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An Extended off-line Least-Squares Method for Parameter Identification and Time Delay Estimation

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Abstract

In this article we propose an off-line extended least-squares method to identify the parameters of a system under closed-loop conditions that satisfy the excitation condition. We also present an off-line time delay estimation method as an application of the proposed method. We show that the performance of this batch extended least-squares approach is better than that of an ordinary least-squares method.

1 Introduction

In order to predict the future response of a dynamic process, or to manipulate its outputs to follow desired trajectories, a mathematical model which describes the dynamics of the underlying process is necessary. In some cases this model can be obtained by analyzing the internal mechanism or physics of the system. In other cases the internal mechanism is not well understood and an identification experiment should be carried out.

The model may be linear or non-linear, though in practice, linear models are more common since they are simple to analyze for identification, prediction or control purposes. Much work has been done in linear identification and control and the theory is well known and documented. The estimation of the system parameters or, equivalently of the process model, can be performed in open-loop by many methods. A detailed description of these techniques is given for example in Ljung [8], Söderström and Stoica [10]. Under certain assumptions these methods can also be used to estimate the parameters of a process operating in closed-loop. If the process is unstable, if the controller is adaptive, or simply if the process is highly non-linear, then the process model must be found by closed loop identification.

Furthermore, it is now known that if the purpose of modeling is to design a controller, then it is advantageous to perform the identification in closed loop.

For these reasons, closed-loop identification has become an important research area, and many methods dedicated to closed-loop identification are now available. Most existing techniques however, are capable of estimating parameters but have difficulty when it comes to identifying the time delay in closed loop.

The most methods of closed loop identification use a sufficiently exciting external signal added to the control signal or to the reference signal. In Van Den Hoff et al.[11] the sensivity function is estimated first, then the process transfer function is determined. A recursive method is proposed by Landau and Karimi[7] using the sensivity function and an external signal.

The use of an external signal to achieve identifiability under feedback conditions was proposed by Gustavsson et al.[6]. An alternative to using an external signal is to switch between different regulators as proposed by Söderström et al. [10].

In this article we propose an off-line extended least-squares method for identifing both the parameters and the time delay of a system under closed loop conditions that satisfy the excitation condition [2]. We show that the performance of this batch extended least-squares approach is better than that of an ordinary least-squares method.

The organization of this article is as follows. In section 2, an ARMA model is discussed for a regulatory closed loop system. In section 3, the proposed extended off-line least-squares method is presented. In section 4, the proposed method is used to determine the parameters and time delay of a system. The same method is used to determine a first-order approximation model for a given process, then in section 5 we draw conclusions.

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2 An ARMA model for a closed-loop system

In most cases, a regulatory closed loop system can be described by an ARMA model. Let us consider the plant model described by the following ARMAX model.

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \quad (1)$$

where $A(q^{-1}) = 1 + a_1q^{-1} + ... + a_{n_A}q^{-n_A}$, $B(q^{-1}) = b_1q^{-1} + ... + b_{n_B}q^{-n_B}$ and $C(q^{-1}) = 1 + c_1q^{-1} + ... + c_{n_C}q^{-n_C}$. $y(t)$, $u(t)$ and $e(t)$ are respectively the process output, the process input and the process disturbance which is assumed to be a sequence of independent and identically distributed random variables with zero mean and variance σ_e.

The process is assumed closed-loop stable under the feedback control law given by

$$u(t) = -\frac{R(q^{-1})}{S(q^{-1})}(y(t) - r(t)), \quad (2)$$

$r(t)$ is a constant reference.

Replacing $u(t)$ in equation (1) by the expression given by equation (2) yields

$$\begin{aligned} y(t) &= \frac{B(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})}r(t) \\ &\quad + \frac{C(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})}e(t). \end{aligned}$$

with

$$\bar{y}(t) = \frac{B(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})}r(t).$$

This equation can be rewritten as

$$\begin{aligned} \tilde{y}(t) &= \frac{C(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})}e(t) \quad (3) \\ &= \frac{\alpha(q^{-1})}{\beta(q^{-1})}e(t) \quad (4) \end{aligned}$$

where $\tilde{y}(t) = y(t) - \bar{y}(t)$. This equation shows that the closed loop system described by equations (1) and (2) can be represented by an autoregressive moving-average model (ARMA) defined by equation (4).

Suppose now that we want to estimate the process parameters in equation (1). This can only be achieved through the noise $e(t)$, when the signal reference $r(t)$ is constant. Therefore we propose to estimate the past noise $e(t)$ and use it as available information to estimate the parameters A, B and C. The estimate of the past noise is obtained once the parameters of the ARMA model of equation (4) are determined. In the ideal case we have

$$\begin{aligned} \alpha(q^{-1}) &= C(q^{-1})S(q^{-1}) \quad \text{and} \quad (5) \\ \beta(q^{-1}) &= A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1}). \quad (6) \end{aligned}$$

ARMA models are widely used in the field of time series analysis because they are the most general representation of a linear, stationary, stochastic process. Their parameters can be determined by a number of ways described in Box and Jenkins[3], Ljung[8] ...etc. In Åström and Söderström[4], the maximum likehood method is used to estimate the polynomials $\alpha(q^{-1})$ and $\beta(q^{-1})$ which are defined as follows: $\alpha(q^{-1}) = 1+\alpha_1q^{-1}+...+\alpha_{n_\alpha}q^{-n_\alpha}$ and $\beta(q^{-1}) = 1+\beta_1q^{-1}+...+\beta_{n_\beta}q^{-n_\beta}$. The cost function to be minimized is given by

$$V(\hat{\alpha}, \hat{\beta}) = \frac{1}{2N}\sum_{i=1}^{N}\varepsilon^2(t) \quad (7)$$

where the residual $\varepsilon(t)$ is a function of the observations $y(1)$, $y(2)$, ..., $y(t)$ defined by

$$\hat{\alpha}(q^{-1})\varepsilon(t) = \hat{\beta}(q^{-1})y(t) \quad (8)$$

where $\hat{\alpha}(q^{-1}) = 1+\hat{\alpha}_1q^{-1}+...+\hat{\alpha}_{n_{\hat{\alpha}}}q^{-n_{\hat{\alpha}}}$ and $\hat{\beta}(q^{-1}) = 1 + \hat{\beta}_1q^{-1} + ... + \hat{\beta}_{n_{\hat{\beta}}}q^{-n_{\hat{\beta}}}$. $\hat{\alpha}(q^{-1})$ and $\hat{\beta}(q^{-1})$ are assumed to be stable. Åström and Söderström [4] showed that $\hat{\alpha}$, $\hat{\beta}$ which minimize $V(\hat{\alpha}, \hat{\beta})$ satisfy

$$\frac{\hat{\alpha}(q^{-1})}{\hat{\beta}(q^{-1})} = \frac{\alpha(q^{-1})}{\beta(q^{-1})} \quad (9)$$

and

$$\sigma_\varepsilon^2 = E(\varepsilon^2(t)) \geq E(e^2(t)) = \sigma_e^2 \quad (10)$$

where $E(.)$ denotes the expectation function, given that

$$n_{\hat{\alpha}} \geq n_\alpha \text{ and } n_{\hat{\beta}} \geq n_\beta. \quad (11)$$

The equality $\sigma_\varepsilon^2 = \sigma_e^2$ is obtained if $n_{\hat{\alpha}} = n_\alpha$ and $n_{\hat{\beta}} = n_\beta$. It is also shown that the residual $\varepsilon(t)$ is a good estimate of $e(t)$ satisfying

$$E(\varepsilon(t)\varepsilon(t-k)) = 0 \text{ for } k \geq 1. \quad (12)$$

The estimation of $e(t)$ can be very accurate. In the following we illustrate this accuracy by mean of an example.

Example 2.1 *[9]* The process considered is described by

$$y(t) = \frac{0.2q^{-1}u(t)}{(1-0.8q^{-1})} + \frac{(1-0.5q^{-1}+0.6q^{-2})e(t)}{(1-0.8q^{-1})} \quad (13)$$

process disturbance $e(t)$ is a sequence of independent and identically distributed random variables with zero mean and variance $\sigma_e = 1$. The control law is given by

$$u(t) = \frac{0.6 - 0.48q^{-1}}{0.2 - 0.2q^{-1}}(1 - y(t)). \quad (14)$$

The estimated ARMA model for this process is given by $\frac{\hat{\alpha}(q^{-1})}{\hat{\beta}(q^{-1})}$ where $\hat{\alpha}(q^{-1}) = 1 - 1.3955q^{-1} + 0.3835q^{-2} + 0.4585q^{-3} - 0.7784q^{-4} + 0.4394q^{-5} - 0.0997q^{-6}$ and $\hat{\beta}(q^{-1}) = 1 - 1.1387q^{-1} - 0.3336q^{-2} + 0.8466q^{-3} + 0.2867q^{-4}$. The residual $\varepsilon(t)$ which is an estimate of $e(t)$ is obtained by

$$\varepsilon(t) = \frac{\hat{\beta}(q^{-1})}{\hat{\alpha}(q^{-1})}(y(t) - 1) \tag{15}$$

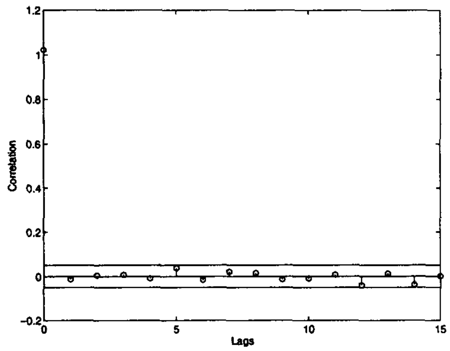

Figure 1: Autocorrelation of the residual.

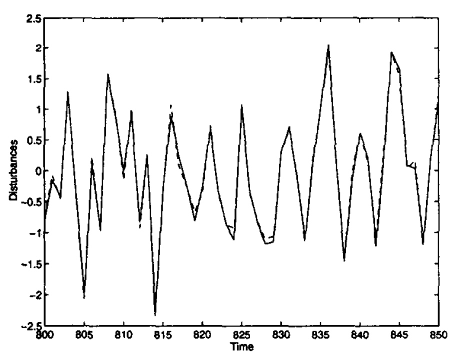

Figure 2: White noise and its estimate.

The variance obtained for $\varepsilon(t)$ is $\sigma_\varepsilon^2 = 1.0186$ which is very close to $\sigma_e^2 = 1$. Its autocorrelation is shown in figure (1). It can be seen that the condition of equation (12) is satisfied. Figure (2) shows the near perfect match between the residual (dashed line) and the white noise (solid line). Clearly the estimate $\varepsilon(t)$ of $e(t)$ can be very accurate, which confirms what we already know from the references cited above. For these reasons we propose in the following an off-line extended least-squares method to estimate the parameters of a process under closed loop. The proposed technique is a batch method, and is based upon the estimate $\varepsilon(t)$ of $e(t)$ as a first step, followed by estimates of the process parameters.

3 An off-line extended least-squares method

As mentioned above, we first estimate the past noise which is obtained by estimating the residual $\varepsilon(t)$, then use it as prior knowledge to determine the process parameters. We first consider a system described by equation (1) with $C(q^{-1}) = 1$. Equation (1) is then also equivalent to

$$y(t) - e(t) = \varphi(t)\theta \tag{16}$$

where

$$\varphi(t) = [-y(t-1)... - y(t-n_A)\ u(t-1)...u(t-n_B)] \tag{17}$$

$$\theta = [a_1\ ...\ a_{n_A}\ b_1\ ...\ b_{n_B}]^T. \tag{18}$$

Given the measurements $y(1)$, ..., $y(t)$ and the estimates $\varepsilon(1)$, ..., $\varepsilon(t)$ of $e(1)$, ..., $e(t)$ the extended least-squares estimate $\hat{\theta}$ of the vector θ is determined by minimizing the following loss function

$$V(\hat{\theta}) = \frac{1}{2}\sum_{i=1}^{N} \epsilon^2(t) \tag{19}$$

where $\epsilon(t)$ is the equation error defined by

$$\epsilon(t) = y(t) - e(t) - \varphi(t)\theta. \tag{20}$$

The estimate $\hat{\theta}$ is given by

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T Y \tag{21}$$

if $\phi^T\phi$ is invertible this is an excitation condition see Åström and Wittenmark[1], where

$$Y = \begin{bmatrix} y(1) - \varepsilon(1) \\ . \\ . \\ . \\ y(N) - \varepsilon(N) \end{bmatrix} \text{ and } \phi = \begin{bmatrix} \varphi(1) \\ . \\ . \\ . \\ \varphi(N) \end{bmatrix}. \tag{22}$$

Notice that $e(t)$ is replaced by its estimate $\varepsilon(t)$ in Y. Some statistical properties of $\hat{\theta}$ are given by the following result

Proposition 3.1 Consider the estimate $\hat{\theta}$ defined by equation (21) Assume that $e(t)$ is white noise and ε is an estimate of e which satisfies equation (12) with zero mean. Then the following properties hold:

1. $\hat{\theta}$ is the unique minimum point of $V(\hat{\theta})$.
2. $E(\hat{\theta}) = \theta$.
3. $cov(\hat{\theta}) = \sigma^2_{(e-\varepsilon)}(\phi^T\phi)^{-1}$.

Proof: To prove this result, we first write $V(\hat{\theta})$ as

$$V(\hat{\theta}) = \frac{1}{2}[Y - \phi\theta]^T[Y - \phi\theta] \qquad (23)$$

then setting the gradient of the loss function $V(\hat{\theta})$ to zero yields

$$0 = \frac{\partial V}{\partial \theta} = -Y^T\phi + \theta^T(\phi^T\phi), \qquad (24)$$

therefore $\hat{\theta}$ is obtained as in equation (21) which is also equivalent to

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T(\phi\theta + e - \varepsilon) \qquad (25)$$

where

$$e = \begin{bmatrix} e(1) \\ \cdot \\ \cdot \\ \cdot \\ e(N) \end{bmatrix} \text{ and } \varepsilon = \begin{bmatrix} \varepsilon(1) \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon(N) \end{bmatrix}. \qquad (26)$$

Therefore

$$\hat{\theta} = \theta + (\phi^T\phi)^{-1}\phi^T(e - \varepsilon), \qquad (27)$$

which implies that $E(\hat{\theta}) = \theta$, because $E(e - \varepsilon) = 0$. Equation (27) shows that if ε is a good estimate of e then $(\phi^T\phi)^{-1}\phi^T(e - \varepsilon) \to 0$ and $\hat{\theta} \to \theta$ which means that the estimate $\hat{\theta}$ is unbiased. Rearranging Equation (27) yields

$$\hat{\theta} - \theta = (\phi^T\phi)^{-1}\phi^T(e - \varepsilon) \qquad (28)$$

Then taking the expectation of both sides of the last equation yields

$$E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = \sigma^2_{e-\varepsilon}(\phi^T\phi)^{-1}. \qquad (29)$$

■

Unlike the ordinary least-squares method where $E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = \sigma_e^2(\phi^T\phi)^{-1}$, the last equation states that the variance of $\hat{\theta}$ can be reduced significantly if the ARMA model for the closed loop system is correctly estimated. In the ideal case $E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = 0$ if $\sigma_e = \sigma_\varepsilon$ which shows that this extended least-squares is better than the normal least-squares. This fact is even clear when we consider the general model i.e. $C(q^{-1}) = 1 + c_1q^{-1} + ... + c_{nC}q^{-nC}$. In this case the off-line extended least-squares is even more appropriate. Equation (1) is also equivalent to

$$y(t) - e(t) = \varphi(t)\theta$$

where $\varphi(t) = [-y(t-1) \; ... \; -y(t-n_A) \; u(t-1) \; ... \; u(t-n_B) \; \varepsilon(t-1) \; ... \; \varepsilon(t-n_c)]$ and $\theta = [a_1 \; ... \; a_{n_A} \; b_1 \; ... \; b_{n_B} \; c_1 \; ... \; c_{n_C}]^T$. Assuming the same conditions as before, the loss function to minimize is

$$V(\hat{\theta}) = \frac{1}{2}\sum_{i=1}^{N}\epsilon^2(t) \qquad (30)$$

where $\epsilon(t)$ is the equation error defined by

$$\epsilon(t) = y(t) - e(t) - \varphi(t)\theta. \qquad (31)$$

The estimate $\hat{\theta}$ is given by

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T Y \qquad (32)$$

if $\phi^T\phi$ is invertible. Y and ϕ are defined in equation (22).

The comparison between the ordinary least-squares, the ordinary recursive extended least-squares and the off-line extended least-squares is illustrated in the following simulation example where we consider the identification of parameters of the process described in example (2.1). The process model determined by the extended off-line least-squares method is $y_{els}(t) = \frac{0.2103q^{-1}u(t)}{(1-0.7610q^{-1})} + \frac{(1-0.3958q^{-1}+0.5868q^{-2})e(t)}{(1-0.7610q^{-1})}$. It can be seen that this model is very close to the process transfer function described in equation (13). Figure (3) shows the comparison between the process output (solid line) and that of the model y_{els} (dashed line).

The process model determined by the ordinary recursive extended least-squares method is $y_{rels}(t) = \frac{0.1749q^{-1}u(t)}{(1-0.6519q^{-1})} + \frac{(1-0.4441q^{-1}+0.5333q^{-2})e(t)}{(1-0.6519q^{-1})}$. This model is also close to the transfer function described in equation (13) though less accurate than the previous one.

The process model determined by the ordinary least-squares method is $(1 - 0.5946q^{-1})y_{ls}(t) = 0.2781q^{-1}u(t)$ It can be seen that this model is a poor estimate of the process transfer function described in equation (13). This is confirmed by Figure (4) where solid line is the process output and the dashed line is the model y_{ls}.

Remark 1. In a recursive least-squares or an ordinary extended least-squares the residual is whitened over time and toward the end of the procedure of identification. However the proposed method begins by determining an estimate ε of the process white noise therefore eliminating one major difficulty of the identification procedure which is to whiten the residual.

In the following section, we use the method introduced here to estimate the parameters of a process with time delay.

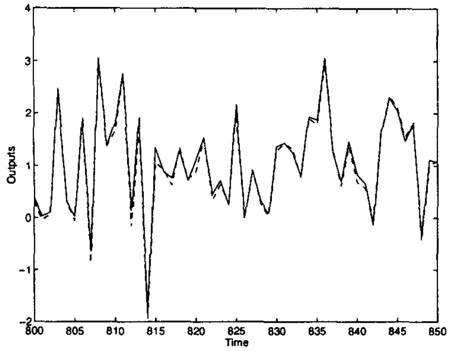

Figure 3: Off-line extended least-squares results.

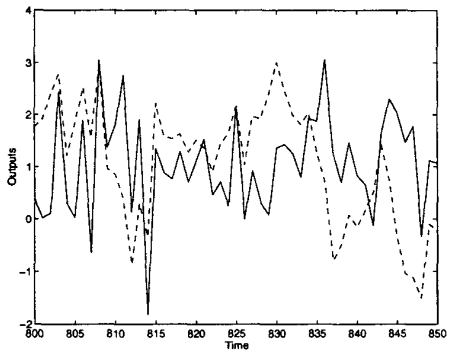

Figure 4: Ordinary least-squares results.

4 Off-line time delay estimation

The process considered is described by the following transfer function where the notations and assumptions are those of the previous sections.

$$A(q^{-1})y(t) \quad = \quad B(q^{-1})q^{-d}u(t) + C(q^{-1})e(t). \tag{33}$$

d is the process time delay for which a maximum and a minimum value are known i.e. $d \in [d_{min} \quad d_{max}]$. $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ as defined in the second section. The control law is given in equation (2). Equation (33) is equivalent to

$$y(t) - e(t) = \varphi(t,d)\theta$$

where $\varphi(t,d) = [-y(t-1) \ldots -y(t-n_A) \; u(t-d) \ldots u(t-d-n_B) \; e(t-1) \ldots e(t-n_c)]$ and $\theta^T = [a_1 \ldots a_{n_A} \; b_1 \ldots b_{n_B} \; c_1 \ldots c_{n_C}]$. Assuming the same conditions as before except for the presence of d, the loss function is

$$V(\hat{\theta}, d) = \frac{1}{2}\sum_{i=1}^{N} \epsilon^2(t), \tag{34}$$

where $\epsilon(t)$ is the equation errors defined by

$$\epsilon(t) = y(t) - e(t) - \varphi(t,d)\theta. \tag{35}$$

The estimates $\hat{\theta}$ and $\hat{d}$ are obtained from the following minimization problem

$$[\hat{\theta},\, \hat{d}] = \min_{d \in [d_{min} \; d_{max}]} V(\hat{\theta}, d). \tag{36}$$

This problem is solved in two stages. We first solve equation (36) for a given $d \in [d_{min} \; d_{max}]$ to obtain an estimate $\hat{\theta}_d$ defined by

$$\hat{\theta}_d = (\phi^T(d)\phi(d))^{-1}\phi^T(d)Y \tag{37}$$

if $\phi^T(d)\phi(d)$ is invertible, where Y and $\phi(d)$ are defined by

$$Y = \begin{bmatrix} y(1) - \varepsilon(1) \\ \cdot \\ \cdot \\ \cdot \\ y(N) - \varepsilon(N) \end{bmatrix} \text{ and } \phi(d) = \begin{bmatrix} \varphi(1,d) \\ \cdot \\ \cdot \\ \cdot \\ \varphi(N,d) \end{bmatrix}, \tag{38}$$

then the estimates $\hat{\theta}$ and $\hat{d}$ are those which minimize $V(\hat{\theta}, d)$ for all $d \in [d_{min} \; d_{max}]$.

Proposition 4.1 Consider the estimated process time delay $\hat{d}$ and the estimated $\hat{\theta}$ defined by equation (37) Assume that $e(t)$ is white noise and $\varepsilon(t)$ is an estimate of $e(t)$ which satisfies equation (12) with zero mean. Then $[\hat{d},\, \hat{\theta}]$ is the unique minimum point of $V(\hat{\theta}, d)$.

Proof: For a fixed d, it is shown in the previous section that $\hat{\theta}_d$ defined in equation (37) is the unique minimum point of $V(\hat{\theta}_d)$. d can only take values between d_{min} and d_{max}, therefore d which minimizes $V(\hat{\theta}_d)$ exists and is included in $[d_{min} \quad d_{max}]$. ■

In the following we illustrate this result by mean of an example.

Example 4.1 *[9]* The process considered is described by

$$(1 - 0.8q^{-1})y(t) \quad = \quad (0.2 + 0.02q^{-1})q^{-6}u(t) \tag{39}$$
$$+(1 - 0.5q^{-1} + 0.6q^{-2})e(t) \tag{40}$$

the process disturbance $e(t)$ is a sequence of independent and identically distributed random variables with zero mean and variance $\sigma_e = 1$. The control law is given by the following Dahlin controller:

$$u(t) = \frac{0.6 - 0.48q^{-1}}{0.2 - 0.08q^{-1} - 0.12q^{-6}}(1 - y(t)). \tag{41}$$

The estimated model is given by
$(1 - 0.7817q^{-1})y(t) = (0.1939 + 0.0225q^{-1})q^{-6}u(t) + (1 - 0.4523q^{-1} + 0.6013q^{-2})e(t)$.
It can be seen that the time delay is exactly identified and the vector $\hat{\theta}$ is reasonably good. The comparison

between the output of the process in solid line and that of the model in dashed line, is shown in Figure (5).

When using the ordinary recursive extended least-squares method to estimate the system parameters, the model determined is: $A(q^{-1}) = 1 - 0.9023q^{-1}$, $B(q^{-1}) = -0.3306 + 0.1704q^{-1} + 0.0146q^{-2} + 0.0126q^{-3} + 0.0101q^{-4} + 0.0084q^{-5} + 0.2095q^{-6} - 0.014q^{-7} - 0.0167q^{-8} - 0.0132q^{-9}$ and $C(q^{-1}) = 1 + 0.8899q^{-1} + 0.7682q^{-2}$. The order considered for $B(q^{-1})$ polynomial is equal to d_{max}. It can be seen that the parameters and time delay were incorrecly estimated by the ordinary extended least-squares. The method by Elnaggar et al. [5] for time delay estimation also provides an incorrect delay estimate of one. Therefore the model for this process cannot be determined using one of the classic methods of identification without considering an external signal.

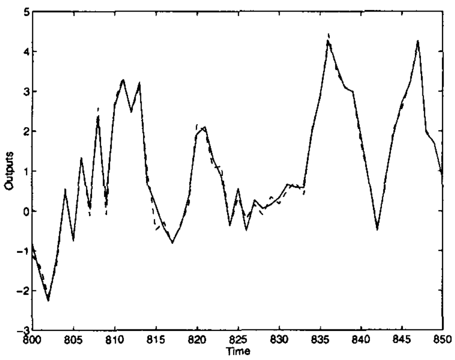

Figure 5: Output of the process and that of the model.

The procedure described above is not only to be used for identification of a first-order-plus-delay system, it can also be used to determine the first order plus delay approximate model for a given process. To illustrate this fact we consider the determination of the first-order-plus-delay approximation model for the system described in example (4.1). The estimated first-order approximation model is $(1 - 0.8433q^{-1})y(t) = (0.1946)q^{-6}u(t) + (1 - 0.5130q^{-1})e(t)$. From the figure (6), which shows the comparison between the process output (solid line) and that of the approximation model y_m (dashed line), we can deduce that the approximated model is a good one.

5 Conclusion

Unlike classic methods of identification where the whitening of the residual is carried out at the same time as the procedure of the determination of process parameters, the method presented here separates the whitening of the residual from the determination of process parameters. Identification starts with residuals which are white, and the focus is on the determination of model structure and process parameters.

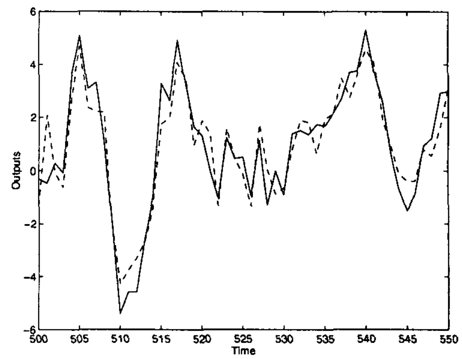

Figure 6: Output of the process and that of the model.

References

[1] K. J. Åström and B. Wittenmark. *"Computer Control System, Theory and Design,"*. Prentice-Hall, Inc, 1986.

[2] K. J. Åström and B. Wittenmark. *"Adaptive Control,"*. Addison-Wesley, 1989.

[3] G. E. P. Box and G. W. Jenkins. *"Time Series Analysis forcasting and Control,"*. Holden-Day, 1976.

[4] K. J. Åström and T. Söderström . "Uniqueness of the Maximum Likelihood Estimates of the Parameters of an ARMA Model". *IEEE Trans. Automat. Contr.*,1974, vol. AC-19, No. 6:pp. 769–773.

[5] A. Elnaggar, G. A. Dumont and A. L. Elshafei. "Delay Estimation Using Variable Regression". *American Control Conference*, 1991, pages pp. 2812–2817.

[6] I. Gustavsson, L. Ljung and T. Söderström . "Survey Paper: Identification of Processes in Closed Loop -Identifiability and Accuracy Aspects". *Automatica*, 1977, vol. 13:pp. 59–75.

[7] I. D. Landau and A. Karimi. "An Output Error Recursive Algorithm for Unbaised Identification in Closed Loop". *Automatica*, 1997, pages Vol. 33, No. 5, pp. 933–938.

[8] L. Ljung *"System Identification,"*. Prentice-Hall, 1987.

[9] C. Lynch "Control Loop Performance Monitoring". *M.A Sc. Thesis, Department of Electrical Engineering, University of British Columbia*, 1992.

[10] T. Söderström and P. Stoica. *"System Identification,"*. Prentice-Hall, 1989.

[11] P. M. Van Den Hof and R. J. P. Schrama. "An Indirect Method for Transfer Function Estimation from Closed Loop Data". *Automatica*, pages Vol. 29, No. 6, pp. 1523–1527, 1993.