Adaptive Dual Control Systems: A Survey

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Abstract

An adaptive dual control system can be defined as an adaptive control system that operates under conditions of uncertainty and incorporates the existing uncertainty into the control strategy with the control signal having the following properties: (i) it cautiously follows the control goal and (ii) excites the plant to improve the estimation.
The adaptive part of the system tries to determine the controller parameters during operation in real-time mode, whereas the dual part of it realises this actively by means of optimal excitation added to the cautious control action. Therefore adaptive dual controllers provide a number of advantages compared to the classical adaptive control schemes: (i) Especially the accuracy of the estimation is taken into account, (ii) an optimal excitation for speeding up the estimation is provided, and (iii) the adaptation time becomes shorter and provides a smooth transitional behaviour.
This paper reviews the historical stages of the development of theory and application of dual adaptive control methods elaborated from the early 60's till today, considering also its close interconnection with the general progress in adaptive control systems. Detailed classifications of stochastic adaptive control methods and their adaptive dual versions are presented. New developments are reported.

1. Introduction

The last 40 years have born witness to the fantastic development and enhancement of adaptive control theory and application, which have been meticulously collected and presented in various scientific publications, as for example (Filatov and Unbehauen, 2000). Many of the developed methods have been successfully applied to adaptive control systems, which find practical applications in a wide range of engineering fields. However, most adaptive control systems are based on the certainty-equivalence (CE) approach, that is, the uncertainty of estimation is not taken into consideration for the controller design, and the parameter estimates are used in the control law as if they were the real values of the unknown parameters. This approach is simple to implement. Unfortunately, the CE assumption appears to be the reason for insufficient control performance in the cases of large uncertainties. These systems suffer from large overshoots during phases of rapid adaptation (at startup and after parameter changes), which limit their acceptance for many practical cases.

In his early works, A. Feldbaum (1960-61, 1965) considered the problem of optimal adaptive control and indicated that systems based on the CE approach are not always optimal but can indeed be far from so. He postulated two main properties that the control signal of an optimal adaptive system should have: it should ensure that (i) the system output *cautiously* tracks the desired reference value and that (ii) it *excites* the plant sufficiently for accelerating the parameter estimation process so that the control quality becomes better in future time intervals. These properties are known as *dual properties* (or *dual features*). Adaptive control systems showing these two properties are named *adaptive dual control systems*.

The formal solution to the optimal adaptive dual control problem in the formulation considered by Feldbaum (1965) can be obtained through the use of dynamic programming, but the equations can neither be solved analytically nor numerically even for simple examples because of the growing dimension of the underlying space (exact solutions to simple dual control problems can be found in (Sternby, 1976) where a system with only a few possible states was considered). These difficulties in finding the optimal solution led to the appearance of various simplified approaches that can be divided into two large groups: those based on various *approximations* of the optimal dual adaptive control problem and those based on the *reformulation* of the problem to obtain a simple solution so that the system maintains its dual properties. These approaches were named *implicit* and *explicit* adaptive dual control methods. The main idea of these adaptive dual control methods lies in the design of adaptive systems that are not optimal but have at least the main dual features of optimal adaptive dual control systems.

The adaptive control approaches that are based on approximations of the stochastic dynamic programming equations are usually complex and require large computational efforts. They are based on rough approximations so that the system loses the dual features and the control performance remains inadequate (Bar- Shalom and Tse, 1976;

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Bayard and Eslami, 1985; Bertsekas, 1976; Birmiwal, 1994), to name a few. The methods of problem reformulation are more flexible and promising. Before the elaboration of the new bicriterial design method for adaptive dual control systems (see, for example, Filatov and Unbehauen, 1995a; Unbehauen and Filatov, 1995; Zhivoglyadov et al. 1993a), the reformulated adaptive dual control problems considered a special cost function with two added parts involving: control losses and an uncertainty measure (the measure of precision of the parameter estimation) (Wittenmark, 1975a; Milito et al., 1982, Lindhoff et al., 1999). With these methods, it is possible to design simple adaptive dual controllers, and the computational complexity of the control algorithms can become comparable to those of the CE controllers generally used. However, the optimization of such cost functions does not guarantee persistent excitation of the control signal, and the control performance of the adaptive dual controllers based on this special cost functions remains, therefore, inadequate. The before mentioned bicritical approach combines the advantages of the methods for both, the approximation and reformulation of the adaptive dual control problem. This design method will be discussed in more detail in this paper.

2. Some Fundamentals of Dual Control

2.1. Statement of Feldbaum's Optimal Dual Control Problem

Adaptive dual control aims at optimally controlling an uncertain stochastic control system, described by the following discrete-time equations of state, parameter and output vectors:

$$x(k+1) = \bar{f}_k[x(k), p(k), u(k), \xi(k)],\ k=0,1,\dots,N-1, \quad (2.1)$$

$$p(k+1) = \upsilon_k\,[p(k), \varepsilon(k)], \quad (2.2)$$

and

$$y(k) = h_k\,[x(k), \eta(k)], \quad (2.3)$$

where $x(k) \in \Re^{n_x}$ is the state vector; $p(k) \in \Re^{n_p}$ the vector of unknown parameters; $u(k) \in \Re^{n_u}$ the vector of control inputs; $y(k) \in \Re^{n_y}$ the vector of system outputs; and $\xi(k) \in \Re^{n_\xi}$, $\varepsilon(k) \in \Re^{n_\varepsilon}$ and $\eta(k) \in \Re^{n_\eta}$ are vectors of independent random white sequences with zero mean and known probability distributions; $\bar{f}_k(\cdot)$, $\upsilon_k(\cdot)$ and $h_k(\cdot)$ are known *simple* vector functions. The function $\upsilon_k(\cdot)$ describes the stochastic time-varying parameters of the system. The probability density for the initial values $\mathrm{p}[x(0), p(0)]$ is assumed to be known.

The set of outputs and control inputs available at time k is denoted as

$$\Im_k = \{y(k),\dots,y(0), u(k-1),\dots,u(0)\}, k = 1,\dots,N-1, \Im_0 = \{y(0)\}. \quad (2.4)$$

The performance index for control optimization has the form

$$J = \mathrm{E}\left\{\sum_{k=0}^{N-1} g_{k+1}[x(k+1), u(k)]\right\}, \quad (2.5)$$

where the $g_{k+1}[\cdot,\cdot]$'s are known positive convex scalar functions. The expectation is taken with respect to all random variables $x(0)$, $p(0)$, $\xi(k)$, $\varepsilon(k)$ and $\eta(k)$ for k = 0, 1, ..., N-1, which act upon the system.

The problem of optimal adaptive dual control consists of finding the control policy $u(k) = u_k(\Im_k) \in \Omega_k$ for k = 0, 1, ..., N-1 that minimizes the performance index of eq. (2.5) for the system described by eqs. (2.1) to (2.3), where Ω_k is the domain in the space $\Re^{n_u}$, which defines the admissible control values.

2.2. Unsolvability of Feldbaum's Optimal Dual Control Problem

Backward recursion of the following stochastic dynamic programming equations can generate the optimal stochastic (dual) control sought for the above problem:

$$J_{N-1}(\Im_{N-1}) = \min_{u(N-1)\in\Omega_{N-1}} \left[\mathrm{E}\{g_N[x(N), u(N-1)] | \Im_{N-1}\}\right], \quad (2.6)$$

$$J_k(\Im_k) = \min_{u(k)\in\Omega_k} \left[\mathrm{E}\{g_{k+1}[x(k+1), u(k)] + J_{k+1}(\Im_{k+1}) | \Im_k\}\right]$$

$$\text{for } k = N\text{-}2, N\text{-}3, \dots, 0. \quad (2.7)$$

It is known that the analytical difficulties in finding simple recursive solutions from the multi-step performance index of eqs. (2.6) and (2.7) and the numerical difficulties caused by the dimensionality of the underlying spaces make this problem practically *unsolvable* even for simple cases (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985). However, the detailed investigation of this problem enables one to find the main dual properties of the control signal in optimal adaptive systems and to use them for other formulations of the adaptive dual control problem. This leads to the elaboration of design methods for adaptive dual controllers and, practically, to the solution of the adaptive dual control problem. A simple example for such a problem is given below to demonstrate the properties of adaptive dual control systems.

A simple example:

Consider the SISO system described by

$$y(k+1) = bu(k) + \xi(k),\ b \neq 0, \quad (2.8)$$

where b is the unknown parameter with initial estimate $b(0)$ and covariance of the estimate $P(0)$, and the disturbance $\xi(k)$ has the variance $\mathrm{E}\{\xi^2(k)\} = \sigma_\xi^2$. This simplified model can be used for the description of a stable plant with unknown amplification b. The cost function

$$J = \mathrm{E}\left\{\sum_{k=1}^{N} [w(k) - y(k)]^2\right\}, \quad (2.9)$$

as a special case of eq. (2.5) with the output signal $y(k)$ and set point $w(k)$ should be minimized. The resulting optimal control problem, $u(k) = f[w(k) - y(k)]$, is unsolvable. Equations (2.6) and (2.7) can be successfully applied only to the multistep control problem with a few steps N to obtain a solution. The optimal parameter estimate for the considered system can , however, be obtained using the Kalman filter in the form

$$\hat{b}(k+1) = \hat{b}(k) + \frac{P(k)u(k)}{P(k)u^2(k) + \sigma_\xi^2}\left(y(k+1) - \hat{b}(k)u(k)\right), \tag{2.10}$$

and

$$P(k+1) = \frac{P(k)\sigma_\xi^2}{P(k)u^2(k) + \sigma_\xi^2} = P(k) - \frac{P^2(k)u^2(k)}{P(k)u^2(k) + \sigma_\xi^2} \tag{2.11}$$

After inspection of eqs. (2.10) and (2.11), the dependence of the estimate and its covariance on the manipulating signal $u(k)$ can be observed for a given σ_ξ : large values of $u(k)$ improve the estimation; and for an unbounded control signal, the exact estimate after only one measurement can already be obtained

$$\lim_{|u(k)| \to \infty} P(k+1) = 0, \tag{2.12}$$

and

$$\lim_{|u(k)| \to \infty} \hat{b}(k+1) = b. \tag{2.13}$$

Therefore, persistent excitation by a large magnitude of $u(k)$ can significantly improve the estimate. The problem is the optimal selection of this excitation so that the total performance of the system is enhanced.

Using the CE approach, it is assumed that all stochastic variables in the system are equal to their expectations. In the considered case, this means that $\xi(k)$=0 and $b(k)=b$. Thus for the CE assumption the optimal control has the simple form

$$u(k) = u_{CE}(k) = \frac{w(k+1)}{\hat{b}(k)}. \tag{2.14}$$

On the other hand, the minimization of the one-step cost function, (instead of the above multi-step performance index) described by eq. (2.9), for the considered example of eq. (2.8)

$$J_k^c = \mathrm{E}\{(w(k+1) - y(k+1))^2 | \Im_k\}, \tag{2.15}$$

leads to the control action given by

$$u(k) = u_c(k) = \frac{\hat{b}(k)w(k+1)}{\hat{b}^2(k) + P(k)} = \frac{1}{1 + P(k)/\hat{b}^2(k)} u_{CE}(k), \tag{2.16}$$

where $\mathrm{E}\{\cdot|\Im_k\}$ is the conditional expectation operator. The controller given by eq. (2.16) has a positive value in the denominator, and it generates the manipulating signal with smaller magnitude than the CE controller given by eq. (2.14). Controllers of this kind are named *cautious* controllers (denoted by u_c). Thus, the indicated two properties (cautious control and excitation) are attributed to the optimal adaptive control in various systems. Systems that are designed to ensure these properties of their control signal are named adaptive dual control systems.

2.3. General Structure of the Adaptive Dual Controller

To summarize the properties of dual control systems the schemes of a conventional adaptive control system and an adaptive dual control system are portrayed in Figure 1. The transmission of the *accuracy* of the parameter estimates from the estimation to the control design algorithm is the main difference between the presented structures. The utilization of the accuracy of the estimation for the controller design allows generating the optimal excitation and cautious control signal for an adaptive dual controller. Thus significant improvements of the control performance in cases of large uncertainties can be achieved, as will be shown later.

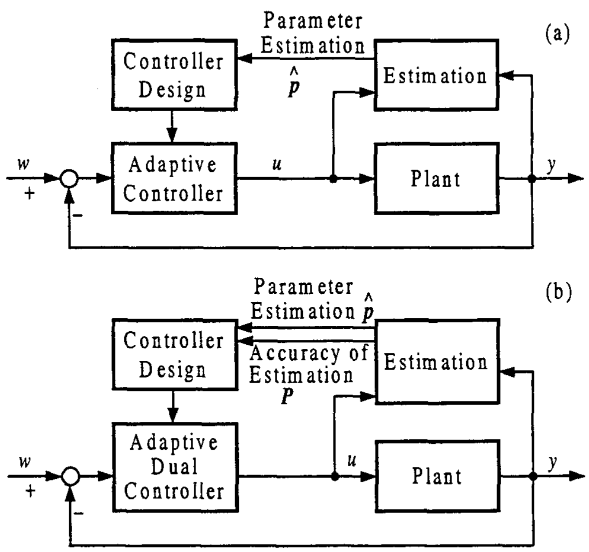

Figure 1: (a) Conventional adaptive control system and (b) Adaptive dual control system ($\hat{\boldsymbol{p}}$ =estimated parameter vector, $\boldsymbol{P}$ = covariance matrix of the estimation error)

3. Classification of Adaptive Dual Control Systems

3.1. Some Useful Definitions

Besides other existing definitions the following ones try to summerize what control engineers usually understand by an adaptive system:

Definition 1: *Adaptive Control System*
A control system operating under conditions of *uncertainty* of the controller that provides the *desired system performance* by changing its *parameters* and/or *structure* in order to reduce the uncertainty and to improve the operation of the desired system is an adaptive control system.

Defintion 2: *Adaptive Dual Control System*
A control system operating under conditions of *uncertainty* of the controller that provides the *desired system performance* by changing its *parameters* and/or stucture in order to reduce the uncertainty and to improve the operation of the desired system by incorperating the existing uncertainty into the control strategy with the control signal having the properties: (i) it follows the control goal and (ii) excites the plant to improve the estimation, is an adaptive dual control system.

It should be noted that dual control systems could also be nonadaptive as well. For example, dual effects can be viewed in stochastic nonlinear systems where the uncertainty consists of the inaccuracy of state estimation (Bar-Shalom and Tse, 1976).
Based on the above presented definition 2 follows the structure of the adaptive dual controller according to Fig. 2. It is important to mention that the performance of almost all

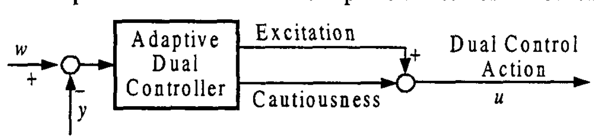

Figure 2: Adaptive dual controller

adaptive control systems can be improved through the application of dual control methods and the replacement of CE controllers (or other non-dual controllers) with dual controllers providing cautions behaviour with optimal excitations. However, the parameters of the adaptive dual controller have to be selected carefully. For instance, the cautiousness can reduce tracking performance but increase robustness in case of slight dynamical changes of the plant.

3.2. Simplificatoin of the Original Dual Control Problem

The practical unsolvability of the original dual control problem led to the development of various suboptimal stochastic adaptive control methods that are based on different approximations and simplifying assumptions concerning the probability measures of the unknown states and parameters of the system. Thus instead of eq. (2.5), the approximated performance index

$$J_k(\Im_k, \rho_k) = \mathrm{E}_{\rho_k}\left\{\sum_{i=k}^{N-1} g_{i+1}[x(i+1), u(i)] \middle| \Im_k\right\} \qquad (3.1)$$

has been introduced, where the expectation $\mathrm{E}_{\rho_k}\{.\}$ is calculated with the approximation ρ_k. At sampling time k, the control policy $u_k(\Im_k)$ is to be found with this ρ-approximation of the conditional probability desities of the system states and parameters for the future steps: $\mathrm{p}[\mathbf{x}(k+i), \mathbf{p}(k+i)|\Im_{k+i}]$, i = 0, 1, ..., N-k-1. Three special cases will be discussed in the following:

a) The *open-loop control* (OLC) (Bertsekas, 1976; Dreyfus, 1962) assumes the system to be without feedback in the future steps k+1, but with feedback at time k. At every time instant k, the observation y(k) is used for the estimation of both, parameters and states. Thus feedback is realized only for the current but not for the future time instants. This simplifying assumption can be described by the following ρ-approximation in eq. (3.1):

$$\begin{aligned}\rho_k = \rho_k^f &= \{\mathrm{p}[x(k+i)p(k+i)|\Im_{k+1}] \\ &= \mathrm{p}[x(k+i), p(k+i)|\Im_k], \quad i = 0, \ldots, N-k-1\}\end{aligned} \qquad (3.2)$$

b) The well-known and generally used *CE approach* can also be interpreted in forms of the ρ-approximation (Filatov and Unbehauen, 1995b). For the considered control problem the ρ-approximation of the probability density for the performance index decribed by eq. (3.1) takes the form

$$\begin{aligned}\rho_k = \rho_k^c &= \{\mathrm{p}[x(k+i), p(k+i)|\Im_{k+i} \\ &= \delta[x(k+i) - \hat{x}(k+i)]\delta[p(k+i) - \hat{p}(k+i)|k] \\ &\quad i = 0, \ldots, N-k-1\}\end{aligned} \qquad (3.3)$$

where

$$\hat{x}(k+i) = \mathrm{E}\{x(k+i)|\Im_{k+i}\}, \quad \hat{p}(k+i|k) = \mathrm{E}\{p(k+i)|\Im_k\} \qquad (3.4)$$

are the estimates and $\delta(\cdot,\cdot)$ is the Dirac function. The estimates are used here in the control law as if they were the real deterministic values of the unknown parameters.

c) A new ρ-approximation of the joint probability measures for both the system states and parameters was suggested in (Filatov and Unbehauen, 1995b). In this approach, adaptive control policies are derived, which are computationally simple, especially for linear systems and which give improved control performance. Consider the extended state vector for the system described by eqs. (2.1) to (2.3)

$$z^T(k) = [x^T(k), p^T(k)], \tag{3.5}$$

the vector $z(k)$ is divided into two separate vectors, $z_1(k)$ and $z_2(k)$. Introduce the following ρ-approximation of the extended state vector of eq. (3.5), which will be used for designing the control law via minimization of the future cost according to eq. (3.1):

$$\rho_k = \rho_k^p = \{ \mathrm{p}[z_1(k+i), z_2(k+i)|\Im_{k+i}] = \delta[z_1(k+i) - \hat{z}_1(k+i)]\,\mathrm{p}(z_2(k+i)|\Im_k) \quad i = 0,\dots,N-k-1 \} \tag{3..6}$$

where

$$\mathrm{p}[z_1(k+i), z_2(k+i)|\Im_{k+i}] = \mathrm{p}[z(k+i)|\Im_{k+i}] = \mathrm{p}[x(k+i), p(k+i)|\Im_{k+i}].$$

For the ρ-approximation ρ_k^p, eq. (3.6), the CE assumption (see eq. (3.3)) is applied to $z_1(k)$, the first part of the extended state vector, and the same simplifying assumption that is used for the OLF control policy given by eq. (3.2) is used for the second part, $z_2(k)$. Thus, it is assumed that at every sampling time k the system operates in closed-loop feedback mode for the future time intervals with respect to the first part of the extended state vector $z_1(k+i)$ and in open-loop feedback mode for the second part $z_2(k+i)$. In such a way this is a special combination of CE and OLF control strategies. It is assumed at the same time that the CE assumption is applied to the first part of the extended state vector, but not to the second one. This partial certainty equivalence (PCE) approach, together with the assumption according to eq. (3.6), allows the design of adaptive controllers that are simple in computation, especially for linear systems.

3.3 Implicit and Explicit Adaptive Dual Control

As already mentioned in the introduction the suboptimal solutions for the optimal adaptive dual control problem are based either on

- approximations of the problem leading to *implicit* adaptive dual control methods, or on
- reformulation of the problem leading to *explicit* adaptive dual control methods.

Some of the contol schemes according to these two categories will briefly discussed in the following.

3.3.1 Implicit Adaptive Dual Control

a) The partial *open-loop feedback* (POLF) policy (Bertsekas, 1976) is based on the assumption that instead of full information $\Im_{k+i}$ in future steps, i = 0, ..., N-k-1, only incomplete information $\Im_{k+i}$ from future measurements will be used. This assumption is equivalent to the ρ-approximation

$$\rho_k = \rho_k^l = \{ \mathrm{p}[x(k+i), p(k+i)|\Im_{k+i}] = \mathrm{p}[x(k+i), p(k+i)|\bar{\Im}_{k+i}],\ i = 0,\dots,N-k-1 \} \tag{3.7}$$

where

$\Im_{k+i} = \{\bar{y}(k+i),\dots,\bar{y}(k+1), y(k),\dots,y(0), u(k+i-1),\dots, u(0)\}$, $\bar{y}(\cdot)$ is the partial observation vector that does not contain all the elements of $y(\cdot)$ and has dimension, $n_{\bar{y}} \le n_y$.

b) The *m-measurement feedback* (m-MF) control policy is based on the assumption that the system operates in feedback mode in the future m time steps and without feedback mode after the time k+m (Curry, 1969). This assumption is equivalent to the approximation of the probability densities for eq. (3.1) by

$$\rho_k = \rho_k^m = \mathrm{p}[x(k+i), p(k+i)|\Im_{k+i}] = \mathrm{p}[x(k+i), p(k+i)|\Im_{k+i}], i = 0,\dots, m;$$
$$\mathrm{p}[x(k+m+j), p(k+m+j)|\Im_{k+m+j}] = \mathrm{p}[x(k+m+j), p(k+m+j)|\Im_{k+m}],$$
$$j = 1,\dots N-k-m-1\},\ \text{for } k < N-m-1, \tag{3.8}$$

and without any approximation for $k \ge N-m$. This assumption results in a suboptimal dual control scheme that is difficult to derive. The ρ-approximation, as in eq. (3.8), also coincides with the one for the OLF policy, eq. (3.2), if $m = 0$.

c) The *wide-sense* dual (WSD) and the *utility-costs* (UC) control policies (Bar-Shalom and Tse, 1976; Bayard and Eslami, 1985) are based on other approximations. The WSD control policy uses a linearization of the system equations around the nominal trajectory of the system where the OLF control policy is used. The utility costs approach where various control policies may be used as a nominal trajectory of the systems for further linearization or other approximations, can be considered as a generalization of the WSD policy. Various other implicit dual control polices were suggested that use different kinds of approximations and linearization. These approaches provide dual control with improved performance, but significant computational requirements in real time restrict their practical applicability.

3.3.2. Explicit Dual Control

Various explicit dual control algorithms have received considerable attention (Allison, et al., 1995a; Chan and Zarrop, 1985; Milito et al., 1982; Wittenmark, 1975a; Wittenmark and Elevitch, 1985). They are based on the minimization of cost functions of the form

$$J_k^e = J_k^c + \lambda J_k^a, \lambda \ge 0, \tag{3.9}$$

where J_k^c is given by eq. (2.15) and J_k^a by

$$J_k^a = P(k+1), \tag{3.10}$$

which stands for parametric uncertainty at the $(k+1)$-th sampling instant, and

$$J_k^a = -E\{[y(k+1) - \hat{b}(k)u(k)]^2 | \Im_k\}, \qquad (3.11)$$

which characterizes the desired increase, in the innovational value of the parameter estimation algorithm of eq. (2.10). In (Zhivoglyadov, 1965) it was suggested to use $\boldsymbol{P}(k)$, the covariance of the unknown parameters. In (Wittenmark, 1975b), the cost function

$$J_k^a = tr\{\boldsymbol{P}(k+1)\} \qquad (3.12)$$

was used, where $\boldsymbol{P}(k)$ is the covariance matrix of the estimation error. In (Goodwin and Payne, 1977) the cost function

$$J_k^a = -\frac{\det\{\boldsymbol{P}(k)\}}{\det\{\boldsymbol{P}(k+1)\}}, \qquad (3.13)$$

was applied, and for experiment design (not for dual control)

$$J_k^a = \log(\det\{\boldsymbol{P}(k+1)\}). \qquad (3.14)$$

In (Wittenmark and Elevitch, 1985) as well as in (Allison et al. 1995a) the covariance of the estimate of the first parameter b_1 of the ARX model of the system was used. Instead of the well-known innovational cost measure of (Milito et al., 1982), the function

$$J_k^a = tr\{\boldsymbol{P}^{-1}(k)\boldsymbol{P}(k+1)\} \qquad (315)$$

can be applied in the bicriterial design method (Unbehauen and Filatov, 1995). In the case of drifting stochastic parameters

$$J_k^a = tr\{\boldsymbol{P}^{-1}(k+1|k)\boldsymbol{P}(k+1)\} \qquad (3.16)$$

can be used instead.

3.4 Stages in the Development of Adaptive Dual Control

The most important stages in the development of adaptive dual control were

- the discovery of the dual effect in stochastic adaptive systems,
- the separation of basic principles for the design of dual controllers into two groups: implicit and explicit dual control, which are based on approximations and reformulations of the dual control problem, and
- the elaboration of the bicriterial design method.

Fig. 3 presents this development in chronological order. Naturally, the presented diagrams cannot include all results, and many different dual adaptive controllers have appeared in the literature since the first work of Feldbaum (1960-61) on dual control. It should be noted that the dvelopment of adaptive dual control for systems with nonstochastic uncertainties was also an important step (Veres, 1995).

4. Bicritirial Synthesis Method for Adaptive Dual Controllers

4.1. The Basic Idea

The basic idea of this method will be demonstrated on the simple *example* of section 2.2. The method is based on the minimization of two cost functions, which correspond to the two control goals of dual control. Within a sequential minimization, first the cost function according to eq. (2.15) is minimized which results in the cautious control action described by eq. (2.16). Then the second cost function

$$J_k^a = -E\{[y(k+1) - \hat{b}(k)u(k)]^2 | \Im_k\}, \qquad (4.1)$$

is minimized. This would lead to unbounded large control actions. Therefore, some constraints Ω_k should be used. To get a reasonable compromise between optimal persistent excitation and cautious control, it would be suitable to define these constraints around the cautious control $u_c(k)$, as defined in eq. (2.16), in the form

$$\Omega_k = [u_c(k) - \theta(k); u_c(k) + \theta(k)]. \qquad (4.2)$$

These constraints limit the magnitude of the excitation symmetrically around the cautious control $u_c(k)$ by the value $\theta(k) \ge 0$. It is easy to see that the optimal controller for the uncertainty indices according to eqs. (4.1) and constraints given by eq. (4.2) can be described by the general form

$$u(k) = u_c(k) + \mathrm{sgn}\{u_c(k)\}\,\theta(k), \qquad (4.3)$$

where

$$\mathrm{sgn}\{\kappa\} = \begin{cases} 1, & \text{if } \kappa \ge 0 \\ -1, & \text{if } \kappa < 0. \end{cases} \qquad (4.4)$$

Eq. (4.3) is derived in the following way. Through substitution of eq. (2.8) into eq. (4.1) it follows that

$$J_k^a = -E\{[y(k+1) - \hat{b}(k)u(k)]^2 | \Im_k\} = -P(k)u^2(k) + \sigma_\xi^2, \qquad (4.5)$$

and from eqs. (4.1) and (4.2), taking into account eq. (4.5), it can be concluded that the optimum for

$$u(k) = \arg\min_{u(k)\in\Omega_k} J_k^a \qquad (4.6)$$

is achieved on the boundary of the domain Ω_k as

$$u(k) = u_c(k) + \mathrm{sgn}\{J_k^a[u_c(k) - \theta(k)] - J_k^a[u_c(k) + \theta(k)]\}\,\theta(k). \qquad (4.7)$$

Therefore, the dual control signal is determined by eq. (4.3) which is obtained after substitution of eqs. (2.11) and (4.5) into eq. (4.7) and some further manipulations. The principle of the bicriterical optimization for the design of the dual controller is portrayed in Figure 4. The magnitude of the excitation can be selected in relation to the uncertainty measure P(k) as

$$\theta(k) = \eta P(k), \quad \eta \ge 0. \qquad (4.8)$$

Thus, the presented dual controller, according to eqs. (2.16), (4.3) and (4.8), minimizes sequentially both cost

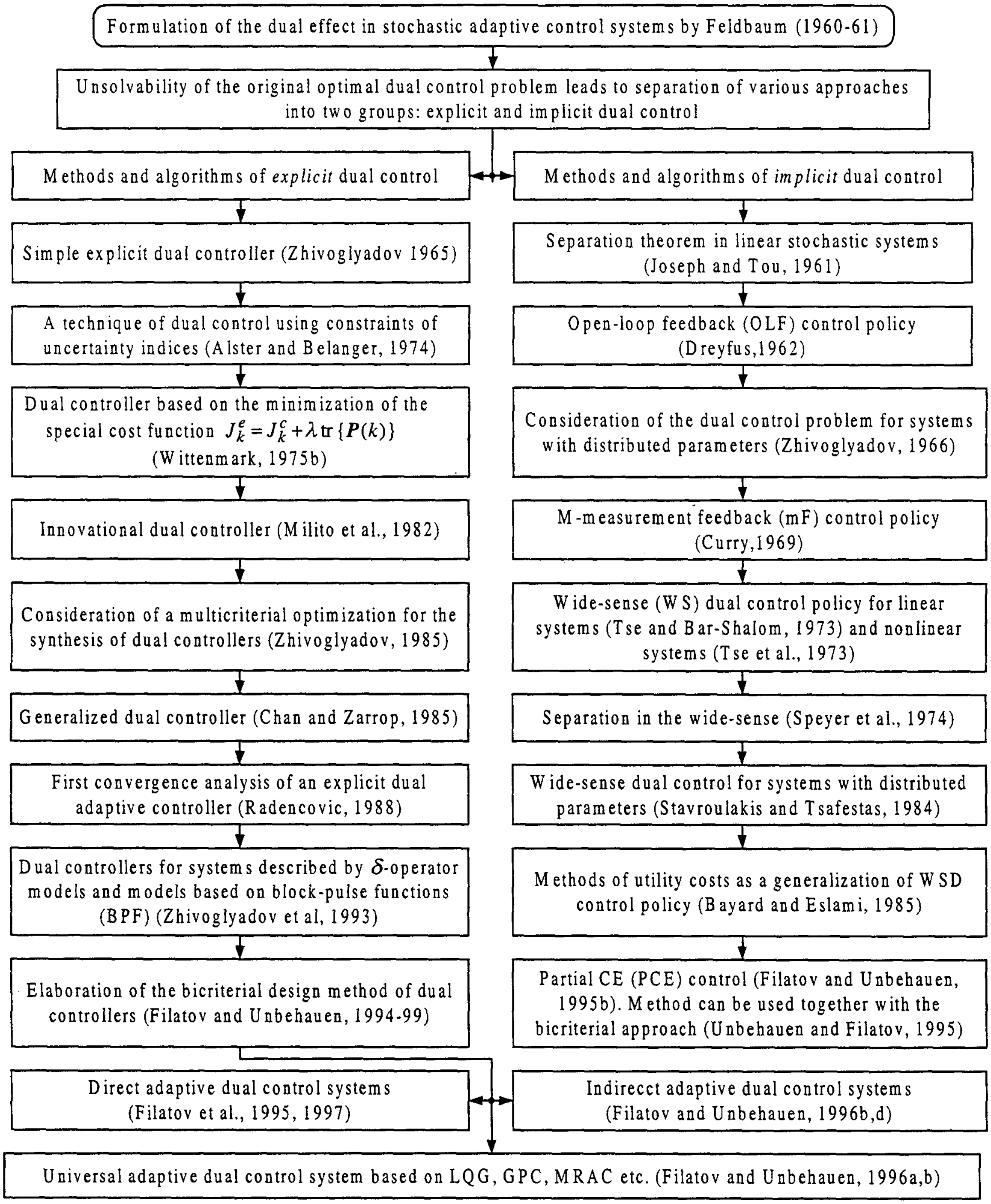

Figure. 3: Development of adaptive dual control

functions, eqs. (2.15) and (4.1), and the parameter, according to eq. (4.8), determines the compromise between these cost functions during the minimization. In contrast to other explicit dual control approaches, the parameter $\theta(k)$ has a clear physical interpretation: the magnitude of the excitations. Therefore, it can easily be selected. The Pareto set, indicated in Fig. 4 is the set where both cost functions cannot be minimized since the minimization of one leads to an increase in the second one. The basic idea demonstrated for this simple example can be easily generalized and applied to nearly all conventional adaptive control schemes (Filatov and Unbehauen, 1994-1998).

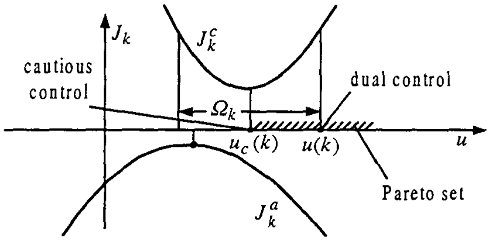

Figure 4. Sequential minimization of the two cost functionals in the bicriterial approach

4.2. Applications of the Bicriterial Approach

As a first example, the adaptive speed control of a thyristor-driven DC-motor is considered. The detailed description of the experimental setup, which was used here had been described in (Filatov, 1998). Two different adaptive schemes, the adaptive pole-placement controller (APPC) and the adaptive LQG-controller (ALQG) had been applied in two versions: (i) CE-approach and (ii) bicriterial dual (BD-) approach. The results are portrayed in Fig. 5 in normalized form. The comparison of the transient behaviour of the CE control with the bicriterial dual one indicates in both cases, the APPC and the LQG-control, a smoother startup and shorter adaptation time for the adaptive dual controller. The behaviour of the system after parameter changes is also smoother and show less overshoot for the dual controller. There is also a larger adaptation error e, which accelerates the estimation in case of the dual controller at the beginning of the process.

As a second example, the application the adaptive roll-angle control of a laboratory scale vertical take-off airplane (Patra et al. 1994) is discussed briefly. This is an unstable nonlinear control object due to its double integral behaviour and the nonlinearity between propeller speed and the generated monomentum induced by the propeller mechanism. Four adaptive control schemes had been applied for stabilisation:

(a) indirect APPC with CE approach,
(b) direct APPC with CE approach,
(c) direct cautious APPC,
(d) direct dual APPC.

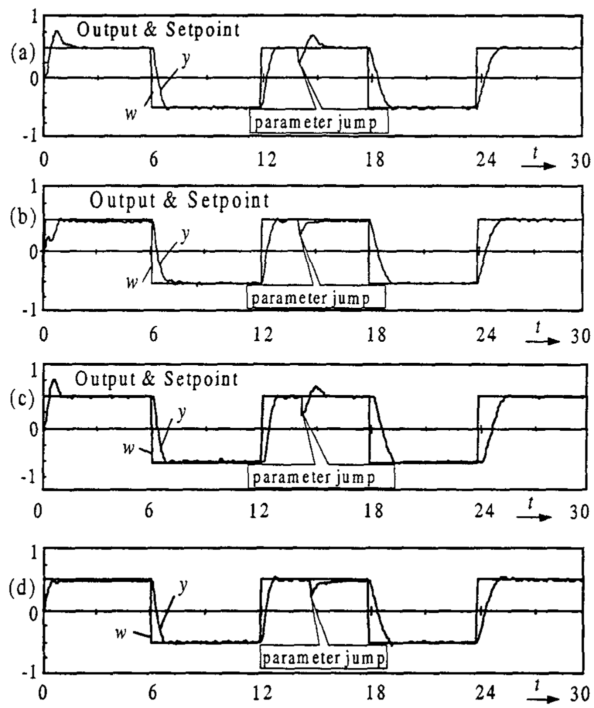

Figure 5.Reference (w) and controlled signal (y) of the rotational speed for
(a) APPC with CE approach
(b) APPC with BD approach
(c) LQG control with CE approach
(d) LQG control with BC approach

The results for the same experiment for all four controllers are presented in Fig. 6, in which the controlled value of case (a) exhibits the typical problems of indirect adaptive control schemes based on the CE assumption during the phases of adaptation. There is a large overshoot (more than 200%) during the adaptation period, and the convergence of parameters is slow mainly because of the missing excitation since the manipulated signal reaches the saturation limits $|u(k)| = 1$. In case (b) the adaptation transients die out significantly faster than in the case of indirect control, but there still is a 40% overshoot after the first setpoint change. In the cases (c) and (d) the bicriterical adaptive dual control stategy with different settings of the tuning parameter η for cautious and dual control action (η=0 and η=0.0001, respectively, had been applied. The results obtained by the direct dual control approach show better control performance at the beginning of the adaptation because of the cautious and probing properties. It should be mentioned that cautious controllers normally give good results (see Fig. 6c) but have not found a broad practical application up to now, because they lead to slow adaptation and sometimes to the"turn off" effect (Wittenmark, 1995) when the estimation process in the adaptive system is interrupted. Only the combination of cautious control with optimal excitation, as in eq. (4.7) provides acceptable control performance as can be seen from Fig. 6d.

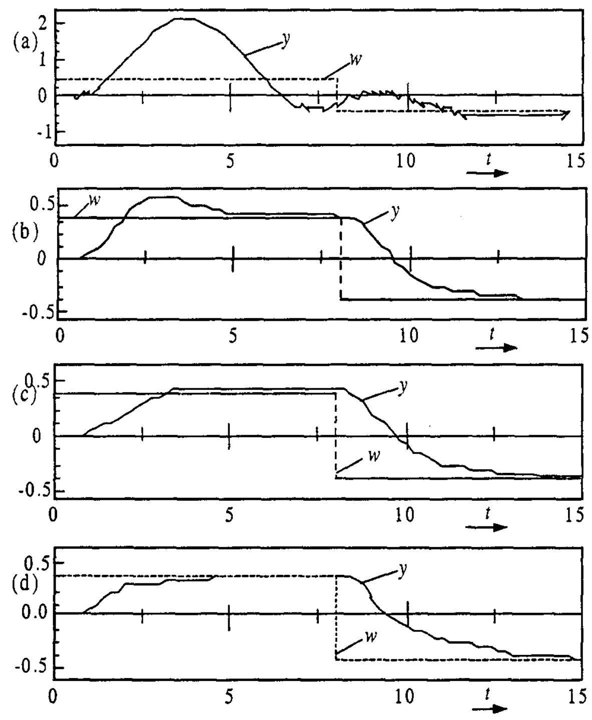

Figure 6.Refernce (w) and controlled signal (y) of the roll-angle for the four discussed cases (a) – (d)

5. Convergence of Adaptive Dual Control Algorithms

During the last 25 years the methods of Lyapunov functions and the methods of Martingale convergence theory (as their stochastic counterpart) have been successfully applied to convergence analysis of various adaptive systems (Goodwin and Sin, 1984). The control goal in these systems was formulated as global stability or asymptotic optimality. Thus, the systems guarantee optimum of the cost function only after adaptation for the considered infinite horizon problems, but the problem of improving the control quality during the adaptation had not been considered for a long time. The systems are usually based on the CE assumption and suffer from insufficient control performance at the beginning of the adaptation and after changes of the system parameters. At the same time, various adaptive dual control problems had been formulated as *finite horizon* optimal control problems but the convergence aspects were not applicable to them. Adaptive dual control methods, based on the problem reformulation, and predictive adaptive dual controllers consider the systems over an infinite control horizon; thus, the convergence properties must be studied for such systems. The difficulties of strict convergence analysis of adaptive dual control systems appear because of the nonlinearity of many dual controllers. The first results on convergence analysis of adaptive dual control systems were obtained by Radenkovic (1988). Based on these results the convergence analysis of various adaptive dual controllers can be performed, as was successfully demonstrated in (Filatov, 1998) for the GMV-controller. Stability of these adaptive dual controllers can be proved for the case of unmodelled effects, which can represent nonlinearities, time variations of parameters or high-order terms.

6. Conclusions

This paper presents the fundamentals for understanding the principle of optimal dual control. It includes the necessary definition of adaptive dual control systems and tries to provide a classification of these systems. The stages of historical development are mentioned. The recently developed bicriterial approach for adaptive dual control systems is mentioned as an important development for dual control systems. This method allows designing various well-known discrete-time adaptive controllers with direct and indirect adaptation. These new adaptive control structures compared to classical adaptive controllers, based on the CE principle, provide a considerably shorter and smoother transitional behaviour.

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