	2	≥ 0.06	0.45
1.75	0.27	0.46	1.75	0.13	0.42	1.75	0.09	0.39
1.75	0.07	0.36	1.5	0.33	0.44	1.5	0.17	0.38
1.5	0.11	0.33	1.5	0.08	0.28	1.0	0.50	0.40
1.0	0.25	0.46	1.0	0.17	0.48	1.0	0.13	0.49

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[17] $$K_c^{(539)} = \frac{0.9666 - 2.3853\xi_m + 2.4096\xi_m^2}{K_m}\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{-1.533 - 0.05442\xi_m + 10.916\xi_m^2};$$

$$T_i^{(539)} = 2\xi_m T_{m1}\left(6.3891 - 15.773\xi_m + 10.916\xi_m^2\right)\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)^{x_1},$$

$$x_1 = 0.04175 - 1.3860\xi_m + 1.4053\xi_m^2;$$

$$T_d^{(539)} = 2\xi_m T_{m1}\exp(x_2),\ x_2 = \left(3.2620 - 10.789\xi_m + 7.6420\xi_m^2\right) + \left(2.277 - 8.9402\xi_m + 7.7176\xi_m^2\right)\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right) + \left(0.3465 - 1.9274\xi_m + 1.8030\xi_m^2\right)\left[\ln\left(\frac{\tau_m}{2\xi_m T_{m1}}\right)\right]^2.$$

[18] $$K_c^{(540)} = \frac{3}{K_m\left(1 + \dfrac{3\tau_m}{T_{m1} + T_{m2}}\right)}.$$

Rule	K_c	T_i	T_d	Comment
Poulin *et al.* (1996). *Model: Method 1*	$\frac{T_{m1}}{K_m(T_{m1}+\tau_m)}$	T_{m1}	T_{m2}	$N = 5$; $T_{m1} \le 5T_{m2}$; Minimum $\phi_m = 55^0$
Schaedel (1997). *Model: Method 1*	[19] $K_c^{(541)}$	$T_i^{(541)}$	$T_d^{(541)}$	N not defined
Panda *et al.* (2004). *Model: Method 7*	[20] $K_c^{(542)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$N = 8$
Panda *et al.* (2004). *Model: Method 1*	$\frac{T_{m1}}{2K_m\tau_m}$	T_{m1}	T_{m2}	N not defined

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[19] $$K_c^{(541)} = \frac{0.375}{K_m}\frac{4\xi_m{}^2T_{m1}{}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m{}^2 - T_{m1}{}^2}{(2\xi_m T_{m1}+\tau_m)^2 + \left(\frac{1}{N}-1\right)(2\xi_m T_{m1}+\tau_m)T_{m1}\sqrt{2(2\xi_m{}^2-1)} - x},$$

$$x = 4\xi_m{}^2T_{m1}{}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m{}^2 - T_{m1}{}^2;$$

$$T_i^{(541)} = \frac{4\xi_m{}^2T_{m1}{}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m{}^2 - T_{m1}{}^2}{2\xi_m T_{m1}+\tau_m};$$

$$T_d^{(541)} = \sqrt{(2\xi_m T_{m1}+\tau_m)^2 - 2\left(T_{m1}{}^2 + 2\xi_m T_{m1}\tau_m + 0.5\tau_m{}^2\right)}.$$

[20] $$K_c^{(542)} = \frac{3}{K_m\left(1+\frac{1.5\tau_m}{\xi_m T_{m1}}\right)}.$$

4.5.10 *Classical controller 2* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+NT_d s}{1+T_d s}\right)$

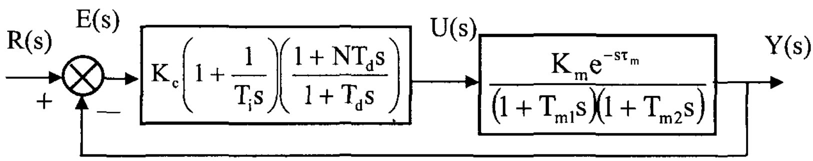

Table 124: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Hougen (1979) – *page 348-349. Model: Method 1*	$\frac{0.8T_{m1}^{0.7}T_{m2}^{0.3}}{\tau_m}$	$0.5T_{m1}+T_{m2}$	$0.1\sqrt[3]{\tau_m T_{m1} T_{m2}}$	N=10
	$\frac{0.84T_{m1}^{0.8}T_{m2}^{0.2}}{\tau_m}$	[21] $T_i^{(543)}$	$T_d^{(543)}$	N=30
Hougen (1988). *Model: Method 1*	[22] $K_c^{(544)}$	$0.5T_{m1}+T_{m2}$	$T_d^{(544)}$	N=10; $\tau_m < T_{m1}$

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[21] $T_i^{(543)} = 0.53T_{m1} + 1.3T_{m2}$, $T_d^{(543)} = 0.08(\tau_m T_{m1} T_{m2})^{0.28}$.

[22] $K_c^{(544)} = \frac{1}{K_m}10^{\left[x_1 - x_2 \log_{10}\left(\frac{T_{m2}}{T_{m1}}\right)\right]}$. Coefficients of $K_c^{(544)}$ deduced from graphs:

τ_m/T_{m2}	0.1	0.3	0.5	1	2
x_1	0.9	0.42	0.24	-0.10	-0.39
x_2	0.7	0.7	0.68	0.70	0.69

$$T_d^{(544)} = (T_{m1}+T_{m2})10^{\left[0.33\log_{10}\left(\frac{\tau_m}{T_{m1}}\right)-1.40\right]}, \quad \frac{T_{m2}}{T_{m1}} = 0.1;$$

$$T_d^{(544)} = (T_{m1}+T_{m2})10^{\left[0.31\log_{10}\left(\frac{\tau_m}{T_{m1}}\right)-1.63\right]}, \quad 0.2 \le \frac{T_{m2}}{T_{m1}} \le 1.$$

4.5.11 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+T_d s)$$

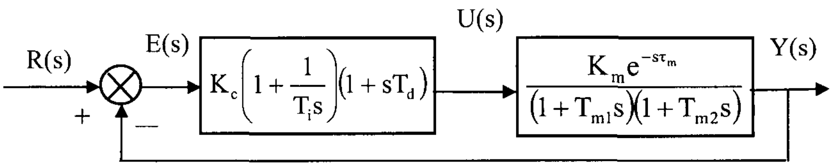

Table 125: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum ISE - Haalman (1965).	$\frac{2T_{m1}}{3\tau_m K_m}$	T_{m1}	T_{m2}	*Model: Method 1*
	$T_{m1} > T_{m2}$. $M_s = 1.9$, $A_m = 2.36$, $\phi_m = 50^0$.			
Bohl and McAvoy (1976b).	[1] $K_c^{(545)}$	$T_i^{(545)}$	$T_d^{(545)}$	Minimum ITAE; *Model: Method 1* $T_d^{(545)} = T_i^{(545)}$

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[1] $\frac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5$; $\frac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9$.

$$K_c^{(545)} = \frac{5.5030}{K_m}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{-1.1445+0.1100\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.1648+0.1709\ln\left(10\frac{T_{m2}}{T_{m1}}\right)-0.2142\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right];$$

$$T_i^{(545)} = 1.8681T_{m1}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{0.6212-0.0760\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.3144-0.0062\ln\left(10\frac{T_{m2}}{T_{m1}}\right)+0.0085\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right].$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Yang and Clarke (1996). *Model: Method 8*	$\frac{\omega_p T_{m1}}{K_m A_m}$	${}^2 T_i^{(546)}$	T_{m1}	$T_{m1} = T_{m2}$
O'Dwyer (2001a). *Model: Method 1*	$\frac{x_1 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = \frac{\pi}{2x_1}$; $\phi_m = 0.5\pi - x_1$.
	Sample result			
	$\frac{0.785 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$; $\phi_m = 45^0$
Skogestad (2003).	$\frac{0.5 T_{m1}}{K_m \tau_m}$	$\min(T_{m1}, 8\tau_m)$	T_{m2}	*Model: Method 18*
Majhi and Litz (2003). *Model: Method 8*	${}^3 K_c^{(547)}$	$T_i^{(547)}$	T_{m1}	$T_{m1} = T_{m2}$
	Ultimate cycle			
Bohl and McAvoy (1976b).	${}^4 K_c^{(548)}$	$T_i^{(548)}$	$T_d^{(548)}$	$T_d^{(548)} = T_i^{(548)}$; *Model: Method 1*

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[2] $$T_i^{(546)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_{m1}}}.$$

[3] $$K_c^{(547)} = \frac{2\phi_m + \pi(A_m - 1)}{2(A_m^2 - 1)} \frac{T_{m1}}{K_m \tau_m},$$

$$T_i^{(547)} = \frac{T_{m1}}{1 + \frac{T_{m1}}{\tau_m} \frac{A_m}{(A_m^2 - 1)} [2\phi_m + \pi(A_m - 1)] \left[1 - \frac{A_m}{\pi(A_m^2 - 1)} (2\phi_m + \pi(A_m - 1))\right]}.$$

[4] $$K_c^{(548)} = \frac{0.5458}{K_m} \left[\left(\frac{\tau_m}{T_u}\right)^{1.2264 + 0.4568 \ln\left(\frac{\tau_m}{T_u}\right)}\right] \left[(K_u K_m)^{1.2190 - 0.0958 \ln(K_u K_m) - 0.1007 \ln\left(\frac{\tau_m}{T_u}\right)}\right],$$

$$T_i^{(548)} = 0.0393 T_u \left[\left(\frac{\tau_m}{T_u}\right)^{-1.9314 - 0.6221 \ln\left(\frac{\tau_m}{T_u}\right)}\right] \left[(K_u K_m)^{-0.0703 - 0.1125 \ln(K_u K_m) - 0.2944 \ln\left(\frac{\tau_m}{T_u}\right)}\right].$$

4.5.12 *Non-interacting controller 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$$

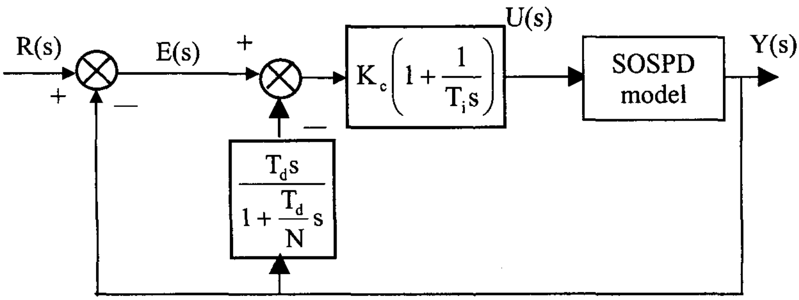

Table 126: PID controller tuning rules – SOSPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s)=\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000). *Model: Method 1*	$K_c^{(549)}$ [5]	$T_i^{(549)}$	$T_d^{(549)}$	
	$\lambda = 0.5, \frac{\tau_m}{T_{m1}}\xi_m \le 1$; $\lambda = 0.6$,$1<\frac{\tau_m}{T_{m1}}\xi_m \le 2$; $\lambda = 0.7, \frac{\tau_m}{T_{m1}}\xi_m > 2$, Servo tuning			
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \le 1$, $\lambda = 0.7,\ 1<\frac{\tau_m}{T_{m1}}\xi_m \le 2$, $\lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2$, Regulator tuning			

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[5] $K_c^{(549)} = \lambda\frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m\tau_m}$, $T_i^{(549)} = 2T_{m1}\xi_m + 0.4\tau_m$,

$$T_d^{(549)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Huang *et al.* (1996).	[6] $K_c^{(550)}$	$T_i^{(550)}$	$T_d^{(550)}$	*Model: Method 1*; N = 10

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[6] Note: equations continued into the footnote on page 356; $0 < T_{m2}/T_{m1} \le 1$; $0.1 \le \tau_m/T_{m1} \le 1$.

$$K_c^{(550)} = \frac{1}{K_m}\left[0.1098 - 8.6290\frac{\tau_m}{T_{m1}} + 76.6760\frac{T_{m2}}{T_{m1}} - 3.3397\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}}\right]$$

$$+\frac{1}{K_m}\left[1.1863\left(\frac{\tau_m}{T_{m1}}\right)^{-0.8058} + 23.1098\left(\frac{\tau_m}{T_{m1}}\right)^{0.6642} + 20.3519\left(\frac{\tau_m}{T_{m1}}\right)^{2.1482}\right]$$

$$+\frac{1}{K_m}\left[-52.0778\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8405} - 12.1033\left(\frac{T_{m2}}{T_{m1}}\right)^{2.1123}\right]$$

$$+\frac{1}{K_m}\left[9.4709\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.5306} + 13.6581\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-1.0781}\right]$$

$$+\frac{1}{K_m}\left[-19.4944\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.4500} - 28.2766\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.1427}\right]$$

$$+\frac{1}{K_m}\left[-19.1463e^{\frac{\tau_m}{T_{m1}}} + 8.8420e^{\frac{T_{m2}}{T_{m1}}} + 7.4298e^{\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}}} - 11.4753\frac{T_{m2}}{\tau_m}\right];$$

$$T_i^{(550)} = T_{m1}\left[-0.0145 + 2.0555\frac{\tau_m}{T_{m1}} + 0.7435\frac{T_{m2}}{T_{m1}} - 4.4805\left(\frac{\tau_m}{T_{m1}}\right)^2 + 1.2069\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}}\right]$$

$$+T_{m1}\left[0.2584\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 7.7916\left(\frac{\tau_m}{T_{m1}}\right)^3 - 6.0330\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[3.9585\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.0626\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 7.0263\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[7.0004\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 2.7755\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^2 - 1.5769\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[3.1663\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 2.4311\left(\frac{\tau_m}{T_{m1}}\right)^5 - 0.9439\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[-2.4506\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4-0.2227\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[1.9228\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3-0.5494\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^4\right];$$

$$T_d^{(550)}=T_{m1}\left[-0.0206+0.9385\frac{\tau_m}{T_{m1}}+0.7759\frac{T_{m2}}{T_{m1}}-2.3820\left(\frac{\tau_m}{T_{m1}}\right)^2+2.9230\frac{\tau_m T_{m2}}{T_{m1}^2}\right]$$

$$+T_{m1}\left[-3.2336\left(\frac{T_{m2}}{T_{m1}}\right)^2+7.2774\left(\frac{\tau_m}{T_{m1}}\right)^3-9.9017\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[2.7095\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2+6.1539\left(\frac{T_{m2}}{T_{m1}}\right)^3-11.1018\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[10.6303\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3+5.7105\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2-2.274\left(\frac{\tau_m}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[-7.9490\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3-6.6597\left(\frac{T_{m2}}{T_{m1}}\right)^4+8.0849\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[-4.4897\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4-7.6469\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[2.2155\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3+5.0694\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4+4.1225\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[0.519\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5-1.1295\left(\frac{T_{m2}}{T_{m1}}\right)^6-1.6307\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[2.2875\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2+0.9524\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3-0.9321\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right].$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Huang *et al.* (1996).	[7] $K_c^{(551)}$	$T_i^{(551)}$	$T_d^{(551)}$	*Model: Method 1*; N = 10.

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[7] Note: equations continued into the footnote on page 358. $0.4 \le \xi_m \le 1$, $0.05 \le \tau_m / T_{m1} \le 1$.

$$K_c^{(551)} = \frac{1}{K_m}\left[-35.7307 + 14.19\frac{\tau_m}{T_{m1}} + 1.4023\xi_m + 6.8618\xi_m\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+\frac{1}{K_m}\left[-0.9773\left(\frac{\tau_m}{T_{m1}}\right)^3 + 55.5898\xi_m\left(\frac{\tau_m}{T_m}\right)^{0.086} - 3.3093\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+\frac{1}{K_m}\left[53.8651\frac{\tau_m}{T_{m1}}\xi_m^{\ 2} + 11.4911\xi_m^{\ 3} + 0.8778\left(\frac{\tau_m}{T_{m1}}\right)^{-1.6624}\right]$$

$$+\frac{1}{K_m}\left[-29.8822\left(\frac{\tau_m}{T_{m1}}\right)^{-0.6951} + 53.535\left(\frac{\tau_m}{T_{m1}}\right)^{-0.4762} - 16.9807\xi_m^{\ 1.1197}\right]$$

$$+\frac{1}{K_m}\left[-25.4293\xi_m^{\ 1.4622} - 0.1671\xi_m^{\ 58981} + 0.0034\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-2.1208}\right]$$

$$+\frac{1}{K_m}\left[-25.0355\frac{\tau_m}{T_{m1}}\xi_m^{\ 3.0836} - 54.9617\frac{\tau_m}{T_{m1}}\xi_m^{\ 1.2103} - 0.1398e^{\frac{\tau_m}{T_{m1}}}\right]$$

$$+\frac{1}{K_m}\left[-8.2721e^{\xi_m} + 6.3542e^{\frac{\tau_m\xi_m}{T_{m1}}} + 1.0479\frac{\xi_m T_{m1}}{\tau_m}\right];$$

$$T_i^{(551)} = T_{m1}\left[0.2563 + 11.8737\frac{\tau_m}{T_{m1}} - 1.6547\xi_m - 16.1913\left(\frac{\tau_m}{T_{m1}}\right)^2 - 9.7061\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+T_{m1}\left[3.5927\xi_m^{\ 2} + 19.5201\left(\frac{\tau_m}{T_{m1}}\right)^3 - 14.5581\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.939\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+T_{m1}\left[-0.4592\xi_m^{\ 3} - 34.6273\left(\frac{\tau_m}{T_{m1}}\right)^4 + 50.5163\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[8.9259\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 8.6966\frac{\tau_m}{T_{m1}}\xi_m^{\ 3} - 6.9436\xi_m^{\ 4}\right]$$

$$+T_{m1}\left[27.2386\left(\frac{\tau_m}{T_{m1}}\right)^5-20.0697\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-42.2833\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[8.5019\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3-12.2957\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4+8.0694\xi_m{}^5-2.7691\xi_m{}^6\right]$$

$$+T_{m1}\left[-7.7887\left(\frac{\tau_m}{T_{m1}}\right)^6+2.3012\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5+4.6594\frac{\tau_m}{T_{m1}}\xi_m{}^5\right]$$

$$+T_{m1}\left[8.8984\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2+10.2494\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3-5.4906\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4\right];$$

$$T_d{}^{(551)}=T_{m1}\left[-0.021+3.3385\frac{\tau_m}{T_{m1}}+0.185\xi_m-0.5164\left(\frac{\tau_m}{T_{m1}}\right)^2-0.9643\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+T_{m1}\left[-0.8815\xi_m{}^2+0.584\left(\frac{\tau_m}{T_{m1}}\right)^3-12.513\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2+1.3468\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+T_{m1}\left[2.3181\xi_m{}^3-5.2368\left(\frac{\tau_m}{T_{m1}}\right)^4+15.3014\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3-3.1988\xi_m{}^4\right]$$

$$+T_{m1}\left[11.9607\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^2-2.0411\frac{\tau_m}{T_{m1}}\xi_m{}^3+3.4675\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[-0.8219\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-15.0718\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3-1.8859\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3\right]$$

$$+T_{m1}\left[0.4841\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4+2.2821\xi_m{}^5-0.9315\left(\frac{\tau_m}{T_{m1}}\right)^6+0.529\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[-0.6772\xi_m{}^6-1.4212\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2+7.1176\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3\right]$$

$$+T_{m1}\left[-2.3636\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4+0.5497\frac{\tau_m}{T_{m1}}\xi_m{}^5\right].$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Minimum IAE - Huang *et al.* (1996).	[8] $K_c^{(552)}$	$T_i^{(552)}$	$T_d^{(552)}$	*Model: Method 1.* N =10

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[8] Note: equations continued into the footnote on page 360; $0 < T_{m2}/T_{m1} \le 1$, $0.1 \le \tau_m/T_{m1} \le 1$.

$$K_c^{(552)} = \frac{1}{K_m}\left[7.0636 + 66.6512\frac{\tau_m}{T_{m1}} - 137.8937\frac{T_{m2}}{T_{m1}} - 122.7832\frac{\tau_m T_{m2}}{T_{m1}^2}\right]$$

$$+\frac{1}{K_m}\left[26.1928\left(\frac{\tau_m}{T_{m1}}\right)^{0.0865} + 33.6578\left(\frac{\tau_m}{T_{m1}}\right)^{2.6405} + 3.0098\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0309}\right]$$

$$+\frac{1}{K_m}\left[-10.9347\left(\frac{T_{m2}}{T_{m1}}\right)^{2.345} + 141.511\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0570} + 29.4068\frac{T_{m2}}{\tau_m}\right]$$

$$+\frac{1}{K_m}\left[34.3156\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9450} - 70.1035\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9282}\right]$$

$$+\frac{1}{K_m}\left[152.6392\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8866} - 47.9791\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8148}\right]$$

$$+\frac{1}{K_m}\left[-57.9370e^{\frac{\tau_m}{T_{m1}}} + 10.4002e^{\frac{T_{m2}}{T_{m1}}} + 6.7646e^{\frac{\tau_m T_{m2}}{T_{m1}^2}} + 7.3453\left(\frac{\tau_m}{T_m}\right)^{-0.4062}\right];$$

$$T_i^{(552)} = T_{m1}\left[0.9923 + 0.2819\frac{\tau_m}{T_{m1}} - 0.2679\frac{T_{m2}}{T_{m1}} - 1.4510\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.6712\frac{\tau_m T_{m2}}{T_{m1}^2}\right]$$

$$+T_{m1}\left[0.6424\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.504\left(\frac{\tau_m}{T_{m1}}\right)^3 + 2.5324\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[2.3641\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.0500\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.8759\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[0.8519\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 1.3496\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 3.4972\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-2.4216\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 3.1142\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 0.5862\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[0.0797\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4+0.985\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[1.2892\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3+1.2108\left(\frac{T_{m2}}{T_{m1}}\right)^5\right];$$

$$T_d^{(552)}=T_{m1}\left[0.0075+0.3449\frac{\tau_m}{T_{m1}}+0.3924\frac{T_{m2}}{T_{m1}}-0.0793\left(\frac{\tau_m}{T_{m1}}\right)^2+2.7495\frac{\tau_m T_{m2}}{T_{m1}{}^2}\right]$$

$$+T_{m1}\left[0.6485\left(\frac{T_{m2}}{T_{m1}}\right)^2+0.8089\left(\frac{\tau_m}{T_{m1}}\right)^3-9.7483\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[3.4679\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2-5.8194\left(\frac{T_{m2}}{T_{m1}}\right)^3-1.0884\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[12.0049\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3-1.4056\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2-3.7055\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[10.0045\left(\frac{T_{m2}}{T_{m1}}\right)^4+0.3520\left(\frac{\tau_m}{T_{m1}}\right)^5-6.3603\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[-3.2980\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3+7.0404\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[1.4294\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4-6.9064\left(\frac{T_{m2}}{T_{m1}}\right)^5+0.0471\left(\frac{\tau_m}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[1.1839\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5+1.7087\left(\frac{T_{m2}}{T_{m1}}\right)^6+1.7444\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[-1.2817\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3-2.1281\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4+1.5121\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right].$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Huang *et al.* (1996).	[9] $K_c^{(553)}$	$T_i^{(553)}$	$T_d^{(553)}$	*Model: Method 1.* N = 10.

[9] Note: equations continued into the footnote on page 362. $0.4 \le \xi_m \le 1$; $0.05 \le \tau_m/T_{m1} \le 1$.

$$K_c^{(553)} = \frac{1}{K_m}\left[-8.1727 - 32.9042\frac{\tau_m}{T_{m1}} + 31.9179\xi_m + 38.3405\xi_m\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+\frac{1}{K_m}\left[+0.2079\left(\frac{\tau_m}{T_{m1}}\right)^{-1.9009} + 29.3215\left(\frac{\tau_m}{T_{m1}}\right)^{0.1571} + 35.9456\left(\frac{\tau_m}{T_{m1}}\right)^{1.2234}\right]$$

$$+\frac{1}{K_m}\left[-21.4045\xi_m^{0.1311} + 5.1159\xi_m^{1.9814} - 21.9381\xi_m^{1.737}\right]$$

$$+\frac{1}{K_m}\left[-17.7448\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.1303} + 26.8655\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{1.2008}\right]$$

$$+\frac{1}{K_m}\left[-52.9156\frac{\tau_m}{T_{m1}}\xi_m^{1.1207} - 22.4297\frac{\tau_m}{T_{m1}}\xi_m^{0.3626} - 3.3331e^{\frac{\tau_m}{T_{m1}}}\right]$$

$$+\frac{1}{K_m}\left[8.5175e^{\xi_m} - 1.5312e^{\frac{\tau_m\xi_m}{T_{m1}}} + 0.8906\frac{\xi_m T_{m1}}{\tau_m}\right] \text{ [Huang (2002)];}$$

$$T_i^{(553)} = T_{m1}\left[1.1731 + 6.3082\frac{\tau_m}{T_{m1}} - 0.6937\xi_m + 8.5271\left(\frac{\tau_m}{T_{m1}}\right)^2 - 24.7291\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+T_{m1}\left[-6.7123\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 7.9559\xi_m^2 - 32.3937\left(\frac{\tau_m}{T_{m1}}\right)^3 + 5.5268\xi_m^6\right]$$

$$+T_{m1}\left[-27.1372\left(\frac{\tau_m}{T_{m1}}\right)^4 + 166.9272\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 36.3954\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+T_{m1}\left[-94.8879\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2 - 22.6065\xi_m^3 - 1.6084\frac{\tau_m}{T_{m1}}\xi_m^3 + 29.9159\xi_m^4\right]$$

$$+T_{m1}\left[49.6314\left(\frac{\tau_m}{T_{m1}}\right)^5 - 84.3776\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4 - 93.8912\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[110.1706\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3-25.1896\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4-19.7569\xi_m{}^5\right]$$

$$+T_{m1}\left[-12.4348\left(\frac{\tau_m}{T_{m1}}\right)^6-11.7589\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5+68.3097\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2\right]$$

$$+T_{m1}\left[-17.8663\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3-22.5926\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4+9.5061\frac{\tau_m}{T_{m1}}\xi_m{}^5\right];$$

$$T_d{}^{(553)}=T_{m1}\left[0.0904+0.8637\frac{\tau_m}{T_{m1}}-0.1301\xi_m+4.9601\left(\frac{\tau_m}{T_{m1}}\right)^2+14.3899\xi_m\frac{\tau_m}{T_{m1}}\right]$$

$$+T_{m1}\left[0.7170\xi_m{}^2-12.5311\left(\frac{\tau_m}{T_{m1}}\right)^3-42.5012\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2+17.0952\xi_m{}^4\right]$$

$$+T_{m1}\left[-21.4907\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)-6.9555\xi_m{}^3-12.3016\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[102.9447\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3+7.5855\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^2+19.1257\frac{\tau_m}{T_{m1}}\xi_m{}^3\right]$$

$$+T_{m1}\left[10.8688\left(\frac{\tau_m}{T_{m1}}\right)^5-17.2130\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-110.0342\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[50.6455\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3-16.7073\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4-16.2013\xi_m{}^5+5.4409\xi_m{}^6\right]$$

$$+T_{m1}\left[-0.0979\left(\frac{\tau_m}{T_{m1}}\right)^6-10.9260\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5+29.4445\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2\right]$$

$$+T_{m1}\left[21.6061\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3-24.1917\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4+6.2798\frac{\tau_m}{T_{m1}}\xi_m{}^5\right].$$

4.5.13 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$

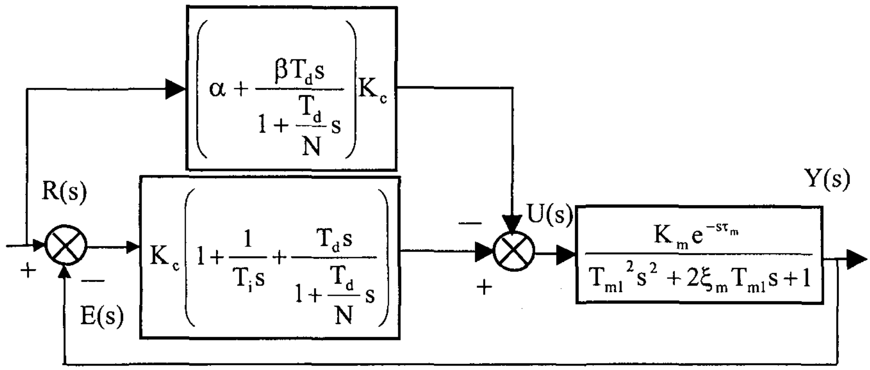

Table 127: PID controller tuning rules – SOSPD model $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IE – Shen (1999).	[1] $K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	*Model: Method 1*; $K_m = 1$

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[1] $x_1 = \exp[a + b\left(\frac{\tau_m}{T_{m1}}\right) + c\left(\frac{\tau_m}{T_{m1}}\right)^2 + d\left(\frac{\tau_m}{T_{m1}}\right)^3 + e\frac{\xi_m \tau_m}{T_{m1}} + f\xi_m + g\xi_m^2$

$+ h\frac{\xi_m^2 \tau_m}{T_{m1}} + i\left(\frac{\tau_m}{T_{m1}}\right)^3 \xi_m + j\left(\frac{\tau_m}{T_{m1}}\right)^2 \xi_m^2 + k\left(\frac{\tau_m}{T_{m1}}\right)^3 \xi_m^2];$

$K_c^{(554)} = \frac{T_{m1}}{\tau_m}x_1$, $T_i^{(554)} = \tau_m x_1$, $T_d^{(554)} = \tau_m x_1$, $\alpha = 1 - x_1$; coefficients of x_1 are given below (and continued into the footnote on page 364). $1.4 \le M_s \le 2$; $\beta = 0$; $N = 0$.

	$K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	α
a	1.40	2.43	1.32	-1.44
b	-11.60	-2.50	-0.51	1.27
c	14.17	-4.16	0.70	1.34

Rule	K_c	T_i	T_d	Comment
Minimum IE – Shen (2000).	$^2 K_c^{(555)}$	$T_i^{(555)}$	$T_d^{(555)}$	*Model: Method 1*; $K_m = 1$

(continued)	$K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	α
d	-6.44	2.85	-1.06	-0.37
e	8.28	-0.30	0.02	1
f	-0.07	0.1	-0.09	-5.97
g	0.13	-1.72	-7.43	-0.78
h	-4.66	15.51	1.77	-10.63
i	-0.99	-7.98	2.64	18.85
j	-3.03	-8.45	4.77	-10.97
k	2.89	4.66	-3.74	6.44

[2] $$x_1 = \exp\left[a + b\left(\frac{\tau_m}{\tau_m + T_{m1}}\right) + c\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 + d\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 + e\xi_m + f\xi_m^{\ 2}\right.$$
$$+ g\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m + h\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 \xi_m + i\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 \xi_m$$
$$\left. + j\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m^{\ 2} + k\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 \xi_m^{\ 2} + l\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 \xi_m^{\ 2}\right];$$

$K_c^{(555)} = \frac{T_{m1}}{\tau_m} x_1$, $T_i^{(555)} = \tau_m x_1$, $T_d^{(555)} = \tau_m x_1$, $\alpha = 1 - x_1$; coefficients of x_1 are given below; $M_s = 2$; $\beta = 0$; $N = 0$; $0 < \xi_m < 1$; $0 \le \frac{\tau_m}{T_{m1}} \le 1$.

	$K_c^{(555)}$	$T_i^{(555)}$	$T_d^{(555)}$	α
a	1.73	2.28	1.43	-1.60
b	-17.51	0.87	-1.94	4.30
c	30.73	-24.00	14.82	-12.16
d	-24.69	15.09	-21.44	26.54
e	0.15	-0.01	-0.25	1.33
f	-0.12	-0.02	0.16	-1.08
g	3.30	-8.03	-1.06	-17.56
h	33.07	45.21	-43.39	38.13
i	-37.64	-22.48	51.16	-56.45
j	2.63	3.89	-3.25	14.80
k	-34.57	-22.39	36.04	-39.33
l	36.88	8.32	-34.95	51.18

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th colspan="2">T_i</th><th colspan="2">T_d</th><th colspan="2">Comment</th></tr>
<tr><td>Minimum IE – Shen (2000). Model: Method 1</td><td colspan="2">[3] $K_c^{(556)}$</td><td colspan="2">$T_i^{(556)}$</td><td colspan="2">$T_d^{(556)}$</td><td colspan="2">$K_m = 1$</td></tr>
<tr><td></td><td colspan="8">Minimum performance index: servo/regulator tuning</td></tr>
<tr><td rowspan="2">Minimum ITAE – Pecharromán and Pagola (2000).</td><td colspan="2">$0.7236K_u$</td><td colspan="2">$0.5247T_u$</td><td colspan="2">$0.1650T_u$</td><td colspan="2">N = 10; Model: Method 11</td></tr>
<tr><td colspan="8">$\alpha = 0.5840$, $\beta = 1$, $\phi_c = -139.65^0$, $K_m = 1$; $T_{m1} = 1$; $\xi_m = 1$</td></tr>
<tr><td rowspan="5">Minimum ITAE – Pecharromán (2000). Model: Method 11</td><td colspan="2">x_1K_u</td><td colspan="2">x_2T_u</td><td colspan="2">x_3T_u</td><td colspan="2"></td></tr>
<tr><td colspan="8">Coefficient values and other data</td></tr>
<tr><td>x_1</td><td colspan="2">x_2</td><td colspan="2">x_3</td><td colspan="2">α</td><td>ϕ_c</td></tr>
<tr><td>0.803</td><td colspan="2">0.509</td><td colspan="2">0.167</td><td colspan="2">0.585</td><td>-146^0</td></tr>
<tr><td>0.727</td><td colspan="2">0.524</td><td colspan="2">0.165</td><td colspan="2">0.584</td><td>-140^0</td></tr>
</table>

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[3] $$x_1 = \exp[a + b\left(\frac{\tau_m}{\tau_m + T_{m1}}\right) + c\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 + d\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 + e\xi_m + f\xi_m^{\ 2}$$
$$+ g\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m + h\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 \xi_m + i\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 \xi_m$$
$$+ j\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m^{\ 2} + k\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 \xi_m^{\ 2} + l\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 \xi_m^{\ 2}];$$

$K_c^{(556)} = \dfrac{T_{m1}}{\tau_m}x_1$, $T_i^{(556)} = \tau_m x_1$, $T_d^{(556)} = \tau_m x_1$, $\alpha = 1 - x_1$; coefficients of x_1 are given below; $M_s = 2$; $\beta = 0$; $N = 0$; $0.4 \le \xi_m \le 1$; $1 < \dfrac{\tau_m}{T_{m1}} \le 15$.

	$K_c^{(556)}$	$T_i^{(556)}$	$T_d^{(556)}$	α
a	128.9	92.8	-3.1	-119.9
b	-560.3	-391	46.3	539.5
c	769.2	521.6	-107.4	-785.1
d	-338.9	-225.5	63.1	371.8
e	-324.8	-224.5	5.2	233.9
f	194.3	137.5	-2.5	-115.7
g	1421	963.9	-79.6	-1068
h	-1981.6	-1310.6	176.0	1560.5
i	895.0	574.4	-103.6	-741.5
j	-848.1	-590.8	36.2	529.2
k	1186.1	808.0	-78.4	-774.6
l	-538.1	-356.7	45.8	369.2

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Pecharromán (2000) – continued. $K_m = 1$; $T_{m1} = 1$; $\beta = 1$; $N = 10$; $\xi_m = 1$; $0.1 < \tau_m < 10$	Coefficient values and other data (continued)			

x_1	x_2	x_3	α	ϕ_c
0.672	0.532	0.161	0.577	-134^0
0.669	0.486	0.170	0.550	-125^0
0.600	0.498	0.157	0.543	-115^0
0.578	0.481	0.154	0.528	-105^0
0.557	0.467	0.149	0.504	-93^0
0.544	0.466	0.141	0.495	-84^0
0.537	0.444	0.144	0.484	-73^0
0.527	0.450	0.131	0.477	-63^0
0.521	0.440	0.129	0.454	-52^0
0.515	0.429	0.126	0.445	-41^0
0.509	0.399	0.132	0.433	-30^0
0.496	0.374	0.123	0.385	-19^0
0.480	0.315	0.112	0.286	-10^0
0.430	0.242	0.084	0.158	-6^0

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000). *Model: Method 1*	[4] $K_c^{(557)}$	$T_i^{(557)}$	$T_d^{(557)}$	$\xi_m = 1.0$

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[4] $$K_c^{(557)} = \frac{1}{K_m}\left(1.389 + \frac{0.6978}{[\frac{\tau_m}{T_{m1}} + 0.02295]^2}\right),$$

$$T_i^{(557)} = T_{m1}\left(0.02453 + 4.104\frac{\tau_m}{T_{m1}} - 3.434\left[\frac{\tau_m}{T_{m1}}\right]^2 + 1.231\left[\frac{\tau_m}{T_{m1}}\right]^3\right),$$

$$T_d^{(557)} = T_{m1}\left(0.03459 + 1.852\frac{\tau_m}{T_{m1}} - 2.741\left[\frac{\tau_m}{T_{m1}}\right]^2 + 2.359\left[\frac{\tau_m}{T_{m1}}\right]^3 - 0.7962\left[\frac{\tau_m}{T_{m1}}\right]^4\right),$$

$$\alpha = 0.6726 - 0.1285\frac{\tau_m}{T_{m1}} - 0.1371\left[\frac{\tau_m}{T_{m1}}\right]^2 + 0.07345\left[\frac{\tau_m}{T_{m1}}\right]^3,$$

$$\beta = 0.8665 - 0.2679\frac{\tau_m}{T_{m1}} + 0.02724\left[\frac{\tau_m}{T_{m1}}\right]^2.$$

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000) – continued.	[5] $K_c^{(558)}$	$T_i^{(558)}$	$T_d^{(558)}$	$\xi_m = 0.5$
	$\tau_m/T_{m1} \le 1.0$; Overshoot (servo step) $\le 20\%$; N not specified			
	Direct synthesis			
Huang *et al.* (2000) - servo tuning. *Model: Method 1*	[6] $K_c^{(559)}$	$T_i^{(559)}$	$T_d^{(559)}$	
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \le 1$, $A_m = 2.83$; $\lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$, $A_m = 2.43$; $\lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2$, $A_m = 2.13$. $\alpha = \frac{2T_{m1}\xi_m}{2T_{m1}\xi_m + 0.4\tau_m} - 1$; $\beta = \frac{0.5T_{m1}^2}{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m} - 1$; $N = \min[20, 20T_d]$			

[5] $$K_c^{(558)} = \frac{1}{K_m}\left(0.3363 + \frac{0.5013}{[\frac{\tau_m}{T_{m1}} + 0.01147]^2}\right),$$

$$T_i^{(558)} = T_{m1}\left(-0.02337 + 4.858\frac{\tau_m}{T_{m1}} - 5.522\left[\frac{\tau_m}{T_{m1}}\right]^2 + 2.054\left[\frac{\tau_m}{T_{m1}}\right]^3\right),$$

$$T_d^{(558)} = T_{m1}\left(0.03392 + 2.023\frac{\tau_m}{T_{m1}} - 1.161\left[\frac{\tau_m}{T_{m1}}\right]^2 + 0.2826\left[\frac{\tau_m}{T_{m1}}\right]^3\right),$$

$$\alpha = 0.6678 - 0.05413\frac{\tau_m}{T_{m1}} - 0.5680\left[\frac{\tau_m}{T_{m1}}\right]^2 + 0.1699\left[\frac{\tau_m}{T_{m1}}\right]^3,$$

$$\beta = 0.8646 - 0.1205\frac{\tau_m}{T_{m1}} - 0.1212\left[\frac{\tau_m}{T_{m1}}\right]^2.$$

[6] $$K_c^{(559)} = \lambda\frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m\tau_m},\quad T_i^{(559)} = 2T_{m1}\xi_m + 0.4\tau_m,$$

$$T_d^{(559)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.5.14 *Non-interacting controller 4*

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$$

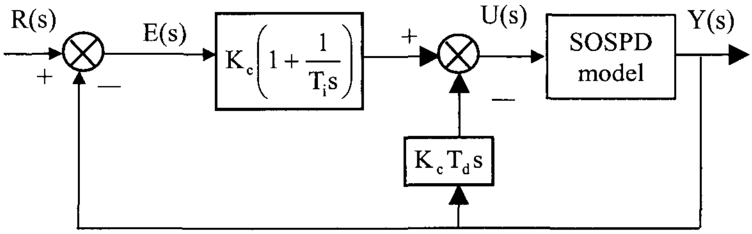

Table 128: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum IAE - Shinskey (1996) – *page 119. Model: Method 1*; $\frac{\tau_m}{T_{m1}} = 0.2$	$\frac{1.18T_{m1}}{K_m \tau_m}$	$2.20\tau_m$	$0.72\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{1.25T_{m1}}{K_m \tau_m}$	$2.20\tau_m$	$1.10\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{1.67T_{m1}}{K_m \tau_m}$	$2.40\tau_m$	$1.65\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{2.5T_{m1}}{K_m \tau_m}$	$2.15\tau_m$	$2.15\tau_m$	$\frac{T_{m2}}{T_{m1}} = 1.0$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Shinskey (1994) – *page 159. Model: Method 1*	[1] $K_c^{(560)}$	$T_i^{(560)}$	$T_d^{(560)}$	$\frac{T_{m2}}{\tau_m} \le 3$
	$\frac{3.33T_{m1}}{K_m \tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\frac{T_{m2}}{\tau_m} > 3$
Minimum IAE - Shinskey (1996) – *page 121. Model: Method 1*	$0.85K_u$	$0.35T_u$	$0.17T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$

[1] $$K_c^{(560)} = \frac{100}{\left(38 + 40\left[1 - e^{-\frac{1.5T_{m1}}{\tau_m}}\right]\right)\frac{K_m \tau_m}{T_{m1}}\left(1 + 0.34\frac{T_{m2}}{\tau_m} - 0.2\left[\frac{T_{m2}}{\tau_m}\right]^2\right)},$$

$$T_i^{(560)} = \tau_m\left(0.5 + 1.4[1 - e^{-\frac{T_{m1}}{1.5\tau_m}}]\right)\left(1 + 0.48\left[1 - e^{-\frac{T_{m2}}{\tau_m}}\right]\right),$$

$$T_d^{(560)} = 0.42\tau_m\left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right) + 0.6T_{m2}.$$

4.5.15 *Non-interacting controller 5*

$$U(s) = K_c\left(b + \frac{1}{T_i s}\right)E(s) - (c + T_d s)Y(s)$$

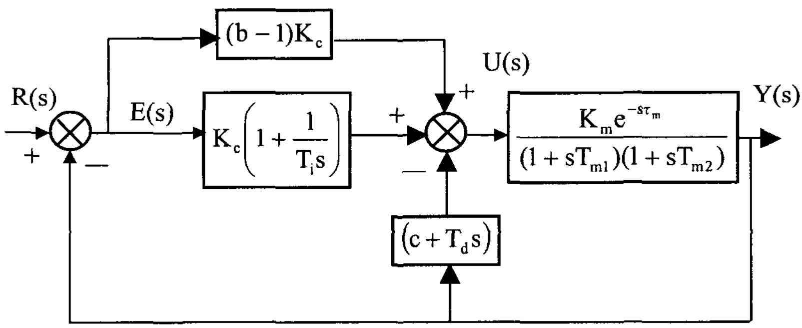

Table 129: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Hansen (1998). *Model: Method 1*	$K_c^{(561)}$ [2]	$1.54T_i^{(561)}$	$T_d^{(561)}$	$b = 0.198$; nearly zero overshoot
		$1.27T_i^{(561)}$		$b = 0.289$; nearly minimum IAE
		$1.75T_i^{(561)}$		$b = 0.143$; conservative tuning

---

[2] $K_c^{(561)} = \frac{2\left[T_{m1}T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2\right]^3}{9K_m\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3\right]^2}$,

$T_i^{(561)} = \frac{3\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3\right]}{\left[T_{m1}T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2\right]}$,

$T_d^{(561)} = \frac{2\left[T_{m1}T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2\right]^2}{3K_m\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3\right]} - \frac{T_{m1} + T_{m2} + \tau_m}{K_m}$,

$c = K_c^{(561)} - \frac{1}{K_m}$.

4.5.16 *Non-interacting controller 6* $U(s) = \frac{K_c}{T_i s} E(s) - K_c(1 + T_d s)Y(s)$

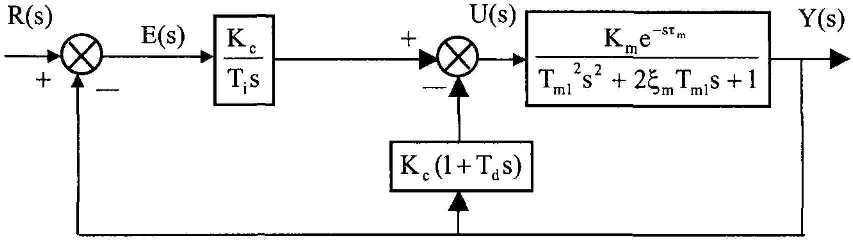

Table 130: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Arvanitis *et al.* (2000). *Model: Method 1*	[3] $K_c^{(562)}$	$T_i^{(562)}$	$\frac{T_{m1}}{\varepsilon\left(2\xi_m + \frac{T_{m1}}{\tau_m}\right)}$	$0 < \varepsilon < 1$; suggested $\varepsilon = 0.5$

---

[3] $K_c^{(562)} = \frac{\varepsilon T_{m1}}{K_m \tau_m}\left(2\xi_m + \frac{T_{m1}}{\tau_m}\right)$,

$$T_i^{(562)} = \frac{\frac{\varepsilon T_{m1}}{\tau_m}\left(2\xi_m + \frac{T_{m1}}{\tau_m}\right)}{\frac{1}{\tau_m}\left[1 + \frac{T_{m1}(2\xi_m\tau_m + T_{m1})\left(4\xi^2(1-\varepsilon)+\varepsilon\right)}{\tau_m^2}\right] - a}$$ with

$$a = \frac{\xi}{\tau_m^2}\sqrt{8(1-\varepsilon)\frac{T_{m1}}{\tau_m}(2\xi_m\tau_m + T_{m1})\left[\left(2\xi^2(1-\varepsilon)+\varepsilon\right)\frac{T_{m1}}{\tau_m}(2\xi_m\tau_m + T_{m1}) + \tau_m\right]},$$

where $\frac{T_{m1}}{K_m^2}(2\xi_m\tau_m + T_{m1}) + \frac{1}{\tau_m^2}\left[\varepsilon + 2\xi^2(1-\varepsilon)^2\left(\frac{T_{m1}}{K_m}\right)^2(2\xi_m\tau_m + T_{m1})^2\right] \ge 0$.

4.5.17 *Alternative controller 4* $U(s) = K_c\left(\dfrac{1+sT_d}{1+\dfrac{sT_d}{N}}\right)E(s) + \dfrac{R(s)}{K_m}$

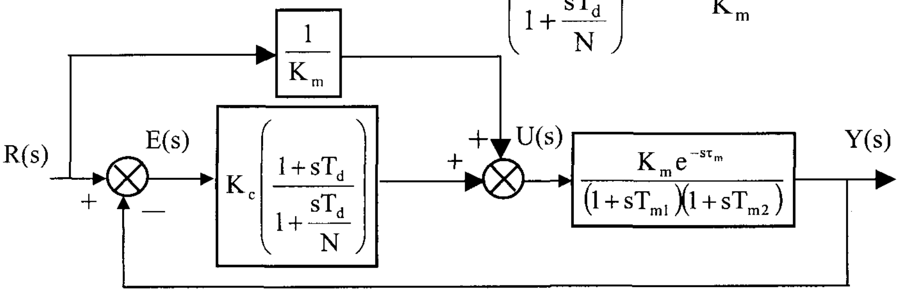

Table 131: PID controller tuning rules – SOSPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_d	Comment
	Direct synthesis: time domain criteria		
Bohl and McAvoy (1976a). *Model: Method 1*	[4] $K_c^{(563)}$	$T_d^{(563)}$	$N = 10$

[4] Response reaches a new steady state in minimum time, to a servo input, with no observable overshoot. Upper and lower limits exist on the manipulated variable.

$\dfrac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5$; $\dfrac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9$.

$$K_c^{(563)} = \frac{4.486}{K_m}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{-1.240+0.1409\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{-0.0252+0.1542\ln\left(10\frac{T_{m2}}{T_{m1}}\right)-0.1616\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right],$$

$$T_d^{(563)} = 0.104T_{m1}\left[\left(10\frac{\tau_m}{T_{m1}}\right)^{0.3664-0.0476\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right]\left[\left(10\frac{T_{m2}}{T_{m1}}\right)^{0.7797-0.0773\ln\left(10\frac{T_{m2}}{T_{m1}}\right)+0.0270\ln\left(10\frac{\tau_m}{T_{m1}}\right)}\right].$$

Rule	K_c	T_d	Comment
	Ultimate cycle		
Bohl and McAvoy (1976a). *Model: Method 1*	[5] $K_c^{(564)}$	$T_d^{(564)}$	N = 10

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[5] $$K_c^{(564)} = \frac{0.2455}{K_m}\left[\left(\frac{\tau_m}{T_u}\right)^{-0.4986-0.7510\ln\left(\frac{\tau_m}{T_u}\right)}\right]\left[\left(K_u K_m\right)^{0.3875-0.2667\ln(K_u K_m)-1.1111\ln\left(\frac{\tau_m}{T_u}\right)}\right],$$

$$T_d^{(564)} = 0.000726T_u\left[\left(\frac{\tau_m}{T_u}\right)^{-6.1523-1.4285\ln\left(\frac{\tau_m}{T_u}\right)}\right]\left[\left(K_u K_m\right)^{-1.4494+0.0402\ln(K_u K_m)-0.4602\ln\left(\frac{\tau_m}{T_u}\right)}\right]$$

4.6 I^2PD Model $G_m(s)=\frac{K_m e^{-s\tau_m}}{s^2}$

4.6.1 *Controller with filtered derivative with set-point weighting 2*

$$G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)\left(\frac{1+b_{f1}s+b_{f2}s^2}{1+a_{f1}s+a_{f2}s^2}R(s)-Y(s)\right)$$

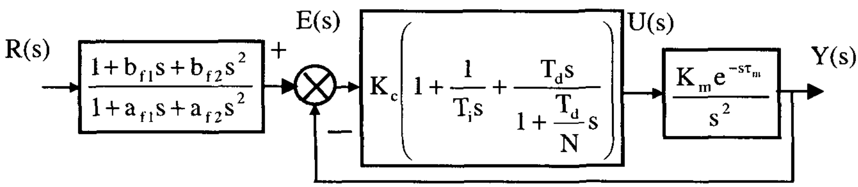

Table 132: PID tuning rules - I^2PD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Liu *et al.* (2003). *Model: Method 1*	[1] $K_c^{(565)}$	$T_i^{(565)}$	$T_d^{(565)}$	

---

[1] $K_c^{(565)}=\frac{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})-\lambda^4}{K_m(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})^2}$,

$T_i^{(565)}=\frac{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})-\lambda^4}{(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})}$,

$T_d^{(565)}=\frac{(6\lambda^2+4\lambda\tau_m+\tau_m^{\ 2})(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})}{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})-\lambda^4}$

$-\frac{\lambda^4}{4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3}}$, $a_{f1}=4\lambda+\tau_m$; $a_{f2}=6\lambda^2+4\lambda\tau_m+\tau_m^{\ 2}$,

$N=\frac{6\lambda^2+4\lambda\tau_m+\tau_m^{\ 2}}{\lambda^3[(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^{\ 2}+\tau_m^{\ 3})-\lambda^4]}-1$, $b_{f1}=2\lambda$, $b_{f2}=\lambda^2$.

Rule	K_c		T_i	T_d		Comment
Liu *et al.* (2003) – continued.	Sample data (deduced from a graph)					
	λ	OS	Rise time	λ	OS	Rise time
	$0.8\tau_m$	$\approx 33\%$	$\approx 4\tau_m$	$2.5\tau_m$	$\approx 0\%$	$\approx 10\tau_m$
	τ_m	$\approx 20\%$	$\approx 5\tau_m$	$3\tau_m$	$\approx 0\%$	$\approx 12.5\tau_m$
	$1.5\tau_m$	$\approx 3\%$	$\approx 7\tau_m$	$3.5\tau_m$	$\approx 0\%$	$\approx 14\tau_m$
	$2\tau_m$	$\approx 0\%$	$\approx 8\tau_m$	$4\tau_m$	$\approx 0\%$	$\approx 16\tau_m$

4.6.2 Controller with filtered derivative with set-point weighting 4

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)\left(\frac{1+b_{f1}s+b_{f2}s^2+b_{f3}s^3+b_{f4}s^4}{1+a_{f1}s+a_{f2}s^2+a_{f3}s^3+a_{f4}s^4}R(s)-Y(s)\right)$$

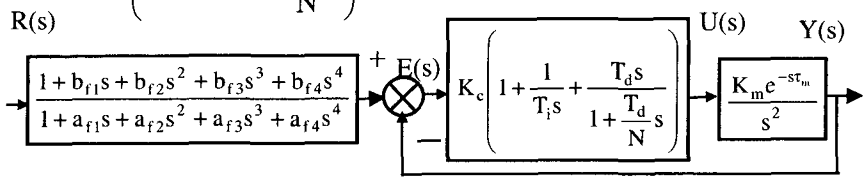

Table 133: PID tuning rules – I^2PD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{s^2}$

<table>
<tr><th>Rule</th><th colspan="2">K_c</th><th>T_i</th><th colspan="2">T_d</th><th>Comment</th></tr>
<tr><td></td><td colspan="6">Robust</td></tr>
<tr><td rowspan="6">Liu et al. (2003).
Model: Method 1</td><td colspan="2">$K_c^{(566)}$ [2]</td><td colspan="2">$T_i^{(566)}$</td><td>$T_d^{(566)}$</td><td></td></tr>
<tr><td colspan="6">Sample data (deduced from a graph)[3]</td></tr>
<tr><td>λ</td><td>OS</td><td>Rise time</td><td>λ</td><td>OS</td><td>Rise time</td></tr>
<tr><td>$0.8\tau_m$</td><td>$\approx 14\%$</td><td>$\approx 3\tau_m$</td><td>$2.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 7\tau_m$</td></tr>
<tr><td>τ_m</td><td>$\approx 4\%$</td><td>$\approx 3.5\tau_m$</td><td>$3\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 8.5\tau_m$</td></tr>
<tr><td>$1.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 4.5\tau_m$</td><td>$3.5\tau_m$</td><td>$\approx 0\%$</td><td>$\approx 10.5\tau_m$</td></tr>
</table>

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[2] $K_c^{(566)}=\frac{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)-\lambda^4}{K_m(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)^2}$,

$T_i^{(566)}=\frac{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)-\lambda^4}{(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)}$,

$T_d^{(566)}=\frac{(6\lambda^2+4\lambda\tau_m+\tau_m^2)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)}{(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)-\lambda^4}$

$-\frac{\lambda^4}{4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3}$, $b_{f1}=4\lambda$, $b_{f2}=6\lambda^2$, $b_{f3}=4\lambda^3$, $b_{f4}=\lambda^4$,

$N=\frac{6\lambda^2+4\lambda\tau_m+\tau_m^2}{\lambda^3[(4\lambda+\tau_m)(4\lambda^3+6\lambda^2\tau_m+4\lambda\tau_m^2+\tau_m^3)-\lambda^4]}-1$.

[3] $a_{f1}=8\lambda+\tau_m$, $a_{f2}=26\lambda^2+8\lambda\tau_m+\tau_m^2$, $a_{f3}=4\lambda(10\lambda^2+20\lambda\tau_m+4\tau_m^2)$,
$a_{f4}=4\lambda^2(6\lambda^2+4\lambda\tau_m+\tau_m^2)$.

Rule	K_c		T_i	T_d	Comment
Liu *et al.* (2003) – continued.	Sample data (deduced from a graph) - continued					
	$2\tau_m$	$\approx 0\%$	$\approx 6\tau_m$	$4\tau_m$	$\approx 0\%$	$\approx 13\tau_m$
	Sample data (deduced from a graph)[4]					
	λ	OS	Rise time	λ	OS	Rise time
	$0.8\tau_m$	$\approx 46\%$	$\approx 6\tau_m$	$2.5\tau_m$	$\approx 4\%$	$\approx 20.5\tau_m$
	τ_m	$\approx 34\%$	$\approx 13.5\tau_m$	$3\tau_m$	$\approx 2\%$	$\approx 24\tau_m$
	$1.5\tau_m$	$\approx 16\%$	$\approx 13\tau_m$	$3.5\tau_m$	$\approx 1\%$	$\approx 28\tau_m$
	$2\tau_m$	$\approx 8\%$	$\approx 16.5\tau_m$	$4\tau_m$	$\approx 0\%$	$\approx 32\tau_m$

[4] $a_{f1} = 5\lambda + \tau_m$, $a_{f2} = 10.25\lambda^2 + 5\lambda\tau_m + {\tau_m}^2$, $a_{f3} = \lambda\left(7\lambda^2 + 4.25\lambda\tau_m + {\tau_m}^2\right)$, $a_{f4} = 0.25\lambda^2\left(6\lambda^2 + 4\lambda\tau_m + {\tau_m}^2\right)$.

4.6.3 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$$

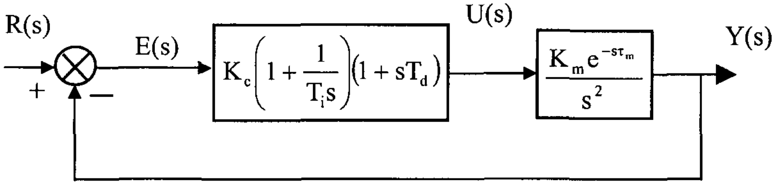

Table 134: PID tuning rules – I^2PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Skogestad (2003). *Model: Method 1*	$\frac{0.0625}{K_m \tau_m^2}$	$8\tau_m$	$8\tau_m$	

4.6.4 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$

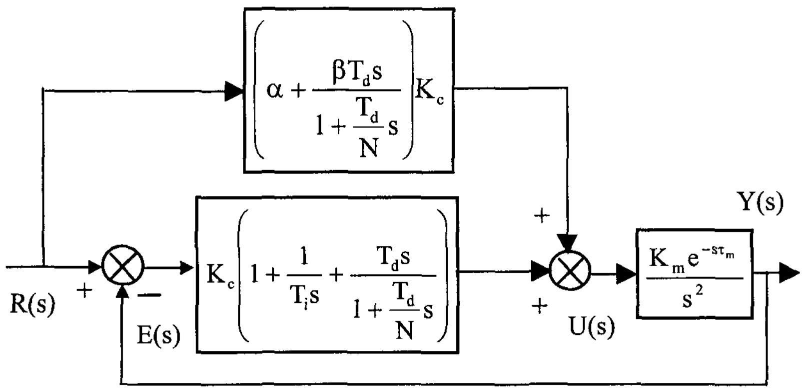

Table 135: PID tuning rules – I^2PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Hansen (2000). *Model: Method 1*	$3.75 K_m \tau_m{}^2$	$5.4\tau_m$	$2.5\tau_m$	$N = 0$; $\beta = 1$; $\alpha = 0.833$

4.6.5 *Industrial controller* $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$

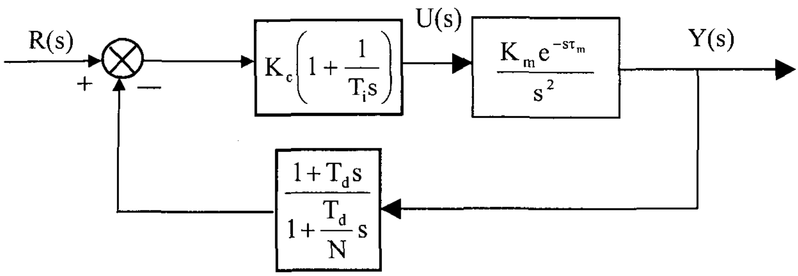

Table 136: PID tuning rules – I^2PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Skogestad (2004b). *Model: Method 1*	$\frac{0.25}{K_m(T_{CL}+\tau_m)^2}$	$4\xi^2(T_{CL}+\tau_m)$	$4(T_{CL}+\tau_m)$	'good' robustness - $T_{CL}=\tau_m$; ξ=0.7 or 1
	$N \in [5,10]$; N = 2 if measurement noise is a serious problem			

4.7 SOSIPD Model (repeated pole) $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

4.7.1 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$

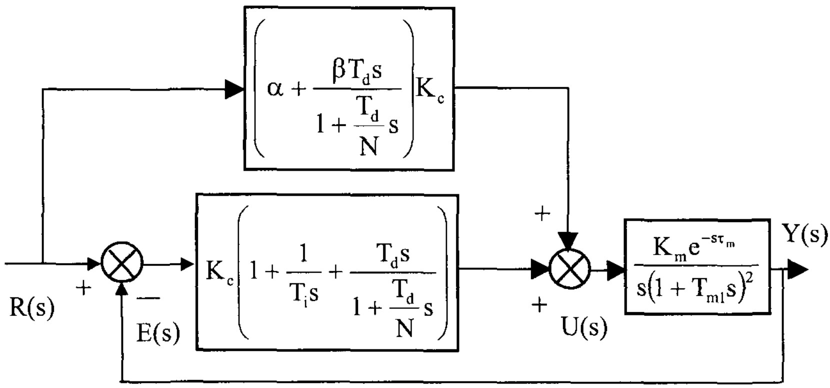

Table 137: PID controller tuning rules – SOSIPD model (repeated pole) $\dfrac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

Rule	K_c	T_i	T_d	Comment	
	Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000). *Model: Method 2* $K_m = 1$; $T_m = 1$; $0.1 < \tau_m < 10$; $\beta = 1$, $N = 10$.	$x_1 K_u$	$x_2 T_u$	$x_3 T_u$		
	Coefficient values				
	x_1	x_2	x_3	α	ϕ_c
	1.672	0.366	0.136	0.601	-164^0
	1.236	0.427	0.149	0.607	-160^0
	0.994	0.486	0.155	0.610	-155^0
	0.842	0.538	0.154	0.616	-150^0
	0.752	0.567	0.157	0.605	-145^0

Rule	K_c	T_i	T_d	Comment
Pecharromán and Pagola (2000) – continued.	Coefficient values (continued)			
	x_1	x_2	x_3	α	ϕ_c
	0.679	0.610	0.149	0.610	-140^0
	0.635	0.637	0.142	0.612	-135^0
	0.590	0.669	0.133	0.610	-130^0
	0.551	0.690	0.114	0.616	-125^0
	0.520	0.776	0.087	0.609	-120^0
	0.509	0.810	0.068	0.611	-118^0
Taguchi and Araki (2000). *Model: Method 1*	[1] $K_c^{(567)}$	$T_i^{(567)}$	$T_d^{(567)}$	$\frac{\tau_m}{T_{m1}} \le 1.0$; Overshoot (servo step) $\le 20\%$; N fixed but not specified

[1] $$K_c^{(567)} = \frac{1}{K_m}\left(0.1778 + \frac{0.5667}{\frac{\tau_m}{T_{m1}} + 0.002325}\right),$$

$$T_i^{(567)} = T_{m1}\left(0.2011 + 11.16\frac{\tau_m}{T_{m1}} - 14.98\left[\frac{\tau_m}{T_{m1}}\right]^2 + 13.70\left[\frac{\tau_m}{T_{m1}}\right]^3 - 4.835\left[\frac{\tau_m}{T_{m1}}\right]^4\right),$$

$$T_d^{(567)} = T_{m1}\left(1.262 + 0.3620\frac{\tau_m}{T_{m1}}\right),\ \alpha = 0.6666,$$

$$\beta = 0.8206 - 0.09750\frac{\tau_m}{T_{m1}} + 0.03845\left[\frac{\tau_m}{T_{m1}}\right]^2.$$

4.8 SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1} \text{ or } G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

4.8.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

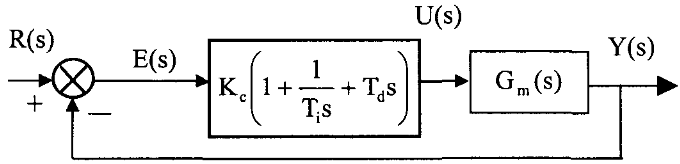

Table 138: PID controller tuning rules – SOSPD model with a positive zero $G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$ or $\frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Minimum IAE – Wang *et al.* (1995a). *Model: Method 1*	[1] $\frac{1}{K_m}\frac{p_0 q_1 - p_1}{p_0^2} r$	$q_1 - \frac{p_1}{p_0}$	$\frac{p_0 q_2 - p_2}{p_0 q_1 - p_1} - \frac{p_1}{p_0}$	

[1] $p_0 = T_{m1} + T_{m2} + T_{m3} + \tau_m$, $p_1 = T_{m1}T_{m2} + 0.5\tau_m(T_{m1} + T_{m2}) - 0.5\tau_m T_{m3}$,
$p_2 = 0.5T_{m1}T_{m2}\tau_m$, $q_1 = T_{m1} + T_{m2} + 0.5\tau_m$, $q_2 = T_{m1}T_{m2} + 0.5\tau_m(T_{m1} + T_{m2})$,

$$r = \delta_0 + \delta_1\frac{\tau_m}{T_{m1}} + \delta_2\left(\frac{T_{m2}}{T_{m1}}\right) + \delta_3\left(\frac{T_{m3}}{T_{m1}}\right) + \left[\delta_4\left(\frac{\tau_m}{T_{m1}}\right) + \delta_7\left(\frac{T_{m2}}{T_{m1}}\right) + \delta_9\left(\frac{T_{m3}}{T_{m1}}\right)\right]\frac{\tau_m}{T_{m1}}$$

$$+\left[\delta_7\left(\frac{\tau_m}{T_{m1}}\right) + \delta_5\left(\frac{T_{m2}}{T_{m1}}\right) + \delta_8\left(\frac{T_{m3}}{T_{m1}}\right)\right]\frac{T_{m2}}{T_{m1}} + \left[\delta_9\left(\frac{\tau_m}{T_{m1}}\right) + \delta_8\left(\frac{T_{m2}}{T_{m1}}\right) + \delta_6\left(\frac{T_{m3}}{T_{m1}}\right)\right]\frac{T_{m3}}{T_{m1}};$$

$\delta_0 = 3.5550$, $\delta_1 = -3.6167$, $\delta_2 = 2.1781$, $\delta_3 = -5.5203$, $\delta_4 = 1.4704$,
$\delta_5 = -0.4918$, $\delta_6 = 2.5356$, $\delta_7 = -0.4685$, $\delta_8 = -0.3318$, $\delta_9 = 1.4746$.

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Wang *et al.* (1995a). *Model: Method 1*	$\frac{1}{K_m}\frac{p_0q_1-p_1}{{p_0}^2}r$ [2]	$q_1-\frac{p_1}{p_0}$	$\frac{p_0q_2-p_2}{p_0q_1-p_1}-\frac{p_1}{p_0}$	
Minimum ITAE – Wang *et al.* (1995a). *Model: Method 1*	$\frac{1}{K_m}\frac{p_0q_1-p_1}{{p_0}^2}r$ [3]	$q_1-\frac{p_1}{p_0}$	$\frac{p_0q_2-p_2}{p_0q_1-p_1}-\frac{p_1}{p_0}$	
	Direct synthesis			
Wang *et al.* (2000b), (2001a). *Model: Method 2*	[4] $K_c^{(568)}$	$2\xi_m T_{m1}$	${T_{m1}}^2$	$M_s = 1.5$

[2] $p_0, p_1, p_2, q_1, q_2, r$ defined as on page 383.
$\delta_0 = 3.9395$, $\delta_1 = -3.2164$, $\delta_2 = 1.6185$, $\delta_3 = -5.8240$, $\delta_4 = 1.0933$,
$\delta_5 = -0.6679$, $\delta_6 = 2.5648$, $\delta_7 = -0.2383$, $\delta_8 = -0.0564$, $\delta_9 = 1.3508$.

[3] $p_0, p_1, p_2, q_1, q_2, r$ defined as on page 383.
$\delta_0 = 3.2950$, $\delta_1 = -3.4779$, $\delta_2 = 2.5336$, $\delta_3 = -5.5929$, $\delta_4 = 1.4407$,
$\delta_5 = -0.3268$, $\delta_6 = 2.7129$, $\delta_7 = -0.5712$, $\delta_8 = -0.5790$, $\delta_9 = 1.5340$.

[4] $K_c^{(568)} = \frac{1}{M_s}\frac{\cos\theta}{\cos\left[\omega\tau_m - \tan^{-1}(-T_{m3}\omega)\right]}\frac{2\xi_m T_{m1}\omega}{K_m\sqrt{{T_{m3}}^2\omega^2+1}}$;

ω and θ are determined by solving the equations:

$\tan^{-1}\left(\frac{\cos\theta}{M_s - \sin\theta}\right) = \tan^{-1}(-T_{m3}\omega) - \omega\tau_m - 0.5\pi$ and

$$\theta = \tan^{-1}\left[\frac{\left(-T_{m3}\tau_m\omega^2 - 1\right)\cos(\omega\tau_m) - \omega\tau_m\sin(\omega\tau_m)}{\left(-T_{m3}\tau_m\omega^2 - 1\right)\sin(\omega\tau_m) + \omega\tau_m\sin(\omega\tau_m)}\right].$$

4.8.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

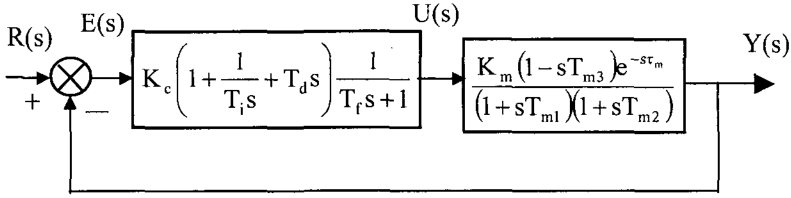

Table 139: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Li *et al.* (2004). *Model: Method 1*	[5] $K_c^{(569)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	

[5] $K_c^{(569)} = \frac{T_{m1} + T_{m2}}{K_m(2T_{CL2} + T_{m3} + \tau_m)}$, $T_f = \frac{T_{CL2}^2 - T_{m3}\tau_m}{2T_{CL2} + T_{m3} + \tau_m}$;

desired closed loop transfer function $= -\frac{(1 - sT_{m3})e^{-s\tau_m}}{(T_{CL2}s + 1)^2}$.

4.8.3 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$

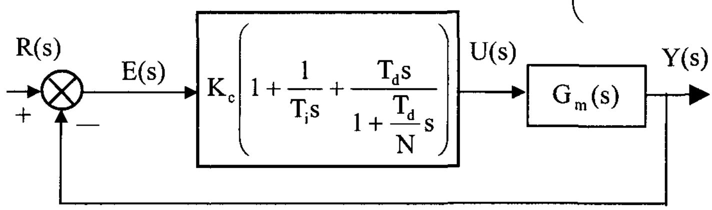

Table 140: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	[6] $K_c^{(570)}$	$T_i^{(570)}$	$T_d^{(570)}$	
	[7] $K_c^{(571)}$	$T_i^{(571)}$	$T_d^{(571)}$	
	$T_{m1} > T_{m2} > T_{m3}$; N=10; $\lambda = [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).			

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[6] $K_c^{(570)} = \frac{T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}{K_m(\lambda + T_{m3} + \tau_m)}$, $T_i^{(570)} = T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}$,

$T_d^{(570)} = \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} + \frac{T_{m1}T_{m2}}{T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}$.

[7] $K_c^{(571)} = \frac{2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}{K_m(\lambda + T_{m3} + \tau_m)}$, $T_i^{(571)} = 2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}$,

$T_d^{(571)} = \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} + \frac{T_{m1}^2}{2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}$.

4.8.4 *Classical controller 1* $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

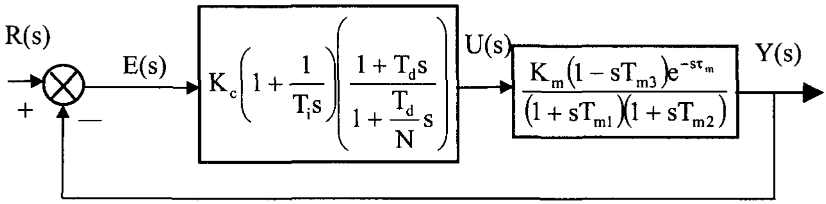

Table 141: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s)=\frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Poulin *et al.* (1996). *Model: Method 1*	[8] $K_c^{(572)}$	T_{m1}	T_{m2}	Minimum $\phi_m=65^0$
O'Dwyer (2001a). *Model: Method 1*	$\frac{x_1 T_{m1}}{K_m\tau_m}$	T_{m1}	T_{m2}	$A_m=0.5\pi/x_1$; $\phi_m=0.5\pi-x_1$; $N=T_{m2}/T_{m3}$
	Sample result			
	$\frac{0.785T_{m1}}{K_m\tau_m}$	T_{m1}	T_{m2}	$A_m=2.0$; $\phi_m=45^0$
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_{m2}}{K_m(\lambda+\tau_m)}$	T_{m2}	T_{m1}	N=10; $\lambda\in[T_{m1},\tau_m]$ (Chien and Fruehauf (1990))
	$\frac{T_{m1}}{K_m(\lambda+\tau_m)}$	T_{m1}	T_{m2}	
	$T_{m1}>T_{m2}>T_{m3}$			

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[8] $K_c^{(572)}=\frac{T_{m1}}{K_m(T_{m1}+2T_{m3}+\tau_m)}$, $N=\frac{T_{m2}(T_{m1}+2T_{m3})}{T_{m1}T_{m3}}$,

$\phi_m=56^0$ when $T_{m3}=T_{m1}=\tau_m$.

4.8.5 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+T_d s)$$

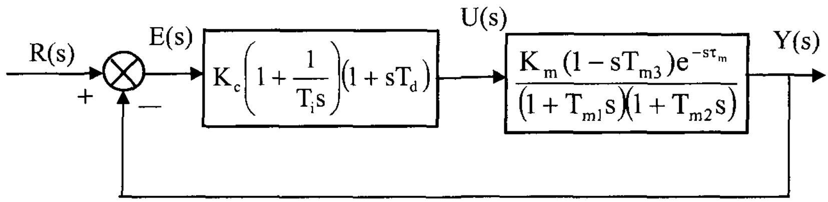

Table 142: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Zhang (1994). *Model: Method 1*	$\frac{T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	"Good" servo response; τ_m is "small".

4.8.6 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$

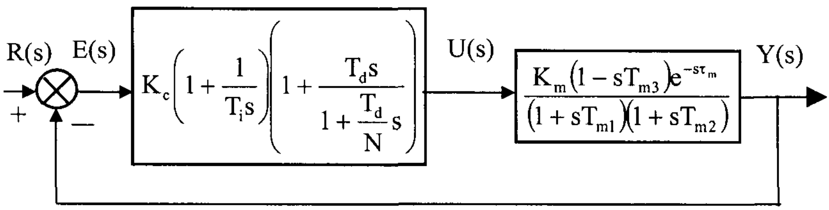

Table 143: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_{m2}}{K_m(\lambda+\tau_m)}$	T_{m2}	[9] $T_d^{(573)}$	
	$\frac{T_{m1}}{K_m(\lambda+\tau_m)}$	T_{m1}	[10] $T_d^{(574)}$	
	N=10; $\lambda \in [T, \tau_m]$, T = dominant time constant (Chien and Fruehauf (1990)).			

[9] $T_d^{(573)} = T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}$.

[10] $T_d^{(574)} = T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}$.

4.8.7 *Non-interacting controller 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$$

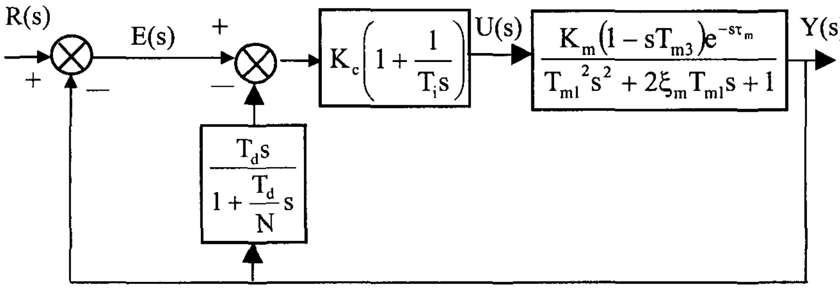

Table 144: PID controller tuning rules – SOSPD model with a positive zero $G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000). *Model: Method 1* N not specified	$K_c^{(575)}$ [11]	$2T_{m1}\xi_m + 0.4\tau_m$	$T_d^{(575)}$	
	$\lambda = 0.5, \frac{\tau_m}{T_{m1}}\xi_m \le 1$; $\lambda = 0.6$, $1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$; $\lambda = 0.7, \frac{\tau_m}{T_{m1}}\xi_m > 2$, servo tuning			
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \le 1$; $\lambda = 0.7$, $1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$; $\lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2$, regulator tuning			

[11] $K_c^{(575)} = \lambda\frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}$, $T_d^{(575)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}$.

4.8.8 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_d s/N}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + T_d s/N}\right)R(s)$$

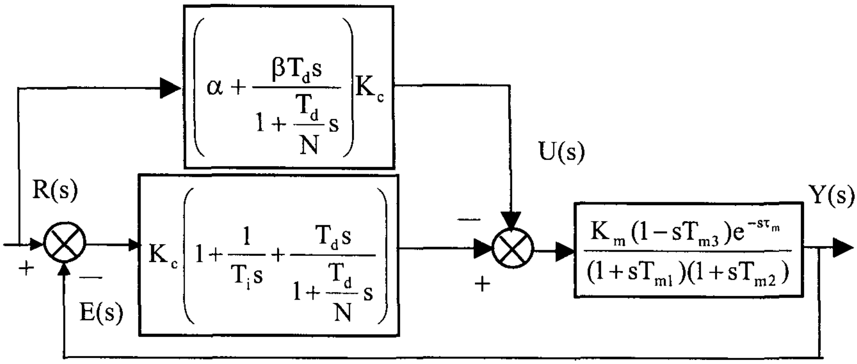

Table 145: PID controller tuning rules – SOSPD model with a positive zero $G_m(s) = \frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000). *Model: Method 1* N = min[20, $20T_d$]	$K_c^{(576)}$ [12]	$T_i^{(576)}$	$T_d^{(576)}$	Servo tuning
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \le 1$, $A_m = 2.83$; $\lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$, $A_m = 2.43$; $\lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2$, $A_m = 2.13$; $\alpha = \frac{2T_{m1}\xi_m}{2T_{m1}\xi_m + 0.4\tau_m} - 1$; $\beta = \frac{0.5T_{m1}^2}{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m} - 1$			

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[12] $K_c^{(576)} = \lambda\frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}$, $T_i^{(576)} = 2T_{m1}\xi_m + 0.4\tau_m$,

$$T_d^{(576)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.9 SOSPD Model with a Negative Zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1} \text{ or } G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

4.9.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

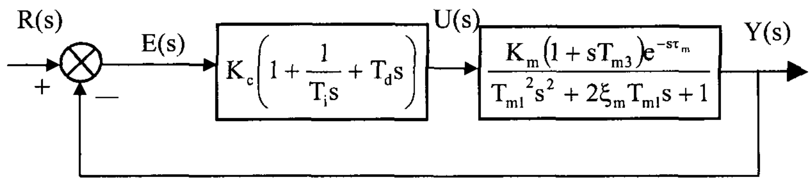

Table 146: PID controller tuning rules – SOSPD model with a negative zero $G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Wang *et al.* (2000b), (2001a). *Model: Method 2*	$K_c^{(577)}$ [1]	$2\xi_m T_{m1}$	T_{m1}^2	$M_s = 1.5$

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[1] $K_c^{(577)} = \frac{1}{M_s} \frac{\cos\theta}{\cos\left[\omega\tau_m - \tan^{-1}(T_{m3}\omega)\right]} \frac{2\xi_m T_{m1}\omega}{K_m\sqrt{T_{m3}^2\omega^2 + 1}}$; ω and θ are determined by solving the equations:

$$\tan^{-1}\left(\frac{\cos\theta}{M_s - \sin\theta}\right) = \tan^{-1}(T_{m3}\omega) - \omega\tau_m - 0.5\pi \text{ and}$$

$$\theta = \tan^{-1}\left[\frac{(T_{m3}\tau_m\omega^2 - 1)\cos(\omega\tau_m) - \omega\tau_m \sin(\omega\tau_m)}{(T_{m3}\tau_m\omega^2 - 1)\sin(\omega\tau_m) + \omega\tau_m \sin(\omega\tau_m)}\right].$$

4.9.2 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$

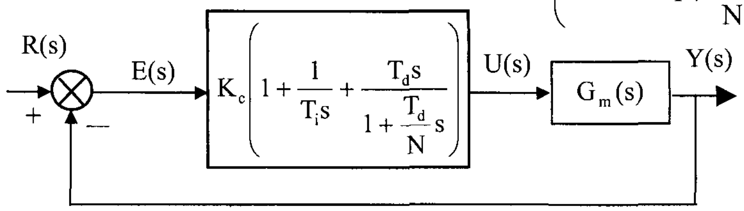

Table 147: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1 + sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1 + sT_{m3})e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	[2] $K_c{}^{(578)}$	$T_i{}^{(578)}$	$T_d{}^{(578)}$	N=10; $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).
	$\frac{2\xi_m T_{m1} - T_{m3}}{K_m(\lambda + \tau_m)}$	$2\xi_m T_{m1} - T_{m3}$	[3] $T_d{}^{(579)}$	

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[2] $K_c{}^{(578)} = \frac{T_{m1} + T_{m2} - T_{m3}}{K_m(\lambda + \tau_m)}$, $T_i{}^{(578)} = T_{m1} + T_{m2} - T_{m3}$,

$$T_d{}^{(578)} = \frac{T_{m1}T_{m2} - (T_{m1} + T_{m2} - T_{m3})T_{m3}}{T_{m1} + T_{m2} - T_{m3}}, \; T_{m1} > T_{m2} > T_{m3}.$$

[3] $T_d{}^{(579)} = \frac{T_{m1}^2 - (2\xi_m T_{m1} - T_{m3})T_{m3}}{2\xi_m T_{m1} - T_{m3}}$.

4.9.3 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

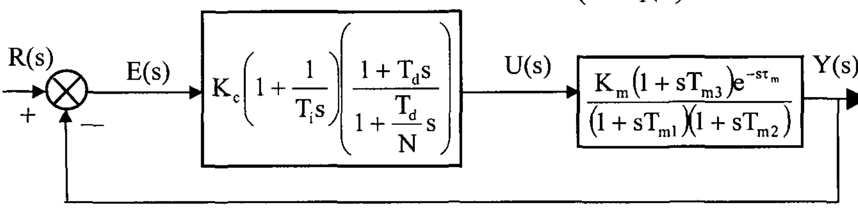

Table 148: PID controller tuning rules – SOSPD model with a negative zero $G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Zhang (1994). *Model: Method 1*	$\frac{T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$N = T_{m2}/T_{m3}$
	"small" τ_m; "good" servo response.			
Poulin *et al.* (1996). *Model: Method 1*	$\frac{T_{m1}}{K_m(T_{m1}+\tau_m)}$	T_{m1}	T_{m2}	$N = T_{m2}/T_{m3}$; Minimum $\phi_m = 90^0$; $\phi_m = 60^0$, $\tau_m = T_{m1}$
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_{m2}}{K_m(\lambda+\tau_m)}$	T_{m2}	T_{m1}	
	$\frac{T_{m1}}{K_m(\lambda+\tau_m)}$	T_{m1}	T_{m2}	
	$T_{m1} > T_{m2}$; N=10; $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).			

4.9.4 *Classical controller 4* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$

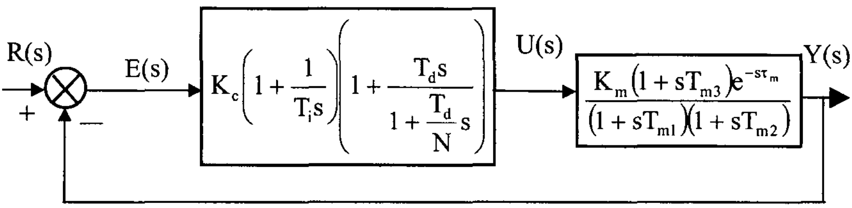

Table 149: PID controller tuning rules – SOSPD model with a negative zero $G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Chien (1988). *Model: Method 1*	$\frac{T_{m2}}{K_m(\lambda+\tau_m)}$	T_{m2}	$T_{m1} - T_{m3}$	
	$\frac{T_{m1}}{K_m(\lambda+\tau_m)}$	T_{m1}	$T_{m2} - T_{m3}$	
	$T_{m1} > T_{m2} > T_{m3}$; N=10; $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).			

4.9.5 *Non-interacting controller 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}} Y(s)\right)$$

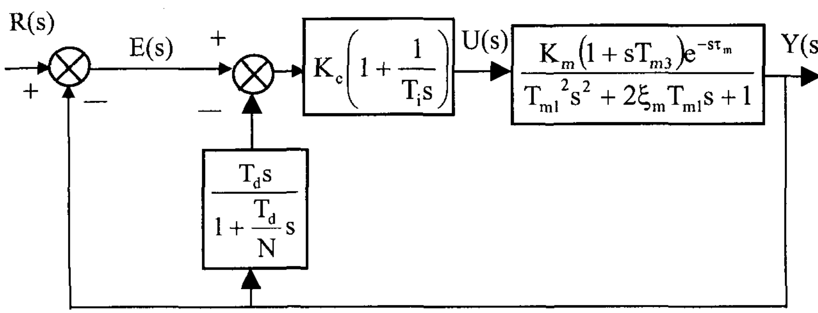

Table 150: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000). *Model: Method 1* N not specified	${}^4 K_c^{(580)}$	$2T_{m1}\xi_m + 0.4\tau_m$	$T_d^{(580)}$	
	$\lambda = 0.5, \frac{\tau_m}{T_{m1}}\xi_m \le 1$; $\lambda = 0.6, 1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$; $\lambda = 0.7, \frac{\tau_m}{T_{m1}}\xi_m > 2$; servo tuning			
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \le 1$; $\lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}}\xi_m \le 2$; $\lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2$; regulator tuning			

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4 $K_c^{(580)} = \lambda\frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}$, $T_d^{(580)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}$.

4.9.6 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$

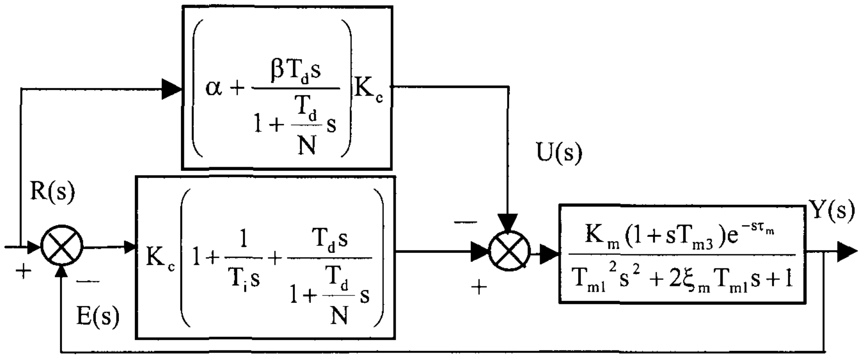

Table 151: PID controller tuning rules – SOSPD model with a negative zero $G_m(s)=\dfrac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2+2\xi_m T_{m1}s+1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Huang *et al.* (2000) - servo tuning. *Model: Method 1*	$K_c^{(581)}$ [5]	$2T_{m1}\xi_m+0.4\tau_m$	$T_d^{(581)}$	$N=\min[20, 20T_d]$
	$\alpha=\dfrac{2T_{m1}\xi_m}{2T_{m1}\xi_m+0.4\tau_m}-1$; $\beta=\dfrac{0.5T_{m1}^2}{T_{m1}^2+0.8T_{m1}\xi_m\tau_m}-1$; $\lambda=0.6, \frac{\tau_m}{T_{m1}}\xi_m\le 1$, $A_m=2.83$; $\lambda=0.7, 1<\frac{\tau_m}{T_{m1}}\xi_m\le 2$, $A_m=2.43$; $\lambda=0.8, \frac{\tau_m}{T_{m1}}\xi_m>2$, $A_m=2.13$.			

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[5] $K_c^{(581)}=\lambda\dfrac{2T_{m1}\xi_m+0.4\tau_m}{K_m(\tau_m+2T_{m3})}$, $T_d^{(581)}=\dfrac{T_{m1}^2+0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}$.

4.10 Third Order System plus Time Delay Model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3} \text{ or } G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

4.10.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

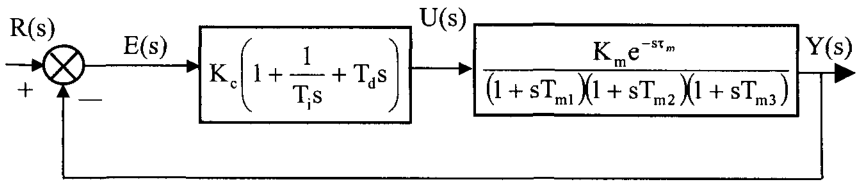

Table 152: PID controller tuning rules – third order system plus time delay model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index			
Standard form optimisation - binomial - Polonyi (1989).	$K_c^{(582)}$ [1]	$T_i^{(582)}$	τ_m	*Model: Method 1*; $T_{m1} > 10(T_{m2} + \tau_m)$
Standard form optimisation – minimum ITAE - Polonyi (1989).	$K_c^{(583)}$ [2]	$T_i^{(583)}$	τ_m	

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[1] $K_c^{(582)} = \left(1 - 4\sqrt{\frac{T_{m2}}{6\tau_m} + \frac{T_{m2}}{T_{m3}}}\right)\frac{T_{m1}}{\tau_m}$, $T_i^{(582)} = 4\sqrt{6T_{m2}\tau_m} + \tau_m$.

[2] $K_c^{(583)} = \left(1 - 2.1\sqrt{\frac{T_{m2}}{3.4\tau_m} + \frac{T_{m2}}{T_{m3}}}\right)\frac{T_{m1}}{\tau_m}$, $T_i^{(583)} = 2.7\sqrt{3.4T_{m2}\tau_m} + \tau_m$.

4.10.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

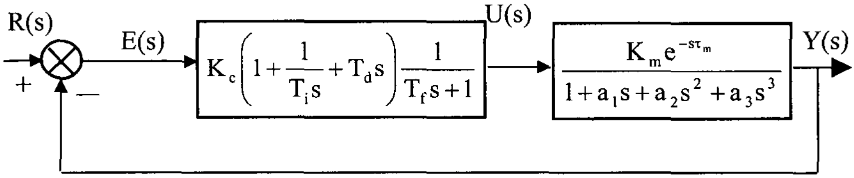

Table 153: PID controller tuning rules –Third order system plus time delay model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Schaedel (1997). *Model: Method 1*	[3] $K_c^{(584)}$	$T_i^{(584)}$	$T_d^{(584)}$	$T_f = T_d^{(584)}/N$; $5 \le N \le 20$

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[3] $K_c^{(584)} = \dfrac{0.375 T_i^{(584)}}{K_m\left(a_1 + \tau_m + \dfrac{T_d^{(584)}}{N} - T_i^{(584)}\right)}$, $T_i^{(584)} = \dfrac{a_1^2 - a_2 + a_1\tau_m + 0.5\tau_m^2}{a_1 + \tau_m - T_d^{(584)}}$,

$T_d^{(584)} = \dfrac{a_2 + a_1\tau_m + 0.5\tau_m^2}{a_1 + \tau_m} - \dfrac{a_3 + a_2\tau_m + 0.5a_1\tau_m^2 + 0.167\tau_m^3}{a_2 + a_1\tau_m + 0.5\tau_m^2}$.

4.10.3 *Controller with filtered derivative* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right)$

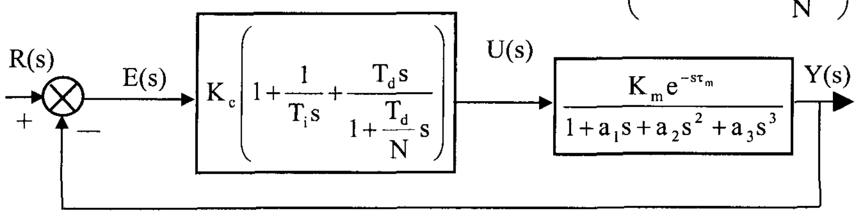

Table 154: PID controller tuning rules – Third order system plus time delay model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Schaedel (1997).	[4] $K_c^{(585)}$	$T_i^{(585)}$	$T_d^{(585)}$	*Model: Method 1*

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[4] $K_c^{(585)} = \frac{0.375 T_i^{(585)}}{K_m\left(a_1 + \tau_m + \frac{T_d^{(585)}}{N} - T_i^{(585)}\right)}$, $T_i^{(585)} = \frac{a_1^2 - a_2 + a_1\tau_m + 0.5\tau_m^2}{a_1 + \tau_m - T_d^{(585)}}$,

$$T_d^{(585)} = \frac{a_2 + a_1\tau_m + 0.5\tau_m^2}{a_1 + \tau_m} - \frac{a_3 + a_2\tau_m + 0.5 a_1\tau_m^2 + 0.167\tau_m^3}{a_2 + a_1\tau_m + 0.5\tau_m^2},$$

$$N = \frac{T_d^{(585)}}{-\left(\frac{a + \tau_m - T_i^{(585)}}{2}\right) + \sqrt{\frac{\left(a + \tau_m - T_i^{(585)}\right)^2}{4} + 0.375\frac{T_i^{(585)} T_d^{(585)}}{K_v}}},$$

$0.05 \le a \le 0.2$, K_v = prescribed overshoot in the manipulated variable for a step response in the command variable.

4.10.4 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$

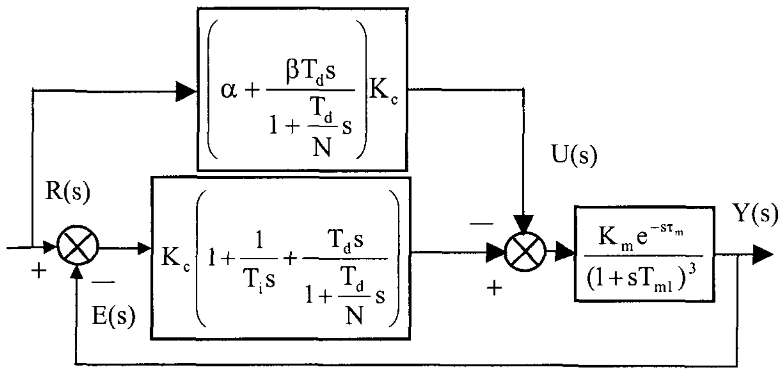

Table 155: PID controller tuning rules – third order system plus time delay model – $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})^3}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo/regulator tuning			
Taguchi and Araki (2000). *Model: Method 1*	[5] $K_c^{(586)}$	$T_i^{(586)}$	$T_d^{(586)}$	Overshoot (servo step) $\le 20\%$; N fixed but not specified

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[5] Equations continued into the footnote on page 402; $\tau_m/T_{m1} \le 1.0$.

$$K_c^{(586)} = \frac{1}{K_m}\left(0.4020+\frac{1.275}{\frac{\tau_m}{T_{m1}}+0.003273}\right),$$

$$T_i^{(586)} = T_{m1}\left(0.3572+7.467\frac{\tau_m}{T_{m1}}-12.86\left[\frac{\tau_m}{T_{m1}}\right]^2+11.77\left[\frac{\tau_m}{T_{m1}}\right]^3-4.146\left[\frac{\tau_m}{T_{m1}}\right]^4\right),$$

$$T_d^{(586)} = T_{m1}\left(0.8335 + 0.2910\frac{\tau_m}{T_{m1}} - 0.04000\left[\frac{\tau_m}{T_{m1}}\right]^2\right),$$

$$\alpha = 0.6661 - 0.2509\frac{\tau_m}{T_{m1}} + 0.04773\left[\frac{\tau_m}{T_{m1}}\right]^2, \ \beta = 0.8131 - 0.2303\frac{\tau_m}{T_{m1}} + 0.03621\left[\frac{\tau_m}{T_{m1}}\right]^2.$$

4.11 Unstable FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

4.11.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

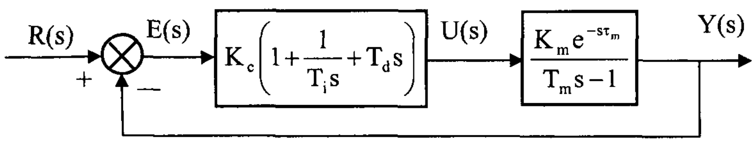

Table 156: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Minimum ISE – Visioli (2001).	$\frac{1.37}{K_m}\left(\frac{\tau_m}{T_m}\right)$	$2.42T_m\left(\frac{\tau_m}{T_m}\right)^{1.18}$	$0.60\tau_m$	*Model: Method 1*
Minimum ITSE – Visioli (2001).	$\frac{1.37}{K_m}\left(\frac{\tau_m}{T_m}\right)$	$4.12T_m\left(\frac{\tau_m}{T_m}\right)^{0.90}$	$0.55\tau_m$	*Model: Method 1*
Minimum ISTSE - Visioli (2001).	$\frac{1.70}{K_m}\left(\frac{\tau_m}{T_m}\right)$	$4.52T_m\left(\frac{\tau_m}{T_m}\right)^{1.13}$	$0.50\tau_m$	*Model: Method 1*
	Minimum performance index: servo tuning			
Minimum ISE – Visioli (2001).	$\frac{1.32}{K_m}\left(\frac{\tau_m}{T_m}\right)^{0.92}$	$4.00T_m\left(\frac{\tau_m}{T_m}\right)^{0.47}$	[1] $T_d^{(587)}$	*Model: Method 1*
Minimum ITSE – Visioli (2001).	$\frac{1.38}{K_m}\left(\frac{\tau_m}{T_m}\right)^{0.90}$	$4.12T_m\left(\frac{\tau_m}{T_m}\right)^{0.90}$	[2] $T_d^{(588)}$	*Model: Method 1*

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[1] $T_d^{(587)} = 3.78T_m\left[1 - 0.84\left(\frac{T_m}{\tau_m}\right)^{0.02}\right]\left(\frac{\tau_m}{T_m}\right)^{0.95}$.

[2] $T_d^{(588)} = 3.62T_m\left[1 - 0.85\left(\frac{T_m}{\tau_m}\right)^{0.02}\right]\left(\frac{\tau_m}{T_m}\right)^{0.93}$.

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Visioli (2001).	$\frac{1.35}{K_m}\left(\frac{\tau_m}{T_m}\right)^{0.95}$	$4.52T_m\left(\frac{\tau_m}{T_m}\right)^{1.13}$	[3] $T_d^{(589)}$	*Model: Method 1*
	Minimum performance index: servo/regulator tuning			
Jhunjhunwala and Chidambaram (2001).	[4] $K_c^{(590)}$	$T_i^{(590)}$	$T_d^{(590)}$	*Model: Method 1*
	Direct synthesis			
De Paor and O'Malley (1989). *Model: Method 1*	[5] $K_c^{(591)}$	$T_i^{(591)}$	$T_d^{(591)}$	$\left\lvert\frac{\tau_m}{T_m}\right\rvert < 1$
Chidambaram (1995b). *Model: Method 1*	[6] $K_c^{(592)}$	$T_m\left(25 - 27\frac{\tau_m}{T_m}\right)$	$0.46\tau_m$	$\frac{\tau_m}{T_m} < 0.6$

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[3] $$T_d^{(589)} = 3.70T_m\left[1 - 0.86\left(\frac{T_m}{\tau_m}\right)^{0.02}\right]\left(\frac{\tau_m}{T_m}\right)^{0.97}.$$

[4] $$K_c^{(590)} = \frac{1}{K_m}\left(13.0 - 39.712\frac{\tau_m}{T_m}\right),\ \frac{\tau_m}{T_m} < 0.2;$$

$$K_c^{(590)} = \frac{1.397}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.769},\ 0.2 \le \frac{\tau_m}{T_m} \le 1.4;$$

$$T_i^{(590)} = 0.856T_m e^{\frac{2.044\tau_m}{T_m}},\ 0 \le \frac{\tau_m}{T_m} \le 1;\ T_i^{(590)} = 0.3444T_m e^{\frac{2.9595\tau_m}{T_m}},\ 1 \le \frac{\tau_m}{T_m} \le 1.4;$$

$$T_d^{(590)} = T_m\left(0.5643\frac{\tau_m}{T_m} + 0.0075\right),\ 0 \le \frac{\tau_m}{T_m} \le 1.4.$$

[5] $$K_c^{(591)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}}\sin\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right],$$

$$T_i^{(591)} = \frac{T_m}{\left[\sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}}\right]\tan(0.75\phi)},\ T_d^{(591)} = T_m\sqrt{\frac{\tau_m/T_m}{1 - \tau_m/T_m}}\tan(0.75\phi),$$

$$\phi = \tan^{-1}\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{T_m}{\tau_m}} - \sqrt{\frac{\tau_m}{T_m}\left(1 - \frac{\tau_m}{T_m}\right)}.$$

[6] $$K_c^{(592)} = \frac{1}{K_m}\left(1.3 + 0.3\frac{T_m}{\tau_m}\right).$$

Rule	K_c	T_i	T_d	Comment
Chidambaram (1997). *Model: Method 1*	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)$	[7] $T_i^{(593)}$	$\frac{T_i^{(593)}}{\alpha}$	
Valentine and Chidambaram (1997b) - dominant pole placement. *Model: Method 1*	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.245}$	[8] $T_i^{(594)}$	$T_d^{(594)}$	$\left\lvert\frac{\tau_m}{T_m}\right\rvert < 0.6$
		[9] $T_i^{(595)}$		$0.6 \le \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 0.8$
		[10] $T_i^{(596)}$		$0.8 < \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 1$

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[7] $T_i^{(593)} = \left(0.176 + 0.360\frac{\tau_m}{T_m}\right)T_m$; $\alpha = 0.179 - 0.324\frac{\tau_m}{T_m} + 0.161\left(\frac{\tau_m}{T_m}\right)^2$, $\frac{\tau_m}{T_m} < 0.6$;

$\alpha = 0.04$, $0.6 < \frac{\tau_m}{T_m} < 0.8$; $\alpha = 0.12 - 0.1\frac{\tau_m}{T_m}$, $0.8 < \frac{\tau_m}{T_m} < 1.0$.

[8] $T_i^{(594)} = \frac{\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m}{0.179 - 0.324\frac{\tau_m}{T_m} + 0.161\left(\frac{\tau_m}{T_m}\right)^2}$, $T_d^{(594)} = \left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m$.

[9] $T_i^{(595)} = \left(0.176 + 0.36\frac{\tau_m}{T_m}\right)25T_m$.

[10] $T_i^{(596)} = \frac{0.176T_m + 0.36\tau_m}{0.12 - 0.1\frac{\tau_m}{T_m}}$.

Rule	K_c	T_i	T_d	Comment
Pramod and Chidambaram (2000). *Model: Method 1*	11 $K_c^{(597)}$	$T_i^{(597)}$	$T_d^{(597)}$	$\left\lvert\frac{\tau_m}{T_m}\right\rvert < 0.6$
		12 $T_i^{(598)}$		$0.6 < \left\lvert\frac{\tau_m}{T_m}\right\rvert < 0.8$
Gaikward and Chidambaram (2000). *Model: Method 1*	13 $K_c^{(599)}$	$T_i^{(599)}$	$T_d^{(599)}$	$0.1 \le \frac{\tau_m}{T_m} \le 0.6$
Sree *et al.* (2004) *Model: Method 1*	14 $K_c^{(600)}$	$T_i^{(600)}$	$0.4917\tau_m$	$0.01 \le \frac{\tau_m}{T_m} \le 0.9$

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11 $$K_c^{(597)} = \frac{1}{K_m}\left[2.121 - 1.906\frac{\tau_m}{T_m} + 0.945\left(\frac{\tau_m}{T_m}\right)^2\right],$$

$$T_i^{(597)} = \frac{\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m}{0.179 - 0.324\frac{\tau_m}{T_m} + 0.161\left(\frac{\tau_m}{T_m}\right)^2},\ T_d^{(597)} = \left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m .$$

12 $$T_i^{(598)} = \left(0.176 + 0.36\frac{\tau_m}{T_m}\right)2.5T_m .$$

13 $$K_c^{(599)} = \frac{1}{K_m}\left[0.299 + 0.726\frac{T_m}{\tau_m}\right],\ T_i^{(599)} = T_m\left[4.104\left(\frac{\tau_m}{T_m}\right)^2 + 2.099\frac{\tau_m}{T_m} + 0.096\right],$$

$$T_d^{(599)} = T_m\left[0.822\left(\frac{\tau_m}{T_m}\right)^2 + 0.419\frac{\tau_m}{T_m} + 0.019\right].$$

14 $$K_c^{(600)} = \frac{1.4183}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.9147},$$

$$T_i^{(600)} = T_m\left[16.327\left(\frac{\tau_m}{T_m}\right)^2 + 5.5778\frac{\tau_m}{T_m} + 0.8158\right],\ 0.01 \le \frac{\tau_m}{T_m} \le 0.6;$$

$$T_i^{(600)} = T_m\left[196\left(\frac{\tau_m}{T_m}\right)^2 - 247.28\frac{\tau_m}{T_m} + 87.72\right],\ 0.6 \le \frac{\tau_m}{T_m} \le 0.9.$$

Rule	K_c	T_i	T_d	Comment
Sree *et al.* (2004) – continued.	[15] $K_c^{(601)}$	$T_i^{(601)}$	$T_d^{(601)}$	$0.01 \le \frac{\tau_m}{T_m} \le 1.2$
	Robust			
Tan *et al.* (1999b). *Model: Method 1*	[16] $K_c^{(602)}$	$T_i^{(602)}$	$T_d^{(602)}$	$\frac{\tau_m}{T_m} < 1.0$
Lee *et al.* (2000) *Model: Method 1*	[17] $K_c^{(603)}$	$T_i^{(603)}$	$T_d^{(603)}$	$0 \le \frac{\tau_m}{T_m} < 2$

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[15] $$K_c^{(601)} = \frac{1.2824}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.8325},$$

$$T_i^{(601)} = T_m\left[5.5734\frac{\tau_m}{T_m} - 0.0063\right],\ 0.01 \le \frac{\tau_m}{T_m} \le 0.5;$$

$$T_i^{(601)} = 0.483T_m e^{3.3739\frac{\tau_m}{T_m}},\ 0.5 \le \frac{\tau_m}{T_m} \le 1.2;\ T_d^{(601)} = T_m\left[0.507\frac{\tau_m}{T_m} + 0.0028\right].$$

[16] $$K_c^{(602)} = \frac{1}{K_m\left\{\left[-\frac{0.0373}{\lambda} + 0.1862\right]\frac{\tau_m}{T_m} + [0.6508 - 0.1498\log\lambda]\right\}},$$

$$T_i^{(602)} = \frac{\left\{\left[\frac{10.5045}{\lambda} + 0.8032\right]\frac{\tau_m}{T_m} + \left[\frac{3.2312}{\lambda} - 0.3445\right]\right\}}{\left\{\left[-\frac{0.0373}{\lambda} + 0.1862\right]\frac{\tau_m}{T_m} + [0.6508 - 0.1498\log\lambda]\right\}}T_m,$$

$$T_d^{(602)} = T_m\left[0.9065\lambda^{0.1632}\frac{\tau_m}{T_m} - \frac{1}{0.2075\lambda + 2.0335}\right.$$
$$\left.\left[\left(\frac{-0.0373}{\lambda} + 0.1862\right)\frac{\tau_m}{T_m} + (0.6508 - 0.1498\log\lambda)\right]\right],\ \lambda \text{ not specified.}$$

[17] $$K_c^{(603)} = \frac{T_i^{(603)}}{-K_m(2T_{CL} + \tau_m - \alpha)},\ \alpha = T_m\left[\left(\frac{T_{CL}}{T_m} + 1\right)^2 e^{\tau_m/T_m} - 1\right]$$

$$T_i^{(603)} = -T_m + \alpha - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$T_d^{(603)} = \frac{-T_m\alpha - \frac{\tau_m^2(0.167\tau_m - 0.5\alpha)}{2T_{CL} + \tau_m - \alpha}}{T_i^{(603)}} - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha}.$$

4.11.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

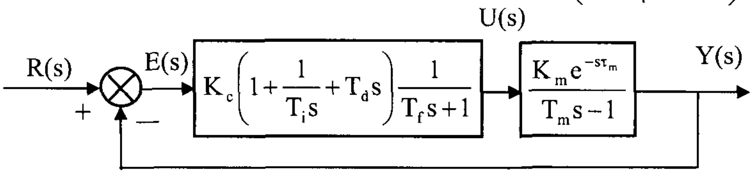

Table 157: PID controller tuning rules – unstable FOLPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Chandrashekar *et al.* (2002).	[18] $K_c^{(604)}$	$T_i^{(604)}$	$0.5\tau_m$	*Model: Method 1*

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[18] $K_c^{(604)} = \dfrac{1}{K_m}\left[4282\left(\dfrac{\tau_m}{T_m}\right)^2 - 1334.6\dfrac{\tau_m}{T_m} + 101\right]$, $0 < \dfrac{\tau_m}{T_m} < 0.2$;

$$K_c^{(604)} = \frac{1.1161}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.9427}, \quad 0.2 \le \frac{\tau_m}{T_m} \le 1.0 .$$

$T_i^{(604)} = T_m\left[36.842\left(\dfrac{\tau_m}{T_m}\right)^2 - 10.3\left(\dfrac{\tau_m}{T_m}\right) + 0.8288\right]$, $0 \le \dfrac{\tau_m}{T_m} \le 0.8$;

$$T_i^{(604)} = 76.241T_m\left(\frac{\tau_m}{T_m}\right)^{6.77}, \quad 0.8 \le \frac{\tau_m}{T_m} \le 1.0 .$$

$T_f = \left(0.1233\dfrac{\tau_m}{T_m} + 0.0033\right)T_m$, $0 \le \dfrac{\tau_m}{T_m} \le 1.0$.

Desired closed loop transfer function $= \dfrac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2}$; $T_{CL2} = 4.5115T_m\left(\dfrac{\tau_m}{T_m}\right)^{5.661}$,

$0.8 \le \dfrac{\tau_m}{T_m} \le 1.0$, $T_{CL2} = \left(0.0326 + 0.4958\dfrac{\tau_m}{T_m} + 2.03\left(\dfrac{\tau_m}{T_m}\right)^2\right)T_m$, $0 \le \dfrac{\tau_m}{T_m} \le 0.8$.

Rule	K_c	T_i	T_d		Comment
	Alternative tuning rules				
Chandrashekar *et al.* (2002). *Model: Method 1*	K_c	T_i	T_d	T_f	T_{CL2}	τ_m / T_m
	$\frac{101.0000}{K_m}$	$0.0404T_m$	0.0	0.0	$0.02T_m$	0.0
	$\frac{10.3662}{K_m}$	$0.3874T_m$	$0.0435T_m$	$0.0134T_m$	$0.10T_m$	0.1
	$\frac{5.3750}{K_m}$	$0.8600T_m$	$0.0884T_m$	$0.0250T_m$	$0.20T_m$	0.2
	$\frac{3.2655}{K_m}$	$1.8018T_m$	$0.1375T_m$	$0.0435T_m$	$0.40T_m$	0.3
	$\frac{2.4516}{K_m}$	$3.0400T_m$	$0.1868T_m$	$0.0581T_m$	$0.60T_m$	0.4
	$\frac{2.0217}{K_m}$	$4.6500T_m$	$0.2366T_m$	$0.0696T_m$	$0.80T_m$	0.5
	$\frac{1.7525}{K_m}$	$6.7286T_m$	$0.2866T_m$	$0.0784T_m$	$1.00T_m$	0.6
	$\frac{1.5458}{K_m}$	$10.337T_m$	$0.3380T_m$	$0.0885T_m$	$1.30T_m$	0.7
	$\frac{1.3723}{K_m}$	$17.693T_m$	$0.3910T_m$	$0.1005T_m$	$1.80T_m$	0.8
	$\frac{1.2420}{K_m}$	$33.610T_m$	$0.4440T_m$	$0.1124T_m$	$2.60T_m$	0.9
	$\frac{1.1400}{K_m}$	$80.620T_m$	$0.4969T_m$	$0.1247T_m$	$4.20T_m$	1.0

4.11.3 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

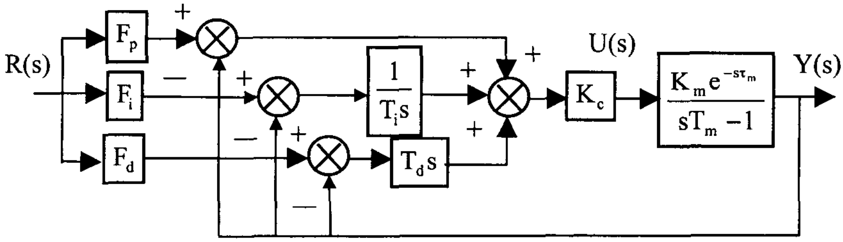

Table 158: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_me^{-s\tau_m}}{T_ms-1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Prashanti and Chidambaram (2000). *Model: Method 1*	[1] $K_c^{(605)}$ $F_p = 0.17$	$T_i^{(605)}$ $F_i = 1$	$T_d^{(605)}$ $F_d = 0.654$	Modified method of De Paor and O'Malley (1989); $\lvert\tau_m/T_m\rvert < 1$

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[1] $K_c^{(605)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1-\frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{1-\tau_m/T_m}{\tau_m/T_m}}\sin\sqrt{\left(1-\frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right]$,

$T_i^{(605)} = \frac{T_m}{\left[\sqrt{\frac{1-\tau_m/T_m}{\tau_m/T_m}}\right]\tan(0.75\phi)}$, $T_d^{(605)} = T_m\sqrt{\frac{\tau_m/T_m}{1-\tau_m/T_m}}\tan(0.75\phi)$,

$\phi = \tan^{-1}\sqrt{\left(1-\frac{\tau_m}{T_m}\right)\frac{T_m}{\tau_m}} - \sqrt{\frac{\tau_m}{T_m}\left(1-\frac{\tau_m}{T_m}\right)}$.

Rule	K_c	T_i	T_d	Comment
Prashanti and Chidambaram (2000) - Alternative tuning 1. *Model: Method 1*; $K_c^{(605)}$, $T_i^{(605)}$, $T_d^{(605)}$ specified on page 410.	$K_c^{(605)}$	$T_i^{(605)}$	$T_d^{(605)}$	$0.05 \le \tau_m/T_m \le 0.90$
	Representative results [2]			
	$F_p = 0.309$	$F_i = 1$	$F_d = 2.834$	$\tau_m/T_m = 0.05$
	$F_p = 0.281$	$F_i = 1$	$F_d = 2.605$	$\tau_m/T_m = 0.10$
	$F_p = 0.244$	$F_i = 1$	$F_d = 2.413$	$\tau_m/T_m = 0.20$
	$F_p = 0.213$	$F_i = 1$	$F_d = 2.346$	$\tau_m/T_m = 0.30$
	$F_p = 0.182$	$F_i = 1$	$F_d = 2.336$	$\tau_m/T_m = 0.40$
	$F_p = 0.150$	$F_i = 1$	$F_d = 2.207$	$\tau_m/T_m = 0.50$
	$F_p = 0.116$	$F_i = 1$	$F_d = 2.093$	$\tau_m/T_m = 0.60$
	$F_p = 0.080$	$F_i = 1$	$F_d = 1.884$	$\tau_m/T_m = 0.70$
	$F_p = 0.040$	$F_i = 1$	$F_d = 1.508$	$\tau_m/T_m = 0.80$
	$F_p = 0.0091$	$F_i = 1$	$F_d = 1.159$	$\tau_m/T_m = 0.90$
Prashanti and Chidambaram (2000) – Alternative tuning 2. *Model: Method 1*; $K_c^{(605)}$, $T_i^{(605)}$, $T_d^{(605)}$ specified on page 410.	$K_c^{(605)}$	$T_i^{(605)}$	$T_d^{(605)}$	
	$F_p = 0.9726 e^{\frac{-6.405\tau_m}{T_m}}$, $F_i = 1$, $F_d = 0$; $0.05 \le \frac{\tau_m}{T_m} \le 0.90$.			
	Representative results			
	$F_p = 0.777$, $\tau_m/T_m = 0.05$		$F_p = 0.046$, $\tau_m/T_m = 0.50$	
	$F_p = 0.457$, $\tau_m/T_m = 0.10$		$F_p = 0.026$, $\tau_m/T_m = 0.60$	
	$F_p = 0.227$, $\tau_m/T_m = 0.20$		$F_p = 0.013$, $\tau_m/T_m = 0.70$	
	$F_p = 0.121$, $\tau_m/T_m = 0.30$		$F_p = 0.0057$, $\tau_m/T_m = 0.80$	
	$F_p = 0.078$, $\tau_m/T_m = 0.40$		$F_p = 0.0022$, $\tau_m/T_m = 0.90$	

[2] $F_p = 0.3127 - 0.3369\frac{\tau_m}{T_m} + 0.0378\left(\frac{\tau_m}{T_m}\right)^2 - 0.045\left(\frac{\tau_m}{T_m}\right)^3$, $F_i = 1$,

$$F_d = 2.779 - 2.44\frac{\tau_m}{T_m} + 5\left(\frac{\tau_m}{T_m}\right)^2 - 4.81\left(\frac{\tau_m}{T_m}\right)^3 .$$

Rule	K_c	T_i	T_d	Comment
Prashanti and Chidambaram (2000). *Model: Method 1*	[3] $K_c^{(606)}$	$T_i^{(606)}$	$\alpha T_i^{(606)}$	
	Representative results			
	$F_p = 0.542$	$F_i = 1$	$F_d = 2.916$	$\tau_m/T_m = 0.05$
	$F_p = 0.437$	$F_i = 1$	$F_d = 2.878$	$\tau_m/T_m = 0.10$
	$F_p = 0.366$	$F_i = 1$	$F_d = 2.525$	$\tau_m/T_m = 0.20$
	$F_p = 0.303$	$F_i = 1$	$F_d = 2.307$	$\tau_m/T_m = 0.30$
	$F_p = 0.263$	$F_i = 1$	$F_d = 1.948$	$\tau_m/T_m = 0.40$
	$F_p = 0.214$	$F_i = 1$	$F_d = 1.729$	$\tau_m/T_m = 0.50$
	$F_p = 0.164$	$F_i = 1$	$F_d = 1.530$	$\tau_m/T_m = 0.60$
	$F_p = 0.151$	$F_i = 1$	$F_d = 1.297$	$\tau_m/T_m = 0.70$
	$F_p = 0.139$	$F_i = 1$	$F_d = 1.093$	$\tau_m/T_m = 0.80$
	$F_p = 0.113$	$F_i = 1$	$F_d = 0.962$	$\tau_m/T_m = 0.90$
Prashanti and Chidambaram (2000) – alternative tuning. *Model: Method 1*	$F_p = 0.7757 - 1.289\frac{\tau_m}{T_m} + 0.6267\left(\frac{\tau_m}{T_m}\right)^2$, $F_i = 1$, $F_d = 0$; $0.05 \le \frac{\tau_m}{T_m} \le 0.90$			
	Representative results			
	$F_p = 0.605$, $\tau_m/T_m = 0.05$		$F_p = 0.322$, $\tau_m/T_m = 0.50$	

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[3] $K_c^{(606)} = \frac{1}{K_m}[2.121 - 1.906\frac{\tau_m}{T_m} + 0.945\left(\frac{\tau_m}{T_m}\right)^2]$, $T_i^{(606)} = \frac{T_m}{\alpha}\left[0.176 + 0.36\frac{\tau_m}{T_m}\right]$;

$$\alpha = 0.179 - 0.324\frac{\tau_m}{T_m} + 0.161\left(\frac{\tau_m}{T_m}\right)^2, \frac{\tau_m}{T_m} < 0.6 \quad \alpha = 0.04, 0.6 < \frac{\tau_m}{T_m} \le 0.8;$$

$$\alpha = 0.12 - 0.1\frac{\tau_m}{T_m}, 0.8 < \frac{\tau_m}{T_m} \le 1.0;$$

$$F_p = 0.5165 - 0.8482\frac{\tau_m}{T_m} + 0.5133\left(\frac{\tau_m}{T_m}\right)^2 - 0.071\left(\frac{\tau_m}{T_m}\right)^3; F_i = 1;$$

$$F_d = 3.206 - 3.461\frac{\tau_m}{T_m} + 0.984\left(\frac{\tau_m}{T_m}\right)^2 + 0.089\left(\frac{\tau_m}{T_m}\right)^3.$$

Rule	K_c	T_i	T_d	Comment
Prashanti and Chidambaram (2000) - continued.	Representative results (continued)			
	$F_p = 0.664$, $\tau_m/T_m = 0.10$		$F_p = 0.248$, $\tau_m/T_m = 0.60$	
	$F_p = 0.550$, $\tau_m/T_m = 0.20$		$F_p = 0.228$, $\tau_m/T_m = 0.70$	
	$F_p = 0.456$, $\tau_m/T_m = 0.30$		$F_p = 0.209$, $\tau_m/T_m = 0.80$	
	$F_p = 0.395$, $\tau_m/T_m = 0.40$		$F_p = 0.169$, $\tau_m/T_m = 0.90$	
Jhunjhunwala and Chidambaram (2001).	[4] $K_c^{(607)}$	$T_i^{(607)}$	$T_d^{(607)}$	*Model: Method 1*
	Robust			
Lee *et al.* (2000). *Model: Method 1*	[5] $K_c^{(608)}$	$T_i^{(608)}$	$T_d^{(608)}$	$0 \le \frac{\tau_m}{T_m} < 2$
	$F_p = F_i = F_d = \frac{1}{\alpha s + 1}$, $\alpha = T_m\left[\left(\frac{T_{CL}}{T_m}+1\right)^2 e^{\tau_m/T_m} - 1\right]$			

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[4] $$K_c^{(607)} = \frac{1}{K_m}\left(13.0 - 39.712\frac{\tau_m}{T_m}\right),\ \frac{\tau_m}{T_m} < 0.2;$$

$$K_c^{(607)} = \frac{1.397}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.769},\ 0.2 \le \frac{\tau_m}{T_m} \le 1.4;$$

$$T_i^{(607)} = 0.856 T_m e^{\frac{2.044\tau_m}{T_m}},\ 0 \le \frac{\tau_m}{T_m} \le 1;\ T_i^{(607)} = 0.3444 T_m e^{\frac{2.9595\tau_m}{T_m}},\ 1 \le \frac{\tau_m}{T_m} \le 1.4;$$

$$T_d^{(607)} = T_m\left(0.5643\frac{\tau_m}{T_m} + 0.0075\right),\ 0 \le \frac{\tau_m}{T_m} \le 1.4;$$

$$F_i = 0;\ F_d = 0;\ F_p = 0.6863,\ \frac{\tau_m}{T_m} = 0.1;\ F_p = 0.4772 e^{\frac{-1.58\tau_m}{T_m}},\ 0.2 \le \frac{\tau_m}{T_m} \le 1.$$

[5] $$K_c^{(608)} = \frac{T_i^{(608)}}{-K_m(2T_{CL} + \tau_m - \alpha)},\ \alpha = T_m\left[\left(\frac{T_{CL}}{T_m}+1\right)^2 e^{\tau_m/T_m} - 1\right];$$

$$T_i^{(608)} = -T_m + \alpha - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$T_d^{(608)} = \frac{-T_m\alpha - \frac{\tau_m^2(0.167\tau_m - 0.5\alpha)}{2T_{CL} + \tau_m - \alpha}}{T_i^{(608)}} - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha}.$$

4.11.4 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

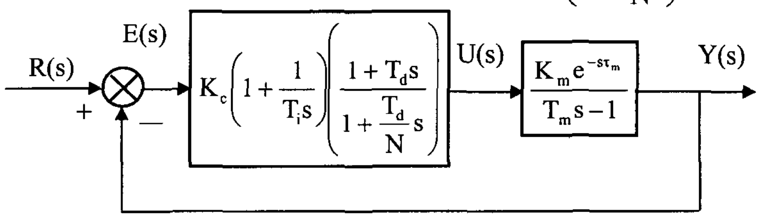

Table 159: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Shinskey (1988) - minimum IAE – *page 381.* *Method: Model 1* N not specified	$\frac{0.91T_m}{K_m\tau_m}$	$1.70\tau_m$	$0.60\tau_m$	$\lvert\tau_m/T_m\rvert = 0.1$
	$\frac{1.00T_m}{K_m\tau_m}$	$1.90\tau_m$	$0.60\tau_m$	$\lvert\tau_m/T_m\rvert = 0.2$
	$\frac{0.89T_m}{K_m\tau_m}$	$2.00\tau_m$	$0.80\tau_m$	$\lvert\tau_m/T_m\rvert = 0.5$
	$\frac{0.86T_m}{K_m\tau_m}$	$2.25\tau_m$	$0.90\tau_m$	$\lvert\tau_m/T_m\rvert = 0.67$
	$\frac{0.83T_m}{K_m\tau_m}$	$2.40\tau_m$	$1.00\tau_m$	$\lvert\tau_m/T_m\rvert = 0.8$

4.11.5 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$$

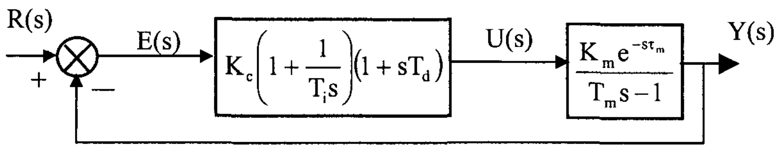

Table 160: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s-1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Huang and Chen (1997), (1999).	[6] $K_c^{(609)}$	$T_i^{(609)}$	$T_d^{(609)}$	*Model: Method 4*

[6] $0 < \frac{\tau_m}{T_m} < 1$; 'Good' servo and regulator response.

$$K_c^{(609)} = \frac{0.5}{K_m}\left[1+1.61519\left(\frac{\tau_m}{T_m}\right)^{-1.02379}\right],\ T_i^{(609)} = \frac{K_c^{(609)}K_m T_m}{K_c^{(609)}K_m - 1}\left(\frac{2x_1}{T_m}+\frac{\tau_m}{T_m}\right),$$

with recommended $x_1 = T_m\left[-1.3735.10^{-3}+0.22091\frac{\tau_m}{T_m}+1.6653\left(\frac{\tau_m}{T_m}\right)^2\right]$;

$$T_d^{(609)} = T_m\left[1.5006.10^{-4}+0.23549\frac{\tau_m}{T_m}+0.16970\left(\frac{\tau_m}{T_m}\right)^2\right].$$

4.11.6 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+(T_d/N)s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+(T_d/N)s}\right)R(s)$$

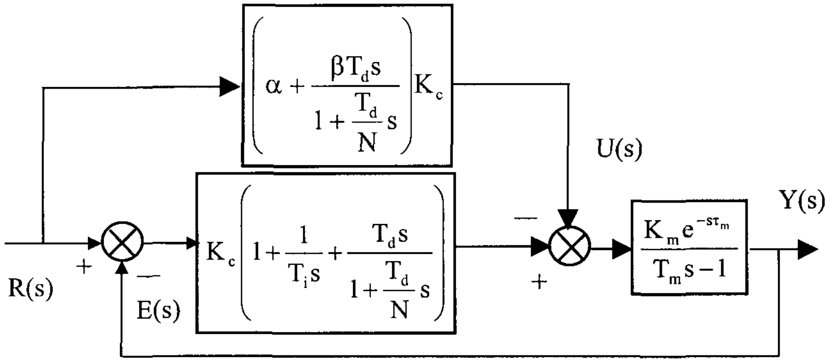

Table 161: PID controller tuning rules – unstable FOLPD model $G_m(s)=\frac{K_m e^{-s\tau_m}}{T_m s-1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Chidambaram (2000c). *Model: Method 1* $\xi=0.333$; $N=\infty$; $\beta=1$	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.245}$	$T_i^{(610)}$ [7]	$T_d^{(610)}$	$\left\|\frac{\tau_m}{T_m}\right\|<0.6$
		$4.4T_m+9\tau_m$		$0.6\le\left\|\frac{\tau_m}{T_m}\right\|\le 0.8$
		$T_i^{(611)}$ [8]		$0.8<\left\|\frac{\tau_m}{T_m}\right\|\le 0.9$

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[7] $T_i^{(610)}=\frac{\left(0.176+0.36\frac{\tau_m}{T_m}\right)T_m}{0.179-0.324\frac{\tau_m}{T_m}+0.161\left(\frac{\tau_m}{T_m}\right)^2}$, $T_d^{(610)}=\left(0.176+0.36\frac{\tau_m}{T_m}\right)T_m$.

[8] $T_i^{(611)}=\frac{0.176T_m+0.36\tau_m}{0.12-0.1\frac{\tau_m}{T_m}}$, $\alpha=0.74+0.42\frac{\tau_m}{T_m}-0.23\left(\frac{\tau_m}{T_m}\right)^2$.

Rule	K_c	T_i	T_d	Comment
Srinivas and Chidambaram (2001). *Model: Method 1*	[9] $K_c^{(612)}$	$T_i^{(612)}$	$T_d^{(612)}$	$N = \infty$
Chandrashekar *et al.* (2002). *Model: Method 1*	[10] $K_c^{(613)}$	$T_i^{(613)}$	$0.5\tau_m$	$N = \infty$; $\beta = 0$

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[9] Sample $K_c^{(612)}$, $T_i^{(612)}$, $T_d^{(612)}$ taken by the authors correspond to the dominant pole placement tuning rules of Valentine and Chidambaram (1997b) (see page 405);

$$\alpha = \frac{\tau_m}{T_i^{(612)}} + \frac{1}{K_c^{(612)}K_m} \; ; \; \beta = \frac{1}{T_d^{(612)}}\left[\tau_m - \frac{T_m}{K_c^{(612)}K_m} - \frac{\tau_m^{\,2}}{2T_i^{(612)}}\right].$$

[10] $$K_c^{(613)} = \frac{1}{K_m}\left[4282\left(\frac{\tau_m}{T_m}\right)^2 - 1334.6\frac{\tau_m}{T_m} + 101\right], \; 0 < \frac{\tau_m}{T_m} < 0.2 \; ;$$

$$K_c^{(613)} = \frac{1.1161}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.9427}, \; 0.2 \le \frac{\tau_m}{T_m} \le 1.0 \, .$$

$$T_i^{(613)} = T_m\left[36.842\left(\frac{\tau_m}{T_m}\right)^2 - 10.3\left(\frac{\tau_m}{T_m}\right) + 0.8288\right], \; 0 \le \frac{\tau_m}{T_m} \le 0.8 \; ;$$

$$T_i^{(613)} = 76.241T_m\left(\frac{\tau_m}{T_m}\right)^{6.77}, \; 0.8 \le \frac{\tau_m}{T_m} \le 1.0 \, .$$

$$\alpha = 0.83, \; \frac{\tau_m}{T_m} \le 0.1 \; ; \; \alpha = 0.8053 - 0.1935\frac{\tau_m}{T_m}, \; 0.1 \le \frac{\tau_m}{T_m} \le 0.7 \, .$$

Desired closed loop transfer function $= \dfrac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2}$;

$$T_{CL2} = \left(0.0326 + 0.4958\frac{\tau_m}{T_m} + 2.03\left(\frac{\tau_m}{T_m}\right)^2\right)T_m, \; 0 \le \frac{\tau_m}{T_m} \le 0.8 \; ;$$

$$T_{CL2} = 4.5115T_m\left(\frac{\tau_m}{T_m}\right)^{5.661}, \; 0.8 \le \frac{\tau_m}{T_m} \le 1.0 \, .$$

Rule	K_c	T_i	T_d	Comment
Chandrashekar *et al.* (2002). *Model: Method 1*	Alternative tuning rules: $K_c = \frac{x_1}{K_m}$, $T_i = x_2 T_m$, $T_d = x_3 T_m$, $T_f = x_4 T_m$, $T_{CL2} = x_5 T_m$			
Wang and Cai (2002). *Model: Method 1*	[11] $K_c^{(614)}$	$T_i^{(614)}$	$T_d^{(614)}$	N=0; $\beta = 0$

Coefficient values (Chandrashekar *et al.* (2002))

x_1	x_2	x_3	x_4	x_5	α	τ_m/T_m
101.000	0.0404	0.0	0.0	0.02	0.505	0.0
10.3662	0.3874	0.0435	0.0134	0.10	0.742	0.1
5.3750	0.8600	0.0884	0.0250	0.20	0.767	0.2
3.2655	1.8018	0.1375	0.0435	0.40	0.778	0.3
2.4516	3.0400	0.1868	0.0581	0.60	0.803	0.4
2.0217	4.6500	0.2366	0.0696	0.80	0.828	0.5
1.7525	6.7286	0.2866	0.0784	1.00	0.851	0.6
1.5458	10.337	0.3380	0.0885	1.30	0.874	0.7
1.3723	17.693	0.3910	0.1005	1.80	0.898	0.8
1.2420	33.610	0.4440	0.1124	2.60	0.923	0.9
1.1400	80.620	0.4969	0.1247	4.20	0.948	1.0

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[11] $K_c^{(614)} = \frac{0.5236 T_m}{K_m \tau_m} - \frac{0.4764}{K_m}\sqrt{\frac{T_m}{\tau_m}}$, $T_i^{(614)} = \frac{0.5236 T_m - 0.4764\sqrt{T_m \tau_m}}{0.5236\left(\sqrt{\frac{T_m}{\tau_m}} - 1\right)}$,

$$T_d^{(614)} = \frac{0.2618 \tau_m \sqrt{T_m \tau_m}}{0.5236 T_m - 0.4764\sqrt{T_m \tau_m}},\ \alpha = \frac{1.9099\sqrt{\frac{\tau_m}{T_m}}}{1 + 0.9099\sqrt{\frac{\tau_m}{T_m}}}\ (A_m = 3,\ \phi_m = 60^0).$$

Rule	K_c	T_i	T_d	Comment
Xu and Shao (2003b). *Model: Method 1*	[12] $K_c^{(615)}$	$T_i^{(615)}$	$T_d^{(615)}$	N=0; $\beta = 0$
Sree and Chidambaram (2005a). *Model: Method 1*	[13] $K_c^{(616)}$	$T_i^{(616)}$	$T_d^{(616)}$	N= ∞ ; $\beta = 1$

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[12] $K_c^{(615)} = \dfrac{0.5}{K_m}\left(\dfrac{T_m}{\tau_m} + \sqrt{\dfrac{T_m}{\tau_m}}\right)$, $T_i^{(615)} = \dfrac{\tau_m\left(\dfrac{T_m}{\tau_m} + \sqrt{\dfrac{T_m}{\tau_m}}\right)}{\sqrt{\dfrac{T_m}{\tau_m}} - 1}$,

$T_d^{(615)} = \dfrac{0.5\sqrt{T_m \tau_m}}{\left(\dfrac{T_m}{\tau_m} + \sqrt{\dfrac{T_m}{\tau_m}}\right)}$, $\alpha = \dfrac{2\sqrt{T_m \tau_m}}{T_m + \sqrt{T_m \tau_m}}$ ($A_m > 2$, $\phi_m > 60^0$).

[13] Sample $K_c^{(616)}$, $T_i^{(616)}$, $T_d^{(616)}$ taken by the authors correspond to the tuning rules of Chandrashekar *et al.* (2002) and Huang and Chen (1999); the latter tuning rule is defined for an unstable SOSPD process model.

Desired closed loop transfer function = $\dfrac{\left[(1-\alpha)T_i^{(616)}s + 1\right]e^{-s\tau_m}}{T_{CL}{}^2 s^2 + 2\xi T_{CL} s + 1}$.

The optimum value of α is chosen by minimizing the ISE (servo response). Then,

$\alpha = 1 - \dfrac{2T_{CL}}{T_i^{(616)}}$, $\xi = 1$; $\alpha = 1 - \dfrac{2T_{CL}\xi}{T_i^{(616)}}$, $\xi < 1$;

$$\alpha = 1 - \frac{T_{CL}}{T_i^{(616)}}\left[\frac{\left(-\xi+\sqrt{\xi^2-1}\right)\left(-\xi-\sqrt{\xi^2-1}\right)^2 - \left(-\xi+\sqrt{\xi^2-1}\right)^3 + x_1}{\left(-\xi+\sqrt{\xi^2-1}\right)\left(-\xi-\sqrt{\xi^2-1}\right)^3 + x_2}\right], \quad \xi > 1$$

with $x_1 = -\left(-\xi-\sqrt{\xi^2-1}\right)^3 + \left(-\xi+\sqrt{\xi^2-1}\right)^2\left(-\xi-\sqrt{\xi^2-1}\right)$ and

$$x_2 = \left(-\xi-\sqrt{\xi^2-1}\right)\left(-\xi+\sqrt{\xi^2-1}\right)^3 - 2\left(-\xi+\sqrt{\xi^2-1}\right)^2\left(-\xi-\sqrt{\xi^2-1}\right)^2.$$

If it is desired to minimize the overshoot (servo response), for $\xi \ge 1$, then $\alpha = 1$.

4.11.7 *Non-interacting controller 3*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+\frac{sT_d}{N}}Y(s)$$

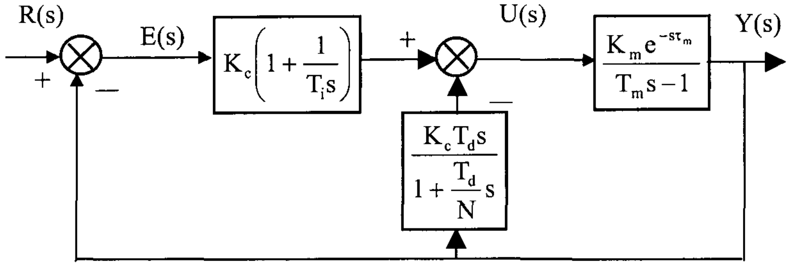

Table 162: PID controller tuning rules – unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index			
Huang and Lin (1995) - minimum IAE. *Model: Method 2*	[1] $K_c^{(617)}$	$T_i^{(617)}$	$T_d^{(617)}$	$0.01 \le \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 0.8$; N=10
	[2] $K_c^{(618)}$	$T_i^{(618)}$	$T_d^{(618)}$	

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[1] Servo response: $K_c^{(617)} = -\frac{1}{K_m}\left(-0.433 + 0.2056\frac{\tau_m}{T_m} + 0.3135\left[\frac{\tau_m}{T_m}\right]^{-1.0251}\right)$,

$$T_i^{(617)} = T_m\left(-0.0018 + 0.8193\frac{\tau_m}{T_m} + 7.7853\left[\frac{\tau_m}{T_m}\right]^{6.6423}\right),$$

$$T_d^{(617)} = T_m\left(-0.0312 + 1.6333\frac{\tau_m}{T_m} + 0.0399e^{7.6983\frac{\tau_m}{T_m}}\right).$$

[2] Regulator response: $K_c^{(618)} = -\frac{1}{K_m}\left(0.2675 + 0.1226\frac{\tau_m}{T_m} + 0.8781\left[\frac{\tau_m}{T_m}\right]^{-1.004}\right)$,

$$T_i^{(618)} = T_m\left(0.0005 + 2.4631\frac{\tau_m}{T_m} + 9.5795\left[\frac{\tau_m}{T_m}\right]^{2.9123}\right),\quad T_d^{(618)} = 0.0011T_m + 0.4759\tau_m .$$

4.11.8 *Non-interacting controller 8*

$$U(s)=K_c\left(1+\frac{1}{T_is}+T_ds\right)E(s)-K_i\left(1+T_{di}s\right)Y(s)$$

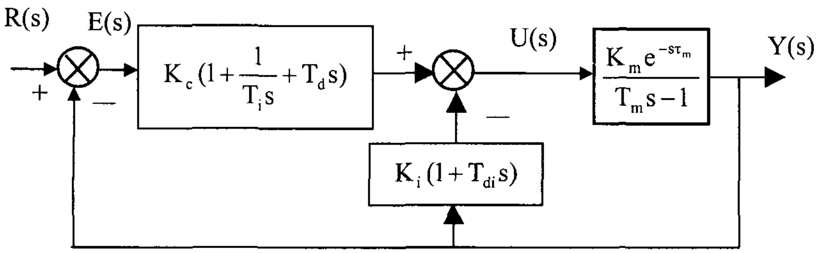

Table 163: PID controller tuning rules – unstable FOLPD model $G_m(s)=\frac{K_me^{-s\tau_m}}{T_ms-1}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Park *et al.* (1998). *Model: Method 1*	[3] $K_c^{(619)}$	$T_i^{(619)}$	$T_d^{(619)}$	$K_i=\frac{1}{K_m}\sqrt{\frac{T_m}{\tau_m}}$ $T_{di}=0$

[3] Apply minimum ITAE – regulator tuning rule or minimum ITAE – servo tuning rule defined by Sung *et al.* (1996) for SOSPD model, ideal PID controller (pages 313, 316). For these formulae, K_m, T_{m1} and ξ_m are replaced by the following parameters from the unstable FOLPD model:

$$\frac{K_m}{\sqrt{\frac{T_m}{\tau_m}-1}}\ \text{(Unstable FOLPD)}\rightarrow K_m\ \text{(SOSPD)}$$

$$\frac{0.71T_m^{0.25}\tau_m^{0.75}}{\sqrt{\left(\frac{T_m}{\tau_m}\right)^{0.5}-1}}\ \text{(Unstable FOLPD)}\rightarrow T_{m1}\ \text{(SOSPD)}$$

$$\frac{0.71\left|T_m^{0.5}-\tau_m^{0.5}\right|T_m^{0.25}\tau_m^{-0.75}}{\sqrt{\left(\frac{T_m}{\tau_m}\right)^{0.5}-1}}\ \text{(Unstable FOLPD)}\rightarrow \xi_m\ \text{(SOSPD)}$$

Rule	K_c	T_i	T_d	Comment
Lee and Edgar (2002). *Model: Method 1*	[4] $K_c^{(620)}$	$T_i^{(620)}$	$T_d^{(620)}$	$T_{di} = 0$

[4] $$K_c^{(620)} = \frac{K_i^{(620)}K_m - 1}{K_m(\lambda + \tau_m)}\left[\frac{T_m - K_i^{(620)}K_m\tau_m}{K_i^{(620)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)}\right],$$

$$T_i^{(620)} = \frac{T_m - K_i^{(620)}K_m\tau_m}{K_i^{(620)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)},$$

$$T_d^{(620)} = \frac{K_i^{(620)}K_m - 1}{K_m(\lambda + \tau_m)}\left\{\frac{K_i^{(620)}K_m\tau_m^2}{2(K_i^{(620)}K_m - 1)} - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_i^{(620)}K_m\tau_m)\tau_m^2}{2(K_i^{(620)}K_m - 1)(\lambda + \tau_m)}\right\},$$

$$K_i^{(620)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)}\sin\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right].$$

4.11.9 *Non-interacting controller 10*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_f\left(\frac{T_d}{T_m}s+1\right)Y(s)$$

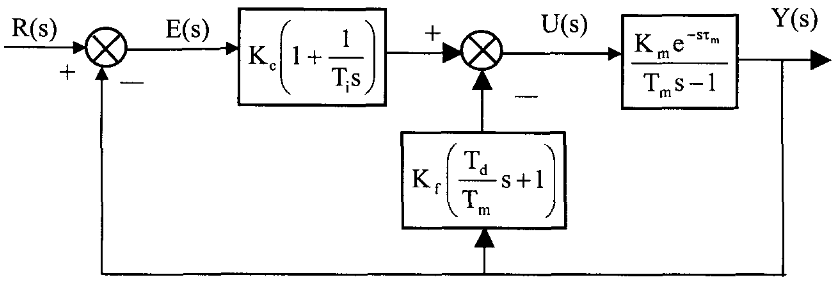

Table 164: PID tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Majhi and Atherton (2000) – minimum ITSE	$K_c^{(621)}$ [5]	$T_i^{(621)}$	$0.5\tau_m$	*Model: Method 1*
Majhi and Atherton (2000) - minimum ITSE. *Model: Method 3*	$K_c^{(622)}$ [6]	$T_i^{(622)}$	$\frac{\ln(1+K)}{4\tanh^{-1}K}T_m$	$0 < \frac{\tau_m}{T_m} < 0.693$

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[5] $K_c^{(621)} = \frac{0.8011T_m\left(1-0.9358\sqrt{\frac{\tau_m}{T_m}}\right)}{K_m\tau_m}$,

$T_i^{(621)} = T_m\left(0.1227+1.4550\frac{\tau_m}{T_m}-1.2711\frac{\tau_m^{\ 2}}{T_m^{\ 2}}\right)$, $K_f = \frac{1}{K_m}\sqrt{\frac{2T_m}{\tau_m}}$.

[6] $K_c^{(622)} = \frac{0.8011\left(1-0.9358\sqrt{\ln(1+K)}\right)}{K_m\ln(1+K)}$, $K_f = \frac{1}{K_m}\sqrt{\frac{2}{\ln(1+K)}}$, $K = A_p/(K_m h)$,

$T_i^{(622)} = \frac{0.1227+1.4550\ln(1+K)-1.2711[\ln(1+K)]^2}{2\tanh^{-1}K}T_m$.

Rule	K_c	T_i	T_d	Comment
Majhi and Atherton (2000) – continued.	[7] $K_c^{(623)}$	$T_i^{(623)}$	$T_d^{(623)}$	$0.693 < \frac{\tau_m}{T_m} < 1$

[7] $K_c^{(623)} = \frac{1}{K_m}\left[\frac{0.8011(K+0.9946)}{K+0.0682} - \frac{0.7497\sqrt{K+0.9946}}{\sqrt{K+0.0682}}\right]$,

$$T_i^{(623)} = \frac{0.0145K^3 + 0.5773K^2 + 2.6554K + 0.3488}{K^3 + 5.2158K^2 + 4.4816K + 0.2817} T_m,$$

$$T_d^{(623)} = \frac{0.0237(K+34.5338)}{K+4.1530} T_m,\ K_f = \frac{1}{K}\sqrt{\frac{2(K+0.9946)}{K+0.0682}}.$$

4.11.10 *Non-interacting controller 12*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_d s + 1}E(s) - K_1 Y(s)$$

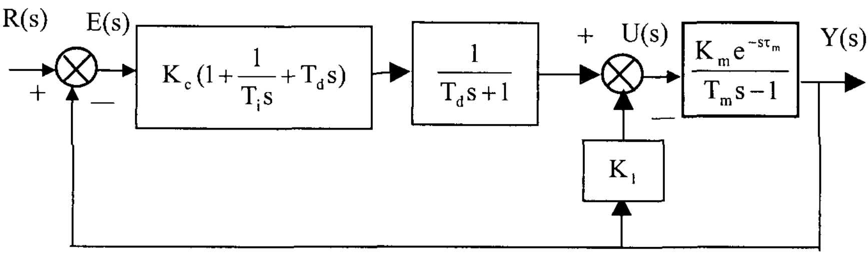

Table 165: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee and Edgar (2002). *Model: Method 1*	[8] $K_c^{(624)}$	$T_i^{(624)}$	$T_d^{(624)}$	

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[8] $$K_c^{(624)} = \frac{K_1^{(624)}K_m - 1}{K_m(\lambda + \tau_m)}\left\{\frac{T_m - K_1^{(624)}K_m\tau_m}{K_1^{(624)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m\left[\frac{K_1^{(624)}K_m}{2(K_1^{(624)}K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)}K_m\tau_m)}{2(K_1^{(624)}K_m - 1)(\lambda + \tau_m)}\right]^{0.5}\right\};$$

$$T_i^{(624)} = \frac{T_m - K_1^{(624)}K_m\tau_m}{K_1^{(624)}K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m\left[\frac{K_1^{(624)}K_m}{2(K_1^{(624)}K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)}K_m\tau_m)}{2(K_1^{(624)}K_m - 1)(\lambda + \tau_m)}\right]^{0.5};$$

$$T_d^{(624)} = \tau_m\left[\frac{K_1^{(624)}K_m}{2(K_1^{(624)}K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)}K_m\tau_m)}{2(K_1^{(624)}K_m - 1)(\lambda + \tau_m)}\right]^{0.5};$$

$$K_1^{(624)} = \frac{1}{K_m}\left[\cos\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m}\left(1 - \frac{\tau_m}{T_m}\right)}\sin\sqrt{\left(1 - \frac{\tau_m}{T_m}\right)\frac{\tau_m}{T_m}}\right].$$

Rule	K_c	T_i	T_d	Comment
Lee and Edgar (2002). *Model: Method 1*	[9] $K_c^{(625)}$	$T_i^{(625)}$	$T_d^{(625)}$	$K_1 = 0.25K_u$

[9] $$K_c^{(625)} = \frac{1+0.25K_uK_m}{K_m(\lambda+\tau_m)}\{\frac{T_m - 0.25K_uK_m\tau_m}{1+0.25K_uK_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)} +$$

$$\tau_m\left[\frac{0.25K_uK_m}{2(1+0.25K_uK_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_uK_m\tau_m)\tau_m^2}{2(1+0.25K_uK_m)(\lambda+\tau_m)}\right]^{0.5}\};$$

$$T_i^{(625)} = \frac{T_m - 0.25K_uK_m\tau_m}{1+0.25K_uK_m} + \frac{\tau_m^2}{2(\lambda+\tau_m)} +$$

$$\tau_m\left[\frac{0.25K_uK_m}{2(1+0.25K_uK_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_uK_m\tau_m)\tau_m^2}{2(1+0.25K_uK_m)(\lambda+\tau_m)}\right]^{0.5};$$

$$T_d^{(625)} =$$

$$\tau_m\left[\frac{0.25K_uK_m}{2(1+0.25K_uK_m} - \frac{\tau_m}{6(\lambda+\tau_m)} + \frac{\tau_m^2}{4(\lambda+\tau_m)^2} + \frac{(T_m - 0.25K_uK_m\tau_m)\tau_m^2}{2(1+0.25K_uK_m)(\lambda+\tau_m)}\right]^{0.5}.$$

4.12 Unstable SOSPD Model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

4.12.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

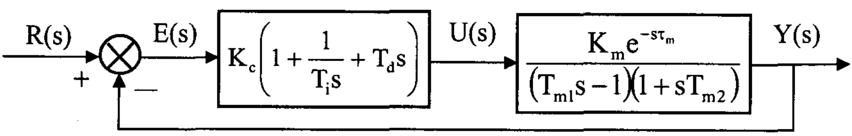

Table 166: PID controller tuning rules – unstable SOSPD model (one unstable pole) $G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Wang and Jin (2004).	[1] $K_c^{(626)}$	$T_i^{(626)}$	$T_d^{(626)}$	*Model: Method 1*

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[1] $K_c^{(626)} = \frac{K_m T_i^{(626)} (T_{m1} - \tau_m)(T_{m2} + \tau_m)}{(T_{CL3} + \tau_m)^3}$,

$T_i^{(626)} = \frac{T_{CL3}^3 + 3\tau_m T_{CL3}^2 + 3(T_{m1}T_{m2} + \tau_m T_{m1} - \tau_m T_{m2})T_{CL3} + \tau_m T_{m2}(T_{m1} - \tau_m)}{(T_{m1} - \tau_m)(T_{m2} + \tau_m)}$,

$T_d^{(626)} = \frac{(T_{m2} + \tau_m - T_{m1})T_{CL3}^3 + 3T_{m1}T_{m2}T_{CL3}(T_{CL3} + \tau_m) + T_{m1}T_{m2}\tau_m^2}{T_i^{(626)}(T_{m1} - \tau_m)(T_{m2} + \tau_m)}$.

Rule	K_c	T_i	T_d	Comment
	Robust			
Rotstein and Lewin (1991). *Model: Method 1*	[2] $K_c^{(627)}$	$T_i^{(627)}$	$T_d^{(627)}$	
	λ determined graphically – sample values provided			
	$\tau_m/T_m = 0.2$, $\lambda \in [0.5T_m, 1.9T_m]$			K_m uncertainty = 50%
	$\tau_m/T_m = 0.4$, $\lambda \in [1.3T_m, 1.9T_m]$			
	$\tau_m/T_m = 0.2$, $\lambda \in [0.4T_m, 4.3T_m]$			K_m uncertainty = 30%
	$\tau_m/T_m = 0.4$, $\lambda \in [1.1T_m, 4.3T_m]$			
	$\tau_m/T_m = 0.6$, $\lambda \in [2.2T_m, 4.3T_m]$			
Lee *et al.* (2000). *Model: Method 1*	[3] $K_c^{(628)}$	$T_i^{(628)}$	$T_d^{(628)}$	$0 \le \frac{\tau_m}{T_m} < 2$

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[2] $K_c^{(627)} = \frac{T_{m1}\left[\lambda\left(\frac{\lambda}{T_{m1}} + 2\right) + T_{m2}\right]}{\lambda^2 K_m}$, $T_i^{(627)} = \lambda\left(\frac{\lambda}{T_{m1}} + 2\right) + T_{m2}$,

$$T_d^{(627)} = \frac{\lambda\left(\frac{\lambda}{T_{m1}} + 2\right)T_{m2}}{\lambda\left(\frac{\lambda}{T_{m1}} + 2\right) + T_{m2}}.$$

[3] $K_c^{(628)} = \frac{T_i^{(628)}}{-K_m(2\lambda + \tau_m - \alpha)}$, λ = desired closed loop dynamic parameter,

$$\alpha = T_{m1}\left[\left(\frac{\lambda}{T_{m1}} + 1\right)^2 e^{\tau_m/T_{m1}} - 1\right];$$

$$T_i^{(628)} = -T_{m1} + T_{m2} + \alpha - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha},$$

$$T_d^{(628)} = \frac{-T_{m1}\alpha + T_{m2}\alpha - T_{m1}T_{m2} - \frac{\tau_m^2(0.167\tau_m - 0.5\alpha)}{2\lambda + \tau_m - \alpha}}{T_i^{(628)}} - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha}.$$

Rule	K_c	T_i	T_d	Comment
	Ultimate cycle			
McMillan (1984). *Model: Method 1*	[4] $K_c^{(629)}$	$T_i^{(629)}$	$T_d^{(629)}$	Tuning rules developed from K_u, T_u

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[4] $$K_c^{(629)} = \frac{1.111}{K_m}\frac{T_{m1}T_{m2}}{\tau_m^2}\left\{\frac{1}{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}}\right\}^2,$$

$$T_i^{(629)} = 2\tau_m\left\{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}\right\},$$

$$T_d^{(629)} = 0.5\tau_m\left\{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}\right\}.$$

4.12.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

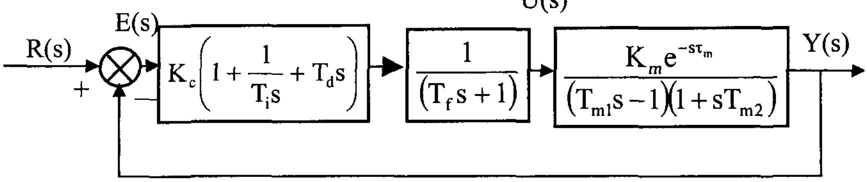

Table 167: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Zhang and Xu (2002). *Model: Method 1*	[5] $K_c^{(630)}$	$T_i^{(630)}$	$T_d^{(630)}$	$\tau_m < T_{m1}$

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[5] $K_c^{(630)} = \frac{T_{m2}T_{m1}(T_{m1} - \tau_m) + \lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2}{K_m(\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}\tau_m + \tau_m^2 T_{m1})}$,

$T_i^{(630)} = T_{m2} + \frac{\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2}{T_{m1}(T_{m1} - \tau_m)}$,

$T_d^{(630)} = \frac{T_{m2}(\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2)}{T_{m1}T_{m2}(T_{m1} - \tau_m) + \lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2}$,

$T_f^{(630)} = \frac{\lambda^3 T_{m1}(T_{m1} - \tau_m)}{T_{m1}(\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}\tau_m + \tau_m^2 T_{m1})}$;

for $\frac{\tau_m}{T_{m1}} \le 0.5$, $\lambda = 2 - 5(\tau_m + T_{m2})$.

4.12.3 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

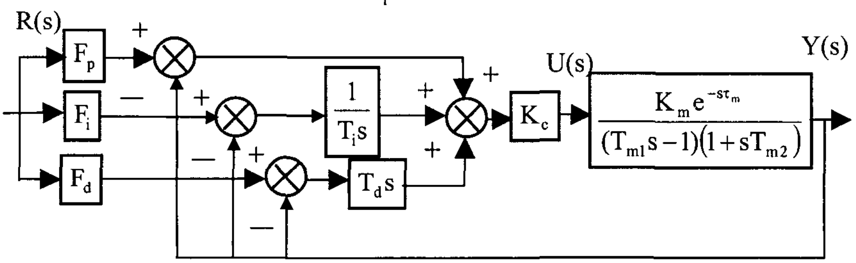

Table 168: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_me^{-s\tau_m}}{(T_{m1}s-1)(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee *et al.* (2000). *Model: Method 1*	$K_c^{(631)}$ [6]	$T_i^{(631)}$	$T_d^{(631)}$	$0 \le \frac{\tau_m}{T_m} < 2$
	$F_p = F_i = F_d = \frac{1}{\alpha s + 1}$, $\alpha = T_{m1}\left[\left(\frac{\lambda}{T_{m1}} + 1\right)^2 e^{\tau_m/T_{m1}} - 1\right]$			

---

[6] $K_c^{(631)} = \frac{T_i^{(631)}}{-K_m(2\lambda + \tau_m - \alpha)}$, λ = desired closed loop dynamic parameter,

$$T_i^{(631)} = -T_{m1} + T_{m2} + \alpha - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^{\ 2}}{2\lambda + \tau_m - \alpha},$$

$$T_d^{(631)} = \frac{-T_{m1}\alpha + T_{m2}\alpha - T_{m1}T_{m2} - \frac{\tau_m^{\ 2}(0.167\tau_m - 0.5\alpha)}{2\lambda + \tau_m - \alpha}}{T_i^{(631)}} - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^{\ 2}}{2\lambda + \tau_m - \alpha}.$$

4.12.4 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d}{N}s}\right)$

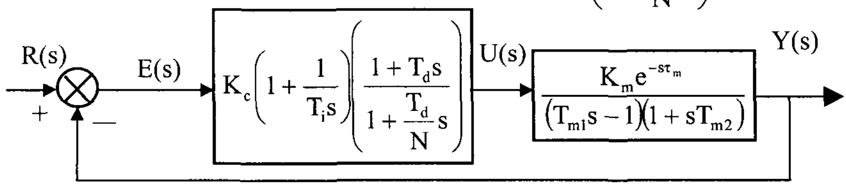

Table 169: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(1+sT_{m2})}$$

Rule	K_c		T_i	T_d		Comment
	Minimum performance index: regulator tuning					
Minimum ITAE – Poulin and Pomerleau (1996), (1997).	[7] $K_c^{(632)}$		$T_i^{(632)}$	T_{m2}		*Model: Method 1*
	Coefficient values (deduced from graph)					
	$\frac{\tau_m + T_{m2}}{T_{m1}}$	x_1	x_2	$\frac{\tau_m + T_{m2}}{T_{m1}}$	x_1	x_2
Process output step load disturbance	0.05	0.9479	2.3546	0.30	1.6163	2.6612
	0.10	1.0799	2.4111	0.35	1.7650	2.7368
	0.15	1.2013	2.4646	0.40	1.9139	2.8161
	0.20	1.3485	2.5318	0.45	2.0658	2.9004
	0.25	1.4905	2.5992	0.50	2.2080	2.9826
Process input step load disturbance	0.05	1.1075	2.4230	0.30	1.6943	2.7007
	0.10	1.2013	2.4646	0.35	1.8161	2.7637
	0.15	1.3132	2.5154	0.40	1.9658	2.8445
	0.20	1.4384	2.5742	0.45	2.1022	2.9210
	0.25	1.5698	2.6381	0.50	2.2379	3.0003

---

[7] $K_c^{(632)} = \frac{x_2 T_{m1}\sqrt{1+\frac{x_1 T_{m2}^2}{4(\tau_m + T_{m2})^2}}}{K_m(x_1 T_{m1} - 4[\tau_m + T_{m2}])}$, $T_i^{(632)} = \frac{4T_{m1}(\tau_m + T_{m2})}{x_1 T_{m1} - 4(\tau_m + T_{m2})}$;

$0 \le \frac{\tau_m}{(T_d/N)} \le 2$; $0.1T_{m2} \le \frac{T_d}{N} \le 0.33T_{m2}$.

4.12.5 *Series controller (classical controller 3)*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + T_d s)$$

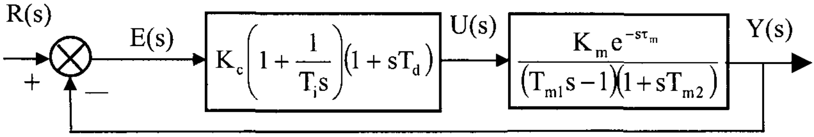

Table 170: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Chidambaram and Kalyan (1996).	[8] $K_c^{(633)}$	$T_i^{(633)}$	T_{m2}	*Model: Method 1*
Huang and Chen (1997), (1999). *Model: Method 4*	[9] $K_c^{(634)}$	$T_i^{(634)}$	T_{m2}	'Good' servo and regulator response
	Direct synthesis: frequency domain criteria			
Ho and Xu (1998). *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	[10] $T_i^{(635)}$	T_{m2}	
	Representative result			
	$\frac{0.7854 T_{m1}}{K_m \tau_m}$	$-T_{m1}$	T_{m2}	$A_m = 2$, $\phi_m = 45^0$

---

[8] $K_c^{(633)}$ and $T_i^{(633)}$ are the proportional gain and integral time defined by De Paor and O'Malley (1989), Rotstein and Lewin (1991) or Chidambaram (1995), for the ideal PI control of an unstable FOLPD model [see Table 31].

[9] $$K_c^{(634)} = \frac{0.5}{K_m}\left[1 + 1.11416\left(\frac{\tau_m}{T_m}\right)^{-1.13076}\right],\ T_i^{(634)} = \frac{K_c^{(634)} K_m T_m}{K_c^{(634)} K_m - 1}\left(\frac{2x_1}{T_m} + \frac{\tau_m}{T_m}\right);$$

Recommended $x_1 = 0.160367 T_m e^{6.06426\frac{\tau_m}{T_{m1}}}$; $0 < \frac{\tau_m}{T_{m1}} < 0.8$.

[10] $$T_i^{(635)} = \frac{1}{1.57\omega_p - \omega_p^2 \tau_m - \frac{1}{T_{m1}}}.$$

4.12.6 *Non-interacting controller 3*

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+\frac{sT_d}{N}}Y(s)$$

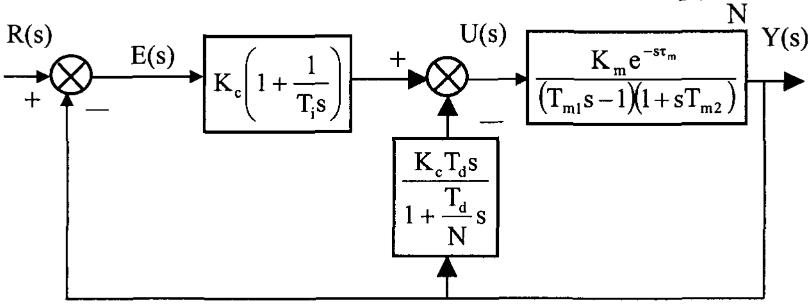

Table 171: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: regulator tuning			
Huang and Lin (1995) – minimum IAE.	[1] $K_c^{(636)}$	$T_i^{(636)}$	$T_d^{(636)}$	*Model: Method 2*; N not specified

[1] Note: equations continued onto page 435. $0.05 \le \tau_m/T_{m1} \le 0.4$; $T_{m2} \le T_{m1}$.

$$K_c^{(636)} = -\frac{1}{K_m}\left[-174.167 - 31.364\frac{\tau_m}{T_{m1}} + 0.4642\left(\frac{\tau_m}{T_{m1}}\right)^{-1.164} - 103.069\frac{T_{m2}}{T_{m1}}\right]$$

$$-\frac{1}{K_m}\left[-83.916\left(\frac{T_{m2}}{T_{m1}}\right)^{2.54} - 66.962\frac{T_{m2}\tau_m}{T_{m1}^2} + 59.496\left(\frac{T_{m1}}{\tau_m}\right)^{1.065}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.014}\right]$$

$$-\frac{1}{K_m}\left[-70.79\frac{T_{m2}}{\tau_m} + 23.121e^{\frac{\tau_m}{T_{m1}}} + 126.924e^{\frac{T_{m2}}{T_{m1}}} + 26.944e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right];$$

$$T_i^{(636)} = T_{m1}\left[0.008 + 2.0718\frac{\tau_m}{T_{m1}} + 6.431\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.4556\frac{T_{m2}}{T_{m1}} + 0.7503\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[2.4484\frac{T_{m2}\tau_m}{T_{m1}^2} - 18.686\left(\frac{\tau_m}{T_{m1}}\right)^3 - 2.9978\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 21.135\left(\frac{T_{m2}\tau_m^2}{T_{m1}^3}\right)\right]$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin (1995) – continued.	[2] $K_c^{(637)}$	$T_i^{(637)}$	$T_d^{(637)}$	N not specified; *Model: Method 2*

$$+T_{m1}\left[12.822\left(\frac{\tau_m T_{m2}^{\ 2}}{T_{m1}^{\ 3}}\right)+39.001\left(\frac{\tau_m}{T_{m1}}\right)^4+22.848\left(\frac{T_{m2}\tau_m^{\ 3}}{T_{m1}^{\ 4}}\right)\right]$$

$$+T_{m1}\left[-4.754\left(\frac{T_{m2}^{\ 3}\tau_m}{T_{m1}^{\ 4}}\right)-0.527\left(\frac{T_{m2}^{\ 2}\tau_m^{\ 2}}{T_{m1}^{\ 4}}\right)+1.64\left(\frac{T_{m2}}{T_{m1}}\right)^4\right];$$

$$T_d^{(636)}=T_{m1}\left[1.1766\frac{\tau_m}{T_{m1}}-4.4623\left(\frac{\tau_m}{T_{m1}}\right)^2+0.5284\frac{T_{m2}}{T_{m1}}-1.4281\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[4.6\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}+11.176\left(\frac{\tau_m}{T_{m1}}\right)^3+1.0886\left(\frac{T_{m2}}{T_{m1}}\right)^3-5.0229\left(\frac{\tau_m^{\ 2}T_{m2}}{T_{m1}^{\ 3}}\right)\right]$$

$$+T_{m1}\left[-0.0301-0.5039\left(\frac{\tau_m T_{m2}^{\ 2}}{T_{m1}^{\ 3}}\right)-9.8564\left(\frac{\tau_m}{T_{m1}}\right)^4-7.528\left(\frac{T_{m2}\tau_m^{\ 3}}{T_{m1}^{\ 4}}\right)\right]$$

$$+T_{m1}\left[-2.3542\left(\frac{\tau_m T_{m2}^{\ 3}}{T_{m1}^{\ 4}}\right)+9.3804\left(\frac{\tau_m^{\ 2}T_{m2}^{\ 2}}{T_{m1}^{\ 4}}\right)-0.1457\left(\frac{T_{m2}}{T_{m1}}\right)^4\right].$$

[2] Note: equations continued onto page 436. $0.05 \le \tau_m/T_{m1} \le 0.25$, $T_{m1} < T_{m2} \le 10T_{m1}$;

$$K_c^{(637)}=-\frac{1}{K_m}\left[1750.08+1637.76\frac{\tau_m}{T_{m1}}+1533.91\left(\frac{\tau_m}{T_{m1}}\right)^{2.1984}-7.917\frac{T_{m2}}{T_{m1}}\right]$$

$$-\frac{1}{K_m}\left[6.187\left(\frac{T_{m2}}{T_{m1}}\right)^{0.791}-6.451\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}+0.002452\left(\frac{T_{m1}}{\tau_m}\right)^{3.2927}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0757}\right]$$

$$-\frac{1}{K_m}\left[1.3729\frac{T_{m2}}{\tau_m}-1739.77e^{\frac{\tau_m}{T_{m1}}}-0.000296e^{\frac{T_{m2}}{T_{m1}}}+2.311e^{\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}}\right];$$

$$T_i^{(637)}=T_{m1}\left[51.678-57.043\frac{\tau_m}{T_{m1}}+1337.29\left(\frac{\tau_m}{T_{m1}}\right)^2+0.1742\frac{T_{m2}}{T_{m1}}-0.1524\left(\frac{T_{m2}}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[7.7266\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}-6011.57\left(\frac{\tau_m}{T_{m1}}\right)^3+0.0213\left(\frac{T_{m2}}{T_{m1}}\right)^3+9135.52\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

Rule	K_c	T_i	T_d	Comment
	Minimum performance index: servo tuning			
Huang and Lin (1995) – minimum IAE.	[3] $K_c^{(638)}$	$T_i^{(638)}$	$T_d^{(638)}$	N not specified; *Model: Method 2*

$$+T_{m1}\left[-65.283\left(\frac{T_{m2}\tau_m^{\ 2}}{T_{m1}^{\ 3}}\right)+0.0645\left(\frac{\tau_m T_{m2}^{\ 2}}{T_{m1}^{\ 3}}\right)+274.851\left(\frac{T_{m2}\tau_m^{\ 3}}{T_{m1}^{\ 4}}\right)\right]$$

$$+T_{m1}\left[0.003926\left(\frac{T_{m2}^{\ 3}\tau_m}{T_{m1}^{\ 4}}\right)-2.0997\left(\frac{T_{m2}^{\ 2}\tau_m^{\ 2}}{T_{m1}^{\ 4}}\right)-0.001077\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[-49.007e^{\frac{\tau_m}{T_{m1}}}+0.000026e^{\frac{T_{m2}}{T_{m1}}}+0.2977e^{\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}}\right];$$

$$T_d^{(637)}=T_{m1}\left[-0.0605+4.6998\frac{\tau_m}{T_{m1}}-29.478\left(\frac{\tau_m}{T_{m1}}\right)^2+0.0117\frac{T_{m2}}{T_{m1}}\right]$$

$$+T_{m1}\left[-0.0129\left(\frac{T_{m2}}{T_{m1}}\right)^2+0.6874\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}+140.135\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[0.002455\left(\frac{T_{m2}}{T_{m1}}\right)^3+1.4712\left(\frac{\tau_m^{\ 2}T_{m2}}{T_{m1}^{\ 3}}\right)-0.1289\left(\frac{\tau_m T_{m2}^{\ 2}}{T_{m1}^{\ 3}}\right)\right]$$

$$+T_{m1}\left[-238.864\left(\frac{\tau_m}{T_{m1}}\right)^4+0.6884\left(\frac{T_{m2}\tau_m^{\ 3}}{T_{m1}^{\ 4}}\right)+0.007725\left(\frac{\tau_m T_{m2}^{\ 3}}{T_{m1}^{\ 4}}\right)\right]$$

$$+T_{m1}\left[-0.1222\left(\frac{\tau_m^{\ 2}T_{m2}^{\ 2}}{T_{m1}^{\ 4}}\right)-0.000135\left(\frac{T_{m2}}{T_{m1}}\right)^4\right].$$

[3] Note: equations continued onto page 437. $0.05\le\tau_m/T_{m1}\le 0.4$, $T_{m2}\le T_{m1}$;

$$K_c^{(638)}=-\frac{1}{K_m}\left[10.741-13.363\frac{\tau_m}{T_{m1}}+0.099\left(\frac{\tau_m}{T_{m1}}\right)^{-1.344}+727.914\frac{T_{m2}}{T_{m1}}\right]$$

$$-\frac{1}{K_m}\left[-708.481\left(\frac{T_{m2}}{T_{m1}}\right)^{0.995}+9.915\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}+84.273\left(\frac{T_{m1}}{\tau_m}\right)^{1.031}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.997}\right]$$

$$-\frac{1}{K_m}\left[-90.959\frac{T_{m2}}{\tau_m}+9.034e^{\frac{\tau_m}{T_{m1}}}-2.386e^{\frac{T_{m2}}{T_{m1}}}-16.304e^{\frac{T_{m2}\tau_m}{T_{m1}^{\ 2}}}\right]$$

$$T_i^{(638)} = T_{m1}\left[-149.685 - 141.418\frac{\tau_m}{T_{m1}} - 88.717\left(\frac{\tau_m}{T_{m1}}\right)^{2.12} - 17.29\frac{T_{m2}}{T_{m1}}\right]$$

$$+T_{m1}\left[20.518\left(\frac{T_{m2}}{T_{m1}}\right)^{0.985} - 12.82\frac{T_{m2}\tau_m}{T_{m1}^2} + 3.611\left(\frac{\tau_m}{T_{m1}}\right)^{0.286}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.988}\right]$$

$$+T_{m1}\left[0.000805\frac{T_{m2}}{\tau_m} + 141.702e^{\frac{\tau_m}{T_{m1}}} - 2.032e^{\frac{T_{m2}}{T_{m1}}} + 10.006e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right];$$

$$T_d^{(638)} = T_{m1}\left[-0.4144 + 15.805\frac{\tau_m}{T_{m1}} - 142.327\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.7287\frac{T_{m2}}{T_{m1}}\right]$$

$$+T_{m1}\left[0.1123\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 18.317\frac{T_{m2}\tau_m}{T_{m1}^2} + 486.95\left(\frac{\tau_m}{T_{m1}}\right)^3 - 10.542\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[204.009\left(\frac{\tau_m^{\,2}T_{m2}}{T_{m1}^{\,3}}\right) + 47.26\left(\frac{\tau_m T_{m2}^{\,2}}{T_{m1}^{\,3}}\right) + 396.349\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[-138.038\left(\frac{\tau_m^{\,3}T_{m2}}{T_{m1}^{\,4}}\right) + 52.155\left(\frac{\tau_m T_{m2}^{\,3}}{T_{m1}^{\,4}}\right) - 646.848\left(\frac{\tau_m^{\,2}T_{m2}^{\,2}}{T_{m1}^{\,4}}\right)\right]$$

$$+T_{m1}\left[19.302\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 4731.72\left(\frac{\tau_m}{T_{m1}}\right)^5 + 425.789\left(\frac{\tau_m^{\,4}T_{m2}}{T_{m1}^{\,5}}\right)\right]$$

$$+T_{m1}\left[-289.476\left(\frac{\tau_m T_{m2}^{\,4}}{T_{m1}^{\,5}}\right) - 841.807\left(\frac{\tau_m^{\,3}T_{m2}^{\,2}}{T_{m1}^{\,5}}\right) + 1313.72\left(\frac{\tau_m^{\,2}T_{m2}^{\,3}}{T_{m1}^{\,5}}\right)\right]$$

$$+T_{m1}\left[-3.7688\left(\frac{T_{m2}}{T_{m1}}\right)^5 + 6264.79\left(\frac{\tau_m}{T_{m1}}\right)^6 - 161.469\left(\frac{\tau_m^{\,5}T_{m2}}{T_{m1}^{\,6}}\right)\right]$$

$$+T_{m1}\left[204.689\left(\frac{\tau_m T_{m2}^{\,5}}{T_{m1}^{\,6}}\right) + 25.706\left(\frac{\tau_m^{\,4}T_{m2}^{\,2}}{T_{m1}^{\,6}}\right) - 791.857\left(\frac{\tau_m^{\,2}T_{m2}^{\,4}}{T_{m1}^{\,6}}\right)\right]$$

$$+T_{m1}\left[648.217\left(\frac{\tau_m T_{m2}}{T_{m1}^{\,2}}\right)^3 - 5.71\left(\frac{T_{m2}}{T_{m1}}\right)^6\right].$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin (1995) – minimum IAE – continued.	[4] $K_c^{(639)}$	$T_i^{(639)}$	$T_d^{(639)}$	N not specified; *Model: Method 2*

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[4] Note: equations continued onto page 439. $0.05 \le \tau_m/T_{m1} \le 0.25$, $T_{m1} < T_{m2} \le 10T_{m1}$;

$$K_c^{(639)} = -\frac{1}{K_m}\left[-130.2 + 85.914\frac{\tau_m}{T_{m1}} + 34.82\left(\frac{\tau_m}{T_{m1}}\right)^{-0.3055} + 10.442\frac{T_{m2}}{T_{m1}}\right]$$

$$-\frac{1}{K_m}\left[-22.547\left(\frac{T_{m2}}{T_{m1}}\right)^{0.5174} - 14.698\frac{T_{m2}\tau_m}{T_{m1}^2} + 52.408\left(\frac{T_{m1}}{\tau_m}\right)^{1.0077}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.9879}\right]$$

$$-\frac{1}{K_m}\left[-51.47\left(\frac{T_{m2}}{\tau_m}\right) + 53.378e^{\frac{\tau_m}{T_{m1}}} - 0.000001e^{\frac{T_{m2}}{T_{m1}}} + 0.286e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right];$$

$$T_i^{(639)} = T_{m1}\left[72.806 - 268.746\frac{\tau_m}{T_{m1}} - 4.9221\left(\frac{T_{m2}}{T_{m1}}\right) + 2468.19\left(\frac{\tau_m}{T_{m1}}\right)^2\right]$$

$$+T_{m1}\left[0.6724\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 151.351\frac{T_{m2}\tau_m}{T_{m1}^2} - 6914.46\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0092\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-795.465\left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3}\right) - 14.27\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) + 5580.17\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[1417.65\left(\frac{\tau_m^3 T_{m2}}{T_{m1}^4}\right) + 0.4057\left(\frac{\tau_m T_{m2}^3}{T_{m1}^4}\right) + 55.536\left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4}\right)\right]$$

$$+T_{m1}\left[-0.001119\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 44.903e^{\frac{\tau_m}{T_{m1}}} + 0.000034e^{\frac{T_{m2}}{T_{m1}}} - 15.694e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right]$$

$$+T_{m1}\left[678778\left(\frac{\tau_m}{T_{m1}}\right)^{19.056}\left(\frac{T_{m2}}{T_{m1}}\right)^{7.3464}\right];$$

$$T_d^{(639)} = T_{m1}\left[175.515 - 86.2\frac{\tau_m}{T_{m1}} + 348.727\left(\frac{\tau_m}{T_{m1}}\right)^{1.1798} - 0.008207\frac{T_{m2}}{T_{m1}}\right]$$

$$+T_{m1}\left[-55.619\left(\frac{T_{m2}}{T_{m1}}\right)^{0.1064} + 0.0418\frac{T_{m2}\tau_m}{T_{m1}^2} + 78.959\left(\frac{\tau_m}{T_{m1}}\right)^{0.0355}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.0827}\right]$$

4.12.7 *Non-interacting controller 8*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)E(s) - K_i(1 + T_{di}s)Y(s)$$

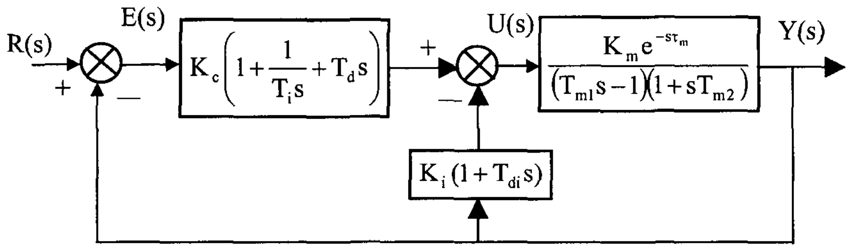

Table 172: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Kwak *et al.* (2000). *Model: Method 3*	[5] $K_c^{(640)}$	$T_i^{(640)}$	$T_d^{(640)}$	Optimal gain margin: regulator response

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$$+ T_{m1}\left[0.005048\left(\frac{T_{m2}}{\tau_m}\right) - 187.01e^{\frac{\tau_m}{T_{m1}}} + 0.000001e^{\frac{T_{m2}}{T_{m1}}} - 0.0149e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right].$$

[5] Note: equations continued onto page 440. $0 \le \frac{T_{m1}}{\tau_m} \le 1 - \frac{T_{m2}}{T_{m1}}$.

$$K_c^{(640)} = \frac{1}{K_m}\left[-0.67 + 0.297\left(\frac{\tau_m}{T_{m1}}\right)^{-2.001} + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} < 0.9 \text{ or}$$

$$K_c^{(640)} = \frac{1}{K_m}\left[-0.365 + 0.260\left(\frac{\tau_m}{T_{m1}} - 1.4\right)^2 + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} \ge 0.9;$$

$$T_i^{(640)} = T_{m1}\left[2.212\left(\frac{\tau_m}{T_{m1}}\right)^{0.520} - 0.3\right],\ \frac{\tau_m}{T_{m1}} < 0.4 \text{ or}$$

Rule	K_c	T_i	T_d	Comment
Kwak *et al.* (2000) - continued.	[6] $K_c^{(641)}$	$T_i^{(641)}$	$T_d^{(641)}$	Optimal gain margin: servo response

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$$T_i^{(640)} = T_{m1}\{-0.975 + 0.910\left(\frac{\tau_m}{T_{m1}} - 1.845\right)^2 + \alpha\},\ \frac{\tau_m}{T_{m1}} \ge 0.4 \text{ with}$$

$$\alpha = \left[1 - e^{-\frac{\xi_m}{0.15 + 0.33\frac{\tau_m}{T_{m1}}}}\right]\left[5.25 - 0.88\left(\frac{\tau_m}{T_{m1}} - 2.8\right)^2\right];$$

$$T_d^{(640)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\xi_m}{-0.15 + 0.939\left(\frac{\tau_m}{T_{m1}}\right)^{-1.121}}}\right]\left[1.45 + 0.969\left(\frac{T_{m1}}{\tau_m}\right)^{1.171}\right] - 1.9 + 1.576\left(\frac{\tau_m}{T_{m1}}\right)^{-0.530}};$$

$$T_{di} = \left(X_1 + X_2\frac{\tau_m}{T_{m1}} + X_3\left[\frac{\tau_m}{T_{m1}}\right]^2\right)T_{m1} \text{ with}$$

$$X_1 = -0.0030 + 0.6482\frac{T_{m2}}{T_{m1}} - 2.2841\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.6221\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 0.9611\left(\frac{T_{m2}}{T_{m1}}\right)^4,$$

$$X_2 = 0.2446 - 1.0410\frac{T_{m2}}{T_{m1}} - 13.6723\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 16.7622\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 5.1471\left(\frac{T_{m2}}{T_{m1}}\right)^4,$$

$$X_3 = 0.1685 + 0.8289\frac{T_{m2}}{T_{m1}} - 9.3630\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.9855\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 7.3803\left(\frac{T_{m2}}{T_{m1}}\right)^4,$$

$$K_i = \frac{1}{\sqrt{|G_m(j\omega_u)(1 + jT_{di}\omega_u)|K_m}}.$$

[6] Note: equations continued onto page 441. $0 \le \frac{T_{m1}}{\tau_m} \le 1 - \frac{T_{m2}}{T_{m1}}$.

$$K_c^{(641)} = \frac{1}{K_m}\left[-0.04 + \left[0.333 + 0.949\left(\frac{\tau_m}{T_{m1}}\right)^{-0.983}\right]\xi_m\right],\ \xi_m \le 0.9 \text{ or}$$

$$K_c^{(641)} = \frac{1}{K_m}\left[-0.544 + 0.308\frac{\tau_m}{T_{m1}} + 1.408\left(\frac{\tau_m}{T_{m1}}\right)^{-0.832}\xi_m\right],\ \xi_m > 0.9\ .$$

$$T_i^{(641)} = T_{m1}\left[2.055 + 0.072\left(\tau_m/T_{m1}\right)\right]\xi_m,\ \tau_m/T_{m1} \le 1 \text{ or}$$

$$T_i^{(641)} = T_{m1}\left[1.768 + 0.329\left(\tau_m/T_{m1}\right)\right]\xi_m,\ \tau_m/T_{m1} > 1;$$

$$T_d^{(641)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\left(\frac{\tau_m}{T_{m1}}\right)^{1.060}\xi_m}{0.870}}\right]\left[0.55 + 1.683\left(\frac{T_{m1}}{\tau_m}\right)^{1.090}\right]};$$

$$T_{di} = \left(X_1 + X_2\frac{\tau_m}{T_{m1}} + X_3\left[\frac{\tau_m}{T_{m1}}\right]^2\right)T_{m1} \text{ with}$$

$$X_1 = -0.0030 + 0.6482\frac{T_{m2}}{T_{m1}} - 2.2841\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.6221\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 0.9611\left(\frac{T_{m2}}{T_{m1}}\right)^4,$$

$$X_2 = 0.2446 - 1.0410\frac{T_{m2}}{T_{m1}} - 13.6723\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 16.7622\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 5.1471\left(\frac{T_{m2}}{T_{m1}}\right)^4,$$

$$X_3 = 0.1685 + 0.8289\frac{T_{m2}}{T_{m1}} - 9.3630\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.9855\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 7.3803\left(\frac{T_{m2}}{T_{m1}}\right)^4;$$

$$K_i = \frac{1}{\sqrt{\left|G_m(j\omega_u)(1 + jT_{di}\omega_u)\right|K_m}}\ .$$

4.13 Unstable SOSPD Model (two unstable poles)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(T_{m2}s-1)}$$

4.13.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

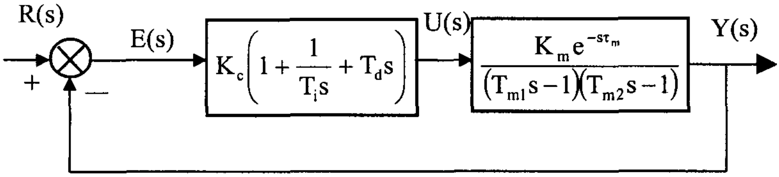

Table 173: PID controller tuning rules – unstable SOSPD model (two unstable poles) $G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(T_{m2}s-1)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Wang and Jin (2004).	[7] $K_c^{(642)}$	$T_i^{(642)}$	$T_d^{(642)}$	*Model: Method 1*

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[7] $K_c^{(642)} = \frac{K_m T_i^{(642)}(T_{m1}-\tau_m)(T_{m2}-\tau_m)}{(T_{CL3}+\tau_m)^3}$,

$$T_i^{(642)} = \frac{T_{CL3}^{\ 3} + 3\tau_m T_{CL3}^{\ 2} + 3(T_{m1}T_{m2} + \tau_m T_{m1} - \tau_m T_{m2})T_{CL3} + \tau_m T_{m2}(T_{m1}-\tau_m)}{(T_{m1}-\tau_m)(\tau_m - T_{m2})},$$

$$T_d^{(642)} = \frac{(T_{m2}+\tau_m-T_{m1})T_{CL3}^{\ 3} + 3T_{m1}T_{m2}T_{CL3}(T_{CL3}+\tau_m) + T_{m1}T_{m2}\tau_m^{\ 2}}{T_i^{(642)}(T_{m1}-\tau_m)(T_{m2}-\tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee *et al.* (2000). *Model: Method 1*	[8] $K_c^{(643)}$	$T_i^{(643)}$	$T_d^{(643)}$	$0 \le \frac{\tau_m}{T_m} < 2$

[8] $K_c^{(643)} = \dfrac{T_i^{(643)}}{K_m(4\lambda + \tau_m - \alpha_1)}$, λ = desired closed loop dynamic parameter.

$$T_i^{(643)} = -T_{m1} - T_{m2} + \alpha_1 - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^{\ 2}}{4\lambda + \tau_m - \alpha_1};$$

$$T_d^{(643)} = \frac{\alpha_2 - (T_{m1} + T_{m2})\alpha_1 + T_{m1}T_{m2} - \dfrac{4\lambda^3 + \alpha_2\tau_m + 0.167\tau_m^{\ 3} - 0.5\alpha_1\tau_m^{\ 2}}{4\lambda + \tau_m - \alpha_1}}{T_i^{(643)}} - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^{\ 2}}{4\lambda + \tau_m - \alpha_1};$$

with α_1, α_2 values determined by solving $1 - \dfrac{\left(\alpha_2 s^2 + \alpha_1 s + 1\right)e^{-\tau_m s}}{(\lambda s + 1)^4}\Big|_{s = \frac{1}{T_{m1}}, \frac{1}{T_{m2}}} = 0$.

4.13.2 *Ideal controller with set-point weighting 1*

$$U(s) = K_c\left(F_pR(s) - Y(s)\right) + \frac{K_c}{T_is}\left(F_iR(s) - Y(s)\right) + K_cT_ds\left(F_dR(s) - Y(s)\right)$$

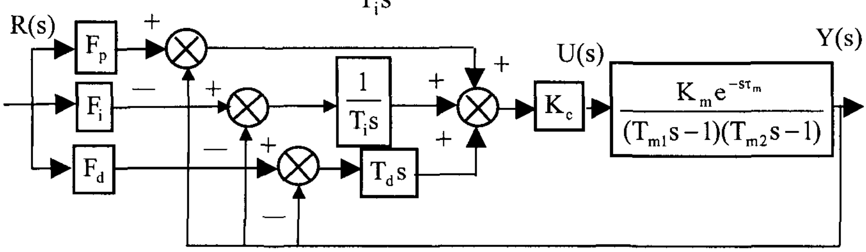

Table 174: PID controller tuning rules – unstable SOSPD model (two unstable poles)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(T_{m2}s-1)}$$

Rule	K_c	T_i	T_d	Comment
	Robust			
Lee *et al.* (2000). *Model: Method 1*	[9] $K_c^{(644)}$	$T_i^{(644)}$	$T_d^{(644)}$	$0 \le \frac{\tau_m}{T_m} < 2$
	$F_p = F_i = F_d = \frac{1}{\alpha_2 s^2 + \alpha_1 s + 1}$, α_1, α_2 determined by $1 - \frac{\left(\alpha_2 s^2 + \alpha_1 s + 1\right)e^{-\tau_m s}}{(\lambda s + 1)^4}\Big\vert_{s=\frac{1}{T_{m1}},\frac{1}{T_{m2}}} = 0$			

[9] $K_c^{(644)} = \frac{T_i^{(644)}}{K_m(4\lambda + \tau_m - \alpha_1)}$, λ = desired closed loop dynamic parameter.

$$T_i^{(644)} = -T_{m1} - T_{m2} + \alpha_1 - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1},$$

$$T_d^{(644)} = \frac{\alpha_2 - (T_{m1} + T_{m2})\alpha_1 + T_{m1}T_{m2} - \dfrac{4\lambda^3 + \alpha_2\tau_m + 0.167\tau_m^3 - 0.5\alpha_1\tau_m^2}{4\lambda + \tau_m - \alpha_1}}{T_i^{(644)}} - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1}.$$

4.14 Unstable SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

4.14.1 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

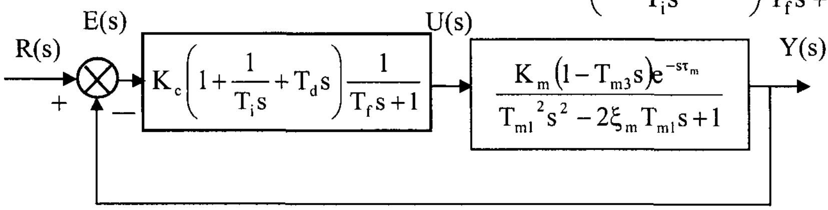

Table 175: PID controller tuning rules – unstable SOSPD model with a positive zero
$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Sree and Chidambaram (2004).	[10] $K_c^{(645)}$	[11] $T_i^{(645)}$	$T_d^{(645)}$	*Model: Method 1*

[11] Desired closed loop transfer function $= \frac{(1 - sT_{m3})(1 + T_i^{(645)}s)e^{-s\tau_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}$.

---

[10] $K_c^{(645)} = \frac{T_i^{(645)}}{K_m\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(645)}\right)}$,

$T_d^{(645)} = \frac{0.5\left(T_i^{(645)} - 0.5\tau_m\right)\tau_m}{T_i^{(645)}}$, $T_f = \frac{0.5\tau_m T_{CL}^{(1)} T_{CL}^{(2)} x_2}{T_{m1}^2\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(645)}\right)}$.

[11] Equations continued into the footnote on page 446.

$T_i^{(645)} = \frac{x_1 - T_{m1}^2\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + \tau_m\right)}{0.5T_{m1}^2 T_{m3}\tau_m - T_{m1}^4}$ with

$x_1 = T_{CL}^{(1)} T_{CL}^{(2)} x_2 + 0.5\tau_m T_{m1}^2\left(T_{CL}^{(1)} T_{CL}^{(2)} + T_{CL}^{(2)} T_{CL}^{(3)} + T_{CL}^{(3)} T_{CL}^{(1)}\right) + \xi_m T_{m1}\tau_m T_{CL}^{(1)} T_{CL}^{(2)} x_2$,

$$x_2 = \frac{T_{m1}{}^4\left(0.5T_{m3}\tau_m - T_{m1}{}^2\right)\left(T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m\left[T_{CL}^{(1)} + T_{CL}^{(2)} - T_{m3}\right]T_{CL}^{(2)} + 2\xi_m T_{m3}x_3\right)}{x_4}$$

$$+\frac{T_{m1}{}^4\left(-2\xi_m T_{m1} + T_{m3} + 0.5\tau_m\right)\left(0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m3}{}^2 x_3\right)}{x_4},$$

$$x_3 = T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m,$$

$$x_4 = -T_{m1}{}^2\left(0.5T_{m3}\tau_m - T_{m1}{}^2\right)\left(T_{m1}{}^2\left[T_{CL}^{(1)} + T_{CL}^{(2)} + 0.5\tau_m + 2\xi_m T_{m1}\right] - 0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)}\right)$$
$$-\left(-2\xi T_{m1} + T_{m3} + 0.5\tau_m\right)$$
$$\left(T_{m1}{}^2\left[T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m\left(T_{CL}^{(1)} + T_{CL}^{(2)}\right)\right] + \xi_m T_{m1}\tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}{}^4\right).$$

4.14.2 *Non-interacting controller based on the two degree of freedom structure 1*

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$

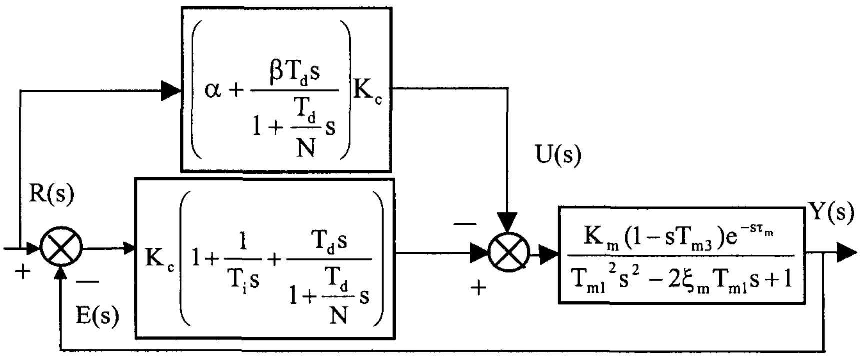

Table 176: PID controller tuning rules — unstable SOSPD model with a positive zero $G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: time domain criteria			
Sree and Chidambaram (2004). *Model: Method 1*	[12] $K_c^{(646)}$	[13] $T_i^{(646)}$	$T_d^{(646)}$	$\beta = 0$; $N = \infty$

[13] Desired closed loop transfer function = $\frac{(1 - sT_{m3})(1 + T_i^{(646)}s)e^{-s\tau_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}$.

---

[12] $K_c^{(646)} = \frac{T_i^{(646)}}{K_m\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(646)}\right)}$,

$T_d^{(646)} = \frac{0.5\left(T_i^{(646)} - 0.5\tau_m\right)\tau_m}{T_i^{(646)}}$, $\alpha = 1 - \frac{T_{m3} + 0.5\left(T_{CL}^{(1)} + T_{CL}^{(2)}\right)}{T_i^{(646)}}$.

[13] Equations continued into the footnote on page 448.

$$T_i^{(646)} = \frac{x_1 - T_{m1}^{\,2}\left(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + \tau_m\right)}{0.5T_{m1}^{\,2}T_{m3}\tau_m - T_{m1}^{\,4}} \text{ with}$$

$$x_1 = T_{CL}^{(1)}T_{CL}^{(2)}x_2 + 0.5\tau_m T_{m1}^{\,2}\left(T_{CL}^{(1)}T_{CL}^{(2)} + T_{CL}^{(2)}T_{CL}^{(3)} + T_{CL}^{(3)}T_{CL}^{(1)}\right) + \xi_m T_{m1}\tau_m T_{CL}^{(1)}T_{CL}^{(2)}x_2,$$

$$x_2 = \frac{T_{m1}^{\,4}\left(0.5T_{m3}\tau_m - T_{m1}^{\,2}\right)\left(T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m\left[T_{CL}^{(1)} + T_{CL}^{(2)} - T_{m3}\right]T_{CL}^{(2)} + 2\xi_m T_{m3}x_3\right)}{x_4}$$
$$+ \frac{T_{m1}^{\,4}\left(-2\xi_m T_{m1} + T_{m3} + 0.5\tau_m\right)\left(0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m3}^{\,2}x_3\right)}{x_4},$$

$$x_3 = T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m,$$

$$x_4 = -T_{m1}^{\,2}\left(0.5T_{m3}\tau_m - T_{m1}^{\,2}\right)\left(T_{m1}^{\,2}\left[T_{CL}^{(1)} + T_{CL}^{(2)} + 0.5\tau_m + 2\xi_m T_{m1}\right] - 0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)}\right)$$
$$-\left(-2\xi T_{m1} + T_{m3} + 0.5\tau_m\right)$$
$$\left(T_{m1}^{\,2}\left[T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m\left(T_{CL}^{(1)} + T_{CL}^{(2)}\right)\right] + \xi_m T_{m1}\tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}^{\,4}\right).$$

4.15 Delay Model $G_m(s) = K_m e^{-s\tau_m}$

4.15.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

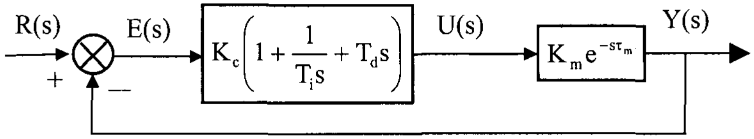

Table 177: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Callender *et al.* (1935/6). *Model: Method 1*	[1] $\frac{0.749}{K_m \tau_m}$	$2.734\tau_m$	$0.303\tau_m$	*Representative results deduced from graphs*
	[2] $\frac{1.24}{K_m \tau_m}$	$1.31\tau_m$	$0.303\tau_m$	

[1] Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

[2] Decay ratio = 0.12; Period of decaying oscillation = $6.28\tau_m$

4.15.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}$$

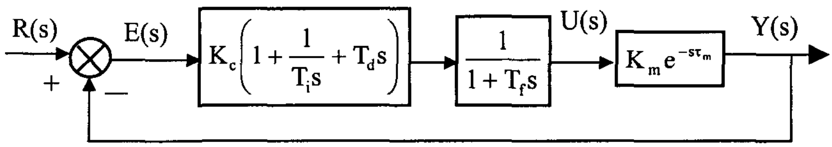

Table 178: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	T_d	Comment
	Robust			
Bequette (2003) – *page 300. Model: Method 1*	$\frac{\tau_m}{K_m(4\lambda+\tau_m)}$	$0.5\tau_m$	$0.167\tau_m$	$T_f = \frac{2\lambda^2 - 0.167\tau_m^{\ 2}}{4\lambda+\tau_m}$

4.15.3 *Classical controller 1* $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$

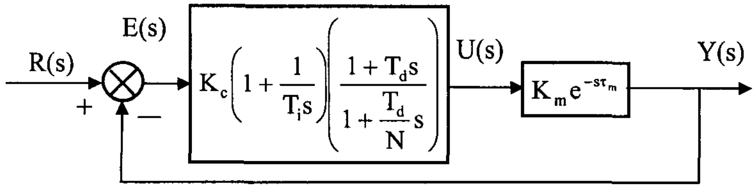

Table 179: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	T_d	Comment
	Process reaction			
Hartree *et al.* (1937). *Model: Method 1*	[3] $\frac{0.70}{K_m \tau_m}$	$2.66\tau_m$	τ_m	*Representative results*
	[4] $\frac{0.78}{K_m \tau_m}$	$2.97\tau_m$	$0.5\tau_m$	

[3] N = 2. Decay ratio = 0.15; Period of decaying oscillation = $4.49\tau_m$

[4] N = 2. Decay ratio = 0.042; Period of decaying oscillation = $5.03\tau_m$

4.16 General Model with a Repeated Pole $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$

4.16.1 *Ideal controller* $G_c(s) = K_c\left(1 + \dfrac{1}{T_i s} + T_d s\right)$

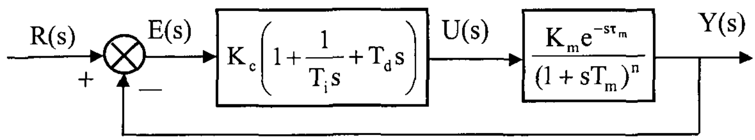

Table 180: PID controller tuning rules – general model with a repeated pole
$G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Skoczowski and Tarasiejski (1996).	[5] $K_c^{(647)}$	T_m	$T_d^{(647)}$	*Model: Method 1*

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[5] $K_c^{(647)} \le \dfrac{\omega_g T_m}{K_m}\sqrt{\dfrac{\left(1+\omega_g{}^2 T_m{}^2\right)^{n-1}}{1+\omega_g{}^2[T_d{}^{(647)}]^2}}$, $T_d^{(647)} = \dfrac{n-1}{n+2}T_m$, $n \ge 2$ with

$$\omega_g = \frac{-T_m\left[n^2 - 2n - 2 + \dfrac{2n+4}{\pi}\dfrac{\tau_m}{T_m} + \dfrac{4n+2}{\pi}\phi_m\right] \pm b}{2T_m{}^2\left[n^2 - 4n + 3 + \dfrac{4n+2}{\pi}\dfrac{\tau_m}{T_m} + \dfrac{2n-2}{\pi}\phi_m\right]}, \text{ and with}$$

$$b = T_m\sqrt{\left[n^2 - 2n - 2 + \frac{2n+4}{\pi}\frac{\tau_m}{T_m} + \frac{4n+2}{\pi}\phi_m\right]^2 + c}, \text{ with}$$

$$c = 4(n+2)\left(1 + \frac{2}{\pi}\phi_m\right)\left[n^2 - 4n + 3 + \frac{4n+2}{\pi}\left(\frac{\tau_m}{T_m}\right) + \frac{2n-2}{\pi}\phi_m\right],$$

ϕ_m = specified phase margin.

4.16.2 *Ideal controller in series with a first order lag*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$$

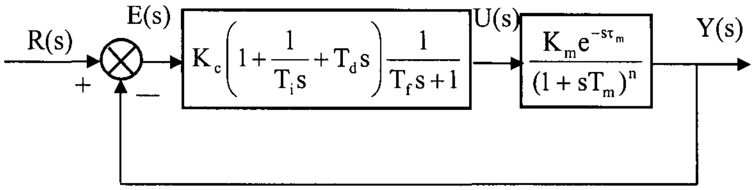

Table 181: PID controller tuning rules – general model with a repeated pole

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis: frequency domain criteria			
Schaedel (1997). *Model: Method 2*	$\frac{0.563}{K_m}\left[\frac{n+1}{n-2}\right]$	[6] $T_i^{(648)}$	$T_d^{(648)}$	$T_f = T_d^{(648)}/N$, $5 \le N \le 20$

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[6] $T_i^{(648)} = \frac{3(n+1)}{5n-1}\left(\tau_m + T_{ar}\right)$, $T_d^{(648)} = \frac{n+1}{6n}\left(\tau_m + T_{ar}\right)$.

4.17 General Stable Non-Oscillating Model with a Time Delay

4.17.1 *Ideal controller* $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$

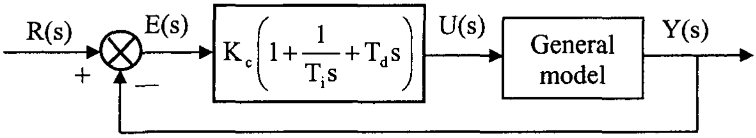

Table 182: PID controller tuning rules – general stable non-oscillating model with a time delay

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Gorez and Klàn (2000). *Model: Method 1*	[7] $K_c^{(649)}$	$T_i^{(649)}$	$T_d^{(649)}$	
			$T_d^{(650)}$	alternative tuning
			$0.25T_i^{(649)}$	alternative tuning

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[7] $K_c^{(649)} = \frac{T_i^{(649)}}{T_i^{(649)} + \tau_m}$, $T_i^{(649)} = T_{ar}\frac{1 + \sqrt{1 + 2\left(\frac{\tau_m}{T_{ar}}\right)^2} - 2\frac{\tau_m}{T_{ar}}}{2}$,

$$T_d^{(649)} = \frac{T_{ar}}{T_i^{(649)}}\left[\left(T_i^{(649)} + T_{cr}\right)\frac{T_{cr}}{T_{ar}} + T_i^{(649)} - T_{aa} - \frac{\tau_m^{\;2}}{2T_{ar}}\left(1 - K_c^{(649)}\left\{1 + \frac{2\tau_m}{3T_i^{(649)}}\right\}\right)\right],$$

$T_d^{(650)} = T_i^{(649)}\frac{\tau_m}{T_{ar}}\left(1 - \frac{\tau_m}{T_{ar}}\right)$, T_{ar} = average residence time of the process (which equals $T_m + \tau_m$ for a FOLPD process, for example); T_{aa} = additional apparent time constant; $T_{cr} = \left[1 - K_c^{(649)}\left(1 + \frac{\tau_m}{2T_i^{(649)}}\right)\right]\tau_m$.

4.18 Fifth Order System plus Delay Model

$$G_m(s)=\frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_s s^5)e^{-s\tau_m}}{\left(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5\right)}$$

4.18.1 *Ideal controller* $G_c(s)=K_c\left(1+\frac{1}{T_is}+T_ds\right)$

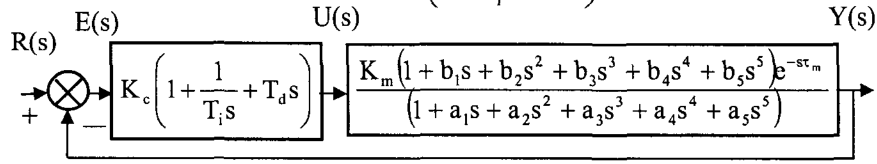

Table 183: PID controller tuning rules – fifth order model with delay $G_m(s)=\frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_s s^5)e^{-s\tau_m}}{\left(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5\right)}$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Vrančić *et al.* (1999).	[8] $K_c^{(651)}$	$T_i^{(651)}$	$T_d^{(651)}$	*Model: Method 1*

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[8] Note: equations continued into the footnote on page 456.

$$K_c^{(651)}=\frac{a_1^3-a_1^2b_1+a_1b_2-2a_1a_2+a_2b_1+a_3-b_3+y_1}{2K_m\left[-a_1^2b_1+a_1a_2+a_1b_1^2-a_3-b_1b_2+b_3+y_2\right]}, \text{ with}$$

$$y_1=\tau_m\left(a_1^2-a_1b_1-a_2+b_2\right)+0.5\left(a_1-b_1\right)\tau_m^2+0.167\tau_m^3 \text{ and}$$

$$y_2=\left(a_1-b_1\right)^2\tau_m+\left(a_1-b_1\right)\tau_m^2+0.333\tau_m^3-\left(a_1-b_1+\tau_m\right)^2T_d^{(651)};$$

$$T_i^{(651)}=\frac{a_1^3-a_1^2b_1+a_1b_2-2a_1a_2+a_2b_1+a_3-b_3+y_1}{\left[a_1^2-a_1b_1-a_2+b_2+\left(a_1-b_1\right)\tau_m+0.5\tau_m^2-\left(a_1-b_1+\tau_m\right)T_d^{(651)}\right]};$$

$T_d^{(651)}$ is found by solving the following equation:

$$T_d^{(651)}x_1+360x_2-360\tau_mx_3+180\tau_m^2x_4-60\tau_m^3x_5-15\tau_m^4x_6$$
$$-3\tau_m^5x_7+7\tau_m^6\left(b_1-a_1\right)-\tau_m^7=0, \text{ with}$$

$$x_1=360[a_1^4b_2-a_1^3(a_2b_1+a_3+b_1b_2+b_3)$$
$$+a_1^2(a_2^2+a_2(b_1^2-2b_2)+3a_3b_1+2a_4+b_1b_3+b_2^2-b_4)$$
$$-a_1(a_2(2a_3-3b_3)+a_3(2b_1^2-b_2)+3a_4b_1+a_5-b_1b_4+2b_2b_3-b_5)$$

$$-a_2b_1b_3+a_3^2+a_3(b_1b_2-2b_3)+a_4b_1^2+a_5b_1-b_1b_5+b_3^2]$$
$$-360\tau_m[a_1^4b_1-a_1^3(a_2+b_1^2+2b_2)+3a_1^2(a_3+b_1b_2)$$
$$+a_1(3a_2b_2-3a_3b_1-3a_4-b_1b_3-2b_2^2+2b_4)$$
$$-a_2(b_1b_2+b_3)+a_3(b_1^2-b_2)+2a_4b_1+a_5-b_1b_4+2b_2b_3-b_5]$$
$$+180\tau_m^2[a_1^4-4a_1^3b_1+3a_1^2(b_1^2+b_2)+a_1(3a_2b_1-3a_3-5b_1b_2+b_3)$$
$$-a_2(b_1^2+2b_2)+a_3b_1+2a_4+b_1b_3+2(b_2^2-b_4)]$$
$$+60\tau_m^3[4a_1^3-9a_1^2b_1-a_1(3a_2-5b_1^2-4b_2)+4a_2b_1-a_3-5b_1b_2+b_3]$$
$$+15\tau_m^4[9a_1^2-14a_1b_1-4a_2+5b_1^2+4b_2]-42\tau_m^5(b_1-a_1)+7\tau_m^6,$$
$$x_2=a_1^4b_3-a_1^3(a_2b_2+a_1b_3+a_4+b_1b_3)$$
$$+a_1^2(a_2^2b_1+a_2(2a_3+b_1b_2-3b_3)+a_3(b_1^2+b_2)+2a_4b_1+a_5+b_2b_3-b_5)$$
$$-a_1(a_2^3+a_2^2(b_1^2-2b_2)+a_2(2a_3b_1-2b_1b_3+b_2^2-b_4)$$
$$+2a_3^2+a_3(2b_1b_2-3b_3)+a_4(b_1^2+b_2)+a_5b_1-b_1b_5+b_3^2)$$
$$+a_2^2a_3+a_2[a_3(b_1^2-2b_2)-a_5-b_1b_4+b_5]+a_3^2b_1$$
$$+a_3(a_4-b_1b_3+b_2^2-b_4)+a_4(b_1b_2-b_3)+a_5b_2-b_2b_5+b_3b_4,$$
$$x_3=a_1^4b_2-a_1^3(a_2b_1+a_3+b_1b_2+b_3)$$
$$+a_1^2(a_2^2+a_2(b_1^2-2b_2)+3a_3b_1+2a_4+b_1b_3+b_2^2-b_4)$$
$$-a_1(a_2(2a_3-3b_3)+a_3(2b_1^2-b_2)+3a_4b_1+a_5-b_1b_4+2b_2b_3-b_5)$$
$$-a_2b_1b_3+a_3^2+a_3(b_1b_2-2b_3)+a_4b_1^2+a_5b_1-b_1b_5+b_3^2,$$
$$x_4=a_1^4b_1-a_1^3(a_2+b_1^2+2b_2)$$
$$+3a_1^2(a_3+b_1b_2)+a_1(3a_2b_2-3a_3b_1-3a_4-b_1b_3-2b_2^2+2b_4)$$
$$-a_2(b_1b_2+b_3)+a_3(b_1^2-b_2)+2a_4b_1+a_5-b_1b_4+2b_2b_3-b_5),$$
$$x_5=a_1^4-4a_1^3b_1+3a_1^2(b_1^2+b_2)+a_1(3a_2b_1-3a_3-5b_1b_2+b_3)$$
$$-a_2(b_1^2+2b_2)+a_3b_1+2a_4+b_1b_3+2(b_2^2-b_4),$$
$$x_6=4a_1^3-9a_1^2b_1-a_1(3a_2-5b_1^2-4b_2)+4a_2b_1-a_3-5b_1b_2+b_3,$$
$$x_7=9a_1^2-14a_1b_1-4a_2+5b_1^2+4b_2.$$

4.18.2 *Controller with filtered derivative*

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + (T_d/N)s}\right)$$

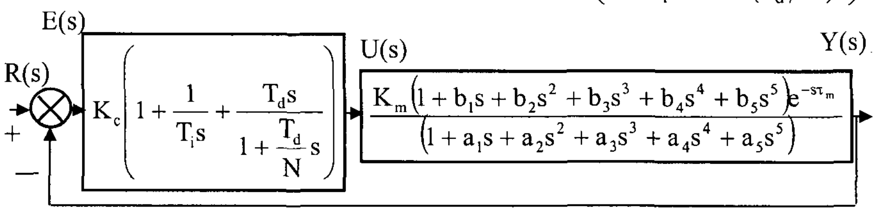

Table 184: PID controller tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5)e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5)}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Vrančić (1996) – *pages 118-119. Model: Method 1*	$\frac{T_i^{(652)}}{2\left(A_1 - T_i^{(652)}\right)}$	[9] $T_i^{(652)}$	$T_d^{(652)}$	$N < 10$

---

[9] $T_i^{(652)} = \dfrac{A_3}{A_2 - T_d^{(652)}A_1 - \dfrac{[T_d^{(652)}]^2}{N}}$,

$$T_d^{(652)} = \frac{-(A_3^2 - A_5A_1) + \sqrt{(A_3^2 - A_5A_1)^2 - \frac{4}{N}(A_3A_2 - A_5)(A_5A_2 - A_4A_3)}}{\frac{2}{N}(A_3A_2 - A_5)},$$

$A_1 = K_m(a_1 - b_1 + \tau_m)$, $A_2 = K_m(b_2 - a_2 + A_1a_1 - b_1\tau_m + 0.5\tau_m^2)$,

$A_3 = K_m(a_3 - b_3 + A_2a_1 - A_1a_2 + b_2\tau_m - 0.5b_1\tau_m^2 + 0.167\tau_m^3)$,

$A_4 = K_m(b_4 - a_4 + A_3a_1 - A_2a_2 + A_1a_3 - b_3\tau_m + 0.5b_2\tau_m^2 + 0.167b_1\tau_m^3 + 0.042\tau_m^4)$

$$A_5 = K_m(a_5 - b_5 + A_4a_1 - A_3a_2 + A_2a_3 - A_1a_4 + b_4\tau_m - 0.5b_3\tau_m^2) + K_m(0.167b_2\tau_m^3 - 0.042b_1\tau_m^4 + 0.008\tau_m^5).$$

Rule	K_c	T_i	T_d	Comment
Vrančić *et al.* (1999). *Model: Method 1*	[10] $K_c^{(653)}$	$T_i^{(653)}$	$T_d^{(653)}$	$8 \le N \le 20$

---

[10] $K_c^{(653)} = \dfrac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{2K_m\left[-a_1^2 b_1 + a_1 a_2 + a_1 b_1^2 - a_3 - b_1 b_2 + b_3 + (a_1 - b_1)^2 \tau_m + y_2\right]}$, with

$y_1 = \tau_m\left(a_1^2 - a_1 b_1 - a_2 + b_2\right) + 0.5\left(a_1 - b_1\right)\tau_m^2 + 0.167\tau_m^3$ and

$y_2 = \left(a_1 - b_1\right)\tau_m^2 + 0.333\tau_m^3 - \left(a_1 - b_1 + \tau_m\right)^2 T_d^{(653)} - \dfrac{[T_d^{(653)}]^2}{N}\left(a_1 - b_1 + \tau_m\right)$;

$$T_i^{(653)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{\left[a_1^2 - a_1 b_1 - a_2 + b_2 + \left(a_1 - b_1\right)\tau_m + 0.5\tau_m^2 - \left(a_1 - b_1 + \tau_m\right)T_d^{(653)} - \dfrac{[T_d^{(653)}]^2}{N}\right]}$$

$T_d^{(653)}$ is found by solving the following equation:

$$[T_d^{(653)}]^4 N^{-3} z_1 + [T_d^{(653)}]^3 N^{-2} z_2 + [T_d^{(653)}]^2 N^{-1} z_3 + T_d^{(653)} x_1 + 360 x_2 - 360\tau_m x_3 + 180\tau_m^2 x_4 - 60\tau_m^2 x_5 - 15\tau_m^4 x_6 - 3\tau_m^5 x_7 + 7\tau_m^6\left(b_1 - a_1\right) - \tau_m^7 = 0$$

with $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 as defined in Table 183; z_1, z_2 and z_3 are defined as follows:

$$z_1 = 360[a_1^3 - a_1^2 b_1 + a_1(b_2 - 2a_2) + a_2 b_1 + a_3 - b_3] + 360\tau_m[a_1^2 - a_1 b_1 - a_2 + b_2] + 180\tau_m^2(a_1 - b_1) + 60\tau_m^3,$$

$$z_2 = 360(a_1 - b_1)[a_1^3 - a_1^2 b_1 + a_1(b_2 - 2a_2) + a_2 b_1 + a_3 - b_3] + 360\tau_m[2a_1^3 - 3a_1^2 b_1 - a_1(3a_2 - b_1^2 - 2b_2) + 2a_2 b_1 + a_3 - b_1 b_2 - b_3] + 180\tau_m^2(3a_1^2 - 4a_1 b_1 - 2a_2 + b_1^2 + 2b_2) + 240\tau_m^3(a_1 - b_1) + 60\tau_m^4,$$

$z_3 = z_4 + z_5\tau_m + z_6\tau_m^2 + z_7\tau_m^3 + 135(a_1 - b_1)\tau_m^4 + 27\tau_m^5$, with

$$z_4 = -360[a_1^4 b_1 - a_1^3(a_2 + b_1^2 + b_2) + a_1^2(2(a_3 + b_1 b_2) - a_2 b_1) + a_1(a_2^2 + a_2(b_1^2 + b_2) - a_3 b_1 - 2a_4 - b_1 b_3 - b_2^2 + b_4) - a_2(a_3 + b_1 b_2) + a_4 b_1 + a_5 + b_2 b_3 - b_5],$$

$$z_5 = 360[a_1^4 - 3a_1^3 b_1 + a_1^2(2(b_1^2 + b_2) - a_2) + a_1(3a_2 b_1 - a_3 - 3b_1 b_2) - a_2(b_1^2 + b_2) + a_4 + b_1 b_3 + b_2^2 - b_4],$$

$z_6 = 540[a_1^3 - 2a_1^2 b_1 - a_1(a_2 - b_1^2 - b_2) + b_1(a_2 - b_2)]$ and

$z_7 = 180[2a_1^2 - 3a_1 b_1 - a_2 + b_1^2 + b_2]$.

Rule	K_c	T_i	T_d	Comment
Vrančić *et al.* (2004a), Vrančić and Lumbar (2004). *Model: Method 1*	[11] $K_c^{(654)}$	$T_i^{(654)}$	$T_d^{(654)}$	$8 \le N \le 20$

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[11] $K_c^{(654)}$, $T_i^{(654)}$ and $T_d^{(654)}$ are determined by solving the following equations:

$$K_c^{(654)} \cdot T_d^{(654)} = \frac{0.5\frac{A_2}{A_1}\left(A_4A_1 - A_5K_m\right) - 0.5\frac{A_4}{K_m}\left(A_1A_2 - A_3K_m\right)}{A_5A_1K_m + A_1A_2A_3 - A_4A_1^2 - A_3^2K_m},$$

$$\frac{K_c^{(654)}}{T_i^{(654)}} = \frac{1 + 2K_mK_c^{(654)} + K_m^2\left[K_c^{(654)}\right]^2}{2\left(A_1 + K_m^2K_c^{(654)}T_d^{(654)}\right)} \text{ and}$$

$$K_c^{(654)} = \frac{A_3K_m - A_1^2K_mK_c^{(654)}T_d^{(654)} - A_2A_1 + A_2K_m^2K_c^{(654)}T_d^{(654)} + x}{2A_1A_2K_m - A_1^3 - A_3K_m^2}, \text{ with}$$

$$x^2 = \left[\left(2A_1K_mK_c^{(654)}T_d^{(654)} + A_2\right)A_2 - A_1A_3 - A_3\left(K_m^2 + A_1^2\right)K_c^{(654)}T_d^{(654)}\right] \left[A_1 + K_m^2K_c^{(654)}T_d^{(654)}\right],$$

and with

$$A_1 = K_m\left(a_1 - b_1 + \tau_m\right), \quad A_2 = K_m\left(b_2 - a_2 + A_1a_1 - b_1\tau_m + 0.5\tau_m^2\right),$$

$$A_3 = K_m\left(a_3 - b_3 + A_2a_1 - A_1a_2 + b_2\tau_m - 0.5b_1\tau_m^2 + 0.167\tau_m^3\right),$$

$$A_4 = K_m\left(b_4 - a_4 + A_3a_1 - A_2a_2 + A_1a_3 - b_3\tau_m + 0.5b_2\tau_m^2 + 0.167b_1\tau_m^3 + 0.042\tau_m^4\right)$$

$$A_5 = K_m\left(a_5 - b_5 + A_4a_1 - A_3a_2 + A_2a_3 - A_1a_4 + b_4\tau_m - 0.5b_3\tau_m^2\right) + K_m\left(0.167b_2\tau_m^3 - 0.042b_1\tau_m^4 + 0.008\tau_m^5\right).$$

4.18.3 *Non-interacting controller 7*

$$U(s)=\frac{K_c}{T_i s}E(s)-K_c\left(1+\frac{T_d s}{1+\frac{T_d}{N}s}\right)Y(s)$$

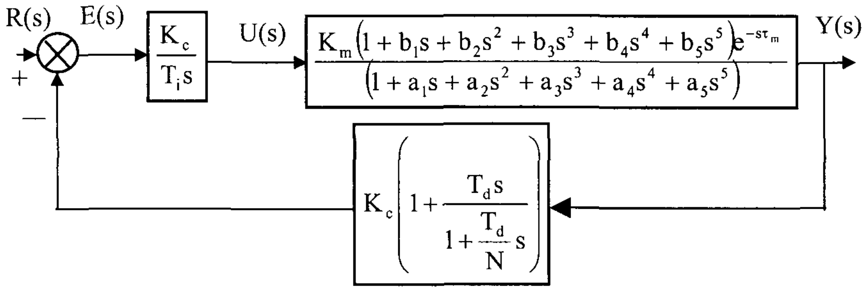


Table 185: PID controller tuning rules – fifth order model with delay

$$G_m(s)=\frac{K_m(1+b_1 s+b_2 s^2+b_3 s^3+b_4 s^4+b_5 s^5)e^{-s\tau_m}}{\left(1+a_1 s+a_2 s^2+a_3 s^3+a_4 s^4+a_5 s^5\right)}$$

Rule	K_c	T_i	T_d	Comment
	Direct synthesis			
Vrančić *et al.* (2004a), Vrančić and Lumbar (2004).	[12] $K_c^{(655)}$	$T_i^{(655)}$	$T_d^{(655)}$	*Model: Method 1*; $8 \le N \le 20$

[12] Note: equations continued into the footnote on page 461. $K_c^{(655)}$, $T_i^{(655)}$ and $T_d^{(655)}$ are determined by solving the following equations:

$$K_c^{(655)}\,.\,T_d^{(655)}=\frac{0.5\frac{A_2}{A_1}\left(A_4A_1-A_5K_m\right)-0.5\frac{A_4}{K_m}\left(A_1A_2-A_3K_m\right)}{A_5A_1K_m+A_1A_2A_3-A_4A_1^{\,2}-A_3^{\,2}K_m},$$

$$\frac{K_c^{(655)}}{T_i^{(655)}}=\frac{1+2K_mK_c^{(655)}+K_m^{\,2}\left[K_c^{(655)}\right]^2}{2\left(A_1+K_m^{\,2}K_c^{(655)}T_d^{(655)}\right)}\text{ and}$$

$$K_c^{(655)}=\frac{A_3K_m-A_1^{\,2}K_mK_c^{(655)}T_d^{(655)}-A_2A_1+A_2K_m^{\,2}K_c^{(655)}T_d^{(655)}+x}{2A_1A_2K_m-A_1^{\,3}-A_3K_m^{\,2}},\text{ with}$$

$$x^2=\left[\left(2A_1K_mK_c^{(655)}T_d^{(655)}+A_2\right)A_2-A_1A_3-A_3\left(K_m^{\,2}+A_1^{\,2}\right)K_c^{(655)}T_d^{(655)}\right]\left[A_1+K_m^{\,2}K_c^{(655)}T_d^{(655)}\right],$$

and with

$$A_1 = K_m(a_1 - b_1 + \tau_m) \text{ , } A_2 = K_m\left(b_2 - a_2 + A_1 a_1 - b_1\tau_m + 0.5\tau_m{}^2\right),$$

$$A_3 = K_m\left(a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2\tau_m - 0.5b_1\tau_m{}^2 + 0.167\tau_m{}^3\right),$$

$$A_4 = K_m\left(b_4 - a_4 + A_3 a_1 - A_2 a_2 + A_1 a_3 - b_3\tau_m + 0.5b_2\tau_m{}^2 + 0.167b_1\tau_m{}^3 + 0.042\tau_m{}^4\right)$$

$$A_5 = K_m\left(a_5 - b_5 + A_4 a_1 - A_3 a_2 + A_2 a_3 - A_1 a_4 + b_4\tau_m - 0.5b_3\tau_m{}^2\right) + K_m\left(0.167b_2\tau_m{}^3 - 0.042b_1\tau_m{}^4 + 0.008\tau_m{}^5\right).$$

Chapter 5

Performance and Robustness Issues in the Compensation of FOLPD Processes with PI and PID Controllers

5.1 Introduction

This chapter will discuss the compensation of FOLPD processes using PI and PID controllers whose parameters are specified using appropriate tuning rules. The gain margin, phase margin and maximum sensitivity of the compensated system as the ratio of time delay to time constant of the process varies, are used as ways of judging the performance and robustness of the system. This work follows the method of Ho *et al.* (1995b), (1996), in which good approximations for the gain and phase margins of the PI or PID controlled system are analytically calculated. The method will be extended to determine analytically the maximum sensitivity of the compensated system. Insight will be obtained into the range of time delay to time constant ratios over which it is sensible to apply various tuning rules, to compensate a FOLPD process.

The chapter is organised as follows. Formulae for calculating analytically the gain margin, phase margin and maximum sensitivity are outlined in Sections 5.2 and 5.3. The performance and robustness of FOLPD processes compensated with a variety of PI and PID tuning rules are evaluated in Section 5.4. In Section 5.5, tuning rules are designed to achieve constant gain and phase margins for all values of delay, for a number of process models and controller structures. Conclusions of the work are drawn in Section 5.6. This work was previously published by O'Dwyer (1998), (2001a).

5.2 The Analytical Determination of Gain and Phase Margin

5.2.1 *PI tuning formulae*

The controller and process model are respectively given by

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right) \tag{5.1}$$

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m} \tag{5.2}$$

Then

$$G_m(j\omega)G_c(j\omega) = \frac{K_m e^{-j\omega\tau_m}}{1 + j\omega T_m}\frac{K_c(j\omega T_i + 1)}{j\omega T_i} \tag{5.3}$$

From the definition of gain and phase margin, the following sets of equations are obtained:

$$\phi_m = \arg\left[G_c(j\omega_g)G_m(j\omega_g)\right] + \pi \tag{5.4}$$

$$A_m = \frac{1}{\left|G_c(j\omega_p)G_m(j\omega_p)\right|} \tag{5.5}$$

where ω_g and ω_p are given by

$$\left|G_c(j\omega_g)G_m(j\omega_g)\right| = 1 \tag{5.6}$$

$$\arg\left[G_c(j\omega_p)G_m(j\omega_p)\right] = -\pi \tag{5.7}$$

From Equation (5.3),

$$G_m(j\omega)G_c(j\omega) = \frac{K_m K_c\sqrt{1 + \omega^2 T_i^{\,2}}}{\omega T_i\sqrt{1 + \omega^2 T_m^{\,2}}} \angle -0.5\pi + \tan^{-1}\omega T_i - \tan^{-1}\omega T_m - \omega\tau_m \tag{5.8}$$

Therefore, from Equation (5.4)

$$\phi_m = \pi - 0.5\pi + \tan^{-1}\omega_g T_i - \tan^{-1}\omega_g T_m - \omega_g\tau_m \tag{5.9}$$

with ω_g given by the solution of Equation (5.6) i.e.

$$\frac{K_m K_c \sqrt{1+\omega_g{}^2 T_i{}^2}}{\omega_g T_i \sqrt{1+\omega_g{}^2 T_m{}^2}} = 1 \tag{5.10}$$

Also, from Equations (5.5) and (5.8),

$$A_m = \frac{1}{\left|G_m(j\omega_p)G_c(j\omega_p)\right|} = \frac{\omega_p T_i}{K_m K_c}\sqrt{\frac{1+\omega_p{}^2 T_m{}^2}{1+\omega_p{}^2 T_i^2}} \tag{5.11}$$

with ω_p given by the solution of Equation (5.7) i.e.

$$-0.5\pi + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \omega_p \tau_m = -\pi \tag{5.12}$$

From Equation (5.10), ω_g may be determined analytically to be

$$\omega_g = \sqrt{\frac{T_i\left(K_c{}^2 K_m{}^2 - 1\right) + \sqrt{\left(K_c{}^2 K_m{}^2 - 1\right)^2 T_i{}^2 + 4K_c{}^2 K_m{}^2 T_m{}^2}}{2T_i T_m{}^2}} \tag{5.13}$$

An analytical solution of Equation (5.12) (to determine ω_p) is not possible. An approximate analytical solution may be obtained if the following approximation for the arctan function is made:

$$\tan^{-1} x \approx \frac{\pi}{4}x, \quad |x| < 1 \quad \text{and} \quad \tan^{-1} x \approx \frac{\pi}{2} - \frac{\pi}{4x}, \quad |x| > 1 \tag{5.14}$$

This is quite an accurate approximation, as is shown in Figures 1a and 1b (page 465). Considering Equation (5.12), four possibilities present themselves if the approximation in Equation (5.14) is to be used. These possibilities, together with the formula for ω_p that may be determined analytically for each of these cases, are

(i) $$\omega_p T_i > 1,\ \omega_p T_m > 1:\ \omega_p = \frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m\left(\frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m} \tag{5.15}$$

(ii) $$\omega_p T_i > 1,\ \omega_p T_m < 1:\ \omega_p = \frac{\pi \pm \sqrt{\pi^2 - \frac{\pi}{T_i}\left(0.25\pi T_m + \tau_m\right)}}{2\left(0.25\pi T_m + \tau_m\right)} \tag{5.16}$$

(iii) $\quad \omega_p T_i < 1, \ \omega_p T_m > 1: \omega_p = \sqrt{\dfrac{\pi / T_m}{4\tau_m - \pi T_i}} \qquad (5.17)$

(iv) $\quad \omega_p T_i < 1, \ \omega_p T_m < 1: \omega_p = \dfrac{2\pi}{4\tau_m + \pi(T_m - T_i)} \qquad (5.18)$

The gain and phase margin of the compensated system, for each of the tuning rules, as a function of τ_m / T_m, may be calculated by applying Equations (5.9), (5.11), (5.13) and the relevant approximation for ω_p from Equations (5.15) to (5.18).

Figure 1a: Arctan x and its approximation (Equation 5.14)

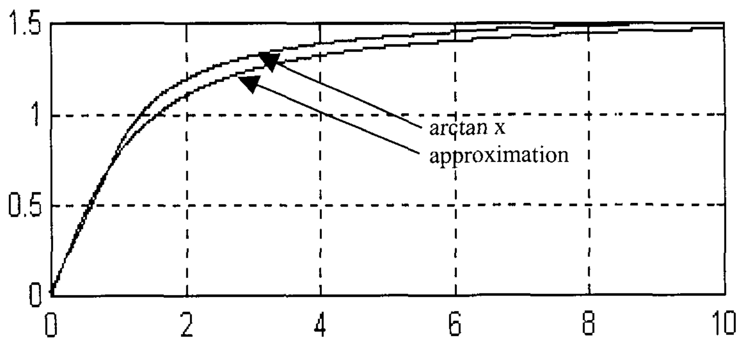
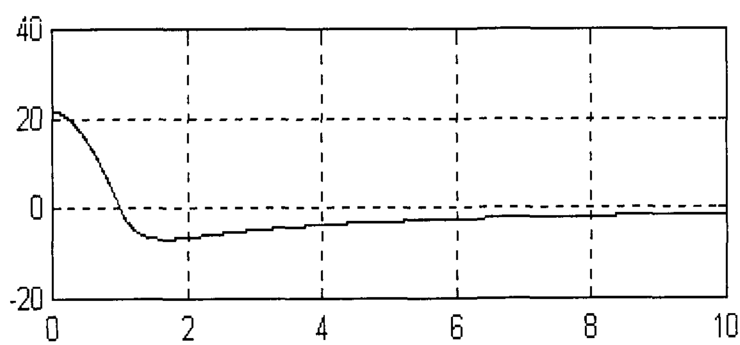


Figure 1b: % error in taking approximation (Equation 5.14) to arctan(x)



5.2.2 PID tuning formulae

The classical PID controller structure is considered, whose transfer function is given as

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+sT_d}{1+s\alpha T_d}\right) \tag{5.19}$$

and the process model is given by Equation (5.2). Substituting Equations (5.2) and (5.19) into Equations (5.4) and (5.5) gives

$$\phi_m = 0.5\pi + \tan^{-1}\omega_g T_i + \tan^{-1}\omega_g T_d - \tan^{-1}\omega_g T_m - \tan^{-1}\omega_g \alpha T_d - \omega_g \tau_m \tag{5.20}$$

and

$$A_m = \frac{\omega_p T_i}{K_c K_m}\sqrt{\frac{\left(1+\omega_p{}^2 T_m{}^2\right)\left(1+\omega_p{}^2\alpha^2 T_d{}^2\right)}{\left(1+\omega_p{}^2 T_i{}^2\right)\left(1+\omega_p{}^2 T_d{}^2\right)}} \tag{5.21}$$

From Equation (5.6), ω_g is given by the solution of

$$\frac{K_m K_c\sqrt{1+\omega_g{}^2 T_i{}^2}\sqrt{1+\omega_g{}^2 T_d{}^2}}{\omega_g T_i\sqrt{1+\omega_g{}^2 T_m{}^2}\sqrt{1+\alpha^2\omega_g{}^2 T_d{}^2}} = 1 \tag{5.22}$$

The equation for ω_g may be determined to be

$$\omega_g{}^6 + a_1\omega_g{}^4 + a_2\omega_g{}^2 + a_3 = 0 \tag{5.23}$$

with $a_1 = \dfrac{T_i{}^2\left(T_m{}^2+\alpha^2 T_d{}^2\right) - K_c{}^2 K_m{}^2 T_i{}^2 T_d{}^2}{T_i{}^2 T_m{}^2\alpha^2 T_d{}^2}$,

$a_2 = \dfrac{T_i{}^2 - K_c{}^2 K_m{}^2 (T_i{}^2 + T_d{}^2)}{T_i{}^2 T_m{}^2\alpha^2 T_d{}^2}$ and $a_3 = -\dfrac{K_c{}^2 K_m{}^2}{T_i{}^2 T_m{}^2\alpha^2 T_d{}^2}$.

Following the procedure outlined by Ho *et al.* (1996), the analytical solution of Equation (5.23) is given as

$$\omega_g = \sqrt{\sqrt[3]{R+\sqrt{Q^3+R^2}} + \sqrt[3]{R-\sqrt{Q^3+R^2}} - \frac{a_1}{3}} \tag{5.24}$$

with $Q = \dfrac{3a_2 - a_1{}^2}{9}$ and $R = \dfrac{9a_1a_2 - 27a_3 - 2a_1{}^3}{54}$.

From Equation (5.7), ω_p is given by the solution of

$$0.5\pi + \tan^{-1}\omega_p T_d + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \tan^{-1}\omega_p \alpha T_d - \omega_p \tau_m = 0 \tag{5.25}$$

As with Equation (5.12), an analytical solution of this equation is not possible. An approximate analytical solution may be obtained if the approximation detailed in Equation (5.14) is made. Looking at Equation (5.25), twelve possibilities present themselves if the approximation in Equation (5.14) is to be used; these possibilities are

(1) $\underline{\omega_p T_i > 1,\ \omega_p T_m > 1}$

(a) $\omega_p T_d \le 1,\ \omega_p \alpha T_d < 1$

(b) $\omega_p T_d > 1,\ \omega_p \alpha T_d < 1$

(c) $\omega_p T_d > 1,\ \omega_p \alpha T_d > 1$

(2) $\underline{\omega_p T_i > 1,\ \omega_p T_m < 1}$

(a) $\omega_p T_d \le 1,\ \omega_p \alpha T_d < 1$

(b) $\omega_p T_d > 1,\ \omega_p \alpha T_d < 1$

(c) $\omega_p T_d > 1,\ \omega_p \alpha T_d > 1$

(3) $\underline{\omega_p T_i < 1,\ \omega_p T_m > 1}$

(a) $\omega_p T_d \le 1,\ \omega_p \alpha T_d < 1$

(b) $\omega_p T_d > 1,\ \omega_p \alpha T_d < 1$

(c) $\omega_p T_d > 1,\ \omega_p \alpha T_d > 1$

(4) $\underline{\omega_p T_i < 1,\ \omega_p T_m < 1}$

(a) $\omega_p T_d \le 1,\ \omega_p \alpha T_d < 1$

(b) $\omega_p T_d > 1,\ \omega_p \alpha T_d < 1$

(c) $\omega_p T_d > 1,\ \omega_p \alpha T_d > 1$

The formulae for ω_p that may be determined for each of these cases are as follows:

(1) $\underline{\omega_p T_i > 1,\ \omega_p T_m > 1}$

(a) $\omega_p T_d \le 1,\ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - \pi(4\tau_m - \pi T_d(1-\alpha))\left(\frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m - \pi T_d(1-\alpha)} \tag{5.26}$$

(b) $\omega_p T_d > 1,\ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 - \pi(4\tau_m + \pi T_d \alpha)\left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m + \pi T_d \alpha} \tag{5.27}$$

(c) $\omega_p T_d > 1,\ \ \omega_p \alpha T_d > 1$:

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m\left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{\alpha T_d} - \frac{1}{T_m}\right)}}{4\tau_m} \quad (5.28)$$

(2) $\omega_p T_i > 1,\ \ \omega_p T_m < 1$:

(a) $\omega_p T_d \le 1,\ \ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 + \frac{\pi}{T_i}\left(\pi T_d - \pi T_m - \pi\alpha T_d - 4\tau_m\right)}}{\pi T_m + 4\tau_m + \pi\alpha T_d - \pi T_d} \quad (5.29)$$

(b) $\omega_p T_d > 1,\ \ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{3\pi \pm \sqrt{9\pi^2 - \pi\left(\frac{1}{T_d} + \frac{1}{T_i}\right)\left(\pi T_m + \pi\alpha T_d + 4\tau_m\right)}}{\pi T_m + 4\tau_m + \pi\alpha T_d} \quad (5.30)$$

(c) $\omega_p T_d > 1,\ \ \omega_p \alpha T_d > 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 + \pi\left(\frac{1}{\alpha T_d} - \frac{1}{T_d} - \frac{1}{T_i}\right)\left(\pi T_m + 4\tau_m\right)}}{\pi T_m + 4\tau_m} \quad (5.31)$$

(3) $\omega_p T_i < 1,\ \ \omega_p T_m > 1$:

(a) $\omega_p T_d \le 1,\ \ \omega_p \alpha T_d < 1$: $\omega_p = \sqrt{\frac{\pi}{T_m\left(4\tau_m + \pi\left[\alpha T_d - T_d - T_i\right]\right)}}$ (5.32)

(b) $\omega_p T_d > 1,\ \ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{-\pi \pm \sqrt{\pi^2 - \pi\left(\frac{1}{T_m} - \frac{1}{T_d}\right)\left(\pi T_i - \pi\alpha T_d - 4\tau_m\right)}}{\pi T_i - 4\tau_m - \pi\alpha T_d} \quad (5.33)$$

(c) $\omega_p T_d > 1,\ \ \omega_p \alpha T_d > 1$: $\omega_p = \sqrt{\frac{\frac{\pi}{T_d} - \frac{\pi}{T_m} - \frac{\pi}{\alpha T_d}}{\pi T_i - 4\tau_m}}$ (5.34)

(4) $\underline{\omega_p T_i < 1, \ \ \omega_p T_m < 1}$:

(a) $\omega_p T_d \le 1, \ \ \omega_p \alpha T_d < 1$: $\omega_p = \dfrac{2\pi}{\pi(\alpha T_d - T_d) + \pi(T_m - T_i) + 4\tau_m}$ (5.35)

(b) $\omega_p T_d > 1, \ \ \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 - \frac{\pi}{T_d}(\pi T_m - \pi T_i + \pi\alpha T_d + 4\tau_m)}}{\pi T_m - \pi T_i + 4\tau_m - \pi\alpha T_d} \quad (5.36)$$

(c) $\omega_p T_d > 1, \ \ \omega_p \alpha T_d > 1$:

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - \pi\left(\frac{1}{\alpha T_d} - \frac{1}{T_d}\right)(\pi T_i - \pi T_m - 4\tau_m)}}{4\tau_m - \pi T_m - \pi T_i} \quad (5.37)$$

The gain and phase margin of the compensated system, for each of the tuning rules, as a function of τ_m / T_m may now be calculated by applying Equations (5.20), (5.21), (5.24) and the relevant approximation for ω_p from Equations (5.26) to (5.37). As Equation (5.19) shows, the procedure applies to tuning rules defined for the classical PID controller structure only; however, Ho *et al.* (1996) suggest that the method may be applied to tuning rules defined for the ideal PID controller structure, by using equations that approximately convert these tuning rules into their equivalent in the classical controller structure.

5.3 The Analytical Determination of Maximum Sensitivity

The maximum sensitivity is the reciprocal of the shortest distance from the Nyquist curve to the (-1,0) point on the Rl-Im axis. It is defined as follows:

$$M_{max} = \underset{\text{all } \omega}{\text{Max}} \left| \frac{1}{1 + G_m(j\omega)G_c(j\omega)} \right| \quad (5.38)$$

For a FOLPD process model controlled by a PI controller,

$$\left| G_m(j\omega)G_c(j\omega) \right| = \frac{\sqrt{1 + \omega^2 T_i^2}}{\sqrt{1 + \omega^2 T_m^2}} \frac{K_c K_m}{\omega T_i} \quad (5.39)$$

and

$$\arg[G_m(j\omega)G_c(j\omega)] = -0.5\pi - \tan^{-1}\omega T_m + \tan^{-1}\omega T_i - \omega\tau_m \qquad (5.40)$$

For a FOLPD process model controlled by a PID controller,

$$|G_m(j\omega)G_c(j\omega)| = \frac{\sqrt{(1+\omega^2 T_i^{\,2})(1+\omega^2 T_d^{\,2})}}{\sqrt{(1+\omega^2 T_m^{\,2})(1+\alpha^2\omega^2 T_d^{\,2})}} \frac{K_c K_m}{\omega T_i} \qquad (5.41)$$

and $\arg[G_m(j\omega)G_c(j\omega)] =$

$$-0.5\pi - \tan^{-1}\omega T_m - \tan^{-1}\omega\alpha T_d + \tan^{-1}\omega T_i + \tan^{-1}\omega T_d - \omega\tau_m \quad (5.42)$$

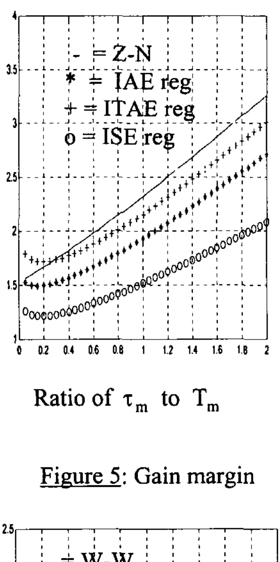
The maximum sensitivity may be calculated over an appropriate range of frequencies corresponding to phase lags of 100^0 to 260^0.

5.4 Simulation Results

Space considerations dictate that only representative simulation results may be provided; an extensive set of simulation results covering 103 PI controller tuning rules and 125 PID controller tuning rules (for the ideal and classical controller structures) are available (O'Dwyer, 2000). The MATLAB package has been used in the simulations. Figures 2 to 7 show how gain margin, phase margin and maximum sensitivity vary as the ratio of time delay to time constant varies, if some PI tuning rules are used (Figures 2 to 4) and corresponding PID tuning rules for the classical controller structure (with $\alpha = 0.1$) are used (Figures 5 to 7). In these results, Z-N refers to the process reaction curve method of Ziegler and Nichols (1942); W-W refers to the process reaction curve method of Witt and Waggoner (1990); IAE reg, ISE reg and ITAE reg refer to the tuning rules for regulator applications that minimise the integral of absolute error criterion, the integral of squared error criterion and the integral of time multiplied by absolute error criterion, respectively, as defined by Murrill (1967) for PI tuning rules and Kaya and Scheib (1988) for PID tuning rules based on the classical controller structure. Figures 8 to 15 show gain and phase margin comparisons between corresponding PI and PID controller tuning rules.

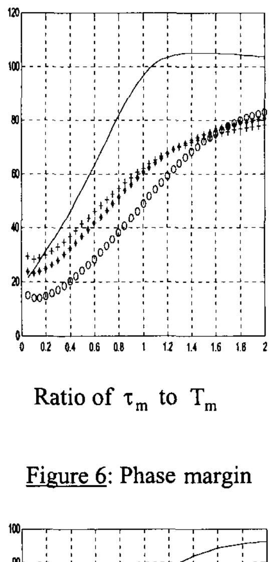
It is clear that the gain margin is generally less when the PID rather than the PI tuning rules are considered, over the ratios of time delay to time constant taken; the difference between the phase margins is less clear cut. This suggests that these PID tuning rules should provide a greater degree of performance than the corresponding PI tuning rules, but may be less robust. Comparing the individual tuning rules, it is striking that the ISE based tuning rules have generally the smallest gain margin and have also a small phase margin, suggesting that this is a less robust tuning strategy. The results in Figures 4 and 7 confirm these comments.

Figure 2: Gain margin



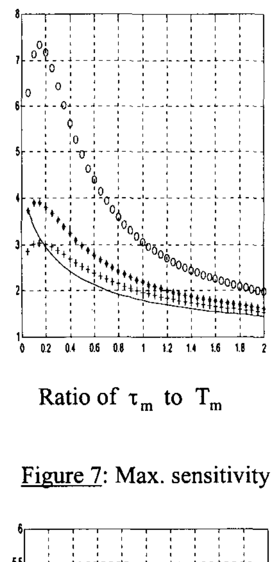
Ratio of τ_m to T_m

Figure 3: Phase margin



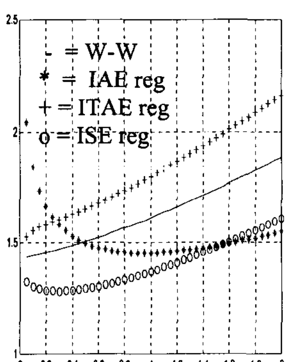
Ratio of τ_m to T_m

Figure 4: Max. sensitivity



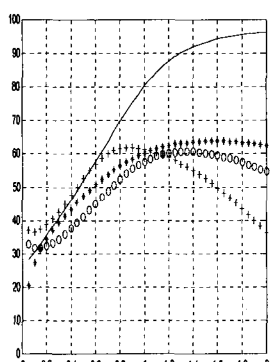
Ratio of τ_m to T_m

Figure 5: Gain margin



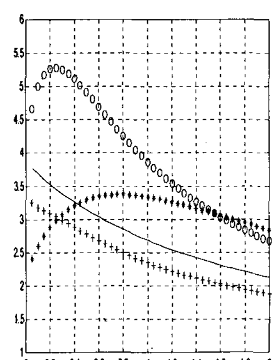
Ratio of τ_m to T_m

Figure 6: Phase margin



Ratio of τ_m to T_m

Figure 7: Max. sensitivity



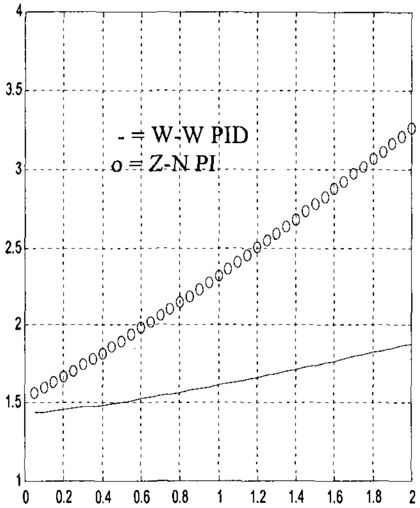
Ratio of τ_m to T_m

No general conclusion can be reached as to the best tuning rule (as expected); it is interesting, though, that a full panorama of simulation results (O'Dwyer, 2000) show that many tuning rules may be applied at ratios of time delay to time constant greater than that normally recommended. One example may be seen in Figures 5 to 7, where the gain margin, phase margin and maximum sensitivity (associated with the use of the PID tuning rule for obtaining minimum IAE in the regulator mode) tends to level out when the ratio of time delay to time constant is greater than 1; normally, the tuning rule is used when the ratio is less than 1 (Murrill, 1967). On the other hand, it is clear from Figures 8 and 9 that there is a significant degradation of performance when using the PID tuning rule of Witt and Waggoner (1990) and the PI tuning rule of Ziegler and Nichols (1942) for large ratios of time delay to time constant, which is compatible with application experience.

The decision between the use of a PI and PID controller to compensate the process, depends on the ratio of time delay to time constant in the FOLPD model, together with the desired trade-off between performance and robustness, as expected. It turns out, however, that the analytical method explored allows the calculation of a far wider range of gain and phase margins for PI controllers; it is also true that stability tends to be assured when a PI controller is used (O'Dwyer, 2000). Thus, a cautious design approach is to use a PI controller, particularly at larger ratios of time delay to time constant.

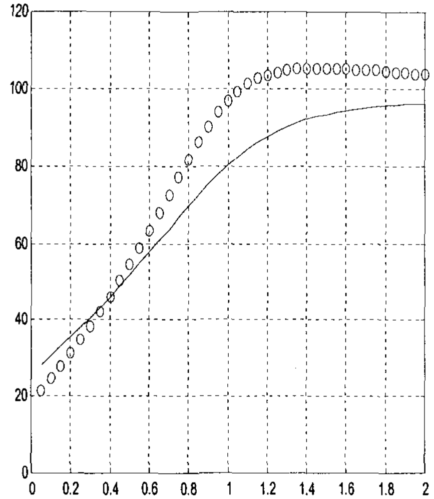
Finally, the volume of tuning rules and data generated means that the use of an expert system to recommend a tuning rule based on user defined requirements is indicated; work is ongoing on such an implementation (Feeney and O'Dwyer, 2002).

Figure 8: Gain margin comparison



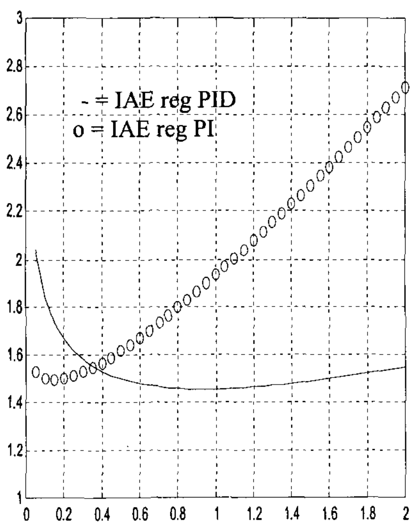
Ratio of τ_m to T_m

Figure 9: Phase margin comparison



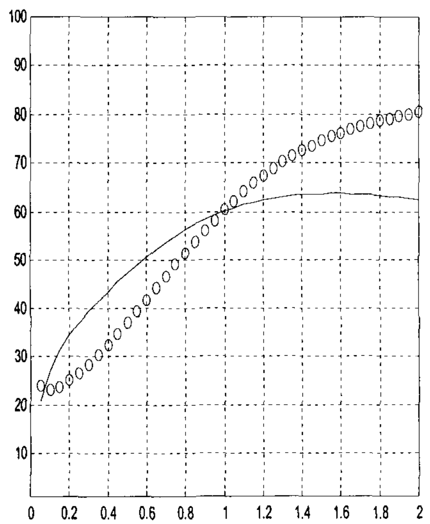
Ratio of τ_m to T_m

Figure 10: Gain margin comparison



Ratio of τ_m to T_m

Figure 11: Phase margin comparison



Ratio of τ_m to T_m

Figure 12: Gain margin comparison

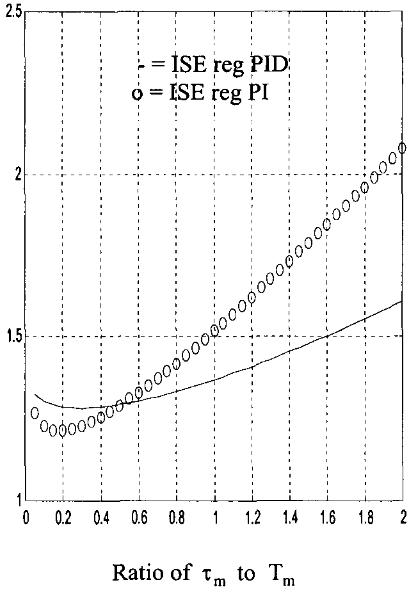


Figure 13: Phase margin comparison

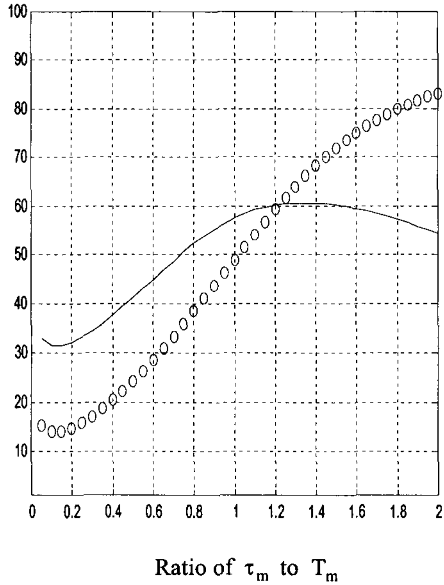


Figure 14: Gain margin comparison

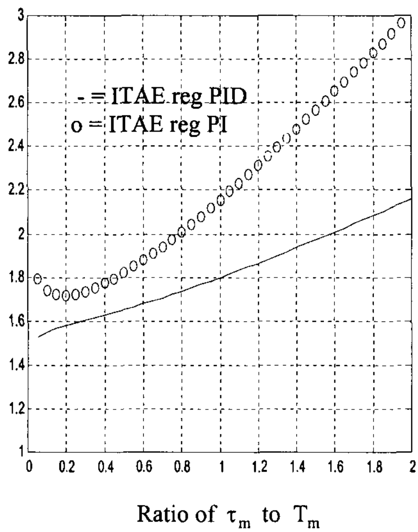
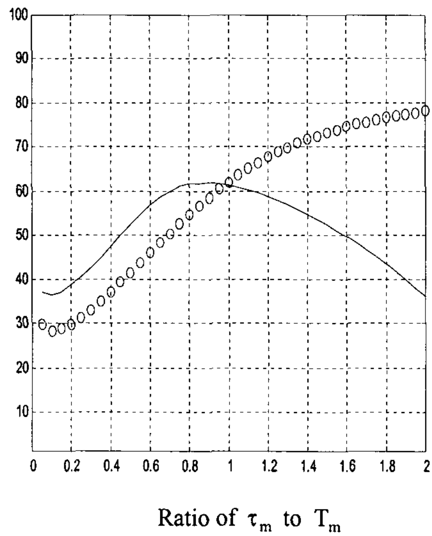


Figure 15: Phase margin comparison



5.5 Design of Tuning Rules to Achieve Constant Gain and Phase Margins, for all Values of Delay

Normally, the gain and phase margins of the compensated systems tend to increase as the time delay increases, supporting the common view that PI and PID controllers are less suitable for the control of dominant time delay processes. However, for the PI control of a FOLPD process model, O'Dwyer (2000) shows that the tuning rules proposed by Chien *et al.* (1952), Haalman (1965) and Pemberton (1972a), among others, facilitate the achievement of a constant gain and phase margin as the time delay of the process model varies. All of these tuning rules have the following structure: $K_c = aT_m/K_m\tau_m$, $T_i = T_m$. Following this observation, original approaches to the design of tuning rules for PI, PD and PID controllers are proposed, for a wide variety of process models, which allow constant gain and phase margins for the compensated system.

This work organised as follows. In Section 5.5.1, PI controller tuning rules are specified for processes modelled in FOLPD form and IPD form. In Section 5.5.2, PID controller tuning rules are described for processes modelled in FOLPD form, SOSPD form and SOSPD form with a negative zero. Section 5.5.3 deals with the design of PD controller tuning rules for the control of processes modelled in FOLIPD form.

5.5.1 *PI controller design*

5.5.1.1 *Processes modelled in FOLPD form*

For such processes and controllers, Equations (5.1) and (5.2) apply i.e.,

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$$

with $$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)$$

From Section 5.2.1,

$$\phi_m = \pi - 0.5\pi + \tan^{-1}\omega_g T_i - \tan^{-1}\omega_g T_m - \omega_g\tau_m \qquad \text{(Equation 5.9)}$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1+\omega_g^2 T_i^2}}{\omega_g T_i \sqrt{1+\omega_g^2 T_m^2}} = 1 \qquad \text{(Equation 5.10)}$$

and

$$A_m = \frac{\omega_p T_i}{K_m K_c} \sqrt{\frac{1+\omega_p^2 T_m^2}{1+\omega_p^2 T_i^2}} \qquad \text{(Equation 5.11)}$$

with ω_p given by the solution of

$$-0.5\pi + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \omega_p \tau_m = -\pi \qquad \text{(Equation 5.12)}$$

If K_c and T_i are designed as follows:

$$K_c = \frac{aT_m}{K_m \tau_m} \qquad (5.43)$$

and

$$T_i = T_m \qquad (5.44)$$

Then Equation (5.12) becomes

$$-0.5\pi - \omega_p \tau_m = -\pi \qquad (5.45)$$

i.e. $$\omega_p = \pi/2\tau_m \qquad (5.46)$$

Substituting Equation (5.46) into Equation (5.11) gives

$$A_m = \pi T_m / 2K_m K_c \tau_m \qquad (5.47)$$

Substituting Equation (5.44) into Equation (5.10) gives

$$K_m K_c / \omega_g T_m = 1 \qquad (5.48)$$

i.e. $$\omega_g = K_m K_c / T_m \qquad (5.49)$$

Substituting Equations (5.44) and (5.49) into Equation (5.9) gives

$$\phi_m = 0.5\pi - K_m K_c \tau_m / T_m \qquad (5.50)$$

Finally, substituting Equation (5.43) into Equation (5.47) gives

$$A_m = \pi/2a \qquad (5.51)$$

and substituting Equation (5.43) into Equation (5.50) gives

$$\phi_m = 0.5\pi - a \qquad (5.52)$$

Some typical tuning rules are shown in Table 186.

Table 186: Typical PI controller tuning rules – FOLPD process model

a	K_c	T_i	A_m	ϕ_m
$\pi/3$	$1.047T_m/K_m\tau_m$	T_m	1.5	$\pi/6$
$\pi/4$	$0.785T_m/K_m\tau_m$	T_m	2.0	$\pi/4$
$\pi/6$	$0.524T_m/K_m\tau_m$	T_m	3.0	$\pi/3$

These rules are also provided in Table 3, Chapter 3. It may also be demonstrated that, if K_c and T_i are designed as follows:

$$K_c = \frac{aT_m}{\tau_m}\frac{T_u K_u}{\sqrt{T_u^2 + 4\pi^2 T_m^2}} \tag{5.53}$$

and

$$T_i = T_m \tag{5.54}$$

where K_u and T_u are the ultimate gain and ultimate period, respectively, then the constant gain and phase margins provided in Equations (5.51) and (5.52) are obtained.

5.5.1.2 *Processes modelled in IPD form*

A similar analysis to that of Section 5.5.1.1 may be done for the design of PI controllers for processes modelled in IPD form. The process is modelled as follows:

$$G_m(s) = K_m e^{-s\tau_m}/s \tag{5.55}$$

Corresponding to Equations (5.4) and (5.9), the phase margin is

$$\phi_m = \pi - 0.5\pi + \tan^{-1}\omega_g T_i - 0.5\pi - \omega_g \tau_m \tag{5.56}$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1+\omega_g^2 T_i^2}}{\omega_g^2 T_i} = 1 \tag{5.57}$$

If K_c and T_i are designed as follows:

$$K_c = \frac{a}{K_m \tau_m} \tag{5.58}$$

and

$$T_i = b\tau_m \tag{5.59}$$

then it may be shown that

$$\omega_g = \left[\frac{a^2}{2\tau_m^{\ 2}} \pm \frac{a}{2b\tau_m^{\ 2}}\sqrt{a^2b^2+4}\right]^{0.5} \tag{5.60}$$

and, substituting Equations (5.59) and (5.60) into Equation (5.56),

$$\phi_m = \tan^{-1}\left[0.5a^2b^2 + 0.5ab\sqrt{a^2b^2+4}\right]^{0.5} - \sqrt{0.5a^2 + 0.5\frac{a}{b}\sqrt{a^2b^2+4}} \tag{5.61}$$

Corresponding to Equations (5.5) and (5.11), the gain margin,

$$A_m = \frac{\omega_p^{\ 2}T_i}{K_m K_c\sqrt{1+\omega_p^{\ 2}T_i^{\ 2}}} \tag{5.62}$$

with ω_p given by the solution of

$$-0.5\pi + \tan^{-1}\omega_p T_i - 0.5\pi - \omega_p\tau_m = -\pi \tag{5.63}$$

i.e. ω_p is given by the solution of

$$\tan^{-1}\omega_p T_i = \omega_p\tau_m \tag{5.64}$$

An analytical solution to this equation is not possible, though if the approximation $\tan^{-1}\omega_p T_i \approx 0.5\pi - \frac{\pi}{4\omega_p T_i}$ (when $|\omega_p T_i| > 1$) is used, simple calculations show that the following analytical solution for ω_p may be obtained:

$$\omega_p = \frac{0.5}{\tau_m}\left[0.5\pi + \sqrt{0.25\pi^2 - \frac{\pi}{b}}\right] \tag{5.65}$$

The inequality $|\omega_p T_i| > 1$ may be shown to be equivalent to $b > 1.273$. Substituting Equations (5.58), (5.59) and (5.65) into Equation (5.62), calculations show that:

$$A_m = \frac{\frac{b}{4a}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{b}}\right]^2}{\sqrt{1+\frac{b^2}{4}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{b}}\right]^2}} \tag{5.66}$$

Some typical tuning rules are shown in Table 187.

Table 187: Typical PI controller tuning rules – IPD process model

K_c	T_i	A_m	ϕ_m
$0.558/K_m\tau_m$	$1.4\tau_m$	1.5	46.2^0
$0.484/K_m\tau_m$	$1.55\tau_m$	2.0	45.5^0
$0.458/K_m\tau_m$	$3.35\tau_m$	3.0	59.9^0
$0.357/K_m\tau_m$	$4.3\tau_m$	4.0	60.0^0
$0.305/K_m\tau_m$	$12.15\tau_m$	5.0	75.0^0

It may also be shown that the maximum sensitivity is a constant if K_c and T_i are specified according to Equations (5.58) and (5.59). The maximum sensitivity may be calculated to be:

$$M_{max} = \left[1 - 2a\cos(\omega_r\tau_m) - \frac{a^2}{\omega_r^{\ 2}\tau_m^{\ 2}}\right]^{-0.5} \tag{5.67}$$

with $\omega_r\tau_m$ being a constant obtained numerically from the solution of the equation $(a^2/\omega_r^{\ 2}\tau_m^{\ 2}) + \cos\omega_r\tau_m = \sin(\omega_r\tau_m)/\omega_r\tau_m$.

It may also be shown that, if K_c and T_i are designed as follows:

$$K_c = aK_u \tag{5.68}$$

and

$$T_i = bT_u \tag{5.69}$$

then the following constant gain and phase margins are determined:

$$A_m = \frac{\frac{2b}{\pi a}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{4b}}\right]^2}{\sqrt{1+\frac{1}{a^2\pi^2}\left[\frac{\pi}{2}+\sqrt{\frac{\pi^2}{4}-\frac{\pi}{4b}}\right]^2}} \tag{5.70}$$

and $$\phi_m = \tan^{-1}\left[2\pi^2a^2b^2 + 2\pi ab\sqrt{4\pi^2a^2b^2+4}\right]^{0.5} - \sqrt{0.125\pi^2a^2 + \frac{\pi a}{16b}\sqrt{4\pi^2a^2b^2+4}} \tag{5.71}$$

5.5.2 *PID controller design*

5.5.2.1 *Processes modelled in FOLPD form - classical controller 1*

The classical PID controller is given by

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+sT_d}{1+s\alpha T_d}\right) \quad \text{(Equation 5.19)}$$

and the process model is given by

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m} \quad \text{(Equation 5.2)}$$

From Section 5.2.2,

$$\phi_m = 0.5\pi + \tan^{-1}\omega_g T_i + \tan^{-1}\omega_g T_d - \tan^{-1}\omega_g T_m - \tan^{-1}\omega_g \alpha T_d - \omega_g \tau_m \quad \text{(Equation 5.20)}$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1+\omega_g{}^2 T_i{}^2}\sqrt{1+\omega_g{}^2 T_d{}^2}}{\omega_g T_i \sqrt{1+\omega_g{}^2 T_m{}^2}\sqrt{1+\alpha^2\omega_g{}^2 T_d{}^2}} = 1 \quad \text{(Equation 5.22)}$$

Also

$$A_m = \frac{\omega_p T_i}{K_c K_m}\sqrt{\frac{\left(1+\omega_p{}^2 T_m{}^2\right)\left(1+\omega_p{}^2\alpha^2 T_d{}^2\right)}{\left(1+\omega_p{}^2 T_i{}^2\right)\left(1+\omega_p{}^2 T_d{}^2\right)}} \quad \text{(Equation 5.21)}$$

with ω_p given by

$$0.5\pi + \tan^{-1}\omega_p T_d + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \tan^{-1}\omega_p \alpha T_d - \omega_p \tau_m = 0$$

(Equation 5.25)

If K_c, T_i and T_d are designed as follows:

$$K_c = \frac{aT_m}{K_m \tau_m} \tag{5.72}$$

$$T_i = \alpha T_m \tag{5.73}$$

and

$$T_d = T_m \tag{5.74}$$

then, Equation (5.25) becomes

$$0.5\pi - \omega_p \tau_m = 0 \tag{5.75}$$

i.e.

$$\omega_p = \pi/2\tau_m \tag{5.76}$$

and Equation (5.21) becomes

$$A_m = \alpha\,\pi T_m / 2K_m K_c \tau_m \tag{5.77}$$

Equation (5.22) becomes

$$K_m K_c / \omega_g T_m = 1 \tag{5.78}$$

i.e.

$$\omega_g = K_m K_c / T_m \tag{5.79}$$

Finally, substituting Equations (5.73), (5.74) and (5.79) into Equation (5.20) gives

$$\phi_m = 0.5\pi - K_m K_c \tau_m / \alpha T_m \tag{5.80}$$

Substituting Equation (5.72), (5.73), (5.74) and (5.76) into Equation (5.21) gives

$$A_m = \pi\alpha/2a \tag{5.81}$$

and substituting Equation (5.72) into Equation (5.80) gives

$$\phi_m = 0.5\pi - a/\alpha \tag{5.82}$$

This design reduces to the PI controller design when $\alpha = 1$.

5.5.2.2 *Processes modelled in SOSPD form – series controller*

For such processes and controllers,

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \tag{5.83}$$

with

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d) \tag{5.84}$$

Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}(1+sT_i)(1+sT_d)}{T_i s(1+sT_{m1})(1+sT_{m2})} \tag{5.85}$$

If T_i and T_d are designed as follows:

$$T_i = T_{m1} \tag{5.86}$$

and

$$T_d = T_{m2} \tag{5.87}$$

then, from Equation (5.85)

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}}{T_i s} \tag{5.88}$$

which is equal to $G_m(s)G_c(s)$ in Section 5.5.1.1 when $T_i = T_m$. Therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \tag{5.89}$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as before.

5.5.2.3 *Processes modelled in SOSPD form with a negative zero - classical controller 1*

For such processes,

$$G_m(s) = \frac{K_m e^{-s\tau_m}(1+sT_{m3})}{(1+sT_{m1})(1+sT_{m2})} \tag{5.90}$$

with $G_c(s)$ given by Equation (5.19). Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}(1+sT_{m3})(1+sT_i)(1+sT_d)}{T_i s(1+sT_{m1})(1+sT_{m2})(1+\alpha s T_d)} \quad (5.91)$$

If T_i, T_d and α are designed as follows:

$$T_i = T_{m1} \quad (5.92)$$

$$T_d = T_{m2} \quad (5.93)$$

and

$$\alpha = T_{m3}/T_{m2} \quad (5.94)$$

therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (5.95)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 5.5.1.1 and 5.5.2.2.

5.5.3 *PD controller design*

In this case, the process is modelled in FOLIPD form, i.e.

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)} \quad (5.96)$$

with

$$G_c(s) = K_c(1+T_d s) \quad (5.97)$$

Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}(1+sT_d)}{s(1+sT_m)} \quad (5.98)$$

Therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (5.99)$$

and

$$T_d = T_m \quad (5.100)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 5.5.1.1, 5.5.2.2 and 5.5.2.3.

5.6 Conclusions

This chapter has considered the performance and robustness of a PI and PID controlled FOLPD process, with the parameters of the controllers determined by a variety of tuning rules. The original contributions of this work are as follows: (a) an expansion of the analytical approach of Ho *et al.* (1995b; 1996) to determine the approximate gain and phase margin analytically under all operating conditions (b) the analytical determination of the approximate maximum sensitivity of the compensated system and (c) the application of the algorithms using a wide variety of PI and PID tuning rules. The implementation of autotuning algorithms in commercial controllers means that choice of a suitable tuning rule is an important issue; the techniques discussed allow an analytical evaluation to be performed of candidate tuning rules.

Finally, the chapter discusses an original approach to design tuning rules for both PI and PID controllers, for a variety of delayed process models, with the objective of achieving constant gain and phase margins for all values of delay. In one of the cases discussed (PI control of an IPD process model), an analytical approximation is used in the development; this approximation may also be used to determine further tuning rules for other process models with delay, and for other PID controller structures.

Appendix 1

Glossary of Symbols Used in the Book

a_{f1}, a_{f2}, b_{f1} = Parameters of a filter in series with some PID controllers

$a_1, a_2, a_3, b_1, b_2, b_3$ = Parameters of a third order process model

$a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ = Parameters of a fifth order process model

$A_1 = y_1(\infty)$, $A_2 = y_2(\infty)$, $A_3 = y_3(\infty)$, $A_4 = y_4(\infty)$, $A_5 = y_5(\infty)$

A_m = Gain margin

A_p = Peak output amplitude of limit cycle determined from relay autotuning

b, c = Weighting factors in some PID controller structures

D_R = Desired closed loop damping ratio

d(t) = Disturbance variable (time domain)

du/dt = Time derivative of the manipulated variable (time domain)

e(t) = Desired variable, r(t), minus controlled variable, y(t) (time domain)

E(s) = Desired variable, R(s), minus controlled variable, Y(s)

FOLPD model = First Order Lag Plus time Delay model

FOLIPD model = First Order Lag plus Integral Plus time Delay model

F_p, F_i, F_d = Weights on the desired variable for one PID controller structure

$G_c(s)$ = PID controller transfer function

$G_{CL}(s)$ = Closed loop transfer function

$G_{CL}(j\omega)$ = Desired closed loop frequency response

$G_{cp}(s)$ = Ideal PID controller transfer function

$G_{cs}(s)$ = Series PID controller transfer function

$G_m(s)$ = Process model transfer function

$G_p(s)$ = Process transfer function

$G_p(j\omega)$ = Process transfer function at frequency ω

$|G_p(j\omega)|$ = Magnitude of $G_p(j\omega)$

$|G_p(j\omega_\phi)|$ = Magnitude of $G_p(j\omega)$, at frequency ω, corresponding to phase lag of ϕ

$\angle G_p(j\omega)$ = Phase of $G_p(j\omega)$

$\pm h$ = Relay amplitude (relay autotuning)

IE = Integral of Error = $\int_0^\infty e(t)dt$

IAE = Integral of Absolute Error = $\int_0^\infty |e(t)|dt$

IMC = Internal Model Controller

IPD model = Integral Plus time Delay model

I^2PD model = Integral Squared Plus time Delay model

ISE = Integral of Squared Error = $\int_0^\infty e^2(t)dt$

ISTES = Integral of Squared Time multiplied by Error, all to be Squared $= \int_0^\infty [t^2 e(t)]^2 dt$

ISTSE = Integral of Squared Time multiplied by Squared Error = $\int_0^\infty t^2 e^2(t)dt$

ITSE = Integral of Time multiplied by Squared Error = $\int_0^\infty te^2(t)dt$

ITAE = Integral of Time multiplied by Absolute Error = $\int_0^\infty t|e(t)|dt$

K_c = Proportional gain of the controller

K_{cp} = Proportional gain of the ideal PID controller

K_{cs} = Proportional gain of the series PID controller

K_f = Feedback gain used in non-interacting PID controller 10

$$K_H = \frac{9}{2\tau_m{}^2 K_m}\left[\frac{\tau_m{}^2}{18} - T_{ml}{}^2 - \frac{\xi_m T_{ml}\tau_m}{9}\right] \text{ (Hwang, 1995)}$$

$$K_{H1} = \frac{9}{2K_m\tau_m{}^2}\left[\frac{\tau_m{}^2}{18} - \frac{\tau_m T_m}{18} + \frac{1.884 K_c K_u \tau_m}{9\omega_u}\right]$$

$$+\frac{9}{2K_m\tau_m}\left[\sqrt{\frac{\tau_m{}^2}{324} + \frac{49T_m{}^2}{324} + \frac{7T_m\tau_m}{162} - \frac{4.71K_uK_c(\tau_m + 1.6T_m)}{81\omega_u} - \frac{1.775K_c{}^2K_u{}^2}{81\omega_u{}^2}}\right]$$

(Hwang, 1995)

$$K_{H2} = \frac{9}{2K_m\tau_m{}^2}\left[\frac{\tau_m{}^2}{18} - T_{ml}{}^2 - \frac{\xi_m\tau_m T_{ml}}{9} + \frac{1.884K_uK_c\tau_m}{9\omega_u}\right] + \frac{9}{2K_m\tau_m{}^2}\alpha$$

with

$$\alpha = \left[\sqrt{T_{ml}{}^4 + \frac{\tau_m{}^4}{324} + \frac{49\tau_m{}^2\xi_m{}^2T_{ml}{}^2}{81} - \frac{\tau_m{}^2T_{ml}{}^2}{9} + \frac{7T_{ml}\xi_m\tau_m{}^3}{81} + \frac{10\tau_mT_{ml}{}^3\xi_m}{9} - a}\right]$$

and

$$a = \frac{0.471K_uK_c}{\omega_u}\left[\frac{10\tau_m{}^3}{81} + \frac{4T_{ml}\tau_m(8\xi_m\tau_m + 9T_{ml})}{81}\right] - \frac{1.775K_u{}^2K_c{}^2\tau_m{}^2}{81\omega_u{}^2}$$

(Hwang, 1995)

K_i = Feedback gain term used in non-interacting PID controller 8, 11

K_I = Integral-only controller gain (Pinnella *et al.*, 1986)

K_L = Gain of a load disturbance process model

K_m = Gain of the process model

K_o = Weighting parameter used in one PID controller structure

K_p = Gain of the process

K_u = Ultimate proportional gain

K_ϕ = Proportional gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of ϕ

$\hat{K}_u$ = Ultimate proportional gain estimate determined from relay autotuning

K_1 = Feedback gain term used in one PI controller structure (labeled controller with proportional term acting on the output 2) and the PID non-interacting controller 12 structure

$K_{25\%}$ = Proportional gain required to achieve a quarter decay ratio

$K_{x\%}$ = Proportional gain required to achieve a decay ratio of 0.01x

K_{135^0} = Controller gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of 135^0

K_{150^0} = Controller gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of 150^0

M_s = Closed loop sensitivity

M_{max} = Maximum value of closed loop sensitivity, M_s

n = Order of a process model with a repeated pole

N = Parameter that determines the amount of filtering on the derivative term on some PID controller structures

OS = Closed loop response overshoot

PI controller = Proportional Integral controller

PID controller = Proportional Integral Derivative controller

r = Desired variable (time domain)

$$r_1 = \frac{0.7071 M_{max}^2 - M_{max}\sqrt{1 - 0.5 M_{max}^2}}{M_{max}^2 - 1}$$ (Chen *et al.*,1999a)

R(s) = Desired variable (Laplace domain)

s = Laplace variable

SOSPD model = Second Order System Plus time Delay model

SOSIPD model = Second Order System plus Integral Plus time Delay model

T_{ar} = Average residence time = time taken for the open loop process step response to reach 63% of its final value

T_d = Derivative time of the controller

T_{di} = Derivative feedback term used in non-interacting PID controller 8

T_{dp} = Derivative time of the ideal PID controller

T_{ds} = Derivative time of the series PID controller

T_{CL} = Desired closed loop system time constant

T_{CL2} = Desired parameter of second order closed loop system response

T_{CL3} = Desired parameter of third order closed loop system response, with a repeated pole

T_f = Time constant of the filter in series with some PI or PID controllers

T_i = Integral time of the controller

T_{ip} = Integral time of the ideal PID controller

T_{is} = Integral time of the series PID controller

T_L = Time constant of a load disturbance FOLPD process model

T_m = Time constant of the process model

T_{m1}, T_{m2}, T_{m3} = Time constants of second or third order process models, as appropriate.

$T_{m_1i}, T_{m_2i}, T_{m_3i}, T_{m_4i}$ = Time constants of a general process model

T_p = Time constant of the process

T_r = Parameter of the filter on the set-point in some PID controller structures

T_R = Closed loop rise time

T_S = Closed loop settling time

T_u = Ultimate period

$\hat{T}_u$ = Ultimate period estimate determined from relay autotuning

$T_{25\%}$ = Period of the quarter decay ratio waveform, when the closed loop system is under proportional control

$T_{x\%}$ = Period of the waveform with a decay ratio of $0.01x$, when the closed loop system is under proportional control

TOLPD model = Third Order Lag Plus time Delay model

u(t) = Manipulated variable (time domain).

u_∞ = Final value of the manipulated variable (time domain)

U(s) = Manipulated variable (Laplace domain)

V = Closed loop response overshoot (as a fraction of the controlled variable final value)

x_1, x_2, x_3, x_4, x_5 = Coefficient values.

y = Controlled variable (time domain)

y_∞ = Final value of the controlled variable (time domain)

Y(s) = Controlled variable (Laplace domain)

$y_1(t) = \int_0^t \left(K_m - \frac{y(\tau)}{\Delta u}\right) d\tau$, $y_2(t) = \int_0^t (A_1 - y_1(\tau)) d\tau$,

$y_3(t) = \int_0^t (A_2 - y_2(\tau)) d\tau$, $y_4(t) = \int_0^t [A_3 - y_3(\tau)] d\tau$,

$y_5(t) = \int_0^t [A_4 - y_4(\tau)] d\tau$

α, β, χ = Weighting factors in some PI or PID controller structures

$\varepsilon = \frac{6T_{m1}^2 + 4\xi_m T_{m1}\tau_m + K_H K_m \tau_m^2}{2T_{m1}^2 \tau_m \omega_H}$ (Hwang, 1995)

$\varepsilon_1 = \frac{2T_m\omega_u + K_{H1}K_m\tau_m\omega_u - 1.884K_uK_c}{0.471K_cK_u\omega_{H1}\tau_m}$ (Hwang, 1995)

$\varepsilon_0 = \frac{6T_{m1}^2 + 4T_{m1}\xi_m\tau_m + K_uK_m\tau_m^2}{2\tau_m T_{m1}^2 \omega_u}$ (Hwang, 1995)

$$\varepsilon_2 = \frac{6T_{m1}^2\omega_u + 4T_{m1}\xi_m\tau_m\omega_u + K_{H2}K_m\tau_m^2\omega_u - 1.884K_uK_c\tau_m}{(0.471K_uK_c\tau_m^2 + 2\tau_m T_{m1}^2\omega_u)\omega_{H2}}$$

(Hwang, 1995)

λ = Parameter that determines robustness of compensated system.

ξ = Damping factor of the compensated system

ξ_m = Damping factor of an underdamped process model

ξ_m^{des} = Desired damping factor of an underdamped process model

$\xi_{m_n i}$, $\xi_{m_d i}$ = Damping factors of a general underdamped process model

$\kappa = 1/K_m K_u$

ϕ = Phase lag

ϕ_c = Phase of the plant at the crossover frequency of the compensated system, with a 'conservative' PI controller (Pecharroman and Pagola, 2000).

ϕ_m = Phase margin

ϕ_ω = Phase lag at an angular frequency of ω

τ_L = Time delay of a load disturbance process model

τ_m = Time delay of the process model

$\tau = \tau_m/(\tau_m + T_m)$

ω = Angular frequency

ω_{bw} = -3dB closed loop system bandwidth (Shi and Lee, 2002)

ω_{-6dB} = Frequency where closed loop system magnitude is –6dB (Shi and Lee, 2004)

ω_c = Maximum cut-off frequency

ω_d = Bandwidth of stochastic disturbance signal (Van der Grinten, 1963)

$\omega_{CL} = 2\pi/T_{CL}$

ω_g = Specified gain crossover frequency (Shi and Lee, 2002)

$$\omega_H = \sqrt{\frac{1 + K_H K_m}{T_{m1}^2 + \frac{2T_{m1}\tau_m\xi_m}{3} + \frac{K_H K_m \tau_m^2}{6}}} \quad \text{(Hwang, 1995)}$$

$$\omega_{H1} = \sqrt{\frac{1 + K_{H1} K_m}{\frac{\tau_m T_m}{3} + \frac{K_{H1} K_m \tau_m^2}{6} - \frac{0.942 K_c K_u \tau_m}{3\omega_u}}} \quad \text{(Hwang, 1995)}$$

$$\omega_{H2} = \sqrt{\frac{1 + K_{H2} K_m}{T_{m1}^2 + \frac{2\xi_m \tau_m T_{m1}}{3} + \frac{K_{H2} K_m \tau_m^2}{6} - \frac{0.942 K_c K_u \tau_m}{3\omega_u}}}$$

(Hwang, 1995)

$\omega_{M_{max}}$ = Frequency where the sensitivity function is maximized (Rotach, 1994)

ω_n = Undamped natural frequency of the compensated system

$\omega_p = \dfrac{A_m\phi_m + 0.5\pi A_m(A_m - 1)}{(A_m{}^2 - 1)\tau_m}$ (Hang *et al.* 1993a, 1993b)

ω_r = Resonant frequency (Rotach, 1994)

ω_u = Ultimate frequency

ω_ϕ = Angular frequency at a phase lag of ϕ

ω_{90^0} = Angular frequency at a phase lag of 90^0

ω_{135^0} = Angular frequency at a phase lag of 135^0

ω_{150^0} = Angular frequency at a phase lag of 150^0

Appendix 2

Some Further Details on Process Modelling

Processes with time delay may be modelled in a variety of ways. The modelling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from the tuning rules. The modelling strategy used in association with each tuning rule, as described in the original papers, is indicated in the tables in Chapters 3 and 4. Some outline details of these modelling strategies are provided, together with the references that describe the modelling method in detail. The references are given in the bibliography. For all models, the label "Model: Method 1" indicates that the model method has not been defined or that the model parameters are assumed known. Of the 1134 tuning rules specified (442 PI controller tuning rules, 692 PID controller tuning rules), it is startling to relate that only 386 (or 34%) of tuning rules have been specified based on a defined process modelling method.

A2.1 FOLPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$

A2.1.1 *Parameters estimated from the open loop process step or impulse response*

Method 2: Parameters estimated using a tangent and point method (Ziegler and Nichols, 1942).

Method 3: K_m, τ_m determined from the tangent and point method of Ziegler and Nichols (1942); T_m determined at 60% of the total process variable change (Fertik and Sharpe, 1979).

Method 4: K_m, τ_m assumed known; T_m estimated using a tangent method (Wolfe, 1951).

Method 5: Parameters estimated using a second tangent and point method (Murrill, 1967).

Method 6: Parameters estimated using a third tangent and point method (Davydov *et al.*, 1995).

Method 7: τ_m and T_m estimated using the two-point method; K_m estimated from the open loop step response (ABB, 2001).

Method 8: τ_m and T_m estimated from the process step response: $T_m = 1.4(t_{67\%} - t_{33\%})$, $\tau_m = t_{67\%} - 1.1T_m$; K_m assumed known (Chen and Yang, 2000). [1]

Method 9: τ_m and T_m estimated from the process step response: $T_m = 1.245(t_{70\%} - t_{33\%})$, $\tau_m = 1.498t_{33\%} - 0.498t_{70\%}$; K_m assumed known (Vítečková *et al.*, 2000b). [2]

Method 10: K_m estimated from the process step response; τ_m = time for which the process variable does not change; T_m determined at 63% of the total process variable change (Gerry, 1999).

Method 11: K_m estimated from the process step response; τ_m = time for which the process variable has to change by 5% of its total value; T_m determined at 63% of the total process variable change (Kristiansson, 2003).

Method 12: Parameters estimated using a least squares method in the time domain (Cheng and Hung, 1985).

Method 13: Parameters estimated from linear regression equations in the time domain (Bi *et al.*, 1999).

Method 14: Parameters estimated using the method of moments (Åström and Hägglund, 1995).

Method 15: Parameters estimated from the process step response and its first time derivative (Tsang and Rad, 1995).

Method 16: Parameters estimated from the process step response using numerical integration procedures (Nishikawa *et al.*, 1984).

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[1] $t_{67\%}$, $t_{33\%}$ are the times taken by the process variable to change by 67% and 33%, respectively, of its total value.

[2] $t_{70\%}$ is the time taken by the process variable to change by 70% of its total value.

Method 17: Parameters estimated from the process impulse response (Peng and Wu, 2000).

Method 18: Parameters estimated using a "process setter" block (Saito *et al.*, 1990).

Method 19: Parameters estimated from a number of process step response data values (Kraus, 1986).

Method 20: Parameters estimated in the sampled data domain using two process step tests (Pinnella *et al.*, 1986).

Method 21: A model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters $K_m^{'}$, $T_m^{'}$ and $\tau_m^{'}$. Subsequently, τ_m is estimated from the process step response; then, a parameter labelled $\tau_m^a = \tau_m^{'} - \tau_m$ (Schaedel, 1997).

Method 22: A model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters $K_m^{'}$, $T_m^{'}$ and $\tau_m^{'}$. Subsequently, τ_m is estimated from the process step response (Henry and Schaedel, 2005).

A2.1.2 *Parameters estimated from the closed loop step response*

Method 23: K_m estimated from the open loop step response. $T_{90\%}$ and τ_m estimated from the closed loop step response under proportional control (Åström and Hägglund, 1988).

Method 24: Parameters estimated using a method based on the closed loop transient response to a step input under proportional control (Sain and Özgen, 1992).

Method 25: Parameters estimated using a second method based on the closed loop transient response to a step input under proportional control (Hwang, 1993).

Method 26: Parameters estimated using a third method based on the closed loop transient response to a step input under proportional control (Chen, 1989; Taiwo, 1993).

Method 27: Parameters estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Åström and Hägglund, 1995).

Method 28: Parameters estimated from the closed loop step response when process is in series with a PID controller (Morilla *et al.*, 2000).

Method 29: Parameters estimated by modelling the closed loop response as a second order system (Ettaleb and Roche, 2000).

A2.1.3 *Parameters estimated from frequency domain closed loop information*

Method 30: Parameters estimated from two points, determined on process frequency response, using a relay and a relay in series with a delay (Tan *et al.*, 1996).

Method 31: T_m and τ_m estimated from the ultimate gain and period, determined using a relay in series with the process in closed loop; K_m assumed known (Hang and Cao, 1996).

Method 32: T_m and τ_m estimated from the ultimate gain and period, determined using a relay in series with the process in closed loop; K_m estimated from the process step response (Hang *et al.*, 1993b).

Method 33: T_m and τ_m estimated from the ultimate gain and period, determined using the ultimate cycle method of Ziegler and Nichols (1942); K_m estimated from the process step response (Hang *et al.*, 1993b).

Method 34: T_m and τ_m estimated from relay autotuning method (Lee and Sung, 1993); K_m estimated from the closed loop process step response under proportional control (Chun *et al.*, 1999).

Method 35: Parameters estimated in the frequency domain from the data determined using a relay in series with the closed loop system in a master feedback loop (Hwang, 1995).

Method 36: K_u and T_u estimated from relay autotuning method; τ_m estimated from the open loop process step response, using a tangent and point method (Wojsznis *et al.*, 1999).

Method 37: Parameters estimated by including a dynamic compensator outside or inside an (ideal) relay feedback loop (Huang and Jeng, 2003).

Method 38: Parameters estimated from measurements performed on the manipulated and controlled variables when a relay with hysteresis is introduced in place of the controller (Zhang *et al.*, 1996).

Method 39: Non-gain parameters estimated using a relay, with hysteresis, in series with the process in closed loop; K_m is estimated with the aid of a small step signal added to the reference (Potočnik *et al.*, 2001).

Method 40: Parameters estimated using a relay in series with the process in closed loop (Perić *et al.*, 1997).

Method 41: Parameters estimated using an iterative method, based on data from a relay experiment (Leva and Colombo, 2004).

Method 42: K_m estimated from relay autotuning method; τ_m, T_m estimated using a least squares algorithm based on the data recorded from the relay autotuning method, with the aid of neural networks (Huang *et al.*, 2005).

A2.1.4 *Other methods*

Method 43: Parameter estimates back-calculated from a discrete time identification method (Ferretti *et al.*, 1991).

Method 44: Parameter estimates determined graphically from a known higher order process (McMillan, 1984).

Method 45: Parameter estimates determined from a known higher order process.

Method 46: The parameters of a second order model plus time delay are estimated using a system identification approach in discrete time; the parameters of a FOLPD model are subsequently determined using standard equations (Ou and Chen, 1995).

Method 47: Parameter estimates back-calculated from a discrete time identification non-linear regression method (Gallier and Otto, 1968).

Table 188 summaries the most common FOLPD process modelling methods.

Table 188: The most common FOLPD process modelling methods

Model Method	PI	PID	Total
Method 1	112	147	259
Method 2	18	36	54
Method 5	12	26	38
Method 31	6	6	12

It is interesting that in 57% of tuning rules based on the FOLPD process model (252 out of 443), the modelling method has not been specified or the model parameters are known *a priori*.

A2.2 FOLPD Model with a Negative Zero $G_m(s) = \frac{K_m(1 + sT_{m2})e^{-s\tau_m}}{1 + sT_{m1}}$

Method 2: Non-delay parameters are estimated in the discrete time domain using a least squares approach; time delay assumed known (Chang *et al.*, 1997).

A2.3 Non-Model Specific

Modelling methods have not been specified in the tables in most cases, as typically the tuning rules are based on K_u and T_u. Modelling methods have been specified whenever relevant data have been estimated using a relay method.

Method 2: K_u and T_u estimated from an experiment using a relay in series with the process in closed loop (Jones *et al.*, 1997).

Method 3: K_u and T_u estimated with the assistance of a relay (Lloyd, 1994).

Method 4: ω_{90^0} and A_p estimated using a relay in series with an integrator (Tang *et al.*, 2002).

A2.4 IPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

A2.4.1 *Parameters estimated from the open loop process step response*

Method 2: τ_m assumed known; K_m estimated from the slope at start of the process step response (Ziegler and Nichols, 1942).

Method 3: K_m, τ_m estimated from the process step response (Hay, 1998).

Method 4: K_m, τ_m estimated from the process step response using a tangent and point method (Bunzemeier, 1998).

A2.4.2 *Parameters estimated from the closed loop step response*

Method 5: Parameters estimated from the servo or regulator closed loop transient response, under PI control (Rotach, 1995).

Method 6: Parameters estimated from the servo closed loop transient response under proportional control (Srividya and Chidambaram, 1997).

Method 7: Parameters estimated from the closed loop response, under the control of an on-off controller (Zou *et al.*, 1997).

Method 8: Parameters estimated from the closed loop response, under the control of an on-off controller (Zou and Brigham, 1998).

A2.4.3 *Parameters estimated from frequency domain closed loop information*

Method 9: Parameters estimated from K_u and T_u values, determined from an experiment using a relay in series with the process in closed loop (Tyreus and Luyben, 1992).

Method 10: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

The most common IPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is unspecified (i.e. Model: Method 1); 46 PI controller tuning rules, and 47 PID controller tuning rules have been specified using the method. Altogether 76% of tuning rules based on the IPD process model (93 out of 122) have been specified using the method.

A2.5 FOLIPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Method 2: Parameters estimated from the open loop process step response and its first and second time derivatives (Tsang and Rad, 1995).

Method 3: Parameters estimated using the method of moments (Åström and Hägglund, 1995).

Method 4: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

Method 5: Parameters estimated from the open loop response of the process to a pulse signal (Tachibana, 1984).

Method 6: Parameters estimated from the waveform obtained by introducing a single symmetrical relay in series with the process in closed loop (Majhi and Mahanta, 2001).

Method 7: K_m estimated from the response of the process to a square wave pulse; other process parameters estimated from the wavefrom obtained by introducing a relay in series with the process in closed loop (Wang *et al.*, 2001a).

Method 8: Parameters estimated using a relay in series with the process in closed loop (Perić *et al.*, 1997).

The most common FOLIPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is unspecified (i.e. Model: Method 1); 25 PI controller tuning rules, and 49 PID controller tuning rules have been specified using the method. Altogether 85% of tuning rules based on the FOLIPD process model (74 out of 87) have been specified using the method.

A2.6 SOSPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ or $\dfrac{K_m e^{-s\tau_m}}{(1+T_{m1}s)(1+T_{m2}s)}$

A2.6.1 *Parameters estimated from the open loop process step response*

Method 2: K_m, T_{m1} and τ_m are determined from the tangent and point method of Ziegler and Nichols (1942) (Shinskey, 1988, page 151); T_{m2} assumed known.

Method 3: FOLPD model parameters estimated using a tangent and point method (Ziegler and Nichols, 1942); corresponding SOSPD parameters subsequently deduced (Auslander *et al.*, 1975).

Method 4: $T_{m1} = T_{m2}$. τ_m, T_{m1} estimated from process step response: $T_{m1} = 0.794(t_{70\%} - t_{33\%})$, $\tau_m = 1.937t_{33\%} - 0.937t_{70\%}$. K_m assumed known (Vítečková *et al.*, 2000b).

Method 5: $T_{m1} = T_{m2}$. K_m, τ_m and T_{m1} estimated from the process step response; τ_m = time for which the process variable does not change; T_{m1} determined at 73% of the total process variable change (Pomerleau and Poulin, 2004).

Method 6: Parameters estimated from the underdamped or overdamped transient response in open loop to a step input (Jahanmiri and Fallahi, 1997).

A2.6.2 *Parameters estimated from the closed loop step response*

Method 7: Parameters estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Åström and Hägglund, 1995).

A2.6.3 *Parameters estimated from frequency domain closed loop information*

Method 8: In this method, $T_{m1} = T_{m2} = T_m$. T_m and τ_m estimated from K_u, T_u determined using a relay autotuning method; K_m estimated from the process step response (Hang *et al.*, 1993a).

Method 9: Parameters estimated using a two-stage identification procedure involving (a) placing a relay and (b) placing a proportional controller, in series with the process in closed loop (Sung *et al.*, 1996).

Method 10: Parameters estimated in the frequency domain from the data determined using a relay in series with the closed loop system in a master feedback loop (Hwang, 1995).

Method 11: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

Method 12: Parameters estimated from data obtained when the process phase lag is -90^0 and -180^0, respectively (Wang *et al.*, 1999).

Method 13: Model parameters estimated by including a dynamic compensator outside or inside an (ideal) relay feedback loop (Huang and Jeng, 2003).

Method 14: K_m estimated from a relay autotuning method; τ_m, T_{m1} and ξ_m estimated using a least squares algorithm based on the data recorded from the relay autotuning method, with the aid of neural networks (Huang *et al.*, 2005).

A2.6.4 *Other methods*

Method 15: Parameter estimates back-calculated from a discrete time identification method (Ferretti *et al.*, 1991).

Method 16: Parameter estimates back-calculated from a second discrete time identification method (Wang and Clements, 1995).

Method 17: Parameter estimates back-calculated from a third discrete time identification method (Lopez *et al.*, 1969).

Method 18: Model parameters estimated assuming higher order process parameters are known.

Method 19: Parameter estimates back-calculated from a discrete time identification non-linear regression method (Gallier and Otto, 1968).

The most common SOSPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is unspecified (i.e. Model: Method 1); 15 PI controller tuning rules, and 93 PID controller tuning rules have been specified using the method. Altogether 68% of tuning rules based on the SOSPD process model (108 out of 160) have been specified using the method.

A2.7 SOSIPD model – Repeated Pole $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)^2}$

Method 2: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

A2.8 SOSPD Model with a Positive Zero $G_m(s) = \dfrac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

Method 2: Parameters estimated from a closed loop step response test using a least squares approach (Wang *et al.*, 2001a).

Method 3: $T_{m1} = T_{m2}$. K_m, τ_m, T_{m1} and T_{m3} estimated from the process step response; the latter three parameters are estimated using a tabular approach (Pomerleau and Poulin, 2004).

A2.9 SOSPD model with a Negative Zero $G_m(s) = \dfrac{K_m(1 + sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

Method 2: Parameters estimated from a closed loop step response test using a least squares approach (Wang *et al.*, 2001a).

Method 3: $T_{m1} = T_{m2}$. K_m, τ_m, T_{m1} and T_{m3} estimated from the process step response; the latter three parameters are estimated using a tabular approach (Pomerleau and Poulin, 2004).

A2.10 Unstable FOLPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{T_m s - 1}$

Method 2: Parameters estimated by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin, 1995; Deshpande, 1980).

Method 3: Parameters estimated using a relay feedback approach (Majhi and Atherton, 2000).

Method 4: Parameters estimated using a biased relay feedback test (Huang and Chen, 1999).

A2.11 Unstable SOSPD Model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(1+sT_{m2})}$ or $\dfrac{K_m e^{-s\tau_m}}{(T_{m1}s-1)(T_{m2}s-1)}$

Method 2: Parameters estimated by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin, 1995; Deshpande, 1980).

Method 3: Parameters estimated by minimising the difference in the frequency responses between the high order process and the model, up to the ultimate frequency. The gain and delay are estimated from analytical equations, with the other parameters estimated using a least squares method (Kwak *et al.*, 2000).

Method 4: Parameters estimated using a biased relay feedback test (Huang and Chen, 1999).

A2.12 General Model with a Repeated Pole $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$

Method 2: A FOLPD model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters K_m', T_m' and τ_m'. Then, $n = 10\dfrac{\tau_m'}{T_m'} + 1$. Subsequently, τ_m is estimated from the open loop step response, and T_{ar} is estimated using "known methods" (Schaedel (1997)).

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Index

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