Non-Linear Programming Optimization Algorithms

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1 Introduction

This report is dedicated to implementing multiple algorithms for solving non-linear programming problems. The algorithms are implemented in *python* programming language. First, five 1D minimization algorithms are implemented. The five algorithms are: Fibonacci method, golden section method, Newton's method, Quasi-Newton (1D) method, and Secant method. Second, three algorithms for solving unconstrained non-linear programming problems, namely, Fletcher-Reeves Conjugate Gradient method, Marquardt method, and Quasi-Newton method. All of the algorithms will be tested on Rosenbrock's parabolic valley function, see Eq. 1.

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{1}$$

In addition, they will be tested on Powell's quartic function, see Eq. 2.

$$f(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4 \tag{2}$$

The initial point for all algorithms is $x_0 = (-1.2, 1)$ for Rosenbrock's function while $x_0 = (3, -1, 0, 1)$ for Powell's function. For the 1D algorithms, the search direction is $-\nabla f$. For both Fibonacci and Golden Section methods, the range is $[0, 1]$, and the accuracy is 0.01. For Newton's, Quasi-Newton (1D), and Secant methods, the starting point, λ_0, is 0 and the accuracy is 0.01. For both Quasi-Newton (1D) and Secant methods, $\Delta x = 0.1$ with a decay factor of value 0.9. Kindly find the colab notebook with the algorithms implementation here

2 Results

2.1 1D Minimization Algorithms

Table 1 summarizes the results of running the five 1D algorithms mentioned previously on Rosenbrock's function, where as Table 2 summarizes the results of running the them on Powell's function.

Method	Optimal solution	Optimal value	Iterations	CPU Time in sec.
Fibonacci	0.0139	100.193	10	0.044
Golden Section	0.011	24.270	11	0.041
Newton	$7.8\text{x}10^{-4}$	4.130	4	0.027
Quasi-Newton (1D)	$7.3\text{x}10^{-4}$	4.210	45	0.099
Secant	0.012	0.195	45	0.136

Table 1: 1D results on Rosenbrock's function.

Method	Optimal solution	Optimal value	Iterations	CPU Time in sec.
Fibonacci	$7\text{x}10^{-3}$	319.210	10	0.057
Golden Section	$3.1\text{x}10^{-3}$	31.337	11	0.041
Newton	$3.6\text{x}10^{-3}$	30.830	8	0.018
Quasi-Newton (1D)	$3.7\text{x}10^{-4}$	147.750	58	0.174
Secant	$3.6\text{x}10^{-3}$	30.830	57	0.157

Table 2: 1D results on Powell's function.

2.2 Minimization Algorithms for Unconstrained Non-Linear Programming Problems

In this section, the results of running the three unconstrained non-linear optimization algorithms (i.e., Fletcher-Reeves CG, Marquardt, Quasi-Newton) on both Rosenbrock's parabolic valley function and Powell's quartic function are presented.

2.2.1 Fletcher-Reeves CG

Based on the set up defined in section 1, the results of running Fletcher-Reeves CG algorithm on Rosenbrock's function are:

- x^*: $(0.99823557, 0.99650502)$
- $f(x^*)$: $3.2\text{x}10^{-6}$
- $\|\nabla f(x^*)\|$: $17\text{x}10^{-3}$
- **Number of iterations**: 116
- **CPU time in sec.**: $5.7\text{x}10^{-3}$

while for Powell's function the results are:

- x^*: $(0.09671869, -0.00952667, 0.04545902, 0.04602491)$
- $f(x^*)$: 172x10^{-6}
- $\|\nabla f(x^*)\|$: 26.4x10^{-3}
- **Number of iterations**: 41
- **CPU time in sec.**: 2.3x10^{-3}

2.2.2 Marquardt

The results for Rosenbrock's function are as follows:

- x^*: $(0.99630927, 0.9926215)$
- $f(x^*)$: 13.6x10^{-6}
- $\|\nabla f(x^*)\|$: 3.8x10^{-3}
- **Number of iterations**: 26
- **CPU time in sec.**: 1.8x10^{-3}

The results for Powell's function are:

- x^*: $(0.08440091, -0.00843456, 0.04186865, 0.04205224)$
- $f(x^*)$: 5922$10^{-6}$
- $\|\nabla f(x^*)\|$: 26.4x10^{-3}
- **Number of iterations**: 19
- **CPU time in sec.**: 3.7x10^{-3}

2.2.3 Quasi-Newoton

To prevent the Quasi-Newton algorithm from diverging, a backtracking line search algorithm was implemented to find an approximately optimal step size along the current search direction. The stopping condition for the line search algorithm is the Armijo-Goldstien condition [1]. The reults for the Rosenbrock function are:

- x^*: $(1.00246043, 1.00495416)$
- $f(x^*)$: 6.1x10^{-6}
- $\|\nabla f(x^*)\|$: 8.1x10^{-3}
- **Number of iterations**: 141
- **CPU time in sec.**: 26.8x10^{-3}

For Powell's function, find the results below:

- x^*: $(-0.01814078, 0.0018444, -0.00171489, -0.00180868)$
- $f(x^*)$: 0.810^{-6}
- $\|\nabla f(x^*)\|$: 6.2x10^{-3}
- **Number of iterations**: 49
- **CPU time in sec.**: 16.8x10^{-3}

References

[1] Wikipedia contributors, "Backtracking line search," *Wikipedia*, Jul. 28, 2024. Available at: https://en.wikipedia.org/wiki/Backtracking$_{l}ine_{s}earch$.