

# Global Asymptotic Saturated PID Control for Robot Manipulators

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**Abstract**—This paper addresses the global asymptotic regulation of robot manipulators under input constraints, both with and without velocity measurements. It is proven that robot systems subject to bounded inputs can be globally asymptotically stabilized via a saturated proportional–integral–derivative (PID) control in agreement with Lyapunov’s direct method and LaSalle’s invariance principle. Advantages of the proposed controller include an absence of modeling parameters in the control law formulation and an ability to ensure actuator constraints are not breached. This is accomplished by selecting control gains *a priori*, removing the possibility of actuator failure due to excessive torque input levels. The effectiveness of the proposed approach is illustrated via simulations.

**Index Terms**—Actuator saturation, asymptotic stability, bounded control, global stability, proportional–integral–derivative (PID) control, robot control.

## I. INTRODUCTION

**D**ESPITE the success of modern control theory, robot manipulator controllers still commonly use classic proportional-derivative (PD) or proportional-integral-derivative (PID) algorithms [2]–[4], [12], [22], due mostly to their conceptual simplicity and explicit tuning procedures. To improve the performance of the classical PD/PID control, most of these controllers have been designed using linear or linearized models. Some interesting nonlinear PID structures have also been proposed [2]–[4], [9], [12], [14], [21], [26], [27].

While these PD/PID-type control schemes are simple, elegant, and intuitively appealing, there is an implicit assumption in the development of these schemes that the manipulator actuators are able to provide any requested joint torque. This assumption can lead to difficulties in practice since the available torque amplitude is limited in actual manipulators. Moreover, it is known that control system design approaches that do not incorporate input constraints directly into the design suffer from important performance limitations [6], [7], [11], [23].

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Recognizing these difficulties, several solutions that take into account actuator constraints have been proposed in the literature. Colbaugh *et al.* [8] designed full-state feedback and output feedback global asymptotic regulating controllers that compensate for uncertainty; however, the control strategy switches between one controller that is used to drive the setpoint error to a small value, and another that is used to drive the setpoint error to zero. Kelly and Santibanez [13] proposed a global asymptotic regulating controller that is composed of a saturated PD feedback plus an exact model knowledge feed-forward gravity compensation term. Recently, Morabito *et al.* [19] integrated a static nonlinear controller into an available PD control plus exact gravity compensation to guarantee the global setpoint control for the Euler–Lagrange system with input saturation. More recently, Zavala–Rio and Santibanez [28] extended this approach to involve only one saturation function at each joint, and showed that the proposed approach can be conceived within the framework of the energy shaping plus damping injection methodology. Loria *et al.* [18] and Burkov [7] designed an output feedback global asymptotic regulating controller; however, exact knowledge of the gravity terms was also required. Providing gravity compensation by including a model of the manipulator gravity torques in the control law is undesirable because this requires precise *a priori* knowledge of both the structure and the parameter values for this model, including the effects of any payload. Particularly restrictive is the need for information concerning the payload, because in typical tasks many different payloads are encountered and it is unrealistic to assume that the properties of all payloads are accurately known [17], [25]. To overcome the parametric uncertainties on the gravity force, Laib [16], and Zergeroglu *et al.* [29] proposed adaptive control to guarantee global asymptotic regulation of a robotic manipulator. A minor weakness for this approach is that the structure of the gravitational torque has to be known. Without the use of model information in the control law formulation, Gorez [9] developed a so-called decoupled PID controller to resolve the global asymptotic regulation of mechanical systems in the presence of actuator constraints, which in particular includes robot manipulators. Although it was based on PID control methodology, the developed approach has the drawbacks that the control law is too complicated and involves nine tunable gains. These disadvantages make it difficult to implement. Recently, Alvarez–Ramirez *et al.* [1] formulated a saturated PID controller by resorting an additional saturated integral term to avoid the evaluation of the gravity term. Unfortunately, only the semi-global stability of the controlled system is proven. As pointed in Gunawardana and Ghorbel [10] and Kasac *et al.* [12], it is often difficult to

explicitly characterize a domain of attraction that may be much smaller than the robot workspace. This means that a global result is always more useful for both theoretical analysis and practical implementation.

In this paper, the global asymptotic regulation of robot manipulators under input constraints, both with and without velocity measurements, by a saturated PID controller is addressed. The proposed controllers do not use the modeling parameters in the control law formulations and thus permit easy implementation. To the best of our knowledge, the proposed approach yields the first global asymptotic regulation of robot manipulators subject to bounded inputs with and without velocity measurements in a three-term PID framework. It is proven that robot systems subject to actuator constraints can be globally asymptotically stabilized via a saturated PID control using Lyapunov's direct method and LaSalle's invariance principle. Simple explicit tuning rules for the controller gains ensuring global asymptotic stability and actuator constraints are not breached are given, requiring only well-known robot manipulator bounds. The fact that the proposed controller can be *a priori* bounded is a significant added advantage. The practical implications are that the actuators can be appropriately sized without an ad hoc saturation scheme to protect the actuator. Simulations are included to illustrate the effectiveness of the proposed approach.

## II. ROBOT MANIPULATOR MODEL AND PROPERTIES

In the absence of disturbances, the dynamics of an  $n$ -DOF robot manipulator can be written as [2], [17], [25]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D\dot{q} + G(q) = \tau \quad (1)$$

where  $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$  denote the link position, velocity, and acceleration, respectively,  $M(q) \in \mathbb{R}^{n \times n}$  represents the inertia matrix,  $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$  denotes the centrifugal-Coriolis matrix,  $D \in \mathbb{R}^{n \times n}$  represents the matrix composed of damping friction coefficients for each joint,  $G(q) = \partial U(q)/\partial q \in \mathbb{R}^n$  is a gravity force,  $U(q)$  is the potential energy due to gravity force, and  $\tau \in \mathbb{R}^n$  denotes the torque input. Recalling that robot manipulators are being considered, the following properties can be established [2], [17], [25].

- 1) *Property 1:* The matrix  $D$  is diagonal positive definite.
- 2) *Property 2:* The matrix  $M(q)$  is symmetric, positive definite and bounded by

$$0 < M_m \leq \|M(q)\| \leq M_M \quad (2)$$

where the norm of a matrix  $A$  is defined as the corresponding induced norm  $\|A\| = \sqrt{\lambda_M(A^T A)}$ , and  $\lambda_M(\cdot)$  denotes the maximum eigenvalue of a matrix.

- 3) *Property 3:* The time derivative of the inertia matrix and the centripetal-Coriolis matrix satisfy the following skew symmetric relationship:

$$\zeta^T (\dot{M}(q) - 2C(q, \dot{q})) \zeta = 0, \quad \forall q, \dot{q}, \zeta \in \mathbb{R}^n \quad (3)$$

which implies that

$$\dot{M}(q) = C(q, \dot{q}) + C^T(q, \dot{q}), \quad \forall q, \dot{q}, \zeta \in \mathbb{R}^n. \quad (4)$$

- 4) *Property 4:* The matrix  $C(q, \dot{q})$  is bounded by

$$0 < C_m \|\dot{q}\|^2 \leq \|C(q, \dot{q})\dot{q}\| \leq C_M \|\dot{q}\|^2, \quad \forall q, \dot{q} \in \mathbb{R}^n \quad (5)$$

where  $C_m$  and  $C_M$  are some positive known constants.

- 5) *Property 5:* There exists a positive definite diagonal matrix  $A$  such that the following two inequalities, with specified constant  $a > 0$ , are satisfied simultaneously for any fixed  $q_d$  and any  $q$ :

$$U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2} \Delta q^T A \Delta q \geq a \|\Delta q\|^2 \quad (6)$$

$$\Delta q^T [G(q) - G(q_d)] + \Delta q^T A \Delta q \geq a \|\Delta q\|^2 \quad (7)$$

where  $\Delta q = q - q_d$  denotes the position error of the actuator, and  $q$  and  $q_d$  denote the actual and desired coordinates, respectively.

- 6) *Property 6:* The gravitational force vector  $G(q)$  is bounded for all  $q \in \mathbb{R}^n$ . That is, there exist finite constants  $\gamma_i \geq 0$  such that  $\sup_{q \in \mathbb{R}^n} \{|G_i(q)|\} \leq \gamma_i, \forall i = 1, 2, \dots, n$ .

We also assume that each joint actuator has a maximum torque  $\tau_{i, \max}$  that satisfies

$$\tau_{i, \max} > \gamma_i \quad \forall i = 1, 2, \dots, n \quad (8)$$

where  $\gamma_i$  was defined in Property 6. This assumption implies that the robot actuators are able to supply requested torques to keep the robot at rest for an arbitrarily desired position  $q_d$ .

## III. CONTROL DEVELOPMENT

### A. Control Formulation

Given a target position  $q_d$ , we consider the control problem of globally regulating robotic manipulator subject to actuator constraints

$$|\tau_i| \leq \tau_{i, \max} \quad (9)$$

and without reference to model parameters, such that  $\Delta q(t) \rightarrow 0$  and  $\dot{q}(t) \rightarrow 0$  as  $t \rightarrow \infty$  for any initial state  $(q(0), \dot{q}(0))$ , where  $\tau_i$  denotes the  $i$ th torque input of the  $i$ th actuator.

To aid the subsequent control design and analysis, we define the vector  $\text{Tanh}(\cdot) \in \mathbb{R}^n$  and diagonal matrices  $\text{Cosh}(\cdot), \text{Sech}(\cdot) \in \mathbb{R}^{n \times n}$  as follows:

$$\text{Tanh}(\xi) = [\tanh(\xi_1), \dots, \tanh(\xi_n)]^T \quad (10)$$

$$\text{Sech}(\xi) = \text{diag}(\text{sech}(\xi_1), \dots, \text{sech}(\xi_n)) \quad (11)$$

$$\text{Cosh}(\xi) = \text{diag}(\cosh(\xi_1), \dots, \cosh(\xi_n)) \quad (12)$$

where  $\xi = [\xi_1, \dots, \xi_n]^T \in \mathbb{R}^n$ ,  $\tanh(\cdot)$ ,  $\text{sech}(\cdot)$  and  $\cosh(\cdot)$  being the standard hyperbolic tangent, secant and cosine functions, respectively, and  $\text{diag}(\cdot)$  denotes a diagonal matrix. Note that  $\tanh(\cdot)$  function serves as a saturation function and other saturation functions can be freely chosen to replace it in the following proposed controller. Based on the definition of (10) and (11), it can easily be shown that the following expressions hold:

$$\frac{1}{2} \tanh^2(\xi_i) \leq \ln(\cosh(\xi_i)) \quad (13)$$

$$\lambda_M(\text{Sech}^2(\xi)) = 1 \quad (14)$$

$$\text{Tanh}(\xi)^T \text{Tanh}(\xi) \leq \xi^T \text{Tanh}(\xi) \quad (15)$$

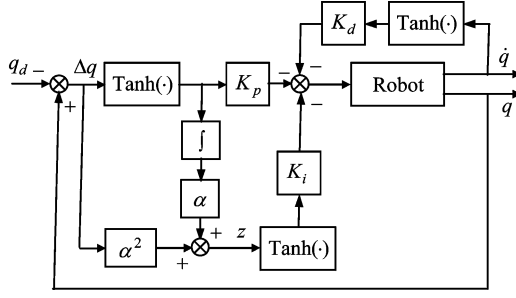


Fig. 1. State feedback saturated PID control system.

where  $\lambda_M(\cdot)$  was defined in (2), and the norm of a vector  $\xi$  is defined as  $\|\xi\| = \sqrt{\xi^T \xi}$ .

Based on the control objective and the subsequent stability analysis, the proposed saturated PID controller is formulated as

$$\tau = -K_p \text{Tanh}(\Delta q) - K_i \text{Tanh}(z) - K_d \text{Tanh}(\dot{q}) \quad (16)$$

$$z(t) = \alpha^2 \Delta q + \alpha \int_0^t \text{Tanh}(\Delta q(\sigma)) d\sigma \quad (17)$$

where  $K_p$ ,  $K_i$ , and  $K_d$  are positive definite constant diagonal proportional, integral, and derivative matrices, respectively, and  $\alpha$  is a large positive constant. Particular concern should be given to the design of  $z$ , which ensures that  $\alpha$  is sufficiently large to satisfy stability condition (24), as presented below in Theorem 1. The schematic diagram of the state feedback saturated PID control system is illustrated in Fig. 1.

The control effort can be upper bounded in terms of *a priori* known terms as

$$|\tau_i| \leq k_{pi} + k_{ii} + k_{di} \quad (18)$$

where  $k_{pi}$ ,  $k_{ii}$ , and  $k_{di}$  denote the  $i$ th diagonal elements of control gain matrices  $K_p$ ,  $K_i$ , and  $K_d$ , respectively. They can be made arbitrarily small provided some relative magnitudes are maintained as subsequently described.

Based on this fact, the actuator constraints expressed in (9) can be satisfied by selecting the control gains *a priori*

$$k_{pi} + k_{ii} + k_{di} < \tau_{i,\max} \quad (19)$$

where  $\tau_{i,\max}$  was defined in (8).

The final stationary state of the system (1) and (16) is  $\Delta q = 0$ ,  $\dot{q} = 0$ ,  $z = z^*$ , where  $z^*$  denotes a final stationary state and satisfies

$$G(q_d) = -K_i \text{Tanh}(z^*). \quad (20)$$

Introducing the following new vector:

$$\phi(z(t)) = \text{Tanh}(z(t)) - \text{Tanh}(z^*) \quad (21)$$

and substituting (16) and (21) into (1), the closed-loop dynamics become

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D\dot{q} + G(q) - G(q_d) + K_p \text{Tanh}(\Delta q) + K_d \text{Tanh}(\dot{q}) + K_i \phi = 0 \quad (22)$$

whose origin  $[\Delta q^T \quad \dot{q}^T \quad \phi^T]^T = 0 \in \mathbb{R}^{3n}$  is the unique equilibrium.

After taking the time derivative of (21) and (17), we have

$$\dot{\phi} = \text{Sech}^2(z)\dot{z}(t) = \alpha^2 \text{Sech}^2(z) \left( \Delta \dot{q} + \frac{1}{\alpha} \text{Tanh}(\Delta q) \right). \quad (23)$$

## B. Stability Analysis

**Theorem 1:** Given the robotic system defined in (1) under the input constraints (19), with the proposed state feedback saturated PID controller (16) and (17), the closed-loop system (22) is globally asymptotically stable, provided that the control gains are chosen as follows:

$$\alpha D \geq (\sqrt{n}C_M + M_M)I \quad (24)$$

$$2\alpha K_d > \lambda_M(K_d)I \quad (25)$$

$$k_{ii} > \gamma_i \quad (26)$$

$$K_p \geq 4\alpha^{-2}M_M I \quad (27)$$

$$U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2} \sum_{i=1}^n k_{pi} \ln(\cosh(\Delta q_i)) \geq a \|\text{Tanh}(\Delta q)\|^2 \quad (28)$$

$$\text{Tanh}^T(\Delta q) (G(q) - G(q_d)) + \text{Tanh}^T(\Delta q) K_p \text{Tanh}(\Delta q) \geq \left( a + \frac{1}{2} \lambda_M(K_d) \right) \|\text{Tanh}(\Delta q)\|^2 \quad (29)$$

where  $k_{pi}$  and  $k_{ii}$  were defined in (18),  $I$  denotes the  $n \times n$  identity matrix, and  $a$  was an arbitrary small positive constant defined in (6) and (7). Note that the inequalities (28) and (29) correspond to inequalities (6) and (7) of Property 5, respectively, and the existence of such a matrix  $K_p$  is confirmed by the same argument given in proposing (6) and (7), since each component  $\tanh(\Delta q_i)$  satisfying (10) is quadratic in the vicinity of  $\Delta q = 0$  and (13) [2], [27].

**Remark 1:** Condition (24) in Theorem 1 is not excessively restrictive and limitative, due to the fact that friction always exists in a practical robot and the friction coefficient matrix  $D$  is diagonal positive definite, as expressed in Property 1.

**Remark 2:** The tuning procedure of the proposed controller can be stated as follows:  $\alpha$  should be determined first to satisfy (24). After determining that  $\alpha$  is sufficiently large, we can determine other gains according to the conditions (25)–(29) and the input constraints (19). There is no conflict in choosing  $K_p$  and  $K_d$ , even though both are dependent on  $\alpha$ . In fact, from conditions (27) and (25), both  $K_p$  and  $K_d$  benefit from a large value for  $\alpha$ . Note that  $K_i$  should satisfy condition (26) so that (20) holds due to  $|\tanh(z_i^*)| \leq 1$  and Property 6, where  $z_i^*$  denotes the  $i$ th element defined by (20).

**Proof:** Theorem 1 is proven following Lyapunov's direct method and can be divided into two parts. First, a positive definite Lyapunov function  $V$  is proposed. Then, the negative semi-definite of Lyapunov function derivative  $\dot{V}$  is shown. Finally, the LaSalle's invariance principle is invoked to guarantee global asymptotic stability.

1) *Lyapunov Function Candidate:* The Lyapunov-like function  $V$  is proposed as follows:

$$\begin{aligned} V = & \frac{1}{2} \dot{q}^T M(q) \dot{q} + \frac{1}{\alpha} \text{Tanh}^T(\Delta q) M(q) \dot{q} + U(q) - U(q_d) \\ & - \Delta q^T G(q_d) + \sum_{i=1}^n \left( k_{pi} + \frac{d_i}{\alpha} \right) \ln(\cosh(\Delta q_i)) \\ & + \frac{1}{\alpha^2} \int_0^\phi \sigma^T K_i \text{Cosh}^2(z) d\sigma \end{aligned} \quad (30)$$

where

$$\int_0^\phi \sigma^T K_i \text{Cosh}^2(z) d\sigma = \sum_{i=1}^n \int_0^{\phi_i} k_{ii} \cosh^2(z_i) \sigma_i d\sigma_i. \quad (31)$$

In this representation,  $k_{ii}$ ,  $\cosh^2(z_i)$ , and  $d_i$  denote the  $i$ th diagonal elements of matrices  $K_i$ ,  $\text{Cosh}^2(z)$ , and  $D$ , respectively. Note that the terms in the proposed Lyapunov function (except for the last one) are quite common for global asymptotic regulation of robot manipulators [2], [12], [14], [26], [27]. Special attention should be given to the final term, which accounts for  $\phi$ , the state variable induced in (21) and (22) to track saturated integral action. It turns out to be useful to cancel cross terms in the time derivative of the Lyapunov function by using the properties of hyperbolic tangent and secant functions.

To show the positive definiteness of the proposed Lyapunov function candidate, we first consider the following:

$$\begin{aligned} & \frac{1}{4} \dot{q}^T M(q) \dot{q} + \frac{1}{2} \sum_{i=1}^n k_{pi} \ln(\cosh(\Delta q_i)) \\ & + \frac{1}{\alpha} \text{Tanh}^T(\Delta q) M(q) \dot{q} \\ & = \frac{1}{4} \left( \dot{q} + \frac{2}{\alpha} \text{Tanh}(\Delta q) \right)^T M(q) \left( \dot{q} + \frac{2}{\alpha} \text{Tanh}(\Delta q) \right) \\ & - \frac{1}{\alpha^2} \text{Tanh}^T(\Delta q) M(q) \text{Tanh}(\Delta q) \\ & + \frac{1}{2} \sum_{i=1}^n k_{pi} \ln(\cosh(\Delta q_i)) \\ & \geq \frac{1}{2} \sum_{i=1}^n k_{pi} \ln(\cosh(\Delta q_i)) \\ & - \frac{1}{\alpha^2} \text{Tanh}^T(\Delta q) M(q) \text{Tanh}(\Delta q) \\ & \geq \frac{1}{4} \sum_{i=1}^n \{ k_{pi} - 4\alpha^{-2} M_M \} \tanh^2(\Delta q_i) \end{aligned} \quad (32)$$

where (2) of Property 2 and (13) have been used.

Substituting (32) into (30), we have

$$\begin{aligned} V \geq & \frac{1}{4} \dot{q}^T M(q) \dot{q} + \frac{1}{4} \sum_{i=1}^n \{ k_{pi} - 4\alpha^{-2} M_M \} \tanh^2(\Delta q_i) \\ & + U(q) - U(q_d) - \Delta q^T G(q_d) \\ & + \sum_{i=1}^n \left( \frac{k_{pi}}{2} + \frac{d_i}{\alpha} \right) \ln(\cosh(\Delta q_i)) \\ & + \frac{1}{\alpha^2} \int_0^\phi \sigma^T K_i \text{Cosh}^2(z) d\sigma. \end{aligned} \quad (33)$$

Furthermore, it can be shown that (31) satisfies

$$\int_0^\phi \sigma^T K_i \text{Cosh}^2(z) d\sigma > 0 \quad \forall \phi \neq 0 \in \mathbb{R}^n \quad (34)$$

because  $K_i$  and  $\text{Cosh}^2(z)$  are diagonal positive definite matrices,  $\sigma_i|_{\sigma_i=0} = 0$ , and  $\sigma_i$  is an increasing function with respect to  $\phi$ . Therefore, this term is positive definite with respect to  $\phi$ .

From (27), (28), (33), and (34), we get

$$\begin{aligned} V \geq & \frac{1}{4} \dot{q}^T M(q) \dot{q} + a \|\text{Tanh}(\Delta q)\|^2 \\ & + \frac{1}{\alpha} \sum_{i=1}^n d_i \ln(\cosh(\Delta q_i)) \\ & + \frac{1}{\alpha^2} \int_0^\phi \sigma^T K_i \text{Cosh}^2(z) d\sigma > 0 \end{aligned} \quad (35)$$

for  $[\Delta q^T \quad \dot{q}^T \quad \phi^T]^T \neq 0$ .

Hence, we can conclude that  $V$  is a positive definite Lyapunov function with respect to  $\Delta q, \dot{q}, \phi$ .

2) *Global Asymptotic Stability:* Differentiating  $V$  with respect to time, we have

$$\begin{aligned} \dot{V} = & \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{q}^T M(q) \ddot{q} + \frac{1}{\alpha} (\text{Sech}^2(\Delta q) \Delta \dot{q})^T M(q) \dot{q} \\ & + \frac{1}{\alpha} \text{Tanh}^T(\Delta q) \dot{M}(q) \dot{q} + \frac{1}{\alpha} \text{Tanh}^T(\Delta q) M(q) \ddot{q} \\ & + \Delta \dot{q}^T K_p \text{Tanh}(\Delta q) + \dot{q}^T G(q) - \Delta \dot{q}^T G(q_d) \\ & + \frac{1}{\alpha} \text{Tanh}^T(\Delta q) D \Delta \dot{q} + \frac{1}{\alpha^2} \phi^T \text{Cosh}^2(z) K_i \dot{\phi}. \end{aligned} \quad (36)$$

Substituting  $M(q) \ddot{q}$  from (22) and (23) into (36), and using (3) and (4) of Property 3, it follows that

$$\begin{aligned} \dot{V} = & -\dot{q}^T K_d \text{Tanh}(\dot{q}) - \dot{q}^T D \dot{q} \\ & - \frac{1}{\alpha} \text{Tanh}^T(\Delta q) K_d \text{Tanh}(\dot{q}) \\ & + \frac{1}{\alpha} \left[ \text{Tanh}^T(\Delta q) C^T(q, \dot{q}) \dot{q} \right. \\ & \quad \left. + (\text{Sech}^2(\Delta q) \Delta \dot{q})^T M(q) \dot{q} \right] \\ & - \frac{1}{\alpha} \text{Tanh}^T(\Delta q) [(G(q) - G(q_d)) \\ & \quad + K_p \text{Tanh}(\Delta q)]. \end{aligned} \quad (37)$$

Using (2) of Property 2, (5) of Property 4 and (14), the fourth term of the right-hand side of (37) can be upper bounded by

$$\begin{aligned} & \frac{1}{\alpha} \left[ \text{Tanh}^T(\Delta q) C^T(q, \dot{q}) \dot{q} + (\text{Sech}^2(\Delta q) \Delta \dot{q})^T M(q) \dot{q} \right] \\ & \leq \frac{1}{\alpha} (\sqrt{n} C_M + M_M) \|\dot{q}\|^2. \end{aligned} \quad (38)$$

Note that in the derivation of the first term of (38) we utilized  $\|\text{Tanh}(\Delta q)\| \leq \sqrt{n}$  according to (10) and  $|\tanh(\Delta q_i)| \leq 1$ .



## IV. EXAMPLES

1) *Example 4.1:* Comparisons with the saturated PD plus exact gravity compensation recently proposed by Zavala-Rio and Santibanez [28] are conducted. The reasoning behind this comparison is that the proposed saturated PID (SPID) controller and the partially model-based control have the same global asymptotic stability. We carried out a comparison using the two degree-of-freedom (2-DOF) direct-drive robot used in [24], [28]. Robot dynamics are described in Appendix.

The partially model-based saturated control algorithm developed by Zavala-Rio and Santibanez [28] is as follows:

$$\tau = s_2(G(q) - K_1\dot{q} - s_1(K_2\Delta q)) \quad (44)$$

where  $K_1$  and  $K_2$  are constant positive definite diagonal matrices, and  $s_j(\cdot) \in \mathbb{R}^n$ ,  $j = 1, 2$  are defined as follows:

$$s_j(\xi) = [\sigma_{j1}(\xi_1), \dots, \sigma_{jn}(\xi_n)]^T \quad \forall \xi = [\xi_1, \dots, \xi_n]^T \in \mathbb{R}^n \quad (45)$$

with  $\sigma_{ji}(\cdot)$ ,  $i = 1, \dots, n$ , being expressed as

$$\sigma_{ji}(x) = M_{ji} \text{sat}\left(\frac{x}{M_{ji}}\right) \quad \forall x \in \mathbb{R} \quad (46)$$

and  $\text{sat}(\cdot)$  is the standard saturation function, i.e.,

$$\text{sat}(x) = \begin{cases} x, & |x| < 1 \\ \frac{x}{|x|}, & |x| \geq 1. \end{cases} \quad (47)$$

The final desired positions were  $q_d = [\pi/4, \pi/2]^T$  (rad). The sampling period was  $T = 1$  ms. All the initial parameters were set as zero. The actuator constraints were assumed as  $\tau_{\max} = [150, 15]$  N · m [28]. The gains for the proposed saturated PID (SPID) controller were chosen in accordance with stability conditions (24)–(29) and the constraints (19), and determined as  $\alpha = 100$ ,  $K_p = \text{diag}(20, 8.5)$ ,  $K_i = \text{diag}(45.0, 2.0)$ , and  $K_d = \text{diag}(32, 2.5)$ . Following the guidelines presented in [28], the gains of the partially model-based saturated control were chosen as  $K_1 = \text{diag}(30, 4.0)$ ,  $K_2 = \text{diag}(3.0, 0.05)$ , and  $M_{11} = 90.0$ ,  $M_{12} = 8.0$ ,  $M_{21} = 115.0$ , and  $M_{22} = 12.0$ .

Figs. 3 and 4 illustrate the position errors and the requested input torques of the two controllers, respectively. It can be seen that the robot successfully completed its movement at the desired final position, and after a transient due to errors in initial condition, the position errors tend asymptotically to zero. Furthermore, a faster response is achieved in comparison with the partially model-based control. Notice that the favorable results are obtained with a very simple saturated PID controller, which does not require any model information in the control law formulation. Note also that the proposed saturated PID control may cause a much larger input torque in the transient response than the partially model-based control. This is due to the fact that the error in estimating the gravitational force is large at the beginning without using any knowledge of the gravity force.

As a result, we can conclude that the proposed model-free saturated PID control obtains a better result over the partially model-based saturated PD control with exact gravity compensation, under the actuator constraints. The gains of the proposed

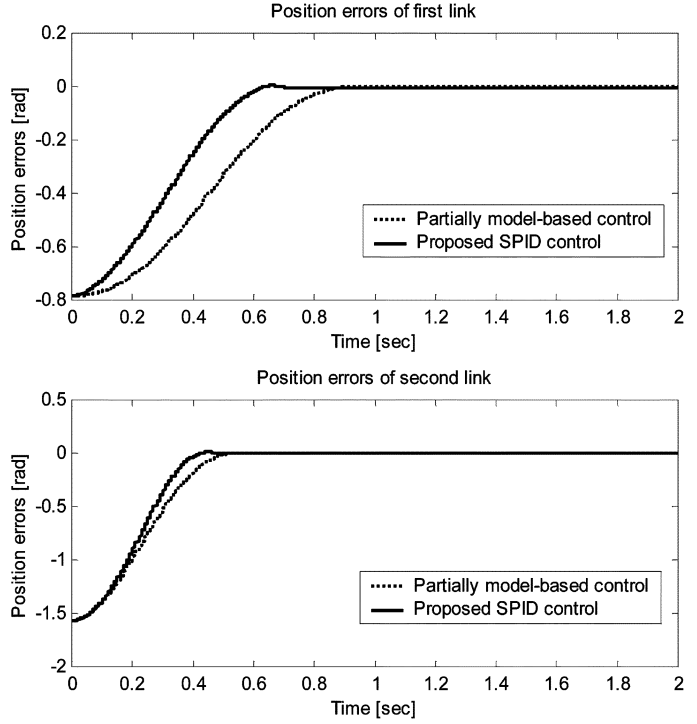


Fig. 3. Position errors.

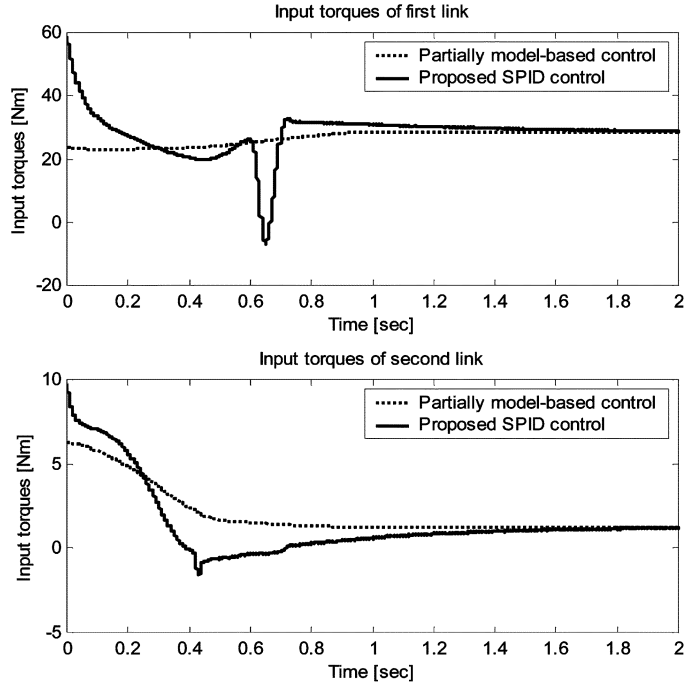


Fig. 4. Input torques.

saturated PID control can be explicitly determined to meet the actuator constraints, depending only on some well-known bounds of the robot manipulator.

2) *Example 4.2:* Output feedback regulation of a three degree-of-freedom (3-DOF) industrial robot manipulator without

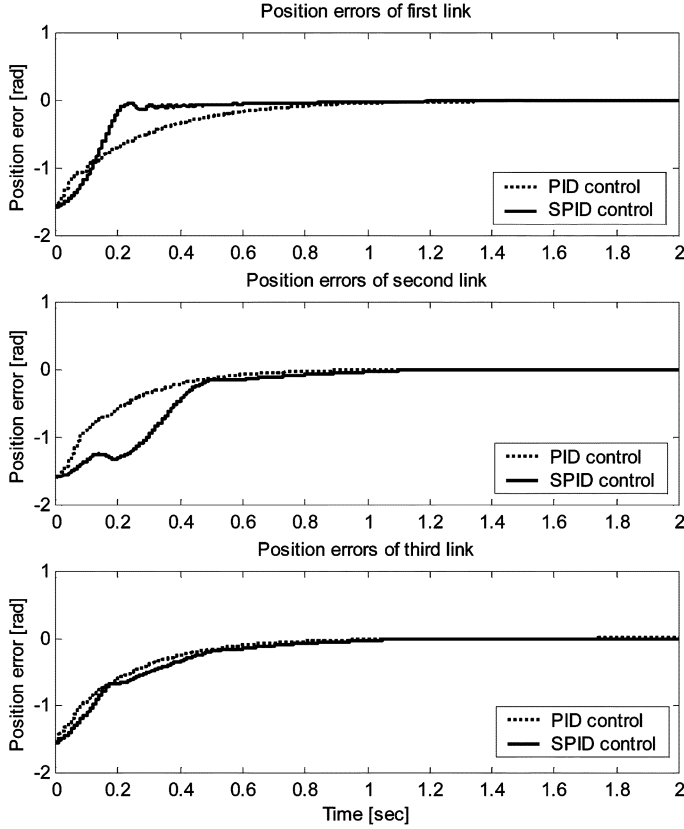


Fig. 5. Position errors of the 3-DOF robot.

velocity measurements is also conducted. 3-DOF robot is described in the Appendix. The position is assumed to be contaminated by a noise with an amplitude of 0.01 rad. The desired positions were set as  $q_d = [\pi/2, \pi/2, \pi/2]^T$  (rad). The following unbounded output feedback PID controller is used:

$$\tau = -K_p \Delta q - K_i \int_0^t \text{Tanh}(\Delta q(\sigma)) d\sigma - K_d v \quad (48)$$

where  $v \in \mathbb{R}^n$  is the output of the filter (40).

The controller parameters were selected by trial-and-error until a good regulation performance was obtained. The gains of the output feedback PID control along with the filter (40) were chosen as:  $K_p = \text{diag}(60, 40, 40)$ ,  $K_i = \text{diag}(1.0, 2.0, 1.0)$ ,  $K_d = \text{diag}(12, 5, 5)$ ,  $A = \text{diag}(40, 30, 30)$ , and  $B = \text{diag}(50, 40, 40)$ . The actuator constraints were assumed as  $\tau_{\max} = [50, 30, 30]$  N · m. The gains for the proposed output feedback SPID controller were chosen in accordance with the constraints (19) and the stability conditions in Theorem 2, and determined as  $\alpha = 100$ ,  $K_p = \text{diag}(28, 18, 18)$ ,  $K_i = \text{diag}(1.0, 3.2, 1.0)$ , and  $K_d = \text{diag}(18, 9, 5)$ . The filter gains were as the same as the ones of the output feedback PID control. Figs. 5 and 6 illustrate the position errors and the requested input torques of the two controllers, respectively. It is seen that the robot also completed its movement at the final desired position asymptotically, after a transient due to errors in initial condition. Furthermore, requested input torques using the proposed SPID control remain uniformly within the stated torque constraints, while the PID control initially requests input torques that exceed the actuator limits.

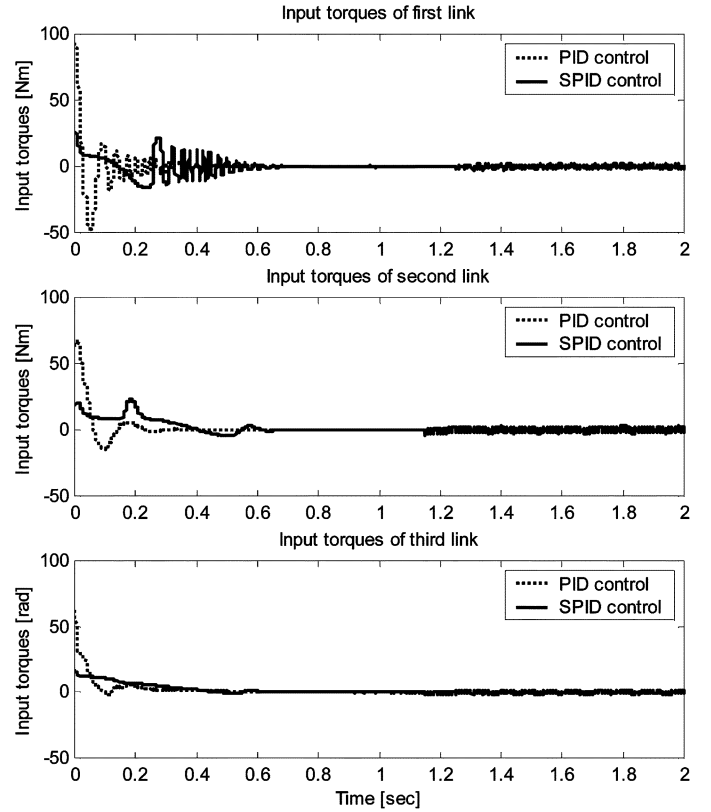


Fig. 6. Input torques of the 3-DOF robot.

## V. CONCLUSION

We have presented a very simple saturated PID control for global asymptotic regulation of robot manipulators subject to actuator constraints, both with and without velocity measurements. It is proven that the closed-loop system formed by the saturated PID controller and robot system is global asymptotically stable. Advantages of the proposed controller include an absence of modeling parameters in the control law formulation and an ability to ensure actuator constraints are not breached. The practical implication is that it can remove the possibility of actuator failure due to excessive torque input levels by selecting control gains *a priori*. Additionally, it is easy to implement. Simulations are included to demonstrate the effectiveness of the proposed saturated PID control.

## APPENDIX DYNAMIC MODEL OF THE ROBOTS

*Dynamic Model of the 2-DOF Robot:* The entries to model the robot are, respectively [28]

$$\begin{aligned} M &= \begin{bmatrix} \theta_1 + 2\theta_2 \cos(q_2) & \theta_3 + \theta_2 \cos(q_2) \\ \theta_3 + \theta_2 \cos(q_2) & \theta_3 \end{bmatrix} \\ C &= \begin{bmatrix} -2\theta_2 \sin(q_2) \dot{q}_2 & -\theta_2 \sin(q_2) \dot{q}_2 \\ \theta_2 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix} \\ G &= \begin{bmatrix} \theta_4 \sin(q_1) + \theta_5 \sin(q_1 + q_2) \\ \theta_5 \sin(q_1 + q_2) \end{bmatrix}. \end{aligned} \quad (49)$$

Furthermore, a Coulomb friction is also considered in the simulations. To keep the notation used for model (1), it is defined  $D = \text{diag}(\theta_6, \theta_7)$ , and

$$f_c(\dot{q}) = [\theta_8 \text{sgn}(\dot{q}_1), \theta_9 \text{sgn}(\dot{q}_2)]^T \quad (50)$$

where  $\text{sgn}(\cdot)$  being the standard signum function. The parameters are given in SI units and summarized as follows:  $\theta_1 = 2.351$ ,  $\theta_2 = 0.084$ ,  $\theta_3 = 0.102$ ,  $\theta_4 = 38.465$ ,  $\theta_5 = 1.825$ ,  $\theta_6 = 2.288$ ,  $\theta_7 = 0.175$ ,  $\theta_8 = 7.170$  if  $\dot{q}_1 > 0$  and  $8.049$  if  $\dot{q}_1 < 0$ , and  $\theta_9 = 1.724$ .

**Dynamic Model of the 3-DOF Robot:** The motion of the 3-DOF industrial robot is given by [20]

$$\begin{aligned} q &= [q_1, q_2, q_3]^T \\ M(q) &= \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix} \\ m_{11} &= m_2 l_1^2 \cos^2(q_2) + 2m_2 l_1 l_2 \cos(q_2) \cos(q_2 + q_3) \\ &\quad + m_2 l_2^2 \cos^2(q_2 + q_3) + m_1 l_1^2 \cos^2(q_2) + I_1 \quad (51) \\ m_{22} &= (m_1 + m_2) l_1^2 + m_2 l_2 (2l_1 \cos(q_3) + l_2) + I_2 + I_3 \\ m_{23} &= m_2 l_1 l_2 \cos(q_3) + m_2 l_2^2 + I_3 \\ m_{33} &= m_2 l_2^2 + I_3 \\ C(q, \dot{q}) &= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix} \\ c_{11} &= -m_2 l_1^2 \sin(q_2) \cos(q_2) \dot{q}_2 - m_2 l_1 l_2 \sin(q_2) \\ &\quad \times \cos(q_2 + q_3) \dot{q}_2 - m_1 l_1^2 \sin(q_2) \cos(q_2) \dot{q}_2 \\ &\quad - m_2 l_1 l_2 \cos(q_2) \sin(q_2 + q_3) (\dot{q}_2 + \dot{q}_3) \\ &\quad - m_2 l_2^2 \sin(q_2 + q_3) \cos(q_2 + q_3) (\dot{q}_2 + \dot{q}_3) \\ c_{12} &= -m_2 l_1 l_2 \cos(q_2) \sin(q_2 + q_3) \dot{q}_1 - m_2 l_2^2 \sin(q_2 + q_3) \\ &\quad \times \cos(q_2 + q_3) \dot{q}_1 - m_2 l_1 l_2 \sin(q_2) \cos(q_2 + q_3) \dot{q}_1 \\ &\quad - m_1 l_1^2 \sin(q_2) \cos(q_2) \dot{q}_1 - m_2 l_2^2 \sin(q_2 + q_3) \\ &\quad \times \cos(q_2 + q_3) \dot{q}_1 \\ c_{13} &= -m_2 (l_1 l_2 \cos(q_2) + l_2^2 \cos(q_2 + q_3)) \sin(q_2 + q_3) \dot{q}_1 \\ c_{21} &= ((m_1 + m_2) l_1^2 \cos(q_2) + m_2 l_1 l_2 \cos(q_2 + q_3)) \\ &\quad \times \sin(q_2) \dot{q}_1 + (m_2 l_1 l_2 \cos(q_2) + m_2 l_2^2 \\ &\quad \times \cos(q_2 + q_3)) \sin(q_2 + q_3) \dot{q}_1 \\ c_{22} &= -m_2 l_1 l_2 \sin(q_3) \dot{q}_3 \\ c_{23} &= -m_2 l_1 l_2 \sin(q_3) (\dot{q}_2 + \dot{q}_3) \\ c_{31} &= m_2 l_2 (l_1 \cos(q_2) + l_2 \cos(q_2 + q_3)) \sin(q_2 + q_3) \dot{q}_1 \\ c_{32} &= m_2 l_1 l_2 \sin(q_3) \dot{q}_2 \quad (52) \\ D &= \text{diag}(d_1, d_2, d_3) \quad (53) \\ G(q) &= \begin{bmatrix} 0 \\ (m_1 + m_2) l_1 g \cos(q_1) + m_2 l_2 g \cos(q_2 + q_3) \\ m_2 l_2 g \cos(q_2 + q_3) \end{bmatrix} \quad (54) \end{aligned}$$

The parameters are given in SI units and summarized as follows:  $l_1 = 0.297$ ,  $l_2 = 0.297$ ,  $m_1 = 0.38$ ,  $m_2 = 0.34$ ,  $I_1 = 0.243 \times 10^{-3}$ ,  $I_2 = 0.068 \times 10^{-3}$ ,  $I_3 = 0.015 \times 10^{-3}$ ,  $d_1 = 0.193$ ,  $d_2 = 0.852$ ,  $d_3 = 1.524$ , and  $g = 9.8$ .

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