Supervised adaptive PID control and time-varying fuzzy identifier of nonlinear dither systems

Z.R. Tsai, J.D. Hwang, Y.Z. Chang, and J. Lee

To enhance system robustness between system parameters and adaptation law, a supervised adaptive PID control scheme and time-varying fuzzy identifier of unknown nonlinear dither systems are proposed. The dither and the proposed time-varying term can obtain the smooth quality of plant for reducing the rule number of the fuzzy identifier. The supervised adaptive PID control law aims to pull the states back to the pre-specified state region and reduce overshoot towards an extra stabilising input.

Introduction: In reducing the nonlinear behaviour of the physical system, the dither [1] is injected into the nonlinear system. To simplify the design of the control law, a time-varying term is proposed in executing system identification. Reference [2] proposed a supervising controller to enhance the robustness of an adaptive controller. However, the control scheme lacks robustness. Also, the plant is identified in a closed-loop manner; there is no assurance of initial behaviour. The PID gains and the parameters required for the supervisory controller can be identified offline in advance, giving the opportunity to ensure initial tracking performance. The model used to identify the plants is in a fuzzily blended and time-varying canonical form [3]. Parameters of the model are identified online by the simplex algorithm. The algorithm shortage can be alleviated by initialising the parameters with accurate results from offline identification, obtained specifically by the parallel genetic algorithms (PGA) proposed in [4].

Preliminaries: The plants under investigation are disturbed unknown SISO nonlinear dither systems, which can be described in canonical form:

$$F : \dot{x}_n = f(x, D(t, \omega)) + \Delta f(x, t) + g(x)u(t) + w(t)$$

where $y = x_1$, $\dot{x}_{n-1} = x_n$, $n = 1,2,\ldots$, and the frequency of dither, $D(t,\omega)$ is ω. If we denote the period of dither as T, and divide each period, $[t,\ t+T]$, into N subintervals, the duration of the mth subinterval can be written as $\alpha_m T$, where the duration parameter $0 \le \alpha_m \le 1$, $\Sigma_{m=1}^{M}\ \alpha_m = 1$, and the magnitude of dither in the subinterval is denoted as β_m. $x = [x_1\ x_2 \ldots x_n]^T$ denotes the state vector, $g(x)$ and $f(x,\ D(t,\ \omega))$ are bounded in x, and $w(t)$ is an external disturbance. $\Delta f\ (x,\ t)$ denotes the unknown system dynamics, and is bounded such that $|\Delta f(x,\ t)| \le \Delta f_U \|x\| + C_0$ with Δf_U and C_0 being positive real constants.

The task is to design a robust adaptive controller that can drive the system output, y, to follow the reference output, y_d, of a reference model: $\dot{y}_d = A_r\ y_d + r(t)$, where r is the reference input. A fuzzily blended time-varying canonical model is used to represent the nonlinear dither plant, with each rule represented by:

Plant rule i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p\ (t)$ is M_{ip}

$$\text{THEN } \dot{x}_n = a_i x + \delta(t) + b_i u + v_i,\ i = 1, 2, \ldots, L.$$

In each rule, $z_1\ (t), \ldots, z_p(t)$ are the p premise variables, M_{ij} is the fuzzy set corresponding to the jth premise variable, a_i is the ith system parameter, $\delta(t)$ is a time-varying term, and b_i denotes the ith control input parameter. Note that v_i is the ith bias, and L is the number of the IF–THEN rules. The nonlinear and time-varying dither plant is then represented as a fuzzy blending of the rules:

$$\begin{aligned}\dot{x}_n &= \sum_{i=1}^{L} h_i \left\{ [a_i\ 1] \begin{bmatrix} x \\ \delta(t) \end{bmatrix} + b_i u + v_i \right\} \\ &= \sum_{i=1}^{L} h_i [a_i x + b_i u + v_i] + \delta(t)\end{aligned}$$

where h_i is the normalised firing strength of the ith rule. In practice, h_i, a_i, b_i, v_i, and parameters in $\delta(t)$, are to be initialised by an evolutionary global optimisation technique, the PGA [4] using offline and batch identification.

Supervisory controller design: First, an ideal feedback control law, u^*, is defined as

$$\begin{aligned}u^* = \left(\sum_{i=1}^{L} h_i b_i\right)^{-1} \Bigg[&- \sum_{i=1}^{L} h_i a_i x - \delta(t) \\ &- \sum_{i=1}^{L} h_i v_i - e_{\text{mod}} + y_d^{(n)} + K^T E \Bigg]\end{aligned} \tag{1}$$

where

$$\begin{aligned} e_{\text{mod}}(t) &= f(x) + \Delta f(x,t) + g(x)u + w \\ &\quad - \sum_{i=1}^{L} h_i \times \{a_i x + \delta(t) + b_i u + v_i\},\ |e_{\text{mod}}| \\ &\le e_U,\ e = (y_d - y) \end{aligned} \tag{2}$$

$E = [e\ \dot{e} \ldots e^{(n-1)}]^T$ and the vector, $K = [k_0 \quad k_1 \ldots k_{n-1}]^T$ is designed such that all roots of $s^n + k_{n-1}\ s^{n-1} + \ldots + k_0 = 0$ are in the open left-half complex plane. Next, let the control input be given by

$$u = u_{PID} + u_S \tag{3}$$

where $u_{PID}(t) = K_P e(t) + K_I \int_0^f e(t)dt + K_D\ (de(t))/(dt)\ \equiv\ \theta^T \xi(e)$ with $\theta = [K_P,\ K_I,\ K_D]^T$, and $\xi(e) = [e(t),\ \int_0^f e(t)dt,\ (de(t))/(dt)]^T \cdot u_S$ is the output of a supervisory controller. Let

$$A_c = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ & -K^T \end{bmatrix},\ B_c = [0 \quad 0 \ldots \sum_{i=1}^{L} h_i b_i]^T \tag{4}$$

which is in companion form. We have, from (1), (3) and (4), $\dot{E} = A_c E + B_c\ (u^* - u_{\text{PID}}\ - u_S)$. Next, let us define the Lyapunov function candidate, $V_E = 2^{-1}\ (E^T PE)$, where $P = P^T > 0$, which satisfies the Lyapunov equation

$$A_c^T P + PA_c = -Q \tag{5}$$

with $Q = Q^T > 0$ also. We define

$$V_M = 2^{-1} \lambda_{\min}(P)(M_x - ||Y_d(t)||_\infty)^2 \tag{6}$$

where $Y_d = [y_d \dot{y}_d \ldots y_d^{(n-1)}]^T$, $M_x > 0$. From (5), we have

$$\dot{V}_E \le 2^{-1}[-E^T QE + 2|E^T PB_c|(|u^*| + |u_{PID}|) - 2|E^T PB_c|u_S] \tag{7}$$

u_S is then designed, inferring (2) and (7), as

$$\begin{aligned} u_S = I^* \text{sgn}(E^T PB_c) \Bigg\{ & \left|u_{PID}\right| + \left| \left(\sum_{i=1}^{L} h_i b_i\right)^{-1} \right| \\ & \times \left[\left| \sum_{i=1}^{L} h_i (a_i x + v_i) + \delta(t) + y_d^{(n)} + K^T E \right| + e_U \right] \Bigg\}, \end{aligned} \tag{8}$$

where the indicator function, I^*, is defined as: $I^* = 1$ if $V_E \ge V_M$ or 0 if $V_E < V$. From (2), (7) and (8), it is guaranteed that $\dot{V}_E < 0$ if $V_E \ge V_M$.

Adaptation law for PID controller: The derivation of the adaptive control law begins with assuming the existence of an optimal parameter vector, θ^*, for the (ideal) PID control law: $u_{PID}{}^* = \theta^{*T}\ \xi(e)$, such that the absolute of the approximation error, $\delta u = u_{PID}{}^*\ - u^*$, is minimised. Next, let us consider a Lyapunov function candidate, V_θ, which contains the quantities of tracking error and the deviation between θ^* and θ:

$$V_\theta = 2V_E + \gamma^{-1}(\theta^* - \theta)^T(\theta^* - \theta) = E^T PE + \gamma^{-1}(\theta^* - \theta)^T(\theta^* - \theta) \tag{9}$$

where $\gamma > 0$ is the adaptation rate, which determines the convergence speed. From (9), we have

$$\dot{V}_\theta \le -E^T QE - 2\gamma^{-1}(\theta^* - \theta)^T(\dot{\theta} - \gamma\ E^T PB_c \xi) - 2E^T PB_c \delta u$$

Hence, the adaptation law is defined as:

$$\begin{cases} \dot{\theta} = \gamma E^T PB_c \xi, & \text{if } (||\theta|| < M_\theta) \text{ or } (||\theta|| = M_\theta \text{ and } \dot{\theta}^T \theta \le 0) \\ \dot{\theta} = \gamma E^T PB_c \xi - \gamma E^T PB_c (\theta / M_\theta^2) \theta^T \xi, & \text{otherwise} \end{cases} \tag{10}$$

where $M_\theta > 0$. We have $\dot{V}_\theta \le -E^T QE - 2\,E^T PB_c\,\delta\,u$. Furthermore, assuming $|E^T PB_c| \le \tilde{e}$, we have $\dot{V}_\theta \le -\lambda_{\min}(Q)\|E\|^2 + 2\delta\tilde{u}\tilde{e}$. This guarantees the UUB stable that $\dot{V}_\theta < 0$ if $\|E\| > \sqrt{(2\delta\tilde{u}\tilde{e})} / \sqrt{(\lambda_{\min}(Q))}$.

Simulation study: The plant is an uncertain Duffing-Holmes dither system as follows: $\dot{x}_2(t) = x_1(t) - 0.25x_2(t) - (x_1(t) + D(t,\omega))^3 + \Delta f(t) + w(t) + u(t)$, where $y = x_1$ is state 1, $\dot{x}_1(t) = x_2(t)$ is state 2, $w \in [-1.2, 1.2]$, $\Delta f(x,t) = 0.3\cos(t) + 0.2\,\zeta(t)\|x\|$ is an unpredictable stochastic term with $\zeta \in [-1, 1]$ being a random number such that $\Delta f(x,t) \le 0.3 + 0.2\|x\|$. $A_r = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$ and $r(t) = [0 \quad 2\cos(t)]^T$ generate the reference signals y_d and $\dot{y}_d$ shown in Fig. 1.

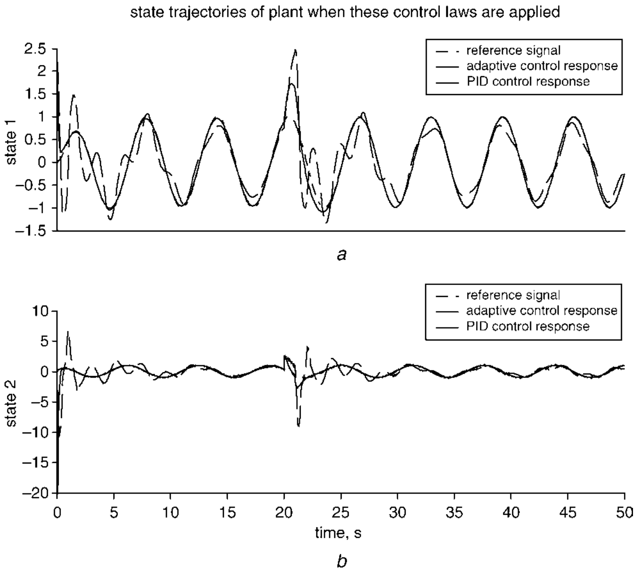

Fig. 1 *Plant disturbed by w = 100 at t = 20 s*

The dither system is first identified as a fuzzy blending of the following two rules.

Rule 1: IF x_1 is A_1, THEN $\dot{x}_2 = a_1x + u + \delta + v_1$

Rule 2: IF x_1 is A_2, THEN $\dot{x}_2 = a_2x + u + \delta + v_2$

where $a_1 = [-5.1222 \quad -0.2669]$, $a_2 = [-0.1465 \quad -0.5792]$, $v_1 = -1.5930$, $v_2 = 0.9725$, and $\delta = 0.301/\cos(t)$. The identification is achieved by PGA when the plant is excited by dither $D(t, \omega)$ with $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = -1$, $\beta_2 = 1$ and $\omega = 100$ Hz (pulse signal). Next, we choose $M_x = 2$, $M_\theta = 900$, and $e_U = 0.2$. Then, we choose $K = [1 \quad 20]^T$, $Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and solve for the solution $P = \begin{bmatrix} 20.1 & 1 \\ 1 & 0.1 \end{bmatrix}$ to construct the u_s according to (8). The initial PID control gains are selected as $K_P = 5$, $K_I = 0$, and $K_D = 0$. The simulation is based on initial states $x_1(0) = 2.2$ and $x_2(0) = 2.2$. From Fig. 1, it is clear that the tracking performance of the supervised adaptive controller significantly outperforms that of a constant gain PID controller in facing the disturbance.

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