Stability Analysis of Social Foraging Swarm With Interaction Time Delays

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Abstract

This paper considers a swarm model with an attraction-repulsion function involving variable communication time lags and an attractant/repellent. It is proved that for quadratic attractant/repellent profiles the members of the swarm with time delays will aggregate and form a cohesive cluster of finite size in a finite time. Moreover, all the swarm members will converge to more favorable areas of the quadratic attractant/repellent profiles under certain conditions in the presence of communication delays.

1. Introduction

In nature, we often observe the swarming in many organisms, such as herds of animals, colonies of bacteria, and flocks of birds or schools of fish move together for finding food and avoiding predators. The principles of cooperative and operational of such systems may be well for developing formation control and cooperative control [1]–[6].

Gazi and Passino [7] proposed a social foraging swarms model with an attractant/repellent profile, and then they analyzed its stability properties for different profiles. In the following, Chu et al [8] investigated continuous time swarm model with coupling weight matrix, but the interaction time delays haven't been considered. However, time delays often degrade performance of system and lead to divergence or instability. Hence, in this paper, we give a new swarm model and analyze swarm dynamics under the interplays of the individuals with interaction time delays and the effect of environment.

Next section, we give the swarm model with interaction time delays. Section 3 gives analytical analysis of aggregation and cohesion of the swarm dynamics under a balance condition of weights and symmetry condition of time delays with a nutrient. Section 4 analyzes the behavior of this swarm for quadratic attractant/repellent profiles. In Section 5 shows some numerical simulations of general swarms. Finally, give our conclusions.

2. Model

In an n-dimensional Euclidean space, the motion of a swarm who contains M individuals (or members) is described by the following delayed differential equations

$$\dot{x}^i(t) = -\nabla_{x^i}\sigma(x^i) + \sum_{j\in N_i} w_{ij} g(x^i(t-\tau_{ij}(t)) - x^j(t-\eta_{ji}(t))), \qquad (1)$$

where $x^i \in \mathbb{R}^n$ represents the position of individual i of the swarm, $i = 1, 2, \cdots, M$; N_i is the set of all those coupling to individual i (referred to as its neighbors); $W = [w_{ij}] \in \mathbb{R}^{M\times M}$ is the coupling weight matrix with $w_{ij} > 0$ and $w_{ii} = 0$ for all i, j; $\tau_{ij}(t), \eta_{ji}(t) \in [0, \tau]$ are time delays due to communication in the swarm, where τ is a constant; and $g(\cdot)$ is the attraction-repulsion function that describes a long-distance attractive and short-distance repulsive interaction between the individuals. $\sigma : \mathbb{R}^n \to \mathbb{R}$ represents the attractant/repellent profile or the "$\sigma-$profile" which can be a profile of nutrients or some attractant/repellent substances.

Next, we consider the following assumptions on the attraction/repulsion function:

Assumption 1: The attraction-repulsion function $g(\cdot)$ is of the form

$$g(y) = -y[g_a(\|y\|) - g_r(\|y\|)], y \in \mathbb{R}^n,$$

where $g_a : \mathbb{R}^+ \to \mathbb{R}^+$ represents (the magnitude of) attraction term and has a long range, and $g_r : \mathbb{R}^+ \to \mathbb{R}^+$ represents (the magnitude of) repulsion term and has a short range, which $\mathbb{R}^+$ stands for the set of nonnegative real numbers, $\|y\| = \sqrt{y^T y}$ is the Euclidean norm.

Assumption 2: There exist positive constants a, b such that for any $y \in \mathbb{R}^n$,

$$g_a(\|y\|) = a \text{ and } g_r(\|y\|) \le \frac{b}{\|y\|}.$$

That is, we assume a fixed linear attraction function and a bounded repulsion function.

One function that satisfies the previous assumptions is [11]

$$g(y) = -y\left[a - bexp\left(-\frac{\|y\|}{c}\right)\right], \qquad (2)$$

which $g_a(\|y\|) = a > 0$ and $g_r(\|y\|) = bexp(-\|y\|/c)$. Other forms of attraction-repulsion function can be seen in [8], [9].

Having these conditions, we will present the analytical results for different profiles.

3. Main Results

In this section, we discuss the delayed swarm model (1) for several different profiles.

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3.1. Relationship between members and center

For the first, in order to simplify the discussion, we need to make some assumptions on the time delays, the connectivity structure of the swarm and the profiles.

Assumption 3: The time delays satisfy: $\tau_{ij}(t) = \eta_{ij}(t), \forall t \geq 0, \forall i, j.$

Assumption 4: The weights satisfy the balance condition: $\sum_{j\in N_i} w_{ij} = \sum_{j\in N_i} w_{ji}, \forall i.$

Assumption 5: There exists a constant $\bar{\sigma} > 0$ such that,

$$\|\nabla_y \sigma(y)\| \leq \bar{\sigma}, \text{ for all y.}$$

Assumption 6: There exists a constant $A_\sigma > \frac{-a\lambda_2}{2}$, such that

$$e^{i^T}\left[\nabla_{x^i}\sigma(x^i) - \frac{1}{M}\sum_{i=1}^{M}\nabla_{x^i}\sigma(x^i)\right] \geq A_\sigma\|e^i\|^2,$$

for all x^i and x^j, where λ_2 is the second smallest real eigenvalue of the symmetric matrix $L+L^T$, $L = [l_{ij}]$ defined as

$$l_{ij} = \begin{cases} -w_{ij}, & i \neq j, \\ \sum_{k\in N_i} w_{ik}, & i = j. \end{cases}$$

Assumption 7: There exists a constant $K > 0$ such that, $\forall t \geq 0, \forall i, j,$

$$\|x^i(t) - x^i(t-\tau_{ij}(t))\|, \|x^j(t) - x^j(t-\eta_{ji}(t))\| \leq K.$$

Before investigating the collective behavior of the delayed swarm, we define the virtual center of swarm as below.

Definition 1: The swarm center is

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x^i.$$

Because of Assumptions 3 and 4, the motion of the center is given by

$$\begin{aligned} \dot{\bar{x}} &= -\frac{a}{M}\sum_{i=1}^{M}\left(\sum_{j\in N_i} w_{ij} - \sum_{j\in N_i} w_{ji}\right)x^i(t-\tau_{ij}(t)) \\ &\quad +\frac{b}{M}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}f_r - \frac{1}{M}\sum_{i=1}^{M}\nabla_{x^i}\sigma(x^i) \\ &= \frac{b}{M}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}f_r - \frac{1}{M}\sum_{i=1}^{M}\nabla_{x^i}\sigma(x^i), \end{aligned}$$

where $f_r = (x^i(t-\tau_{ij}(t)) - x^j(t-\eta_{ji}(t)))E_{ij}$,

$$E_{ij} = exp\left(-\frac{\|x^i(t-\tau_{ij}(t)) - x^j(t-\eta_{ji}(t))\|}{c}\right).$$

Obviously, the center of the swarm will move in time. But whether the members will move to the center and form a cluster is the key issue which we are concentrate on. Therefore, we will discuss this issue in the following theorem.

Theorem 1: Consider the swarm described by the model in (1) with g(·) as given in (2) with linear attraction and bounded repulsion, satisfy Assumptions 1–4 and 7, then as $t \to \infty$, we have $x^i(t) \to B_\sigma(\bar{x}(t))$, where $x = (x^{1^T}, x^{2^T}, \cdots, x^{M^T})^T \in \mathbb{R}^{Mn}$

$$B_\sigma(\bar{x}(t)) = \{x(t) : \sum_{i=1}^{M}\|x^i - \bar{x}\|^2 \leq \sigma^2\},$$

and

- if Assumption 5 is satisfied, then

$$\sigma = \sigma_1 = \frac{4\left[(aK + bce^{-1})|||W||| + \bar{\sigma}(M-1)\right]}{a\lambda_2};$$

- if Assumption 6 is satisfied, then

$$\sigma = \sigma_2 = \frac{4\left(aK + bce^{-1}\right)|||W|||}{a\lambda_2 + 2A_\sigma},$$

where $|||W||| = \sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}$ and λ_2 are defined as before.

Proof: Case 1) : Define $e^i = x^i - \bar{x}$, and choose a Lyapunov function for estimating e^i as

$$V = \frac{1}{2}\sum_{i=1}^{M} e^{i^T}e^i.$$

Differentiating, and noticing that the function- $\|y\|exp(-\|y\|/c)$ is a bounded function, that is, when $\|y\| = c$, the maximum is ce^{-1}. Using the reality $\|e^i\| \leq \sqrt{2V(e)}\forall i$, and Assumption 7, substituting these into the following inequality , we can obtain

$$\begin{aligned} \dot{V} &= -a\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}e^{i^T}(x^i(t-\tau_{ij}(t)) - x^j(t-\eta_{ji}(t))) \\ &\quad +b\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}e^{i^T}f_r - \frac{b}{M}\sum_{i=1}^{M}e^{i^T}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}f_r \\ &\quad -\sum_{i=1}^{M}e^{i^T}\left[\nabla_{x^i}\sigma(x^i) - \frac{1}{M}\sum_{i=1}^{M}\nabla_{x^i}\sigma(x^i)\right] \\ &\leq -a\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}e^{i^T}(e^i - e^j) + 2\sqrt{2}\bar{\sigma}(M-1)\sqrt{V(e)} \\ &\quad +2\sqrt{2}(aK + bce^{-1})|||W|||\sqrt{V(e)} \\ &\leq -ae^T(L\otimes I)e + 2\sqrt{2}\bar{\sigma}(M-1)\sqrt{V(e)} \\ &\quad +2\sqrt{2}(aK + bce^{-1})|||W|||\sqrt{V(e)} \\ &= -\frac{a}{2}e^T((L+L^T)\otimes I)e + 2\sqrt{2}[\bar{\sigma}(M-1) \\ &\quad +(aK + bce^{-1})|||W|||]\sqrt{V(e)}, \end{aligned}$$

where $e = (e^{1^T}, e^{2^T}, \cdots, e^{M^T})^T$, $L\otimes I$ is the Kronecker product of L which is defined in Assumption 6 and I which is the identity matrix of order n. For getting further estimate of $\dot{V}(e)$, we only need to estimate the term $e^T((L+L^T)\otimes I)e$.

For Assumption 4, we can easily conclude that $\lambda = 0$ is an eigenvalue of $L+L^T$ and $u = (l, l, \cdots, l)^T$ with $l \neq 0$ is the associated eigenvector. Moreover, because $L+L^T$ is symmetric and $W+W^T$ (so is $L+L^T$) is irreducible, it follows from nonnegative matrix theory [10] that $\lambda = 0$ is a simple eigenvalue and all the rest eigenvalues of $L+L^T$ are real and positive. Therefore, we can order the eigenvalues of $L+L^T$ as $0=\lambda_1<\lambda_2\leq\cdots\leq\lambda_n$. Also, it is known that the identity matrix I has n multiple eigenvalues $\mu=1$ and n independent eigenvectors $h^1=[1,0,\cdots,0]^T,\cdots,h^n=[0,0,\cdots,1]^T$.

By matrix theory [10], the eigenvalues of $(L+L^T)\otimes I$ are $\lambda_i\mu=\lambda_i$ (n multiple for each i) and the corresponding eigenvectors are $u^i\otimes h^j$. It is clear that, because $(L+L^T)\otimes I$ is symmetric, the n^2 eigenvectors $u^i\otimes h^j$ are linearly independent. So, if $e^T((L+L^T)\otimes I)e=0$ then e must lie in the eigenspace of $(L+L^T)\otimes I$ spanned by eigenvectors $u\otimes h^j$ corresponding to the zero eigenvalue, that is, $e^1=e^2=\cdots=e^M$. This happens only when $e^1=e^2=\cdots=e^M=0$. But this is impossible for the swarm model under consideration, because it implies that the M members occupy the same position at the same time. Hence, for any solution x of system (1), e must be in the subspace spanned by eigenvectors of $(L+L^T)\otimes I$ corresponding to the nonzero eigenvalues. Then,

$$e^T((L+L^T)\otimes I)e\geq\lambda_2\|e\|^2=2\lambda_2V(e).$$

From this, we have

$$\begin{aligned}\dot V(e)\quad&\leq\quad -a\lambda_2V(e)+2\sqrt{2}[(aK+bce^{-1})|||W|||\\&\qquad+\bar\sigma(M-1)]\sqrt{V(e)}<0,\end{aligned}$$

whenever

$$V(e)>\left(\frac{2\sqrt{2}[(aK+bce^{-1})|||W|||+\bar\sigma(M-1)]}{a\lambda_2}\right)^2.$$

This implies that each agent i will finally enter and remain in $B_{\sigma_1}(\bar x)$, where $\sigma_1=4[(aK+bce^{-1})|||W|||+\bar\sigma(M-1)]/a\lambda_2$. From the analysis above, we have

$$\dot V\leq -a\lambda_2V+\frac{a\lambda_2\sigma_1}{\sqrt{2}}\sqrt{V}\tag{3}$$

for $\sum_{i=1}^M\|e^i\|^2\geq\sigma_1^2$. Suppose that a member i enters into the region $B_{\sigma_1+\varepsilon}(\bar x)$ at time T. Moreover, from (3), we can gain

$$T\leq\frac{2}{a\lambda_2}\ln\left(\frac{\sigma_1^0-\sigma_1}{\varepsilon}\right),\ \text{where}\ \sigma_1^0=\sqrt{2V(0)}>\sigma_1+\varepsilon.$$

Case 2) : Similarly using Assumption 6 one can show that $\dot V$ satisfies

$$\begin{aligned}\dot V(e)\quad&\leq\quad -(a\lambda_2+2A_\sigma)V(e)\\&\qquad+2\sqrt{2}(aK+bce^{-1})|||W|||\sqrt{V(e)}<0.\end{aligned}$$

Therefore, we conclude that all the members of the swarm described in Eq. (1) will enter into and remain in a bounded region $B_{\sigma_2}(\bar x(t))$, where $x=(x^{1^T},x^{2^T},\cdots,x^{M^T})^T\in\mathbb{R}^{Mn}$ and

$$\sigma_2=\frac{4(aK+bce^{-1})|||W|||}{(a\lambda_2+2A_\sigma)}.$$

Moreover, all the members who are out of $B_{\sigma_2}(\bar x(t))$ will enter into its $\varepsilon-$neighborhood (i.e., $B_{\sigma_2+\varepsilon}(\bar x(t))$) for an arbitrary $\varepsilon>0$) in a finite time which is bounded by

$$T=\frac{2}{a\lambda_2+2A_\sigma}\ln\left(\frac{\sigma_2^0-\sigma_2}{\varepsilon}\right),$$

where $\sigma_2^0=\sqrt{2V(0)}$ and $\sigma_2^0>\sigma_2+\varepsilon$. This completes the proof. □

Theorem 1 shows that the delayed swarm individuals aggregate and form a bounded cluster around the swarm center in a finite time.

Remark 1: Attention that $w_{ij}=w_{ji},\forall i,j$ is a special case of $\sum_{j\in N_i}w_{ij}=\sum_{j\in N_i}w_{ji},\ i=1,\cdots,M$ which is given in Assumption 4. So we can obtain the similar conclusion as Theorem 1.

3.2. Special Case

Theorem 1 gives the general case. Now we consider a special case when $\tau_{ij}(t)=0$ and the swarm model (1) is rewritten as

$$\dot x^i(t)=-\nabla_{x^i}\sigma(x^i)+\sum_{j\in N_i}W_{ij}g(x^i(t)-x^j(t-\eta_{ji}(t))),\tag{4}$$

where $i=1,2,\cdots,M$, x^i, $g(\cdot)$ and $\eta_{ji}(t)$ are defined as before. Similarly, we can establish the next theorem.

Theorem 2: Consider the swarm described by the model in (4) with g$(\cdot)$ as given in (2), and the Assumptions $1,2,4,7$ hold, then all the members of the swarm will access into $B_{\sigma^*}(\bar x(t))$, where $x=(x^{1^T},x^{2^T},\cdots,x^{M^T})^T\in\mathbb{R}^{Mn}$

$$B_{\sigma^*}(\bar x(t))=\{x(t):\sum_{i=1}^M\|x^i-\bar x\|^2\leq\sigma^{*2}\},$$

and if Assumption 5 is satisfied, then

$$\sigma^*=\sigma_1^*=\frac{4[(aK+bce^{-1})|||W|||+\bar\sigma(M-1)]}{a\lambda_2};$$

if Assumption 6 is satisfied, then

$$\sigma^*=\sigma_2^*=\frac{4(aK+bce^{-1})|||W|||}{a\lambda_2+2A_\sigma}$$

with $|||W|||$ and λ_2 are defined as before.

The proof is analogous to that of Theorem 1, so we don't repeat here.

4. Quadratic Attractant/repellent Profiles

Consider now a quadratic profile of the swarm for model (1) represented by

$$\sigma(y)=\frac{A_\sigma}{2}\|y-c_\sigma\|^2+b_\sigma,\tag{5}$$

where $A_\sigma\in\mathbb{R},b_\sigma\in\mathbb{R}$ and $c_\sigma\in\mathbb{R}^n$. It is easily seen that at $y=c_\sigma$, the profile described in Eq. (5) achieves either a

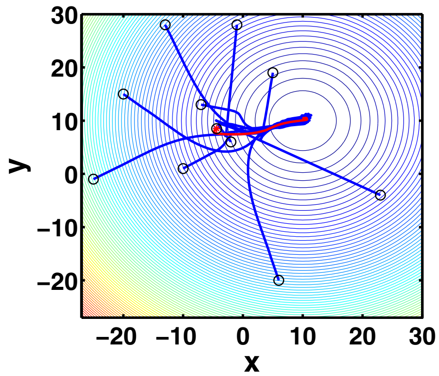

Figure 1. The swarm member and center trajectories with $\tau_{ij}(t) = 0.1$, $\eta_{ji}(t) = 0.3$, $A_\sigma = 0.5$.

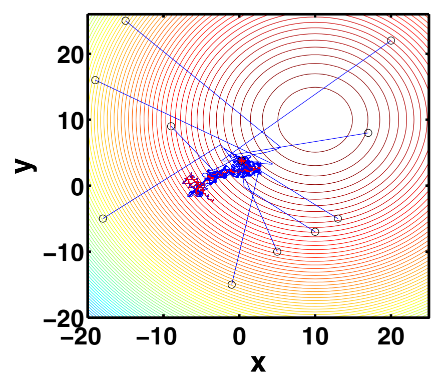

Figure 2. The swarm member and center trajectories with $\tau_{ij}(t) = 0.4$, $\eta_{ji}(t) = 2$, $A_\sigma = -0.01$.

minimum or a maximum which is decided by the sign of A_σ. Its gradient at $y \in \mathbb{R}^n$ is

$$\nabla y\sigma(y) = A_\sigma(y - c_\sigma).$$

Based on the defined $e_\sigma = \bar{x} - c_\sigma$, then

$$\dot{e_\sigma} \quad = \quad -A_\sigma e_\sigma + \frac{b}{M}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij} f_r.$$

Through $\dot{e_\sigma}$, we have interest on the relationship between $\bar{x}$ and c_σ, i.e., as $t \to \infty$, whether there exists a constant $\rho > 0$ such that $\bar{x}$ enters into $B_\rho(c_\sigma)$ or moves far away from c_σ. Therefore, we have the following results.

Lemma 1: Consider the swarm described by the model in (1) with g$(\cdot)$ as given in (2), suppose that $\sigma-$profile of the environment is given by (5) and Assumption 6 holds, we have as $t \to \infty$ that

- if $A_\sigma > 0$, then $\bar{x}(t)$ will enter $B_\rho(c_\sigma)$, where $\rho = \frac{b|||W|||}{MA_\sigma}$;
- if $A_\sigma < 0$ and $\bar{x}(0) \neq c_\sigma$, then $\bar{x}(t) \to \infty$, i.e., the center of the swarm diverges from the global maximum c_σ of the profile.

Proof: Case 1) : Let $V_\sigma = \frac{1}{2}e_\sigma^T e_\sigma$, differentiating is given by

$$\begin{aligned}\dot{V}_\sigma &= -A_\sigma\|e_\sigma\|^2 + \frac{b}{M}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij} e_\sigma^T f_r \\ &\leq -A_\sigma\|e_\sigma\|^2 + \frac{b}{M}\sum_{i=1}^{M}\sum_{j\in N_i} w_{ij}\|e_\sigma\| \\ &= -A_\sigma\|e_\sigma\|\left[\|e_\sigma\| - \frac{b|||W|||}{MA_\sigma}\right].\end{aligned}$$

The above inequality shows that if $\|e_\sigma(t)\| > \frac{b|||W|||}{MA_\sigma}$, then the center of the swarm will be moving toward it. Therefore, as $t \to \infty$, we can asymptotically have $\|e_\sigma(t)\| \leq \frac{b|||W|||}{MA_\sigma}$, i.e., c_σ will be within the swarm.

Case 2): When $A_\sigma < 0$, with analysis similar to the Case 1 above it can be shown that

$$\dot{V}_\sigma \geq |A_\sigma|\|e_\sigma\|\left[\|e_\sigma\| - \frac{b|||W|||}{M|A_\sigma|}\right],$$

which implies that we have $\dot{V}_\sigma > 0$, in other words, if $\|e_\sigma\| > \frac{b|||W|||}{M|A_\sigma|}$, then $\|e_\sigma\|$ will increase. Therefore, as $t \to \infty$, $\|e_\sigma\| \to \infty$(i.e., $\bar{x}(t) \to \infty$). The proof is finished. □

From the results of Lemma 1 and Theorem 1 and the above observation, we can obtain the following theorem.

Theorem 3: Consider the swarm described by the model in (1) with g$(\cdot)$ as given in (2), assumed that $\sigma-$profile of the environment is given by (5) and $A_\sigma > -\frac{a\lambda_2}{2}$, then the following results hold

- if $A_\sigma > 0$, then $\forall\varepsilon^* > \sigma_2 + \rho$, all individuals $i = 1, 2, \cdots, M$, will enter into $B_{\varepsilon^*}(c_\sigma)$ in a finite time.
- if $A_\sigma < 0$ and $\bar{x}(0) \neq c_\sigma$, then $\forall D < \infty$, all individuals $i = 1, 2, \cdots, M$, will exit $B_D(c_\sigma)$ in a finite time.

From Lemma 1, we notice that when $A_\sigma > 0$, $\forall\varepsilon > \rho$, $\|\bar{x}(t) - c_\sigma\| < \varepsilon$ in a finite time sets up. While the result of Theorem 1 shows that all individuals will enter into the σ_2-neighborhood of $\bar{x}$ in a finite time. Then all individuals will enter $B_{\varepsilon^*}(c_\sigma)(\forall\varepsilon^* > \sigma_2 + \rho)$in a finite time. In contrast, when $A_\sigma < 0$ and $\bar{x}(0) \neq c_\sigma, \|\bar{x} - c_\sigma\| > D$ $(\forall D > 0)$ in a finite time. i.e., $\|\bar{x}\|$ exits any bounded $D-$neighborhood of c_σ in a finite time.

5. Numerical Simulations

In this section, we will give some simulation results for illustrating the analytical results. The blue and the red trajectories show the paths of the swarm members and the swarm center when $M = 10$ and $n = 2$. In these simulations, the attraction-repulsion function is taken in the form of (2) with parameters $a = 1, b = 20, c = 0.2$, and we select a profile with extremum at $c_\sigma = [10, 10]^T$, the coupling

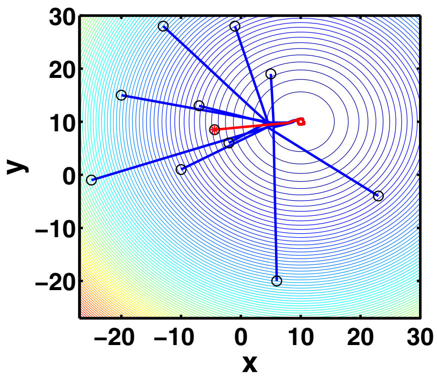

Figure 3. The swarm member and center trajectories with $\tau_{ij}(t) = 0.2$, $\eta_{ji}(t) = 0.8, A_\sigma = 2$.

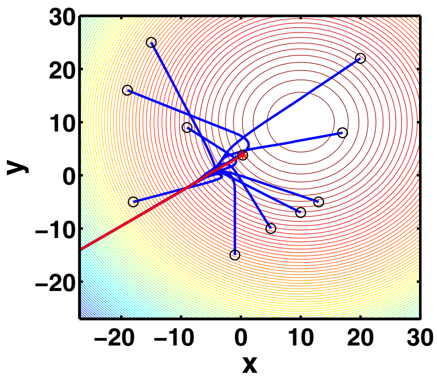

Figure 4. The swarm member and center trajectories with $\tau_{ij}(t) = 0.2$, $\eta_{ji}(t) = 1, A_\sigma = -0.2$.

matrix satisfies the assumption which is made in the previous sections.

Figs $1,2$ show the case that the coupling matrix of the model (1) for quadratic profile is asymmetric, and Figs $3,4$ show that the coupling matrix is symmetrical. When $A_\sigma > 0$, apparently through Figs $1,3$, we can clearly see that all the members move around the swarm center, and at the same time they together go toward to the c_σ. However, when magnitude $A_\sigma < 0$, in Figs $2,4$ we can observer that all the members and the swarm center move far from c_σ.

6. Conclusion

We have present a swarm model with an attraction-repulsion function in the presence of variable communication time lags and a nutrient. Moreover, for quadratic attractant/repellent profiles, we show that all the swarm individuals move around the swarm center or go away from c_σ. At the same time, our numerical simulations indicate the theories presented in the previous section.

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