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Impact Angle Control Guidance Law Using Lyapunov Function and PSO method

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Abstract: New guidance law, based on Lyapunov stability theory and applied parameter optimization method, is proposed, which can minimize miss distance and adjust impact angle to predefined value. Using LOS angle and predefine impact angle, we define the state variables and propose a lyapunov function candidate for driving structure of the guidance law and conditions of gain parameters. The PSO algorithm is used for selecting the guidance gains, which satisfies the necessary condition of stability of the guidance loop. This new guidance law has simple structure, shows robustness about different engagement conditions, and provides wider capture area than conventional algorithm considering impact angle only.

Keywords: missile, nonlinear guidance law, parameter optimization, PSO.

1. INTRODUCTION

The object of missile guidance is hit the target, or locate the missile in appropriate position and attitude at final impact time for satisfying the mission requirements.

Since World War II, Many missile systems have developed and the required performance of guidance algorithm of each missile system becomes more difficult and complex, and so the guidance laws developed more precision, robust and sophisticated to meet these requirements. The proportional navigation guidance (PNG) and their variants have satisfied those requirements [1].

Excepting the PNG method, many new guidance laws have been proposed based on optimal control theory [2], differential game theory, geometric approach, variable structure control etc.

For some cases, anti-tank, air to ground or sea target missile, an impact angle at final time becomes the most important performance index for maximizing the system's performance. Many researchers proposed the guidance laws considering the final impact angle. Recently, Ryoo [3] propose the analytic solution using optimal control theory with final impact angle constraints. This guidance law show good result with various missile and target engagement conditions but the solutions of the result needs to calculate complex equation for guidance command, so it seems hard to use in real applications. Yanushevsky [4] proposed new approach to guidance law design using lyapunov function and stability theory in planer engagement condition. In this work, he shows the Lyapunov method approach to design guidance law can improve the performance of general PN guidance law. And Lechevin [5] consider final impact angle and show the bound of the guidance parameters for ensure the stability of guidance system.

We define the line of sight angle as a state variable of system and modify this state with adding predefined impact angle. Then applying Lyapunov redesign concept [4], we can get the guidance command structure with three parameters with range for satisfy this guidance loop.

To define the guidance parameters, we use particle swarm optimization (PSO) method. It was firstly introduced by Kennedy and Eberhart[6] and Eberhart and Kennedy[7]. Since, this algorithm was introduced, many researchers have studied the PSO algorithm. Venter and Sobieszczanski-Sobieski [8] survey this algorithm from basic to enhancement method and present some examples.

We combine Lyapunov stability theory and the PSO algorithm. So that the proposed guidance law has simple structure and shows some robustness.

This article is organized as follows: in the first section, we present the background and main idea of this research. The second section defines the kinematics and assumptions used in this article. The third section deals how to obtain the impact-angle control guidance law. The forth section explains the PSO algorithm briefly. Then, in the fifth section, we show the numerical simulation result. The final section summarizes the overall discussion.

2. PROBLEM FORMULATION

2.1 Missile-Target Kinematics

We deal two-dimensional horizontal plain motion only in this article, because, in anti-ship missile, the target altitude is sea level; and so the guidance channel can consist of lateral and longitudinal direction channel. For subsonic anti-ship missile, the longitudinal guidance channel focuses on holding the altitude as low as possible not detected by target's surveillance system, but not to hit the sea wave, during mid-course flight region, and, at terminal homing flight region, guide to hit near the seal level to maximizing weapon's effect. Therefore, the lateral guidance channel could consider plain motion independently to vertical motion.

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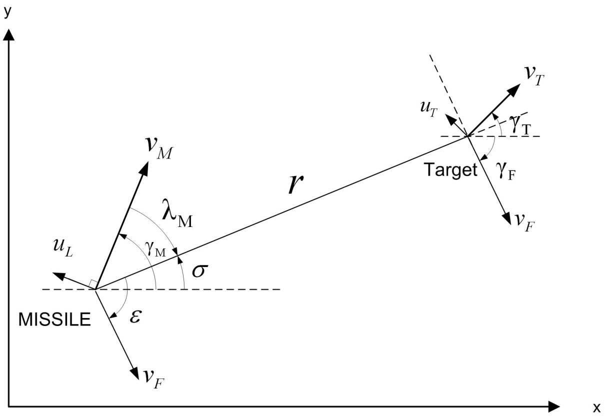

Fig. 1. Missile-Target engagement geometry

In Fig. 1, the LOS (Line of Sight) angle σ defines between the missile and target's position. v_M is the velocity vector of missile. γ_M is flight path angle and γ_F is the missile's flight path angle at impact time. λ_M is the measured angle between missile and target by missile's seeker, and becomes zero at impact time. The LOS angle and the flight path angle of missile have relationship as below.

$$\sigma - \lambda_M = \gamma_M \tag{1}$$

$$\dot{\sigma} - \dot{\lambda}_M = \dot{\gamma}_M \tag{2}$$

The relation of LOS angle and range between missile and target can display as

$$\sin\sigma = \frac{y}{r} \tag{3}$$

The rate of LOS angle shows as below.

$$\dot{\sigma}\cos\sigma = \frac{\dot{y}r - y\dot{r}}{r^2} \tag{4}$$

2.2 Assumptions and simulation model

We assume missile and target move in constant speed (the speed of missile assumes 300m/sec and that of target is 20m/sec, which is equivalent to 40knots) and the guidance acceleration applied on its center of mass point. So the guidance command effect only the missile's flight path angel and target (ship)'s path angle. This has relationship as below.

$$\dot{\gamma}_M = \frac{u_L}{v_M} \tag{5}$$

$$\dot{\gamma}_T = \frac{u_T}{v_T} \tag{6}$$

In addition, we assume that the missile has perfect seeker that has no time delay but we consider glint noise of target only, which is one of the major error source of radar seeker. And we consider the dynamics and controller of missile as a single lag system, or

$$u_L = \frac{1}{sT+1}u_c \tag{7}$$

where u_L is achieved missile acceleration, n_c the commanded missile acceleration, and T the missile system's time constant is define 0.3sec.

Fig 2 shows the organization of simulation program, which is used verifying the designed guidance law.

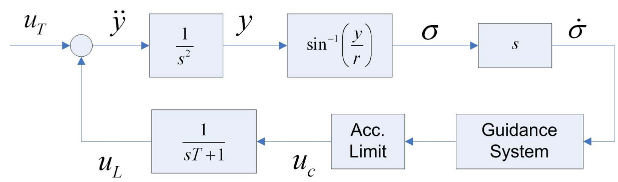

Fig 2 Missile control system used in simulations

3. IMPACT ANGLE CONTROL GUIDANCE LAW

The guidance law, we present in this paper, has two objectives: minimize the miss distance and archives the final impact angle. In Ref [4], Yanushevsky and Boord present lyapunov function based guidance law. We apply this concept to impact angle constrained missile-target engagement case.

We define the state variables as below

$$\begin{aligned} x_1 &= \sigma - \sigma_f \\ x_2 &= \dot{x}_1 \end{aligned} \tag{8}$$

σ is LOS(line-of –sight) angle between missile and target at inertial coordinate system(Fig. 1). σ_f is impact angle at final time.

To minimize the miss distance, the LOS rate ($\dot{\sigma}$) should be kept zero with the additional requirement $\dot{r} < 0$ (Ref [9]), and to accomplish predefined final impact angle, the flight path angle γ_M should be γ_F at impact time. As a missile homing to the target, the angle λ_M becomes zero, so that the final flight path angle γ_F becomes same to σ_f. To achieve the objectives of missile guidance we defined, the state variable x_1 and x_2 should be zero at final time.

To make the state variables zero, we define lyapunov function candidate as below

$$V = \frac{1}{2}\left(x_1^2 + x_2^2\right) \tag{9}$$

As $V > 0$ for $\forall$ $[x_1, x_2] \subseteq R^2$ and $V(0,0) = 0$, Eq.(9) satisfies the lyapunov function condition (Ref [10]), then the point $x = [x_1, x_2]^T = [0,0]^T$ is asymptotically stable, and this condition also satisfies the missile hit the target with impact angle γ_F.

$$\dot{V} = (\sigma - \sigma_f)\dot{\sigma} - \frac{u_c}{r}\dot{\sigma}(\cos\lambda_M + \tan\sigma \cdot \sin\lambda_M)$$
$$- \frac{\ddot{r}}{r}\dot{\sigma}\tan\sigma - 2\frac{\dot{r}\dot{\sigma}^2}{r} + \dot{\sigma}^3\tan\sigma \quad \le 0 \tag{10}$$

if the target's speed is very slow with respect to the missile's and both are constant, we can assumed the closing speed between missile and target is constant, and the rate of closing speed is 0, or $\ddot{r} \approx 0$. We can drive the guidance law, which satisfy the negative definiteness of Eq.(10).

$$u_c = N_0 V_c \dot{\sigma} + N_1 r(\sigma - \sigma_f) + N_2 r\dot{\sigma}^2 \tan\sigma$$
$$N_0 > 2$$
$$N_1 \begin{cases} >1 & (\sigma - \sigma_f)\dot{\sigma} \ge 0 \\ <1 & (\sigma - \sigma_f)\dot{\sigma} < 0 \end{cases} \tag{11}$$
$$N_2 \begin{cases} >1 & \dot{\sigma}\cdot\sigma_f \ge 0 \\ <1 & \dot{\sigma}\cdot\sigma_f < 0 \end{cases}$$

Now we need to define the guidance parameter N_0, N_1 and N_2 for use this guidance law to some applications.

There are various way to define these parameter to satisfies the boundary condition defined at Eq.(11). We consider this as parameter optimization problem.

We use particle swarm optimization (PSO) method to solve this problem.

4. PARAMETER OPTIMIZATION

The particle swarm optimization (PSO) has successfully applied in parameter optimization problem. In this paper, we attempted to obtain the gain of guidance command by converting the guidance-law design problem to parameter optimization problem. In a parameter optimization problem, the optimization solver is the most critical factor to determine the convergence speed and the ability to search the global optimal solution.

4.1 Particle Swarm Optimization Algorithm

The PSO is one of the evolutionary computation techniques introduced by Kennedy and Eberhart in 1995 [6]. The PSO algorithm is similar to evolutionary computation in producing a random population initially and generating the next population based on current cost, but it does not need reproduction or mutation to produce the next generation. Thus, PSO is faster in finding solution compared to any other evolutionary computation technique. In the PSO algorithm, each particle is moving, and hence has a velocity. In addition, each particle remembers the position it was in where, and it had its best result so far. Moreover, the particles in the swarm co-operate exchanging information about what they have discovered in the search region they have visited. The basic PSO algorithm can be summarized as follows;

4.2 The basic PSO algorithm

- Step 1: Initialize a population of particles with random position and velocity.

$$x_0^i = x_{\min} + rand\cdot(x_{\max} - x_{\min})$$
$$v_0^i = v_{\min} + rand\cdot(v_{\max} - v_{\min}) \tag{12}$$

- Step 2. : For each particle, evaluate the fitness value.

-Step 3: Compare a particle's fitness evaluation with particle's pbest (p_i^k). Exchange the particle's fitness value and position with pbest, if it is better.

Compare a particle's fitness evaluation with the population's overall previous best, gbest (p_k^g). Exchange the particle's fitness value and position with gbest, if it is better.

Step 4: Update the velocity and the position of the particle according to following update equations.

$$v_{k+1}^i = wv_k^i + c_1 r_1(p_k^i - x_k^i) + c_2 r_2(p_k^g - x_k^i)$$
$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{13}$$

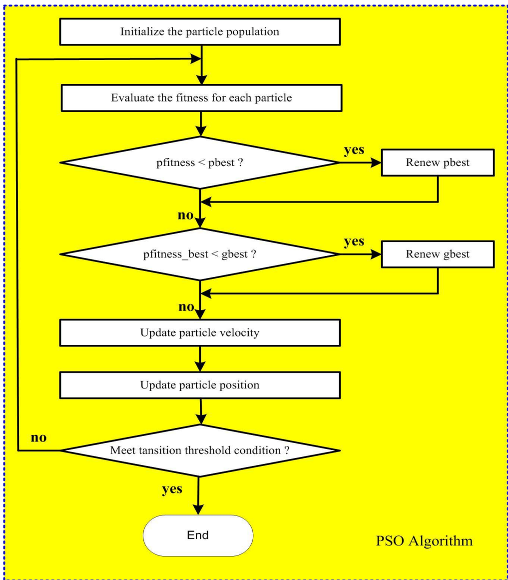

Fig. 3. Flowchart of PSO algorithm

Loop to step 2 until the given criterion is met.

In the velocity update equation of Eq.(13), w, c1, and c2 mean inertia weight, self and swarm confidence factors, respectively. In addition, r1, and r2 are random numbers on the interval [0, 1].

5. NUMERICAL SIMULATION

To verify the performance of the guidance law, we construct two-dimensional, point mass missile-target engagement simulation program based on Ref [1], and this simulation program needs guidance gains for performing simulation. The guidance law are already defined at Eq.(11).

We can consider this guidance gain-selecting problem as parameter optimization problem, which minimizes the cost function defined as,

$$J = J_1\left(\sigma - \sigma_f\right)^2 + J_2 R_{MISS}^2 \tag{14}$$

here R_{MISS} is miss distance, $J_1 = 100000$ and $J_2 = 1000$.

As the cost function value J minimized, the miss distance between missile and target are minimized and the flight path angle become predefined final impact angle σ_f.

We apply PSO algorithm, which explained ahead section, with parameter dimension 5 and population number 50. Iteration number defined 4000.

We choose the engagement condition is that heading angle error (HE) is defined -30deg and impact angle is defined 20deg. the position of target is (10000m, 0m). The resulted trajectory is displayed at **Fig 4**.

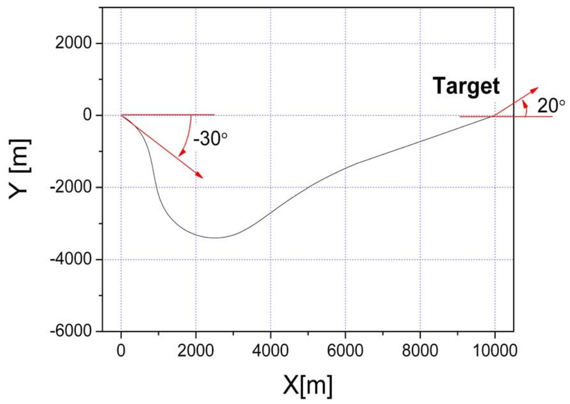

Fig 4 The trajectory used for guidance gain selection

Table. 1 Guidance gain (PSO algorithm)

Guidance Gain		Value
N_0		2.351033
N_1	$\left(\sigma - \sigma_f\right)\dot{\sigma} \geq 0$	6.99113
	$\left(\sigma - \sigma_f\right)\dot{\sigma} < 0$	0.0511189
N_2	$\dot{\sigma} \cdot \sigma_f \geq 0$	23.6124313
	$\dot{\sigma} \cdot \sigma_f < 0$	-74.044826

About 1,500 iteration of PSO algorithm, we obtain the gains with the minimum global cost value (gbest = 0.000097984.).

The gain displayed at Table. 1 is optimized result for specific engagement condition (**Fig 4**), in addition this guidance law shows good result about the other engagement conditions, that target position is reduced and impact angle and heading error at initial time is changed (Fig. 5)

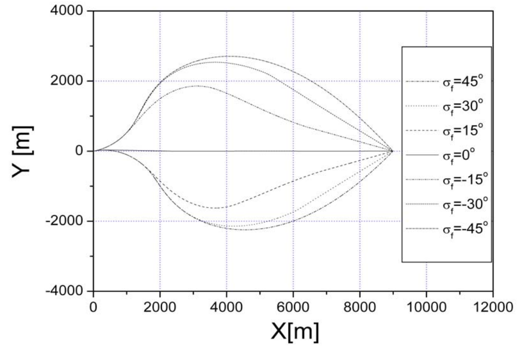

Fig. 5 Result for non-optimized engagement conditions

In Ref [3], Ryoo propose the optimal guidance law. This guidance law show good result, but this

Zarchan [1] introduces another guidance law, which called the trajectory shaping guidance as below.

$$n_c = 4V_c\dot{\sigma} + \frac{2V_c\left(\sigma - \sigma_F\right)}{t_{go}} + n_{T_{PLOS}} \tag{15}$$

This guidance law focuses on impact angle at final time only so that this law shows good result for impact angle, but using this guidance law need to estimate the target's acceleration exactly and need minimum range to hit the target.

We define missile's acceleration limit as 5g and target ship's maximum acceleration as 0.25g. When the target engagement starts at shot range (here homing start range defend as 4000m). The target applies its maximum acceleration to stir its bearings to the missile's approaching direction. Even if there are no sensor noise the trajectory shaping guidance law cannot hit the target, because this guidance law's objective focused on maintain the impact angle. Therefore, the flight path angle becomes predefined impact angle, but the miss distance is not satisfied small enough to consider hit the target.

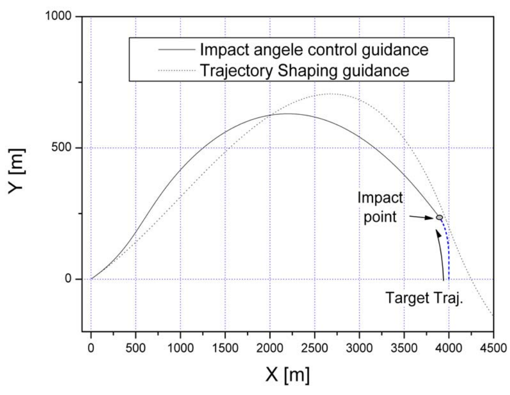

Fig. 6 Compare with the trajectory shaping guidance.

In same engagement condition, the impact-angle control guidance law, which we propose, hit the target but the impact angle is not satisfied. The defined impact angle is -45deg. but accomplished impact angle at final time is -24deg. However, the proposed guidance law tries to satisfy both objectives at same time, the result shows that minimizing miss distance is satisfied firstly. This nature provides more wide capture area than trajectory shaping guidance.

6. CONCLUSION

New guidance law proposed which minimize miss distance and can adjust the missile's flight path angle to predefined impact angle. The structure of this guidance law developed based on Lyapunov stability theory, and the guidance gain obtained solving parameter optimization problem by PSO method.

Carefully choosing the state variable and Lyapunov function candidate, we can drive new guidance law structure and gain conditions, which has characteristics what we wanted, then applies this guidance law to parameter optimization problem and get the optimized guidance gains.

The proposed guidance law has simple structure and has robustness for different impact angle and target positions that did not optimized. The guidance law provides more wide capture area than other guidance method considering impact angle.

REFERENCES

[1] Zarchan, P., *Tactical and Strategic Missile Guidance 4th Ed.*, Vol. 199, Progress in Astronautics and Aeronautics, AIAA, Reston, VA, 2002

[2] Bryson, A. E., and Ho, Y. C., *Applied Optimal Control*, Hemisphere Publishing Corp., 1975

[3] Ryoo, C., Cho, H., and Tahk, M., "Optimal Guidance Laws with Terminal Impact angle Constraint," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 4, 2005

[4] Yanushevsky, R.T., and Boord, W.J., "New Approach to Guidance Law Design," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 1, 2005

[5] Lechevin, N., and Rabbath, C.A., "Lyapunov-Based Nonlinear Missile Guidance," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 6, 2004

[6] Kennedy,J., and Eberhart, R. C., "Particle Swarm Optimization," *Proceedings of the 1995 IEEE International Conference on Neural Networks,* Vol. 4, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 1995, pp. 1942~1948

[7] Eberhart, R. C., and Kennedy, J., "A New Optimizer Using Particle Swarm Theory," *Proceedings of the Sixth International Symposium on Micro Machine and Human Science, Inst. of Electrical and Electronics Engineers*, Piscataway, NJ, 1995, pp. 39~43

[8] Venter, Gerhard, and Sobiezanski-Sobieski, Jaroslaw, "Particle Swarm Optimization," *AIAA Journal*. Vol. 41, No. 8, Aug. 2003

[9] Shneydor, N. A., *Missile Guidance and Pursuit Kinematics, Dynamics and Control*, Horwood Pub., 1998, pp78

[10] Khalil, Hassan k., *Nonlinear Systems 3rd Ed.*, Prentice Hall 2002, pp.114