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Adaptive Dual Control

Theory and Applications

With 83 Figures

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PREFACE

Adaptive control systems have been developed considerably during the last 40 years. The aim of this technique is to adjust automatically the controller parameters both in the case of unknown and time-varying process parameters such that a desired degree of the performance index is met. Adaptive control systems are characterised by their ability to tune the controller parameters in real-time from the measurable information in the closed-loop system. Most of the adaptive control schemes are based on the separation of parameter estimation and controller design. This means that the identified parameters are used in the controller as if they were the real values of the unknown parameters, whereas the uncertainty of the estimation is not taken into consideration. This approach according to the certainty-equivalence (CE) principle is mainly used in adaptive control systems still today. Already in 1960 A. Feldbaum indicated that adaptive control systems based on the CE approach are often far away to be optimal. Instead of the CE approach he introduced the principle of adaptive dual control (Feldbaum 1965). Due to numerical difficulties in finding simple recursive solutions for Feldbaum's stochastic optimal adaptive dual control problem, many suboptimal and modified adaptive dual control schemes had been proposed. One of the most efficient approaches under those is given by the bicriterial synthesis method for dual adaptive controllers. This bicriterial approach developed essentially by the authors of this book during the last 10 years and presented in detail herein is appropriate for adaptive control systems of various structures. The main idea of the bicritical approach consists of introducing two cost functions that correspond to the two goals of dual control: (i) the system output should track cautiously the desired reference signal; (ii) the control signal should excite the plant sufficiently for accelerating the parameter estimation process.

The main aim of this book is to show how to improve the performance of various well-known adaptive controllers using the dual effect without complicating the algorithms and also how to implement them in real-time mode. The considered design methods allow improving the synthesis of dual versions of various known adaptive controllers: linear quadratic controllers, model reference controllers, predictive controllers of various kinds, pole-placement controllers with direct and indirect adaptation, controllers based on Lyapunov functions, robust controllers and nonlinear controllers. The modifications to incorporate dual control are realized separately and independently of the main adaptive controller. Therefore, the designed dual control modifications are unified and can easily be introduced in many certainty equivalence adaptive control schemes for performance improvement. The theoretical aspects concerning convergence and comparisons of various controllers are also discussed. Further, the book contains descriptions and the text of several computer programs in the MATLAB/SIMULINK environment for simulation studies and direct implementation of the controllers in real-time, which can be used for many practical control problems.

This book consists of sixteen chapters, each of which is devoted to a specific problem of control theory or its application. Chapter 1 provides a short introduction to the dual control problem. The fundamentals of adaptive dual control, including the dual control problem considered by A. Feldbaum, its main features and a simple example of a dual control system are presented in Chapter 2. Chapter 3 gives a detailed survey of adaptive dual control methods. The bicriterial synthesis method for dual controllers is introduced in Chapter 4. Chapter 5 provides an analysis of the convergence properties of the adaptive dual version of Generalized Minimum Variance (GMV) controllers. Applications of the bicriterial approach to the design of direct adaptive control systems are described in Chapter 6. In this chapter, also a special cost function is introduced for the optimization of the adaptive control system. Chapter 7 describes the adaptive dual version of the Model Reference Adaptive Control (MRAC) scheme with improved performance. Multivariable systems in state space representation will be considered in Chapter 8. The partial-certainty-equivalence approach and the combination of the bicriterial approach with approximate dual approaches, are also presented in Chapter 8. Chapter 9 deals with the application of the Certainty Equivalence (CE) assumption to the approximation of the nominal output of the system. This provides the basis for further development of the bicriterial approach and the design of the adaptive dual control unit. This general method can be applied to various adaptive control systems with indirect adaptation. Adaptive dual versions of the well known pole-placement and Linear Quadratic Gaussian (LQG) controllers are highlighted in Chapter 10. Chapters 11 and 12 present practical applications of the designed controllers to several real-time computer control problems. Chapter 13 considers the issue of robustness of the adaptive dual controller in its pole-placement version with indirect adaptation. Continuous-time dual control systems appear in Chapter 14. Chapter 15 deals with different real-time dual control schemes for a hydraulic positioning system, using SIMULINK and software for AD/DA converters. General conclusions about the problems, results presented and discussions are offered in Chapter 16.

The organization of the book is intended to be user friendly. Instead reducing the derivation of a novel adaptive dual control law by permanent refering to controller types presented in previous chapters, the development of each new controller is discussed in all important steps such that the reader needs not to jump between different chapters. Thus the presented material is characterized by enough redundancy.

The main part of the results of this book were obtained during the intensive joint research of both authors at the “Control Engineering Lab” in the Faculty of Electrical Engineering at Ruhr-University Bochum, Germany, during the years from 1993 to 2000. Also some very new results concerning the application of the previous results to neural network based “intelligent” control systems have been included. During the preparation of this book we had the helpful support of Mrs. P. Kiesel who typed the manuscript and Mrs. A. Marschall who was responsible for the technical drawings. We would like to thank both of them.

This is the first book that provides a complete exposition on the dual control problem from the inception in the early '60s to the present state of research in this field. This book can be helpful for the design engineers as well as undergraduate, postgraduate and PhD students interested in the field of adaptive real-time control. The reader needs some preliminary knowledge in digital control systems, adaptive control, probability theory and random variables.

Bochum, Dezember 2003

CONTENTS

ABBREVIATIONS AND ACRONYMS

A/D:	Analog / Digital
ANN:	Artificial Neural Network
APCC:	Adaptive Pole Placement Controller
ARIMA:	AutoRegressive Integrated Moving – Average
ARMA:	AutoregRessive Moving - Average
a.s.:	asymptotically stable
CAR:	Controlled AutoRegressive
CARIMA:	Controlled AutoRegressive Integrated Moving - Average
CARMA:	Controlled AutoRegressive Moving - Average
CE:	Certainly Equivalence
CLO:	Closed-Loop Optimal
D/A:	Digital / Analog
DMC:	Dynamic Matrix Control
DMRAC:	Dual Model Reference Adaptive Control
FIR:	Finite Impulse Response
FSR:	Finite Step Response
GDC:	Generalized Dual Control
GMV:	Generalized Minimum Variance
GPC:	Generalized Predictive Control
LFC:	Ljapunov Function Controller
LQ:	Linear Quadratic
LQR:	Linear Quadratic Gaussian
LS:	Least Squares
MAC:	Model Algorithmic Control
MATLAB:	Computer program
MIMO:	Multi-Input / Multi-Output
MISO:	Multi-Input / Single-Output
MF:	Measurement Feedback
MRAC:	Model Refence Adaptive Control
MUSMAR:	Multistep - Multivariable Adaptive Regulator

MV:	Minimum Variance
ND:	Nondual
OL:	Open-Loop
OLF:	Open-Loop Feedback
PCE:	Partial Certainty Equivalence
POLF:	Partial Open-Loop Feedback
PZPC:	Pole-Zero Placement Controller
RBF:	Radial Basis Function
RLS:	Recursive Least Squares
SIMULINK:	Simulation program
SISO:	Single-Input / Single-Output
STR:	Self-Tunning Regulator
UC:	Utility Cost
WSD:	Wide Sence Dual

1. INTRODUCTION

Most adaptive controllers are based on the separation of parameter estimation and controller design. In such cases, the certainty-equivalence (CE) approach is applied, that is, the uncertainty of estimation is not taken into consideration for the controller design, and the parameter estimates are used in the control law as if they were the real values of the unknown parameters. This approach is simple to implement. It has been used in many adaptive control schemes from the beginning of the development of adaptive control theory (the mid-'50s) and is still being used today. In his early works, A. Feldbaum (1960-61, 1965) considered the problem of optimal adaptive control and indicated that systems based on the CE approach are not always optimal but can indeed be far from so. He postulated two main properties that the control signal of an optimal adaptive systems should have: it should ensure that (i) the system output *cautiously* tracks the desired reference value and that (ii) it *excites* the plant sufficiently for accelerating the parameter estimation process so that the control quality becomes better in future time intervals. These properties are known as *dual properties* (or *dual features*). Adaptive control systems showing these two properties are named adaptive dual *control systems*.

The formal solution to the optimal adaptive dual control problem in the formulation considered by Feldbaum (1965) can be obtained through the use of dynamic programming, but the equations can neither be solved analytically nor numerically even for simple examples because of the growing dimension of the underlying space (exact solutions to simple dual control problems can be found in the paper of Sternby (1976) where a system with only a few possible states was considered). These difficulties in finding the optimal solution led to the appearance of various simplified approaches that can be divided into two large groups: those based on various *approximations* of the optimal adaptive dual control problem and those based on the *reformulation* of the problem to obtain a simple solution so that the system maintains its dual properties. These approaches were named implicit and explicit adaptive dual control methods. The main idea of these adaptive dual control methods lies in the design of adaptive systems that are not optimal but have at least the main dual features of optimal adaptive control systems. The adaptive control approaches that are based on approximations of the stochastic dynamic programming equations are usually complex and require large computational efforts. They are based on rough approximations so that the system loses the dual features and the control performance remains inadequate: Bar-Shalom and Tse (1976), Bayard and Eslami (1985), Bertsekas (1976), Birmiwal (1994), to name a few. The methods of problem reformulation are more flexible and promising. Before the elaboration of the bicriterial design method for adaptive dual control systems (see, for example, Filatov and Unbehauen, 1995a; Unbehauen and Filatov, 1995; Zhivoglyadov et al. 1993a), the reformulated adaptive dual control problems considered a special cost function with two added parts involving: control losses and an uncertainty measure (the measure of precision of the parameter estimation) (Wittenmark, 1975a; Milito et al., 1982). With these methods,

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it is possible to design simple dual controllers, and the computational complexity of the control algorithms can become comparable to those of the CE controllers generally used. However, the optimization of such cost functions does not guarantee persistent excitation of the control signal, and the control performance of the dual controllers based on this special cost functions remains, therefore, inadequate. A detailed survey of adaptive dual control and suboptimal adaptive dual control methods is given below.

Most adaptive control systems cannot operate successfully in situations when the controlled systems has come to a standstill, that is when a controlled equilibrium is reached, and no changes of the reference signal occur. Adaptive dual control, however, is able to release the system from such a situation due to its optimal excitations and cautious behavior of the controller. In cases without adaptive dual control the parameter estimation and hence the adaptation is stopped, also denoted as "turn-off" effect (Wittenmark, 1975b). In the absence of movements of the states of the system, whose unknown parameters have to be estimated, the "turn-off" effect causes the determinant of the information matrix of the parameter estimation algorithm to accept values close to zero, and when inverting this matrix a significant computing error may arise. This results in the subsequent burst of parameters, where the estimates take very large unrealistic values, and the output of the system reaches inadmissable large absolute values. In such cases the adaptive control system becomes unacceptable for practical applications. For eliminating these undesirable effects, in many adaptive control systems special noise or test signals for complimentary excitation are added to the reference signal, thus providing efficiency of the adaptation algorithm. In adaptive dual control systems, however, there is no necessity for introducing such additional excitation signals since they provide cautious excitation of the system such that the determinant of the information matrix above mentioned never takes values close to zero.

The last 40 years have born witness to the fantastic development and enhancement of adaptive control theory and application, which have been meticulously collected and presented in various scientific publications. Many of the developed methods have been successfully applied to adaptive control systems, which find practical applications in a wide range of engineering fields. However, most adaptive control systems are based on the CE assumption, which appears to be the reason for insufficient control performance in the cases of large uncertainty. These systems suffer from large overshoots during phases of rapid adaptation (at startup and after parameter changes), which limit their acceptance for many practical cases. In accordance with this, an important and challenging problem of modern adaptive control engineering is the improvement of various presently recognized adaptive controllers with the help of the dual approaches rather than the design of completely new adaptive dual control systems. The newly elaborated bicriterial synthesis method for adaptive dual controllers (hereafter, bicriterial approach), offered in this book, is primarily aimed at meeting this challenge.

In this book the bicriterial approach is developed in detail for adaptive control systems of various structures, and its fundamental principles are analysed and studied through several simulations and applied to many practical real-time control problems. It is demonstrated how the suggested bicriterial approach can be used to improve various well-known adaptive controllers. This method was originally developed on the basis of the reformulation of the adaptive dual control problem (Filatov and Unbehauen, 1995a; Zhivoglyadov et al., 1993a), but it is shown that the method can also combine the advantages of the methods for both the approximation and reformulation approaches to the dual control problem. The main idea of the bicriterial approach consists of introducing two cost functions that correspond to the two goals of dual control: (i) to track the plant output according to the desired reference signal and (ii) to induce the excitation for speeding up the parameter estimation. These two cost functions represent control losses and an uncertainty index for the parameter estimates, respectively. Minimization of the first one results in cautious control action. Minimization of the uncertainty index provides the system with optimal persistent excitation. It should be noted that the minimization of the uncertainty index is realized in the domain around the optimal solution for the first criterion and the size of this domain determines the magnitude of the excitation. Therefore, the designed systems clearly achieve the two main goals of dual control. Moreover, the designed controllers are usually computationally simple to implement. The resulting dual controllers have one additional parameter that characterizes the magnitude of the persistent excitation and can easily be selected because of its clear physical interpretation.

The problem of selecting cost functions for control optimization demands special consideration. Many processes of nature realize themselves by minimizing various criteria; therefore, they are optimal in the sense of a specific cost functional. A diverse variety of criteria is indeed available to be used in engineering problems, and one should be chosen depending on the nature of the system and its required performance. At the same time, the quadratic cost function of weighted squared system states and control input, which is usually used for optimization of control systems, has no physical interpretation (excluding several specific cases where this cost function represents the energy losses of the system). From the control engineering point of view, the desired system behavior, in many cases, can be defined using pole assignment of the closed-loop system rather than using the calculated parameters of the aforementioned cost function. On the other hand, certain criteria are also used for determining the specific system structure or the optimal pole location. For example, the criterion for the generalized minimum-variance controller (Chan and Zarrop, 1985; Clarke and Gawthrop, 1975) establishes the structure of the system but has no clear engineering meaning. The minimization of the derivative of a Lyapunov function (or first difference for discrete-time systems) is applied to ensure system stability only (Unbehauen and Filatov, 1995); and the parameters of the above mentioned quadratic criterion provide certain pole locations of the closed-loop systems and are frequently used for this purpose (Keuchel and Stephan, 1994; Yu et al., 1987). Therefore, for many practical cases, it is not necessary to seek approximate solutions to the originally considered unsolvable optimal adaptive control problem with the quadratic cost function. The other formulations (reformulations) of the dual control problem consider the control optimization problem formulation with clearer engineering contents and result in the design of computationally simple adaptive controllers with improved control performance. Introducing two cost functions in the bicriterial synthesis method corresponds to the two goals of the control signal in adaptive systems. At the same time, the elaborated method is flexible, offering the freedom of choosing various possible cost functions for both two aims. Thus, the uncertainty index could be represented by a scalar function of the covariance matrix of the unknown parameters, and in this book it is shown how the different kinds of control losses can be used for design of various adaptive controllers to provide improved control performance and smooth transient behavior even in the presence of system uncertainty.

Introducing the new cost function in the bicriterial approach as squared deviation of the system output from its desired (nominal) value allows designing various dual versions with improved performance for various well-known discrete-time adaptive controllers such as model reference adaptive control (MRAC) (Landau and Lozano, 1981), adaptive pole-placement controllers (APPC) with direct and indirect adaptation (Filatov et al., 1995; Filatov and Unbehauen, 1996b), pole-zero placement controllers (PZPC) (Filatov and Unbehauen, 1994; Filatov et al., 1996), self-tuning regulators (STR) (Filatov and Unbehauen, 1995a; Zhivoglyadov et al., 1993a), generalized minimum variance (GMV) controllers (Filatov and Unbehauen, 1996c), linear quadratic Gaussian (LQG) and other various predictive controllers. The minimization of the control losses, which are suggested in the bicriterial approach, brings the system output closer to the output of the undisturbed system with selected structure and without uncertainty (closer to the system with unknown parameters that would be obtained after the adaptation is finished), which has been named nominal output. This nominal output would be provided by the desired system with adjusted parameters. Therefore, this cost function is independent of the structure and principles of designing the original adaptive system and can be applied to any system only during the adaptation time. After finishing the adaptation, the system takes its final form with fixed parameters, and the suggested performance index assumes the lowest possible value. This cost function corresponds to the true control aim at the time of adaptation, which has a clear engineering interpretation, and is generally applicable to all adaptive control systems. Minimization of this cost function does not change the structure of the system that will be obtained after finishing the adaptation; therefore, it can be applied to various adaptive control systems. The bicriterial dual approach can be used not only for the improvement of the performance of the above-mentioned systems but also for many other adaptive controllers, for example, nonparametric adaptive controllers. Further development of the bicriterial approach, originally considered by Filatov and Unbehauen (1996a), has opened the possibility for design of a universal adaptive dual control algorithm that improves various CE-based systems, or other nondual (ND) controllers, with indirect adaptation and can immediately be applied in various adaptive control systems. More elaboration of this method allows separating the ND controller from its uniform dual controller (dual modification of the ND controller). Thus, the dual controller can be inserted in various well-known adaptive control schemes.

Dual control systems of all kinds require an uncertainty description and an uncertainty measure for evaluating their estimation accuracy. The theory of stochastic processes, bounded estimation (or set-membership estimation), and the theory of fuzzy sets can be used for the representation and description of the uncertainty. However, most systems exploit a stochastic uncertainty description, and some of the recently developed methods are based on bounded estimation (Veres and Norton, 1993; Veres, 1995). Dual control systems based on fuzzy-set uncertainty representation are not known up to now. The stochastic approach to uncertainty representation is the most well-developed and accepted one because of its numerous practical applications. Therefore, it is used in the present book.

The advantages of adaptive dual control can be observed especially in cases of large uncertainties and swift parameter drift, and during startup of the adaptation process (Wittenmark 1975a, 1995). It is necessary to point out that in some cases it is important to finish the adaptation quickly, while in other cases the adaptation cannot be finished in a short time, and cautious properties of the dual controller become important for smooth transient behavior in the presence of significant uncertainties. From two examples presented by Bar-Shalom (1976) for problems of soft landing and interception, the advantages of dual control can be clearly observed where the large excitations are important for successful control. Therefore, it is more important to have an adaptive control law that envisages large excitations for problems of soft landing and interception because the terminal term of the cost function contains the goal of the control, with the behavior at the beginning of the process being unimportant. On the other hand, it had been demonstrated (Filatov et al., 1995) that for smooth transient behavior the cautious properties are more important than large excitation.

The problem of convergence of adaptive dual control systems necessitates special consideration. During the last 25 years the methods of Lyapunov functions and the methods of martingale convergence theory (as their stochastic counterpart) have been successfully applied to convergence analysis of various adaptive systems (Goodwin and Sin, 1984). The control goal in these systems was formulated as global stability or asymptotic optimality. Thus, the systems guarantee optimum of the cost function only after adaptation for the considered infinite horizon problems, but the problem of improving the control quality during the adaptation had not been considered for a long time. The systems are usually based on the CE assumption and suffer from insufficient control performance at the beginning of the adaptation and after changes of the system parameters. At the same time, various adaptive dual control problems had been formulated as *finite horizon* optimal control problems, and the convergence aspects were not applicable to them. Adaptive dual control methods, based on the problem reformulation, and predictive adaptive dual controllers consider the systems over an infinite control horizon; thus, the convergence properties must be studied for such systems. The difficulties of strict convergence analysis of adaptive dual control systems appear because of the nonlinearity of many dual controllers. The first results on convergence analysis of adaptive dual control systems were obtained by Radenkovic (1988). These problems are thoroughly investigated in Chapter 5.

The problems of convergence of adaptive control under the conditions of unstructured uncertainty, such as unmodeled dynamics, and the design of robust adaptive control systems (Ortega and Tang, 1989) have to be considered here. Stability of the suggested dual controller, coupled with the robust adaptation scheme, is proved for systems with unmodeled effects, which can represent nonlinearities, time variations of parameters or high-order terms. It is demonstrated that after the insertion of the dual controller the robust adaptive system maintains its stability, but some known assumptions about the nonlinear and unmodeled residuals of the plant model should be modified.

2. FUNDAMENTALS OF DUAL CONTROL

The formulation of the optimal dual control problem is presented in this chapter. The main features of dual control systems and fundamentals of the bicriterial synthesis method are discussed by means of simple examples.

2.1. Dual Control Problem of Feldbaum

The unsolvable stochastic optimal adaptive dual control problem was originally formulated by Feldbaum (1960-61, 1965). This problem is described in a more general form below. A model with time-varying parameters in state-space representation will be employed.

2.1.1. Formulation of the Optimal Dual Control Problem

Consider the system described by the following discrete-time equations of state, parameter and output vectors:

$$\boldsymbol{x}(k+1) = \bar{\boldsymbol{f}}_k[\boldsymbol{x}(k), \boldsymbol{p}(k), \boldsymbol{u}(k), \boldsymbol{\xi}(k)],\ k = 0, 1, ..., N-1\,, \tag{2.1}$$

$$\boldsymbol{p}(k+1) = \boldsymbol{\upsilon}_k\ [\boldsymbol{p}(k), \boldsymbol{\varepsilon}(k)]\,, \tag{2.2}$$

and

$$\boldsymbol{y}(k) = \boldsymbol{h}_k[\boldsymbol{x}(k), \boldsymbol{\eta}(k)]\,, \tag{2.3}$$

where $\boldsymbol{x}(k) \in \Re^{n_x}$ is the state vector; $\boldsymbol{p}(k) \in \Re^{n_p}$ the vector of unknown parameters; $\boldsymbol{u}(k) \in \Re^{n_u}$ the vector of control inputs; $\boldsymbol{y}(k) \in \Re^{n_y}$ the vector of system outputs; and $\boldsymbol{\xi}(k) \in \Re^{n_\xi}$, $\boldsymbol{\varepsilon}(k) \in \Re^{n_\varepsilon}$ and $\boldsymbol{\eta}(k) \in \Re^{n_\eta}$ are vectors of independent random white noise sequences with zero mean and known probability distributions; $\bar{\boldsymbol{f}}_k(\cdot)$, $\boldsymbol{\upsilon}_k(\cdot)$ and $\boldsymbol{h}_k(\cdot)$ are known *simple* vector functions. The function $\boldsymbol{\upsilon}_k(\cdot)$ describes the stochastic time-varying parameters of the system. The probability density for the initial values $\mathrm{p}[\boldsymbol{x}(0), \boldsymbol{p}(0)]$ is assumed to be known.

The set of outputs and control inputs available at time k is denoted as

$$\Im_k = \{\boldsymbol{y}(k),...,\boldsymbol{y}(0), \boldsymbol{u}(k-1),...,\boldsymbol{u}(0)\}, \quad k = 1,...,N-1, \Im_0 = \{\boldsymbol{y}(0)\}\,. \tag{2.4}$$

The performance index for control optimization has the form

$$J = \mathrm{E}\left\{\sum_{k=0}^{N-1} g_{k+1}[\boldsymbol{x}(k+1), \boldsymbol{u}(k)]\right\}, \tag{2.5}$$

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where the $g_{k+1}[\cdot,\cdot]$'s are known positive convex scalar functions. The expectation is taken with respect to all random variables $\boldsymbol{x}(0)$, $\boldsymbol{p}(0)$, $\boldsymbol{\xi}(k)$, $\boldsymbol{\varepsilon}(k)$ and $\boldsymbol{\eta}(k)$ for k = 0, 1, ..., N-1, which act upon the system.

The problem of optimal adaptive dual control consists of finding the control policy $\boldsymbol{u}(k) = u_k(\Im_k) \in \overline{\Omega_k}$ for k = 0, 1, ..., N-1 that minimizes the performance index of eq. (2.5) for the system described by eqs. (2.1) to (2.3), where $\overline{\Omega}_k$ is the domain in the space $\Re^{n_{\boldsymbol{u}}}$, which defines the admissible control values.

2.1.2. Formal Solution Using Stochastic Dynamic Programming

Backward recursion of the following stochastic dynamic programming equations can generate the optimal stochastic (dual) control sought for the above problem:

$$J_{N-1}^{\mathrm{CLO}}(\Im_{N-1}) = \min_{\boldsymbol{u}(N-1)\in\Omega_{N-1}} \left[\mathrm{E}\left\{ g_N[\boldsymbol{x}(N),\, \boldsymbol{u}(N-1)] \middle| \Im_{N-1} \right\} \right], \tag{2.6}$$

$$J_k^{\mathrm{CLO}}(\Im_k) = \min_{\boldsymbol{u}(k)\in\Omega_k} \left[\mathrm{E}\left\{ g_{k+1}[\boldsymbol{x}(k+1),\, \boldsymbol{u}(k)] + J_{k+1}^{\mathrm{CLO}}(\Im_{k+1}) \middle| \Im_k \right\} \right],$$

for $k = N$-2, N-3, ..., 0, (2.7)

where the superscript CLO denotes 'closed-loop optimal' according to the terminology suggested by Bar-Shalom and Tse (1976).

It is known that the analytical difficulties in finding simple recursive solutions from eqs. (2.6) and (2.7) and the numerical difficulties caused by the dimensionality of the underlying spaces make this problem practically unsolvable even for simple cases (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985). However, the detailed investigation of this problem enables one to find the main dual properties (Wittenmark, 2003) of the control signal in optimal adaptive systems and to use them for other formulations of the adaptive dual control problems. This leads to the elaboration of design methods for adaptive dual controllers and, practically, to the solution of the adaptive dual control problem. A simple example for such a problem is given below to demonstrate the properties of adaptive dual control systems.

2.2. Features of Adaptive Dual Control Systems

Consider a simple discrete-time single input / single output (SISO) system described by

$$y(k+1) = bu(k) + \xi(k)\,,\ b \neq 0\,, \tag{2.8}$$

where b is the unknown parameter with initial estimate $\hat{b}(0)$ and covariance of the estimate $P(0)$, and the disturbance $\xi(k)$ has the variance $\mathrm{E}\{\xi^2(k)\} = \sigma_\xi^2$. This simplified model can be used for the description of a stable plant with unknown amplification b. The cost function

$$J = \mathrm{E}\left\{\sum_{k=1}^{N}[w(k) - y(k)]^2\right\}, \tag{2.9}$$

as a special case of eq. (2.5) with the output signal $y(k)$ and set point $w(k)$, should be minimized. The resulting optimal control problem, $u(k) = f[w(k) - y(k)]$, is unsolvable. Equations (2.6) and (2.7) can be successfully applied only to the multi-step control problem with a few steps N to obtain a solution. The optimal parameter estimate for the considered system can be obtained, however, using the Kalman filter in the form

$$\hat{b}(k+1) = \hat{b}(k) + \frac{P(k)u(k)}{P(k)u^2(k) + \sigma_\xi^2}[y(k+1) - \hat{b}(k)u(k)], \tag{2.10}$$

and

$$P(k+1) = \frac{P(k)\sigma_\xi^2}{P(k)u^2(k) + \sigma_\xi^2} = P(k) - \frac{P^2(k)u^2(k)}{P(k)u^2(k) + \sigma_\xi^2}. \tag{2.11}$$

It should be noted that for the case of Gaussian probability densities the Bayesian estimation (Feldbaum, 1965; Saridis, 1977) and the recursive least squares (RLS) approach give the same equations for the parameter estimation in this example. After inspection of eqs. (2.10) and (2.11), the dependence of the estimate and its covariance on the manipulating signal $u(k)$ can be observed for a given σ_ξ (large values of $u(k)$ improve the estimation); and for an unbounded control signal, the exact estimate after only one measurement can already be obtained

$$\lim_{|u(k)|\to\infty} P(k+1) = 0, \tag{2.12}$$

and

$$\lim_{|u(k)|\to\infty} \hat{b}(k+1) = b. \tag{2.13}$$

Therefore, persistent excitation by a large magnitude of $u(k)$ can significantly improve the estimate. The problem is the optimal selection of this excitation so that the total performance of the system is enhanced.

Using the CE approach, it is assumed that all stochastic variables in the system are equal to their expectations. In the considered case, this means that $\xi(k) = 0$ and $\hat{b}(k) = b$. It is easy to see that for the CE assumption the optimal control has the simple form

$$u(k) = u_{\mathrm{CE}}(k) = \frac{w(k+1)}{\hat{b}(k)}. \tag{2.14}$$

On the other hand, the minimization of the one-step cost function

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[w(k+1) - y(k+1)]^2 \middle| \Im_k\right\}, \tag{2.15}$$

instead of the multi-step performance index described by eq. (2.9), leads for the considered example of eq. (2.8) to the control action given by

$$u(k) = u_{\mathrm{c}}(k) = \frac{\hat{b}(k)w(k+1)}{\hat{b}^2(k) + P(k)} = \frac{1}{1 + P(k)/\hat{b}^2(k)} u_{\mathrm{CE}}(k) \tag{2.16}$$

where $\mathrm{E}\{\cdot|\Im_k\}$ is the conditional expectation operator with the set $\Im_k$ being defined according to eq. (2.4). The controller given by eq. (2.16) has a positive value in the denominator, and it generates the manipulating signal with smaller magnitude than the CE controller given by eq. (2.14). Controllers of this kind are named *cautious controllers* (denoted by u_{c}) because of this property. Thus, the indicated two properties (cautious control and excitation) are attributed to the optimal adaptive control in various systems. Systems that are designed to ensure these properties of their control signal are named adaptive dual control systems.

2.3. Simple Example of Application of the Bicriterial Approach

Further consideration of the above simple example is given below. Various cost functions for optimization of the excitation can be considered. The most prominent ones among a host of such cost functions are

$$J_k^{\mathrm{a}} = P(k+1), \tag{2.17}$$

which stands for parametric uncertainty at the $(k+1)-$th sampling instant, and

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[y(k+1) - \hat{b}(k)u(k)]^2 \middle| \Im_k\right\}. \tag{2.18}$$

The last one was suggested by Milito et al. (1982). It characterizes the desired increase in the innovational value of the parameter estimation algorithm of eq. (2.10). Minimization of any of these cost functions leads to unbounded large control values; therefore, some constraints Ω_k should be used. To get a reasonable compromise between optimal persistent excitation and cautious control, it would be suitable to define these constraints around the cautious control $u_{\mathrm{c}}(k)$, as defined in eq. (2.16), in the form

$$\Omega_k = \left[u_{\mathrm{c}}(k) - \theta(k); u_{\mathrm{c}}(k) + \theta(k)\right]. \tag{2.19}$$

These constraints limit the magnitude of the excitation symmetrically around the cautious control $u_{\mathrm{c}}(k)$ by the value $\theta(k) \geq 0$. It is easy to see that the optimal controller for the uncertainty indices according to eqs. (2.17) or (2.18) and constraints eq. (2.19) can be described by the general form

$$u(k) = u_{\mathrm{c}}(k) + \mathrm{sgn}\{u_{\mathrm{c}}(k)\}\ \theta(k), \tag{2.20}$$

where

$$\operatorname{sgn}\{\kappa\} = \begin{cases} 1, & \text{if } \kappa \geq 0 \\ -1, & \text{if } \kappa < 0. \end{cases} \tag{2.21}$$

Equation (2.20) is derived in the following way. Through substitution of eq. (2.8) into eq. (2.18) it follows that

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[y(k+1) - \hat{b}(k)u(k)]^2 \middle| \Im_k\right\} = -P(k)u^2(k) + \sigma_\xi^2, \tag{2.22}$$

and from eqs. (2.17) or (2.18), and eq. (2.19), taking into account eq. (2.22), it can be concluded that the optimum for

$$u(k) = \arg\min_{u(k)\in\Omega_k} J_k^{\mathrm{a}} \tag{2.23}$$

is achieved on the boundary of the domain Ω_k as

$$u(k) = u_{\mathrm{c}}(k) + \operatorname{sgn}\{J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) - \theta(k)] - J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) + \theta(k)]\}\theta(k)\,. \tag{2.24}$$

Therefore, the dual control signal is determined by eq. (2.20), which is obtained after substitution of eqs. (2.11) and (2.17), or eq. (2.22), into eq. (2.24) and some further manipulations. The bicriterial optimization for the design of the dual controller is portrayed in Figure 2.1. The magnitude of the excitation can be selected in relation to the uncertainty measure according to eq. (2.17) as

$$\theta(k) = \eta P(k), \eta \geq 0\,. \tag{2.25}$$

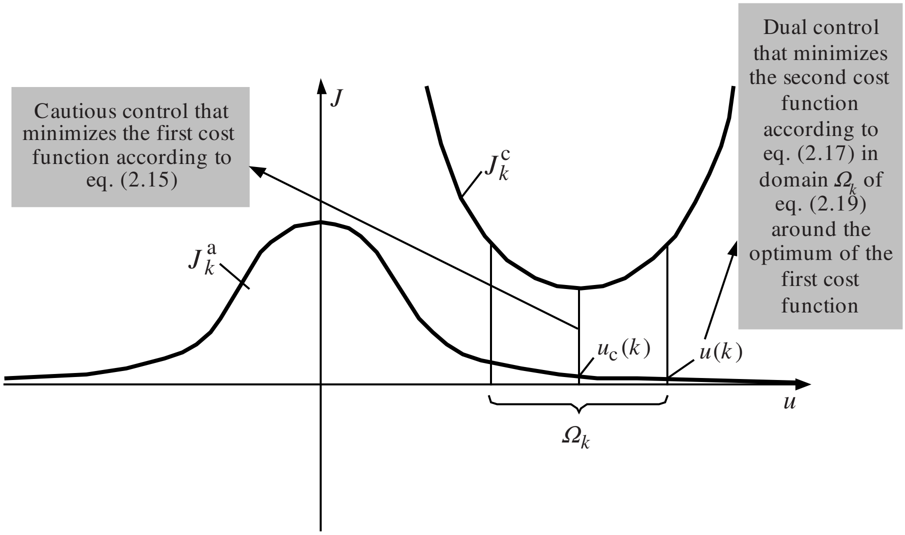

Figure 2.1. Sequential minimization of two cost functions for dual control.

Therefore, the presented dual controller, according to eqs. (2.16), (2.20) and (2.25), minimizes sequentially both cost functions, eqs. (2.15) and (2.17), or eq. (2.18), and the parameter, according to eq. (2.25), determines the compromise between these cost functions during the minimization. In contrast to other explicit dual control approaches, as for example (Milito et al., 1982), the parameter $\theta(k)$ has a clear physical interpretation: the magnitude of the excitations. Therefore, it can easily be selected.

2.4. Simple Example of a Continuous-Time Dual Control System

As pointed by Feldbaum (1960-61), the dual effect can appear not only in discrete-time systems but also in continuous-time ones. However, the solution of the dual control problem for continuous-time systems can prove to be complex and cumbersome indeed. Below, a simple (deterministic) continuous-time system with nonstochastic uncertainty is considered, where a simple dual controller is derived using the bicriterial approach and a heuristic understanding of the uncertainty in such systems as its integral-square-error.

Consider the simple continuous-time SISO static plant

$$y(t) = bu(t)\,, \tag{2.26}$$

with the unknown parameter b. The estimate $b(t)$ can be obtained from the differential equation (Narendra and Annaswamy, 1989)

$$\dot{\hat{b}}(t) = -u^2(t)\hat{b}(t) + u(t)y(t) = -u^2(t)[\hat{b}(t) - b] \tag{2.27}$$

whose equilibrium state is $b(t) \equiv b$. The right-hand side of eq. (2.27) can be considered as the negative gradient of the cost function

$$J_1 = \frac{1}{2}[y(t) - \hat{b}(t)u(t)]^2 \tag{2.28}$$

with respect to $\hat{b}(t)$. Substitution of eq. (2.26) into eq. (2.27) leads to

$$\dot{\hat{b}}(t) = -u(t)\hat{y}(t) + u(t)y(t) = -u(t)[\hat{y}(t) - y(t)]\,, \tag{2.29}$$

where

$$\hat{y}(t) = \hat{b}(t)u(t)\,. \tag{2.30}$$

Introducing an uncertainty cost function analogously to eq. (2.18) for the parameter estimation as

$$J^{\mathrm{a}} = -[\hat{y}(t) - y(t)]^2\,, \tag{2.31}$$

which should be minimized for $u \in \Omega(t)$ where

$$\Omega(t) = [u_{\mathrm{c}}(t) - \theta(t); u_{\mathrm{c}}(t) + \theta(t)] \tag{2.32}$$

and selecting $\theta(t) > 0$ leads to the magnitude of the excitation signal. The increase in the absolute value of the error $[\hat{y}(t) - y(t)]$ in eq. (2.29) makes the system adapt faster. The certainty equivalence (CE) control law is determined by

$$u_{\mathrm{CE}}(t) = \frac{w(t)}{\hat{b}(t)} . \tag{2.33}$$

The following cost function, or uncertainty measure

$$I(t) = \frac{1}{t}\int_0^t \bar{e}^2(\tau)d\tau \, , \; \bar{e}(t) = \hat{y}(t) - y(t) \, , \tag{2.34}$$

can be used for the control optimization. The caution and excitation factors α and η, respectively, are also introduced. Then the amplitude of the excitation is defined as

$$\theta(t) = \eta I(t) . \tag{2.35}$$

The cautious control is determined similarly to eq. (2.16) through the parameter α, uncertainty measure $I(t)$ and CE control action $u_{\mathrm{CE}}(t)$ as

$$u_{\mathrm{c}}(t) = \frac{1}{1+\alpha I(t)} u_{\mathrm{CE}}(t) . \tag{2.36}$$

Using the bicriterial approach, the minimum of eq. (2.31), in the domain described by eq. (2.32), is achieved as

$$u(t) = u_{\mathrm{c}}(t) + \operatorname{sgn}\left\{ J^{\mathrm{a}}[u_{\mathrm{c}}(t) - \theta(t)] - J^{\mathrm{a}}[u_{\mathrm{c}}(t) + \theta(t)] \right\} \eta I(t) . \tag{2.37}$$

After inserting eqs. (2.26), (2.30) and (2.31) in eq. (2.37), we have

$$u(t) = u_{\mathrm{c}}(t) + \operatorname{sgn}\{u_{\mathrm{c}}(t)\} \, \eta I(t) . \tag{2.38}$$

Thus, eqs. (2.36) and (2.38) determine the simple dual control law for the considered continuous-time adaptive dual control system. This controller provides optimal excitation added to the cautious control. The uncertainty measure is taken into account; and after finishing the adaptation, the control signal is equivalent to the one of the CE-controller according to eq. (2.33).

2.5. General Structure of the Adaptive Dual Control System

To summarize the properties of dual control systems presented in this chapter the following schemes of a conventional adaptive control system and an adaptive dual control system are portrayed in Figures 2.2. and 2.3. The transmission of the accuracy of the parameter estimates from the estimation to the control design algorithm is the main difference between the presented structures. The utilization of the accuracy of the estimation for the controller design allows generating the optimal excitation and cautious control signal for an adaptive dual controller. Thus significant improvements of the control performance in cases of large uncertainty can be achieved.

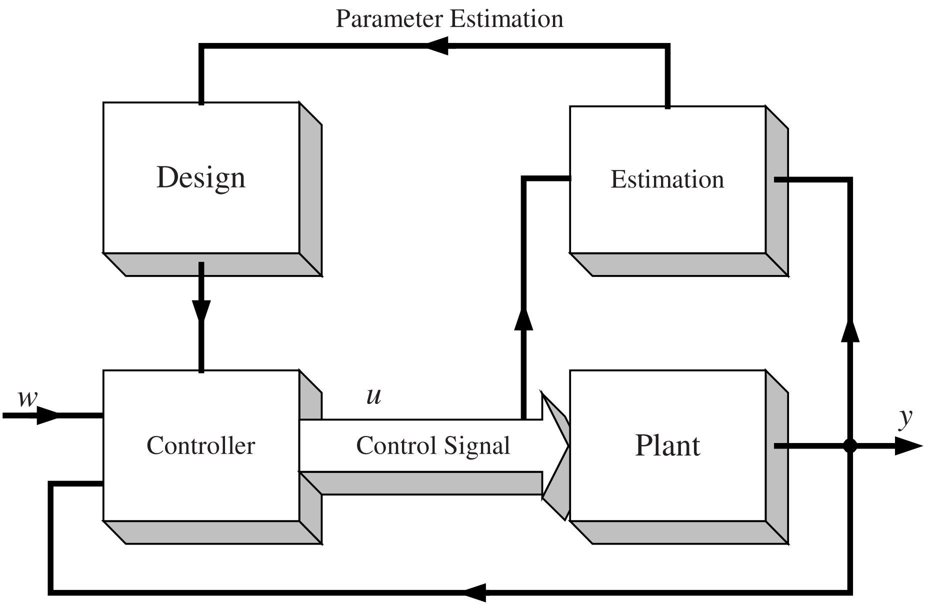

Figure 2.2. Adaptive control system based on the CE assumption.

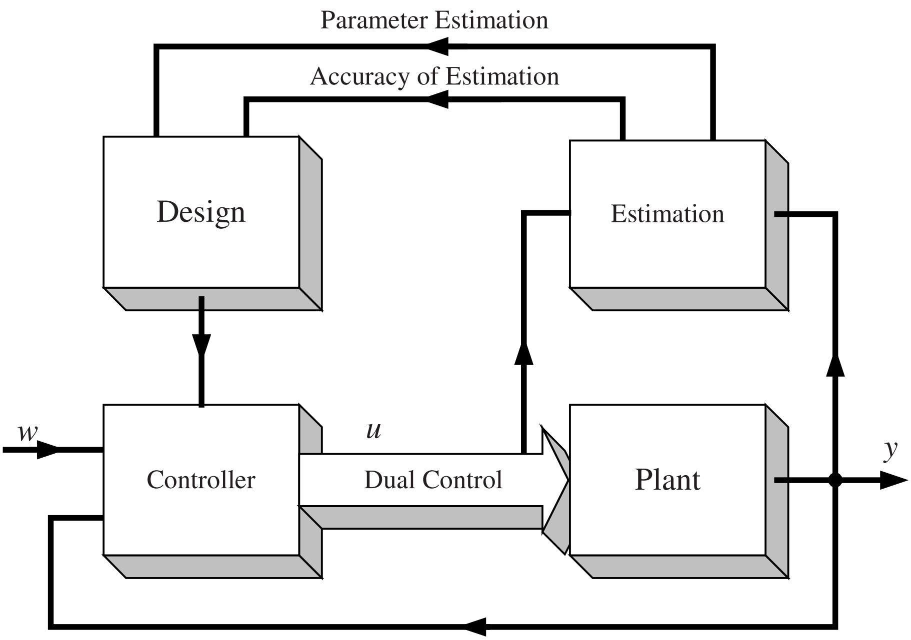

Figure 2.3. Adaptive dual control system.

3. SURVEY OF DUAL CONTROL METHODS

3.1. Classification of Adaptive Controllers

The sheer number of different adaptive control approaches presented in the literature makes a survey of this field a cumbersome and formidable task. Before the adaptive controllers are classified, it is natural to give a definition of adaptive controllers and to draw a line between adaptive and nonadaptive controllers. Attempts to define adaptive control systems strictly were made by Saridis (1977) and Åström and Wittenmark (1989). Definitions of adaptive control and adaptation are also cited in Tsypkin's work (1971). However, there is no definition available by which adaptive systems can be strictly separated from nonadaptive ones. The difficulties lie in the destinction of nonlinear and time-varying control laws from adaptive control ones. The following definition summarizes what control engineers usually understand by an adaptive control system:

> **Definition 1:** Adaptive Control System
>
> A control system operating under conditions of *uncertainty* 1) of the controller that provides the *desired system performance* 2) by changing its *parameters* 3) and/or *structure* 4) in order to reduce the uncertainty and to improve the approximation of the desired system is an adaptive control system.

1. *Uncertainty* comprises unknown parameters and characteristics of the plant, environment or an unknown controller.
2. *Desired system performance* is the specification that the controlled system should satisfy when uncertainty is removed. The goal of adaptation is to reduce the uncertainty and give the possibility to achieve this desired system performance. For example, in the case of convergence, the desired system performance will be achieved by a system with fixed parameters after finishing the adaptation.
3. *Parameters* are the values that determine system components or connections between the components. Engineers separate parameters from states of the system: the states of the system are to be controlled explicitely, and they generally change more rapidly than parameters.
4. *Structure* of the system represents the system components and connections between them in the aggregate.

Therefore, the notion of an *adaptive control system* that is presented stems from the notions of *uncertainty*, *unknown desired system* as well as from the notions of *parameters* and *structure*. With this definition, adaptive systems can be separated from systems with time-varying, nonlinear and robust controllers and from systems with controllers that have changing structure, where the structures and parameters are known. The criterion to separate an adaptive system from a non-adaptive one is the following:

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 14–32, 2004.

> In an adaptive system the parameters and/or the structure of the controller that provide the desired performance for a given unknown plant are unknown, and the superimposed adaptive system tries to find these parameters and/or the structure of the controller during operation in real-time mode.

Therefore, it is desired to switch off the adaptation as soon as the system uncertainty is reduced sufficiently and to use the adjusted controller with a fixed structure and fixed parameters from then on. The adaptive system aims at on-line identification and correction of the structure and the parameters of a standard control loop and the switch-off of the adaptation loop once convergence has occurred. However, this goal of a one-shot adaptation can only be achieved in cases of limited variations of the plant or the environment; otherwise, the adaptation does not terminate, and the desired controller is sought continuously (continuous adaptation), or the adaptation will be restarted after every change in the operating conditions. It should be noted that in adaptive dual control a main objective is to ensure that the adaptation is accelerated and finishes early.

The above definition of an adaptive control system does not contradict the known and accepted ones (Åström and Wittenmark, 1989; Saridis, 1977; Tsypkin, 1971) - including also self-organizing systems, which can be considered complex cases of adaptive control systems. This new definition of an adaptive control system, with the clearly indicated goal of adaptation, implies that a new cost functional for the deviation of the system output from an unknown nominal output of the system can be introduced. This cost functional can be used to modify many control systems with direct and indirect adaptation through the addition of a dual control component. This topic is thoroughly considered in the sequel, especially in Chapters 6 and 9 to 13.

It should be noted that dual control systems could be adaptive as well as nonadaptive. For example, dual effects can be viewed in stochastic nonlinear systems where the uncertainty consists of the inaccuracy of state estimation (Bar-Shalom and Tse, 1976). The definition of a dual control system is as follows:

> **Definition 2:** Dual Control System
>
> A control system that operates under conditions of *uncertainty* and incorporates the existing uncertainty in the control strategy with the control signal having the following properties: (i) it cautiously follows the control goal and (ii) excites the plant to improve the estimation, is a dual control system.

The meaning of item (i) “cautiously follows the control goal” was explained, for example, by Bar-Shalom and Tse (1976). In the case of uncertain parameters of the system, it means the control signal should be smaller (cautious) than the control signal in the system with known parameters and after adaptation. This is also defined as cautious control.

The structures of a standard adaptive control system and adaptive dual control system are portrayed in Figures 3.1 and 3.2, respectively. In these structures, the goal of adaptation, that is, determining the unknown parameters of the controller, is emphasized by the irregular form of the controller block. The adaptive system tries to determine the controller parameters during operation in real-time mode, whereas the adaptive dual control system realizes this actively by means of optimal excitation added to the cautious control action.

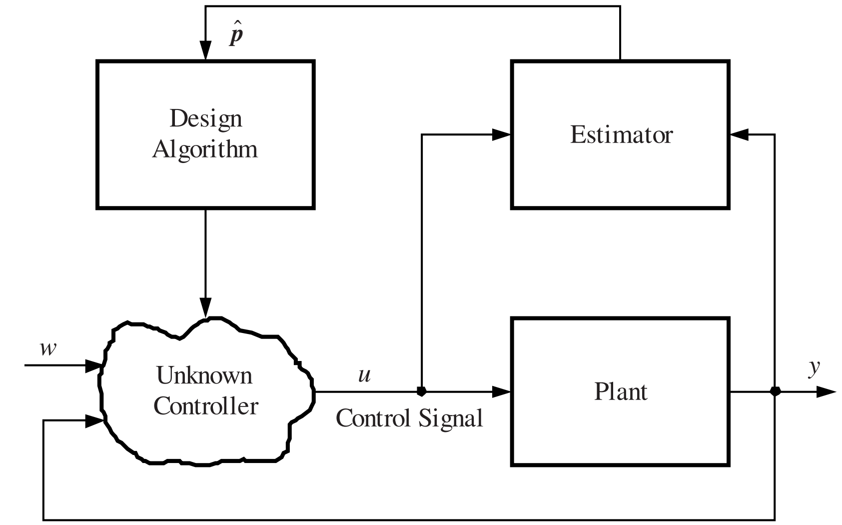

Figure 3.1. Adaptive nondual control system (e.g., based on the CE assumption) (The goal of adaptive controller design is to determine the control signal)

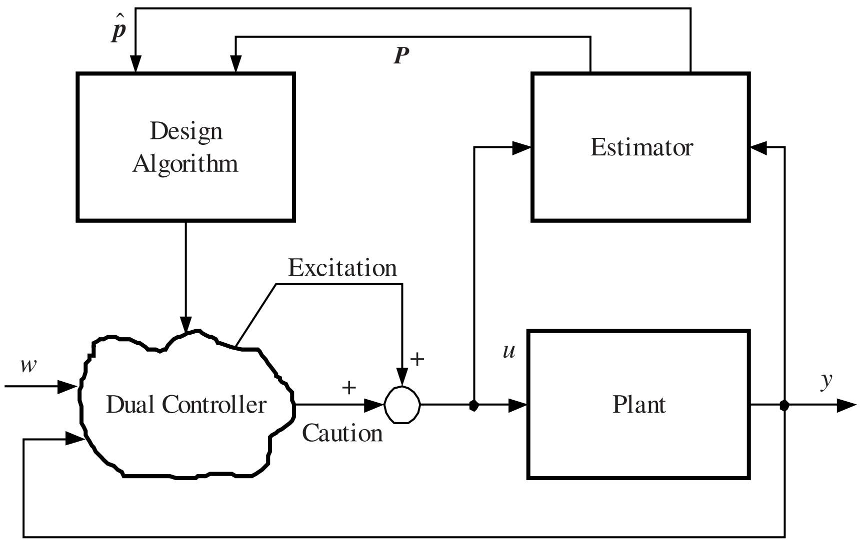

Figure 3.2. Adaptive dual control system ($\boldsymbol{P}$ is the covariance matrix of the estimation error)

Control systems, adaptive and nonadaptive, may be classified in three large groups, which generate the control or manipulating signal in different ways, as indicated in Table 3.1. These types of control systems determine the corresponding control methods that have been developed for different groups of controllers. For example, almost all suboptimal stochastic approaches have appeared as a result of considering control problems for systems of type I. Methods of predictive control consider type II systems. Many controllers belong to type III. It should be noted that, methods of implicit (*direct*) dual control were originally elaborated for systems of type I, whereas the explicit (*indirect*) dual controllers were developed for systems of type III.

The classification of stochastic control approaches of type I and their main characteristics are portrayed in Figure 3.3. These approaches are based on various simplifications. Many approaches, indicated in Figure 3.3, can also be applied to predictive control systems of type II. The detailed description of these methods, as simplified approaches for solving the stochastic control problem described in Section 2.1, is given in Sections 3.3 and 3.4. Presently, the differences between stochastic control polices with the CE assumption, separation and wide sense separation are emphasized.

To find a CE control law for the problem described by eqs (2.1) to (2.3) and (2.5), all stochastic variables should be replaced by their expectations. The resulting deterministic feedback controller is denoted as

$$\boldsymbol{u}(k) = \varphi_k[\boldsymbol{x}(k)]\,. \tag{3.1}$$

Table 3.1. Classification of discrete-time controllers for various types of control signals

All discrete-time controllers can be divided into the three following groups:		
Type	Description of the group	Examples
I	A sequence of control signals $u(k)$, ..., $u(N$-1) or control policies $u_k(\Im_k)$, ..., $u_{N-1}(\Im_{N-1})$ is generated, where k=0, 1, ..., N-1; N can assume values from the set {1, ..., ∞}.	Optimal control problems with finite and infinite horizon.
II	At every control instant k, a sequence of control signals $u(k)$, ..., $u(k+N)$ that optimizes a cost function is generated, but only $u(k)$ is applied, where k=0, 1, ..., ∞, ; N can assume values from the set {1, ..., ∞}.	All predictive controllers. In the case of $N \to \infty$, the controllers coincide with type I.
III	At every control instant, only $u(k)$ is generated, where k=0, 1, ..., ∞. Knowledge of the future reference signal is not required.	*STR, GMV, various MRAC and APPC, *etc.*, also controllers of type I that generate constant feedback.

*STR: self-tuning regulators; GMV: generalized minimum-variance controller; MRAC: model reference adaptive control; APPC: adaptive pole-placement controller

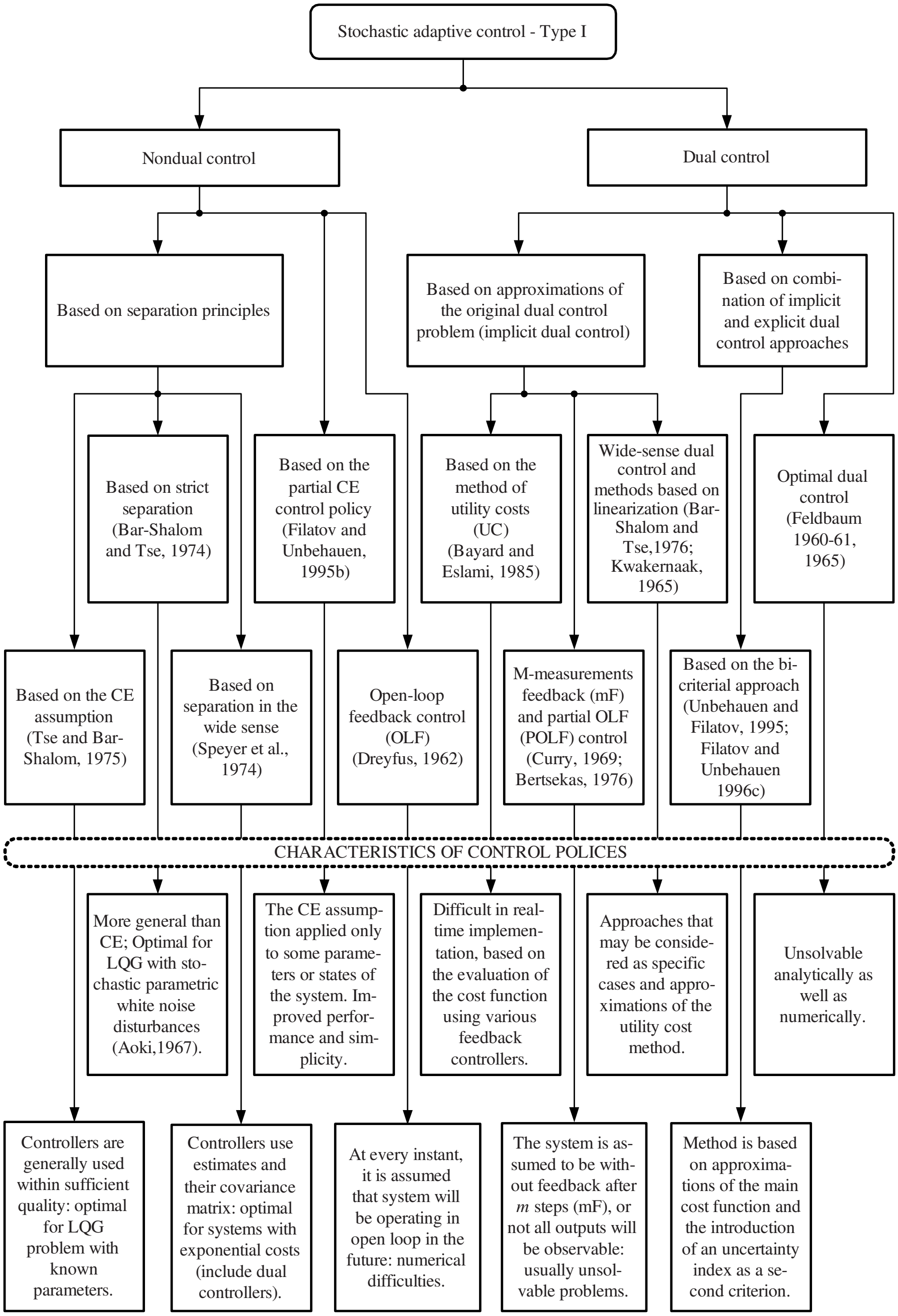

Figure 3.3. Classification of stochastic adaptive control systems of type I (Some of the considered methods of feedback and dual control can be directly applied to systems of type II)

Especially in case of the CE controller the unknown system state $\boldsymbol{x}(k)$ is replaced by its estimate $\hat{\boldsymbol{x}}(k)$

$$\boldsymbol{u}(k) = \varphi_k[\hat{\boldsymbol{x}}(k)] . \tag{3.2}$$

The CE controller is the optimal solution of the LQG control problem (Bar-Shalom and Tse, 1976). The notion of *separation* is more general than the CE assumption and is given below. The control is generated using the state estimate as

$$\boldsymbol{u}(k) = \psi_k[\hat{\boldsymbol{x}}(k)] , \tag{3.3}$$

where the function ψ_k differs from the optimal deterministic feedback φ_k. Here only the separation of the estimator and the controller is important. This control law is optimal for the linear quadratic control problem and for systems with stochastic parametric disturbances of white noise (Aoki, 1967). Sometimes the definition of the control law with separation in the "wide sense" is used. The controller then depends not only on the estimate but also on the covariance matrix of the estimation $\boldsymbol{P}(k)$ as

$$\boldsymbol{u}(k) = \phi_k[\hat{\boldsymbol{x}}(k), \boldsymbol{P}(k)] . \tag{3.4}$$

This control law is optimal for a linear stochastic system with known parameters and exponential cost function (Speyer et al., 1974). Many of the dual controllers are based on the separation in the wide sense, and the parameter estimates and their covariance matrix are used in the controller.

The classification of the type III controllers is presented in Figure 3.4. It should be noted that well-know adaptive controllers such as STR, GMV, LQG, APPC, MRAC and the controller based on Lyapunov functions (LFC) (see Unbehauen and Filatov, 1995, and Chapter 8) were originally developed with the CE assumption. In a system with *indirect* adaptation, the controller parameters are calculated using estimates of the plant parameters, whereas in the systems with *direct* adaptation the controller parameters themselves are estimated directly from the input and output data without parameter estimation the of the plant model, as indicated in Figures 3.5 and 3.6, respectively. The application of the bicriterial approach to the systems presented in Figure 3.4, with indirect as well as direct adaptation, allows to design dual versions with improved control performance for all these systems, as will be shown later.

Adaptive control systems can also be classified according to the types of models used. Likewise, various predictive controllers are based on nonparametric models. For example, dynamic matrix control (DMC) (Cutler and Ramaker, 1980; Garcia et al., 1989; De Keyser et al., 1988) uses a step response model, and model algorithmic control (MAC) (Richalet et al., 1978) is based on an impulse response model. Various nonparametric controllers also use frequency-domain plant models. Many dual controllers have been developed for least-squares (LS) and state-space models, and the results can be extended to CARMA and CARIMA models. This general classification is given in Figure 3.7. Unbehauen (1985) presented more general linear models.

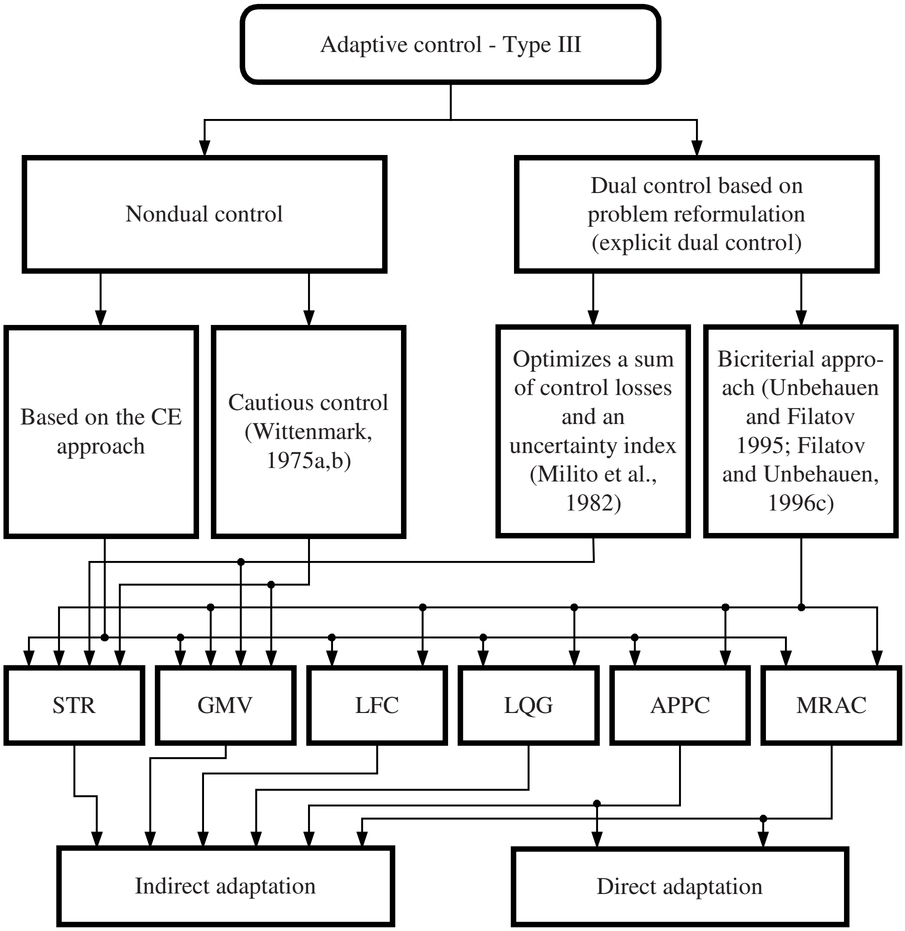

Figure 3.4. Classification of adaptive controllers of type III (The bicriterial approach allows the design of dual versions of all type III controllers. The complexity of the controllers increases from left to right)

3.2. Dual Effect and Neutral Systems

An example of a system where the dual effect cannot appear is considered below. A SISO system of such kind is described by the equation

$$y(k+1) = y(k) + bu(k) + c + \xi(k)\,,\ \ b \neq 0\,, \tag{3.5}$$

where b is a known parameter (in contrast to the previously considered example in Section 2.2), c is an unknown parameter with the initial estimate $\hat{c}(0)$ and variance of the estimate $P(0)$, and the disturbance $\xi(k)$ has the properties as in eq. (2.8). This simplified model is an integrator with an unknown additive parameter. Consider the cost function for control optimization, eq. (2.15), with the setpoint $w(k)$. This optimal control problem has a simple solution. The optimal estimate $\hat{c}(k)$ can be obtained using a Kalman filter in the form

$$\hat{c}(k+1) = \hat{c}(k) + \frac{P(k)}{P(k)+\sigma_\xi^2}\left[y(k+1) - y(k) - bu(k) - \hat{c}(k)\right], \tag{3.6}$$

and

$$P(k+1) = \frac{P(k)\sigma_\xi^2}{P(k)+\sigma_\xi^2} = P(k) - \frac{P^2(k)}{P(k)+\sigma_\xi^2}, \tag{3.7}$$

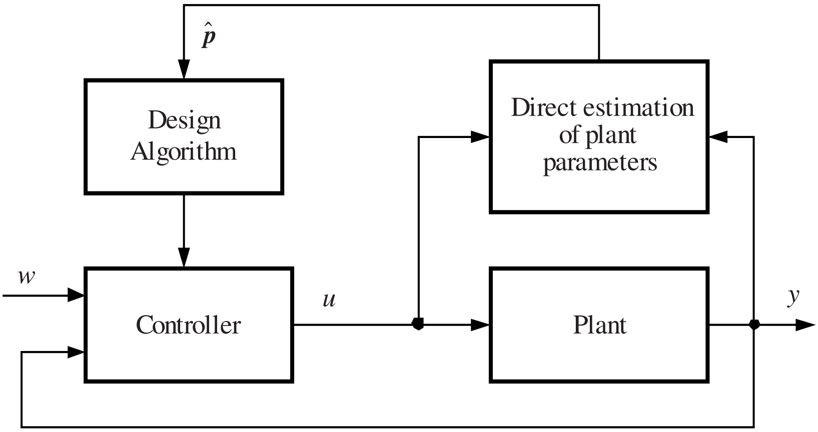

Figure 3.5. System with indirect adaptation (with direct identification)

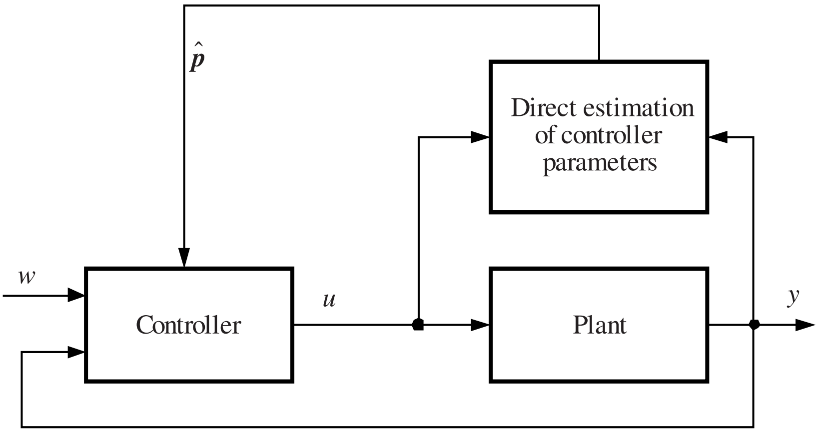

Figure 3.6. System with direct adaptation (with indirect identification)

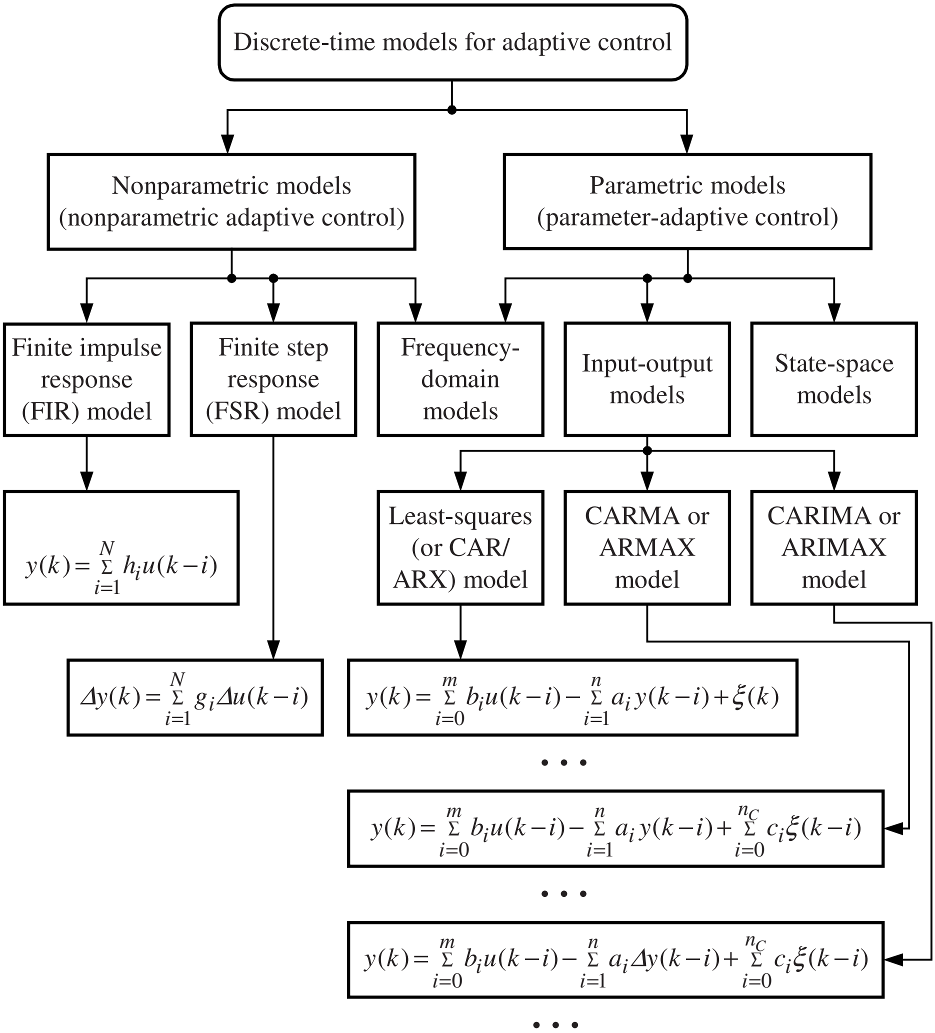

Figure 3.7. Classification of important linear discrete-time models used in adaptive control

CAR	– controlled autoregressive	ARX	– autoregressive with auxiliary input
CARMA	– controlled autoregressive moving-average	ARMAX	– autoregressive moving-average with auxiliary input
CARIMA	– controlled autoregressive integrated moving- average	ARIMAX	– autoregressive integrated moving-average with auxiliary input

$$\Delta x(k)=x(k)-x(k-1)$$

where the covariance of the estimation error is determined as

$$P(k) = \mathrm{E}\left\{(c - \hat{c}(k))^2 \middle| \Im_k\right\}. \tag{3.8}$$

Utilizing the CE approach, one assumes that all stochastic variables in the system are equal to their expectations, that is, $\xi(k)$=0 and $P(k)$=0 in the considered case. Thus, for the CE assumption, the optimal control has the simple form

$$u(k) = \frac{w(k+1) - y(k) - \hat{c}(k)}{b}. \tag{3.9}$$

It is easily seen that the optimal adaptive control for the cost function, eq. (2.9), is also described by eq. (3.9). In such systems, the estimation is independent of the control action. Feldbaum (1960-61) has named them *neutral* systems or control systems with independent (passive) accumulation of information. The strict definition is given as follows:

> **Definition 3:** Neutral Control System
>
> A control system that operates under conditions of uncertainty so that any excitations added to the control signal cannot improve the accuracy of the estimation, is a neutral control system.

It is necessary to point out that adaptive control systems are usually not neutral, and the "additive" uncertainty of systems, similar to the above presented system of eq. (3.5), can be compensated when an integral feedback control law without any adaptation is used.

Almost all adaptive systems have uncertain parameters or states that are multiplicatively connected to the control signal or state variables; therefore, they are not neutral. The dual effect can be used for improvement of the performance of such control systems. Exceptions are systems with additive uncertainty, like the system described by eq. (3.5). Therefore, the above discussion can be concluded with the following statement:

> The performance of almost all *adaptive* control systems can be improved through the application of dual control methods and the replacement of CE controllers (or other non-dual controllers) with dual controllers providing cautious behavior with optimal excitations.

Some simple possibilities of replacing nondual controllers with dual ones are considered and supported by examples in Chapter 9.

Remark: The present remark is given here for better understanding the statement framed above. It is important to mention that in modern control system theory the measure of performance can be understood in different ways. For instance, if the tracking error is the measure of performance, then cautiousness can reduce tracking performance but increase robustness in case of slight dynamical changes of the plant. Furthermore, there can be a mismatch between estimated and actual stochastic uncertainty of the plant model, which might make the dual controller perform poorer than CE control. At the same time, the choice of the parameters for the dual controller is important for the improvement of the control performance. A dual controller with uncommonly large excitation or too cautious behaviour might also perform poorer than the CE controller.

3.3. Simplifications of the Original Dual Control Problem

The stochastic dynamic programming equations, eqs. (2.6) and (2.7), formally give a solution to the considered stochastic optimal control problem (Bar-Shalom and Tse, 1976; Bertsekas, 1976; Feldbaum, 1965). But it is well known that the analytical difficulties in finding simple recursive solutions and the numerical difficulties caused by the dimensionality of the underlying spaces make this problem practically unsolvable, even for simple cases (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985). This has led to the development of various suboptimal stochastic adaptive control methods (Åström, 1987; Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985; Bertsekas, 1976; Curry, 1969; Dreyfus, 1962; Filatov and Unbehauen, 1995b) that are based on different approximations and simplifying assumptions, presented in the classification of Figure 3.3. Many of these simplifications can be interpreted as approximations of the probability measures of the unknown states and parameters of the system as described below. Thus, the suboptimal adaptive control policies are based on the minimization of the following remaining part (cost-to-go) of the general performance index given by eq. (2.5):

$$J_k(\Im_k, \rho_k) = \mathrm{E}_{\rho_k}\left\{ \sum_{i=k}^{N-1} g_{i+1}[\boldsymbol{x}(i+1), \boldsymbol{u}(i)] \middle| \Im_k \right\}. \tag{3.10}$$

At sampling time k, the control policy $u_k(\Im_k)$ is to be found with the approximation of the conditional probability densities of the system states and parameters for the future steps: $\mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) | \Im_{k+i}]$, i = 0, 1, ..., N-k-1. This approximation is denoted as ρ_k (the notion of ρ-approximation has been introduced by Filatov and Unbehauen, 1995b). In eq. (3.10), the expectation $\mathrm{E}_{\rho_k}\{\cdot\}$ is calculated with the approximation ρ_k. The various suboptimal stochastic adaptive control approaches are based on different approximations ρ_k (ρ-approximations) in eq. (3.10), as shown below:

- For the open-loop (OL) control policy, the system is assumed to be without feedback, and the optimal control is found from a priori information about the system parameters and states. This simplifying assumption is equivalent to the following approximation of the probability densities for eq. (3.10):

$$\rho_k = \rho_k^{\mathrm{o}} = \left\{ \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) | \Im_{k+i}] = \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) | \Im_0],\ i = 0, \ldots, N-k-1 \right\}, \tag{3.11}$$

  where $\mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) | \Im_{k+i}]$ depends on the deterministic sequence {$\boldsymbol{u}$(0), ..., $\boldsymbol{u}$(N-1)} for this case.

- To find the control input for the known open-loop feedback (OLF) control policy, the system is assumed to be without feedback in the future steps (from time k+1 to N), but with feedback at time k. At every time instant k, the observation $\boldsymbol{y}(k)$ is used for the estimation of both parameters and states; and then the probability measures are corrected (Bertsekas, 1976; Dreyfus, 1962). Therefore the feedback is realized only for the current time k but not for the future time instants. This simplifying assumption can be described by the following ρ-approximation in eq. (3.10)

$$\rho_k = \rho_k^{\mathrm{f}} = \left\{ \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \middle| \Im_{k+i}] = \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \middle| \Im_k],\ i = 0,...,N-k-1 \right\} . \tag{3.12}$$

In the case of the approximate assumption described by eq. (3.12), the system is designed with feedback, but at every time instant k the OLF control policy is calculated. It is known that the OLF control policy provides a superior control performance compared with the OL control using the ρ-approximation from eq. (3.11) (Bertsekas, 1976).

- The well-known and generally used CE approach can also be interpreted in terms of the ρ-approximation. For the considered control problem the ρ-approximation (Filatov and Unbehauen, 1995b) of the probability densities for the performance index according to eq. (3.10) takes the form

$$\begin{aligned} \rho_k = \rho_k^c = \Big\{ & \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \big| \Im_{k+i}] \\ & = \delta[\boldsymbol{x}(k+i) - \hat{\boldsymbol{x}}(k+i)]\ \delta[\boldsymbol{p}(k+i) - \hat{\boldsymbol{p}}(k+i|k)],\ i = 0, ..., N-k-1 \Big\} , \end{aligned} \tag{3.13}$$

where

$$\hat{\boldsymbol{x}}(k+i) = \mathrm{E}\left\{ \boldsymbol{x}(k+i) \middle| \Im_{k+i} \right\},\ \ \hat{\boldsymbol{p}}(k+i|k) = \mathrm{E}\left\{ \boldsymbol{p}(k+i) \middle| \Im_k \right\} \tag{3.14}$$

are the estimates and $\delta(\cdot,\cdot)$ is the Dirac function. The estimates are used here in the control law as if they were the real deterministic values of the unknown parameters.

- It should be noted that only the OLF and the CE policies, with the approximations described by eqs. (3.11) and (3.13), are used in practice. The CE control policy is simple to implement but provides an insufficient control performance in many cases, because the inaccuracy of the estimates is not taken into account (Unbehauen and Filatov, 1995). The OLF control policy gives better control performance but requires a numerical optimization routine in real time, because of the difficulties in finding an analytical solution and complex equations obtained after taking the expectation in the cost function. It should be also noted that the approximation using the Dirac function $\delta(\cdot,\cdot)$ is actually a substitution of the stochastic variables by deterministic values (CE-assumption). Apparently it can be named ρ-substitution instead of ρ-approximation for the case of CE assumption.

- A new ρ-approximation of the joint probability measures for both the system states and parameters was suggested by Filatov and Unbehauen (1995b). In this approach, adaptive control policies are derived, which are computationally simple, especially for linear systems and which give improved control performance.

Consider the extended state vector for the system described by eqs. (2.1) to (2.3)

$$\boldsymbol{z}^{\mathrm{T}}(k) = [\boldsymbol{x}^{\mathrm{T}}(k)\ \ \boldsymbol{p}^{\mathrm{T}}(k)] , \tag{3.15}$$

the vector $z(k)$ is divided into two separate vectors, $z_1(k)$ and $z_2(k)$. Introduce the following ρ-approximation of the extended state vector of eq. (3.15), which will be used for designing the control law via minimization of the future cost according to eq. (3.10):

$$\rho_k = \rho_k^{\mathrm{p}} = \{\mathrm{p}[z_1(k+i),\, z_2(k+i)\big|\Im_{k+i}] = \delta[z_1(k+i) - \hat{z}_1(k+i)]\,\mathrm{p}(z_2(k+i)\big|\Im_k)\ ,$$

$$i = 0,\ldots,N-k-1\}\ , \quad (3.16)$$

where $\mathrm{p}[z_1(k+i), z_2(k+i)\big|\Im_{k+i}] = \mathrm{p}[z(k+i)\big|\Im_{k+i}] = \mathrm{p}[\boldsymbol{x}(k+i),\boldsymbol{p}(k+i)\big|\Im_{k+i}]$.

For the ρ-approximation according to eq. (3.16), the CE assumption (see eq. (3.13)) is applied to $z_1(k)$, the first part of the extended state vector, and the same simplifying assumption that is used for the OLF control policy given by eq. (3.12) is used for the second part, $z_2(k)$. Thus, it is assumed that at every sampling time k the system operates in closed-loop feedback mode for the future time intervals with respect to the first part of the extended state vector $z_1(k+i)$ and in open-loop feedback mode for the second part $z_2(k+i)$. In such a way this is a special combination of the CE and OLF control policies. It is assumed at the same time that the CE assumption is applied to the first part of the extended state vector, but not to the second one. This partial certainty equivalence (PCE) approach, together with the assumption according to eq. (3.16), allows the design of adaptive controllers that are simple in computation, especially for linear systems. Moreover, it is possible to estimate an upper bound of the cost function for this control policy (Filatov and Unbehauen, 1995b) when the first part of the extended state vector $z_1(k)$ is exactly observable and as analytically shown (Filatov and Unbehauen, 1995b), the performance of the PCE policy is superior to that of the OL policy in this case. It should be mentioned that the suggested PCE approach proposes a separation of the extended state vector into its two parts $z_1(k)$ and $z_2(k)$, which should be realized in accordance with the specific structure of the system, described in general by eqs. (2.1) to (2.3). Depending on this separation, the PCE control policy can be dual or nondual. An example of this separation for linear systems with unknown stochastic parameters was given by Filatov and Unbehauen (1995b). The PCE control policy can be used, together with the bicriterial approach, to design a dual controller that combines implicit and explicit dual control methods; this is demonstrated in Section 8.6.

3.4. Implicit Dual Control

The implicit dual control methods are based on various approximations that still maintain the dual properties of the system and are generally complex. Some of them, for example, the original dual control problem, are even unsolvable in spite of the approximations.

The partial open-loop feedback (POLF) policy (Bertsekas, 1976; see also Figure 3.1) is based on the assumption that instead of full information $\Im_{k+i}$ in future steps, i = 0, ..., N-k-1, only incomplete information $\overline{\Im}_{k+i}$ from future measurements will be used. This assumption is equivalent to the ρ-approximation

$$\rho_k = \rho_k^l = \left\{ \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \middle| \Im_{k+i}] = \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \middle| \overline{\Im}_{k+i}],\ i = 0,\ldots,N-k-1 \right\} \tag{3.17}$$

where $\overline{\Im}_{k+i} = \{\overline{\boldsymbol{y}}(k+i), \ldots, \overline{\boldsymbol{y}}(k+1), \boldsymbol{y}(k), \ldots, \boldsymbol{y}(0), \boldsymbol{u}(k+i-1), \ldots, \boldsymbol{u}(0)\}$, $\overline{\boldsymbol{y}}(\cdot)$ is the partial observation vector that does not contain all the elements of $\boldsymbol{y}(\cdot)$ and has dimension $n_{\overline{y}} \le n_y$.

The m-measurement feedback (m-MF) control policy is based on the assumption that the system operates in feedback mode in the future m time steps and without feedback mode after the time $k+m$ (Curry, 1969). This assumption is equivalent to the approximation of the probability densities for eq. (3.10) by

$$\rho_k = \rho_k^m = \left\{ \mathrm{p}[\boldsymbol{x}(k+i),\ \boldsymbol{p}(k+i) \middle| \Im_{k+i}] = \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) \middle| \Im_{k+i}],\ i = 0,\ldots,m \right\},$$

where

$$\mathrm{p}[\boldsymbol{x}(k+m+j), \boldsymbol{p}(k+m+j) \middle| \Im_{k+m+j}] = \mathrm{p}[\boldsymbol{x}(k+m+j), \boldsymbol{p}(k+m+j) \middle| \Im_{k+m}],$$
$$j = 1, \ldots, N-k-m-1, \text{ for } k < N-m-1, \tag{3.18}$$

and without any approximation for $k \ge N - m$. This assumption results in a suboptimal dual control scheme that is difficult to derive. The ρ-approximation, as in eq. (3.18), also coincides with the one for the OLF policy, eq. (3.12), if $m = 0$.

The wide-sense dual (WSD) and the utility-costs (UC) control policies (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985) are based on other approximations. The WSD control policy uses a linearization of the system equations around the nominal trajectory of the system where the OLF control policy is used. The utility costs approach (Bayard and Eslami, 1985), where various control policies may be used as a nominal trajectory of the systems for further linearization or other approximations, can be considered as a generalization of the WSD policy. Various implicit dual control polices were suggested that use different kinds of approximations and linearization (Kwakernaak, 1965; MacRae, 1975; Norman, 1976; Birmiwal, 1994; Pronzato et al., 1996; Kulcsar et al., 1996; Lindoff et al., 1999). These approaches provide dual control with improved performance, but significant computational requirements in real time restrict their practical applicability.

3.5. Explicit Dual Control

Various explicit dual control algorithms have received considerable attention (Allison, et al., 1995a; Chan and Zarrop, 1985; Milito et al., 1982; Wittenmark, 1975a; Wittenmark and Elevitch, 1985) as can be seen in Figure 3.8. They are based on the minimization of cost functions of the form

$$J_k^{\mathrm{e}} = J_k^{\mathrm{c}} + \lambda J_k^{\mathrm{a}},\ \lambda \ge 0, \tag{3.19}$$

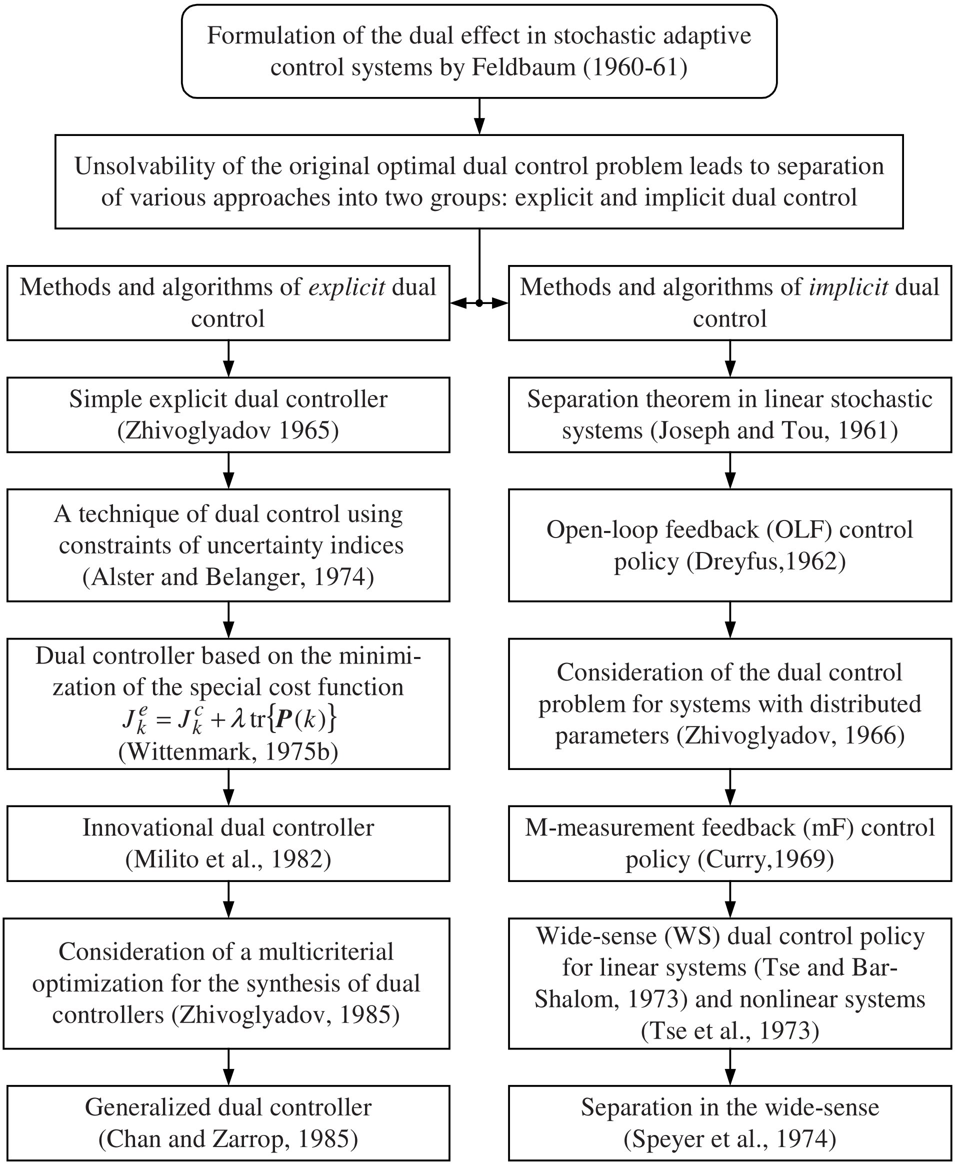

Figure 3.8. Historical stages of the development of dual control theory

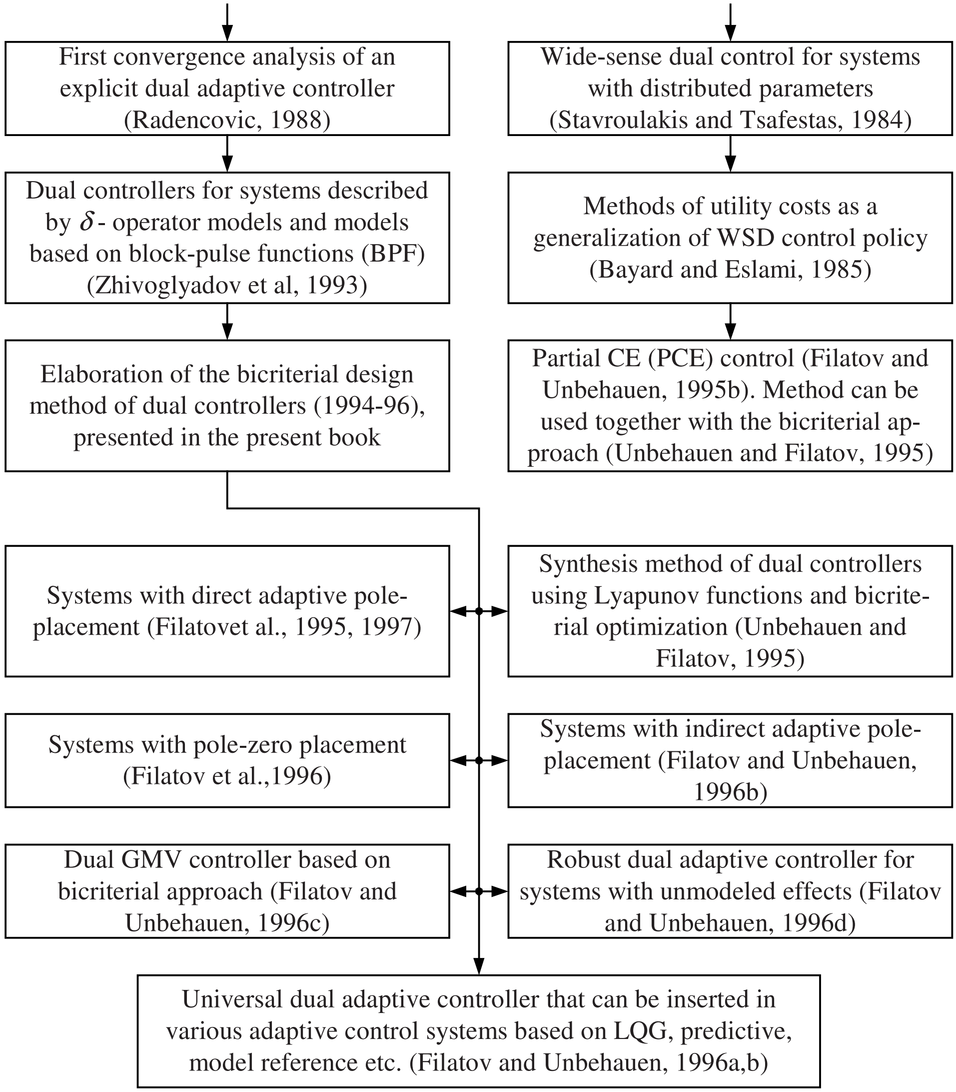

Figure 3.8. (continued)

where J_k^{c} is given by eq. (2.15) and J_k^{a} by eq. (2.17) or (2.18). It should be noted that the requirement of the sufficiency condition for the optimum of the cost function as in eq. (3.19),

$$\frac{\partial^2 J_k^{\mathrm{e}}}{\partial u(k)^2} > 0 \tag{3.20}$$

constrains the parameter λ in many cases. A simple application example for this approach was given by Milito et al. (1982). These algorithms were developed for simple system models and a structure of STR type. An application of these approaches to design a dual version of the GMV controller was given by Chan and Zarrop (1985). The shortcoming of this approach arises from the fact that the magnitude of the excitation cannot be controlled by means of the parameter λ. The amplitude of excitations varies significantly, and the control signal takes values in an interval between cautious and CE control, as shown by Milito et al. (1982). This property leads to unsatisfactory performance and is investigated in detail in Chapter 4. The bicriterial approach, which is introduced in Chapter 4 (an example has already been given in Sec. 2.3), is free from this shortcoming, and the magnitude of excitations is determined directly by the parameter $\theta(k)$, as indicated in eq. (2.20).

Some dual controllers based on minimization of one cost function with constraints for other cost function were presented by Alster and Belanger (1974) and Bodyansky and Solovjova (1987), but the advantages of these approaches over the ones with the cost functions as in eq. (3.19) are not clearly indicated.

It should be noted that dual controllers could be derived with various uncertainty indices J_k^{a} instead of eq. (2.22), for the construction of the cost function, eq. (3.19). It was suggested by Zhivoglyadov (1965) to use $\boldsymbol{P}(k)$, the covariance of the unknown parameters. In the paper of Wittenmark (1975b), the cost function

$$J_k^{\mathrm{a}} = tr\{\boldsymbol{P}(k+1)\} \tag{3.21}$$

was used, where $\boldsymbol{P}(k)$ is the covariance matrix of the estimation error. Goodwine and Payne (1977) applied

$$J_k^{\mathrm{a}} = -\frac{\det\{\boldsymbol{P}(k)\}}{\det\{\boldsymbol{P}(k+1)\}}, \tag{3.22}$$

and for experiment design (not for dual control)

$$J_k^{\mathrm{a}} = \log(\det\{\boldsymbol{P}(k+1)\}), \tag{3.23}$$

while Wittenmark and Elevitch (1985) as well as Allison et al. (1995a) used the covariance of the estimate of the first parameter b_1 of the ARX model of the system. Instead of the well-known innovational cost measure of Milito et al. (1982), see also eq. (2.22), the function

$$J_k^{\mathrm{a}} = tr\left\{\boldsymbol{P}^{-1}(k)\boldsymbol{P}(k+1)\right\} \tag{3.24}$$

can be used in the bicriterial design method. In the case of drifting stochastic parameters

$$J_k^{\mathrm{a}} = tr\left\{\boldsymbol{P}^{-1}(k+1|k)\boldsymbol{P}(k+1)\right\} \tag{3.25}$$

can be used instead. The cost function according to eq. (3.23) can be easily used for the controller design in the framework of the bicriterial design method, whereas this cost function results in great computational difficulties for the approach according to eq. (3.19).

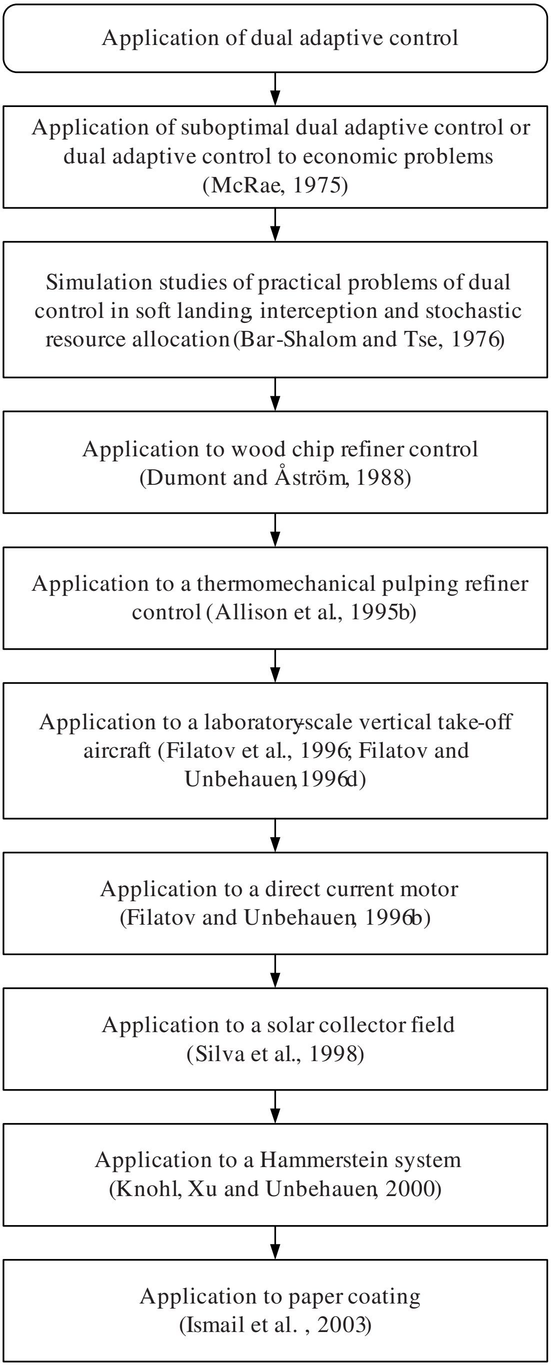

Figure 3.9. Historical stages of the application of adaptive dual control

3.6. Brief History of Dual Control and its Applications

Important stages of the development of adaptive dual control, and its application, are presented in chronological order in Figures 3.8 and 3.9, respectively. Naturally, the presented diagrams cannot include all results, and many different adaptive dual controllers have appeared in the literature since the first work of Feldbaum (1960-61) on dual control. It should be noted that this book presents results in the development and application of adaptive dual control that have been obtained during the last few years. Special attention is given to practical aspects of adaptive and dual control as well as to applications and development of modern software.

The most important stages in the development of adaptive dual control were

- the discovery of the dual effect in stochastic adaptive systems,
- the separation of basic principles for the design of dual controllers into two groups: implicit and explicit dual control, which are based on approximations and reformulations of the dual control problem, and
- the elaboration of the bicriterial design method.

The development of adaptive dual control in systems with nonstochastic uncertainties was also an important step (Veres and Norton, 1993; Veres, 1995). It should be also noted that the block-pulse function (BPF) models and δ-operator models are important for the development of modern software for real-time adaptive dual controllers with a high sampling rate.

4. BICRITERIAL SYNTHESIS METHOD FOR DUAL CONTROLLERS

A simple example that demonstrates the bicriterial approach was given in Section 2.3. Here, the bicriterial approach is applied to the derivation of the dual versions of the self-tuning regulator (STR) and the adaptive generalized minimum variance (GMV) controller. The designed controllers are compared with standard versions of the controllers based on the CE assumption as well as with known explicit dual controllers. At first, some remarks about least-squares parameter estimation for SISO systems with time-varying parameters are given.

4.1. Parameter Estimation

4.1.1. Algorithms for Parameter Estimation

Consider a discrete-time SISO system with time-varying parameters described by

$$y(k+1) = b_1(k)u(k) + \ldots + b_m(k)u(k-m+1) - a_1(k)y(k) + \ldots$$
$$-a_n(k)y(k-n+1) + \xi(k)\,, \tag{4.1}$$

where $y(k)$ is the system output and $u(k)$ the control signal. The disturbance $\xi(k)$ is a white noise sequence with zero mean and variance σ_ξ^2; σ_ξ is the standard deviation; k is the discrete time index; $a_i(k)$ and $b_j(k)$ for $i = 1,\ldots,n,$ and $j = 1,\ldots,m$ are the unknown time-varying plant parameters. Equation (4.1) can be written in vector form as

$$y(k+1) = \boldsymbol{p}^{\mathrm{T}}(k)\boldsymbol{m}(k) + \xi(k)\,, \tag{4.2}$$

where

$$\boldsymbol{p}(k) = \left[b_1(k)\ldots b_m(k) \;\vdots\; -a_1(k)\ldots -a_n(k)\right]^{\mathrm{T}} \tag{4.3}$$

and

$$\boldsymbol{m}(k) = \left[u(k)\ldots u(k-m+1) \;\vdots\; y(k)\ldots y(k-n+1)\right]^{\mathrm{T}}. \tag{4.4}$$

Consider the case when an additional stochastic parameter drift representing a Wiener process (see, for example, Alster and Belanger, 1974) occurs, described by

$$\boldsymbol{p}(k+1) = \boldsymbol{p}(k) + \boldsymbol{\varepsilon}(k)\,, \tag{4.5}$$

where the noise vector $\boldsymbol{\varepsilon}(k)$ and the scalar $\xi(k)$ in eqs. (4.5) and (4.2), respectively, are independent variables and have the same properties as the corresponding terms in eqs. (2.1) and (2.2). The covariance matrix for the noise $\boldsymbol{\varepsilon}(k)$ is denoted by $\boldsymbol{Q}_\varepsilon(k)$. A Kalman filter can be used for the estimation of the parameter vector $\boldsymbol{p}(k)$ in the form

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 33–54, 2004.

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1)e(k+1)\,, \tag{4.6}$$

$$e(k+1) = y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)\,, \tag{4.7}$$

$$\boldsymbol{q}(k+1) = \boldsymbol{P}(k)\boldsymbol{m}(k)[\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}(k) + \sigma_{\xi}^{2}]^{-1}\,, \tag{4.8}$$

$$\boldsymbol{P}(k+1) = \boldsymbol{P}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k) + \boldsymbol{Q}_{\varepsilon}(k)\,, \tag{4.9}$$

where

$$\hat{\boldsymbol{p}}(k) = \mathrm{E}\left\{ \boldsymbol{p}(k) \middle| \Im_k \right\}\,, \tag{4.10}$$

$$\boldsymbol{P}(k) = \mathrm{E}\left\{ [\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]\,[\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}} \middle| \Im_k \right\}\,, \tag{4.11}$$

$$\Im_k = \{y(0)...y(k)\vdots u(0)...u(k-1)\},\ \Im_0 = \{y(0)\}. \tag{4.12}$$

$\Im_k$ is the set of input and output values available at time k, see also eq. (2.4). It is assumed here that the initial values $\hat{\boldsymbol{p}}(0)$ and $\boldsymbol{P}(0)$ for eqs. (4.6) to (4.9) are given.

After introducing the matrices

$$\overline{\boldsymbol{P}}(k) = \sigma_{\xi}^{-2}(k)\boldsymbol{P}(k), \quad \overline{\boldsymbol{Q}}_{\varepsilon}(k) = \sigma_{\xi}^{-2}(k)\boldsymbol{Q}_{\varepsilon}(k), \tag{4.13}$$

eqs. (4.8) and (4.9) become

$$\boldsymbol{q}(k+1) = \overline{\boldsymbol{P}}(k)\boldsymbol{m}(k)\ [\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\boldsymbol{m}(k) + 1]^{-1} \tag{4.14}$$

and

$$\overline{\boldsymbol{P}}(k+1) = \overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k) + \overline{\boldsymbol{Q}}_{\varepsilon}(k). \tag{4.15}$$

The above parameter estimation algorithm, according to eqs. (4.6), (4.7), (4.10), (4.14) and (4.15), is the generalization of the RLS algorithm and its well-known versions for systems with time-varying parameters: RLS with forgetting and constant trace modification. It can easily be verified that eq. (4.15) is identical to the RLS algorithm when $\overline{\boldsymbol{Q}}_{\varepsilon}(k) \equiv 0$. The RLS algorithm with forgetting factor $0 < \alpha \le 1$ will be obtained when eq. (4.15) takes the form

$$\overline{\boldsymbol{P}}(k+1) = \frac{1}{\alpha}\ [\overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)]\ . \tag{4.16}$$

Thus eq. (4.15) has the same form as eq. (4.15) if

$$\overline{\boldsymbol{Q}}_{\xi}(k) = \frac{1-\alpha}{\alpha}\,[\overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\,\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)]\ . \tag{4.17}$$

The constant trace modification, where $\mathrm{tr}[\overline{\boldsymbol{P}}(k+1)] = \mathrm{tr}[\overline{\boldsymbol{P}}(k)]$, can be obtained by appropriately choosing the matrix $\overline{\boldsymbol{Q}}_{\xi}(k)$ in eq. (4.15). In this case, the formula is

$$\overline{\boldsymbol{Q}}_{\xi}(k) = \frac{\mathrm{tr}\,[\boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)]}{\mathrm{tr}\,[\overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)]}\,[\overline{\boldsymbol{P}}(k) - \boldsymbol{k}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)]\,. \tag{4.18}$$

From the above discussion follows that the RLS algorithm, with a forgetting factor or a constant trace modification, provides the optimal Kalman filter estimates for systems with *Wiener parameter drift* described by eq. (4.5), and with the covariance matrices for the drift noise $\varepsilon(k)$ as defined by eqs. (4.17), (4.18) and (4.13). From the fact that the RLS algorithm with forgetting or constant trace modification has been successfully applied to numerous practical problems, it can be concluded that modeling the parameter drift as a Wiener process is indeed a useful assumption, wherever feasible.

4.1.2. Simulation Example of Parameter Estimation

Consider the following stable second-order plant

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}\ ,$$

where T_2 is constant and K and T_1 are time-varying as follows:

$$K(t) = 13 - \frac{1}{75}t\ ,$$

$$T_1(t) = 1.6\ \sin\left(\frac{2\pi}{900}\ t\right),$$

$$T_2(t) = \mathrm{const} = 4\ .$$

This time-varying plant can also be represented in the time domain as

$$T_1(t)T_2(t)\ddot{y}(t) + [T_1(t) + T_2(t)]\,\dot{y}(t) + y(t) = K(t)u(t)\ .$$

After discretizing with a sampling time of 3 seconds using a zero-order hold, the discrete transfer function

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

is obtained, where the parameter values are as shown in Figure 4.1. The parameters correspond to the parameters of the model, according to eqs. (4.1) to (4.5). The input and output signals are indicated in Figure 4.2. As initial conditions and disturbances, the following values have been selected:

$$\hat{\boldsymbol{p}}(0) = [0\ \ 0\ \ 0\ \ 0],\ \ \boldsymbol{P}(0) = \mathbf{I},\ \ \sigma_{\xi}^{2}(k) \equiv 0.1\,,\ \boldsymbol{Q}_{\varepsilon}(k) \equiv 0.001\mathbf{I}\ .$$

The simulation results for the estimated parameters are shown in Figure 4.3 obtained by the linear estimator, according to eqs. (4.6) to (4.9).

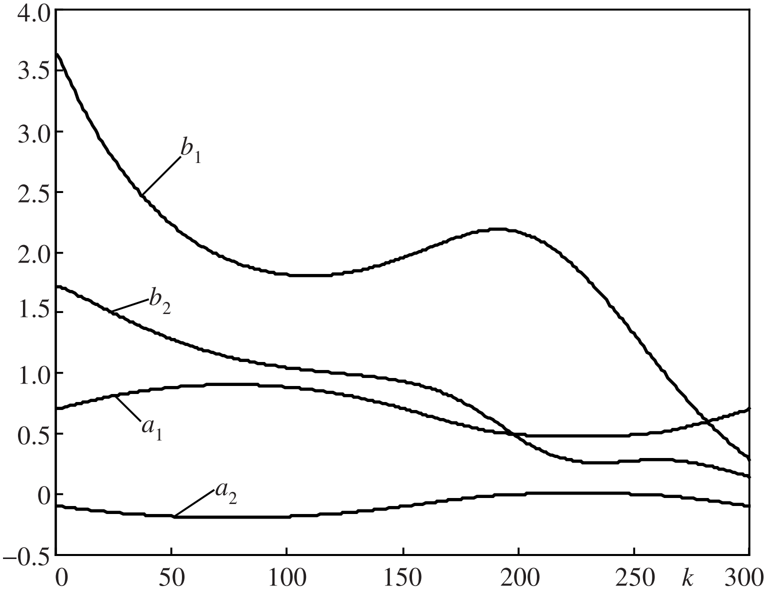

Figure 4.1. Parameter drift of the corresponding discrete-time model (Δt=3 seconds)

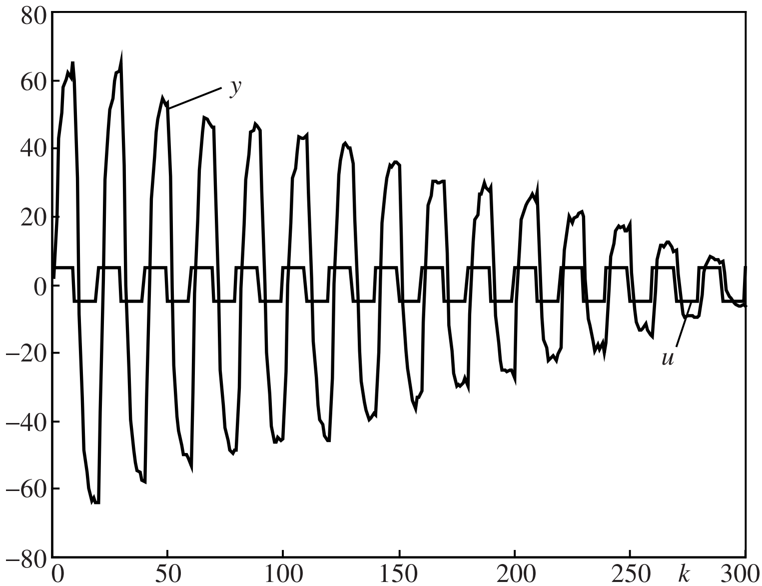

Figure 4.2. Input and output signals

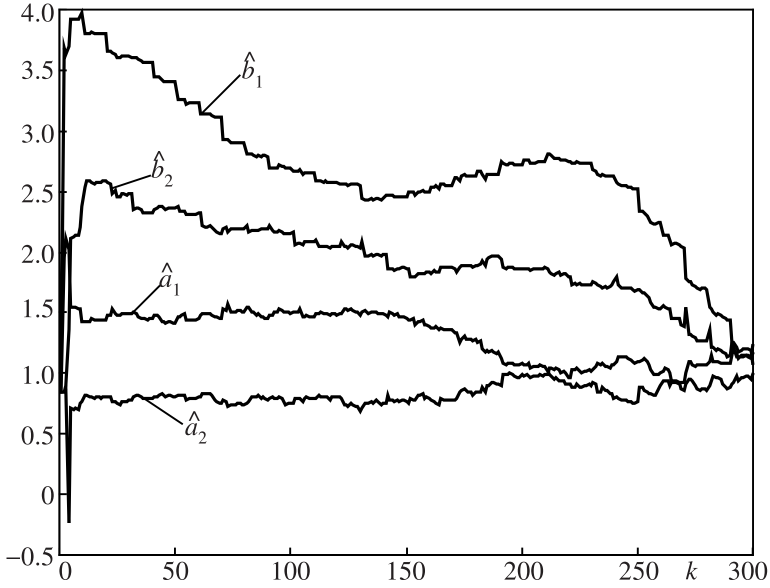

Figure 4.3. Estimates of the linear estimator

4.2. The Bicriterial Synthesis Method and the Dual Version of the STR

As mentioned in Section 2.3, the bicriterial approach is based on the minimization of two cost functions, which correspond to the two control goals of dual control. This sequential minimization is depicted in Figure 4.4. First, the control losses J_k^{c} are minimized, and the result is the cautious control action $u_{\mathrm{c}}(k)$. Then the second cost function J_k^{a} is minimized in the domain Ω_k around the cautious control. The resulting value is the dual control $u(k)$ and is based on the compromise of minimizing these two cost functions; the magnitude of the excitation component in the control signal is determined by the size of the domain Ω_k. The Pareto set, indicated in Figure 4.4, is the set where both cost functions cannot be minimized since the minimization of one leads to an increase in the second cost.

In accordance with the bicriterial approach, the following two performance indices are introduced to derive the dual control law for the system described by eqs (4.1) and (4.5):

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{ [w(k+1) - y(k+1)]^2 + r[u(k) + q_1 u(k-1) + \ldots + q_{n_Q} u(k-n_Q)]^2 \middle| \Im_k \right\},$$
$$r \geq 0\,, \tag{4.19}$$

and

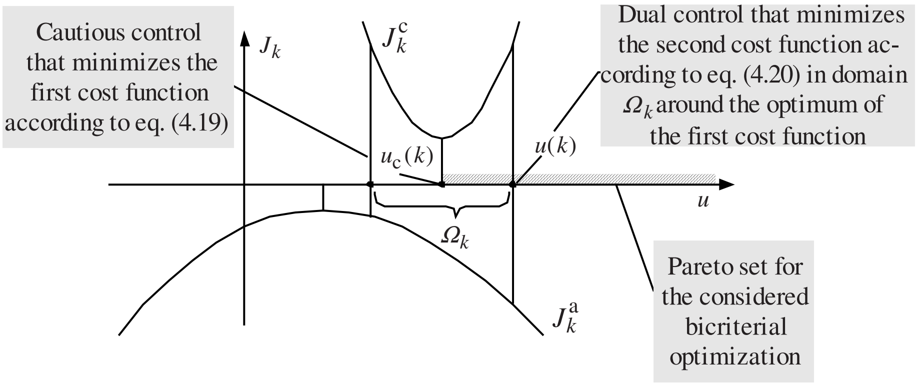

Figure 4.4. Sequential minimization of two cost functions in the bicriterial approach.

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{ [y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 \middle| \Im_k \right\}, \tag{4.20}$$

where in the special case of the STR q_i are the coefficients of the polynomial

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \ldots + q_{n_Q} z^{-n_Q}, \ n_Q \geq 0, \tag{4.21}$$

with $q_0 = 1$. The polynomial $Q(z^{-1})$ is introduced to control nonminimum phase plants by appropriate selection of its parameters q_i and the parameter r, as shown below (Clarke and Gawthrop, 1975). The considered performance index is more general than the one that was used for the derivation of the original self-tuning regulator (Åström et al., 1977). The first performance index, eq. (4.19), is used for control purposes to minimize the deviation of the system output $y(k)$ from the prescribed value $w(k)$ with weighted control effort, whereas the second one, according to eq. (4.20), is used for the acceleration of the parameter estimation process (Filatov and Unbehauen, 1994, 1995a) by increasing the innovating value of eq. (4.6). The dual controller is obtained after the bicriterial optimization problem is solved with the constraints Ω_k for eq. (4.20):

$$u(k) = \underset{u(k)\in\Omega_k}{\operatorname{argmin}}\ J_k^{\mathrm{a}}, \tag{4.22}$$

$$\Omega_k = [u_{\mathrm{c}}(k) - \theta(k); u_{\mathrm{c}}(k) + \theta(k)], \tag{4.23}$$

$$\theta(k) = \eta \ \operatorname{tr}\{\boldsymbol{P}(k)\}, \eta \geq 0, \tag{4.24}$$

$$u_{\mathrm{c}}(k) = \underset{u(k)}{\operatorname{argmin}}\, J_k^{\mathrm{c}}, \tag{4.25}$$

where $u_{\mathrm{c}}(k)$ is the cautious control, which is obtained after the minimization of eq. (4.19). The constraints Ω_k define the value of the excitation signal for each time step k. They are symmetrically located around the optimal solution according to eq. (4.25), as indicated in Figure 4.4. Therefore, according to eqs. (4.23) and (4.24), the amplitude of the excitation depends on the value of the parameter η and the trace of the covariance matrix $\boldsymbol{P}(k)$.

The following notations is introduced

$$\boldsymbol{p}(k) = [b_1(k) \vdots \boldsymbol{p}_0^{\mathrm{T}}(k)]^{\mathrm{T}}, \quad \boldsymbol{m}(k) = [u(k) \vdots \boldsymbol{m}_0^{\mathrm{T}}(k)]^{\mathrm{T}}, \tag{4.26}$$

and

$$\boldsymbol{P}(k) = \begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1 p_0}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix}. \tag{4.27}$$

After substituting eq. (4.2) into eq. (4.19), taking the expectation (compare Appendix D) and using the terms of eqs. (4.26) and (4.27), the cost function can be transferred in the following form

$$\begin{aligned} J_k^{\mathrm{c}} &= [\hat{b}_1^2(k) + p_{b_1}(k)]u^2(k) - 2\hat{b}_1(k)w(k+1)u(k) \\ &\quad + 2(\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\hat{b}_1(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k))\boldsymbol{m}_0(k)u(k) \\ &\quad + ru^2(k) + 2r\bar{u}(k-1)u(k) + \bar{c}_1, \end{aligned} \tag{4.28}$$

where

$$\bar{u}(k-1) = q_1 u(k-1) + \ldots + q_{n_Q} u(k - n_Q) \tag{4.29}$$

and $\bar{c}_1$ is independent of $u(k)$. Differentiation of the eq. (4.28) gives the condition for the minimum as

$$\begin{aligned} \frac{\partial J_k^{\mathrm{c}}}{\partial u(k)} &= 2[\hat{b}_1^2(k) + p_{b_1}(k)]u(k) - 2\hat{b}_1(k)w(k+1) \\ &\quad + 2(\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\hat{b}_1(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k))\boldsymbol{m}_0(k) \\ &\quad + 2ru(k) + 2r\bar{u}(k-1) = 0. \end{aligned}$$

Thus the minimization of eq. (4.28) gives the cautious control action as

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_1(k)w(k+1) - [\hat{b}_1(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)]\,\boldsymbol{m}_0(k) + r\bar{u}(k-1)}{\hat{b}_1^2(k) + p_{b_1}(k) + r}. \tag{4.30}$$

Minimization of eq. (4.22), with constraints according to eq. (4.23), leads directly to

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\operatorname{sign}\left\{J_k^a[u_{\mathrm{c}}(k) - \theta(k)] - J_k^a[u_{\mathrm{c}}(k) + \theta(k)]\right\}, \tag{4.31}$$

see also eqs. (2.20) and (2.21). After substituting of eq. (4.2) into eq. (4.20) and taking the expectation (compare Appendix D) using eqs. (4.26) and (4.27), we have as

$$J_k^{\mathrm{a}}[u(k)] = -p_{b_1}(k)u^2(k) - 2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) - \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{p_0}(k)\boldsymbol{m}_0(k) - \sigma_\xi^2, \tag{4.32}$$

$$
\begin{aligned}
&J_k^{\mathrm{a}}(u_{\mathrm{c}}(k)-\theta(k))-J_k^{\mathrm{a}}(u_{\mathrm{c}}(k)+\theta(k)) \\
&\qquad=-p_{b_1}(k)[u_{\mathrm{c}}(k)-\theta(k)]^2-2\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)(u_{\mathrm{c}}(k)-\theta(k)) \\
&\qquad\quad+p_{b_1}(k)[u_{\mathrm{c}}(k)+\theta(k)]^2+2\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)(u_{\mathrm{c}}(k)+\theta(k)) \\
&\qquad=4p_{b_1}(k)\theta(k)u_{\mathrm{c}}(k)+4\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\theta(k)\,.
\end{aligned}
\tag{4.33}
$$

Substituting eq. (4.33) into (4.31) provides the adaptive dual control law

$$
u(k)=u_{\mathrm{c}}(k)+\theta(k)\operatorname{sgn}[p_{b_1}(k)u_{\mathrm{c}}(k)+\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)]\,. \tag{4.34}
$$

Thus, the complete adaptive dual controller is determined by eqs. (4.6) to (4.9), (4.30) and (4.34).

It should be noted that the stability of the designed self-tuning control system depends on the roots of the characteristic polynomial (Clarke and Gawthrop, 1975)

$$
B(z^{-1})+rQ(z^{-1})A(z^{-1})\,, \tag{4.35}
$$

where

$$
B(z^{-1})=b_1z^{-1}+\ldots+b_mz^{-m}\,, \tag{4.36}
$$

$$
A(z^{-1})=1+a_1z^{-1}+\ldots+a_nz^{-n}\,, \tag{4.37}
$$

and the selected value for the parameter of $Q(z^{-1})$ and for r.

The difference between the controllers obtained from the bicriterial approach and the explicit dual control approach is illustrated in Figure 4.5. The explicit dual control approach was described in Section 3.5. In Figure 4.5, three cases of possible locations of the optima of the cost functions J_k^{c} and J_k^{a} and the corresponding location of the uncertainty index J_k^{e}, see eq. (3.19), are shown (close, distant and the same locations). It can be clearly seen that the excitations (the deviation of the control signal from cautious control $u_{\mathrm{c}}(k)$ to minimize J_k^{a}) of the explicit dual control $u_{\mathrm{e}}(k)$ are different in every case (zero in the third case), but the bicriterial dual control $u(k)$ provides the same excitation in all cases. The cause of this is the irregular change of the locations of the optimum points at every control instant: the explicit dual controller provides worse performance as can be seen from the computer simulation given below in the Section 4.4. It should be noted that both of these approaches provide a solution to the bicriterial optimization problem, and both solutions belong to the Pareto set of solutions.

4.3. Design of the Dual Version of the GMV Controller

The adaptive generalized minimum-variance (GMV) controller was suggested by Clarke and Gawthrop (1975). This controller can be applied to nonminimum phase plants with colored noise disturbances and time delay. The approach is flexible and makes synthesizing systems with various structures possible. The explicit dual control approach was applied to the GMV controller by Chan and Zarrop (1985). The dual version of an adaptive GMV controller will be designed below using the bicriterial approach.

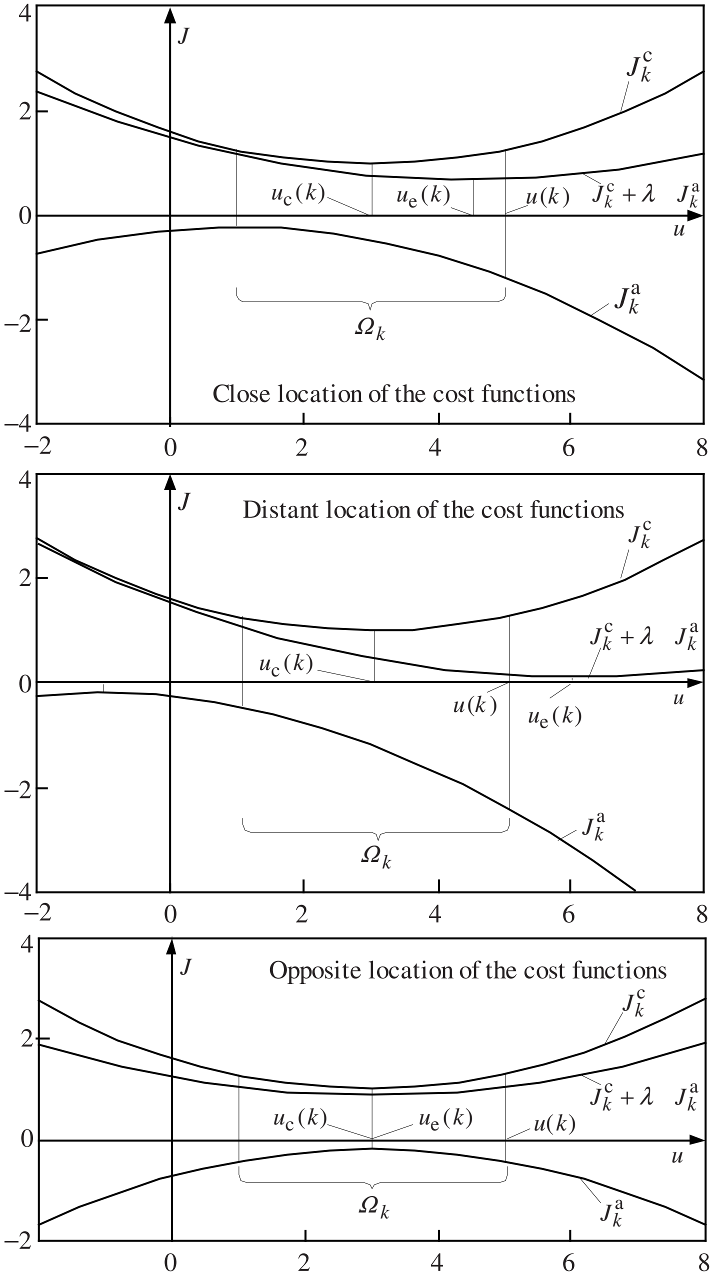

Figure 4.5. Comparison of bicriterial and explicit dual control approaches

Consider the system described by the following CARMA model with time delay and constraint parameters:

$$A(z^{-1})Y(z) = z^{-d}B(z^{-1})U(z) + C(z^{-1})\Xi(z)\,,\ d \geq 1\,, \tag{4.38}$$

where $Y(z)$, $U(z)$ and $\Xi(z)$ are the z-transforms of the output $y(t)$, the control $u(t)$ and the disturbance $\xi(t)$, respectively, d being the time delay, the polynomials $A(z^{-1})$, $B(z^{-1})$ are as in eqs. (4.36) and (4.37), and

$$C(z^{-1}) = 1 + c_1 z^{-1} + ... + c_{n_\mathrm{C}} z^{-n_\mathrm{C}}\,. \tag{4.39}$$

So far as the time delay d is not zero, $B(z^{-1})$ assumes the polynomial

$$B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_{n_\mathrm{B}} z^{-n_\mathrm{B}}\,,\ b_0 \neq 0\,. \tag{4.40}$$

The auxiliary output of the system can be represented as

$$Y_\mathrm{a}(z) = P(z^{-1})Y(z) + z^{-d}Q(z^{-1})U(z) - R(z^{-1})W(z)\,, \tag{4.41}$$

where $Y_\mathrm{a}(z)$ and $W(z)$ are the z-transforms of the auxiliary signal $y_\mathrm{a}(k)$ and the setpoint $w(k)$, respectively,

$$P(z^{-1}) = p_0 + p_1 z^{-1} + ... + p_{n_\mathrm{P}} z^{-n_\mathrm{P}}\,,\ n_\mathrm{P} \geq 0\,, \tag{4.42}$$

$$R(z^{-1}) = 1 + r_1 z^{-1} + ... + r_{n_\mathrm{R}} z^{-n_\mathrm{R}}\,,\ n_\mathrm{R} \geq 0\,, \tag{4.43}$$

and the polynomial $Q(z^{-1})$ is described by eq. (4.21) with an arbitrary q_0. It is known that for stability of the closed-loop system, which minimizes the cost function

$$J_k^\mathrm{c} = \mathrm{E}\left\{[y_\mathrm{a}(k+d)]^2 \middle| \Im_k\right\}, \tag{4.44}$$

the roots of the characteristic polynomial

$$D(z^{-1}) = P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1}) \tag{4.45}$$

must be located inside the unit circle on the z-plane. Therefore, stability of the system can be provided by an appropriate selection of the coefficients of the polynomials $P(z^{-1})$ and $Q(z^{-1})$.

It follows from eqs. (4.38) and (4.41) that

$$\begin{aligned} z^d Y_\mathrm{a}(z) = {} & \frac{A(z^{-1})Q(z^{-1}) + B(z^{-1})P(z^{-1})}{A(z^{-1})} U(z) + z^d \frac{P(z^{-1})C(z^{-1})}{A(z^{-1})} \Xi(z) \\ & - R(z^{-1})W(z). \end{aligned} \tag{4.46}$$

Let us introduce the identity

$$P(z^{-1})C(z^{-1}) = A(z^{-1})L(z^{-1}) + z^{-d}G(z^{-1})\,, \tag{4.47}$$

where the orders of the polynomials L (with parameters l_i) and G (with parameters g_i) are $n_{\mathrm{L}} = d-1$ and $n_{\mathrm{G}} = n-1$, respectively. Substituting eq. (4.47) into eq. (4.41) results in

$$z^d C(z^{-1}) Y_{\mathrm{a}}(z) = F(z^{-1})U(z) + G(z^{-1})Y(z) + H(z^{-1})W(z) + z^d C(z^{-1})\bar{\Xi}(z), \tag{4.48}$$

where $q_0 + b_0 l_0 \neq 0$,

$$F(z^{-1}) = Q(z^{-1})C(z^{-1}) + B(z^{-1})L(z^{-1}), \tag{4.49}$$

$$H(z^{-1}) = -C(z^{-1})R(z^{-1}), \tag{4.50}$$

$$\bar{\Xi}(z) = L(z^{-1})\Xi(z); \tag{4.51}$$

$\bar{\Xi}(z)$ is the z-transform of the filtered disturbance $\bar{\xi}(k)$ with variance $\sigma_{\bar{\xi}}^2$. The orders of the polynomials $F(z^{-1})$ and $H(z^{-1})$ are $n_{\mathrm{F}} = n_{\mathrm{B}} + d - 1$ and $n_{\mathrm{H}} = n_{\mathrm{C}} + n_{\mathrm{R}}$ with $h_0 = -1$. First, the case of $C(z^{-1}) = 1$ is considered for controller synthesis; and, afterwards, the convergence of the desired dual controller is shown for the case of $C(z^{-1}) \neq 1$. Since the system output $y(k)$ is independent from $\bar{\xi}(k+d)$ for $C(z^{-1}) = 1$, the RLS estimator can be applied. Introducing the notations

$$\boldsymbol{p} = [f_0 \ldots f_{n_F}\, g_0 \ldots g_{n_G}\, h_2 \ldots h_{n_H}]^{\mathrm{T}} = [f_0 \vdots \boldsymbol{p}_0^{\mathrm{T}}]^{\mathrm{T}}, \tag{4.52}$$

$$\boldsymbol{m}(k) = [u(k) \ldots u(k-n_{\mathrm{F}})\, y(k) \ldots y(k-n_{\mathrm{G}})\, w(k-1) \ldots w(k-n_{\mathrm{H}})]^{\mathrm{T}} = [u(k) \vdots \boldsymbol{m}_0^{\mathrm{T}}(k)]^{\mathrm{T}} \tag{4.53}$$

and rewriting eq. (4.48) in the time domain taking into account that $h_0 = -1$, gives

$$y_{\mathrm{a}}(k+d) = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k) - w(k) + \bar{\xi}(k+d). \tag{4.54}$$

The RLS estimator with the initial covariance matrix $\boldsymbol{P}(0)$, initial estimate $\hat{\boldsymbol{p}}(0)$ and variance $\sigma_{\bar{\xi}}$ of the noise $\bar{\xi}(k)$ can be applied for calculating the parameter estimate $\hat{\boldsymbol{p}}(k)$ and covariance matrix $\boldsymbol{P}(k)$

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1)e(k+1), \tag{4.55}$$

where

$$e(k+1) = y_{\mathrm{a}}(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k-d+1) + w(k-d+1), \tag{4.56}$$

$$\begin{aligned} \boldsymbol{q}(k+1) &= \boldsymbol{P}(k-d+1)\boldsymbol{m}(k)\left[\boldsymbol{m}^{\mathrm{T}}(k-d+1)\boldsymbol{P}(k)\boldsymbol{m}(k-d+1) + \sigma_{\bar{\xi}}^2\right]^{-1} \\ &= \bar{\boldsymbol{P}}(k+1)\boldsymbol{m}(k-d+1), \end{aligned} \tag{4.57}$$

$$\boldsymbol{P}(k+1) = \boldsymbol{P}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k-d+1)\boldsymbol{P}(k) , \tag{4.58a}$$

$$\overline{\boldsymbol{P}}(k+1) = \overline{\boldsymbol{P}}(k) + \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k-d+1)\overline{\boldsymbol{P}}(k) , \tag{4.58b}$$

and the covariance matrix has the structure

$$\boldsymbol{P}(k) = \begin{bmatrix} p_{f_0}(k) & \boldsymbol{p}^{\mathrm{T}}_{f_0 p_0}(k) \\ \boldsymbol{p}_{f_0 p_0}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix} = \sigma_{\xi}^2 \overline{\boldsymbol{P}}(k) = \sigma_{\xi}^2 \begin{bmatrix} \overline{p}_{f_0}(k) & \overline{\boldsymbol{p}}^{\mathrm{T}}_{f_0 p_0}(k) \\ \overline{\boldsymbol{p}}_{f_0 p_0}(k) & \overline{\boldsymbol{P}}_{p_0}(k) \end{bmatrix}. \tag{4.59}$$

Similar to eq. (4.17), the uncertainty index for the considered GMV system

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{ [y_{\mathrm{a}}(k+d) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\,\boldsymbol{m}(k) + w(k)]^2 \middle| \Im_k \right\} \tag{4.60}$$

is introduced. The bicriterial approach is applied here according to eqs. (4.22) to (4.25) using the cost functions of eqs. (4.44) and (4.60). After substituting eq. (4.54) and taking the expectation (compare Appendix D) the minimization of eq. (4.44) gives the cautious GMV controller

$$u_{\mathrm{c}}(k) = \frac{\hat{f}_0(k)w(k) - [\hat{f}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}^{\mathrm{T}}_{f_0 p_0}(k)]\,\boldsymbol{m}_0(k)}{\hat{f}_0^2(k) + p_{f_0}(k)} \quad . \tag{4.61}$$

The dual controller will be obtained in the form of eq. (4.31). The obtained transformation of the cost function, eq. (4.60), as completed in eqs. (4.32) and (4.33), finally leads to the dual GMV controller

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\ \mathrm{sgn}[p_{f_0}(k)u_{\mathrm{c}}(k) + \boldsymbol{p}^{\mathrm{T}}_{f_0 p_0}(k)\boldsymbol{m}_0(k)] . \tag{4.62}$$

Thus, the adaptive dual GMV controller is determined by eqs. (4.55) to (4.58), (4.61) and (4.62) and has the structure of a cautious controller with the added optimal excitation of the dual controller designed in Section 4.2.

Compared with the *explicit* dual controller, also denoted as *generalized dual controller* (GDC) (Chan and Zarrop, 1985), the elaborated controller is a nonlinear one. The GDC is obtained after minimizing a cost function according to eqs. (3.19), (4.44) and (4.60), and has the form

$$u_{\mathrm{e}}(k) = \frac{\hat{f}_0(k)w(k) - [\hat{f}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + (1-\lambda)\,\boldsymbol{p}^{\mathrm{T}}_{f_0 p_0}(k)]\,\boldsymbol{m}_0(k)}{\hat{f}_0^2(k) + (1-\lambda)\,p_{f_0}(k)} \quad . \tag{4.63}$$

This controller coincides with the cautious controller having the covariance matrix reduced by the factor $(1-\lambda)$.

4.4. Computer Simulations

4.4.1. The Plant without Time Delay d=1

In the following discussion, the results of computer simulations for the suggested adaptive dual controller, according to eqs. (4.6) to (4.9), (4.30) and (4.34), are highlighted for comparison with those of an standard adaptive controller based on the CE assumption. A second-order unstable nonminimum phase plant with the discrete transfer function

$$G_y(z) = \frac{-0.0104z^{-1} - 0.0107z^{-2}}{1 - 2.1262z^{-1} + 1.1052z^{-2}}$$

is used in the simulation. The plant parameters are assumed to be unknown and constant. For the algorithm, the following values have been chosen:

$$\hat{\boldsymbol{p}}(0) = [1 \quad 1 \quad 1 \quad 1]^{\mathrm{T}}, \quad \boldsymbol{P}(0) = 0.5\mathbf{I}, \quad \sigma_\xi^2 \equiv 0.007, \quad \boldsymbol{Q}_\varepsilon(k) \equiv \mathbf{0}, \quad \eta = 0.1, \quad r = 7 \cdot 10^{-6}.$$

A dynamic filter with the transfer function

$$G_w(z) = \frac{0.2z^{-1}}{1 - 0.8z^{-1}}, \quad W(z) = G_w(z) W_1(z)$$

is used for the reference signal $W(z)$ to provide smooth system behavior, where $W_1(z)$ is the z-transform of the input signal. The polynomial for the performance index, according to eq. (4.21), is assumed to be $Q(z^{-1}) = 1 - z^{-1}$. For the control signal, upper and lower limits of ±500 have been chosen. The controller based on the CE-assumption can be obtained from eq. (4.30), when $\boldsymbol{P}(k)$ is set to $\mathbf{0}$.

The simulation results of the adaptive dual controller are shown in Figure 4.6. The simulation of the adaptive standard controller, based on the CE assumption, is portrayed in Figure 4.7. It is clear from Figures 4.6 and 4.7 that the parameter estimates converge in both cases. The adaptive dual controller provides, however, a smooth startup of the process and better control performance. It should be kept in mind that the noise component has a significant variance of σ_ξ =0.084 in the considered example.

4.4.2. GMV Controller for the Plant with Time Delay d=4

The example used for the computer simulation of the plant is the one considered in Section 4.4.1 but with a time delay of d=4, according to eq. (4.38). The adaptive dual GMV controller, according to eqs. (4.55) to (4.58), (4.61) and (4.62), is applied as well as the standard adaptive GMV controller based on the CE assumption for comparison. The plant is given by the discrete transfer function

$$G_y(z) = z^{-3} \frac{-0.0104z^{-1} - 0.0107z^{-2}}{1 - 2.1262z^{-1} + 1.1052z^{-2}}.$$

The plant parameters are assumed to be unknown and constant. The following values have been chosen for the GMV algorithm:

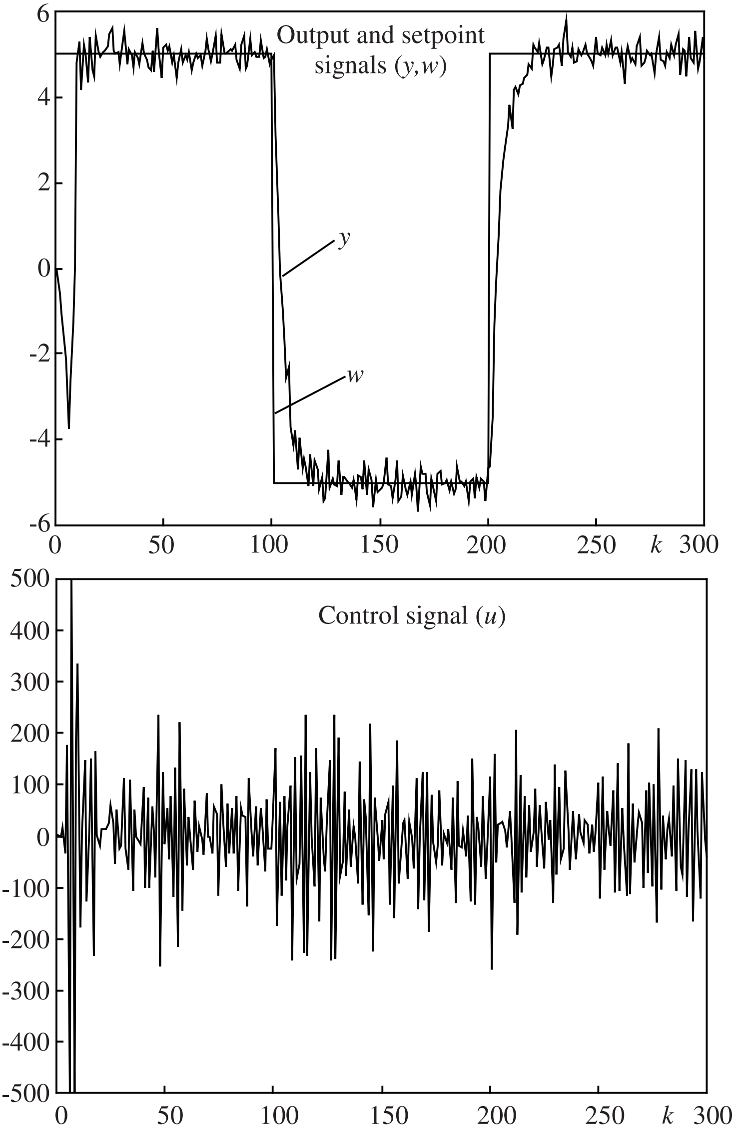

Figure 4.6. Simulation results for the adaptive dual controller

$$\hat{\boldsymbol{p}}(0)=\begin{bmatrix}0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1\end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{P}(0)=\mathbf{I}, \quad \sigma_{\xi}^{2}\equiv 0.0001,$$

$$\eta=0.5, \quad C(z^{-1})=R(z^{-1})=1, \quad Q(z^{-1})=-0.18z^{-1}+0.023z^{-2},$$

$$P(z^{-1})=6.4z^{-1}-5.4z^{-2}.$$

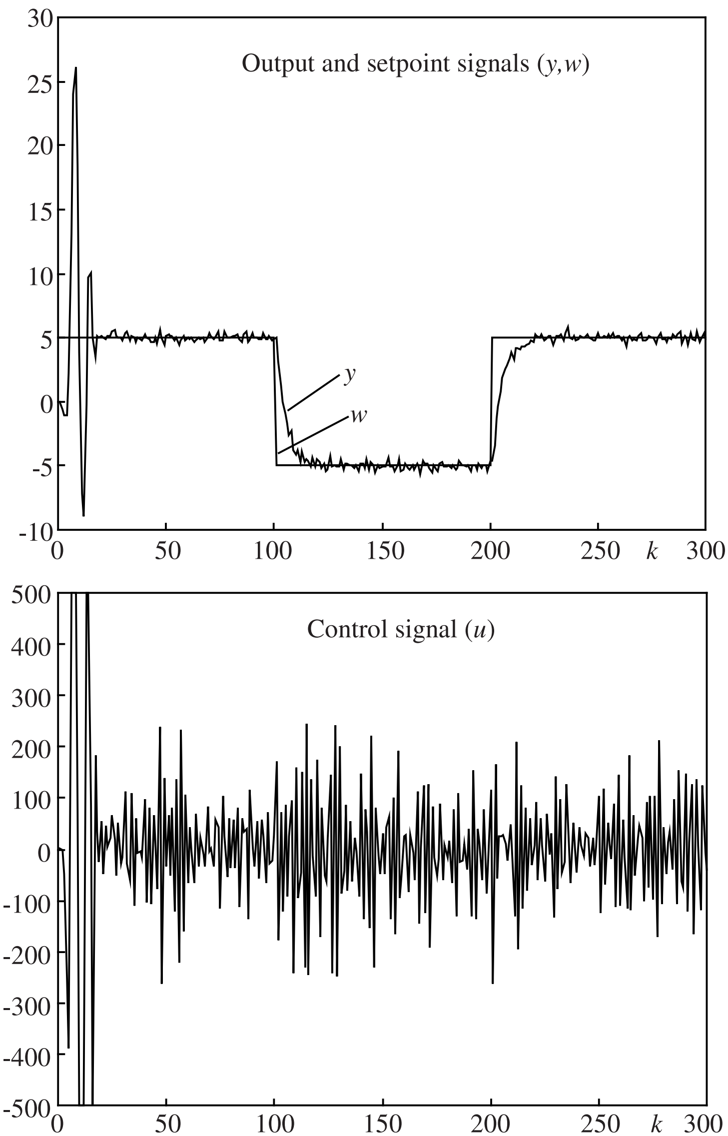

Figure 4.7. Simulation results for the adaptive controller based on the CE assumption

Upper and lower limits of ±500 are used for the control signal. The controller based on the CE assumption can be obtained from eq. (4.61), with $\boldsymbol{P}(k)$ set to $\mathbf{0}$.

The simulation of the standard adaptive GMV controller based on the CE assumption is displayed in Figure 4.8. The results of the simulation of the designed adaptive dual version of the GMV controller are shown in Figure 4.9. The simulation results for the GDC according to eq. (4.63) with parameter $\lambda = 0.5$ are shown in Fig. 4.10. As can be seen in Figures 4.8, 4.9 and 4.10, the adaptive dual controller provides better control performanc and a smaller overshoot. The control of the same plant with a larger time delay is considered in the next section.

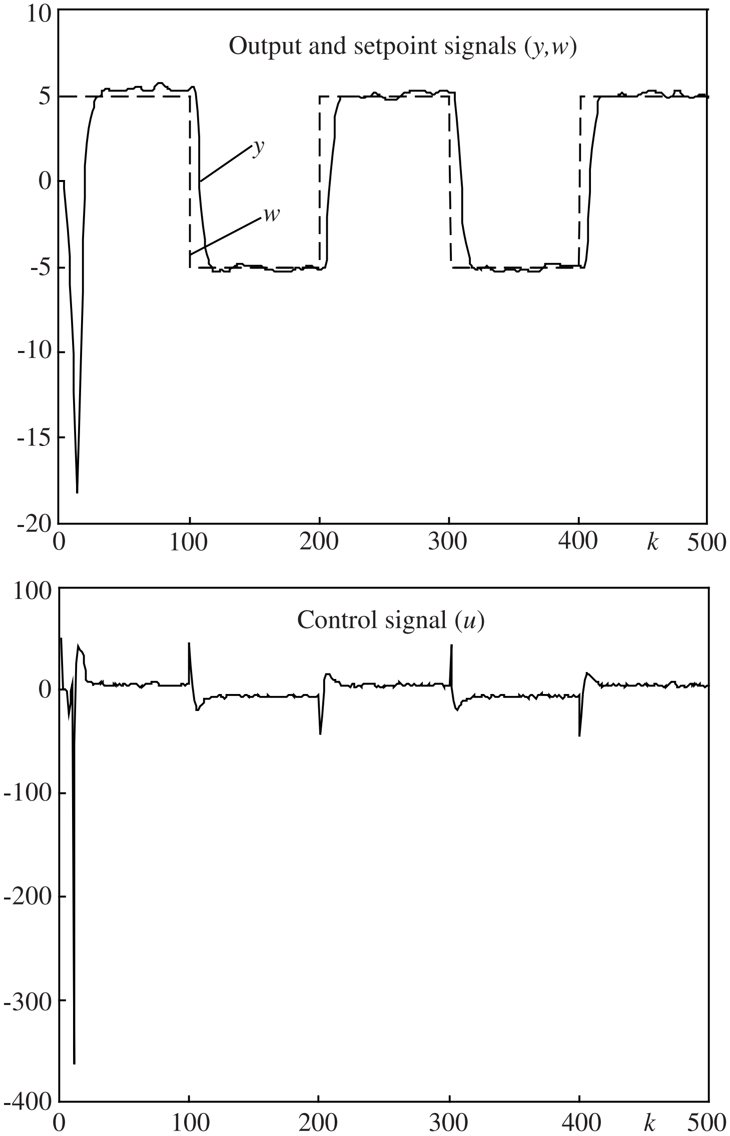

Figure 4.8. Simulation results for the standard adaptive GMV controller based on the CE assumption (d=4)

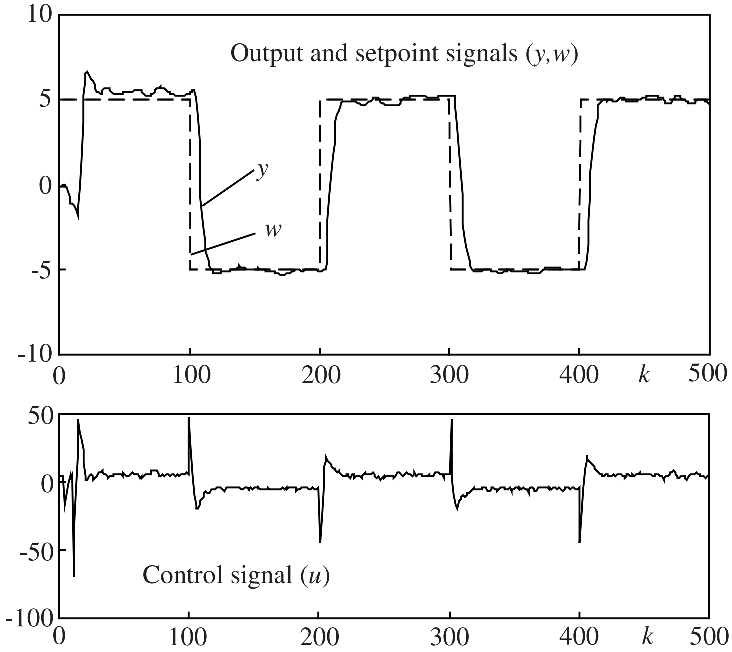

Figure 4.9. Simulation results for the adaptive dual GMV controller (d=4).

4.4.3. GMV Controller for the Plant with Time Delay d=7

Now the same plant as described above by eq. (4.38) is considered but with a larger time delay (d=7). Again, the adaptive dual GMV controller, according to eqs. (4.55) to (4.58), (4.61) and (4.62), will be applied and compared to the adaptive standard GMV controller based on the CE assumption. The plant is described by the discrete transfer function

$$G_y(z) = z^{-6} \frac{-0.0104z^{-1} - 0.0107z^{-2}}{1 - 2.1262z^{-1} + 1.1052z^{-2}}.$$

The plant parameters are assumed to be unknown and constant. The following values are chosen for the GMV controller design:

$$\hat{\boldsymbol{p}}(0) = \left[0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\ 0.5\right], \quad \boldsymbol{P}(0) = \mathbf{I}, \quad \sigma_\xi^2 \equiv 0.00001,$$

$$\eta = 0.5, \quad C(z^{-1}) = R(z^{-1}) = 1, \quad Q(z^{-1}) = -0.17z^{-1} + 0.026z^{-2},$$

$$P(z^{-1}) = 6.1z^{-1} - 5.1z^{-2}.$$

For the control signal, upper and lower limits of ±500 have again been chosen. The controller based on the CE-assumption can be obtained from eq. (4.61), with $\boldsymbol{P}(k)$ set to $\mathbf{0}$.

The simulation results for the designed standard adaptive dual GMV controller are presented in Figure 4.11. The simulation results for the adaptive dual version of the GMV controller based on the CE-assumption are shown in Figure 4.12. In Figure 4.13 the simulation results for the GDC according to eq. (4.63) with parameter $\lambda = 0.5$ are depicted. As seen in Figures 4.11 to 4.13 the dual adaptive controller provides again better control performance and a smaller overshoot than both the other two controllers.

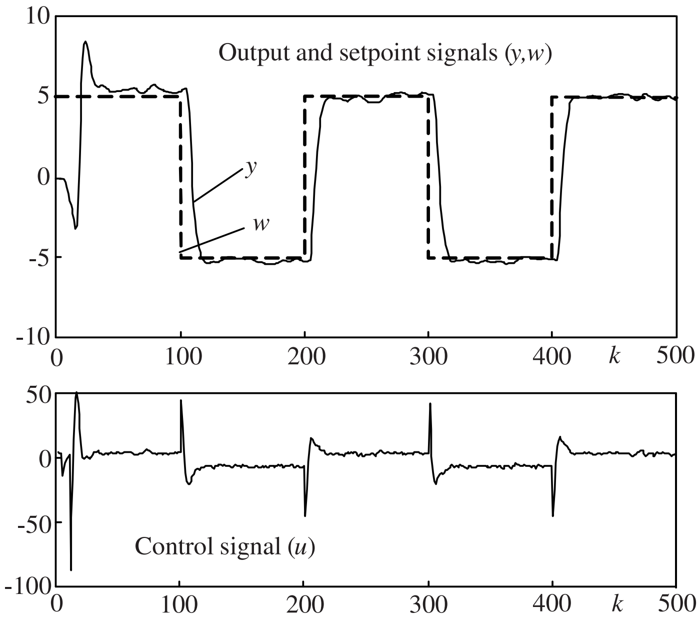

Figure 4.10. Results for the GDC (d=4)

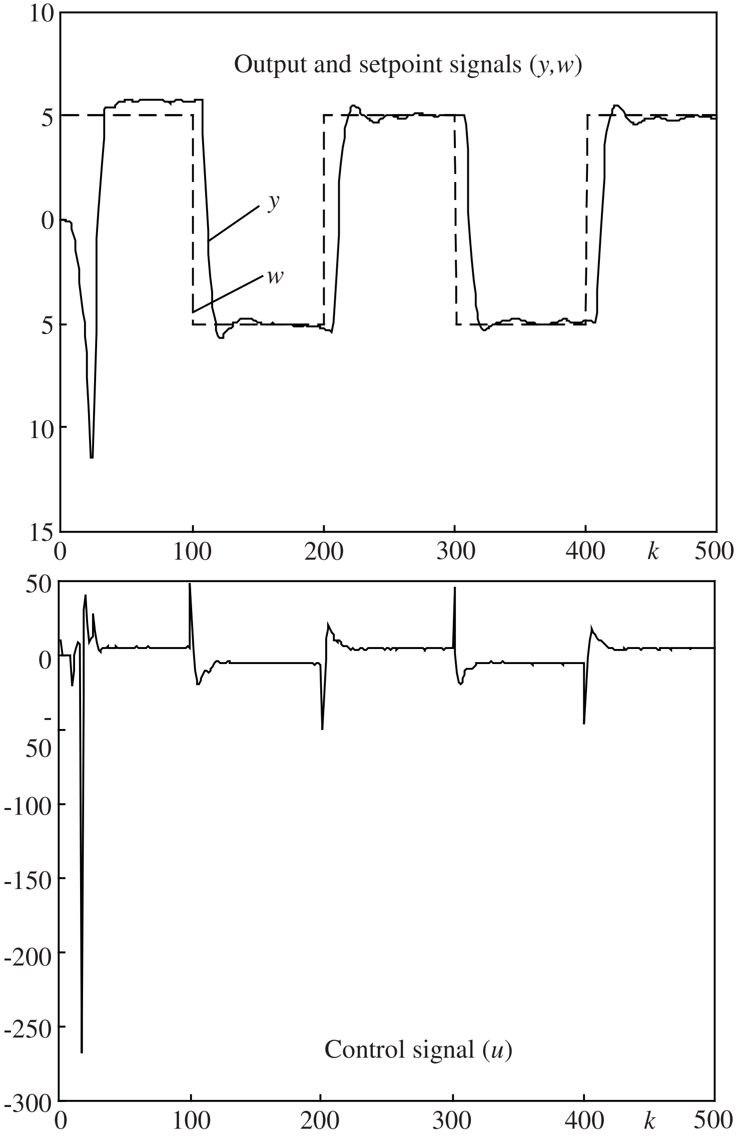

Figure 4.11. Simulation results for the standard adaptive GMV controller based on the CE assumption (d=7)

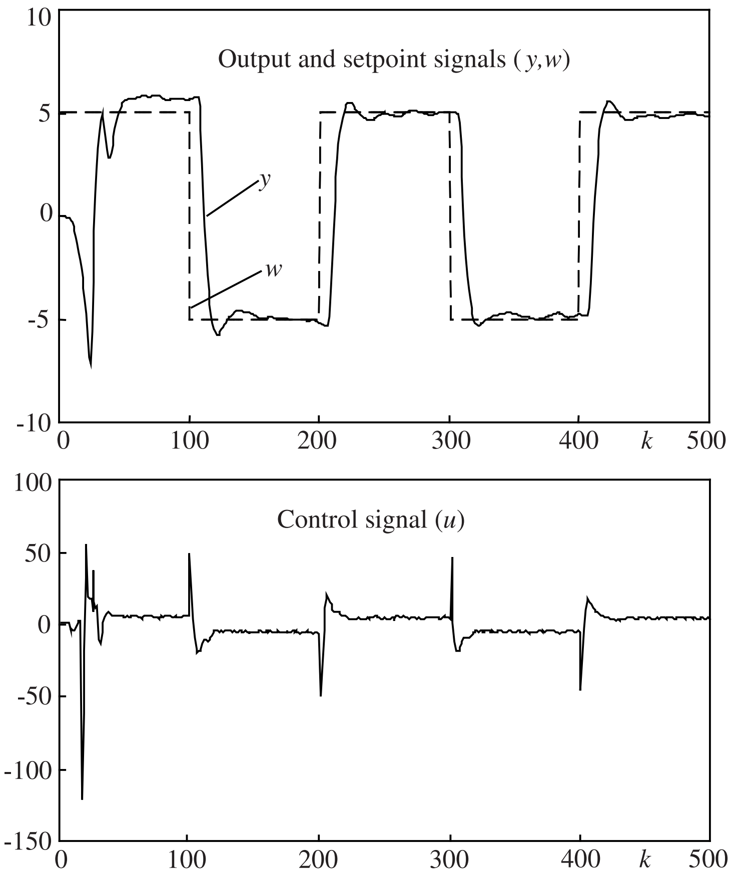

Figure 4.12. Simulation results for the adaptive dual GMV controller (d=7)

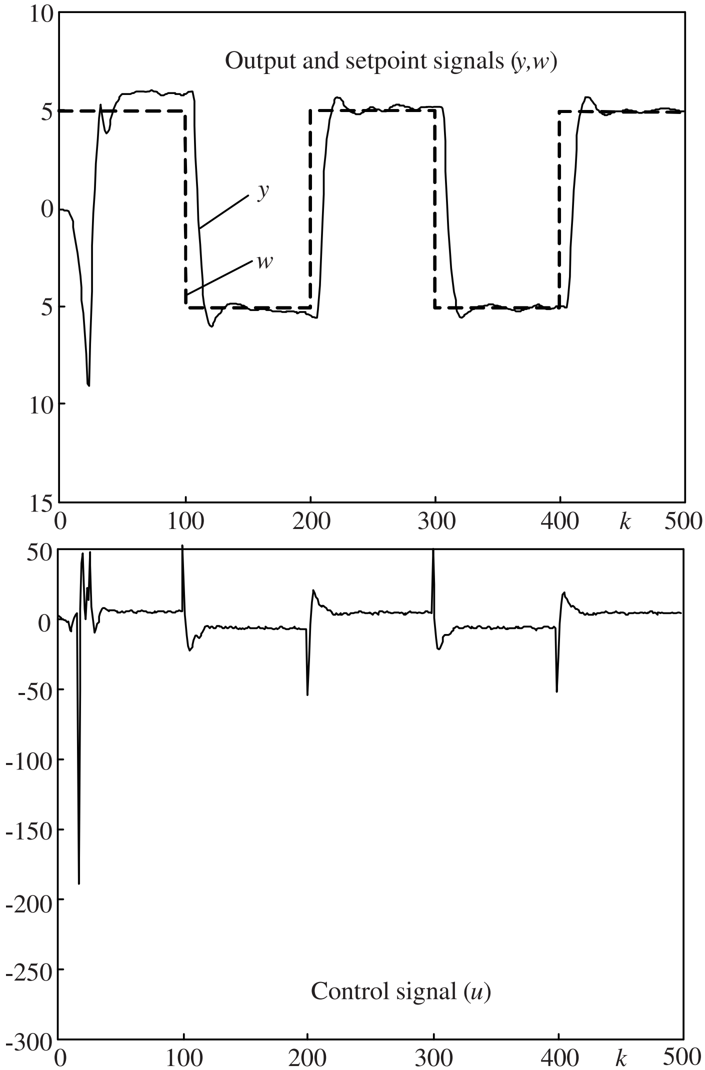

Figure 4.13. Results for the GDC (d=7)

4.5. Summary

In the present chapter the bicriterial synthesis method has been described and applied for the synthesis of the dual version of the self-tuning regulator (STR) and the generalized minimum variance (GMV) controller. The advantages of the designed controllers and the comparison with the explicit dual control approach have been illustrated. The convergence properties of the derived dual controller has been investigated by application of the martingale convergence theory discussed in the next chapter. The improved performance of the bicriterial adaptive dual controller has been demonstrated using simulation examples. The simulation example for the plant of second order with unknown and time-varying parameters have been used to test the new controller. It is necessary to point out, that the elaborated controllers have been tested and compared using an unstable nonminimum phase plant, although it is well known that the application of adaptive controllers to unstable nonminimum phase plants causes difficulties. The designed controllers demonstrated, however, acceptable performance for this difficult problem.

5. CONVERGENCE AND STABILITY OF ADAPTIVE DUAL CONTROL

5.1. The Problem of Convergence Analysis

The investigations in the field of dual controllers concentrated for many years on the design of various adaptive dual controllers using approximative approaches. However, problems of convergence and stability were not considered. The main reasons for this are the complexity and nonlinearity of many adaptive controllers as well as insufficient experience in the application of stochastic methods to convergence and stability analysis of adaptive systems. This experience was accumulated during the 1970s and 1980s and the first convergence analysis of an explicit dual controller that uses the martingale convergence theory was presented by Radenkovic (1988). This approach was extended for convergence analysis and stability of dual controllers with direct adaptation (Filatov and Unbehauen, 1994) and will be applied below with some modifications to analyse convergence and stability of the adaptive dual controllers discussed in Chapter 4.

5.2. Preliminary Assumptions for the System

In the following the system described by eq. (4.38) is considered together with the adaptive dual controller given by eqs. (4.55) to (4.58), (4.61) and (4.62). In order to simplify the mathematical manipulations, the case without delay d=1 and $C(z^{-1})=1$ is treated for the convergence and stability analysis. At the same time, the approaches presented by Radenkovic (1988), and Goodwin and Sin (1984) can be applied for the generalization of the results that are presented here for the cases of general delay and a noise polynomial. It should be mentioned that the case of $C(z^{-1})\neq 1$ has been considered in many papers (see Goodwin and Sin, 1984) and convergence and stability has been shown for various adaptive controllers. It is obvious that the systems converge for weaker assumptions than those considered below, but the convergence cannot be proved under weaker assumptions without further investigations. In other words, the strict convergence and stability analysis of the considered adaptive systems presents a difficult problem that cannot be solved analytically for the general case presently. However, convergence and stability of adaptive controllers can be checked by computer simulations and experimental applications in almost every practical case.

The convergence of the GMV adaptive dual controller designed in Section 4.3 will be proved under the following assumptions:

Assumption 1: $1-\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)>0$ (with probability 1), $\forall k$. (5.1)

Assumption 2: $|w(k)|<\infty, \ \forall k$. (5.2)

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Assumption 3: $$\sum_{k=1}^{\infty}\theta^2(k) < \infty\,, \quad \forall k\,, \tag{5.3}$$

where $\theta(k)$ is chosen such that $\theta^2(k) = \frac{\eta}{d(k)}, \eta < \infty, d(k) = k^{\mu}$, $0 \le \mu < \frac{1}{2}$.

Assumption 4: The roots of the characteristic equation of the polynomial $D(z^{-1})$, according to eq. (4.45), are strictly inside the unit disk of the *z*-domain.

Inequality (5.1) can be interpreted as follows. Using eqs. (4.57) and (4.58) the following relation holds:

$$\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k) = \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\boldsymbol{m}(k)}{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\boldsymbol{m}(k)+1} \le 1\,. \tag{5.4}$$

According to Assumption 1, the strict inequality of eq. (5.4) is required. Therefore, it is assumed that the corresponding elements of the matrix $\overline{\boldsymbol{P}}(k)$ take zero values in the cases of infinite norm of the vector $\boldsymbol{m}(k)$. Assumption 2 holds as usual for control systems. Assumption 3 requires bounded magnitude of the optimal excitations of the dual controller, according to eq. (4.62), which should converge to zero. In addition the controller of eq. (4.61) is considered with $\hat{f}_0(k) \ne 0$. But this controller is capable to operate under real practical conditions even if the value of the estimate $\hat{f}_0(k)$ is close to zero because of the positive value $p_{f_0}(k)$ in the denominator of eq. (4.61).

From eqs. (4.61) and (4.62) follow the equations

$$[\hat{f}_0^2(k) + p_{f_0}(k)]u(k) = \hat{f}_0(k)w(k) - [\hat{f}_0(k)\boldsymbol{p}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{f_0p_0}^{\mathrm{T}}(k)]\,\boldsymbol{m}_0(k)$$
$$+[\hat{f}_0^2(k) + p_{f_0}(k)]\,u_{\mathrm{a}}(k)$$

or

$$\hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k) = w(k) - \frac{1}{\hat{f}_0(k)}\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}(k) + \frac{\hat{f}_0^2(k) + p_{f0}(k)}{\hat{f}_0(k)}u_a(k)$$

$$\hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k) = w(k) + \alpha(k)\,, \tag{5.5}$$

where

$$\alpha(k) = -\frac{1}{\hat{f}_0(k)}\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}(k) + \frac{\hat{f}_0^2(k) + p_{f_0}(k)}{\hat{f}_0(k)}u_a(k)\,, \tag{5.6}$$

$$\boldsymbol{p}_1^{\mathrm{T}}(k) = [p_{f_0}(k) \vdots \boldsymbol{p}_{f_0p_0}^{\mathrm{T}}(k)] \tag{5.7}$$

and

$$u_a(k) = \operatorname{sgn}\{p_{f_0}(k)u_c(k) + \boldsymbol{p}^T_{f_0 p_0}(k)\boldsymbol{m}_0(k)\}\theta(k)\,. \tag{5.8}$$

Let us introduce the notations

$$\omega(k) = y_a(k+1) + w(k) - \hat{\boldsymbol{p}}^T(k)\boldsymbol{m}(k) + \alpha(k) - \overline{\xi}(k+1)\,, \tag{5.9}$$

$$h(k) = [\ln r(k)]^{(1+\delta_1)},\ 0 < \delta_1 < \frac{1}{2}, \tag{5.10}$$

$$r(k) = r(k-1) + \boldsymbol{m}^T(k)\boldsymbol{m}(k),\ r(0) > 1\,, \tag{5.11}$$

$$V(k) = \frac{\tilde{\boldsymbol{p}}^T(k)\overline{\boldsymbol{P}}^{-1}(k)\tilde{\boldsymbol{p}}^T(k)}{h(k)}\,, \tag{5.12}$$

$$\overline{\boldsymbol{P}}^{-1}(k+1) = \overline{\boldsymbol{P}}^{-1}(k) + \boldsymbol{m}(k)\boldsymbol{m}^T(k)\,. \tag{5.13}$$

When we use eq. (4.54), eq. (5.9) takes the form

$$\omega(k) = -\tilde{\boldsymbol{p}}^T(k)\boldsymbol{m}(k) + \alpha(k) = -\eta(k) + \alpha(k)\,, \tag{5.14}$$

where

$$\tilde{\boldsymbol{p}}(k) = \hat{\boldsymbol{p}}(k) - \boldsymbol{p} = \tilde{\boldsymbol{p}}(k-1) + \overline{\boldsymbol{P}}(k)\boldsymbol{m}(k-1)e(k) \tag{5.15}$$

and

$$\eta(k) = \tilde{\boldsymbol{p}}^T(k)\boldsymbol{m}(k)\,. \tag{5.16}$$

5.3. Global Stability and Convergence of the System

Using the martingale convergence theory (see, for example, Elliott 1982b), the global stability and convergence of the auxiliary system output, eqs. (4.41) and (4.54), to the minimum possible variance σ_e^2 in the closed-loop feedback mode are presented in the following proposition. Only the case of d=1 and $C(z^{-1}) = 1$ is considered.

Proposition 5.1. Let Assumptions 1-4 hold. Then for the adaptive controller according to eqs. (4.55) to (4.58), (4.61) and (4.62) it holds with probability 1 that,

$$0 < \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \boldsymbol{m}^T(k)\boldsymbol{m}(k) < \infty \tag{5.17}$$

and

$$\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} y^2(k) = \sigma_e^2\,. \tag{5.18}$$

Proof. Starting from eq. (5.12) and using eqs. (5.13), (5.15) and (5.16) results in

$$V(k+1) = V(k) + \frac{\eta^2(k)}{h(k)} + \frac{2\eta(k)e(k+1)}{h(k)} + \frac{\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k+1)\boldsymbol{m}(k)e^2(k+1)}{h(k)} \text{ (a.s.)}, \tag{5.19}$$

where the notation (a.s.) means "as surely" or "with probability 1". From eqs. (4.54), (4.56) and (5.16) follows that

$$e(k+1) = -\eta(k) + \overline{\xi}(k+1). \tag{5.20}$$

Substituting the last equation into eq. (5.19)

$$V(k+1) = V(k) + [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\,\frac{-\eta^2(k)}{h(k)} + \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\overline{\xi}^2(k+1)$$

$$+ \frac{2\eta(k)\overline{\xi}(k+1)}{h(k)} - \frac{2\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\eta(k)\overline{\xi}(k+1),$$

and taking the expectation with respect to random noise provides

$$\mathrm{E}_{\xi}\left\{V(k+1)\middle|\Im_k\right\} \le V(k) + [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{-\eta^2(k)}{h(k)}$$

$$+ \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\sigma_{\overline{\xi}}^2 \quad \text{(a.s.)}. \tag{5.21}$$

When $\eta(k) = \alpha(k) - \omega(k)$ according to eq. (5.14) is used, it is easy to see that

$$-\eta^2(k) \le \alpha^2(k) - \omega^2(k).$$

Substitution of this into eq. (5.21) gives

$$\mathrm{E}_{\xi}\left\{V(k+1)\middle|\Im_k\right\} \le V(k) + [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{-\omega^2(k)}{h(k)}$$

$$+ [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{\alpha^2(k)}{h(k)} + \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\sigma_{\overline{\xi}}^2 \quad \text{(a.s.)}. \tag{5.22}$$

Introducing a small value $\overline{\rho} > 0$ $(\overline{\rho} << 1)$ the last inequality can be rewritten as

$$\mathrm{E}_{\xi}\left\{V(k+1)\middle|\Im_k\right\} \le V(k) - \frac{\overline{\rho}\omega^2(k)}{h(k)} - [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k) - \overline{\rho}]\frac{\omega^2(k)}{h(k)}$$

$$+ [1 - \boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{\alpha^2(k)}{h(k)} + \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\sigma_{\overline{\xi}}^2 \quad \text{(a.s.)}. \tag{5.23}$$

The introduction of

$$g(k) = \sum_{i=1}^{k} \omega^2(i)\,[1 - \boldsymbol{m}^{\mathrm{T}}(i)\overline{\boldsymbol{P}}(i+1)\boldsymbol{m}(i) - \overline{\rho}] + K \ge 0,\ \forall k, \tag{5.24}$$

where $0 \le K < \infty$, indicates positive semidefiniteness of $g(k)$ because of Assumption 1. Thus, from eqs. (5.24) and (5.23) follows

$$\mathrm{E}_{\xi}\{V(k+1)|\Im_k\} + \frac{g(k)}{h(k)} \le V(k) + \frac{g(k-1)}{h(k-1)} - \frac{\bar{\rho}\omega^2(k)}{h(k)}$$

$$+[1-\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{\alpha^2(k)}{h(k)} + \frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)}{h(k)}\sigma_{\xi}^2 \quad \text{(a.s.)}. \tag{5.25}$$

Taking into account eqs. (5.2), (5.3), (5.6), (5.8) and Lemma A.1 of Radenkovic (1988) the following inequality is obtained:

$$\sum_{k}^{\infty}[1-\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}(k)]\frac{\alpha^2(k)}{h(k)} < \infty . \tag{5.26}$$

Lemma 3 of Chen and Guo (1986) provides

$$\frac{\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)}{h(k)}\sigma_{\xi}^2 < \infty. \tag{5.27}$$

On the basis of the martingale convergence theorem follows from eqs. (5.25) to (5.27)

$$\sum_{k=1}^{\infty}\frac{\omega^2(k)}{h(k)} < \infty \quad \text{(a.s.)} . \tag{5.28}$$

From substituting eq. (4.38) in eq. (4.45) and using at first eq. (4.41) and then eqs. (4.54), (5.9), and (5.14) it is easy to show that

$$\begin{aligned} D(z^{-1})Y(z) &= P(z^{-1})B(z^{-1})Y(z) + Q(z^{-1})A(z^{-1})Y(z) \\ &= B(z^{-1})[Y_{\mathrm{a}}(z) - R(z^{-1})W(z)] + Q(z^{-1})C(z^{-1})\,\Xi(z) \\ &= z^{-1}B(z^{-1})\,\Xi(z) + z^{-1}B(z^{-1})\,\Omega(z) - B(z^{-1})R(z^{-1})W(z) \\ &\qquad + Q(z^{-1})C(z^{-1})\,\Xi(z), \\ &= z^{-1}B(z^{-1})\,\Omega(z) + [Q(z^{-1})\,C(z^{-1}) \\ &\qquad + B(z^{-1})]\,\Xi(z) - R(z^{-1})B(z^{-1})W(z) . \end{aligned} \tag{5.29}$$

Similarly introducing eq. (4.41) into eq. (4.45) and using thereafter eqs. (4.38), (4.47), (4.51), (5.5), (5.9), and (5.14) leads to

$$\begin{aligned} D(z^{-1})U(z) &= P(z^{-1})B(z^{-1})U(z) + Q(z^{-1})A(z^{-1})U(z) \\ &= z\,P(z^{-1})C(z^{-1})\;\Xi(z) + z\,A(z^{-1})Y_{\mathrm{a}}(z) + z\,A(z^{-1})R(z^{-1})W(z) \\ &= A(z^{-1})\,\Omega(z) + z\,R(z^{-1})\,A(z^{-1})W(z) - G(z^{-1})\;\Xi(z), \end{aligned} \tag{5.30}$$

where $\Omega(z)$ is the z-transform of $\omega(k)$. The boundedness of the input and output variables in the mean square sense, according to eq. (5.17), and the statement, that

$$\lim_{N\to\infty} \frac{1}{(\ln N)^{1+\delta_1}} \sum_{k=1}^{N} \omega^2(k) = 0 \qquad \text{(a.s.)}, \tag{5.31}$$

follow from assumptions 1 and 2 and eqs. (5.28) to (5.30), as described by Radenkovic (1988). Asymptotic optimality of the algorithm, eq. (5.18), as well as the first inequality of eq. (5.17) are resulting from eq. (5.31), as in the paper of Caines and Lafortune (1984). It should be noted that a special noise signal for complimentary excitation is added to be reference signal *w(k)* that provides efficiency of the adaptation algorithm and guarantees the properties of the system according to eqs. (5.17) and (5.18). However, in case of adaptive dual control considered here with the optimal excitations according to eq. (5.8) there is no necessity for introducing such additional excting disturbances, since the algorithm of dual control provides optimal excitation of the system which supports the convergence. The system noise *ξ(k)* acts through feedback on the exciter according to eq. (5.8) which is transferred there to the more effective stochastic excitation signal. Thus, the proof is completed. ■

The general case of the controller according to eqs. (4.61) and (4.62) will now be considered with the excitation $u_a(k)$ in the control action

$$u(k) = u_c(k) + u_a(k), \tag{5.32}$$

chosen in a different way. The following proposition states the requirements for the excitations to satisfy the system convergence as in eqs. (5.17) and (5.18). Again, the case of d=1 and $C(z^{-1}) = 1$ is considered.

Proposition 5.2. Let the Assumptions 1,2 and 4 hold, $f_0(k) \neq 0$, and the excitation be bounded by the inequality

$$\sum_{k}^{\infty} u_a^2(k) < \infty, \tag{5.33}$$

then the conditions according to eqs. (5.17) and (5.18) hold with probability 1 for the adaptive controller given by eqs. (4.55) to (4.58), (4.61) and (5.32). In contrast with proposition 5.1 the above statement holds for any excitation according to eqs. (5.32) and (5.33), not only for the excitations according to eqs. (4.62) and (5.3).

Proof. First the manipulations according to eqs. (5.19) to (5.25) are considered with $u_a(k)$ in eq. (5.6) arbitrarily chosen. Inequality eq. (5.26) holds in the case of condition eq. (5.33); therefore, inequality (5.28) follows from eq. (5.25) when eq. (5.27), which provides eqs. (5.17) and (5.18) with probability 1, is taken into account. In this way the proposition is proved. ■

Thus, according to the result of Proposition 5.2, the stability of any adaptive controller for the considered GMV system will be guaranteed if the excitations added to the cautious controller according to eq. (4.61) are selected in the form of eq. (5.32) to satisfy the inequality (5.33).

5.4. Conclusion

The problem of convergence of adaptive dual control algorithms has been considered as an important and very difficult mathematical problem. This problem was not investigated for a long time because of the difficulties connected with the nonlinearity and complexity of the adaptive control laws. First these difficulties were overcome by Radenkovic (1988) for the generalized dual controller suggested by Chan and Zarrop (1985). In this chapter the bicriterial adaptive dual controller has been investigated with respect to convergence. Furthermore, the convergence of the dual controller with the excitation of the general form has been studied.

6. DUAL POLE-PLACEMENT CONTROLLER WITH DIRECT ADAPTATION

The following two problems have to be solved in order to extend the adaptive dual control approach to direct adaptive pole-placement control systems: (i) selecting an appropriate performance index for control optimization and (ii) describing the uncertainty of the controller parameters in the direct adaptive pole-placement control systems as well as defining a measure for the uncertainty. Both these problems will be dealt with in detail in this chapter.

Below the synthesis of dual control systems with pole placement and direct adaptation is described. A dual version of the well-known direct adaptive pole-placement controller (APPC) suggested by Elliott (1982a) (see also Elliott et al., 1984) is designed. The bicriterial approach discussed in the previous chapters is extended to the synthesis of dual control systems with direct adaptation. Therefore a new optimization criterion is introduced for this system type, where the cost function is defined as the conditional expectation of the squared error of a nominal system output. This optimization criterion can be generally applied also for designing other adaptive dual controllers, as shown in the next chapters. The covariance matrix of the unknown parameters of the regulator is used as uncertainty measure. This uncertainty measure enters the new control algorithm which results in robust and cautious properties for the developed control system.

There is neither an explicit reference model nor an error between the reference model output and system output in APPC systems. The desired dynamical behaviour of the closed-loop system (reference model) is used in these systems implicitly in the process of indirect identification. Normally, no optimization criterion is used in these kinds of adaptive systems. The control performance is determined only by the structure of the controller, and the adaptation mechanism aims to provide the desired dynamical behaviour of the closed-loop system as determined by the selected pole-placement, when indirect identification is used. Below a new optimization criterion for adaptive pole-placement control is proposed. At the outset, the nominal output will be defined as the desired system output when no disturbances act upon the system. Thus, the nominal output is the desired output at every moment, and the cost function is determined as the conditional expectation of the square of the system output deviation from the nominal desired output. In contrast with the usual criterion, the nominal desired output of the system at every moment is unknown, because the parameters of the desired regulator are unknown. It will be shown that the application of this criterion leads to an optimal control algorithm, which has the structure of the desired pole-placement controller. Contrary to other well-known controllers, this one depends on the covariance matrix of the uncertain parameters. Thus, the existing parameter uncertainty level is taken into consideration by this controller. It should be mentioned that, unlike the suggested approach, the system synthesis using the usual criterion based on the error between the outputs of the system and the reference model leads to an adaptive system with explicit model reference.

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The derivation of the algorithm for the direct APPC, using the well-known approach suggested by Elliott (1982a), which is based on the CE assumption, is shown in Section 6.1. In contrast to Elliott's controller, the one considered here is synthesized for systems with stochastic disturbances and time-varying parameters. The dual version of this controller is derived in Section 6.2, using a bicriterial approach. Two simulated examples are presented in Section 6.3 to demonstrate the potential of the proposed algorithms, and the properties of the new controller are discussed and compared with the well-known, aforementioned one.

6.1. Design of a Direct Adaptive Pole-Placement Controller Using the Standard Approach

A discrete-time SISO plant with coloured disturbance noise given by

$$A(z^{-1})Y(z) = z^{-1}B(z^{-1})U(z) + \Psi(z)\,, \tag{6.1}$$

is considered here, where $\Psi(z)$ is the z-transform of the white noise disturbance $\psi(k)$, and $A(z^{-1})$ and $B(z^{-1})$ are polynomials of the form

$$A(z^{-1}) = (1+\bar{a}_1 z^{-1} + \ldots + \bar{a}_{n_\mathrm{A}} z^{-n_\mathrm{A}})(1-z^{-1})^{\kappa} = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}\,, \tag{6.2}$$

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \ldots + b_{n_\mathrm{B}} z^{-n_\mathrm{B}+1}\,, \tag{6.3}$$

where κ integral actions are incorporated into the system, $\kappa \geq 0, n = n_\mathrm{A} + \kappa$. It is assumed that only the orders of the plant polynomials are known, whereas the coefficients of the polynomials according to eqs. (6.2) and (6.3) are unknown. Consider the following control law for the plant (6.1):

$$U(z) = \frac{1}{r_0}[W(z) - S(z^{-1})Y(z) - z^{-1}R(z^{-1})U(z)]\,,\ r_0 \neq 0, \tag{6.4}$$

where $W(z)$ is the z-transform of the reference signal $w(k)$ and

$$R(z^{-1}) = r_1 + r_2 z^{-1} + \ldots + r_{n_\mathrm{R}} z^{-n_\mathrm{R}+1},\ n_\mathrm{R} = n_\mathrm{A} - 1, \tag{6.5}$$

$$S(z^{-1}) = s_0 + s_1 z^{-1} + \ldots + s_{n_\mathrm{S}} z^{-n_\mathrm{S}},\ n_\mathrm{S} = n_\mathrm{A} + \kappa - 1 = n - 1, \tag{6.6}$$

are controller polynomials. After transformation of eq. (6.4) to the discrete time domain, the control law can be expressed in vector form as

$$u(k) = \frac{1}{r_0}[w(k) - \boldsymbol{p}_0^\mathrm{T}\boldsymbol{m}_0(k)]\,,\quad r_0 \neq 0 \tag{6.7}$$

where

$$\boldsymbol{p}_0^\mathrm{T} = [s_0 \ldots s_{n_\mathrm{S}} \vdots r_1 \ldots r_{n_\mathrm{R}}]\,, \tag{6.8}$$

and

$$\boldsymbol{m}_0(k) = [y(k)...y(k-n_S) \vdots u(k-1)...u(k-n_R)]^{\mathrm{T}}. \tag{6.9}$$

Consider the asymptotically stable monic polynomial

$$C(z^{-1}) = 1 + c_1 z^{-1} + ... + c_{n_{\mathrm{C}}} z^{-n_{\mathrm{C}}},\ n_{\mathrm{C}} \leq n_{\mathrm{A}} + n_{\mathrm{B}} + \kappa - 1. \tag{6.10}$$

Its zeros represent the desired location of the poles of the transfer function between the bounded external input $w(k)$ and the output $y(k)$. So the z-transform of the closed-loop output $y(k)$ takes the form

$$Y(z) = \frac{z^{-1}B(z^{-1})}{r_0 C(z^{-1})} W(z), \tag{6.11}$$

where

$$r_0 C(z^{-1}) = A(z^{-1})[r_0 + z^{-1}R(z^{-1})] + z^{-1}B(z^{-1})S(z^{-1}). \tag{6.12}$$

Equation (6.12) follows from eqs. (6.1) and (6.4). eqs. (6.11) and (6.12) define the implicit reference model.

To derive an appropriate model for estimating the controller parameter vector, eq. (6.8), (see Elliott (1982a)) the Bezout identity

$$A(z^{-1})D(z^{-1}) + B(z^{-1})F(z^{-1}) = r_0 z^{-l+2},\ l = n_{\mathrm{A}} + n_{\mathrm{B}} + \kappa \tag{6.13}$$

is used, where

$$D(z^{-1}) = d_0 + d_1 z^{-1} + ... + d_{n_{\mathrm{D}}} z^{-n_{\mathrm{D}}}, \quad d_0 \neq 0,\ n_{\mathrm{D}} = n_{\mathrm{B}} - 2, \tag{6.14}$$

and

$$F(z^{-1}) = 1 + f_1 z^{-1} + ... + f_{n_{\mathrm{F}}} z^{-n_{\mathrm{F}}},\ n_{\mathrm{F}} = n_{\mathrm{A}} + \kappa - 1. \tag{6.15}$$

Multiplication of eqs. (6.12) and (6.13) gives

$$C(AD + BF) = z^{-l+2}\,[A(r_0 + z^{-1}R) + z^{-1}BS]. \tag{6.16}$$

The arguments of the polynomials are omitted here for notational simplicity. After multiplying eq. (6.16) by $Y(z)$ and introducing eq. (6.1), the following equation can be obtained:

$$z^{-1}DCU(z) + FCY(z) - z^{-l}RU(z) - z^{-l+1}SY(z) = r_0 z^{-l+1}U(z) + \Xi(z), \tag{6.17}$$

where $\Xi(z)$ is the z-transform of the colored noise disturbance $\xi(k)$, which is obtained from $\Psi(z)$ after the above algebraic manipulations. In order to obtain the vector form of eq. (6.17), the following filtered values of the input and output signals are introduced:

$$\bar{Y}(z) = C(z^{-1})Y(z),\ \bar{y}(k+1) = y(k+1) + \sum_{i=1}^{m} c_i y(k-i+1) \tag{6.18}$$

and

$$\overline{U}(z) = C(z^{-1})U(z), \quad \overline{u}(k) = u(k) + \sum_{i=1}^{m} c_i u(k-i) . \tag{6.19}$$

Using eqs. (6.18) and (6.19) one can represent eq. (6.17) in the form

$$F\overline{Y}(z) = -z^{-1}D\overline{U}(z) + r_0 z^{-l+1} U(z) + z^{-l} RU(z) + z^{-l+1} SY(z) + \Xi(z) .$$

This equation can be written in the time domain as

$$\begin{aligned}\overline{y}(k) = &-d_0 \overline{u}(k-1) - \dots - d_{n_\mathrm{D}} \overline{u}(k-n_\mathrm{D}-1) - f_1 \overline{y}(k-1) - \dots - f_{n_\mathrm{F}} \overline{y}(k-n_F) \\ &+ r_0 u(k-l+1) + r_1 u(k-l) + \dots r_{n_\mathrm{R}} u(k-l-n_\mathrm{R}+1) \\ &+ s_0 y(k-l+1) + \dots + s_{n_\mathrm{S}} y(k-l-n_\mathrm{S}+1) + \xi(k) .\end{aligned}$$

In vector notation this equation yields

$$\overline{y}(k) = \boldsymbol{p}^\mathrm{T} \boldsymbol{m}(k-1) + \xi(k), \tag{6.20}$$

where

$$\boldsymbol{p}^\mathrm{T} = [-d_0 \dots -d_{n_\mathrm{D}} \vdots - f_1 \dots - f_{n_\mathrm{F}} \vdots r_0 \vdots r_1 \dots r_{n_\mathrm{R}} \vdots s_0 \dots s_{n_\mathrm{S}}] \tag{6.21}$$

and

$$\begin{aligned}\boldsymbol{m}(k-1) = [&\overline{u}(k-1) \dots \overline{u}(k-n_\mathrm{D}-1) \vdots \overline{y}(k-1) \dots \overline{y}(k - n_\mathrm{F}) \vdots u(k-l+1) \vdots \\ &u(k-l) \dots u(k-l-n_\mathrm{R}+1) \vdots y(k-l+1) \dots y(k-l-n_\mathrm{S}+1)]^\mathrm{T}.\end{aligned} \tag{6.22}$$

The signals $\overline{u}(k)$ and $\overline{y}(k)$ represent $\overline{U}(z)$ and $\overline{Y}(z)$, respectively, in the discrete-time domain.

In the case of time-varying plant parameters, the parameter vector according to eq. (6.19) including the parameters of the controller, eq. (6.5), and the polynomials given by eqs. (6.14) and (6.15) also become time-varying. If the case of stochastic parameter drift, described by a Wiener process as in eq. (4.5) is considered, where $\boldsymbol{\varepsilon}(k)$ is a white noise drift vector with zero mean and covariance matrix $\boldsymbol{Q}_\varepsilon(k)$, then in the case when $\xi(k)$ is a white noise with zero mean and variance σ_ξ^2, the following Kalman filter approach, according to eqs. (4.6) to (4.12), is applied for the estimation of the parameters in eq. (6.18):

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1) \, [\overline{y}(k+1) - \hat{\boldsymbol{p}}^\mathrm{T}(k)\boldsymbol{m}(k)], \tag{6.23}$$

where

$$\boldsymbol{q}(k+1) = \boldsymbol{P}(k)\boldsymbol{m}(k) \, [\boldsymbol{m}^\mathrm{T}(k)\boldsymbol{P}(k)\boldsymbol{m}(k) + \sigma_\xi^2]^{-1}, \tag{6.24}$$

and

$$\boldsymbol{P}(k+1) = \boldsymbol{P}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^\mathrm{T}(k)\boldsymbol{P}(k) + \boldsymbol{Q}_\varepsilon(k) . \tag{6.25}$$

It is assumed here that the initial values $\hat{\boldsymbol{p}}(0)$ and $\boldsymbol{P}(0)$ for eqs. (6.23) and (6.25) are given. When the CE approach for the controller according to eq. (6.7) and the adaptation algorithm of eqs. (6.23) to (6.25) is used, the direct APPC takes the form

$$u(k) = \frac{1}{\hat{r}_0(k)}[w(k) - \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)] \tag{6.26}$$

with $\hat{\boldsymbol{p}}_0$ being an estimate of $\boldsymbol{p}_0$.

In contrast to the regulator suggested by Elliott (1982a), the direct APPC, eqs. (6.23) to (6.25) and (6.26), has been derived for systems with noise and time-varying parameters. Moreover, the structure of this controller is suitable for the development of its dual version, as shown in the next section.

6.2. Design of Dual Pole-Placement Controller with Direct Adaptation

Define the nominal output of the system $y_{\mathrm{n}}(k+1)$ as the response to the input signal

$$u_{\mathrm{n}}(k) = \frac{1}{r_0}[w(k) - \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k)] \tag{6.27}$$

of the unknown controller that provides the desired system dynamics according to the transfer function of eq. (6.11), when the state of the system is $\boldsymbol{m}_0(k)$ and no disturbances act upon the system ($\xi(k) \equiv 0$). Then the dependence of $y_{\mathrm{n}}(k+1)$ on $u_{\mathrm{n}}(k)$, eq. (6.20), takes the form

$$\bar{y}_{\mathrm{n}}(k+1) = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}_{\mathrm{n}}(k), \tag{6.28}$$

where $\boldsymbol{p}^{\mathrm{T}}$ is defined according to eq. (6.21), and

$$\boldsymbol{m}_{\mathrm{n}}(k) = [\bar{u}_{\mathrm{n}}(k)\,\bar{u}(k-1)\ldots\bar{u}(k-n_{\mathrm{D}}) \;\vdots\; \bar{y}(k)\ldots\bar{y}(k-n_{\mathrm{F}}+1) \;\vdots\; u(k-l+2)$$
$$u(k-l+1)\ldots u(k-l-n_{\mathrm{R}}+2) \;\vdots\; y(k-l+2)\ldots y(k-l-n_{\mathrm{S}}+2)]^{\mathrm{T}}, \tag{6.29}$$

$$\bar{y}_{\mathrm{n}}(k+1) = y_{\mathrm{n}}(k+1) + \sum_{i=1}^{m} c_i y(k-i+1),$$

$$\bar{y}(k+1) = y(k+1) + \sum_{i=1}^{m} c_i y(k-i+1), \tag{6.30}$$

and

$$\bar{u}_{\mathrm{n}}(k) = u_{\mathrm{n}}(k) + \sum_{i=1}^{m} c_i u(k-i), \quad \bar{u}(k) = u(k) + \sum_{i=1}^{m} c_i u(k-i). \tag{6.31}$$

It is clear that the control performance would be improved if, in case of disturbances and parameter uncertainties, the controller tried to bring the system output as close as possible to the nominal output, which is attained only for the controller with known parameters, after complete noise compensation. In accordance with this and the bicriterial approach, the following two cost functions to be minimised are introduced in order to derive the control law:

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{\beta^2[y_n(k+1) - y(k+1)]^2 \middle| \Im_k\right\}, \ \beta^2 = \frac{r_0^2}{d_0^2} \tag{6.32}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[\bar{y}(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 \middle| \Im_k\right\}. \tag{6.33}$$

The first cost function, eq. (6.32), is used for control purposes to minimize the deviation of the system output from the unknown nominal output, which would be obtained by the adjusted unknown regulator. The coefficient β^2 is introduced for the simplification of further algebraic manipulations. The second cost function, eq. (6.33), is used for the acceleration of the parameter estimation process (as described in Section 4) by increasing the predictive error value, shown in the square brackets of eq. (6.23). According to the bicriterial approach, these two criteria correspond to the two goals of adaptive dual control: to control the system output and to accelerate the estimation for future control improvement. The direct adaptive dual controller will be obtained after solving the bicriterial optimization problem, given by minimization of the two eqs. (6.32) and (6.33) for the system described by eqs. (6.20) and (6.27), under the constraints Ω_k as given by eqs. (4.22) to (4.25).

Now the minimization of the cost function according to eq. (6.32) is considered. Due to eq. (6.30), we have

$$y_{\mathrm{n}}(k+1) - y(k+1) = \bar{y}_{\mathrm{n}}(k+1) - \bar{y}(k+1)$$

for the deviation of the system output from the nominal output in eq. (6.32). Further manipulations of the last equation using eqs. (6.20) to (6.22) and (6.28) to (6.31) result in

$$\begin{aligned}\bar{y}_{\mathrm{n}}(k+1) - \bar{y}(k+1) &= \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}_{\mathrm{n}}(k) - \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k) - \xi(k+1) \\ &= d_0\bar{u}(k) - d_0\bar{u}_{\mathrm{n}}(k) - \xi(k+1).\end{aligned}$$

Using eq. (6.31), the last two equations can be represented in the form

$$y_{\mathrm{n}}(k+1) - y(k+1) = d_0 u(k) - d_0 u_{\mathrm{n}}(k) - \xi(k).$$

Direct substitution of the last equation into eq. (6.32) yields

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[r_0 u_n(k) - r_0 u(k)]^2 \middle| \Im_k\right\} + \sigma_\xi^2. \tag{6.34}$$

Inserting eq. (6.27) into eq. (6.34) gives

$$
\begin{aligned}
J_k^{\mathrm{c}} &= \mathrm{E}\left\{[w(k) - \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k) - r_0 u(k)]^2 \middle| \Im_k \right\} + \sigma_\xi^2 \\
&= \mathrm{E}\left\{-2[w(k) - \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k)] r_0 u(k) + [r_0 u(k)]^2 \middle| \Im_k \right\} + \bar{c}_1(k) \\
&= -2\hat{r}_0(k) w(k) u(k) + 2[\hat{r}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)]\,\boldsymbol{m}_0(k) u(k) \\
&\qquad + [\hat{r}_0^2(k) + p_{r_0}(k)] u^2(k) + \bar{c}_1(k)\,,
\end{aligned}
\tag{6.35}
$$

where the expectation have been taken similar as in Appendix D, and the following elements of the covariance matrix $\boldsymbol{P}(k)$ have been used:

$$
p_{r_0}(k) = \mathrm{E}\left\{[r_0 - \hat{r}_0(k)]^2 \middle| \Im_k \right\}, \tag{6.36}
$$

and

$$
\boldsymbol{p}_{r_0 p_0}(k) = \mathrm{E}\left\{[r_0 - \hat{r}_0(k)]\,[\boldsymbol{p}_0 - \hat{\boldsymbol{p}}_0(k)] \middle| \Im_k \right\}. \tag{6.37}
$$

The term $\bar{c}_1(k)$ is equal to $\bar{c}_1(k) = \mathrm{E}\left\{[w(k) - \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k)]^2 \middle| \Im_k \right\} + \sigma_\xi^2$ and it does not depend on $u(k)$. Minimization of eq. (6.35) results in

$$
\frac{\partial J_k^{\mathrm{c}}}{\partial u(k)} = -2\hat{r}_0(k) w(k) + 2[\hat{r}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)]\boldsymbol{m}_0(k) + 2[\hat{r}_0^2(k) + p_{r_0}(k)] u(k) = 0\,.
$$

Solving this equation according to $u(k) \equiv u_{\mathrm{c}}(k)$ gives the cautious control action described by

$$
u_{\mathrm{c}}(k) = \frac{\hat{r}_0(k) w(k) - [\hat{r}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)]\boldsymbol{m}_0(k)}{\hat{r}_0^2(k) + p_{r_0}(k)}\,. \tag{6.38}
$$

In Chapter 4 it has been demonstrated in detail that the minimization of eq. (4.22) with the constraints of eqs. (4.23) and (4.24) leads to eq. (4.31). These properties of the cost function are also valid for the system considered here with the cost function according to eq. (6.33). Therefore, the second cost function is met through substitution of eq. (6.20) into eq. (6.33):

$$
J_k^{\mathrm{a}}[u(k)] = -\mathrm{E}\left\{[(\boldsymbol{p}(k) - \hat{\boldsymbol{p}}(k))^{\mathrm{T}}\boldsymbol{m}(k) + \xi(k)]^2 \middle| \Im_k \right\}.
$$

Further manipulations using eqs. (6.21) and (6.22) lead to

$$
\begin{aligned}
J_k^{\mathrm{a}}[u(k)] &= -\mathrm{E}\left\{[[d_0(k) - \hat{d}_0(k)]\,\bar{u}(k) + [\boldsymbol{p}_1(k) - \hat{\boldsymbol{p}}_1(k)]^{\mathrm{T}}\boldsymbol{m}_1(k)]^2 \middle| \Im_k \right\} + \bar{c}_2(k) \\
&= -p_{d_0}(k)\bar{u}^2(k) - 2\boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k)\boldsymbol{m}_1(k))\bar{u}(k) + \bar{c}_3(k)\,,
\end{aligned}
\tag{6.39}
$$

where $\bar{c}_2(k)$ and $\bar{c}_3(k)$ stands for the terms that do not contain $u(k)$, and

$$
\bar{u}(k) = u(k) + \sum_{i=1}^{m} c_i u(k-i)\,, \tag{6.40}
$$

$$\boldsymbol{P}(k) = \mathrm{E}\left\{[\boldsymbol{p} - \hat{\boldsymbol{p}}(k)][\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}} \middle| \Im_k \right\} = \begin{bmatrix} p_{d_0}(k) & \boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{d_0 p_1}(k) & \boldsymbol{P}_{p_1}(k) \end{bmatrix}, \tag{6.41}$$

$$\hat{\boldsymbol{p}}^{\mathrm{T}}(k) = [-\hat{d}_0(k) \;\vdots\; \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)], \tag{6.42}$$

$$\boldsymbol{m}(k) = [\overline{u}(k) \;\vdots\; \boldsymbol{m}_1^{\mathrm{T}}(k)]^{\mathrm{T}}. \tag{6.43}$$

It should be mentioned that eq. (6.40) leads to

$$\overline{u}(k) = u(k) + \overline{c}_4(k), \tag{6.44}$$

$$\overline{u}_{\mathrm{c}}(k) = u_{\mathrm{c}}(k) + \overline{c}_4(k), \tag{6.45}$$

where $\overline{c}_4(k)$ represents a term that does not contain $u(k)$ or $u_{\mathrm{c}}(k)$. Introducing eqs. (6.40) to (6.43) into eq. (6.39) for J_k^{a}, provides

$$\begin{aligned} J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) - \theta(k)] - J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) + \theta(k)] \\ = -\, p_{d_0}(k)[u_{\mathrm{c}}(k) - \theta(k) + \overline{c}_4(k)]^2 \\ - 2\boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k)\, \boldsymbol{m}_1(k)[u_{\mathrm{c}}(k) - \theta(k) + \overline{c}_4(k)] \\ + p_{d_0}(k)[u_{\mathrm{c}}(k) + \theta(k) + \overline{c}_4(k)]^2 \\ + 2\boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k)\boldsymbol{m}_1(k)\, [u_{\mathrm{c}}(k) + \theta(k) + \overline{c}_4(k)] \\ = 4 p_{d_0}(k)\overline{u}_{\mathrm{c}}(k)\theta(k) + 4\boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k)\boldsymbol{m}_1(k)\theta(k). \end{aligned} \tag{6.46}$$

Substituting eq. (6.46) into eq. (4.31)

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\ \mathrm{sgn}\left\{J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) - \theta(k)] - J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) + \theta(k)]\right\}$$

and using eq. (4.24)

$$\theta(k) = \eta\ \mathrm{tr}\{\boldsymbol{P}(k)\},\ \eta \geq 0$$

provides the adaptive dual control law

$$u(k) = u_{\mathrm{c}}(k) + \eta\ \mathrm{tr}\{\boldsymbol{P}(k)\}\,\mathrm{sgn}\left\{p_{d_0}(k)\, \overline{u}_{\mathrm{c}}(k) + \boldsymbol{p}_{d_0 p_1}^{\mathrm{T}}(k)\, \boldsymbol{m}_1(k)\right\}. \tag{6.47}$$

Thus, the adaptive dual controller, with pole-placement and direct adaptation, is determined by eqs. (6.38) and (6.47) together with the estimation algorithm of eqs. (6.23) to (6.25).

6.3. Simulation Examples

The simulation examples discussed below illustrate the behaviour of the adaptive dual controller and allow its properties to be compared with those algorithms based on the CE assumption. In each example, both the system with disturbances and the controller have been digitally simulated, and the polynomial orders appropriate for the plant have been correctly chosen. The figures associated with each example portray the time histories of the setpoint (w), the system output (y) and the controller output (u). The initial parameters were all taken as 0.01 and the initial covariance matrix as $\boldsymbol{P}(0) = 0.5\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Different second-order plants with four unknown parameters were considered in both examples. Therefore, six parameters of the vector, according to eq. (6.21), have to be estimated. The model of the plants with disturbances has the form

$$y(k+1) = b_1 u(k) + b_2 u(k-1) + a_2 y(k) + a_2 y(k-1) + \psi(k)\,, \tag{6.48}$$

where $\psi(k)$ is a white noise sequence with zero mean and small variance $\sigma_\psi^2 = 0.00001$. Parameters for the estimation algorithm of eqs. (6.23) to (6.25) have been chosen as $\sigma_\xi^2 = 0.00001$ and $\boldsymbol{Q}_\varepsilon(k) \equiv 0$, i.e. no parameter drift is considered. The polynomial shown in eq. (6.10) was chosen to provide the desired location of the poles of the closed-loop system, given in Table 1.

Table 6.1. Pole location

Re	Im
0.6	+0.1
0.6	-0.1
0.2	0

It should be mentioned that in case of κ=0 (plant without integral behaviour) the steady-state gain of the closed-loop system, according to eq. (6.11), has to be estimated to correct the steady state behavior, or integral action has to be incorporated in the system $(\kappa > 0)$ as part of the control law. In the following examples, the steady state error is corrected with the information about the plant steady-state gain; the plant steady-state gain is assumed to be known.

6.3.1 Example 1: Unstable Minimum Phase Plant

The plant involved is unstable, minimum phase, with integral behaviour (κ=1) and can be described by a model according to eqs. (6.2) and (6.3)

$$A(z^{-1}) = 1 - 1.9418z^{-1} + 0.9418z^{-2},$$

$$B(z^{-1}) = 0.0088 + 0.0086z^{-1}.$$

The parameter for excitation (probing) η, see eq. (6.47) was set to 4.5. Figure 6.1 shows a typical run with the designed adaptive dual controller according to eqs. (6.38) and (6.47), and Figure 6.2 presents a typical run with the standard Elliott controller (APPC) of eq. (6.26), which is based on the CE assumption and uses the estimated parameters as if they were the true values. This standard controller gives a large overshoot at the beginning of the process, while the dual controller provides a smooth start up due to its cautious and probing properties.

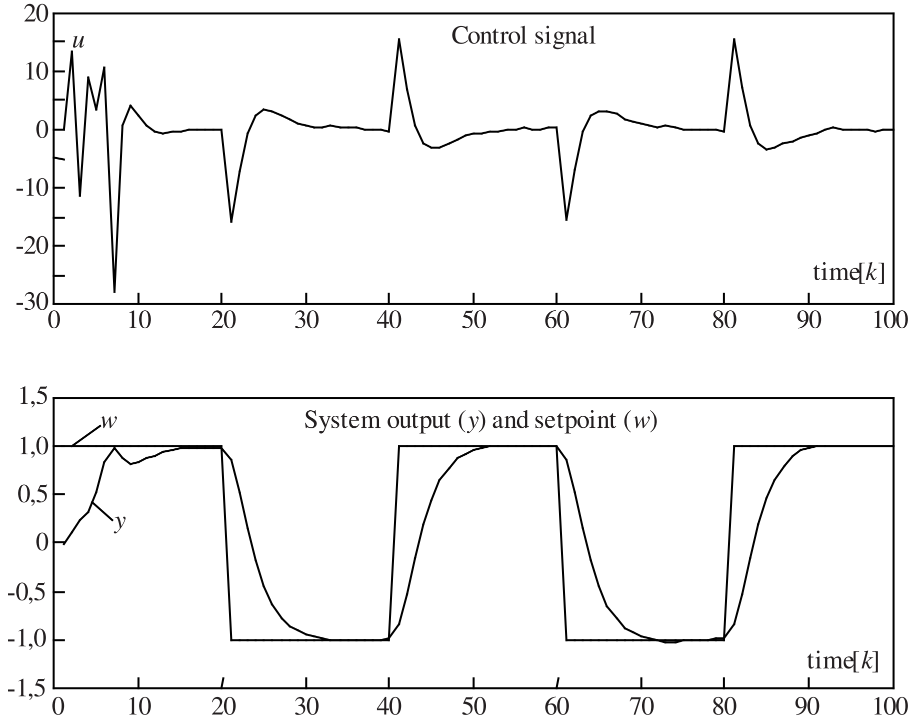

Figure 6.1. Direct adaptive dual control for an integral plant (Example 1)

6.3.2. Example 2: Unstable Nonminimum Phase Plant

Figures 6.3 and 6.4 show the simulation results for both controllers, namely the adaptive dual controller and the standard APPC controller, respectively, for an unstable overdamped and nonminimum phase plant (κ=0), described by a discrete model with the polynomials of eqs. (6.2) and (6.3) as

$$A(z^{-1}) = 1 - 2.1889z^{-1} + 1.1618z^{-2},$$

and

$$B(z^{-1}) = -0.0132 + 0.0139z^{-1}.$$

For this example, the parameter for probing is $\eta = 2.2$. The superiority of the designed dual controller can again be observed in this case, as in the first example.

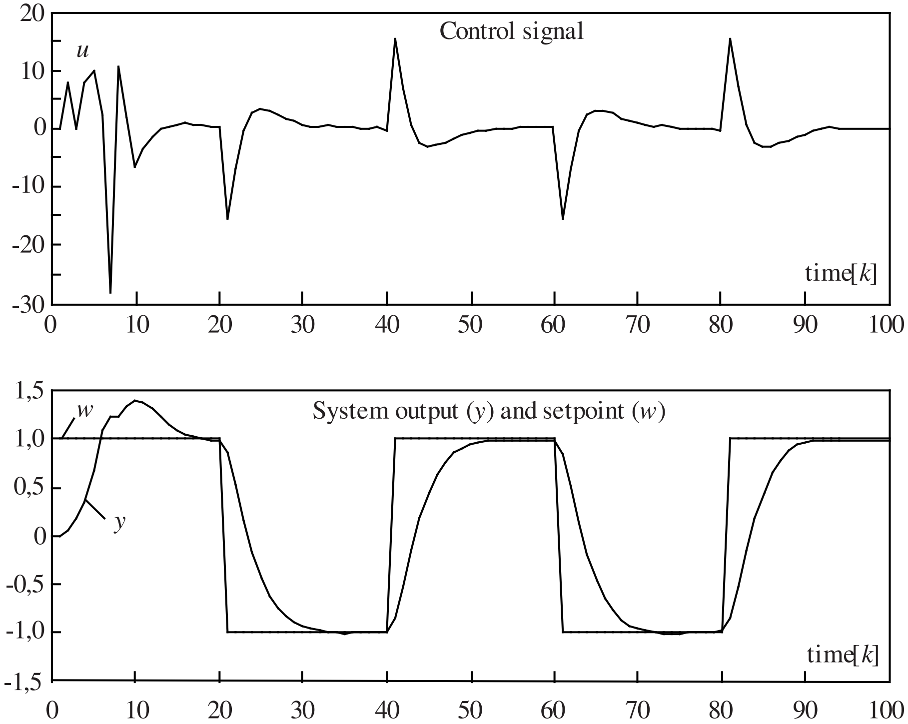

Figure 6.2. Standard APPC for an integral plant (Example 1)

6.3.3. Comparison of Controllers Based on Standard and Adaptive Dual Approaches

It is well known that adaptive dual controllers give improved control performance compared with standard adaptive controllers based on the CE assumption (Zhivoglyadov et al., 1993a; Chan and Zarrop, 1985; Bayard and Eslami, 1985; Milito et al., 1982; Filatov et al., 1994). This improvement of control performance was shown above by simulations and is achieved as a result of cautious and probing (stimulating the estimation) properties of the dual control algorithms. The cautious properties of the controller are due to the fact that the control law includes not only the parameter estimates but also the variances of their errors, see eq. (6.38). The presence of variances and covariances of parameter estimates provides the cautious behaviour of the control system when the uncertainty in the system is large. The adaptive dual system tries to concentrate its efforts on the plant excitation for accelerating the parameter estimation. When the parameter estimates are more precise (the variances being small), the efforts of the adaptive dual control are concentrated more on the control goal, and the probing signal becomes small, see eqs. (6.38) and (6.47). It should be noted that in the case of independent cautious controllers, according to eq. (6.38), better control performance is provided compared with adaptive control based on the CE assumption. But these controllers do not provide probing signals (optimal excitation), and they have not found broad applications because of their slow identification, that sometimes leads to turn-off effects (Zhivoglyadov et al., 1993; Åström and Wittenmark, 1971) whose presence usually interrupts the identification of the plant model. The direct adaptive dual control based on eqs. (6.38) and (6.47), as suggested here, gives the same improvement of the control performance as already pointed out in other publications (Åström, 1987; Zhivoglyadov et al., 1993; Chan and Zarrop, 1985; Bayard and Eslami, 1985; Milito et al., 1982) and indicated in Chapter 4.

The solution to the problem of strong convergence analysis for the control algorithm of eqs. (6.38) and (6.47) can be obtained using the approaches suggested by Filatov and Unbehauen (1994), Radenkovic (1988) and Janecki (1988).

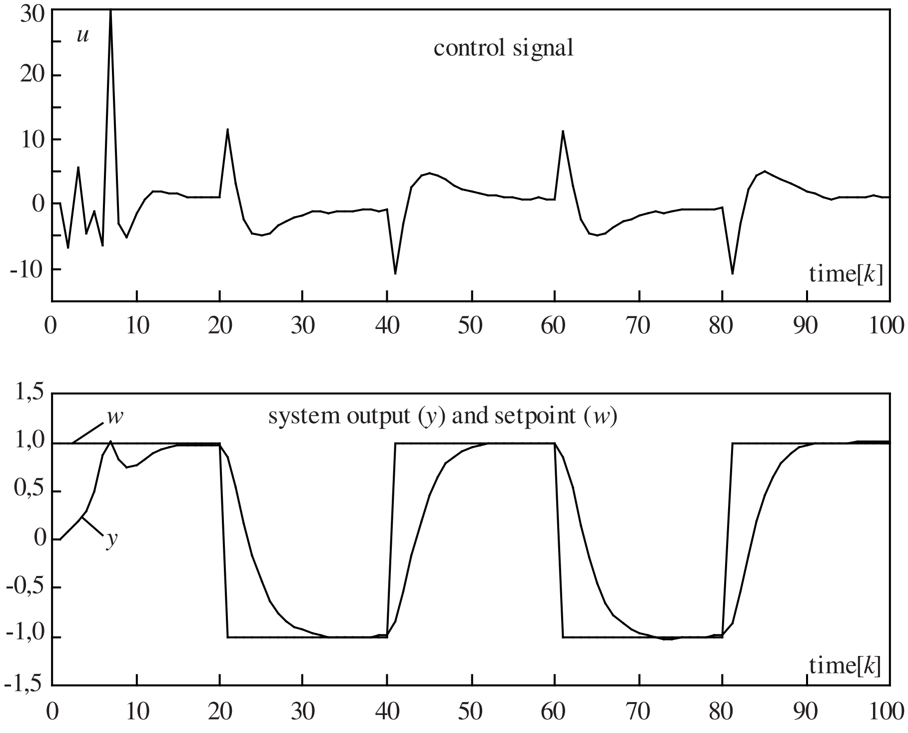

Figure 6.3. Direct adaptive dual control for an unstable plant (Example 2)

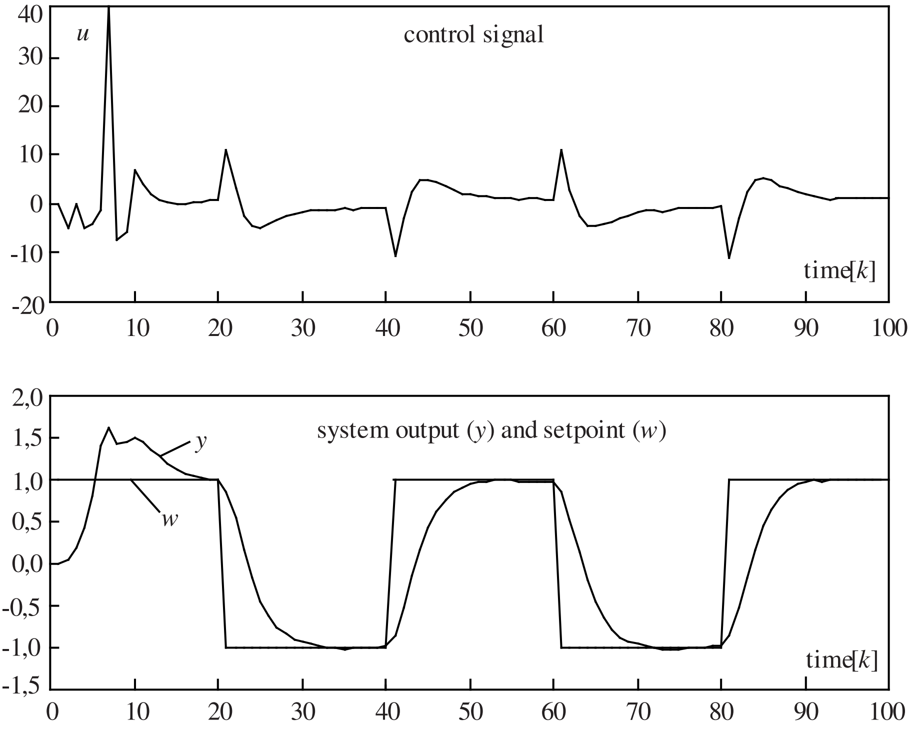

Figure 6.4. Standard APPC for an unstable plant (Example 2)

7. DUAL MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

In this chapter, the bicriterial dual control approach is used for developing the dual version of the model reference adaptive control (MRAC) scheme proposed by Landau and Lozano (1981). The formulation of the synthesis problem based on a bicriterial approach is given in Section 7.1. The dual MRAC (DMRAC) is described in Section 7.2. The modification of the approach for nonminimumphase systems through the introduction of the weighted control signal into the cost functional is given in Section 7.3. Schemes for MRAC and DMRAC are presented in Section 7.4. Simulations for the new controller and a comparison with the classical MRAC based on the CE assumption are given in Section 7.5.

7.1. Formulation of the Bicriterial Synthesis Problem for Dual MRAC

At first the case will be considered that the model of a SISO plant with unknown parameters, which is described by eqs. (4.37) to (4.40)

$$A(z^{-1})Y(z) = z^{-d} B(z^{-1})U(z) + C(z^{-1})\Xi(z), \tag{7.1}$$

is minimum phase, where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{n_\mathrm{A}} z^{-n_\mathrm{A}} \quad , \tag{7.2}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_{n_\mathrm{B}} z^{-n_\mathrm{B}}, \; b_0 \neq 0, \tag{7.3}$$

and

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_{n_\mathrm{C}} z^{-n_\mathrm{C}} . \tag{7.4}$$

$Y(z)$ and $U(z)$ are the z-transforms of the output signal $y(k)$ and the control signal $u(k)$, $\Xi(z)$ is the z-transform of a white noise sequence $\xi(k)$ with zero mean and covariance σ_ξ^2, k is the discrete time, and $d \geq 1$ represents the plant dead time. The case of a nonminimum phase plant described by eq. (7.1) is considered in Section 7.3. Following Landau and Lozano (1981) the desired input-output relationship between setpoint $w(k)$ and reference model output $y_\mathrm{m}(k)$ (whose z-transformed equivalents are $W(z)$ and $Y_\mathrm{m}(z)$, respectively) is defined by the stable explicit reference model

$$A_\mathrm{m}(z^{-1})Y_\mathrm{m}(z) = z^{-d} B_\mathrm{m}(z^{-1})W(z), \tag{7.5}$$

where

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 75–84, 2004.

$$A_{\mathrm{m}}(z^{-1}) = 1 + a_1^{\mathrm{m}} z^{-1} + \ldots + a_{n_{A_{\mathrm{m}}}}^{\mathrm{m}} z^{-n_{A_{\mathrm{m}}}} , \tag{7.6}$$

and

$$B_{\mathrm{m}}(z^{-1}) = b_0^{\mathrm{m}} + b_1^{\mathrm{m}} z^{-1} + \ldots + b_{n_{B_{\mathrm{m}}}}^{\mathrm{m}} z^{-n_{B_{\mathrm{m}}}} . \tag{7.7}$$

In the case of $w(k) \equiv 0$, in contrast to the above mentioned system structure of Landau and Lozano (1981), another reference model has to be used to eliminate initial disturbances with the dynamics defined by an asymptotically stable polynomial $D(z^{-1})$. In this case, the control objective is accomplished if

$$D(z^{-1}) z^d E_{\mathrm{m}}(z) = 0 , \tag{7.8}$$

where

$$E_{\mathrm{m}}(z) = Y(z) - Y_{\mathrm{m}}(z) , \; D(z^{-1}) = 1 + d_1 z^{-1} + \ldots + d_{n_{\mathrm{D}}} z^{-n_{\mathrm{D}}} . \tag{7.9}$$

The well-known identity (Aström 1970)

$$D(z^{-1}) = A(z^{-1}) S(z^{-1}) + z^{-d} R(z^{-1}) \tag{7.10}$$

has a unique solution $S(z^{-1})$ and $R(z^{-1})$:

$$S(z^{-1}) = 1 + s_1 z^{-1} + \ldots + s_{n_{\mathrm{S}}} z^{-n_{\mathrm{S}}} , \tag{7.11}$$

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \ldots + r_{n_{\mathrm{R}}} z^{-n_{\mathrm{R}}} , \tag{7.12}$$

where $n_{\mathrm{S}} = d - 1$ and $n_{\mathrm{R}} = \max(n_{\mathrm{A}} - 1, n_{\mathrm{D}} - d)$. Therefore, eq. (7.8) can be written in the form

$$\begin{aligned} D(z^{-1}) z^d E_{\mathrm{m}}(z) = {} & B(z^{-1}) S(z^{-1}) U(z) + R(z^{-1}) Y(z) + C(z^{-1}) S(z^{-1}) z^d \Xi(z) \\ & - D(z^{-1}) z^d Y_{\mathrm{m}}(z). \end{aligned} \tag{7.13}$$

After introducing the filtered signals

$$\overline{E}_{\mathrm{m}}(z) = D(z^{-1}) E_{\mathrm{m}}(z) , \tag{7.14a}$$

$$\overline{Y}(z) = D(z^{-1}) Y(z) , \tag{7.14b}$$

$$\overline{Y}_{\mathrm{m}}(z) = D(z^{-1}) Y_{\mathrm{m}}(z) , \tag{7.14c}$$

$$\overline{\Xi}(z) = C(z^{-1}) S(z^{-1}) \Xi(z) , \tag{7.14d}$$

which represent the corresponding signals $\overline{e}_{\mathrm{m}}(k)$, $\overline{y}(k)$, $\overline{y}_{\mathrm{m}}(k)$ and $\overline{\xi}(k)$ in the time domain, respectively, and taking the inverse z-transformation, eq. (7.13) can be written as

$$\begin{aligned} \overline{e}_{\mathrm{m}}(k+d) &= b_0 u(k) + \boldsymbol{p}_0^{\mathrm{T}} \boldsymbol{m}_0(k) - \overline{y}_{\mathrm{m}}(k+d) + \overline{\xi}(k+d) \\ &= \boldsymbol{p}^{\mathrm{T}} \boldsymbol{m}(k) - \overline{y}_{\mathrm{m}}(k+d) + \overline{\xi}(k+d), \end{aligned} \tag{7.15}$$

where

$$\boldsymbol{m}^{\mathrm{T}}(k) = [u(k) \ldots u(k-n_{\mathrm{S}}-n_{\mathrm{B}}),\ y(k) \ldots y(k-n_{\mathrm{R}})] = [u(k) \vdots \boldsymbol{m}_0^{\mathrm{T}}(k)], \tag{7.16}$$

and

$$\boldsymbol{p}^{\mathrm{T}} = [b_0\ \ b_0 s_1 + b_1\ \ b_0 s_2 + b_1 s_1 + b_2\ \ldots\ b_{n_{\mathrm{B}}} s_{n_{\mathrm{S}}}\ \ r_0\ \ldots\ r_{n_{\mathrm{R}}}] = [b_0 \vdots \boldsymbol{p}_0^{\mathrm{T}}]. \tag{7.17}$$

The equation for estimating the parameter vector described by eq. (7.17) can be obtained from eq. (7.15) in the form

$$\bar{y}(k+d) = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k) + \bar{\xi}(k+d). \tag{7.18}$$

It is easy to see that for $C(z^{-1}) = 1$, the scalar $\bar{\xi}(k+d)$ and the vector $\boldsymbol{m}(k)$ are stochastically independent and therefore, the following algorithm of recursive least squares can be used for unbiased and minimum variance estimation of $\boldsymbol{p}$ in eq. (7.18) (see, for example, Durbin, 1960):

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1)\,[\bar{y}(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k-d+1)], \tag{7.19}$$

$$\boldsymbol{q}(k+1) = \boldsymbol{P}(k)\boldsymbol{m}(k-d+1)[\boldsymbol{m}^{\mathrm{T}}(k-d+1)\boldsymbol{P}(k)\boldsymbol{m}(k-d+1) + \bar{\sigma}_{\xi}^2]^{-1}, \tag{7.20}$$

and

$$\boldsymbol{P}(k+1) = \boldsymbol{P}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k-d+1)\boldsymbol{P}(k), \tag{7.21}$$

where $\hat{\boldsymbol{p}}(k) = \mathrm{E}\{\boldsymbol{p}(k)|\Im_k\}$, $\boldsymbol{P}(k) = \mathrm{E}\{[\boldsymbol{p} - \hat{\boldsymbol{p}}(k)][\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}}|\Im_k\}$, $\Im_k$ is defined by eq. (4.12) and $\bar{\sigma}_{\xi}^2$ is the variance of the filtered disturbance $\bar{\xi}(k)$

$$\bar{\sigma}_{\xi}^2 = (1 + \sum_{i=1}^{n_{\mathrm{S}}} s_i^2)\sigma_{\xi}^2. \tag{7.22}$$

In the case of time-varying plant parameters, the parameter drift model and estimator described in Section 4 can be used.

To accomplish the control objective

$$\bar{e}_{\mathrm{m}}(k+d) = 0, \tag{7.23}$$

the following two performance indices for control optimization are again introduced to derive the dual controller:

$$J_k^{\mathrm{c}} = \mathrm{E}\{\bar{e}_{\mathrm{m}}^2(k+d)|\Im_k\}, \tag{7.24}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\{[\bar{y}(k+d) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2|\Im_k\}. \tag{7.25}$$

The second performance index according to eq. (7.25) is used for the acceleration of the parameter estimation by increasing the innovation value in eq. (7.19) (see Chapter 4). By this bicriterial approach, the dual controller is derived after performing the optimization according to eqs. (4.22) to (4.25) with the cost functions determined by eqs. (7.24) and (7.25). As described in Chapter 4, Ω_k is a bounded compact domain symmetrically located around the optimal solution $u_c(k)$ for the first criterion, where $u_c(k)$ is the cautious control. In this domain, the minimum for the second criterion should be found according to eq. (4.22). In this way, the amplitude of the optimal active excitation, which accelerates the parameter estimation, depends on the scalar parameter η and the accuracy of the estimation (trace of the covariance matrix) according to eq. (4.24).

7.2. Design of Dual MRAC (DMRAC)

After introducing eq. (7.15) into eq. (7.24) and taking the expectation for $C(z^{-1}) = 1$, the cost function takes the following form:

$$\begin{aligned} J_k^{\mathrm{c}} &= \mathrm{E}\left\{2[\boldsymbol{p}_0^{\mathrm{T}} \boldsymbol{m}_0(k) - y_{\mathrm{m}}(k+d)]\, b_0 u(k) + b_0^2 u^2(k) \mid \Im_k \right\} + \bar{c}_1(k) \\ &= 2[\hat{b}_0(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)]\, \boldsymbol{m}_0(k) u(k) - 2\hat{b}_0(k) \bar{y}_{\mathrm{m}}(k+d) u(k) \\ &\quad + [\hat{b}_0^2(k) + p_{b_0}(k)]\, u^2(k) + \bar{c}_1(k)\,, \end{aligned} \tag{7.29}$$

where

$$\begin{aligned} \bar{c}_1(k) &= \bar{\sigma}_\xi^2 + \boldsymbol{m}_0^{\mathrm{T}}(k)(\hat{\boldsymbol{p}}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{P}_{p_0}(k))\boldsymbol{m}_0(k) + \bar{y}_{\mathrm{m}}^2(k+d) \\ &\quad - 2\bar{y}_{\mathrm{m}}(k+d)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) \end{aligned}$$

does not depend on $u(k)$, and the covariance matrix of eq. (7.21) has the form

$$\boldsymbol{P}(k) = \begin{bmatrix} p_{b_0}(k) & \boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_0 p_0}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix}. \tag{7.30}$$

Only the case when $C(z^{-1}) = 1$ is considered here. The first derivative of eq. (7.29) with respect to $u(k)$ takes the form

$$\begin{aligned} \frac{\partial J_k^{\mathrm{c}}}{\partial u} &= 2[\hat{b}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)]\, \boldsymbol{m}_0(k) - 2\hat{b}_0(k)\bar{y}_{\mathrm{m}}(k+d) \\ &\qquad + 2[\hat{b}_0^2(k) + p_{b_0}(k)]\, u(k). \end{aligned} \tag{7.31}$$

Equating the first derivative to zero and taking into account that the second derivative

$$\frac{\partial^2 J_k^{\mathrm{c}}}{\partial u(k)^2} = 2\,[\hat{b}_0^2(k) + p_{b_0}(k)] \geq 0\,, \tag{7.32}$$

is non-negative (i.e. the minimum of J_k^{c} is achieved), we obtain the cautious controller

$$u_{\mathrm{c}}(k)=\frac{\hat{b}_0(k)\bar{y}_{\mathrm{m}}(k+d)-[\hat{b}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)+\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)]\boldsymbol{m}_0(k)}{\hat{b}_0^2(k)+p_{b_0}(k)}, \tag{7.33}$$

which provides the minimum of eq. (7.29). The minimum for the second cost function, eq. (4.19), is obtained by eq. (4.31), (see also eqs. (2.20) and (2.21)).

Substitution of eq. (7.18) into eq. (7.25) leads to the considered cost function

$$J_k^{\mathrm{a}}(u(k))=-p_{b_0}(k)u^2(k)-2\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k)+\bar{c}_2(k), \tag{7.34}$$

where

$$\bar{c}_2(k)=-[\bar{\sigma}_\xi^2+\boldsymbol{m}_0^T(k)\boldsymbol{P}_{\boldsymbol{p}_0}(k)\boldsymbol{m}_0(k)]. \tag{7.35}$$

Therefore, the difference between the values of the cost function according to eq. (7.34) on the boundaries of the domain of eq. (4.23) is defined by

$$\begin{aligned} &J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)-\theta(k)]-J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)+\theta(k)] \\ &\quad=-p_{b_0}(k)[u_{\mathrm{c}}(k)-\theta(k)]^2-2\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)[u_{\mathrm{c}}(k)-\theta(k)] \\ &\qquad+p_{b_0}(k)[u_{\mathrm{c}}(k)+\theta(k)]^2+2\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)[u_{\mathrm{c}}(k)+\theta(k)]. \end{aligned} \tag{7.36}$$

Further manipulations of eq. (7.36) lead to

$$J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)-\theta(k)]-J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)+\theta(k)]=4p_{b_0}(k)u_{\mathrm{c}}(k)\theta(k)+4\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\theta(k). \tag{7.37}$$

After substitution of eq. (7.37) into eq. (4.31) and with the help of some simple manipulations using eq. (4.21) the following dual control law can be obtained:

$$u(k)=u_{\mathrm{c}}(k)+\eta\,\mathrm{tr}\{\boldsymbol{P}(k)\}\,\mathrm{sgn}\{p_{b_0}(k)u_{\mathrm{c}}(k)+\boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\}. \tag{7.38}$$

In comparison with eq. (7.38) the adaptive controller obtained by the standard approach based on the CE assumption, results from the minimization of the cost function, similar to eq. (7.24)

$$J_k^{\mathrm{c}}=\mathrm{E}_{\rho_k}\left\{\bar{e}_{\mathrm{m}}^2(k+d)\middle|\Im_k\right\}, \tag{7.39}$$

where the expectation is taken under the CE assumption

$$\rho_k=\{\boldsymbol{p}=\hat{\boldsymbol{p}}(k)\}. \tag{7.40}$$

Minimization of the cost function, eq. (7.39), leads for the considered system with the error $\bar{e}_{\mathrm{m}}(k+d)$ described by eq. (7.15) to the controller suggested by Landau and Lozano (1981)

$$u(k)=\frac{1}{\hat{b}_0(k)}[\bar{y}_{\mathrm{m}}(k+d)-\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)]. \tag{7.41}$$

The accuracy of the estimation is not taken into account in the controller described by eq. (7.41), and inadequate control performance is attributed to controllers of this kind when the values of the estimates are far from the true ones, especially at the beginning of the adaptation. The presented approach can only be applied to minimum phase systems. To extend it to nonminimum phase systems, a special criterion is introduced in the next section. A correction/compensation network as suggested by Unbehauen and Keuchel (1992) can also be used for this purpose. For the proof of convergence properties of the considered system, the approaches presented by Landau and Lozano (1981) and Filatov and Unbehauen (1994) can be applied.

7.3. Controller for Nonminimum Phase Plants

Consider the case in which the first performance index, described by eq. (7.24), is selected in the more general form

$$J_k^{\mathrm{c}} = \mathrm{E}_{\rho_k} \left\{ \bar{e}_{\mathrm{m}}^2(k+d) + \bar{u}^2(k) \middle| \Im_k \right\}, \tag{7.42}$$

where

$$\bar{u}(k) = \sum_{i=0}^{n_{\mathrm{G}}} g_i u(k-i) = g_0 u(k) + \bar{u}_0(k-1) \; . \tag{7.43}$$

The coefficients g_i determine the polynomials

$$G(z^{-1}) = g_0 + g_1 z^{-1} + \ldots + g_{n_{\mathrm{G}}} z^{-n_{\mathrm{G}}} \quad \text{and} \quad G_0(z^{-1}) = G(z^{-1}) - g_0 \, . \tag{7.44}$$

Minimization of the cost function, eq. (7.39), using eqs. (7.15) and (7.40) leads to the cautious control

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_0(k)\bar{y}_{\mathrm{m}}(k+d) - [\hat{b}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) + \boldsymbol{p}_{b_0 p_0}^{\mathrm{T}}(k)]\boldsymbol{m}_0(k) - g_0 \bar{u}_0(k-1)}{\hat{b}_0^2(k) + p_{b_0}(k) + g_0^2}, \tag{7.45}$$

which can be obtained in the same way as in eq. (7.33) starting from eq. (7.29). Equation (7.45) together with eq. (7.38) define the dual controller for nonminimum phase plants. By introducing the squared weighted control signal into eqs. (7.42) and (7.43) the magnitude of the control signal can be limited and large deviations of the control signal be avoided. The system provides a closed-loop behavior, which differs from the reference model according to eqs. (7.5) and (7.8). The stability of the closed-loop system depends on the location of the roots of the polynomial $b_0 B(z^{-1}) D(z^{-1}) + g_0 G(z^{-1}) A(z^{-1})$, as shown in the next section. Applying the CE approach as in eq. (7.40) during the minimization of the cost function, eq. (7.42), provides the control law

$$u(k) = \frac{1}{\hat{b}_0^2(k) + g_0^2} [\hat{b}_0(k)\bar{y}_{\mathrm{m}}(k+d) - \hat{b}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) - g_0 \bar{u}_0(k-1)] \, . \tag{7.46}$$

The structures of the designed adaptive control systems and stability conditions are considered in the next section.

7.4. Standard and Dual MRAC Schemes (DMRAC)

After z-transformation and the introduction of the notation

$$B^{\mathrm{s}}(z^{-1}) = B(z^{-1})S(z^{-1}) - b_0 \ , \tag{7.47}$$

the cautious control law of eq. (7.45) can be written as

$$U(z) = \frac{1}{\hat{b}_0^2 + p_{b_0} + g_0^2}\Big[\hat{b}_0 z^d \overline{Y}_{\mathrm{m}}(z) - \mathrm{E}\big\{b_0 R(z^{-1})\big|\Im_k\big\}Y(z) - \mathrm{E}\big\{b_0 B^{\mathrm{s}}(z^{-1})\big|\Im_k\big\}U(z) - g_0 G_0(z^{-1})U(z)\Big], \tag{7.48}$$

and the controller based on the CE assumption, eq. (7.46), takes the form

$$U(z) = \frac{1}{\hat{b}_0^2 + g_0^2}\Big[\hat{b}_0 z^d \overline{Y}_{\mathrm{m}}(z) - \hat{b}_0 \hat{R}_k(z^{-1})Y(z) - \hat{b}_0 \hat{B}_k^{\mathrm{s}}(z^{-1})U(z) - g_0 G(z^{-1})U(z)\Big], \tag{7.49}$$

where

$$\hat{R}_k(z^{-1}) = \mathrm{E}\big\{R(z^{-1})\big|\Im_k\big\},\ \hat{B}_k^{\mathrm{s}}(z^{-1}) = \mathrm{E}\big\{B^{\mathrm{s}}(z^{-1})\big|\Im_k\big\}. \tag{7.50}$$

The schemes of the MRAC according to eqs. (7.46) or (7.49) and the new DMRAC according to eqs. (7.45) or (7.48) and (7.38) with the adaptation mechanism described by eqs. (7.19) to (7.21) have the block diagram structures shown in Figures 7.1 and 7.2, respectively. It is obvious that the main difference between these adaptive control schemes is that the covariance matrix $\boldsymbol{P}(k)$ is used for forming the cautious control and the optimal excitation in the DMRAC. Therefore, the DMRAC takes into account the accuracy of the estimates. When the uncertainty is large, this controller provides a larger excitation and a smaller cautious control signal. This property of the DMRAC improves the adaptive control performance, decreases the adaptation time and provides a smoother adaptive transient behaviour.

After adaptation and in the case of known parameters and without noise $\xi(k) \equiv 0$, the z-transformed output signals of both the considered adaptive control schemes, shown in Figures 7.1 and 7.2, are given by

$$Y(z) = \frac{b_0 B(z^{-1})D(z^{-1})}{b_0 B(z^{-1})D(z^{-1}) + g_0 G(z^{-1})A(z^{-1})} Y_{\mathrm{m}}(z) \, . \tag{7.51}$$

Therefore, the stability of the nonminimum phase system can be enforced by appropriate selection of the polynomials $G(z^{-1})$ and $D(z^{-1})$. All poles of the transfer function according to eq. (7.51) must be allocated inside the unit circle in the z-plane, and their desired distribution can be determined by choosing appropriate polynomials $G(z^{-1})$ and $D(z^{-1})$ (see, for example, Unbehauen, 1985).

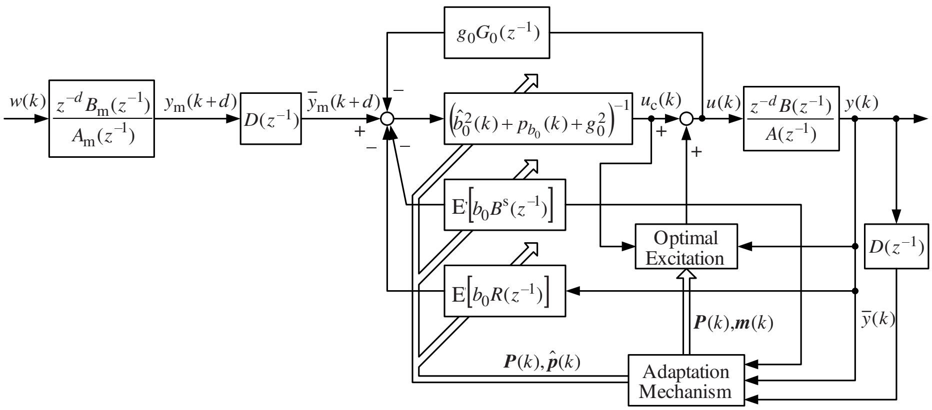

Figure 7.1. Scheme of DMRAC

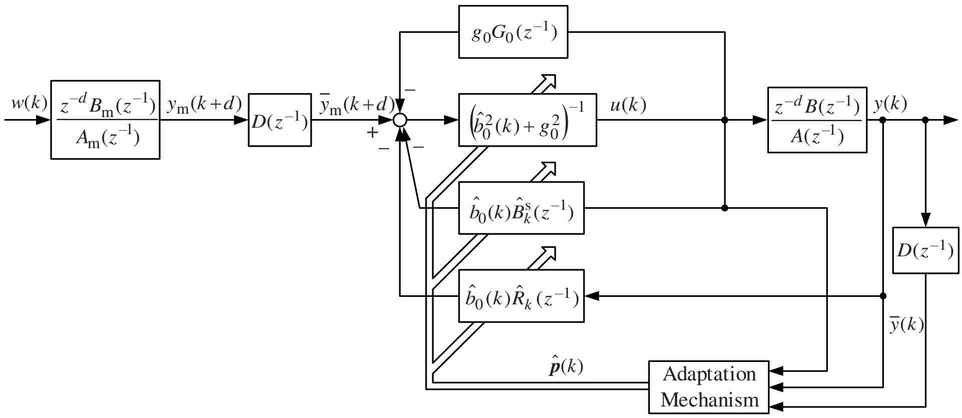

Figure 7.2. Scheme of CE-based MRAC

7.5. Simulations and Comparisons

Consider the third-order plant with an unstable oscillatory behavior and a time delay described by eq. (7.1) in the form

$$(1-2.93z^{-1}+2.866z^{-2}-0.9277z^{-3})Y(z)=z^{-2}(0.1961+0.0001z^{-1}$$
$$-0.1892z^{-2})U(z)+\Xi(z)\,. \qquad (7.52)$$

The reference model described by eqs. (7.5) and (7.9) is given as

$$\frac{z^{-d}B_{\rm m}(z^{-1})}{A_{\rm m}(z^{-1})}=\frac{0.5z^{-2}}{1-0.5z^{-1}},$$
$$. \qquad (7.53)$$
$$D(z^{-1})=(1-0.2z^{-1})(1-(0.6+0.1{\rm j})z^{-1})(1-(0.6-0.1{\rm j})z^{-1})$$

The other parameters for this simulation example are chosen as

$$\sigma_\xi^2 = 0.00005,\ \boldsymbol{P}(0) = \mathbf{I},\ \hat{\boldsymbol{p}}^{\mathrm{T}}(0) = [0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1]\,,$$
$$G(z^{-1}) = 0.1(1 - z^{-1}),\ \eta = 0.01, C(z^{-1}) = 1\,. \tag{7.54}$$

The reference signal is a square wave with an amplitude of ±1 and a period of 20 sampling steps. The simulation results for the DMRAC, given by eqs. (7.45) and (7.38), as well as for the cautious MRAC, eq. (7.45), when η = 0 and the MRAC based on the CE assumption, eq. (7.46), are shown in Figures 7.3, 7.4 and 7.5, respectively. From these figures it can be seen that the differences between the control actions are observable only at the beginning of the adaptation. The DMRAC provides the best control performance. Cautious control usually gives better control performance than the generally used CE approach, but the time for adaptation is longer, and small values of the control signals can sometimes lead to a "turn-off" effect (Zhivoglyadov *et al.*, 1993a) when the parameter estimation process is almost interrupted. Therefore, the dual controller provides the best control performance, a fast adaptation with a smooth startup because of its cautious and active learning properties. Moreover, the designed DMRAC is computationally simple and can be easily applied. In the considered example, the polynomial $G(z^{-1})$ of eq. (7.44) provides better control performance than when $G(z^{-1})$ = 0. The parameter $\eta = 0.01$ can easily be selected because of its clear physical interpretation (i.e. the amplitude of the active learning signals).

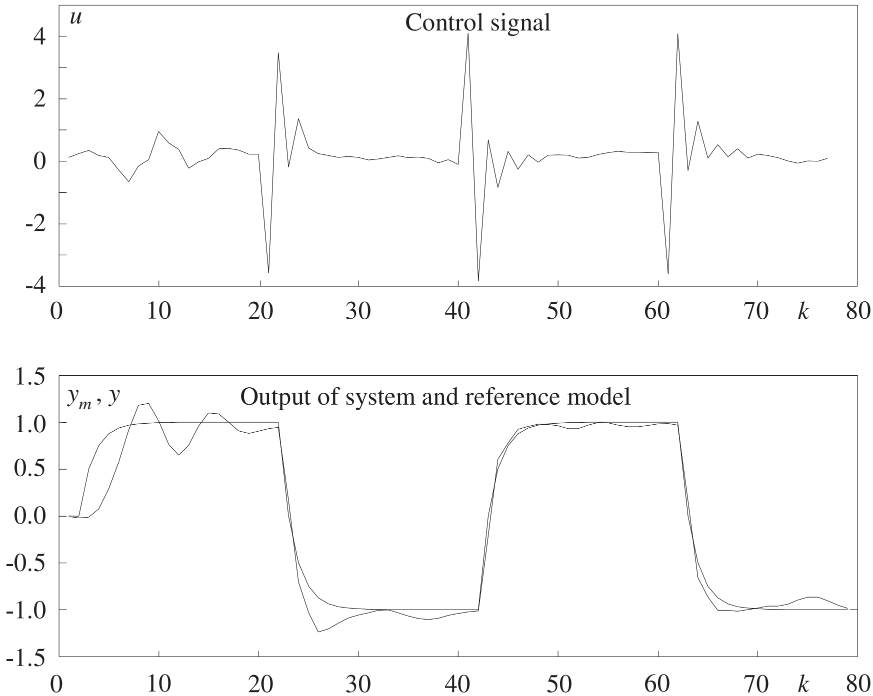

Figure 7.3. Dual control

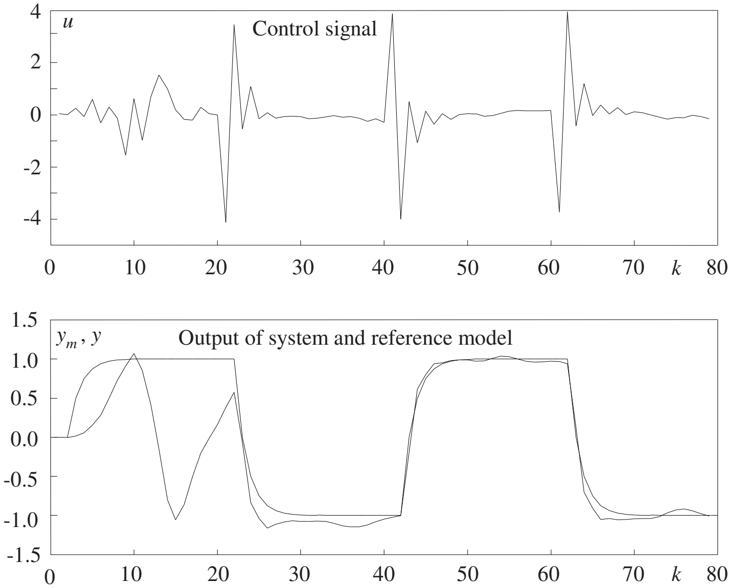

Figure 7.4. Cautious control

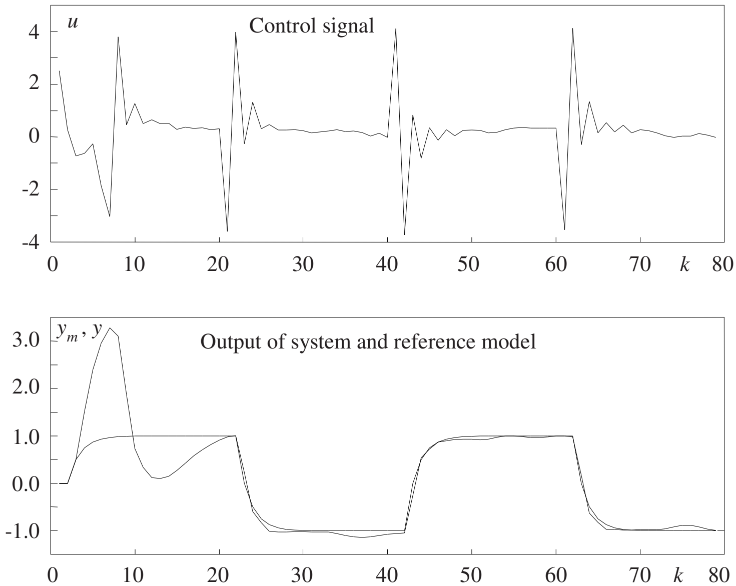

Figure 7.5. Adaptive control based on the CE assumption

8. DUAL CONTROL FOR MULTIVARIABLE SYSTEMS IN STATE SPACE REPRESENTATION

Applications of the bicriterial approach to various Single-Input Single-Output (SISO) adaptive systems of type III (according to Table 3.1 and classification suggested in Chapter 3) have been presented above. However, some difficulties may arise during the consideration of multivariable systems and control systems of type I.

In this chapter, the bicriterial approach will be developed, based on Lyapunov functions, for the design of multivariable adaptive controllers applied to plants described by linear models with unknown parameters. The resulting control algorithms for multivariable systems of type I are computationally simple and provide improved performance because of the attributed dual properties. The partial certainty equivalence (PCE) control policy, considered in Chapter 3 (Filatov and Unbehauen, 1995b), will be used here in combination with the bicriterial approach to derive a computationally simple adaptive dual controller. In Section 8.1, the general synthesis problem will be formulated using bicriterial optimization and Lyapunov functions applied to linear system models in state space representation. The synthesis procedure of dual controllers will be described in Section 8.2. Implementation details and interconnections with the well-known linear quadratic control problem will be discussed in Section 8.3. Statistical experimental and simulation results obtained from the application of the elaborated approach to control a helicopter will be used to demonstrate the potential of the method and to compare it with the standard approach based on the CE assumption in Section 8.4. The PCE control policy for linear MIMO systems will be presented in Section 8.5. The detailed mathematical derivation of the PCE control is given in Appendix A. The dual control policy will be derived in Section 8.6 using the PCE approximation. The simulation results as well as the comparison of the PCE, CE and dual controllers will be presented in Section 8.7.

8.1. Synthesis Problem Formulation by Applying Lyapunov Functions

The following linear dynamic multivariable system in discrete time representation with unknown parameters is considered:

$$\begin{aligned} &\boldsymbol{x}(k+1) = \boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}(k) + \boldsymbol{B}(\boldsymbol{p})\boldsymbol{u}(k) + \boldsymbol{\xi}(k), \;\; k = 0, 1, \ldots, \\ &\boldsymbol{x}(0) = \boldsymbol{x}_0 \, , \end{aligned} \tag{8.1}$$

where $\boldsymbol{x}(k) \in \Re^{n_x}$ is the state vector, $\boldsymbol{u}(k) \in \Re^{n_u}$ is the control input vector, $\boldsymbol{p} \in \Re^{n_p}$ is the vector of unknown parameters, $\{\boldsymbol{\xi}(k) \in \Re^{n_\xi}\}$ is the sequence of independent random

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vectors with zero mean and known covariance matrix $\boldsymbol{Q}_\xi$, $\boldsymbol{A}(\boldsymbol{p})$ and $\boldsymbol{B}(\boldsymbol{p})$ are matrices of corresponding dimensions, being linear functions of the vector $\boldsymbol{p}$ of unknown parameters, and $\boldsymbol{x}_0$ is the initial system state. The system, according to eq. (8.1), is assumed to be controllable and parametrically identifiable and the state vector is assumed to be fully accessible. In contrast with the problem considered in Section 2.1, the linear system with constant parameters, a completely accessible state vector and an infinite horizon are considered.

The set of state and control values at time k differs slightly from eq. (2.4) and is denoted as

$$\Im_k = \{\boldsymbol{x}(k), ..., \boldsymbol{x}(0), \boldsymbol{u}(k-1), ..., \boldsymbol{u}(0)\},\ k = 1, ...,\ \Im_0 = \{\boldsymbol{x}(0)\}. \tag{8.2}$$

After the introduction of the Lyapunov function

$$v_k = \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_k\boldsymbol{x}(k)\ , \tag{8.3}$$

where $\boldsymbol{P}_k$ is a positive definite symmetric matrix, the first difference of this Lyapunov function is defined by

$$\Delta v_k = \boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{P}_k\boldsymbol{x}(k+1) - \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_k\boldsymbol{x}(k)\ . \tag{8.4}$$

Obviously, the stability of the system can be guaranteed if $v_k > 0$ and $\Delta v_k < 0$ for $\boldsymbol{x}(k) \neq \boldsymbol{0}$ (Kuo, 1992). To derive the control law that would stabilize the system, the performance index

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{\Delta v_k + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k)\middle|\Im_k\right\}, \tag{8.5}$$

has to be minimized, where $\boldsymbol{R}$ is a positive definite symmetric matrix of corresponding dimensions and $\mathrm{E}\{\cdot|\Im_k\}$ is the conditional expectation operator when the set $\Im_k$ according to eq. (8.2) is available. It is suggested here to choose the matrix $\boldsymbol{P}_k$ of the Lyapunov function to satisfy the inequality

$$\mathrm{E}_{\rho_k}\left\{\Delta v_k + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k)\middle|\Im_k\right\} \leq 0\,, \tag{8.6}$$

where

$$\rho_k = \{\boldsymbol{A}(\boldsymbol{p}) = \boldsymbol{A}(\hat{\boldsymbol{p}}(k)),\ \boldsymbol{B}(\boldsymbol{p}) = \boldsymbol{B}(\hat{\boldsymbol{p}}(k)),\ \boldsymbol{\xi}(k) = \boldsymbol{0}\} \tag{8.7}$$

is the CE assumption when all the random values in the system are assumed to be equal to their expectations and

$$\hat{\boldsymbol{p}}(k) = \mathrm{E}\{\boldsymbol{p}|\Im_k\} \tag{8.8}$$

is the estimate (conditional expectation) of the unknown parameter vector. A stabilizing feedback control of the closed-loop system is guaranteed by inequality (8.6) after adaptation, because the value of the first difference of the Lyapunov function according to eq. (8.4) is negative. Thus, the CE assumption of the form ρ_k is used here only for the derivation of the matrix $\boldsymbol{P}_k$.

Following the bicriterial approach to design a dual controller, the second performance index

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))^{\mathrm{T}} \boldsymbol{W}(\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k))|\Im_k\right\}, \tag{8.9}$$

is introduced, where

$$\hat{\boldsymbol{x}}(k+1|k) = \mathrm{E}\left\{\boldsymbol{x}(k+1)|\Im_k\right\} = \boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k) + \boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}(k) \tag{8.10}$$

and $\boldsymbol{W}$ is a positive semidefinite symmetric weighting matrix. Minimization of eq. (8.9) will increase the expected error of the one-step-ahead prediction $\hat{\boldsymbol{x}}(k+1|k)$. This error contains the information that is used for parameter estimation (innovational value) and its covariance matrix and, in this way, accelerates the parameter estimation process. Minimization of the first performance index, eq. (8.5), results in a control signal which may be named "cautious control" $\boldsymbol{u}_{\mathrm{c}}(k)$ in analogy with the practice adopted in dual control. The second performance index, eq. (8.9), is minimized in the compact domain Ω_k, and this domain is symmetrically located around $\boldsymbol{u}_{\mathrm{c}}(k)$. The value of the optimal excitation will depend on the size of Ω_k. The domain Ω_k for multivariable systems is defined as

$$\Omega_k = \left\{\boldsymbol{u}(k) : \boldsymbol{u}(k) = \boldsymbol{u}_c(k) + \boldsymbol{u}_a(k), \boldsymbol{u}_a^{\mathrm{T}}(k)\boldsymbol{W}_a\boldsymbol{u}_a(k) \le f_k(\boldsymbol{P}(k))\right\}, \tag{8.11}$$

where $\boldsymbol{u}_{\mathrm{a}}(k)$ is interpreted as an excitation signal, $\boldsymbol{W}_{\mathrm{a}}$ is a positive definite weighting matrix of corresponding dimensions,

$$\boldsymbol{P}(k) = \mathrm{E}\left\{(\boldsymbol{p} - \hat{\boldsymbol{p}}(k))(\boldsymbol{p} - \hat{\boldsymbol{p}}(k))^{\mathrm{T}}|\Im_k\right\} \tag{8.12}$$

is the covariance matrix of the estimation errors of the unknown parameters and $f_k(\cdot)$ is a positive definite scalar function of the covariance matrix. Function f_k, which, together with the matrix $\boldsymbol{W}_{\mathrm{a}}$, determines the amplitude of the excitation signal, may be chosen in one of the following forms:

$$f_k(\boldsymbol{P}(k)) = \eta\,\mathrm{tr}\{\boldsymbol{P}(k)\},\ \eta \ge 0, \tag{8.13}$$

$$f_k(\boldsymbol{P}(k)) = \eta(\mathrm{tr}\{\boldsymbol{P}(k)\})^2. \tag{8.14}$$

Thus the size of Ω_k and, therefore, the magnitude of the excitation depend on the accuracy of estimation, which is characterized by $\boldsymbol{P}(k)$, as in eq. (4.24). The function according to eq. (8.14) may be used to provide a faster decrease in the excitation than the previous one described by eq. (8.13).

The synthesis problem for deriving the control law as described above is summarized in the following bicriterial optimization:

$$\boldsymbol{u}(k) = \arg\min_{\boldsymbol{u}(k)\in\Omega_k} J_k^{\mathrm{a}}, \tag{8.15}$$

where, according to eq. (8.11), Ω_k is an ellipsoid centred at $\boldsymbol{u}_{\mathrm{c}}(k)$ with

$$\boldsymbol{u}_{\mathrm{c}}(k) = \arg\min_{\boldsymbol{u}(k)} J_k^{\mathrm{c}} . \tag{8.16}$$

At the same time, the matrix $\boldsymbol{P}_k$ in eq. (8.4) must be chosen to fulfil inequality (8.6).

8.2. Synthesis of Adaptive Dual Controllers

Substitution of eq. (8.1) into eq. (8.4) leads to

$$\Delta v_k = \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})\boldsymbol{u}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k(\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}(k) + \xi(k)) + c_1(k), \tag{8.17}$$

where $c_1(k)$ does not depend on $\boldsymbol{u}(k)$. Using the last equation, eq. (8.5) can be expressed as

$$J_k^{\mathrm{c}} = \boldsymbol{u}^{\mathrm{T}}(k)\mathrm{E}\left\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})\middle|\Im_k\right\}\boldsymbol{u}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\mathrm{E}\left\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{A}(\boldsymbol{p})\middle|\Im_k\right\}\boldsymbol{x}(k)$$
$$+\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k) + c_2(k), \tag{8.18}$$

where $c_2(k)$ does not contain $\boldsymbol{u}(k)$. Taking into account the positive definiteness of $\boldsymbol{R}$, it follows that the cautious control

$$\boldsymbol{u}_{\mathrm{c}}(k) = -[\mathrm{E}\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})|\Im_k\} + \boldsymbol{R}]^{-1}\mathrm{E}\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{A}(\boldsymbol{p})|\Im_k\}\ \boldsymbol{x}(k) \tag{8.19}$$

minimizes the performance index according to eq. (8.18). The second performance index, eq. (8.9), can be represented in the following form after substituting of eq. (8.1) and taking the expectation:

$$J_k^{\mathrm{a}} = -[\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{A}}(k)\boldsymbol{x}(k) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{B}}(k)\boldsymbol{u}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k) + \mathrm{tr}\{\boldsymbol{W}\boldsymbol{Q}_{\xi}\}], \tag{8.20}$$

where

$$\boldsymbol{P}_{\mathrm{A}}(k) = \mathrm{E}\{\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{W}\boldsymbol{A}(\boldsymbol{p})|\Im_k\} - \boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{W}\boldsymbol{A}(\hat{\boldsymbol{p}}(k)), \tag{8.21}$$

$$\boldsymbol{P}_{\mathrm{B}}(k) = \mathrm{E}\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{W}\boldsymbol{B}(\boldsymbol{p})|\Im_k\} - \boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{W}\boldsymbol{B}(\hat{\boldsymbol{p}}(k)), \tag{8.22}$$

and

$$\boldsymbol{P}_{\mathrm{BA}}(k) = \mathrm{E}\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{W}\boldsymbol{A}(\boldsymbol{p})|\Im_k\} - \boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{W}\boldsymbol{A}(\hat{\boldsymbol{p}}(k)). \tag{8.23}$$

According to eq. (8.11), the ellipsoid, which defines the constraint for the amplitude of the excitation, is described by

$$g_k = [\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)]^{\mathrm{T}}\boldsymbol{W}_{\mathrm{a}}[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)] - f_k(\boldsymbol{P}(k)). \tag{8.24}$$

Then, taking into account the convexity of the last constraint and the concavity of the second performance index, eq. (8.20), the necessary conditions for minimization of this performance index can be written as

$$\lambda_k \nabla J_k^{\mathrm{a}} = -\nabla g_k\,, \tag{8.25}$$

where ∇ is the gradient vector operator with respect to $\boldsymbol{u}(k)$ and $\lambda_k \geq 0$ is a scalar parameter to be defined. The sufficient condition for the minimum is the positive definiteness of the second derivative (Hessian) matrix of the cost functional, that is,

$$\frac{\partial^2(\lambda_k J_k^{\mathrm{a}} + g_k)}{\partial \boldsymbol{u}^2(k)} > 0\,. \tag{8.26}$$

Substituting eqs. (8.20) and (8.24) into eq. (8.25) gives

$$-\lambda_k \boldsymbol{P}_{\mathrm{B}}(k)\boldsymbol{u}(k) + \boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}(k) - \boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{c}}(k) - \lambda_k \boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k) = \boldsymbol{0}\,. \tag{8.27}$$

After solving the last equation, the control law can finally be written as

$$\boldsymbol{u}(k) = [\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k)]^{-1}\,[\boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{c}}(k) + \lambda_k \boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k)] \tag{8.28}$$

with the conditions

$$[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)]^{\mathrm{T}}\boldsymbol{W}_{\mathrm{a}}(\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)) - f_k(\boldsymbol{P}(k)) = 0 \tag{8.29}$$

and

$$\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k) > \boldsymbol{0}\,. \tag{8.30}$$

Therefore, to find the dual control signal, eqs. (8.28) and (8.29) should be solved together after calculating the cautious control and fulfilling inequality (8.30).

Consider inequality (8.6) to determine the matrix $\boldsymbol{P}_k$. After using eqs. (8.19), (8.28) and (8.29) and taking the expectation with the CE assumption described by eq. (8.7), the inequality (8.6) takes the form

$$\begin{aligned}&\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k)\\ &+\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k) - \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_k\boldsymbol{x}(k) \leq 0\end{aligned} \tag{8.31}$$

or

$$\begin{aligned}&-\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\,[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k)) + \boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k)\\ &+\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k) - \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_k\boldsymbol{x}(k) \leq 0\,.\end{aligned} \tag{8.32}$$

From the last inequality, it follows that the matrix $\boldsymbol{P}_k$ must satisfy the inequality

$$\begin{aligned}&-\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\,[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k)) + \boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\\ &+\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k)) - \boldsymbol{P}_k \leq \boldsymbol{0}\,.\end{aligned} \tag{8.33}$$

Some aspects for the implementation of the designed dual controller, given by eqs. (8.19), (8.28) and (8.29) with the conditions of inequalities (8.30) and (8.33), as well as their relationship to the linear quadratic approach, are discussed in the next section.

8.3. Implementation of the Designed Controller and the Relation to the Linear Quadratic Control Problem

It is easy to see that inequality (8.33) will be satisfied if the matrix $\boldsymbol{P}_k$ is defined by the equation

$$-\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\,[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k))+\boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))$$
$$+\boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))-\boldsymbol{P}_k+\boldsymbol{V}=\boldsymbol{0}, \tag{8.34}$$

where $\boldsymbol{V}$ is a positive definite matrix of dimensions $n_{\boldsymbol{x}}\times n_{\boldsymbol{x}}$. Therefore, eq. (8.34) and the additional matrix $\boldsymbol{V}$ can be used to resolve ambiguities resulting from nonunique solutions of inequality (8.33). Moreover, the matrices $\boldsymbol{R}$ and $\boldsymbol{V}$ are closely related to the linear quadratic control problem as will be shown below.

Consider the system after adaptation when the estimates have converged to their true values, this is $\hat{\boldsymbol{p}}(k)\equiv\boldsymbol{p}$. In this case, the control law obtained from eqs. (8.19), (8.28), (8.29) and (8.34) takes the form

$$\boldsymbol{u}(k)=-[\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})+\boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}(k), \tag{8.35}$$

where $\boldsymbol{P}_k=const$, satisfies the discrete (algebraic) matrix Riccati equation resulting from eq. (8.34):

$$-\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})\,[\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{B}(\boldsymbol{p})+\boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{A}(\boldsymbol{p})+\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{P}_k\boldsymbol{A}(\boldsymbol{p})-\boldsymbol{P}_k+\boldsymbol{V}=\boldsymbol{0}. \tag{8.36}$$

It is obvious that the control law, eq. (8.35) in connection with eq. (8.36), is the optimal solution to the linear quadratic control problem for the system described by eq. (8.1) with the performance index (Phillips and Nagle, 1984)

$$J=\lim_{N\to\infty}\frac{1}{N}\mathrm{E}\left\{\sum_{k=0}^{N}[\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{V}\boldsymbol{x}(k)+\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k)]\right\}. \tag{8.37}$$

To calculate the estimate $\hat{\boldsymbol{p}}(k)$ and covariance matrix $\boldsymbol{P}(k)$ during the implementation of the derived adaptive dual controller, the extended Kalman filter (Ljung, 1979) or other suitable methods of parameter estimation can be used. The controller is implemented by the following sequence of actions:

1. Measuring the state vector $\boldsymbol{x}(k)$.
2. Calculating the estimate of the parameter vector $\hat{\boldsymbol{p}}(k)$ and the covariance matrix $\boldsymbol{P}(k)$.
3. Computing of $\boldsymbol{P}_k$ from eq. (8.34) and $\boldsymbol{u}_c(k)$ from eq. (8.19).
4. Determining λ_k and $\boldsymbol{u}(k)$ from eqs. (8.28) and (8.29) using condition (8.30).
5. If the covariance matrix is small enough, then switching off the adaptation mechanism and using a controller with fixed parameters ($\boldsymbol{P}_k=const$) according to eqs. (8.35) and (8.36) in further steps.

6. Otherwise, repeating from step 1 for the next time interval k+1.

It should be noted that in many cases of time-varying parameters in the system described by eq. (8.1), the stochastic model of a Wiener process according to eq. (4.5) can be used successfully. In this case, the extended Kalman filter can also be applied for parameter estimation, and it is possible to estimate the state vector together with the parameters when measurement noise acts upon the system.

8.4. Simulation Results for Controllers Based on Lyapunov Functions

Below, the problem of helicopter control (Su and Fong, 1994) is considered in order to compare the adaptive dual controller with the standard one based on the CE assumption. The linearized dynamic equation of the helicopter is

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681+k_1 & -0.7070 & 1.42+k_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446+k_3 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix} \boldsymbol{u}(t), \tag{8.38}$$

where the bounds for the uncertain parameters are

$$|k_1| \le 0.05, \quad |k_2| \le 0.01 \quad \text{and} \quad |k_3| \le 0.04 \,. \tag{8.39}$$

The discrete-time model with sampling time 0.1 sec for the nominal parameters ($k_1 = k_2 = k_3 = 0$) is

$$\boldsymbol{x}(k+1) = \begin{bmatrix} -0.9964 & 0.0026 & -0.0004 & -0.0460 \\ 0.0045 & 0.9037 & -0.0188 & -0.3834 \\ 0.0098 & \underline{0.0339} & 0.9383 & \underline{0.1302} \\ 0.0005 & \underline{0.0017} & 0.0968 & 1.0067 \end{bmatrix} \boldsymbol{x}(k) + \begin{bmatrix} 0.0445 & 0.0167 \\ 0.3407 & -0.7249 \\ \underline{-0.5278} & \underline{0.4214} \\ -0.0268 & \underline{0.0215} \end{bmatrix} \boldsymbol{u}(k) + \boldsymbol{\xi}(k)$$

$$= \boldsymbol{A}(\boldsymbol{p})\boldsymbol{x}(k) + \boldsymbol{B}(\boldsymbol{p})\boldsymbol{u}(k) + \boldsymbol{\xi}(k), \tag{8.40}$$

where $\boldsymbol{\xi}(k)$ represents disturbances and unmodeled dynamics and its covariance matrix $\boldsymbol{Q}_{\xi}$ has been selected as $\boldsymbol{Q}_{\xi}=0.0001\,\mathbf{I}$, $\mathbf{I}$ is the identity matrix. Variation of the parameters of the continuous-time model described by eq. (8.39) within the intervals leads to the variation of the six underlined parameters of the discrete-time model, eq. (8.40), that generate the vector of unknown parameters

$$\boldsymbol{p} = [a_{3,2} \quad a_{3,4} \quad a_{4,2} \quad b_{3,1} \quad b_{3,2} \quad b_{4,2}]^{\mathrm{T}}, \tag{8.41}$$

where $a_{i,j}$ and $b_{i,j}$ are the elements of the i-th row and the j-th column of the matrices $\boldsymbol{A}(\boldsymbol{p})$ and $\boldsymbol{B}(\boldsymbol{p})$, in eq. (8.40). The other parameters of the model described by eq. (8.40) do not change significantly. The following parameters have been selected for the simulation:

$$\boldsymbol{x}(0) = [-1 \quad -1 \quad -1 \quad -1]^{\mathrm{T}},\ \boldsymbol{P}(0) = \mathbf{I},\ \boldsymbol{p}(0) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}},$$

$$\boldsymbol{R} = 0.5\mathbf{I},\ \boldsymbol{V} = \mathbf{I},\ \boldsymbol{W} = \mathbf{I}, \tag{8.42}$$

where the plant model, eq. (8.40), is taken with changes from the nominal parameter values as

$$\boldsymbol{p} = [a_{3,2} - 0.004 \quad a_{3,4} + 0.001 \quad a_{4,2} - 0.0002 \quad b_{3,1} - 0.0008 \quad b_{3,2} + 0.0018$$
$$b_{4,2} + 0.0001]^{\mathrm{T}}. \tag{8.43}$$

It is assumed here that the state vector is available from exact measurements and the extended Kalman filter is used for the estimation of the parameter vector $\boldsymbol{p}$. For the applied dual controller, the function f_k has been chosen in the form of eq. (8.14) with $\eta = 0.0001$. The parameter η can be chosen by simulation to provide good system behavior. The designed dual controller is compared with the corresponding control law, eq. (8.35), based on the CE assumption

$$\boldsymbol{u}(k) = -[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{B}(\hat{\boldsymbol{p}}(k)) + \boldsymbol{R}]^{-1}\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\boldsymbol{P}_k\boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k), \tag{8.44}$$

where $\boldsymbol{P}_k$ is determined from eq. (8.34).

The simulation results are shown in Fig. 8.1 (dual control) and Fig. 8.2 (CE control). In Fig. 8.3, the simulation results for the optimal controller with known parameters according to eqs. (8.35) and (8.36) are presented to indicate the best control behavior, which cannot be achieved in the case of unknown parameters. For the comparison the average-type of performance indices

$$\bar{J}_x = \frac{1}{N_r(N_k+1)}\sum_{i=1}^{N_r}\sum_{k=0}^{N_k+1}\boldsymbol{x}_i^{\mathrm{T}}(k)\boldsymbol{x}_i(k),\ \bar{J}_u = \frac{1}{N_r N_k}\sum_{i=1}^{N_r}\sum_{k=0}^{N_k}\boldsymbol{u}_i^{\mathrm{T}}(k)\boldsymbol{u}_i(k), \tag{8.45}$$

$$\bar{J} = \bar{J}_x + 0.5\bar{J}_u \tag{8.46}$$

have been used, where $N_k = 60$ is the number of sampled data for the simulation and $N_r = 50$ is the number of simulation runs with different parameter values taken from a normally distributed random number generator. The values of the performance indices, eqs. (8.45) and (8.46), are listed in Table 8.1 and displayed in the graph of Fig. 8.4.

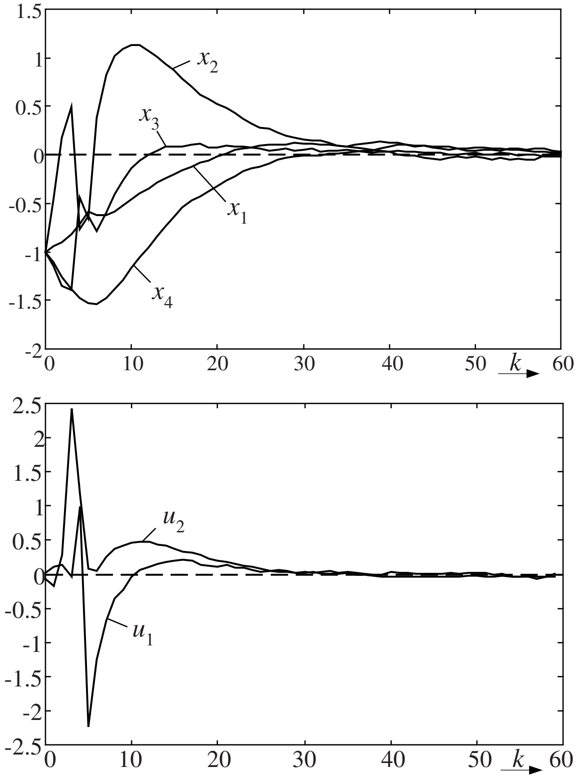
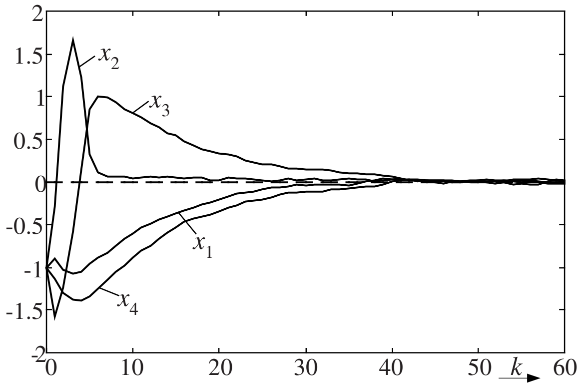


Fig. 8.1. Simulation results of the adaptive dual controller

The simulation results indicate that the proposed adaptive dual controller can improve the control performance and is superior to the adaptive standard controller based on the CE assumption. Moreover, the synthesized dual control algorithm is computationally simple and has an analytical formulation. This contrasts other dual control methods for multivariable systems (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985; Birmiwal, 1994).

Table 8.1. Average values of performance indices

Performance indices	Dual control	Control based on the CE assumption	Optimal control with known parameters
Total cost $\overline{J}$	0.8601	1.0901	0.5952
State cost J_x	0.7685	0.8098	0.5387
Control cost $\overline{J}_u$	0.1847	0.5605	0.1129



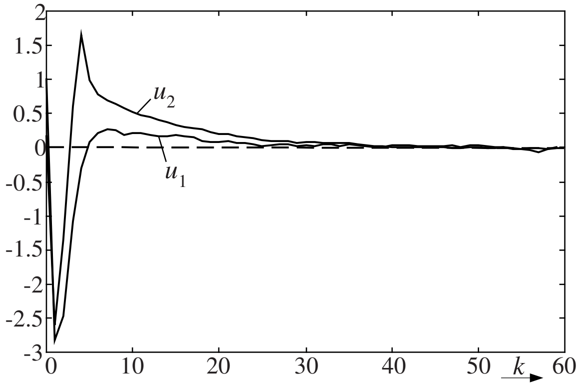


Fig. 8.2. Simulation results of the adaptive standard controller based on the certainty equivalence assumption

8.5. Partial Certainty Equivalence Control for Linear Systems

The linear version of the system described by eqs. (2.1)-(2.3)

$$\boldsymbol{x}(k+1)=\boldsymbol{A}_k(\boldsymbol{p}(k))\boldsymbol{x}(k)+\boldsymbol{B}_k(\boldsymbol{p}(k))\boldsymbol{u}(k)+\boldsymbol{\xi}(k),\ k=0,1,\ldots,N-1, \tag{8.47}$$

$$\boldsymbol{p}(k+1)=\boldsymbol{\upsilon}_k(\boldsymbol{p}(k),\boldsymbol{\varepsilon}(k)), \tag{8.48}$$

$$\boldsymbol{y}(k)=\boldsymbol{C}_k\boldsymbol{x}(k)+\boldsymbol{\eta}(k) \tag{8.49}$$

is considered, where $\boldsymbol{A}_k(\boldsymbol{p}(k))$ and $\boldsymbol{B}_k(\boldsymbol{p}(k))$ are matrices of corresponding dimensions that are linear functions of the vector $\boldsymbol{p}(k)$ of unknown parameters, and $\boldsymbol{C}_k$ is the known measurement matrix. The covariance matrices for the disturbances $\boldsymbol{\xi}(k)$ and $\boldsymbol{\eta}(k)$ are denoted by $\boldsymbol{Q}_{\xi,k}$ and $\boldsymbol{Q}_{\eta,k}$, respectively. The criteria according to eqs. (8.5) and (2.6) have the form

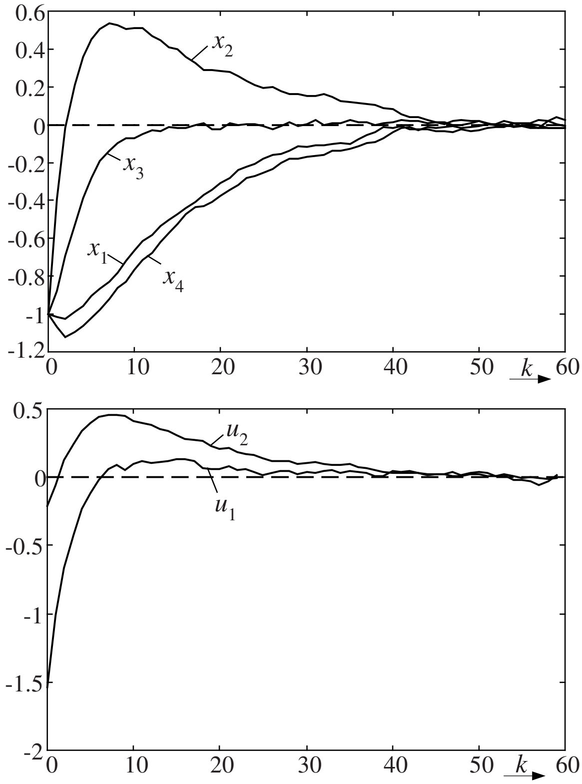


Fig. 8.3. Simulation results for the optimal LQ controller with known parameters

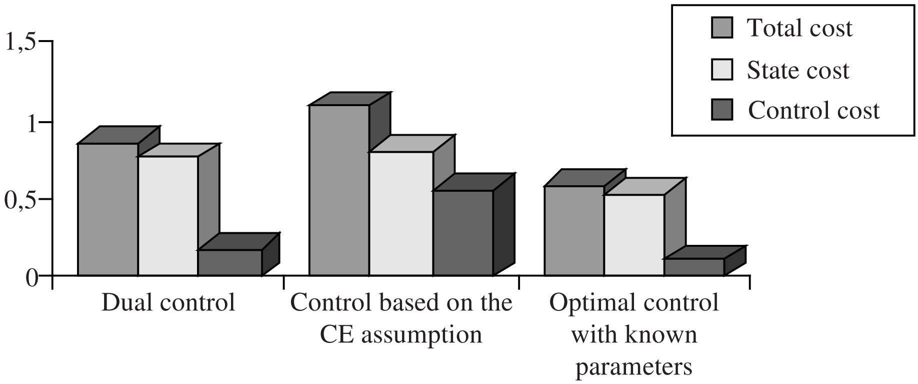


Figure 8.4. Performance indices

$$J = \mathrm{E}\left\{ \sum_{k=0}^{N-1} [\boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{V}_{k+1}\boldsymbol{x}(k+1) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}_k\boldsymbol{u}(k)] \right\}, \tag{8.50}$$

and in the case of an infinite moving horizon correspondingly

$$J(k_p) = \mathrm{E}\left\{ \sum_{k=k_p}^{k_p+l-1} [\boldsymbol{x}^{\mathrm{T}}(k+1)\boldsymbol{V}_{k+1}\boldsymbol{x}(k+1) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}_k\boldsymbol{u}(k)] \right\}, \tag{8.51}$$

where $\boldsymbol{V}_{k+1}$ and $\boldsymbol{R}_k$ are the sequences of non-negative definite symmetric matrices, and l is the receding horizon. For the considered problem of linear systems, it is assumed that the input signal is unconstrained, that is, $\overline{\Omega}_k = \Re^{n_u} \forall k$.

The partial-certainty-equivalence (PCE) controller will be derived here for the linear system where the CE assumption is applied only to the system state $\boldsymbol{x}$ and the accuracy of the estimates of the parameters $\boldsymbol{p}$ is taken into account.

Analog to eqs. (3.11) and (3.10), the ρ-approximation and the cost function in the present case are given by

$$\rho_k = \rho_k^p = \{ \mathrm{p}[\boldsymbol{x}(k+i), \boldsymbol{p}(k+i) | \Im_{k+i}] = \delta[\boldsymbol{x}(k+i) - \hat{\boldsymbol{x}}(k+i)]\mathrm{p}[\boldsymbol{p}(k+i) | \Im_k], \quad i = 0, ..., N-k-1 \}, \tag{8.52}$$

and

$$J_k(\Im_k, \rho_k^p) = \mathrm{E}_{\rho_k^p}\left\{ \sum_{i=k}^{N-1} [\boldsymbol{x}^{\mathrm{T}}(i+1)\boldsymbol{V}_{i+1}\boldsymbol{x}(i+1) + \boldsymbol{u}^{\mathrm{T}}(i)\boldsymbol{R}_i\boldsymbol{u}(i)] \Big| \Im_k \right\}, \tag{8.53}$$

respectively, where $\Im_k$ is defined by eq. (2.4). For the control problem with the model described by eqs. (8.29) to (8.31) and the criterion according to eq. (8.50), the partial open-loop control (POLF, see Fig. 3.3) policy will be defined after minimization of eq. (8.53) under the conditions of the ρ-approximation, eq. (8.52). This leads to the control law (see Appendix A)

$$\boldsymbol{u}_p(k) = -\boldsymbol{K}_k \hat{\boldsymbol{x}}(k), \ k = 0, 1, ..., N-1 , \tag{8.54}$$

where

$$\boldsymbol{K}_k = [\boldsymbol{R}_k + \mathrm{E}\{\boldsymbol{B}_k^{\mathrm{T}}(\boldsymbol{p}(k))(\boldsymbol{V}_{k+1} + \boldsymbol{L}_{N-k-1})\boldsymbol{B}_k(\boldsymbol{p}(k)) | \Im_k\}]^{-1} \cdot \mathrm{E}\{\boldsymbol{B}_k^{\mathrm{T}}(\boldsymbol{p}(k))(\boldsymbol{V}_{k+1} + \boldsymbol{L}_{N-k-1})\boldsymbol{A}_k(\boldsymbol{p}(k)) | \Im_k\}, \tag{8.55}$$

$$\boldsymbol{L}_{N-i} = \mathrm{E}\{\boldsymbol{A}_i^{\mathrm{T}}(\boldsymbol{p}(i))(\boldsymbol{V}_{i+1} + \boldsymbol{L}_{N-i-1})\boldsymbol{A}_i(\boldsymbol{p}(i)) | \Im_k\} - \boldsymbol{F}_{N-i} , \tag{8.56}$$

and

$$\begin{aligned}\boldsymbol{F}_{N-i} = \mathrm{E}\Big\{&\boldsymbol{A}_i^{\mathrm{T}}(\boldsymbol{p}(i))(\boldsymbol{V}_{i+1}+\boldsymbol{L}_{N-i-1})\boldsymbol{B}_i(\boldsymbol{p}(i))\Big|\Im_k\Big\}\\ &\cdot\Big[\boldsymbol{R}_i+\mathrm{E}\Big\{\boldsymbol{B}_i^{\mathrm{T}}(\boldsymbol{p}(i))(\boldsymbol{V}_{i+1}+\boldsymbol{L}_{N-i-1})\boldsymbol{B}_i(\boldsymbol{p}(i))\Big|\Im_k\Big\}\Big]^{-1}\\ &\cdot\mathrm{E}\Big\{\boldsymbol{B}_i^{\mathrm{T}}(\boldsymbol{p}(i))(\boldsymbol{V}_{i+1}+\boldsymbol{L}_{N-i-1})\boldsymbol{A}_i(\boldsymbol{p}(i))\Big|\Im_k\Big\}, i=N-1, N-2, \ldots, k+1, \boldsymbol{L}_0=\boldsymbol{0}.\end{aligned} \tag{8.57}$$

In case of a singular matrix under the inversion operator in eqs. (8.55) and (8.57), the pseudoinverse operation can be used instead (Aoki, 1967). The detailed synthesis of the control policy described by eqs. (8.54)-(8.57) is presented in Appendix A.

The designed PCE control policy, eqs. (8.54) to (8.57), for linear systems is computationally simple and, in contrast to the known OLF control policy, does not require any numerical optimization in real-time operation. At the same time, this control law takes into account the uncertainty level in the system using the covariances of the unknown parameters and, therefore, it will provide better control performance than the controllers generally used that are based on the CE assumption. Moreover, the PCE control policy demonstrates superior control performance and a smooth transitional behavior, as is shown for the simulated examples in section 8.7.

8.6. Design of Dual Controllers Using the Partial Certainty Equivalence Assumption and Bicriterial Optimization

Similar to the approach presented in Section 8.1, the synthesis problem can be formulated as a bicriterial optimization task

$$\boldsymbol{u}(k) = \underset{\boldsymbol{u}(k)\in\Omega_k}{\arg\min}\ J_k^{\mathrm{a}}, \tag{8.58}$$

and

$$\boldsymbol{u}_p(k) = \underset{\boldsymbol{u}(k),\ldots,\boldsymbol{u}(N-1)}{\arg\min}\ J_k(\Im_k, \rho_k^p), \tag{8.59}$$

where, like in eq. (8.11), Ω_k is an ellipsoid with center $\boldsymbol{u}_p(k)$

$$\Omega_k = \Big\{\boldsymbol{u}(k) : \boldsymbol{u}(k) = \boldsymbol{u}_{\mathrm{p}}(k) + \boldsymbol{u}_{\mathrm{a}}(k),\ \boldsymbol{u}_{\mathrm{a}}^{\mathrm{T}}(k)\boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{a}}(k) \le f_k(\boldsymbol{P}(k))\Big\}, \tag{8.60}$$

with $f_k(\cdot)$ and $\boldsymbol{W}_a$ as defined in Section 8.1, and the cost functions J_k^{a} and $J_k(\Im_k, \rho_k^p)$ are described by eqs. (8.9) and (8.54), respectively. Using the constraints

$$g_k = [\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{p}}(k)]^{\mathrm{T}}\boldsymbol{W}_{\mathrm{a}}[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{p}}(k)] - f_k(\boldsymbol{P}(k)), \tag{8.61}$$

the necessary and sufficient conditions for a minimum of the problem according to eqs. (8.58) and (8.59) are determined by eqs. (8.25) and (8.26). Similar to the derivation of the dual controller presented in Section 8.2, eqs. (8.61) and (8.20) are used to obtain the dual control law with the PCE assumption

$$\boldsymbol{u}(k) = [\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k)]^{-1}[\boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{p}}(k) + \lambda_k \boldsymbol{P}_{\mathrm{BA}}(k)\hat{\boldsymbol{x}}(k)] \tag{8.62}$$

under the conditions

$$[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{p}}(k)]^{\mathrm{T}} \boldsymbol{W}_{\mathrm{a}} [\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{p}}(k)] - f_k(\boldsymbol{P}(k) = 0 \tag{8.63}$$

and

$$\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k) > \mathbf{0} \tag{8.64}$$

which are obtained from the minimization of eq. (8.58). It is worth noting that the cost function, eq. (8.9), takes the form of eq. (8.20) after calculation of the expectation in eq. (8.9) using the PCE assumption, eq. (8.53).

An example for the implementation of the derived PCE controller and the dual controller with PCE assumption will be shown below. These two examples deal with SISO systems. However, the PCE controller presented here has been applied for both systems in state space representation, that is the systems are considered similar to multivariable ones.

8.7. Simulation Examples

8.7.1. Example 1: Underdamped Plant

For the simulation, the control of a stable underdamped third-order plant with the transfer function

$$G(s) = \frac{0.33s + 1}{0.35s^3 + 1.15s^2 + 0.33s + 1} \tag{8.65}$$

is considered. The discrete-time model obtained with a zero-order hold for the sampling time 0.5 sec has the output sequence

$$\begin{aligned} q(k+1) &= 1.65q(k) - 1.02q(k-1) + 0.202q(k-2) + 0.11u(k) \\ &\quad + 0.08u(k-1) - 0.023u(k-2) + \xi(k),\ k = 0, \ldots, N-1, \end{aligned} \tag{8.66}$$

and the noise corrupted observation model is given by

$$y(k) = q(k) + \eta(k), \tag{8.67}$$

The white noise disturbance sequences $\xi(k)$ and $\eta(k)$ have the variances Q_ξ=0.0006 and Q_η=0.006, respectively, and zero mean values. The system is simulated for N=50 time steps. The initial conditions for the considered plant are zero and known. The initial estimates for all six parameters are equal to 0.1. The covariance matrix of the errors of the initial parameter estimates is $\boldsymbol{P}(0) = 0.3\mathbf{I}_6$, where $\mathbf{I}_6$ is the identity matrix of dimension 6×6. The cost function has the form

$$J = \mathrm{E}\left\{\sum_{k=0}^{N-1}[w-q(k+1)]^2 + 0.0005u^2(k)\right\}, \tag{8.68}$$

where w=5 is the setpoint (being constant in the considered example). An extended Kalman filter is used to estimate the unknown parameters and states of the system.

The simulation results for this example are presented in Fig. 8.5 for the PCE policy, in Fig. 8.6 for the CE policy, in Fig. 8.7 for the closed loop optimal control policy with known parameters and in Fig. 8.8 for the dual controller with parameter η =4. This simulation clearly demonstrates that the PCE-based control policy described by eqs. (8.54) to (8.57) provides a smooth start up of the process without an overshoot. The CE-based control policy that is usually applied does not take into account the uncertainty level in the system and, therefore, has no cautious properties (Bar-Shalom and Tse, 1976; Feldbaum, 1965). A large overshoot of the system output at the beginning of the adaptation is attributed to this control policy. The optimal control law with known parameters is also given in Fig. 8.7 to show the best control behavior that cannot be achieved in the case of unknown parameters and that will be provided by the derived controller after adaptation. It should be noted that an optimal excitation signal can be used together with the derived PCE-based control law for speeding up the adaptation and improving the system behavior, as done, for example, by Filatov and Unbehauen (1995a). Simulations for the OLF control policy may furnish results comparable with the PCE-based policy, but the computational difficulties are significant. The dual controller provides the best results, which are portrayed in Fig. 8.8.

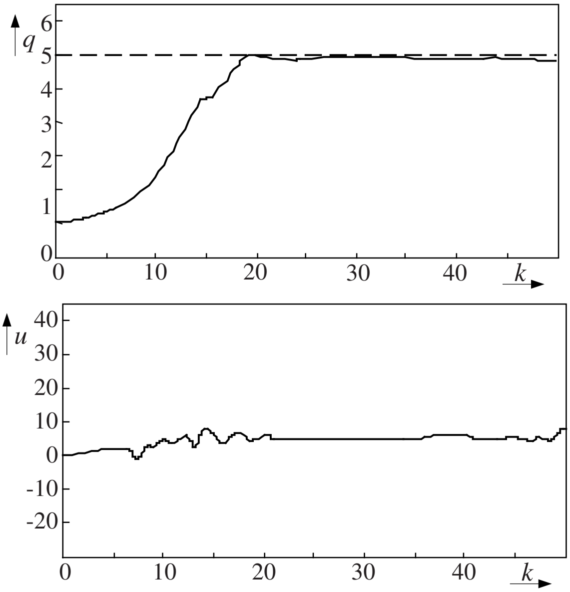


Fig. 8.5. Simulation results for the PCE-based adaptive control strategy

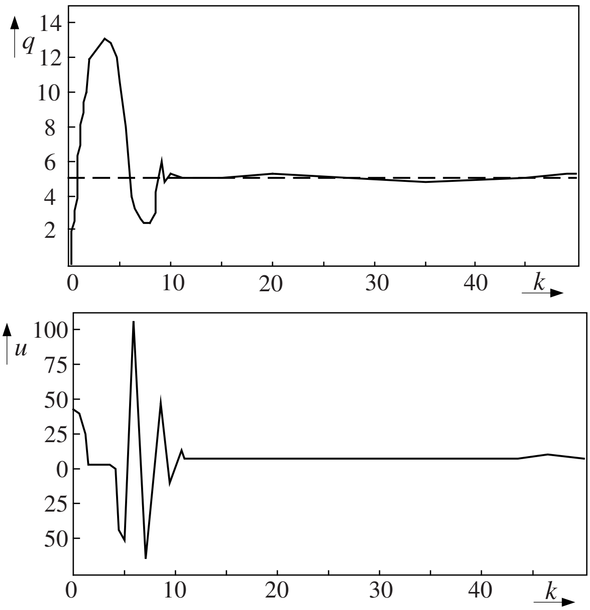


Fig. 8.6. Simulation results for the CE-based adaptive control strategy

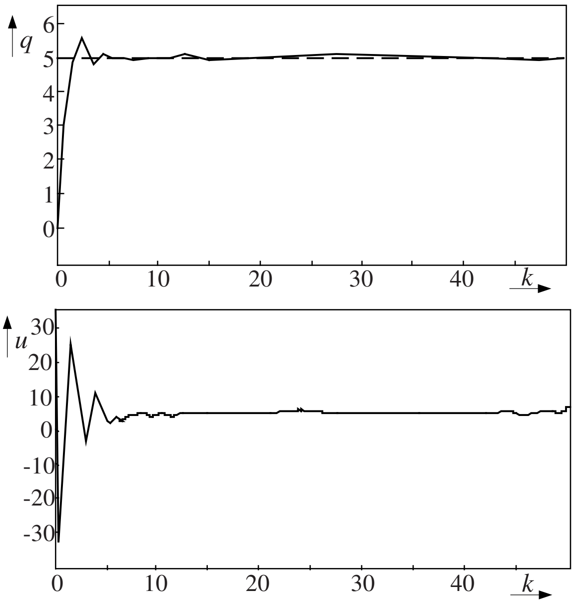


Fig. 8.7. Simulation results for the optimal controller with known parameters

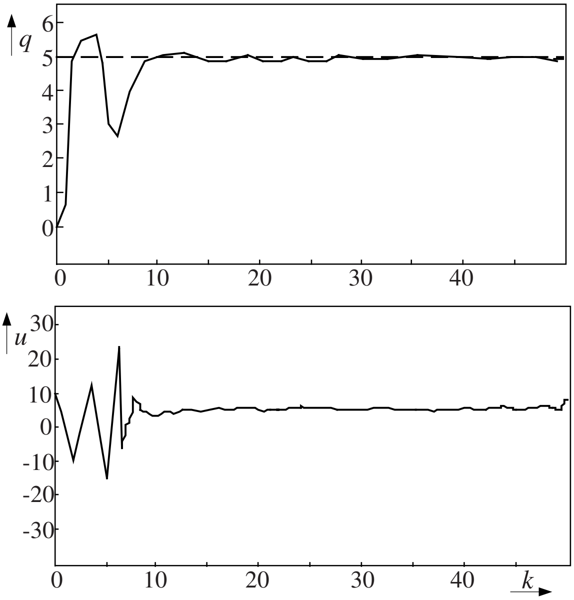


Fig. 8.8. Simulation results for the adaptive dual controller

8.7.2. Example 2: Nonminimum Phase Plant

For the next simulation example consider the control of a stable nonminimum phase second order plant with the transfer function

$$G(s) = \frac{2-2s}{(1+2s)(1+5s)} . \tag{8.69}$$

The discrete-time model obtained with zero-order hold for a sampling time of 1.0 sec and incorporating disturbances has the form

$$q(k+1) = 1.43q(k) - 0.5q(k-1) - 0.2u(k) + 0.35u(k-1) + \xi(k),\ k = 0,..., N-1 , \tag{8.70}$$

and the observation model is as given in eq. (8.67). The disturbances $\xi(k)$ and $\eta(k)$ have the variances Q_ξ=0.00004 and Q_η=0.0002, respectively. The system is again simulated for N=50 time steps. The initial conditions for the considered plant are zero and known. The initial estimates for all four parameters are equal to 0.1. The covariance matrix of the errors of the initial parameter estimates is $\boldsymbol{P}(0) = 0.4\mathbf{I}_4$, where $\mathbf{I}_4$ is the identity matrix of dimension 4×4. The cost function has the form

$$J = \mathrm{E}\left\{\sum_{k=0}^{N-1}[w - q(k+1)]^2 + 0.05[\overline{u} - u(k)]^2\right\}, \tag{8.71}$$

where w=1 is the setpoint (being constant in this example). The parameter η of the dual controller has been selected as 2. An extended Kalman filter is used to estimate the unknown parameters and states of the system. As in the previous example, the PCE-based adaptive control strategy demonstrates a smooth transitional behavior (Fig. 8.9). The results for the CE-based controller, indicated in Fig. 8.10, show worse performance than those of the designed dual controller (Fig. 8.12). The resulting process for the adaptive dual controller is similar to the optimal controller with known parameters as can be seen from Fig. 8.11. At the same time, large excitations of the adaptive dual controller can be observed at the beginning of the adaptation process (Fig. 8.12).

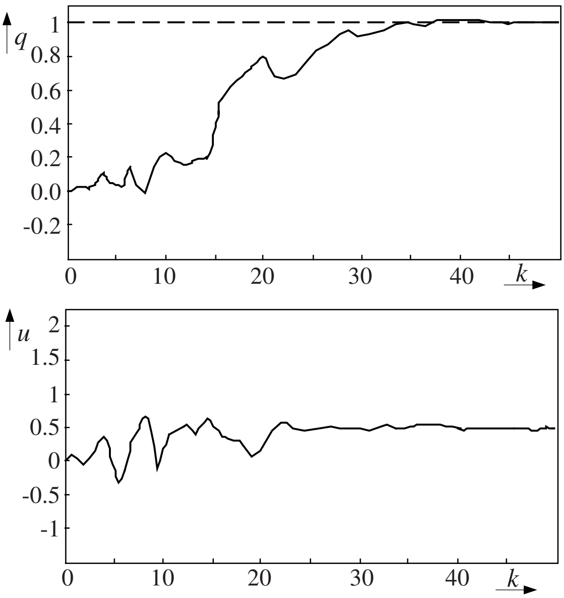


Fig. 8.9. Simulation results for the PCE-based adaptive controller

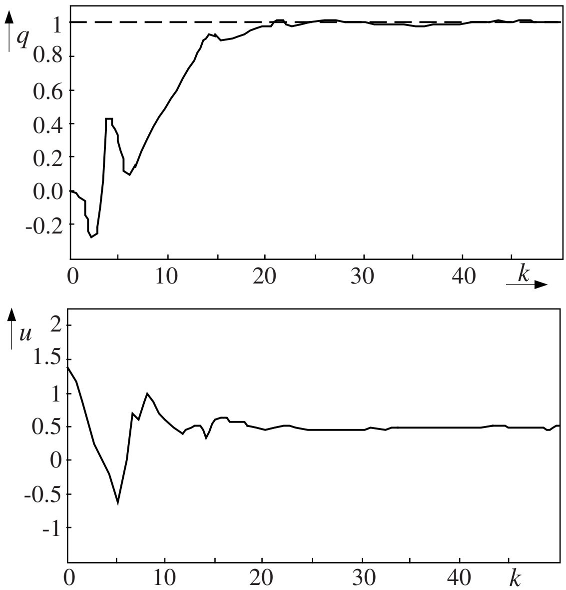


Fig. 8.10. Simulation results for the CE-based adaptive controller

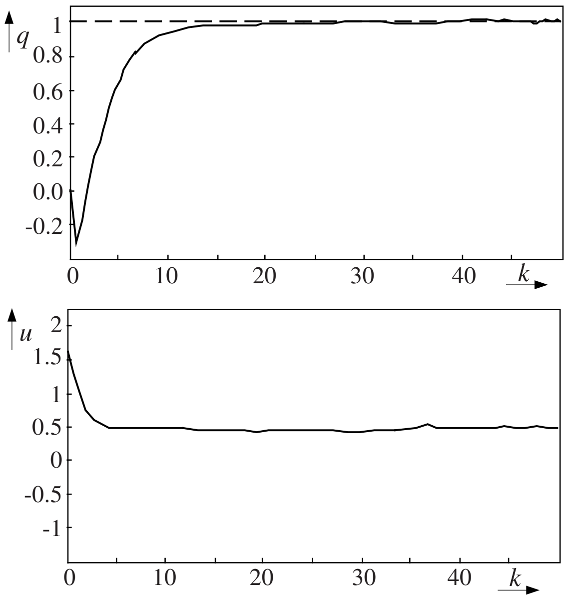


Fig. 8.11. Simulation results for the optimal controller with known parameters

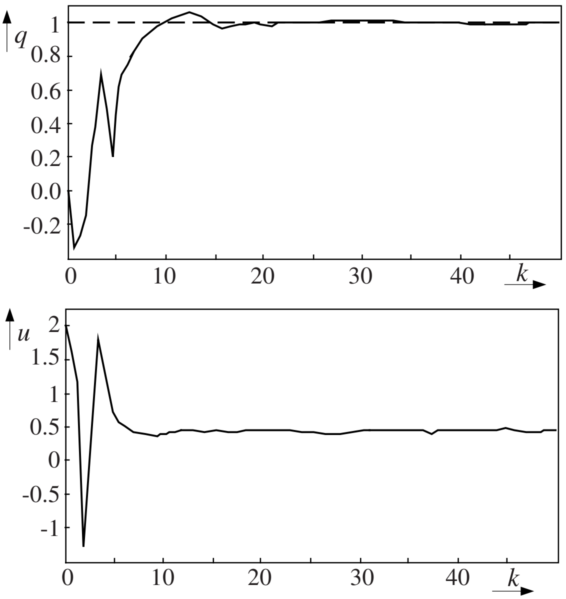


Fig. 8.12. Simulation results for the adaptive dual controller

9. A SIMPLIFIED APPROACH TO THE SYNTHESIS OF DUAL CONTROLLERS WITH INDIRECT ADAPTATION

The cost function of the deviation between the system output and its nominal value (system response to the unknown desired controller), suggested by Filatov and Unbehauen (1994), Filatov et al. (1995) and applied in Chapter 6, can also be used successfully for the derivation of dual controllers with direct adaptation, but requires a modification for control optimization in indirect adaptive control systems. Difficulties in computing the covariance matrix of the controller parameter estimates arise during the application of this cost function in the optimization of control systems with indirect adaptation. In this chapter, a simplified computation of the nominal output will be presented. Dual versions with improved control performance can be designed by the suggested approach for various existing indirect adaptive controllers, including such important and generally-used controllers like linear quadratic Gaussian (LQG), self-tuning, adaptive pole placement, and predictive controllers. The dual controller will be synthesized independently of the structure of the CE controller and can be used together with any of the CE discrete-time controllers with indirect adaptation as an additional unit (device) that modifies the CE control signal to dual control action. It will be shown that some already known dual versions of self-tuning regulators (STR) can also be obtained by the elaborated method. The improvement of the control performance will be demonstrated by computer simulation.

This chapter is organized as follows. In section 9.1, the approach for multivariable systems in the state-space representation will be introduced. Sections 9.2 and 9.3, respectively, will deal with the derived controllers for *single-input / multi-output (SIMO) and single-input / single-output (SISO) systems*. A simple example for the implementation of the suggested approach to the modification of STR will be given in section 9.4. The simulation results will be presented in section 9.5.

9.1. Modification of Certainty-Equivalence Adaptive Controllers

Consider the linear discrete-time system in state-space representation described by eq. (8.1). In addition to the assumptions described in section 8.1 for the given system, it is also assumed that the adaptive controller has already been designed, using the CE assumption

$$\boldsymbol{u}_{\mathrm{CE}}(k) = \boldsymbol{u}_{\mathrm{CE}}(\hat{\boldsymbol{p}}(k), \Im_k), \tag{9.1}$$

which is a vector function of the available observations $\Im_k$ [eq. (8.2)] and the parameter estimate $\hat{\boldsymbol{p}}(k) = \mathrm{E}\{\boldsymbol{p} | \Im_k\}$ at time k.

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 105–118, 2004.

Following the bicritirial approach, again the two cost functions for control optimization are introduced

$$J_k^{\mathrm{c}} = \mathrm{E}\{[\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_n(k+1)]^{\mathrm{T}} \overline{\boldsymbol{W}} [\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}_n(k+1)] | \Im_k\}, \tag{9.2}$$

$$J_k^{\mathrm{a}} = -\mathrm{E}\{[\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k)]^{\mathrm{T}} \boldsymbol{W} [\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1|k)] | \Im_k\}, \tag{9.3}$$

where $\hat{\boldsymbol{x}}(k+1|k)$ is determined by eq. (8.10), $\overline{\boldsymbol{W}}$ and $\boldsymbol{W}$ are positive semi-definite symmetric weighting matrices and $\hat{\boldsymbol{x}}_{\mathrm{n}}(k+1)$ is an estimate of the nominal output of the system. A cost function according to eq. (9.2) is introduced to minimize the derivation of the system state from the nominal state $\boldsymbol{x}_{\mathrm{n}}(k+1)$, which is the response of the system to the nominal control signal $\boldsymbol{u}_{\mathrm{n}}(k)$, generated by the unknown desired controller (Filatov and Unbehauen, 1994; Filatov et al., 1995; see also Chapter 6). It is clear that in the case of exact estimates $\hat{\boldsymbol{p}}(k) = \boldsymbol{p}$ the CE controller, eq. (9.1), generates the nominal control signal. Therefore, the nominal control signal can be determined only after finishing the adaptation when the exact values of the controller or plant parameters are available. The second cost function, according to eq. (9.3), is introduced for the acceleration of the parameter estimation as in eq. (8.9). Minimization of the cost functional in eq. (9.3) increases the expectation of the squared one-step-ahead prediction error, which is used in the estimation algorithm [recursive least squares (RLS) and similar algorithms] for updating the parameter estimation. To derive a general adaptive dual controller with simplifications, the nominal state is defined here using the CE assumption

$$\hat{\boldsymbol{x}}_{\mathrm{n}}(k+1) = \boldsymbol{A}(\hat{\boldsymbol{p}}(k))\boldsymbol{x}(k) + \boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}_{\mathrm{CE}}(k). \tag{9.4}$$

In the system based on the CE assumption, all random variables are assumed to be equal to their expectations. In the present approach, the CE assumption is applied only to calculate the estimate of the nominal system output according to eq. (9.4).

Adopting the bicriterial design method, the dual control law is obtained after the minimization of the cost functions in eqs. (9.2) and (9.3) in the form of eqs. (8.15) and (8.16) with constraints described by eq. (8.11). The non-negative scalar function f_k of the covariance matrix can be defined, for example, in one of the forms of eq. (8.13) or (8.14). Therefore, the magnitude of the excitation depends on the scalar parameter η and the function f_k of the covariance matrix, that is, the magnitude of the excitation depends on the uncertainty measure, and the size of the domain Ω_k, according to eq. (8.11).

Substituting eqs. (8.1) and (9.4) into eq. (9.2) gives

$$\begin{aligned} J_k^{\mathrm{c}} = \mathrm{E}\{ & \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\overline{\boldsymbol{W}}\boldsymbol{B}(\boldsymbol{p})\boldsymbol{u}(k) - 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}_{\mathrm{CE}}(k) \\ & + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(p)\overline{\boldsymbol{W}}\left[\boldsymbol{A}(p) - \boldsymbol{A}(\hat{\boldsymbol{p}}(k))\right]\boldsymbol{x}(k) | \Im_k\} + c_1(k), \end{aligned} \tag{9.5}$$

where $c_1(k)$ is independent of $\boldsymbol{u}(k)$. As in eqs. (8.21)-(8.23), the covariance matrices

$$\boldsymbol{P}_{\mathrm{A}\overline{\mathrm{W}}}(k) = \mathrm{E}\{\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{p})\overline{\boldsymbol{W}}\boldsymbol{A}(\boldsymbol{p}) | \Im_k\} - \boldsymbol{A}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\overline{\boldsymbol{W}}\boldsymbol{A}(\hat{\boldsymbol{p}}(k)), \tag{9.6}$$

$$\boldsymbol{P}_{\mathrm{B}\overline{\mathrm{W}}}(k) = \mathrm{E}\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\overline{\boldsymbol{W}}\boldsymbol{B}(\boldsymbol{p}) | \Im_k\} - \boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k)), \tag{9.7}$$

and

$$\boldsymbol{P}_{\mathrm{B\overline{W}A}}(k) = \mathrm{E}\left\{\boldsymbol{B}^{\mathrm{T}}(\boldsymbol{p})\overline{\boldsymbol{W}}\boldsymbol{A}(\boldsymbol{p})\middle|\Im_k\right\} - \boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\overline{\boldsymbol{W}}\boldsymbol{A}(\hat{\boldsymbol{p}}(k)) \tag{9.8}$$

are introduced. These matrices can be calculated by the covariance matrix of the parameter vector according to eq. (8.12). Then taking the expectation of eq. (9.5), similar as in Appendix D, we obtain

$$\begin{aligned} J_k^{\mathrm{c}} &= \boldsymbol{u}^{\mathrm{T}}(k)[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k)) + \boldsymbol{P}_{\mathrm{B\overline{W}}}(k)]\boldsymbol{u}(k) \\ &\quad - 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k))\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}_{\mathrm{CE}}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{B\overline{W}A}}\boldsymbol{x}(k) + c_1(k). \end{aligned} \tag{9.9}$$

The minimum of the last equation with respect to the control value is provided by the cautious control law

$$\begin{aligned} \boldsymbol{u}_{\mathrm{c}}(k) &= [\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k)\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k)) + \boldsymbol{P}_{\mathrm{B\overline{W}}}(k)]^{-1} \\ &\quad \cdot[\boldsymbol{B}^{\mathrm{T}}(\hat{\boldsymbol{p}}(k)\overline{\boldsymbol{W}}\boldsymbol{B}(\hat{\boldsymbol{p}}(k))\boldsymbol{u}_{\mathrm{CE}}(k) - \boldsymbol{P}_{\mathrm{B\overline{W}A}}(k)\boldsymbol{x}(k)]. \end{aligned} \tag{9.10}$$

The second performance index according to eq. (9.3) after the substituting of eq. (9.1) and taking the expectation can be represented in the form

$$J_k^{\mathrm{a}} = -[\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{A}}(k)\boldsymbol{x}(k) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{B}}(k)\boldsymbol{u}(k) + 2\boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k) + \mathrm{tr}\{\boldsymbol{W}\boldsymbol{Q}_{\xi}\}], \tag{9.11}$$

where the covariance matrices $\boldsymbol{P}_{\mathrm{A}}$, $\boldsymbol{P}_{\mathrm{B}}$ and $\boldsymbol{P}_{\mathrm{BA}}$ are defined as in eqs. (8.21) to (8.23).

The ellipsoid, which defines the amplitude constraint of the excitation according to eq. (8.11), has the form given in eq. (8.24). Taking into account the convexity of the constraint and concavity of the performance index according to eq. (9.11), the necessary conditions for minimization of the second performance index can be written as in eqs. (8.25) and (8.26). Substituting eqs. (9.11) and (8.24) in eq. (8.25), we have analog to eq. (8.27)

$$-\lambda_k \boldsymbol{P}_{\mathrm{B}}(k)\boldsymbol{u}(k) + \boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}(k) - \boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{c}}(k) - \lambda_k \boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k) = \boldsymbol{0}. \tag{9.12}$$

After solving the last equation, the dual control law is finally obtained as

$$\boldsymbol{u}(k) = [\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k)]^{-1}[\boldsymbol{W}_{\mathrm{a}}\boldsymbol{u}_{\mathrm{c}}(k) + \lambda_k \boldsymbol{P}_{\mathrm{BA}}(k)\boldsymbol{x}(k)]. \tag{9.13}$$

The parameter λ_k has to be chosen such that

$$[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)]^{\mathrm{T}}\boldsymbol{W}_{\mathrm{a}}[\boldsymbol{u}(k) - \boldsymbol{u}_{\mathrm{c}}(k)] - f_k(\boldsymbol{P}(k)) = 0, \tag{9.14}$$

and

$$\boldsymbol{W}_{\mathrm{a}} - \lambda_k \boldsymbol{P}_{\mathrm{B}}(k) > \boldsymbol{0} \tag{9.15}$$

are satisfied.

Therefore, the dual controller can be computed from the CE control using eqs. (9.10), (9.13), (9.14) and (9.15). The additional parameter η and weighting matrix $\boldsymbol{W}_{\mathrm{a}}$ should be selected such that the optimal value of the excitation $\boldsymbol{u}_{\mathrm{a}}(k)$ is obtained.

These parameters may be determined using computer simulations of the control process. It should be noted that in the cases of SIMO and SISO systems only the scalar parameter η has to be selected because the matrix W_a may be assumed as the constant 1 and an on-line solution of eqs. (9.14) and (9.15) to determine the optimal λ_k is not required. This is shown in detail in the next chapter. The dual controller is derived here independently of the structure of the CE adaptive systems; therefore, it can be used as an additional unit, which transforms the CE control signal to a dual control one. The suggested dual unit can modify and improve the performance of various CE adaptive controllers, for example, the LQG, pole-placement, predictive, generalized minimum-variance, self-tuning controllers, etc. The standard structures of an adaptive control system and an adaptive dual control system (with a dual control unit) are portrayed in Fig. 9.1 and Fig. 9.2, respectively.

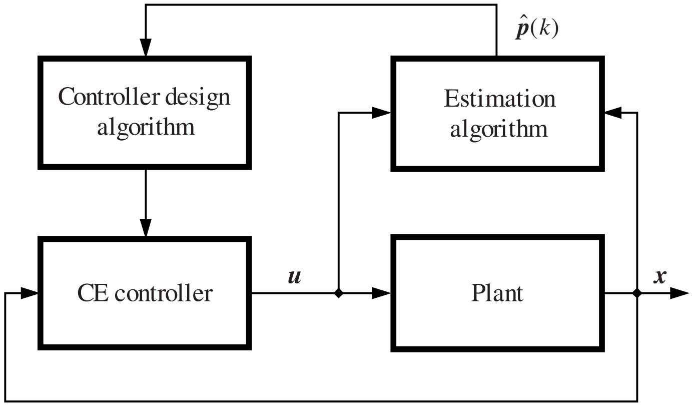


Fig. 9.1. Standard adaptive control system based on the CE assumption

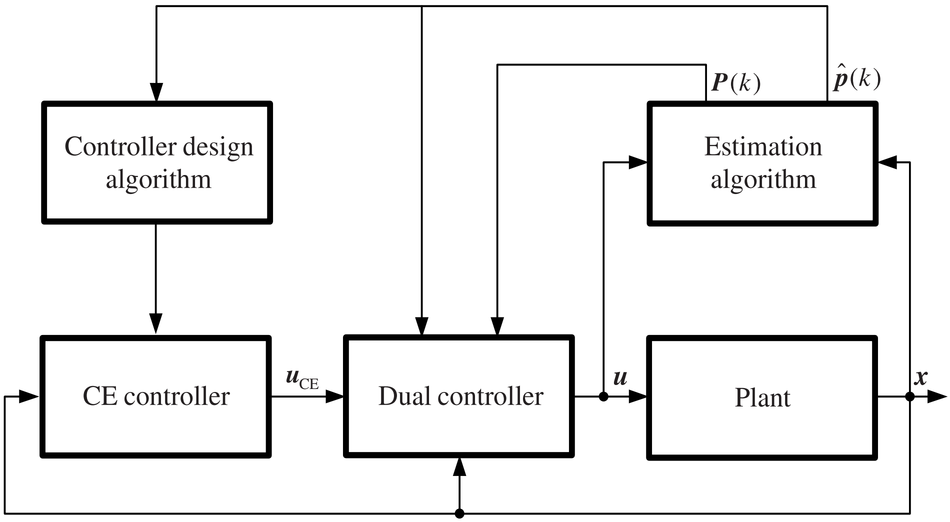


Fig. 9.2a. Adaptive dual control system (modification of the CE adaptive system)

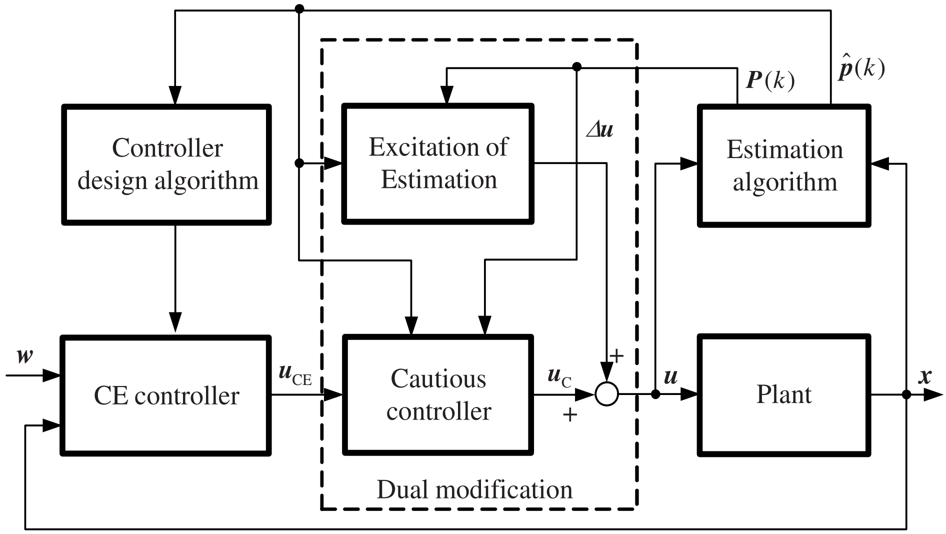


Fig. 9.2b. Detailed scheme of an adaptive dual control system ($\Delta \boldsymbol{u}$ represents the optimal excitation)

9.2. Controllers for SIMO Systems

Consider the special case of a system, according to eq. (8.1), with the scalar input signal $u(k) \in \Re^1$. In this case, the domain Ω_k takes the form of a closed interval according to eqs. (4.23) and (4.24). Eq. (4.31) defines the dual control that minimizes the second cost function according to eq. (9.3) in the domain described by eq. (4.23).

Substituting eq. (9.11) into eq. (4.31) provides the following expression for the term within the brackets {.} of eq. (4.31):

$$\begin{aligned} J[u_c(k)-\theta(k)] - J[u_c(k)+\theta(k)] &= -P_B(k)[u_c(k)-\theta(k)]^2 \\ &\quad -2\boldsymbol{P}_{BA}(k)\boldsymbol{x}(k)[u_c(k)-\theta(k)] \\ &\quad +P_B(k)[u_c(k)+\theta(k)]^2 + 2\boldsymbol{P}_{BA}(k)\boldsymbol{x}(k)[u_c(k)+\theta(k)] \\ &\quad +4P_B(k)u_c(k)\theta(k) + 4\boldsymbol{P}_{BA}(k)\boldsymbol{x}(k)\theta(k)\,, \end{aligned} \tag{9.16}$$

where $P_B(k)$ is a scalar and $\boldsymbol{P}_{BA}(k)$ a $1 \times n_x$ row vector, according to eq. (8.22) and (8.23). Substitution of eq. (9.16) into eq. (4.31) results in the dual control law

$$u(k) = u_c(k) + \theta(k)\,\mathrm{sgn}\{P_B(k)u_c(k) + \boldsymbol{P}_{BA}(k)\boldsymbol{x}(k)\}\,. \tag{9.17}$$

Therefore, the dual controller can be computed easily from the CE control using eqs. (9.10), (9.17), (4.23) and (8.13) or (8.14). One additional parameter η should be selected to determine the amplitude of the excitation $u_{\mathrm{a}}(k)$ according to eqs. (9.13), (8.13) or (8.14).

9.3. Controllers for SISO Systems with Input-Output Models

Consider a SISO system described by the discrete-time input-output model

$$y(k+1) = b_1 u(k) + \ldots + b_n u(k-n+1) + a_1 y(k) + \ldots + a_n y(k-n+1) + \xi(k)$$

$$= b_1 u(k) + \boldsymbol{p}_0^{\mathrm{T}} \boldsymbol{m}_0(k) + \xi(k) = \boldsymbol{p}^{\mathrm{T}} \boldsymbol{m}(k) + \xi(k)\,, \tag{9.18}$$

where

$$\boldsymbol{p}^{\mathrm{T}} = [b_1, \ldots, b_n, a_1, \ldots a_n] = [b_1 \vdots \boldsymbol{p}_0^{\mathrm{T}}]\,, \tag{9.19}$$

$$\boldsymbol{m}^{\mathrm{T}}(k) = [u(k),\ \ldots,\ u(k-n+1),\ y(k),\ \ldots,\ y(k-n+1)] = [u(k) \vdots \boldsymbol{m}_0^{\mathrm{T}}(k)] \tag{9.20}$$

and $y(k)$ is the system output. The noise sequence $\xi(k)$ has the variance σ_ξ^2. To estimate the plant parameters for eq. (9.18), the RLS algorithm can be applied in the form

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1) e(k+1)\,, \tag{9.21}$$

$$e(k+1) = y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k) \boldsymbol{m}(k)\,, \tag{9.22}$$

$$\boldsymbol{q}(k+1) = \boldsymbol{P}(k) \boldsymbol{m}(k) [\boldsymbol{m}^{\mathrm{T}}(k) \boldsymbol{P}(k) \boldsymbol{m}(k) + \sigma_\xi^2]^{-1}\,, \tag{9.23}$$

and

$$\boldsymbol{P}(k+1) = \boldsymbol{P}(k) - \boldsymbol{q}(k+1) \boldsymbol{m}^{\mathrm{T}}(k) \boldsymbol{P}(k)\,, \tag{9.24}$$

where

$$\boldsymbol{P}(k) = \mathrm{E}\{[\boldsymbol{p} - \hat{\boldsymbol{p}}(k)][\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}} | \Im_k\} = \begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix}, \tag{9.25}$$

and $\boldsymbol{P}(0)$ is assumed to be determinable and $\Im_k$ is defined by eq. (4.12). The parameters of the time-varying plant can be estimated using an RLS algorithm with a forgetting factor or a constant trace of the covariance matrix. The parameter drift model, eq. (4.5), can also been used.

A dual controller with indirect adaptation is obtained using the CE assumption to compute the nominal output $y_{\mathrm{n}}(k+1)$ in the same way as in eq. (9.4), with the CE assumption the nominal system output becomes

$$\hat{y}_{\mathrm{n}}(k+1) = \hat{b}_1(k) u_{\mathrm{CE}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k)\,. \tag{9.26}$$

The cost functions of eqs. (9.2) and (9.3) can be written as

$$J_k^{\mathrm{c}} = \mathrm{E}\{[\hat{y}_{\mathrm{n}}(k+1) - y(k+1)]^2 | \Im_k\} \tag{9.27}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\{[y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 | \Im_k\}. \tag{9.28}$$

After substituting eqs. (9.18) and (9.26) into eq. (9.27) and taking the expectation, we obtain

$$\begin{aligned} J_k^{\mathrm{c}} &= \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{p_0}(k)\boldsymbol{m}_0(k) + \hat{b}_1^2(k)u_{\mathrm{CE}}^2(k) + [\hat{b}_1^2(k) + p_{b_1}(k)]u^2(k) \\ &\quad - 2\hat{b}_1^2(k)u_{\mathrm{CE}}(k)u(k) - 2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) + \sigma_\xi^2. \end{aligned} \tag{9.29}$$

The minimum of the last equation leads to the cautious control law

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_1^2(k)u_{\mathrm{CE}}(k) - \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)}{\hat{b}_1^2(k) + p_{b_1}(k)}. \tag{9.30}$$

At the same time, after insertion of eq. (9.18) it follows from eq. (9.28) that

$$\begin{aligned} J_k^{\mathrm{a}}(u(k)) &= -\mathrm{E}\left\{[[\boldsymbol{p}(k) - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}}\boldsymbol{m}(k) + \xi(k)]^2 | \Im_k\right\} \\ &= -\mathrm{E}\left\{[(b_1(k) - \hat{b}_1(k))u(k) + (\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k))^{\mathrm{T}}\boldsymbol{m}_0(k)]^2 | \Im_k\right\} + \bar{c}_2(k) \\ &= -p_{b_1}(k)u^2(k) - 2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) + \bar{c}_3(k), \end{aligned} \tag{9.31}$$

where $\bar{c}_2(k)$ and $\bar{c}_3(k)$ do not contain $u(k)$. From eq. (9.31), we find that

$$\begin{aligned} &J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) - \theta(k)] - J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) + \theta(k)] \\ &\qquad = 4p_{b_1}(k)u_{\mathrm{c}}(k)\theta(k) + 4\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\theta(k). \end{aligned} \tag{9.32}$$

Substituting the last equation into eq. (4.31) gives the dual control law

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\mathrm{sgn}\{p_{b_1}(k)u_{\mathrm{c}}(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\}. \tag{9.33}$$

The dual control algorithm is finally given by eqs. (9.30), (9.33), (4.24) and (8.13) or (8.14).

9.4. An Example for Applying the Method to Derive the Dual Version of an STR

Consider the application of the suggested approach to derive the dual version of an STR. A simple STR, which minimizes the cost function

$$J_k = \mathrm{E}\{[w(k+1) - y(k+1)]^2 \mid \Im_k\}, \tag{9.34}$$

using the CE assumption for the system described by eq. (9.18), is given by (Filatov and Unbehauen, 1995a; Wittenmark, 1995) in the form

$$u_{\mathrm{CE}}(k)=\frac{w(k+1)-\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)}{\hat{b}_1(k)+p_{b_1}(k)}. \tag{9.35}$$

At the same time, the cautious controller

$$u_{\mathrm{c}}(k)=\frac{\hat{b}_1(k)w(k+1)-[\hat{b}_1(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)+\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)]\boldsymbol{m}_0(k)}{\hat{b}_1^2(k)+p_{b_1}(k)} \tag{9.36}$$

is obtained after minimizing the cost function according to eq. (9.34) without application of the CE assumption (Filatov and Unbehauen, 1995a; Zhivoglyadov et al., 1993). It can be seen easily that the previously derived cautious controller, eq. (9.30), after substituting eq. (9.35), is identical to the one given by eq. (9.36). Therefore, the method discussed here not only allows the dual version of a STR, as given by (Filatov and Unbehauen, 1995a; Zhivoglyadov et al., 1993 to be designed), but provides a far wider range of dual controllers with indirect adaptation.

9.5. Simulation Examples for Controllers with Dual Modification

9.5.1. Example 1: LQG Controller

Consider the continuous-time SISO system described by the transfer function

$$G(s)=\frac{-0.5-s}{-0.54+0.3s+s^2}\cdot\frac{1}{s}. \tag{9.37}$$

After z-transformation, using a sampling time of 0.5 sec and a zero-order hold, we obtain

$$G(z)=\frac{-0.535z^{-1}+0.4176z^{-2}}{1-1.987z^{-1}+0.8607z^{-2}}\cdot\frac{1}{1-z^{-1}}. \tag{9.38}$$

The cost function

$$J=\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}y^2(k)+u^2(k) \tag{9.39}$$

is used for the derivation of an optimal LQG controller based on the CE assumption. The initial values of the parameter estimates of the model, according to eq. (9.38), are taken as $\hat{\boldsymbol{p}}^{\mathrm{T}}=[0.1\quad 0.1\quad 0.1\quad 0.1]$ with the covariance matrix $\boldsymbol{P}(0)=0.4\,\mathbf{I}$. The other parameters for the dual controller, according to eqs. (4.24), (8.14), (9.30) and (9.33) are selected as $\sigma_\xi^2=0.01$ and $\eta=0.00025$. The control signal is assumed to be unconstrained. The simulation results are depicted in Figs. 9.3 and 9.4. They demonstrate the improvement of the transient behavior at the beginning of the adaptation for the system with dual modification. The adaptation error e is increased at the beginning which stimulates the estimation and decreases the adaptation time.

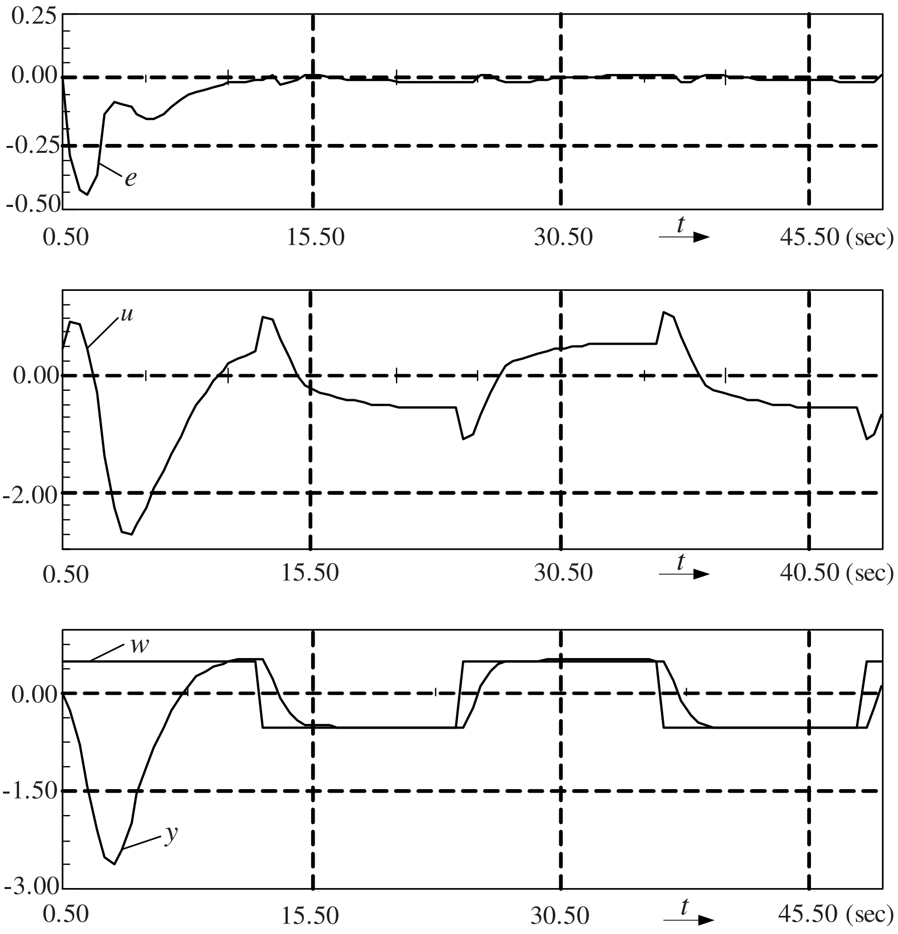


Figure 9.3. Simulation results for adaptive LQG controller based on the CE assumption

9.5.2. Example 2: Pole-Placement Controller

Continuous-time and discrete-time models, respectively, of a plant are given by

$$G(s) = \frac{-1}{-1.2s + s^2} \cdot \frac{1}{s}, \tag{9.40}$$

and

$$G(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \cdot \frac{1}{1 - z^{-1}}. \tag{9.41}$$

The parameters of the model transfer function, according to eq. (9.41), for the sampling time 0.05 sec are

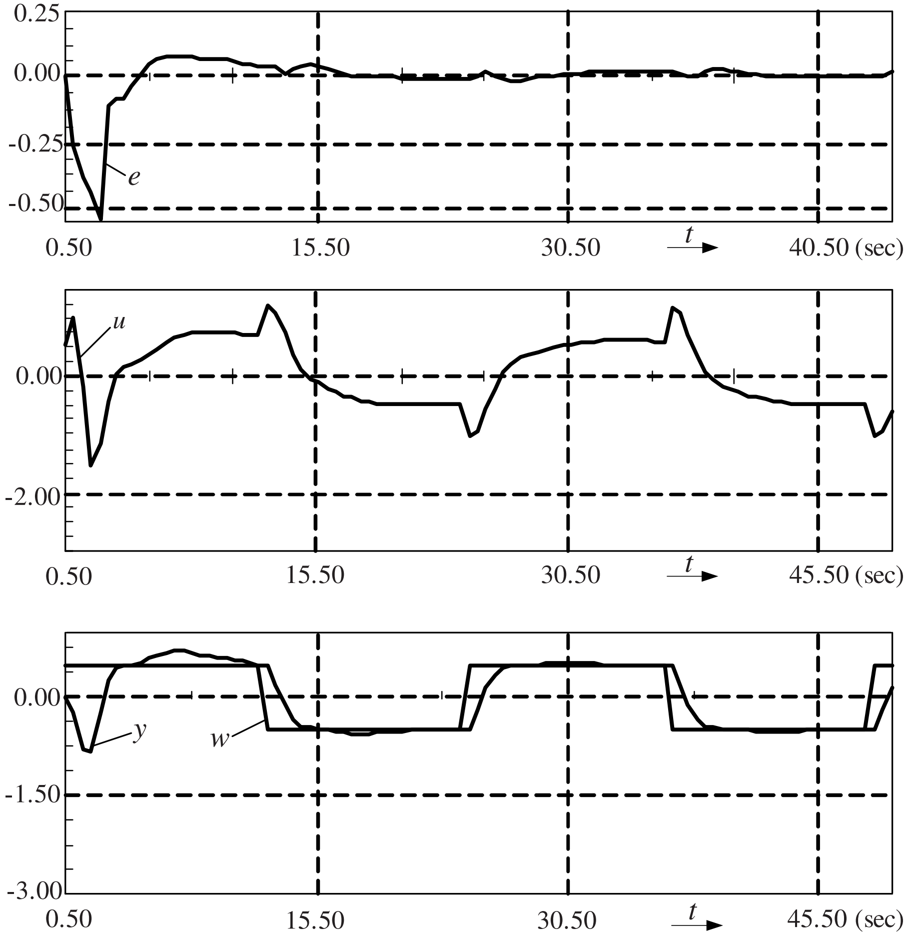


Figure 9.4. Simulation results for the adaptive LQG controller with dual modification

$$b_1 = -1.275 \cdot 10^{-3},\ b_2 = -1.3 \cdot 10^{-3},$$

$$a_1 = -2.062,\ a_2 = +1.062\ .$$

The adaptive pole-placement controller described by Keuchel and Stephan (1994) is used here as the main CE controller. The following pole locations for the closed-loop system and the observer have been selected:

System poles		Observer poles	
Re	Im	Re	Im
0.75	0	0.52	0
0.83	-0.2	0.60	0
0.83	0.2		

The initial values for the adaptive control unit described by eqs. (4.24), (8.14), (9.30) and (9.33) and the estimation algorithm according to eqs. (9.21) to (9.24) are:

- initial estimates: $\hat{\boldsymbol{p}}^{\mathrm{T}}(0) = [0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0.01]$
- initial covariance matrix: $\boldsymbol{P}(0) = 0{,}4\ \mathbf{I}$

and

- other parameters: σ_{ξ}^{2}=0.01, η=0.002.

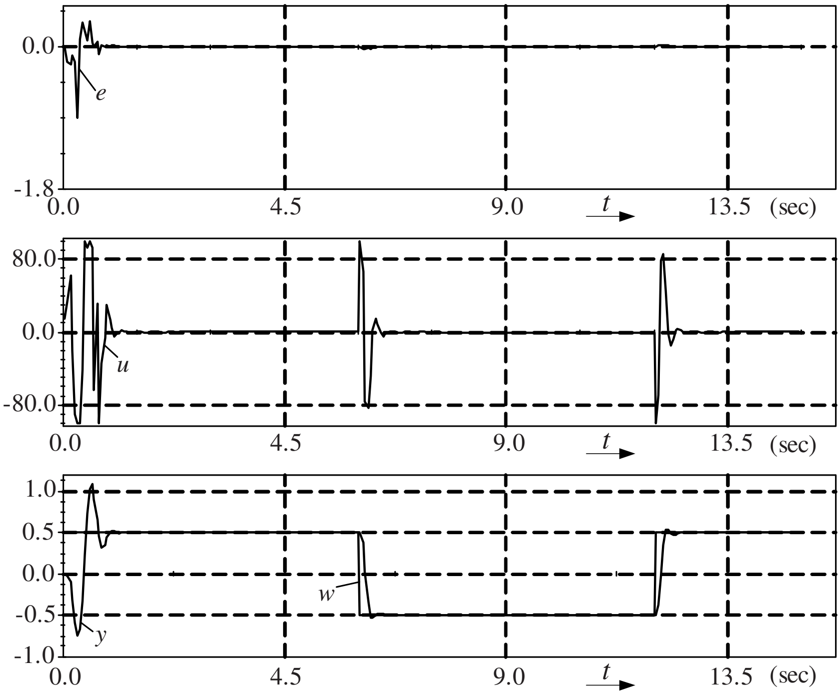


Fig. 9.5. Simulation results for the adaptive pole-placement controller based on the CE assumption

The results of the simulations are depicted in Fig. 9.5 for the adaptive pole-placement controller based on the CE assumption and in Fig. 9.6 for the adaptive dual pole-placement controller. The comparison of the transient behaviour of the CE and dual controller indicates a smoother startup without large overshoot and a shorter adaptation time for the dual controller. At the same time, we can see a larger adaptation error e for the dual controller at the beginning of the adaptation.

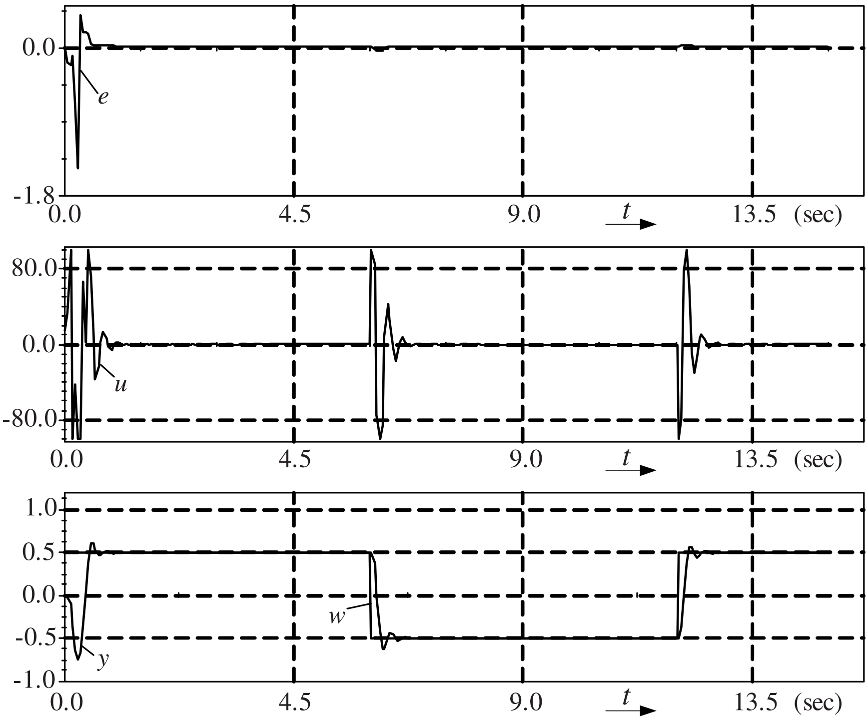


Figure 9.6. Simulation results for the adaptive pole-placement controller with dual modification

9.5.3. Example 3: Pole-Placement Controller for a Plant with Integral Behaviour

In this example, the adaptive pole-placement controller with the dual modification is applied to control an unstable second order plant described by

$$G(s) = \frac{1}{s(1+0.83s)}. \tag{9.42}$$

The discrete-time model is obtained using a zero-order hold and a sampling time of 0.05 sec

$$G(z) = \frac{0.0088z^{-1} + 0.0086z^{-2}}{1-1.942z^{-1} + 0.942z^{-2}}. \tag{9.43}$$

The parameters of the controller described by eqs. (4.24), (8.14), (9.30) and (9.33) are selected as

- initial estimates: $\hat{\boldsymbol{p}}^{\mathrm{T}}(0) = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}$,
- initial covariance matrix: $\boldsymbol{P}(0) = 0.4\,\mathbf{I}$,
- other parameters: $\sigma_{\xi}^{2} = 0.01$, $\eta = 0.003$.

The simulation results for the considered example are shown in Fig. 9.7 (CE control) and in Fig. 9.8 (dual control). In the case of dual control in Fig. 9.8, we can observe a large excitation at the beginning of the adaptation phase. This excitation of the dual controller provides highly accurate estimates of the system parameters within the first control steps. This gives improved control performance (especially important is the estimation accuracy of the numerator parameters of the transfer function). We can also observe the strong decrease in the excitations with the increase in the accuracy of the estimation.

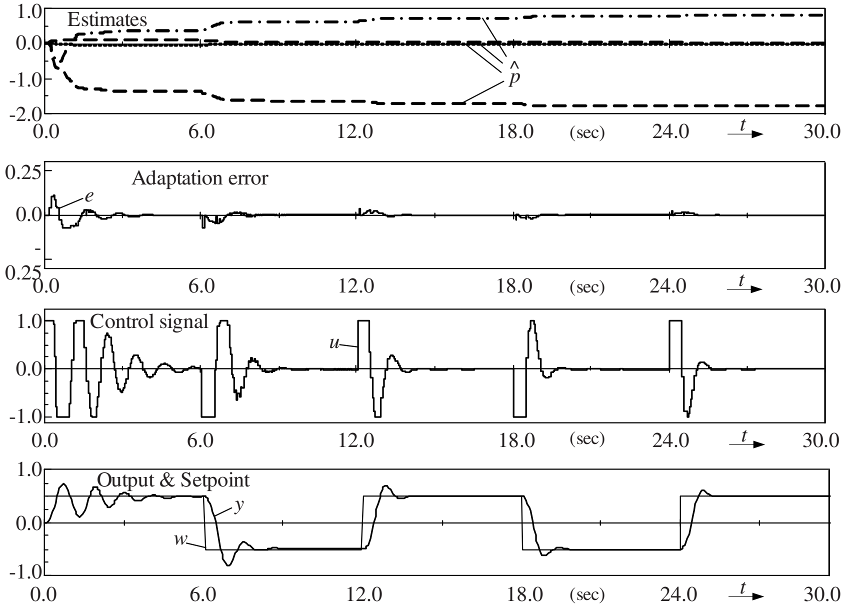


Figure 9.7. Standard adaptive pole-placement controller

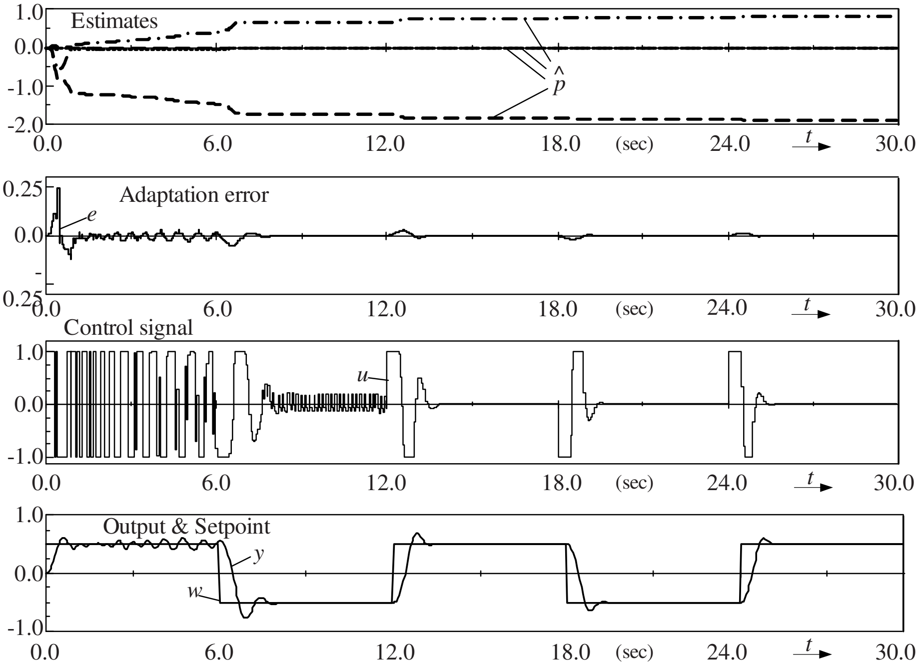


Fig. 9.8. Adaptive pole-placement controller with the dual modification

10. DUAL POLE-PLACEMENT AND LQG CONTROLLERS WITH INDIRECT ADAPTATION

Adaptive pole-placement controllers (APPC) exhibit important properties and provide closed-loop system dynamics according to the desired position of the poles. Moreover, the pole locations can be determined to ensure optimal or robust properties of the system (see, for example, Yu et al., 1987). In systems with indirect adaptation, the plant parameters are estimated and used for computation of the controller parameters, whereas in systems with direct adaptation the controller parameters are estimated directly (usually together with some additional parameters) without estimation of the plant parameters. Usually the indirect APPC requires the estimation of less parameters in comparison with the direct APPC, but it is necessary to solve the Diophantine equation online in real time. Therefore, the indirect APPC can provide a more rapid adaptation and may be applied in cases where this requirement is important. The bicriterial approach for the synthesis of the direct dual APPC was already presented in Chapter 6, whereas the dual version of the indirect APPC was derived only for the simplified approach presented in Chapter 9. Below, this problem will be investigated first without the simplified calculation of the nominal output. The importance of APPC is also displayed in their direct application for the LQG control law (Keuchel and Stephan, 1994; Yu et al., 1987). The LQG control law can be obtained using a spectral factorization for the pole placement. Therefore, in the following at first the dual APPC is derived, and then the LQG controller can be obtained formally by application of the spectral factorization mentioned above (Keuchel and Stephan, 1994; Yu et al., 1987). Both controller types, the indirect APPC and the dual LQG controller, will be applied to a thyristor-driven DC motor in Chapter 11.

The indirect APPC based on the CE assumption and the corresponding LQG controller obtained by spectral factorization will be considered in Section 10.1 for systems with time-varying and suddenly changing parameters. In Section 10.2, the dual modification of the controller based on bicriterial optimization will be derived. Section 10.3, presents the calculation of the covariance matrix of the controller parameter estimates using the Diophantine equation. The simplified approach for the design of the dual indirect APPC, suggested in Chapter 9, will also be considered, where the calculation of the covariance matrix of the controller parameters is not required.

10.1. Indirect Adaptive Pole-Placement Controller and the Corresponding LQG Controller

A system with the plant model

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 119–129, 2004.

$$Y(z) = \frac{B(z^{-1})}{A(z^{-1})} U(z) + \frac{C(z^{-1})}{A(z^{-1})} \Xi(z) \tag{10.1}$$

is considered, where $U(z)$ and $Y(z)$ are the z-transforms of the control signal $u(k)$ and the output signal $y(k)$, $\Xi(z)$ is the z-transform of the disturbance $\xi(k)$, representing a white noise with zero mean and variance σ_ξ^2, k is the discrete time index, $A(z^{-1})$ and $B(z^{-1})$ are polynomials of the form

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_n z^{-n}, \tag{10.2}$$

$$B(z^{-1}) = b_1 z^{-1} + ... + b_n z^{-n}, \; b_1 \neq 0, \tag{10.3}$$

$C(z^{-1})$ is the noise polynomial of order n describing the noise dynamics. The controller with the inputs $W(z)$ (reference signal) and Y(z) (controlled signal) and the output $U(z)$ (control signal) is given by

$$L(z^{-1})\overline{U}(z) = G_\mathrm{v}(z^{-1})Q(z^{-1})W(z) - H(z^{-1})Y(z) \tag{10.4a}$$

with

$$(1 - z^{-1})\overline{U}(z) = U(z), \tag{10.4b}$$

where

$$L(z^{-1}) = l_0 + l_1 z^{-1} + ... + l_n z^{-n}, \; l_0 \neq 0, \tag{10.5}$$

$$H(z^{-1}) = h_0 + h_1 z^{-1} + ... + h_n z^{-n} \tag{10.6}$$

and

$$Q(z^{-1}) = 1 + q_1 z^{-1} + ... + q_n z^{-n}, \tag{10.7}$$

$G_\mathrm{v}(z^{-1})$ is the transfer function of an additional prefilter for the reference signal. For simplicity, and without loss of generality, it is assumed that $G_\mathrm{v}(z^{-1})$ =1. The closed-loop system is obtained from eqs. (10.1) and (10.4) as

$$Y(z) = \frac{Q(z^{-1})B(z^{-1})}{L(z^{-1})A(z^{-1})(1-z^{-1}) + H(z^{-1})B(z^{-1})} W(z)$$

$$+ \frac{C(z^{-1})L(z^{-1})(1 - z^{-1})}{L(z^{-1})A(z^{-1})(1 - z^{-1}) + H(z^{-1})B(z^{-1})} \Xi(z) \tag{10.8}$$

or, when the noise dynamics are compensated by selecting $C(z^{-1}) = Q(z^{-1})$

$$Y(z) = \frac{B(z^{-1})}{P(z^{-1})} W(z) + \frac{L(z^{-1})(1-z^{-1})}{P(z^{-1})} \Xi(z), \tag{10.9}$$

where

$$Q(z^{-1})P(z^{-1}) = L(z^{-1})A(z^{-1})(1-z^{-1}) + H(z^{-1})B(z^{-1}). \tag{10.10}$$

The parameters of the controller polynomials $L(z^{-1})$ and $H(z^{-1})$ can be determined by solving the Diophantine equation (10.10). The observer polynomial $Q(z^{-1})$ is freely selectable, as long as it remains stable. The structure of the considered system without adaptation is shown in Fig. 10.1.

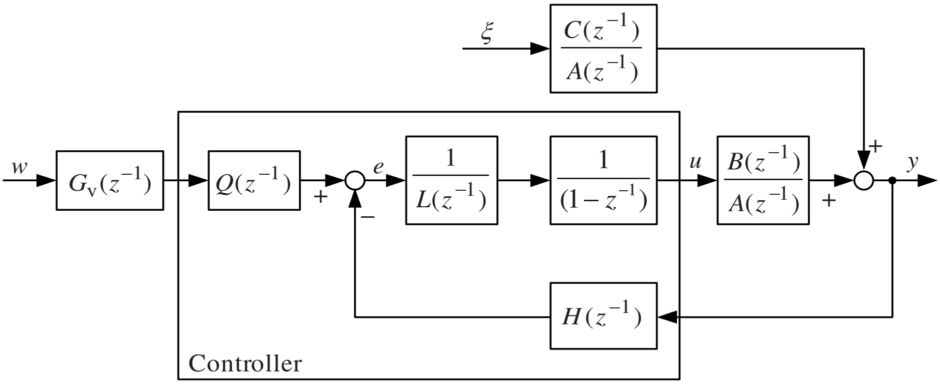


Fig. 10.1. The structure of the closed loop system

In case of $C(z^{-1}) = 1$, the plant model, according to eq. (10.1), can be represented in the discrete-time domain as

$$\begin{aligned} y(k+1) &= b_1 u(k) + \ldots + b_n u(k-n) - a_1 y(k) - \ldots - a_n y(k-n) + \xi(k) \\ &= \boldsymbol{p}^{\mathrm{T}} \boldsymbol{m}(k) + \xi(k), \end{aligned} \tag{10.11}$$

where

$$\boldsymbol{p}^{\mathrm{T}} = [b_1 \ldots b_n \,\vdots\, a_1 \ldots a_n] = [b_1 \,\vdots\, \boldsymbol{p}_0^{\mathrm{T}}], \tag{10.12}$$

and

$$\boldsymbol{m}^{\mathrm{T}}(k) = [u(k) \ldots u(k-n) \,\vdots\, -y(k) \ldots -y(k-n)] = [u(k) \,\vdots\, \boldsymbol{m}_0^{\mathrm{T}}(k)]. \tag{10.13}$$

To estimate the plant parameters of eq. (10.11), the following recursive least squares (RLS) algorithm can be applied:

$$\hat{\boldsymbol{p}}(k+1) = \hat{\boldsymbol{p}}(k) + \boldsymbol{q}(k+1)e(k+1)\,, \tag{10.14}$$

$$e(k+1) = y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)\,, \tag{10.15}$$

$$\boldsymbol{q}(k+1) = \overline{\boldsymbol{P}}(k)\boldsymbol{m}(k)[\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\boldsymbol{m}(k)+1]^{-1}\,, \tag{10.16}$$

$$\overline{\boldsymbol{P}}(k+1) = \overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\,, \tag{10.17}$$

where $\hat{\boldsymbol{p}}(k) = \mathrm{E}\{\boldsymbol{p}(k)|\Im_k\}$,

$$\overline{\boldsymbol{P}}(k) = \sigma_\xi^{-2}\mathrm{E}\left\{[\boldsymbol{p}-\hat{\boldsymbol{p}}(k)][\boldsymbol{p}-\hat{\boldsymbol{p}}(k)]^{\mathrm{T}}\middle|\Im_k\right\} = \sigma_\xi^{-2}\boldsymbol{P}(k)\,, \tag{10.19}$$

$\Im_k$ is the set of the plant inputs and outputs available at time k according to eq. (4.12), $\boldsymbol{P}(k)$ and $\overline{\boldsymbol{P}}(k)$ are the covariance matrices, $\boldsymbol{P}(0)$ and $\overline{\boldsymbol{P}}(0)$ are assumed to be known. The parameters of time-varying plants can be estimated, using in eq. (10.17) the forgetting factor $0 < \alpha \le 1$ as in eq. (4.16), by

$$\overline{\boldsymbol{P}}(k+1) = \frac{1}{\alpha}\left[\overline{\boldsymbol{P}}(k) - \boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\overline{\boldsymbol{P}}(k)\right]. \tag{10.21}$$

Moreover, sudden parameter jumps can be detected by the controller in real-time mode by comparing the one-step-ahead prediction error $e(k)$, according to eq. (10.15), to its theoretical variance

$$\sigma_e^2(k+1) = \mathrm{E}\left\{e^2(k+1)\middle|\Im_k\right\} = \boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}(k) + \sigma_\xi^2\,. \tag{10.22}$$

The controller detects a parameter jump in the case of $|e(k+1)| >> \sigma_e(k+1)$ and updates the covariance matrix $\boldsymbol{P}(k)$ with its initial value $\boldsymbol{P}(0)$ after a jump has been detected.

The estimates of the polynomials $\hat{L}_k(z^{-1})$ and $\hat{H}_k(z^{-1})$ can be computed from the estimates of the plant polynomials $\hat{A}_k(z^{-1})$ and $\hat{B}_k(z^{-1})$ using eq. (10.10) at the time k in real-time mode

$$Q(z^{-1})P(z^{-1}) = \hat{L}_k(z^{-1})\hat{A}_k(z^{-1})(1-z^{-1}) + \hat{H}_k(z^{-1})\hat{B}_k(z^{-1})\,. \tag{10.23}$$

The control law based on the CE principle and the estimation algorithm can, respectively, be realized using eqs. (10.4) and (10.23) as

$$u(k) = u_{\mathrm{CE}}(k) = \frac{1}{\hat{l}_0(k)}[\boldsymbol{p}_2^{\mathrm{T}}\boldsymbol{m}_2(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)] + u(k-1)\,, \tag{10.24}$$

where

$$\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k) = [\hat{l}_1(k) \ldots \hat{l}_n(k) \vdots \hat{h}_1(k) \ldots \hat{h}_n(k)]\,, \tag{10.25}$$

$$\boldsymbol{m}_1^{\mathrm{T}}(k) = [u'(k-1) \ldots u'(k-n) \vdots - y(k) \ldots - y(k-n)], \quad u'(k) = u(k) - u(k-1), \tag{10.26}$$

$$\boldsymbol{p}_2^{\mathrm{T}} = [1 \;\; q_1 \ldots q_n], \tag{10.27}$$

$$\boldsymbol{m}_2^{\mathrm{T}}(k) = [w(k) \ldots w(k-n)], \tag{10.28}$$

$\hat{l}_i(k)$ and $\hat{h}_i(k)$ are estimates of the coefficients of the polynomials described by eqs. (10.5) and (10.6). It is further assumed that the controller parameter vector

$$\overline{\boldsymbol{p}}_1^{\mathrm{T}}(k) = [l_0 l_1 \ldots l_n \vdots h_1 \ldots h_n] = [l_0 \vdots \boldsymbol{p}_1^{\mathrm{T}}], \tag{10.29}$$

has the expectation

$$\hat{\overline{\boldsymbol{p}}}_1(k) = \mathrm{E}\left\{\overline{\boldsymbol{p}}_1(k) \middle| \Im_k\right\} \tag{10.30}$$

and the covariance matrix

$$\tilde{\boldsymbol{P}}(k) = \mathrm{E}\left\{[\overline{\boldsymbol{p}}_1 - \hat{\overline{\boldsymbol{p}}}_1(k)][\overline{\boldsymbol{p}}_1 - \hat{\overline{\boldsymbol{p}}}_1(k)]^{\mathrm{T}} \middle| \Im_k\right\} = \begin{bmatrix} \tilde{p}_{l_0}(k) & \tilde{\boldsymbol{p}}_{l_0 p_1}^{\mathrm{T}}(k) \\ \tilde{\boldsymbol{p}}_{l_0 p_1}(k) & \tilde{\boldsymbol{P}}_{p_1}(k) \end{bmatrix}. \tag{10.31}$$

The expressions for computation of the covariance matrix, eq. (10.31), will be derived in Section 10.3.

The APPC described above provides a corresponding LQG controller version if the pole locations are determined by the LQG cost function through spectral factorization (Keuchel and Stephan, 1994; Yu et al., 1987). If the cost function

$$J = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \left[w(k+1) - y(k+1)\right]^2 + r u^2(k), \;\; r > 0, \tag{10.32}$$

is used for the derivation of the LQG controller with the CE assumption, the pole placement of the closed loop system according to the polynomial $P(z^{-1})$ is determined using spectral factorization (Keuchel and Stephan, 1994; Yu et al., 1987) as

$$\varphi P(z) P(z^{-1}) = r A(z) A(z^{-1}) + B(z) B(z^{-1}), \tag{10.33}$$

where the parameter φ is introduced for the normalization of the leading coefficient of the polynomial $P(z^{-1})$. Therefore, the polynomial $P(z^{-1})$ can be calculated from eq. (10.23) after estimation of the plant parameters in real time, and it can be used then immediately in the proposed pole-placement controller. Therefore, in the next sections, the derivation of the dual version of the indirect APPC will be emphasized, whereas the LQG variant can be obtained easily using the above-mentioned approach.

10.2. Dual Modification of the Controller

According to the method presented in Chapter 6 the nominal output of the system $y_{\mathrm{n}}(k+1)$ is defined as the response to its nominal input signal

$$u_{\mathrm{n}}(k) = \frac{1}{l_0}[\boldsymbol{p}_2^{\mathrm{T}}\boldsymbol{m}_2(k) - \boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{m}_1(k)] + u(k-1)\,. \tag{10.34}$$

The signal $u_{\mathrm{n}}(k)$ is taken to be generated by the controller with the unknown parameters that provide the desired system pole placement and the closed-loop system according to eq. (10.9), when the signal vector of the system is $\boldsymbol{m}_0(k)$ and no disturbances act upon the system ($\xi(k) \equiv 0$). Then from eq. (10.11) follows the dependence of $y_{\mathrm{n}}(k+1)$ on $u_{\mathrm{n}}(k)$ in the form

$$y_{\mathrm{n}}(k+1) = b_1 u_{\mathrm{n}}(k) + \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k)\,, \tag{10.35}$$

where $\boldsymbol{p}_0$ and $\boldsymbol{m}_0(k)$ are defined by eqs. (10.12) and (10.13). The control performance can be improved if the controller tries to bring the system output as close as possible to the nominal output, which is attained only for the controller with the known parameters, after full noise-compensation. In accordance with this and the bicriterial approach (Filatov et al., 1995; Zhivoglyadov, et al., 1993), the following two cost functions

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{\gamma^2[y_{\mathrm{n}}(k+1) - y(k+1)]^2 \middle| \Im_k\right\},\ \gamma^2 = \frac{l_0^2}{b_1^2}, \tag{10.36}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 \middle| \Im_k\right\} \tag{10.37}$$

are introduced for the derivation of the control law.

The first cost function, eq. (10.36), is used to minimize the deviation of the system output from the unknown nominal output. The positive coefficient γ^2 simplifies further algebraic manipulations. The second cost function, eq. (10.37), is used for accelerating the parameter estimation by increasing the prediction error according to eq. (10.15). The dual APPC will be obtained then by solving the bicriterial optimization problem given by eqs. (10.36) and (10.37) for the system described by eqs. (10.11) and (10.35), under the constraints given by eqs. (4.22) till (4.25).

The size of the domain Ω_k, described by eq. (4.23), defines the magnitude of the excitation. The selection of this domain is strongly related to the covariance matrix (uncertainty of the system model), where the parameter $\theta(k)$ in eq. (4.23) may be chosen in a more general form than in eq. (4.24) using the function

$$\theta = f\left(\overline{\boldsymbol{P}}(k)\right) = \eta\left(\mathrm{tr}\left\{\overline{\boldsymbol{P}}(k)\right\}\right)^{\delta},\ \eta \ge 0\,,\ \delta \in \{1,2,4\}. \tag{10.38}$$

Here, the parameter η determines the magnitude of the excitation, and the parameter δ leads to a decrease in excitation while increasing the accuracy of the estimation.

Substituting eqs. (10.11) and (10.35) into eq. (10.36) yields

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[l_0 u_n(k) - l_0 u(k)]^2 \middle| \Im_k \right\}. \tag{10.39}$$

Inserting eq. (10.27) in eq. (10.37) gives

$$\begin{aligned} J_k^{\mathrm{c}} &= \mathrm{E}\left\{[\boldsymbol{p}_2^{\mathrm{T}} \boldsymbol{m}_2(k) - \boldsymbol{p}_1^{\mathrm{T}} \boldsymbol{m}_1(k) + l_0 u(k-1) - l_0 u(k)]^2 \middle| \Im_k \right\} \\ &= \mathrm{E}\{2[\boldsymbol{p}_1^{\mathrm{T}} \boldsymbol{m}_1(k) - \boldsymbol{p}_2^{\mathrm{T}} \boldsymbol{m}_2(k)] l_0 u(k) \\ &\quad + l_0^2 u^2(k) - 2 l_0^2 u(k-1) u(k) | \Im_k \} + \bar{c}_1(k), \end{aligned} \tag{10.40}$$

where $\bar{c}_1(k)$ is independent of $u(k)$. The minimization of the last equation gives the cautious control action described by

$$u_{\mathrm{c}}(k) = \frac{\boldsymbol{p}_2^{\mathrm{T}} \boldsymbol{m}_2(k) \hat{l}_0(k) - [\hat{l}_0(k) \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k) + \tilde{\boldsymbol{p}}_{l_0 p_1}^{\mathrm{T}}(k)] \boldsymbol{m}_1(k)}{\hat{l}_0^2(k) + \tilde{p}_{l_0}(k)} + u(k-1). \tag{10.41}$$

Minimization of the second cost function, eq. (10.37), over the domain Ω_k, eq. (4.23), is achieved as shown by eq. (4.31). At the same time eq. (10.37), after the insertion of eq. (10.11), takes the form

$$\begin{aligned} J_k^{\mathrm{a}}(u(k)) &= -\mathrm{E}\left\{[(\boldsymbol{p}(k) - \hat{\boldsymbol{p}}(k))^{\mathrm{T}} \boldsymbol{m}(k) + \xi(k)]^2 \middle| \Im_k \right\} \\ &= -\mathrm{E}\{([(b_1(k) - \hat{b}_1(k)] u(k) \\ &\quad + [(\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k))^{\mathrm{T}} \boldsymbol{m}_0(k)]^2 | \Im_k \} + \bar{c}_2(k) \\ &= -p_{b_1}(k) u^2(k) - 2 \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \boldsymbol{m}_0(k) u(k) + \bar{c}_3(k), \end{aligned} \tag{10.42}$$

where $\bar{c}_2(k)$ and $\bar{c}_3(k)$ do not contain $u(k)$ and the covariance matrix included in eq. (10.19) is structured as

$$\boldsymbol{P}(k) = \begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix}. \tag{10.43}$$

From eq. (10.42) follows

$$
\begin{aligned}
&J_k^{\mathrm{a}}[u_c(k)-\theta(k)]-J_k^{\mathrm{a}}[u_c(k)+\theta(k)] \\
&\qquad =-p_{b_1}(k)[u_c(k)-\theta(k)]^2-2\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)[u_c(k)-\theta(k)] \\
&\qquad\quad +p_{b_1}(k)[u_c(k)+\theta(k)]^2+2\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)[u_c(k)+\theta(k)] \\
&\qquad =4p_{b_1}(k)u_c(k)\theta(k)+4\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\theta(k)\,.
\end{aligned}
\tag{10.44}
$$

Substitution of the last equation into eq. (4.31) gives the dual control law

$$
u(k)=u_{\mathrm{c}}(k)+\theta(k)\operatorname{sgn}\{p_{b_1}(k)u_c(k)+\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\}\,. \tag{10.45}
$$

The dual APPC strategy is determined finally by eqs. (10.41) and (10.45).

10.3. Computation of the Covariance Matrix of the Controller Parameters

The covariance matrix $\tilde{\boldsymbol{P}}(k)$ of the controller parameters in eq. (10.31) can be computed from the covariance matrix $\boldsymbol{P}(k)$ of the plant parameter estimates, eq. (10.19), by solving the Diophantine equation (10.23). Simplified equations for computation of the covariance matrix $\tilde{\boldsymbol{P}}(k)$ from the matrix $\boldsymbol{P}(k)$ are derived below. First, we introduce the following notations:

$$
P(z^{-1})Q(z^{-1})=a_0^*+a_1^*z^{-1}+\ldots+a_{2n+1}^*z^{-2n-1}\,, \tag{10.46}
$$

$$
\boldsymbol{a}^{\mathrm{T}}=[a_0^*\ldots a_{2n+1}^*]\,, \tag{10.47}
$$

$$
\overline{A}(z^{-1})=A(z^{-1})(1-z^{-1})=1+\overline{a}_1z^{-1}+\ldots+\overline{a}_{n+1}z^{-n-1}\,, \tag{10.48}
$$

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 & \cdots & & & & & \\ \bar{a}_1 & 1 & 0 & \cdots & b_1 & & & \mathbf{0} & \\ \bar{a}_2 & \bar{a}_1 & 1 & \cdots & b_2 & b_1 & & & \\ \bar{a}_3 & \bar{a}_2 & \bar{a}_1 & \cdots & \vdots & \vdots & \ddots & & \\ \vdots & \vdots & \vdots & & b_{n-1} & \vdots & & \ddots & \\ \vdots & \vdots & \vdots & & 1 & & & & \\ \bar{a}_{n+1} & \cdots & \cdots & & \bar{a}_1 & b_n & \cdots & b_1 & \\ & \ddots & & & \vdots & & \ddots & & b_1 \\ & & \ddots & & \vdots & & & \ddots & \vdots \\ & \mathbf{0} & & & & & \mathbf{0} & & \\ & & & & \bar{a}_{n+1} & & & & b_n \end{bmatrix} \tag{10.49}$$

$$\hat{\boldsymbol{S}}(k) = \mathrm{E}\left\{\boldsymbol{S}(k) \middle| \Im_k\right\} = \boldsymbol{S} - \tilde{\boldsymbol{S}}(k)\,, \tag{10.50}$$

and

$$\hat{\bar{\boldsymbol{p}}}_1(k) = \bar{\boldsymbol{p}}_1 - \tilde{\bar{\boldsymbol{p}}}_1(k)\,, \tag{10.51}$$

where $\tilde{\boldsymbol{S}}(k)$ and $\tilde{\bar{\boldsymbol{p}}}_1(k)$ are the errors of the corresponding estimates and the matrix $\boldsymbol{S}$ has the dimension $(2n+2)\times(2n+2)$. Using these notations, eqs. (10.10) and (10.23) take the vector-matrix form

$$\boldsymbol{S}\bar{\boldsymbol{p}}_1 = \boldsymbol{a}\,, \tag{10.52}$$

and

$$\hat{\boldsymbol{S}}(k)\bar{\boldsymbol{p}}_1(k) = \boldsymbol{a}\,, \tag{10.53}$$

respectively. Substituting eqs. (10.50) and (10.51) into eq. (10.53) yields

$$[\hat{\mathbf{S}}(k) + \tilde{\mathbf{S}}(k)][\hat{\bar{\mathbf{p}}}_1(k) + \tilde{\bar{\mathbf{p}}}_1(k)] = \mathbf{a} \tag{10.54}$$

or, after further manipulations follows

$$\hat{\boldsymbol{S}}(k)\tilde{\bar{\boldsymbol{p}}}_1(k) + \tilde{\boldsymbol{S}}(k)\hat{\bar{\boldsymbol{p}}}_1(k) + \tilde{\boldsymbol{S}}(k)\tilde{\bar{\boldsymbol{p}}}_1(k) = \mathbf{0}\,, \tag{10.55}$$

where $\mathbf{0}$ is the zero matrix. A simplification is obtained by the omission of the second-order term $\tilde{\boldsymbol{S}}(k)\tilde{\bar{\boldsymbol{p}}}_1(k)$ in eq. (10.55). In this case, eq. (10.55) takes the form

$$\hat{\boldsymbol{S}}(k)\tilde{\bar{\boldsymbol{p}}}_1(k) = -\tilde{\boldsymbol{S}}(k)\hat{\bar{\boldsymbol{p}}}_1(k)\,. \tag{10.56}$$

The covariance matrix

$$\boldsymbol{P}_S(k) = \mathrm{E}\left\{\tilde{\boldsymbol{S}}(k)\hat{\bar{\boldsymbol{p}}}_1(k)\hat{\bar{\boldsymbol{p}}}_1^{\mathrm{T}}(k)\tilde{\boldsymbol{S}}^{\mathrm{T}}(k) \middle| \Im_k\right\} \tag{10.57}$$

can be calculated easily from $\overline{\boldsymbol{P}}(k)$ given by eq. (10.19). Thus after inserting eq. (10.56) into eq. (10.57), we have

$$\boldsymbol{P}_S(k) = \hat{\boldsymbol{S}}(k)\tilde{\boldsymbol{P}}(k)\hat{\boldsymbol{S}}^{\mathrm{T}}(k), \tag{10.58}$$

where

$$\tilde{\boldsymbol{P}}(k) = \hat{\boldsymbol{S}}^{-1}(k)\boldsymbol{P}_S(k)\hat{\boldsymbol{S}}^{-1^{\mathrm{T}}}(k). \tag{10.59}$$

10.4. Simplified Dual Versions of the Controllers

The simplified dual version of the adaptive controllers with indirect adaptation is obtained here using the CE assumption for computation of the nominal output $y_{\mathrm{n}}(k+1)$ according to eq. (10.35) in the same way as in Chapter 9. This simplification allows us to design dual controllers when the calculation of the covariance matrix $\tilde{\boldsymbol{P}}(k)$ of the controller parameters is not required. Thus, the nominal system output takes the form for the CE assumption

$$\hat{y}_{\mathrm{n}}(k+1) = \hat{b}_1(k)u_{\mathrm{CE}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k). \tag{10.60}$$

The cost function according to eq. (10.36) can be written as

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[\hat{y}_{\mathrm{n}}(k+1) - y(k+1)]^2 \middle| \Im_k\right\}. \tag{10.61}$$

After substituting eqs. (10.11) and (10.60) into eq. (10.61) and taking the expectation, we arrive at

$$\begin{aligned} J_k^{\mathrm{c}} = {} & \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{p_0}(k)\boldsymbol{m}_0(k) + \hat{b}_1^2(k)u_{\mathrm{CE}}^2(k) + [\hat{b}_1^2(k) + p_{b_1}(k)]u^2(k) \\ & - 2\hat{b}_1^2(k)u_{\mathrm{CE}}(k)u(k) - 2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) + \sigma_\xi^2. \end{aligned} \tag{10.62}$$

The minimisation of this cost function provides the cautious control law

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_1^2(k)u_{\mathrm{CE}}(k) - \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)}{\hat{b}_1^2(k) + p_{b_1}(k)}. \tag{10.63}$$

The second cost function, according to eq. (10.37), is not affected by the above simplification; therefore, the dual APPC is described by eqs. (10.24), (10.40), (10.45) and (10.63). It should be noted that an integrator can be placed in the control loop after calculation of the control signal. In this case, the CE controller, according to eq. (10.24), assumes the form

$$u_{\mathrm{CE}}(k)=\frac{1}{\hat{l}_0(k)}[\boldsymbol{p}_2^{\mathrm{T}}\mathbf{m}_2(k)-\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)], \tag{10.64}$$

and the cautious controller according to eq. (10.62) is given by

$$u_{\mathrm{c}}(k)=\frac{\hat{b}_1^2(k)u_{\mathrm{CE}}(k)-\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)}{\hat{b}_1^2(k)+p_{b_1}(k)}+u(k-1). \tag{10.65}$$

The derived simplified dual controller can be computed easily from the standard CE controller. This property of the dual controller is attractive for practical applications. Various standard CE controllers with indirect adaptation can be modified easily to their simplified dual versions using the suggested approach. The block diagrams of the standard CE adaptive controller and its dual modification have already been shown in Figures 9.1 and 9.2, respectively.

11. APPLICATION OF DUAL CONTROLLERS TO THE SPEED CONTROL OF A THYRISTOR-DRIVEN DC-MOTOR

Control engineers are often faced with the problem of insufficient control performance, such as large overshoots in the transient stages during the practical application of many digital adaptive controllers in real-time operation (Åström and Wittenmark, 1989; Filatov et al., 1994; Filatov et al., 1995; Filatov and Unbehauen, 1995a; Wittenmark, 1995). Large overshoots and oscillations of the system state can result in untimely wear and tear of the equipment. Moreover this leads to a general lack of acceptance of adaptive control. These shortcomings of adaptive control arise especially in cases of significant uncertainty of the plant parameters. These imperfections of adaptive controllers result from using the CE assumption, where the uncertainties of the estimated values are not taken into account and the parameter estimates are used as if they were the real values of the unknown parameters (Åström and Wittenmark, 1989). In dual controllers, however, the uncertainties of the estimates are taken into account, and the control system tries to track cautiously the reference signal and, at the same time, improves the estimation of the process parameters by optimal persistent excitation.

The dual versions of the indirect adaptive pole-placement controller and the dual LQG controller presented in Chapter 10 will be applied in this chapter to the speed control of a thyristor-driven DC motor. It should be noted that in the past many problems have been experienced during the attempts of practical applications of dual controllers based on the CE assumption, which are either computationally complex or provide only insufficient control performance (Åström and Wittenmark, 1989). However, the dual controllers based on the bicriterial approach are appropriate for practical applications and, thus, can also replace their CE counterparts.

11.1. Speed Control of a Thyristor-Driven DC Motor

The dual controllers derived in Chapter 10 have been successfully applied to control the speed of a thyristor-driven DC motor. The speed control scheme of the DC motor is indicated in Fig. 11.1. This cascaded control scheme consists of two control loops: an analog PI control of armature current and a digital adaptive speed control. The detailed description of the experimental setup, which is used here for testing the dual controller, was described by Keuchel and Stephan (1994). The following parameters of the 1.1 kW DC motor have been taken: current for the field excitation 600 mA, speed setpoint ± 1000 rpm, which corresponds to a voltage at the input of the A/D converter of ± 0.5 V. The normalized manipulating signal u is limited to ± 1.0, and it is determined by the limited valid value of the armature current. The control loop for the DC motor speed can be described approximately by the linear continuous-time model of a second order transfer function

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$$G(s)=\frac{K}{s(1+Ts)}, \tag{11.1}$$

where the parameters depend on the operating point and the continuous or discontinuous current modes of the motor. Therefore, the corresponding discrete-time model is

$$G(z)=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \tag{11.2}$$

and has four unknown parameters. The following initial values for the estimation algorithm according to eqs. (10.14)-(10.17) have been selected: initial estimates $\hat{\boldsymbol{p}}^{\mathrm{T}}(0)$ =[0.01 0.01 0.01 0.01], initial covariance matrix $\boldsymbol{P}(0)$ =diag[40], noise variance σ_ξ^2=10^{-6}. The parameters of the dual controller with an integrator according to eqs. (10.38), (10.45), (10.64) and (10.65) have been selected as $\eta=5\cdot10^{-10}$ and $\delta=4$, which corresponds to the initial value of the normalized excitation of $|u_a(0)|=0.32768$. The parameters of the plant model are changed stepwise by changing the load on the rotating shaft of the DC motor using a clutch and a large mass as shown in Fig. 11.1. These parameter jumps are detected by the controller in real-time by comparing the one-step-ahead prediction error $e(k)$, according to eq. (10.15), with its variance according to eq. (10.22). The controller detects a parameter jump in the case of $|e(k+1)|>>\sigma_e(k+1)$ and updates the covariance matrix $\boldsymbol{P}(k)$ with its initial value $\boldsymbol{P}(0)$ after every jump detection.

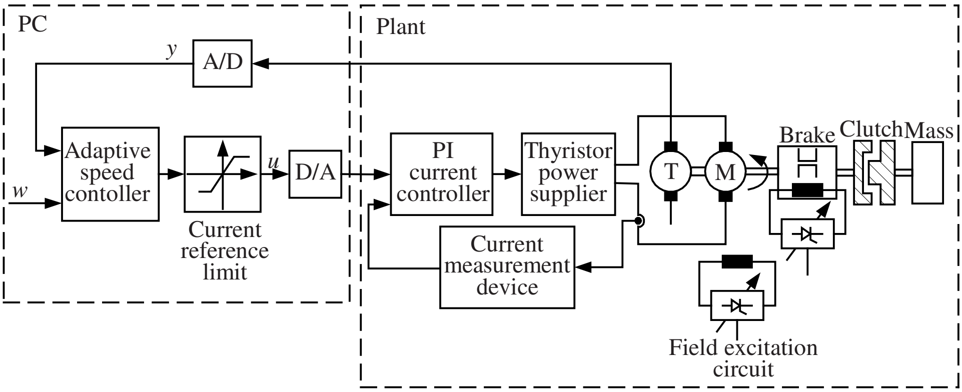

Fig. 11.1. Cascaded speed control scheme of the thyristor-driven DC motor (D/A $\hat{=}$ digital-analog converter, A/D $\hat{=}$ analog-digital converter, T $\hat{=}$ tacho-generator, M $\hat{=}$ DC-motor)

The results of real-time speed control of the considered plant for the pole-placement and LQG controllers with the dual and the CE approaches will be described in the following section.

11.2. Application Results for the Pole-Placement Controller

In the z-domain the following poles for the closed-loop system and the observer are assigned as shown in Table 11.1. The number of the poles (n=3) is determined by the order of the plant plus one because of the additional integrator. The results of the real-time control are presented for the simplified dual controller in Fig. 11.2 and for the corresponding CE controller in Fig. 11.3. The comparison of the transient behavior of the CE control with the dual APPC indicates a smoother startup and shorter adaptation time for the dual controller. The behavior of the system after parameter changes is also smoother and shows less overshoot for the dual controller. We can also see a larger adaptation error e, which accelerates the estimation in case of the dual controller at the beginning of the process.

Table 11.1. Assigned poles of the closed-loop system and the observer

System poles		Observer Poles	
Re	Im	Re	Im
-0.4	0	0.6	0
0.8	-0.1	0.2	0
0.8	0.1		

11.3. Application Results for the LQG Controller

The cost function with r=1 according to eq. (10.32) has been used for the application of the LQG controller. The pole location of the closed loop system according to polynomial $P(z^{-1})$ is determined by the cost function described by eq. (10.32) using a spectral factorization (Keuchel and Stephan, 1994; Yu et al., 1987) of eq. (10.33). Then, the polynomial $P(z^{-1})$ can be calculated from eq. (10.33) after estimation of the plant parameters in real-time and can be used immediately in the proposed pole-placement of the dual or CE controller.

The results of the LQG controller with CE approach are shown in Fig. 11.4 and Fig. 11.5 for the corresponding simplified dual version. The comparison of the behavior of the CE control law with the dual control law indicates the same improvement of the control performance for the dual controller as already demonstrated for the APPC.

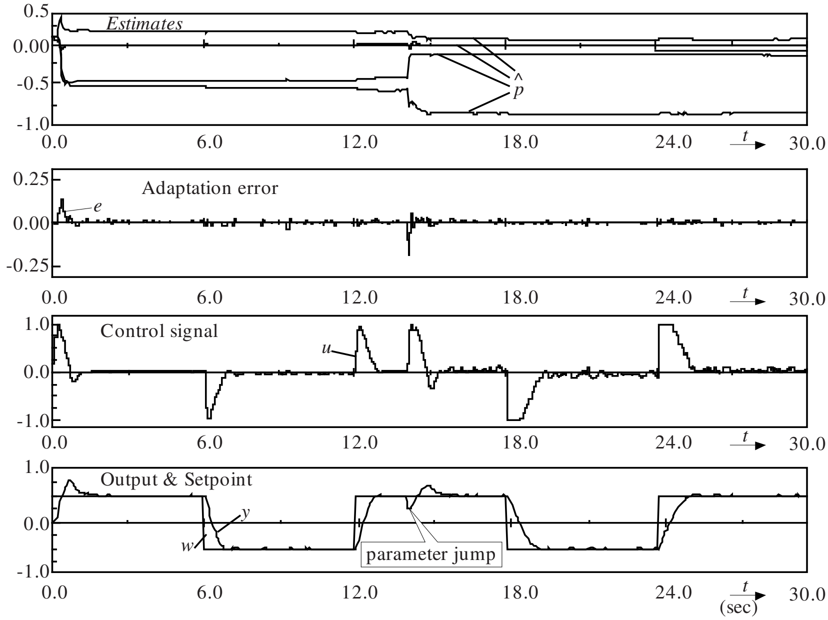

Fig. 11.2. Application results of the standard APPC with the CE approach

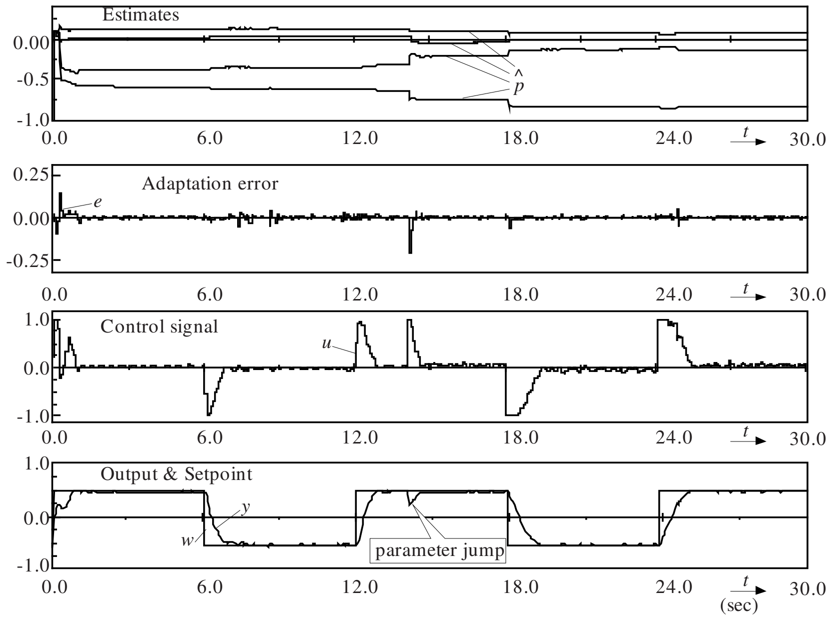

Fig. 11.3. Application results of the dual APPC

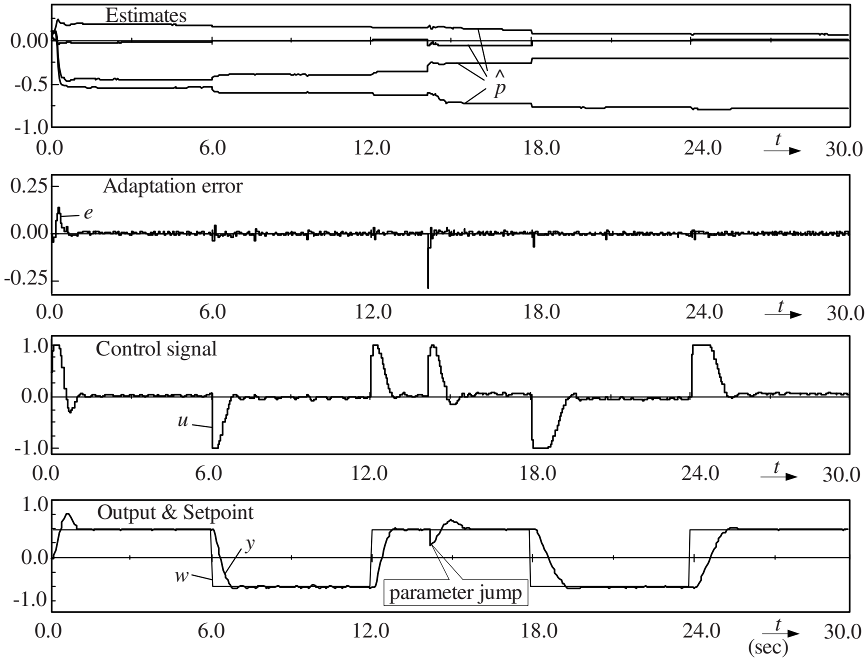

Fig. 11.4. Application results of the LQG controller with the CE approach

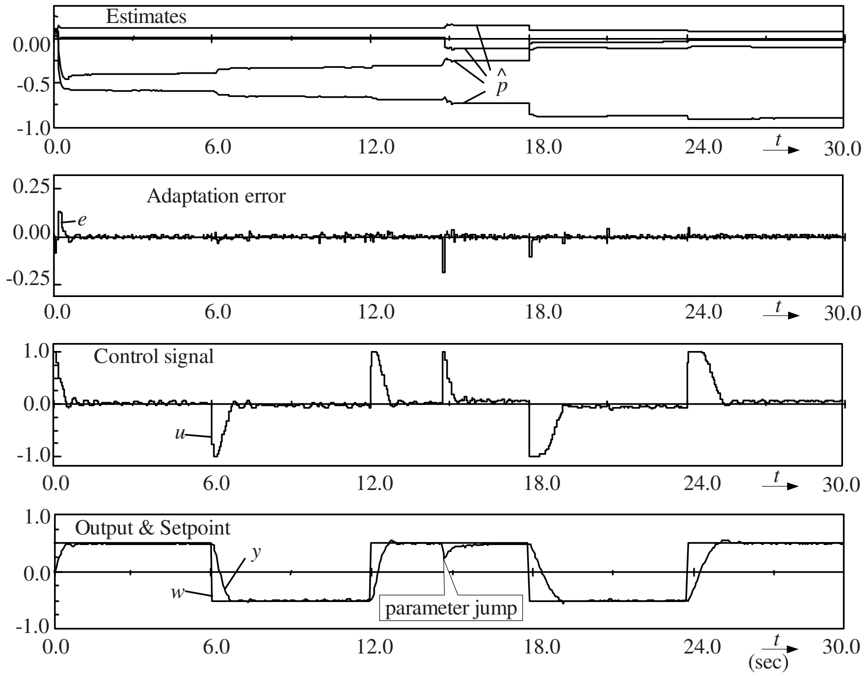

Fig. 11.5. Application results of the dual LQG controller

12. APPLICATION OF DUAL CONTROLLERS TO A LABORATORY SCALE VERTICAL TAKE-OFF AIRPLANE

The application of dual control with direct adaptation to the roll-angle control of a laboratory scale vertical take-off airplane (Patra et al., 1994) will be described below. The bicriterial approach will be applied for the synthesis of a dual controller based on pole-zero-placement. At first, the pole-zero placement adaptive control law for linear SISO plants will be discussed. After describing the experimental setup, the results of an adaptive controller for the mechanical laboratory pilot-plant will be presented. It will be pointed out that adaptive control is necessary and presents the most suitable approach for this plant. The dual control algorithm with direct adaptation will be compared to indirect and direct adaptive control strategies based on the standard CE assumption.

12.1. Pole-Zero-Placement Adaptive Control Law

In the case under consideration, the general structure of the discrete-time controller is chosen as

$$U(z)=\frac{S(z^{-1})}{R(z^{-1})}E_{\mathrm{c}}(z)=\frac{1+s_1 z^{-1}+\ldots+s_{n_S} z^{-n_{\mathrm{S}}}}{r_0+r_1 z^{-1}+\ldots+r_{n_{\mathrm{R}}} z^{-n_{\mathrm{R}}}}E_{\mathrm{c}}(z),\ r_0\neq 0, \tag{12.1}$$

and the model equation of the desired closed-loop system is

$$A_{\mathrm{m}}(z^{-1})Y(z)=B_{\mathrm{m}}(z^{-1})W(z), \tag{12.2}$$

where $E_{\mathrm{c}}(z)=W(z)-Y(z)$ is the control error, $W(z)$, $Y(z)$ and $U(z)$ are the z-transforms of the setpoint $w(k)$, output $y(k)$ and input $u(k)$ signals of the plant, respectively, k is the discrete time index and the polynomials $A_{\mathrm{m}}(z^{-1})$ and $B_{\mathrm{m}}(z^{-1})$ determine the desired tracking behavior of the closed-loop system. These polynomials are specified by

$$B_{\mathrm{m}}(z^{-1})=b_{\mathrm{m}1}z^{-1}+\ldots+b_{\mathrm{m}n_{\mathrm{Bm}}}z^{-n_{\mathrm{Bm}}} \tag{12.3a}$$

and

$$A_{\mathrm{m}}(z^{-1})=1+a_{\mathrm{m}1}z^{-1}+\ldots+a_{\mathrm{m}n_{\mathrm{Am}}}z^{-n_{\mathrm{Am}}}. \tag{12.3b}$$

The parameters of the controller, eq. (12.1), are assumed to be unknown, according to the unknown plant model. For the direct dual control the controller parameters have to be estimated in real time. To find an appropriate error equation for estimation, the model equation (12.2) is inserted into eq. (12.1). After some algebraic manipulations, it follows that

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 135–147, 2004.

$$S(z^{-1})[A_{\mathrm{m}}(z^{-1})-B_{\mathrm{m}}(z^{-1})]Y(z)=R(z^{-1})B_{\mathrm{m}}(z^{-1})U(z). \tag{12.4}$$

Introducing filtered values of the input and output signals $\overline{u}(k)$ and $\overline{y}(k)$, defined by

$$\overline{Y}(z)=[A_{\mathrm{m}}(z^{-1})-B_{\mathrm{m}}(z^{-1})]\,Y(z), \quad \text{and} \quad z^{-1}\overline{U}(z)=B_{\mathrm{m}}(z^{-1})U(z)\,, \tag{12.5}$$

the correspondence between these auxiliary signals is given by

$$S(z^{-1})\overline{Y}(z)=z^{-1}R(z^{-1})\overline{U}(z)\,, \tag{12.6}$$

where $\overline{Y}(z)$ and $\overline{U}(z)$ are the z-transforms of the filtered signals $\overline{y}(k)$ and $\overline{u}(k)$, respectively. After inverse z-transformation of eq. (12.6), we obtain in vector form

$$\overline{y}(k+1)=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k)\ , \tag{12.7}$$

where

$$\boldsymbol{p}=[r_0 \dots r_{n_R} \vdots -s_1 \dots -s_{n_S}]^{\mathrm{T}}=[r_0 \vdots \boldsymbol{p}_0^{\mathrm{T}}]^{\mathrm{T}} \tag{12.8}$$

and

$$\boldsymbol{m}(k)=[\overline{u}(k)\dots\overline{u}(k-n_R)\vdots\overline{y}(k)\dots\overline{y}(k-n_S+1)]^{\mathrm{T}}=[\overline{u}(k)\vdots\boldsymbol{m}_0^{\mathrm{T}}(k)]^{\mathrm{T}}. \tag{12.9}$$

In the case of time-varying parameters with a stochastic parameter drift according to eq. (4.5), the controller parameter vector described by eq. (12.8) can be estimated with a Kalman filter given as

$$\hat{\boldsymbol{p}}(k+1)=\hat{\boldsymbol{p}}(k)+\boldsymbol{q}(k+1)[\overline{y}(k+1)-\hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]\ , \tag{12.11}$$

$$\boldsymbol{q}(k+1)=\boldsymbol{P}(k)\boldsymbol{m}(k)[\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}(k)+\sigma_\xi^2]^{-1} \tag{12.12}$$

and

$$\boldsymbol{P}(k+1)=\boldsymbol{P}(k)-\boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)+\boldsymbol{Q}_\varepsilon(k)\ , \tag{12.13}$$

where the matrix $\boldsymbol{Q}_\varepsilon(k)$ is time-varying.

It is further assumed that some disturbances act on the system and σ_ξ^2 in eq. (12.12) corresponds to the additive white noise with zero mean and covariance σ_ξ^2, which has to be added on the right-hand side of eq. (12.7), similar as in eq. (4.2). In this case the closed-loop adaptive system takes the structure shown in Fig. 12.1. Up to now the direct adaptive control strategy is based on the CE assumption because the uncertainty in the estimated parameters is not taken into consideration.

12.1.1. Modification for Cautious and Dual Control

In many cases, controllers designed under the CE assumption result in insufficient control performance, because large overshoots and oscillations occur at the start-up of the adaptation process and after parameter changes, i.e. in phases where the parameter estimation error is large. To derive the dual control law for the adaptive system with

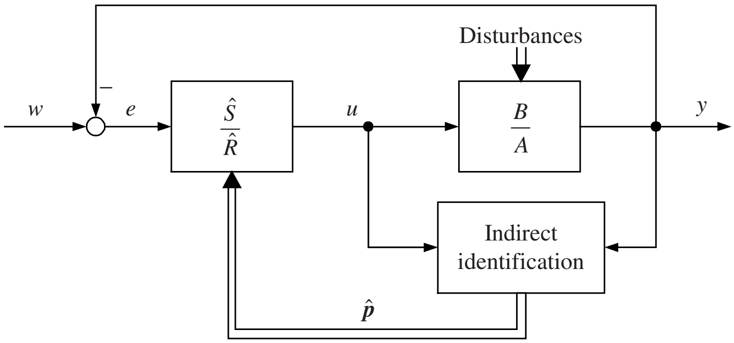

Fig. 12.1. Block diagram of the direct adaptive control algorithm based on the CE assumption ($\hat{S}$ and $\hat{R}$ are the estimates of the polynomials S and R, respectively)

direct adaptation, the two performance indices according to eqs. (6.32) and (6.33)

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[y_{\mathrm{n}}(k+1) - y(k+1)]^2 \middle| \Im_k\right\} \tag{12.16}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[\bar{y}(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 \middle| \Im_k\right\} \tag{12.17}$$

are introduced, where $y_{\mathrm{n}}(k)$ is the nominal output of the system in response to the input signal $u_{\mathrm{n}}(k)$ of the unknown controller with non adjusted parameters. This yields to

$$u_{\mathrm{n}}(k) = \mathcal{Z}^{-1}\left\{\frac{S(z^{-1})}{R(z^{-1})} E_{\mathrm{c}}(z)\right\} = -\frac{1}{r_0}[\boldsymbol{p}_0^{\mathrm{T}} \tilde{\boldsymbol{m}}_0(k) - e_{\mathrm{c}}(k)] \tag{12.18}$$

with

$$\boldsymbol{p}_0^{\mathrm{T}} = [r_1 \; r_2 \; \ldots - s_{n_s}\,],$$

$$\tilde{\boldsymbol{m}}_0(k) = [u(k-1) \ldots u(k-n_{\mathrm{R}}) \vdots e_{\mathrm{c}}(k-1) \ldots e_{\mathrm{c}}(k-n_{\mathrm{S}})]^{\mathrm{T}} \;, \tag{12.19}$$

$$\bar{y}_{\mathrm{n}}(k) = r_0 \bar{u}_{\mathrm{n}}(k-1) + \ldots + r_{n_{\mathrm{R}}} \bar{u}(k-n_{\mathrm{R}}-1) - s_1 \bar{y}(k-1) - \ldots - s_{n_{\mathrm{S}}} \bar{y}(k-n_{\mathrm{S}}), \tag{12.20}$$

$$\bar{y}_{\mathrm{n}}(k) = y_{\mathrm{n}}(k) + \sum_{i=1}^{n_{\mathrm{m}}} (a_i - b_i) y(k-i), \; n_{\mathrm{m}} = \max(n_{\mathrm{A_m}}, n_{\mathrm{B_m}}) \;, \tag{12.21}$$

and

$$\bar{u}_{\mathrm{n}}(k-1) = b_1 u_{\mathrm{n}}(k-1) + \sum_{i=2}^{n_{\mathrm{B}}} b_i u(k-i) \;, \tag{12.22}$$

where $\mathcal{Z}^{-1}\{.\}$ denotes the inverse z-transform.

The cost function by eq. (12.17) is used for accelerating the parameter estimation process (Milito et al., 1982; Wittenmark, 1975b) by increasing the innovation value in eq. (12.11). For the considered adaptive system with indirect identification, the dual control algorithm will be obtained from the solution of a bicriterial optimization problem (see, for example, Wittenmark, 1995). To derive the direct dual control, the second cost function, eq. (12.17), must be minimized over the bounded domain Ω_k, which is symmetrically located around the optimal solution $u_{\mathrm{c}}(k)$ of the first cost function, eq. (12.16), as described by eq. (4.23).

After substituting eq. (12.20) into eq. (12.16), we obtain

$$\begin{aligned} J_k^{\mathrm{c}} &= \mathrm{E}\left\{[y_{\mathrm{n}}(k+1)-y(k+1)]^2 \middle| \Im_k\right\} = \mathrm{E}\left\{[\bar{y}_{\mathrm{n}}(k+1)-\bar{y}(k+1)]^2 \middle| \Im_k\right\} \\ &= \mathrm{E}\left\{[r_0\bar{u}_{\mathrm{n}}(k)-r_0\bar{u}(k)]^2 \middle| \Im_k\right\} = \mathrm{E}\left\{[r_0 b_1 u_{\mathrm{n}}(k)-r_0 b_1 u(k)]^2 \middle| \Im_k\right\}. \end{aligned} \tag{12.23}$$

Taking into account eq. (12.18) and calculating the expectation, eq. (12.23) leads to

$$\begin{aligned} J_k^{\mathrm{c}} &= \mathrm{E}\left\{[-b_1(\boldsymbol{p}_0^{\mathrm{T}}\tilde{\boldsymbol{m}}_0(k)-e(k))-r_0 b_1 u(k)]^2 \middle| \Im_k\right\} \\ &= b_1^2[\hat{r}_0^2(k)+p_{r_0}(k)]u^2(k)-2b_1^2\hat{r}_0(k)e(k)u(k) \\ &\qquad + 2b_1^2[\hat{r}_0(k)\hat{\boldsymbol{p}}_0(k)+\boldsymbol{p}_{r_0p_0}(k)]^{\mathrm{T}}\tilde{\boldsymbol{m}}_0(k)u(k)+\bar{c}_1(k), \end{aligned} \tag{12.24}$$

where $\bar{c}_1(k)$ does not depend on $u(k)$ and the covariance matrix has the structure

$$\boldsymbol{P}(k) = \begin{bmatrix} p_{r_0}(k) & \boldsymbol{p}_{r_0p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{r_0p_0}(k) & \boldsymbol{P}_{p_0}(k) \end{bmatrix}.$$

Minimization of eq. (12.24) with respect to $u(k)$ provides the cautious control law

$$u_{\mathrm{c}}(k) = \frac{\hat{r}_0(k)e(k)-[\hat{r}_0(k)\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)+\boldsymbol{p}_{r_0p_0}^{\mathrm{T}}(k)]\tilde{\boldsymbol{m}}_0(k)}{\hat{r}_0^2(k)+p_{r_0}(k)}. \tag{12.25}$$

Substituting eq. (12.7) into eq. (12.17) and calculating the expectation gives

$$J_k^{\mathrm{a}}[u(k)] = -\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}(k) = -p_{r_0}(k)\bar{u}^2(k)-2\boldsymbol{p}_{r_0p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\bar{u}(k)+\bar{c}_2(k), \tag{12.26}$$

where $\bar{c}_2(k)$ does not contain $u(k)$. It is easy to see that the minimization of eq. (12.17) over the bounded domain according to eq. (4.23) leads to the solution of eq. (4.31).

From the performance index according to eq. (12.17) we obtain for $u_{\mathrm{c}}(k)$

$$\begin{aligned} J_k^{\mathrm{a}}[u_c(k)] &= -p_{r_0}(k)\left(b_1 u_{\mathrm{c}}(k-1)+\sum_{i=2}^{n_{\mathrm{B}}} b_i u(k-i)\right)^2 \\ &\qquad - 2\boldsymbol{p}_{r_0p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\left(b_1 u_c(k-1)+\sum_{i=2}^{n_{\mathrm{B}}} b_i u(k-i)\right)+\bar{c}_2(k) \end{aligned}$$

or

$$
\begin{aligned}
J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)-\theta(k)]-J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)+\theta(k)] \\
&= -p_{r_0}(k)[b_1 u_{\mathrm{c}}(k)-b_1\theta(k)+\bar{c}_3(k)]^2 \\
&\quad -2\boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\left[b_1 u_{\mathrm{c}}(k)-b_1\theta(k)+\bar{c}_3(k)\right] \\
&\quad + p_{r_0}(k)[b_1 u_{\mathrm{c}}(k)+b_1\theta(k)+b_1\bar{c}_3(k)]^2 \\
&\quad +2\boldsymbol{p}_{r_0 p_{01}}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)[b_1 u_{\mathrm{c}}(k)+b_1\theta(k)+\bar{c}_3(k)] \\
&= 4p_{r_0}(k)\bar{u}_{\mathrm{c}}(k)b_1\theta(k)+4\boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)b_1\theta(k)\,, \qquad (12.27)
\end{aligned}
$$

where

$$
\bar{u}_{\mathrm{c}}(k)=b_1 u_{\mathrm{c}}(k)+\sum_{i=2}^{n_{\mathrm{B}}} b_i u(k-i+1)\ , \qquad \bar{c}_3(k)=\sum_{i=2}^{n_{\mathrm{B}}} b_i u(k-i+1)\ . \qquad (12.28)
$$

After inserting eq. (4.24) into eq. (4.31) and performing several simple algebraic manipulations, leads to

$$
u(k)=u_{\mathrm{c}}(k)+\eta\,\mathrm{tr}\{\boldsymbol{P}(k)\}\mathrm{sign}\left\{J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)-\theta(k)]\;-J_k^{\mathrm{a}}[u_{\mathrm{c}}(k)+\theta(k)]\right\}.
$$

Substituting of eq. (12.27) in the last equation, we finally obtain the adaptive dual control law

$$
u(k)=u_{\mathrm{c}}(k)+\eta\,\mathrm{tr}\{\boldsymbol{P}(k)\}\mathrm{sign}\left\{b_1 p_{r_0}(k)\bar{u}_{\mathrm{c}}(k)+b_1\boldsymbol{p}_{r_0 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\right\}. \qquad (12.29)
$$

It should be mentioned that the dual control law given by eqs. (12.25) and (12.29) depends not only on the parameter estimates but also on their covariance. Thus, the controller provides the necessary cautious and excitation properties, which are crucial for the performance of the adaptive control. The parameter η in eq. (12.29) is proportional to the amplitude of the additive excitation signal and, therefore, has a clear interpretation. To implement this algorithm, eqs. (12.11) to (12.13) have to be used for the parameter estimation with the filtered signals according to eq. (12.5) and the control law according to eqs. (12.25) and (12.29).

12.1.2. Modification for Nonminimum Phase Systems

With well-adjusted controller parameters, the control law of eq. (12.1) provides stability of the feedback system if the roots of the characteristic closed-loop polynomial

$$
P_{\mathrm{c}}(z^{-1})=R(z^{-1})A(z^{-1})+S(z^{-1})B(z^{-1}) \qquad (12.30)
$$

are located inside the unit circle of the z-plane, where

$$
A(z^{-1})Y(z)=B(z^{-1})U(z) \qquad (12.31)
$$

describes the plant model. The desired closed-loop dynamics according to eq. (12.2) can be specified for a nonminimum phase plant only if the numerator polynomial $B_{\mathrm{m}}(z^{-1})$ of the closed-loop system model can be factorized in the form

$$B_{\mathrm{m}}(z^{-1}) = S(z^{-1})B(z^{-1}) \ , \tag{12.32}$$

which leads to

$$\frac{B_{\mathrm{m}}(z^{-1})}{A_{\mathrm{m}}(z^{-1})} = \frac{S(z^{-1})B(z^{-1})}{R(z^{-1})A(z^{-1}) + S(z^{-1})B(z^{-1})} \ . \tag{12.33}$$

In the case of a minimum phase plant, the closed-loop transfer function can be chosen as

$$\frac{B_{\mathrm{m}}(z^{-1})}{A_{\mathrm{m}}(z^{-1})} = \frac{S(z^{-1})}{P(z^{-1})A(z^{-1}) + S(z^{-1})} \ , \tag{12.33}$$

where the factorisation $R(z^{-1}) = B(z^{-1})P(z^{-1})$ must hold. It is easy to see that in the case of a unit delay model, $A(z^{-1}) = 1,\ B(z^{-1}) = z^{-1}$, the dual control described by eqs. (12.25) and (12.29) takes the form of the controller with indirect adaptation (Wittenmark, 1995). For the derivation of the desired dynamics of a system with nonminimum phase behavior, the correction network approach (Unbehauen and Keuchel, 1992) may be used in combination with the standard adaptive control loop.

12.2. Experimental Setup and Results

12.2.1. Description of the Plant

An interesting and nontrivial example to test new control strategies is the positioning of a heavy beam by means of two propellers, as shown in Figs. 12.2 and 12.3. The practical problem that motivated this laboratory plant was the roll-angle control of a vertical take-off airplane. The controlled variable is the angle φ formed with the horizontal axis. The input value which is also the manipulated signal, will be the command voltage u for the power amplifiers of the DC motors driving the propellers. In a linearized description the dynamic behavior between the input voltage of the amplifier and the speed n of the propeller, attached to the shaft of the motor, can be described by a first-order lag (representing the dynamic behavior of a separately excited DC-machine). Neglecting friction, integration of the angular acceleration of the beam yields the angular velocity and a further integration generates the angle φ. The nonlinear characteristics between the motor speed n and the generated moment M are introduced by the propeller mechanism.

Ignoring the nonlinearity in Fig. 12.4, the system can be described by the differential equation

$$\dddot{y}(t) + a\ddot{y}(t) = bu(t) \ , \tag{12.34}$$

where $b = K_{\mathrm{A}} K_{\varphi} K_{\mathrm{M}} / (\Theta T_{\mathrm{M}}), a = 1/T_{\mathrm{M}}$, K_{A} is a factor proportional to the gain of the power amplifier, K_{φ} represents the gain of the measurement interface for the angle, K_{M} and T_{M} are the gain and the time constant of the propeller engines, and Θ represents the moment of inertia of the complete setup. If the motor time constant T_{M} is small, then the nonlinearity can be transferred to the input side, and the system can then be approximately described by

$$\dddot{y}(t)+a\ddot{y}(t)=b_1u(t)+b_3u^3(t)\ ,$$

where the parameters have been estimated as $a = 3.02$, $b_1 = 7.1$ and $b_2 = -0.83$.

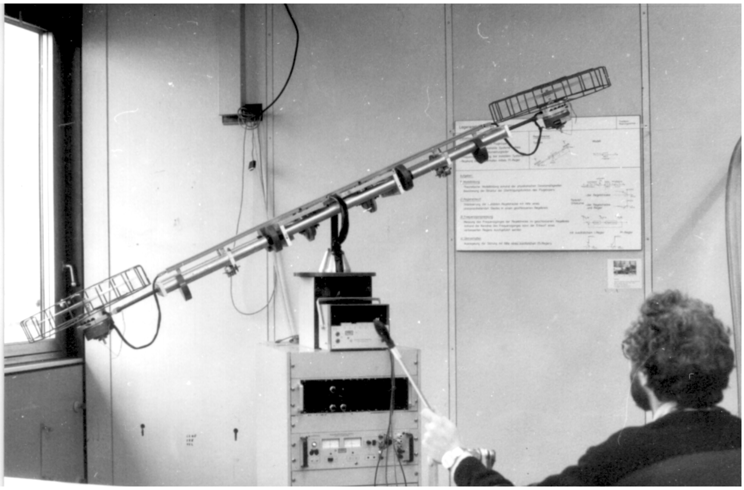

Fig. 12.2. The pilot-plant

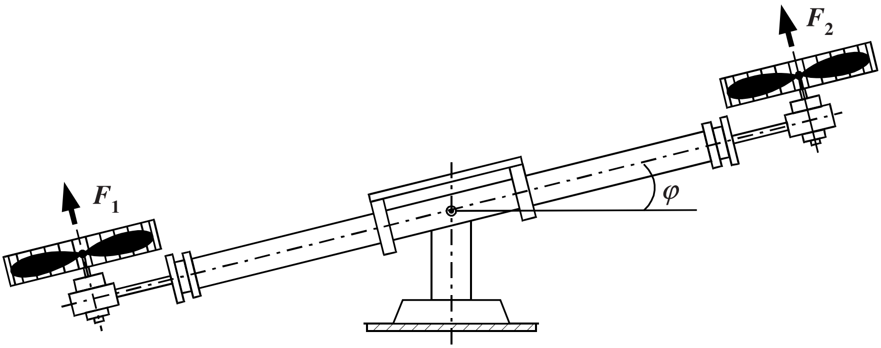

Fig. 12.3. Schematic diagram of the pilot-plant

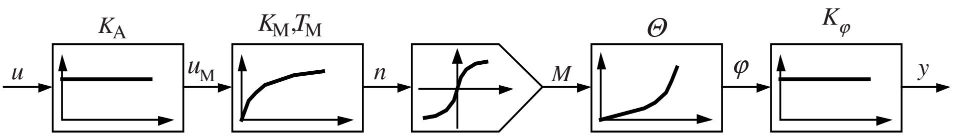

Fig. 12.4. Input-output block diagram of the plant

Thus, after taking the Laplace transform of eq. (12.34), the transfer function of this system is given by

$$G(s) = \frac{b}{s^2(s+a)} \quad . \tag{12.35}$$

The parameters of the model can be estimated based on the measured signals u and y. There is no way to measure the angular speed or acceleration directly, hence a separation of the pure double integrator is not possible. Thus, the corresponding discrete transfer function

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \tag{12.36}$$

contains six parameters that have to be estimated for indirect adaptive approaches. Moreover, the numerator polynomial contains roots outside the unit circle of the z-plane in all operating points for the chosen sampling time.

We can summarize the properties of this system as follows:

- double integral action and unstable,
- nonminimum phase discrete-time transfer function for sampling rates of interest,
- nonlinearities, especially strong cubic nonlinear characteristics between n and M,
- load disturbance acting upon the moment of inertia, so that integral control action is necessary to reach a stationary vanishing control error,
- not stabilizable by a PI controller and unsatisfactory control performance with a usual PID controller, lead compensation necessary.

All above-mentioned items lead to the necessity of using adaptive control for this control problem.

12.2.2. Comparison of Standard and Dual Control

The parameters of the discrete transfer function, eq. (12.36), were preliminarily estimated (Patra, et al., 1994). This was necessary for the determination of the discrete-time closed-loop model according to eqs. (12.2) and (12.3). For a nominal plant model according to eq. (12.34), the continuous-time parameters are estimated as a=3.02 sec^{-1}, b=6.84, where the motor time constant and overall gain parameter are obtained by simple measurements; the third order discrete-time model in eq. (12.36) has the parameters given in Table 12.1 which are obtained after z-transformation for the sampling time 0.2 sec applying a zero-order hold.

Table 12.1. Parameters of the model according to eq. (12.36) after preliminary identification

Numerator		Denominator	
b_1	$7.89 \cdot 10^{-3}$	a_1	-2.5466
b_2	$2.73 \cdot 10^{-2}$	a_2	2.0932
b_3	$5.84 \cdot 10^{-3}$	a_3	-0.5466

This model is nonminimum phase and unstable, and the parameters are changing depending on the setpoint. According to eqs. (12.30) and (12.32), the closed-loop model was chosen to give the pole locations specified in Table 12.2.

Table 12.2. Closed-loop pole locations in the *z*-domain for the experiment

Real part	Imaginary part
0.829	0.2
0.829	-0.2
0.72	0
0.45	0
0.2	0

Obviously the controller given by eq. (12.1) has the polynomial orders $n_S = 2$ and $n_R = 2$. Hence, there are five parameters to be estimated. The startup-parameters for the adaptive controller are chosen as follows:

$$\hat{\boldsymbol{p}}(0) = [5.8 \quad -7.7 \quad 2.8 \quad -0.51 \quad 1.4]^{\mathrm{T}},$$
$$\boldsymbol{P}(0) = \mathrm{diag}[10 \quad 10 \quad 10 \quad 10 \quad 10],$$
$$\sigma_\xi^2 = 0.1, \;\; \eta = 0.0001, \;\; u(k) = 0 \;\text{ and }\; y(k) = 0 \;\text{ for }\; k \le 0,$$

where diag[·] denotes a diagonal matrix with the indicated diagonal elements. The controller parameter values for the considered plant model according to eq. (12.36) and Table 12.1 can be calculated by eqs. (12.32) and (12.33) and the initial values in $\hat{\boldsymbol{p}}(0)$ have been chosen arbitrarily. These parameters are assumed to be unknown, and they depend on the operating point of the system.

Fig. 12.5 shows the results of the indirect adaptive pole-placement controller of Keuchel et al. (1987). In contrast to Keuchel et al. (1987), who used a robust and an LQ optimal pole-placement design, the pole positions according to Table 12.1 have been chosen for the experiment plotted in Fig. 12.5. Besides the parameters, the manipulated and controlled signals of the experiment, Fig. 12.5 also shows the prediction error

$$e(k) = \hat{y}(k) - \hat{\tilde{\boldsymbol{p}}}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}(k),$$

where $\hat{\tilde{\boldsymbol{p}}}$ and $\tilde{\boldsymbol{m}}$ contain the estimated plant parameters and input/output signals for direct estimation of the plant, respectively. The controlled variable $y(t)$ in Fig. 12.5 exhibits the typical problems of indirect adaptive control schemes based on the CE assumption during the phases of adaptation. There is a large overshoot (more than 200 %) during the adaptation period, and the convergence of parameters is slow mainly due to

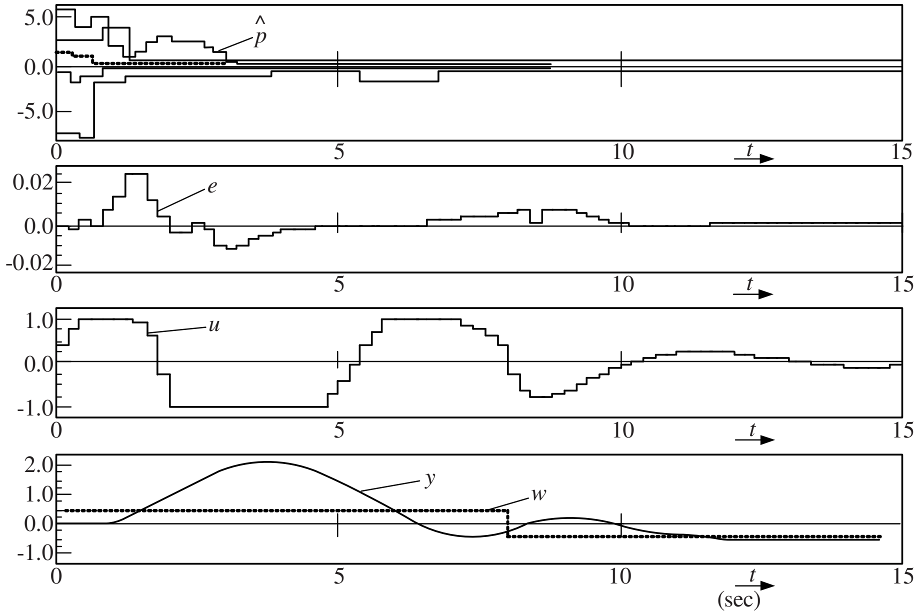

Fig. 12.5. Indirect adaptive pole-placement control

the missing excitation since the normalized manipulated signal reaches the normalized saturation limits, $|u(k)| = 1$.

Fig. 12.6 shows the results for the direct adaptive pole-placement control law based on the standard CE assumption. The adaptation transients die out significantly faster than in the case of indirect control, but there is still a 40 % overshoot after the first setpoint step. Here and in all further diagrams

$$e(k) = \bar{y}(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)$$

is the equation error as introduced in eq. (12.11).

Figs. 12.7 and 12.8 display the results for the same experiment applying the proposed direct adaptive dual control strategy with different settings of the tuning parameter

η for cautious and dual control action ($\eta = 0$ and $\eta = 0.0001$, respectively). The results obtained by the direct dual control approach as illustrated in Fig. 12.8 show better control performance at the beginning of the adaptation because of the cautious and probing properties. The adaptive control based on the CE assumption (Fig. 12.6) gives a non-negligible overshoot at the beginning of the adaptation, because the estimated controller parameters are far from the desired values and the accuracy of the estimates is not taken into account. The presence of the term representing the covariance of the estimates in eq. (12.25) for the direct dual control (Fig 12.8) avoids this undesired large control deviation. It is necessary to point out that cautious controllers normally provide good results (see Fig. 12.7) but have not found a broad practical application up to now, because they lead to slow adaptation and sometimes to the "turn off" effect (Wittenmark, 1995) when the estimation process in the adaptive system is interrupted. Only the combination of cautious control with optimal excitation, as in eq. (12.29), provides acceptable control performance as can be seen from Fig. 12.8.

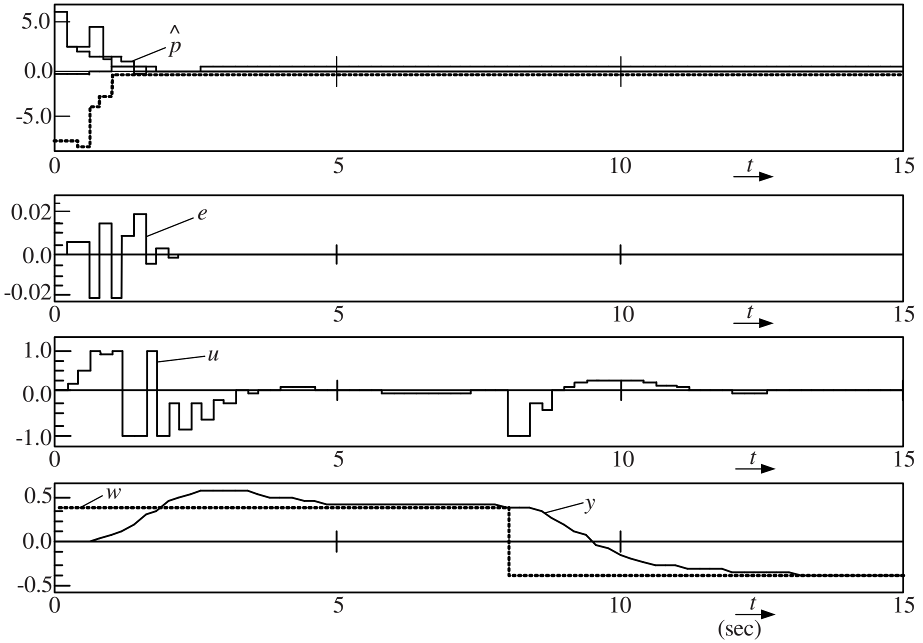

Fig. 12.6. Direct adaptive pole-placement control

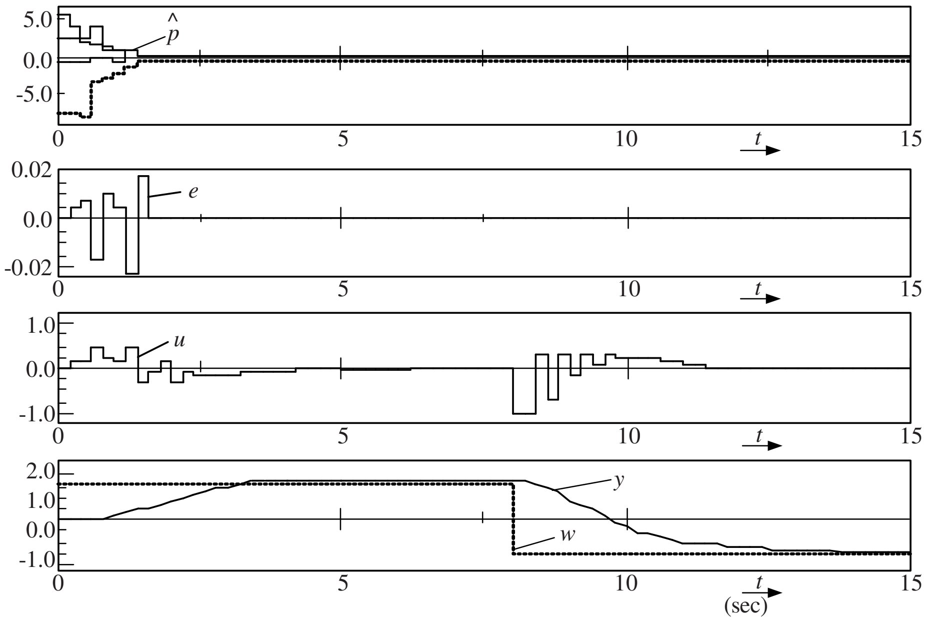

Fig. 12.7. Direct cautious adaptive pole-placement control

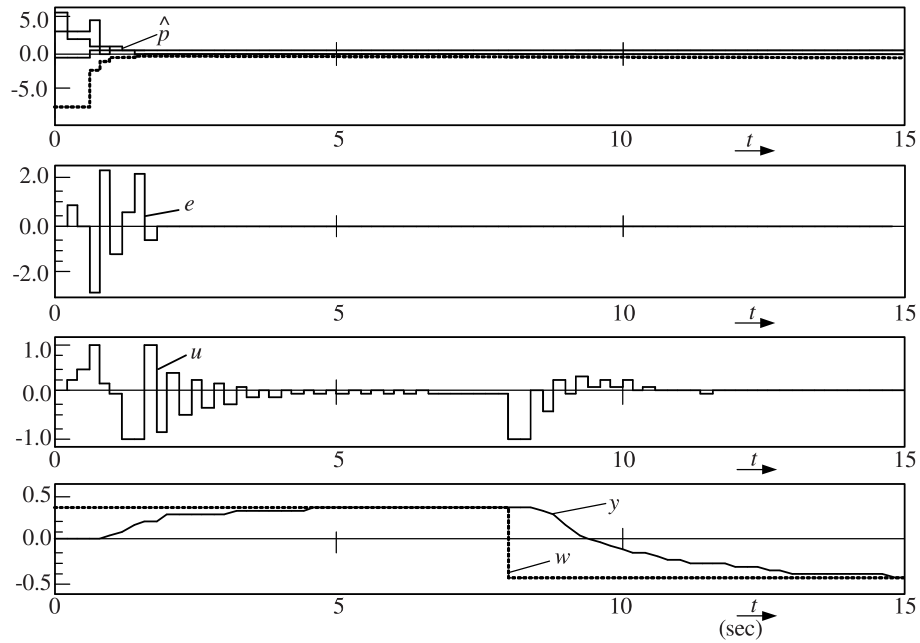

Fig. 12.8. Direct dual pole-placement control

It should be emphasized that the direct dual approach presented above may be extended to other types of adaptive pole-placement controllers, for example, to the direct adaptive pole-placement controller suggested by Elliott (1982), as discussed in Chapter 6. Contrary to innovational dual control (Milito et al., 1982), the proposed dual control according to eqs. (12.25) and (12.29) uses the parameter θ to define the amplitude of the excitation. This clear physical interpretation of the controller parameter θ makes direct dual control even more attractive for applications. The pole-zero placement system considered here is characterized by a small number of parameters to be estimated. The coefficients of $B_{\mathrm{m}}(z^{-1})$ and $A_{\mathrm{m}}(z^{-1})$ can be obtained easily from standard polynomials of the continuous-time case provided in literature (Unbehauen, 1985), and the selection of an appropriate sampling time.

13. ROBUSTNESS AGAINST UNMODELED EFFECTS AND SYSTEM STABILITY

Various engineering systems are described by complex dynamic models that include nonlinearities, time-varying parameters and high-order terms. Because of the difficulties connected with the structural identification and parameter estimation of such complex models, simplified and linearized models are commonly used for the controller design. The theory of robust adaptive control considers the problems of system synthesis based on simplified models taking into account various unmodeled effects, such as nonlinearities, time-varying parameters and dynamics of high order (Ortega and Tang, 1989; Koust et al., 1992; Lozano-Leal, 1989; Praly, 1983). Ortega and Tang (1989) gave a survey of the design of various robust adaptive controllers. Robust adaptive control can be based on different theoretical concepts. In the paper of Koust et al. (1992), the system uncertainty is described and bounded by the H^{∞}-norm. Lozano-Leal (1989) proposed another approach, where the uncertainty is represented by bounds that depend on the system state. A similar approach was considered by Praly (1983), where the uncertainty bounds depend dynamically on the system state.

In this chapter, an adaptive dual pole placement controller is applied together with a robust adaptation scheme to a dynamic plant described by an unknown nonlinear model. The considered approach is based on the robust adaptive pole-placement control (APPC) scheme (Praly, 1983). Stability of the suggested dual modification of the controller applied to a plant with unmodeled nonlinearity is proved. It is shown that the dual modification improves the performance of adaptive controllers. However, the problems of robustness of adaptive dual controllers and stability for the case of unmodeled nonlinearities have not been investigated up to now. To the best knowledge of the authors, this chapter presents the first stability analysis results for robust adaptive dual controllers. It is demonstrated that after the insertion of the dual controller the robust adaptive system maintains its stability (in the sense of boundedness of the input and output signals), if some assumptions about the nonlinear and unmodeled residuals of the plant model are modified. A strict stability analysis is complicated because of the nonlinear character of the dual controller. It will also be shown that in the case of no residuals in the plant model the closed-loop system converges to the model defined by the desired pole locations.

13.1. Description of the Plant with Unmodeled Effects

The original system is nonlinear and can be represented in the form

$$y(k+1) = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k) + f[\boldsymbol{m}(k)] + \bar{\xi}(k)\,, \tag{13.1}$$

where

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, pp. 148–159, 2004.

$$\boldsymbol{p}^{\mathrm{T}} \quad = [b_1 \dots b_n \;\vdots\; a_1 \dots a_n] = [b_1 \;\vdots\; \boldsymbol{p}_0^{\mathrm{T}}], \tag{13.2}$$

$$\begin{aligned}\boldsymbol{m}^{\mathrm{T}}(k) &= [u(k) \dots u(k-n+1) \;\vdots\; y(k) \dots y(k-n+1)] \\ &= [u(k) \;\vdots\; \boldsymbol{m}_0^{\mathrm{T}}(k)],\end{aligned} \tag{13.3}$$

$f[\boldsymbol{m}(k)]$ represents a nonlinear term and $\xi(k)$ includes disturbances and other unmodeled effects. Let us denote

$$\xi(k) = f[\boldsymbol{m}(k)] + \overline{\xi}(k) \tag{13.4}$$

and assume

$$\left|\overline{\xi}(k)\right| \le \eta_{\xi}\, s(k), \tag{13.5}$$

where

$$s(k) = \sigma\, s(k-1) + \left\|\boldsymbol{m}(k)\right\| + s, \tag{13.6}$$

$$0 < \sigma < 1, \;\; s > 0, \;\; \eta_{\xi} \ge 0. \tag{13.7}$$

Inequality (13.5) can be used for the description of various unmodeled effects, such as nonlinearities, time-varying parameters and high-order terms. The problem consists in designing an adaptive dual controller that provides system stability (boundedness of $y(k)$ and $u(k)$), for the considered plant with $\eta_{\xi} \ne 0$, and an exact desired closed-loop system behavior (pole placement determined by the polynomial $P(z^{-1})$) in the case of $\eta_{\xi} = 0$.

13.2. Design of the Dual Controller

13.2.1. Adaptive Pole-Placement Controller Based on the CE Assumption

For the design of an indirect APPC, a linear simplified plant model with $\xi(k) \equiv 0$ is considered:

$$Y(z) = \frac{B(z^{-1})}{A(z^{-1})} U(z), \tag{13.8}$$

where $U(z)$ and $Y(z)$ are the z-transforms of the control signal $u(k)$ and the output signal $y(k)$, k is the discrete time index, $A(z^{-1})$ and $B(z^{-1})$ are polynomials of the form

$$A(z^{-1}) = 1 + a_1 z^{-1} + \dots + a_n z^{-n}, \tag{13.9}$$

and

$$B(z^{-1}) = b_1 z^{-1} + \dots + b_n z^{-n}, \;\; b_1 \ne 0. \tag{13.10}$$

The system with a disturbance $\xi(k) \ne 0$, which represents noise, nonlinearities, parameter drift or unmodeled dynamics, will be considered in Section 13.3. The controller's input/output behavior is described similar to eq. (10.4a) by

$$L(z^{-1})U(z) = W(z) - H(z^{-1})Y(z)\,, \tag{13.11}$$

where

$$L(z^{-1}) = 1 + l_1 z^{-1} + \ldots + l_{n-1} z^{-(n-1)}\,, \tag{13.12}$$

$$H(z^{-1}) = h_0 + h_1 z^{-1} + \ldots + h_{n-1} z^{-(n-1)}\,, \tag{13.13}$$

and $W(z)$ is the z-transform of the reference signal $w(k)$. As described in Chapter 10, an integrator can be included easily in the controller, which would define a special structure of the polynomial according to eq. (13.12). For the simplification of the further considerations, and without loss of generality of the main results, a system without a prefilter is considered. The closed-loop system according to eqs. (13.8) and (13.11) is then described by

$$\begin{aligned} Y(z) &= \frac{B(z^{-1})}{L(z^{-1})A(z^{-1}) + H(z^{-1})B(z^{-1})} W(z) \\ &= \frac{B(z^{-1})}{P(z^{-1})} W(z)\,, \end{aligned} \tag{13.14}$$

with the characteristic polynomial

$$P(z^{-1}) = L(z^{-1})A(z^{-1}) + H(z^{-1})B(z^{-1})\,, \tag{13.15}$$

$$P(z^{-1}) = 1 + p_1 z^{-1} + \ldots + p_{2n-1} z^{-(2n-1)}\,, \tag{13.16}$$

which is stable. This polynomial represents the desired pole locations of the system. An observer polynomial is assumed to be included in $P(z^{-1})$. The parameters of the controller polynomials $L(z^{-1})$ and $H(z^{-1})$ can be determined by solving the Diophantine equation (13.15).

The recursive least squares (RLS) algorithm is applied to estimate the plant parameters of eq. (13.1) with $\xi(k) \equiv 0$, and for the calculation of the covariance matrix:

$$\hat{\boldsymbol{p}}(k) = \mathrm{E}\left\{\boldsymbol{p}(k) \middle| \Im_k\right\}, \tag{13.17}$$

$$\boldsymbol{P}(k) = \mathrm{E}\left\{[\boldsymbol{p} - \hat{\boldsymbol{p}}(k)][\boldsymbol{p} - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}} \middle| \Im_k\right\}, \tag{13.18}$$

$$\Im_k = \left\{ y(0) \ldots y(k) \vdots u(0) \ldots u(k-1) \right\},\ \Im_0 = \left\{ y(0) \right\}, \tag{13.19}$$

where $\Im_k$ is the set of the plant inputs and outputs available at time k and the adaptation error has the form

$$e(k+1) = y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)\,. \tag{13.20}$$

The parameter estimation algorithm of the linear part of the plant will be considered in Section 13.3.

The estimates of the controller polynomials, $\hat{L}_k(z^{-1})$ and $\hat{H}_k(z^{-1})$, at time k, have to be computed from the estimates of the plant polynomials $\hat{A}_k(z^{-1})$ and $\hat{B}_k(z^{-1})$ at the same time by solving eq. (13.15) in real-time. The non dual (ND) controller based on the CE assumption can be realized through eq. (13.11) and the solution of eq. (13.15) and transforming the solution into the time-domain

$$u(k) = u_{\mathrm{ND}}(k) = w(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_0(k), \tag{13.21}$$

where

$$\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k) = [\hat{l}_1(k), ..., \hat{l}_{n-1}(k), \hat{h}_0(k), ..., \hat{h}_{n-1}(k)], \tag{13.22}$$

and $\boldsymbol{m}_0(k)$ as given by eq. (13.3), furthermore,

$$\bar{\boldsymbol{p}}_1^{\mathrm{T}} = [1, l_1, ..., l_{n-1}, h_0, ..., h_{n-1}] = [1 \,\vdots\, \boldsymbol{p}_1^{\mathrm{T}}], \tag{13.23}$$

and $\hat{l}_i(k)$ and $\hat{h}_i(k)$ are the estimates of the parameters of the polynomial described by eqs. (13.12) and (13.13).

13.2.2. Incorporation of the Dual Controller

In this section, the adaptive dual controller is derived in a similar way as described in Section 10.2. If the reader is familiar with the derivation of the adaptive dual controller presented in Section 10.2, he may proceed to Section 13.3.

According to the bicriterial approach, let us define the nominal output $y_{\mathrm{n}}(k+1)$ of the system as the response to its input signal as in eq. (13.21)

$$u_{\mathrm{n}}(k) = w(k) - \boldsymbol{p}_1^{\mathrm{T}}\boldsymbol{m}_0(k). \tag{13.24}$$

Therefore, the signal $u_{\mathrm{n}}(k)$ is generated by the unknown controller that corresponds to the unknown plant model of eq. (13.8) and the desired system pole locations according to eq. (13.15) when no disturbances act upon the system ($\xi(k) \equiv 0$). From eq. (13.8) introducing eqs. (13.2) and (13.3), the dependence of $y_{\mathrm{n}}(k+1)$ on $u_{\mathrm{n}}(k)$ takes the form

$$y_{\mathrm{n}}(k+1) = b_1 u_{\mathrm{n}}(k) + \boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k). \tag{13.25}$$

The control performance can be improved if the controller brings the system output as close as possible to the nominal output, which is attained only for the controller with the known parameters after full noise compensation. In accordance with this and the bicriterial approach, the two cost functions according to eqs. (9.26) and (9.22)

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[\hat{y}_{\mathrm{n}}(k+1) - y(k+1)]^2 \middle| \Im_k\right\}, \tag{13.26}$$

and

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2 \middle| \Im_k\right\} \tag{13.27}$$

are used for the derivation of the control law.

The first cost function, eq. (13.26), is used to minimize the deviation between the system output and its estimate $\hat{y}_{\mathrm{n}}(k+1)$ for the unknown nominal output. The simplified dual version of the APPC with indirect adaptation will be obtained here by using the CE assumption for the computation of the nominal output $y_{\mathrm{n}}(k+1)$ according to eq. (13.25). Thus, for the ND controller, based on the CE assumption, the nominal system output is given by

$$\hat{y}_{\mathrm{n}}(k+1)=\hat{b}_1(k)u_{\mathrm{ND}}(k)+\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\,. \tag{13.28}$$

It should be noted that in eqs. (9.26) and (10.60) $u_{\mathrm{ND}}(k)=u_{\mathrm{CE}}(k)$. The second cost function according to eq. (13.27) is used for the acceleration of the parameter estimation by means of increasing the prediction error according to eq. (13.20). In other words, these two cost functions represent both goals of dual control: (i) tracking the system output and (ii) accelerating the estimation for future control improvement. After solving the bicriterial optimization problem given by eqs. (13.26) and (13.27) for the system described by eqs. (13.8) and (13.24), using the constraints described below, the dual APPC will be obtained in the form

$$u(k)=\arg\min_{u(k)\in\Omega_k} J_k^{\mathrm{a}}\quad, \tag{13.29}$$

where

$$\Omega_k=[u_{\mathrm{c}}(k)-\theta(k);u_{\mathrm{c}}(k)+\theta(k)]\,, \tag{13.30}$$

$$\theta(k)=f_k\{\boldsymbol{P}(k)\}, \tag{13.31}$$

$$u_{\mathrm{c}}(k)=\arg\min_{u(k)} J_k^{\mathrm{c}}, \tag{13.32}$$

$u_{\mathrm{c}}(k)$ is the cautious control that is obtained after minimization of the cost function according to eq. (13.20), $f_k\{\cdot\}$ is a differentiable, bounded, positive scalar function of the matrix $\boldsymbol{P}(k)$ such that $\lim_{k\to\infty}\theta(k)=0$ if $\eta_\xi=0$. The second performance index, eq. (13.27), is minimized over the domain Ω_k, which is symmetrically located around the cautious control according to eq. (13.30). Therefore, the size of this domain defines the amplitude of the excitation given by eq. (13.31).

After substituting eqs. (13.1) and (13.28) with $\xi(k)=0$ into eq. (13.26) and taking the expectation (similar as in Appendix D), we finally arrive at

$$\begin{aligned}J_k^{\mathrm{c}}&=\boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{p_0}(k)\boldsymbol{m}_0(k)+\hat{b}_1^2(k)u_{\mathrm{ND}}^2(k)+[\hat{b}_1^2(k)+p_{b_1}(k)]u^2(k)\\&\quad-2\hat{b}_1^2(k)u_{\mathrm{ND}}(k)u(k)+2\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k)\,,\end{aligned} \tag{13.33}$$

where the covariance matrix according to eq. (13.18) has the structure

$$\boldsymbol{P}(k)=\begin{bmatrix}p_{b_1}(k) & \boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)\\ \boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k) & \boldsymbol{P}_{p_0}(k)\end{bmatrix},\ \boldsymbol{p}_{b_1p}^{\mathrm{T}}(k)=[p_{b_1}(k)\vdots\,\boldsymbol{p}_{b_1p_0}^{\mathrm{T}}(k)]\,. \tag{13.34}$$

The minimization of eq. (13.33) with respect to $u(k)$ provides the cautious control law

$$u_{\rm c}(k)=\frac{\hat{b}_1^2(k)u_{\rm ND}(k)-\boldsymbol{p}_{b_1p_0}^{\rm T}(k)\boldsymbol{m}_0(k)}{\hat{b}_1^2(k)+p_{b_1}(k)}\,. \tag{13.35}$$

Minimization of eq. (13.29) over the domain Ω_k described by eq. (13.30) leads to

$$\begin{aligned} u(k) &= u_{\rm c}(k)+\theta(k)\operatorname{sgn}\left\{J_k^{\rm a}[u_{\rm c}(k)-\theta(k)]-J_k^{\rm a}[u_{\rm c}(k)+\theta(k)]\right\} \\ &= u_{\rm c}(k)+u_{\rm a}(k)\,, \end{aligned} \tag{13.36}$$

where $u_{\rm a}(k)$ is the optimal excitation of the dual controller and

$$\operatorname{sgn}\{\tilde{\alpha}\}=\begin{cases} 1, & \text{if } \tilde{\alpha}\ge 0, \\ -1, & \text{if } \tilde{\alpha}<0. \end{cases} \tag{13.37}$$

At the same time, eq. (13.27), after insertion of eq. (13.1) with $\xi(k)=0$, leads to

$$\begin{aligned} J_k^{\rm a}(u(k)) &= -{\rm E}\left\{[(\boldsymbol{p}(k)-\hat{\boldsymbol{p}}(k))^{\rm T}\boldsymbol{m}(k)]^2\middle|\Im_k\right\} \\ &= -{\rm E}\left\{[(b_1(k)-\hat{b}_1(k))u(k)+[\boldsymbol{p}_0(k)-\hat{\boldsymbol{p}}_0(k)]^{\rm T}\boldsymbol{m}_0(k)]^2\middle|\Im_k\right\}+\bar{c}_2(k) \\ &= -p_{b_1}(k)u^2(k)-2\boldsymbol{p}_{b_1p_0}^{\rm T}(k)\boldsymbol{m}_0(k)u(k)+\bar{c}_3(k)\,, \end{aligned} \tag{13.38}$$

where $\bar{c}_2(k)$ and $\bar{c}_3(k)$ are terms that do not contain $u(k)$.

For the partial expression $\{\cdot\}$ of eq. (13.36) inserting of eq. (13.38) directly yields that

$$\begin{aligned} & J_k^{\rm a}[u_{\rm c}(k)-\theta(k))-J_k^{\rm a}(u_{\rm c}(k)+\theta(k)] \\ &\qquad = 4p_{b_1}(k)u_c(k)\theta(k)+4\boldsymbol{p}_{b_1p_0}^{\rm T}(k)\boldsymbol{m}_0(k)\theta(k)\,. \end{aligned} \tag{13.39}$$

Substitution of the last equation into eq. (13.36) finally gives the dual control law

$$u(k)=u_{\rm c}(k)+\theta(k)\operatorname{sgn}\left\{p_{b_1}(k)u_{\rm c}(k)+\boldsymbol{p}_{b_1p_0}^{\rm T}(k)\boldsymbol{m}_0(k)\right\}=u_{\rm c}(k)+u_{\rm a}(k)\,. \tag{13.40}$$

Thus, the dual APPC is determined by eqs. (13.21), (13.31), (13.35) and (13.40).

The dual APPC above derived can be combined easily with the conventional ND controller based on the CE assumption. This property of the dual controller is attractive for practical applications. Standard adaptive ND controllers can be modified easily to their simplified dual versions using the suggested approach. The schemes of the standard CE adaptive controller and its dual modification are illustrated in Figures 9.1 and 9.2, respectively.

By application of eqs. (13.34), (13.23) and (13.3) it is possible to represent the dual controller given by eqs. (13.21), (13.35) and (13.40) in the form

$$u(k)=u_{\rm ND}(k)+\alpha(k)\,,$$

$$\alpha(k)=u(k)-u_{\rm ND}(k)=-\frac{1}{\hat{b}_1^2(k)}\boldsymbol{p}_{b_1p}^{\rm T}(k)\boldsymbol{m}(k)+\frac{1}{\hat{b}_1^2(k)}[\hat{b}_1^2(k)+p_{b_1}(k)]\,u_{\rm a}(k),$$

and

$$\begin{aligned}\hat{\bar{\boldsymbol{p}}}_1^{\mathrm{T}}(k)\boldsymbol{m}(k) &= w(k) - \frac{1}{\hat{b}_1^2(k)}\boldsymbol{p}_{b_1 p}^{\mathrm{T}}(k)\boldsymbol{m}(k) + \frac{1}{\hat{b}_1^2(k)}[\hat{b}_1^2(k) + p_{b_1}(k)]\, u_{\mathrm{a}}(k) \\ &= w(k) + \alpha(k)\,,\end{aligned} \tag{13.41}$$

where $\alpha(k)$ is the correction of the ND controller introduced by the dual controller. The notation $\boldsymbol{m}^{\mathrm{T}}(k) = [u_{\mathrm{ND}}(k) \,\vdots\, \boldsymbol{m}_0^{\mathrm{T}}(k)]$ is used here instead of eq. (13.3).

13.3. Robustness against Unmodeled Nonlinearity and System Stability

The main purpose of this section is to introduce an adaptation algorithm for calculating the matrix $\boldsymbol{P}(k)$ and the vector $\hat{\boldsymbol{p}}(k)$ such that the closed-loop system with a dual controller, as derived in Section 13.2, remains stable even in the presence of nonlinearities and unmodeled dynamics described by eqs. (13.1) to (13.6). In other words, it is shown that after the insertion of a dual controller the adaptive pole-placement control system maintains its robust properties. It turns out that the robust adaptation scheme proposed by Praly (1983) is applicable for the suggested dual control system and guarantees that the input and output signals $u(k)$ and $y(k)$ are uniformly bounded if some special assumptions hold. Moreover, the system asymptotically approaches the closed-loop behavior described by eq. (13.14) in the absence of unmodeled or nonlinear residuals in eq. (13.1), i.e., as $\eta_\xi \to 0$.

13.3.1. Adaptation Scheme

Introduction of the matrix

$$\boldsymbol{A}[\hat{\boldsymbol{p}}(k)] = \left[\begin{array}{cccccccc} 1 & & & & 0 & & & \\ \hat{a}_1 & 1 & \boldsymbol{0} & & \hat{b}_1 & & \boldsymbol{0} & \\ \hat{a}_2 & \hat{a}_1 & \ddots & & \vdots & \ddots & & 0 \\ \vdots & & \ddots & 1 & & & & \hat{b}_1 \\ \hat{a}_n & \dots & & \hat{a}_1 & \hat{b}_n & & & \vdots \\ & \ddots & & \vdots & & \ddots & & \\ & \boldsymbol{0} & & & \boldsymbol{0} & & & \\ & & & \hat{a}_n & & & & \hat{b}_n \end{array}\right] \tag{13.42}$$

and the vector

$$\boldsymbol{\pi}^{\mathrm{T}} = [1 \;\; p_1 \dots p_{2n-1}]\,, \tag{13.43}$$

allows the Diophantine equation (13.15) to be written as

$$\boldsymbol{A}[\hat{\boldsymbol{p}}(k)]\, \hat{\bar{\boldsymbol{p}}}_1(k) = \boldsymbol{\pi}\,, \tag{13.44}$$

where $\hat{\bar{\boldsymbol{p}}}_1(k)$ is given by the estimate of eq. (13.23). It is assumed that

$$\left|\det \boldsymbol{A}[\hat{\boldsymbol{p}}(k)]\right| \geq \delta \text{, i.e. not singular,} \tag{13.45}$$

where $\delta > 0$ is a small value and

$$\hat{b}_1(k) \neq 0 \,. \tag{13.46}$$

Applying the approach to the robust adaptive control problem suggested by Praly (1983) for APPC systems, the following adaptation scheme is obtained:

$$e(k) = y(k) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k-1)\boldsymbol{m}(k-1)\,, \tag{13.47}$$

$$g(k) = \frac{\overline{\alpha}(k)}{\mu\, s^2(k) + \mathbf{m}^{\mathrm{T}}(k-1)\mathbf{P}(k-1)\mathbf{m}(k-1)}\,, \tag{13.48}$$

$$\hat{\bar{\boldsymbol{p}}}(k) = \hat{\boldsymbol{p}}(k-1) + g(k)\boldsymbol{P}(k-1)\boldsymbol{m}(k-1)e(k)\,, \tag{13.49}$$

$$\overline{\boldsymbol{P}}(k) = \boldsymbol{P}(k-1) - g(k)\boldsymbol{P}(k-1)\boldsymbol{m}(k-1)\boldsymbol{m}^{\mathrm{T}}(k-1)\boldsymbol{P}(k-1)\,, \tag{13.50}$$

$$\hat{\boldsymbol{p}}(k) = \hat{\boldsymbol{p}}(0) + \min\left\{1, \frac{\rho(k)}{\left\|\hat{\bar{\boldsymbol{p}}}(k) - \hat{\boldsymbol{p}}(0)\right\|}\right\}[\hat{\bar{\boldsymbol{p}}}(k) - \hat{\boldsymbol{p}}(0)]\,, \tag{13.51}$$

$$\boldsymbol{P}(k) \geq \overline{\boldsymbol{P}}(k)\,, \tag{13.52}$$

$$\hat{\bar{\boldsymbol{p}}}_1(k) = \boldsymbol{A}^{-1}(\hat{\boldsymbol{p}}(k))\pi\,, \tag{13.53}$$

where the initial estimate $\hat{\boldsymbol{p}}(0)$ and a parameter vector $\boldsymbol{p}$ satisfying the model according to eqs. (13.1) to (13.6) are bounded by the inequality

$$\left\|\boldsymbol{p} - \hat{\boldsymbol{p}}(0)\right\| \leq \rho_0\,. \tag{13.54}$$

The smallest and largest eigenvalues $\lambda_{\min}$ and $\lambda_{\max}$ of $\boldsymbol{P}(k)$ satisfy the inequalities

$$0 < \lambda_0 \leq \lambda_{\min} \leq \lambda_{\max} \leq \lambda_1\,, \tag{13.55}$$

and $\overline{\alpha}(k)$, $\rho(k)$ and μ are chosen such that

$$0 < \overline{\alpha} \leq \overline{\alpha}(k) \leq 1\,, \tag{13.56}$$

$$\max\left\{\frac{\lambda_1}{\lambda_0}, \rho(k-1) - \tau\left\|\hat{\bar{\boldsymbol{p}}}(k) - \hat{\boldsymbol{p}}(k-1)\right\|\right\} \leq \rho(k) \leq \rho\,, \tag{13.57}$$

$$\tau > 0\,,\ \mu > 0\,. \tag{13.58}$$

The constants τ, μ and $\overline{\alpha}$ should be chosen beforehand. To satisfy inequality (13.52), $\boldsymbol{P}(k)$ can, for instance, be chosen according to Praly (1983) such that

$$\boldsymbol{P}(k) = \beta\overline{\boldsymbol{P}}(k) + (1-\beta)\lambda_1\boldsymbol{I},\ 0 \leq \beta \leq 1\,. \tag{13.59}$$

13.3.2. Stability of the Adaptive Control System

The stability proof of the suggested robust adaptive controller applied to a nonlinear plant is based on the approaches that are standard by now. However, these approaches cannot be applied directly to the adaptive dual controller without a special investigation of the influence on the stability caused by the dual modification (the term $\alpha(k)$ of eq. (13.41)). In the present section it is proven that the system with the inserted dual controller (Fig. 9.2) remains robust and stable. First, the dependence of the term $\alpha(k)$ on the adaptation error according to eq. (13.20) is investigated. Let us state the following results:

Lemma 13.1 (Praly, 1983). There exist positive bounds M_p, $\overline{M}_p$, M_e, M_{p_1} and positive gains L_p, L_e and L_{p_1} for the adaptive control system described by eqs. (13.46) to (13.53), which are independent of η_ξ, such that the following inequalities hold:

$$\left\|\hat{\boldsymbol{p}}(k)\right\| \le M_p, \tag{13.60}$$

$$\sum_{k=q+1}^{q+m} \frac{|e(k)|}{s(k)} \le \sqrt{m} M_e + m L_e \eta_\xi, \ \forall(q,m), \tag{13.61}$$

$$\sum_{k=q+1}^{q+m} \left\|\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-1)\right\| \le \sqrt{m}\ \overline{M}_p + m L_p \eta_\xi, \ \forall(q,m), \tag{13.62}$$

$$\left\|\hat{\boldsymbol{p}}_1(k)\right\| \le M_{p_1}, \tag{13.63}$$

$$\left\|\hat{\bar{\boldsymbol{p}}}_1(k) - \hat{\bar{\boldsymbol{p}}}_1(k-i)\right\| \le L_{p_1} \left\|\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-i)\right\|, \ \forall i \le 1, \tag{13.64}$$

where

$$L_e^2 = 1 + \frac{\lambda_1}{\mu}, \tag{13.65}$$

$$L_p^2 = (2+\tau)^2 \frac{\lambda_1^2}{(\mu + \lambda_1)\mu}. \tag{13.66}$$

The proof is given in Appendix B.

Lemma 13.2. There exist positive numbers $L_{\alpha 1}, L_{\alpha 2}$, and $M_{\alpha 2}$ for any M_p such that

$$M_p \left\|\alpha(k)\right\| \le L_{\alpha 1}\left\|\Delta(k)\right\| + L_{\alpha 2}\left\|u_a(k)\right\| \le L_{\alpha 1}\left\|\Delta(k)\right\| + M_{\alpha 2}, \tag{13.67}$$

where

$$\Delta^{\mathrm{T}}(k) = [e(k) \ldots e(k-n+1)]. \tag{13.68}$$

The proof is also given in Appendix B.

According to Praly (1983), the relations between the usual Euclidean norm $\|\cdot\|$ and another equivalent norm $\|\cdot\|_\eta$ are introduced as

$$\gamma_1\|\cdot\| \le \|\cdot\|_\eta \le \gamma_2\|\cdot\|, \quad \gamma_1 > 0,\ \gamma_2 > 0,\ \gamma = \frac{\gamma_2}{\gamma_1}, \tag{13.69}$$

and the system state vector

$$\boldsymbol{x}(k) = [\,y(k) \ldots y(k-2n+2)\ u(k) \ldots u(k-2n+2)]^{\mathrm{T}}. \tag{13.70}$$

Let us represent the closed-loop adaptive dual control system in state-space form using the state vector of eq. (13.70). After multiplication of the Diophantine equation (13.44) by the vectors $[y(k)\ \ y(k\text{-}1)\ \ldots]^{\mathrm{T}}$ and $[u(k)\ u(k\text{-}1)\ \ldots]^{\mathrm{T}}$, we have

$$y(k) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k-1) + \sum_{i=1}^{n-1} \hat{l}_i(k)\,[\,y(k-i) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k-i)]$$

$$+\sum_{i=1}^{n} \hat{b}_i(k)\hat{\bar{\boldsymbol{p}}}_1^{\mathrm{T}}(k)\boldsymbol{m}(k-i) = y(k) + \sum_{i=1}^{2n-1} p_i(k)\,y(k-i)\,, \tag{13.71}$$

$$-\sum_{i=0}^{n-1} \hat{h}_i(k)\,[\,y(k-i) - \boldsymbol{p}^{\mathrm{T}}(k)\boldsymbol{m}(k-i-1)] + \sum_{i=0}^{n} \hat{a}_i(k)\hat{\bar{\boldsymbol{p}}}_1^{\mathrm{T}}(k)\boldsymbol{m}(k+1-i)$$

$$= u(k) + \sum_{i=1}^{2n-1} p_i(k)u(k-i)\,, \tag{13.72}$$

and the insertion of eqs. (13.41) and (13.47) leads to

$$e(k) + \sum_{i=1}^{n-1} \hat{l}_i(k)e(k-i) + \sum_{i=0}^{n} \hat{l}_i(k)\,[\hat{\boldsymbol{p}}(k-i-1) - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}}\boldsymbol{m}(k-i-1)$$

$$+\sum_{i=1}^{n}[\hat{b}_i(k)w(k-i) - \alpha(k-i)] + \sum_{i=1}^{n} \hat{b}_i(k)\,[\hat{\bar{\boldsymbol{p}}}_1(k) - \hat{\bar{\boldsymbol{p}}}_1(k-i)]^{\mathrm{T}}\boldsymbol{m}(k-i)$$

$$= y(k) + \sum_{i=1}^{2n-1} p_i(k)\,y(k-i)\,, \tag{13.73}$$

$$-\sum_{i=0}^{n-1} \hat{h}_i(k)e(k-i) + \sum_{i=0}^{n-1} \hat{h}_i(k)[\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-i-1)]^{\mathrm{T}}\boldsymbol{m}(k-i-1)$$

$$+\sum_{i=0}^{n}[\hat{a}_i(k)w(k-i) - \alpha(k-i)] + \sum_{i=0}^{n} \hat{a}_i(k)[\hat{\bar{\boldsymbol{p}}}_1(k) - \hat{\bar{\boldsymbol{p}}}_1(k-i)]^{\mathrm{T}}\boldsymbol{m}(k-i)$$

$$= u(k) + \sum_{i=1}^{2n-1} p_i(k)u(k-i)\,. \tag{13.74}$$

The parameters $\hat{a}_i(k)$ and the closed-loop adaptive system can be represented in the following state space form:

$$\boldsymbol{x}(k+1) = (\boldsymbol{F} + \Delta\boldsymbol{F}_k)\boldsymbol{x}(k) + \overline{\boldsymbol{P}}_k \Delta(k) + \boldsymbol{P}_k[\overline{\boldsymbol{w}}(k) + \overline{\overline{\boldsymbol{\alpha}}}(k)], \tag{13.75}$$

where

$$\overline{\boldsymbol{w}}(k) = [w(k)...w(k-n+1)]^{\mathrm{T}}, \tag{13.76}$$

and

$$\overline{\overline{\boldsymbol{\alpha}}}(k) = [\alpha(k)...\alpha(k-n+1)]^{\mathrm{T}}. \tag{13.77}$$

The term $\boldsymbol{F}$ is a companion matrix with the characteristic polynomial $P(z^{-1})$ as in eq. (13.16), and $\Delta\boldsymbol{F}_k$ incorporates the differences

$$\begin{aligned}
&\hat{l}_i(k)\,[\hat{\boldsymbol{p}}(k-i-1) - \hat{\boldsymbol{p}}(k)],\\
&\hat{b}_i(k)\,[\hat{\overline{\boldsymbol{p}}}_1(k) - \hat{\overline{\boldsymbol{p}}}_1(k-i)],\\
&\hat{h}_i(k)\,[\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-i-1)],\\
&\hat{a}_i(k)\,[\hat{\overline{\boldsymbol{p}}}_1(k) - \hat{\overline{\boldsymbol{p}}}_1(k-i)],\ i = 1,...,n.
\end{aligned} \tag{13.78}$$

The matrix $\overline{\boldsymbol{P}}_k$ consists of parameter estimates $\hat{l}_i(k)$ and $\hat{h}_i(k)$; $\boldsymbol{P}_k$ includes the estimates $\hat{a}_i(k)$ and $\hat{b}_i(k)$.

With the strict stability of $P(z^{-1})$ there exists a norm $\|\cdot\|_\eta$, equivalent to inequalities (13.69), such that from eq. (13.75) it follows that

$$\|\boldsymbol{x}(k+1)\|_\eta \le \zeta\|\boldsymbol{x}(k)\|_\eta + \left\|\Delta\boldsymbol{F}_k\boldsymbol{x}(k) + \overline{\boldsymbol{P}}_k(k)\Delta(k) + \boldsymbol{P}_k(\overline{w}(k) + \overline{\overline{\alpha}}(k))\right\|_\eta. \tag{13.79}$$

Let us introduce the relations between the parameters of the robust adaptive dual control system and the bounds defined above

$$[(M_{p_1} + L_{p_1}M_p)L_p + \Pi\ L_e(M_{p_1} + L_{\alpha 1})]\,\eta_\xi < \frac{(1-\zeta)(1-\sigma)}{\gamma n^2}, \tag{13.80}$$

where

$$\Pi = \frac{1-\sigma^n}{\sigma^{n-1}(1-\zeta)(1-\sigma)\gamma n^2}. \tag{13.81}$$

The main results for the stability of the adaptive control system can be formulated now in the following theorem:

Theorem 13.1. If the assumptions given in eqs. (13.5) to (13.7), (13.45), (13.46), (13.54), (13.55) and the condition according to eq. (13.80) are valid for the system described by eqs. (13.1), (13.4), (13.41) to (13.53), then the plant input and output sequences $u(k)$ and $y(k)$, respectively, are bounded, and if $\eta_\xi = 0$, then

$$\lim_{k\to\infty}\left[\sum_{i=1}^{n} b_i w(k-i) - \left(y(k) + \sum_{i=1}^{2n-1} p_i y(k-i) \right)\right] = 0\,, \tag{13.82}$$

so that the closed-loop system is described by eq. (13.14) after finishing the adaptation.

Proof: see Appendix B.

According to the results of Theorem 13.1 and Lemmas 13.1, 13.2 and the relations (13.80) and (13.81), the polynomial $P(z^{-1})$ should be selected with the smallest possible spectral radius ζ, furthermore parameter λ_1 for matrix $\boldsymbol{P}(k)$ should be selected smaller and parameter μ larger for the improvement of the robustness of the suggested approach. At the same time, a small λ_1 guarantees a small value of $L_{\alpha 1}$ (see the proof of the Lemma 13.2 in Appendix B), which is important for the robustness of the dual controller. It should be noted that a small λ_1 reduces the adaptation ability of the algorithm and a large μ makes the adaptation slow. Since the dual controller speeds up the adaptation, the parameter μ can be selected large for the considered modified system without decreasing the adaptation speed of the adaptive dual controller. It should also be noted that the condition of inequality (13.80) differs, from the analogous one suggested by Praly (1983), by the term $L_{\alpha 1}$ which represents the influence of the dual controller.

Here, the stability problem of a robust adaptive dual controller for systems with nonlinearities has been considered. It has been demonstrated that the stability of the closed-loop system under modified conditions (see eq. (13.80)) for the parameters of the adaptive dual control law, unmodeled plant nonlinearity and dynamics, is guaranteed. It has been shown that the convergence of the closed-loop system to the desired behavior, determined by the pole locations, is provided by the suggested adaptive dual controller for the case that no unmodeled residuals are present.

14. DUAL MODIFICATION OF PREDICTIVE ADAPTIVE CONTROLLERS

14.1. Model Algorithmic Control (MAC)

The Model Algorithmic Control (MAC) approach was suggested by Richalet et al. (1976). Comparative studies of various predictive controllers including MAC can be found in the contributions of De Keyser et al. (1988) and Krämer (1992). Below, the dual version of MAC will be derived using the bicriterial approach.

14.1.1. Modelling of the Plant and Parameter Estimation

Richalet et al. (1976) considered the finite impulse response model of a stable plant

$$y(k+\ell)=\sum_{j=1}^{n} g(j)u(k+\ell-j)+\xi(k+\ell) \tag{14.1}$$

where $g(j)$ represents samples of the finite impulse response, n is the model order, and ℓ is some discrete time shift. Eq. (14.1) can be expressed in vector form as

$$y(k+\ell)=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{m}(k)+\xi(k+\ell)=b_1 u(k)+\boldsymbol{p}_0^{\mathrm{T}}\boldsymbol{m}_0(k)+\xi(k+\ell), \tag{14.2}$$

where

$$\boldsymbol{p}^{\mathrm{T}}=[b_1\ b_2 \ldots b_n]=[b_1 \vdots \boldsymbol{p}_0^{\mathrm{T}}], \tag{14.3}$$

and

$$\boldsymbol{m}^{\mathrm{T}}(k)=[u(k)\ u(k-1)\ \ldots\ u(k-n)]=[u(k) \vdots \boldsymbol{m}_0^{\mathrm{T}}(k)]. \tag{14.4}$$

The RLS algorithm can be used for parameter estimation of the model according to eq. (14.2)

$$\hat{\boldsymbol{p}}(k+1)=\hat{\boldsymbol{p}}(k)+\boldsymbol{q}(k+1)\ [y(k+1)-\hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)], \tag{14.5}$$

$$\boldsymbol{q}(k+1)=\boldsymbol{P}(k)\boldsymbol{m}(k)[\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}(k)+\sigma_\xi^2]^{-1}, \tag{14.6}$$

and

$$\boldsymbol{P}(k+1)=\boldsymbol{P}(k)-\boldsymbol{q}(k+1)\boldsymbol{m}^{\mathrm{T}}(k)\boldsymbol{P}(k), \tag{14.7}$$

where

$$\boldsymbol{P}(k)=\mathrm{E}\left\{[\boldsymbol{p}-\hat{\boldsymbol{p}}(k)][(\boldsymbol{p}-\hat{\boldsymbol{p}}(k)]^{\mathrm{T}}\middle|\Im_k\right\}=\begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1 p_0}^{T}(k) & \boldsymbol{P}_{p_0}(k)\end{bmatrix}, \tag{14.8}$$

$\boldsymbol{P}(0)$ is assumed to be known and $\Im_k$ is defined by eq. (4.12).

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14.1.2. Cautious and Dual MAC

Following De Keyser et al. (1988), let us introduce a vector of the internal model outputs as

$$\boldsymbol{y}_{\text{im}}(k+1)=\begin{bmatrix} y_{\text{im}}(k+1) \\ y_{\text{im}}(k+2) \\ \vdots \\ y_{\text{im}}(k+l) \end{bmatrix}=\begin{bmatrix} u(k-1) & u(k-1) & u(k-2) & u(k-3) & \cdots & u(k-n+1) \\ u(k-1) & u(k-1) & u(k-1) & u(k-2) & \cdots & u(k-n+2) \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ u(k-1) & u(k-1) & u(k-1) & u(k-1) & \cdots & u(k-n+l) \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$+\begin{bmatrix} \xi(k+1) \\ \xi(k+2) \\ \vdots \\ \xi(k+l) \end{bmatrix}=\boldsymbol{U}(k)\boldsymbol{p}+\boldsymbol{\Xi}(k+1)\,, \tag{14.9}$$

and the following vectors, matrix and scalars, respectively:

$$\boldsymbol{w}^{\text{T}}(k+1)=[w(k+1)\ w(k+2)...w(k+l)]\,, \tag{14.10}$$

$$\boldsymbol{e}^{\text{T}}(k+1)=[e(k+1)\ e(k+2)...e(k+l)]\,, \tag{14.11}$$

$$u(k-1+i)=u(k-1)+\Delta u(k-1+i),\ \ i=1,2,...,l\,, \tag{14.12}$$

$$\Delta\boldsymbol{u}^{\text{T}}(k)=[\Delta u(k)\ \Delta u(k+1)\ ...\Delta u(k+l-1)]\,, \tag{14.13}$$

and

$$\boldsymbol{B}=\begin{bmatrix} b_1 & \cdots & \cdots & \boldsymbol{0} \\ b_2 & b_1 & \iddots & \vdots \\ \vdots & \vdots & & \vdots \\ b_l & b_{l-1} & \cdots & b_1 \end{bmatrix}, \tag{14.14}$$

where the prediction error $\boldsymbol{e}(k+1)$ can be described by

$$\boldsymbol{e}(k+1)=[\boldsymbol{w}(k+1)-\boldsymbol{y}_{\text{im}}(k+1)]-\boldsymbol{B}\Delta\boldsymbol{u}\,. \tag{14.15}$$

The cost function for the cautious control is given by

$$J_k^{\text{c}}=\text{E}\left\{\boldsymbol{e}^{\text{T}}(k+1)\boldsymbol{e}(k+1)+r\Delta\boldsymbol{u}^{\text{T}}(k)\Delta\boldsymbol{u}(k)\middle|\Im_k\right\}, \tag{14.16}$$

where $r>0$ is a scalar parameter. Substituting eq. (14.15) into eq. (14.16) we obtain

$$\begin{aligned} J_k^{\text{c}}&=\text{E}\left\{[\boldsymbol{w}(k+1)-\boldsymbol{y}_{\text{im}}(k+1)]^{\text{T}}[\boldsymbol{w}(k+1)-\boldsymbol{y}_{\text{im}}(k+1)]\middle|\Im_k\right\} \\ &+\Delta\boldsymbol{u}^{\text{T}}(k)(\hat{\boldsymbol{B}}^{\text{T}}(k)\hat{\boldsymbol{B}}(k)+\boldsymbol{P}_B(k))\Delta\boldsymbol{u}(k) \\ &-2\Delta\boldsymbol{u}^{\text{T}}(k)[-(\boldsymbol{B}^{\text{T}}(k)\boldsymbol{U}(k)\hat{\boldsymbol{p}}(k)+\boldsymbol{P}_{Bp}(k))+w(k+1)]+r\Delta\boldsymbol{u}^{\text{T}}(k)\Delta\boldsymbol{u}(k)\,, \end{aligned} \tag{14.17}$$

where

$$\boldsymbol{P}_B(k) = \mathrm{E}\left\{[\hat{\boldsymbol{B}}(k) - \boldsymbol{B}]^{\mathrm{T}}[\hat{\boldsymbol{B}}(k) - \boldsymbol{B}]\middle|\Im_k\right\}, \tag{14.18}$$

and

$$\boldsymbol{P}_{Bp}(k) = \mathrm{E}\left\{[\hat{\boldsymbol{B}}(k) - \boldsymbol{B}]^{\mathrm{T}}\boldsymbol{U}(k)[\hat{\boldsymbol{p}}(k) - \boldsymbol{p}]\middle|\Im_k\right\}. \tag{14.19}$$

The gradient of eq. (14.17) with respect to $\Delta\boldsymbol{u}^{\mathrm{T}}(k)$ is given by

$$\begin{aligned}\frac{\partial J_k^c}{\partial \Delta\boldsymbol{u}(k)} &= 2\left(\hat{\boldsymbol{B}}^{\mathrm{T}}(k)\hat{\boldsymbol{B}}(k) + \boldsymbol{P}_B(k)\right)\Delta\boldsymbol{u}(k) \\ &\quad - 2[w(k+1) - \left(\boldsymbol{B}^{\mathrm{T}}(k)\boldsymbol{U}(k)\hat{\boldsymbol{p}}(k) + \boldsymbol{P}_{Bp}(k)\right)] + 2r\Delta\boldsymbol{u}(k).\end{aligned} \tag{14.20}$$

By setting eq. (14.20) equal to zero, we find that the control law

$$\Delta\boldsymbol{u}(k) = -\left(\hat{\boldsymbol{B}}^{\mathrm{T}}(k)\hat{\boldsymbol{B}}(k) + \boldsymbol{P}_B(k) + 2r\mathbf{I}\right)^{-1}[\left(\boldsymbol{B}^{\mathrm{T}}(k)\boldsymbol{U}(k)\hat{\boldsymbol{p}}(k) + \boldsymbol{P}_{Bp}(k)\right) - w(k+1)]. \tag{14.21}$$

From the last equation, the cautious control law $u_{\mathrm{c}}(k)$ can be derived using eqs. (14.13) and (14.12) by setting

$$\Delta u_{\mathrm{c}}(k) = u(k) - u(k-1),\ u_{\mathrm{c}}(k) = \Delta u_{\mathrm{c}}(k) + u(k-1). \tag{14.22}$$

As in Section 9.3, the dual control law is obtained after minimizing the cost function

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}(k)]^2\middle|\Im_k\right\} \tag{14.23}$$

over the domain as in eqs. (4.22) to (4.24). This minimization leads, after manipulations according to eqs. (4.31) and (4.33), to

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\mathrm{sgn}\left\{p_{b_1}(k)u_{\mathrm{c}}(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\right\}. \tag{14.24}$$

Thus, the dual MAC is described by eqs. (14.21), (14.22) and (14.24). In the same way, the proposed approach can be applied to the dual modification of Dynamic Matrix Control (DMC) of Cutler and Ramaker (1980), see also De Keyser et al. (1988). It should be noted that the nonparametric adaptive controllers, MAC and DMC, are applicable only to stable plants.

14.2. Generalized Predictive Control (GPC)

14.2.1. Equations for the Plant Model and Parameter Estimation

Consider the plant described by the special CARIMA model

$$\Delta y(k+1) = y(k+1) - y(k) = \boldsymbol{p}^{\mathrm{T}}\tilde{\boldsymbol{m}}(k) + \xi(k) = b_1\Delta u(k) + \boldsymbol{p}_0^{\mathrm{T}}\tilde{\boldsymbol{m}}_0(k) + \xi(k), \tag{14.25}$$

where

$$\boldsymbol{p}^{\mathrm{T}} = [b_1 ... b_n \vdots a_1 ... a_n] = [b_1 \vdots \boldsymbol{p}_0^{\mathrm{T}}], \tag{14.26}$$

$$\tilde{\boldsymbol{m}}^{\mathrm{T}}(k)=[\Delta u(k)\ldots\Delta u(k-n)\vdots\Delta y(k)\ldots\Delta y(k-n)]=[u(k)\vdots\tilde{\boldsymbol{m}}_0^{\mathrm{T}}(k)]\,, \tag{14.27}$$

$$\Delta u(k)=u(k)-u(k-1)\,, \tag{14.28}$$

and $\xi(k)$ is discrete white noise with variance σ_ξ^2. The parameter estimation using RLS is obtained from

$$\hat{\boldsymbol{p}}(k+1)=\hat{\boldsymbol{p}}(k)+\boldsymbol{q}(k+1)\;[\Delta y(k+1)-\hat{\boldsymbol{p}}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}(k)]\,, \tag{14.29}$$

$$\boldsymbol{q}(k+1)=\boldsymbol{P}(k)\tilde{\boldsymbol{m}}(k)[\tilde{\boldsymbol{m}}^{\mathrm{T}}(k)\boldsymbol{P}(k)\tilde{\boldsymbol{m}}(k)+\sigma_\xi^2]^{-1}\,, \tag{14.30}$$

$$\boldsymbol{P}(k+1)=\boldsymbol{P}(k)-\boldsymbol{q}(k+1)\tilde{\boldsymbol{m}}^{\mathrm{T}}(k)\boldsymbol{P}(k)\,, \tag{14.31}$$

where the matrix $\boldsymbol{P}(k)$ has the structure given in eq. (14.8).

14.2.2. Generalized Predictive Controller (GPC)

The GPC is based on the transformation of the parametric model to a non-parametric one described by eq. (14.25). Following Clarke et al. (1987), the predictive model has the form

$$\hat{\boldsymbol{y}}=\boldsymbol{G}\tilde{\boldsymbol{u}}+\boldsymbol{f}\,, \tag{14.32}$$

where

$$\hat{\boldsymbol{y}}^{\mathrm{T}}=[y(k+1)\ldots y(k+N)]\,, \tag{14.33}$$

$$\tilde{\mathbf{u}}^{\mathrm{T}}=[\Delta u(k)\ldots\Delta u(k+N-1)]\,, \tag{14.34}$$

and

$$\boldsymbol{f}^{\mathrm{T}}=[f(k+1)\ldots f(k+N)]\,. \tag{14.35}$$

Here $f(k+j)$ is a component of the output $y(k+j)$ composed of signals, which are known at time k (see Clarke et al., 1987). The matrix $\boldsymbol{G}$ consists of the parameters of the impulse response of the plant as follows:

$$\boldsymbol{G}=\begin{bmatrix} g_{(0)} & 0 & \cdots & 0 \\ g_{(1)} & g_{(0)} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ g_{(N-1)} & g_{(N-2)} & \cdots & g_{(0)} \end{bmatrix}. \tag{14.36}$$

Let us introduce a vector of reference signals

$$\boldsymbol{w}^{\mathrm{T}}=[w(k+1)\ldots w(k+N)]\,. \tag{14.37}$$

The GPC controller, based on the certainty equivalence principle (CE), is then determined (Clarke et al., 1987) after minimization of the cost function

$$J=\mathrm{E}_{\mathrm{CE}}\left\{[\boldsymbol{G}\tilde{\boldsymbol{u}}+\boldsymbol{f}-\boldsymbol{w}]^{\mathrm{T}}[\boldsymbol{G}\tilde{\boldsymbol{u}}+\boldsymbol{f}-\boldsymbol{w}]+r\tilde{\boldsymbol{u}}^{\mathrm{T}}\tilde{\boldsymbol{u}}\middle|\Im_k\right\} \tag{14.38}$$

by

$$\tilde{\boldsymbol{u}} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + r\mathbf{I})\boldsymbol{G}^{\mathrm{T}}(\boldsymbol{w} - \boldsymbol{f}), \tag{14.39}$$

where from eq. (14.34) we have

$$\Delta u(k) = \bar{\boldsymbol{g}}^{\mathrm{T}}(\boldsymbol{w} - \boldsymbol{f}),$$

or finally by

$$u_{\mathrm{CE}}(k) = u(k-1) + \bar{\boldsymbol{g}}^{\mathrm{T}}(\boldsymbol{w} - \boldsymbol{f}), \tag{14.40}$$

where $\bar{\boldsymbol{g}}$ is the first row of $(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G} + r\mathbf{I})\boldsymbol{G}^{\mathrm{T}}$. The last equation determines the GPC law based on the CE assumption.

14.2.3. Dual Modification of the GPC

The controller design presented in Section 9.3, is used here to develop the dual modification of the GPC. As suggested in Section 9.3, the first cost function is given by eq. (9.27)

$$J_k^{\mathrm{c}} = \mathrm{E}\left\{[\hat{y}_n(k+1) - y(k+1)]^2 \middle| \Im_k\right\}, \tag{14.41}$$

where $y(k+1)$ is determined by eq. (14.25) and $\hat{y}_{\mathrm{n}}(k+1)$ by eq. (10.60)

$$\Delta\hat{y}_{\mathrm{n}}(k+1) = \hat{y}_{\mathrm{n}}(k+1) - y(k) = \hat{b}_1(k)u_{\mathrm{CE}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k). \tag{14.42}$$

Thus, we have

$$\hat{y}_{\mathrm{n}}(k+1) = y(k) + \hat{b}_1(k)\Delta u_{\mathrm{CE}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k), \tag{14.43}$$

and

$$y(k+1) = y(k) + b_1\Delta u(k) + \boldsymbol{p}_0^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k) + \xi(k). \tag{14.44}$$

Substituting eqs. (14.43) and (14.44) into eq. (14.41), and taking the expectation (similar as in Appendix D) we obtain

$$\begin{aligned} J_k^{\mathrm{c}} &= \hat{b}_1^2(k)\Delta u_{\mathrm{CE}}^2(k) + [\hat{b}_1^2(k) + \boldsymbol{p}_{b_1}(k)]\Delta u^2(k) - 2\hat{b}_1^2(k)\Delta u_{\mathrm{CE}}(k)\Delta u(k) \\ &\quad + 2\tilde{\boldsymbol{m}}_0^{\mathrm{T}}(k)\boldsymbol{P}(k)\boldsymbol{m}_0(k) + 2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}(k)\Delta u(k) + \sigma_\xi^2. \end{aligned} \tag{14.45}$$

The corresponding gradient of eq. (14.45) with respect to $\Delta u(k)$ leads to

$$\frac{\partial J_k^{\mathrm{c}}}{\partial \Delta u(k)} = -2\hat{b}_1^2(k)\Delta u_{\mathrm{CE}}(k) + 2[\hat{b}_1^2(k) + \boldsymbol{p}_{b_1}(k)]\Delta u(k) + 2\boldsymbol{p}_{b_1 p}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k) = 0. \tag{14.46}$$

From the last equation follows the control action

$$\Delta u_{\mathrm{c}}(k) = \frac{\hat{b}_1^2(k)\Delta u_{\mathrm{CE}}(k) - \boldsymbol{p}_{b_1 p}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k)}{\hat{b}_1^2(k) + \boldsymbol{p}_{b_1}(k)}, \tag{14.47}$$

where $\Delta u_{\mathrm{CE}}(k)$ is determined by eq. (14.40).

The second cost function is determined as in eq. (9.28) by

$$J_k^{\mathrm{a}} = -\mathrm{E}\left\{[\Delta y(k+1) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}(k)]^2 \middle| \Im_k\right\}. \tag{14.48}$$

Minimizing this cost function over the domain according to eq. (4.24) and recalling that

$$\Omega_k = [\Delta u_{\mathrm{c}}(k) - \theta(k); \Delta u_{\mathrm{c}}(k) + \theta(k)] \tag{14.49}$$

leads to the dual controller

$$\Delta u(k) = \Delta u_{\mathrm{c}}(k) + \theta(k)\mathrm{sgn}\left\{p_{b_1}(k)\Delta u_{\mathrm{c}}(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k)\right\}. \tag{14.50}$$

Equivalently, using eq. (14.28) gives

$$u(k) = u_{\mathrm{c}}(k) + \theta(k)\mathrm{sgn}\left\{p_{b_1}(k)\Delta u_{\mathrm{c}}(k) + \boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k)\tilde{\boldsymbol{m}}_0(k)\right\}. \tag{14.51}$$

14.3. Other Predictive Controllers

Various adaptive predictive controllers, known at present (see, for example, De Keyser et al., 1988) can be extended to their dual versions in the same way as demonstrated above. The approach presented in Section 9.1 can be directly applied, for example, to derive a dual version of the well-known multistep-multivariable adaptive regulator (MUSMAR), which is based on the general plant model of eq. (8.1). The MUSMAR controller was originally presented by Menga and Mosca (1980) (see also Greco et al. (1984)). Successful applications of the adaptive dual version of this controller for a solar collector field have been reported by Silva et al. (1998a,b).

15. SIMULATION STUDIES AND REAL-TIME CONTROL USING MATLAB/SIMULINK

15.1. Simulation Studies of Adaptive Dual Controllers Using MATLAB

15.1.1. Generalized Minimum Variance Controller

The MATLAB programs for simulating the generalized minimum variance (GMV) controller with the CE assumption as well as its dual version are presented below. The program simulates a system with a time delay d=4 which is described in Section 4.4.2. Parameters `c` and `nu` should be set to zero to get a certainty equivalence controller.

```
clear
randn('seed',1)
nn=500;
n=2;
n1=8;
np=3;
nq=3;
d=3;
csi=zeros(1,nn+n1+d+1);
a0=[-2.1262  1.1052];
b0=[-0.0104 -0.0107];
u=zeros(1,nn+n1+d+1);
y=zeros(size(u));
csi=randn(size(csi));
scsiq=0.0001;
scsi=sqrt(scsiq);
lim=500;
c=1;  %1;       % cautious controller (c=0 - CE-controller)
nu=2;  %0.5;    % excitations
fs=[0.1 0.1 0.1 0.1 0.1 0.1];
gs=[0.1 0.1];
ps=[fs gs];
```

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```
r=[1];
pp=[-35.75 29.78]*(-0.18);
qq=[1 -0.1543]*(-0.18);
ppqq=[pp qq -r];
kam=zeros(1,n1);
prom=kam;
P=eye(n1);
po=0.15;    %1
P=P*po;
p1=zeros(1,n1);
w=ones(1,nn+n1+d+1)*5;
w(101+n1+d:200+n1+d)=-5*ones(1,100);
w(301+n1+d:400+n1+d)=-5*ones(1,100);
w(1:n1+d)=zeros(1,n1+d);
par=[b0 -a0]
for i=n1+d:nn+n1+d,
 k=i;
 fi=[u(k-3) u(k-4) y(k) y(k-1)];
 y(k+1)=par*fi'+scsi*csi(k);
fia=[y(k+1) y(k) u(k-3) u(k-4) w(k+1)];
ya(k+1)=ppqq*fia';
%
f1=[u(k-d:-1:k-d-5) y(k-d:-1:k-d-1)];
en=ya(k+1)-ps*f1'+w(k-d);
prom=(P*f1')';
f=f1*prom'+scsiq;
kam=prom/f;
P=P-kam'*prom;
ps=ps+kam*en;
fii=[u(k:-1:k-4) y(k+1:-1:k)];
p1=P(1,2:n1);
wk=w(k+1);
uc=ps(1)*wk-(ps(2:n1)*ps(1)+c*p1)*fii';
uc=uc/(c*P(1,1)+ps(1)^2);
```

```
tr=0;
for ii=1:n1,
 tr=tr+P(ii,ii);
end
tr;
te=nu*c*tr;
u(k+1)=uc+te*sign(P(1,1)*uc+p1*fii');
 if abs(u(k+1))>lim,
   u(k+1)=lim*sign(u(k+1));
 end
end
ps
tr
i=(1:nn);
plot(i,u(n1+d+1:nn+n1+d))
xlabel('Discrete time k')
ylabel('Control signal (u)')
figure(2)
plot(i,y(n1+d+1:nn+n1+d),'-',i,w(n1+d+1:nn+n1+d),'--')
xlabel('Dual control')
ylabel('Output and setpoint signals (y,w)')
```

The simulation program for a system with a time delay d=7 described in Section 4.4.3 are presented below. The program is identical, but the number of estimated parameters is larger for the case of a larger delay.

```
clear
randn('seed',0)
nn=500;
n=2;
n1=11;
np=3;
nq=3;
d=6;
csi=zeros(1,nn+n1+d+1);
a0=[-2.1262  1.1052];
```

```
b0=[-0.0104 -0.0107];
u=zeros(1,nn+n1+d+1);
y=zeros(size(u));
csi=randn(size(csi));
scsiq=0.00001;
scsi=sqrt(scsiq);
lim=500;
c=1;  %1;
nu=0.01;  %0.01;
fs=[0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5];
gs=[0.5 0.5];
ps=[fs gs];
r=[1];
pp=[-35.75 29.78]*(-0.17);
qq=[1 -0.1543]*(-0.17);
ppqq=[pp qq -r];
kam=zeros(1,n1);
prom=kam;
P=eye(n1);
po=1;
P=P*po;
p1=zeros(1,n1);
w=ones(1,nn+n1+d+1)*5;
w(101+n1+d:200+n1+d)=-5*ones(1,100);
w(301+n1+d:400+n1+d)=-5*ones(1,100);
w(1:n1+d)=zeros(1,n1+d);
par=[b0 -a0]
for i=n1+d:nn+n1+d,
 k=i;
 fi=[u(k-6) u(k-7) y(k) y(k-1)];
 y(k+1)=par*fi'+scsi*csi(k);
fia=[y(k+1) y(k) u(k-6) u(k-7) w(k+1)];
ya(k+1)=ppqq*fia';
f1=[u(k-d:-1:k-d-8) y(k-d:-1:k-d-1)];
```

```
en=ya(k+1)-ps*f1'+w(k-d);
prom=(P*f1')';
f=f1*prom'+scsiq;
kam=prom/f;
P=P-kam'*prom;
ps=ps+kam*en;
fii=[u(k:-1:k-7) y(k+1:-1:k)];
p1=P(1,2:n1);
wk=w(k+1);
uc=ps(1)*wk-(ps(2:n1)*ps(1)+c*p1)*fii';
uc=uc/(c*P(1,1)+ps(1)^2);
tr=0;
for ii=1:n1,
 tr=tr+P(ii,ii);
end
tr;
te=nu*c*tr;
u(k+1)=uc+te*sign(P(1,1)*uc+p1*fii');
 if abs(u(k+1))>lim,
   u(k+1)=lim*sign(u(k+1));
 end
end
ps
tr
i=(1:nn);
plot(i,u(n1+d+1:nn+n1+d))
xlabel('Discrete time k')
ylabel('Control signal (u)')
figure(2)
plot(i,y(n1+d+1:nn+n1+d),'-',i,w(n1+d+1:nn+n1+d),'--')
xlabel('Dual control')
ylabel('Output and setpoint signals (y,w)')
```

15.1.2. Direct Adaptive Pole-Placement Controller

This section describes the program for simulating the dual version of a pole-placement controller with direct adaptation. The description of the controller is presented in Chapter 6. The program simulates the dual controllers according to the example presented in Section 6.3.2.

The included program `polydiop` solves the Diophantine equation. The text of the program `polydiop` is separately presented in Appendix C.

```
clear
rand('seed',1)
rand('normal')
nn=100;
n=2;
na=2*n+1;
csi= zeros(1,nn+na);
a0=[1.1618 -2.1889 1];          %  a3 a2 a1 1
b0=[-0.0139 -0.0132 0];         %  a3 a2 a1 1
cor=[0.6+0.1*j 0.6-0.1*j 0.2];
d1=POLY(cor)
for i=1:2*n,
 d(i)=d1(2*n+1-i);
end
cor1=roots(d)
c=d./d(2*n)
[r1,s1]=polydiop(a0,b0,c)
cz=d1./d(2*n)
u=zeros(1,nn+na);
y=zeros(u);
csi=rand(csi);
scsiq=0.00001;
scsi=sqrt(scsiq);
bs=sum(b0);
cs=sum(cz);
gf=bs/cs
nu=2.2;
cf=1;
```

```
d=[0.01];
f=[0.01];
s=[0.01 0.01];
r=[0.01 0.01];
nc=2*n;
ne=4*n-2;
yk=zeros(1,n);
uk=zeros(1,n-1);
ykk=yk;
ukk=uk;
ps=[d f s r];
kam=zeros(1,ne);
prom=kam;
P=eye(ne);
Q=P;
po=0.5;
P=P*po;
Q=Q*0.0;
p1=zeros(1,2*n-1);
w=ones(1,nn+na+1)*1;
w(21+na:40+na)=-1*ones(1,20);
w(61+na:80+na)=-1*ones(1,20);
w(1:na)=zeros(1,na);
pause(3)
par=[b0(2) b0(1) -a0(2) -a0(1) scsi]
for i=na:nn+na-1,
 k=i;
 fi=[u(k) u(k-1) y(k) y(k-1) csi(k)];
 y(k+1)=par*fi';
ykfi=y(k+1:-1:k-n*2+2);
ukfi=u(k:-1:k-n*2+1);
ykk(2:n)=yk(1:n-1);
%ukk(2:n-1)=uk(1:n-2);
yk=ykk;
```

```
uk=ukk;
yk(1)=ykfi*cz';
uk(1)=ukfi*cz';
fifi=[uk yk(2:n) u(k+2-2*n:-1:k+3-3*n) y(k+2-2*n:-1:k+3-3*n)];
en=yk(1)-ps*fifi';
prom=(P*fifi')';
f=fifi*prom'+scsiq;
kam=prom/f;
P=P-kam'*prom+Q;
ps=ps+kam*en;
r=ps((n-1)*2+1:(n-1)*2+n);
s=ps((n-1)*2+n+1:ne);
pr=cf*P(2*(n-1)+1,2*(n-1)+1);
p1=[P(1,(n-1)*2+n+1:ne) P(1,(n-1)*2+2:(n-1)*2+n)]*cf;
pp=pr+r(1)^2;
uc=r(1)*r(1)*w(k+2)*(1/gf)-([s r(2:n)]*r(1)+p1)*[y(k+1:-1:k-n+2) u(k:-1:k-n+2)]';
uc=uc/pp;
tr=0;
for ii=(n-1)*2-1:ne,
 tr=tr+P(ii,ii);
end
if k==10+na
tr10=tr
end
te=nu*tr;
uck=cz*[uc u(k:-1:k-n*2+2)]';
u(k+1)=uc+te*sign(P(1,1)*uck+P(1,2:ne)*fifi(2:ne)');
end
y1=y;
y1([1:na])=[];
u([1:na-1 nn+na])=[];
plot(u)
z=w;
z([1:na nn+na+1])=[];
```

```
figure(2)
i=(1:nn);
plot(i,y1,'-',i,z,'--')
```

The version of the program for simulating the direct adaptive pole-placement controller designed with the certainty equivalence assumption is given as follows.

```
clear
rand('seed',1)
rand('normal')
nn=100;
n=2;
na=2*n+1;
csi= zeros(1,nn+na);
a0=[1.1618 -2.1889 1];    %  a3 a2 a1 1
b0=[-0.0139 -0.0132 0];  %  a3 a2 a1 1
cor=[0.6+0.1*j 0.6-0.1*j 0.2];
d1=POLY(cor)
for i=1:2*n,
 d(i)=d1(2*n+1-i);
end
cor1=roots(d)
c=d./d(2*n)
[r1,s1]=polydiop(a0,b0,c)
cz=d1./d(2*n)
u=zeros(1,nn+na);
y=zeros(u);
csi=rand(csi);
scsiq=0.00001;
scsi=sqrt(scsiq);
bs=sum(b0);
cs=sum(cz);
gf=bs/cs
nu=0;
cf=0;
d=[0.01];
```

```
f=[0.01];
s=[0.01 0.01];
r=[0.01 0.01];
nc=2*n;
ne=4*n-2;
yk=zeros(1,n);
uk=zeros(1,n-1);
ykk=yk;
ukk=uk;
ps=[d f s r];
kam=zeros(1,ne);
prom=kam;
P=eye(ne);
Q=P;
po=0.5;
P=P*po;
Q=Q*0.0;
p1=zeros(1,2*n-1);
w=ones(1,nn+na+1)*1;
w(21+na:40+na)=-1*ones(1,20);
w(61+na:80+na)=-1*ones(1,20);
w(1:na)=zeros(1,na);
pause(3)
par=[b0(2) b0(1) -a0(2) -a0(1) scsi]
for i=na:nn+na-1,
 k=i;
 fi=[u(k) u(k-1) y(k) y(k-1) csi(k)];
 y(k+1)=par*fi';
ykfi=y(k+1:-1:k-n*2+2);
ukfi=u(k:-1:k-n*2+1);
ykk(2:n)=yk(1:n-1);
%ukk(2:n-1)=uk(1:n-2);
yk=ykk;
uk=ukk;
```

```
yk(1)=ykfi*cz';
uk(1)=ukfi*cz';
fifi=[uk yk(2:n) u(k+2-2*n:-1:k+3-3*n) y(k+2-2*n:-1:k+3-3*n)];
en=yk(1)-ps*fifi';
prom=(P*fifi')';
f=fifi*prom'+scsiq;
kam=prom/f;
P=P-kam'*prom+Q;
ps=ps+kam*en;
r=ps((n-1)*2+1:(n-1)*2+n);
s=ps((n-1)*2+n+1:ne);
pr=cf*P(2*(n-1)+1,2*(n-1)+1);
p1=[P(1,(n-1)*2+n+1:ne) P(1,(n-1)*2+2:(n-1)*2+n)]*cf;
pp=pr+r(1)^2;
uc=r(1)*r(1)*w(k+2)*(1/gf)-([s r(2:n)]*r(1)+p1)*[y(k+1:-1:k-n+2) u(k:-1:k-n+2)]';
uc=uc/pp;
tr=0;
for ii=(n-1)*2-1:ne,
 tr=tr+P(ii,ii);
end
if k==10+na
tr10=tr
end
te=nu*tr;
uck=cz*[uc u(k:-1:k-n*2+2)]';
u(k+1)=uc+te*sign(P(1,1)*uck+P(1,2:ne)*fifi(2:ne)');
end
y1=y;
y1([1:na])=[];
u([1:na-1 nn+na])=[];
plot(u)
z=w;
z([1:na nn+na+1])=[];
figure(2)
```

```
i=(1:nn);
plot(i,y1,'-',i,z,'--')
```

15.1.3. Model Reference Adaptive Controller

The program for simulating the MRAC with the CE assumption, and its dual version presented in Chapter 7, is listed below.

```
rand('seed',2)
rand('normal')
nn=80;
n=3;
dz=2;
nd=4;
la=0.01;
na=2*n+1+dz;
csi=zeros(1,nn+na);
a0=[ -0.9277 2.866 -2.93 1];     %  a3 a2 a1 1
b0=[-0.1892 0.0001 0.1964 0];  %  a3 a2 a1 1
cor=[0.6+0.1*j 0.6-0.1*j 0.2];
d1=POLY(cor)
for i=1:nd,
 d(i)=d1(nd+1-i);
end
cor1=roots(d)
c=d./d(nd);
d=c
cz=d1./d(nd)
u=zeros(1,nn+na);
y=zeros(u);
ym=zeros(u);
csi=rand(csi);
scsiq=0.00005;
scsi=sqrt(scsiq);
nu=0.01;   % parameter of excitation
cf=1;     % parameter of dual control (cf=0 - cautious control)
```

```
bs=[0.1 0.1 0.1 0.1];
r=[0.1 0.1 0.1];
nc=2*n;
nr=max([n-1 nd-1-dz])
ne=2*n+1;
ps=[bs r];
kam=zeros(1,ne);
prom=kam;
P=eye(ne);
Q=P;
po=1;
P=P*po;
Q=Q*0.0;
p1=zeros(1,ne-1);
w=ones(1,nn+na+1)*1;
w(21+na:40+na)=-1*ones(1,20);
w(61+na:80+na)=-1*ones(1,20);
w(1:na)=zeros(1,na);
par=[b0(3) b0(2) b0(1) -a0(3) -a0(2) -a0(1) scsi]
for i=na:nn+na-1,
 k=i;
 fi=[u(k-dz+1) u(k-1-dz+1) u(k-2-dz+1) y(k) y(k-1) y(k-2) csi(k)];
 y(k+1)=par*fi';
 syk=y(k+1);
 ym(k+1+dz)=0.5*ym(k+dz)+0.5*w(k);
ykfi=y(k+1:-1:k-nd+2);
ykmfi=ym(k+1+dz:-1:k+dz-nd+2);
yk=ykfi*cz';
ykm=ykmfi*cz';
fifi=[u(k-dz+1:-1:k-2*dz-n+3) y(k-dz+1:-1:k-dz-nr+1)];
en=yk-ps*fifi';
prom=(P*fifi')';
f=fifi*prom'+scsiq;
kam=prom/f;
```

```
P=P-kam'*prom+Q;
ps=ps+kam*en;
pb=cf*P(1,1);
p1=P(1,2:ne)*cf;
pp=pb+ps(1)^2;
fifi1=[u(k:-1:k-dz-n+3) y(k+1:-1:k-nr+1)];
uc=ps(1)*ykm-(ps(2:ne)*ps(1)+p1)*fifi1'+la*u(k);
uc=uc/(pp+la);
tr=0;
for ii=1:ne,
 tr=tr+P(ii,ii);
end
if k==10+na
tr10=tr
end
te=nu*tr*cf;
u(k+1)=uc+te*sign(pb*uc+p1*fifi1');
suk=u(k+1);
end
y1=y;
u1=u(na+2:na+nn);
y1=y(na+2:na+nn);
plot(u1)
figure(2)
z=ym(na+2:na+nn);
i=(1:nn-1);
plot(i,y1,'-',i,z,'--')
```

15.2. Simulation Studies of Adaptive Controllers Using MATLAB/ SIMULINK

The adaptive dual pole-placement controller is realized here as an S-function in SIMULINK using the MATLAB program language (see Moscinski and Zbigniew, 1995). This adaptive pole-placement controller with indirect adaptation and its simplified dual modification are described in Chapter 10. As an example, the second-order plant described in Section 9.5.3 in continuous and discrete-time form is considered for simulation. The simulation schemes using SIMULINK are presented in Figures 15.1 and 15.2 for discrete-time and continuous-time plant models, respectively.

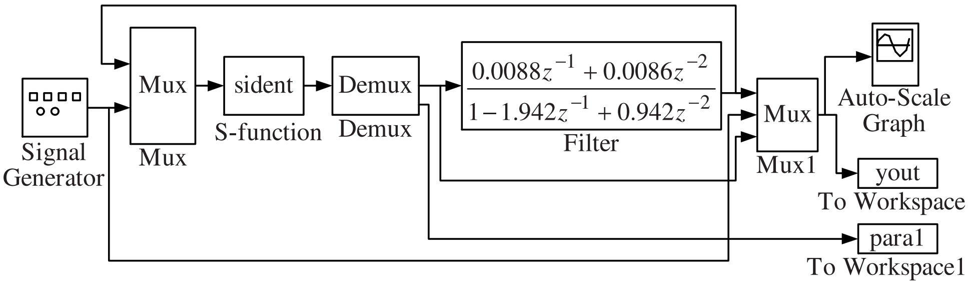


Figure 15.1. SIMULINK scheme for simulating a discrete-time plant

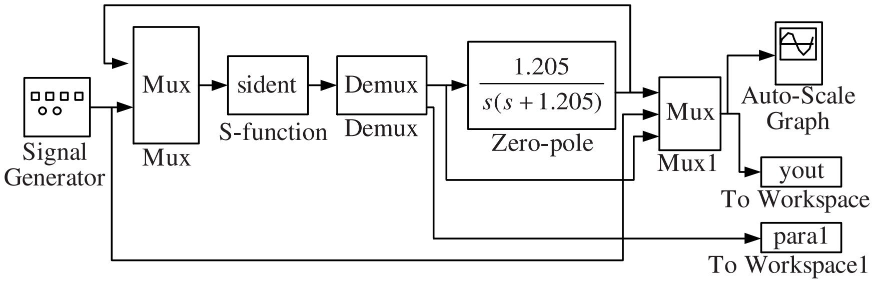


Figure 15.2. SIMULINK scheme for simulating a continuous-time plant

In Figures 15.1 and 15.2, Mux and Demux represent multiplexers and demultiplexers of the signals. The signal generator generates the setpoint signal for the system, and the auto-scale graph draws the graphics of the setpoint, the control signal and the output signal. To realise the adaptive controller in SIMULINK according to the presented schemes, the S-function `sident` needs to be realized in MATLAB code. The text of the program `sident` with the corresponding routine `ident1` is presented below. It is necessary to point out that the controller contains an integrator as described in Chapter 10.

```
function [sys, x0, str, ts]=sident(t,x,u,flag,sample)  % S-function
n=2;                                         % plant order
if abs(flag)==2
   xn=ident1(x);
   m2=[u(2) xn(((2*n)^2+2*n+2):((2*n)^2+3*n+1))'];
   m1y=[u(1) xn(((2*n)^2+3*n+3):((2*n)^2+4*n+2))'];
   sys=[xn(1:((2*n)^2+2*n+1))' m2 m1y xn(((2*n)^2+4*n+4):((2*n)^2+6*n+5))']';
```

```
elseif abs(flag)==3
   sys=[x((2*n)^2+5*n+4)' x(1)]';%  output of S-function =
                                 %  [Control, Parameter a1]
elseif flag==0
   K=1;
   P=100*eye(2*n);
   pp=reshape(P,1,(2*n)^2);
   theta=0.1*ones(1,2*n);   % initial value of the parameter vector
   s=0.001;
   m2=[-1 zeros(1,n)];           % setpoint
   m1y=zeros(1,n+1);             % output
   m1us=[-1 zeros(1,n-1)];       % before the integrator
   uu=[-1 zeros(1,n)];           % after the integrator
   sys=[0,(2*n)^2+6*n+5,2,2,0,0,1];
   x0=[theta pp s m2 m1y m1us uu K]';
   ts=[sample, 0];               % sampling time of the system
else
   sys=[];
end;

function xn=ident1(x)
n=2;                                          % plant order
                                              % pole-placement
p2=conv([1 -0.8],[1 -0.8]);                   % observer poles Q
pol=conv([1 0.9],conv([1 -0.8+0.1*j],[1 -0.8-0.1*j])); % system poles
p2=[p2 0.5*ones(1, n+1-length(p2))]';    %
AM=conv(p2',pol);                             % polynomial P*Q
AM=[AM 0.5*ones(1, 2*n+2-length(AM))]';
%
theta=x(1:2*n);
pp=x(2*n+1:((2*n)^2+2*n));
P=reshape(pp,2*n,2*n);
s=x((2*n)^2+2*n+1);
m2=x(((2*n)^2+2*n+2):((2*n)^2+3*n+2));
```

```
K=x((2*n)^2+6*n+5);
m2=[1/K*m2(1) m2(2:n+1)']';
m1y=x(((2*n)^2+3*n+3):((2*n)^2+4*n+3))';
m1us=x(((2*n)^2+4*n+4):((2*n)^2+5*n+3))';
m1=[m1us m1y]';
uu=x(((2*n)^2+5*n+4):((2*n)^2+6*n+4));

alpha1=1;
alpha2=0.00;
%
mu=0.01;                          % parameter for adaptation
%
PHIu=[uu(2:n+1)]';                % input after integrator
PHIy=[m1y(2:n+1)];                % output
PHI=[PHIu -PHIy]';
                                  % identification
v=m1y(1)-theta'*PHI;
g=alpha1/(mu+PHI'*P*PHI);
theta=theta+g*P*PHI*v;
P=P-g*P*PHI*PHI'*P;

if trace(P) < alpha2
   P=P+alpha2/n*eye(size(P));
end;
                                  % controller
denominator=conv([1 theta(n+1: 2*n)'], [1 -1]);     % A
nominator=[0 theta(1: n)'];                         % B
[L, H]=polydiop(denominator', nominator', AM);      % Diophantine equation
p1=[L(2: n+1)' H']';              % system amplification
Za=conv(p2, nominator);
n1=conv(L', denominator);
n2=conv(H', nominator);
Ne=polyadd(n1, n2);
K=real(sum(Za)/sum(Ne));
```

```
us=real(1/L(1)*(p2'*m2-p1'*m1));
uu=[us uu(1:n)'];
                                        % integrator
uu(1)=uu(2)+uu(1);
if abs(uu(1))>10, uu(1)=10*sign(uu(1)); end;   % limits for control
pp=reshape(P,1,(2*n)^2);
us=uu(1);
m1us=[us m1us(1:n-1)];

xn=[theta' pp s m2' m1y m1us uu K]';
```

In the case of dual control the function `identd` is used in the S-function `sident` instead of `ident1`. Program `identd` for dual control is listed below.

```
function xn=identd(x)
n=2;                        % plant order
                          % pole-placement
p2=conv([1 -0.8],[1 -0.8]);   % observer poles Q
pol=conv([1 0.9],conv([1 -0.8+0.1*j],[1 -0.8-0.1*j])); % system poles
p2=[p2 0.5*ones(1, n+1-length(p2));
AM=conv(p2',pol);                        % polynomial P*Q
AM=[AM 0.5*ones(1, 2*n+2-length(AM))]';
%
theta=x(1:2*n);
pp=x(2*n+1:((2*n)^2+2*n));
P=reshape(pp,2*n,2*n);
s=x((2*n)^2+2*n+1);
m2=x(((2*n)^2+2*n+2):((2*n)^2+3*n+2));
K=x((2*n)^2+6*n+5);
m2=[1/K*m2(1) m2(2:n+1)']';
m1y=x(((2*n)^2+3*n+3):((2*n)^2+4*n+3))';
m1us=x(((2*n)^2+4*n+4):((2*n)^2+5*n+3))';
m1=[m1us m1y]';
uu=x(((2*n)^2+5*n+4):((2*n)^2+6*n+4));
```

```
alpha1=1;
alpha2=0.00;

nu=0.0001;
mu=0.01;
co=0.01;                          % parameter for adaptation

PHIu=[uu(2:n+1)]';                % input after integrator
PHIy=[m1y(2:n+1)];                % output
PHI=[PHIu -PHIy]';

                          % identification

v=m1y(1)-theta'*PHI;
g=alpha1/(mu+PHI'*P*PHI);
theta=theta+g*P*PHI*v;
P=P-g*P*PHI*PHI'*P;

if trace(P) < alpha2
  P=P+alpha2/n*eye(size(P));
end;
                          % controller
denominator=conv([1 theta(n+1: 2*n)'], [1 -1]); % A
nominator=[0 theta(1: n)'];           % B
[L, H]=polydiop(denominator', nominator', AM);   % Diophantine equation
p1=[L(2: n+1)' H']';
                      % system amplification
Za=conv(p2, nominator);
n1=conv(L', denominator);
n2=conv(H', nominator);
Ne=polyadd(n1, n2);
K=real(sum(Za)/sum(Ne));
us=real(1/L(1)*(p2'*m2-p1'*m1));
                          % cautious control
```

```
us=(theta(1)^2*us-co*P(1,2:2*n)*PHI(2:2*n))/(theta(1)^2+co*P(1,1));
uu=[us uu(1:n)'];
te=nu*trace(P);
                                  % integrator
uu(1)=uu(2)+uu(1);
uu(1)=uu(1)+te*sign(P(1,1)*uu(1)+P(1,2:2*n)*PHI(2:2*n));%dual control
if abs(uu(1))>10, uu(1)=10*sign(uu(1)); end;   % limits for control
pp=reshape(P,1,(2*n)^2);
us=uu(1);
m1us=[us m1us(1:n-1)];

xn=[theta' pp s m2' m1y m1us uu K]';
```

Simulation results for the cases of CE and dual controllers are presented in Figures 15.3 and 15.4, respectively. The dual controller with the parameters η=0.0001 and σ_{ξ} =0.01 demonstrates better transients at the beginning of the adaptation. It should be noted that the case of discrete-time simulation according to the scheme presented in Fig. 15.1 is used. Other parameters of the controllers, pole positioning and initial conditions of the system can be seen in the text of the programs listed above. Before starting the simulation, it is necessary to select a sampling time for the system; in the considered case `sample`=0.05 needs to be added. A limit of ±10 is applied for the control signal in the programs `ident1` and `identd`.

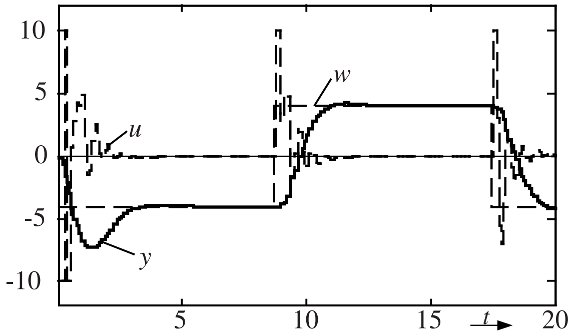


Figure 15.3. Simulation results for the CE controller

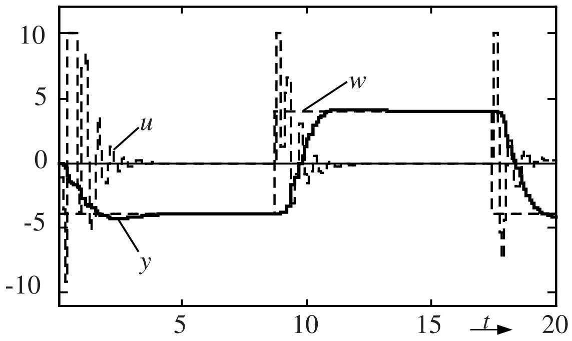


Figure 15.4. Simulation results for the adaptive dual controller

It should be noted that before starting the simulation study for the new plant it is better to test first the pole-placement controller without adaptation. In this case, it is necessary to assign the real values of plant parameters to the parameter estimation vector `theta`. Thus, the following program `ident2` with adaptation switched off can be used for a first simulation and validation of the selected pole locations of the closed loop system.

```
function xn=ident2(x)
n=2;                                                  % plant order

                                                  % pole-placement
p2=conv([1 -0.8],[1 -0.8]);                           % observer poles Q
pol=conv([1 0.9],conv([1 -0.8+0.1*j],[1 -0.8-0.1*j]));   % system poles P
p2=[p2 0.5*ones(1, n+1-length(p2))]';
AM=conv(p2',pol);                                     % polynomial
P*Q
AM=[AM 0.5*ones(1, 2*n+2-length(AM))]';
%
theta=x(1:2*n);
pp=x(2*n+1:((2*n)^2+2*n));
P=reshape(pp,2*n,2*n);
s=x((2*n)^2+2*n+1);
m2=x(((2*n)^2+2*n+2):((2*n)^2+3*n+2));
K=x((2*n)^2+6*n+5);
m2=[1/K*m2(1) m2(2:n+1)']';
m1y=x(((2*n)^2+3*n+3):((2*n)^2+4*n+3))';
```

```
m1us=x(((2*n)^2+4*n+4):((2*n)^2+5*n+3))';
m1=[m1us m1y]';
uu=x(((2*n)^2+5*n+4):((2*n)^2+6*n+4));

alpha1=1;
alpha2=0.00;

mu=0.01;                    % parameter for adaptation

PHIu=[uu(2:n+1)]';          % input after integrator
PHIy=[m1y(2:n+1)];          % output
PHI=[PHIu -PHIy]';
                            % identification
v=m1y(1)-theta'*PHI;
g=alpha1/(mu+PHI'*P*PHI);
theta=theta+g*P*PHI*v;
P=P-g*P*PHI*PHI'*P;

if trace(P) < alpha2
   P=P+alpha2/n*eye(size(P));
end;
theta(1)=0.0088;
theta(2)=0.0086;
theta(3)=-1.942;
theta(4)=0.942;
                                                    % controller
denominator=conv([1 theta(n+1: 2*n)'], [1 -1]);     % A
nominator=[0 theta(1: n)'];                         % B
[L, H]=polydiop(denominator', nominator', AM);      % Diophantine eq.
p1=[L(2: n+1)' H']';
                            % system amplification
Za=conv(p2, nominator);
n1=conv(L', denominator);
n2=conv(H', nominator);
```

```
Ne=polyadd(n1, n2);
K=real(sum(Za)/sum(Ne));

us=real(1/L(1)*(p2'*m2-p1'*m1));
uu=[us uu(1:n)'];

                                   % integrator
uu(1)=uu(2)+uu(1);
if abs(uu(1))>10, uu(1)=10*sign(uu(1)); end; % limits for control signal
pp=reshape(P,1,(2*n)^2);
us=uu(1);
m1us=[us m1us(1:n-1)];

xn=[theta' pp s m2' m1y m1us uu K]';
```

15.3. Real-Time Robust Adaptive Control of a Hydraulic Positioning System Using MATLAB/SIMULINK

15.3.1. Description of the Laboratory Equipment

In the present section, a real-time robust adaptive dual control scheme is applied to a laboratory test bed consisting of a hydraulic positioning system. For the controller realization MATLAB/SIMULINK and a PC with special support by SIMULINK AD/DA-converters are used. The scheme of this plant is portrayed in Fig. 15.5. The hydraulic system moves a mass M. From the other side the reaction forces of a spring and a damper act upon the mass. The manipulating variable is given by the control current I, which activates the hydraulic control valve. The difference of the pressure Δp activates the piston and changes the position y of the mass. For testing the robust adaptive controller presented in Chapter 13, two different control valves are used: a servo-valve with linear characteristic and a 4/3-proportional direction valve with nonlinear characteristic as depicted in Fig 15.6.

The plant is approximately described by the linear transfer function

$$G(s) = \frac{Y(s)}{W(s)} = \frac{K\omega_0}{T_0 s(s^2 + 2\delta\omega_0 s + \omega_0^2)}, \tag{15.1}$$

where for the nominal model $K/T_0 = 1.7$, $\omega_0 = 100\text{sec}^{-1}$, $\delta = 0.25$, and $w(t) = \mathscr{L}^{-1}\{W(s)\}$ is the reference signal. To realize the real-time control using the modern software SIMULINK, the simulation schemes of Figures 15.1 and 15.2 can be modi fied by replacing the simulated plant model through programs supporting the AD/DA-converters as presented in Fig. 15.7. The block "Interactive Realtime1 DAS1601" represents the AD/DA-converters of the PC.

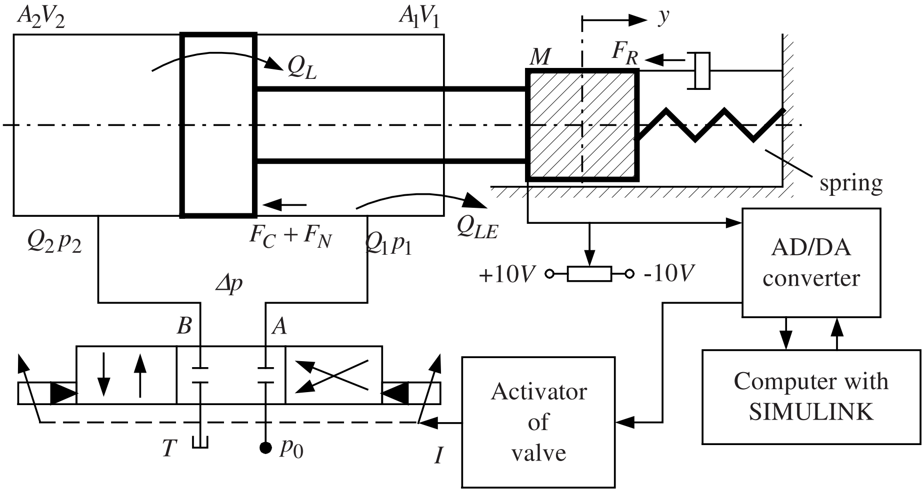


Figure 15.5. Scheme of hydraulic positioning test bed
$Q_{1,2}$ - flow [l/min], Q_{LE} - external flow of oil [l/min],
Q_L - internal flow of oil [l/min], $A_{1,2}$ - piston area [cm^2],
$V_{1,2}$ - volume of the cylinder [cm^3], I - current of activator [mA],
Δp - $p_2 - p_1$ [N/cm^2], y - position of the mass [mm],
F_C - friction of the cylinder [N], F_N - viscose friction [N],
F_R - reaction force [N], $p_{1,2}$ pressure [N/cm^2],
A, B, T, p_0 - connections of valve, M - mass [kg].

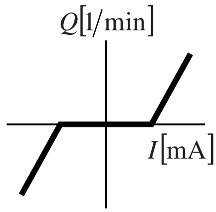


Figure 15.6. Scheme of the nonlinearity of the 3/4-proportional direction valve

The results of real-time robust adaptive dual control are displayed in Figures 15.8 and 15.9 for the servo-valve and in Figures 15.10 and 15.11 for the nonlinear 4/3-proportional direction valve for the case of rectangular changes of the reference signal $w(t)$. As can be seen from Figures 15.9 and 15.11 the robust adaptive controller provides an excellent tracking behavior of the reference signal after a relatively very short transient time. In the case when a nonlinear 4/3-proportional direction valve is a applied, the transient time is a bit longer due to the specific nonlinearity. It is quite obvious from Figures 15.9 and 15.11 that the reference signal is cautiously followed by the controlled variable. It can also be seen from Figures 15.8 and 15.10 that the control signal u is strongly excited at the beginning such that the parameter estimation process is speeded up considerably.

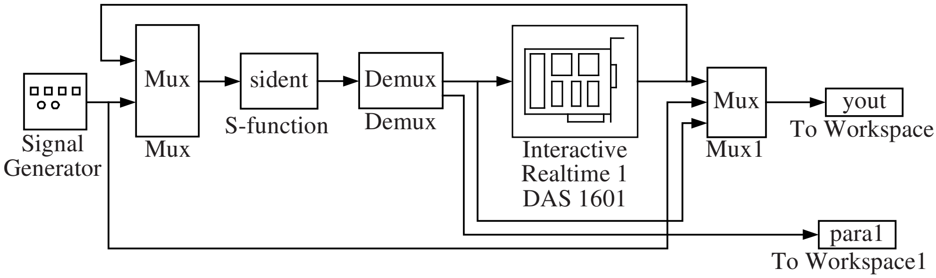


Figure 15.7. SIMULINK scheme for real-time control

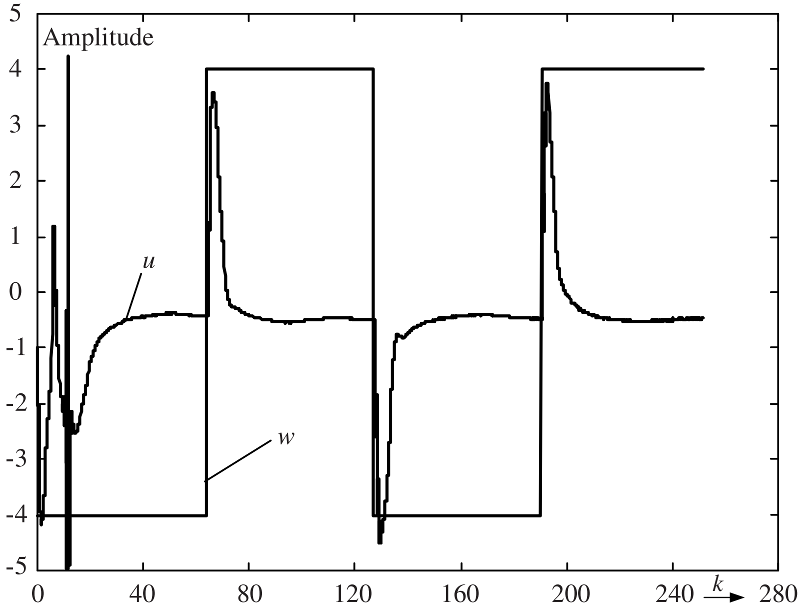


Figure 15.8. Manipulating signal u of the robust adaptive controller for the hydraulic positioning system with servo-valve (sampling time, $\Delta t = 0.04$ *sec*)

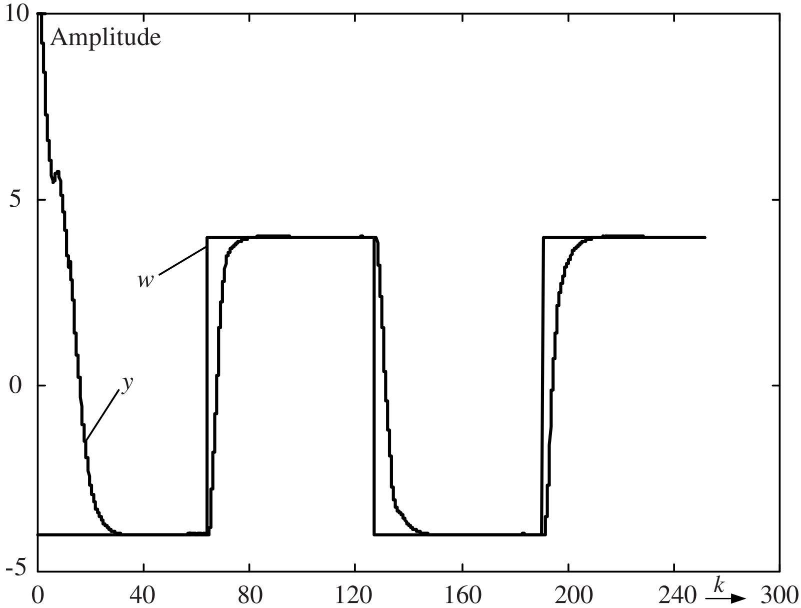


Figure 15.9. Controlled signal y of the robust adaptive controller for the hydraulic positioning system with servo-valve (sampling time, $\Delta t = 0.04\,sec$)

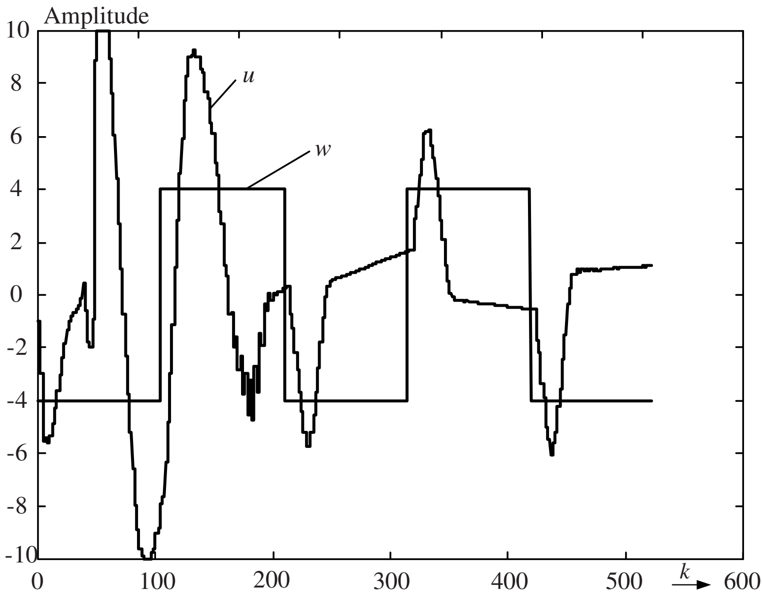


Figure 15.10. Manipulating signal u of the robust adaptive controller for the hydraulic positioning system with 4/3-proportional direction valve ($\Delta t = 0.1\,sec$)

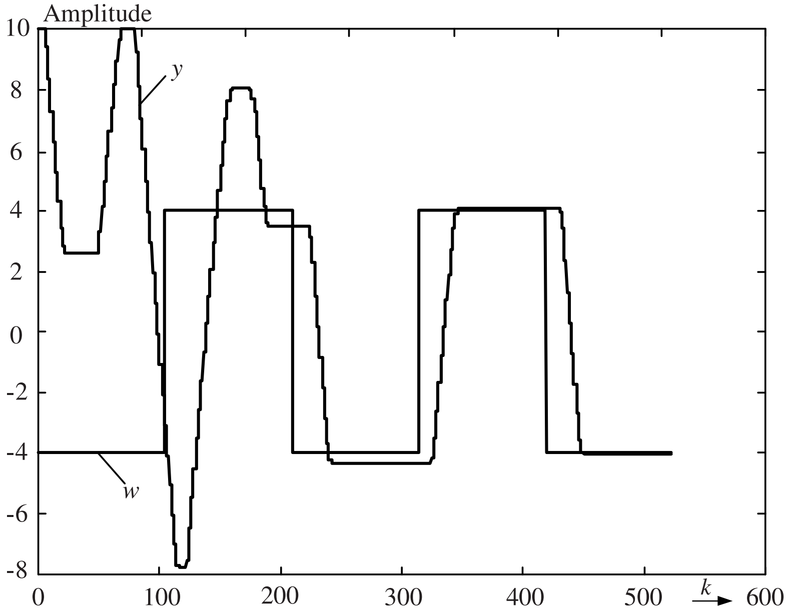


Figure 15.11. Controlled signal y of the robust adaptive controller for hydraulic positioning system with 4/3-proportional direction valve ($\Delta t = 0.1\,sec$)

15.3.2. Program Listing

Below, the program of the real-time robust adaptive dual controller is listed. The parameters of the controller and the pole locations that have been chosen for the real-time control experiments, indicated in Figures 15.8 to 15.11, are defined in this program listing. The program `sident` of the S-function is the same as described in Section 15.2 for the simulation. The robust adaptive controller used here is realized in the following routine `ident`.

```
function xn=ident(x)
n=3;                          % order of the plant

%                        pole-placement
p2=[0.5];                                  % observer polynomial
pol=conv([1 -0.3], conv([1 -0.685], [1 0.56])); % pole-placement
p2=[p2 zeros(1, n+1-length(p2))]';
AM=conv(p2',pol);
AM=[AM zeros(1, 2*n+2-length(AM))]';
%
```

```
theta=x(1:2*n);
pp=x(2*n+1:((2*n)^2+2*n));
P=reshape(pp,2*n,2*n);
s=x((2*n)^2+2*n+1);
m2=x(((2*n)^2+2*n+2):((2*n)^2+3*n+2));
K=x((2*n)^2+6*n+5);
m2=[1/K*m2(1) m2(2:n+1)']';
m1y=x(((2*n)^2+3*n+3):((2*n)^2+4*n+3))';
m1us=x(((2*n)^2+4*n+4):((2*n)^2+5*n+3))';
m1=[m1us m1y]';
uu=x(((2*n)^2+5*n+4):((2*n)^2+6*n+4));

alpha1=1;
alpha2=4;                    % limit for the trace of P
rho=5;
lambda=10;                   % limit for Pdach
sigma=0.00096;           % parameters for adaptation
mu=0.48;
thetanull=[0.2*ones(1,2*n-1) 0.3]';

PHIu=[uu(2:n+1)]';
PHIy=[m1y(2:n+1)];
PHI=[-PHIy PHIu]';

%                 identification
hilf2=norm(PHI);
if hilf2 > s
  s=sigma*s+hilf2;
else
  s=s*sigma+s;
end;
v=m1y(1)-theta'*PHI;
g=alpha1/(mu*s^2+PHI'*P*PHI);
thetadach=theta+g*P*PHI*v;
```

```
Pdach=P-g*P*PHI*PHI'*P;
eigen=eig(Pdach);
mx=max(eigen);                              % the largest eigenvalue
beta=lambda/mx;
if mx > lambda
   P=beta*Pdach;
else
   P=Pdach;
end;
if trace(P) < alpha2
   trace(P)
   P=P+alpha2/n*eye(size(P));
end;
hilf1=rho/norm(thetadach-thetanull);
if hilf1 < 1
   theta=thetanull+(thetadach-thetanull)*hilf1;
else
   theta=thetadach;
end;

                          % controller
denominator=conv([1 theta(1: n)'], [1 -1]);
nominator=[0 theta(n+1: 2*n)'];
[L, H]=polydiop(denominator', nominator', AM);  % Diophantine equation
p1=[L(2: n+1)' H']';

                          % system amplification K
Za=conv(p2, nominator);
n1=conv(L', denominator);
n2=conv(H', nominator);
Ne=polyadd(n1, n2);
K=real(sum(Za)/sum(Ne));

us=real(1/L(1)*(p2'*m2-p1'*m1));
```

```
uu=[us uu(1:n)'];

% Integrator
uu(1)=uu(2)+uu(1);
if abs(uu(1))>10, uu(1)=10*sign(uu(1)); end;          % control signal
pp=reshape(P,1,(2*n)^2);
m1us=[us m1us(1:n-1)];

xn=[theta' pp s m2' m1y m1us uu K]';
```

15.4. Real-Time ANN-Based Adaptive Dual Control of a Hy-draulic Drive

In this section a new adaptive control scheme for nonlinear systems of Hammerstein type is presented. Two artificial neural networks (ANN) are used to compensate the input nonlinearity of the plant. To improve the control performance the dual control strategy will be applied (Knohl, Xu and Unbehauen 2000; Knohl 2001).

15.4.1. Plant Identification Using an ANN

As already mentioned in Section 15.3.1, the hydraulic drive operating with a $3/4$-proportional valve is characterized by a dead zone nonlinearity depicted in Fig. 15.6. A Hammerstein system, as portrayed in Fig. 15.12, consisting of two separate partial systems, a static memoryless nonlinear static subsystem followed in series connection by a linear subsystem, can be applied for describing the static and dynamic behavior of this hydraulic positioning system. The input/output behavior of such a Hammerstein system is described by

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n} b_i \bar{v}(k-i) \tag{15.2}$$

where a_i and b_i are the coefficients of the linear system and $\bar{v} = f[v(k)]$ is the output of a nonlinear static function. The unknown static nonlinearity can be modeled by an RBF network in the form

$$\bar{v} = w_0 v(k) + \sum_{j=1}^{m} w_j \Phi_j(| v(k) - c_j |) \; . \tag{15.3}$$

This network is a very powerful class of ANN. It consists of three layers that are interconnected feed-forward from the input to the output layer. The name is motivated from the activation function of the hidden layer neurons $\Phi_j(| v(k) - c_j |)$, which are realized

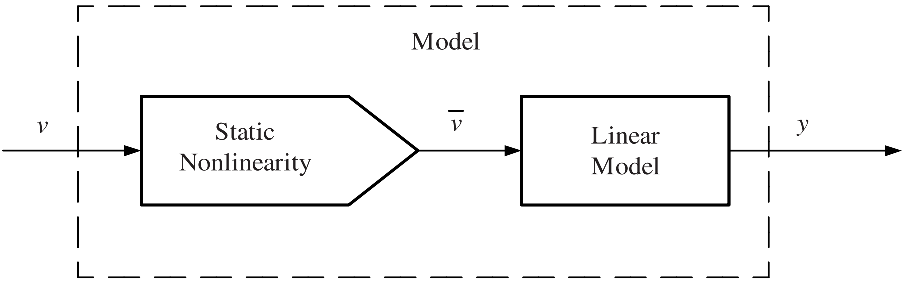


Figure 15.12. Block structure of a Hammerstein system

each by a radial symmetric function. The most popular RBF is the Gaussian function, which is chosen for the further considerations. The network weights $w_j,\ j=0,\dots,m$ are the on-line adjustable parameters and for the sake of simplicity the centers $c_j,\ j=1,\dots,m$ and the design parameters $\beta_{ij},\ i=1,\dots,n$ and $j=1,\dots,m$ defined below are determined and fixed at the beginning.

Combining eqs. (15.2) and (15.3) we get a linear-in-parameter type of model

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n}\sum_{j=0}^{m} \beta_{ij} x_j(k-i) \tag{15.4}$$

and rewritten according to eq. (4.2) in vector form and extended to an error equation it follows

$$y(k) = \boldsymbol{m}^{\mathrm{T}}(k)\,\boldsymbol{p} + \xi(k)\,, \tag{15.5}$$

where the equation error $\xi(k)$ is a sequence which is assumed to be a zero-mean white noise, and

$$\beta_{ij} = b_i w_j\,,\ x_0(k) = v(k) \tag{15.6}$$

and

$$x_j(k) = \Phi_j(|\,v(k) - c_j\,|),\ j=1,\dots m\,. \tag{15.7}$$

The parameter and regression vectors are given as

$$\boldsymbol{p} = [a_1 \dots a_n \ \vdots\ \beta_{10} \dots \beta_{n0} \dots \beta_{1m} \dots \beta_{nm}]^{\mathrm{T}}\,, \tag{15.8}$$

$$\boldsymbol{m}^{\mathrm{T}}(k) = [-y(k-1)\dots y(k-n) \ \vdots\ x_0(k-1)\dots x_m(k-n)]^{\mathrm{T}}\,. \tag{15.9}$$

For estimating the parameter vector $\boldsymbol{p}$ the weighted RLS-method with constant trace (Mosca 1995) had been applied for avoiding sluggishing of the estimation.

The next step consists in determining the $2\,n$ parameters of the linear subsystem out of the $n(m+2)$ estimated parameters $\hat{p}_i$. Defining $w_0 = 1$ yields directly

$$\hat{a}_i = \hat{p}_i \text{ and } \hat{b}_i = \hat{p}_{n+i} \text{ for } i=1,\dots,n\,. \tag{15.10}$$

For determining the network weights w_j there exists redundancy in the system of equations as follows:

$$\begin{aligned} \hat{\beta}_{1j} &= \hat{b}_1 \hat{w}_j \\ &\vdots \qquad\qquad \text{for } j = 1, \ldots, m . \\ \hat{\beta}_{nj} &= \hat{b}_n \hat{w}_j \end{aligned} \tag{15.11}$$

As solution we obtain

$$\hat{w}_j = \left[\sum_{i=1}^{n} \hat{\beta}_{ij} \right] \Big/ \sum_{i=1}^{n} \hat{b}_1^2 , \quad j = 1, \ldots, m . \tag{15.12}$$

15.4.2. ANN-Based Design of a Standard Adaptive LQ Controller

The main idea of the adaptive control scheme introduced below is on the one-hand side to apply two ANN's for compensating the input nonlinearity of the real system by an appropriate and efficient inverse operator and on the other hand to design a standard adaptive controller for the remaining linear subsystem. The basic structure of such an adaptive control system is portrayed in Fig. 15.13.

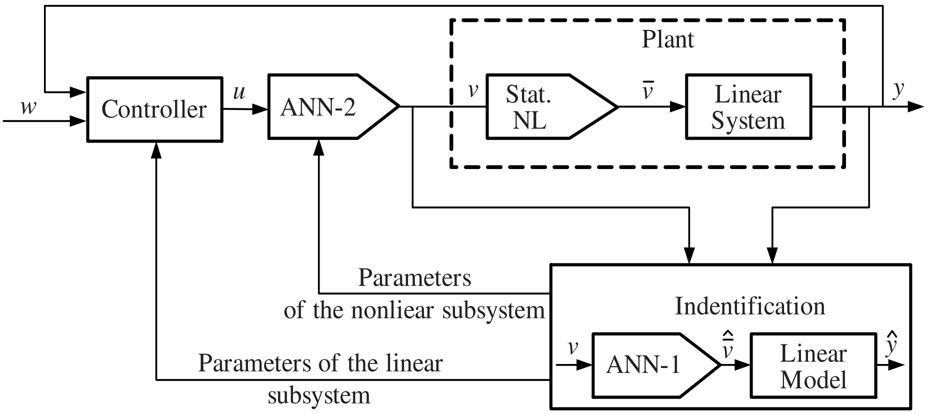


Figure 15.13. Block diagram of the ANN-based Standard adaptive LQ controller

It is important to mention that the direct inversion of ANN-1 in the form

$$v = f_{ANN-2}(u) = f_{ANN-1}^{-1}(u) \tag{15.13}$$

is not possible under real-time operation due to time-critical problems and numerical instability. An alternative to avoid these problems is to take the identified (trained) ANN - 1 for generating a set of input/output data. Swapping the inputs and outputs the second ANN-2 is trained to simultaneously approximate the inverse, as discussed in detail in Section 15.4.3. The remaining step then consists in designing an indirect linear standard adaptive controller, as for example, an optimal LQ-regulator.

The design is based on the minimization of the quadratic performance index

$$J = \mathrm{E}\{[w(k) - y(k)]^2 + \rho u^2(k)\} \tag{15.14}$$

which represents the expectation of the sum of squared control errors and square of control action weighted by ρ . The well-known solution (Aström and Wittenmark 1996) for a SISO plant is given in the z-domain by the control law

$$U(z) = \frac{t_0}{R(z^{-1})} W(z) - \frac{S(z^{-1})}{R(z^{-1})} Y(z)\,, \tag{15.15}$$

where t_0 is a controller parameter, and $R(z^{-1})$ as well as $S(z^{-1})$ are controller polynomials in z^{-1}. By spectral factorization the characteristic polynomial $P(z^{-1})$ of the closed-loop system is obtained with known (or estimated) polynomials $A(z^{-1})$ and $B(z^{-1})$ from

$$P(z)\,P(z^{-1}) = \rho(z-1)\,A(z)\,A(z^{-1})(z^{-1}-1) + B(z)\,B(z^{-1})\,, \tag{15.16}$$

where the factor $(z^{-1}-1)$ is an integral term for avoiding a bias in the controlled variable. The solution $R(z^{-1}) = R^*(z^{-1})(1-z^{-1})$ and $S(z^{-1})$ follows by solving the Diophantine equation

$$A(z^{-1})(1-z^{-1})R^*(z^{-1}) + B(z^{-1})\,S(z^{-1}) = P(z^{-1}) \tag{15.17}$$

and

$$t_0 = \frac{P(1)}{B(1)}\,. \tag{15.18}$$

Eq. (15.15) represents a simplified control law which can be extended also for dead-time behaviour and for inclusion of an observer polynomial. The polynomials $A(z^{-1})$ and $B(z^{-1})$ are estimated with the assumption that the CE-principle holds.

Basically the stability of such a nonlinear adaptive control system can be investigated either via local linearization or by selection of an appropriate Lyapunov function or by application of the hyperstability theory. However, all these approaches are rather tedious and provide only limited usable stability conditions. Therefore, a more heuristic procedure (Goodwin and Sin 1984) seems more realistic. If $A(z^{-1})$ and $B(z^{-1})$ are known and have no common roots and if the estimated parameters are bounded, then also $u(k)$ and $y(k)$ are bounded in case of an ideal compensation of the nonlinear characteristic of the Hammerstein-type plant. In the case of uncomplete compensation, an input disturbance of the linear subsystem of the plant would occur. This disturbance signal is, however, identical to the approximation error of ANN-2 and can be reduced by changing the number of neurons or the network parameters. Thus stability can always be guaranteed.

15.4.3. Extension to the Design of the ANN-Based Adaptive Dual Controller

The controller discussed above is based on the CE principle. For avoiding the effect of uncertain estimates, as discussed in the preceding chapters, this controller is extended by a dual controller block providing a cautions control action u_c and an excitation u_e to improve the convergence of the parameter estimation. The extended block structure of this control system is portrayed in Fig. 15.14.

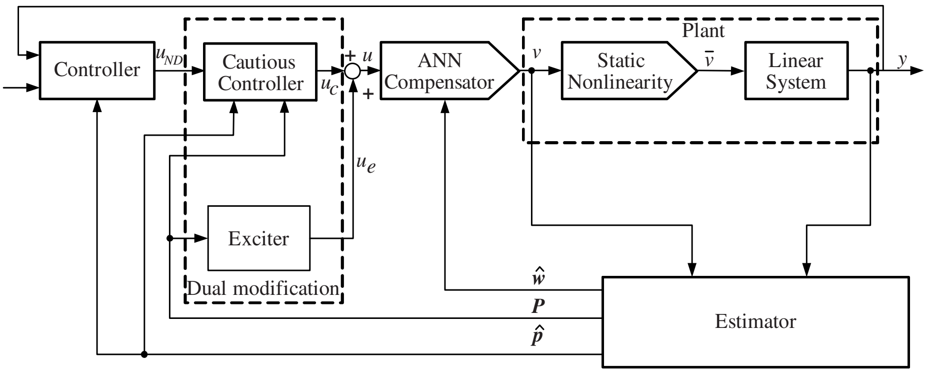


Figure 15.14. Nonlinear adaptive neural control with dual modifiation

Similar to eq. (15.5) we define the error equation

$$y(k+1) = \boldsymbol{m}^{\mathrm{T}}(k)\,\boldsymbol{p} + \xi(k+1) \tag{15.19}$$

and together with eqs. (15.6), (15.7) and $w_0 = 1$ we have the parameter and regressor vectors

$$\begin{aligned}\boldsymbol{p}^{\mathrm{T}} &= [b_1 \;\dots\; b_n, a_n \;\vdots\; \beta_{11} \dots \beta_{nm}] \\ &= [\boldsymbol{p}_1^{\mathrm{T}} \mid \boldsymbol{p}_2^{\mathrm{T}}] = [b_1 \mid \boldsymbol{p}_0^{\mathrm{T}} \;\vdots\; \boldsymbol{p}_2^{\mathrm{T}}]\end{aligned} \tag{15.20}$$

and

$$\begin{aligned}\boldsymbol{m}^{\mathrm{T}}(k) &= [u(k)\dots u(k-n+1),\; y(k)\dots y(k-n+1) \mid x_1(k)\dots x_m(k-n+1)] \\ &= [\boldsymbol{m}_1^{\mathrm{T}}(k) \;\vdots\; \boldsymbol{m}_2^{\mathrm{T}}(k)] = [u(k) \mid \boldsymbol{m}_0^{\mathrm{T}}(k) \;\vdots\; \boldsymbol{m}_2^{\mathrm{T}}(k)]\,.\end{aligned} \tag{15.21}$$

$\boldsymbol{p}_0, \boldsymbol{m}_0(k)$ will be used later to design the dual controller.

A standard RLS estimation algorithm can again be applied to estimate the plant parameters in eq. (15.19). The calculation of the covariance matrix is defined as

$$\boldsymbol{P}(k) = \mathrm{E}\{[\boldsymbol{p}(k) - \hat{\boldsymbol{p}}(k)]\,[\boldsymbol{p}(k) - \hat{\boldsymbol{p}}(k)]^{\mathrm{T}} \mid \Im_k\} \tag{15.22}$$

with

$$\Im_k = [y(0)\,y(k), \;\dots\; u(0)\,u(k-1)],\; \Im_0 = [y(0)] \tag{15.23}$$

as the set of the plant inputs and outputs available at time *k*.

The covariance matrix $\boldsymbol{P}(k)$ can be rewritten in the following form

$$\boldsymbol{P}(k)=\begin{bmatrix} \boldsymbol{P}_1(k) & \boldsymbol{P}_{12}^{\mathrm{T}}(k) \\ \boldsymbol{P}_{12}(k) & \boldsymbol{P}_2(k) \end{bmatrix}, \tag{15.24}$$

where $\boldsymbol{P}_1(k)$ represents the covariance matrix of the parameters of the linear subsystem and $\boldsymbol{P}_2(k)$ is the covariance matrix of the network weights.

With the above definitions the dual modification of the controller can now be obtained. Transforming the control law of eq. (15.15) into the time-domain the *nominal* non-dual (ND) control action based on the CE assumption becomes

$$u_{\mathrm{n}}(k)\equiv u_{\mathrm{ND}}(k)=t_0 w(k)-\boldsymbol{p}_3^{\mathrm{T}}(k)\,\boldsymbol{m}_3(k), \tag{15.25}$$

where

$$\boldsymbol{p}_3^{\mathrm{T}}(k)=[r_1(k)\ldots r_{n-1}(k)\ \vdots\ s_0(k)\ldots s_{n-1}(k)] \tag{15.26}$$

and

$$\boldsymbol{m}_3=[u(k-1)\ldots u(k-n+1)\ \vdots\ y(k)\ldots y(k-n+1]^{\mathrm{T}}. \tag{15.27}$$

The corresponding nominal output signal of the controlled process assumes the form

$$y_{\mathrm{n}}(k+1)=b_1 u_{\mathrm{n}}(k)+\boldsymbol{p}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k). \tag{15.28}$$

To improve the control performance the bicriterial approach is used here for the derivation the control law, with the two cost functions

$$J_k^{\mathrm{a}}=-\mathrm{E}\{[y(k+1)-\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)]^2\mid\Im_k\} \tag{15.29}$$

and

$$J_k^{\mathrm{c}}=\mathrm{E}\{[y_{\mathrm{n}}(k+1)-y(k+1)]^2\mid\Im_k\}, \tag{15.30}$$

to be minimized, where the system output after compensation of the static nonlinearity is given as

$$y(k+1)-\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)+\xi(k). \tag{15.31}$$

The first cost function is used to improve the parameter estimation by increasing the predictive error, and the second is applied for driving the system output to follow a given reference signal. The simplified dual version of the adaptive controller can be obtained by using the CE assumption for computation of the nominal output $y_{\mathrm{n}}(k+1)$, according to eq. (15.28). Thus, the estimated nominal system output for the non-dual controller based on the CE assumption is

$$\hat{y}_{\mathrm{n}}(k+1)=\hat{b}_1 u_{\mathrm{n}}(k)+\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k). \tag{15.32}$$

The dual controller will be obtained after solving the bicriterial optimization problem, given by eqs. (15.29) and (15.30), using the constraints described as following:

$$u(k) = \arg \min_{\mathrm{u(k)} \in \Omega_{\mathrm{k}}} \{J_k^{\mathrm{a}}\}, \tag{15.33}$$

where

$$\Omega_k = [u_{\mathrm{c}}(k) - \theta(k); u_{\mathrm{c}}(k) + \theta(k)], \tag{15.34}$$

$$\theta(k) = h_k\{\boldsymbol{P}(k)\}, \tag{15.35}$$

and $h_k(\cdot)$ is a differentiable bounded scalar positive function of the covariance matrix, so that $\lim_{k\to\infty}\theta(k) = 0$, if $\mathrm{tr}(\boldsymbol{P}(k)) = 0$. A common choice for $h_k(\boldsymbol{P}(k))$ is $h_k(\boldsymbol{P}(k)) = \eta\, \mathrm{tr}\,(\boldsymbol{P}(k))$, where $\eta > 0$ sets the amplitude of the probing signal. The control signal $u_{\mathrm{c}}(k)$ is obtained after minimization of the cost function (15.30). Substituting eqs. (15.31) and (15.32) into eq. (15.30) and taking the expectation leads to (see Appendix D)

$$\begin{aligned} J_k^{\mathrm{c}} &= [\hat{b}_1^2(k) + p_{b_1}(k)]\, u^2(k) + [2\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}} \boldsymbol{m}_0(k) - 2\hat{b}_1^2(k) u_{\mathrm{ND}}(k)] u(k) \\ &\quad + b_1^2(k) u_{\mathrm{ND}}^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k) \boldsymbol{P}_{\boldsymbol{p}_0}(k) \boldsymbol{m}_0(k) + \sigma_\xi^2 , \end{aligned} \tag{15.36}$$

where the covariance matrix $\boldsymbol{P}_1(k)$ in to eq. (15.24) has the structure

$$\boldsymbol{P}_1(k) = \begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1 \boldsymbol{p}_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1 \boldsymbol{p}_0}(k) & \boldsymbol{P}_{\boldsymbol{p}_0}(k) \end{bmatrix} . \tag{15.37}$$

The minimization of eq. (15.36) results in setting

$$\left.\frac{\partial J_k^{\mathrm{c}}}{\partial \mathrm{u}}\right|_{u=u_{\mathrm{c}}} = 2[\hat{b}_1^2(k) + p_{b_1}(k)] u(k) + z[\boldsymbol{p}_{b_1 p_0}^{\mathrm{T}}(k) \boldsymbol{m}_0(k) - b_1^2(k) u_{\mathrm{ND}}(k)]\Big|_{u=u_{\mathrm{c}}} = 0$$

and leads to the cautious control law

$$u_{\mathrm{c}}(k) = \frac{\hat{b}_1^2(k) u_{\mathrm{ND}}(k) - \boldsymbol{p}_{b_1 \boldsymbol{p}_0}^{\mathrm{T}}(k) \boldsymbol{m}_0(k)}{\hat{b}_1^2(k) + p_{b_1}(k)} . \tag{15.38}$$

Minimizing eq. (15.33), in the domain Ω_k yields the solution

$$\begin{aligned} u(k) &= u_{\mathrm{c}}(k) + u_{\mathrm{e}}(k) \\ &= u_{\mathrm{c}}(k) + \theta(k)\ \mathrm{sgn}\{J_k^{\mathrm{a}}(u_{\mathrm{c}}(k) - \theta(k)) - J_k^{\mathrm{a}}(u_{\mathrm{c}}(k) + \theta(k))\} \end{aligned} \tag{15.39}$$

with

$$\mathrm{sgn}\{x\} = \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{if } x < 0. \end{cases} \tag{15.40}$$

According to eq, (15.39) for the dual controller, the estimation exciter $u_{\mathrm{c}}(k)$ is not yet defined. Inserting eq. (15.31), the cost function J_k^{a} of eq. (15.29) takes the form

$$J_k^{\mathrm{a}}[u(k)] = -\mathrm{E}\{[(\boldsymbol{p}_1(k) - \hat{\boldsymbol{p}}(k))^{\mathrm{T}} \boldsymbol{m}_1(k) + \xi(k)]^2 \mid \Im_k\}$$

and after taking the expectation we obtain (see Appendix D)

$$J_k^{\mathrm{a}}[u(k)] = -p_{b_1}(k)u^2(k) - 2\boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}\boldsymbol{m}_0(k)u(k) + \bar{c}_1(k) \ . \tag{15.41}$$

The term $\bar{c}_1(k)$, which does not depend on $u(k)$, is introduced for simplification of further computations. Thus

$$J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) - \theta(k)] - J_k^{\mathrm{a}}[u_{\mathrm{c}}(k) + \theta(k)] = 4p_{b_1}u_{\mathrm{c}}(k)\theta(k) + 4\boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}\boldsymbol{m}_0(k)\theta(k) \, . \tag{15.42}$$

Substituting the last equation into eq. (15.39), finally we get the dual control law

$$u(k) = u_{\mathrm{c}}(k) + \underbrace{\theta(k)\,\mathrm{sgn}\{p_{b_1}(k)u_{\mathrm{c}}(k) + \boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}\boldsymbol{m}_0(k)\}}_{u_{\mathrm{e}}(k)} \ . \tag{15.43}$$

From eq. (15.43) it becomes obvious that the control law $u(k)$ consists o the sum of the cautious control action $u_{\mathrm{c}}(k)$ and the excitation $u_{\mathrm{e}}(k)$. In case of disturbances or modeling errors these might deteriorate the control behavior. This can be avoided by weighting $u_{\mathrm{e}}(k)$ with the low pass filtered magnitude of the prediction error

$$\varepsilon(k) = y(k) - \boldsymbol{m}^{\mathrm{T}}\,\hat{\boldsymbol{p}}(k-1) \tag{15.44}$$

in the form

$$u(k) = u_{\mathrm{c}}(k) + u_{\mathrm{e}}(k)\bar{\varepsilon}(k) \, , \tag{15.45}$$

where

$$\bar{\varepsilon}(k) = \mathfrak{z}^{-1}\left\{\frac{1 - T_f z^{-1}}{1 - T_f z^{-1}} \, |\, \varepsilon(z) \,|\right\} \, . \tag{15.46}$$

This extension provides the decrease of the amplitude of the excitation with decreasing prediction error.

Another modification of eq. (15.43) is recommended for simple nonlinear systems by replacing the cautions control signal as follows (Filatov and Unbehauen, 1999):

$$u_{\mathrm{c}}(k) = \frac{1}{1 + \lambda^* \, |\, \varepsilon(k-1) \,|} u_{\mathrm{ND}}(k) \, . \tag{15.47}$$

The factor λ^* would be denoted as cautious factor and may be optimized depending on the plant dynamics.

The last step of the controller design is to train the ‘inverse’ network ANN-2 denoted in Fig. 15.14 as ANN compensator in the feed-forward path. Similarly, as already mentioned in Section 15.4.2, it is assumed that the inverse of the plant input nonlinearity exists and that this nonlinearity has been learned by the network ANN-1 included in the estimator block of Fig. 15.14. the trained network ANN-1 is used to generate the input/output data set

$$[\boldsymbol{u}_{\mathrm{ANN-1}}(N),\ \boldsymbol{y}_{\mathrm{ANN-1}}(N)=\boldsymbol{f}_{\mathrm{ANN-1}}(\boldsymbol{u}_{\mathrm{ANN-1}}(N))], \tag{15.48}$$

where

$$\boldsymbol{u}_{\mathrm{ANN-1}}(N)=[u_{\mathrm{ANN-1}}(1)\ldots u_{\mathrm{ANN-1}}(N)]^{\mathrm{T}}$$
$$\boldsymbol{y}_{\mathrm{ANN-1}}(N)=[y_{\mathrm{ANN-1}}(1)\ldots y_{\mathrm{ANN-1}}(N)]^{\mathrm{T}}.$$

The values of $u_{\mathrm{ANN-1}}(k)$ with $k=1, 2, \ldots, N$ are selected such that the whole range of the control signal $u_{\min}(k)\le u(k)\le u_{\max}(k)$ is covered. After swapping the input/output data set the network function which is linear in the network weights can be rewritten, by additionally introducing a white noise zero-mean equation error $\xi(k)$ with variance σ_{ξ}^{2}, in the vector form

$$\boldsymbol{y}_{\mathrm{ANN-2}}(N)-\boldsymbol{u}_{\mathrm{ANN-2}}(N)=\boldsymbol{M}(N)\,\boldsymbol{p}+\boldsymbol{\xi}(N), \tag{15.49}$$

where

$$\boldsymbol{M}=\begin{pmatrix} \phi_1(|u_{\mathrm{ANN-2}}(1)-c_1|) & \ldots & \phi_m(|u_{\mathrm{ANN-2}}(1)-c_m|) \\ \vdots & \vdots & \vdots \\ \phi_1(|u_{\mathrm{ANN-2}}(N)-c_1|) & \ldots & \phi_m(|u_{\mathrm{ANN-2}}(N)-c_m|) \end{pmatrix} \tag{15.50}$$

and

$$\boldsymbol{p}=[w_1\ \ldots\ w_m]^{\mathrm{T}}. \quad (15.51$$

A LS estimation can be used as *learning* rule, either leading to the direct solution

$$\hat{\boldsymbol{p}}(N)=[\boldsymbol{M}^{\mathrm{T}}(N)\boldsymbol{M}(N)]^{-1}\boldsymbol{M}(N)[\boldsymbol{y}_{\mathrm{ANN-1}}(N)-\boldsymbol{u}_{\mathrm{ANN-2}}(N)] \tag{15.52}$$

or to a faster recursive solution if eq. (15.52), implemented in a real-time surrounding, leads to time problems. This step concludes the controller design.

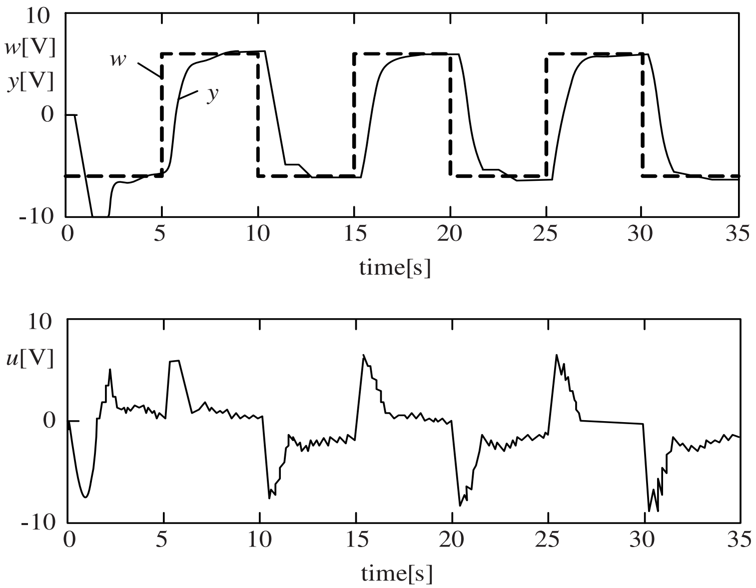
15.4.4. Real-Time Experiments

Both controllers described in Sections 15.4.2 and 15.4.3 had been realized in *Matlab/Simulink* programs and been connected via the *InteractiveRealtime*-Toolbox with the hydraulic positioning system. For the realization of both controllers the sampling time $T=60$ ms was selected. Both, the positioning system output signal $y(k)$ operating from 0 mm to 175 mm and the control action $u(k)$ were normalized each in the range of ± 10 V. The weighting factor of the LQ controller was set to 120. For the dual adaptive controller the low pass filter coefficient of eq. (15.46) was selected as $T_f=0.93$. The network parameters were fixed to the values shown in Table 15.1, where γ is the form parameter of the radial basis functions.

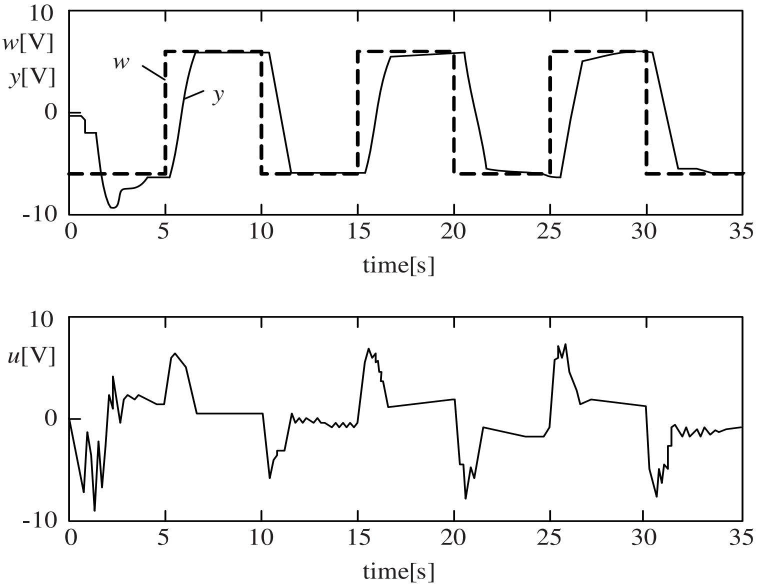
Table 15.1. Network parameters for both controllers

Network	γ	Centers of Basis Functions
ANN-1	2.5	-10 -8.5 -7.5 -6 -5 -4 -3 -2 2 3 4 5 6 7.5 8.5 10
ANN-2	1.6	-5 -2.5 -1.5 -1 -0.5 -0.2 0.2 0.5 1 1.5 2.5 5

The experiments are portrayed in Fig. 15.15. This figure shows for both controllers the most important three signals: The rectangular wave reference signal $w(k)$, the controlled signal (poston position) $y(k)$, and the control action $u(k)$. Both controller types provide good control performance. The non-dual controller, however, reaches for the duration of 1 s at the beginning of the experiment its boundaries. The dual controller is more cautious and reaches faster and without limitation the reference signal due to the higher excitation during the first 5 seconds. After 15 s the behaviour of both controllers is very similar. In conclusion it becomes quite obvious that the control performance of the non-dual controller was considerably improved by extension to the described dual version.



(a)



(b)

Figure 15.15. Dynamic behaviour of the non-dual (a) and the dual adaptive controller (b) at the hydraulic positioning system

16. CONCLUSION

Feldbaum discovered that the behavior of optimal adaptive control is characterized by dual properties: cautious control and active excitation. In other words, this means that the best behavior of adaptive control must not hastily follow the reference signal if the plant parameters are not exactly known (property of cautious control), and, at the same time, the control signal must excite the system to speed up the parameter estimation process for improvement of the future control performance (active excitation). Difficulties in finding the optimal dual-control policy led to the appearance of various suboptimal dual controllers. Most of them are not suitable for practical application because of their complexity and computational difficulties for operation in real time or insufficient control performance. Therefore, simple adaptive controllers that are based on the certainty equivalence (CE) approach, have been commonly used up to now. CE adaptive controllers are computationally simple because the parameter estimation can be separated from the control law. The accuracy of the estimates is not taken into account in these controllers, and the parameter estimates are used as if they were the real values of the unknown parameters. Insufficient control performance and large overshoots are often attributed to these types of adaptive controllers, especially at the beginning of the adaptation and in cases of inaccurate estimation.

The bicriterial design method of adaptive controllers with dual properties has been comprehensively developed and investigated in the present work as an alternative approach to the CE assumption. This approach can be widely used in adaptive control. It is based on a compromised minimization of two performance indices (control losses and uncertainty index) corresponding to the two goals of optimal adaptive dual control and leads to computationally simple control algorithms with improved performance for systems of various structures and different adaptation laws. The results presented include the synthesis of various adaptive controllers, convergence analysis and comparisons of the adaptive dual control algorithms, numerous simulations as well as practical applications.

Further development of the bicriterial approach in the direction of improvement of robust adaptive controllers for systems with unmodeled dynamics and of various continuous-time adaptive systems should be the field of future research.

N.M. Filatov and H. Unbehauen: Adaptive Dual Control, LNCIS 302, p. 206, 2004.

APPENDIX A

Derivation of the PCE Control for Linear Systems

To derive the PCE control law according to eqs. (8.54) to (8.57) for linear systems described by eqs. (8.47) to (8.49), the minimum the cost functional of eq. (8.50) under the conditions of ρ-approximation (Filatov and Unbehauen, 1995b) according to eq. (8.52) has to be found. The system is considered at the sampling instant k as in eq. (8.53), and the minimization of the performance index is started from the last stage N-1:

$$\begin{aligned}
J_{N-1}(\Im_{N-1},\rho_k^p) = \min_{\boldsymbol{u}_{N-1}} \Big[& \mathrm{E}_{\rho_k^p}\big\{[\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\boldsymbol{x}(N-1) \\
& +\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\boldsymbol{u}(N-1)]^{\mathrm{T}}\boldsymbol{V}_{N-1}[\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\boldsymbol{x}(N-1) \\
& +\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\boldsymbol{u}(N-1)]+\boldsymbol{u}^{\mathrm{T}}(N-1)\boldsymbol{R}_{N-1}\boldsymbol{u}(N-1)\big|\Im_{N-1}\big\} \\
& +\mathrm{tr}\big\{\boldsymbol{V}_N\boldsymbol{Q}_{\xi,N-1}\big\}\Big] \\
= \min_{\boldsymbol{u}_{N-1}} \Big[& \mathrm{E}_{\rho_k^p}\Big\{\boldsymbol{x}^{\mathrm{T}}(N-1)\mathrm{E}\big\{\boldsymbol{B}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\big|\Im_k\big\} \\
& \cdot\boldsymbol{x}(N-1) \\
& +2\boldsymbol{u}^{\mathrm{T}}(N-1)\mathrm{E}_{\rho_k^p}\big\{\boldsymbol{B}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\big|\Im_k\big\}\boldsymbol{x}(N-1) \\
& +\boldsymbol{u}^{\mathrm{T}}(N-1)\Big[\boldsymbol{R}_{N-1}+\mathrm{E}_{\rho_k^p}\big\{\boldsymbol{B}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\big|\Im_k\big\}\Big] \\
& \cdot\boldsymbol{u}(N-1)+\mathrm{tr}\big\{\boldsymbol{V}_N\boldsymbol{Q}_{\xi,N-1}\big\}\Big],
\end{aligned} \tag{A.1}$$

where $\boldsymbol{Q}_{\xi,i}=\mathrm{E}\{\xi(i)\xi^{\mathrm{T}}(i)\}$. After the minimization of eq. (A.1), it follows that

$$\boldsymbol{u}(N-1,\rho_k^p) = -\boldsymbol{K}_{N-1}(\rho_k^p)\hat{\boldsymbol{x}}(N-1,\rho_k^p)\,, \tag{A.2}$$

where

$$\hat{\boldsymbol{x}}(N-1,\rho_k^p) = \mathrm{E}_{\rho_k^p}\big\{\boldsymbol{x}(N-1)\big|\Im_{N-1}\big\}, \tag{A.3}$$

and

$$\begin{aligned}
\boldsymbol{K}_{N-1}(\rho_k^p) = & \Big[\boldsymbol{R}_{N-1}+\mathrm{E}_{\rho_k^p}\big\{\boldsymbol{B}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\big|\Im_k\big\}\Big]^{-1} \\
& \cdot\mathrm{E}_{\rho_k^p}\big\{\boldsymbol{B}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\big|\Im_k\big\}.
\end{aligned} \tag{A.4}$$

The equation of the performance index can be obtained after introducing eqs. (A.2) and (A.4) into eq. (A.1)

$$J_{N-1}(\Im_{N-1},\rho_k^p)=\mathrm{E}_{\rho_k^p}\{\boldsymbol{x}^{\mathrm{T}}(N-1)\boldsymbol{L}_1(\rho_k^p)\boldsymbol{x}(N-1)+g_1(\rho_k^p)|\Im_{N-1}\}, \tag{A.5}$$

where

$$\boldsymbol{L}_1(\rho_k^p)=\mathrm{E}_{\rho_k^p}\{\boldsymbol{A}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\mid\Im_k\}-\boldsymbol{F}_1(\rho_k^p) \tag{A.6}$$

$$\begin{aligned}\boldsymbol{F}_1(\rho_k^p)=\mathrm{E}_{\rho_k^p}&\left\{\boldsymbol{A}_{N-1}^{\mathrm{T}}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\middle|\Im_k\right\}\\&\cdot\left[\boldsymbol{R}_{N-1}+\mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-1}^{T}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{B}_{N-1}(\boldsymbol{p}(N-1))\middle|\Im_k\right\}\right]^{-1}\\&\cdot\mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-1}^{T}(\boldsymbol{p}(N-1))\boldsymbol{V}_N\boldsymbol{A}_{N-1}(\boldsymbol{p}(N-1))\middle|\Im_k\right\},\end{aligned} \tag{A.7}$$

and

$$\begin{aligned}g_1(\rho_k^p)=\mathrm{tr}\{\boldsymbol{V}_N\boldsymbol{Q}_{\xi,N-1}\}+\mathrm{E}_{\rho_k^p}&\left\{[\boldsymbol{x}(N-1)-\hat{\boldsymbol{x}}(N-1,\rho_k^p)]^{\mathrm{T}}\boldsymbol{F}_1(\rho_k^p)(\boldsymbol{x}(N-1)\right.\\&\left.-\hat{\boldsymbol{x}}(N-1,\rho_k^p))\middle|\Im_{N-1}\right\}.\end{aligned} \tag{A.8}$$

There is no dependence on the control inputs and observations in eq. (A.8) due to the conditions of the ρ-approximation according to eq. (8.52). The next stage is considered in back-shifted time:

$$\begin{aligned}J_{N-2}(\Im_{N-2},\rho_k^p)=\min_{\boldsymbol{u}_{N-2}}\mathrm{E}_{\rho_k^p}&\left\{\boldsymbol{x}^{\mathrm{T}}(N-1)(\boldsymbol{V}_{N-1}+\boldsymbol{L}_1(\rho_k^p))\boldsymbol{x}(N-1)\right.\\&\left.+\boldsymbol{u}^{\mathrm{T}}(N-2)\boldsymbol{R}_{N-2}\boldsymbol{u}(N-2)+g_1(\rho_k^p)\middle|\Im_{N-1}\right\}.\end{aligned} \tag{A.9}$$

Taking the expectation in the last equation and minimizing the cost functional finally leads to

$$\boldsymbol{u}(N-1,\rho_k^p)=-\boldsymbol{K}_{N-2}(\rho_k^p)\hat{\boldsymbol{x}}(N-2,\rho_k^p), \tag{A.10}$$

where

$$\hat{\boldsymbol{x}}(N-2,\rho_k^p)=\mathrm{E}_{\rho_k^p}\{\boldsymbol{x}(N-2)|\Im_{N-2}\}, \tag{A.11}$$

$$\begin{aligned}\boldsymbol{K}_{N-2}(\rho_k^p)=&\left[\boldsymbol{R}_{N-2}+\mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))[\boldsymbol{V}_{N-1}+\boldsymbol{L}_1(\rho_k^p)]\boldsymbol{B}_{N-2}(\boldsymbol{p}(N-2))\middle|\Im_k\right\}\right]^{-1}\\&\cdot\mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))[\boldsymbol{V}_{N-1}+\boldsymbol{L}_1(\rho_k^p)]\boldsymbol{A}_{N-2}(\boldsymbol{p}(N-2))\middle|\Im_k\right\}.\end{aligned} \tag{A.12}$$

The performance index is defined in the same way as for eq. (A.5)

$$J_{N-2}(\Im_{N-2},\rho_k^p)=\mathrm{E}_{\rho_k^p}\{\boldsymbol{x}^{\mathrm{T}}(N-2)\boldsymbol{L}_2(\rho_k^p)\boldsymbol{x}(N-2)+g_2(\rho_k^p)|\Im_{N-2}\}, \tag{A.13}$$

where

$$\boldsymbol{L}_2(\rho_k^p) = \mathrm{E}_{\rho_k^p}\{\boldsymbol{A}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))[\boldsymbol{V}_{N-1} + \boldsymbol{L}_1(\rho_k^p)]\boldsymbol{A}_{N-2}(\boldsymbol{p}(N-2)) \mid \Im_k\} - \boldsymbol{F}_2(\rho_k^p), \tag{A.14}$$

$$\begin{aligned} \boldsymbol{F}_2(\rho_k^p) = {} & \mathrm{E}_{\rho_k^p}\left\{\boldsymbol{A}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))[\boldsymbol{V}_{N-1} + \boldsymbol{L}_1(\rho_k^p)]\boldsymbol{B}_{N-2}(\boldsymbol{p}(N-2)) \middle| \Im_k\right\} \\ & \cdot\left[\boldsymbol{R}_{N-2} + \mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))[\boldsymbol{V}_{N-1} + \boldsymbol{L}_1(\rho_k^p)]\boldsymbol{B}_{N-2}(\boldsymbol{p}(N-2)) \middle| \Im_k\right\}\right]^{-1} \\ & \cdot \mathrm{E}_{\rho_k^p}\left\{\boldsymbol{B}_{N-2}^{\mathrm{T}}(\boldsymbol{p}(N-2))(\boldsymbol{V}_{N-1} + \boldsymbol{L}_1(\rho_k^p))\boldsymbol{A}_{N-2}(\boldsymbol{p}(N-2)) \middle| \Im_k\right\}, \end{aligned} \tag{A.15}$$

and

$$\begin{aligned} g_2(\rho_k^p) = {} & g_1(\rho_k^p) + \mathrm{tr}\left\{(\boldsymbol{V}_{N-1} + \boldsymbol{L}_1(\rho_k^p))\boldsymbol{Q}_{\xi,N-2}\right\} \\ & + \mathrm{E}_{\rho_k^p}\left\{[\boldsymbol{x}(N-2) - \hat{\boldsymbol{x}}(N-2,\rho_k^p)]^{\mathrm{T}}\boldsymbol{F}_1(\rho_k^p)[\boldsymbol{x}(N-2) - \hat{\boldsymbol{x}}(N-2,\rho_k^p)] \middle| \Im_{N-2}\right\}. \end{aligned} \tag{A.16}$$

The minimization is continued in the same way for the next stages up to the *k*-th back-shifted time instant. Thus, the control algorithm according to eqs. (8.54) to (8.57) is obtained.

APPENDIX B

Proof of Lemmas and Theorem of Stability of Robust Adaptive Dual Control

Proof of Lemma 13.1

Proof of the statements of this lemma is given by Praly (1983). It is based on the Lyapunov function for the estimated parameters

$$V(k) = [\hat{\boldsymbol{p}}(k) - \boldsymbol{p}]^{\mathrm{T}} \boldsymbol{P}^{-1}(k)[\hat{\boldsymbol{p}}(k) - \boldsymbol{p}] \tag{B.1}$$

and the representation of eq. (13.50) in the form

$$\overline{\boldsymbol{P}}^{-1}(k) = \boldsymbol{P}^{-1}(k-1) + \frac{g(k)\boldsymbol{m}^{\mathrm{T}}(k-1)\boldsymbol{m}(k-1)}{1 - g(k)\boldsymbol{m}^{\mathrm{T}}(k-1)\boldsymbol{P}(k-1)\boldsymbol{m}(k-1)}. \tag{B.2}$$

From eqs. (13.5), (13.8), (13.50), (13.54) and (B.2), the relations

$$\overline{V}(k) = V(k-1) + g(k)\left(\frac{\xi^2(k)}{1 - \boldsymbol{m}^{\mathrm{T}}(k-1)\boldsymbol{P}(k-1)\boldsymbol{m}(k-1)} - e^2(k)\right), \tag{B.3}$$

$$V(k) \le \overline{V}(k), \tag{B.4}$$

$$\left\|\hat{\boldsymbol{p}}(k) - \boldsymbol{p}\right\| \le \rho(k) + \rho_0, \tag{B.5}$$

and

$$\left\|\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-1)\right\| \le (2+\tau)g(k)\left|e(k)\right|\left\|\boldsymbol{P}(k-1)\boldsymbol{m}(k-1)\right\| \tag{B.6}$$

are obtained. The above relations follow directly from eqs. (13.5), (13.8), (13.50), (13.54) and (B.2). The boundedness of the estimates according to eq. (13.60) follows from inequality (B.5). Thus, taking also into account the boundedness of $\boldsymbol{P}(k)$ as in eq. (13.55), we can conclude that $V(k)$ is bounded. Then, using the last relations of eq. (13.58), we get the results of eqs. (13.61) and (13.62), as shown by Praly (1983).

The proof of eqs. (13.63) and (13.64) is based on the differentiability of eq. (13.53) under the assumption of eq. (13.44).

Proof of Lemma 13.2

From eq. (13.46) it follows that

$$a(k) = -\frac{1}{\hat{b}_1^2(k)}\; p_{b_1 p}^{\mathrm{T}}(k)\; m(k) + \frac{1}{\hat{b}_1^2(k)}\; [\hat{b}_1^2(k) + p_{b_1}(k)]\; u_a(k)\,.$$

This equation results in the inequality

$$\|\alpha(k)\| \leq \left\| \frac{1}{\hat{b}_1^2(k)} \boldsymbol{p}_{b_1 p}^{\mathrm{T}}(k)\boldsymbol{m}(k) \right\| + \left\| \frac{1}{\hat{b}_1^2(k)} (\hat{b}_1^2(k) + p_{b_1}(k)) u_a(k) \right\| . \tag{B.7}$$

From relation (13.55) and the boundedness of $u_a(k)$, inequalitiy (13.67) follows directly from (B.7).

Proof of Theorem 13.1.

The proof of this theorem is based on the approach presented by Praly (1983) with some modifications. These modifications are due to the nonlinear dual controller applied here according to eq. (13.40) and the results of Lemma 13.2. Utilization of the results of Lemma 13.1, inequalities (13.69), (13.79) and the definitions of $\boldsymbol{m}(k)$ and $\boldsymbol{x}(k)$ yields

$$\|\boldsymbol{\Delta F}_k \boldsymbol{x}(k)\|_\eta \leq \frac{1}{\gamma_1} \|\boldsymbol{\Delta F}_k \boldsymbol{x}(k)\|_\eta \leq \frac{\gamma_2}{\gamma_1} \|\boldsymbol{\Delta F}_k\| \cdot \|\boldsymbol{x}(k)\|_\eta , \tag{B.8}$$

$$\|\overline{\boldsymbol{P}}_k \Delta(k)\|_\eta \leq \gamma_2 M_{p_1} \|\Delta(k)\| , \tag{B.9}$$

$$\|\boldsymbol{P}_k \overline{w}(k)\|_\eta \leq \gamma_2 M_p \|\overline{w}(k)\| , \tag{B.10}$$

$$\|\boldsymbol{P}_k \overline{\overline{\alpha}}(k)\|_\eta \leq \gamma_2 M_p \|\overline{\overline{\alpha}}(k)\| , \tag{B.11}$$

and

$$\|\boldsymbol{x}(k)\|_\eta \geq \gamma_1 \|\boldsymbol{m}(k)\| . \tag{B.12}$$

Here, a norm of a matrix is a sum of norms of matrix elements. From inequality (13.61) and eq. (13.68), it is clear that

$$\sum_{k=q+1}^{q+m} \frac{\|\Delta(k)\|}{s(k)} \leq \frac{1-\sigma^n}{\sigma^{n-1}(1-\sigma)} \left(\sqrt{m} M_e + m L_e \eta_\xi \right), \forall (q,m) . \tag{B.13}$$

Introducing the notation

$$x(k) = \|\boldsymbol{x}(k)\|_\eta \tag{B.14}$$

and applying the inequalities (B.8) to (B.12) in eqs. (13.6) and (13.79) leads to

$$x(k+1) \leq \left(\zeta + \gamma \|\boldsymbol{\Delta F}_k\|\right) x(k) + \gamma_2 \left(M_{p_1} \frac{\|\Delta(k)\|}{s(k)} s(k) + M_p \|\overline{w}(k)\| + M_p \|\overline{\overline{\alpha}}(k)\| \right), \tag{B.15}$$

and

$$s(k) \leq \sigma s(k-1) + \frac{1}{\gamma_1} x(k) + s . \tag{B.16}$$

Boundedness of $\|\Delta F_k\|$ and $\dfrac{\|\Delta(k)\|}{s(k)}$ follows from the inequalities (B.13), (13.78), Lemma 13.1 and eqs. (13.60) to (13.63). Introducing the results of Lemma 13.2 to inequality (B.15) results in

$$x(k+1) \le \left(\zeta + \gamma\|\Delta F_k\|\right)x(k) + \gamma_2\left(M_{p_1}\frac{\|\Delta(k)\|}{s(k)}s(k) + M_p\|\overline{w}(k)\| + L_{\alpha 1}\frac{\|\Delta(k)\|}{s(k)}s(k) + M_{\alpha 2}\right). \tag{B.17}$$

After insertion of eq. (B.12) into (B.13), it can be shown that there exists a positive M_x such that

$$\begin{aligned} x(k+1) \le{}& \left(\zeta + \gamma\|\Delta F_k\| + \gamma(M_{p_1} + L_{\alpha 1})\frac{\|\Delta(k)\|}{s(k)}\right)x(k) + \sigma\gamma_2(M_{p_1} \\ &+ L_{\alpha 1})\frac{\|\Delta(k)\|}{s(k)}s(k-1) + M_x . \end{aligned} \tag{B.18}$$

Now inequality (B.18) can be written in the form

$$\varphi(k+1) \le \psi(k)\varphi(k) + M_\varphi , \tag{B.19}$$

where M_φ is a positive scalar,

$$\varphi(k) = x(k) + \gamma_1\sigma(1-\zeta)s(k-1) , \tag{B.20}$$

and

$$\psi(k) = \zeta + \gamma\|\Delta F_k\| + \frac{\gamma}{1-\zeta}(M_{p_1} + L_{\alpha 1})\frac{\|\Delta(k)\|}{s(k)} + \sigma(1-\zeta). \tag{B.21}$$

Using eq. (13.78) and inequalities (13.60) and (13.63), it can be concluded that

$$M_{p_1}\sum_{i=0}^{n-1}\|\hat{\boldsymbol{p}}(k) - \hat{\boldsymbol{p}}(k-i-1)\| + M_p\sum_{i=1}^{n}\left\|\hat{\overline{\boldsymbol{p}}}_1(k) - \hat{\overline{\boldsymbol{p}}}_1(k-i)\right\| \ge \|\Delta F_k\| . \tag{B.22}$$

From inequalities (13.62), (13.64) and (B.22) it follows that

$$\sum_{k=q+1}^{q+m}\|\Delta F_k\| \le n^2\left(M_{p_1} + L_{\alpha 1} + L_{p_1}M_p\right)\left(\sqrt{m}\overline{M}_p + mL_p\eta_\xi\right). \tag{B.23}$$

According to the results presented by Praly (1983), the sequence $\varphi(k)$ described by (B.19) is bounded if

$$\sum_{k=q+1}^{q+m}\psi(k) \le \sqrt{m}M_\psi + m\eta_\psi, \forall(q,m) \tag{B.24}$$

and

$$0 \le \eta_\zeta < 1. \tag{B.25}$$

It can be shown that $\psi(k)$ described by eq. (B.21) satisfies the conditions of inequalities (B.20) and (B.21) if

$$M_\zeta = \gamma n^2 (M_{p_1} + L_{p_1} M_p)\overline{M}_p + \frac{\gamma(1-\sigma^n)}{\sigma^{n-1}(1-\zeta)(1-\sigma)}(M_{p_1} + L_{\alpha 1})M_e, \tag{B.26}$$

and

$$1 > \eta_\psi = \zeta + \sigma(1-\zeta) + \gamma\eta_\xi\left[(M_{p_1} + L_{p_1} M_p)n^2 L_p + \frac{(1-\sigma^n)}{\sigma^{n-1}(1-\zeta)(1-\sigma)}(M_{p_1} + L_{\alpha 1})L_e\right]. \tag{B.27}$$

Therefore, from the inequalities (13.80) and (B.27) it follows that the conditions (B.24) and (B.25) are met and that $\varphi(k)$ given in (B.19) is bounded. Thus, $x(k)$ and $s(k)$ are bounded and if $\eta_\xi = 0$ Eq. (13.82) is obtained from eq. (13.74) using inequalities (13.61) and (13.62) and the fact that $\lim_{k \to \infty} \theta(k) = 0$. In this way, the theorem is proved.

APPENDIX C

MATLAB Programs for Solving the Diophantine Equation

The programs for solving the Diophantine equation, which are used in the adaptive controllers presented in Chapter 15, are listed below.

```
function [X,Y] = polydiop(A,B,C)

% -----------------------------------------------------
% [X,Y] = polydiop(A,B,C)
%
% POLYDIOP calculates X(1/z) and Y(1/z) that satisfy the
Diophantine equation
%
%        A(1/z) X(1/z) + B(1/z) Y(1/z) = C(1/z).
%
% The orders of the polymonials are given as
%
%        nY = nA -1,
%        nX = max(nC-nA, nB-nA).
%
% Note that  A, B and C must be independent
% -----------------------------------------------------

% All polynomials will be stored as vectors
zeile = 0;
[nA1,tmp] = size(A); if (nA1 < tmp), A = A'; nA1 = tmp;
line=1; end;
[nB1,tmp] = size(B); if (nB1 < tmp), B = B'; nB1 = tmp;
line=1; end;
[nC1,tmp] = size(C); if (nC1 < tmp), C = C'; nC1 = tmp;
line=1; end;
nA = nA1 -1;
```

```
nB = nB1 -1;
nC = nC1 -1;

if nA == 0,
   % Y-polynomial is not necessary
   X = C/A;
   Y = 0;
else
   % Solve for the minimal order Y(1/z)
   nY1 = nA;
   nX1 = max([nC-nA nB-1]) + 1;

   % System of equations to be solved
   % D [x y]' = C
   nD = nX1 + nY1;
   mD = nD;
   toepl(A,nD,nX1);
   toepl(B,nD,nY1);
   D  = [toepl(A,nD,nX1) toepl(B,nD,nY1)];
   C = [C; zeros(mD-nC1,1)];

   % Solve the Diophantine equation
   x = D\C;
   X = x(1:nX1);
   Y = x(nX1+1:nX1+nY1);
end

% error = C - conv(A,X) - [conv(B,Y); zeros(nC1-nB-
nY1)];
% error = sqrt(error '* error)

if line,
   X = X';
   Y = Y';
end
```

```
function t = toepl(c,n,m)

%  TOEPL(c,n,m) is a non symmetric Toeplitz matrix hav-
ing C as its
%  first column/row. The matrix is expanded to n columns
and m rows.
%
%  RUB-ESR

t = zeros(n,m);
if is empty(c),
   return;
end;
[nc,mc] = size(c);

if (nc >= mc)
   %  c is a column vector
   for i=1:min(n,m)
      k = min(n-i+1,nc);
      t(i:i+k-1,i) = c(1:k);    % Fill in the i.th col-
umn
   end;
else
   %  c is a row vector
   for i=1:min(n,m)
      k = min(m-i+1,mc);
      t(i,i:i+k-1) = c(1:k);    % Fill in the i.th row
   end;
end
```

APPENDIX D

Calculation of Mathematical Expectation

The calculation of the mathematical expectation (or simply the expectation) has been used several times in this book, especially when derivating the control laws based on the specific formulation of the cost functions or performance indices. Two typical examples, eqs. (15.36) and (15.41) are presented in the following.

Proof of Equation (15.36)

$$
\begin{aligned}
J_k^{\mathrm{c}} &= \mathrm{E}\{(\hat{y}_{\mathrm{n}}(k+1) - y(k+1))^2 \mid \Im_k\} \\
&= \mathrm{E}\{(\hat{b}_1(k) u_{\mathrm{ND}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) - y(k+1))^2 \mid \Im_k\} \\
&= \mathrm{E}\{\hat{b}_1^2(k) u_{\mathrm{ND}}^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k) \hat{\boldsymbol{p}}_0(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) + y^2(k+1) \\
&\quad + 2\hat{b}_1(k) u_{\mathrm{ND}}(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) - 2\hat{b}_1(k) u_{\mathrm{ND}}(k)\, y(k+1) \\
&\quad - 2\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k)\, y(k+1) \mid \Im_k\}
\end{aligned}
\tag{D.1}
$$

$$
\begin{aligned}
&= \mathrm{E}\{\hat{b}_1^2(k) u_{\mathrm{ND}}^2(k) \mid \Im_k\} + \mathrm{E}\{\boldsymbol{m}_0^{\mathrm{T}}(k) \hat{\boldsymbol{p}}_0(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) \mid \Im_k\} \\
&\quad + \mathrm{E}\{y^2(k+1) \mid \Im_k\} + \mathrm{E}\{2\hat{b}_1(k)\, u_{\mathrm{ND}}(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) \mid \Im_k\} \\
&\quad + \mathrm{E}\{-2\hat{b}_1(k) u_{\mathrm{ND}}(k)\, y(k+1) \mid \Im_k\} \\
&\quad + \mathrm{E}\{-2\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k)\, y(k+1) \mid \Im_k\}.
\end{aligned}
\tag{D.2}
$$

These terms can be calculated independently from each other as follows:

$$
\mathrm{E}\{\hat{b}_1^2(k) u_{\mathrm{ND}}^2(k) \mid \Im_k\} = \hat{b}_1^2(k) u_{\mathrm{ND}}^2(k)\,, \tag{D.3}
$$

$$
\mathrm{E}\{\boldsymbol{m}_0^{\mathrm{T}}(k) \hat{\boldsymbol{p}}_0(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k) \mid \Im_k\} = \boldsymbol{m}_0^{\mathrm{T}}(k) \hat{\boldsymbol{p}}_0(k) \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k)\,, \tag{D.4}
$$

$$
\mathrm{E}\{2\hat{b}_1(k) u_{\mathrm{ND}}(k) \hat{\boldsymbol{p}}_0(k) \boldsymbol{m}_0(k) \mid \Im_k\} = 2\hat{b}_1(k) u_{\mathrm{ND}}(k) \hat{\boldsymbol{p}}_0(k) \boldsymbol{m}_0(k)\,, \tag{D.5}
$$

$$
\begin{aligned}
\mathrm{E}\{-2\hat{b}_1(k) u_{\mathrm{ND}}(k)\, y(k+1) \mid \Im_k\} &= -2\hat{b}_1(k) u_{\mathrm{ND}}(k) \mathrm{E}\{\mathrm{y(k+1)}\} \\
&= -2\hat{b}_1(k) u_{\mathrm{ND}}(k) \mathrm{E}\{\boldsymbol{p}_1^{\mathrm{T}}(k) \boldsymbol{m}_1(k)\} \\
&= -2\hat{b}_1(k) u_{\mathrm{ND}}(k)\, \boldsymbol{p}_1^{\mathrm{T}}(k) \boldsymbol{m}_1(k) \\
&= -2\hat{b}_1(k) u_{\mathrm{ND}}(k) [\hat{b}_1(k) \mid \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)] \begin{bmatrix} u(k) \\ \boldsymbol{m}_0^{\mathrm{T}}(k) \end{bmatrix} \\
&= -2\hat{b}_1^2(k) u_{\mathrm{ND}}(k) u(k) - 2\hat{b}_1(k) u_{\mathrm{ND}}(k)\, \boldsymbol{p}_0^{\mathrm{T}}(k) \boldsymbol{m}_0(k)\,,
\end{aligned}
\tag{D.6}
$$

$$
\begin{aligned}
\mathrm{E}\{-2\hat{\boldsymbol{p}}_0(k)\boldsymbol{m}_0(k)\,y(k+1)\,|\,\Im_k\} &= -2\hat{\boldsymbol{p}}_0(k)\boldsymbol{m}_0(k)\mathrm{E}\{\,y(k+1)\} \\
&= -2\hat{\boldsymbol{p}}_0(k)\boldsymbol{m}_0(k)\mathrm{E}\{\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)\} \\
&= -2\hat{\boldsymbol{p}}_0(k)\boldsymbol{m}_0(k)\,\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k) \\
&= -2\hat{\boldsymbol{p}}_0(k)\boldsymbol{m}_0(k)\hat{b}_1(k)u(k) - 2\boldsymbol{m}_0^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_0(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k),
\end{aligned}
\tag{D.7}
$$

$$
\begin{aligned}
\mathrm{E}\{\,y^2(k+1)\,|\,\Im_k\} &= \mathrm{var}\{\,y(k+1)\} + \mathrm{E}^2\{\,y(k+1)\} \\
&= \mathrm{E}\{(\,y(k+1) - \mathrm{E}\{\,y(k+1)\})^2\} + \mathrm{E}^2\{\,y(k+1)\} \\
&= \mathrm{E}\{[\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k) + \xi(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)] \\
&\quad \cdot [\boldsymbol{p}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k) + \xi(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)]\} + \mathrm{E}^2\{\,y(k+1)\} \\
&= \boldsymbol{m}_1^{\mathrm{T}}(k)\mathrm{E}\{[\boldsymbol{p}_1^{\mathrm{T}}(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)]^{\mathrm{T}}[\boldsymbol{p}_1^{\mathrm{T}}(k) - \hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)]\}\boldsymbol{m}_1(k) \\
&\quad + \sigma_\xi^2 + \mathrm{E}^2\{\,y(k+1)\} \\
&= \boldsymbol{m}_1^{\mathrm{T}}(k)\boldsymbol{P}_1(k)\boldsymbol{m}_1(k) + \sigma_\xi^2 + \mathrm{E}^2\{\,y(k+1)\} \\
&= [u(k) \,\vdots\, \boldsymbol{m}_0^{\mathrm{T}}(k)]\begin{bmatrix} p_{b_1}(k) & \boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k) \\ \boldsymbol{p}_{b_1\boldsymbol{p}_0}(k) & \boldsymbol{P}_{\boldsymbol{p}_0}(k) \end{bmatrix}\begin{bmatrix} u(k) \\ \boldsymbol{m}_0(k) \end{bmatrix} + \sigma_\xi^2 + \mathrm{E}^2\{\,y(k+1)\} \\
&= [p_{b_1}(k)u(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\boldsymbol{p}_{b_1\boldsymbol{p}_0}(k) \,\vdots\, \boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k)u(k) \\
&\quad + \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{\boldsymbol{p}_0}(k)]\begin{bmatrix} u(k) \\ \boldsymbol{m}_0(k) \end{bmatrix} + \sigma_\xi^2 + \mathrm{E}^2\{\,y(k+1)\} \\
&= p_{b_1}(k)u^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\boldsymbol{p}_{b_1\boldsymbol{p}_0}(k)u(k) + \boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) \\
&\quad + \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{\boldsymbol{p}_0}(k)\boldsymbol{m}_0(k) + \sigma_\xi^2 + \boldsymbol{m}_1^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_1\,\hat{\boldsymbol{p}}_1^{\mathrm{T}}\boldsymbol{m}_1(k)\,.
\end{aligned}
\tag{D.8}
$$

The signal $\xi(k)$ can be interpreted as a zero-mean Gaussian white measuring noise with variance σ_ξ^2. Substituting eqs. (D.3) to (D.8) into eq. (D.2) we obtain

$$
\begin{aligned}
J_k^{\mathrm{c}} &= \hat{b}_1^2(k)u_{\mathrm{ND}}^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_0(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) + 2\hat{b}_1(k)u_{\mathrm{ND}}(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) \\
&\quad - 2\hat{b}_1^2(k)u_{\mathrm{ND}}(k)\,u(k) - 2\hat{b}_1(k)u_{\mathrm{ND}}(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) \\
&\quad - 2\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\hat{b}_1(k)u(k) - 2\boldsymbol{m}_0^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_0(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) \\
&\quad + p_{b_1}(k)u^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\boldsymbol{p}_{b_1\boldsymbol{p}_0}(k)u(k) + \boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) \\
&\quad + \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{\boldsymbol{p}_0}(k)\boldsymbol{m}_0(k) + \sigma_\xi^2 + \boldsymbol{m}_1^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_1\,\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k)
\end{aligned}
\tag{D.9}
$$

and recalling

$$\boldsymbol{m}_1^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_1\,\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k) = \hat{b}_1^2(k)u^2(k) + \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k)\hat{b}_1(k)u(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_0(k)\hat{b}_1(k)u(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\hat{\boldsymbol{p}}_0(k)\,\hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)\boldsymbol{m}_0(k) \tag{D.10}$$

yields the solution for the performance index

$$\begin{aligned} J_k^{\mathrm{c}} &= \hat{b}_1^2(k)u_{\mathrm{ND}}^2(k) - 2\hat{b}_1^2(k)u_{\mathrm{ND}}(k)\,u(k) + p_{b_1}u^2(k) + 2\boldsymbol{p}_{b_1\boldsymbol{p}_0}(k)\boldsymbol{m}_0(k)\,u(k) \\ &\quad + \hat{b}_1^2(k)u^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\,\boldsymbol{P}_{P_0}(k)\boldsymbol{m}_0(k) + \sigma_\xi^2 \\ &= [p_{b_1}(k) + \hat{b}_1^2(k)]\,u^2(k) + [2\boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k) - 2\hat{b}_1^2(k)u_{\mathrm{ND}}(k)]\,u(k) \\ &\quad + \hat{b}_1^2(k)u_{\mathrm{ND}}^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)\boldsymbol{P}_{\boldsymbol{p}_0}(k)\boldsymbol{m}_0(k) + \sigma_\xi^2\,, \end{aligned} \tag{D.11}$$

where the covariance matrix $\boldsymbol{P}_1(k)$ is defined by eq. (15.36).

Proof of Equation (15.40)

$$J_k^{\mathrm{a}}\{u(k)\} = -\mathrm{E}\{[\hat{\boldsymbol{p}}_1^{\mathrm{T}}(k)\boldsymbol{m}_1(k) + \xi(k) - \hat{\boldsymbol{p}}^{\mathrm{T}}(k)\boldsymbol{m}_1(k)]^2 \mid \Im_k\} \tag{D.12}$$

$$\begin{aligned} &= -\mathrm{E}\{[(b_1(k) - \hat{b}_1(k))u(k) + (\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k))^{\mathrm{T}}\boldsymbol{m}_0(k) + \xi(k)]^2 \mid \Im_k\} \\ &= -\mathrm{E}\{[b_1(k) - \hat{b}_1(k)]^2u^2(k) + \boldsymbol{m}_0^{\mathrm{T}}(k)[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)] \\ &\qquad \cdot[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k) + \xi^2(k) + 2[b_1(k) - \hat{b}_1(k)]u(k) \\ &\qquad \cdot[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k) + 2[b_1(k) - \hat{b}_1(k)]u(k)\xi(k) \\ &\qquad + 2[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k)\xi(k) \mid \Im_k\} \\ &= -\mathrm{E}\{[b_1(k) - \hat{b}_1(k)]^2u^2(k) \mid \Im_k\} \\ &\qquad - \mathrm{E}\{2[b_1(k) - \hat{b}_1(k)]u(k)\,[\boldsymbol{p}_0^{\mathrm{T}}(k) - \hat{\boldsymbol{p}}_0^{\mathrm{T}}(k)]\boldsymbol{m}_0(k)\} \\ &\qquad - \underline{\mathrm{E}\{\boldsymbol{m}_0^{\mathrm{T}}(k)[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]\,[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k)\}} - \underline{\mathrm{E}\{\xi^2(k)\}} \\ &\qquad - \underline{\mathrm{E}\{2[b_1(k) - \hat{b}_1(k)]u(k)\xi(k)\}} \\ &\qquad - \underline{\mathrm{E}\{2[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k)\xi(k) \mid \Im_k\}} \\ &= -\mathrm{E}\{[b_1(k) - \hat{b}_1(k)]^2u^2(k) \mid \Im_k\} \\ &\qquad - \mathrm{E}\{2[b_1(k) - \hat{b}_1(k)]u(k)\,[\boldsymbol{p}_0(k) - \hat{\boldsymbol{p}}_0(k)]^{\mathrm{T}}\boldsymbol{m}_0(k)\} + \bar{c}_1(k) \\ &= -p_{b_1}(k)u^2(k) - 2\boldsymbol{p}_{b_1\boldsymbol{p}_0}^{\mathrm{T}}(k)\boldsymbol{m}_0(k)u(k) + \bar{c}_1(k)\,. \end{aligned} \tag{D.13}$$

The above underlined terms are independent of $u(k)$ and are summarized in the variable $\bar{c}_1(k)$.

REFERENCES

Alster J. and P.R. Belanger (1974). A technique of adaptive dual control. *Automatica*, **10**, 627-634.

Allison B.J., P.J.-C. Tessier and G.A. Dumont (1995a). Comparison of suboptimal dual adaptive controllers. *Proc. 3rd European Control Conferences ECC 95*, Rome, 36-43.

Allison B.J., J.E. Ciarniello, P.J.-C. Tessier and G.A. Dumont (1995b). Dual adaptive control of chip refiner motor load. *Automatica*, **31**, 1169-1184.

Aoki M. (1967). *Optimization of Stochastic Systems.* New York: Academic Press.

Åström K.J. (1987). Adaptive feedback control. *Proc. of the IEEE*, **75**, 2, 185-217.

Åström K.J., Borrison U., Ljung L. and B. Wittenmark (1977). Theory and applications of self-tuning regulators. *Automatica*, **17**, 457-476.

Åström K.J. and B. Wittenmark (1971). Problems of identification and control. *J. Math. Anal.*, **34**, 90-113.

Åström K.J. and B. Wittenmark (1989). *Adaptive Control*. New York: Addison-Wesley.

Åström K.J. and B. Wittenmark (1996). *Computer-Controlled Systems: Theory and Design*. London: Prentice-Hall.

Bar-Shalom Y. (1981). Stochastic dynamic programming: caution and probing. *IEEE Trans. Autom. Control*, **26**, 1184-1194.

Bar-Shalom Y. and E. Tse (1974). Dual effect, certainty equivalence and separation in stochastic control. *IEEE Trans. Autom. Control*, **AC-19**, 494-500.

Bar-Shalom Y. and E. Tse (1976). Concept and methods in stochastic control. In C.T. Leondes (ed.), *Control and Dynamic Systems.* New York: Academic Press, **12**, 99-172.

Bayard D.S. and M. Eslami (1985). Implicit dual control for general stochastic systems. *Optimal Control Applications and Methods,* **6**, 286-279.

Bertsekas D.P. (1976). *Dynamic Programming and Stochastic Control*. New York: Academic Press.

Birmiwal K. (1994). A new adaptive LQG control algorithm. *Int. J. Adapt. Control Signal Process.*, **8**, 287-295.

Bodyanskiy E.B. and T.V. Solovjova (1987). Synthesis of pseudodual adaptive controllers. *Dokljady AN Ukraine SSR*. Ser. A, **11**, 69-71.

Caines P.E. and S. Lafortune (1984). Adaptive control with recursive identification for stochastic linear systems. *IEEE Trans. Autom. Control*, **29**, 312-321.

Chan S.S. and M.B. Zarrop (1985). A suboptimal dual controller for stochastic systems with unknown parameters. *Int. J. Control,* **41**, 507-524.

Chen H.F. and L. Guo. (1986). Convergence rate of least-squares identification and adaptive control for stochastic systems. *Int. J. Control,* **44,** 1459-1476.

Clarke D.W. and P.J. Gawthrop (1975). Self-tuning controller. *Proc. IEE*, **122**, 929-934.

Clarke D.W., C. Mohtadi and P.S. Tuffs (1987). Generalized predictive control - Part I. The basic algorithm. *Automatica*, **23**, 137-148.

Curry R.E. (1969). A new algorithm for suboptimal stochastic control. *IEEE Trans. Autom. Control*, **14**, 533-536.

Cutler C.R. and B.C. Ramaker (1980). Dynamic matrix control - a computer control algorithm. *Proc. Joint Automatic Control Conference*, San Francisco, paper WP5-B.

De Keyser R.M.C., Ph. G.A. Van de Velde and F.A.G. Dumortier (1988). A comparative study of self-adaptive long-range predictive control. *Automatica*, **24**, 149-163.

Dreyfus S.E. (1962). Some types of optimal control of stochastic systems. *SIAM J. Control. Ser. A.*, **2**, 120-134.

Dumont and Åström K.J. (1988). Wood chip refiner control. *IEEE Control System Magazine*, 38-43.

Durbin J. (1960). Estimation of parameters in time series regression models. *J. R. Stat. Soc. B*, **22**, 139-153

Egardt B. (1979). Stability of adaptive controllers. In: *Lecture Notes in Control and Information Sciences,* New York: Springer.

Elliott H. (1982a). Direct adaptive pole placement with application to nonminimum phase systems. *IEEE Trans. Autom. Control*, **27**, 720-722.

Elliott H., W.A. Wolovich and M. Das (1984). Arbitrary adaptive pole placement for linear multivariable s stems. *IEEE Trans. Autom. Control* **29**

Elliott R.J. (1982b). *Stochastic Calculus and Applications*. New York: Springer.

Feldbaum A.A. (1960-61). Dual control theory I-IV. *Automation and Remote Control*, **21**, 874-880; **21**, 1033-1039; **22**, 1-12; **22**, 109-121.

Feldbaum A.A. (1965). *Optimal Control Systems*. New York: Academic Press.

Filatov N.M. (1996). Introduction to adaptive dual control. *Proc. EURACO Workshop on Control of Nonlinear Systems: Theory and Applications*, Algarve, Portugal, 97-112.

Filatov N.M., U. Keuchel and H. Unbehauen (1994). Application of active adaptive control to an unstable mechanical plant. *Prep. of 2nd IEEE Conference on Control Applications*, Glasgow, 1994, 989-994.

Filatov N.M., H. Unbehauen and U. Keuchel (1995). Dual version of direct adaptive pole placement controller, *Proc. 5th IFAC Symposium on Adaptive Systems in Control and Signal Processing*, Budapest, 449-454.

Filatov N.M., U. Keuchel and H. Unbehauen (1996). Dual control for unstable mechanical plant, *IEEE Control System Magazine*, **16**, 31-37.

Filatov N.M., H. Unbehauen and U. Keuchel (1997). Dual pole placement controller with direct adaptation. *Automatica*, **33**, 113-117.

Filatov N.M. and H. Unbehauen (1994). Strategies of model reference adaptive control with active learning properties. *Proc. of 2nd IFAC Symposium on Intelligent Components and Instruments in Control Applications*, Budapest, 305-310.

Filatov N.M. and H. Unbehauen (1995a). Estimation and adaptive control in systems with additional disturbances. *Proc. 3rd European Control Conferences ECC 95*, Rome, 65-70.

Filatov N.M. and H. Unbehauen (1995b). Adaptive predictive control policy for nonlinear stochastic systems. *IEEE Trans. Automatic Control*, **AC-40**, 1943-1949.

Filatov N.M. and H. Unbehauen (1996a). Indirect adaptive dual controllers. *Proc. World Automation Congress WAC*, Montpellier, France, **4** (Intelligent Automation and Control), 217-222.

Filatov N.M. and H. Unbehauen (1996b). Real-time dual control with indirect adaptive pole-placement. *Proc. UKACC International Conference CONTROL'96*, Exeter, 668-673.

Filatov N.M. and H. Unbehauen (1996c). Improved adaptive dual version of generalized minimum variance (GMV) controller. *Proc. 11th Yale Workshop on Application of Adaptive Systems Theory*, Yale University, 137-142.

Filatov N.M. and H. Unbehauen (1996d). Nonlinear adaptive control of a laboratory-scale vertical take-off aircraft. *Proc. EURACO Workshop II on Robust and Adaptive Control of Integrated Systems*, Herrsching, Germany, 27-34.

Filatov N.M. and H. Unbehauen (1999). Adaptive dual control for continuous-time systems: A simple example. *Proc. of IEE International Conference on Control*, Swansea, UK, Vol. 1, 39-43.

Filatov N.M. and H. Unbehauen (1999). Adaptive dual controller for systems with unmodelled effects. *IEE Proc. Control and Application*, **146**, 301-309.

Filatov N.M. and H. Unbehauen (2000). Survey of adaptive dual control methods. *IEE Proc. Control Theory and Applications*, **147**, 118-128.

Garcia C.E., D.M. Prett and M. Morari (1989). Model predictive control: theory and practice - a survey. *Automatica*, **25**, 335-348.

Goodwin G.C., P.J. Ramadge and P.E. Caines (1980). Discrete time mulivariable adaptive control. *IEEE Trans. Autom. Control*, **AC-25**, 449-456.

Goodwin G.C. and R. Payne (1977). *Dynamic System Identification: Experiment Design and Data Analysis*. New York: Academic Press.

Goodwin G.C. and K.S. Sin (1984). *Adaptive Filtering Prediction and Control.* Englewood Cliffs, New Jersey: Prentice Hall.

Greco C., G. Menga, E. Mosca and G. Zappa (1984). Performance improvements of self-tuning controllers by multistep horizons: the MUSMAR approach. *Automatica*, **20**, 681-699.

Ismail A., G. Dumont and J. Backstrom (2003). Dual adaptive control of a paper coating. *IEEE Trans. Systems Technol.*, **11**, 289-309.

Janecki D. (1988). Stability analysis of Elliott‘s direct adaptive pole placement. *Systems & Control Letters*, **11**, 19-26.

Joseph P.D. and J.T. Tou (1961). On linear control theory. *AIEE Trans. Applications in Industry*, **80**, 193-196.

Keuchel U. and R.M. Stephan (1994). *Microcomputer-Based Adaptive Control Applied to Thyristor Driven DC-motors*, Berlin: Springer Verlag.

Keuchel U., J. Yu and H. Unbehauen (1987). Adaptive pole placement control of a dead-time process with an integrator and of a double integral plant. *Proc. 5th Yale Workshop on Application of Adaptive System Theory*, Yale, 90-95.

Knohl T., W. Xu and H. Unbehauen (2000). Dual adaptive neural control and its application to Hammerstein systems. *Proc. Internat. Conf. on Control, Automation, Robotics and Vision*, Singapore, paper 116.

Knohl T. (2001). *Application of ANN for adaptive control of nonlinear systems* (In German). Düsseldorf: VDI-Verlag (Reihe 8, No. 885).

Koust R.L., M.K. Lau and S.P. Boyd (1992). Set-membership identification of system with parametric and nonparametric uncertainty. *IEEE Trans. Autom. Control*, **37**, 929-941.

Krämer K. (1992). Ein Beitrag zur Analyse und Synthese adaptiver prädiktiver Regler. *Dr.-Ing. Dissertation*, VDI-Verlag, Reihe 8, Nr. 288.

Kulcsar C., L. Prozano and E. Walter (1996). Dual control of linearly parameterised models via prediction of posterior densities. *European Journal of Control*, **2**, 135-143.

Kuo B. C. (1992) *Digital Control Systems* (2nd Ed.) Orlando: Saunders College Publishing.

Kwakernaak H. (1965). On-line iterative optimization of stochastic control systems. *Automatica*, **2**, 195-208.

Landau I.D. and R. Lozano (1981). Unification of discrete-time explicit model reference adaptive control design. *Automatica*, **17**, 593-611.

Lindoff B., J. Holst and B. Wittenmark (1999). Analysis of approximations of dual control. *Int. J. Adaptive Control and Signal Process.*, **13**, 593-620.

Ljung L. (1979). Asymptotic behaviour of the extended Kalman filter as a parameter estimator for linear systems. *IEEE Trans. Autom. Control,* **24**, 36-50.

Lazono-Leal R. (1989). Robust adaptive regulation without persistent excitation. *IEEE Trans. Autom. Control*, **34**, 1260-1267.

McRae E.C. (1975). An adaptive learning role for multiperiod decision problems. *Econometrica,* **43**, 893-906.

Menga G. and E. Mosca (1980). MUSMAR: Multivariable Adaptive Regulator Based on Multistep Cost Functionals. In D.G. Lainiotis and N.S. Tzannes (eds) *Advances in Control*. Dordrecht: D. Reidel, 334-341.

Middleton, R.H., G.C. Goodwin, D.J. Hill and D.Q. Mayne (1988). Design issues in adaptive control. *IEEE Trans. Autom. Control*, **AC-33**, 50-58.

Milito R., C.S. Padilla, R.A. Padilla and D. Cadorin (1982). An innovations approach to dual control. *IEEE Trans. Autom. Control*, **AC-27**, 132-137.

Morse A.S. (1980). Global stability of parameter adaptive control system. *IEEE Trans. Autom. Control*, **AC-25**, 433-440.

Mosca E. (1995). *Optimal, Predictive and Adaptive Control*. Englewood Cliffs, New Jersey: Prentice Hall.

Moscinski J. and O. Zbigniew (1995). *Advanced Control with MATLAB and SIMULINK*. New York: Prentice Hall.

Narendra K. and A. Annaswamy (1989). *Stable Adaptive Systems*., Englewood Cliffs, New Jersey: Prentice-Hall.

Narendra K.S., Y.H. Lin and L.S. Valavani (1980). Stable adaptive controller design - part II: proof of stability. *IEEE Trans. Autom. Control*, **AC-25**, 440-448.

Norman A.L. (1976). First order dual control. *Ann. Econ. Soc. Meas.*, **5**, 311-322.

Ortega R. and Y. Tang (1989). Robustness of adaptive controllers - a survey. *Automatica,* **25**, 651-677.

Patra A., U. Keuchel, U. Kiffmeier and H. Unbehauen (1994). Identification for robust control of an unstable plant. *Proc. 10th IFAC Symposium on System Identification*, Copenhagen, 3589-3594.

Phillips C. L. and H. T. Nagle (1984). *Digital Control Analysis and Design*. Englewood Cliffs, New Jersey: Prentice Hall.

Praly L. (1983). Robustness of indirect adaptive control based on pole-placement design. *IFAC Workshop on Adaptive Control*, San Francisco, CA, June 1983.

Pronzato L., C. Kulcsar and E. Walter (1996). An actively adaptive control policy for linear models. *IEEE Trans. Autom. Control*, **AC-41**, 855-858.

Radenkovic, M.S. (1988). Convergence of the generalised dual control algorithm. *Int. J. Control,* **47**, 1419-1441.

Richalet J., A. Rault, J.L. Testud and Papon (1978). Model predictive heuristic control: applications to industrial processes. *Automatica*, **14**, 413-428.

Saridis G.N. (1977). *Self-Organized Control of Stochastic Systems*. New York: Marcel Dekker.

Shaked U. (1979). A transfer function approach to the linear discrete stationary filtering and the steady-state discrete optimal control problems. *Int. J. Control*, **29**, 279-291.

Silva R., N.M. Filatov, J. Lemos and H. Unbehauen (1998a). Feedback/feedforward dual adaptive control of a solar collector field. *Proc. IEEE Internat. Conf. on Control Applications*. Trieste, 309-313.

Silva R., N.M. Filatov, J. Lemos and H. Unbehauen (1998b). Dual adaptive predictive controller. *Proc. Control 98*, Coimbra (Portugal), 250-254.

Speyer J., J. Deyst and D. Jacobson (1974). Optimization of stochastic linear systems with additive measurement and process noise using exponential performance criteria. *IEEE Trans. Autom. Control*, **AC-19**, 358-366.

Stavroulakis P. and S.G. Tsafestas (1984). Adaptive dual control of discrete-time distributed-parameter stochastic systems. *Int. J. System Sci.*, **15**, 459

Sternby J. (1976). A simple dual control problem with an analytical solution. *IEEE Trans. Autom. Control*, **AC-21**, 840-844.

Su J.-H. and I.-K. Fong (1994). Controller robustification via optimization techniques for linear uncertain systems. *Control - Theory and Advanced Technology,* **10**, 154-160.

Tse E., Y. Bar-Shalom and L. Meier (1973). Wide-sense adaptive dual control for nonlinear systems. *IEEE Trans. Autom. Control*, **AC-18**, 98-108.

Tse E. and Y. Bar-Shalom (1973). An actively adaptive control for linear systems with random parameters via the dual control approach. *IEEE Trans. Autom. Control*, **AC-18**, 109-117.

Tse E. and Y. Bar-Shalom (1975). Generalized certainty equivalence and dual effect in stochastic control. *IEEE Trans. Autom. Control*, **AC-20**, 817-819.

Tsypkin Ya. (1971). *Adaptation and Learning in Adaptive Control*. New York: Academic Press.

Unbehauen H. (1993). Some critical remarks to acceptance of adaptive control in industrial practice, *Proc. of 1993, IEEE Conference on Systems, Man and Cybernetics,* Le Touquet - France, **3**, 447-451.

Unbehauen H., (1985) *Regelungstechnik III*, Braunschweig, Friedr. Vieweg & Sohn.

Unbehauen H. and N.M. Filatov (1995). Synthesis of adaptive controllers using bicriterial optimization and Lyapunov functions. In *World Scientific Series in Applicable Analysis* (ed. R.P. Agarwal), **5**: Recent Trends in Optimization Theory and Applications, World Scientific, London, 435-445.

Unbehauen H. and U. Keuchel (1992). Model reference adaptive control applied to electrical machines. *Int. J. Adapt. Control Signal Process.,* **6**, 95-109.

Veres S.M. and J.P. Norton (1993). Predictive self-tuning control by parameter bounding and worst-case design. *Automatica*, **29**, 911-928.

Veres S.M. (1995). Identification by parameter bounds in adaptive control. *Int. J. Adapt. Control Signal Process.,* **9**, 33-46.

Vidisagar M. (1985). *Control System Synthesis: a Factorization Approach*. MIT Press, Cambridge, Massachusetts.

Wittenmark B. (1975a). Stochastic adaptive control method: a survey, *Int. J. Control*, **21**, 705-730.

Wittenmark B. (1975b). An active suboptimal dual controller for systems with stochastic parameters, *Auto. Contr. Theory and Appl.*, **3**, 13-19.

Wittenmark B. (1995). Adaptive dual control methods: an overview, *Proc. 5th IFAC Symposium on Adaptive Systems in Control and Signal Processing*, Budapest, 67-72.

Wittenmark B. (2003). Adaptive dual control, UNESCO Encyclopedia of Life Support Systems, 6.43.15.6 EOLSS Publishers, Oxford, UK [http://www.eolss.net].

Wittenmark B. and C. Elevitch (1985). An adaptive control algorithm with dual features. *Proc. 7th IFAC Symp. on Identification and System Parameter Estimation,* York, UK, 587-591.

Yu J., U. Keuchel and H. Unbehauen (1987). Adaptive pole placement control of a dead time process with an integrator and of a double integrator plant, Proc. 5th Yale Workshop on Applications of Adaptive Systems Theory, Yale, 90-95.

Zhivoglyadov V.P. (1965). Synthesis method of suboptimal dual control systems. *Automation and Remote Control,* **26**, 58-66.

Zhivoglyadov V.P. (1966). Dual control of distributed plants in discrete-continuous systems. *Izvestija AN Kirgizskoj SSR* (in Russian)*,* No. 2.

Zhivoglyadov V.P. (1985). Active adaptation and identification in adaptive control systems. *Proc. 7th IFAC Symp. on Identification and System Parameter Estimation,* York, UK, 593-597.

Zhivoglyadov V.P., G.P. Rao and N.M. Filatov (1993a). Application of δ-operator models to active adaptive control of continuous-time plants. *Control-Theory and Advanced Technology,* **9**, 1, 127-137.

Zhivoglyadov V.P., G.P. Rao and N.M. Filatov (1993b). Active adaptive control algorithm with BPF. *Proc. 12th IFAC World Congress*, Sydney, Australia, **10**, 401-404.

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