REVISITING THE ZIEGLER-NICHOLS TUNING RULES FOR PI CONTROL — PART II THE FREQUENCY RESPONSE METHOD

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ABSTRACT

This paper presents an analysis of the Ziegler-Nichols frequency response method for tuning PI controllers, showing that this method has severe limitations. The limitations can be overcome by a simple modification for processes where the time delay is not too short. By a major modification it is possible to obtain new tuning rules that also cover processes that are lag dominated.

KeyWords: PID control, design, tuning, optimization, process control.

I. INTRODUCTION

The simple PI controller is the most common control algorithm. Simple methods for tuning this controller were developed by Ziegler and Nichols in the seminal paper [1]. The methods were based on two ideas: to characterize process dynamics by two parameters, that are easily determined experimentally, and to calculate controller parameters from the process characteristics by a simple formula. The methods of Ziegler and Nichols have had a very strong impact on the practice of control. Practically all vendors and users of PI controllers apply the methods or some simple modifications of them in controller tuning.

There are two essential drawbacks with the Ziegler-Nichols method. Too little information is used to characterize process dynamics. The design method, quarter amplitude damping, chosen by Ziegler and Nichols gives closed-loop systems with poor damping and poor robustness. These drawbacks have been known for a long time and there have been many attempts to overcome them. Because of their simplicity the Ziegler Nichols methods have, however, remained very popular.

In this paper we have revisited the Ziegler Nichols methods from the perspective of 60 years of development of automatic control. There are several reasons for doing this. Since the PI controller is so widely used it is

important to have simple tuning rules that are better than the Ziegler-Nichols tuning rules. Such rules are also important for automatic tuning of controllers. This has given new insights and tuning methods that give significantly better performance for a wide range of processes. Ziegler and Nichols presented two tuning methods, one is based on a step response experiment and another is based on a frequency response experiment. The step response method was discussed in [2]. In this paper we discuss the frequency response method.

When Ziegler and Nichols developed their methods they first selected a number of processes that represented typical systems encountered in process control. Controllers were tuned manually for these systems and it was then attempted to find suitable relations between controller parameters and process characteristics in terms of features of the frequency response. The processes were simulated on the differential analyzer at MIT, see [3], and on a pneumatic process simulator constructed from components used in the pneumatic controllers. We have followed a similar approach, but we have used computers for simulation and robust methods for control system design.

The starting point is a method for controller design based on robust loop shaping, see [4] and [5]. This method is briefly described in Section 2. It is is called MIGO (M_s -constrained Integral Gain Optimization) because it is based on maximization of integral gain subject to a constraint on the maximum sensitivity. The maximum sensitivity is a design parameter. In Section 2 we also present a test batch of processes used in the study. The notion of processes with essentially monotone frequency responses is also introduced. This con-

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tains transfer functions of processes typically encountered in process control. The batch contains processes with pure time delay as well as processes with integral action. It does, however, not contain processes with highly oscillatory dynamics. In Section 2 we also give a short review of the approximate MIGO design method based on step response experiments (AMIGOs) presented in [2]. In Sections 3 and 4, the Ziegler-Nichols frequency response method is evaluated, and it is shown that it is not possible to derive efficient tuning rules based on their experiment for all processes in the test batch. Very simple rules can, however, be obtained for processes where the time delay is not too short, typically for time delays larger than a quarter of the time constant. It is also shown that efficient rules for the full test batch can be obtained if other frequency points are considered. This insight is used to derive new simple tuning rules called AMIGOf (Approximate MIGO based on frequency response data) in Section 5. The AMIGOf rules are investigated in Section 6 and examples are given in Section 7.

II. TEST BATCH AND DESIGN METHOD

To develop tuning rules for PI control we will first consider a reasonable class of processes. Some processes which are well suited for PI control have frequency responses of the type shown in Fig. 1. This means that both the gain and the phase of their transfer function are monotonic functions. Many frequency responses are monotone as is shown in A, C, and E in the figure. Process C has a pure time delay and processes B and D have inverse responses that are associated with a right half plane zero. Process F is moderately oscillatory. Processes D and E have integration. With a moderate abuse of language we say that these processes have essentially monotone frequency responses. PI control will work very well for these processes. Notice that oscillatory systems with low damping are excluded.

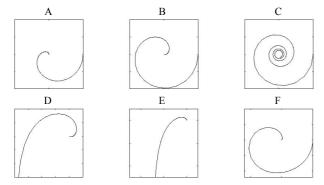


Fig. 1. Nyquist curves for typical processes with essentially monotone frequency responses.

In [2], the notion of essentially monotone was defined for step responses. It is useful to have a working definition also for the frequency response. Let f be a function. The parameter

$$m_f = \frac{|\int df|}{\int |df|} \tag{1}$$

is called the index of monotonicity. The index is between 0 and 1, and it is 1 if the function is monotone. Since a frequency response is characterized by two functions the gain and the phase we define the index as

$$m = \min(m_{g}, m_{p}) \tag{2}$$

where g is the gain and p is the phase of the transfer function and the integration is taken over the essential frequency range from 0 to ω_{180} , *i.e.*, the frequency where the process lag is -180° . A process is *monotone* if m=1 and essentially monotone if the index is larger than a specified value, say m=0.8.

2.1 The MIGO design method

The design method is based on loop shaping, see [4]. It gives the parameters of a PI controller that maximizes integral gain subject to a robustness constraint. Robustness is specified in terms of the maximum value of the sensitivity function (M_s) . The method can also be interpreted as a minimization of the integrated error at step load disturbances subject to constraints on the maximum sensitivity. This gives a controller that reduces load disturbances effectively while maintaining a good robustness. We call the design method MIGO which is short for M_s -constrained Integral Gain Optimization.

The M_s value is a good design parameter. A reasonable range is $M_s = 1.2$ to 2.0, lower values give better robustness but poorer rejection of load disturbances. The value $M_s = 1.4$ is found to give a good compromise between performance and robustness in many applications. With this value the responses typically do not have oscillations.

2.2 The test batch

The MIGO design method presented in [4] requires that the transfer function of the process is known. The method will be applied to a large number of processes that represent the cases typically found in process control. Systems with the following transfer functions have been used

$$G_1(s) = \frac{e^{-s}}{1+sT},$$

 $T = 0.01, 0.05, 0.1, 0.3, 0.5, 1, 2, 3, 5, 10, 20, 100$

$$G_{2}(s) = \frac{e^{-s}}{(1+sT)^{2}},$$

$$T = 0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.3, 1.5,$$

$$2, 4, 6, 8, 10, 20, 100$$

$$G_{3}(s) = \frac{1}{(s+1)(1+sT)},$$

$$T = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5$$

$$G_{4}(s) = \frac{1}{(s+1)^{n}},$$

$$n = 2, 3, 4, 5, 6, 7, 8$$

$$G_{5}(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^{2}s)(1+\alpha^{3}s)},$$

$$\alpha = 0.1, 0.2, 0.5, 0.7$$

$$G_{6}(s) = \frac{1-\alpha s}{(s+1)^{3}},$$

$$\alpha = 0.1, 0.2, 0.5, 1, 2$$

$$G_{7}(s) = \frac{1}{s^{2} + 2\zeta s + 1},$$

$$\zeta = 0.5, 0.7, 0.9$$
(3)

The first five systems have essentially monotone frequency responses. The system G_1 is the standard model that has been used in many investigations of PI tuning. This system does, however, give controller gains that are a little different from the other systems, see [6]. The system G_2 is similar to G_1 but it has two lags. The system G_3 is a system with two lags. The systems G_4 and G_5 are multi-lag systems that have been used for a long time in work on controller tuning. The system G_6 has a zero in the right-half plane, resulting in an inverse step response. This system does not have a strictly monotone step response. For small values of α the step response is, however, essentially monotone. Requiring that the monotonicity index (1) is greater than 0.8 we find that α must be less than 1.1. The system G_7 is a simple oscillatory system. In Fig. 2 we show the monotonicity index as a function of relative damping. Requiring that the monotonicity index is greater than 0.8 we find that the relative damping must be greater than 0.6.

All systems are normalized to have unit steady-state gain. The models have a parameter that can be changed to influence the response of the system. The parameter ranges have been chosen to give a wide range of responses. For the system G_6 we do not allow very large values of α and for the oscillatory system G_7 we have only used values of the damping that gives a moderate oscillation. Our new tuning rules capture models with relative damping larger than $\zeta \approx 0.5$, but processes with lower damping require other tuning rules or other controller structures.

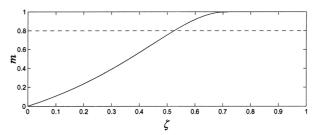


Fig. 2. Monotonicity index (1) as a function of ζ for a system with the transfer function $G(s) = 1/(s^2 + 2\zeta s + 1)$.

The transfer functions (3) are stable. Integrating processes are obtained by adding an integrator to all the transfer functions given in (3).

2.3 The AMIGOs tuning rules

The test batch (3) was used in [2] to investigate the Ziegler-Nichols step response method and to derive new simple tuning rules, the AMIGOs rules. The results are summarized here for further comparison.

The process dynamics for stable processes is first approximated by the simple *KLT* model

$$G_p(s) = \frac{K_p}{1 + sT} e^{-sL} \tag{4}$$

where K_p is the static gain, T the time constant (also called lag), and L the time delay. Processes with integration are approximated by the model

$$G_p(s) = \frac{K_v}{s} e^{-sL} \tag{5}$$

where K_{ν} is the velocity gain and L the time delay. The model (5) can be regarded as the limit of (4) as K_p and T go to infinity an such a way that $K_p/T = K_{\nu}$ is constant. The parameters in (4) and (5) can be determined from a step response experiment, see [2].

The AMIGOs tuning rules were obtained in the following way. The MIGO design method with $M_s = 1.4$ was applied to all processes in the test batch (3), both stable and integrating. This gave the PI controller parameters K and T_i , as well as the process parameters K_p , L and T (K_v and L for processes with integration) for each process. The AMIGOs rules where then obtained by finding relations between the controller parameters and the process parameters.

The AMIGOs rules presented in [2] are simple, but they suffer from the drawback that they are split into different regions depending on the ratio between L and T. A parameter fit shows that the formula in [2] can be replaced by

$$K = \frac{0.15}{K_p} + \left(0.35 - \frac{LT}{(L+T)^2}\right) \frac{T}{K_p L}$$

$$T_i = 0.35L + \frac{6.7LT^2}{T^2 + 2LT + 10L^2} \tag{6}$$

For integrating processes, K_p and T go to infinity and $K_p/T = K_y$. Therefore, the AMIGOs tuning rules (6) can be simplified to

$$K = \frac{0.35}{K_{\nu}L}$$

$$T_i = 7L \tag{7}$$

for integrating processes.

Figure 3 illustrates the relations between the controller parameters and the process parameters for all stable processes in the test batch. The controller gain is normalized by multiplying it either with the static process gain K_p or with the parameter $\alpha = K_p L/T = K_v L$. The integral time is normalized by dividing it by T or by L. The controller parameters in Fig. 3 are plotted versus the relative dead time $\tau = L/(L + T)$.

The solid lines in Fig. 3 correspond to the AMIGOs tuning formula (6), and the dotted lines show the limits for 15% variations in the controller parameters. Almost all processes included in the test batch fall within these limits. Integrating processes have a relative time delay $\tau = 0$. Most of the integrating processes in the test batch also fall within these limits. This means that for most processes, the simple AMIGOs rules give controller parameters that differ less than 15% from the controller parameters obtained using the MIGO method presented in [4].

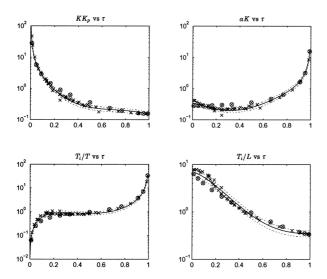


Fig. 3. Normalized PI controller parameters plotted versus relative time delay τ for stable processes. The solid lines correspond to the AMIGOs design rule (6), and the dotted lines indicate 15% parameter variations. Processes with the structure (4) are marked with \circ .

2.4 Parameterization

Ziegler-Nichols characterized the processes by two parameters K_{180} and T_{180} when they developed their frequency response method for controller tuning. The frequency ω_{180} is the frequency where the phase lag of the process is 180° and the parameters are given by $T_{180} = 2\pi/\omega_{180}$ and $K_{180} = |G(i\omega_{180})$.

They define the point where the Nyquist curve of the process transfer function intersects the negative real axis. The parameters can be determined by a simple manual experiment. In the Ziegler-Nichols tuning rules the controller gain is inversely proportional to K_{180} and the integral time is proportional to T_{180} .

The Ziegler-Nichols tuning rules do not use sufficient information and they give too aggressive tuning which does not give robust closed loop systems. When investigating the step response method in [2] it was found that significant improvement could be obtained by including an additional third process parameter, the static process gain. In this paper it will be investigated if similar improvements can be obtained for the frequency domain method.

For the step response method we used the relative time delay $\tau = L/(L+T)$ as a parameter to characterize the process. The corresponding frequency domain parameter is the gain ratio $\kappa = K_{180}/K_p$, where K_p is the static process gain. The relative time delay τ and the gain ratio κ are closely related, see [7]. They are both small for lag dominated processes, and close to one for delay dominated processes.

III. A FIRST ATTEMPT

To evaluate the Ziegler-Nichols rules and to develop new tuning rules we applied the MIGO design method to the processes in the test batch. The processes G_3 , G_4 with n=2, and G_7 have to be excluded because ω_{180} is infinitely large for these processes. The design calculations gave the parameters K and T_i of PI controllers. It is then attempted to correlate the controller parameters with the process parameters K_p , K_{180} , and T_{180} .

3.1 Stable processes

Figure 4 shows the controller parameters obtained for $M_s = 1.4$ and $M_s = 2.0$. The figure shows that there are significant differences between controller parameters obtained for process G_1 and the other processes. A comparison with the corresponding results for the step response method in Fig. 3 shows that the spread in numerical values of the normalized parameters are much smaller for the frequency response method. Notice

however that for the step response method there were not large differences between the process G_1 and the other processes.

The Ziegler-Nichols tuning rules have constant values $KK_{180} = 0.4$ and $= T_i/T_{180} = 0.8$, for all values of κ . Figure 4 shows that it may be reasonable to have a constant value KK_{180} for $\kappa > 0.5$, but not for smaller values of κ . The gain $KK_{180} = 0.4$ suggested by Ziegler and Nichols is clearly too high which explains the poor robustness of their method. The integral time suggested by Ziegler and Nichols, $T_i = 0.8T_{180}$, is too high except for processes with very small values of κ . The figure also shows that it is reasonable to use a constant value of T_i/T_{180} only for $\kappa > 0.5$.

3.2 Integrating processes

Many classical tuning rules treat stable processes and processes with integration separately. Since a process with integration can be regarded as the limiting case of a process with a very large lag it seems attractive to try to unify the tuning rules for stable systems and systems with integration.

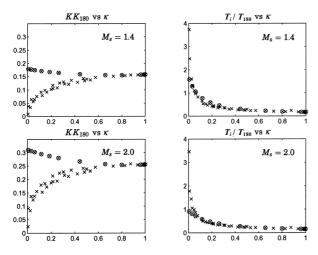


Fig. 4. Normalized controller parameters plotted versus gain ratio κ for stable processes for $M_s = 1.4$ and $M_s = 2.0$. The circles mark data from process G_1 .

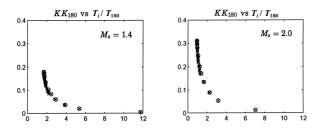


Fig. 5. Normalized controller parameters for integrating processes obtained with $M_s = 1.4$ and $M_s = 2.0$. The circles mark data from process G_1 .

Processes with integration correspond to $\kappa=0$. The results in Fig. 4 indicate that there are substantial variations in the normalized controller parameters for small values of κ . In Fig. 5 we show the normalized controller parameters for processes with integration. The figures show that there are significant variations in the parameters. The normalized gain varies with an order of magnitude, and the normalized integration time varies with a factor of 3. The ranges of the variations matches the data in Fig. 4 for $\kappa=0$.

3.3 Tuning rules for balanced and delay-dominated processes

Even if Fig. 4 indicates that it is not possible capture all data by one tuning rule it is clear that a good tuning rule can be found for delay dominated processes.

For a pure time delay we have $K_{180} = K_p$ and $T_{180} = 2L$. Designing a controller for $M_s = 1.4$ we find $KK_p = 0.15$ and $T_i = 0.35$ L. These parameters are captured by the rule

$$KK_{180} = 0.15$$
 (0.4)
 $\frac{T_i}{T_{180}} = 0.17$ (0.8) (8)

This is similar to the Ziegler-Nichols rule but with different parameters, given in parentheses. Notice, however, that the differences in integral gain is less 0.88 versus 0.5. The rule above thus has higher integral gain but much lower proportional gain.

Figure 4 indicates that the rule designed for a pure time delay process will work at least for $\kappa \ge 0.5$. Figure 4 indicates also that we can find a rule for the integration time at least down to $\kappa = 0.1$ but that it is not possible to find a good rule for the gain for such low values of κ . It may however be possible to obtain a conservative rule. Figure 6 shows the controller parameters obtained for $M_s = 1.4$ and graphs corresponding to the following tuning rule.

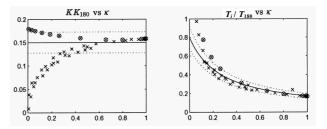


Fig. 6. Normalized controller parameters plotted versus gain ratio κ for stable processes for $M_s = 1.4$. The solid lines correspond to the tuning rule (9), and the dotted lines indicate 15% variations from the rule. The circles mark data from process G_1 .

$$KK_{180} = 0.15$$

$$\frac{T_i}{T_{180}} = \frac{0.8}{1 + 3.7\kappa}$$
(9)

The tuning rule (9) is not appropriate for lag-dominant processes, but it gives controller parameters that are fairly close to the optimal for processes with $\kappa > 0.2$. Notice in particular that the ratio T_i/T_{180} is reduced by a factor of three when κ increases from 0.2 to 1. Figure 7 shows the Nyquist curves obtained for all processes in the test batch with $\kappa > 0.2$ when the tuning rule (9) is used. The figure shows that all loop transfer functions are close to the M_s -circle.

The conclusion is that a small modification of the Ziegler-Nichols method can give good results. This rule gives close to optimal tuning for $\kappa \geq 0.5$ and robust controllers for $\kappa \geq 0.2$. For lag dominant processes, *i.e.* small κ , it is possible to get controllers with larger values of integral gain but these controllers cannot be obtained from tuning rules based on K_p , T_{180} and K_{180} . Controllers for lag dominated processes which are tuned for best reduction of load disturbances have high gain. Because of process noise and excessive controller action it may be better to have detuned controllers with lower gain.

3.4 Summary

The results show that there is some merit in using the normalized parameters KK_{180} and T_i/T_{180} . For delay dominated processes it is indeed possible to find tuning

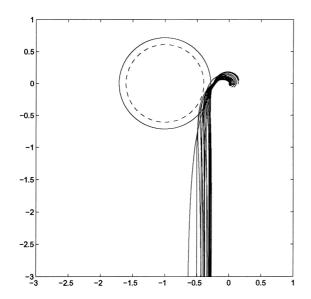


Fig. 7. Nyquist curves of loop transfer functions obtained when PI controllers tuned according to (9) are applied to processes in the test batch with $\kappa > 0.2$. The solid circle corresponds to $M_s = 1.4$ and the dashed to a circle where M_s is increased by 15%.

rules where these parameters are constant, but for lag dominated processes it is necessary to use more information about the process for example in terms of the steady state gain K_p or κ . This is similar to the results in [2] where the relative time delay τ was used as an additional process parameters.

The results in Figs. 4 and 5 are also disappointing because they indicate that it is not possible to get tuning rules for all processes in the test batch. This is surprising because simple tuning rules for the test batch could be developed based on the Ziegler-Nichols step response method, see [2].

It can also be observed that the process G_1 is different from the other processes. This is particularly disconcerting because so much of the work on tuning of PI controller is based on the assumption that process dynamics can be approximated by G_1 .

In the next section we will analyze the results to get clues for improvements.

IV. ANALYSIS

We will now analyze why the controllers obtained for process G_1 are different from those obtained for the other processes when κ is small, *i.e.* when the processes are lag-dominant.

The process G_1 has the transfer functions

$$G_1(s) = K_p \frac{e^{-sL}}{1 + sT}$$

and the frequency ω_{180} is given by

$$\omega_{180}L + \arctan(\omega_{180}T) = \pi$$

For lag-dominant processes, when $L \ll T$, this gives approximately $\omega_{180} \approx \pi/(2L)$, $T_{180} \approx 4L$, and $K_{180} \approx 2LK_p/(\pi T)$.

The AMIGOs tuning rule (7) gives

$$KK_{180} = \frac{0.35T}{K_p L} \cdot \frac{2LK_p}{\pi T} \approx 0.22$$

$$T_i / T_{180} = \frac{7L}{4L} = 1.75 \tag{10}$$

These parameter values agree well with the data in Fig. 4. The process G_2 has the transfer function

$$G_2(s) = K_p \frac{e^{-sL}}{(1+sT)^2}$$

For lag-dominant processes we have approximately

$$\pi = \omega_{180}L + 2\arctan(\omega_{180}T) \approx \omega_{180}L + 1\left(\frac{\pi}{2} - \frac{1}{\omega_{180}T}\right)$$

Hence $\omega_{180} \approx \sqrt{2/(LT)}$ and $K_{180} \approx K_p L/(2T)$. The process G_2 has the apparent time constant $T_{eff} \approx 1.864T$ and the apparent time delay $L_{eff} \approx L + 0.282T \approx 0.282T$ for lag-dominant processes. The tuning rule AMIGOs (7) then gives

$$KK_{180} = \frac{0.35T_{eff}}{K_p L_{eff}} \cdot \frac{K_p L}{2T} \approx 1.2 \frac{L}{T}$$

$$T_i / T_{180} = 7L_{eff} \cdot \frac{\sqrt{2}}{2\pi\sqrt{LT}} \approx 0.44 \sqrt{\frac{T}{L}}$$
(11)

which agrees well with the results in Fig. 4.

A comparison of Eqs. (10) and (11) shows that for processes with lag dominated dynamics the controller for G_2 has smaller gains and larger integration times than the controller for G_1 . The differences are very large for small κ .

The time delay is the parameter that limits the achievable performance. Backtracking we find that the reason for the large differences between the processes G_1 and G_2 is that the frequency ω_{180} is directly related to the time delay for the process G_1 but not for the process G_2 . Notice that for G_2 we have $\omega_{180} \sim \sqrt{1/(LT)}$ but for G_1 ω_{180} is inversely proportional to L.

The difficulty discussed above does not arise for delay-dominant processes. For such processes we have $\kappa \approx 1$ and $\omega_{180} \approx \pi/L$ for G_1 and G_2 .

4.1 Modified tuning procedures

Having found that the straight forward generalization of the Ziegler-Nichols tuning method does not give good tuning rules we will now investigate if is possible to modify the procedure. The main difficulty is that the parameters K_{180} and T_{180} for the process G_2 do not give the relevant information. One possibility is to investigate what happens if the tuning is based on another angle instead of 180° . This has been used very successfully for a long time in relay auto-tuning, see [7]. A relay with hysteresis gives the frequency where the process has a phase lag that is less than 180° . The hysteresis also has other advantages. Tuning based on other angles than 180° have also been suggested in [8-10].

Let ω_{ϕ} denote the frequency where the process has phase lag ϕ and let K_{ϕ} be the process gain at that frequency. We start by investigating the tuning methods for the processes G_1 and G_2 and we will explore if there are some good choices of the angle ϕ for processes with lag-dominated dynamics

Process G₁

For process G_1 we have

$$\omega L + \arctan(\omega T) = \phi$$

For lag-dominant processes, when T >> L, this gives $\omega_{\phi} = (\phi - \pi/2)/L$ and the corresponding period is

$$T_{\phi} = \frac{2\pi L}{\phi - \pi/2}$$

The process gain at frequency φ becomes

$$K_{\phi} = \frac{K_p}{\sqrt{1 + \omega_{\phi}^2 T^2}} \approx \frac{K_p}{\omega_{\phi} T} = \frac{Kp}{\phi - \pi/2} \cdot \frac{L}{T}$$

Assuming that the controller is tuned using AMIGOs, it follows from (7) that $KK_p = 0.35T/L$ and $T_i = 7L$. The tuning rules based on data obtained at angle ϕ becomes

$$K_{G1}^* = KK_{\phi} = \frac{0.35}{\phi - \pi/2}$$

$$T_{G1}^* = \frac{T_i}{T_{\phi}} = \frac{7(\phi - \pi/2)}{2\pi}$$
(12)

Process G₂

For process G_2 we have

$$\omega L + 2 \arctan(\omega T) = \phi$$

For lag-dominant processes, when T >> L, this gives

$$\omega_{\phi} = \frac{1}{T} \tan(\phi/2)$$

with the corresponding period

$$T_{\phi} = \frac{2\pi T}{\tan(\phi/2)}$$

The process gain at frequency ϕ is

$$K_{\phi} = \frac{K_p}{1 + \omega_{\phi}^2 T^2} \approx \frac{K_p}{1 + \tan^2(\phi/2)}$$

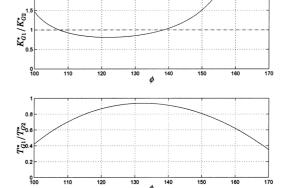


Fig. 8. Ratios of normalized controller gains K_{G1}^*/K_{G2}^* (top) and integral times T_{G1}^*/T_{G2}^* (below) for processes G_1 and G_2 as a function of the angle ϕ for lag-dominated processes.

For the process G_2 we have $L_{eff} \approx L + 0.282T$ and $T_{eff} \approx 1.864T$. For $L \ll T$ the tuning rule (7) gives $KK_p \approx 2.31$ and $T_i \approx 1.97T$. Hence

$$K_{G2}^* = KK_{\phi} = \frac{2.31}{1 + \tan^2(\phi/2)}$$

$$T_{G2}^* = \frac{T_i}{T_{\phi}} = 0.31 \tan(\phi/2)$$
(13)

Is there a ϕ that works for both Processes?

It follows from Eqs. (12) and (13) that

$$\frac{K_{G1}^*}{K_{G2}^*} = 0.15 \frac{1 + \tan^2(\phi/2)}{\phi - \pi/2}$$
$$\frac{T_{G1}^*}{T_{G2}^*} = 3.6 \frac{\phi - \pi/2}{\tan \phi/2}$$

The results in Fig. 4 are clearly explained by these equations. Notice that the ratio K_{G1}^*/K_{G2}^* goes to infinity and that the ratio T_{G1}^*/T_{G2}^* goes to zero as ϕ goes to $\pi/2$ or π . The question is now if there is an angle ϕ such that the ratios are close to one. The ratios are plotted in Fig. 8. The curves are based on results in [2] which are more accurate and more complicated than (12) and (13). Fig. 8 confirms the disappointing results in Fig. 4, but more importantly it shows that there is a range of angles ϕ where the ratios are close to one.

V. THE AMIGOF TUNING RULES

Encouraged by the analysis we will now present tuning rules based on different angles ϕ . Figure 9 shows the normalized controller parameters obtained for ϕ = 125°, 130° and 135° and M_s = 1.4, and in Fig. 10 we show the corresponding results for processes having integration. The gain ratio is defined as $\kappa_{\phi} = K_{\phi}/K_p$. The figures indicate that it is indeed possible to find good tuning rules that work for the full test batch including processes with integration if the angle is chosen appropriately.

The figures indicate that $\phi = 130^{\circ}$ is a reasonable choice. Figure 10 shows that for processes with integration the tuning rule may be obtained as the limit of stable processes as κ_{ϕ} goes to zero.

The controller parameters presented in Figs. 9 and 10 can be approximated by the following tuning rule, which we call AMIGOf (Approximate MIGO based on frequency response data):

$$KK_{\phi} = \frac{a}{1 + b\kappa_{\phi}}$$

$$T_i / T_{\phi} = \frac{c}{\left(1 + d\kappa_{\phi}\right)^2} \tag{14}$$

where the parameters a, b, c and d are given in Table 1.

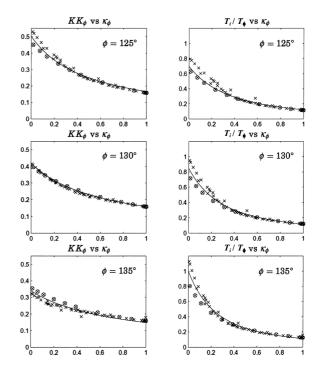


Fig. 9. Normalized controller parameters for design with M_s = 1.4 based on the angles ϕ = 125°, 130°, and 135° for the test batch (3). The circles mark data from process G_1 , and the solid lines correspond to the AMIGOf tuning rule (14).

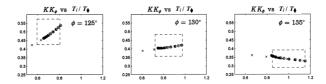


Fig. 10. Normalized controller parameters for design with M_s = 1.4 based on the angles ϕ = 125°, 130°, and 135° for the test batch (3) with integration. The circles mark data from process G_1 and the boxes mark regions where the parameters differ less than 15% from the AMIGOf tuning rule (14).

Table 1. Controller parameters in the AMIGOf tuning rule (14)

	$M_s = 1.2$	$M_s = 1.4$	$M_s = 2.0$
a	$2.01 - 0.80\phi[rad]$	$2.50 - 0.92\phi[rad]$	$3.43 - 1.15\phi[rad]$
b	$15.45 - 6.30\phi[rad]$	$10.75 - 4.01\phi[rad]$	$11.30 - 4.01\phi[rad]$
c	$-2.55 + 1.72\phi[rad]$	$-3.05 + 1.72\phi[rad]$	$-1.50 + 0.86\phi$ [rad]
d	$-5.20 + 3.44\phi$ [rad]	$-6.10 + 3.44\phi$ [rad]	$-2.90 + 1.72\phi[rad]$
Range	110° < φ < 135°	120° < φ < 140°	135° < φ < 155°

The rules for integrating processes are obtained by setting $\kappa_{\phi} = 0$ in Eq. (14), *i.e.*

$$KK_{\phi} = a$$

$$T_i/T_{\phi} = c \tag{15}$$

The AMIGOf tuning rules have been tested on other values of ϕ . For $M_s = 1.4$ the rules give controller parameters that are reasonable close to those given by the underlying MIGO tuning rules for angles in the interval $120^{\circ} < \phi < 140^{\circ}$.

5.1 Other values of M_s

It is interesting to have tuning rules for other values of the design parameter M_s . In this section we present results for $M_s = 2$ and $M_s = 1.2$. It turns out that the angle ϕ depends to the design variable. The results also show that the fit is worse for larger values of M_s .

Figure 11 shows the normalized parameters for stable processes obtained for $M_s = 2.0$, and Fig. 12 shows the corresponding results for processes having integration. Controller parameters given by the AMIGOf tuning rules (14) are also shown in the figures.

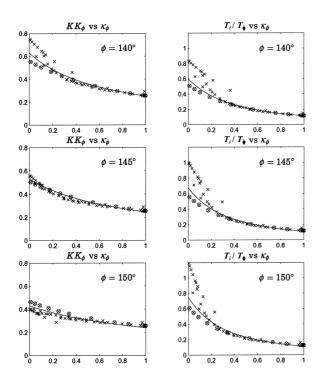


Fig. 11. Normalized controller parameters for design with M_s = 2 based on the angles ϕ = 140°, 145°, and 150° for the test batch (3) with integration. The circles mark data from process G_1 , and the boxes mark regions where the parameters differ less than 15% from the AMIGOF tuning rule (14).

For M_s = 2.0, a good choice of angle is ϕ = 145. An analysis of the figures shows a very good correlation for the controller gain, but there are deviations in the integration time for smaller values of κ_{ϕ} . Apart from this irregularity the AMIGOf rules give controller parameters that are close to those given by the underlying MIGO tuning rules for angles in the interval 135° < ϕ < 150°.

Figure 13 shows the normalized parameters for stable processes obtained for $M_s = 1.2$, and Fig. 14 shows the corresponding results for processes having integration. The AMIGOf tuning rules (14) are also shown in the figures.

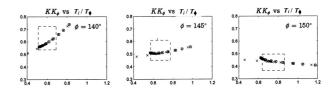


Fig. 12. Normalized controller parameters for design with M_s = 2 based on the angles ϕ = 140°, 145°, and 150° for the test batch (3). The circles mark data from process G_1 , and the solid lines correspond to the AMIGOF tuning rule (14).

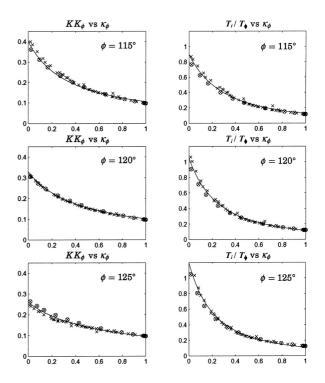


Fig. 13. Normalized controller parameters for design with M_s = 1.2 based on the angles ϕ = 115°, 120°, and 125° for the test batch (3). The circles mark data from process G_1 , and the solid lines correspond to the AMIGOf tuning rule (14).

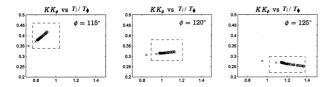


Fig. 14. Normalized controller parameters for design with M_s = 1.2 based on the angles ϕ = 115°, 120° and 125° for the test batch (3) with integration. The circles mark data from process G_1 and the boxes mark regions where the parameters differ less than 15% from the AMIGOf tuning rule (14).

For $M_s = 1.2$, a good choice of angle seems to be $\phi = 120$. The figures show that the correlations are much better for smaller values of the design parameter M_s , *i.e.* more conservative design. A good correspondence with the underlying MIGO tuning rules is also obtained for a larger range of angles, $110^{\circ} < \phi < 135^{\circ}$.

5.2 How to find the frequency ω_{ϕ} ?

The frequency ω_{180} can be determined simply by using proportional control and increasing the gain until an oscillation is obtained, as suggested by Ziegler and Nichols [1]. There are several ways to determine the ω_{ϕ} for other angles; phase-locked loops, relay feedback and manual tuning.

Use of phase-locked loops is discussed in [11] and [12]. This will give data for any angle ϕ .

It follows from describing function theory that a relay with hysteresis inserted in the feedback loop gives an oscillation that corresponds to the frequency where

$$G(i\omega_{\phi}) = -\frac{\pi a}{4d} \left(\sqrt{1 - \left(\frac{\varepsilon}{a}\right)^2} + i \cdot \frac{\varepsilon}{a} \right)$$

where a is the oscillation amplitude, d is the relay amplitude, and ϵ is the relay hysteresis. The oscillation frequency ω_{ϕ} is the frequency where the phase lag is

$$\phi = \pi - \arcsin \frac{\varepsilon}{a}$$

The angle ϕ can be assigned desired values by proper choices of the relay amplitude and the relay hysteresis. See [13] and [7]. In [14] it is suggested to adjust the the relay hysteresis iteratively so that

$$\varepsilon = a \sin \phi$$

It is also possible to find the desired frequency using manual tuning by using a PI controller and choosing the integration time so that $\omega_{\phi}T_i = \tan \phi$ and increasing

the gain until the system oscillates. This requires knowledge about ω_{ϕ} which can be obtained using the following iterative procedure.

- 1. Make a step response experiment and determine the controller parameters using *e.g.* the AMIGOs rules.
- 2. Use a PI controller with integration time determined from step 1.
- 3. Increase the gain until the system reaches the stability boundary.
- 4. Determine PI parameters from the AMIGOf rules.
- 5. Iterate if necessary.

5.3 Summary

In this section we have presented the AMIGOf tuning rules, which give controller parameters that are close to those obtained by the MIGO rules. The AMIGOf rules require process knowledge in terms of the static gain K_p and the gain K_{ϕ} and frequency ω_{ϕ} where the process phase lag is ϕ . This angle ϕ must, however, be chosen with care. It depends on the value of the design parameter M_s .

The fact that the angle ϕ depends on M_s is a disadvantage. Design for other values of M_s requires new experiments. Reasonable values of ϕ are 120°, 130° and 145° for 1.2, 1.4 and 2 respectively. From the experiments it is, however, seen that if $\phi = 135^\circ$ it is possible to change the design parameter in the region $1.2 \le M_s \le 2.0$.

The AMIGOf tuning rules are very well suited for auto-tuning with relay feedback. The angle ϕ can be give a desired values by proper choices of the relay amplitude and the relay hysteresis.

The methods can be used for manual tuning but it is more complicated than for the standard Ziegler-Nichols procedure because it is necessary to iterate.

VI. AN INTERPRETATION OF THE RESULTS

In [7] is shown that the standard Ziegler-Nichols procedure can be interpreted as a loop-shaping procedure where the point where the Nyquist curve of the process intersects the negative real axis is moved to the point -0.4 + 0.08i. We will now give a similar interpretation of the AMIGOf tuning rules. The loop transfer function under PI control is

$$L(s) = K \left(1 + \frac{1}{sT_i} \right) G(s) = \left(K + \frac{k_i}{s} \right) G(s)$$

We have

$$L(i\omega_{\phi}) = \left(K + \frac{k_i}{i\omega_{\phi}}\right) G(i\omega_{\phi}) = \left(K + \frac{k_i}{i\omega_{\phi}}\right) K_{\phi} e^{-i\phi}$$
$$= (\alpha - i\beta) e^{-i\phi} = \sqrt{\alpha^2 + \beta^2} e^{-\phi - \arctan(\beta/\alpha)}$$
(16)

where

$$\alpha = KK_{\phi}$$

$$\beta = \frac{k_i K_{\phi}}{\omega_{\phi}}$$

The parameters α and β can be interpreted as some kind of stability margins. When $\alpha=1$ the loop transfer function intersects the unit circle at $e^{-i\phi}$ and when $\beta=1$ the loop transfer function intersects the unit circle at $e^{-i(\phi+\pi/2)}$. The parameters can also be interpreted as normalized coefficients of proportional and integral action.

Figure 15 shows the parameters α and β as functions of the gain ratio. Notice that normalized proportional action α decreases and normalized integral action β increases as κ_{ϕ} goes from 0 to 1.

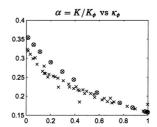
It follows from (16) that the AMIGOf design can be interpreted as moving the point where the open loop system has a phase lag $-\phi$ to the point at a distance $\sqrt{\alpha^2 + \beta^2}$ from the origin and the angle $-\phi$ – arctan (β) A). Figure 16 illustrates how the points where $\phi = 135^{\circ}$ are moved to new positions for the whole test batch in the case when $M_s = 1.4$.

Figure 17 shows the distance and the angle for designs based on $\phi = -135^{\circ}$. The figure shows that the distance from the origin decreases as κ_{ϕ} increases. It is 0.35 for lag-dominant processes and 0.25 for delay-dominant processes. The angle also decreases with increasing κ_{ϕ} from -145° for lag-dominant processes to -185° for delay-dominant processes.

VII. EXAMPLES

We will give a few examples that illustrate the new design method AMIGOf. The method will be compared with the underlying MIGO method to illustrate how well the approximate method corresponds to the original one. A comparison with the Ziegler-Nichols method is also made. Three examples are given, for a lag-dominant process, a process with balanced lag and delay, and a delay-dominant process. The MIGO and AMIGOf methods are based on the design parameter $M_s = 1.4$ in all examples.

Example 1. LAG DOMINATED DYNAMICS Consider a process with the transfer function



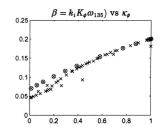


Fig. 15. Parameters α and β representing normalized proportional and integral action as functions of the gain ratio κ_{ϕ} for processes in the test batch (3). The plots are based on designs with $M_s = 1.4$ and $\phi = 135^{\circ}$. The circles denote data from process G_1 .

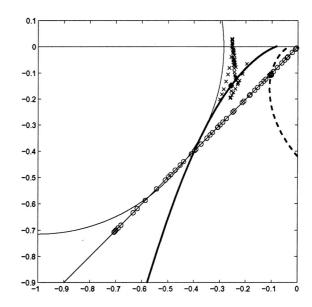


Fig. 16. Frequency points of the processes in the test batch where $\phi = 135^{\circ}$, marked with circles, are moved to new positions, marked with crosses, when $M_s = 1.4$. Nyquist diagrams corresponding to one processes in the test batch are also shown.

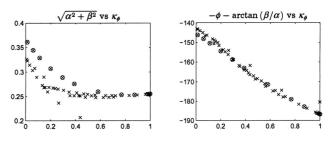


Fig. 17. The distance from the origin (left) and the phase angle (right) of the loop transfer function obtained for $M_s = 1.4$ and $\phi = 135^{\circ}$, plotted versus gain ratio κ_{ϕ} for processes in the test batch (3). The circles mark data from process G_1 .

$$G(s) = \frac{e^{-s}}{1 + 20s}$$

The ultimate frequency is $\omega_{180} = 1.6$, and the corresponding gain $K_{180} = 0.031$. The gain ratio is $\kappa = K_{180}/K_p = 0.031$, which shows that the process is highly lag-dominant. Based on this frequency point, the Ziegler-Nichols tuning rule gives the controller parameters $K = 0.4/K_{180} = 13$ and $T_i = 0.8T_{180} = 3.1$.

For the AMIGOf tuning rule, we choose the frequency point where the phase lag is -135° . The frequency becomes $\omega_{135} = 0.85$ with the corresponding gain $K_{135} = 0.059$. The gain ratio is $\kappa_{\phi} = K_{135}/K_p = 0.059$. The AMIGOf tuning rules (14) gives the controller parameters K = 5.4 and $T_i = 5.9$. The MIGO controller parameters are K = 5.7 and $T_i = 5.1$. The AMIGOs tuning rules (6) give the parameters K = 6.2 and $T_i = 6.3$.

The controller parameters of AMIGOf and MIGO are close to each other, which means that with the given design criteria very little is lost by not using the full transfer function. The AMIGOf parameters are more close to MIGO than the AMIGOs parameters. The Ziegler-Nichols tuning rule gives a gain that is more than twice as large and an integral time that is almost half as long compared to AMIGOf.

Figure 18 shows the responses of the system to changes in set point and load disturbances. The figure shows that AMIGOf gives very reasonable responses. The Ziegler-Nichols method gives a very rapid compensation of the load disturbance, although at the price of a slightly oscillatory response.

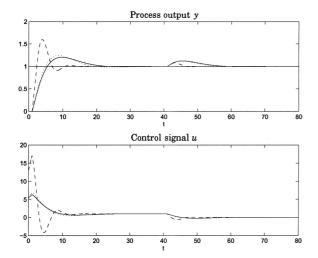


Fig. 18. Responses to step changes in set point and load for PI controllers designed by AMIGOf (full line), Ziegler-Nichols (dashed), and AMIGO (dotted) for a process with the transfer function $G(s) = e^{-s}/(1 + 20s)$. The dynamics of the process is lag dominated with $\kappa = 0.031$

Next we will consider a process where the lag and the delay are balanced.

Example 2. BALANCED LAG AND DELAY Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^4}$$

The ultimate frequency is $\omega_{180} = \tan(\pi/4) = 1$, and the corresponding gain is $K_{180} = 0.25$. The gain ratio is $\kappa = 0.25$, which shows that the process is neither lag-dominant nor delay-dominant. The Ziegler-Nichols tuning rule gives the controller parameters $K = 0.4/K_{180} = 1.6$ and $T_i = 0.8T_{180} = 5.0$.

The AMIGOf tuning rule is based on the frequency point where the phase lag is -135° . The frequency becomes $\omega_{135} = 0.67$ with the corresponding gain $K_{135} = 0.48$ and gain ratio $\kappa_{\phi} = K_{135}/K_p = 0.48$. The AMIGOf tuning rules (14) gives the controller parameters K = 0.44 and $K_i = 0.44$ and

The controller parameters obtained by AMIGOf and MIGO almost the same, but the gain obtained by the Ziegler-Nichols method is about four times larger and the integral time is twice as long. The AMIGOs method gives a gain that is about 20% higher than the MIGO design.

Figure 19 shows the responses of the system to changes in set point and load disturbances. The figure

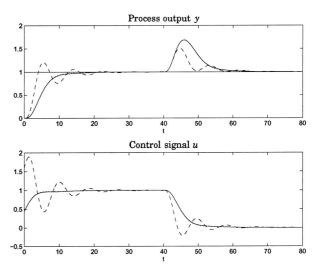


Fig. 19. Responses to step changes in set point and load for PI controllers designed by AMIGOf (full line), Ziegler-Nichols (dashed), and AMIGO (dotted) for a process with the transfer function $G(s) = 1/(s+1)^4$. The dynamics of the process is characterized by $\kappa = 0.25$.

shows that the responses obtained by MIGO and AMIGof are almost identical and reasonable, whereas the response obtained by the Ziegler-Nichols method is highly oscillatory as expected.

Finally we will consider an example where the dynamics is dominated by the time delay.

Example 3. DELAY DOMINATED DYNAMICS Consider a process with the transfer function

$$G(s) = \frac{1}{(0.05s+1)^2} e^{-s}$$

The ultimate frequency is $\omega_{180} = 2.9$, and the corresponding gain $K_{180} = 0.98$. The gain ratio is $\kappa = K_{180}/K_p = 0.98$, which shows that the process is highly delay-dominant. Based on this frequency point, the Ziegler-Nichols tuning rule gives the controller parameters $K = 0.4/K_{180} = 0.41$ and $T_i = 0.8T_{180} = 1.7$.

For the AMIGOf tuning rule, we choose the frequency point where the phase lag is -135° . The frequency becomes $\omega_{135} = 2.1$ with the corresponding gain $K_{135} = 0.99$ and gain ratio $\kappa_{\phi} = K_{135}/K_p = 0.99$. The AMIGOf tuning rules (14) gives the controller parameters K = 0.15 and $T_i = 0.34$. The MIGO controller parameters are K = 0.16 and $T_i = 0.37$ and the AMIGOs parameters are K = 0.16 and $K_i = 0.35$.

The parameters obtained by AMIGOf, AMIGOs, and MIGO are very close which indicates that little is lost by not using the full transfer function. The gain obtained by Ziegler-Nichols is almost three times as large as for AMIGOf and the integral time is almost five times longer than for AMIGOf. Since integral action gives the key contribution to the control for delay dominated processes it can be expected that Ziegler-Nichols gives very poor control.

Figure 20 shows the responses of the system to changes in set point and load disturbances. The figure shows that AMIGOf gives very reasonable responses. The responses obtained with the Ziegler-Nichols method are extremely sluggish because of the low integral gain. Summarizing the results of the example we can make the following conclusions.

- Ziegler-Nichols tuning gives oscillatory responses for lag dominated processes and processes with balanced lag and delay, and very sluggish responses for delay dominated processes.
- The AMIGOf tuning rule gives good results for lag dominated processes, processes with balanced lag and delay, as well as delay dominated processes.
- The AMIGOf tuning rules are close to the MIGO tuning rules, which means that almost the same results are obtained by describing the process with three

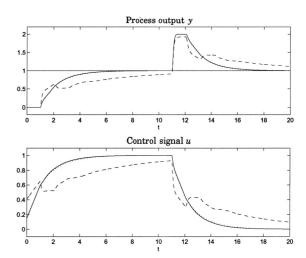


Fig. 20. Responses to step changes in set point and load for PI controllers designed by AMIGOf (full line), Ziegler-Nichols (dashed), and AMIGO (dotted) for a process with transfer function $G(s) = e^{-s}/(0.05s + 1)^2$. The dynamics of the system is delay dominated with $\kappa = 0.98$.

parameters, K_p , K_{ϕ} , and T_{ϕ} instead of the full transfer function.

 The AMIGOf tuning rules are more close to the MIGO tuning rules than the AMIGOs rules for lagdominant processes.

VIII. CONCLUSION

Ziegler-Nichols frequency response method is based on a simple method to manually obtain the process dynamics at the frequency ω_{180} . It is known to provide poor controller tuning in many cases. The goal of this paper has been to investigate the reason for the poor performance and to suggest improvements.

There are several reasons for the poor performance of the Ziegler-Nichols method. One reason is the design goal, quarter amplitude damping, which gives too small robustness margins in most applications. Another reason is that too little process information is obtained. It is not possible to find accurate tuning procedures based on one frequency point only.

A study of the step response method in [2] showed that good tuning rules could be obtained by introducing a third parameter, the static gain of the process. The investigation of the frequency response method in Section 3 shows that it is possible to obtain good tuning rules for delay dominated and balanced processes if the static gain is included as a third parameter. However it is not possible to obtain good tuning rules for all processes, particularly not for integrating processes. The process G_1 , whose dynamics consists of a first order

dynamics and a time delay, is significantly different from the other processes. This is particularly disconcerting because this process is the one predominantly used to develop tuning rules. Our results indicate that such rules may not be well suited for other processes.

We have, however, shown that it is possible to obtain tuning rules in the Ziegler-Nichols spirit based on the static gain and the value of the process function where the phase lag is less than 180°, typically 135°. Such tuning rules, called AMIGOf, have been presented. These rules give controller parameters which are close to those obtained by a full knowledge of the process transfer function. The particular design used is the robust loop-shaping method (MIGO) presented in [4]. A nice property of the tuning rules is that they work for stable processes as well for processes with integration. The method is well suited for automatic tuning based on the relay method. It is also possible to adapt it to manual tuning. Such procedures are, however, more complicated than the original Ziegler-Nichols method.

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