Tutorial Machine Learning in Solid Mechanics (Winter term 2024–2025)

TECHNISCHE UNIVERSITÄT DARMSTADT

Cyber-Physical Simulation TU Darmstadt

Task 3: Viscoelasticity

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In Task 2 of this tutorial, neural networks were used to model hyperelastic material behavior. In the framework of hyperelasticity, no energy is dissipated. This is a strong assumption, as inelastic effects are not considered. A very important class of inelasticity is viscoelasticity, which is used in, e.g., rubber dampers, and includes time-dependence of the stress-strain relation [4]. In the last task of this tutorial, viscoelastic material behavior is modelled. While in the previous task we described material behavior in three dimensions, here, the model is restricted to 1D. This last task gives an outlook on another class of neural networks, namely: Recurrent Neural Networks (RNNs) [1], as well as their usage for the modeling of inelastic material behavior.

For the following tasks, data was generated by applying the harmonic oscillation

$$\varepsilon(t) = A\sin(\omega t)\,, \qquad \dot{\varepsilon}(t) \quad = A\,\omega\cos(\omega t)\,, \qquad t \in [0, 2\pi]\,, \tag{1}$$

on a Maxwell element, where the parameters are set to $\{E^{\infty}, E, \eta\} = \{0.5, 2, 1\}$ [3, Table 1], see Fig. 1. In addition, relaxation tests with different amplitudes are applied. For the relaxation tests, the strain as in Eq. (1) is applied until the first maximum of ε is reached, and is then fixed. For the following investigations, set $(A, \omega) \in \{(1, 1), (1, 2), (2, 3)\}$. For each of the following model calibrations, use different load paths for the calibration dataset and the rest for the test dataset. After solving the following tasks, answer these questions:

- Which of the models are able to interpolate / extrapolate to the given data?
- Which load paths do you have to use for a good model generalization?
- What happens when you change the number of time steps for the datasets (=change the size of the time increments)?
- What is the difference between the different models?
- What happens if you use other values for (A, ω) for calibration / test of the models?
- In what way is physics included in the models?

1 Simple RNN

Visit the GitHub repository CPShub/TutorialMLinSolidMechanics and go to the folder “Winter_term_24_25/viscoelasticity”. Run the main file. This imports and visualizes the data generated with the harmonic oscillation, see Eq. (1), for arbitrary (A, ω). Furthermore, running the main file imports and calibrates a “naive” RNN which does not include any physical information. Investigate the code of this RNN. Is the model able to interpolate / extrapolate the different cases?

2 Maxwell model

Implement the Maxwell model, using the parameters introduced above. This model is not trainable, as all functional relationships and parameters are fixed. For solving the evolution equation, use an explicit Euler scheme. Implement this model in TensorFlow by adapting the RNN from Sec. 1. Check your implementation with the data provided in the GitHub repository. ~~Then~~, adapt the Maxwell model, so that the evolution equation for the internal variable is not a fixed function, but represented by a feed-forward neural network (FFNN).

3 Generalized standard materials (GSM) model

Implement the GSM model [2]. For this, set $g := \eta^{-1} = \text{const}$, with the value for η as introduced above. Use a FFNN to represent the energy function $e = e(\varepsilon, \gamma)$, and calculate the derivatives by using “GradientTape”.

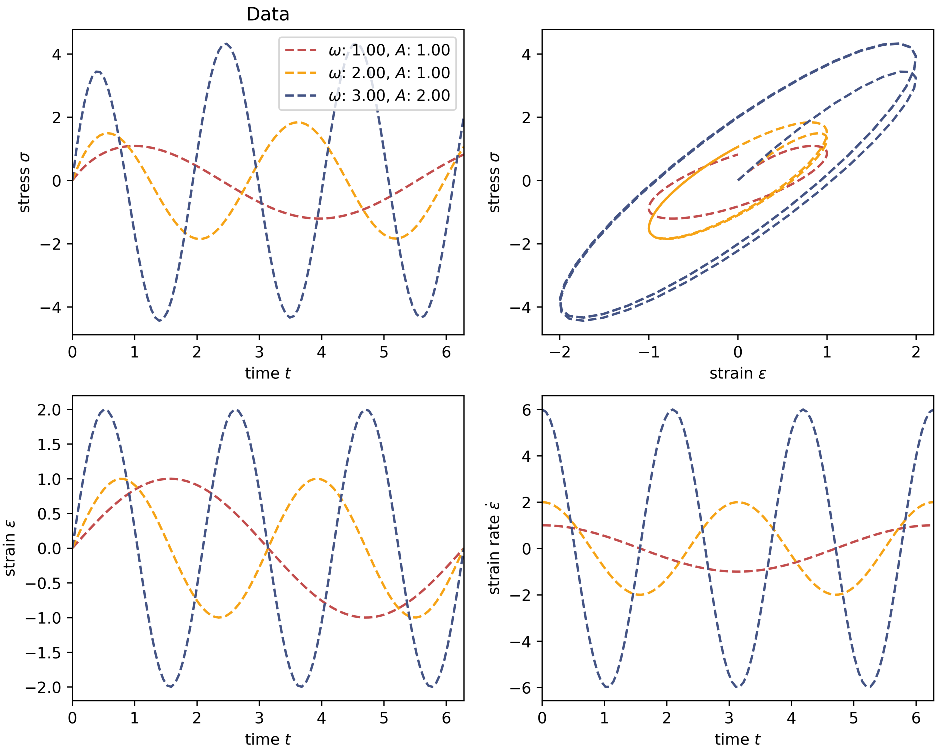

Figure 1: Data for harmonic oscillations.

References

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