

# PID Parameters Tuning Method by Particle Swarm Optimization with Chaotic Disturbance

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**Abstract**—This paper proposes a new PID parameter tuning method using Chaotic Particle Swarm Optimization (CPSO) with ICMIC. The method is implemented into the PID tuning of the position control system, where the transfer function represents a plant model of a servo system. The PID parameter is computed by CPSO-based, PSO-based and ZN-based PID tuning method according to the obtained model, respectively. The simulation results show the effectiveness of the proposed tuning method, and are compared with the classical PSO and Ziegler-Nichols.

**Keywords**- Particle Swarm Optimization(PSO); servo system; chaotic optimization; PID controller

## I. INTRODUCTION

The Proportional-Integral-Derivative (PID) controller has been usually utilized for motion control system because of its simple structure and stable characteristic. The most critical step in application of PID controller is parameters tuning. Recently, as an alternative to the classical mathematical approaches, modern heuristic optimization techniques have been given much attention by many researchers due to their ability to find an almost global optimal solution in PID tuning[1]. One of these modern heuristic optimization paradigms is the Particle Swarm Optimization(PSO) approach[2]. Another reason for us to utilize PSO algorithm is that is no need of gradient information or calculation of gradient. PSO has a unique ability for optimization of non-linearly function and multi-dimensional function. In this context, the literature contains several algorithms using PSO for solving optimization problems in control system [3][4].

Inspired by the Particle Swarm Optimization and the concept of chaotic optimization [5], this work presents a new chaotic Particle Swarm Optimization (CPSO) to tune the PID control gains. The contribution

of this paper is the performance analysis of classical PSO and CPSO methods in tuning of a PID controller applied in servo system. Numerical simulations in tuning of PID controller based on PSO approaches for servo system demonstrate the effectiveness and robustness of optimization method.

In this paper our research is focused on how to improve the performance of auto-tuning digital PID controller based on PSO algorithm. The remainder of this paper organized as follows. Section 2 describes the auto-tuning PID controller framework. Section 3 introduces the chaotic particle swarm optimization algorithm and choosing of fitness function, and provides our solution to the problem. The numerical result and the experimental result show the effectiveness of the proposed PID parameter tuning method in section 4. Finally conclusion is represented in section 5.

## II. PROBLEM STATEMENT

### A. Fundamentals of the PID Control

The PID controllers have been widely applied to solve various control problems for decades in industry. As modeled in this paper, the transfer function of PID controller is described by the following equation in the continuous  $s$ -domain.

$$G(s) = K_p \left( 1 + \frac{1}{T_i \cdot s} + T_d \cdot s \right) \quad (1)$$

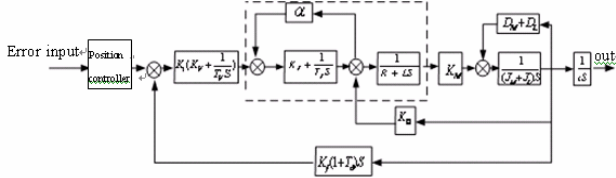
Where  $U(s)$  and  $E(s)$  are the control (controller output) and tracking error signals in  $s$ -domain, respectively;  $K_p$  is the proportional gain,  $K_i$  is the integration gain, and  $K_d$  is the derivative gain;  $T_i$  is the integral action time or reset time and  $T_d$  is referred to as the derivation action time or rate time.

The proportional part of the PID controller reduces error responses to disturbances. The integral term of the error eliminates steady state error and the derivative term of error dampens the dynamic response and thereby improves stability of the system.

The parameters settings of a PID controller for optimal control of a plant depend on the plant's behavior. To design the PID controller the engineer can appropriately choose the combination of the parameters and to simultaneously take care of the transient response as well as the steady-state error. In the design of a PID controller, the three gains of PID must be selected in such a way that the closed loop system has to give the desired response. The desired response should have minimal setting time with a small or no overshoot in the step response of the closed loop system.

#### B. Description of a Servo System

The block diagram of an AC servo system with speed loop is as follows.



**Figure 1. Block diagram of PWSM servo system with PID controller**

The dashed part is current loop, the transfer function of which is described by the following equation.

$$G_I(S) = \frac{(K_I + \frac{1}{T_I S})(\frac{1}{R + LS})}{1 + \alpha(K_I + \frac{1}{T_I S})(\frac{1}{R + LS})} \quad (2)$$

The time constant of the current loop is usually small, which can be treated as the proportional part. Besides, the effect of the parameters  $D_M$ ,  $D_L$ ,  $T_L$  on control system is ignored. Thus, the transfer function of the speed loop is given by the following equation.

$$G_V(S) = \frac{K_I(K_V + \frac{1}{T_V S}) \frac{K_I K_M}{(J_M + J_L)S}}{1 + K_I(K_V + \frac{1}{T_V S}) \frac{K_I K_M}{(J_M + J_L)S} K_f(1 + T_d S)} \quad (3)$$

The parameters of the executive electromotor and driver are substituted in Eq.(3). Thus, the transfer function of the object control model is obtained as follows:

$$G(S) = \frac{2.67(S + 502.78)}{S^3 + 184.8S^2 + 9197.3S} \quad (4)$$

### III. THE PID CONTROLLER TUNING STRATEGY USING CPSO ALGORITHM

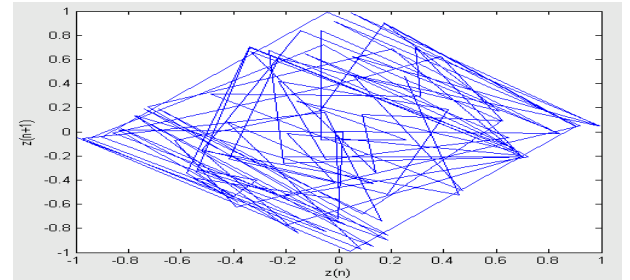
In this section, a novel chaotic PSO is proposed by incorporating The Iterative Chaotic Map with Infinite Collapses (ICMIC) [6] into the velocity updating Equation of the PSO. Then, the optimization problem to determine the optimal parameters ( $K_p, K_i, K_d$ ) of PID controller in section 2 utilizing the proposed CPSO is explicitly formulated.

#### A. The Iterative Chaotic Map with Infinite Collapses (ICMIC)

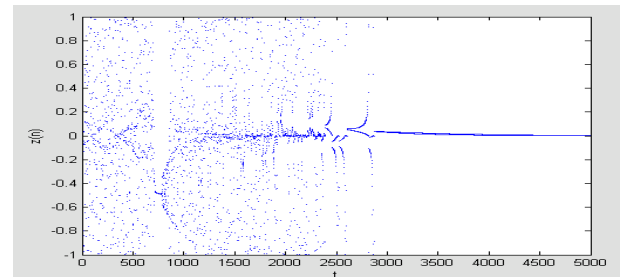
The ICMIC mapping model is defined as follows:

$$z_{k+1} = \eta \sin(\gamma_1/z_k) \exp(\gamma_2 z_k^2) \quad k = 0, 1, 2, \dots, K \quad (5)$$

where  $z_k$  is the variable ( $-1 \leq z_k \leq 1$ ) and  $z_k \neq 0$ ,  $\gamma_1$  and  $\gamma_2$  are two adjustable parameters,  $\eta$  is a damping factor ( $0 \leq \eta \leq 1$ ). Fig. 2 gives the trajectory of ICMIC in phase plan where  $\gamma_1 = 8$ ,  $\gamma_2 = -3$ ,  $\eta = 1$ . We can see ICMIC has infinite collapses in the range of  $[-1, 1]$ , which means it has infinite fixed points and zero points. Fig. 3 shows the chaotic trajectories of the map with the coefficient  $\gamma_1$  exponential decaying ( $\beta = \exp(-0.003)$ ), where  $\gamma_2 = -3$ ,  $\eta = 1$ ,  $K = 5000$ . Fig. 4 shows the chaotic graphs of the map with the coefficient  $\eta$  exponential decaying ( $\beta = \exp(-0.001)$ ), where  $\gamma_1 = 8$ ,  $\gamma_2 = -3$ ,  $K = 5000$ . Compared with Logistic map, Chebyshev map, Hénon map which have finite collapses, the new ICMIC can produce chaotic sequence which has good performance.



**Figure 2. Iterative map with infinite collapses**



**Figure 3. Trajectories of ICMIC with  $\gamma_1$  decaying**

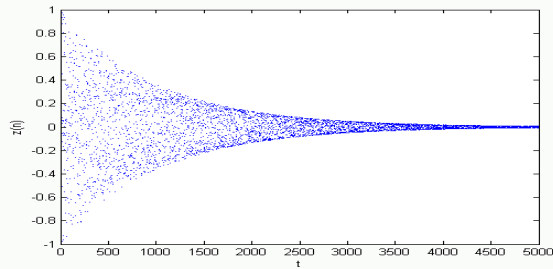


Figure 4. Trajectories of ICMIC with  $\eta$  decaying

### B. The Chaotic PSO with ICMIC

PSO emulates the swarm behavior and the individuals represent points in the dimensional search space. A particle represents a potential solution. Throughout the migrating process, each particle monitors three values: its current position( ); the best position it reached in previous cycles( ); its flying velocity( ); gbest is the best position discovered by the whole population. Each particle adjusts its position in a direction toward its own personal best position and the neighborhood best position. For particle, its position is changed according to

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (6)$$

where the velocity of the particle is calculated using [2]:

$$V_i(t+1) = w \times V_i(t) + c_1 \times \text{rand}() \times (pbest_i - X_i(t)) + c_2 \times \text{Rand}() \times (gbest - X_i(t)) \quad (7)$$

In this study, the velocity of the particle is determined according to the relative location between pbest<sub>i</sub> and gbest. Then, the updating formula of can be converted from Eq. (8) into the following equation:

$$V_i(t+1) = wV_i(t) - C(X_i(t) - p_i) \quad (8)$$

where  $p_i$  and  $C$  are defined as  $p_i = \frac{c_1 r_1 p_{li} + c_2 r_2 p_g}{c_1 r_1 + c_2 r_2}$ ,  $C = c_1 r_1 + c_2 r_2$ ,  $p_{li} = pbest_i$ ,  $p_g = gbest$ .

Here  $r_1$  and  $r_2$  are two random functions in the range of [0, 1].

If the  $V_i(t+1)$  term in Eq.(7) is replaced with Eq.(9), the position updating formula can be derived as;

$$X_i(t+1) = X_i(t) - C(X_i(t) - p_i) + w(X_i(t) - X_i(t-1)) \quad (9)$$

The Eq.(10) shows that the search by the particle can be regarded as a stochastic steepest descent with an inertial term.

By adding the ICMIC to Eq.(8), we propose a concept of the chaotic velocity to encourage individual particles to escape from the local minimum region in a more efficient manner. The velocity updating equation of CPSO is as follows:

$$V_i(t+1) = w \times V_i(t) - C(X_i(t) - p_i) - \eta \alpha x_i^2 \sin(\pi x_i) \quad (10)$$

### C. The Scheme of PID Controller Parameters Tuning with CPSO

The order of types of controller positioning is depicted in Fig.5. The duty of control engineering is to adjust the coefficients of gain to attain the error reduction and fast responses simultaneously.

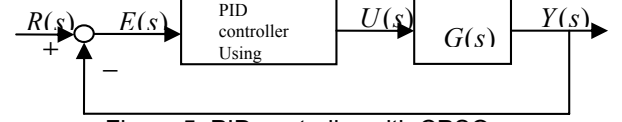


Figure 5. PID controller with CPSO

In the design of a PID controller, the objective function is first defined based on our desired specifications and constraints under input testing signal. In this work, the objective function is defined as integral square error as follows:

$$F = \int_0^{\infty} e(t)^2 dt \quad (11)$$

Where  $F$  and  $e(t)$  are fitness function and error, respectively.

- Each particle in swarm is 3-dimensional that represented parameters  $K_p, K_i, K_d$ .

## IV. SIMULATION RESULTS

In this section, the classical PSO and the proposed CPSO algorithm are applied to single loop PID parameters optimization system and obtained results compare with together.

### A. Performance comparison

At first PID parameters are evaluated roughly using conventional tuning method to get a smaller search space, for example  $K'_p = 1201.3$ ,  $K'_i = 46026.82$  and  $K'_d = 190682.54$  can be got with Ziegler-Nichols (ZN) experimental method[7]. Therefore, the searching ranges are set as follows:

$$K_p \in [K'_p(1-30\%), K'_p(1+30\%)],$$

$$K_i \in [K'_i(1-30\%), K'_i(1+30\%)],$$

$$K_d \in [K'_d(1-30\%), K'_d(1+30\%)].$$

Other parameters used in CPSO algorithm are initialized as follows. The group size of particles swarm is set to 10, and the maximum generation iteration number is set to 30. The inertia weight  $w$  is decreased linearly (from 0.9 to 0.1).

**Table 1. Comparison of two optimization methods with same generation**

Method	$K_p$	$K_i$	$K_d$	$t_r$	$t_s$	$M_p$ [%]
CPSO	175.9	8614.2	1.2649	0.0534	0.4425	32.42
PSO	150.08	828.66	0.1778	0.2475	4.2739	46.72
Ziegler-Nichols	1201.3	46026.82	7.5682	0.0319	0.1736	62.10

The single loop PID parameter tuning for above-mentioned case study system is accomplished by classical PSO algorithm and CPSO algorithm. The statistical results for the case study are given in Table 1, which shows that the CPSO succeeded in finding the best solution for the case study. However, the classical PSO and Ziegler-Nichols method did not outperform the CPSO method.

#### B. Diversity of swarm

Different position to which the particle of swarm flies in searching space represents different solutions. That is, the diversity of particle position in searching space is called diversity of swarm, which can be used for evaluating ability of CPSO. Determining the diversity within the swarm is based on the “distance-to-average -point” measure  $\delta_{avg}$  given by Eq.(12), where  $S$  is the swarm and  $n=|S|$  is the size of  $S$ . The problem dimension is denoted by  $d$  and  $x_{ij}$  is the  $j$ -th component of  $x_i$ . The average point of the particles in  $s$  at iteration  $t$  is given as  $\bar{X}$ , dimension  $j$  of which is  $\bar{X}_j$ .

$$\delta_{avg} = \frac{1}{n} \sum_{i=1}^n \sqrt{\frac{1}{d} \sum_{j=1}^d (X_{ij} - \bar{X}_j)^2} \quad (12)$$

The “distance-to-average-point” measure returns an unbounded absolute value that depends on the considered test function and the state of the optimization process. Therefore,  $\delta_{avg}$  is scaled by the diameter of the swarm. The diameter is determined as the maximum distance  $\delta_{max}$  between any two particles in the entire swarm. As diversity measure  $\delta_{avg}$  of particles in the swarm at iteration  $t$ , Fig. 6 gives the comparison of diversity curves of PSO (blue line) and proposed ICMICPSO (red line) for the functions.

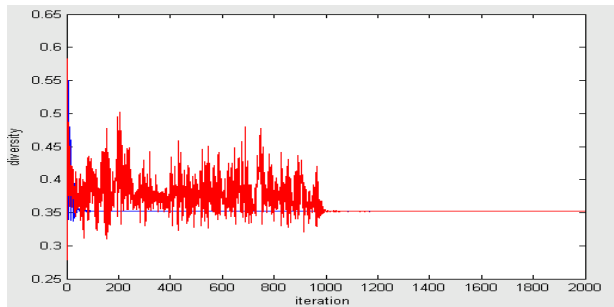


Figure 6. Diversity for the objective function

#### V. CONCLUSIONS

This paper presents a new chaotic PSO with ICMIC to search for optimal parameters in a servo system. The CPSO, classical PSO and ZN methods are implemented into the PID tuning. The numerical results demonstrate the effectiveness of the proposed CPSO-based PID tuning method. The CPSO reduces the rate of overshoot in step response curve in comparison PSO and ZN. Also, from rising time point of view, the CPSO is better than PSO. Future work is to develop the method to obtain the optimal PID parameter to disturbance.

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