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Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins*

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Simple robust estimation techniques provide new methods for automatic tuning of PID regulators which easily can be incorporated in single loop controllers.

Key Words—Adaptive control; control nonlinearities; describing function; identification; limit cycles; Nyquist criterion; PID control; relay control.

Abstract—The paper describes procedures for automatic tuning of regulators of the PID type to specifications on phase and amplitude margins. The key idea is a simple method for estimating the critical gain and the critical frequency. The procedure will automatically generate the appropriate test signal. The method is not sensitive to modelling errors and disturbances. It may be used for automatic tuning of simple regulators as well as initialization of more complicated adaptive regulators.

1. INTRODUCTION

THE MAJORITY of the regulators used in industry are of the PID type. A large industrial plant may have hundreds of regulators. Many instrument engineers and plant personnel are used to select, install and operate such regulators. Several different methods have been proposed for tuning PID regulators. The Ziegler–Nichols (1943) method is one of the more popular schemes. In spite of this, it is common experience that many regulators are in practice poorly tuned. One reason is that simple robust methods for tuning the regulators have not been available. This paper addresses the problem of finding automatic tuning methods. The methods proposed are simple to implement using microprocessors. They offer the possibilities to provide automatic tuning tools for a large class of common control problems.

The methods are based on a simple identification method which gives critical points on the Nyquist curve of the open loop transfer function. The key idea is a scheme which provides automatic excitation of the process which is nearly optimal for estimating the desired process characteristics.

The methods proposed are primarily intended to tune simple regulators of the PID type. In such applications they will of course inherit the limitations of the PID algorithms. They will not work well for problems where more complicated regulators are required. The technique may, however, also be applied to more complicated regulators and the experiences obtained so far from experimentation, in laboratory and industry, indicate that the simple versions of the algorithms work very well and in addition that they are robust.

The proposed algorithms may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide a solution to the long-standing problem of safe initialization of more complicated adaptive or self-tuning schemes. When combined with a bandwidth self-tuner it is, for example, possible to obtain an adaptive regulator which may set a suitable closed loop bandwidth automatically.

There are other alternatives for tuning regulators automatically. Self-tuning regulators based on minimum variance, pole placement or LQG design methods may be configured to give PID control. Such approaches have, e.g., been considered by Wittenmark and Åström (1980) and Gawthrop (1982). These regulators have the disadvantage that some information about the time scale of the process must be provided *a priori* to give a reasonable estimate of the sampling period in the regulator. There are some possibilities to tune the sampling period automatically. Different schemes have been proposed by Kurz (1979) and Åström and Zhaoying (1981). These methods will, however, only work for moderate changes in the process time constants. The method proposed in this paper does not suffer from this disadvantage. It may be applied to processes having widely different time scales.

Conventional self-tuning regulators based on

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recursive estimation of a parametric model requires a computer code of a few kilobytes. The algorithms proposed in this paper which are based on determination of zero-crossings and peak detection may be programmed in a few hundred bytes.

The paper is organized as follows: the estimation method is described in Section 2 and analysed in Section 3. Simple algorithms for automatic tuning to amplitude margin and phase margin specifications are given in Sections 4 and 5. Results from laboratory and industrial experiments with the algorithms are presented in Section 6. In Section 7, the use of the new algorithms to initialize conventional adaptive controllers is discussed.

2. THE BASIC IDEA

The Ziegler–Nichols rule for tuning PID regulators is based on the observation that the regulator parameters can be determined from knowledge of one point on the Nyquist curve of the open loop system. This point is the intersection of the Nyquist curve with the negative real axis, which is traditionally described in terms of the critical gain, k_c, and the critical period, t_c.

In the original Ziegler–Nichols scheme, described in Ziegler and Nichols (1943), the critical gain and the critical period are determined in the following way: a proportional regulator is connected to the system; the gain is gradually increased until an oscillation is obtained; the gain when this occurs is the critical gain and the period of the oscillation is the critical period. It is difficult to automatize this experiment, and perform it in such a way that the amplitude of the oscillation is kept under control. Another method for automatic determination of specific points on the Nyquist curve is therefore proposed.

The method is based on the observation that a system with a phase lag of at least π at high frequencies may oscillate with period t_c under relay control. To determine the critical gain and the critical period, the system is connected in a feedback loop with a relay as is shown in Fig. 1. The errror e is then a periodic signal with the period t_c. If d is the relay amplitude, it follows from a Fourier series

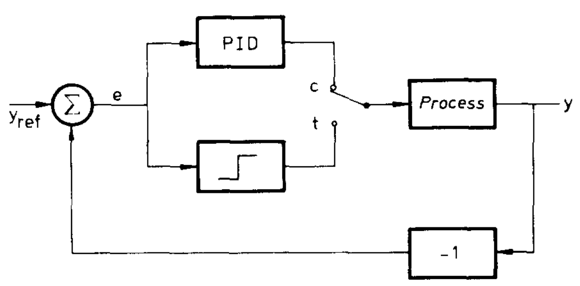

FIG. 1. Block diagram of the auto-tuner. The system operates as a relay controller in the tuning mode (t) and as an ordinary PID regulator in the control mode (c).

expansion that the first harmonic of the relay output has the amplitude $4d/\pi$. If the process output is a, the critical gain is thus approximately given by

$$k_c = \frac{4d}{\pi a}. \tag{1}$$

This result also follows from the describing function approximation. Notice that the describing function $N(a)$ for an ideal relay is given by

$$N(a) = \frac{4d}{\pi a}. \tag{2}$$

It may be advantageous to use other non-linearities than the pure relay. A relay with hysteresis gives a system which is less sensitive to measurement noise. This case is discussed in more detail below.

A simple relay control experiment thus gives the information about the process which is needed in order to apply the design methods. This method has the advantage that it is easy to control the amplitude of the limit cycle by an appropriate choice of the relay amplitude. Notice also that the estimation method will automatically generate an input signal to the process which has a significant frequency content at $\omega_c = 2\pi/t_c$. This ensures that the critical point can be determined accurately.

When the critical point on the Nyquist curve is known, it is straightforward to apply the classical Ziegler–Nichols tuning rules. It is also possible to devise many other design schemes that are based on the knowledge of one point on the Nyquist curve. Algorithms for automatic tuning of simple regulators based on the amplitude and phase margin criteria will be given in Sections 4 and 5.

It is possible to modify the procedure to determine other points on the Nyquist curve. An integrator may be connected in the loop after the relay to obtain the point where the Nyquist curve intersects the negative imaginary axis. Other points on the Nyquist curve can be determined by repeating the procedure with linear systems introduced into the loop. New design methods which are based on such data are described in Åström and Hägglund (1984b).

Determination of amplitude and period

Methods for automatic determination of the frequency and the amplitude of the oscillation will be given to complete the description of the estimation method. The period of an oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values of the output. These estimation methods are easy to implement because they are based on counting and comparisons only. Since the describing function analysis is based on the first harmonic of the oscillation, the simple estimation techniques require that the first harmonic dominates. If this is not the case, it may be necessary to filter the signal before measuring.

More elaborate estimation schemes like least squares estimation and extended Kalman filtering may also be used to determine the amplitude and the frequency of the limit cycle oscillation. Simulations and experiments on industrial processes have indicated that little is gained in practice by using more sophisticated methods for determining the amplitude and the period.

3. ANALYSIS

The reasoning in Section 2 is purely heuristic. Analysis is needed to understand when the method works and when it does not work. Natural questions are: When will there be limit cycle oscillations? When are those oscillations stable? How accurate is the describing function approximation? What happens if the Nyquist curve intersects the negative real axis at several points? Partial answers to these questions are given below.

Exact expressions for the period of oscillation were originally derived by Hamel and Tsypkin. An exposition of the results are also given in the textbooks by Tsypkin (1958), Gille, Pelegrin and Decaulne (1959), Gelb and Vander Belde (1968), and Atherton (1975). Conditions for oscillations are given below.

Theorem 1. Consider the closed loop system obtained by a feedback connection of a linear system having the transfer function $G(s)$ with a relay having hysteresis. Let $H(\tau, z)$ be the pulse transfer function of the series combination of a sample and hold with period τ and $G(s)$. If there is a periodic oscillation, then the period T is given by

$$H(T/2, -1) = -\frac{\varepsilon}{d} \tag{3}$$

where ε is the hysteresis width of the relay and d is the relay output. □

The characteristics of the relay with hysteresis are shown in Fig. 2. The result of Theorem 1 is easily understood by assuming that there exists a periodic oscillation with period T. Sampling the system with period $T/2$ at sampling instants which are synchronized to the relay switches then gives the sampled input and output signals

$$y(kT/2) = \begin{cases} \varepsilon & k \text{ even} \\ -\varepsilon & k \text{ odd} \end{cases}$$

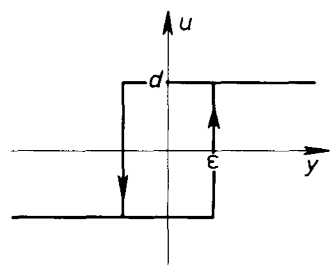

FIG. 2. Characteristics of a relay with hysteresis.

$$u(kT/2) = \begin{cases} -d & k \text{ even} \\ d & k \text{ odd} \end{cases}.$$

The steady state transmission of the sequence $\{u(kT/2)\}$ through the sampled system is characterized by the gain $H(T/2, -1)$. The condition (3) is thus obtained by tracing the propagation of square wave signals around the closed loop.

A formal proof of Theorem 1 is found in Atherton (1982) and in Åström and Hägglund (1984a), which also gives conditions for the stability of the limit cycle. The latter paper also covers the more general case of asymmetric oscillations.

It follows from well-known series expansions of the pulse transfer function that

$$H(\tau, -1) = \sum_{n=0}^{\infty} \frac{4}{\pi[1+2n]} Im\, G\left(\frac{\pi + 2n\pi}{\tau} i\right). \tag{4}$$

The describing function approximation (2) is obtained simply by using the first term in this series expansion. The validity of the describing function approximation (2) can thus be evaluated from this formula. In many cases it gives the period of the oscillation with an error of a few per cent, which is accurate enough for the intended purpose. It is easily shown that the describing function approximation gives the exact period for an integrator with time delay. Another example illustrates the precision that is typically obtained.

Example 1. Consider the linear system

$$G(s) = \frac{1}{s(s+1)(s+a)}.$$

From the describing function approximation, the period of oscillation is

$$T = \frac{2\pi}{\sqrt{s}} \approx 6.3\frac{1}{\sqrt{a}}.$$

The value of the pulse transfer function for $z = -1$ becomes

$$H(\tau, -1) = -\frac{h}{2a} + \frac{1}{a-1}\left[\frac{1-e^{-\tau}}{1+e^{-\tau}} - \frac{1}{a^2}\frac{1-e^{-a\tau}}{1+e^{-a\tau}}\right].$$

For large values of a, the period of oscillation is approximately given by

$$T \approx \frac{4\sqrt{3}}{\sqrt{a}} \approx 6.9 \frac{1}{\sqrt{a}}. \qquad \square$$

The describing function approximation may, however, give misleading results as is seen by the following example.

Example 2. Consider a linear system with the transfer function

$$G(s) = \frac{b}{s+a} e^{-st_0} \quad a, b, t_0 > 0.$$

Since the Nyquist curve intersects the negative real axis at many points, the describing function analysis predicts several possible limit cycles. The value of the pulse transfer function for $z = -1$ is

$$H(\tau, -1) = \frac{b}{a} \cdot \frac{e^{-a\tau}(2e^{at_0} - 1) - 1}{1 + e^{-a\tau}}.$$

The period of the oscillation is given by

$$T = 2\tau = -\frac{2}{a} \ln \left| \frac{bd - a\varepsilon}{bd(2e^{at_0} - 1) + a\varepsilon} \right|. \tag{5}$$

It is shown in Åström and Hägglund (1984a) that the limit cycle is stable. $\square$

The transfer function $G(s)$ in Example 2 becomes strictly positive real if the time delay goes to zero. The describing function approximation then predicts that there will not be any oscillation. The system will however exhibit a stable periodic solution under relay control. The period is obtained by letting t_0 in equation (5) go to zero.

Stable periodic solutions will not be obtained for all systems. A double integrator under pure relay control will give, for example, periodic solutions with an arbitrary period.

It would be highly desirable to give a complete characterization of the systems for which there will be a unique stable limit cycle. Theorem 1 and the stability conditions in Åström and Hägglund (1984a) give some guidance, but the general conditions are still unknown. Consider, for example, stable systems. It follows from sampled data theory that

$$\lim_{\tau \to \infty} H(\tau, -1) = -G(0) = -K$$

where $G(0)$ is the DC gain of the process. Furthermore, if $G(s)$ goes to zero as $s \to \infty$ it also follows that $H(0, -1) = 0$. Provided that εK is larger than d, equation (3) thus always has at least one solution. There may of course be several solutions. It follows from Theorem 3 of Åström and Hägglund (1984a) and Theorem 2 of Åström *et al.* (1984) that the periodic solution is stable at least if ε is sufficiently large. It is conjectured that there is a unique stable limit cycle for stable systems.

4. AMPLITUDE MARGIN AUTO-TUNERS

When the critical point is known, it is straightforward to find a regulator which gives a desired amplitude margin. A simple way is to choose a proportional regulator with the gain

$$k = k_c / A_m \tag{6}$$

where A_m is the desired amplitude margin and k_c is the critical gain.

Sometimes this solution is not satisfactory because integral action may be required. Since the frequency response of a PID regulator can be written as

$$G_R(i\omega) = k \left(1 + \frac{1}{i\omega T_i} (1 - \omega^2 T_i T_d) \right) \tag{7}$$

it follows that any PID regulator with the gain given by (6) and

$$T_d = \frac{1}{\omega_c^2 T_i} \tag{8}$$

where $\omega_c = 2\pi / t_c$ also gives the desired amplitude margin. The integration time can then be chosen arbitrarily, and the derivative time is given by equation (8).

5. PHASE MARGIN AUTO-TUNERS

Consider a situation when one point on the Nyquist curve for the open loop system is known. With PI, PD or PID control it is possible to move the given point on the Nyquist curve to an arbitrary position in the complex plane, as is indicated in Fig. 3. The point A may be moved in the direction of

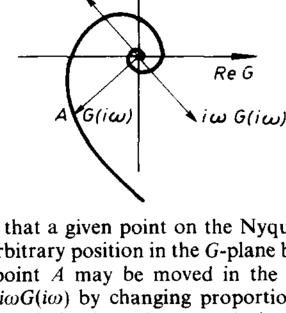

FIG. 3. Shows that a given point on the Nyquist curve may be moved to an arbitrary position in the G-plane by PI, PD or PID control. The point A may be moved in the directions $G(i\omega)$, $G(i\omega)/i\omega$ and $i\omega G(i\omega)$ by changing proportional, integral and derivative gain respectively.

$G(i\omega)$ by changing the gain and in the orthogonal direction by changing the integral or the derivative gain. It is thus possible to move a specified point on the Nyquist curve to an arbitrary position. This idea can be used to obtain design methods. Systems with a prescribed phase-margin are obtained by, e.g. moving A to a point on the unit circle. An example is given below.

Example 3. Consider a process with the transfer function $G(s)$. The loop transfer function with PID control is

$$k\left(1 + sT_d + \frac{1}{sT_i}\right)G(s).$$

Assume that the point where the Nyquist curve of G intersects the negative real axis is known. Let this point correspond to $\omega = \omega_c$. The following condition is obtained from the condition that the argument of the loop transfer function at ω_c is $\phi_m - \pi$.

$$\omega_c T_d - \frac{1}{\omega_c T_i} = \tan\phi_m. \tag{9}$$

There are many T_d and T_i which satisfy this condition. One possibility is to choose T_i and T_d so that

$$T_i = \alpha T_d. \tag{10}$$

Equation (9) then gives a second order equation for T_d which has the solution

$$T_d = \frac{\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}}{2\omega_c}. \tag{11}$$

Simple calculations show that the loop transfer function has unit gain at ω_c if the regulator gain is chosen as

$$k = \frac{\cos\phi_m}{|G(i\omega_c)|} = k_c \cos\phi_m \tag{12}$$

where k_c is the critical gain. The design rules are thus given by the equations (9)–(12).

There are many other possibilities, e.g. the parameter T_i may be chosen so that $\omega_c T_i$ has a given value. □

A point on the Nyquist curve which is different from the critical point is obtained when the relay has hysteresis. The design method in Example 3 can be extended to cover this case too. The negative reciprocal of the describing function of a relay with hysteresis is

$$-\frac{1}{N(a)} = -\frac{\pi}{4d}\sqrt{a^2 - \varepsilon^2} - i\frac{\pi\varepsilon}{4d} \tag{13}$$

where d is the relay amplitude and ε is the hysteresis width. This function may be described as a straight line parallel to the real axis, in the complex plane. See Fig. 4. By choosing the relation between ε and d it is therefore possible to determine a point on the Nyquist curve with a specified imaginary part. In the next example, this property is used to obtain a regulator which gives a desired phase margin of a system.

Example 4. Consider a process with transfer function $G(s)$, controlled by a proportional regulator. The loop transfer function is thus $kG(s)$. Assume that the design goal is to obtain a closed loop system with the phase margin ϕ_m. Choose the relay characteristics so that the negative reciprocal of the describing function goes through the point P defined in Fig. 4. The parameters are then

$$d = \frac{\pi a^*}{4} \qquad \varepsilon = a^* \sin(\phi_m)$$

where a^* is the desired amplitude of the oscillations. The desired phase margin is obtained if the Nyquist curve goes through the point P in Fig. 4. Since the intersection between $-1/N(a)$ and $kG(i\omega)$ can be determined from the amplitude of the oscillation, this point can be reached, e.g. by iteratively changing the gain k. The formula

$$k_{n+1} = k_n - (a_n - a^*)\frac{k_n - k_{n-1}}{a_n - a_{n-1}} \tag{14}$$

has a quadratic convergence rate near the solution. Integral and derivative action can be included, using the methods proposed in Example 3.

There are many possible variations of the given design methods for PID regulators. All methods are closely related because they are based on information about the process dynamics in terms of one point on the Nyquist curve. The points where the Nyquist curve intersects the real axis or straight lines parallel to the real axes are simple choices. The

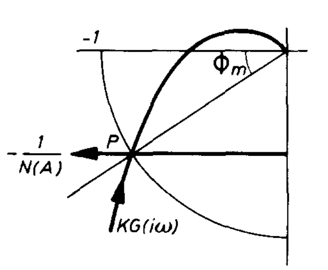

FIG. 4. The negative reciprocal of the describing function $N(a)$ and the Nyquist curve of $G(s)$.

design methods may be modified. Other relations between T_i and T_d than those given by (10) may be used. Other criteria like damping or bandwidth may be chosen instead of the phase or amplitude margins. It is also possible to have design methods which are based on knowledge of more points on the Nyquist curve. See Åström and Hägglund (1984b).

6. EXPERIMENTS

A large number of simulations and experiments on laboratory processes and industrial plants have been performed in order to find out if a useful auto-tuner can be designed based on the ideas described in the previous sections. The results of the experiments are briefly summarized in this section.

Practical aspects

There are several practical problems which must be solved in order to implement an auto-tuner. It is, for example, necessary to account for measurement noise, level adjustment, saturation of actuators and automatic adjustment of the amplitude of the oscillation.

Measurement noise may give errors in detection of peaks and zero-crossings. A hysteresis in the relay is a simple way to reduce the influence of measurement noise. Filtering is another possibility. The estimation schemes based on least squares and extended Kalman filtering can be made less sensitive to noise. Simple detection of peaks and zero-crossings in combination with an hysteresis in the relay has worked very well in practice. See, e.g. Åström (1982).

The process output may be far from the desired equilibrium condition when the regulator is switched on. In such cases it would be desirable to have the system reach its equilibrium automatically. For a process with finite low-frequency gain there is no guarantee that the desired steady state will be achieved with relay control unless the relay amplitude is sufficiently large. To guarantee that the output actually reaches the reference value, it may be necessary to introduce manual or automatic reset.

It is also desirable to adjust the relay amplitude automatically. A reasonable approach is to require that the oscillation is a given percentage of the admissible swing in the output signal.

Different estimation schemes have been explored by simulations covering wide ranges of process dynamics and different types of auto-tuners. The effects of measurement noise and load disturbances have been investigated. The experiments indicate that the simple estimation method based on zero-crossing and peak detection works very well. The experiments also indicate that simple minded level adjustment methods often are satisfactory.

Implementations

The auto-tuners have been implemented on several different computers. A DEC LSI 11/03 was used in some early experiments. The algorithms were coded in Pascal using a real time kernel. Small laboratory processes were controlled. The experiments showed that the simple algorithms were robust and that they worked well. The algorithms were also coded in Basic using the Apple II computer. This implementation was very easy to use because of the graphics and the interactive user interface. Experiments have also been performed using the IBM PC and dedicated micro processors.

Experiment on a laboratory process

An experiment made with the Apple II implementation will now be presented. Figure 5 shows the result when the auto-tuner was applied to level control in a tank with a pump and a free outlet. The pump was controlled from measurements of the water level. The tuning procedure can be divided into two phases. The first phase is an initial phase which brings the process to equilibrium, i.e. to the desired reference level. The second phase is the final tuning phase. The two phases are described in more detail below.

Phase 1. When the process dynamics is totally unknown, the relay feedback is used with a setpoint half way between the current and the desired setpoint. A crude estimate of critical gain and critical period is made based on one period of oscillation. This is done in the first phase. Based on this rough characterization of the process, a conservative PI controller is designed which ramps the system to the equilibrium with a slope determined from the estimated time constant. This first phase can be omitted if the process is manually moved to the equilibrium.

Phase 2. When the desired level is reached, the estimation procedure starts. A relay with a small

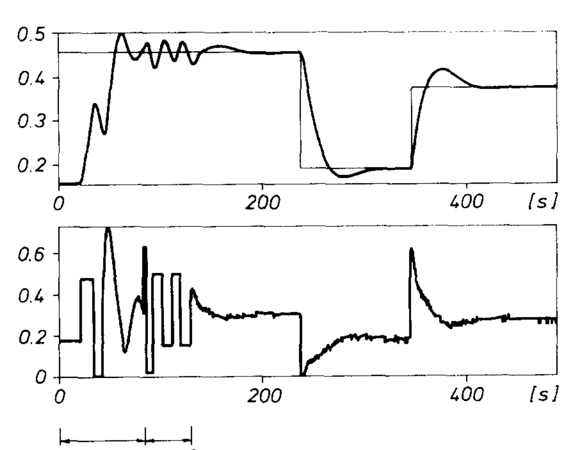

FIG. 5. Experiments made on the tank process.

hysteresis is introduced in the loop as shown in Fig. 1. The relay amplitude is adjusted automatically so that an oscillation with desired amplitude is obtained. The amplitude and the frequency of the oscillation are estimated by peak detection and determination of the times between zero-crossings of the control error.

The design method was based on a combination of phase and amplitude margin specification. It was required that the Nyquist curve intersects the circle with radius 0.5 at an angle of 225°. Two step responses are shown in Fig. 5. The lack of symmetry depends on the nonlinearity of the pump. The high frequency disturbance in the control signal is caused by round-off errors in the AD-converter, eight bits only.

7. INITIALIZATION OF ADAPTIVE CONTROLLERS

The new estimation procedure presented in this paper has been used to derive a technique to tune simple regulators automatically. Initialization of conventional adaptive controllers is another important application. Adaptive and self-tuning controllers based on parameter estimation require prior knowledge of the magnitude of the time delay of the process. This is needed to select the sampling period.

An upper bound of the time delay is given by $t_c/2$, which follows from Example 2 for first order systems. It is also true for systems having monotone step responses. A suitable sampling period for a self-tuner can thus easily be determined from the upper bound of the time delay. Having obtained a suitable sampling period, parameter estimation may also be applied to the signals obtained during the auto-tuning to give good initial parameter estimates for a self-tuner. By combining the auto-tuner with a self-tuner of the type discussed in Åström and Wittenmark (1973), it is possible to obtain an adaptive regulator which can work for processes having a wide range of time delays and time constants.

Another interesting system is obtained by combining an auto-tuner with the bandwidth self-tuner discussed in Åström (1983). A reasonable estimate of the desired bandwidth can be obtained from the critical period. It is then possible to design an adaptive regulator which by itself can determine a reasonable value of the closed loop bandwidth and then execute a control law which gives this. More elaborate combinations of control algorithms are suggested in Åström and Anton (1984).

8. CONCLUSIONS

Simple methods for tuning PID regulators have been proposed. The methods have been investigated theoretically and experimentally. The methods are robust and easy to use. In contrast to other methods based on self-tuning control they do not require *a priori* information about time scales. The methods will of course inherit the limitations of the PID algorithms. They will not work well in situations where more complicated regulators are required.

The algorithms may be used in many different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to initialize more sophisticated adaptive algorithms.

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