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Identification for Hammerstein Systems Using Extended Least Squares Algorithm*

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Abstract: The extended least squares (ELS) algorithm is applied to identifying the Hammerstein system, where the nonlinear static function $f(\cdot)$ is expressed as a linear combination of basic functions with unknown coefficients. Strong consistency of the estimates is established and their convergent rates are obtained as well.

Key Words: Hammerstein system, ARMAX, extended least squares, strong consistency.

1 INTRODUCTION

The Hammerstein system consisting of a static function $f(\cdot)$ followed by a linear dynamics is a commonly used model in engineering practice. Its identification issue naturally receives much attention from researchers, [1], [2], [3], [4], [5], [6], [7], [8] et al.

Consider the following single-input single-output (SISO) Hammerstein system with linear subsystem described by an ARMAX model:

$$A(z)y_{k+1} = B(z)f(u_k) + C(z)\omega_{k+1}, \quad k \geqslant 0 \tag{1}$$

$$A(z) \triangleq 1+a_1z+\cdots+a_pz^p, \; B(z) \triangleq b_1+b_2z+\cdots+b_qz^{q-1}$$

$$C(z) \triangleq 1 + c_1z + \cdots + c_rz^r$$

where z is the backward-shift operator, u_k and y_k denote the system input and output, respectively, ω_k is the driven noise, $f(\cdot)$ is the unknown static function and $v_k \triangleq f(u_k)$ is the unavailable internal signal.

The methods for identifying Hammerstein systems can roughly be divided into two types, the parametric approach [1-4] and the nonparametric approach [5-7], according to the description for $f(\cdot)$. This work belongs to the former type. As to be seen, in this case the identification problem for Hammerstein systems is equivalent to identifying a linear ARMAX system. Hence the well-known results for identifying linear systems, e.g. [9], can be used. The unknown nonlinear function $f(\cdot)$ is expressed as a combination of basic functions with unknown coefficients in several papers, e.g. [1], [2], [3], [4] and [8]. The estimation algorithms in [1], [2], [4] are iterative, i.e. for a given data set with fixed size, the estimates for unknown parameters are iteratively derived by minimizing some cost function. So the estimates cannot be updated on-line. In [3] an instrumental variable identification method is introduced, but how to choose the instrumental variables is not an easy task. The recursive ELS algorithm is used in [8] to identify the Hammerstein system (1), but the proof in [8] is incorrect as pointed by [11]. In this work, we give an analysis of the ELS identification algorithm for Hammerstein systems, prove the strong consistency of the estimates and establish their convergent rates. The main result of the paper is Theorem 2, for which the proof is inspired by Theorem 6.2 in [9].

The rest of the paper is arranged as follows. The identification algorithm is defined in Section 2 and its property is analyzed in Section 3. An illustructive example is given in Section 4. Some concluding remarks are addressed in Section 5.

2 ESTIMATION ALGORITHM

Let us formulate the identification problem precisely. Assume $\{g_1(x), \cdots, g_s(x)\}, x \in \mathbf{R}$ are the known basic functions, and $f(x) = \sum_{j=1}^{s} d_jg_j(x)$, where $d_1, \cdots, d_s$ are the unknown coefficients. The considered Hammerstein system is expressed as:

$$\begin{aligned}
& y_{k+1} + a_1y_k + \cdots + a_py_{k+1-p} \\
= \;& b_1\sum_{j=1}^{s} d_jg_j(u_k) + b_2\sum_{j=1}^{s} d_jg_j(u_{k-1}) + \cdots \\
& +b_q\sum_{j=1}^{s} d_jg_j(u_{k+1-q}) + C(z)\omega_{k+1} \\
= \;& \begin{bmatrix} b_1d_1 \cdots b_1d_s \end{bmatrix} \begin{bmatrix} g_1(u_k) \\ \vdots \\ g_s(u_k) \end{bmatrix} + \cdots \\
& + \begin{bmatrix} b_qd_1 \cdots b_qd_s \end{bmatrix} \begin{bmatrix} g_1(u_{k+1-q}) \\ \vdots \\ g_s(u_{k+1-q}) \end{bmatrix} + C(z)\omega_{k+1} \\
= \;& \begin{bmatrix} b_1d_1 \cdots b_1d_s \cdots b_qd_1 \cdots b_qd_s \end{bmatrix} \begin{bmatrix} g_1(u_k) \\ \vdots \\ g_s(u_k) \\ \vdots \\ g_1(u_{k+1-q}) \\ \vdots \\ g_s(u_{k+1-q}) \end{bmatrix} \\
& +\omega_{k+1} + c_1\omega_k + \cdots + c_r\omega_{k+1-r}
\end{aligned}$$

By setting

$$\begin{aligned}
\theta^{\mathrm{T}} = \;& [-a_1 \cdots - a_p \; (b_1d_1) \cdots (b_1d_s) \cdots \\
& (b_qd_1) \cdots (b_qd_s) \; c_1 \cdots c_r]
\end{aligned} \tag{2}$$

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$$\varphi_k^{0\mathrm{T}} = [y_k \cdots y_{k+1-p}\; g_1(u_k) \cdots g_s(u_k) \cdots g_1(u_{k+1-q}) \cdots g_s(u_{k+1-q})\; \omega_k \cdots \omega_{k+1-r}] \quad (3)$$

the system is written in a compact form:

$$y_{k+1} = \theta^{\mathrm{T}} \varphi_k^0 + \omega_{k+1} \quad (4)$$

Thus, we have transformed the Hammerstein system into a linear ARMAX system, so various identification methods for linear systems including the extended least squares (ELS) method, can be used. With an arbitrary θ_0 and $P_0 = \alpha_0 I$ for some $\alpha_0 > 0$, the ELS method defines $\{\theta_k\}_{k\geqslant 1}$ and $\{P_k\}_{k\geqslant 1}$ by the following algorithm.

$$\theta_{k+1} = \theta_k + a_k P_k \varphi_k (y_{k+1} - \theta_k^{\mathrm{T}} \varphi_k) \quad (5)$$

$$P_{k+1} = P_k - a_k P_k \varphi_k \varphi_k^{\mathrm{T}} P_k \quad (6)$$

$$\hat{\omega}_{k+1} = y_{k+1} - \theta_{k+1}^{\mathrm{T}} \varphi_k,\ a_k = \frac{1}{1 + \varphi_k^{\mathrm{T}} P_k \varphi_k} \quad (7)$$

$$\varphi_k^{\mathrm{T}} = [y_k \cdots y_{k+1-p}\; g_1(u_k) \cdots g_s(u_k) \cdots g_1(u_{k+1-q}) \cdots g_s(u_{k+1-q})\; \hat{\omega}_k \cdots \hat{\omega}_{k+1-r}] \quad (8)$$

According to (6), $P_{k+1}^{-1} = \sum_{i=0}^{k} \varphi_i \varphi_i^{\mathrm{T}} + \frac{1}{\alpha_0} I$.

Notice that for identifiability, we need either $b_1 = 1$ or $d_1 = 1$. Denote $\theta_k^{\mathrm{T}} \triangleq [-a_{1,k} \cdots - a_{p,k}\ (b_1 d_1)_k \cdots (b_1 d_s)_k \cdots (b_q d_1)_k \cdots (b_q d_s)_k\ \ c_{1,k} \cdots c_{r,k}]$. If $b_1 = 1$, then $(b_1 d_1)_k \cdots (b_1 d_s)_k$ serve as the estimates for $d_1, \cdots, d_s$ and $(b_j d_l)_k/(b_1 d_l)_k, j = 2, \cdots, q$ serve as the estimates for $b_j, j = 2, \cdots, q$ whenever $(b_1 d_l)_k \neq 0, l = 1, \cdots, s$. If $d_1 = 1$, then estimates for $b_1, \cdots, b_q$ and $d_2, \cdots, d_s$ are derived from θ_k in a similar way.

3 STRONG CONSISTENCY OF ESTIMATES

Denote by $\lambda_{\max}^0(n)$ and $\lambda_{\min}^0(n)$ the maximal and minimal eigenvalues of $\sum_{i=0}^{n} \varphi_i^0 \varphi_i^{0\mathrm{T}} + \frac{1}{\alpha_0} I$. Introduce the following conditions:

A1 $C^{-1}(z) - \frac{1}{2}$ is strictly positive real (SPR), i.e. $C^{-1}(e^{i\lambda}) + C^{-1}(e^{-i\lambda}) - 1 > 0,\ \forall\, \lambda \in [0, 2\pi]$;

A2 $\{\omega_n, \mathcal{F}_n\}_{n\geqslant 0}$ is a martingale difference sequence, and $\sup_{n\geqslant 0} E[|\omega_{n+1}|^\beta | \mathcal{F}_n] < \infty$ a.s. for some $\beta \geqslant 2$, where $\{\mathcal{F}_n\}_{n\geqslant 0}$ is a sequence of nondecreasing σ-algebras; u_n is $\mathcal{F}_n$-measurable.

According to Theorem 4.2 in [9] we have the following result:

Theorem 1 *For the algorithm defined by* (5)-(8), *if* **A1** *and* **A2** *hold and if*

$$\log \lambda_{\max}^0(n) (\log\log \lambda_{\max}^0(n))^{\delta(\beta-2)} = o(\lambda_{\min}^0(n)) \ \ a.s. \quad (9)$$

then

$$\|\theta_{n+1} - \theta\|^2 = O\Big(\frac{\log \lambda_{\max}^0(n) (\log\log \lambda_{\max}^0(n))^{\delta(\beta-2)}}{\lambda_{\min}^0(n)}\Big) \quad (10)$$

a.s. as $n \to \infty$, i.e., the estimates are strongly consistent.

Remark 1 In order the ELS estimates to be strongly consistent, we need only to find conditions guaranteeing (9). It is worth noticing that (9) is the possibly weakest excitation condition for strong consistency of the ELS estimates in a certain sense (see [9], [10]). We further impose the following conditions:

A3 $A(z)$ is stable, i.e., $A(z) \neq 0, \forall\, |z| \leqslant 1$;

A4 $A(z)$, $zB(z)$ and $C(z)$ have no common factor and $[a_p\ b_q\ c_r] \neq 0$;

A5 $\{1, g_1(x), \cdots, g_s(x)\}$ is linearly independent over some interval $[a, b]$; $\{u_k\}_{k\geqslant 0}$ is a sequence of i.i.d. random variables with density $p(x)$, which is positive and continuous over $[a, b]$ and $0 < Eg_i^2(u_k) < \infty$, $i = 1, \cdots, s$; $\{u_k\}_{k\geqslant 0}$ and $\{\omega_k\}_{k\geqslant 0}$ are mutually independent;

A6 $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \omega_i^2 = R_\omega > 0$ a.s.

A7 $d_1^2 + \cdots + d_s^2 \neq 0$

We have the following results.

Lemma 1 *Denote $\mu_i \triangleq Eg_i(u_k)$, $i = 1, \cdots, s$ and*

$$R \triangleq E \begin{bmatrix} g_1(u_k) - \mu_1 \\ \vdots \\ g_s(u_k) - \mu_s \end{bmatrix} \begin{bmatrix} g_1(u_k) - \mu_1 & \cdots & g_s(u_k) - \mu_s \end{bmatrix} \quad (11)$$

If **A5** *holds, then $R > 0$.*

Theorem 2 *For the algorithm defined by* $(5) - (8)$, *if* **A1**-**A7** *hold, then we have*

$$\|\theta_{n+1} - \theta\|^2 = O\Big(\frac{\log n (\log\log n)^{\delta(\beta-2)}}{n}\Big) \ \ a.s.\ as\ n \to \infty \quad (12)$$

Proof: We first prove

$$\liminf_{n\to\infty} \frac{1}{n} \lambda_{\min}^0(n) > 0 \ \text{ a.s.} \quad (13)$$

Denote $f_i \triangleq A(z)\varphi_i^0 = \varphi_i^0 + a_1 \varphi_{i-1}^0 + \cdots + a_p \varphi_{i-p}^0$. According to the definition of f_i, we have

$$\lambda_{\min}\{\sum_{i=0}^{n} f_i f_i^{\mathrm{T}}\} = \inf_{\|x\|=1} \sum_{i=0}^{n} (x^{\mathrm{T}} f_i)^2 \leqslant (1 + p)(1 + a_1^2 + \cdots + a_p^2) \lambda_{\min}^0(n) \quad (14)$$

So to prove (13), it suffices to show

$$\liminf_{n\to\infty} \frac{1}{n} \lambda_{\min}\{\sum_{i=0}^{n} f_i f_i^{\mathrm{T}}\} > 0 \ \text{ a.s.} \quad (15)$$

If (15) were not held for some sample path ω, then there would exist a subsequence $\{n_k\}_{k\geqslant 1}$ and a vector sequence $\{\eta_{n_k}\}_{k\geqslant 1}$ with $\|\eta_{n_k}\| = 1$, $\eta_{n_k}^{\mathrm{T}} = [\alpha_{n_k}^{(0)} \cdots \alpha_{n_k}^{(p-1)} \beta_{n_k}^{(11)} \cdots \beta_{n_k}^{(1s)} \cdots \beta_{n_k}^{(q1)} \cdots \beta_{n_k}^{(qs)} \gamma_{n_k}^{(0)} \cdots \gamma_{n_k}^{(r-1)}]$, such that for this sample ω,

$$\lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\eta_{n_k}^{\mathrm{T}} f_i)^2 = 0 \quad (16)$$

Since $\{\eta_{n_k}\}_{k\geqslant 1}$ is bounded, there exists a convergent subsequence of $\{\eta_{n_k}\}$, denoted also by $\{\eta_{n_k}\}$ for simplicity of notations: $\eta_{n_k} \to \eta$ as $k \to \infty$. Let $\eta^{\mathrm{T}} \triangleq [\alpha^{(0)} \cdots \alpha^{(p-1)} \beta^{(11)} \cdots \beta^{(1s)} \cdots \beta^{(q1)} \cdots \beta^{(qs)} \gamma^{(0)} \cdots \gamma^{(r-1)}]$, $\|\eta\| = 1$.

From the definition of f_i, we denote

$$\begin{aligned} & \eta_{n_k}^{\mathrm{T}} f_i \\ \triangleq\ & [\sum_{j=0}^{\mu} h_{n_k}^{(1j)} z^j \cdots \sum_{j=0}^{\mu} h_{n_k}^{(sj)} z^j \ \sum_{j=0}^{\mu} h_{n_k}^{(0j)} z^j] \\ & \cdot [g_1(u_i) \ \cdots \ g_s(u_i) \ \omega_i]^{\mathrm{T}} \end{aligned} \tag{17}$$

where for $l = 1, \cdots, s$,

$$\sum_{j=0}^{\mu} h_{n_k}^{(lj)} z^j = \sum_{j=0}^{p-1} \alpha_{n_k}^{(j)} B(z) z^{j+1} d_l + \sum_{j=0}^{q-1} \beta_{n_k}^{(j+1,l)} A(z) z^j \tag{18}$$

and

$$\sum_{j=0}^{\mu} h_{n_k}^{(0j)} z^j = \sum_{j=0}^{p-1} \alpha_{n_k}^{(j)} C(z) z^j + \sum_{j=0}^{r-1} \gamma_{n_k}^{(j)} A(z) z^j \tag{19}$$

From (17) and **A5** we have

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\eta_{n_k}^{\mathrm{T}} f_i)^2 \\ =\ & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) \cdots g_s(u_{i-j})]^{\mathrm{T}} + \sum_{j=0}^{\mu} h_{n_k}^{(0j)} \omega_{i-j})^2 \\ =\ & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} [(\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) \cdots g_s(u_{i-j})]^{\mathrm{T}})^2 + (\sum_{j=0}^{\mu} h_{n_k}^{(0j)} \omega_{i-j})^2] \\ =\ & 0 \end{aligned}$$

where the cross terms are eliminated by using Theorem 2.8 in [9].

From here it follows that

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) \cdots g_s(u_{i-j})]^{\mathrm{T}})^2 = 0 \end{aligned} \tag{20}$$

and

$$\lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} h_{n_k}^{(0j)} \omega_{i-j})^2 = 0 \tag{21}$$

From (20), by the law of large numbers, we have,

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) - \mu_1 \cdots g_s(u_{i-j}) - \mu_s]^{\mathrm{T}} \\ & + \sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}][\mu_1 \cdots \mu_s]^{\mathrm{T}})^2 \\ =\ & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} [(\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) - \mu_1 \cdots g_s(u_{i-j}) - \mu_s]^{\mathrm{T}})^2 \\ & + (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}][\mu_1 \cdots \mu_s]^{\mathrm{T}})^2] \\ =\ & 0 \end{aligned}$$

So we have

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) - \mu_1 \cdots g_s(u_{i-j}) - \mu_s]^{\mathrm{T}})^2 = 0 \end{aligned} \tag{22}$$

From (22), also by Theorem 2.8 in [9], it follows that

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} [h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) - \mu_1 \cdots g_s(u_{i-j}) - \mu_s]^{\mathrm{T}})^2 \\ =\ & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} \sum_{j=0}^{\mu} ([h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \\ & \cdot [g_1(u_{i-j}) - \mu_1 \cdots g_s(u_{i-j}) - \mu_s]^{\mathrm{T}})^2 = 0 \end{aligned} \tag{23}$$

By ergodicity and Lemma 1, from (23) we derive

$$[h_{n_k}^{(1j)} \cdots h_{n_k}^{(sj)}] \underset{k\to\infty}{\longrightarrow} [0 \cdots 0], \ j = 0, \cdots, \mu$$

and hence

$$[\sum_{j=0}^{\mu} h_{n_k}^{(1j)} z^j \cdots \sum_{j=0}^{\mu} h_{n_k}^{(sj)} z^j] \underset{k\to\infty}{\longrightarrow} [0 \cdots 0]$$

Noting that $\eta_{n_k} \to \eta$ as $k \to \infty$, by (18) we have for $l = 1, \cdots, s$,

$$\sum_{j=0}^{p-1} \alpha^{(j)} B(z) z^{j+1} d_l + \sum_{j=0}^{q-1} \beta^{(j+1,l)} A(z) z^j = 0 \tag{24}$$

From (21), by Theorem 2.8 in [9] and **A6**, a similar treatment leads to

$$\begin{aligned} & \lim_{k\to\infty} \frac{1}{n_k} \sum_{i=0}^{n_k} (\sum_{j=0}^{\mu} h_{n_k}^{(0j)} \omega_{i-j})^2 \\ =\ & \lim_{k\to\infty} \sum_{j=0}^{\mu} (h_{n_k}^{(0j)})^2 \frac{1}{n_k} \sum_{i=0}^{n_k} \omega_{i-j}^2 = 0 \end{aligned}$$

which yields $h_{n_k}^{(0j)} \to 0$, as $k \to \infty$, for $j = 0, \cdots, \mu$. By (19), we have

$$\sum_{j=0}^{p-1} \alpha^{(j)} C(z) z^j + \sum_{j=0}^{r-1} \gamma^{(j)} A(z) z^j = 0 \tag{25}$$

By **A4**, there exist polynomials $M(z)$, $N(z)$ and $L(z)$ such that $A(z)M(z) + zB(z)N(z) + C(z)L(z) = 1$. By **A7**, $d_1^2 + \cdots + d_s^2 \neq 0$, without losing generality, we may assume $d_1 \neq 0$. From (24) and (25) it follows that

$$\begin{aligned}
&\sum_{j=0}^{p-1} \alpha^{(j)} z^j (A(z)M(z) + zB(z)N(z) + C(z)L(z)) \\
= \quad & A(z)(\sum_{j=0}^{p-1} \alpha^{(j)} z^j M(z) - \frac{1}{d_1} \sum_{j=0}^{q-1} \beta^{(j+1,1)} z^j N(z) \\
& - \sum_{j=0}^{r-1} \gamma^{(j)} z^j L(z)) \triangleq A(z) \sum_{j=0}^{\lambda} \mu^{(j)} z^j \qquad (26)
\end{aligned}$$

Multiplying $zB(z)$ and $C(z)$ to both side of (26) respectively, by (24) for $l = 1$ and (25), we obtain

$$\sum_{j=0}^{p-1} \alpha^{(j)} z^j = \sum_{j=0}^{\lambda} \mu^{(j)} z^j A(z) \qquad (27)$$

$$-\frac{1}{d_1} \sum_{j=0}^{q-1} \beta^{(j+1,1)} z^j = \sum_{j=0}^{\lambda} \mu^{(j)} z^{j+1} B(z) \qquad (28)$$

$$-\sum_{j=0}^{r-1} \gamma^{(j)} z^j = \sum_{j=0}^{\lambda} \mu^{(j)} z^j C(z) \qquad (29)$$

By **A4**, $[a_p \; b_q \; c_r] \neq 0$, from (27), (28) and (29) we conclude $\mu^{(j)} = 0$, $j = 0, \cdots, \lambda$, and hence $\alpha^{(0)} = \cdots = \alpha^{(p-1)} = 0$, $\gamma^{(0)} = \cdots = \gamma^{(r-1)} = 0$ and $\beta^{(11)} = \cdots = \beta^{(q1)} = 0$. Then from (24), we have

$$\sum_{j=0}^{q-1} \beta^{(j+1,l)} z^j = 0, \; l = 2, \cdots, s$$

and hence $\beta^{(1l)} = \cdots = \beta^{(ql)} = 0$. Thus, we have proved $\eta^{\mathrm{T}} = [\alpha^{(0)} \cdots \alpha^{(p-1)} \beta^{(11)} \cdots \beta^{(1s)} \cdots \beta^{(q1)} \cdots \beta^{(qs)} \gamma^{(0)} \cdots \gamma^{(r-1)}] = 0$, which contradicts with $\|\eta\| = 1$. Hence, we obtain (15) and (13).
By **A3**, **A5**, and **A6**, we can prove

$$\lambda_{\max}^0(n) = O(n)$$

which incorporating with (13) implies (9). The conclusion of the theorem follows by Theorem 1. □

4 EXAMPLE

Consider the following single-input single-output Hammerstein system,

$$y_{k+1} + a_1 y_k = b_1(d_1 u_k + d_2 u_k^2 + d_3 u_k^3) + \omega_{k+1} + c_1 \omega_k \quad (30)$$

where $a_1 = 0.8$, $d_1 = 0.2$, $d_2 = 0.4$, $d_3 = 0.6$ and $c_1 = 0.5$. For identifiability, we assume $b_1 = 1$.
Suppose $\{\omega_k\}_{k \geqslant 0}$ is a sequence of i.i.d. Gaussian $\mathcal{N}(0, 2)$ random variables. Let $\{u_k\}_{k \geqslant 0}$ be mutually independent uniformly distributed over$[-2, 2]$ and independent of $\{\omega_k\}$. It is clear that **A1-A7** hold. Fig.1 shows the estimates for a_1 and c_1, while Fig.2 for d_1, d_2 and d_3. In the figures, the solid lines denote the true values of the parameters and the dotted lines denote the estimates. From the figures we see that the computer simulation results are consistent with the theoretical analysis.

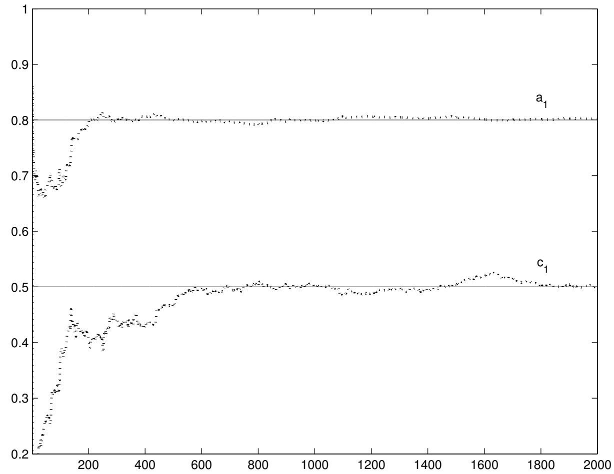

Fig. 1 Estimates for a_1, c_1

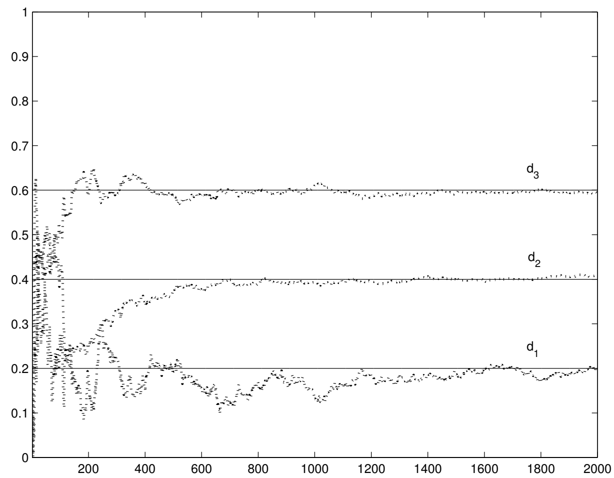

Fig. 2 Estimates for d_1, d_2, d_3

5 CONCLUDING REMARKS

In this work, the ELS algorithms are used to identify the Hammerstein system, where the nonlinear function is presented as a linear combination of basic functions with unknown coefficients. All estimates are proved converging to the true values with probability one, and their convergent rates are also given. For further research, it is of interest to consider the order estimation of the linear subsystem, and control problems including adaptive tracking and quadratic cost problem.

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