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Automatica 39 (2003) 309–315

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Brief Paper

Relay-based closed loop transfer function frequency points estimation☆

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Received 5 April 2001; received in revised form 16 April 2002; accepted 16 September 2002

Abstract

In this paper, the following frequency response estimation problem is addressed: *Given a closed loop transfer function*, *such as the loop transfer function or the sensitivity function*, *find a frequency for which it achieves a user-defined magnitude*. The procedure presented here is based on a relay feedback, which makes it suitable to apply to actual systems. In addition to the frequency, the system phase at that frequency is computed simultaneously. The procedure can be used to evaluate and redesign controllers.

Keywords: Relay feedback; Transfer function estimation; Frequency-domain identification

1. Introduction

In practical applications, sometimes it is desirable to determine the frequency at which a given closed loop system achieves a certain magnitude or phase. For instance, the phase and frequency at which the loop transfer function has unitary magnitude yields the phase margin. Although many loop-shaping techniques assume that the process transfer function is known, there also exist several loop-shaping techniques which use a few points of the frequency response of the loop transfer function to design, for instance, PID controllers (see Åström & Hägglund, 1995). By estimating a few frequency points, one can gather enough information to redesign a controller using loop shaping techniques (Doyle, Francis, & Tannenbaum, 1995). The sensitivity function brings relevant information on the closed loop such as disturbance attenuation level and stability margins (Doyle et al., 1995). With a few frequency points, sensitivity function shaping can be used for controller design (Doyle et al., 1995; Langer & Landau, 1999; Barros & Wittenmark, 1997).

In this paper, a relay experiment that yields a controlled (stable) limit cycle at the frequency where a closed loop transfer function magnitude is equal to a defined value, r, is presented. The use of a relay feedback makes it possible to perform the experiments under well-defined operating conditions.

The relay experiment was introduced in Åström and Hägglund (1984) and is widely used as an estimation tool for controller tuning. There exist several extensions to the original experiment which resulted in a variety of techniques for process transfer function estimation (see Åström & Hägglund, 1995; Tan, Wang, Hang, & Hägglund, 1999, and references therein). Relay tests related to the results presented here are Schei (1992, 1994) and Tan et al. (1999). In Schei (1994), a new relay experiment is presented such that the closed loop system oscillates approximately at the crossover frequency. This turns out to be a particular case when $r = 1$. Finally, in Tan et al. (1999), new relay-based experiments were introduced in order to estimate a frequency point of the process transfer function where an arbitrary phase lag is achieved.

In Section 2 the problem is formally stated and solved. In the sequel the basic procedure is applied to the following transfer functions: loop transfer function (Section 3) and sensitivity function (Section 4). Practical aspects such as signal magnitude and experiment stability are discussed here. The potential for application of the method to actual processes is in fact quite wide. Finally, simulation results are presented in Section 5.

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Tor Arne Johansen under the direction of Editor Sigurd Skogestad.

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PII: S0005-1098(02)00205-4

2. Transfer function frequency response estimation

In this section the problem is formally stated and its general solution is presented. Stability conditions and improvement of the estimate are discussed.

2.1. Problem statement

In this paper, and without loss of generality, the closed loop shown in Fig. 1 is considered. The unknown time-invariant SISO process transfer function is given by $G(s)$ while the controller is given by $C(s)$. For this closed loop configuration, the loop transfer function is defined as $L(s)=G(s)C(s)$. The closed loop transfer function from reference $y_r(t)$ to output $y(t)$ is given by

$$\frac{Y(s)}{Y_r(s)}=T(s)=\frac{L(s)}{1+L(s)}, \tag{1}$$

where $T(s)$ is also known as the complementary sensitivity function. Finally,

$$S(s)=\frac{E(s)}{Y_r(s)}=\frac{1}{1+L(s)} \tag{2}$$

is the sensitivity function.

The problem posed in this paper can be stated as: Given a closed loop transfer function of interest $H(s)$, find a frequency where the magnitude of $H(\mathrm{j}\omega)$ achieves a defined gain r.

It should be noticed that this problem, although related, is not equivalent to the general frequency response estimation problems as presented in Wang, Hang, and Bi (1999) and Guillaume, Pintelon, and Schoukens (1992). In fact, with the data generated by the experiment presented here, one can apply the frequency estimation techniques therein to extract additional information on other frequency points.

2.2. General problem solution

A general solution to the problem posed in this paper can be obtained by using a relay feedback with an integrator, if the transfer function $F(s)$ has a particular form. This is shown in the following proposition.

Proposition 1. *Consider the closed loop system shown in Fig. 2. Assume that for a stable transfer function $H(s)$ and for a real positive number r, transfer function $F(s,r)$ is defined as*

$$F(s,r)=\frac{H(s)-r}{H(s)+r}, \tag{3}$$

which, in addition, is also assumed stable. Then if a limit cycle is present the oscillation occurs at a frequency ω_0 such that

$$|H(\mathrm{j}\omega_0)|\approx r. \tag{4}$$

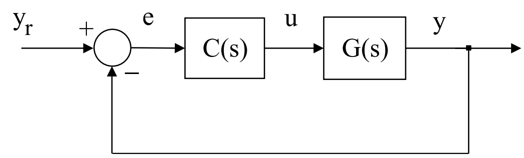

Fig. 1. Closed loop system.

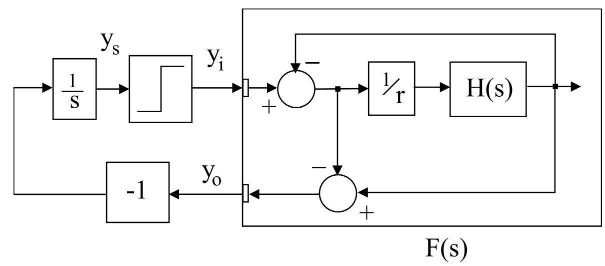

Fig. 2. Relay feedback structure for Proposition 1.

Proof. If a limit cycle is present in the relay closed loop system, then from Åström and Hägglund (1995) $\angle F(\mathrm{j}\omega_0,r)\approx -90°$ at the frequency of the oscillation. Thus,

$$F(\mathrm{j}\omega_0,r)=\frac{H(\mathrm{j}\omega_0)-r}{H(\mathrm{j}\omega_0)+r}\approx -kj \tag{5}$$

for some real $k>0$. Rewrite this expression in order to obtain

$$H(\mathrm{j}\omega_0)\approx r\,\frac{1-kj}{1+kj}\;\Rightarrow |H(\mathrm{j}\omega_0)|\approx r. \qquad \square \tag{6}$$

The structure with $F(s,r)$ given by Eq. (3), shown in Fig. 2, is named in this paper *the frequency response estimation relay test*.

Now consider the transfer function $H(\mathrm{j}\omega)$ and its $\mathscr{H}_\infty$ norm, $\|H(\mathrm{j}\omega)\|_\infty$. What would happen if $r>\|H(\mathrm{j}\omega)\|_\infty$? This question is addressed in the sequel.

Proposition 2. *Consider a stable transfer function $H(s)$ and define $F(s,r)$ as in Eq. (3). If $r>\|H(\mathrm{j}\omega)\|_\infty$, then the describing function method points out no conditions for the relay feedback in Fig. 2 to develop a limit cycle.*

Proof. The condition for oscillation is $\angle F(\mathrm{j}\omega)=-90°$, which means that $F(\mathrm{j}\omega_0)=-\mathrm{j}k$ for some k and ω_0.

Thus, the Nyquist plot of $F(\mathrm{j}\omega)$ should cross the negative imaginary axis at the frequency ω_0. The aim of the proof is to show that this situation never happens if $r>\|H(\mathrm{j}\omega)\|_\infty$. Write

$$F(\mathrm{j}\omega)=\frac{H(\mathrm{j}\omega)-r}{H(\mathrm{j}\omega)+r}=\frac{\rho(\omega)\mathrm{e}^{\mathrm{j}\phi(\omega)}-r}{\rho(\omega)\mathrm{e}^{\mathrm{j}\phi(\omega)}+r}, \tag{7}$$

where $\rho(\omega)=|H(\mathrm{j}\omega)|$ and $\phi(\omega)=\angle H(\mathrm{j}\omega)$. As $\mathrm{e}^{\mathrm{j}\phi(\omega)}=\cos\phi(\omega)+\mathrm{j}\sin\phi(\omega)$, then

$$F(\mathrm{j}\omega)=\frac{\rho^2(\omega)-r^2+\mathrm{j}\,2r\rho(\omega)\sin\phi(\omega)}{\rho^2(\omega)+2\rho(\omega)\cos\phi(\omega)r+r^2}. \tag{8}$$

The real part of $F(j\omega)$ is zero only if $\rho^2(\omega) - r^2 = 0 \Rightarrow \rho(\omega) = r$, for some ω, as $r > 0$ and $\rho(\omega) > 0$. Since $\sup_\omega \rho(\omega) < r$, there is no solution in ω such that Re $F(j\omega) = 0$. Therefore, from the describing function method, no limit cycle is expected. □

Remark 1. Note that from Eq. (8) the magnitude of $F(j\omega)$ at the frequency of the limit cycle, ω_0, depends only on the phase of $H(j\omega)$ at that frequency, i.e., $\phi(\omega_0)$. To see this, assume that at ω_0, $\rho(\omega_0) = r$, then

$$|F(j\omega_0)| = \frac{2r^2 \sin\phi(\omega_0)}{2r^2 + 2r^2\cos\phi(\omega_0)} = \frac{\sin\phi(\omega_0)}{1 + \cos\phi(\omega_0)}. \qquad (9)$$

It can also be seen that $|F(j\omega_0)|$ approaches infinity as $\angle H(j\omega_0) \to (2n+1)\pi$ and zero as $\angle H(j\omega_0) \to 2n\pi$, with $n = 0, \pm 1, \pm 2, \ldots$.

A related question which arises is what will happen in the case where there are several frequencies for which $|H(j\omega)| = r$. This analysis is considered in detail in the related paper de Arruda, Barros, and Bazanella (2002) based on Poincaré maps.

2.3. Stability

Consider the feedback structure shown in Fig. 2 with $F(s,r)$ given by Eq. (3). It can be easily verified that all signals will be bounded if $F(s,r)$ is stable, i.e., has no right-half-plane poles. Just notice that the input signal to $F(s,r)$ is $y_i(t) = \pm d$, which is bounded and nonzero for all $t > 0$. Therefore, if $F(s,r)$ is stable, then the output signal is bounded for all $t > 0$.

The closed loop stability is given as follows. For simplicity $H(s)$ is assumed to be stable.

Proposition 3. *Consider transfer function $F(s,r)$ defined in Eq.* (3). *Suppose that the computation of the critical point of $H(j\omega)$ (where $\angle H(j\omega_c^H) = -180°$) yields*

$$|H(j\omega_c^H)| = \frac{1}{K_c^H}. \qquad (10)$$

Then if $r > 1/K_c^H$, transfer function $F(s,r)$ has no right-half-plane pole.

Proof. Follows from direct application of the relay test presented in Åström and Hägglund (1995). □

2.4. Improving the transfer function estimation

The describing function method applied to a relay feedback system assumes that only the first-order harmonic is present at the process output. If the relay feedback is used to estimate the ultimate gain, the error due to describing function method is relatively small since the limit cycle normally occurs at a frequency larger than the process crossover frequency (Åström & Hägglund, 1984). But, when high-order harmonics are present, the approximation becomes worse as they become more relevant in the signal formation.

One way to deal with this problem is to use the discrete Fourier transform (DFT) of the data to get the true gain and phase lag at the frequency of the oscillation. Given the N-point (DFTs) $Yo_N(\omega)$ and $Yi_N(\omega)$ for $y_o(t)$ and $y_i(t)$, respectively, the following result can be found in standard textbooks such as Ljung (1999).

Proposition 4. *Suppose that $y_o(k)$ and $y_i(k)$ shown in Fig. 2 have the same period N_0. Consider the periodograms $Yo_N(\omega)$ and $Yi_N(\omega)$ calculated over $N = sN_0$ samples, respectively. Then the estimate*

$$\hat{F}(\omega) = \frac{Yo_N(\omega)}{Yi_N(\omega)} \qquad (11)$$

satisfies

- *$\hat{F}(\omega)$ is defined only for a fixed number of frequencies such that $Yi_N(\omega) \neq 0$, and*
- *at these frequencies, the estimates are unbiased and its variance decays with $1/N$.*

Proof. Note that from Fig. 2 the input signal $y_i(t)$ is a square wave from the relay output, which can be considered uncorrelated with the noise at the output signal, $y_o(t)$. The rest follows as in Ljung (1999). □

An estimate of $H(\omega)$ can now be obtained from Eq. (3) as

$$\hat{H}(\omega) = r\,\frac{1 + \hat{F}(\omega)}{1 - \hat{F}(\omega)}. \qquad (12)$$

The DFT can be also used to estimate the gain and phase at the other harmonic frequencies. □

Remark 2. Note that practically the N_o points are not obtained over exactly one period, which causes a leakage error on the DFT of the signals. However, if the signals are smooth and the sampling period is fast, the error is small. For comments on how to overcome the leakage error, refer to Pintelon and Schoukens (1996).

3. Loop transfer function estimation

Now, the basic procedure is applied to estimate the loop transfer function frequency response at a user-defined magnitude. The novelty in this section is that the experiment is performed without breaking the feedback loop and under controlled operating conditions.

3.1. The relay structure

Proposition 5. *Consider the closed loop system shown in Fig. 2. Assume that for a stable $T(s)$ and a real positive*

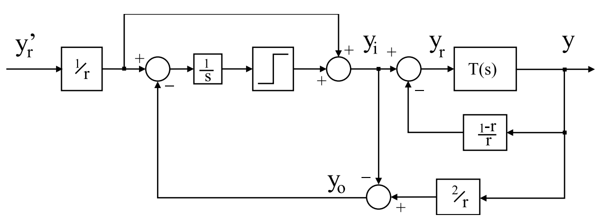

Fig. 3. Relay feedback for loop transfer function estimation.

number r, transfer function

$$F(s,r)=\frac{2}{r}\,\frac{T(s)}{T(s)\left(\frac{1-r}{s}\right)+1}-1 \tag{13}$$

is also stable. If a limit cycle is present the oscillation occurs at a frequency ω_0, then

$$|L(\mathrm{j}\omega_0)|\approx r. \tag{14}$$

Proof. Replacing Eq. (1) in Eq. (13) results in

$$F(s,r)=\frac{2}{r}\,\frac{\dfrac{L(s)}{1+L(s)}}{\dfrac{L(s)}{1+L(s)}\left(\dfrac{1-r}{r}\right)+1}-1=\frac{L(s)-r}{L(s)+r}. \tag{15}$$

Then it follows from Prop. 1 with $H(s)=L(s)$ that $|L(\mathrm{j}\omega_0)|\approx r$. □

The closed loop system which implements the procedure is presented in Fig. 3. Note that the new reference signal, y_r', is weighted by $1/r$ in order to yield unitary gain between y_r' and y, assuming that $T(0)=1$.

Remark 3. The relay feedback structure presented in Schei (1994) can be seen as a particular case of Proposition 1 with $r=1$, resulting in $F(s)=2T(s)-1$. The closed loop system oscillates at frequency ω_{lc} where $|G(\mathrm{j}\omega_{\mathrm{lc}})C(\mathrm{j}\omega_{\mathrm{lc}})|\approx 1$. To see this write

$$F(s)=2T(s)-1=\frac{L(s)-1}{L(s)+1}, \tag{16}$$

and compare to Eq. (3). This structure is stable by construction, since the closed loop $T(s)$ is assumed to be stable.

3.2. Signal boundness and stability

The closed loop experiment should be performed under well-defined operating conditions. The closed loop should be stable, the output $y(t)$ bounded and the limit cycle within feasible limits. Proposition 3 can be used to obtain range for r such that the closed loop relay feedback of Fig. 3 is stable. In this case, $H(s)=L(s)$ so that the critical gain of $L(\mathrm{j}\omega)$ must be estimated. But this means that the standard relay test must be performed by opening the feedback loop, which is not desirable. However, that can be easily avoided as follows.

Proposition 6. *Consider the feedback loop shown in Fig. 1 with $T(s)$ stable. Perform a standard relay test on $T(s)$ and compute the associated critical gain K_c^T. Then, the feedback loop shown in Fig. 3 is stable if*

$$r>\frac{1}{1+K_c^T}. \tag{17}$$

Proof. Stability of feedback loop shown in Fig. 3 reduces to stability of the feedback loop formed by $T(s)$ and $(1-r)/r$. The closed loop is stable if the feedback gain, $(1-r)/r$ is smaller than the critical gain of transfer function $T(s)$, namely K_c^T. Then, for stability

$$\frac{1-r}{r}<K_c^T\Rightarrow r>\frac{1}{1+K_c^T}. \qquad \square \tag{18}$$

Remark 4. If one chooses

$$d=\frac{\pi\Delta y_{\max}}{4}\frac{1}{r}, \tag{19}$$

that is, the relay amplitude is weighted by $1/r$, the output oscillation amplitude remains around the same level for different r (de Arruda & Barros, 2000).

4. Sensitivity function estimation

Now the basic procedure of Section 2 is applied for estimating the sensitivity function. Again, the experiment is performed without breaking the feedback loop and under controlled operating conditions.

4.1. The relay structure

Proposition 7. *Consider the closed loop system shown in Fig. 2. Assume that for a stable transfer function $T(s)$ and a positive real number r, transfer function*

$$F(s,r)=2\,\frac{\frac{1}{r+1}}{1-\frac{r}{r+1}T(s)}-1 \tag{20}$$

is also stable. If a limit cycle is present, the oscillation occurs at a frequency ω_0 such that

$$|S(\mathrm{j}\omega_0)|\approx\frac{1}{r}. \tag{21}$$

Proof. Now,

$$F(s,r)=\frac{2}{r}\,\frac{\frac{r}{r+1}}{1-\frac{r}{r+1}\frac{L(s)}{1+L(s)}}-1=\frac{1+L(s)-r}{1+L(s)+r}. \tag{22}$$

It follows from Proposition 1 with $H(s)=1+L(s)$ that

$$|1+L(\mathrm{j}\omega_0)|\approx r\Rightarrow\frac{1}{|1+L(\mathrm{j}\omega_0)|}=|S(\mathrm{j}\omega_0)|\approx\frac{1}{r}. \qquad \square \tag{23}$$

The closed loop system used to estimate the sensitivity function is shown in Fig. 4.

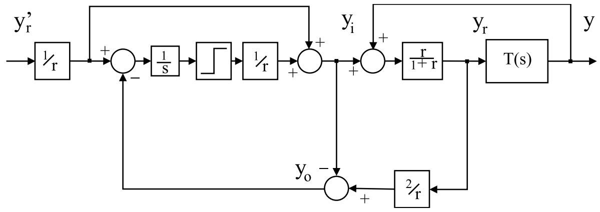

Fig. 4. Relay feedback for sensitivity function estimation.

4.2. Signal boundness and stability

Proposition 3 can be used to obtain a range for r such that the closed loop relay feedback of Fig. 4 is stable. In this case, $H(s)=1+L(s)$ so that the critical gain of $1+L(j\omega)$ must be estimated. An alternative procedure to approach this problem is to go back to the closed loop transfer function $T(s)$.

Proposition 8. *The relay feedback experiment shown in Fig. 4 is stable for all* $0\leqslant k<1, k=r/(1+r)$, *whenever*

$$1+k(-T(s))=0. \tag{24}$$

has its roots in the complex left half plane.

Proof. Follows from inspection of the closed loop formed by the positive feedback of $T(s)$ and $r/(1+r)$ in Fig. 4, and the fact that $T(s)$ is stable. □

Remark 5. As in the loop transfer function case, the relay amplitude should be proportional to $1/r$, which is already included in Fig. 4.

5. Simulation examples

5.1. First-order transfer function with time delay

In this simulation example all the relay experiments presented in this paper are verified. A first-order plus dead time process is considered and a PI controller is designed for the closed loop system.

5.1.1. Process description

Consider now the process given by

$$G(s)=\frac{1}{s+1}\,e^{-s}. \tag{25}$$

The relay test is applied to the process in order to obtain the critical point. For this case,

$$\hat{K}_c^G\cong 2.0483 \quad \text{and} \quad \hat{T}_c^G=2.97. \tag{26}$$

This estimate is now used to design a controller according to the Ziegler–Nichols frequency domain table (see Åström & Hägglund, 1995), yielding

$$K_p=0.4\hat{K}_c^G=0.8193 \quad \text{and} \quad T_i=0.8\hat{T}_c^G=2.3775 \tag{27}$$

Table 1
Loop transfer function estimation results

$\hat{\omega}_i$	r	$\lvert L(j\hat{\omega}_i)\rvert$	$\lvert\hat{L}(j\hat{\omega}_i)\rvert$	$\angle L(j\hat{\omega}_i)$	$\angle\hat{L}(j\hat{\omega}_i)$
1.3370	0.5	0.5144	0.5135	−146.74°	−147.23°
0.3069	1	1.3287	1.3259	−88.53°	−88.52°
0.1454	2	2.4809	2.4823	−87.53°	−87.51°
0.0714	4	4.8801	4.8733	−88.54°	−88.53°
0.0089	32	38.8835	38.8071	−89.81°	−89.78°

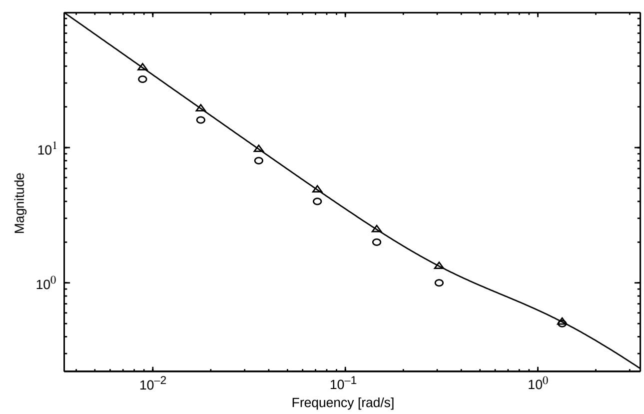

Fig. 5. Bode magnitude plot of the loop transfer function: real curve (−), r at $\hat{\omega}_i$ (○) and estimates using DFT (Δ).

such that,

$$C(s)=0.8193\,\frac{s+0.4206}{s}. \tag{28}$$

This controller is used in the sequel for estimating the loop transfer function and the sensitivity function of the corresponding closed loop.

5.1.2. Loop transfer function estimation

Consider the loop transfer function experiment, shown in Fig. 3. The relay feedback method as presented in Åström and Hägglund (1995) is applied to the closed loop in order to obtain the loop critical point. As before, this information is used to define the range for r such that the experiment is stable. The estimates are

$$\hat{K}_c^L=2.1050 \quad \text{and} \quad \hat{T}_c^L=3.34. \tag{29}$$

Since the integrator in the controller yields a loop transfer function with infinite gain at zero frequency, this relay test is stable for any value of r such that $r>K_u^{-1}=0.475$. The chosen gains are

$$r=[0.5\ 1\ 2\ 4\ 8\ 16\ 32]. \tag{30}$$

The obtained estimates for a few points are shown in Table 1. The Bode magnitude plot of these values is shown in Fig. 5, while the Bode phase plot is shown in Fig. 6.

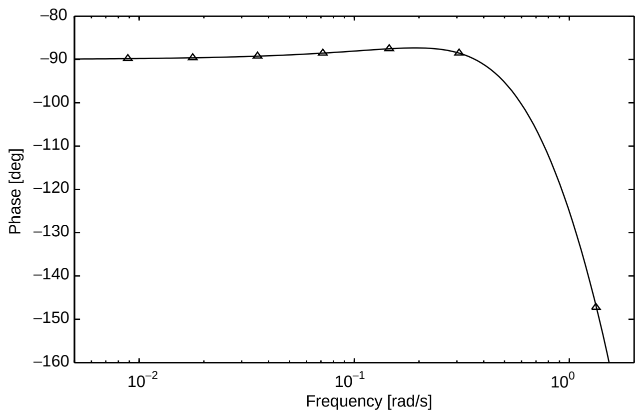

Fig. 6. Bode phase plot of the loop transfer function: real curve (−) and estimates using DFT (Δ).

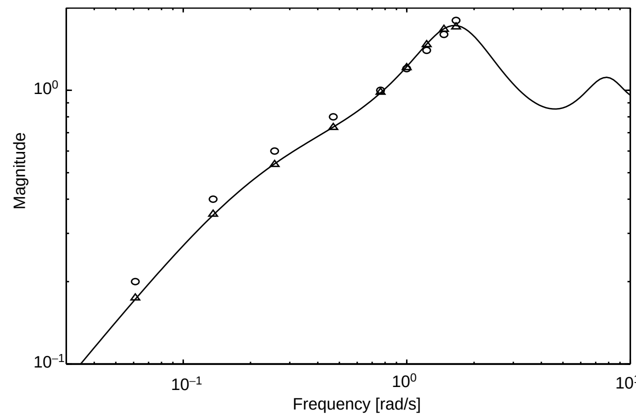

Fig. 7. Bode magnitude plot of the sensitivity function: real curve (−), r^{-1} at $\hat{\omega}_i$ (○) and estimates using DFT (Δ).

Table 2
Sensitivity function estimation results

$\hat{\omega}_i$	r^{-1}	$\lvert S(j\hat{\omega}_i)\rvert$	$\lvert \hat{S}(j\hat{\omega}_i)\rvert$	$\angle S(j\hat{\omega}_i)$	$\angle \hat{S}(j\hat{\omega}_i)$
0.0611	0.2	0.1725	0.1744	78.80°	79.41°
0.1361	0.4	0.3498	0.3529	67.16°	67.53°
0.7636	1.0	0.9783	0.9848	43.12°	43.21°
1.4673	1.6	1.6787	1.6716	19.30°	19.41°
1.6622	1.8	1.7245	1.7056	8.95°	9.50°
—	2.0	—	—	—	—

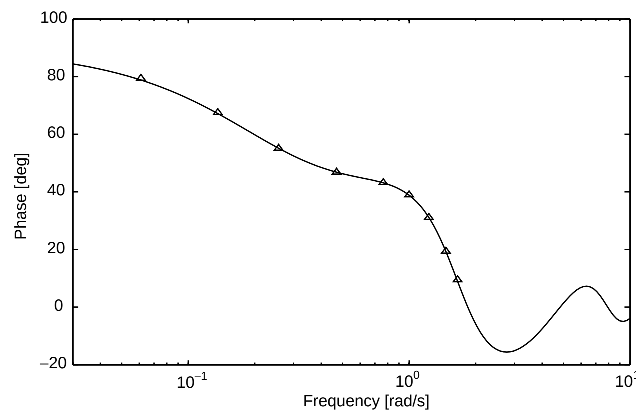

Fig. 8. Bode phase plot of the sensitivity function: real curve (−) and estimates using DFT (Δ).

5.1.3. Sensitivity function estimation

Application of the relay test results in that the sensitivity experiment is stable for all $r > 0$. In this case, the following set of values is used,

$$r^{-1} = [0.2\ 0.4\ 0.6\ 0.8\ 1.0\ 1.2\ 1.4\ 1.6\ 1.8\ 2.0]. \tag{31}$$

For $r^{-1} = 2.0$ the limit cycle vanishes at process output, which means that the maximum sensitivity peak is reached. Therefore, no limit cycle is developed for any values of r such that $r^{-1} > 2.0$. For this closed loop, the maximum sensitivity peak is given by the infinite norm of $S(j\omega)$,

$$\|S\|_\infty = \max_\omega |S(j\omega)| = 1.7254. \tag{32}$$

A few of the obtained estimates are shown in Table 2, and the Bode magnitude plot of these values and the true curve is shown in Fig. 7. The Bode phase plot of the estimates and the true curve is shown in Fig. 8.

6. Conclusions

In this paper, a new technique for transfer function magnitude estimation is presented. A basic procedure is developed and new relay experiments are obtained for estimating frequency points in the loop transfer function and the sensitivity function. Stability conditions are presented which make the technique reliable for application to actual processes. Simulation examples illustrate the capabilities of the identification procedures.

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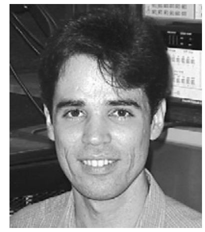

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