What's the role of higher-order *******?

An introduction

G. Petri

ISI Foundation & ISI Global Science Foundation

TDA Crash Course, Complex Networks 2021

Madrid, 29th Dec 2021

Why are we talking about *****?

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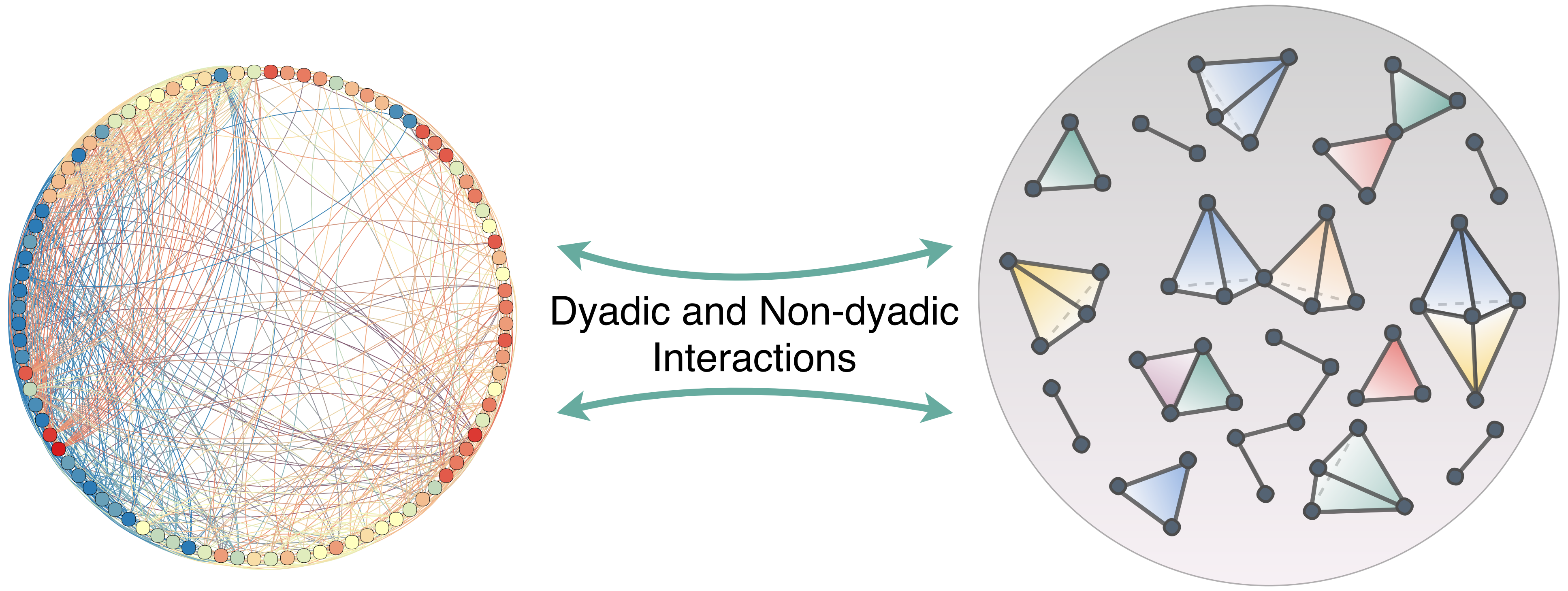
1. **Higher-order interactions:**
    1. Paradigmatic example: collaboration
    2. complex contagion / local environment
    3. gene-gene interactions
    4. Some type of neuronal interactions



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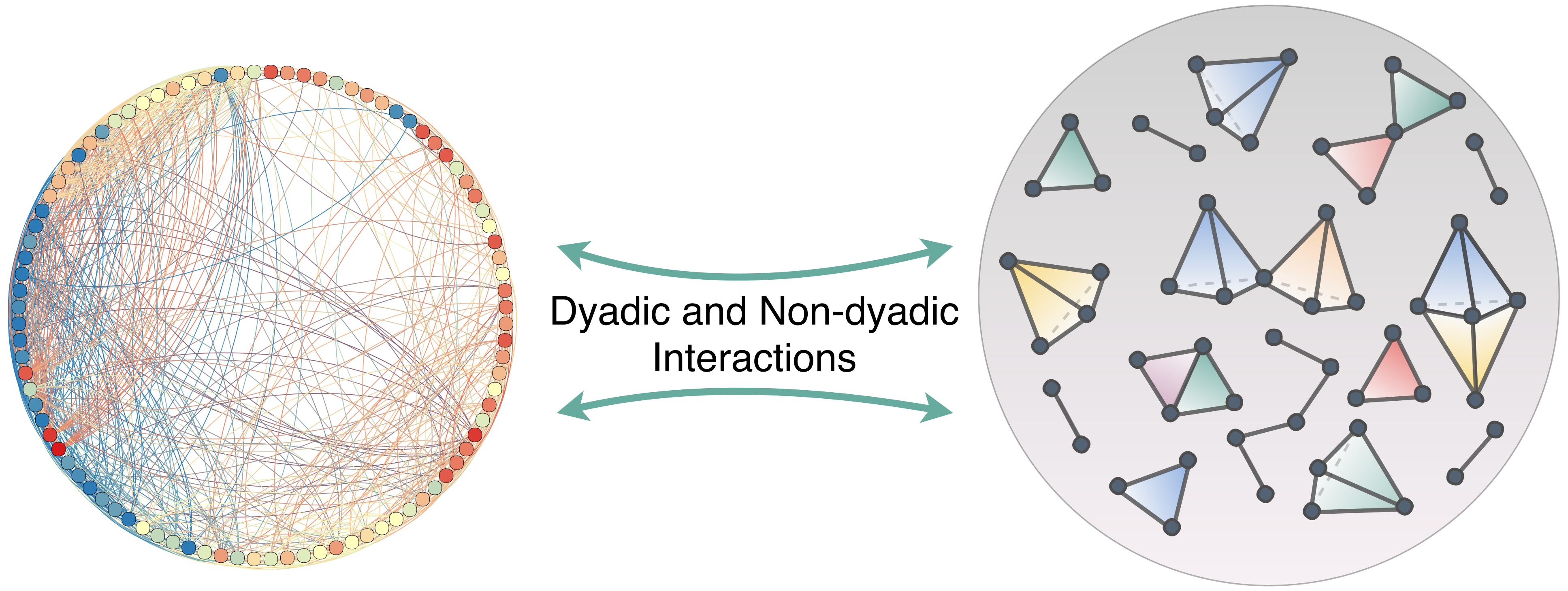
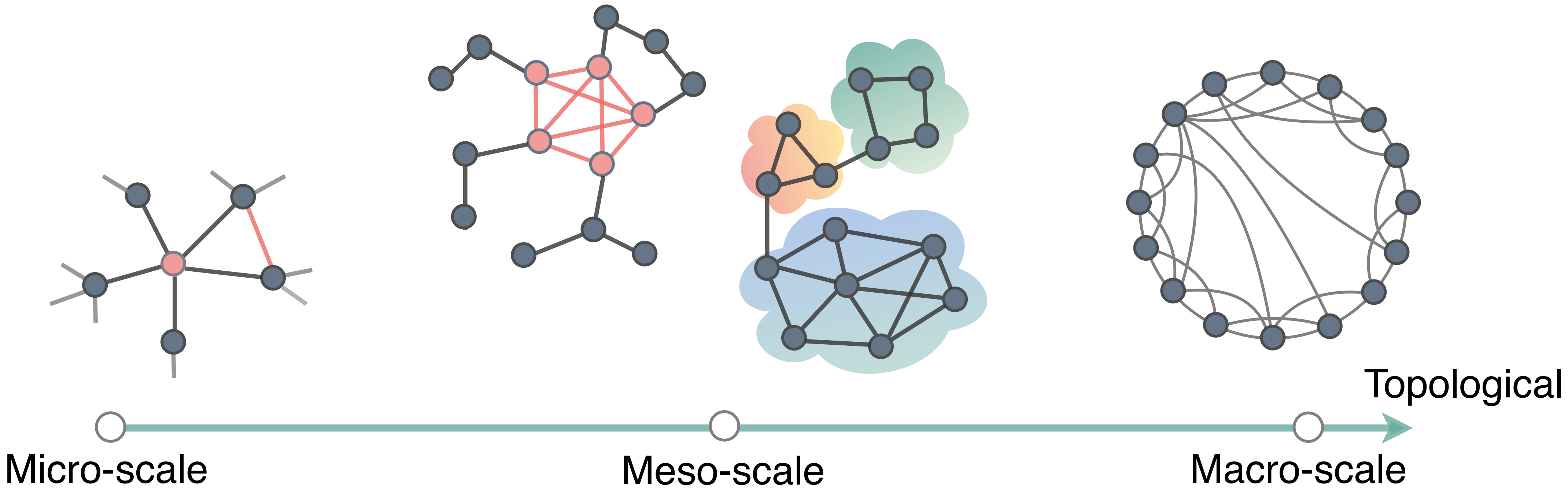


2. **Shapes / Mesoscale structures**
    1. Organisation on intermediate scales
    2. Non-local features



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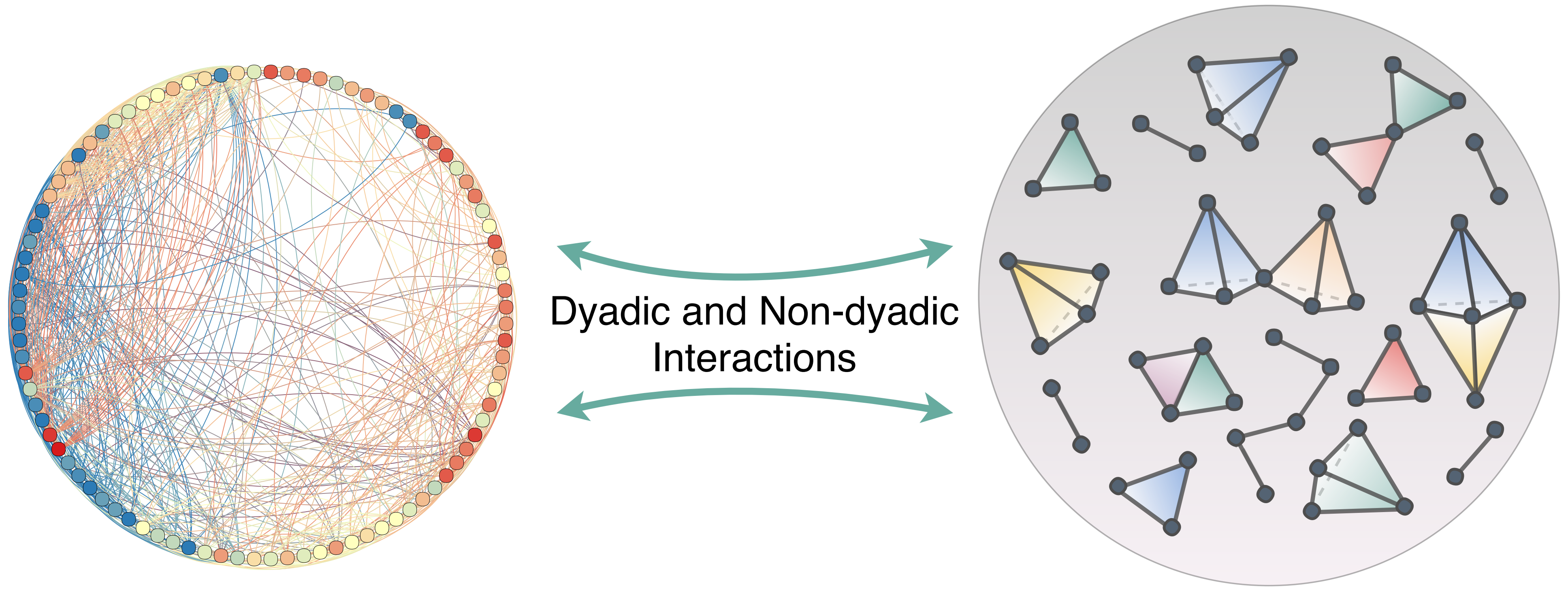

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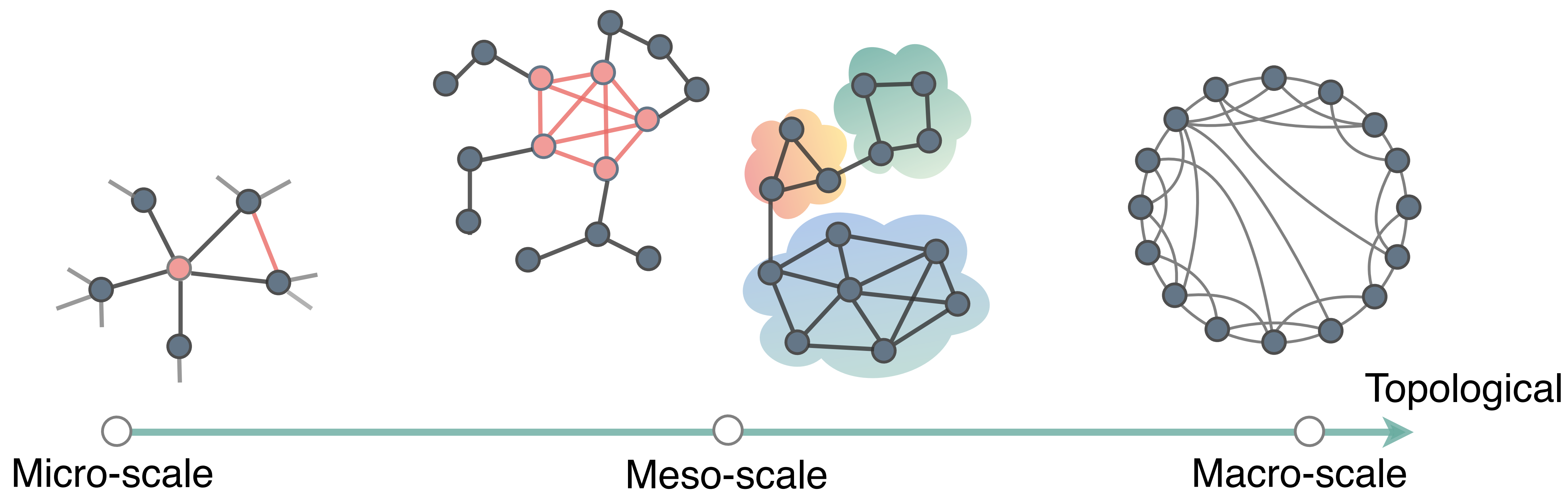

3. General comparison of spaces
1. Distance manifold to manifold
2. Compare networks with different structures

What does it mean in practice?

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1. **(Persistent) homology**
   1. Multiscale description of a network/space
   2. Complementary to communities/blocks
   3. Categorical foundation

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   1. Configuration models (SCM, ERSC)
   2. Generative models (Network geo+flavour)
   3. Dynamics on/off models (SAD)

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TDA

“Netsci”

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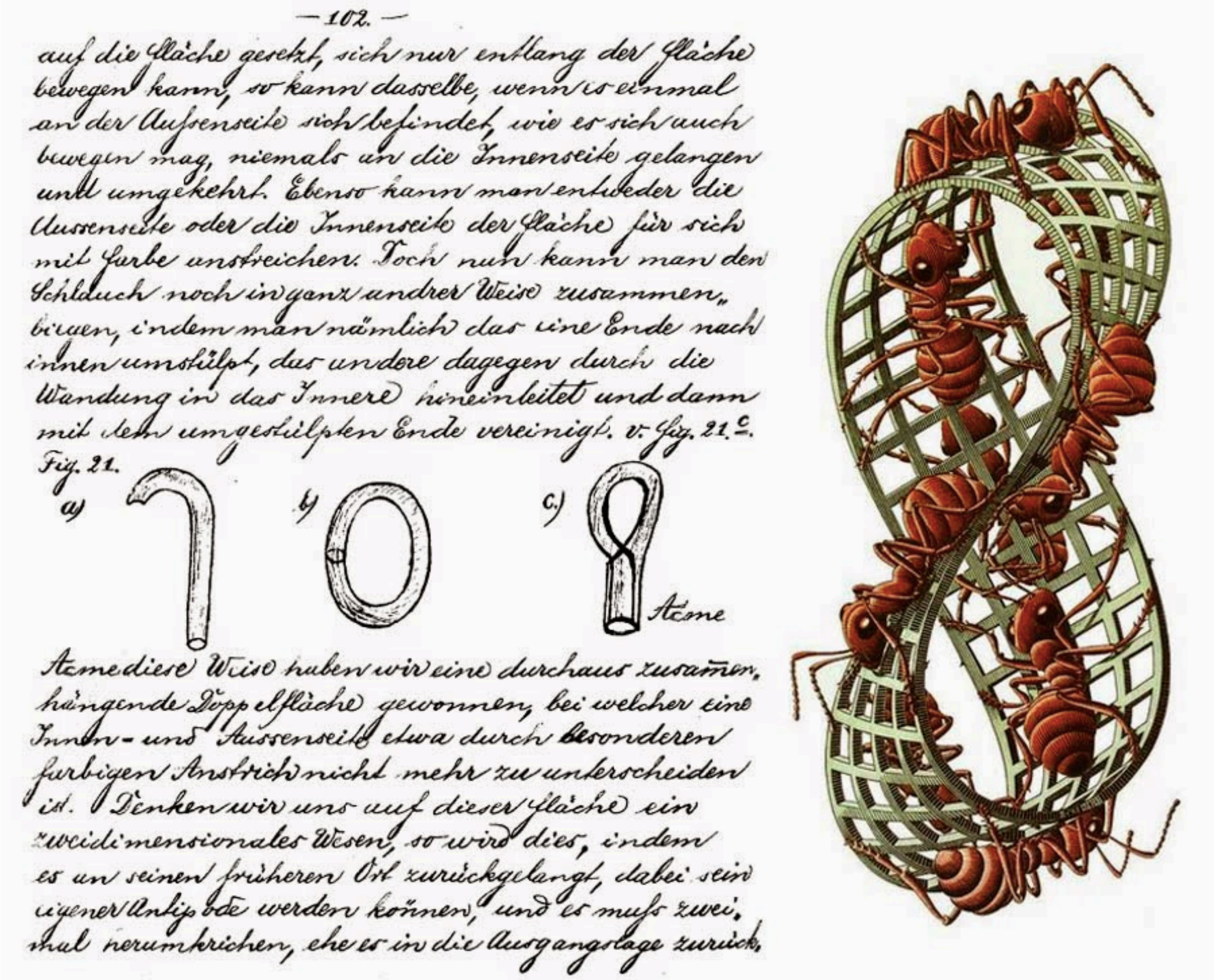
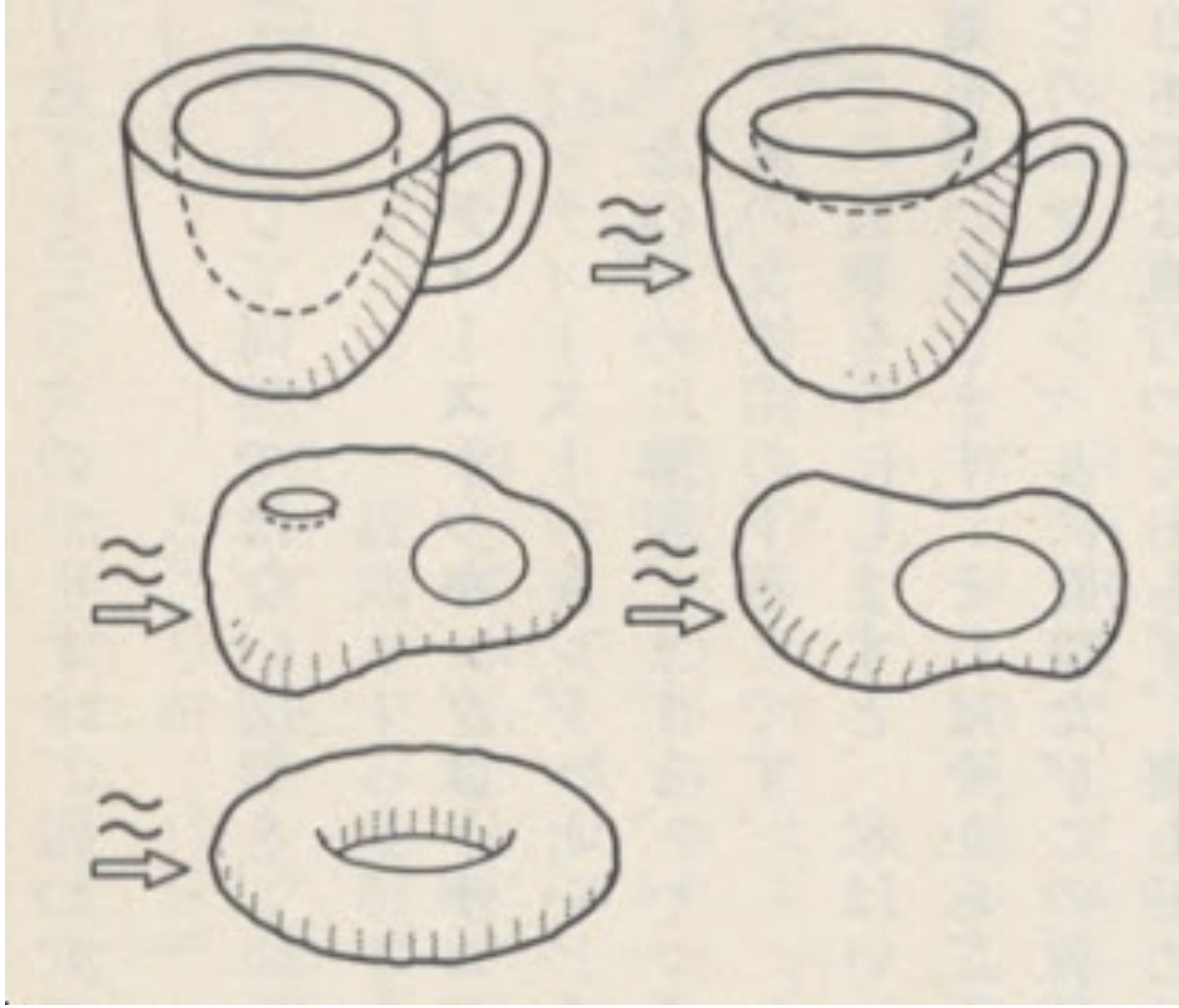
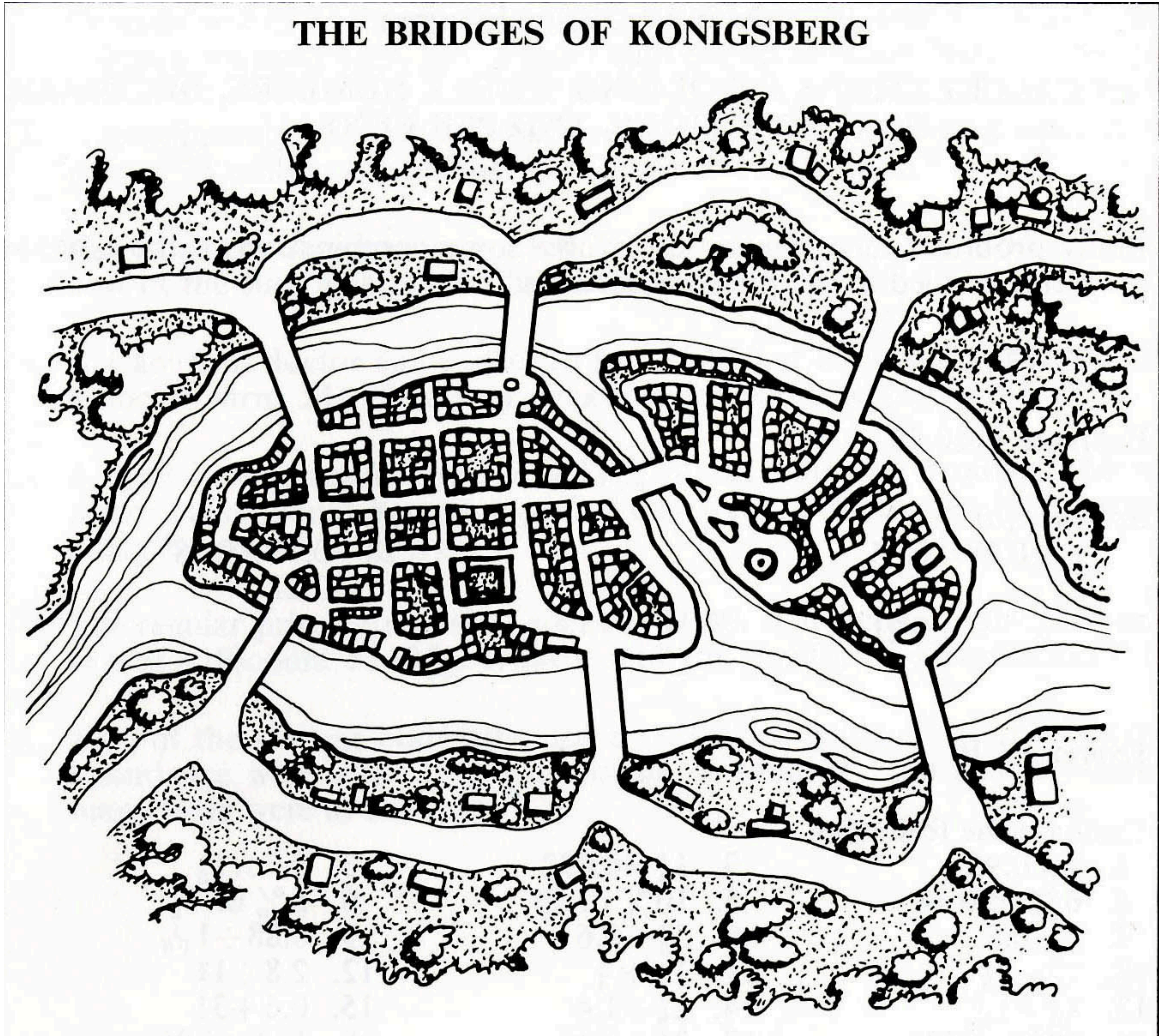
TDA

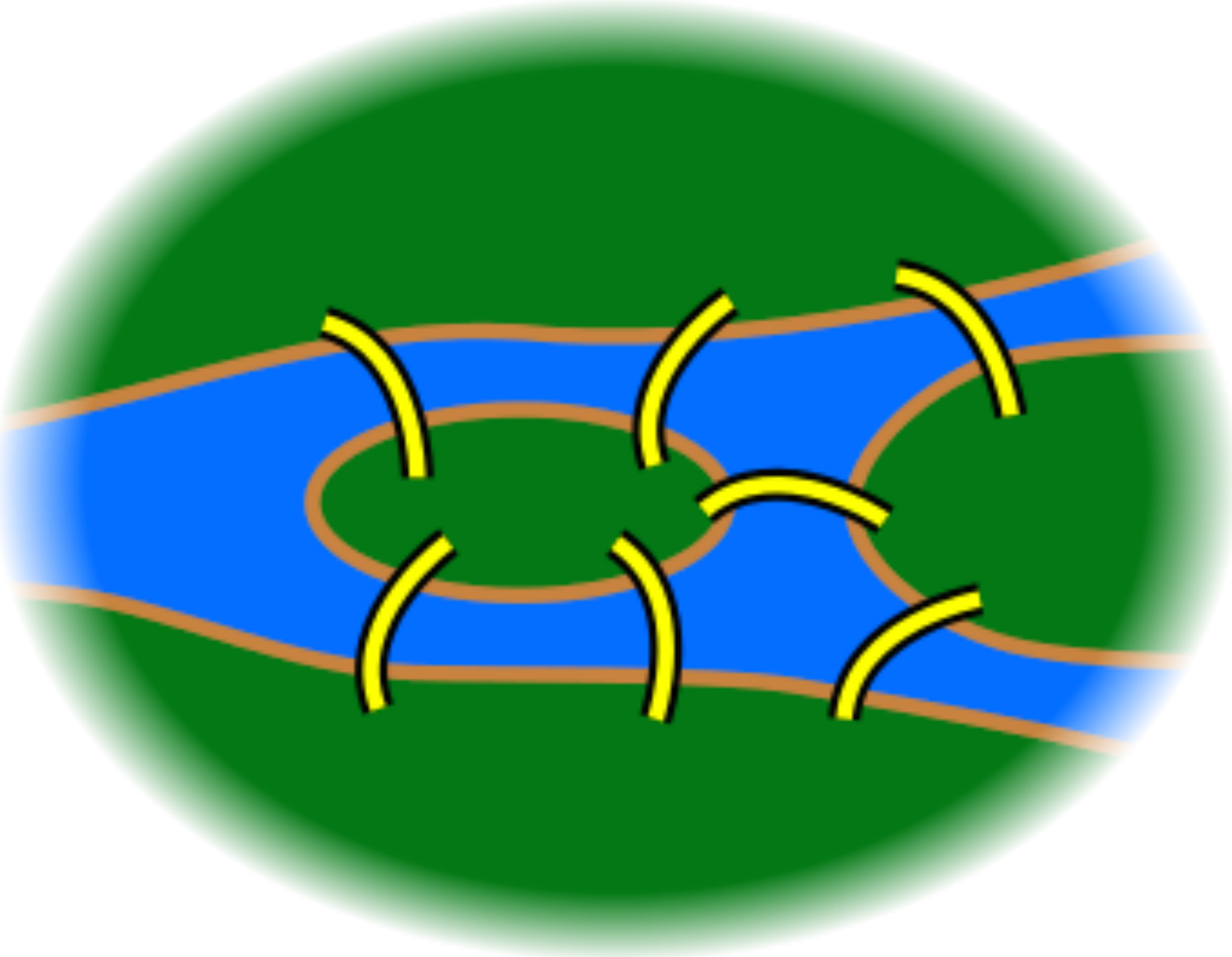
"Netsci"

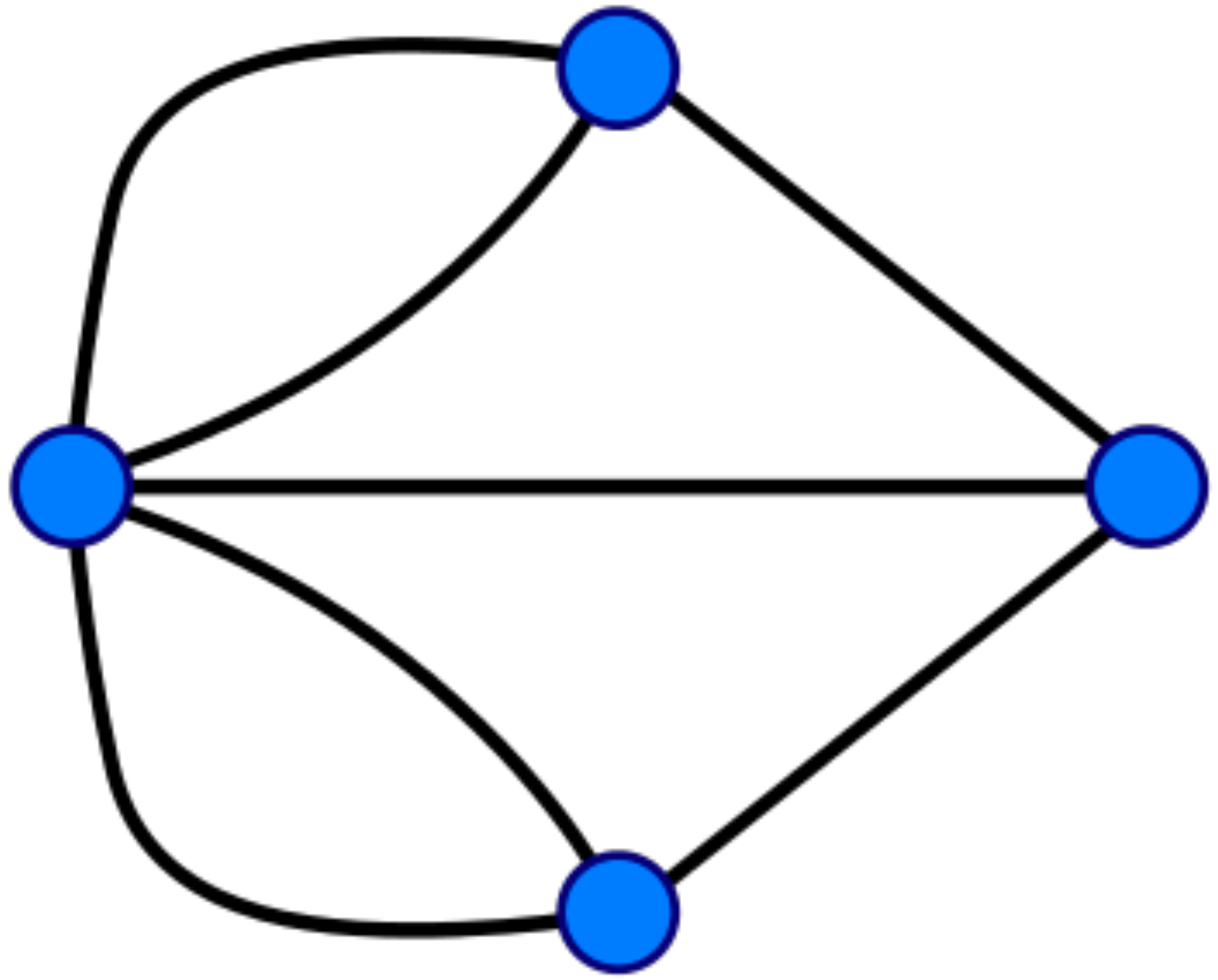
Why Topology?






THE BRIDGES OF KONIGSBERG





Northwood
Northwood Hills
Ruislip
Ruislip Manor
Pinner
North Harrow
Eastcote
Rayners Lane
West Harrow
Harrow-on-the-Hill
Harrow & Wealdstone
Hatch End
Headstone Lane
Stanmore
Canons Park
Queensbury
Kingsbury
Kenton
Northwick Park
Preston Road
South Kenton
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Paddington
Edgware Road
Marylebone
South Harrow
Sudbury Hill
Sudbury Town
Alperton
Perivale
Hanger Lane
Park Royal
North Ealing
Ealing Broadway
West Ealing
Acton Main Line
West Acton
North Acton
East Acton
White City
Ealing Common
Acton Central
South Acton
Acton Town
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow Central
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Neasden
Dollis Hill
Willesden Green
Wembley Park
Kilburn
West Hampstead
Kensal Rise
Brondesbury Park
Brondesbury
Finchley Road & Frognal
Finchley Road
Swiss Cottage
South Hampstead
St John's Wood
Kilburn High Road
Hampstead Heath
Belsize Park
Chalk Farm
Camden Town
Mornington Crescent
Gospel Oak
Kentish Town West
Kentish Town
Camden Road
Mill Hill East
West Finchley
Finchley Central
East Finchley
Highgate
Archway
Upper Holloway
Tufnell Park
Crouch Hill
Euston
Euston Square
King's Cross St Pancras
Great Portland Street
Baker Street
Regent's Park
Warren Street
Goodge Street
Russell Square
Royal Oak
Westbourne Park
Ladbroke Grove
Latimer Road
Shepherd's Bush
Wood Lane
Shepherd's Bush Market
Goldhawk Road
Hammersmith
Barons Court
Kensington (Olympia)
Holland Park
Notting Hill Gate
Bayswater
Queensway
Lancaster Gate
Marble Arch
Bond Street
Oxford Circus
Tottenham Court Road
Holborn
Chancery Lane
High Street Kensington
Gloucester Road
Earl's Court
West Kensington
South Kensington
Sloane Square
Knightsbridge
Hyde Park Corner
Green Park
Piccadilly Circus
Leicester Square
Covent Garden
Charing Cross
Victoria
St James's Park
Westminster
Embankment
Temple
Waterloo
Southwark
Lambeth North
Elephant & Castle
Kennington
Oval
Stockwell
Vauxhall
Pimlico
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfields
Wimbledon Park
Wimbledon
Imperial Wharf
Clapham Junction
Wandsworth Road
Clapham High Street
Clapham North
Clapham Common
Amos Grove
Bounds Green
Wood Green
Turnpike Lane
Manor House
Harringay Green Lanes
Arsenal
Holloway Road
Caledonian Road
Finsbury Park
Highbury & Islington
Caledonian Road & Barnsbury
Canonbury
Dalston Kingsland
Dalston Junction
Haggerston
Hoxton
Shoreditch High Street
Angel
Old Street
Farringdon
Barbican
Moorgate
Liverpool Street
Aldgate
Aldgate East
Bank
St Paul's
Cannon Street
Mansion House
Blackfriars
Monument
Tower Hill
Fenchurch Street
Tower Gateway
London Bridge
Borough
Bermondsey
Canada Water
Rotherhithe
Surrey Quays
Queens Road Peckham
Peckham Rye
Denmark Hill
New Cross
New Cross Gate
Brockley
Honor Oak Park
Forest Hill
Sydenham
River Thames
Silver Street
White Hart Lane
Bruce Grove
South Tottenham
Seven Sisters
Tottenham Hale
Blackhorse Road
Walthamstow Central
Stamford Hill
Walthamstow Queen's Road
Stoke Newington
Rectory Road
Clapton
Hackney Downs
Hackney Central
Homerton
Hackney Wick
London Fields
Cambridge Heath
Bethnal Green
Mile End
Stepney Green
Whitechapel
Shadwell
Wapping
Limehouse
Westferry
West India Quay
Canary Wharf
Heron Quays
South Quay
Crossharbour
Mudchute
Island Gardens
Cutty Sark for Maritime Greenwich
Greenwich
Deptford Bridge
Elverson Road
Lewisham
Highams Park
Wood Street
Leyton Midland Road
St James Street
Leytonstone High Road
Leyton
Stratford International
Stratford
Stratford High Street
Pudding Mill Lane
Abbey Road
Bow Road
Bow Church
Bromley-by-Bow
West Ham
Star Lane
Devons Road
Langdon Park
All Saints
Poplar
Blackwall
East India
Canning Town
North Greenwich
Emirates Greenwich Peninsula
Emirates Royal Docks
Royal Victoria
Custom House for ExCeL
Prince Regent
Royal Albert
West Silvertown
Beckton Park
Pontoon Dock
Cyprus
London City Airport
King George V
Woolwich Arsenal
Chigwell
Hainault
Woodford
Fairlop
South Woodford
Barkingside
Snaresbrook
Newbury Park
Redbridge
Wanstead
Gants Hill
Leytonstone
Wanstead Park
Forest Gate
Maryland
Manor Park
Woodgrange Park
Ilford
Seven Kings
Goodmayes
Chadwell Heath
Romford
Gidea Park
Emerson Park
Harold Wood
Upminster
Hornchurch
Elm Park
Dagenham East
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Becontree
Upney
Barking
East Ham
Upton Park
Plaistow

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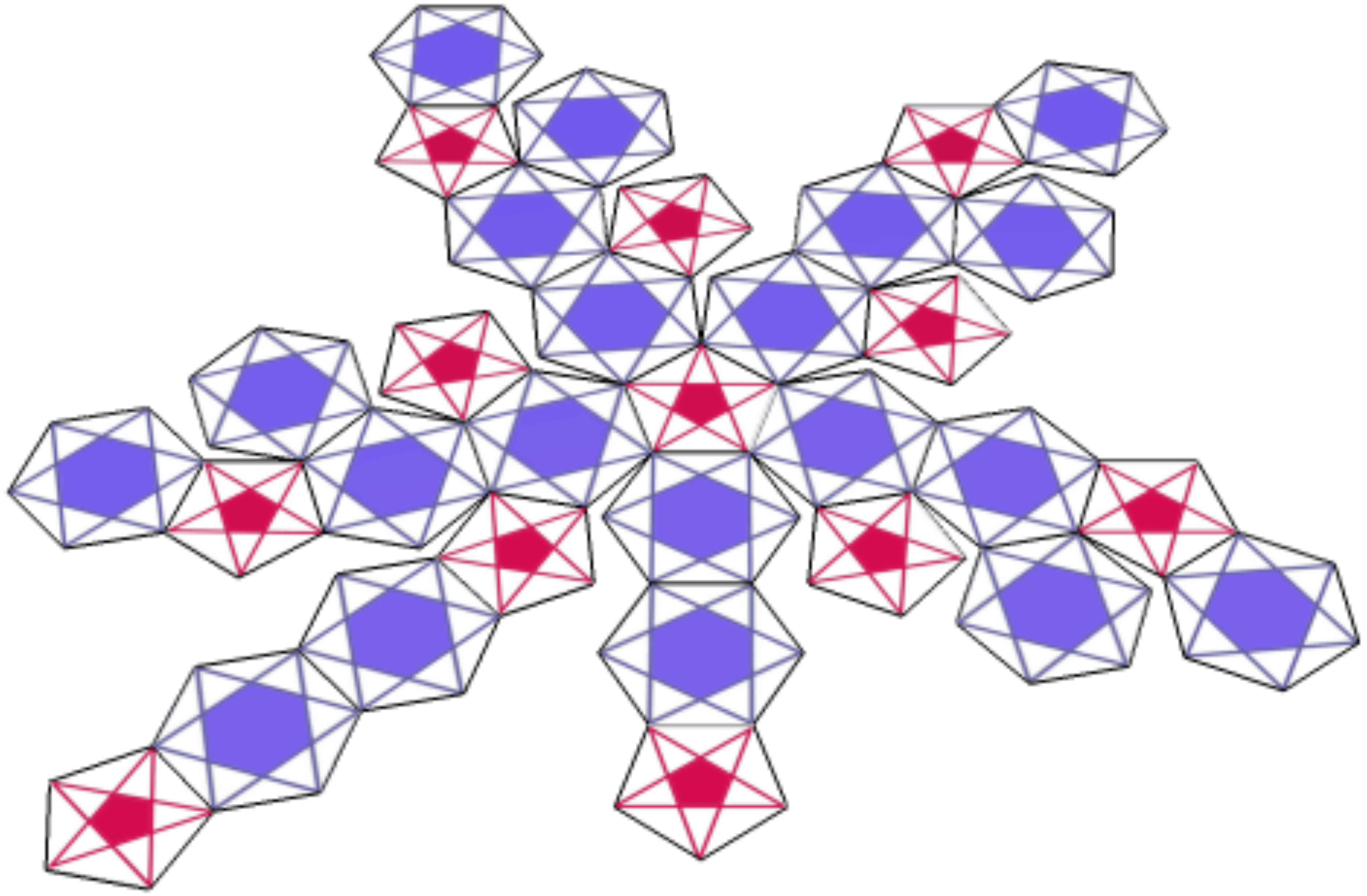

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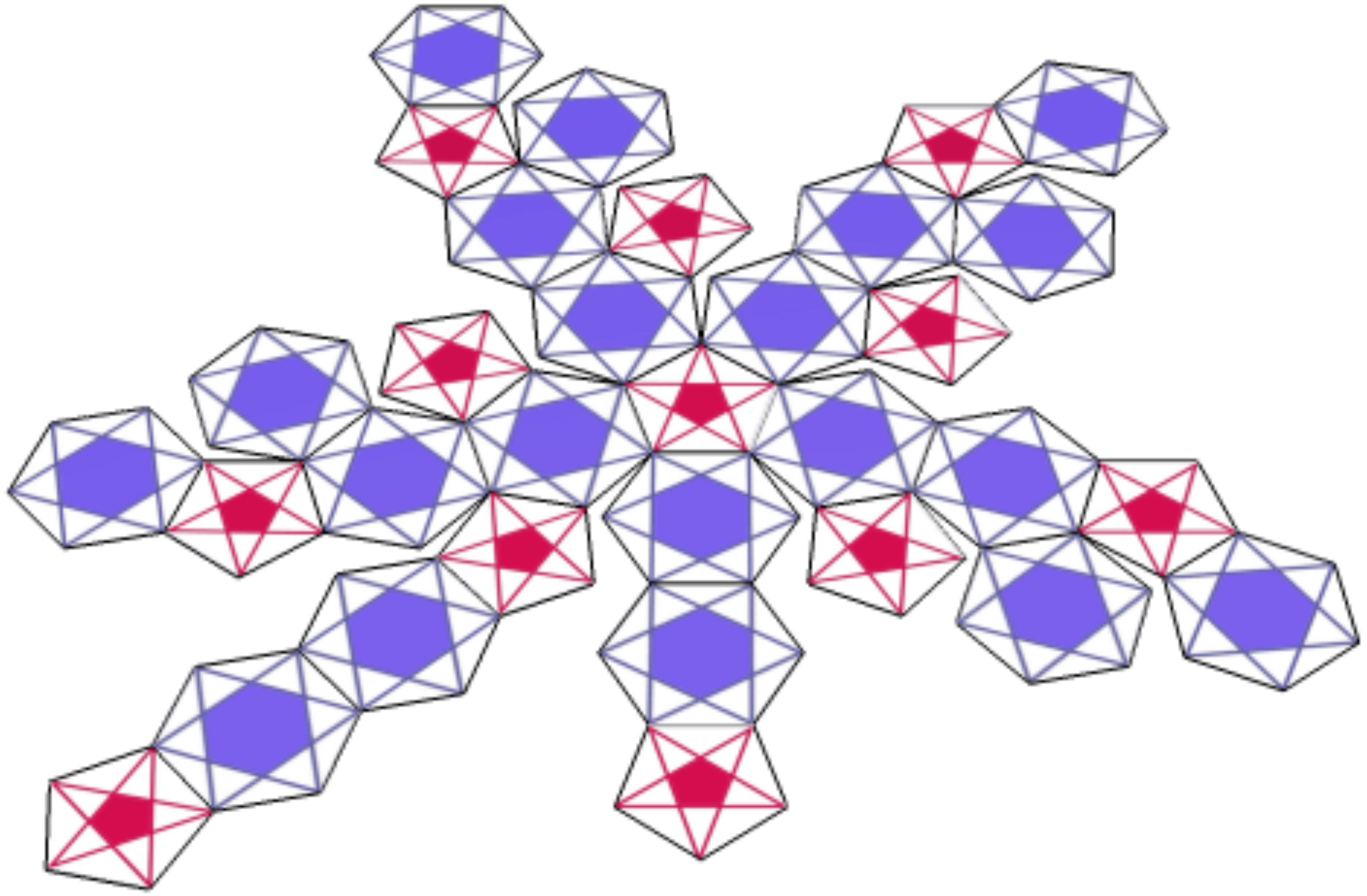

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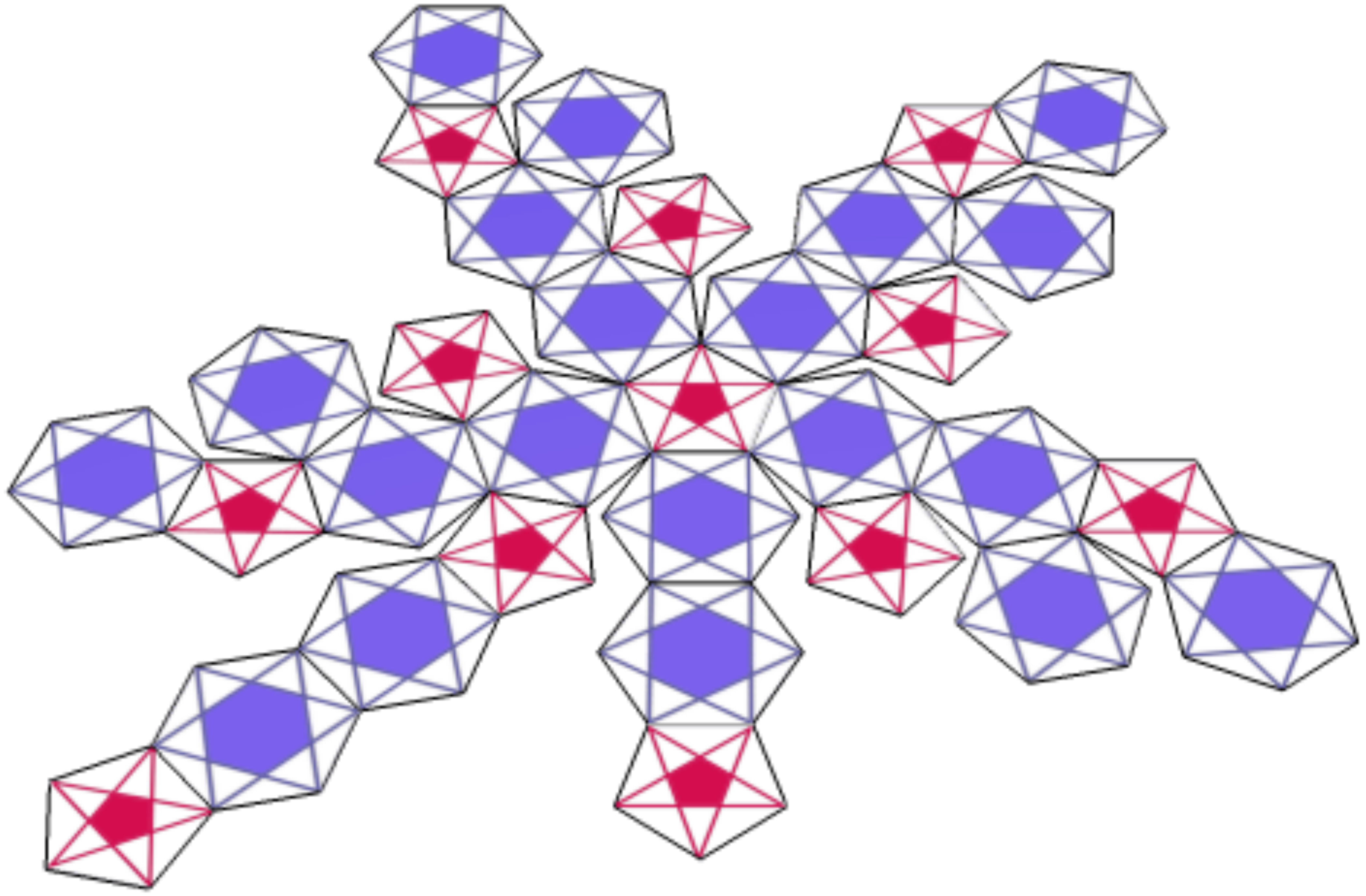

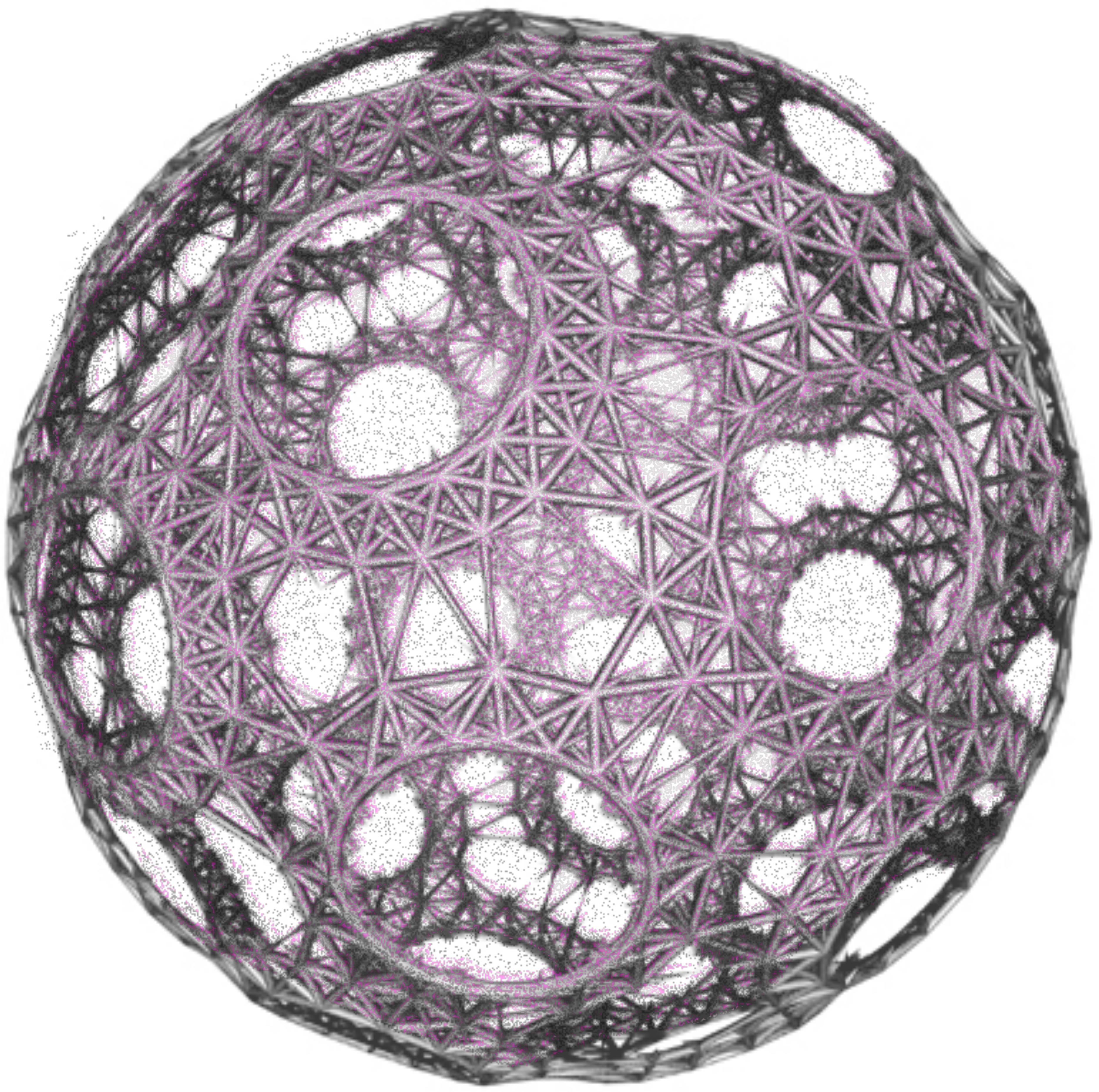

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Persistent homology pipeline (Ghrist 2008)

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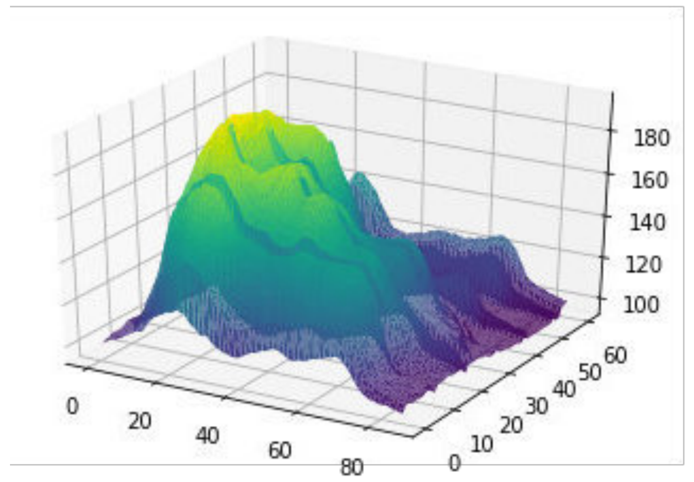

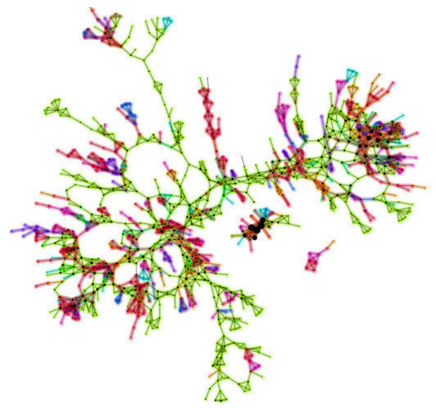

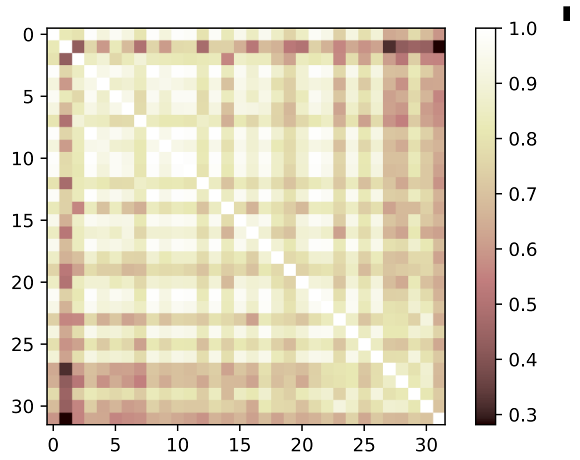

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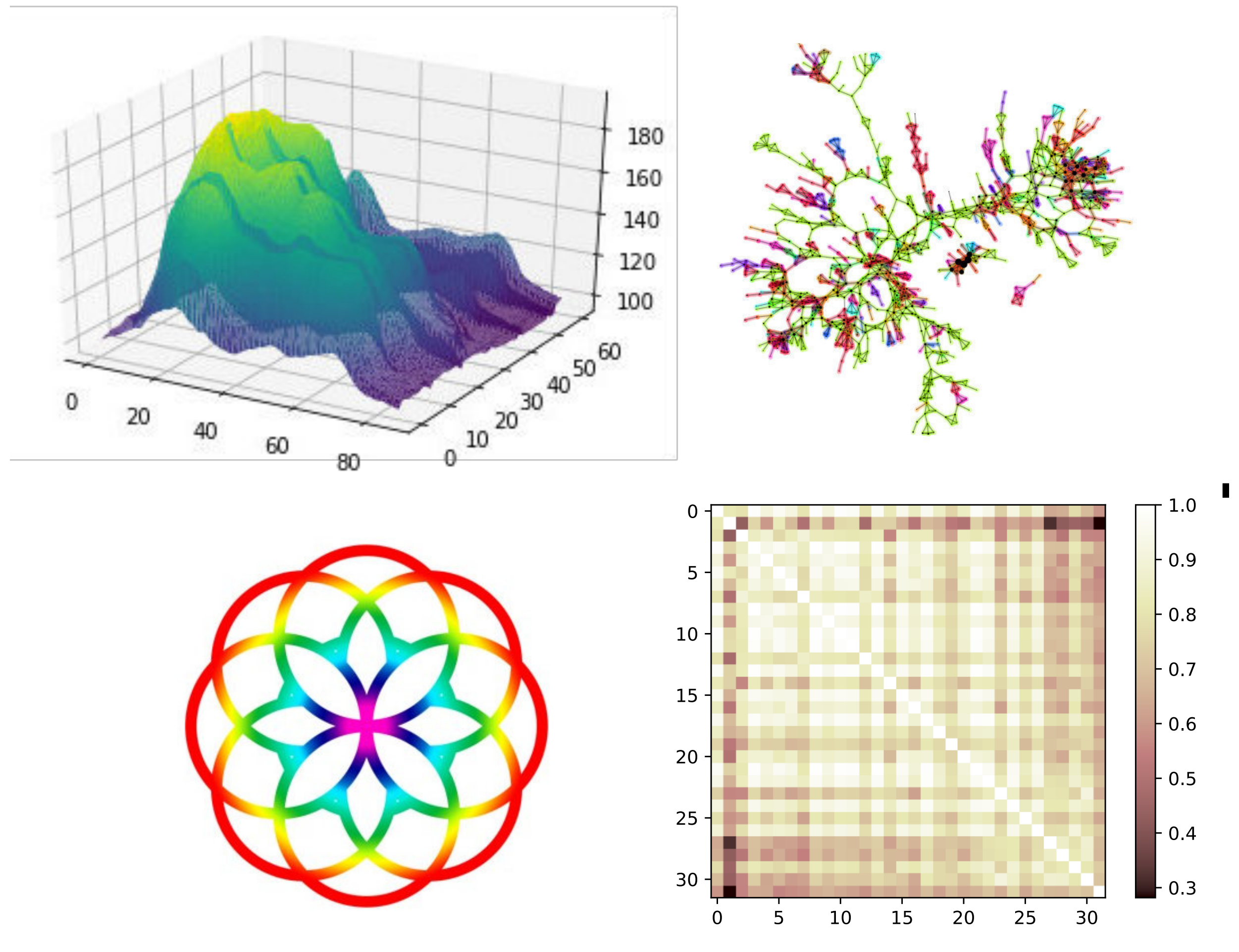

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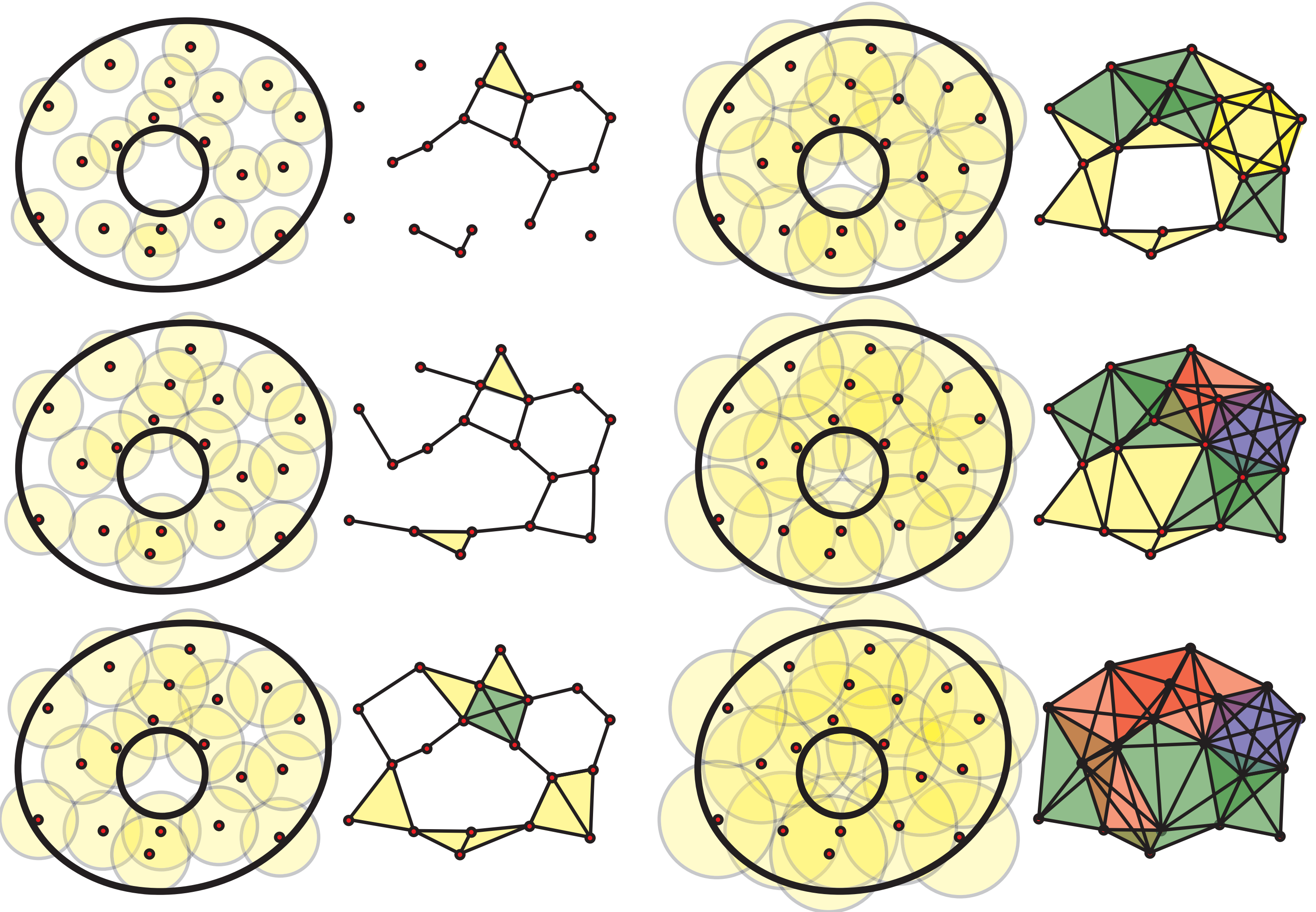

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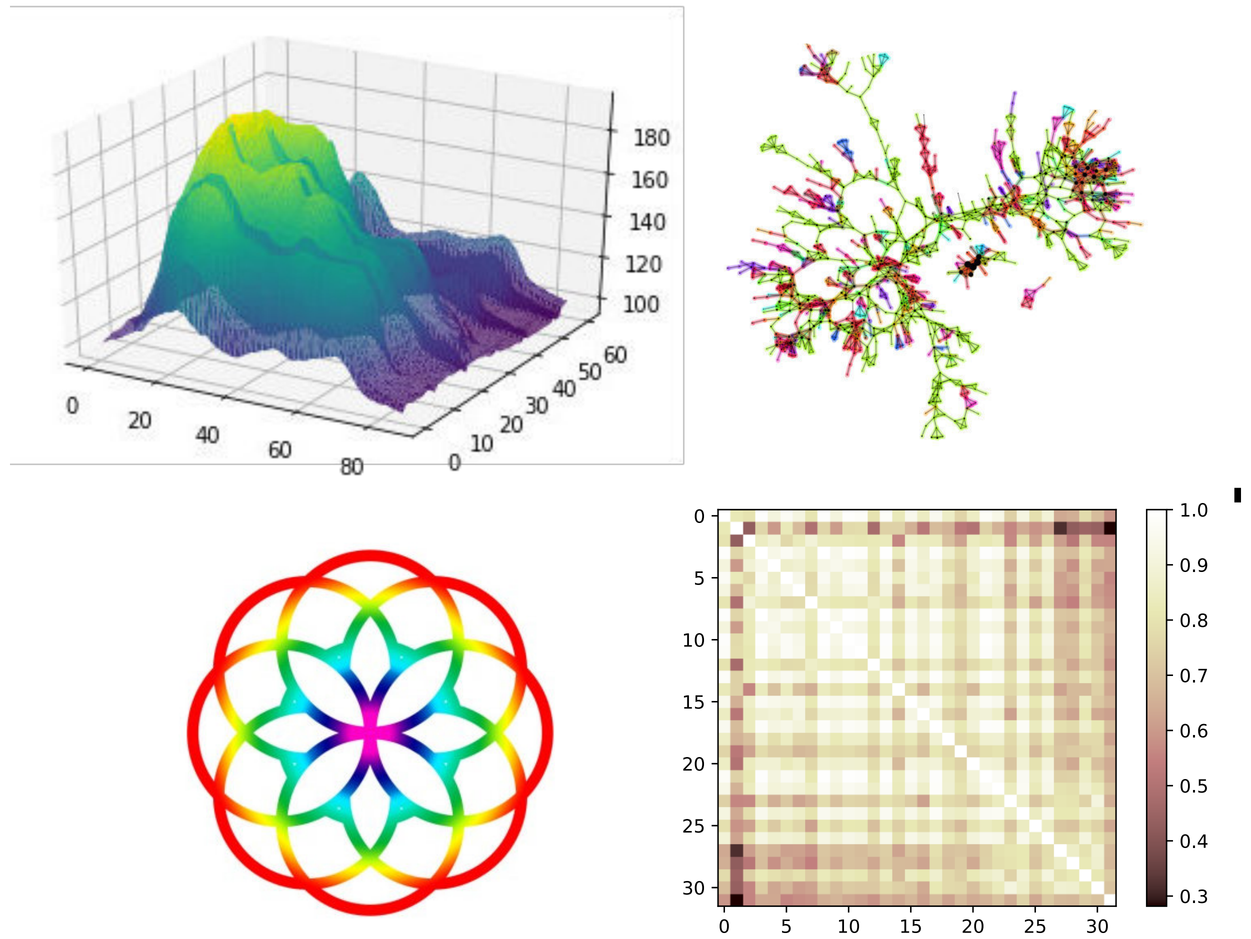

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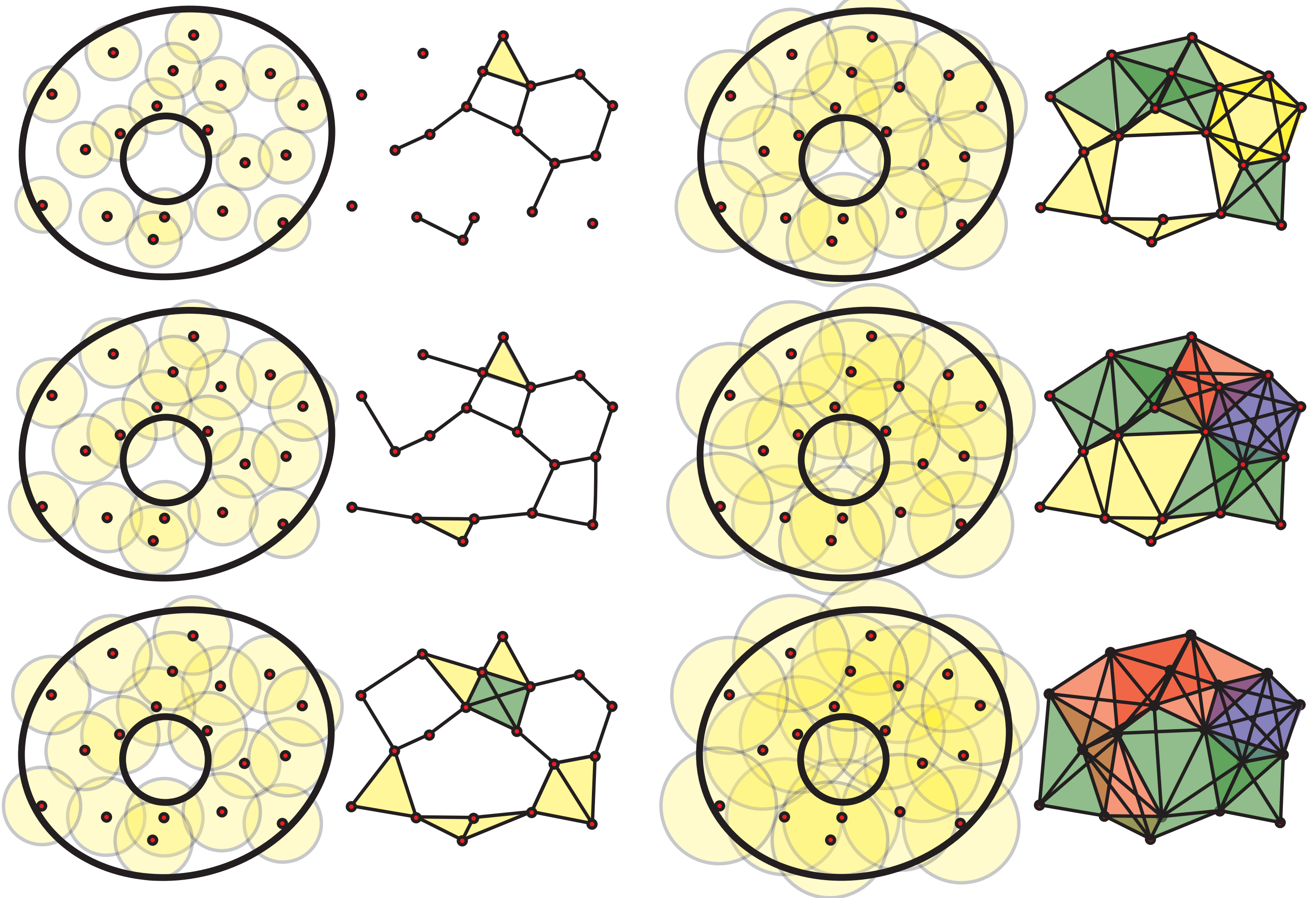
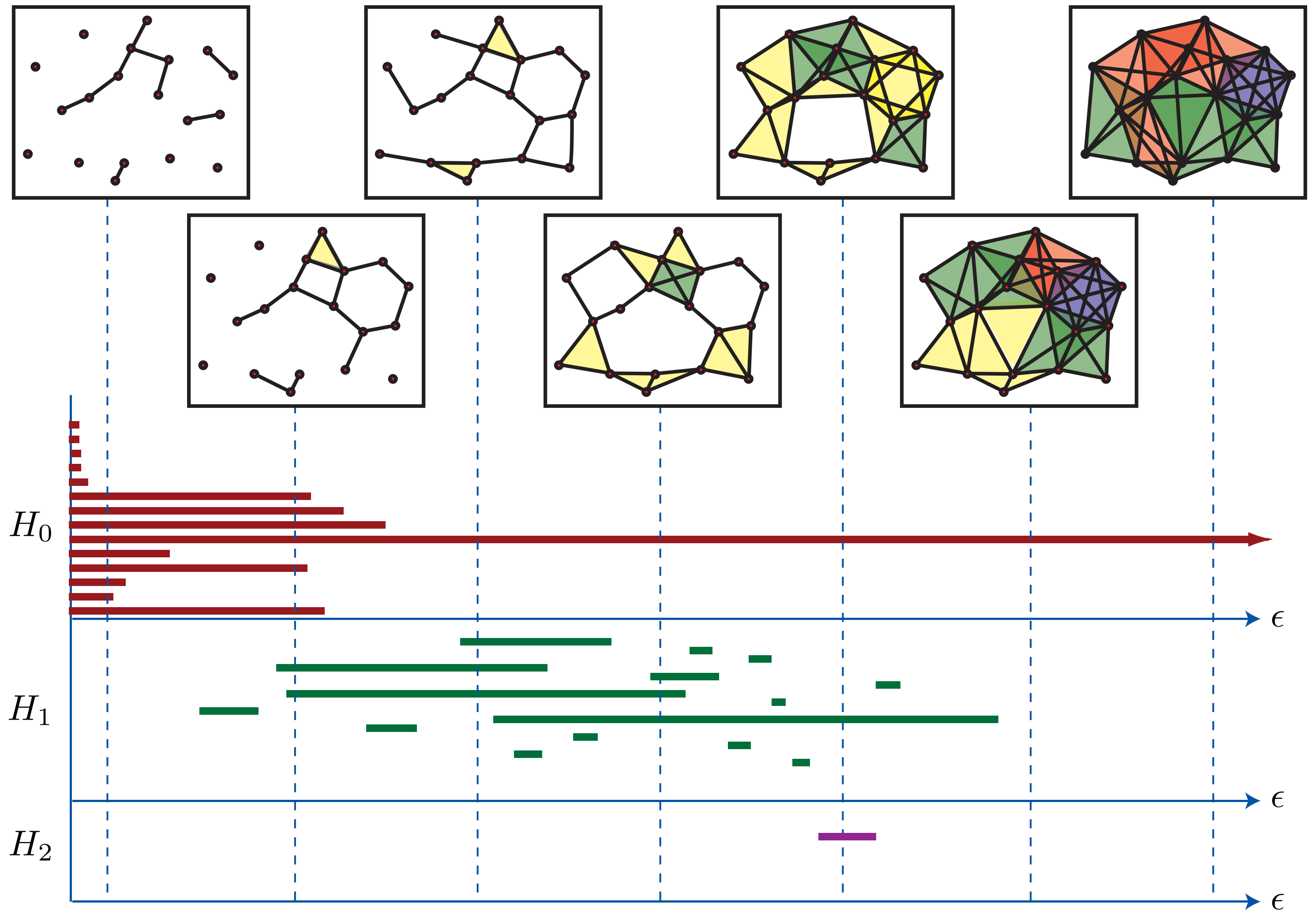


Homological properties



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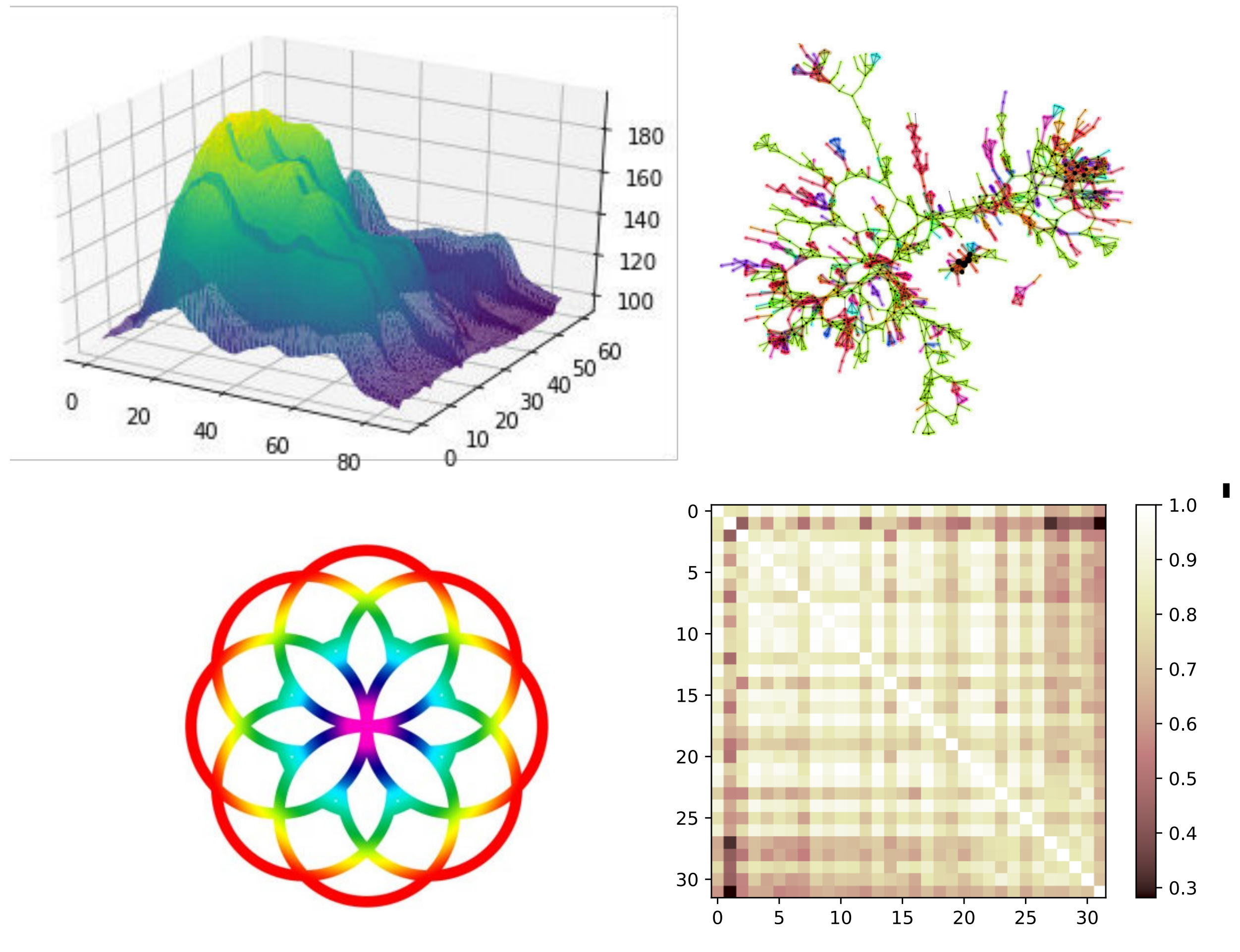

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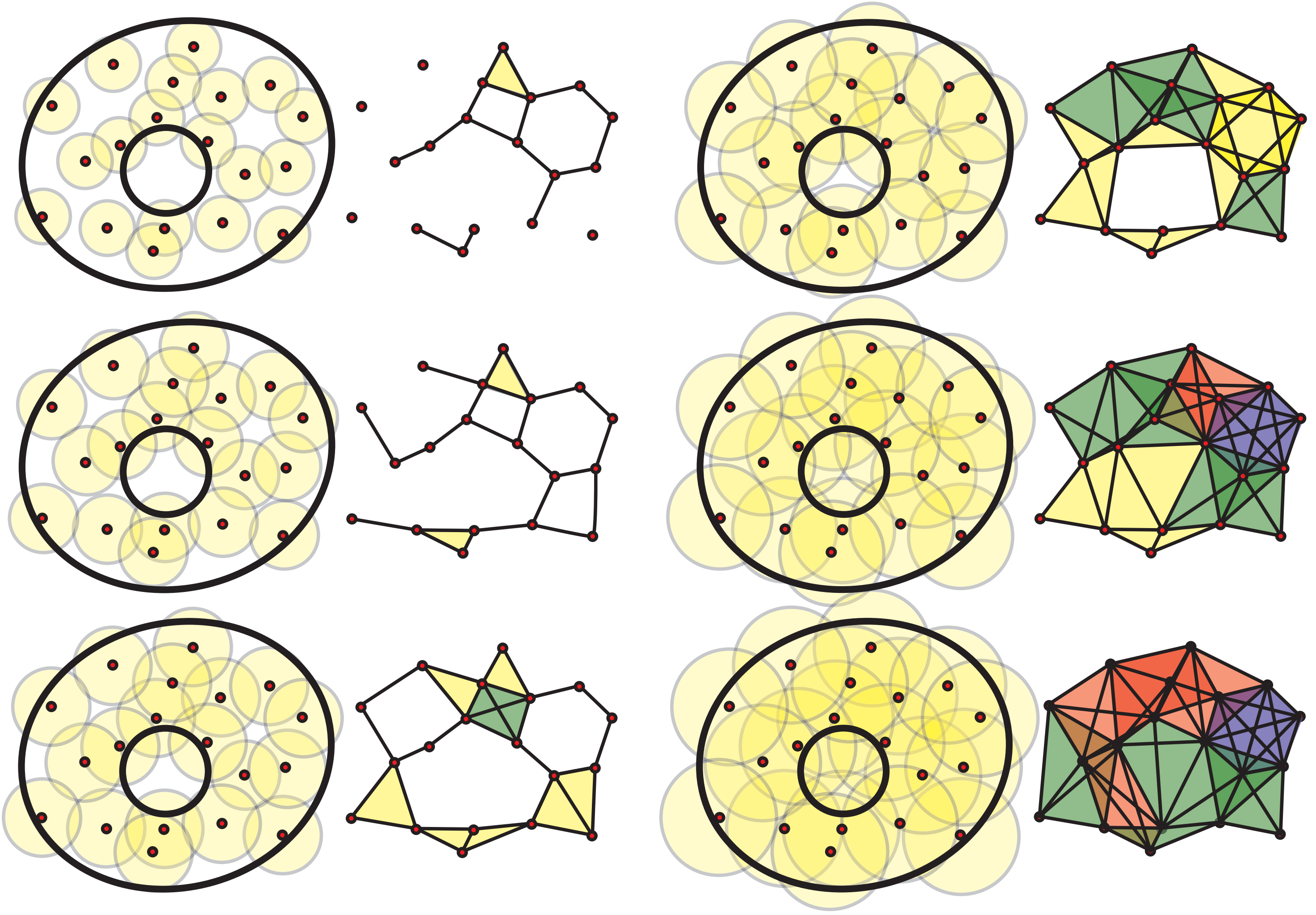

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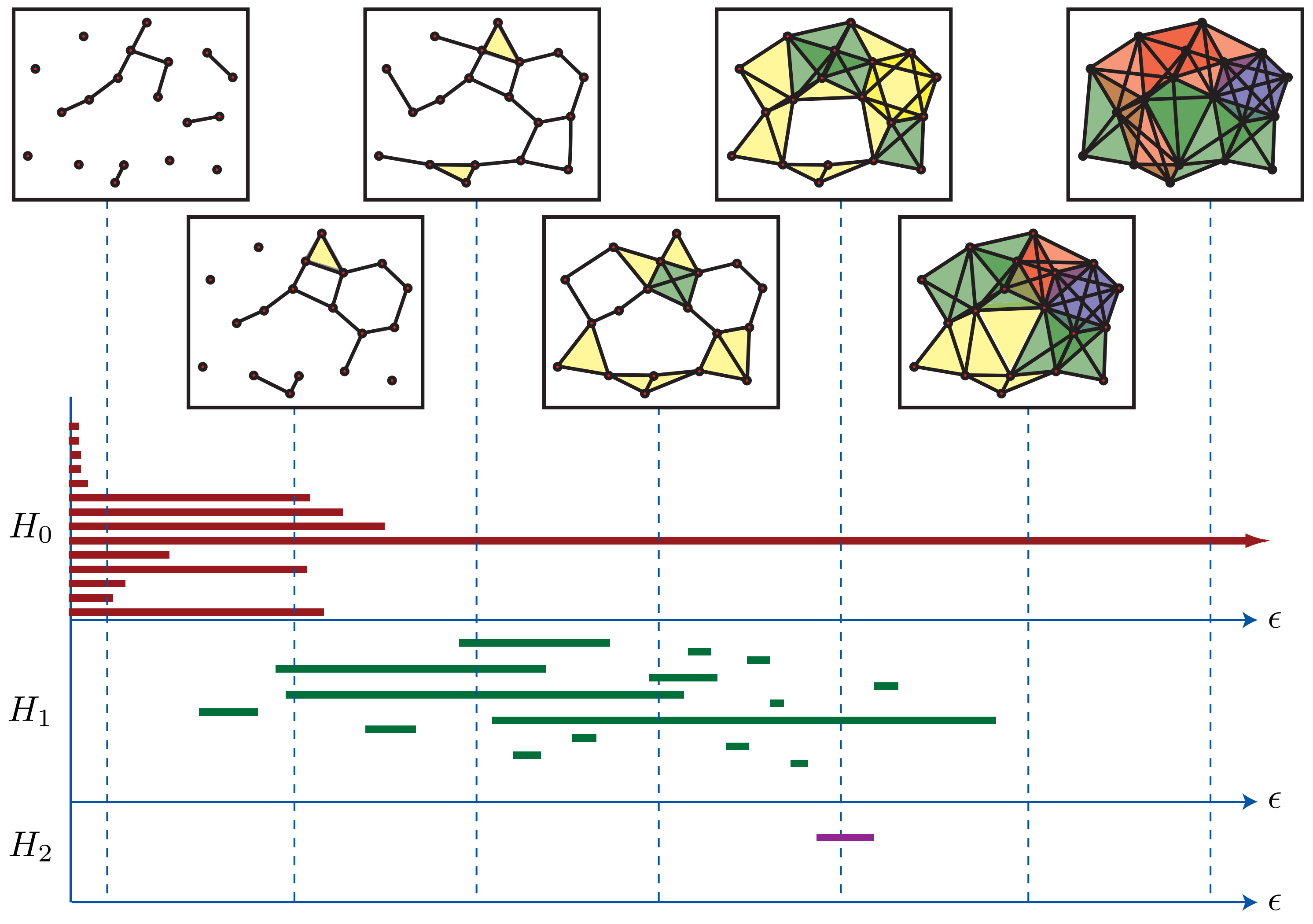

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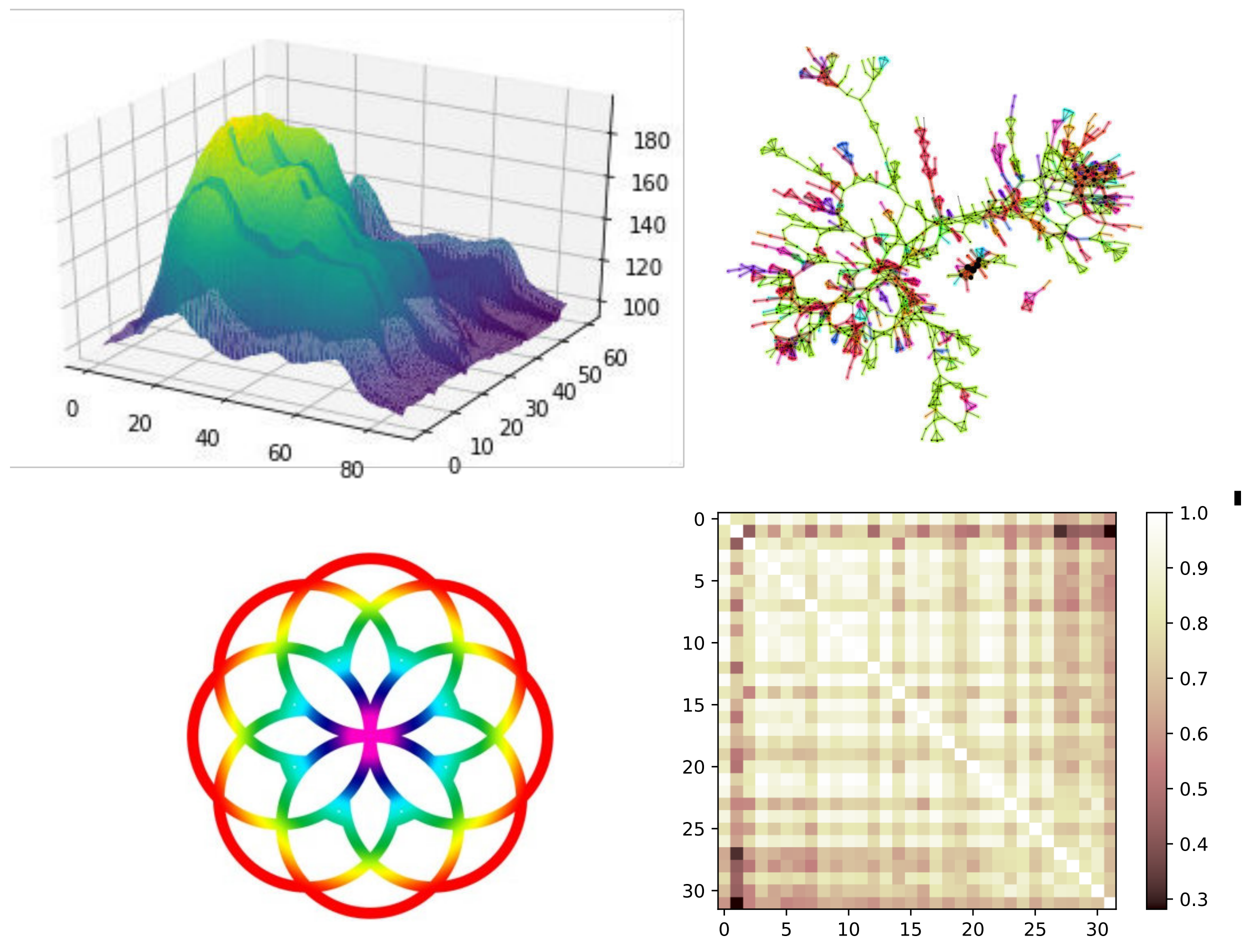

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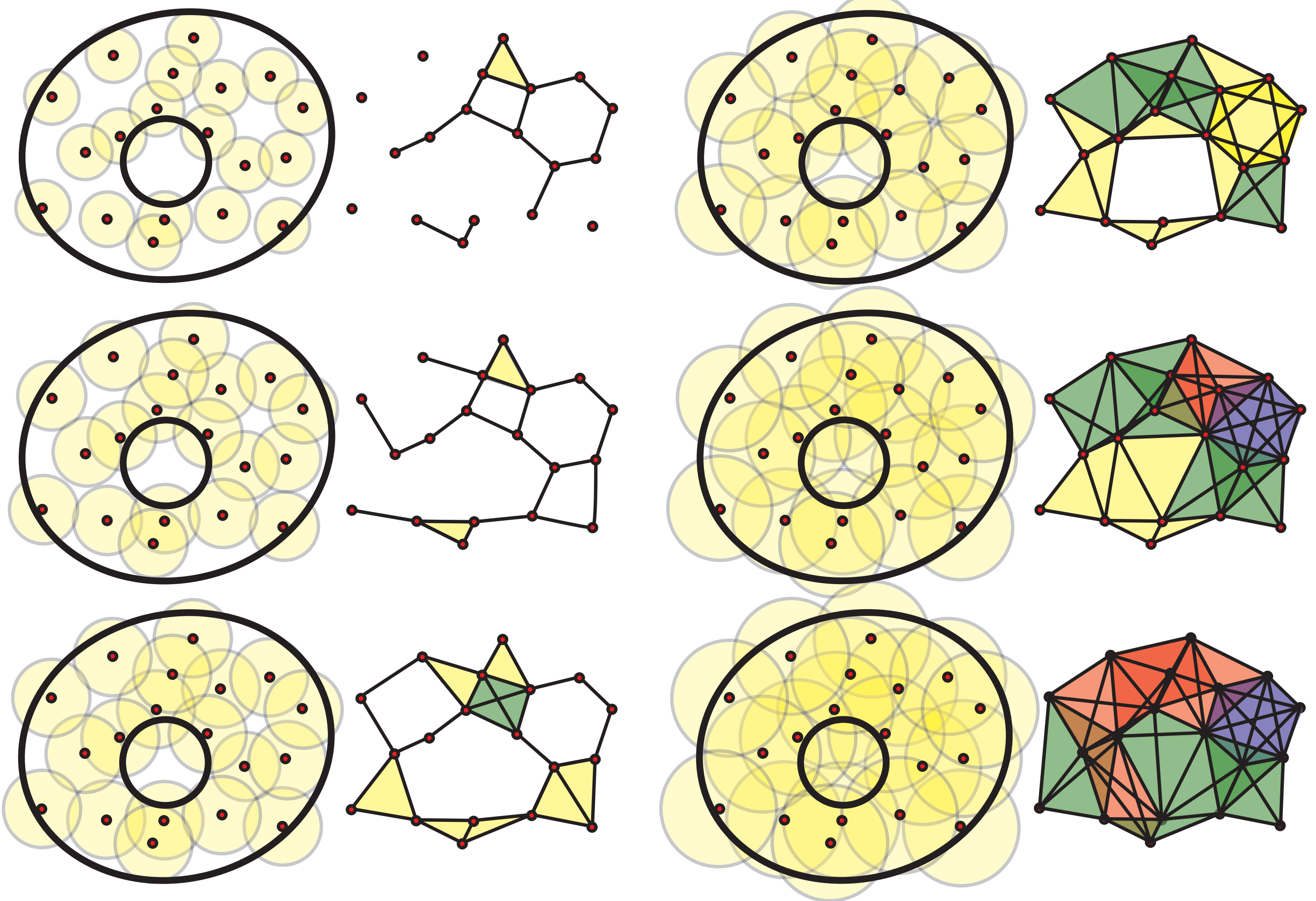

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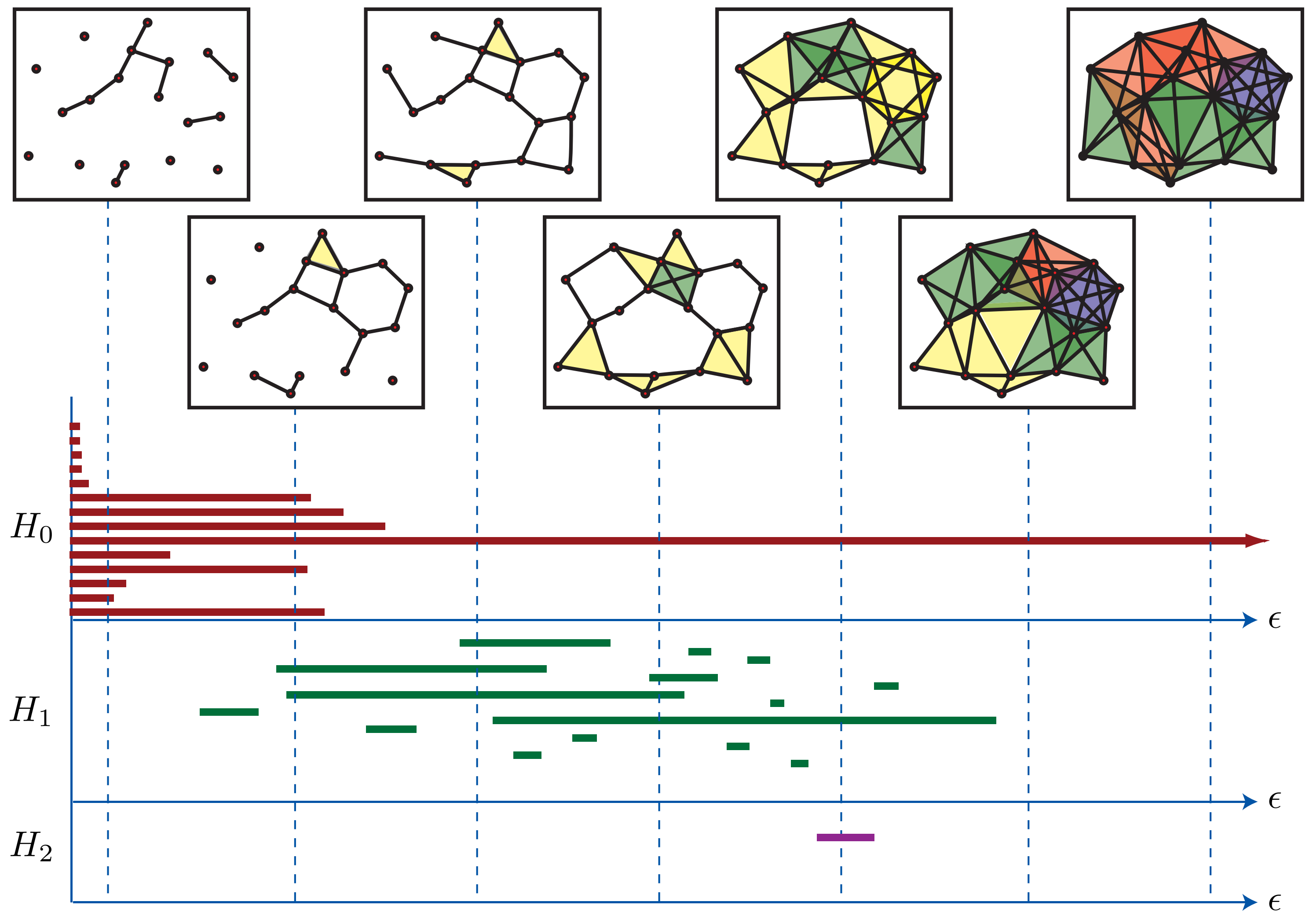

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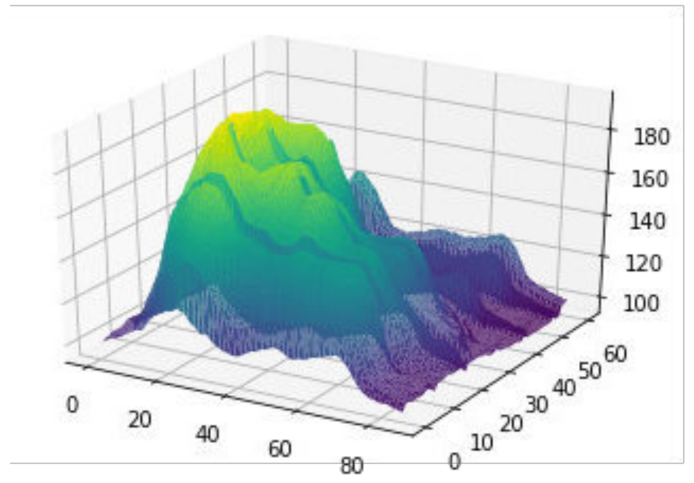

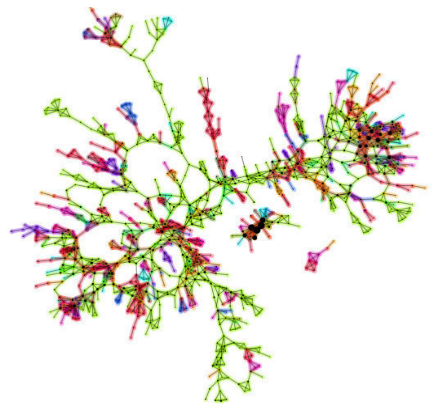

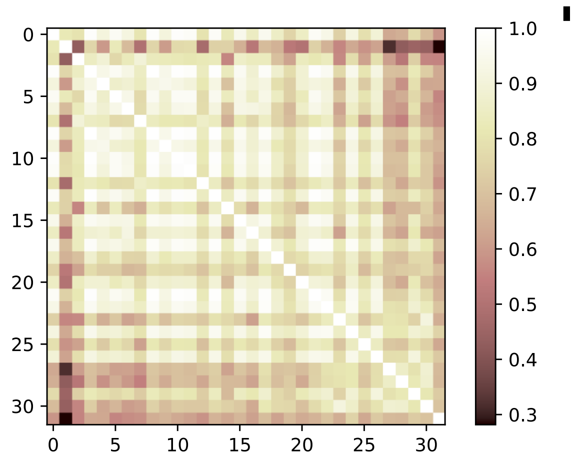

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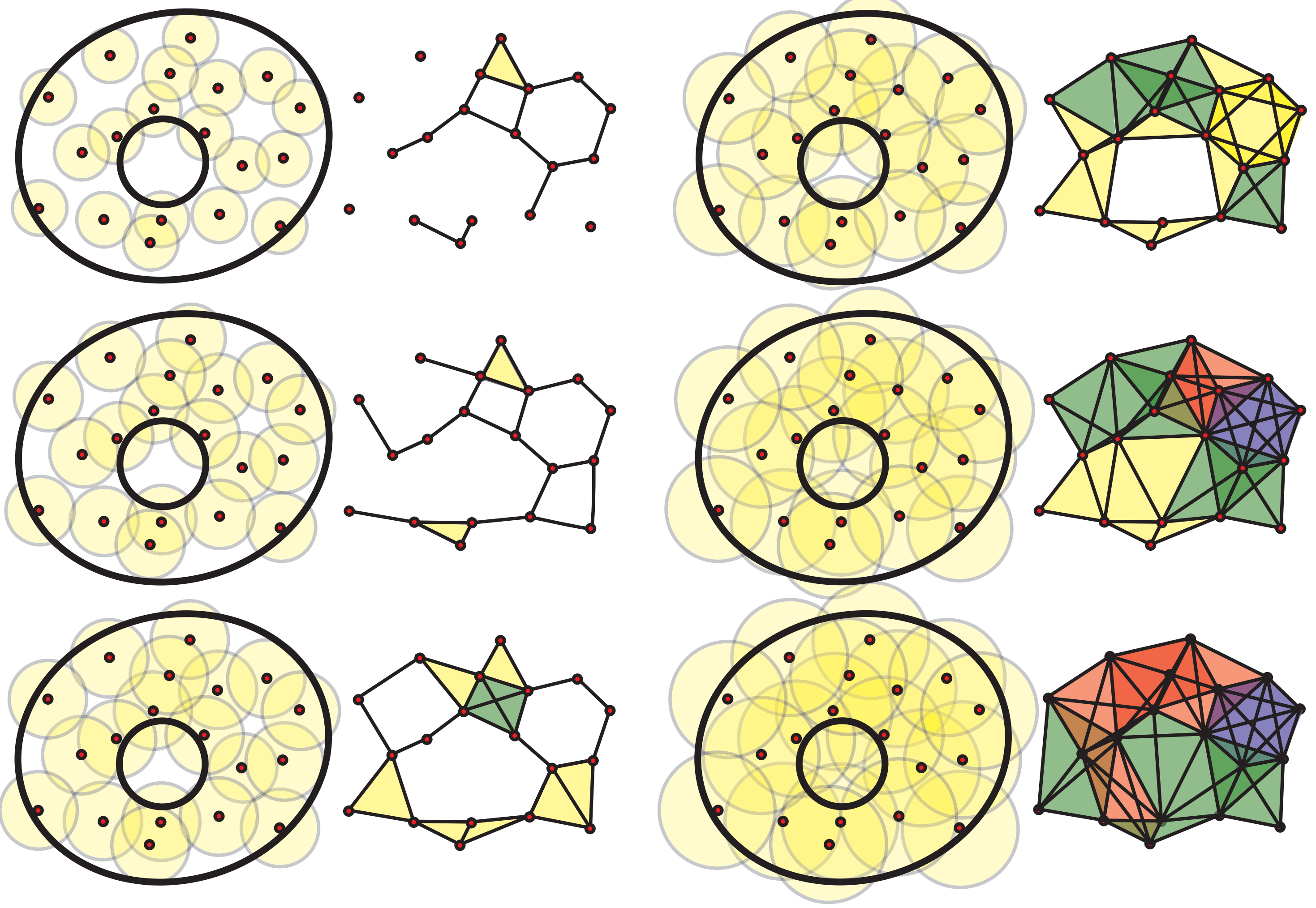

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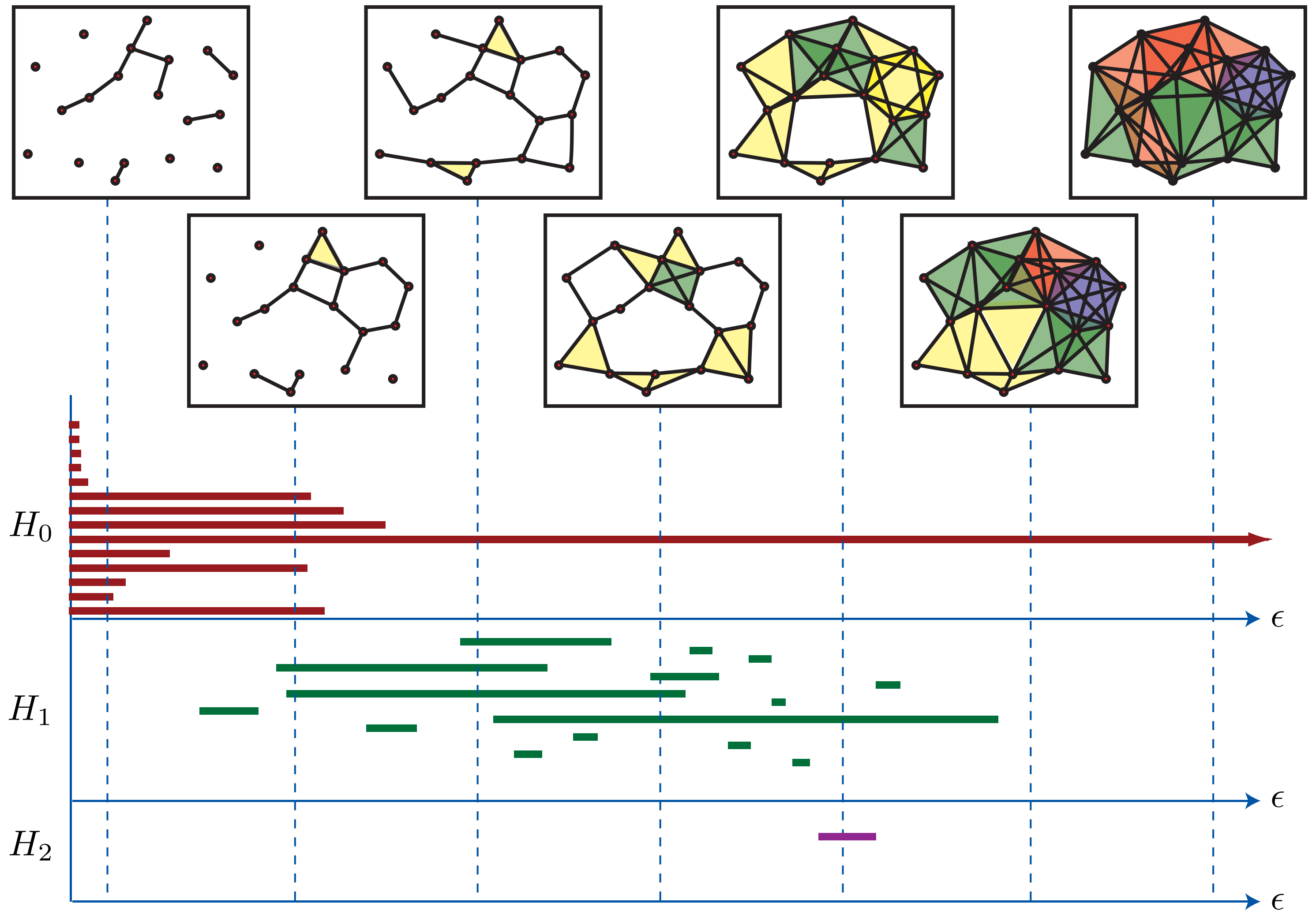

Mapper Pipeline (Singh et al 2007)

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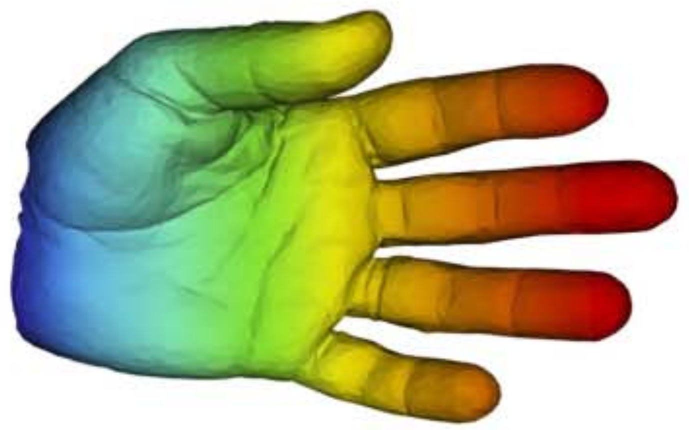
colouring (projecting)
using geometric filters



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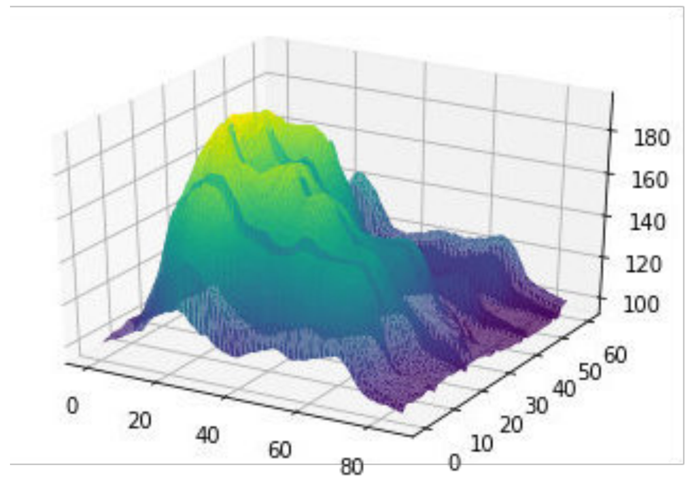

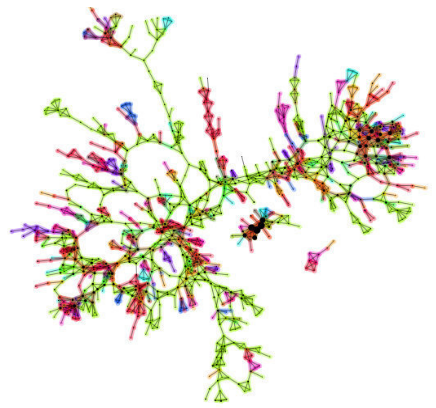

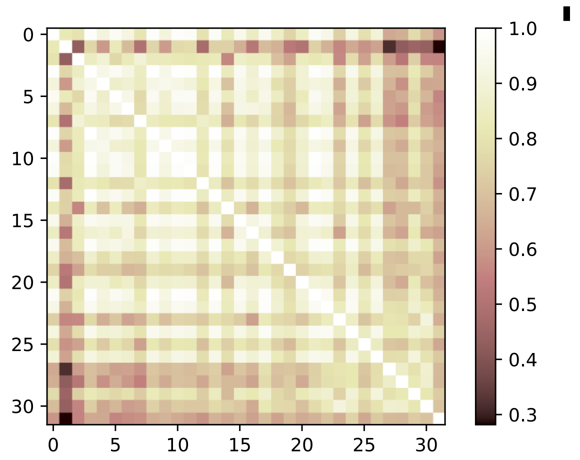

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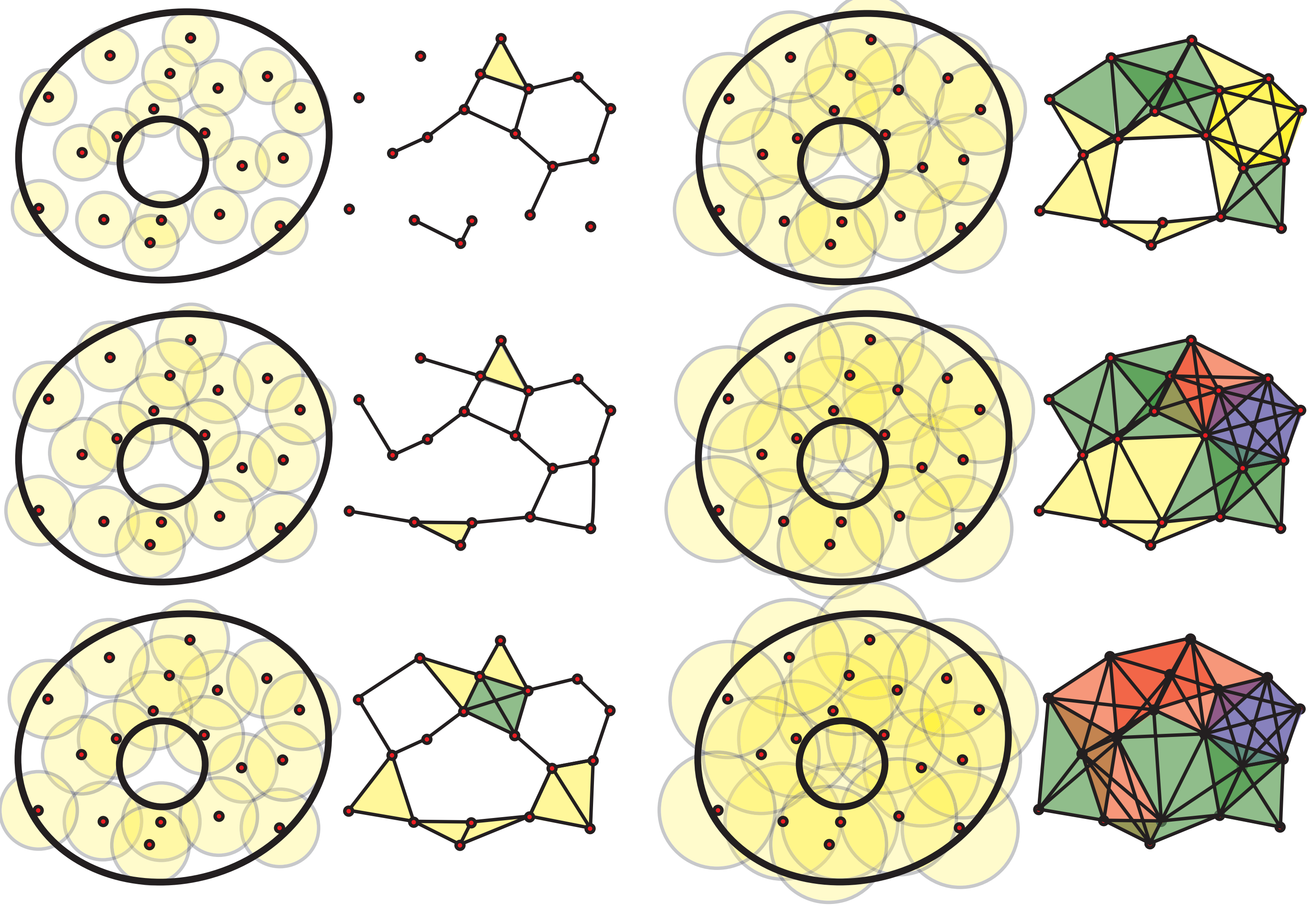

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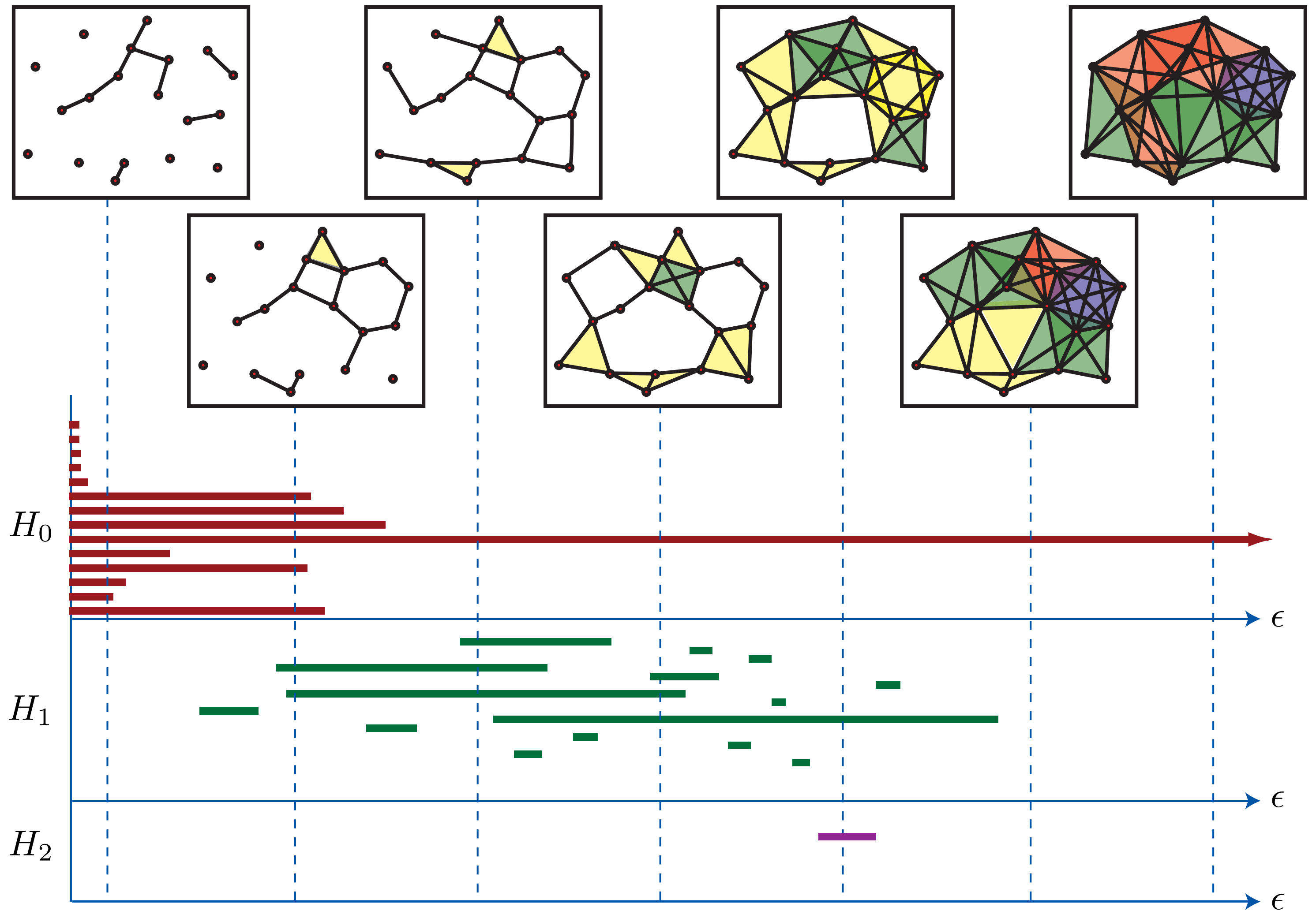
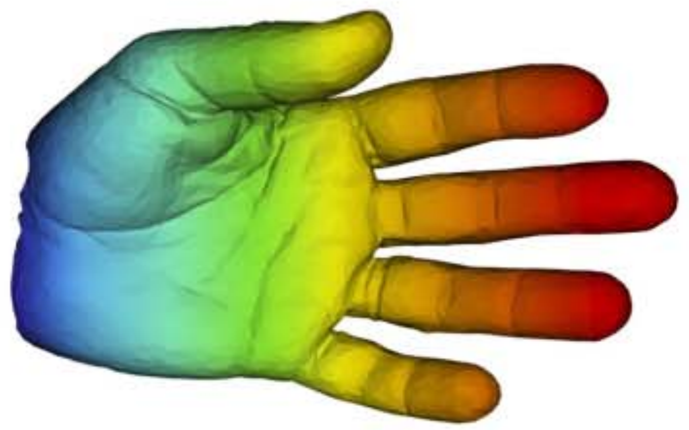


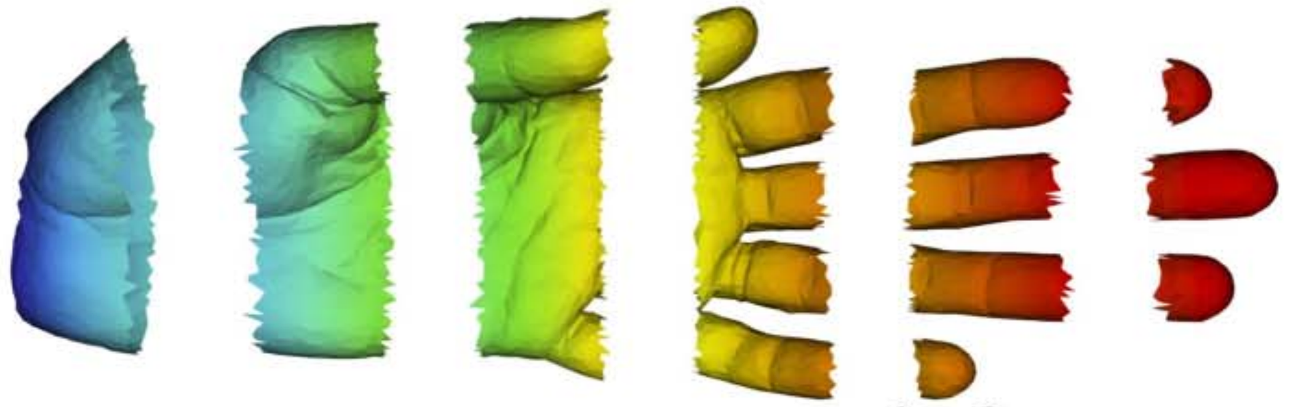
Mapper Pipeline (Singh et al 2007)

Point cloud

colouring (projecting) using geometric filters



overlapped binning



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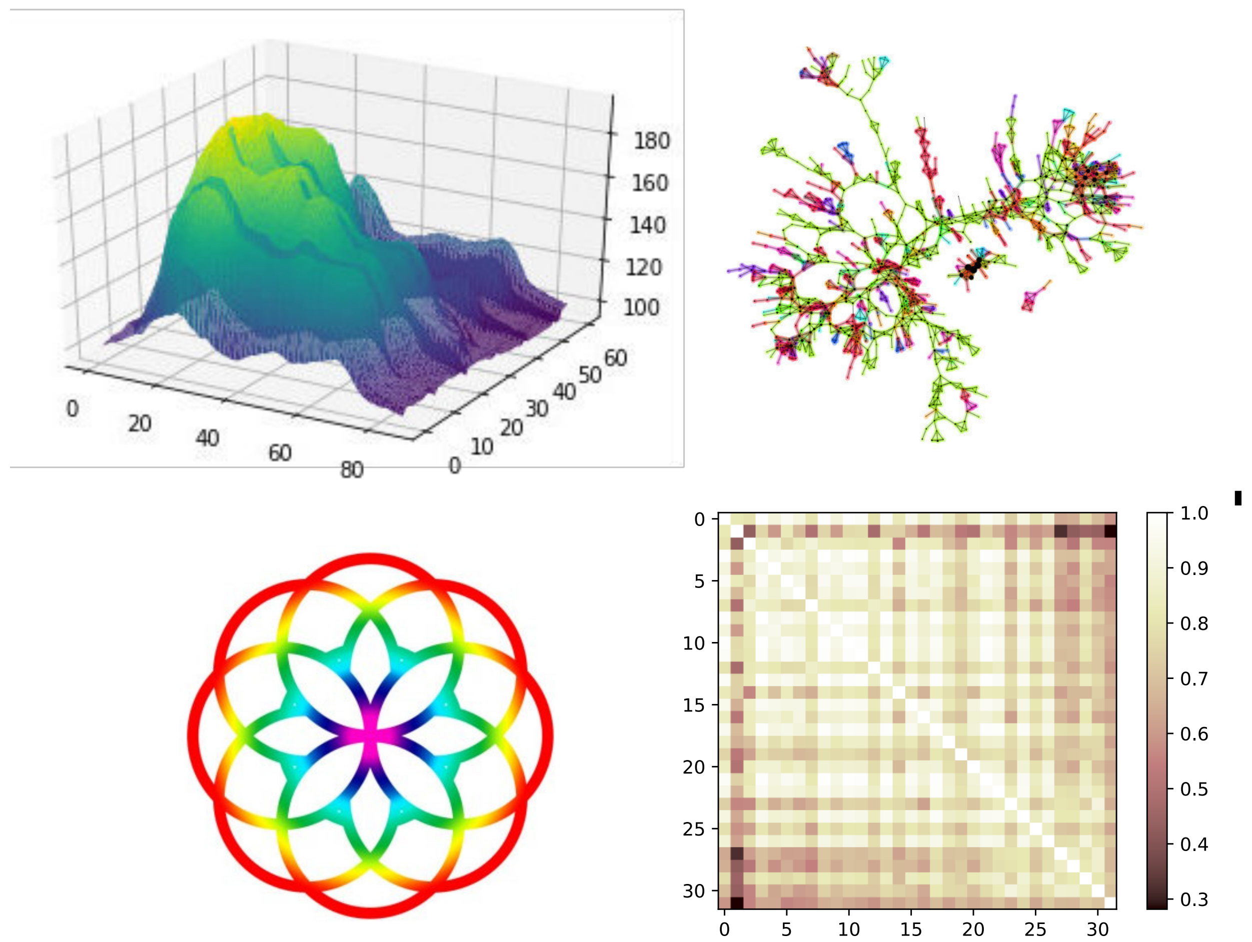

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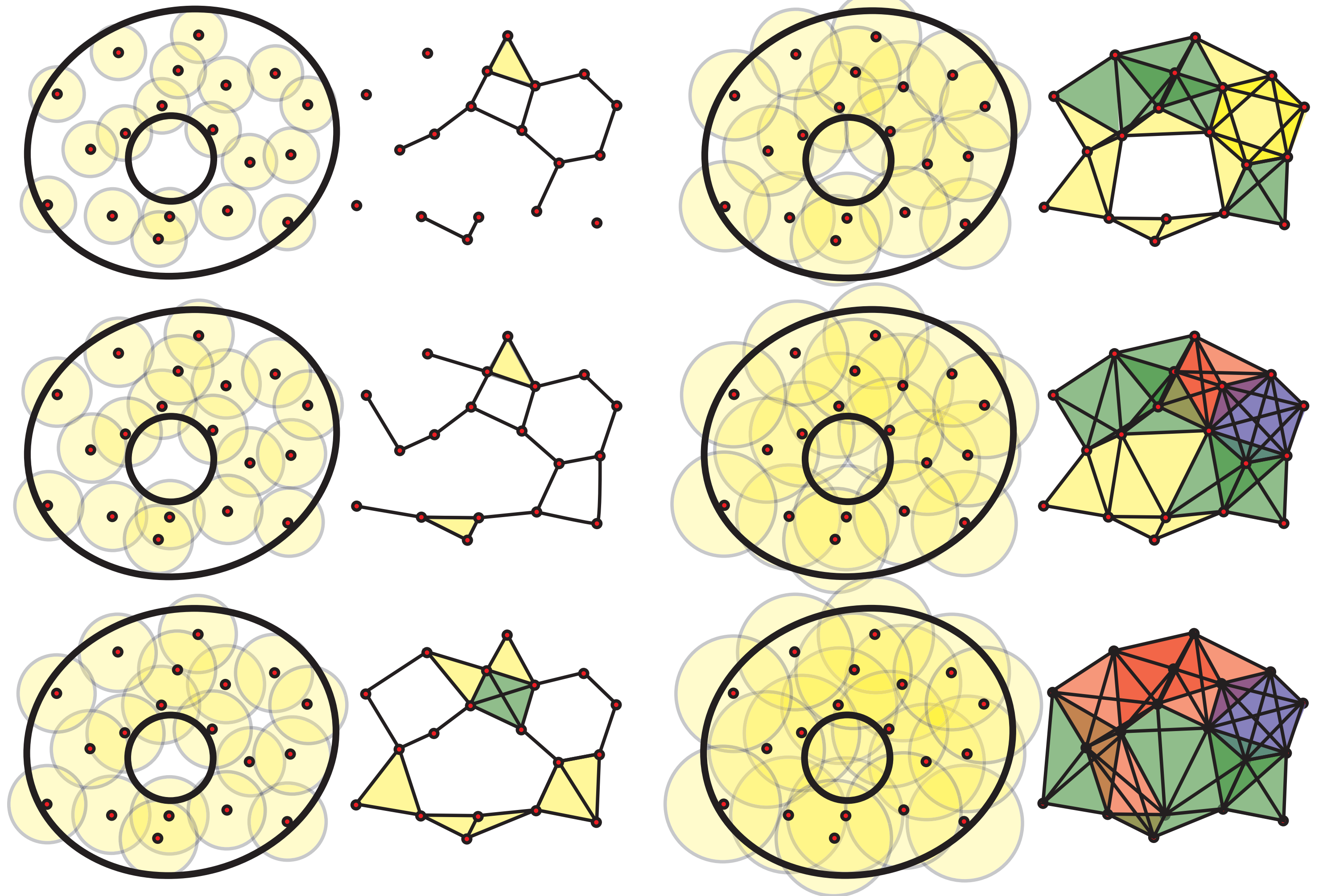

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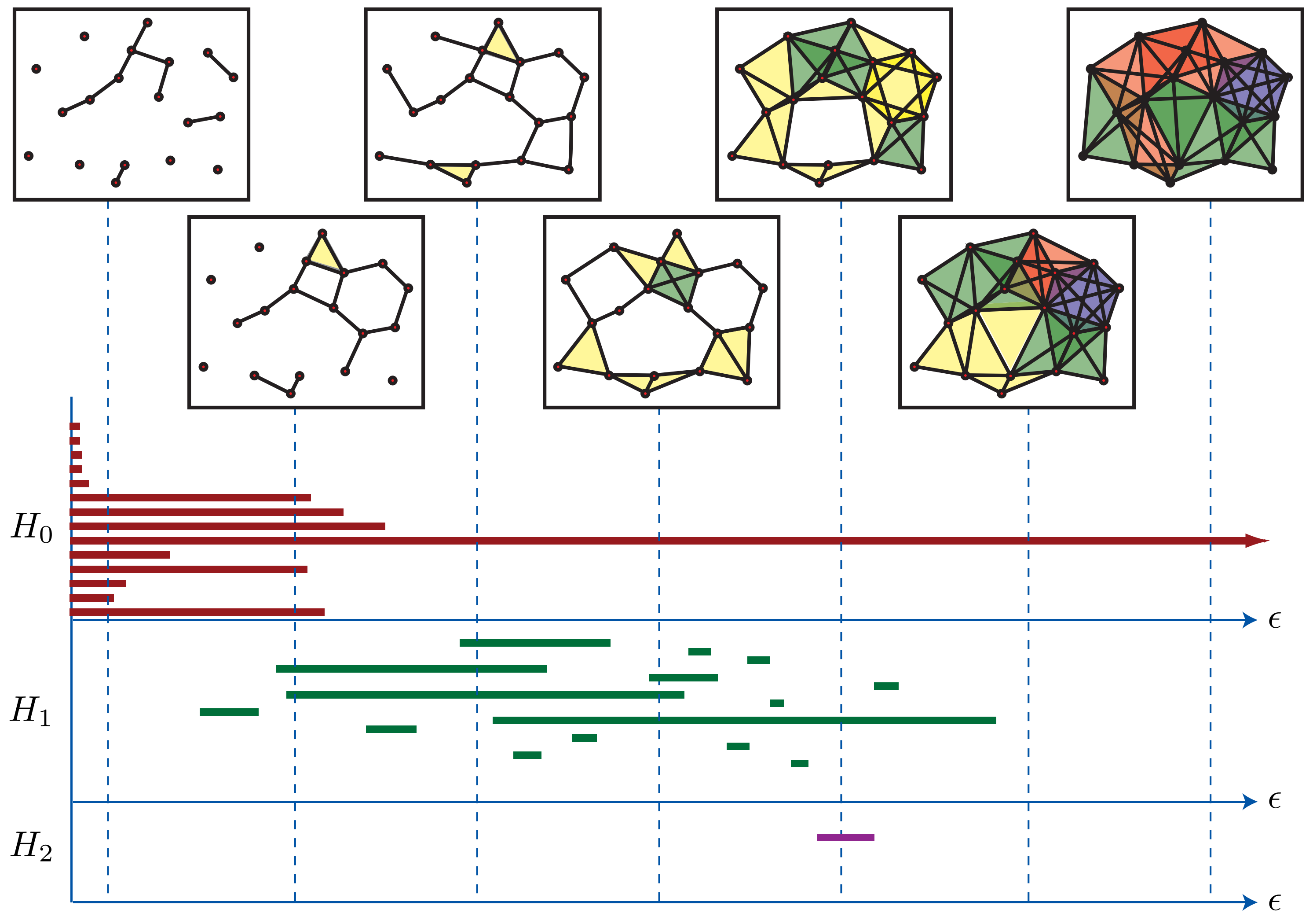

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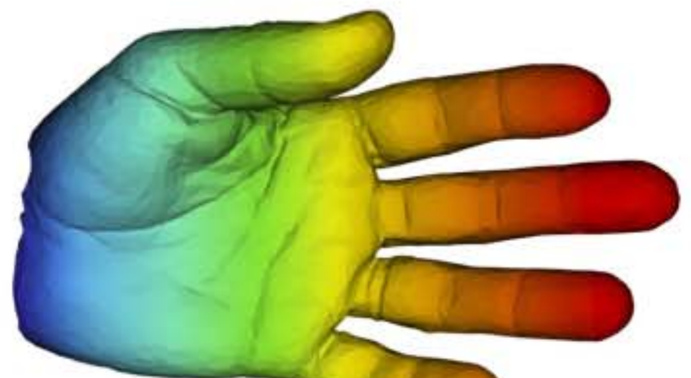
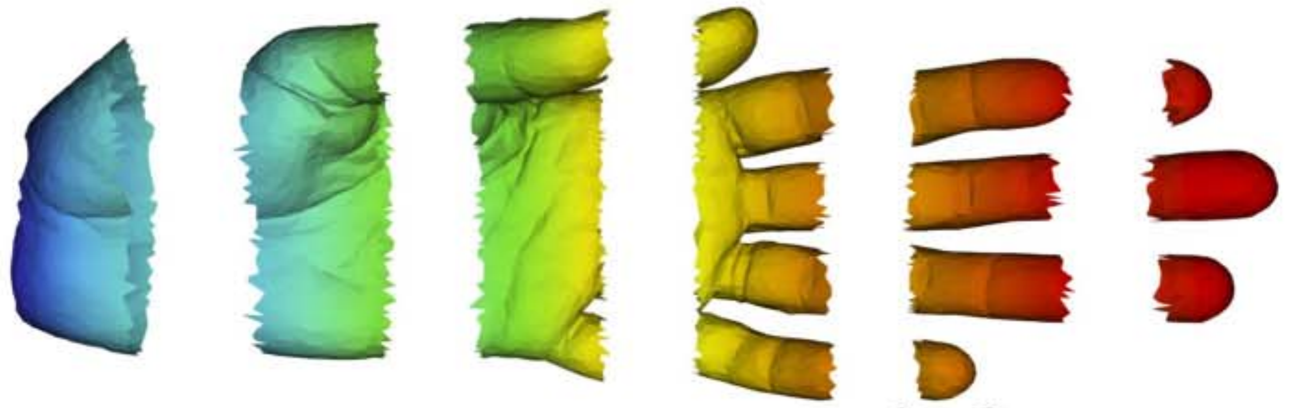
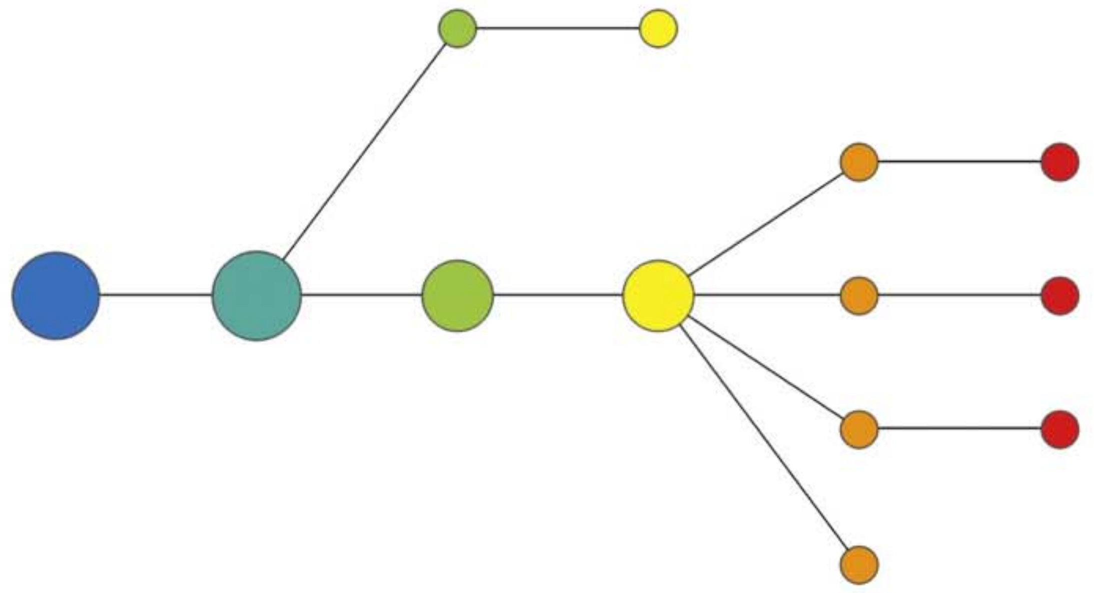


overlapped binning



Clustering and network construction binning



What does it mean in practice?

Persistent homology pipeline (Ghrist 2008)

Data of sorts

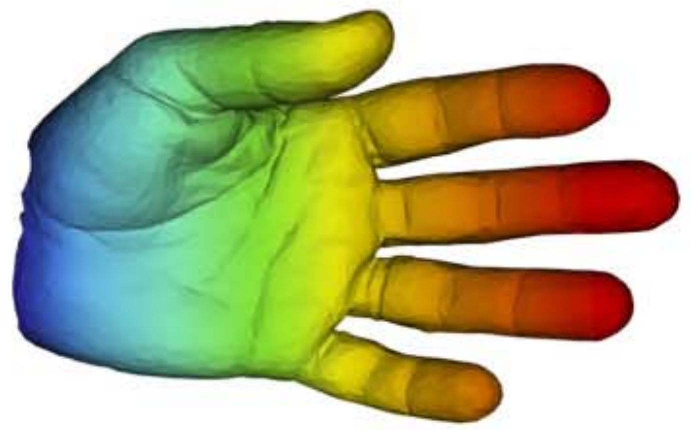
Filtration over distance/density/weights

Homological properties

Mapper Pipeline (Singh et al 2007)

Point cloud

colouring (projecting) using geometric filters



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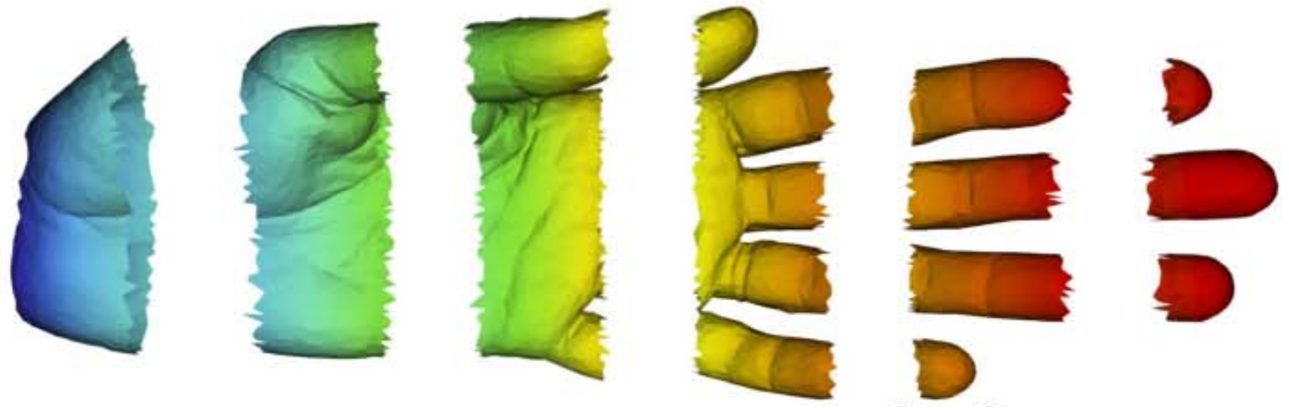

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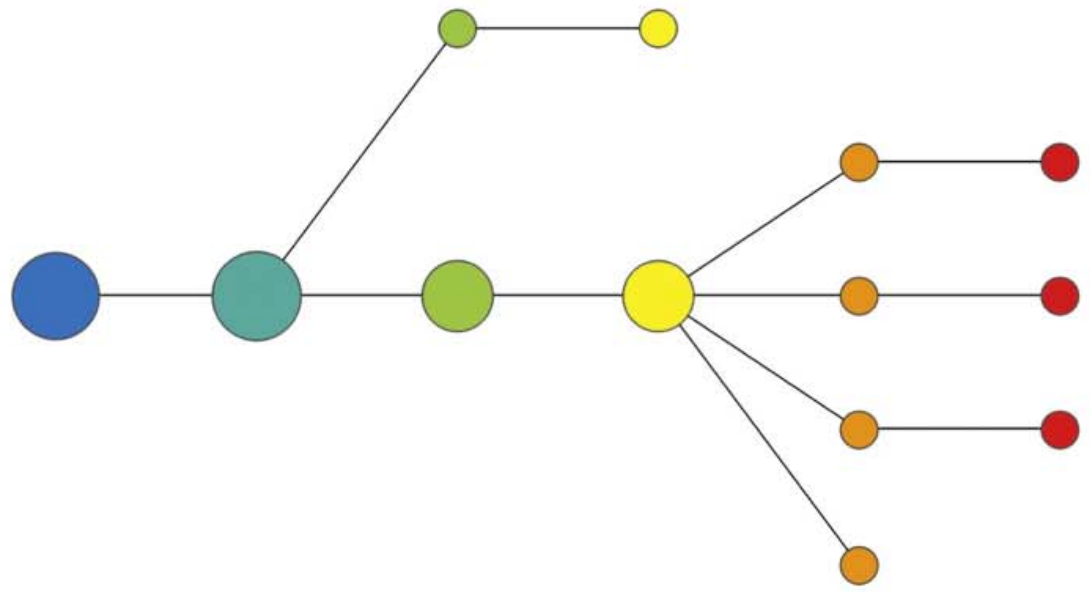
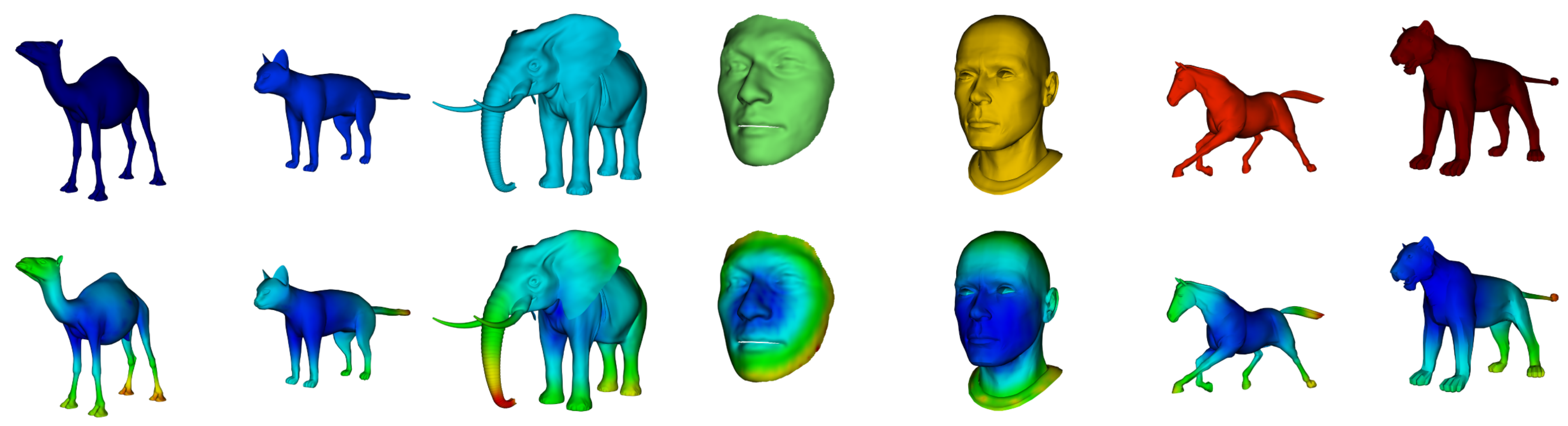
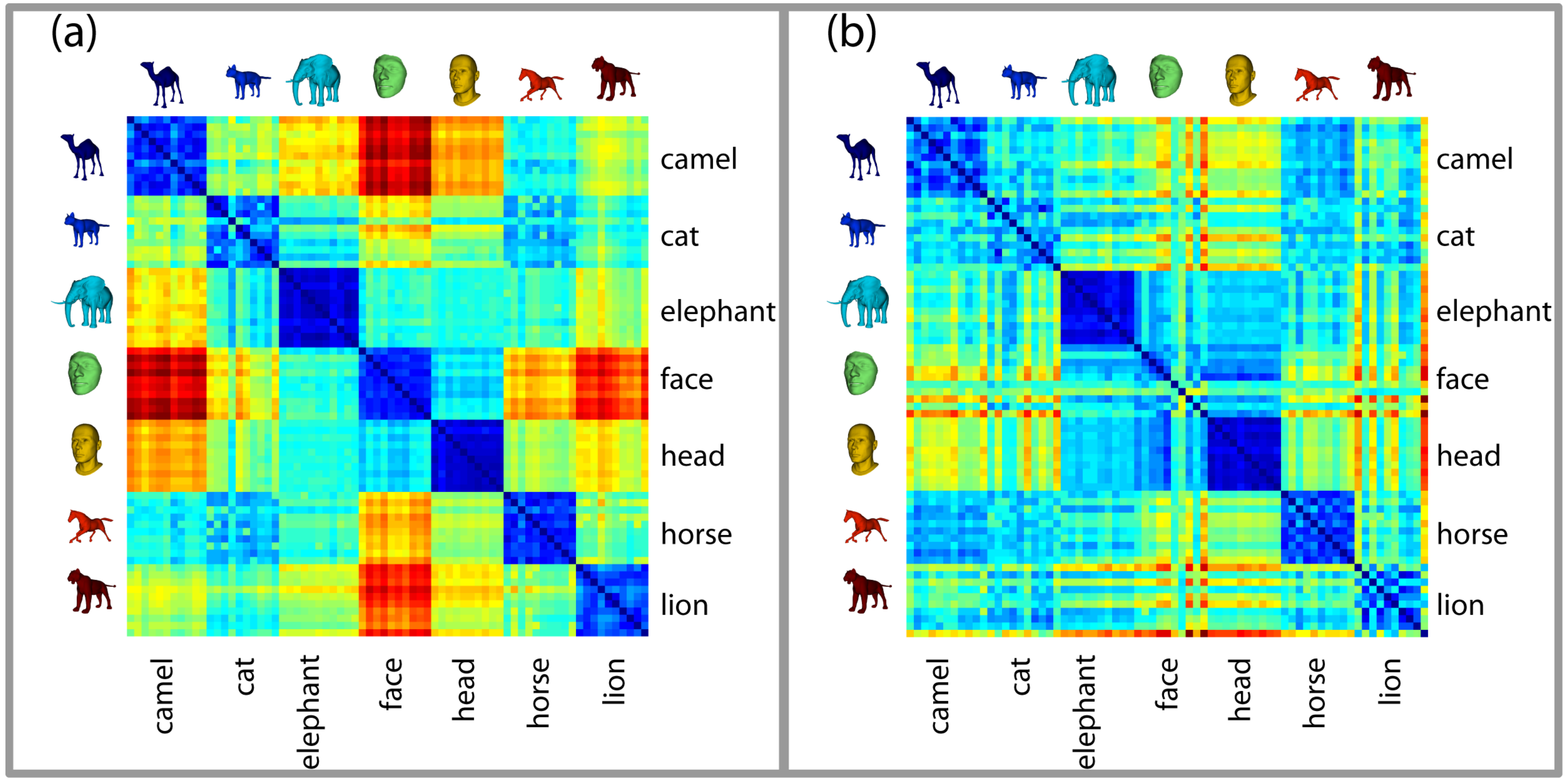


TDA as topological simplification

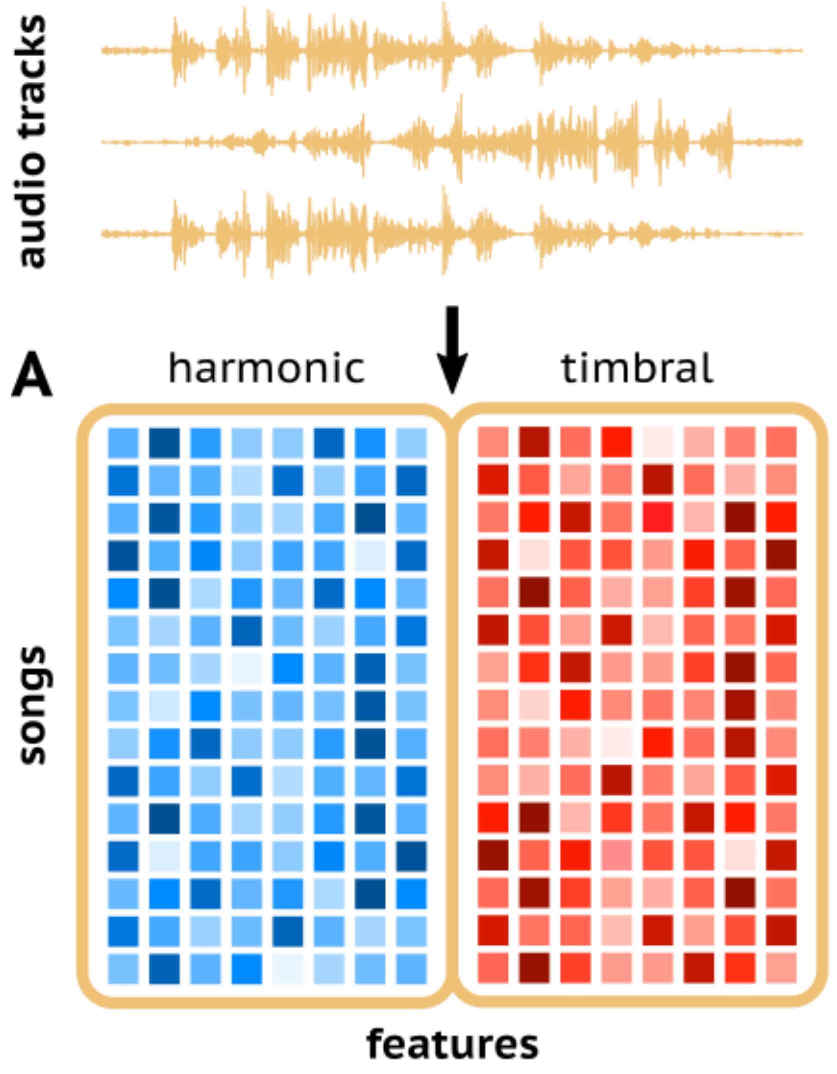


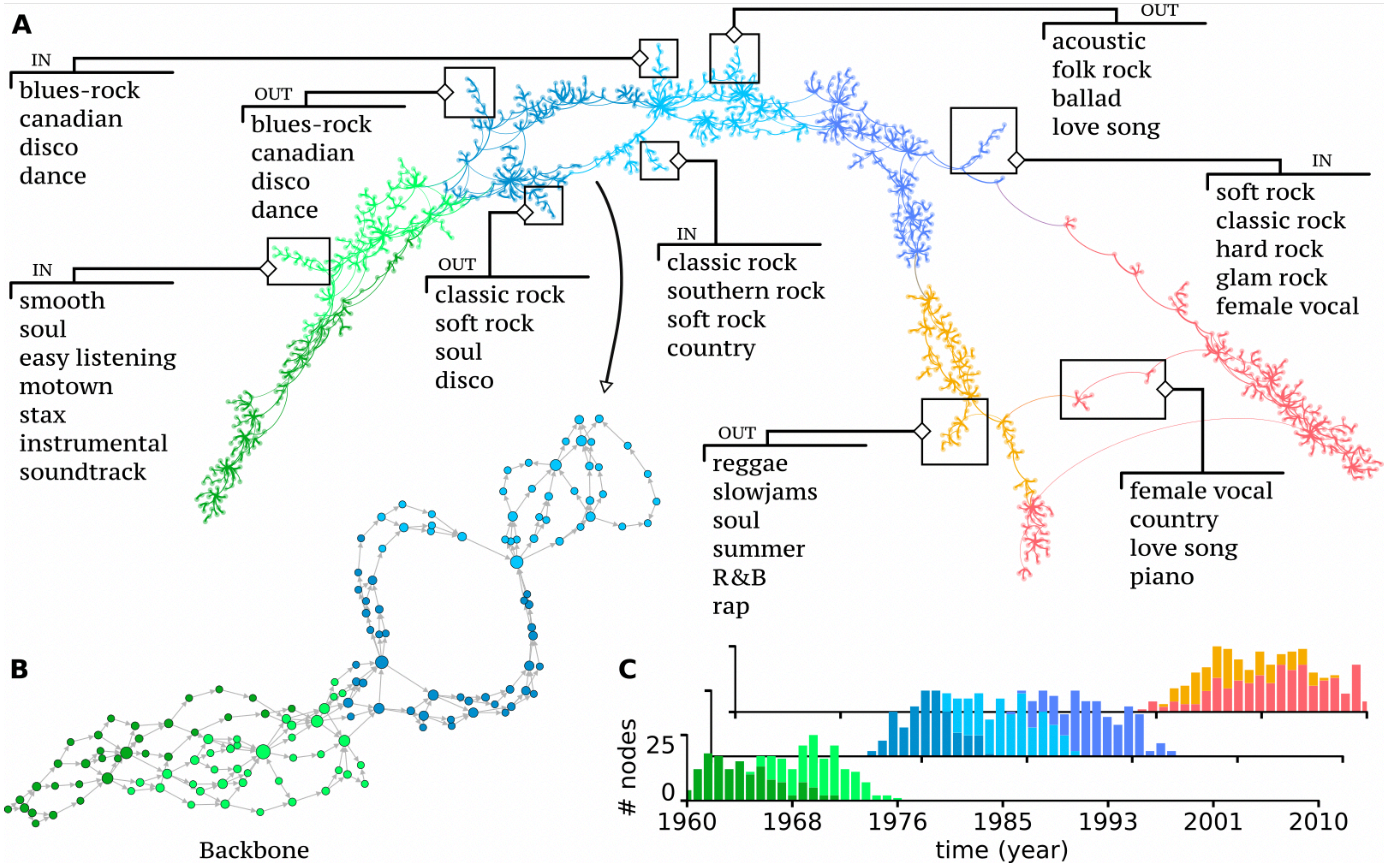


Topological Methods for the Analysis of High Dimensional Data Sets and 3D Object Recognition

Gurjeet Singh[1] , Facundo Mémoli[2] and Gunnar Carlsson[†2]

What does it mean in practice?





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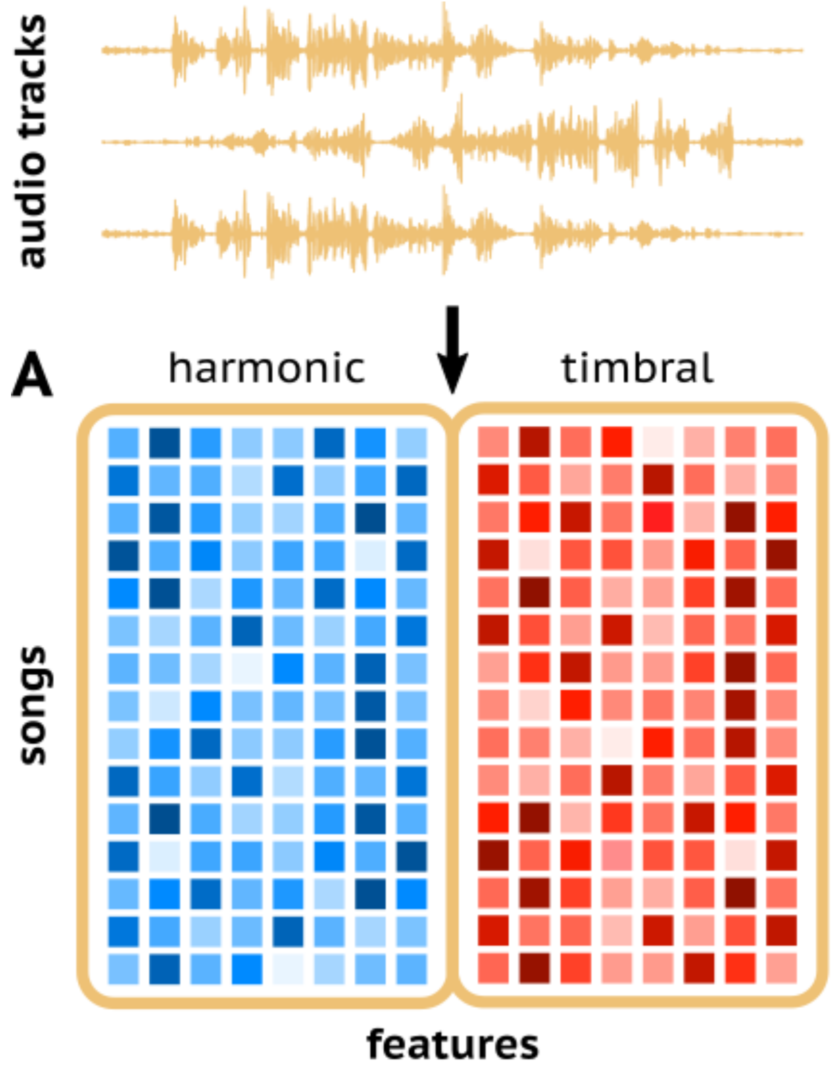
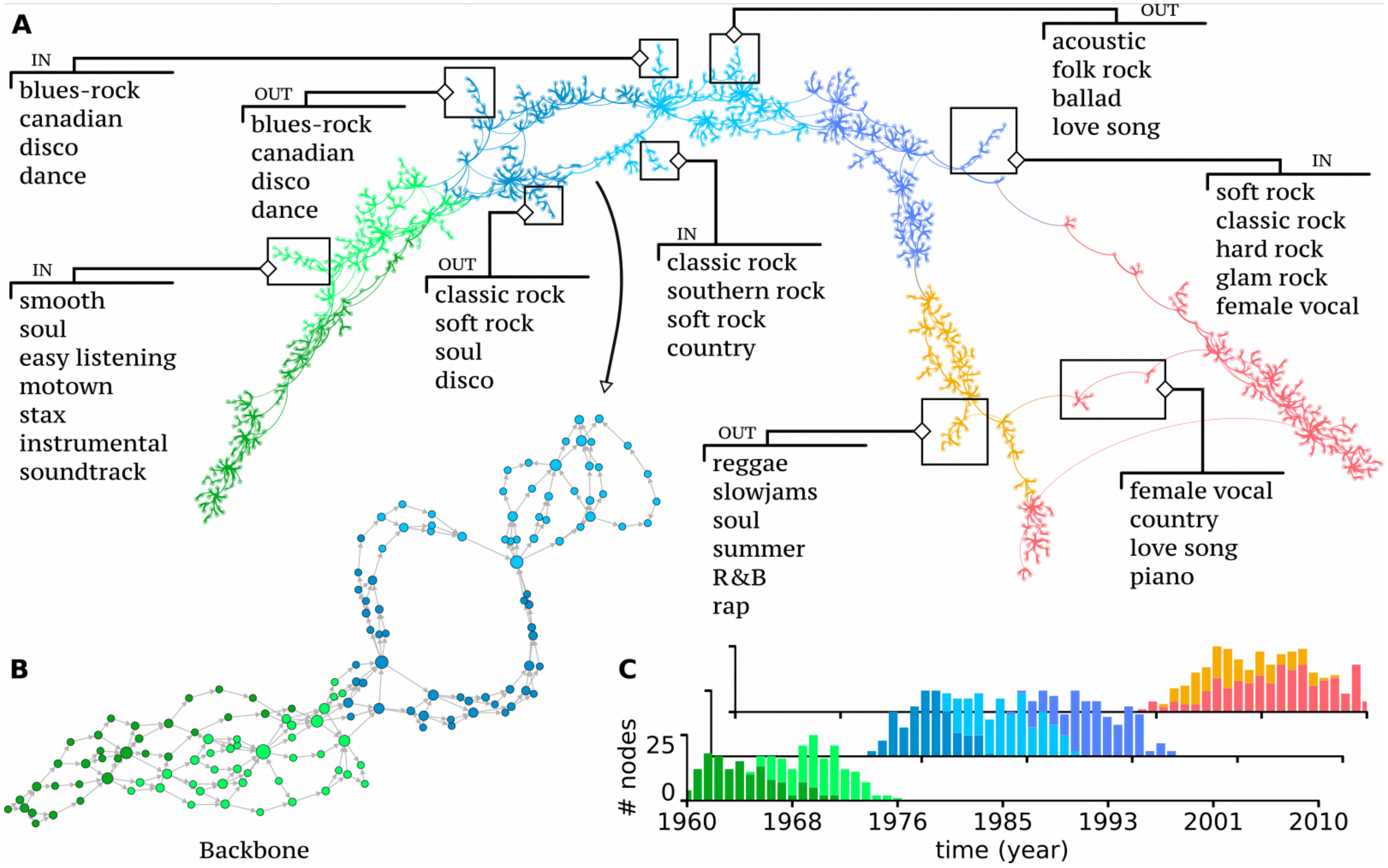




Iacopini, Petri, Latora, (2018) in preparation

Notebook 01

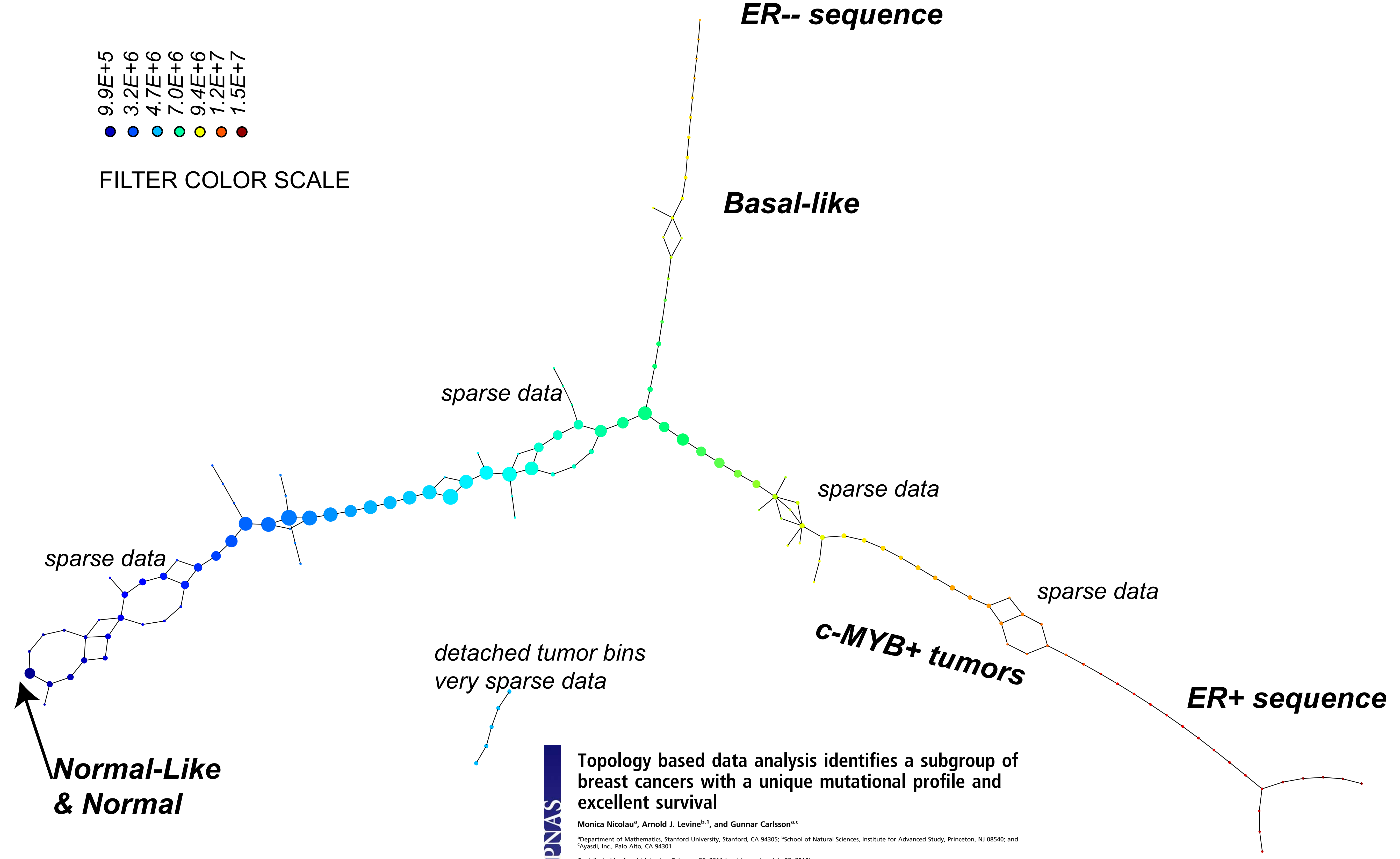
Do topological gene-backbones carry information?

SPACE

MAPPER

SPACE

Do topological gene-backbones carry information?



PNAS

Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival

Monica Nicolau[a], Arnold J. Levine[b,1], and Gunnar Carlsson[a,c]

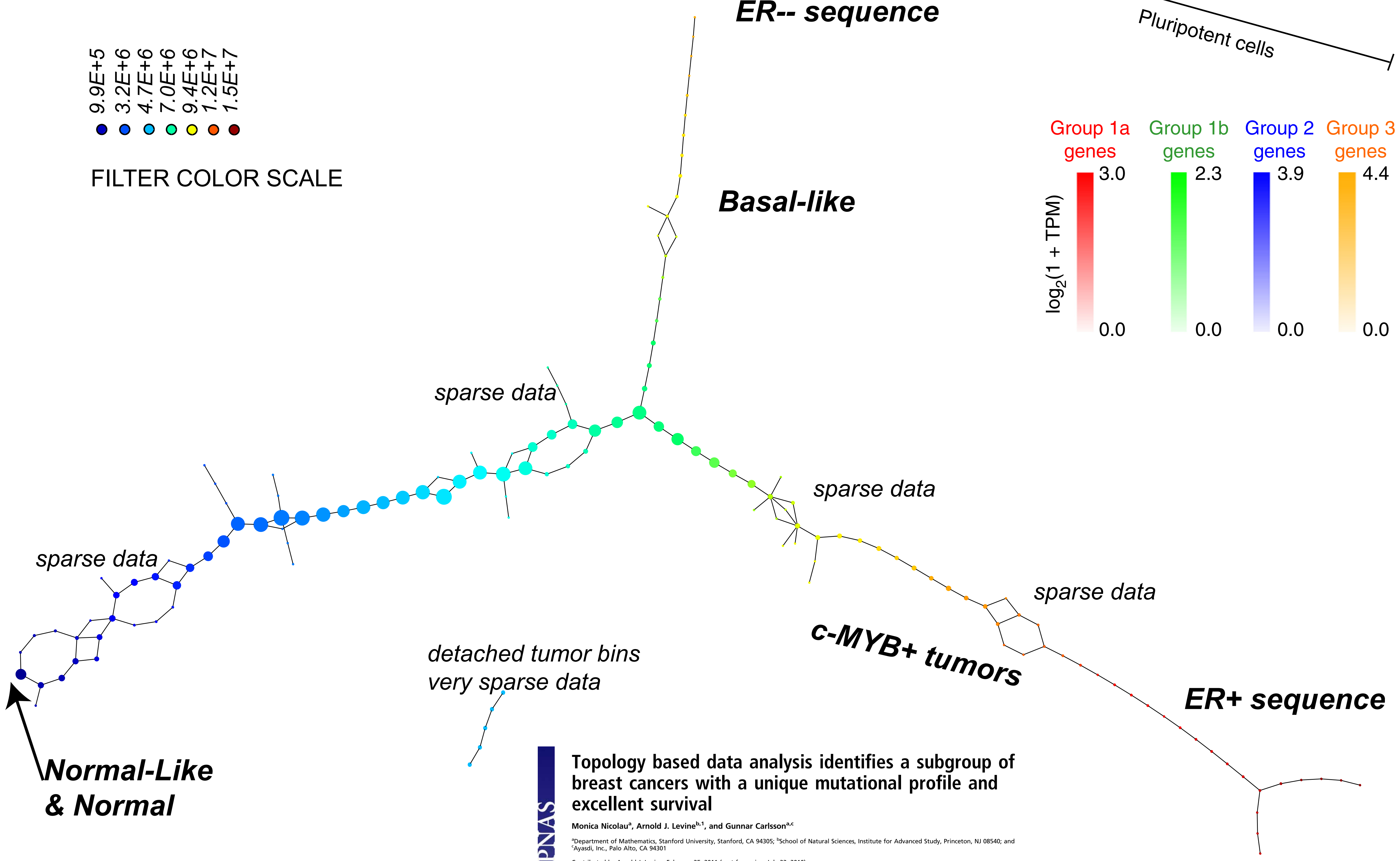
[a]Department of Mathematics, Stanford University, Stanford, CA 94305; [b]School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540; and [c]Ayasdi, Inc., Palo Alto, CA 94301

Contributed by Arnold J. Levine, February 25, 2011 (sent for review July 23, 2010)

MAPPER

Do topological gene-backbones carry information?

SPACE



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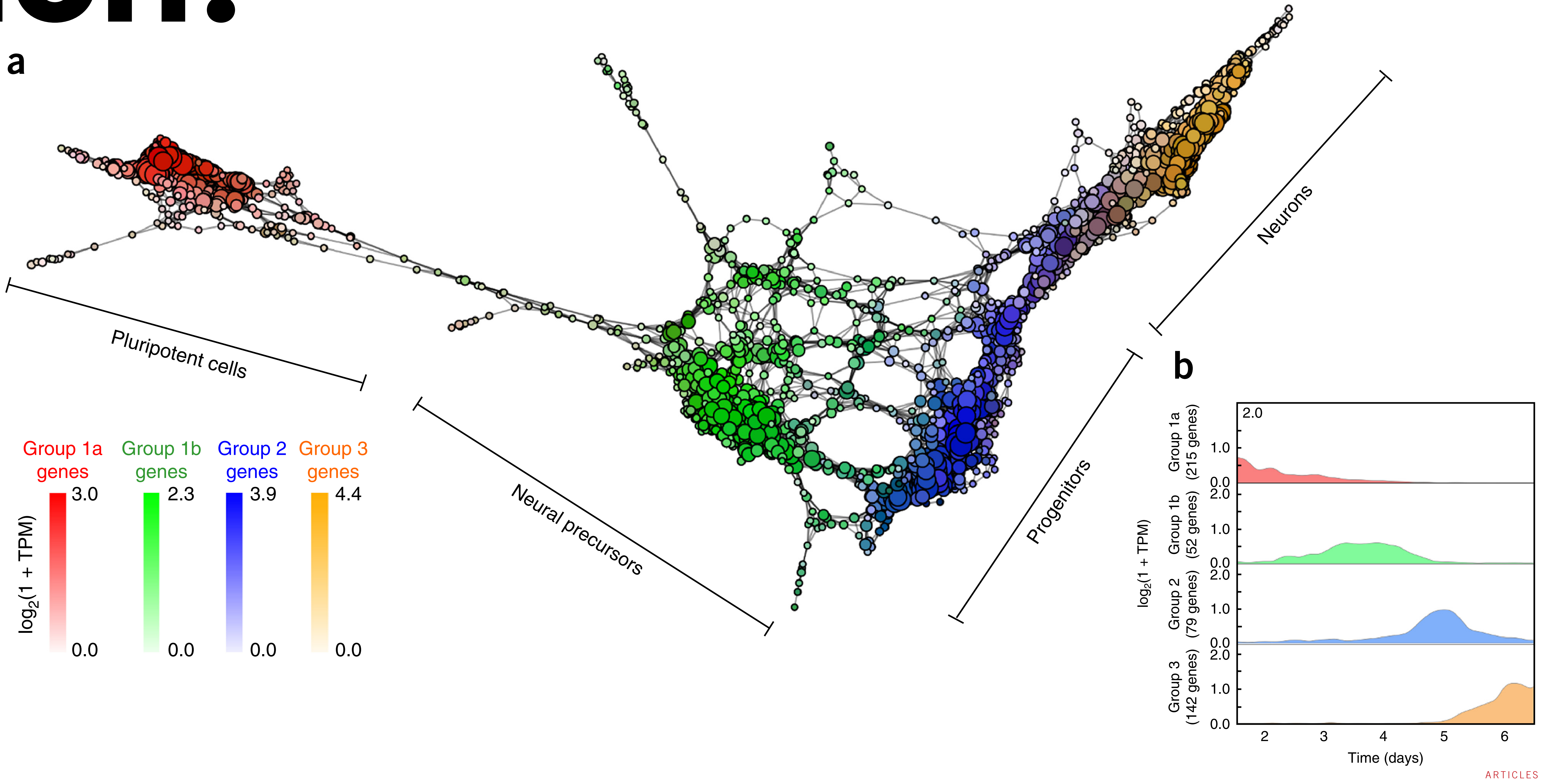


nature biotechnology

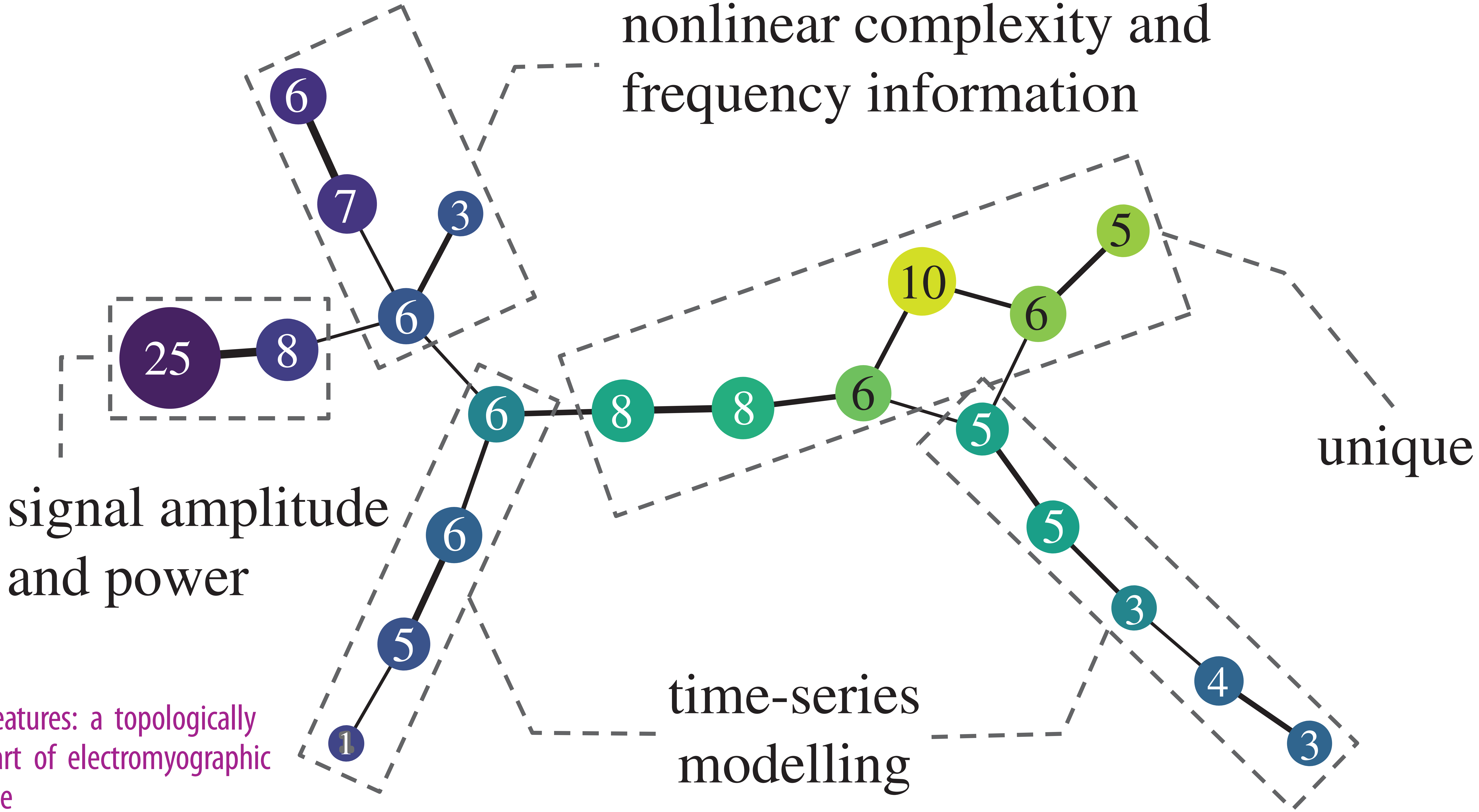
Single-cell topological RNA-seq analysis reveals insights into cellular differentiation and development

Abbas H Rizvi[1,2,6], Pablo G Camara[3,4,6], Elena K Kandror[1,2], Thomas J Roberts[1,2,4], Ira Schieren[2,5], Tom Maniatis[1,2] & Raul Rabadan[3,4]

MAPPER

Notebook 02

Can we build topological feature charts?



INTERFACE

rsif.royalsocietypublishing.org

Research

Check for updates

Navigating features: a topologically informed chart of electromyographic features space

Angkoon Phinyomark[1,2], Rami N. Khushaba[3], Esther Ibáñez-Marcelo[1], Alice Patania[4], Erik Scheme[2] and Giovanni Petri[1]

(ISI+Fredericton University+Sydney)

Notebook 03

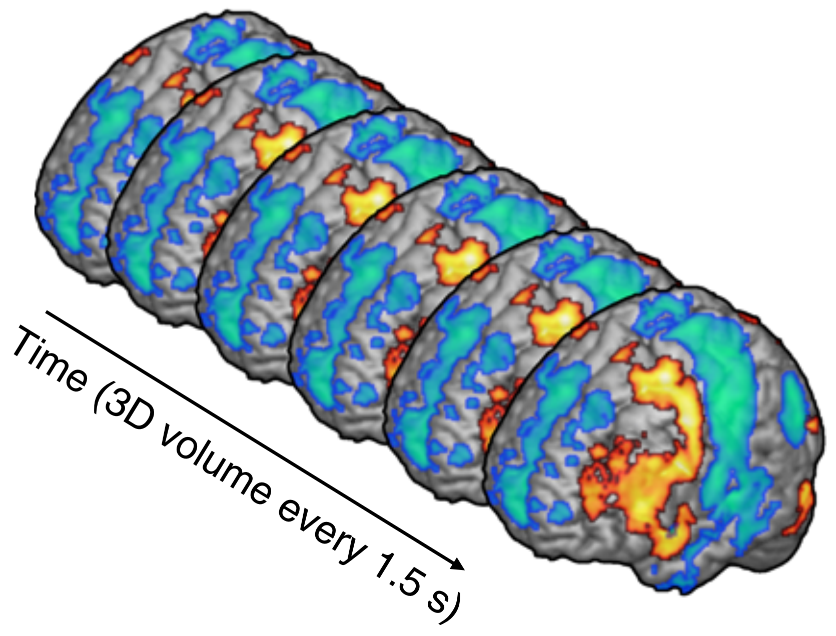
Approximate activity landscapes using topology

Saggar, Manish, et al. "Towards a new approach to reveal dynamical organization of the brain using topological data analysis." *Nature communications* 9.1 (2018): 1399.

Approximate activity landscapes using topology

a

Task brain activation data



Task-fMRI dataset from one CMP participant S_{01}

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Approximate activity landscapes using topology

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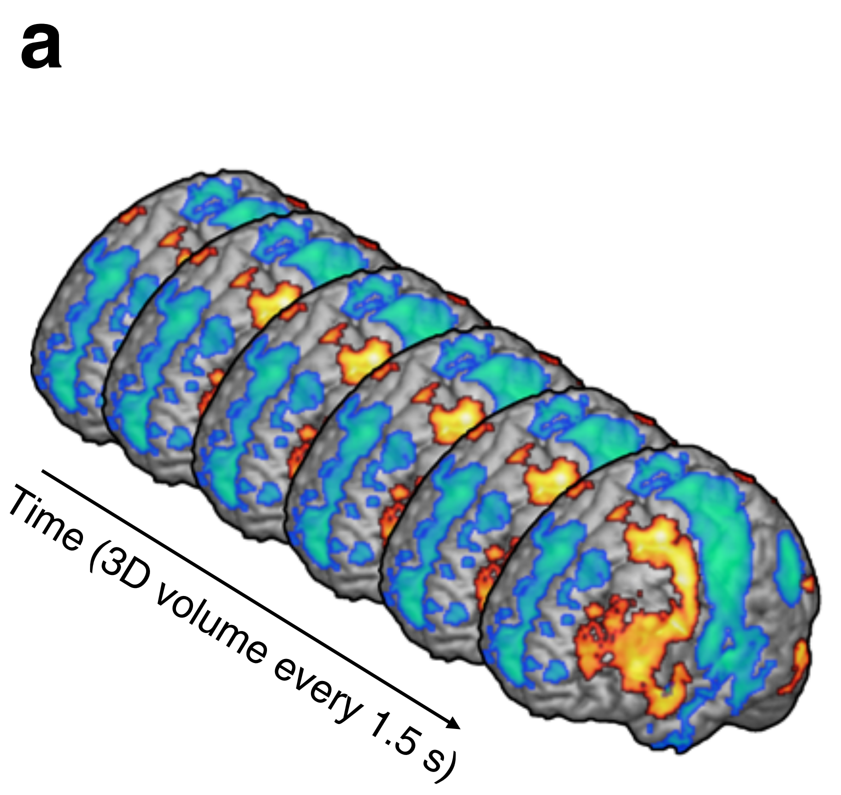

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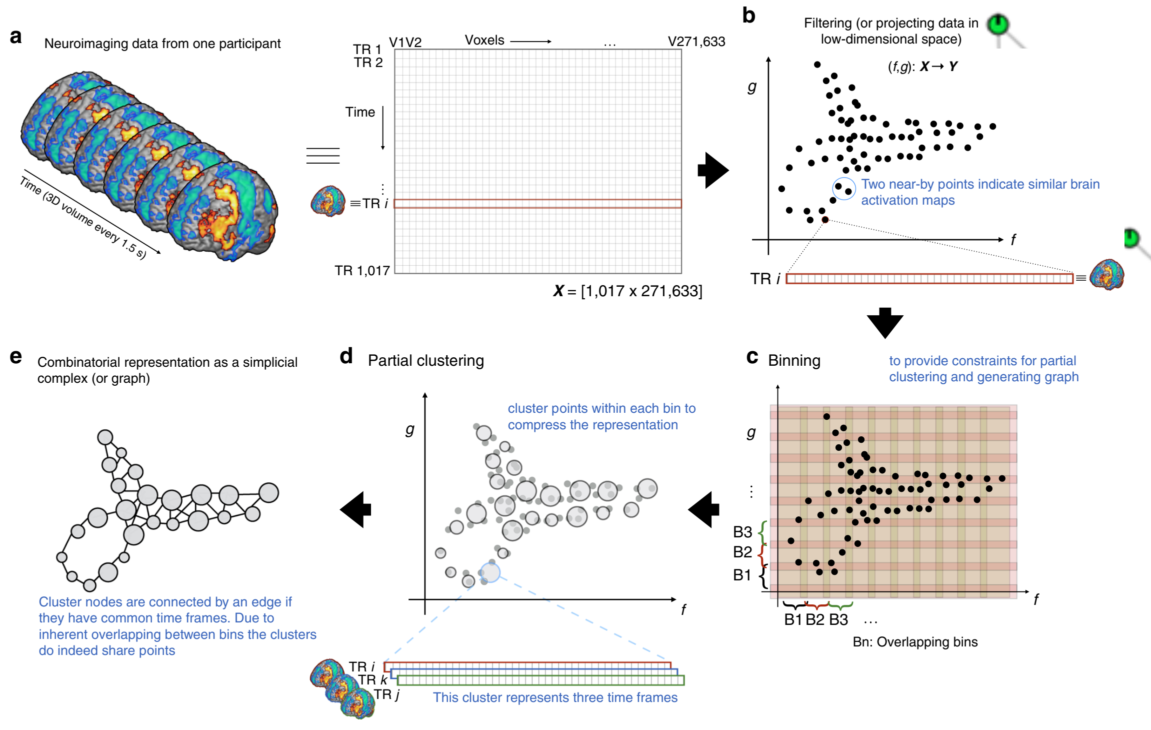

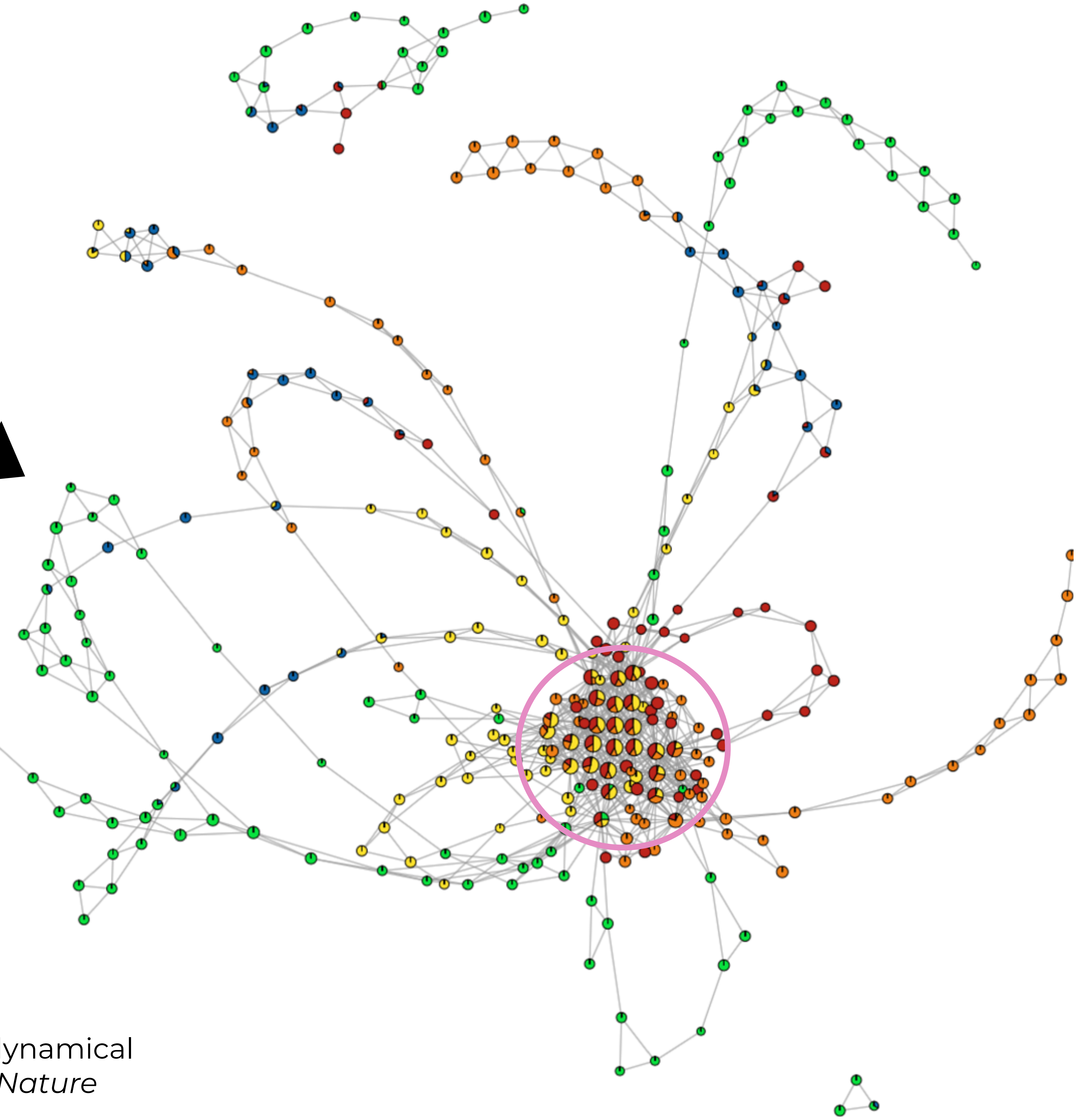

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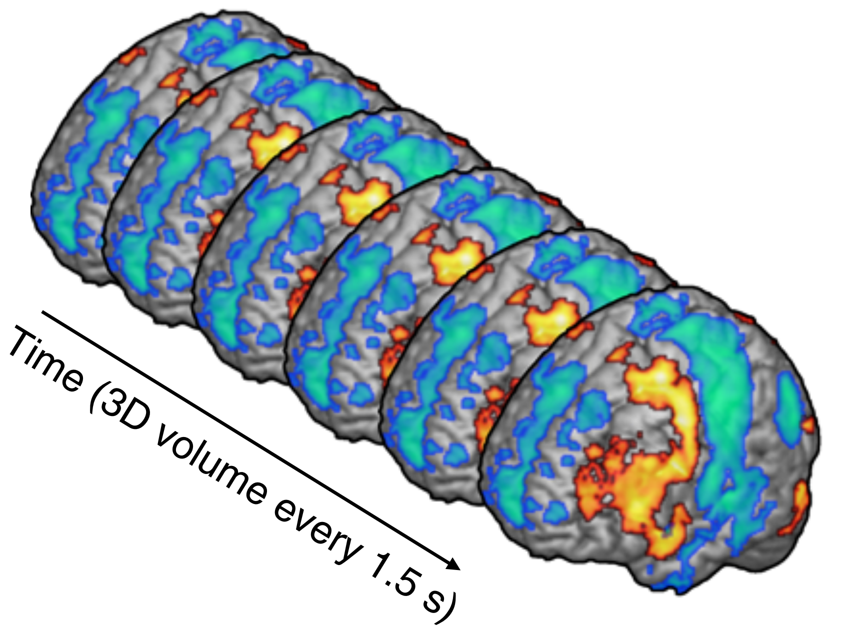

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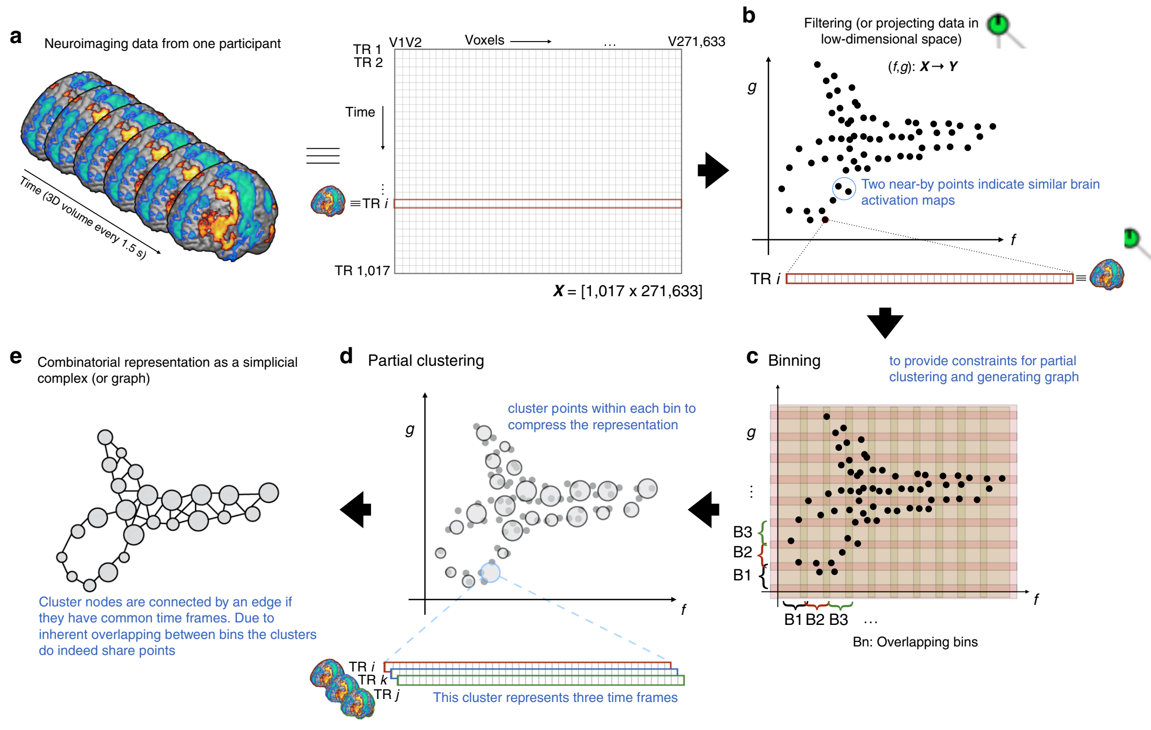
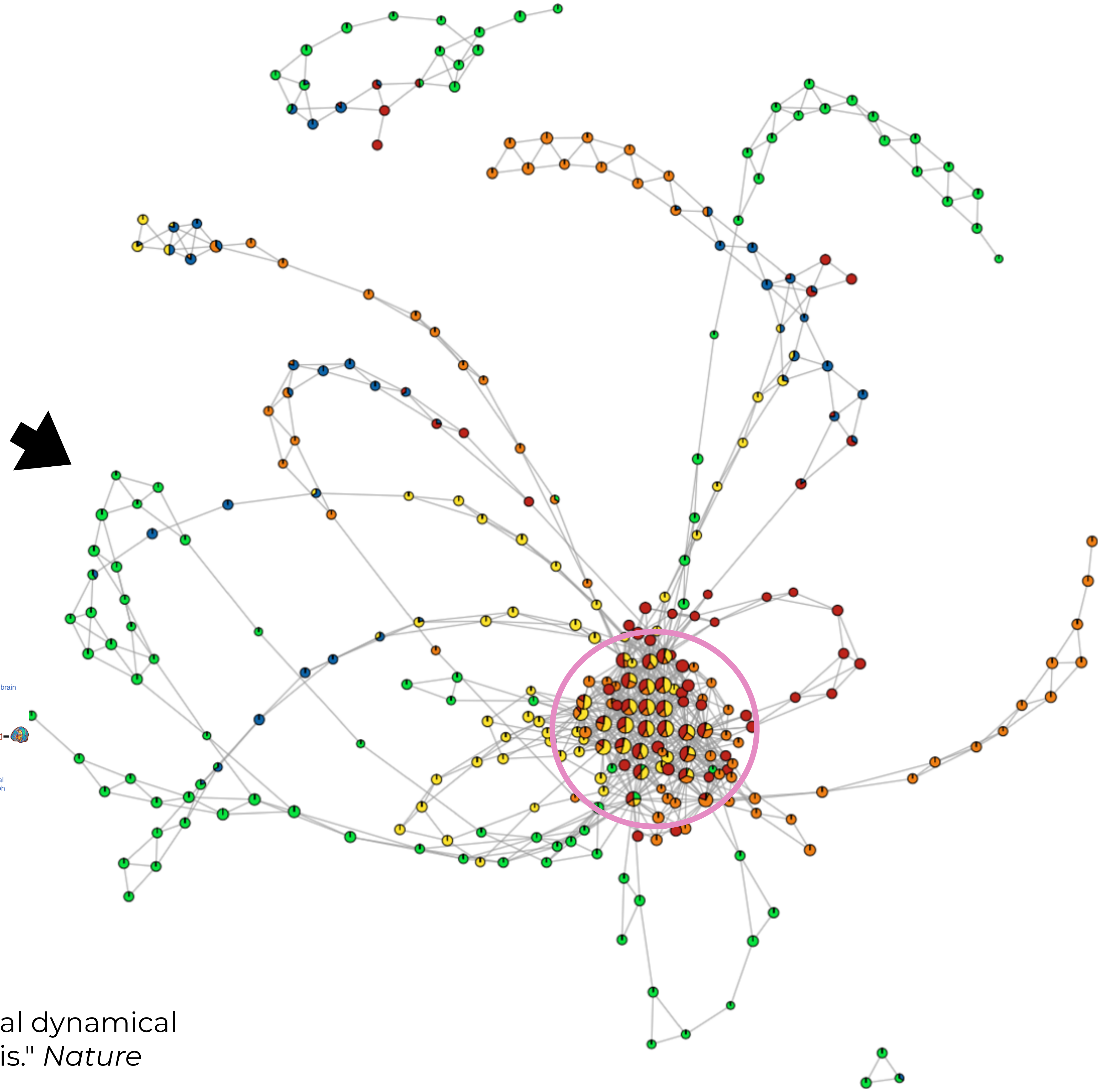
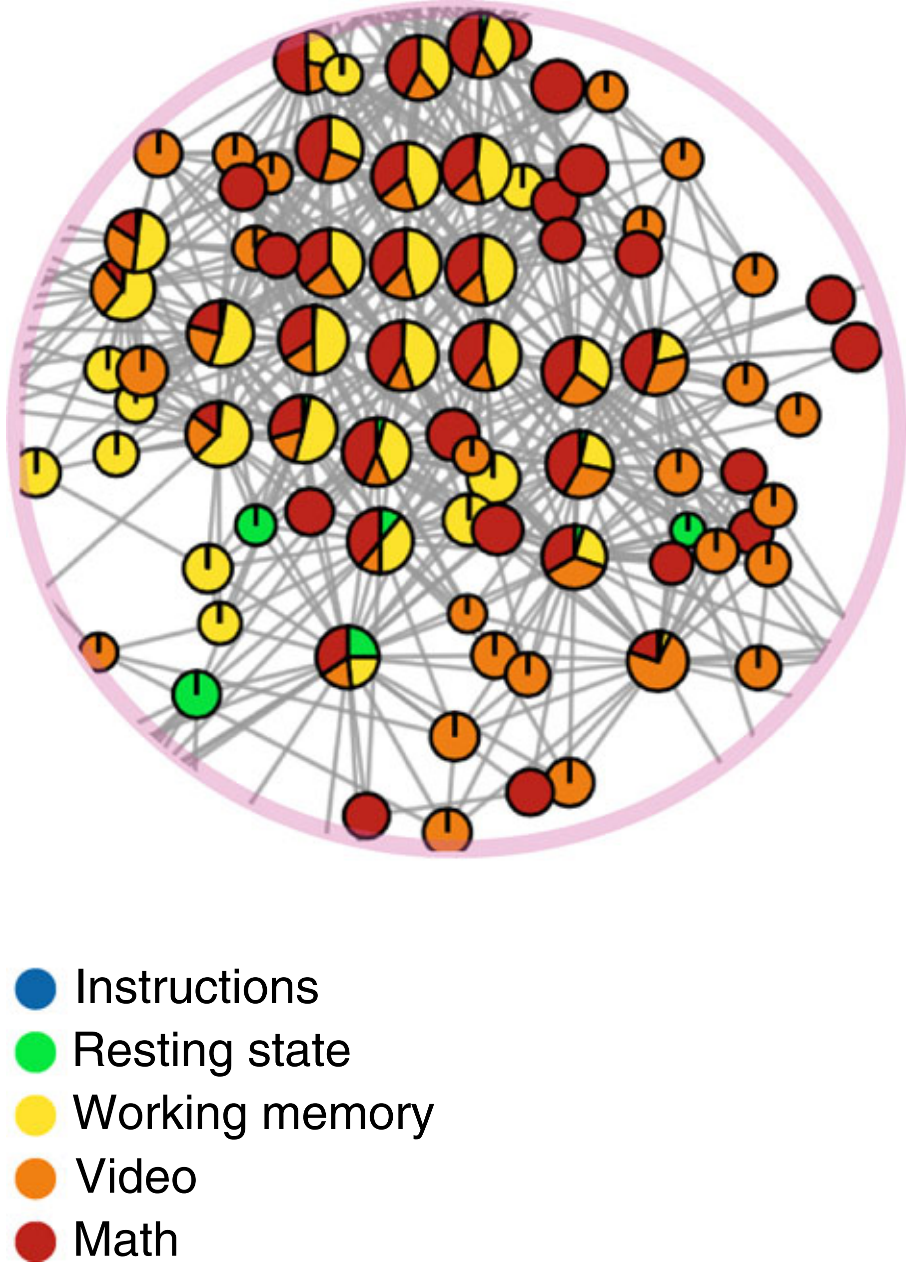




b



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Approximate activity landscapes using topology

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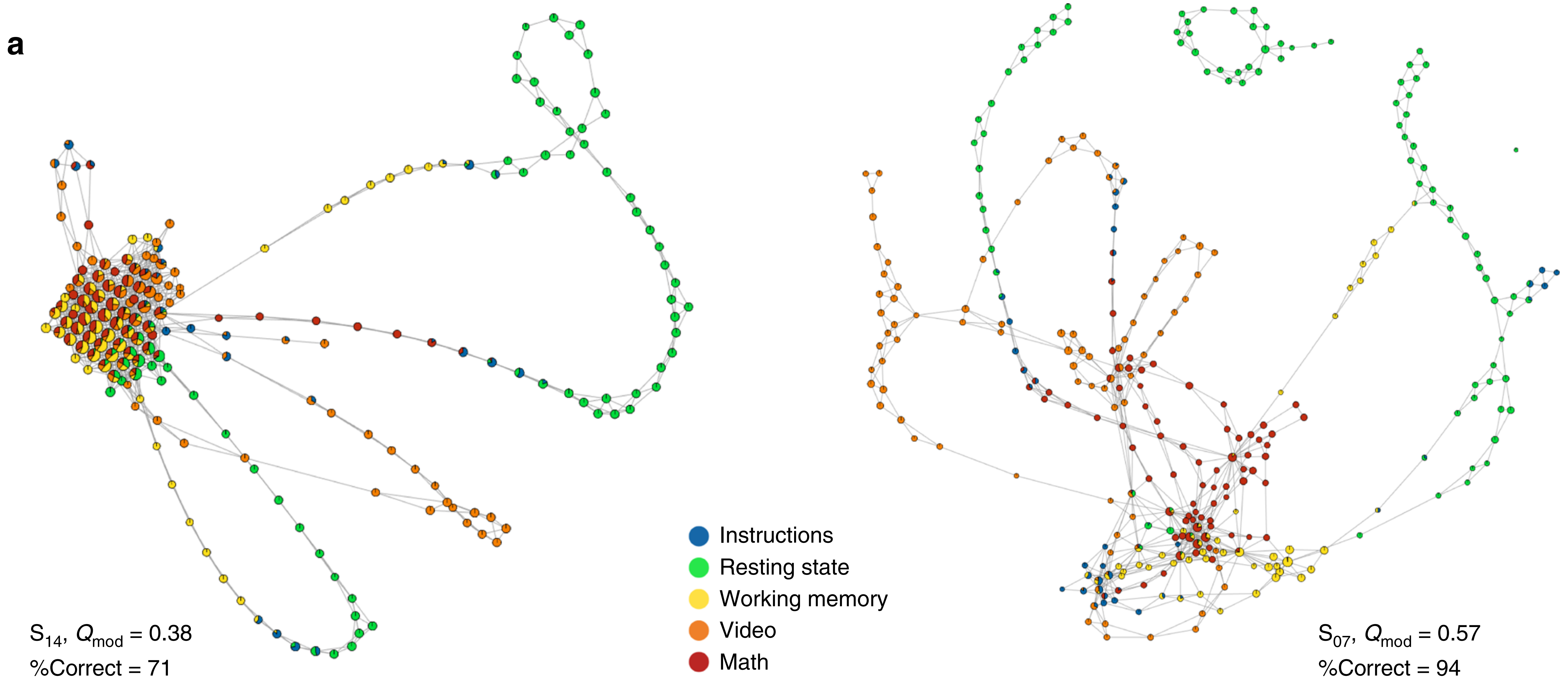
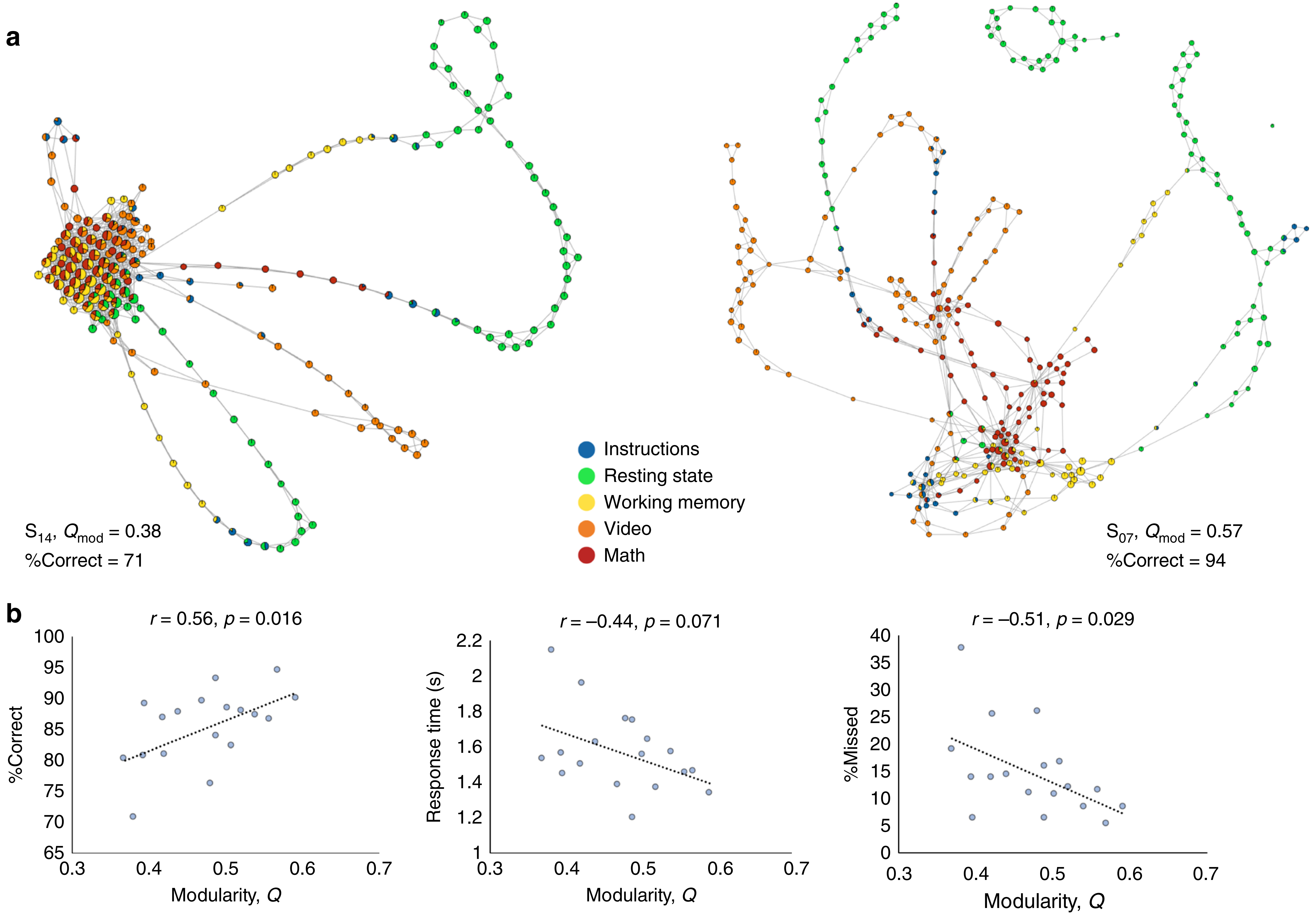
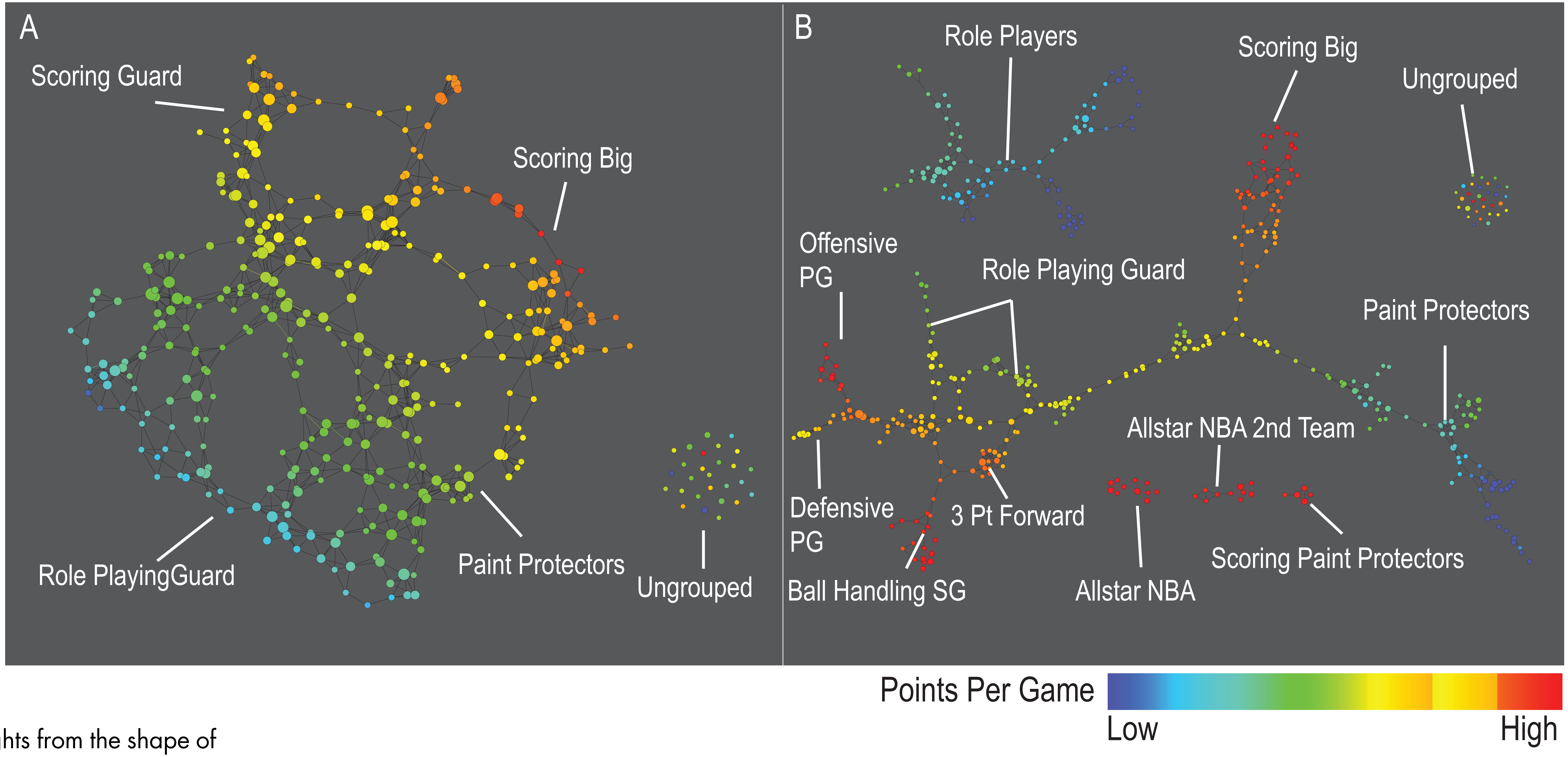


Approximate activity landscapes using topology



Multi-resolution analysis?

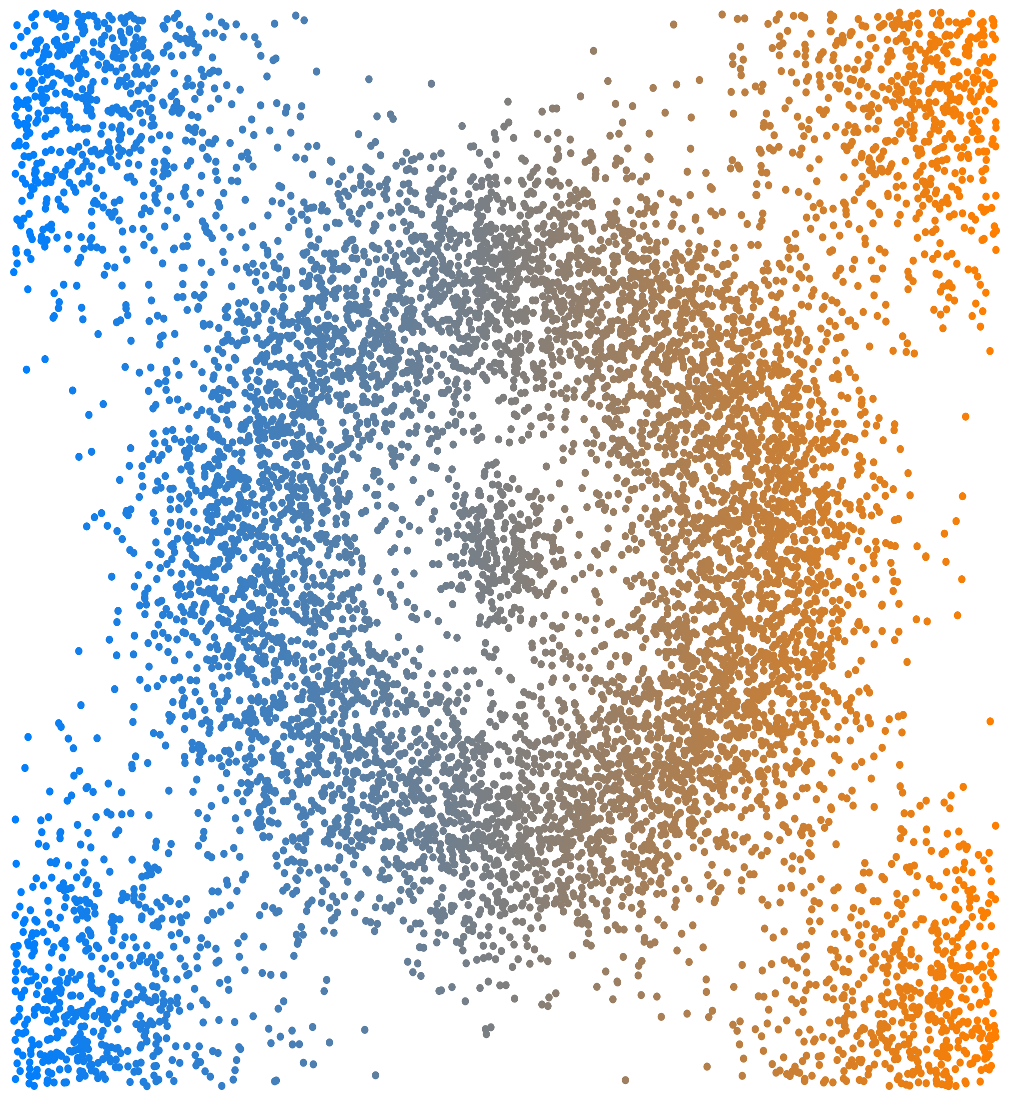


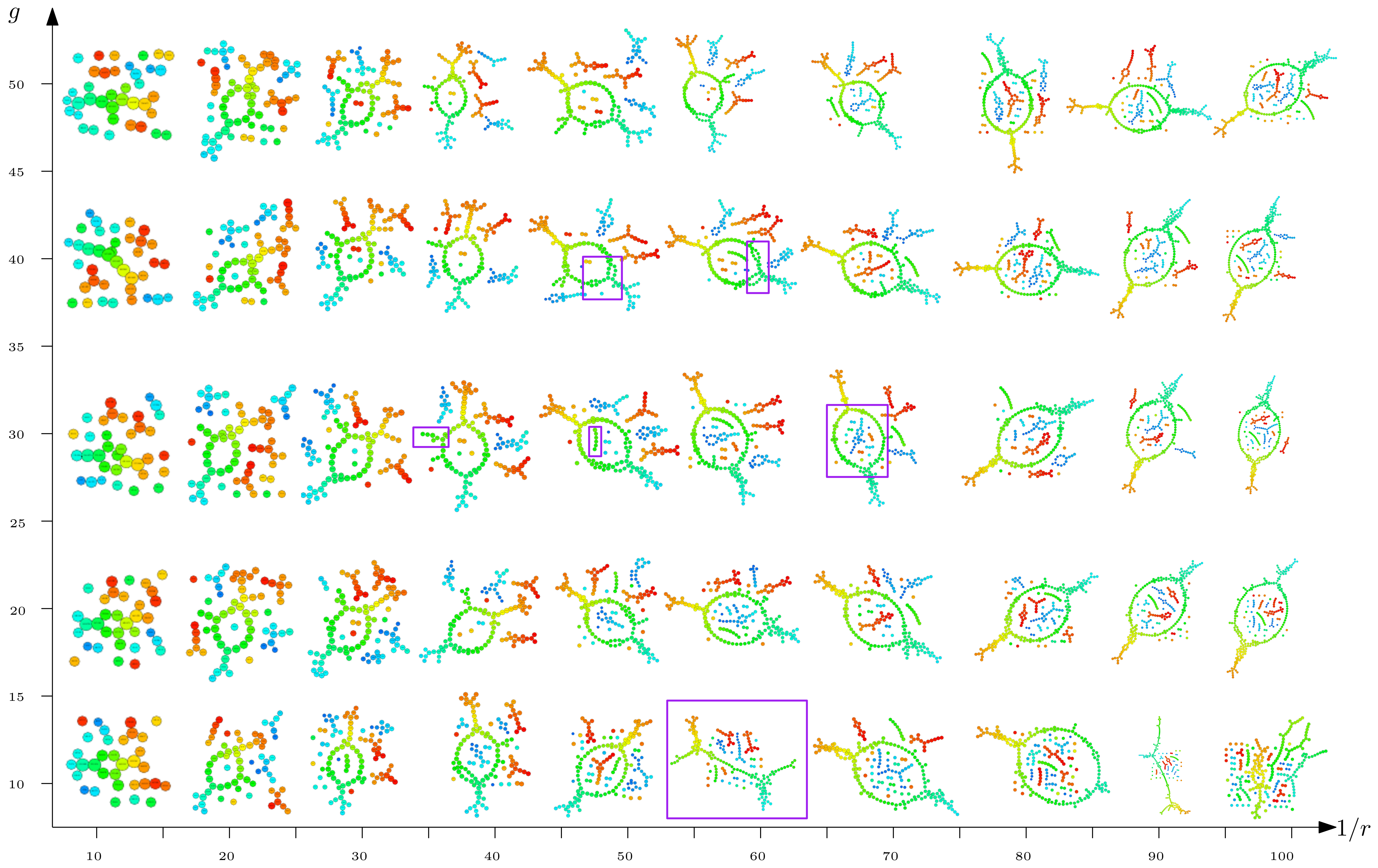
Extracting insights from the shape of complex data using topology

P. Y. Lum[1], G. Singh[1], A. Lehman[1], T. Ishkanov[1], M. Vejdemo-Johansson[2], M. Alagappan[1], J. Carlsson[3] & G. Carlsson[1,4]

https://www.wired.com/2012/04/analytics-basketball/

Automatic Mapper parameter choice





Statistical Analysis and Parameter Selection for Mapper

Mathieu Carrière MATHIEU.CARRIERE@INRIA.FR
Inria Saclay
91120 Palaiseau, France

Bertrand Michel BERTRAND.MICHEL@EC-NANTES.FR
LMJL UMR 6629– Ecole Centrale Nantes
44322 Nantes, France

Steve Oudot STEVE.OUDOT@INRIA.FR
Inria Saclay
91120 Palaiseau, France

Selection based on “extended” persistent homology

Can topology quantify shapes?

Functional, structural, you name it…

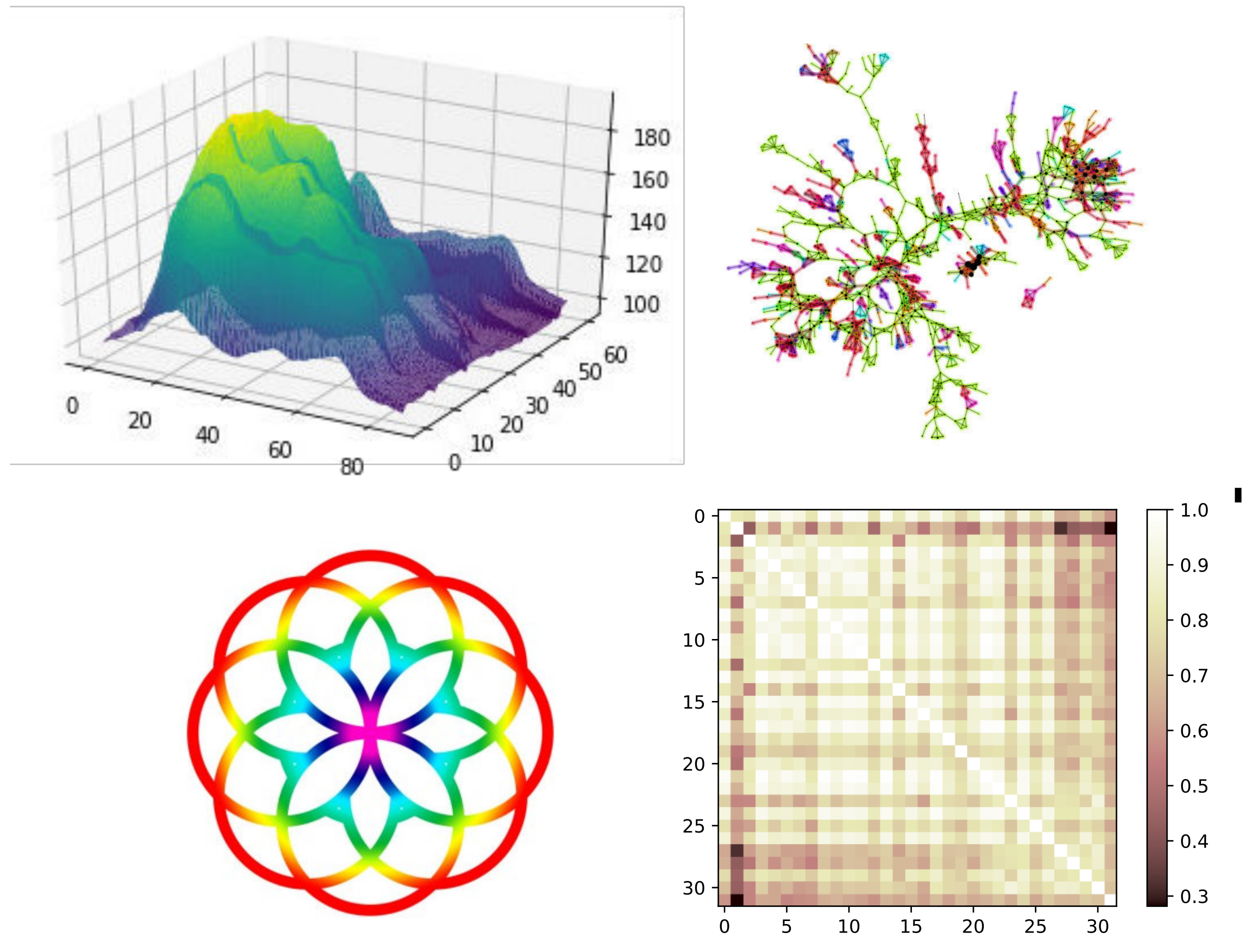
Can topology quantify shapes?

Functional, structural, you name it…

What does it mean in practice?

Persistent homology pipeline (Ghrist 2008)

Data of sorts



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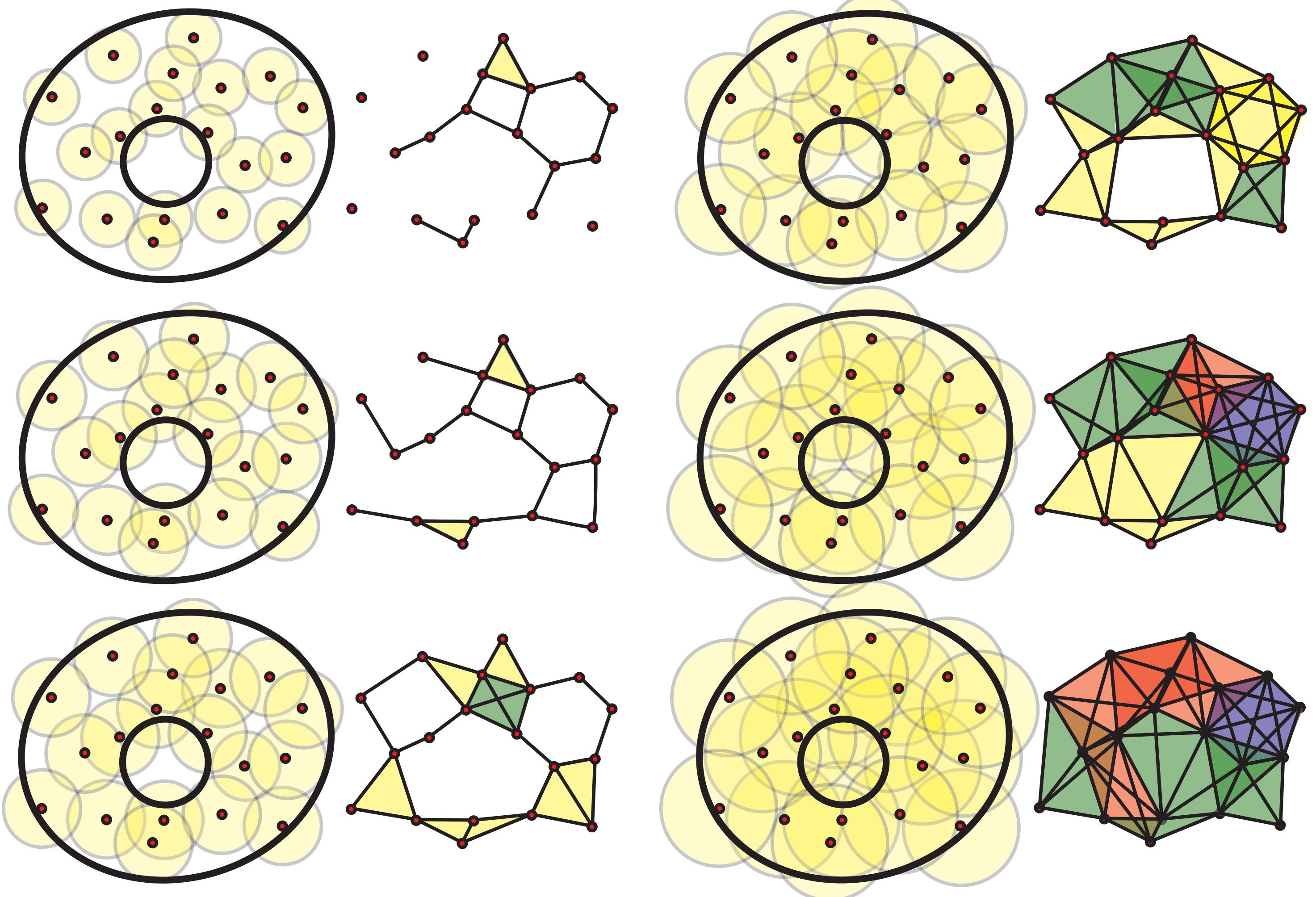
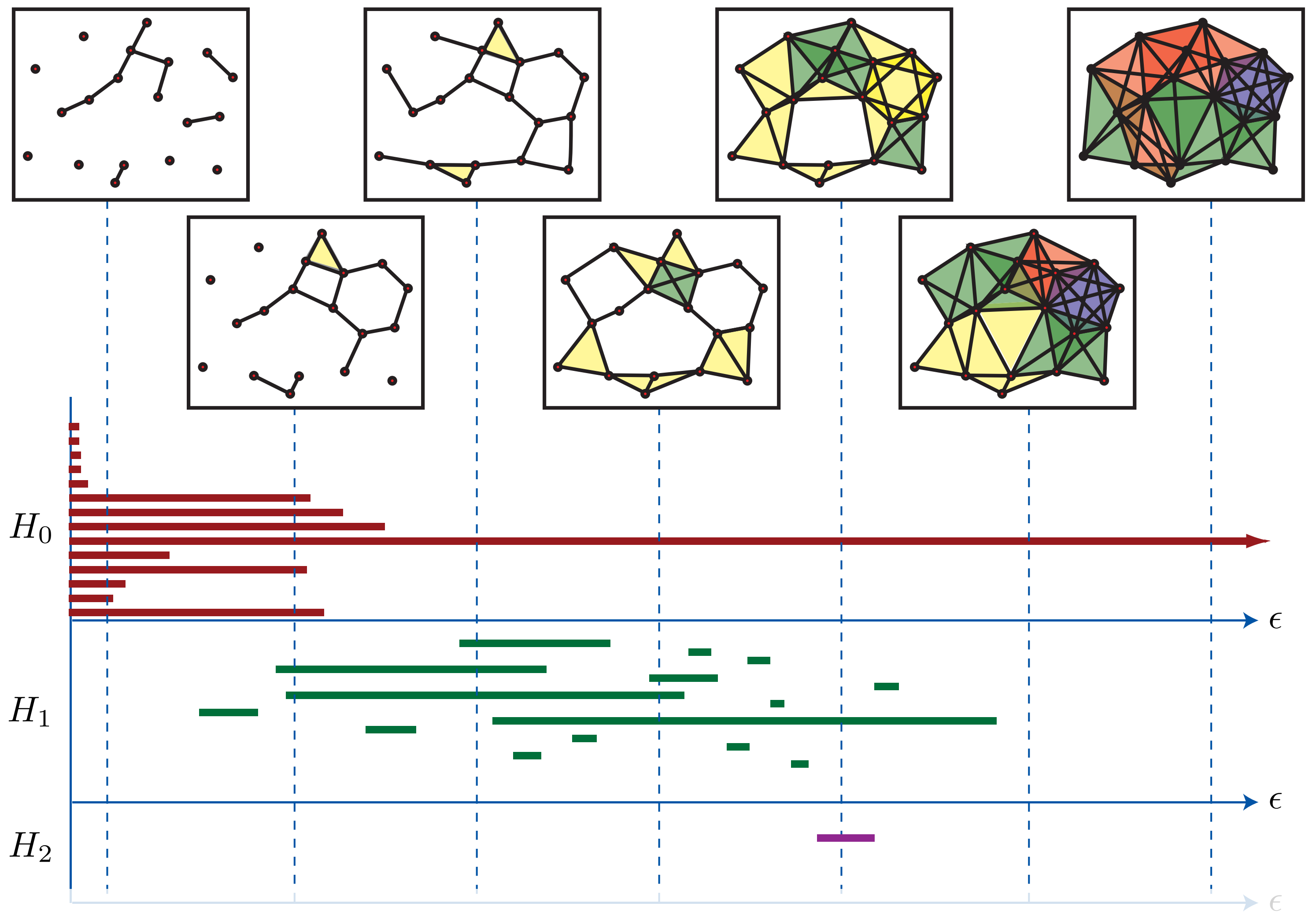


Homological properties



Mapper Pipeline (Singh et al 2007)

Point cloud

colouring (projecting) using geometric filters

overlapped binning

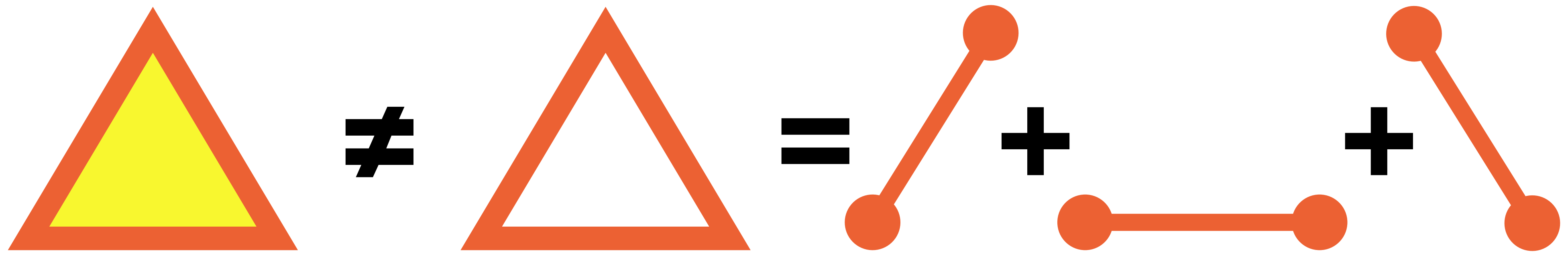
Clustering and network construction binning

Basic simplicial dictionary

DOT
= 0-simplex

EDGE =
1-simplex

TRIANGLE
= 2-simplex



Ghrist, R. (2008). Barcodes: The persistent topology of data.

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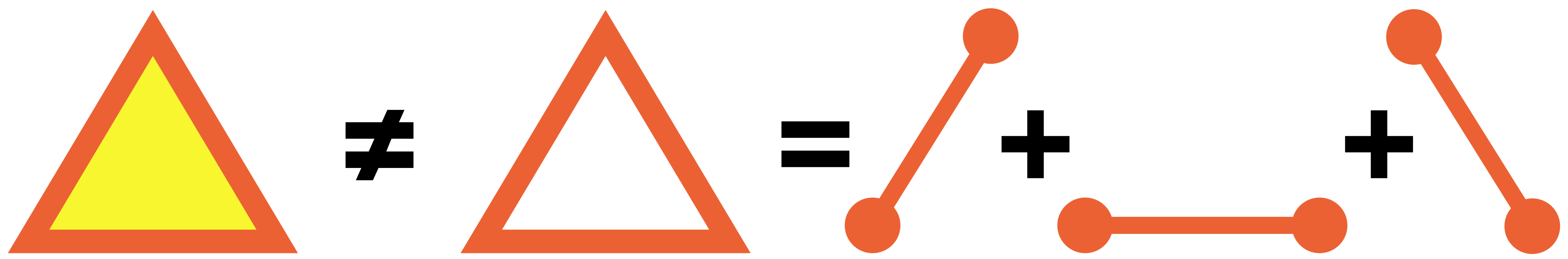
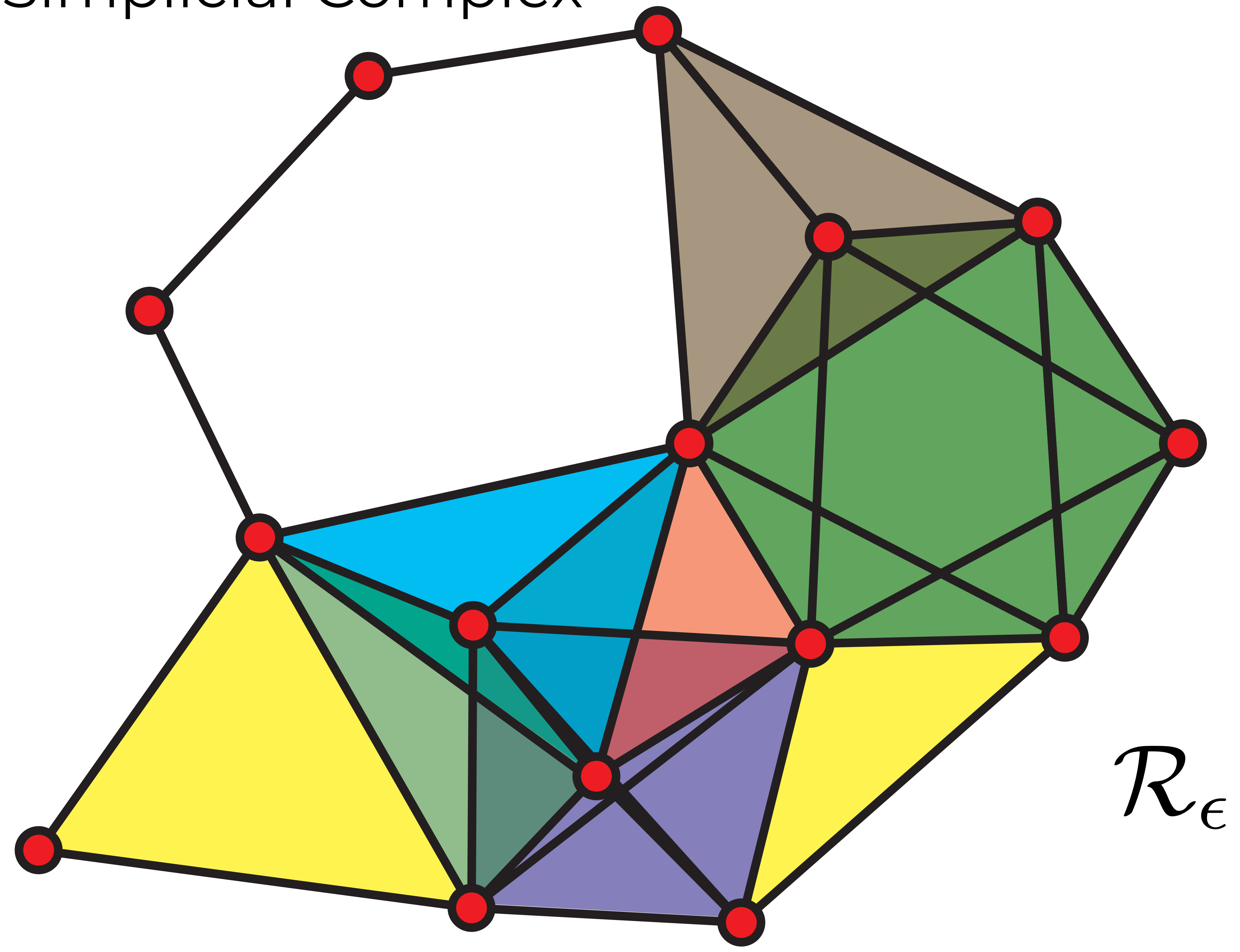


Simplicial Complex



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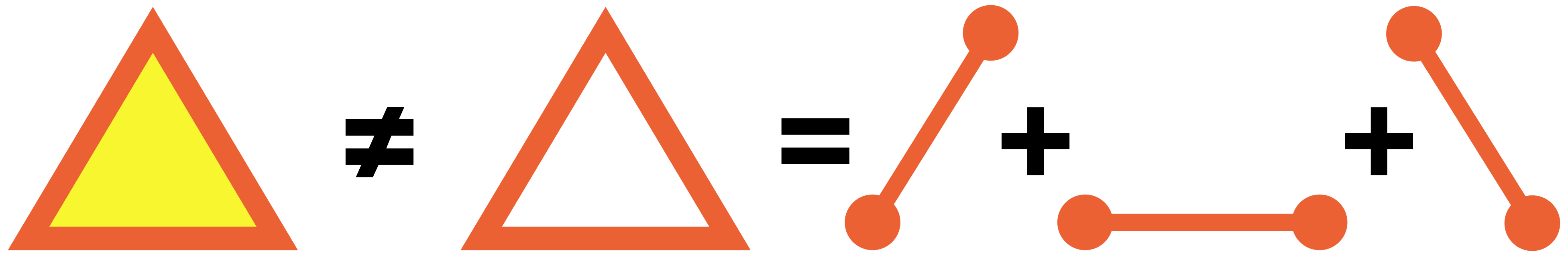

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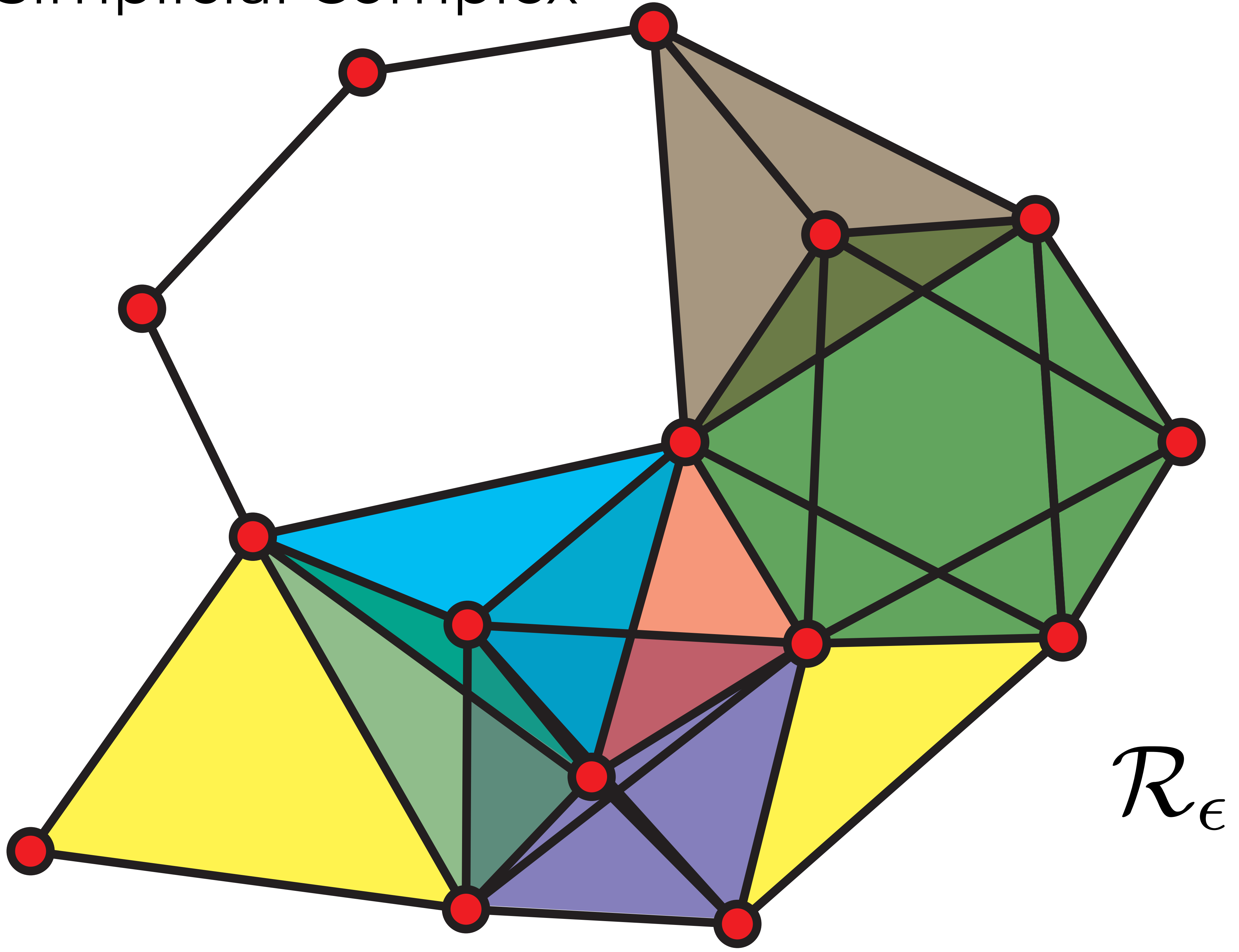

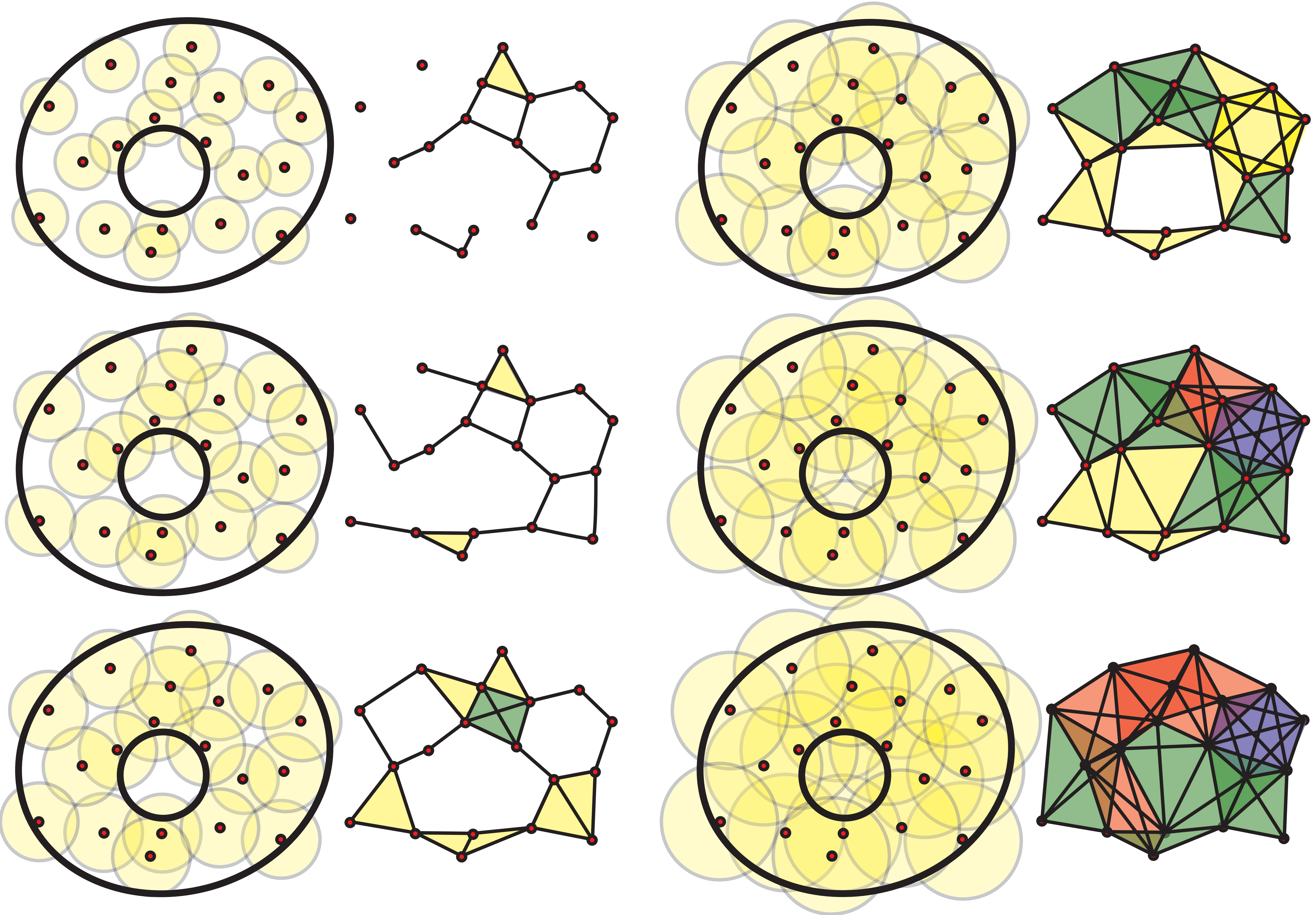

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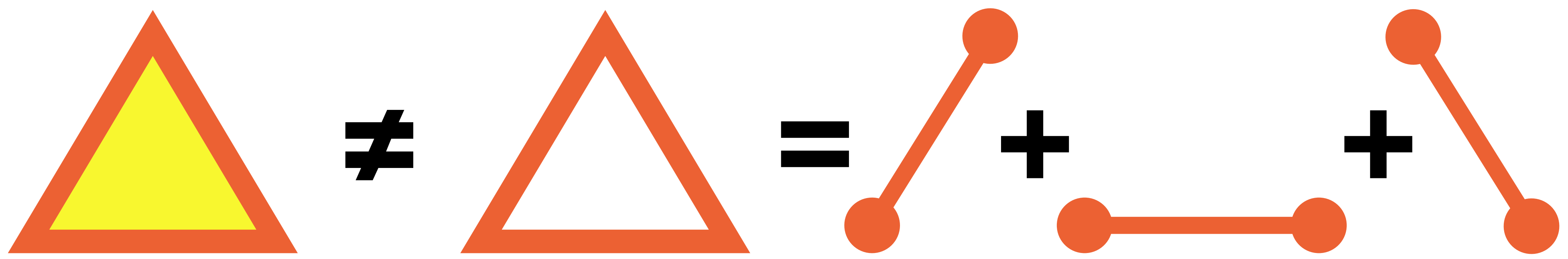

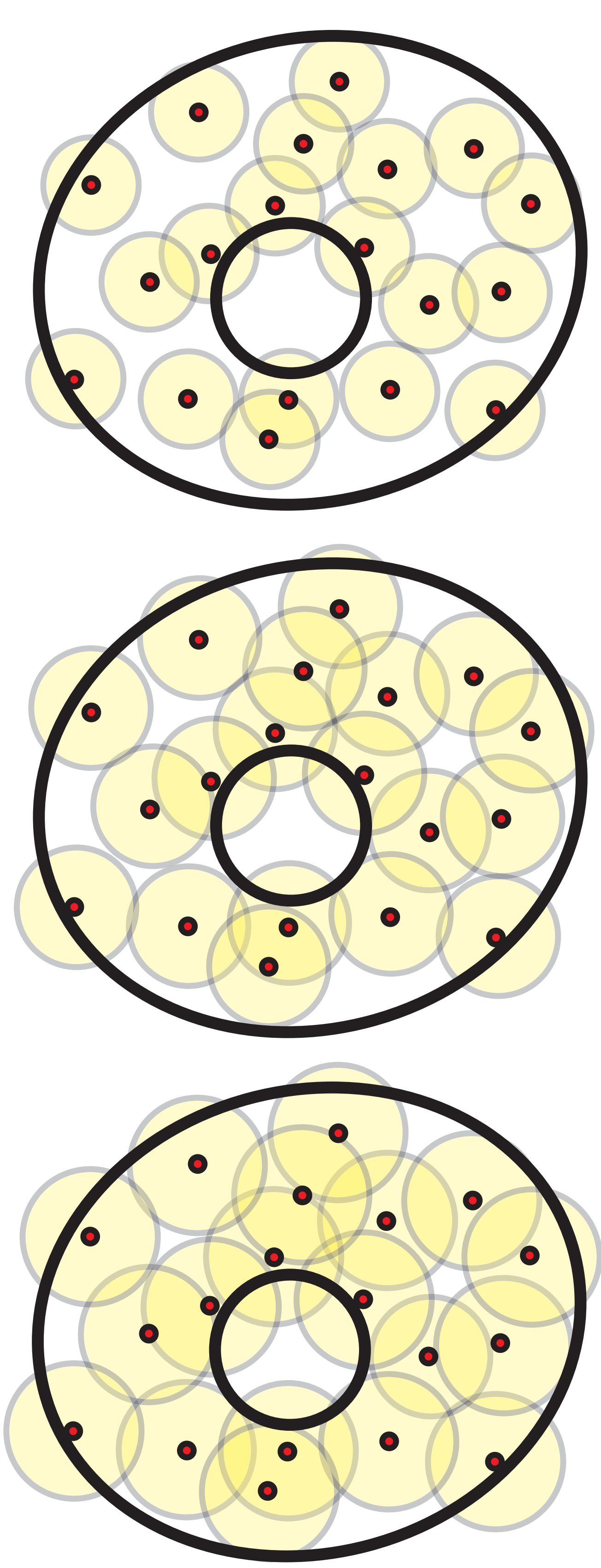

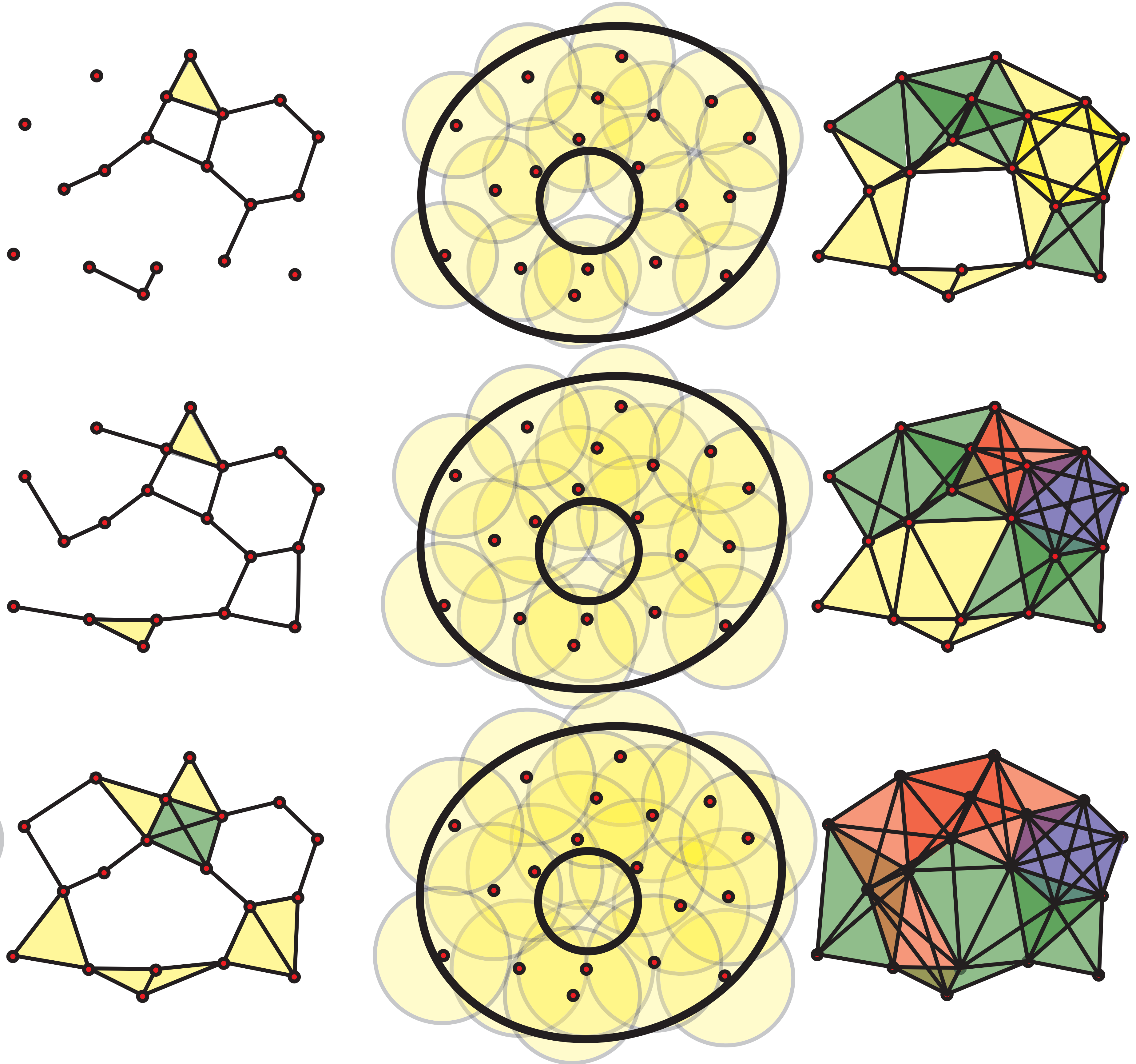

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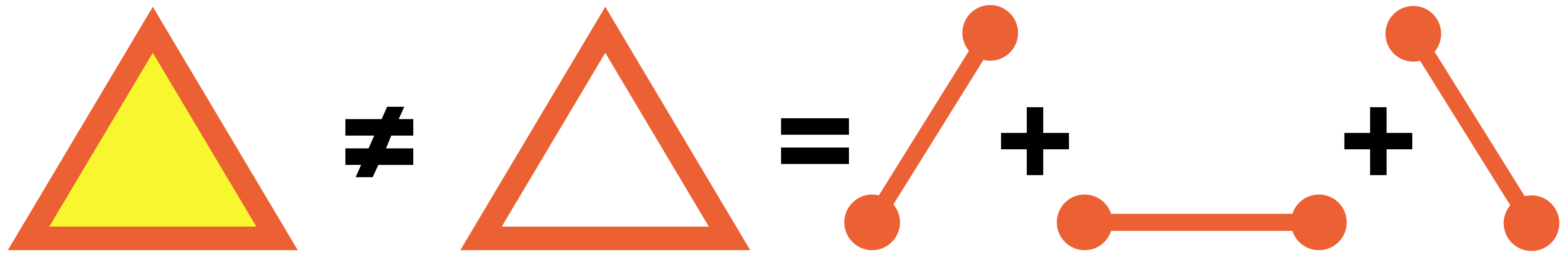
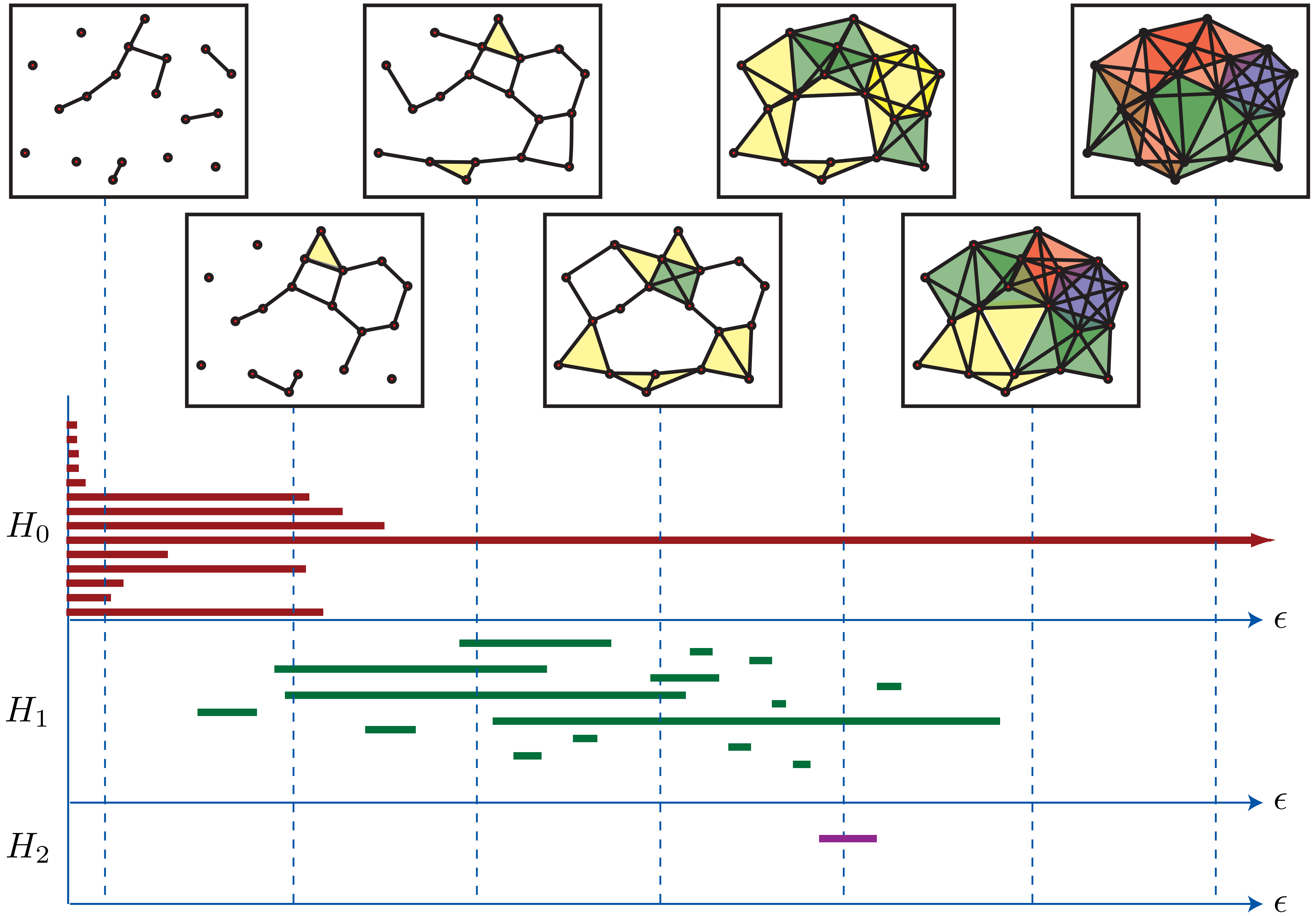
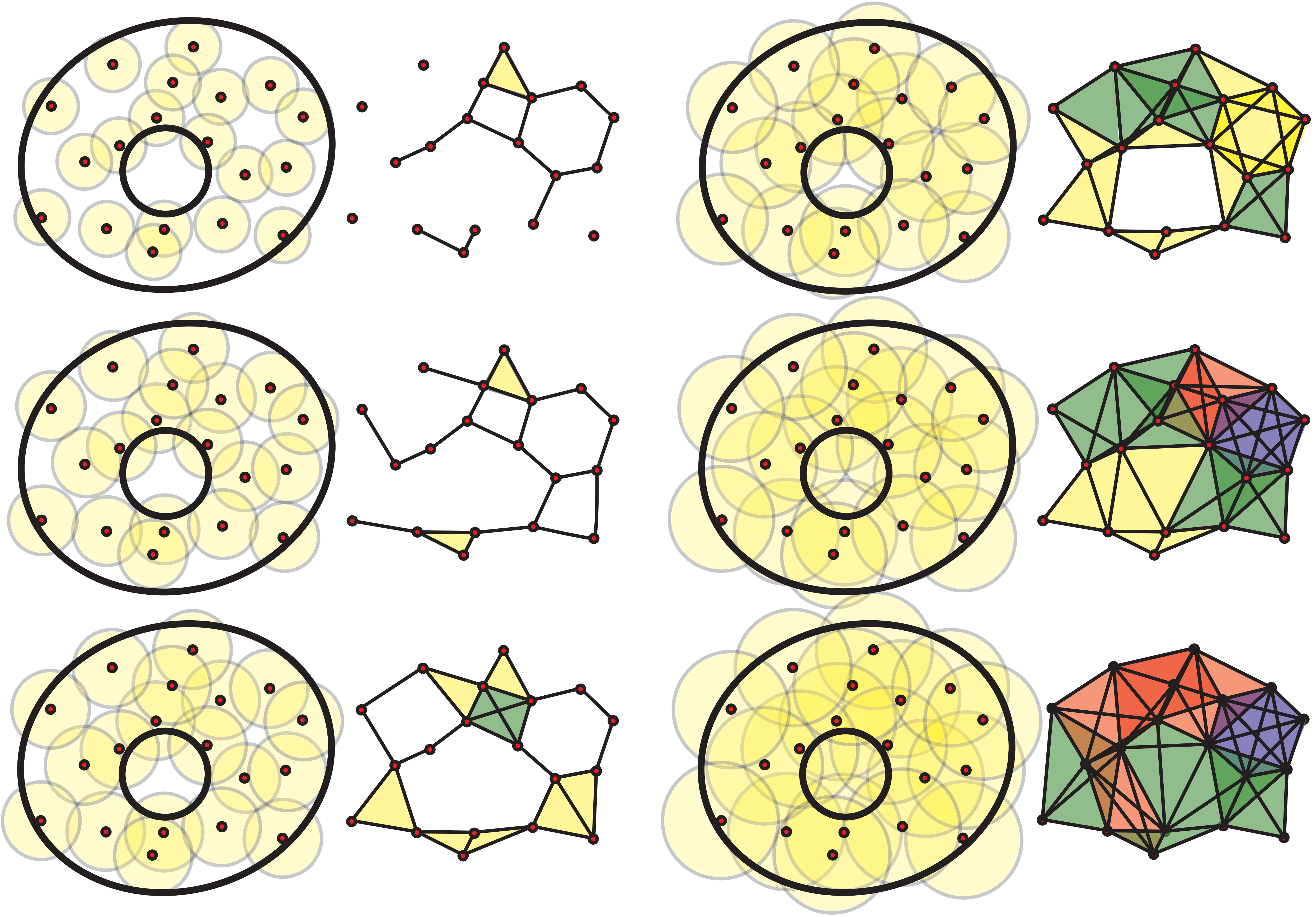




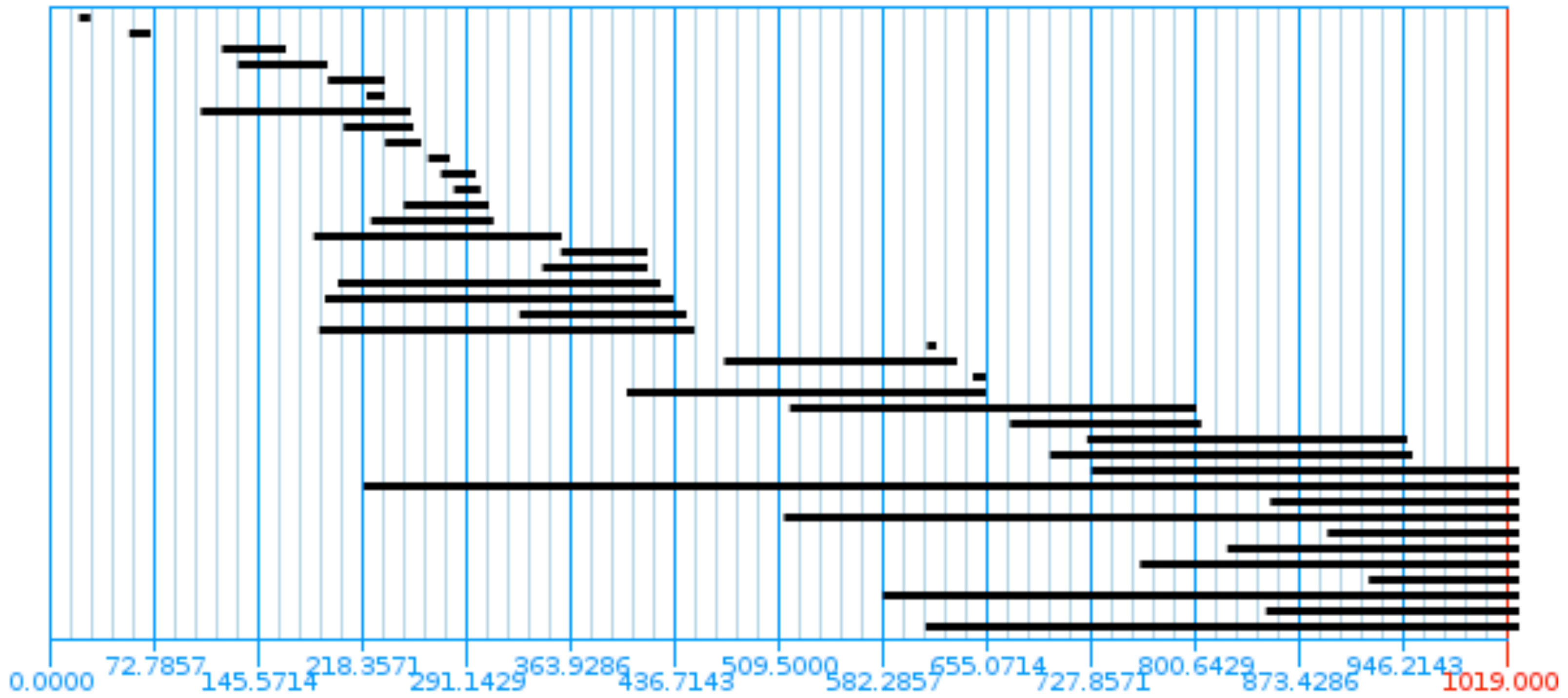


Ghrist, R. (2008). Barcodes: The persistent topology of data.

Notebook 04

How to compare shapes?

How to compare shapes?



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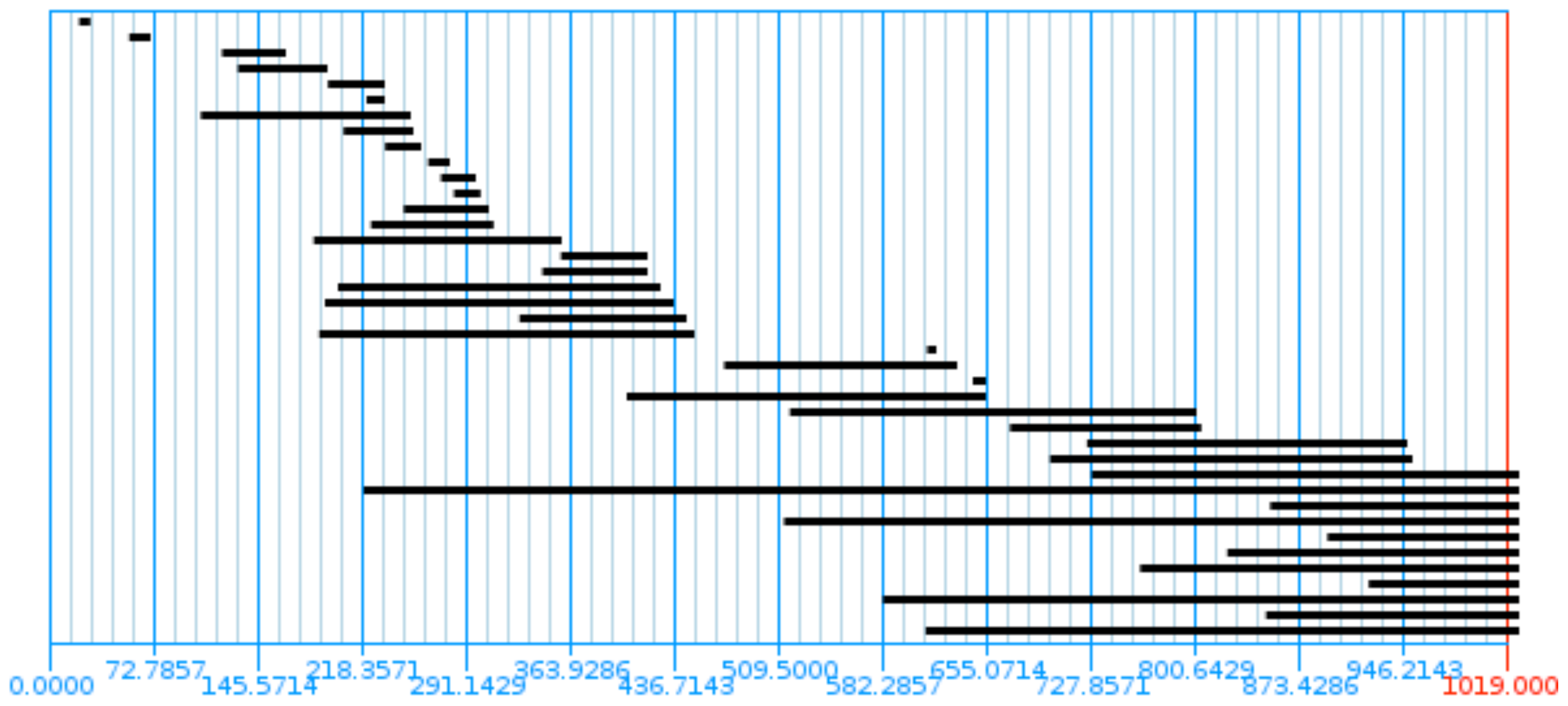

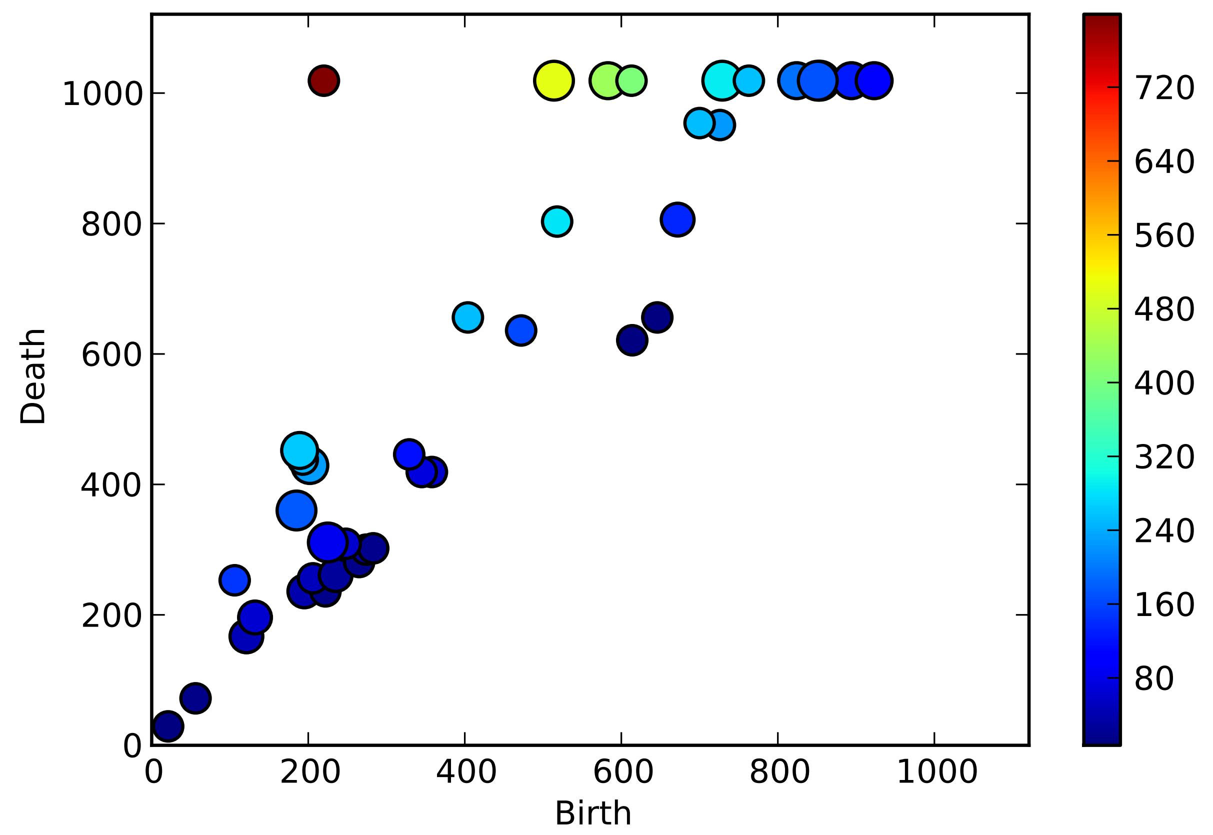

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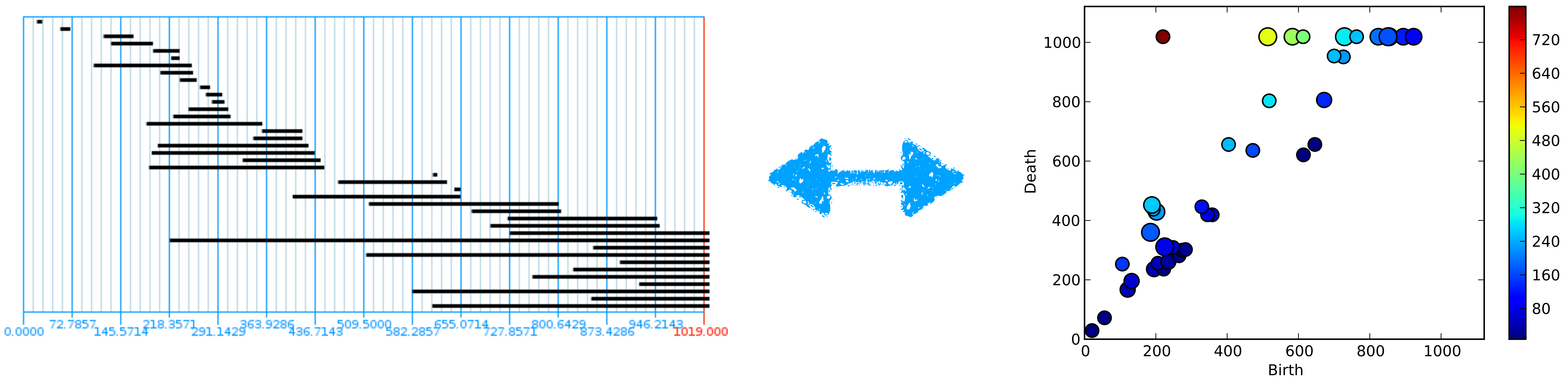
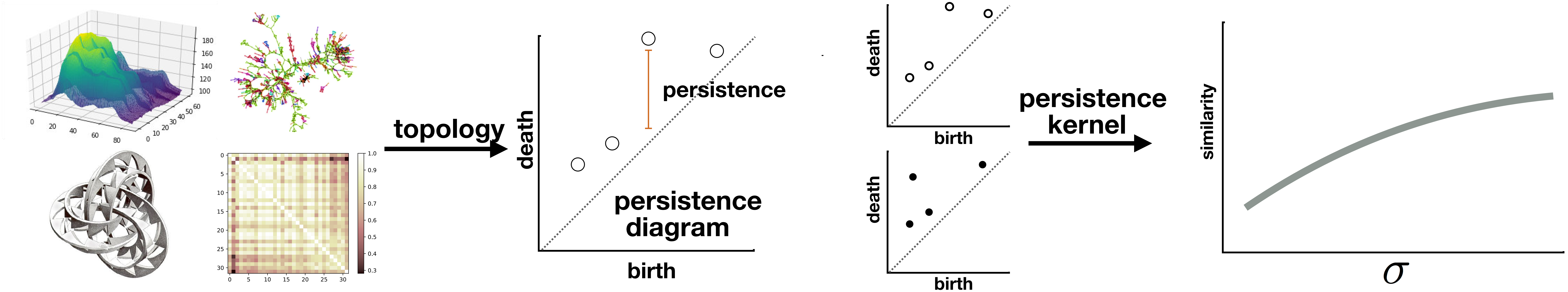
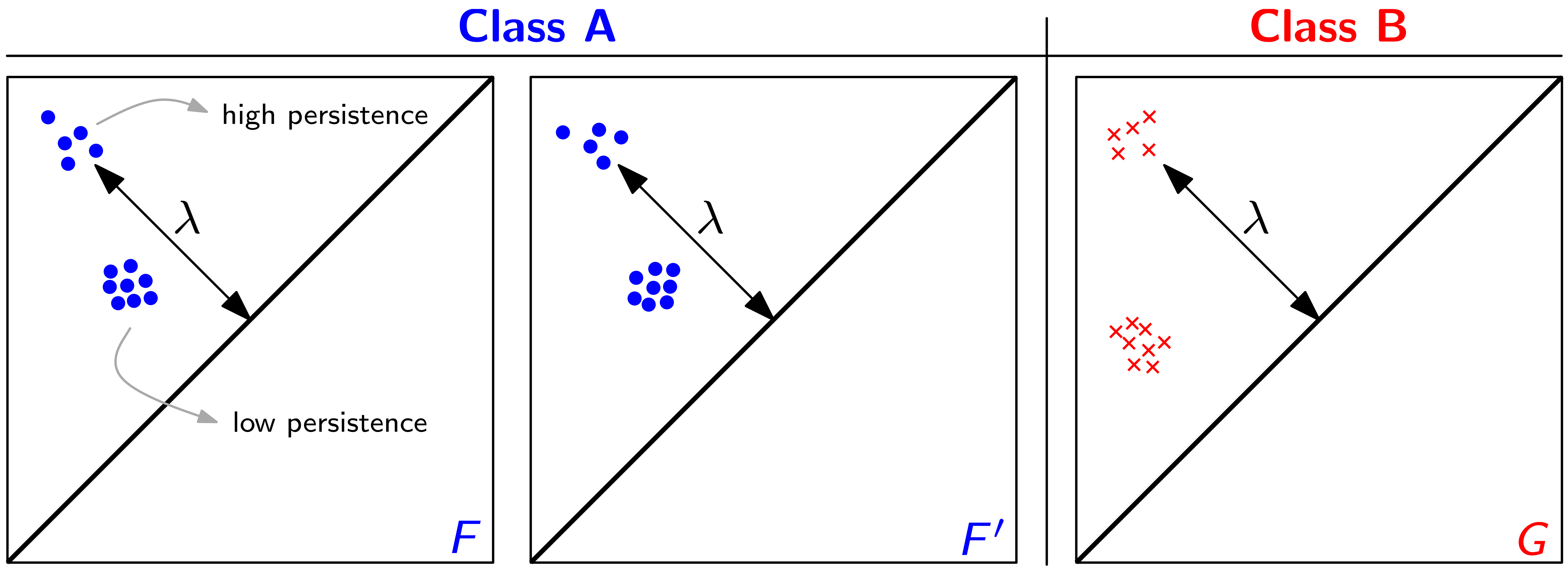




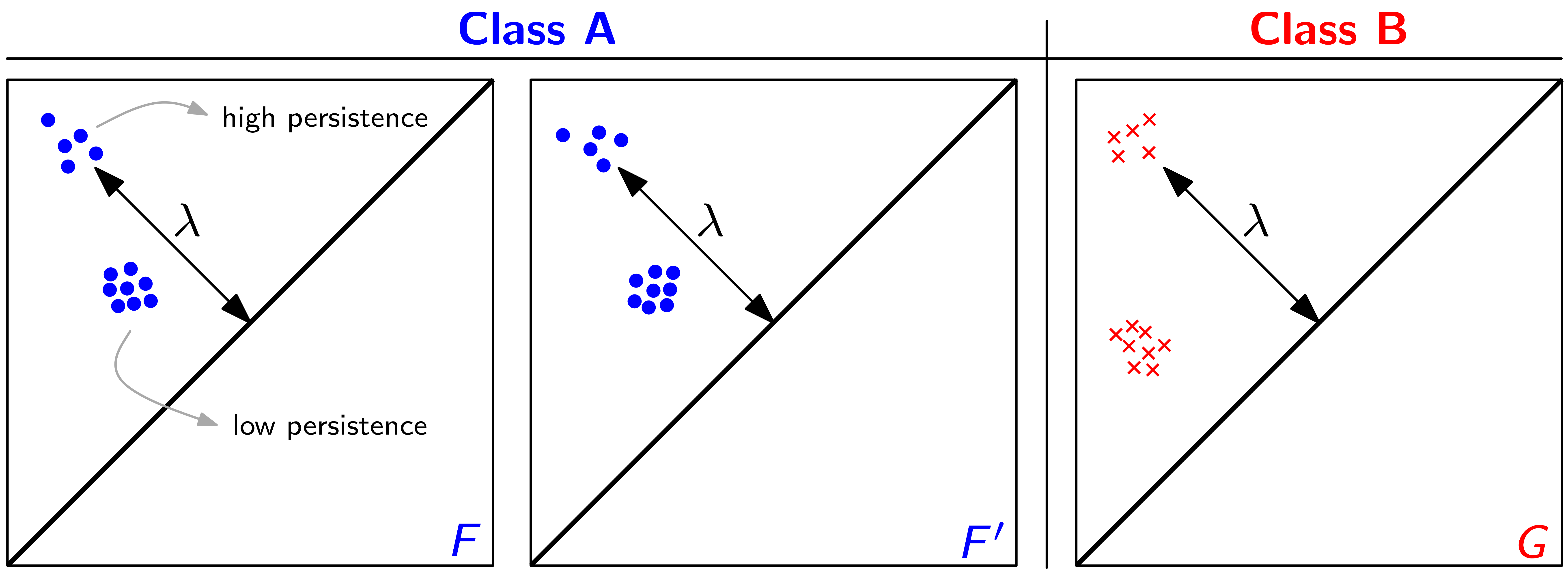
Distances

Distances



Class A
Class B
high persistence
low persistence
λ
λ
λ
F
F'
G

Distances



Distances and Kernels:

- Bottleneck Distance
- (sliced) Wasserstein distance
- Persistence Scale Kernel (Reininghaus et al 2015)
- Weighted Persistence Kernel (Kusano et al, 2016)

Distances

Bottleneck Distance

$$\delta_\infty(D_1, D_2) = \inf_{\gamma} \sup_{z \in D_1} ||z - \gamma(z)||_\infty$$

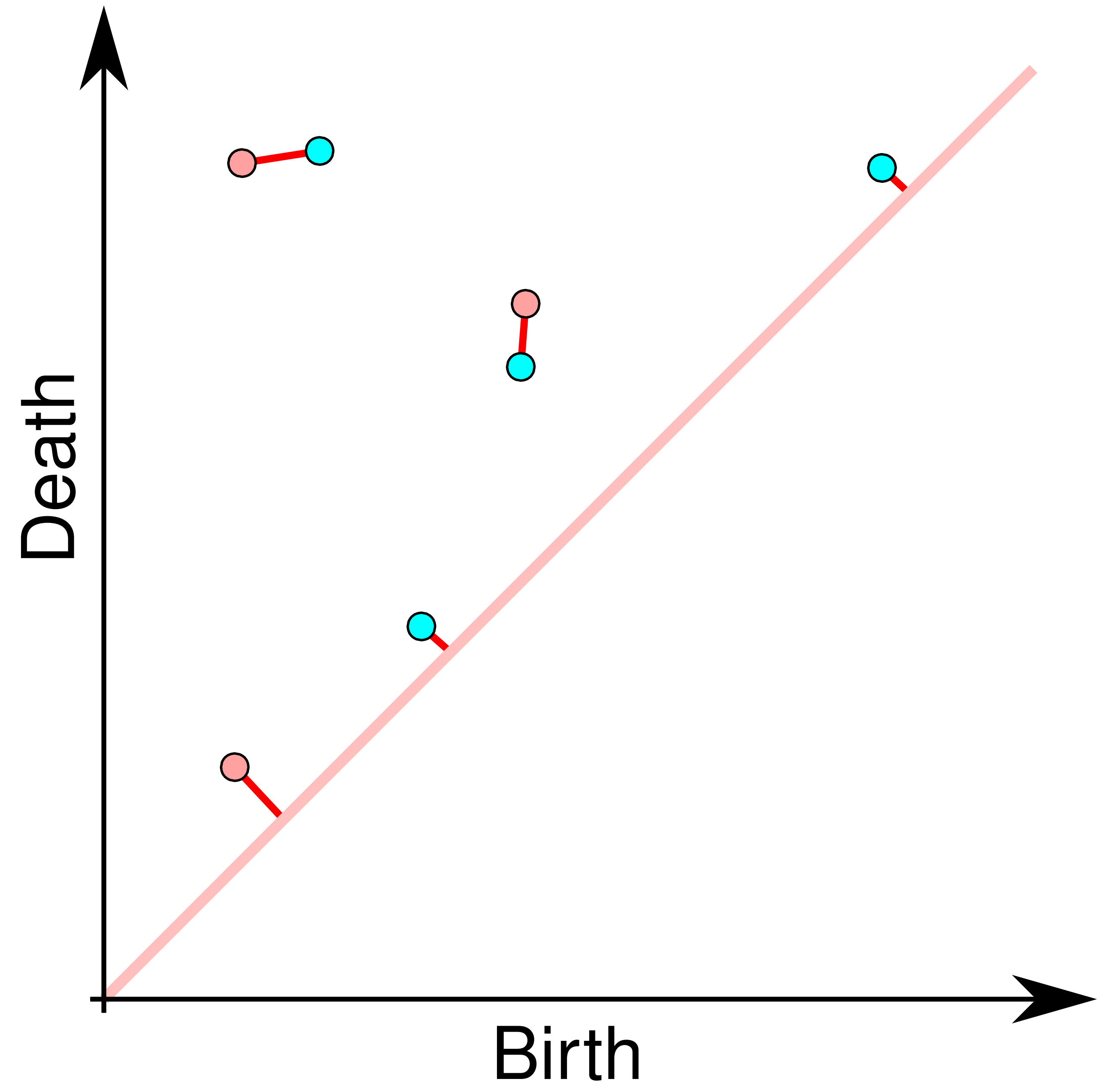
Wasserstein Distance

$$W_p\,(D_1, D_2) \;=\; \inf_{\gamma} \Big(\sum_{u \in D_1} \|u - \gamma(u)\|_\infty^p \Big)^{1/p}$$

Sliced Wasserstein Distance

$$\mathrm{SW}(\mathrm{Dg}_1, \mathrm{Dg}_2) \stackrel{\text{def.}}{=} \frac{1}{2\pi} \int_{\mathbb{S}_1} \mathcal{W}(\mu_1^\theta + \mu_{2\Delta}^\theta, \mu_2^\theta + \mu_{1\Delta}^\theta) \mathrm{d}\theta.$$

$$k_{\mathrm{SW}}(\mathrm{Dg}_1, \mathrm{Dg}_2) \stackrel{\text{def.}}{=} \exp\left(-\frac{\mathrm{SW}(\mathrm{Dg}_1, \mathrm{Dg}_2)}{2\sigma^2} \right).$$



Persistence Scale Kernel

Distances

Bottleneck Distance

$$\delta_\infty(D_1, D_2) = \inf_{\gamma} \sup_{z \in D_1} ||z - \gamma(z)||_\infty$$

Wasserstein Distance

$$W_p(D_1, D_2) = \inf_{\gamma} \Big(\sum_{u \in D_1} \|u - \gamma(u)\|_\infty^p \Big)^{1/p}$$

Sliced Wasserstein Distance

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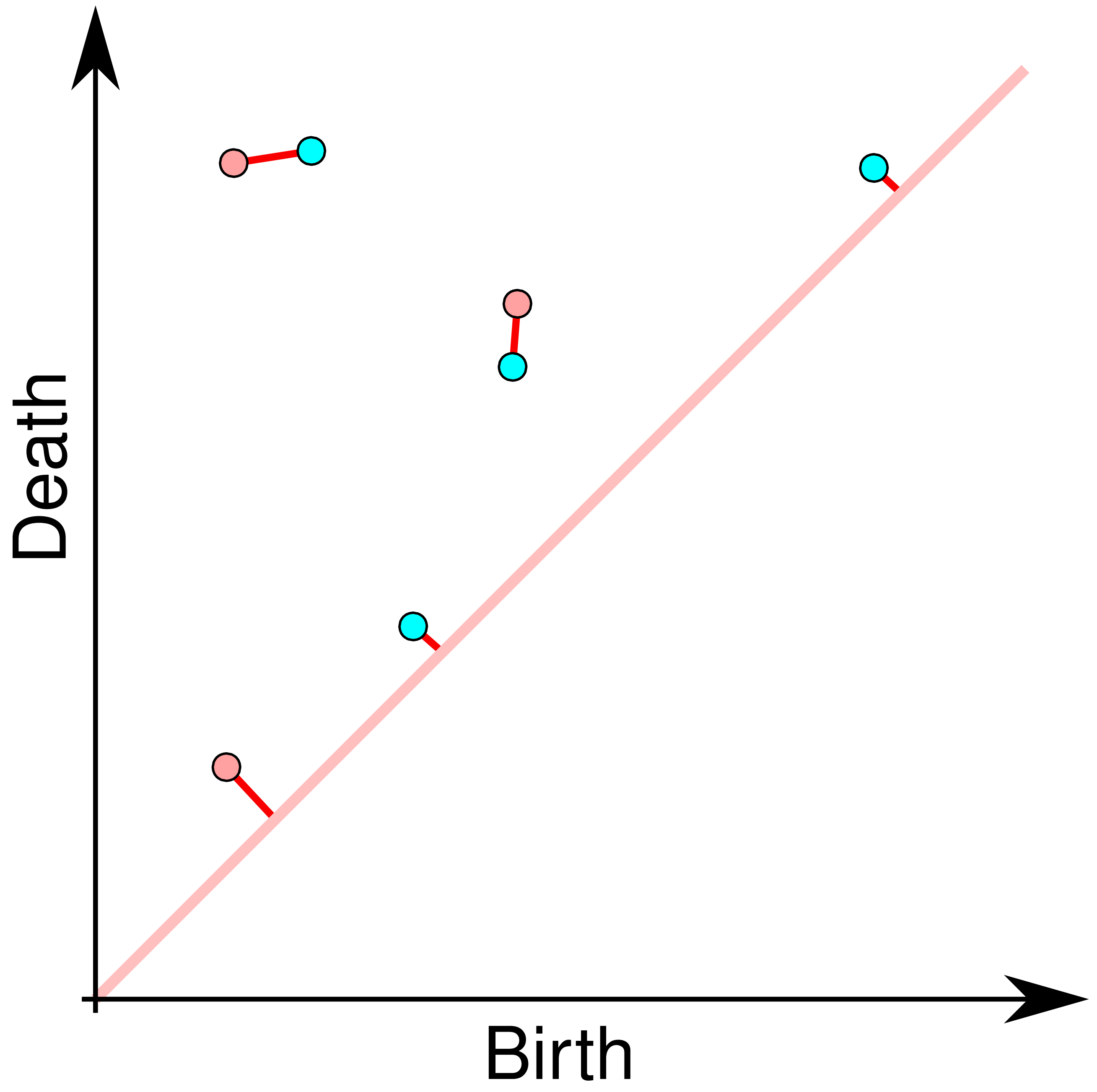


Persistence Scale Kernel

$$k_\sigma(F, G) = \frac{1}{8\pi\sigma} \sum_{\substack{p \in F \\ q \in G}} e^{-\frac{\|p-q\|^2}{8\sigma}} - e^{-\frac{\|p-\bar{q}\|^2}{8\sigma}}.$$

Notebook 05-06-gtda

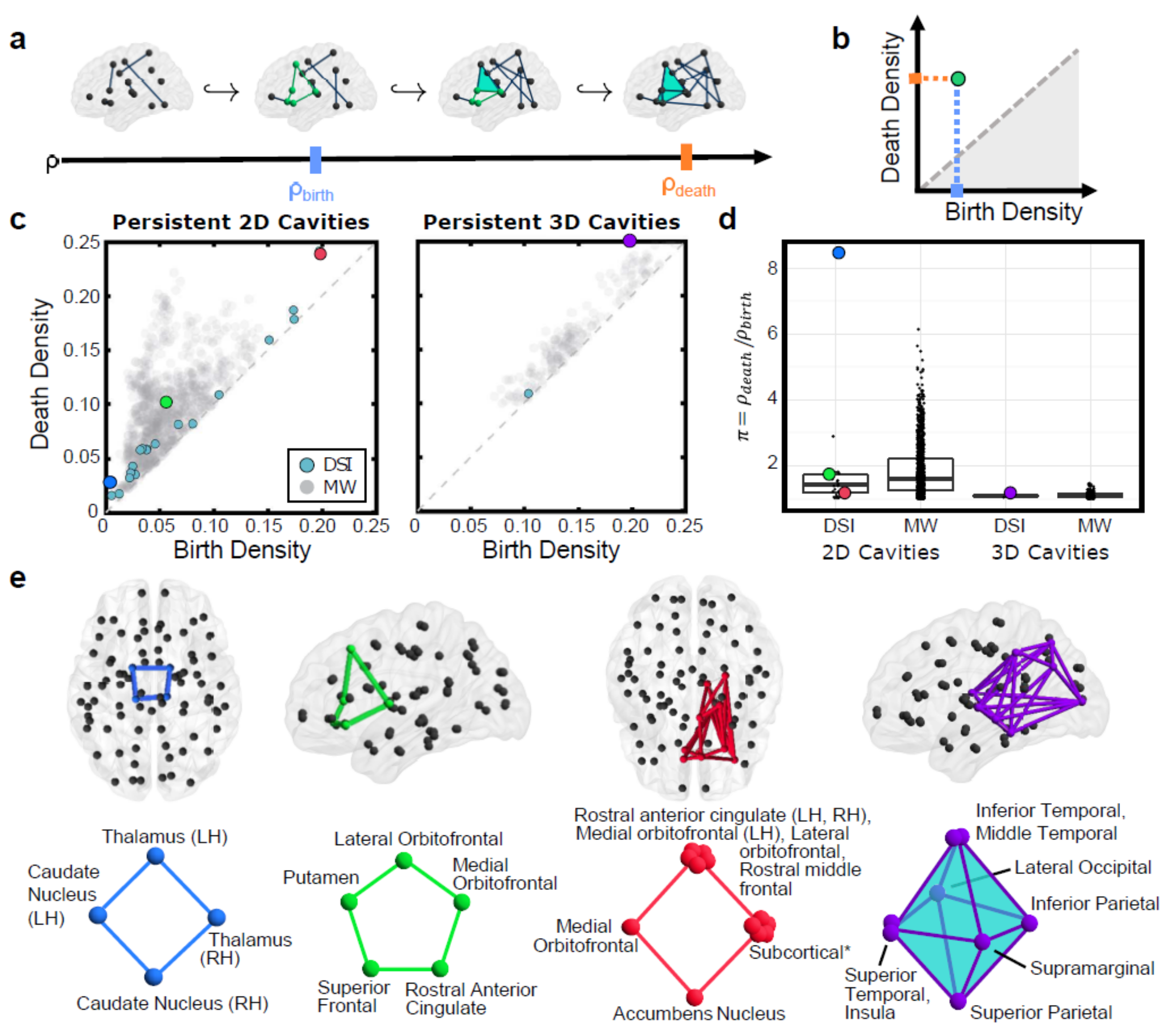
Do homological mesoscales characterise brain networks?

LANGUAGE

PHOM

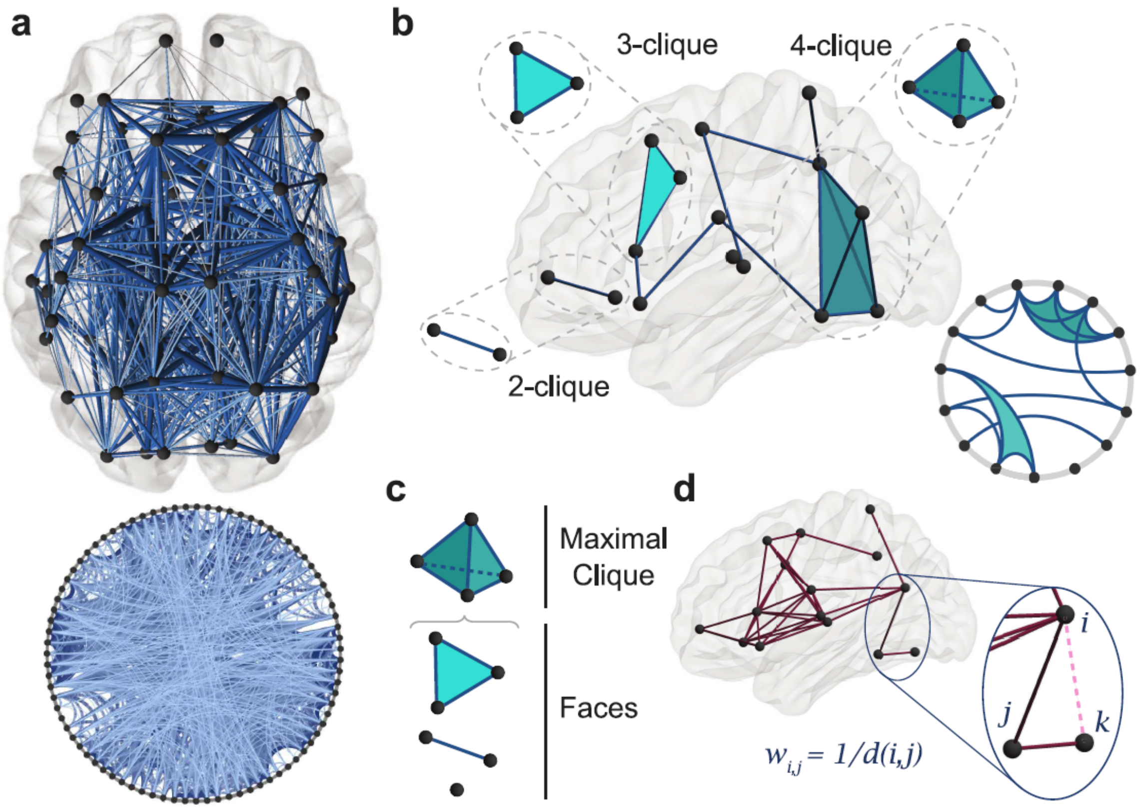
Do homological mesoscales characterise brain networks?

LANGUAGE



Structural brain cavities

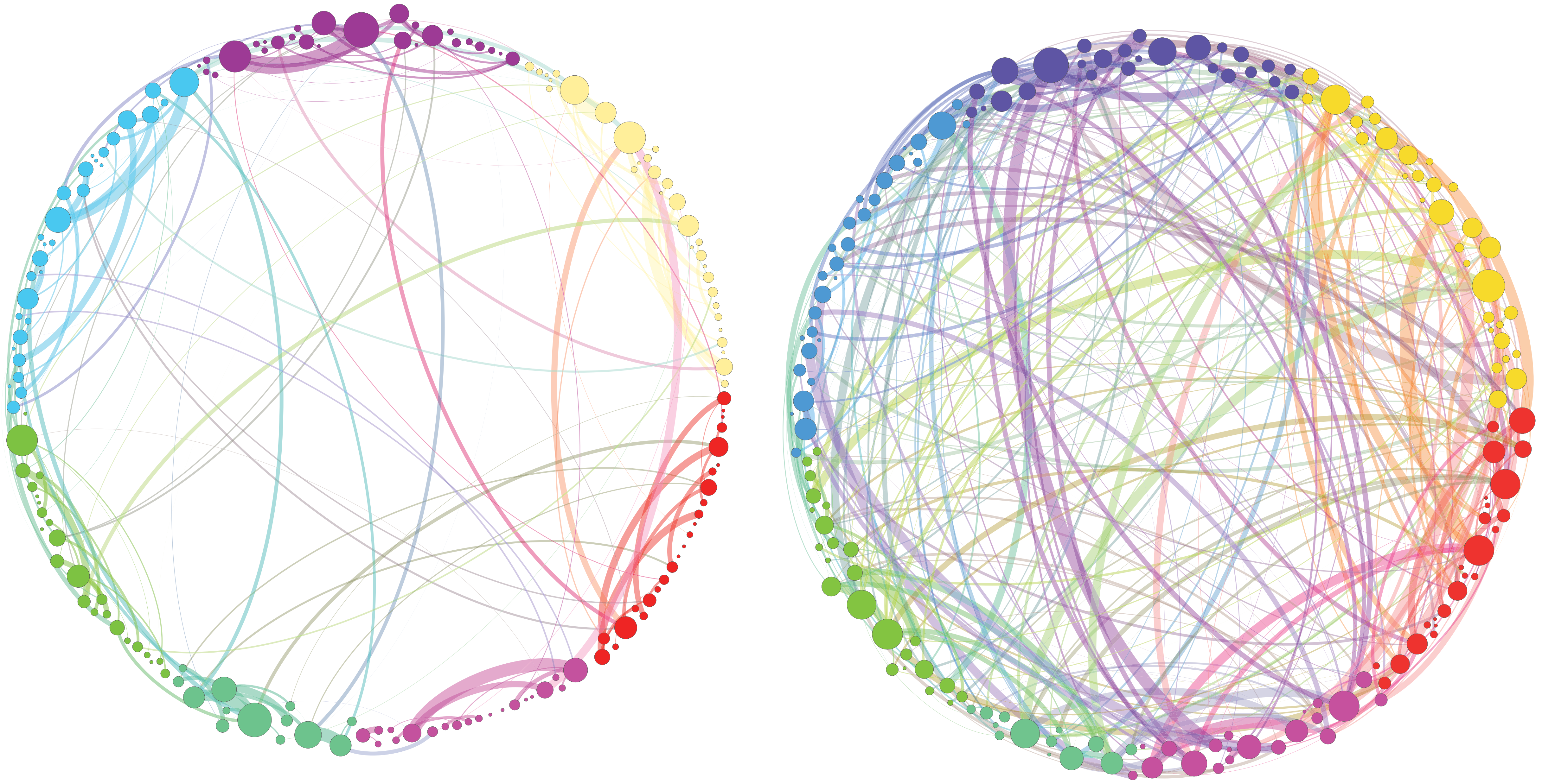
Sizemore, Ann, et al. arXiv:1608.03520 (2016).



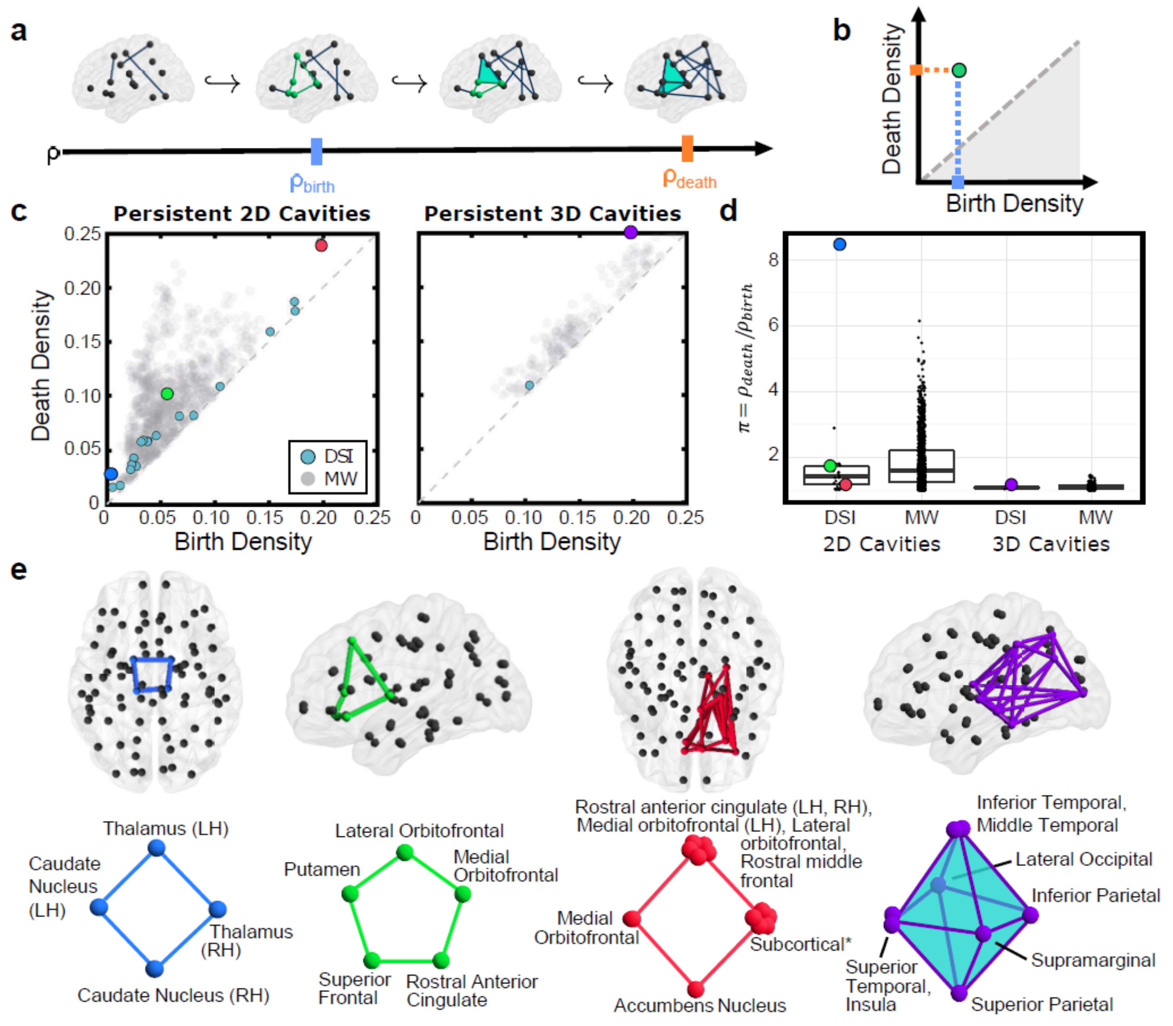
PHOM

Do homological mesoscales characterise brain networks?

Graph-representations of 1-dimensional hole structure



Petri, Giovanni, et al. "Homological scaffolds of brain functional networks." *Journal of The Royal Society Interface* 11.101 (2014): 20140873.



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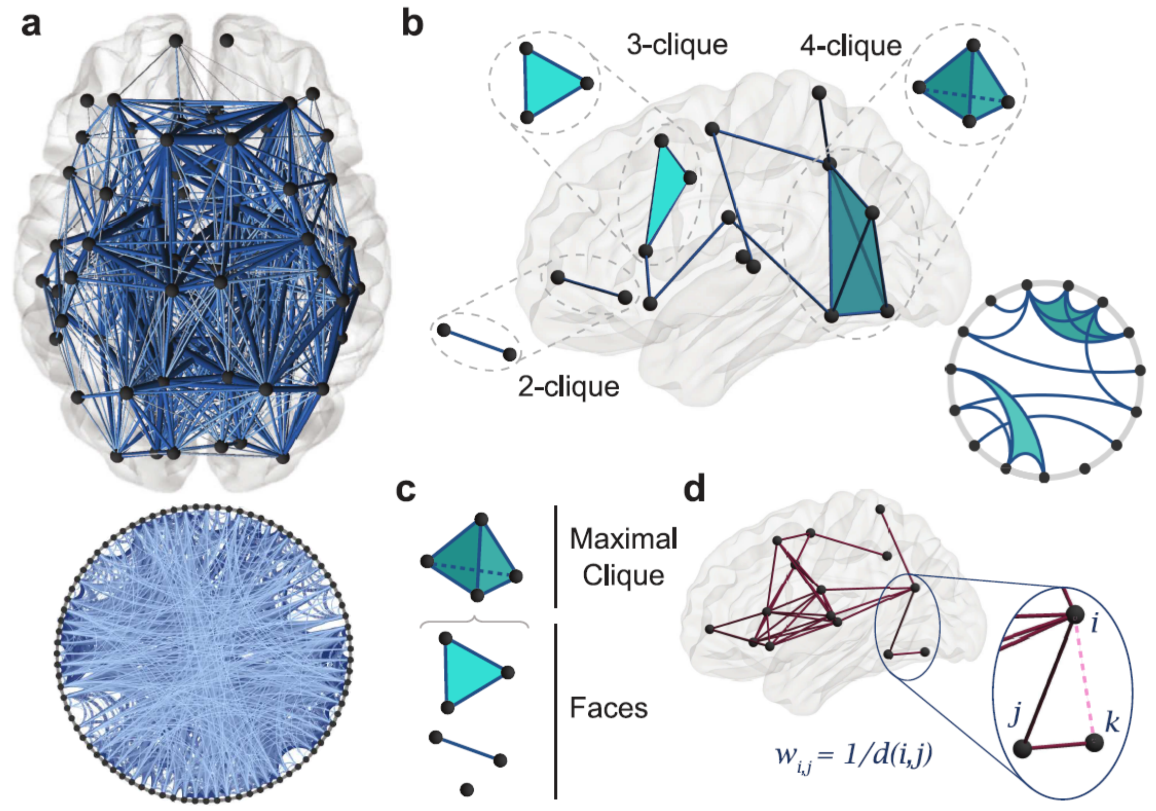

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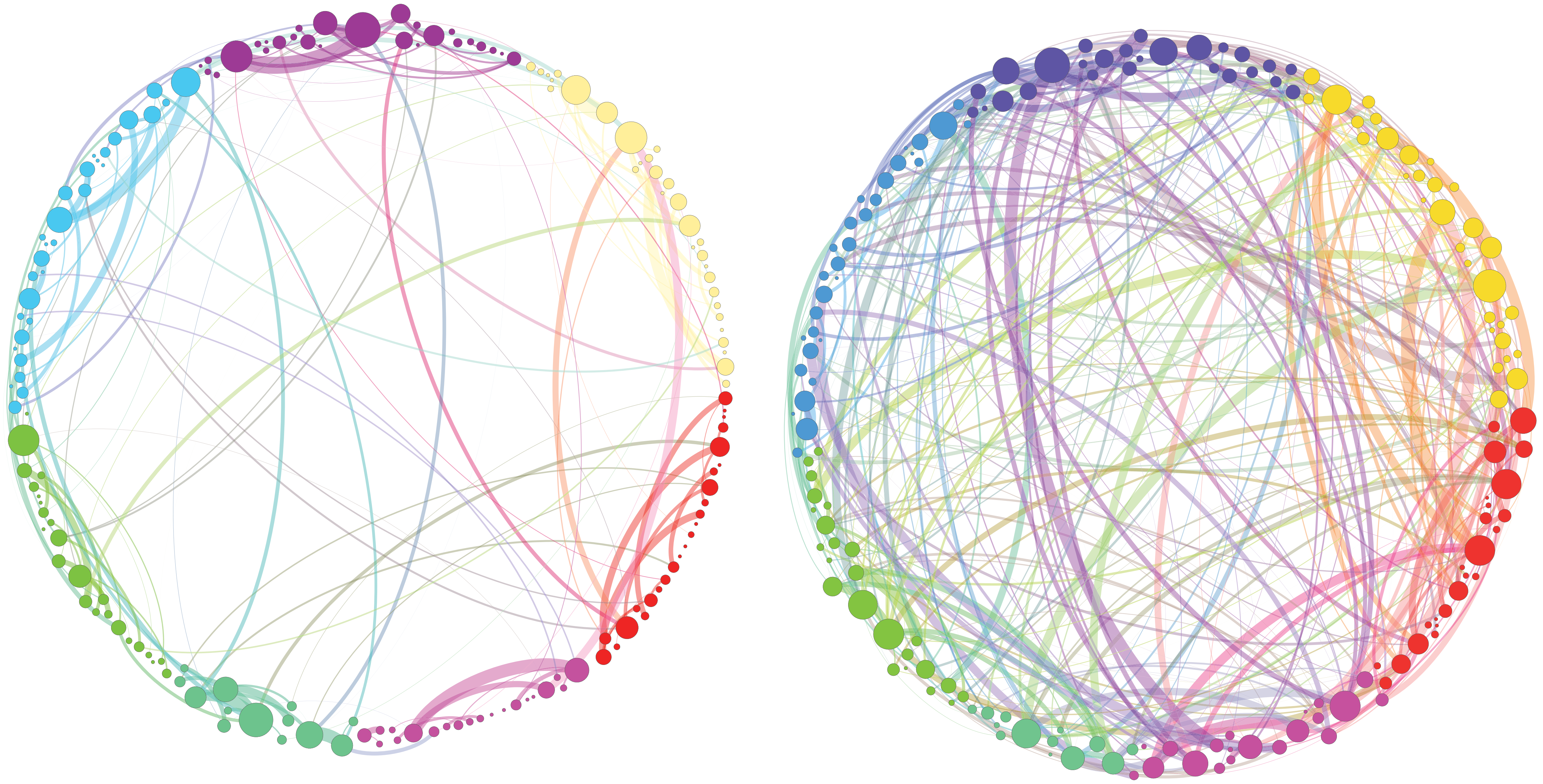

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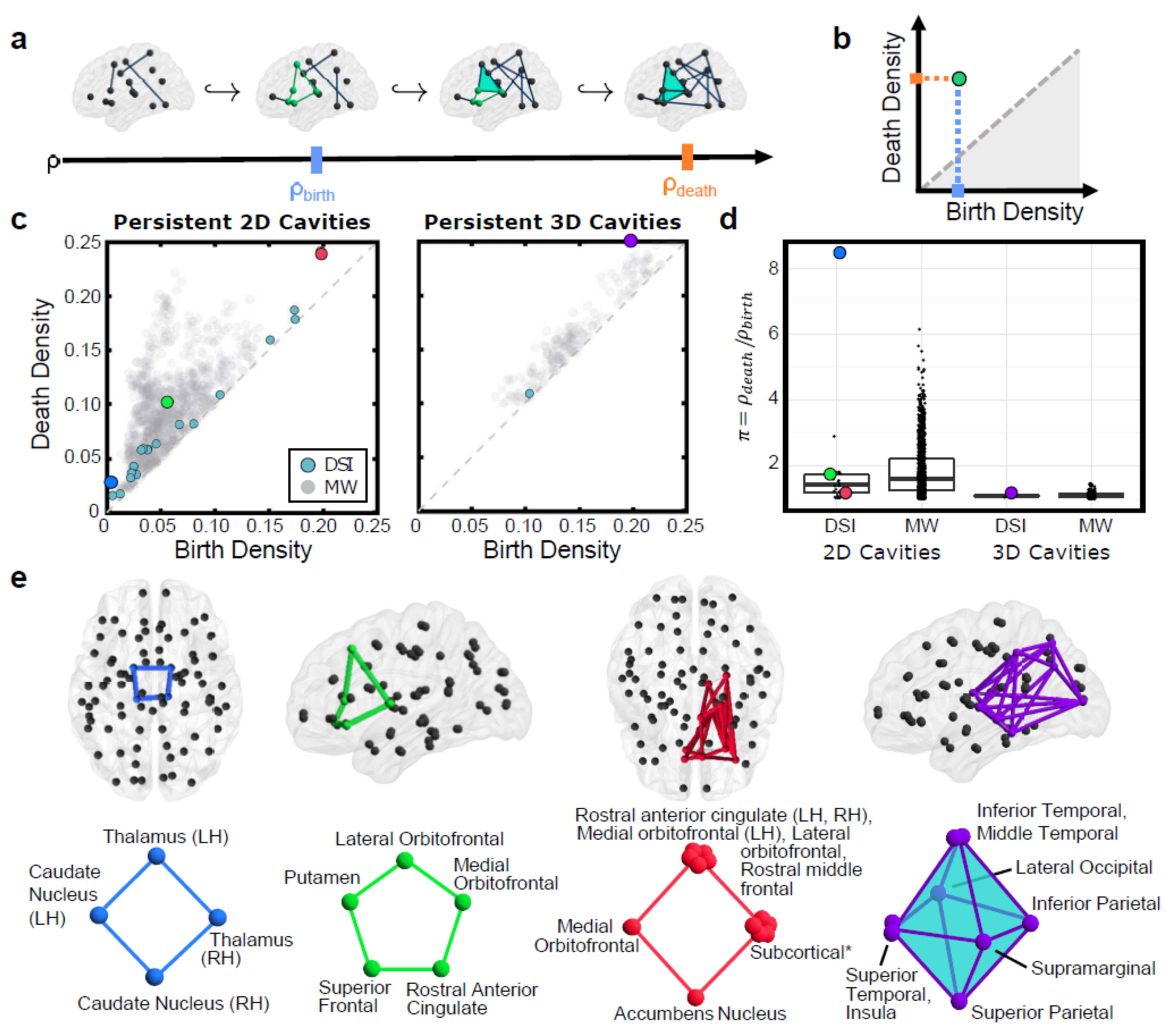

Structural brain cavities

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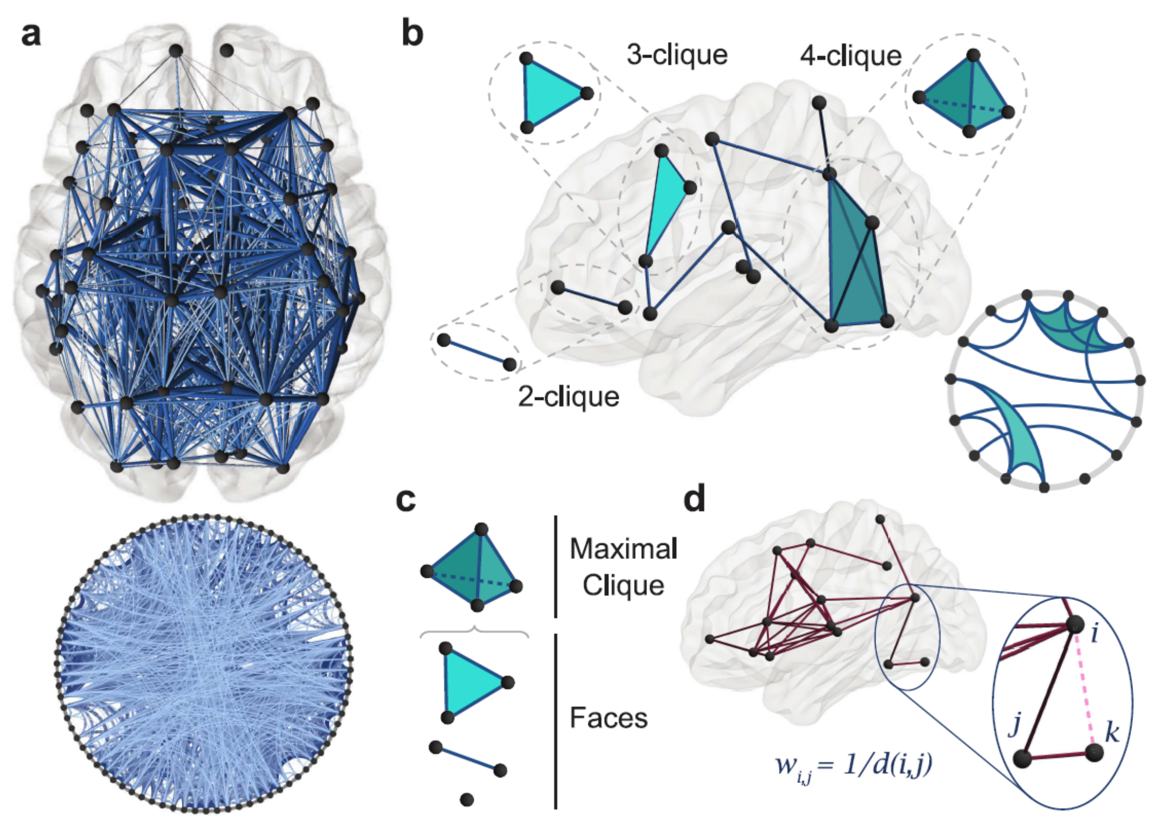
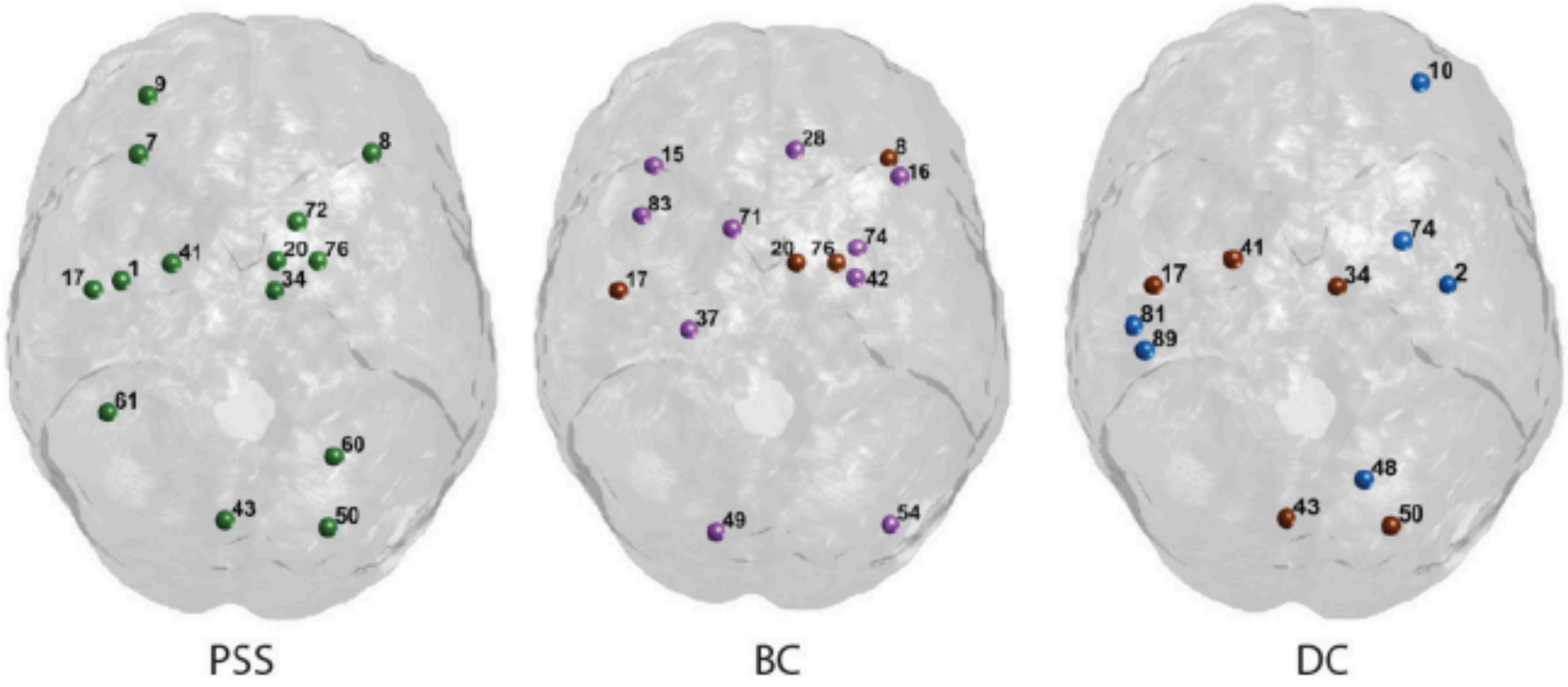


Homological persistence centrality



Lord, Louis-David, et al. "Insights into brain architectures from the homological scaffolds of functional connectivity networks." *Frontiers in systems neuroscience* 10 (2016).

PHOM

Notebook 07

Do homological shapes capture transitions?

LANGUAGE

Ibanez-Marcelo, Esther, et al. "Topology highlights mesoscopic functional equivalence between imagery and perception." *bioRxiv* (2018): 268383.

PHOM

Do homological shapes capture transitions?

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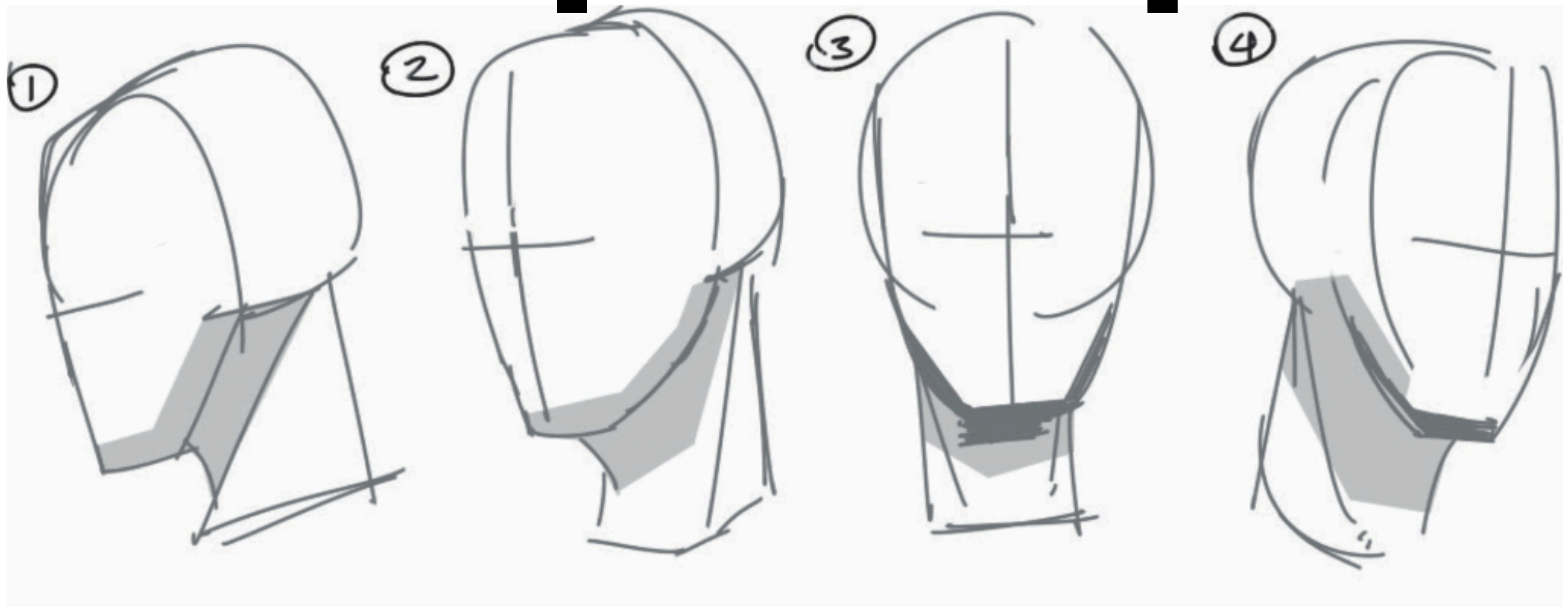

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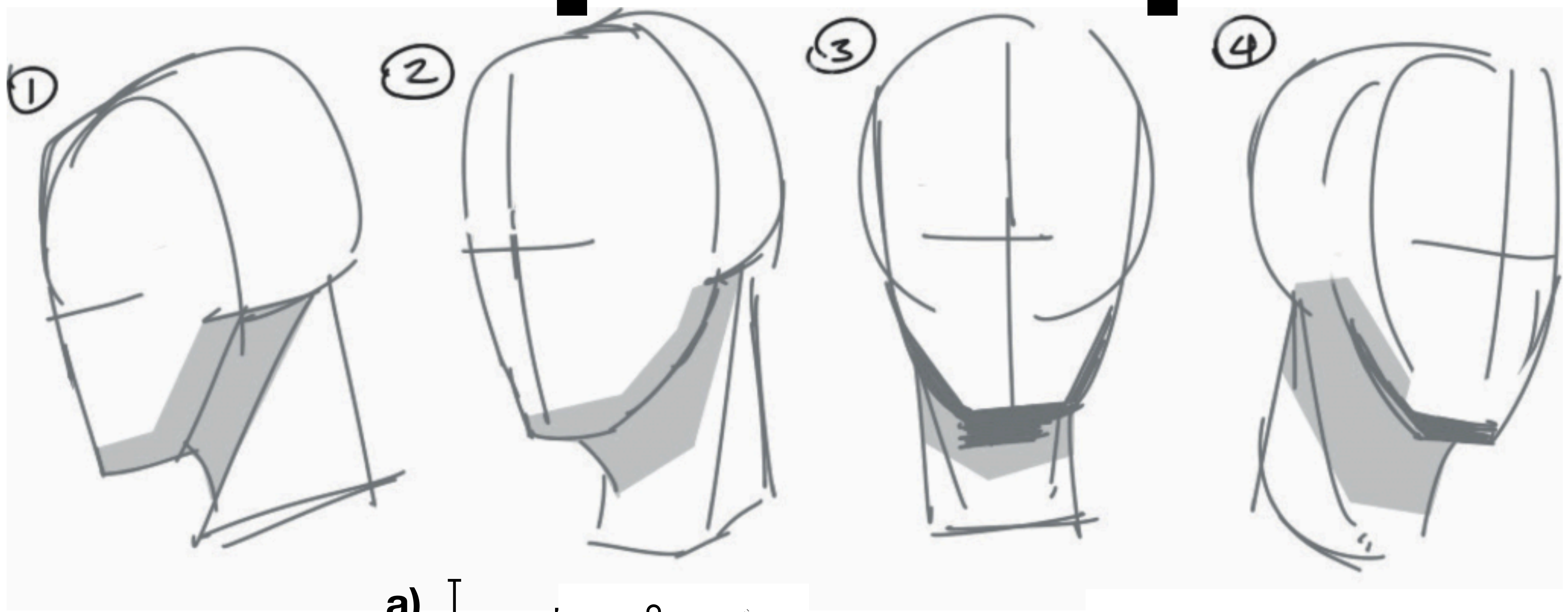
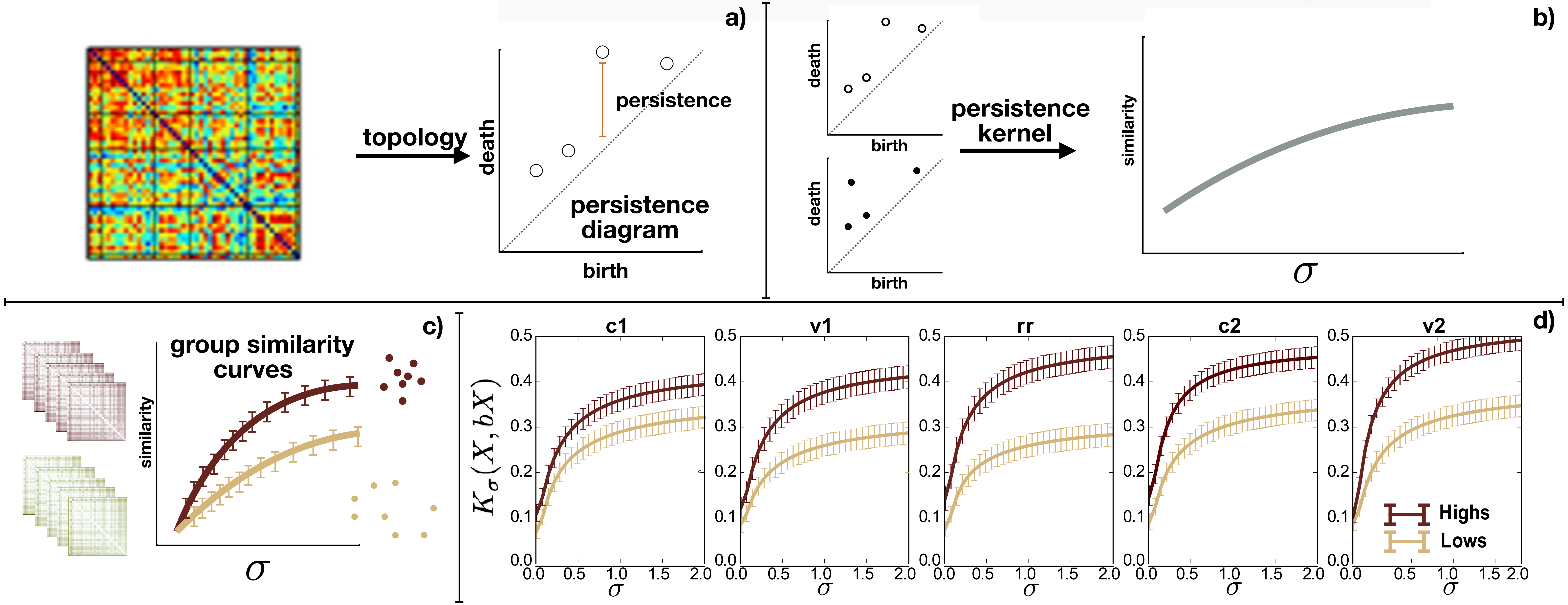


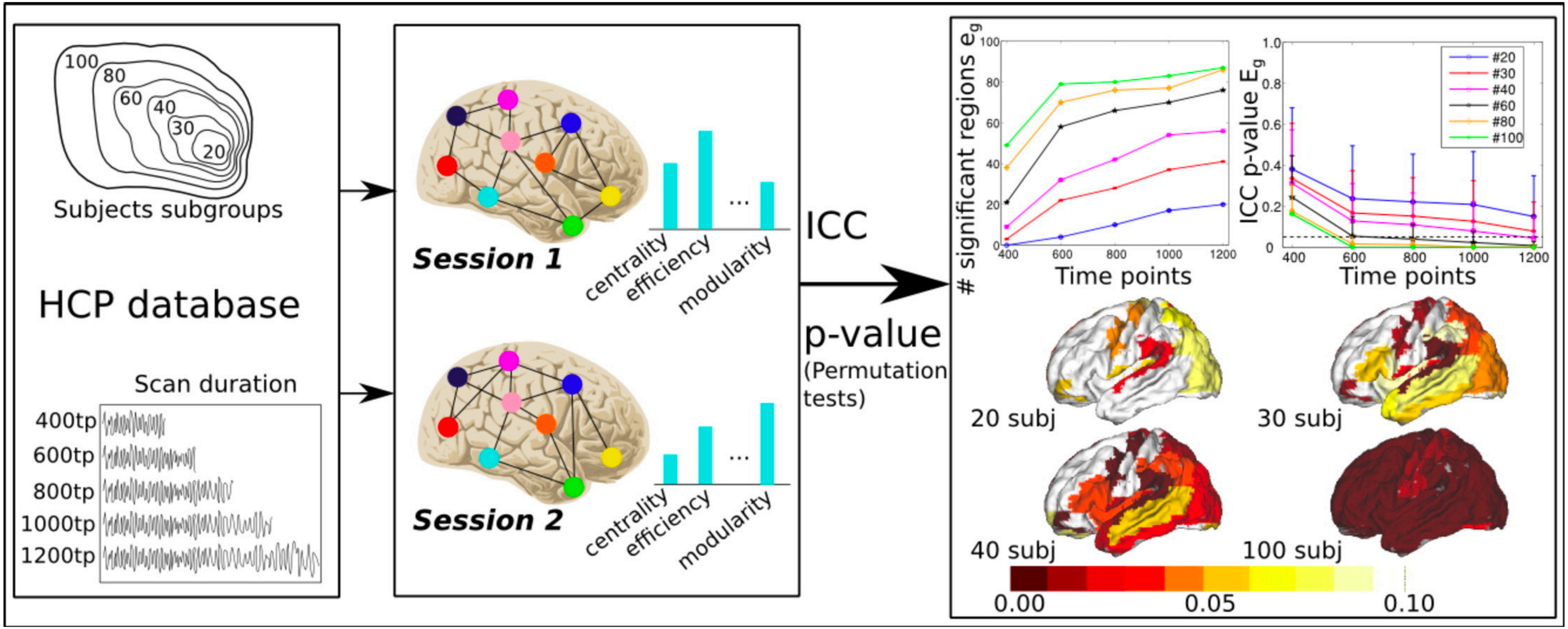


Ibanez-Marcelo, Esther, et al. "Topology highlights mesoscopic functional equivalence between imagery and perception." *bioRxiv* (2018): 268383.

PHOM

Dataset

100 subjects, 2 sessions, long recordings (~1000 TRs)



Termenon, Maïté, et al. "Reliability of graph analysis of resting state fMRI using test-retest dataset from the Human Connectome Project." *NeuroImage* 142 (2016): 172-187.

Can we fingerprint with it?

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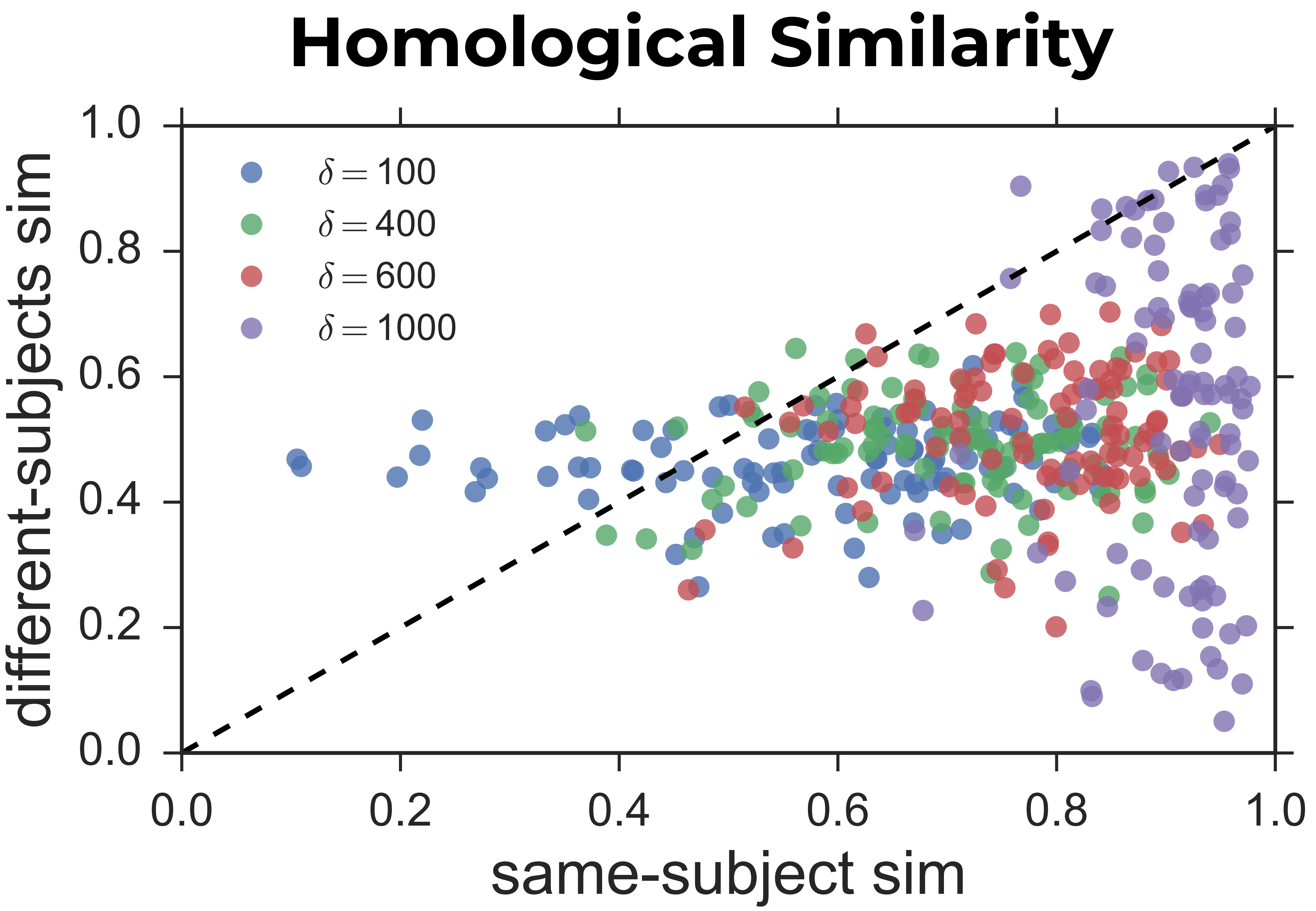

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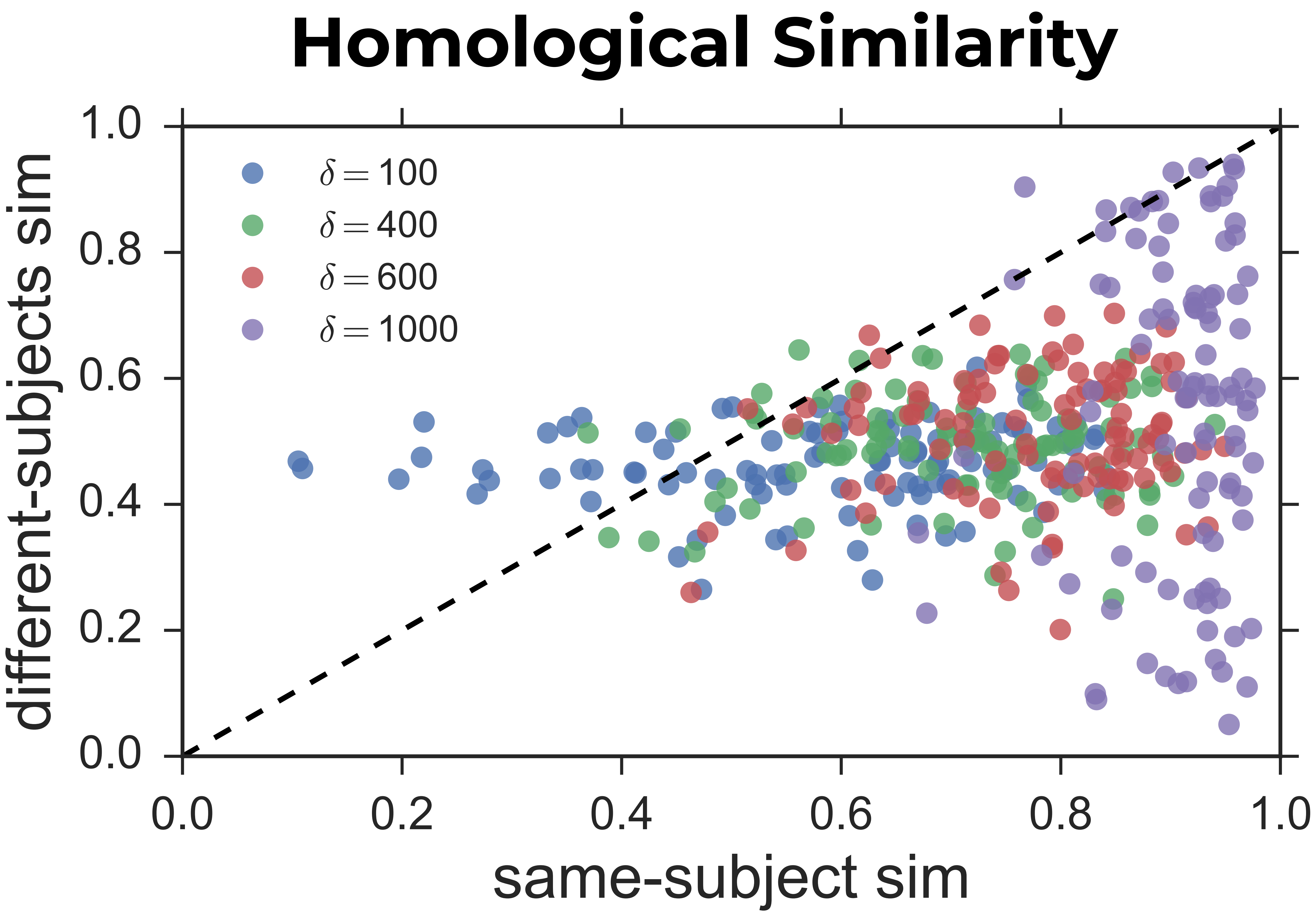
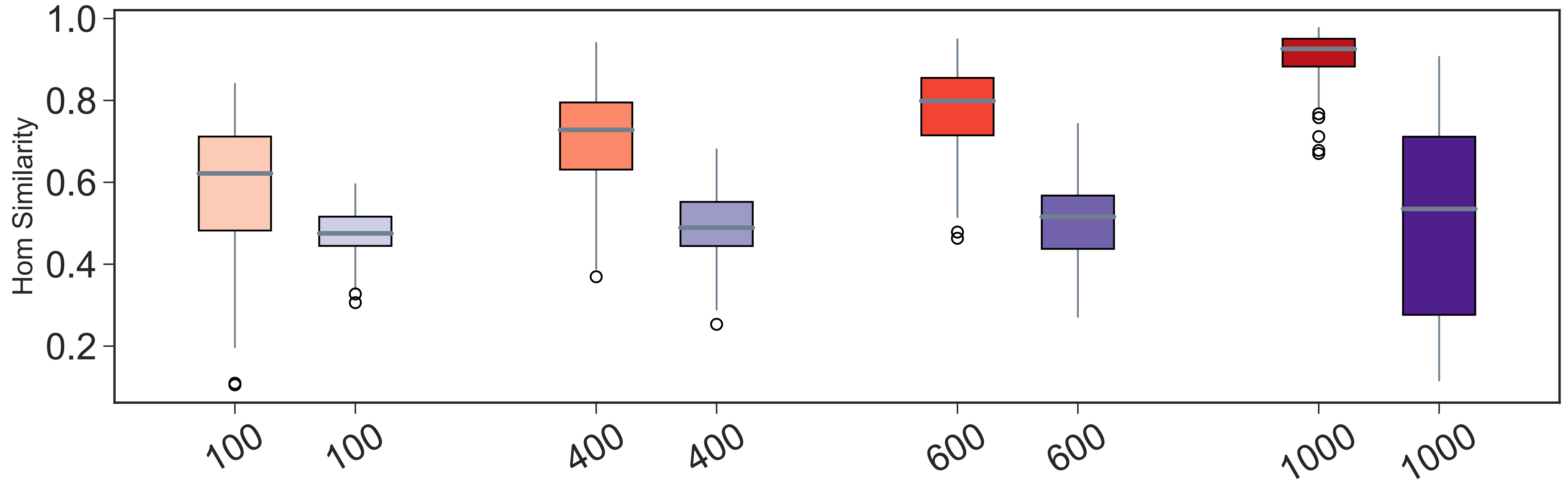
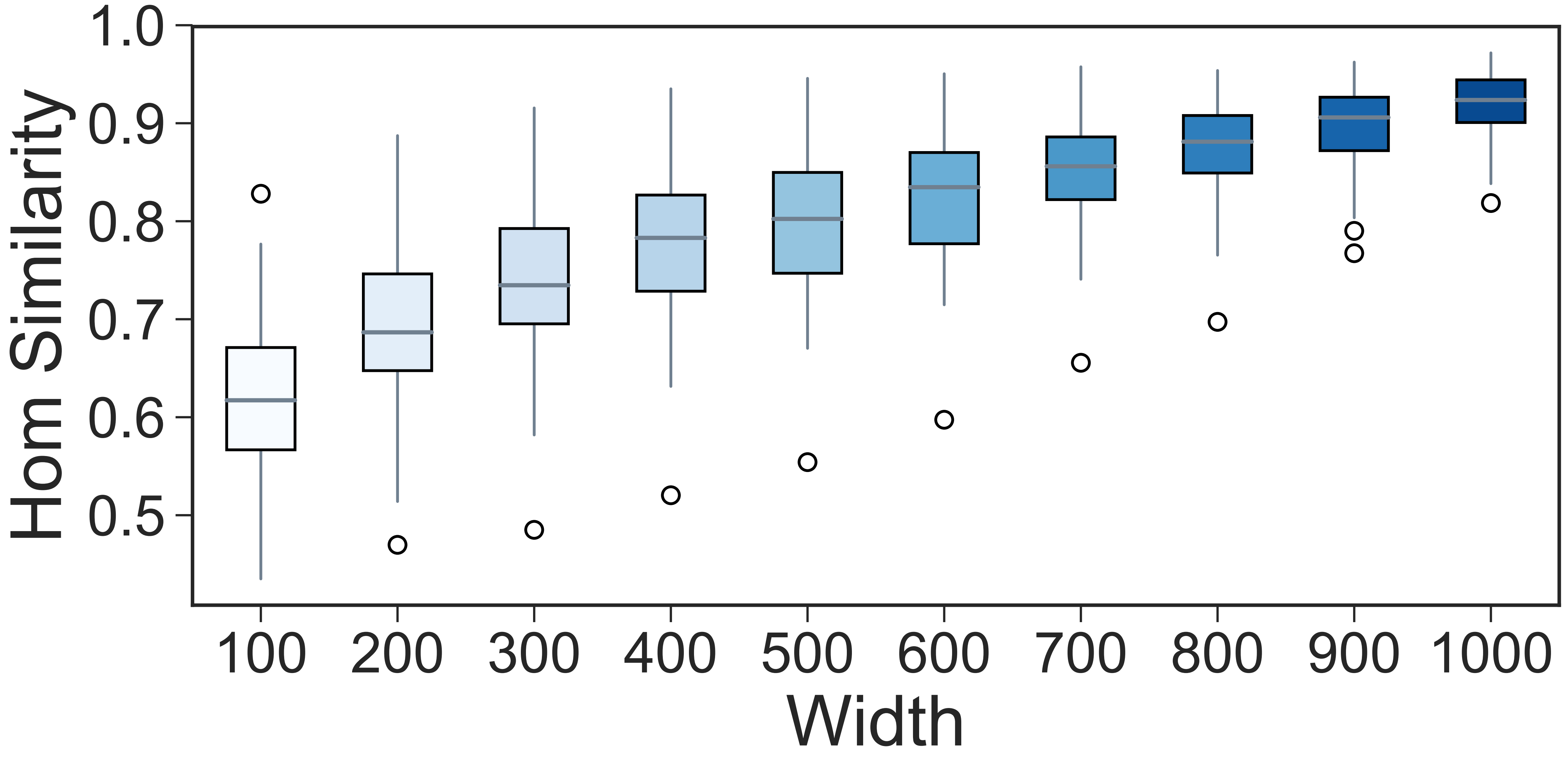
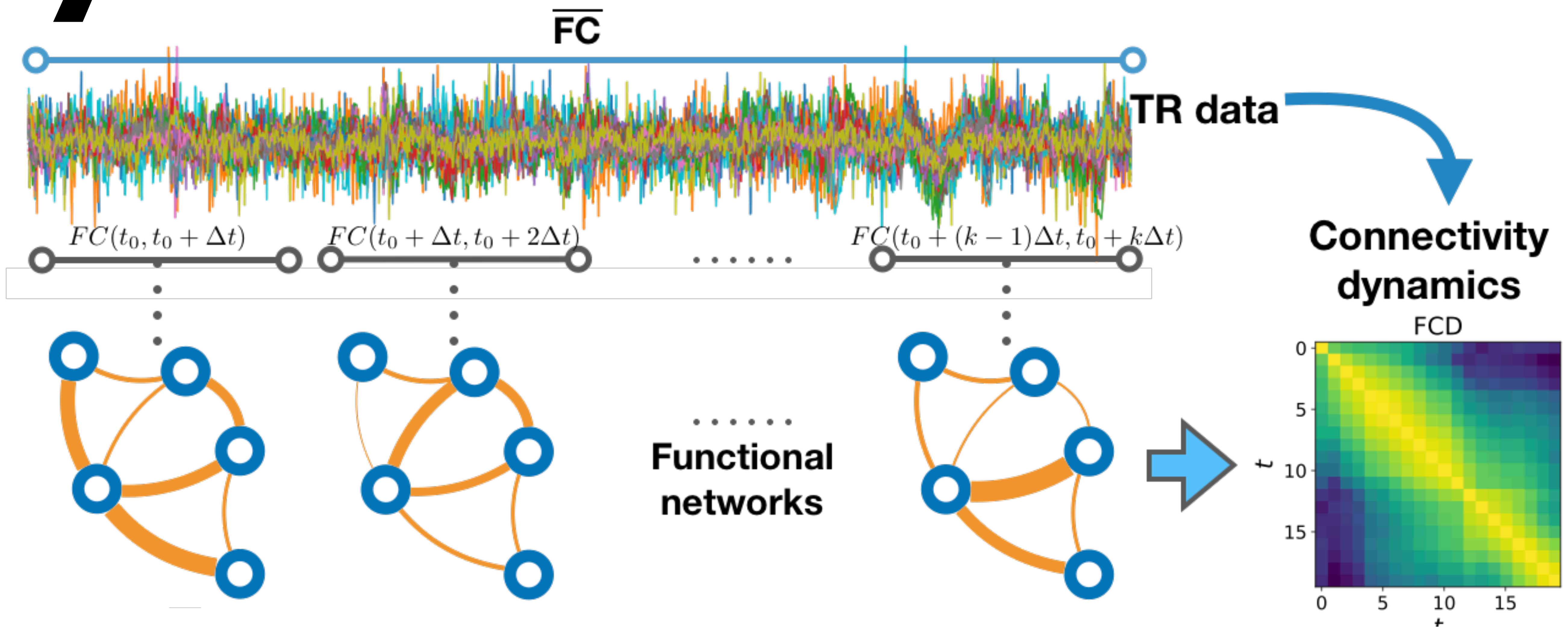






What about homological FC dynamics?



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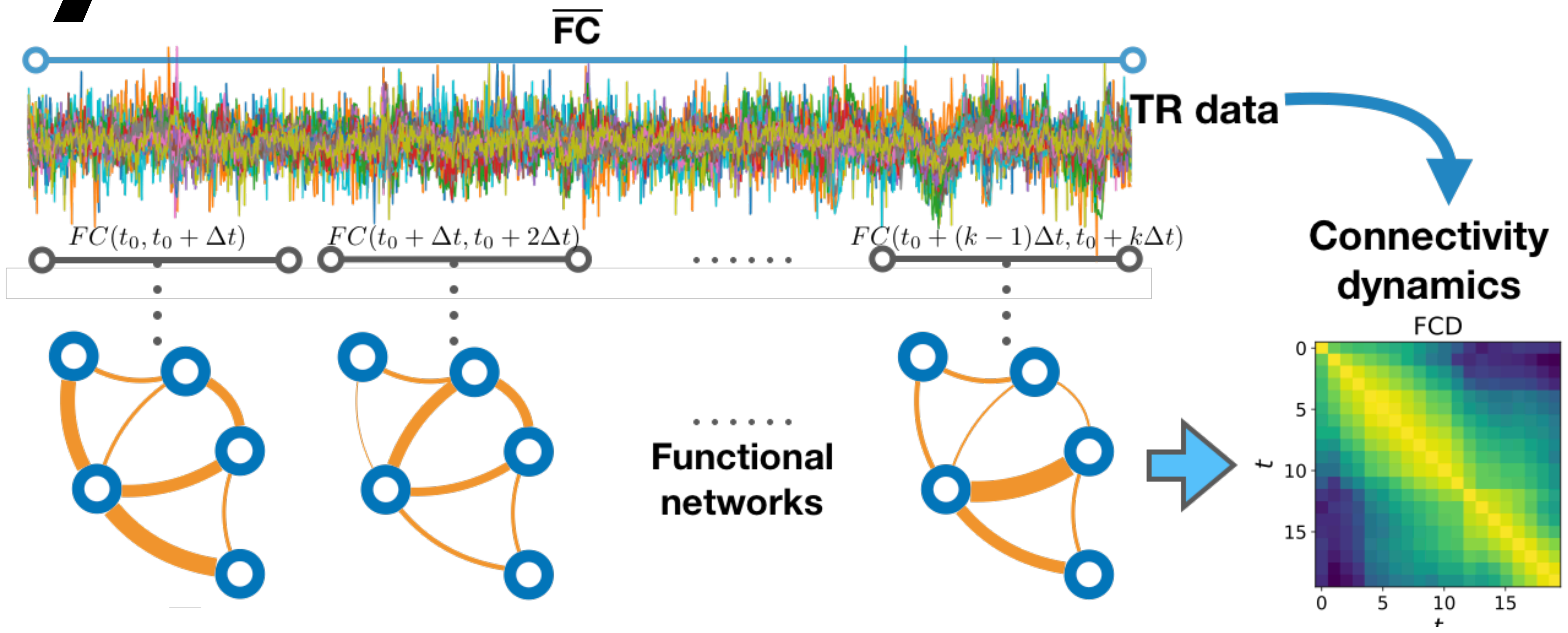
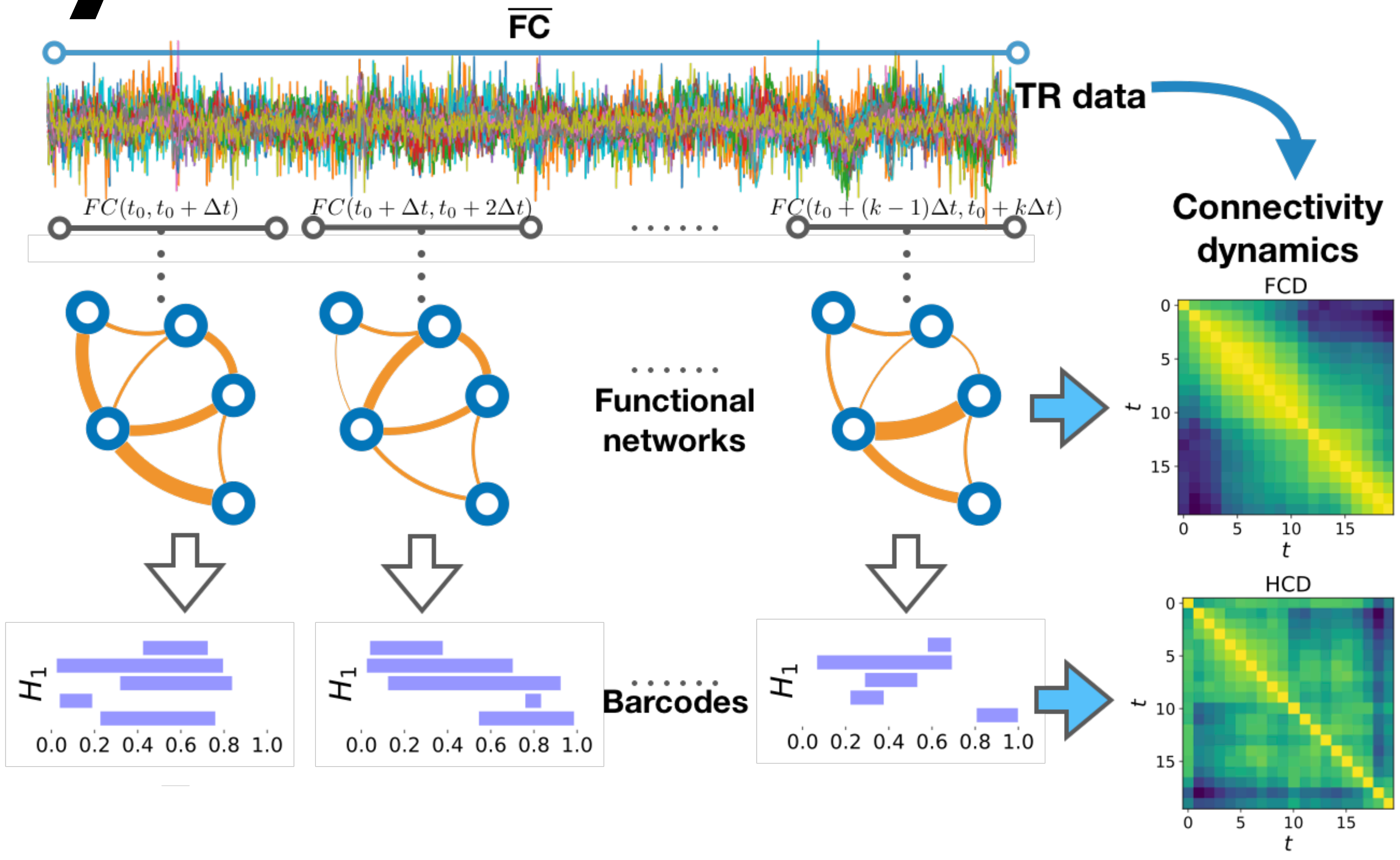


Functional connective dynamics

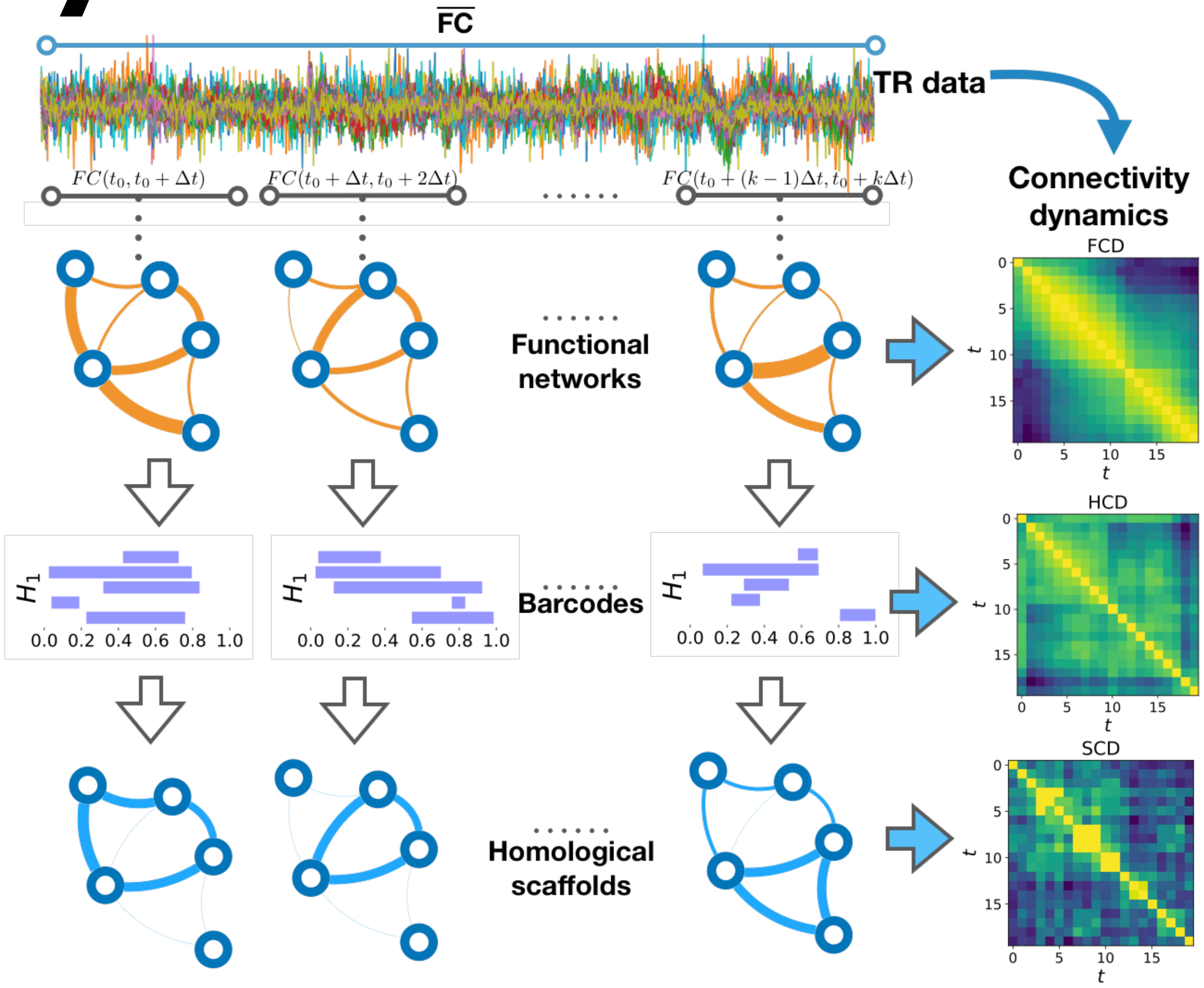
What about homological FC dynamics?



Functional connective dynamics

Homological connective dynamics

What about homological FC dynamics?



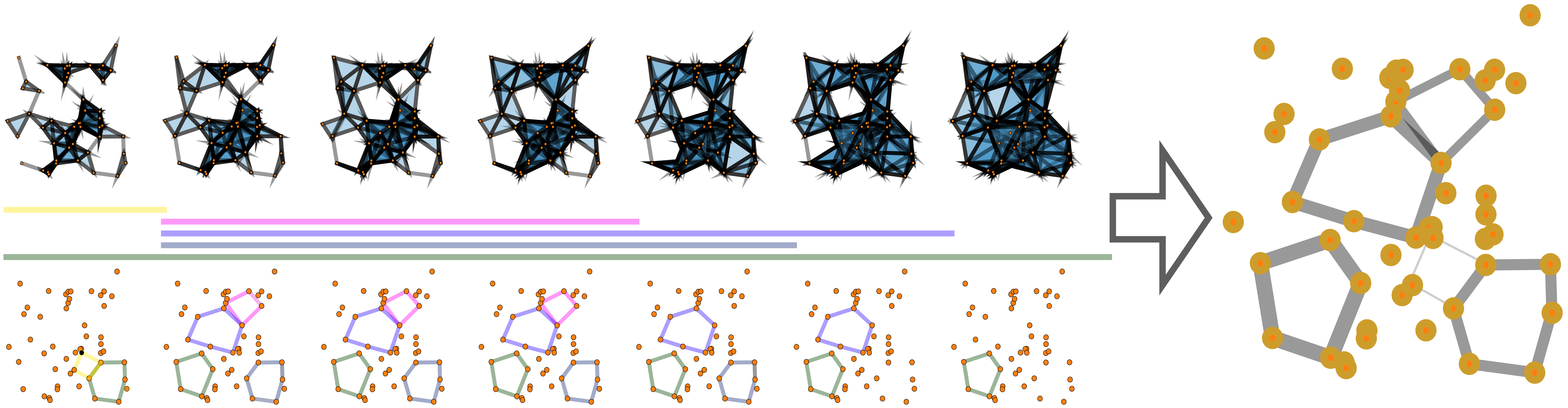
Functional connective dynamics

Homological connective dynamics

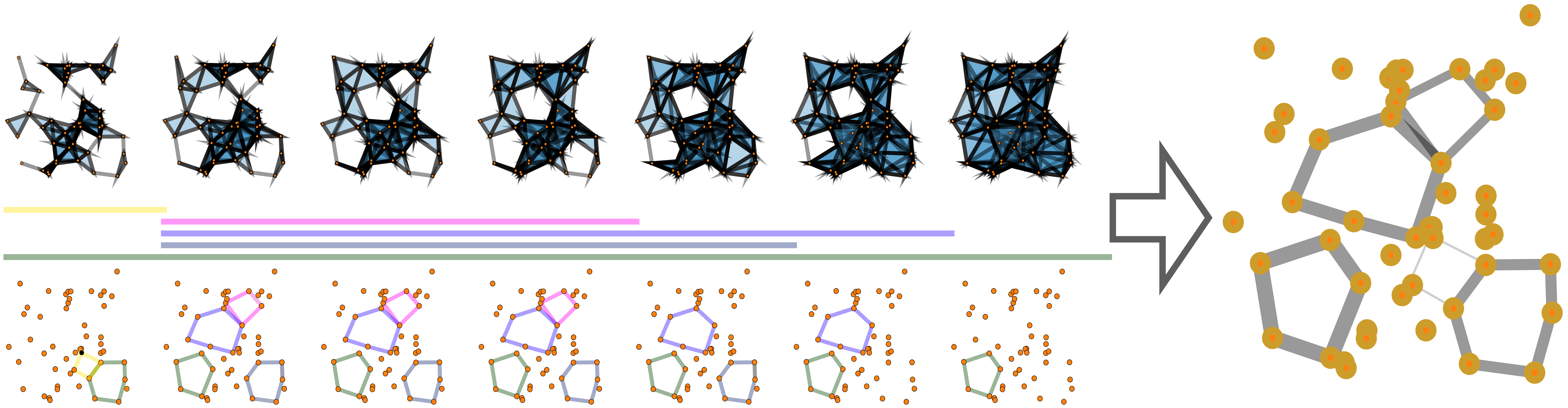
What the...?

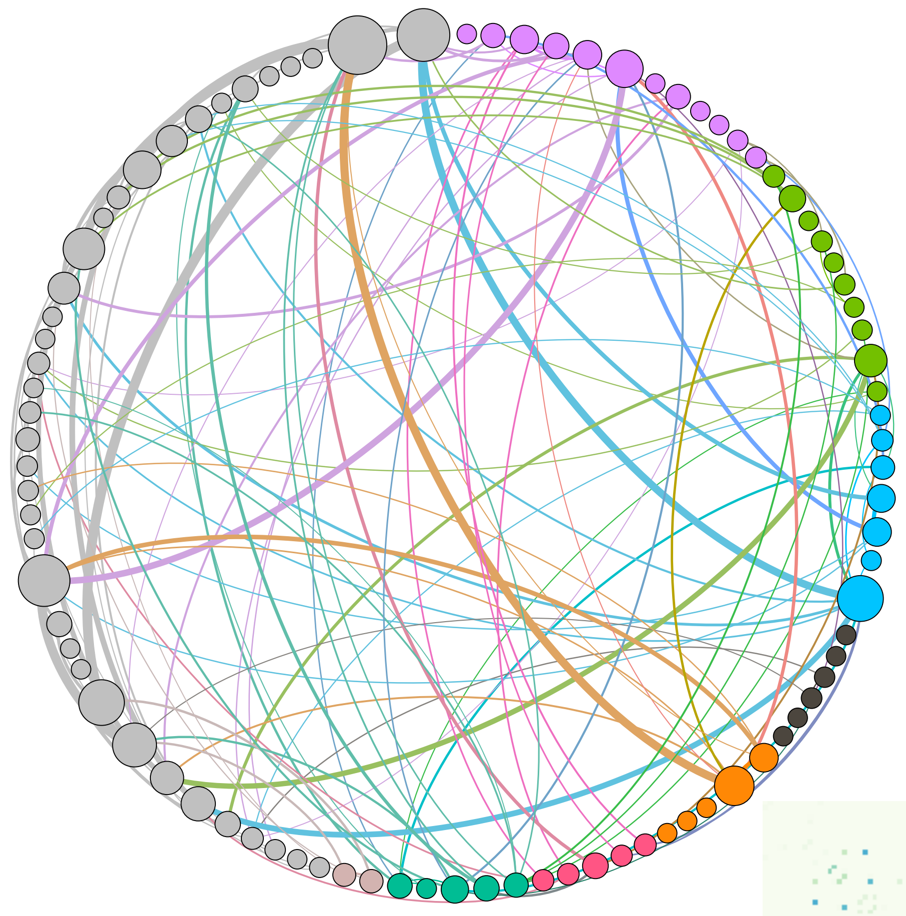
Scaffolds in one slide

Scaffolds in one slide

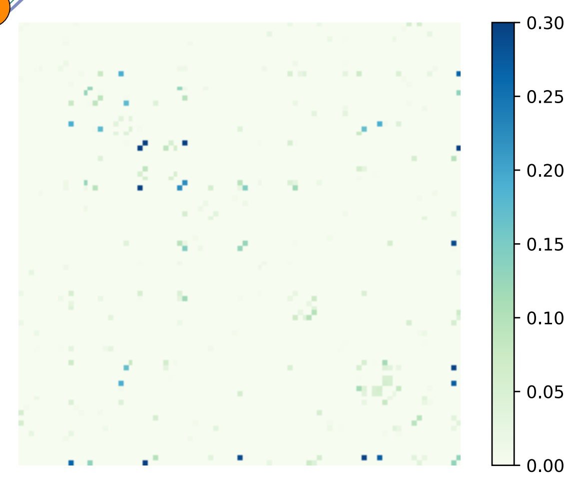


Scaffolds in one slide

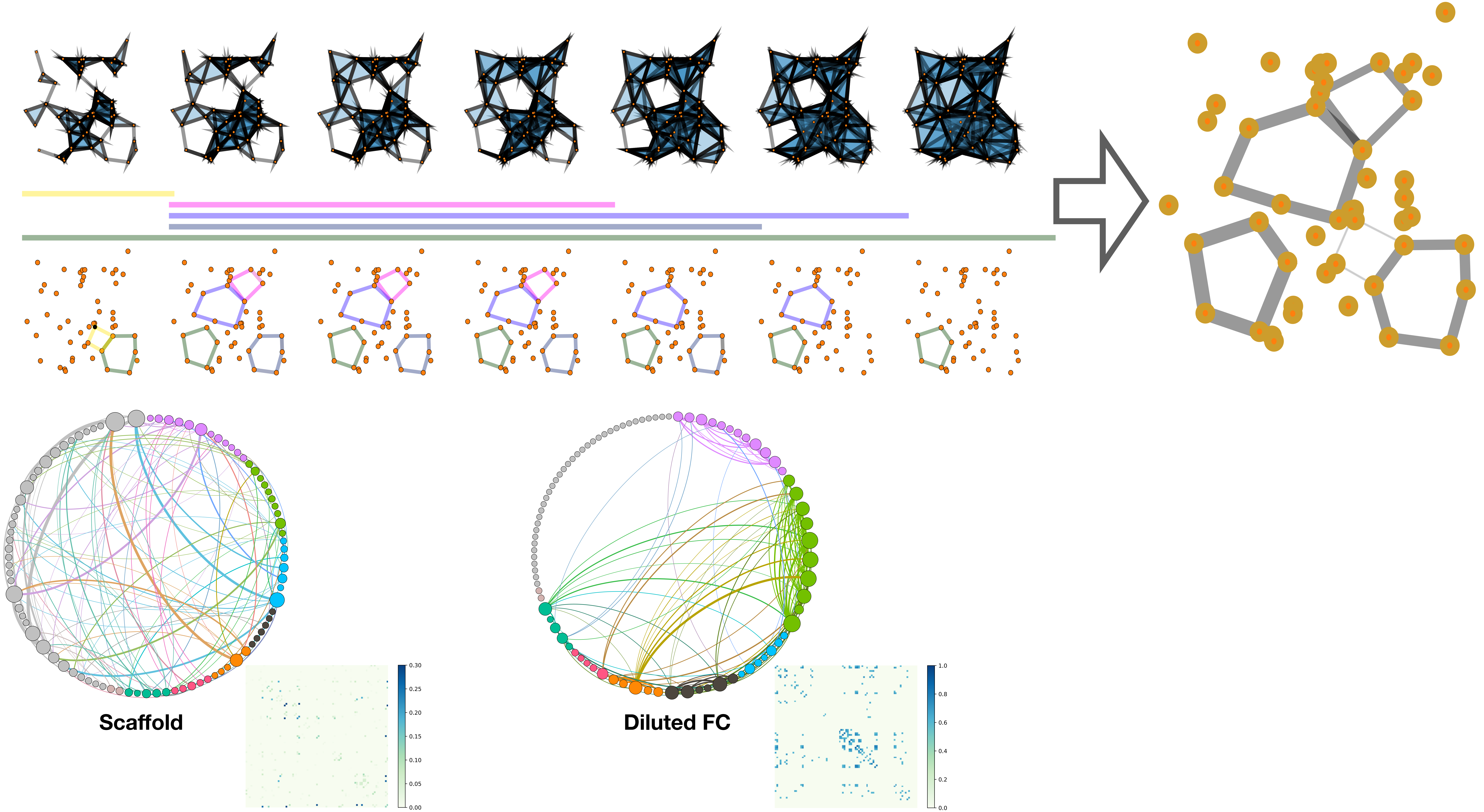




Scaffold



Scaffolds in one slide



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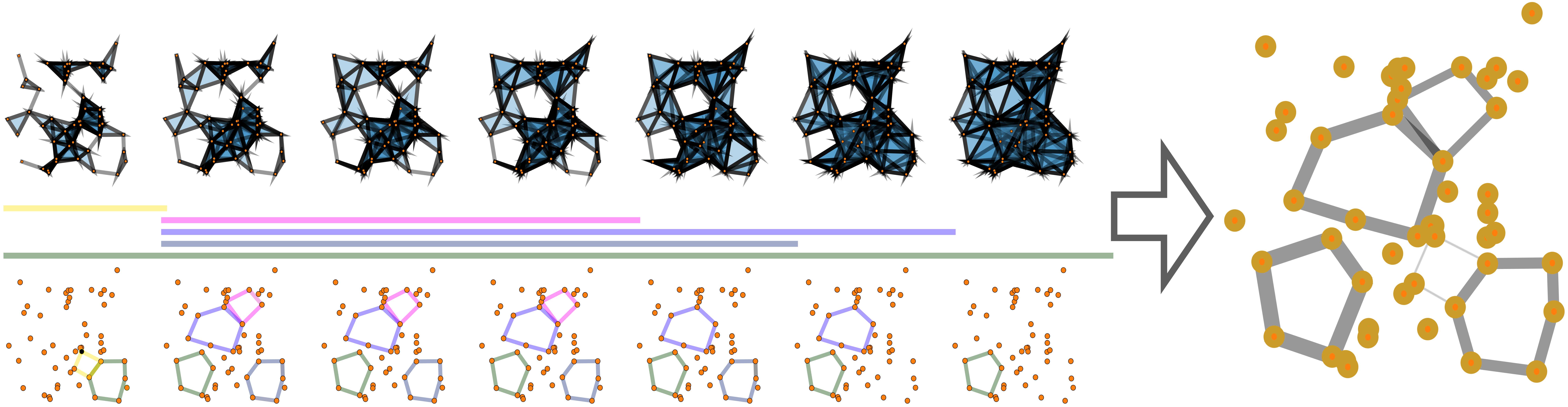

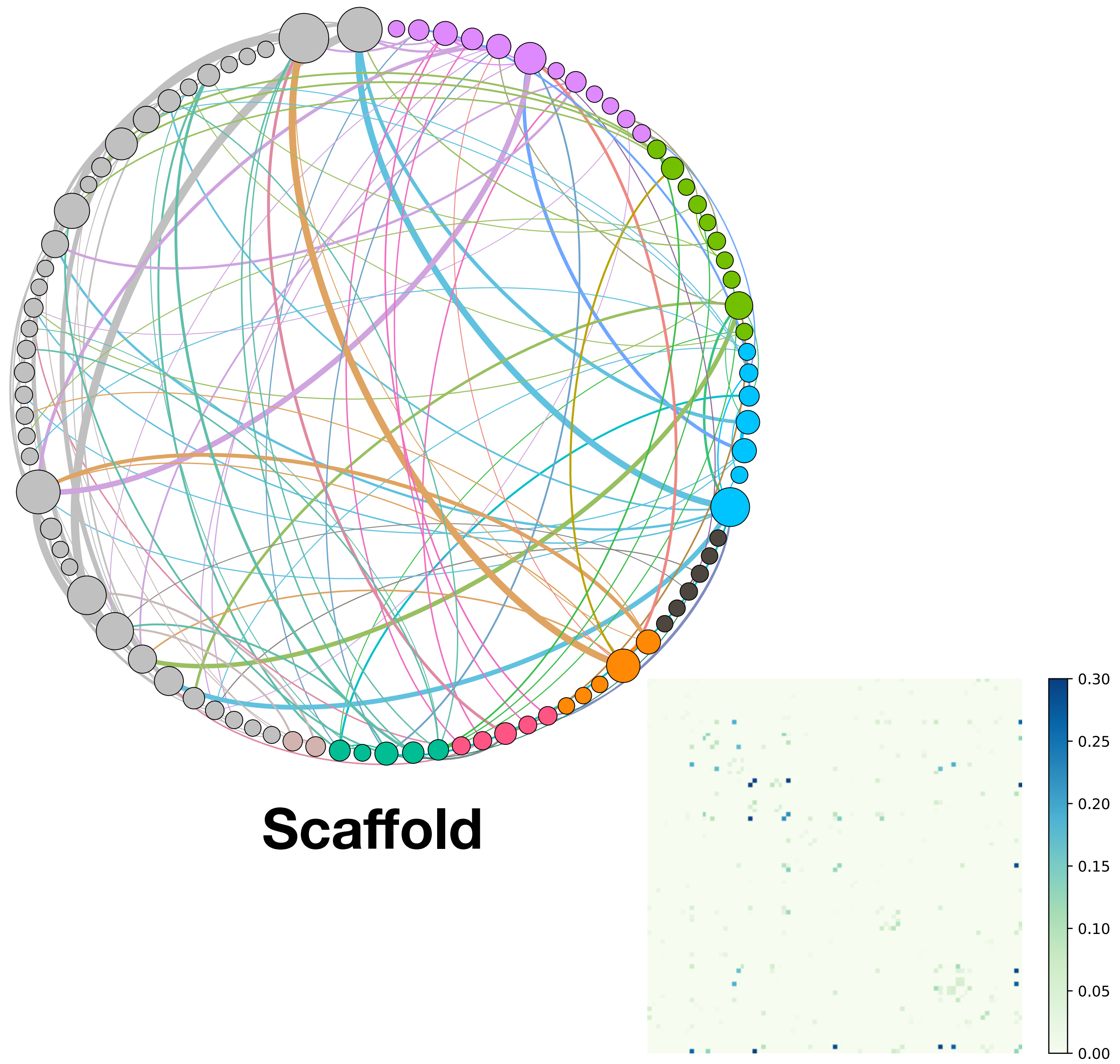

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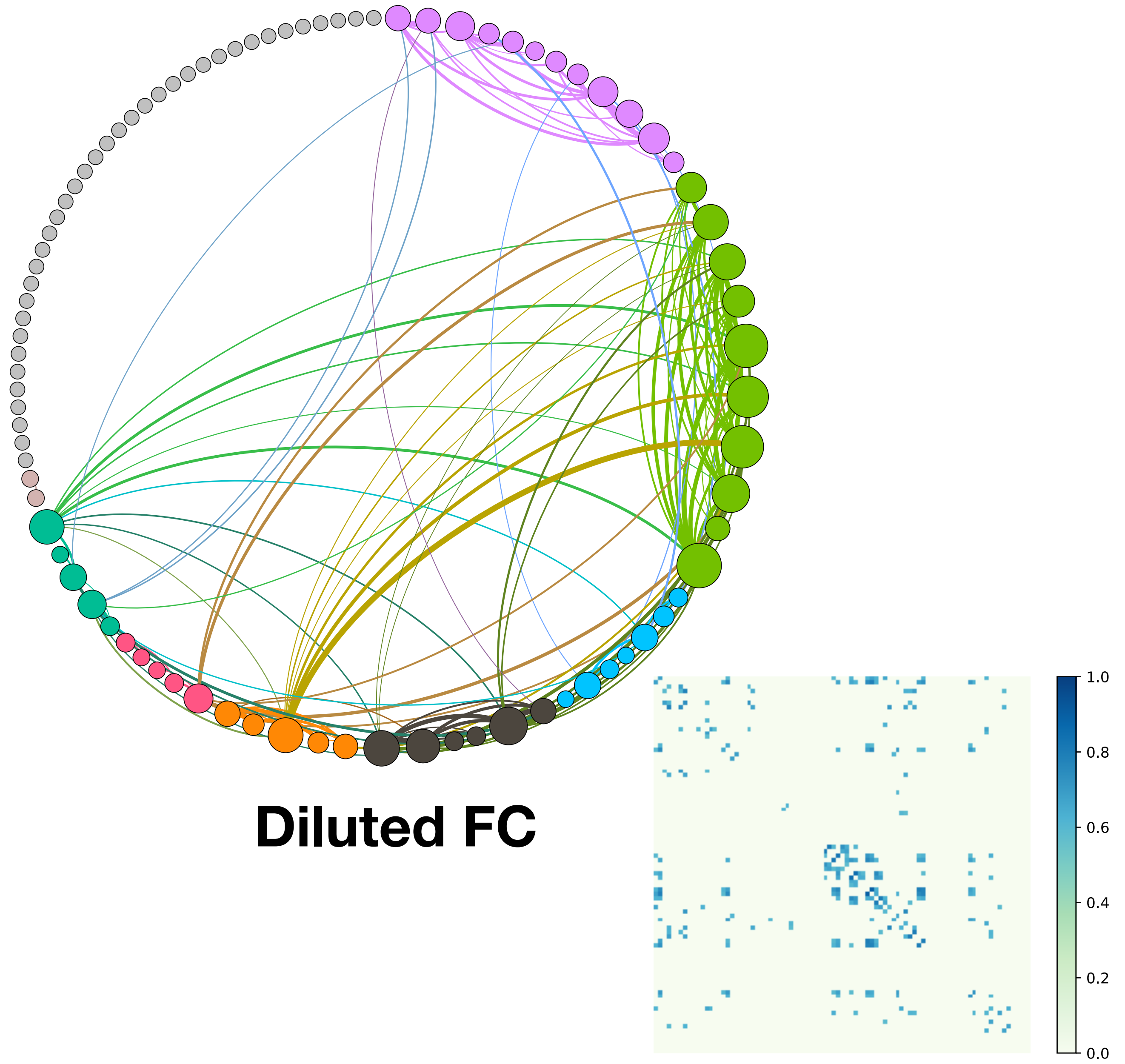
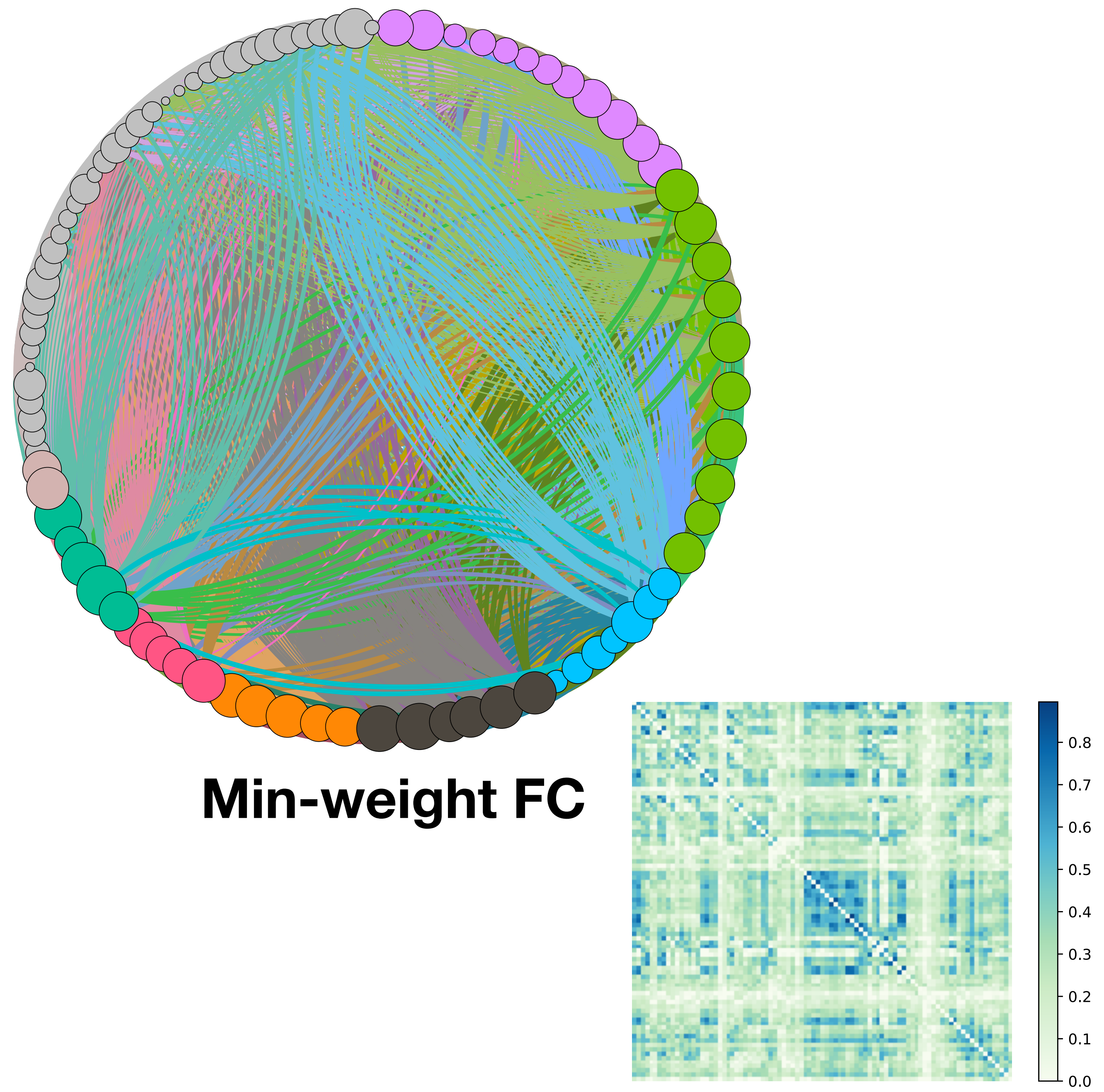
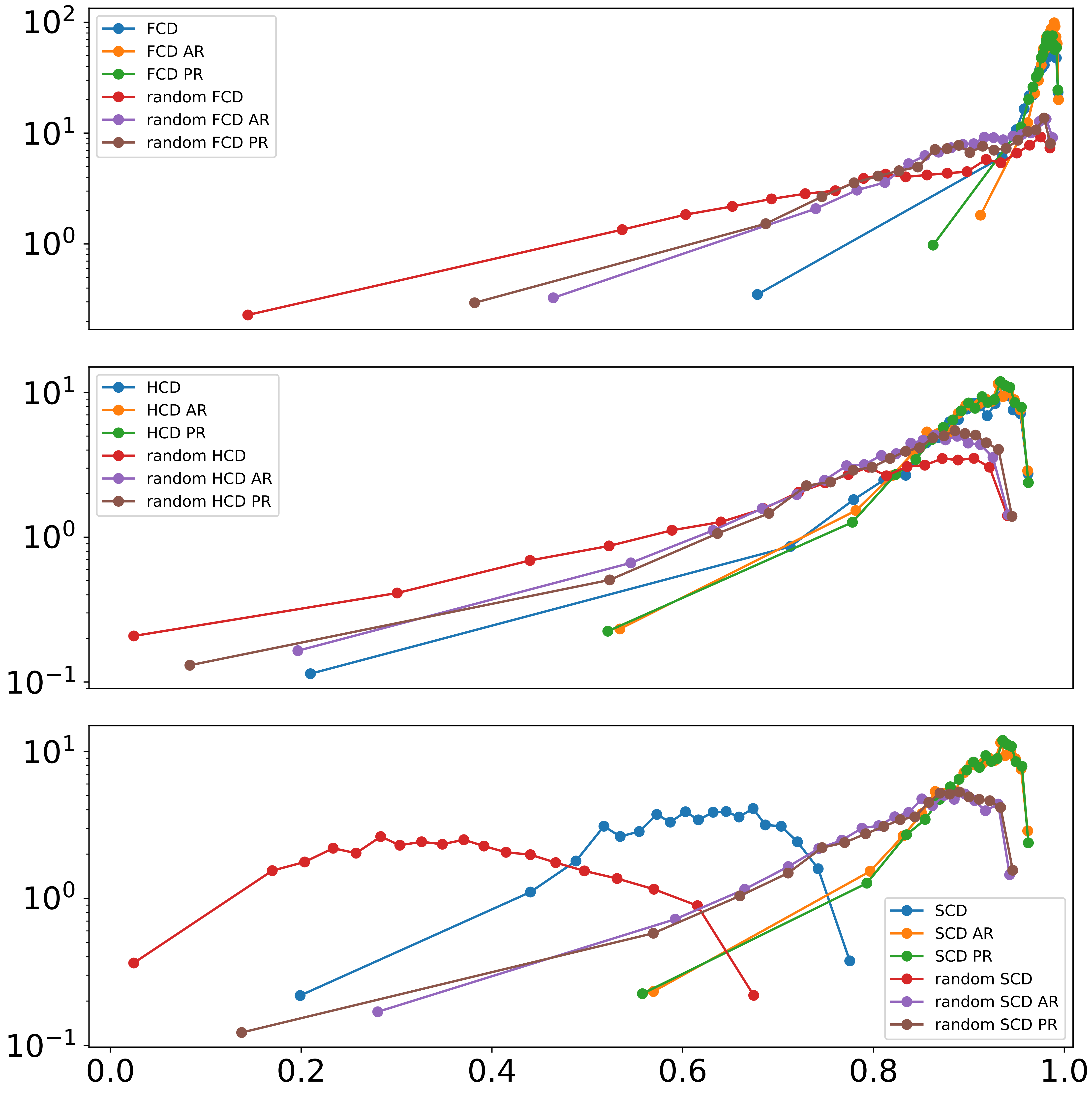


Diluted FC



Min-weight FC

What about homological FC dynamics?



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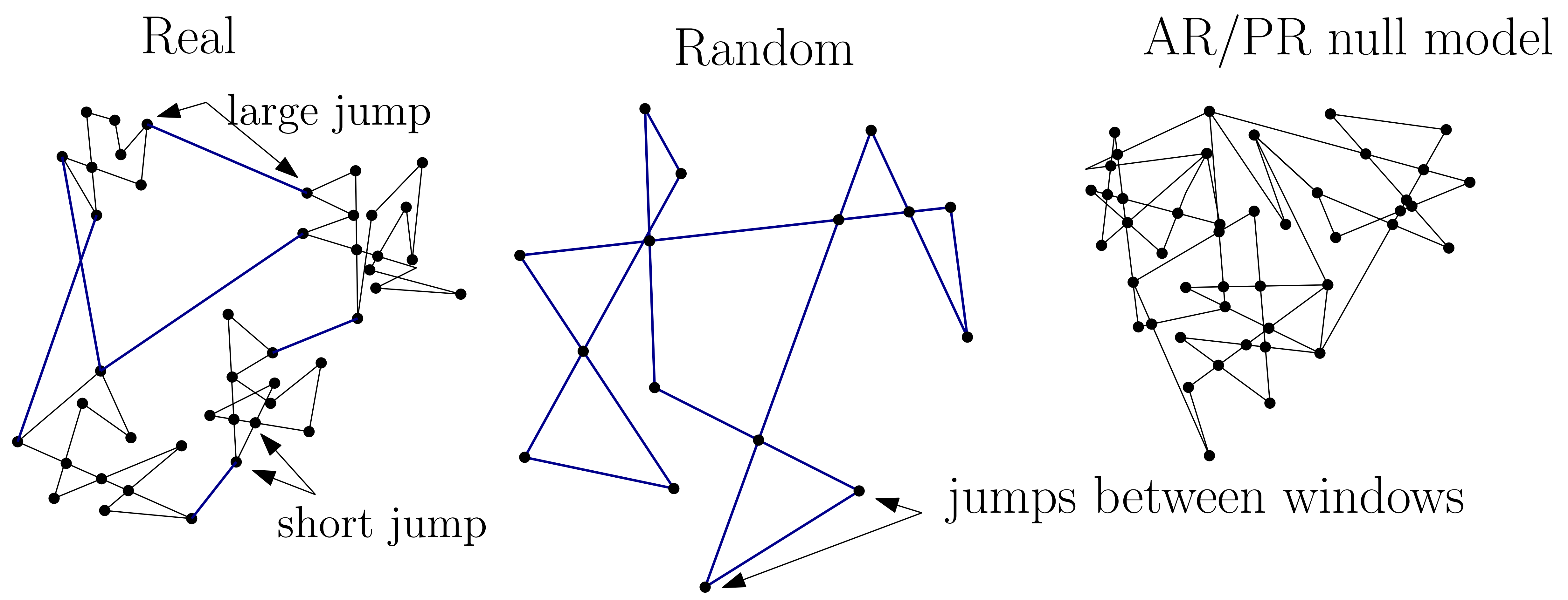

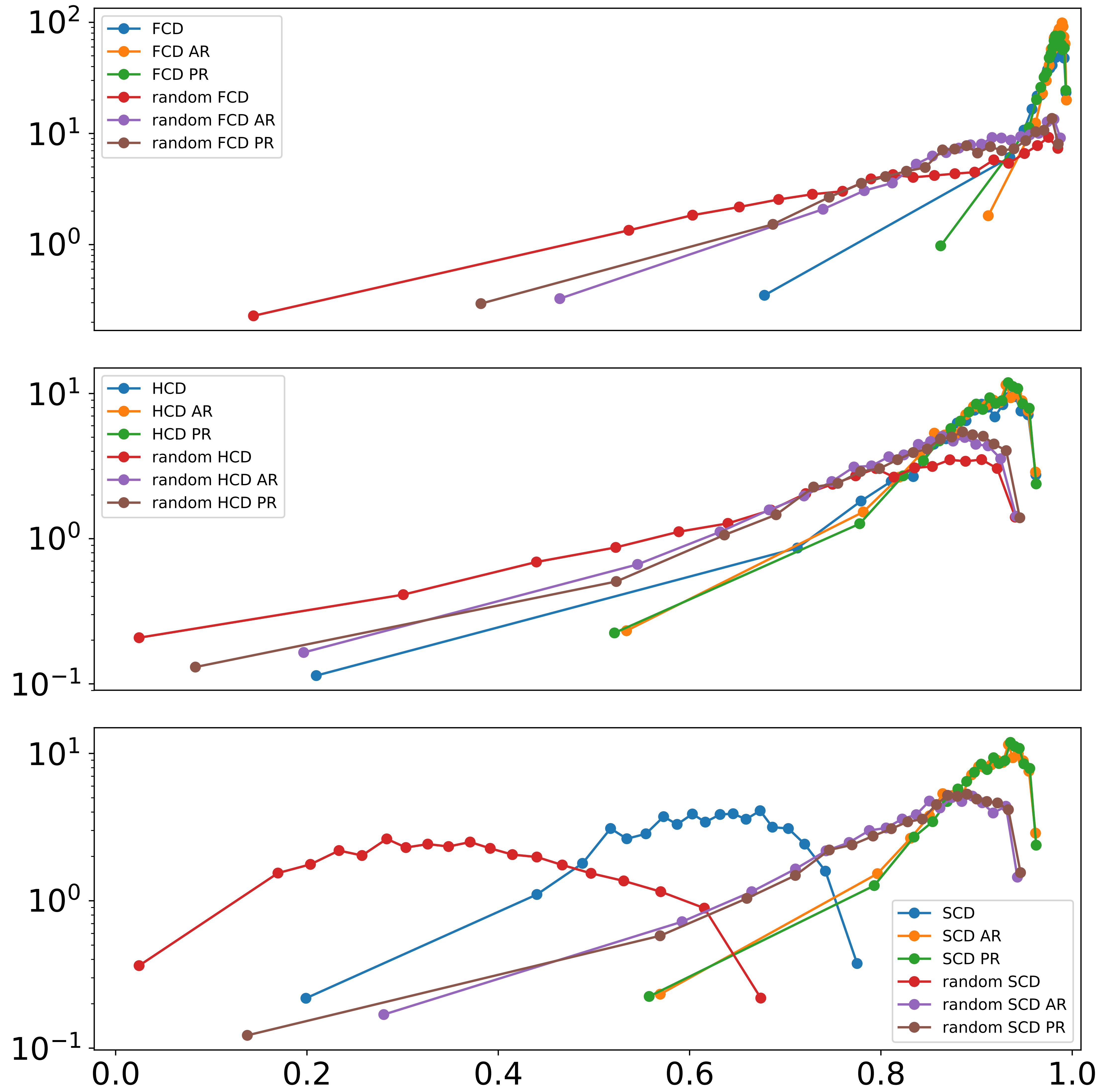

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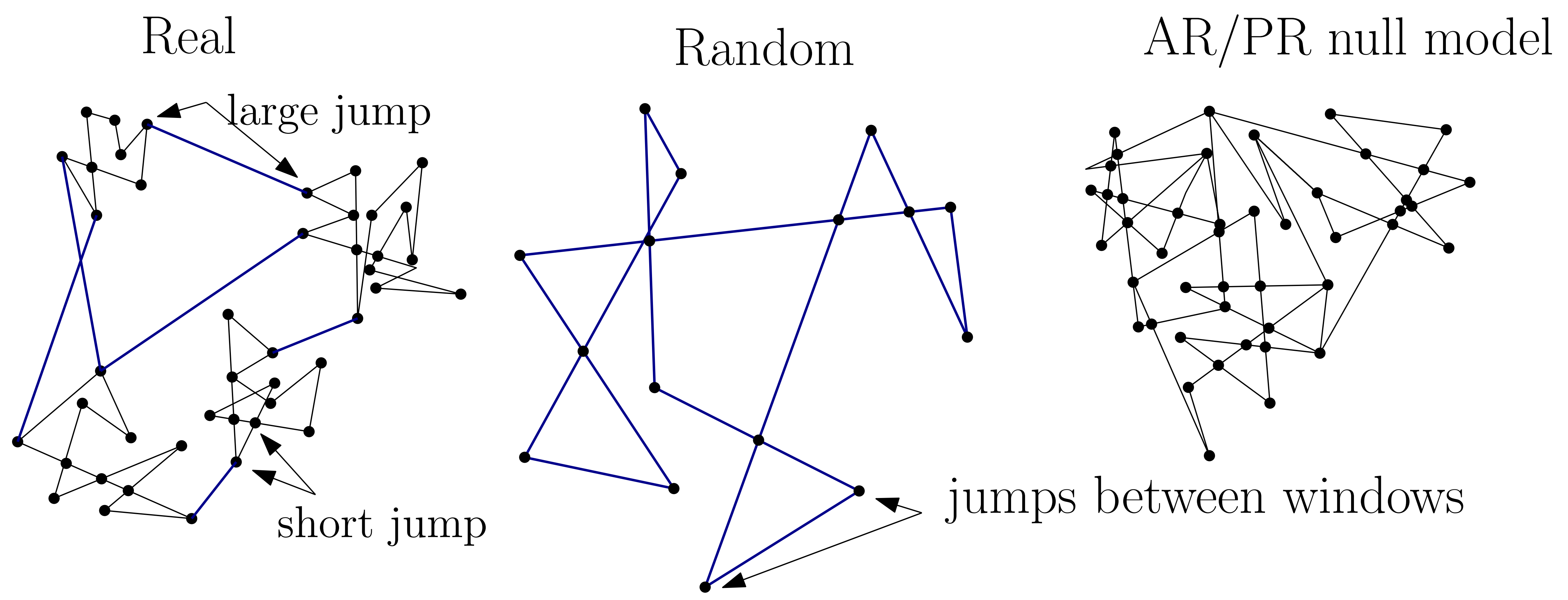
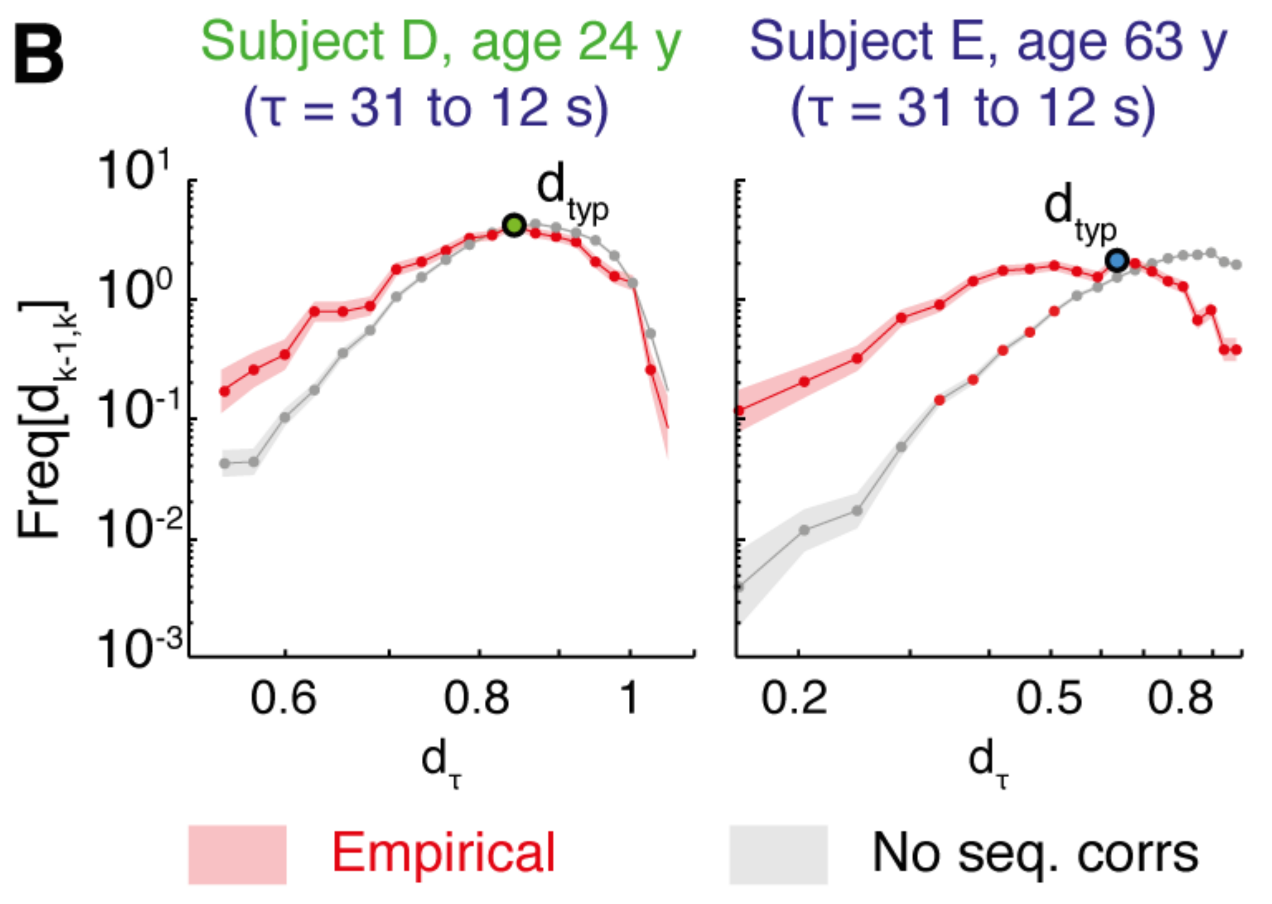
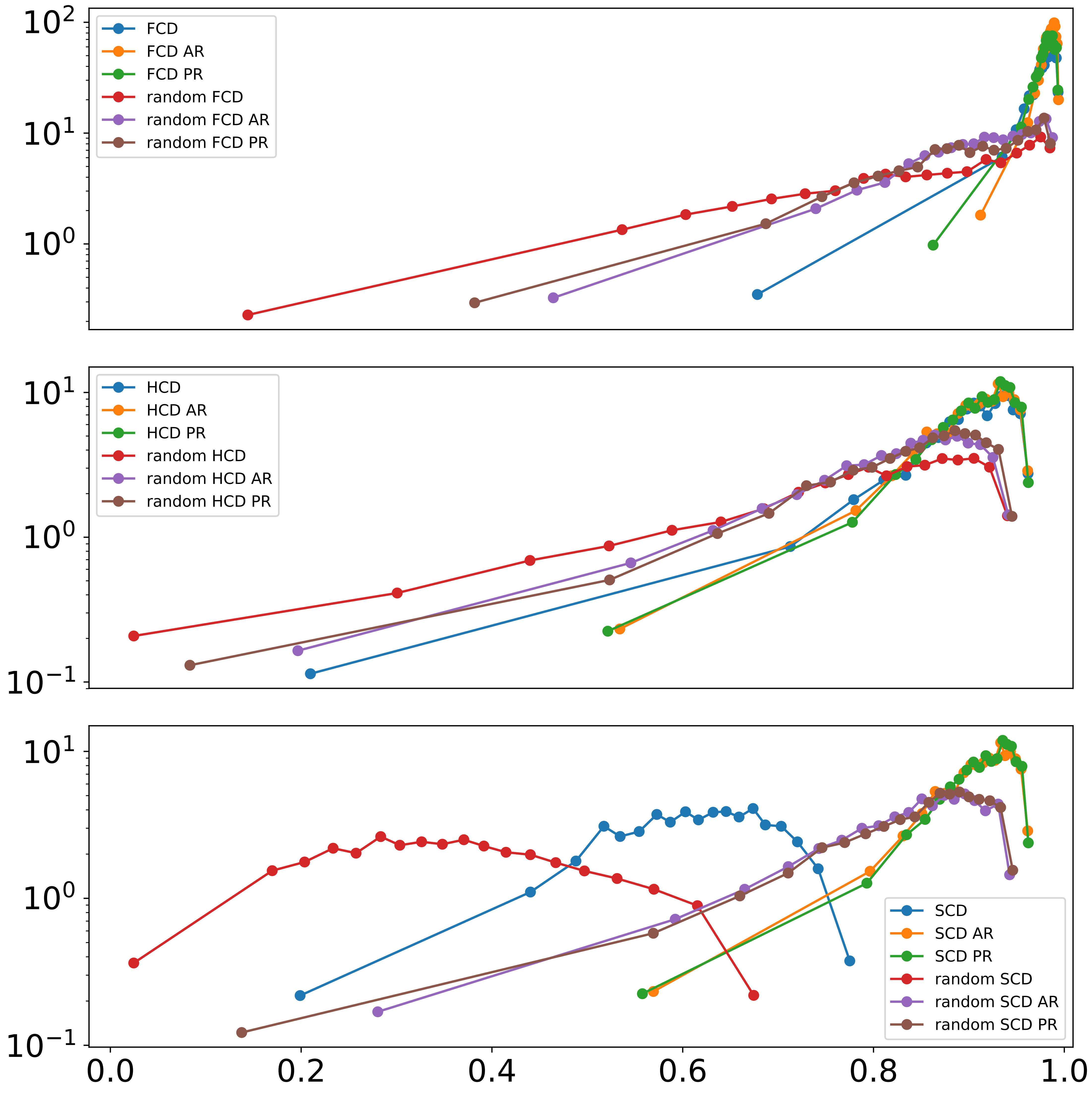




From Battaglia et al 2017



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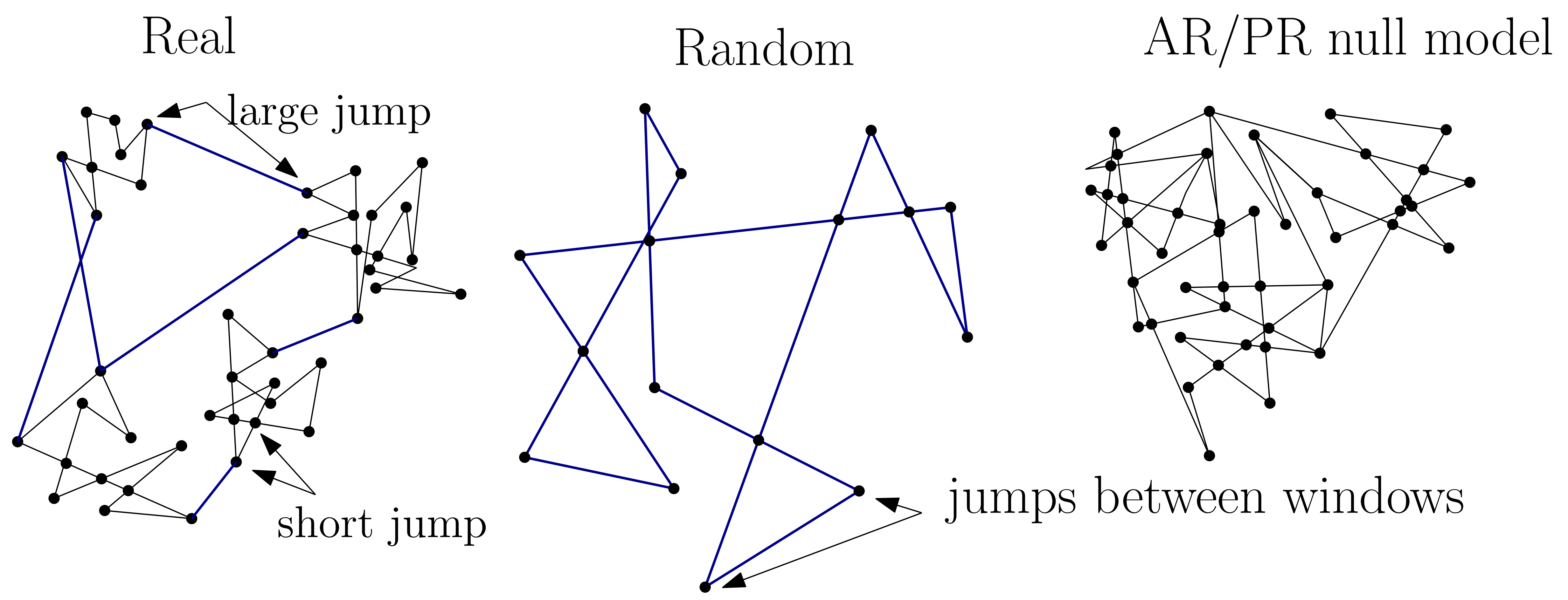

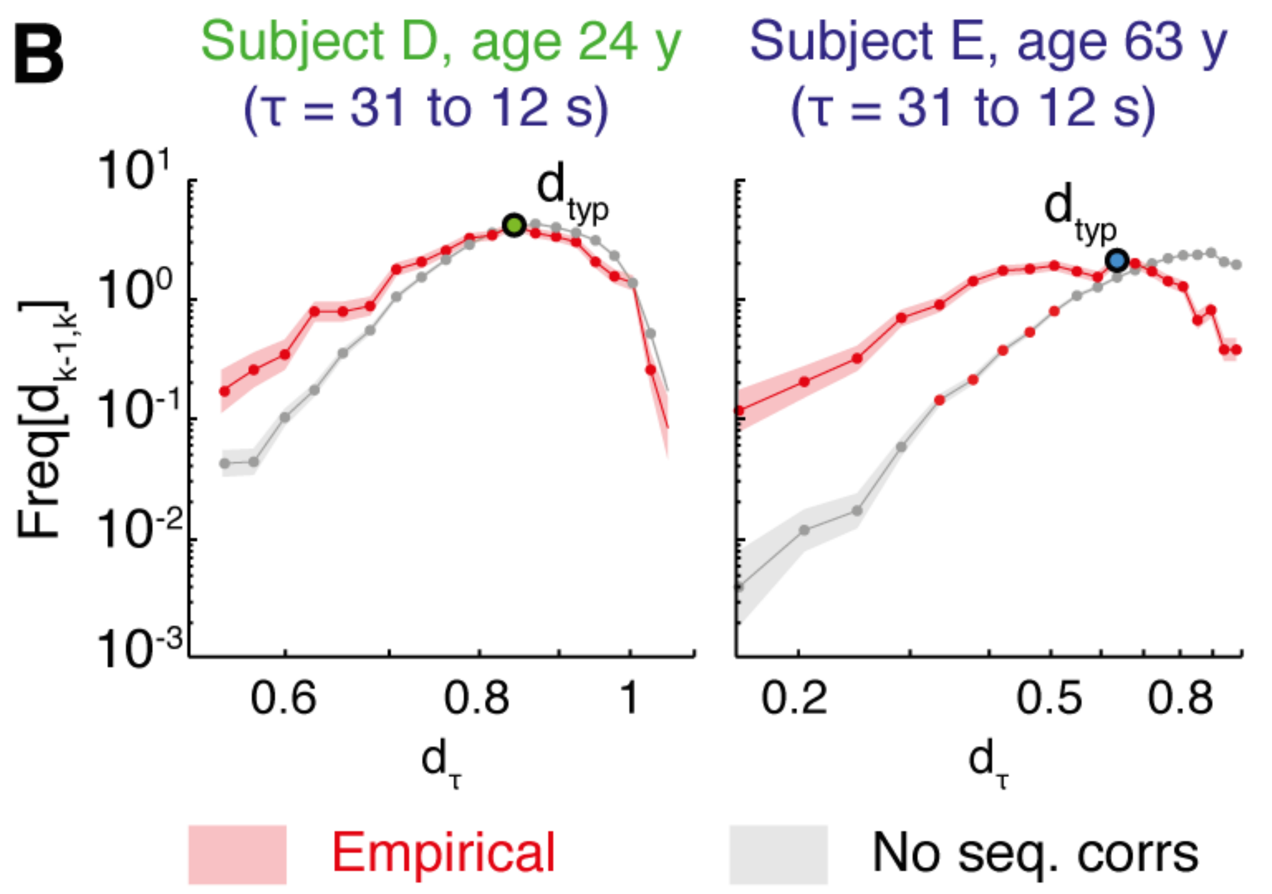

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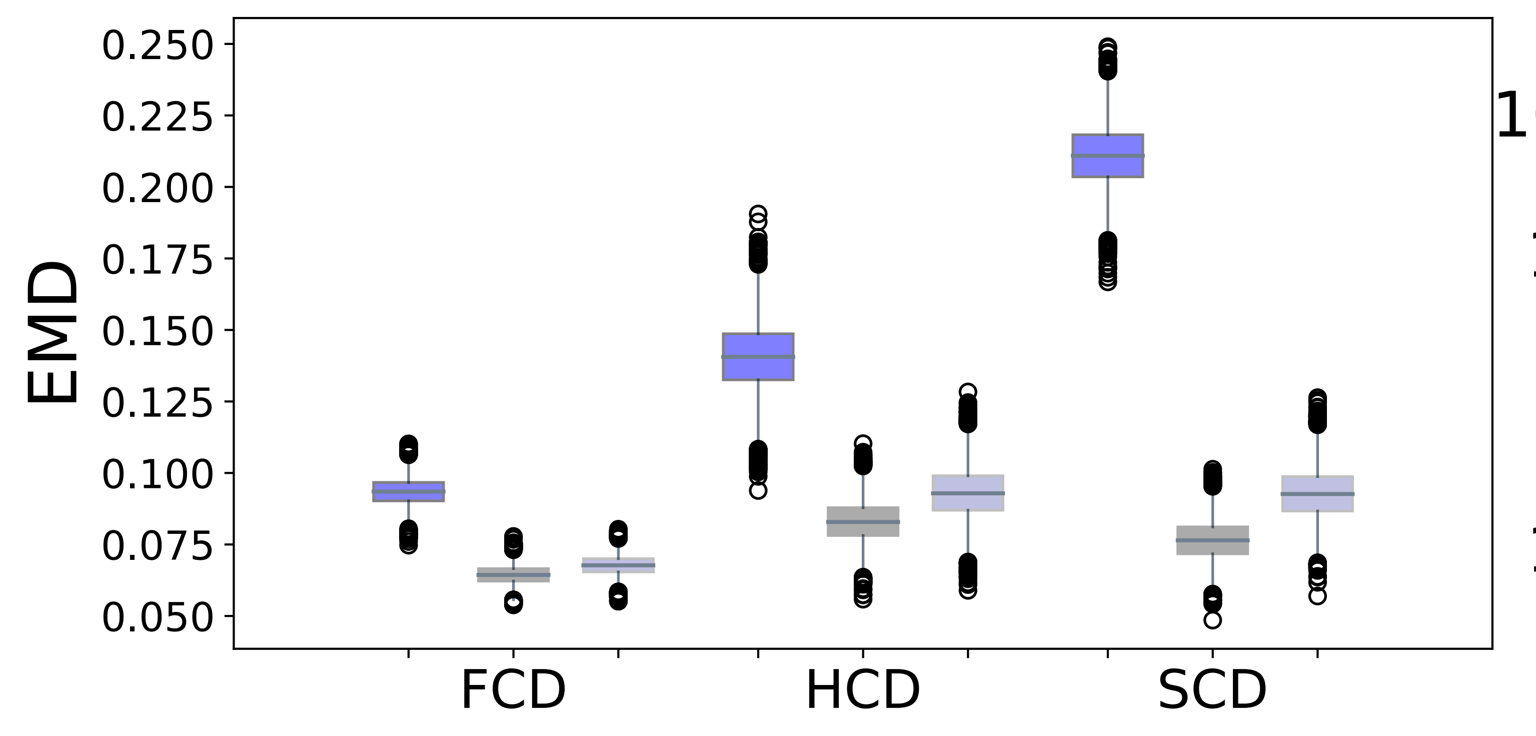
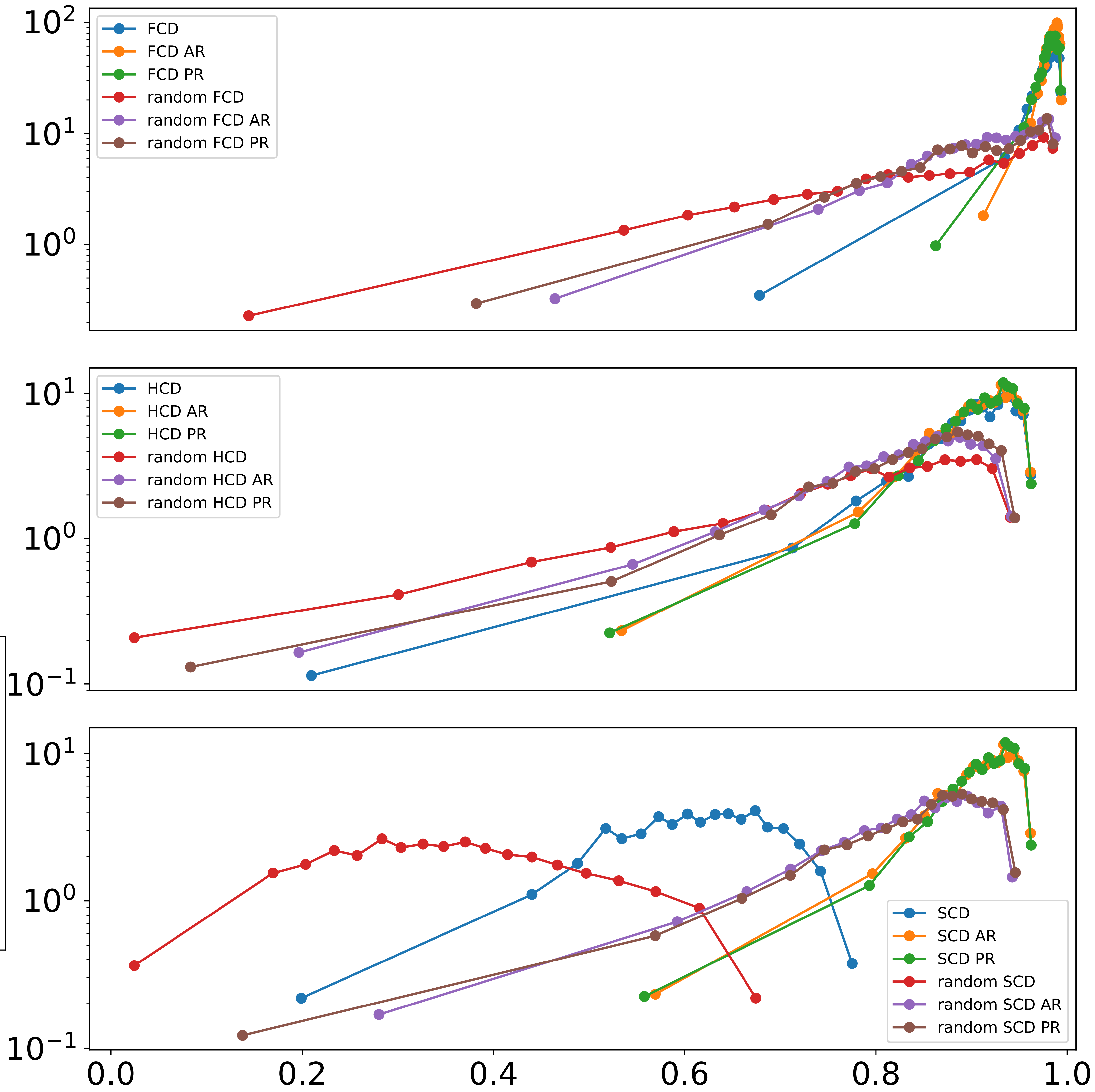


Better discrimination in Homology



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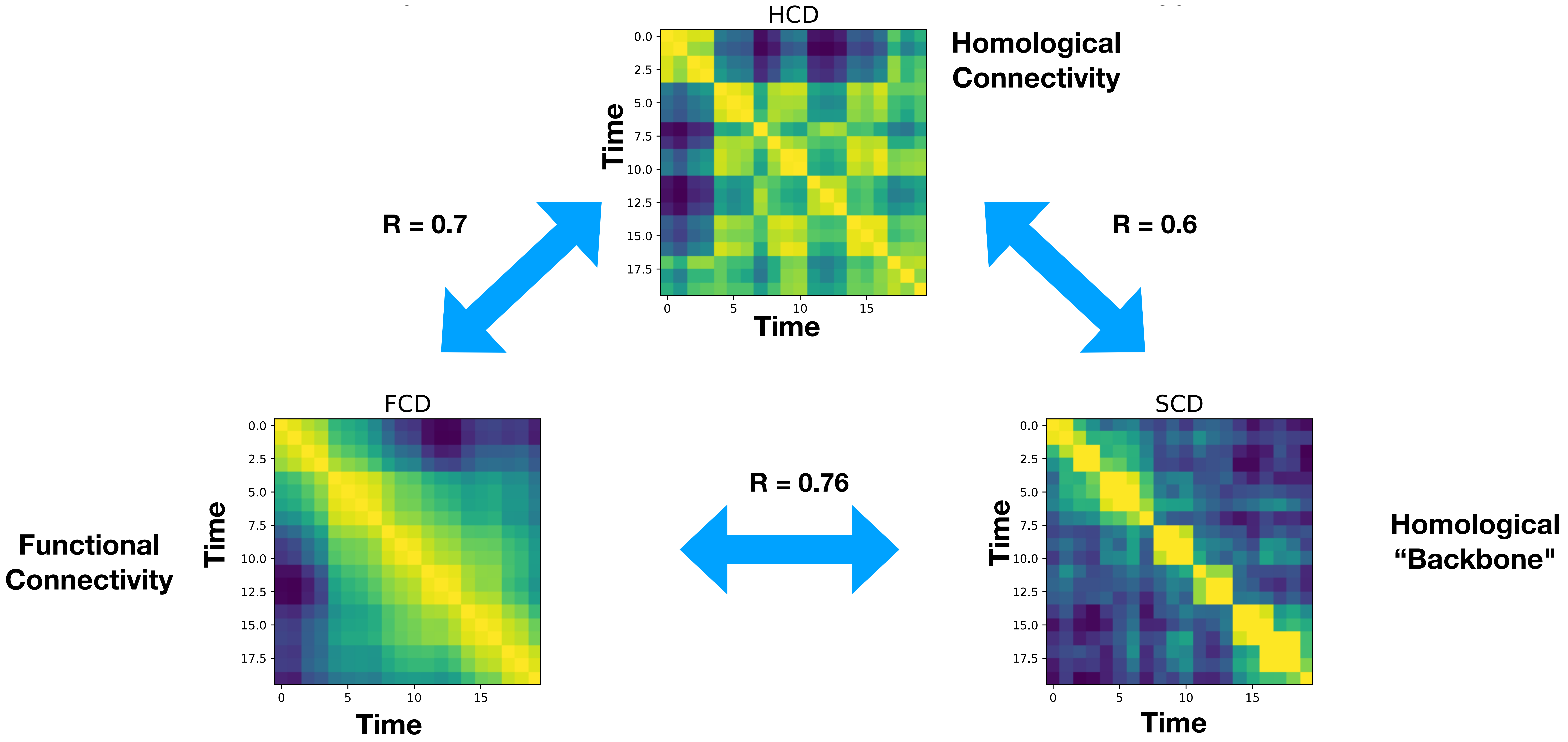
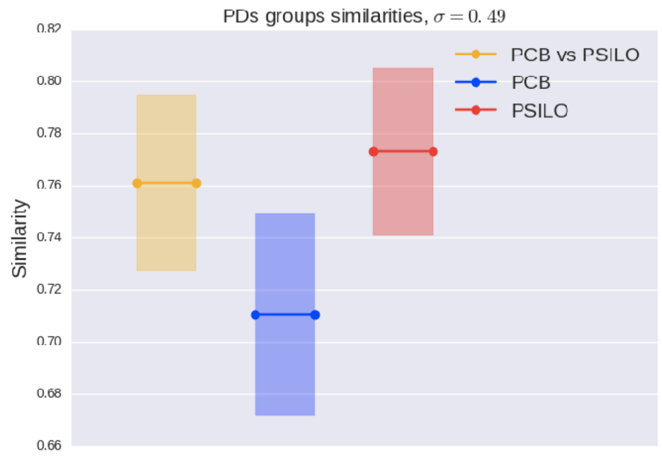


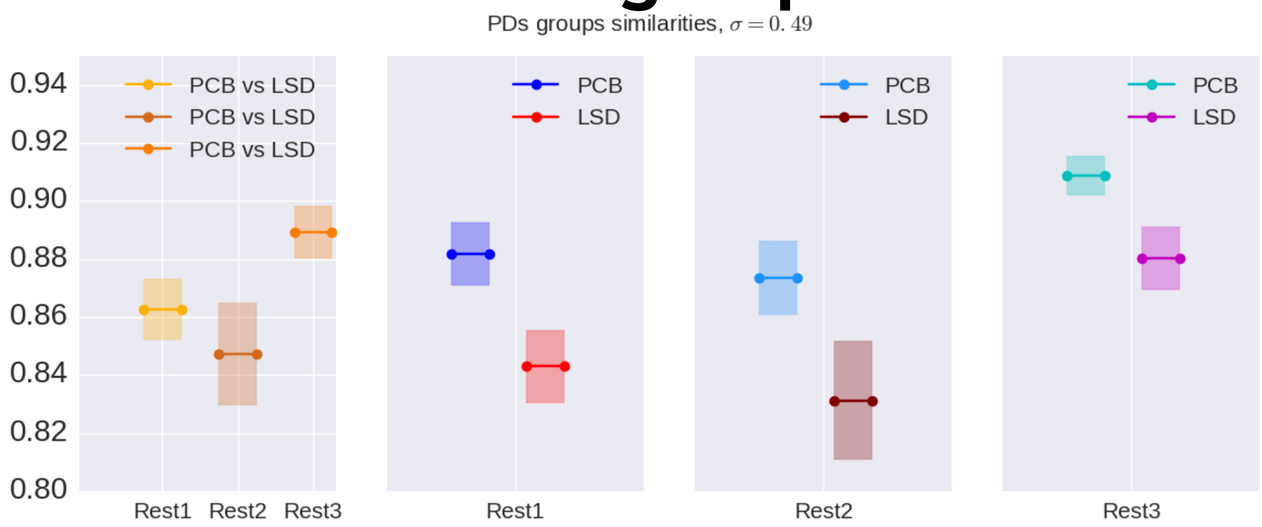
Does this generalize?

Does this generalize?

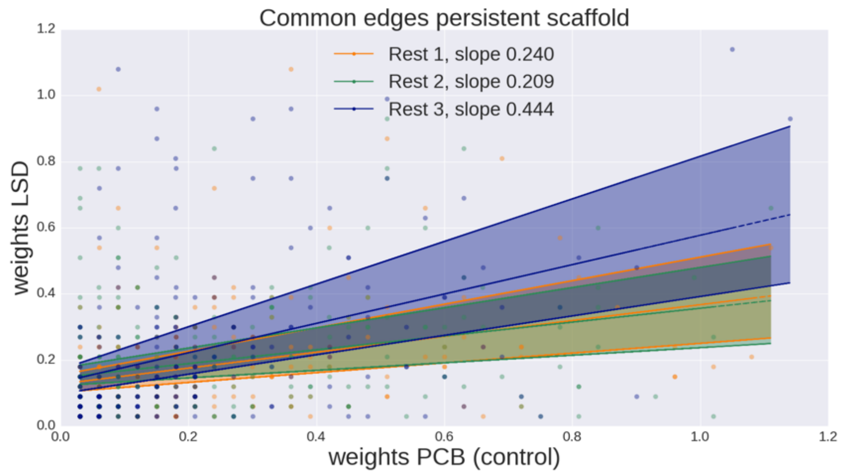
LSD vs Psilo

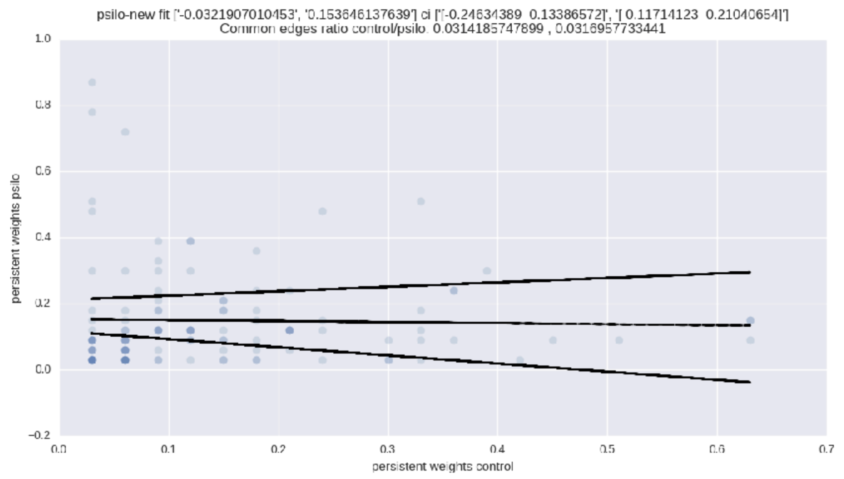
Similarity within and across groups

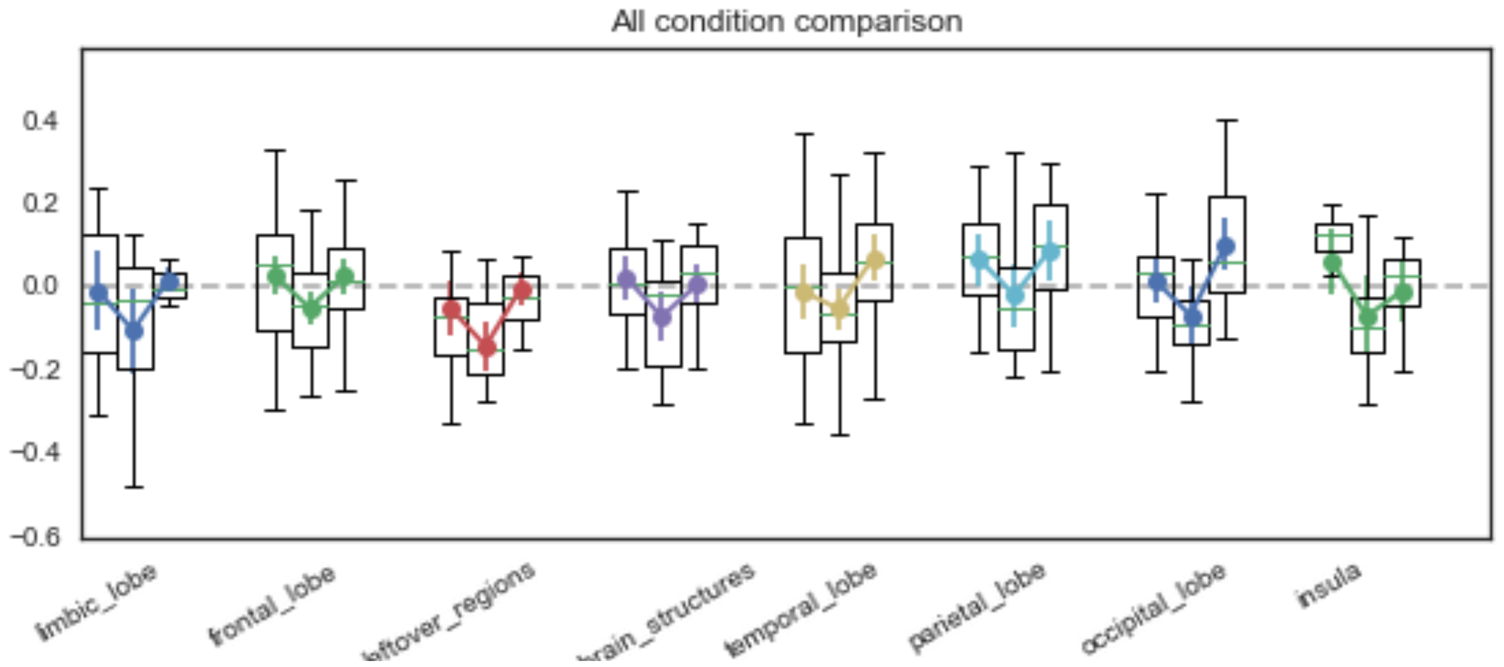




weight localization in condition vs placebo







Distribution of persistence scores across conditions and macro-regions

(ISI+Imperial+KCL)

Does this generalize?

LSD vs Psilo

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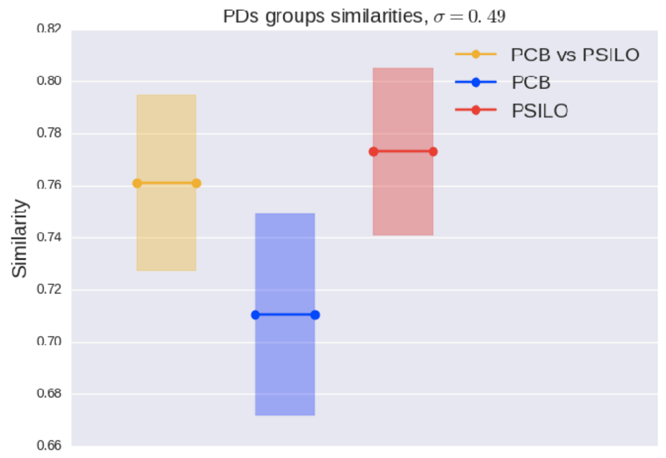

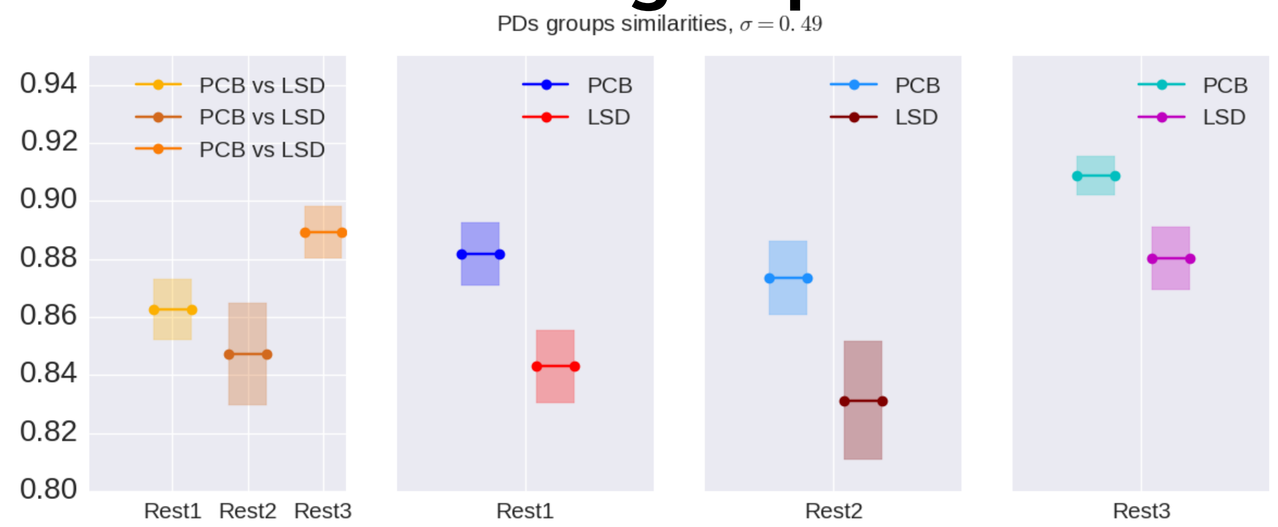

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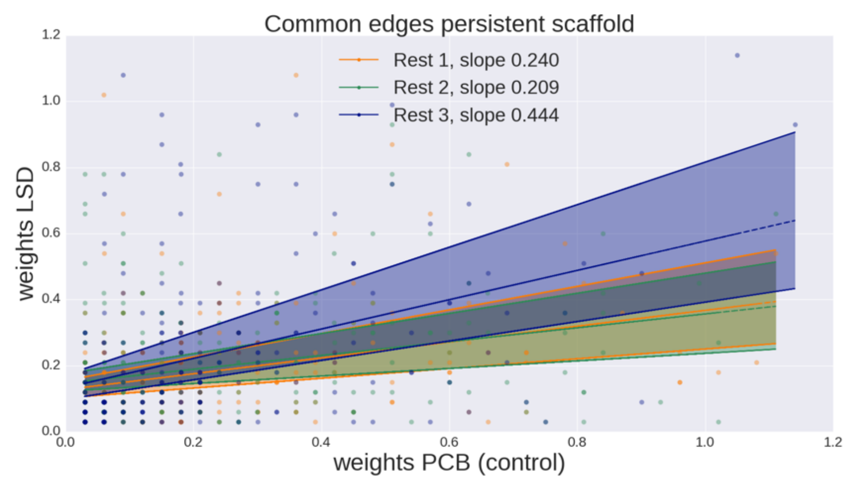
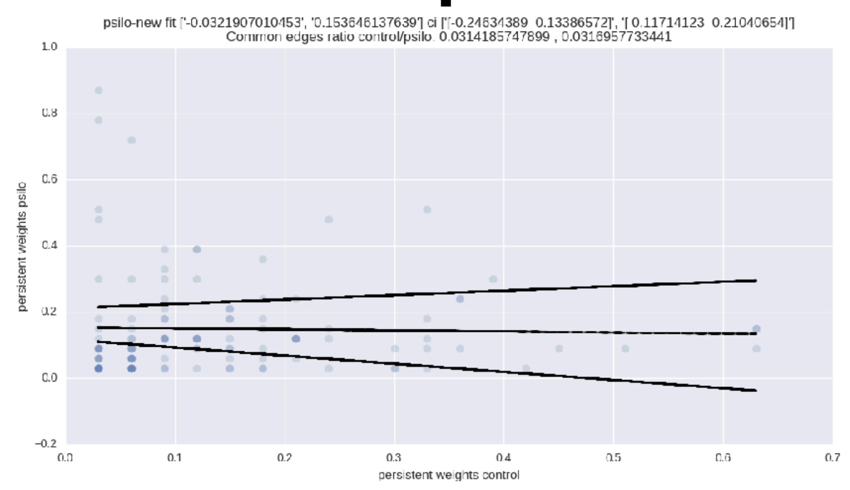
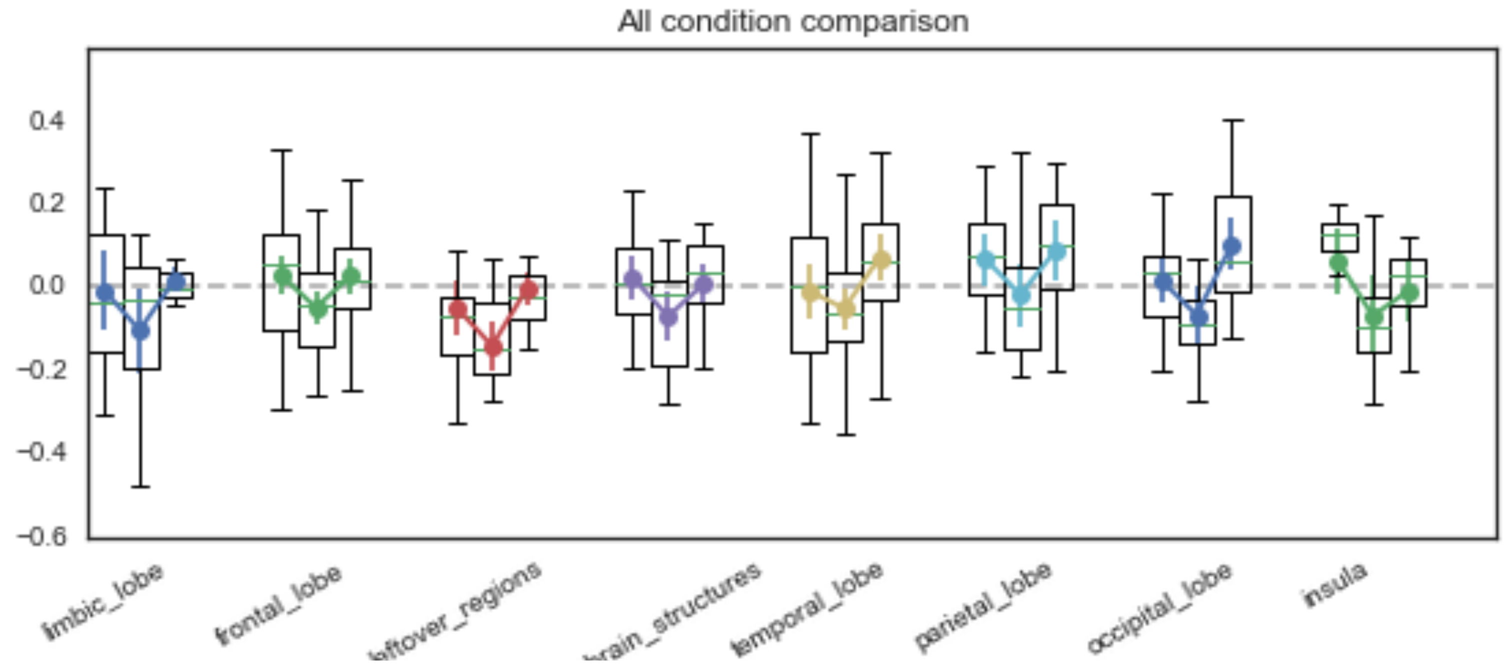


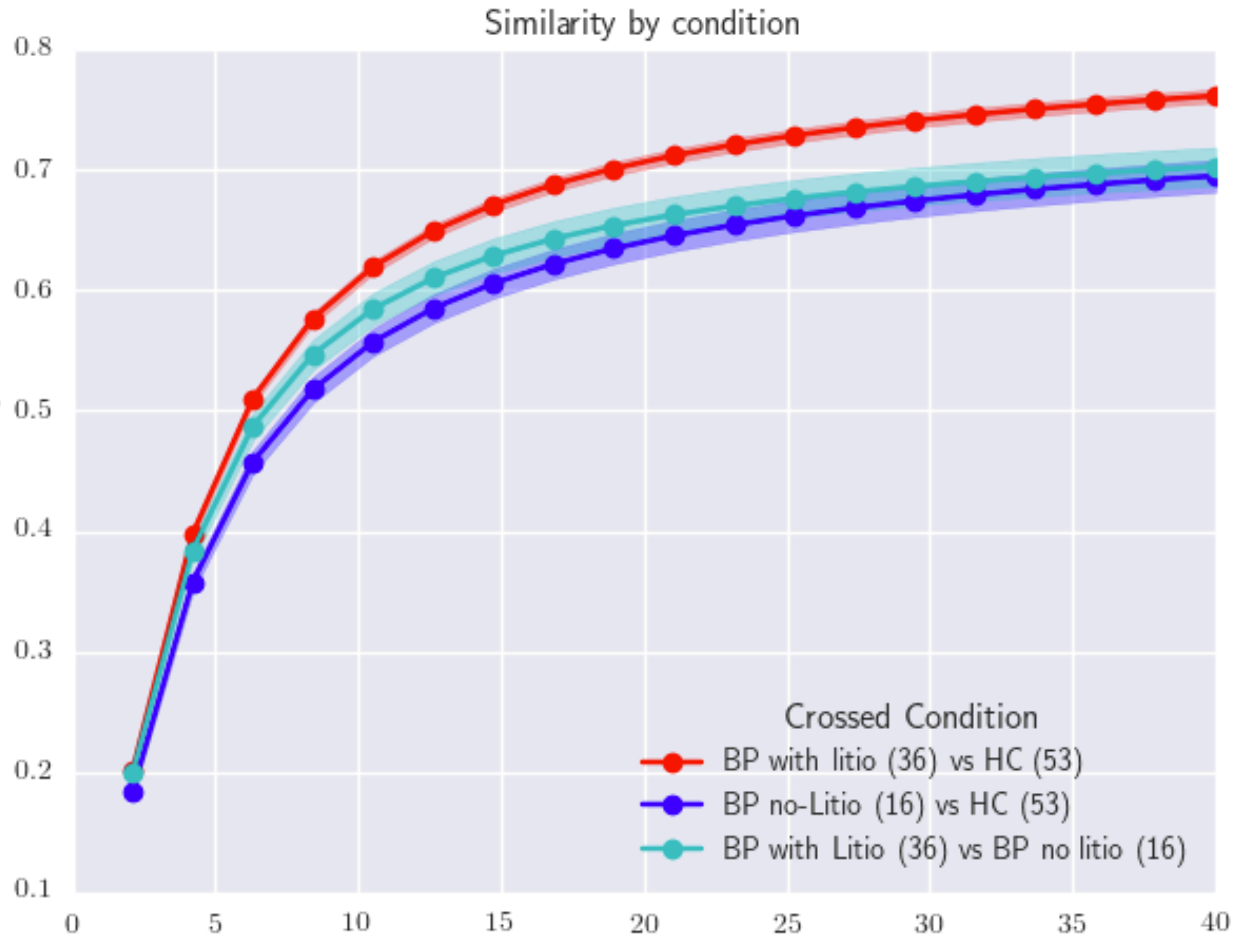


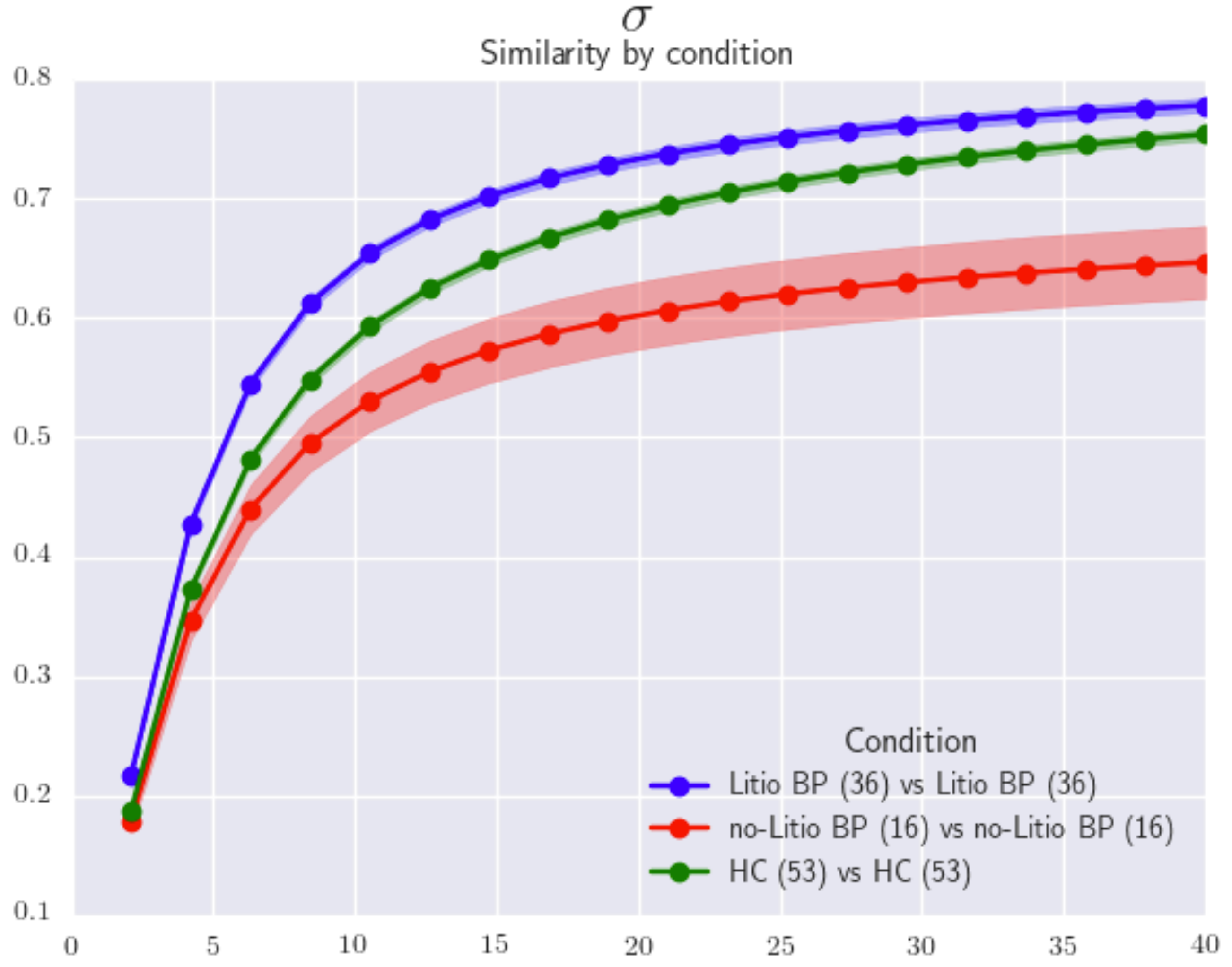


Distribution of persistence scores across conditions and macro-regions

(ISI+Imperial+KCL)

Bipolar





(ISI+IMT+S.Lucia)

Does this generalize?

LSD vs Psilo

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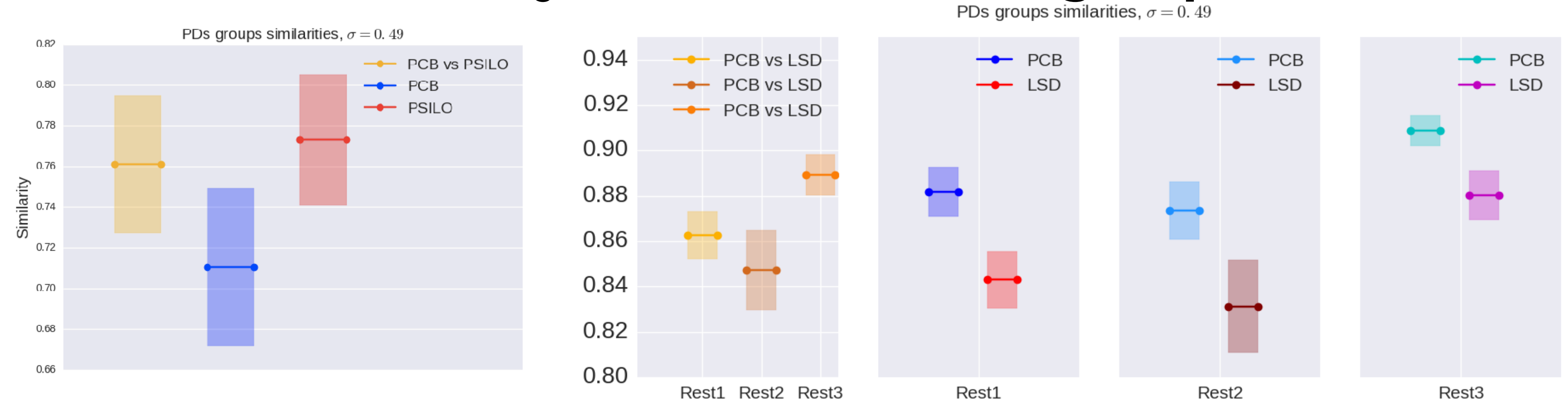

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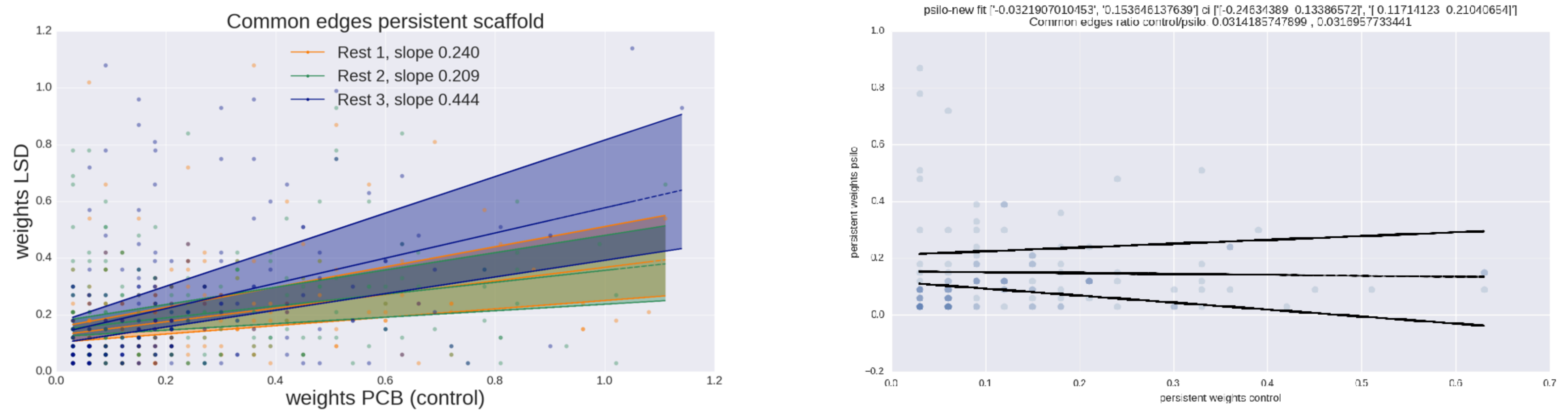

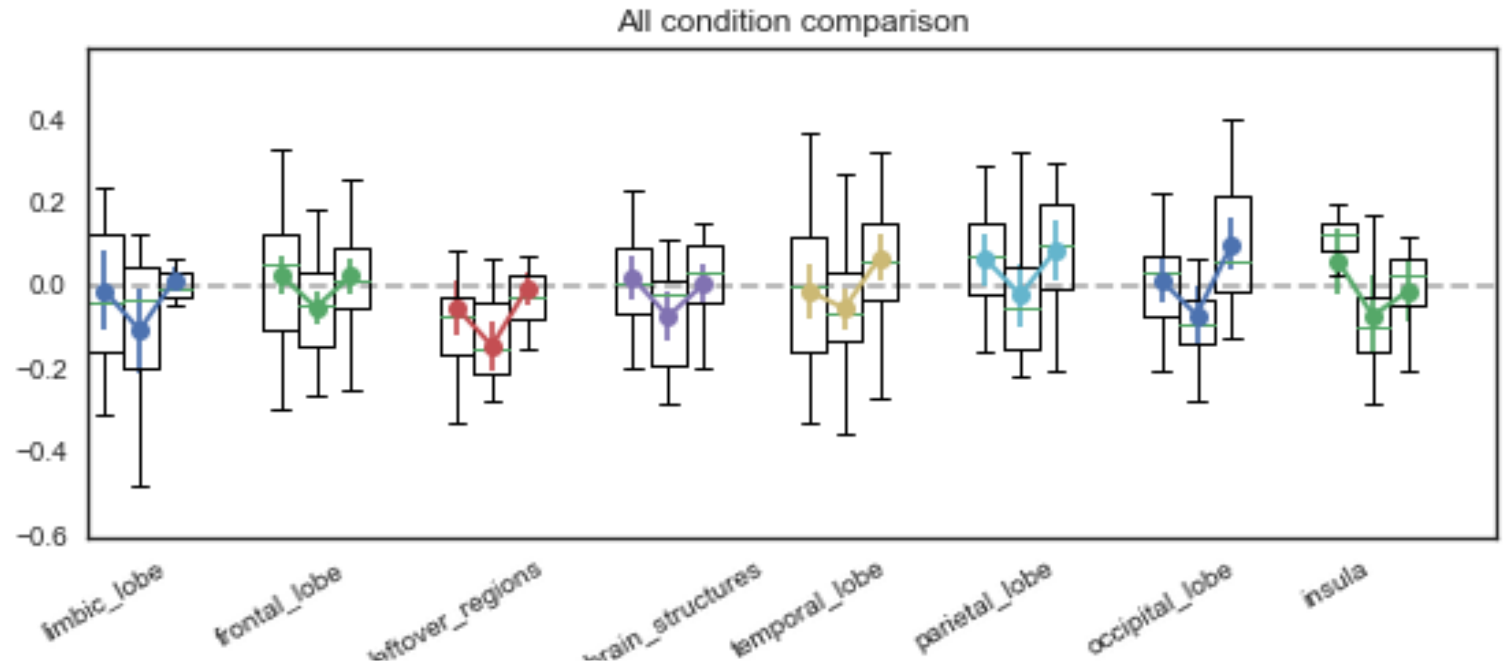

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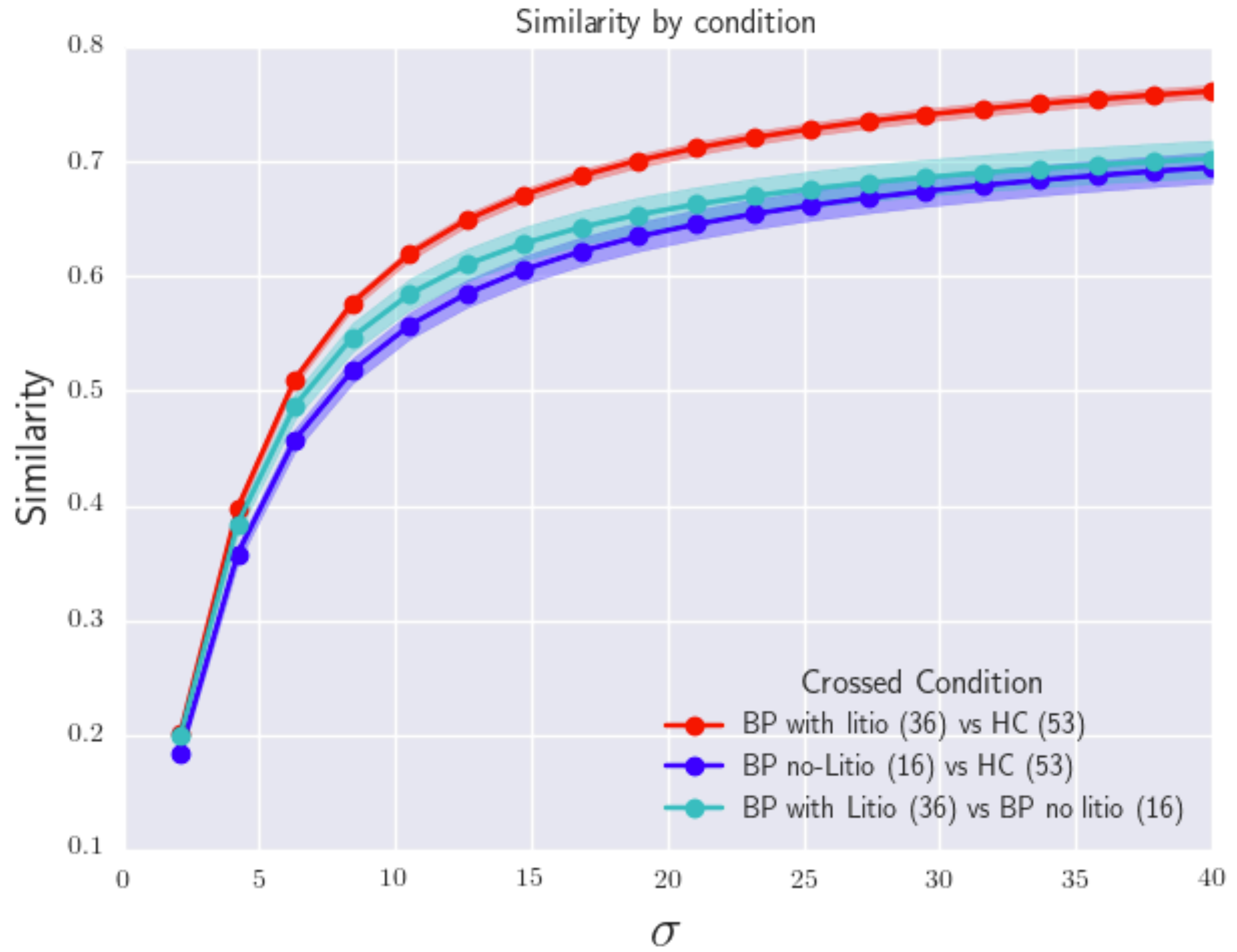
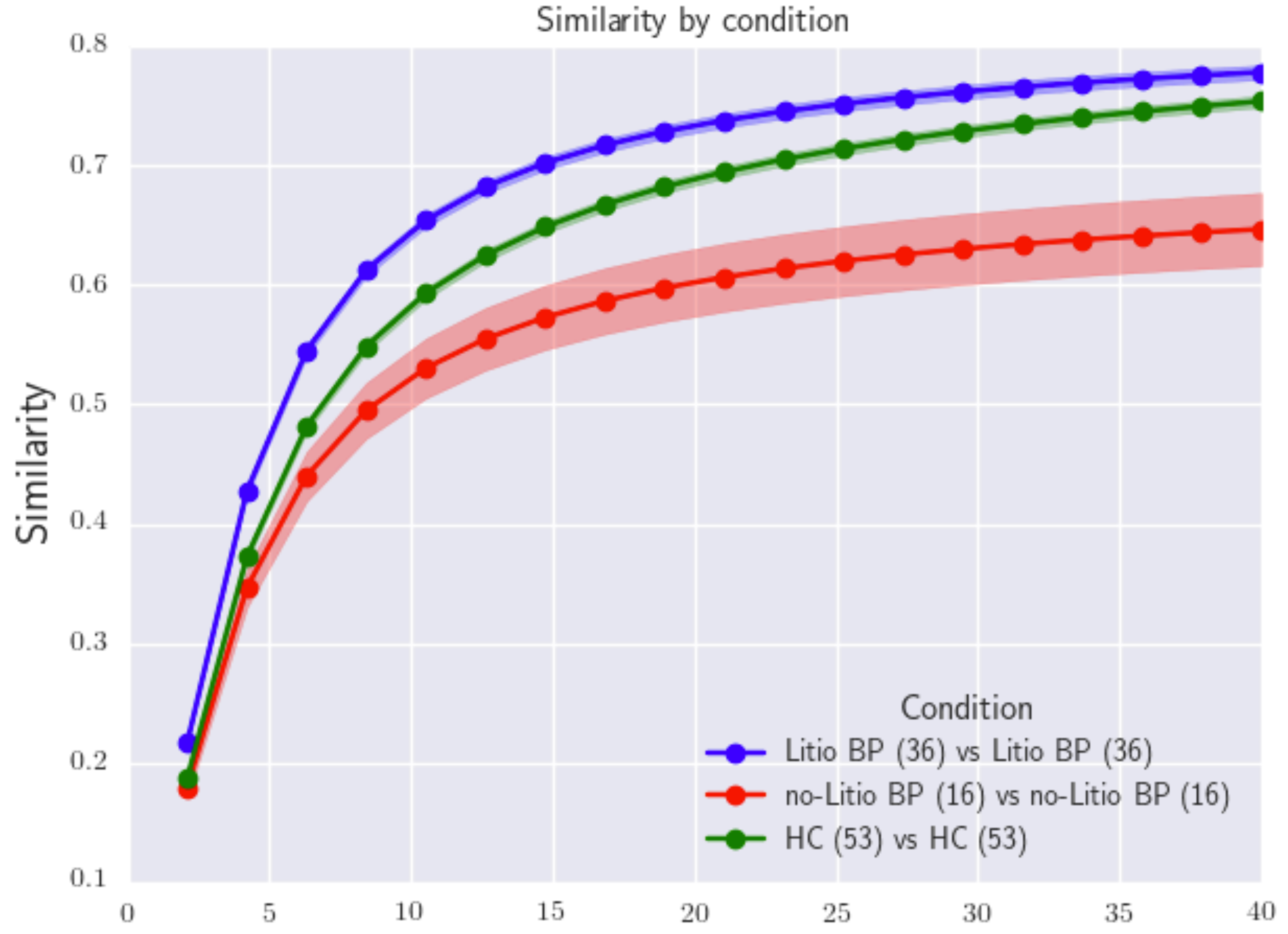
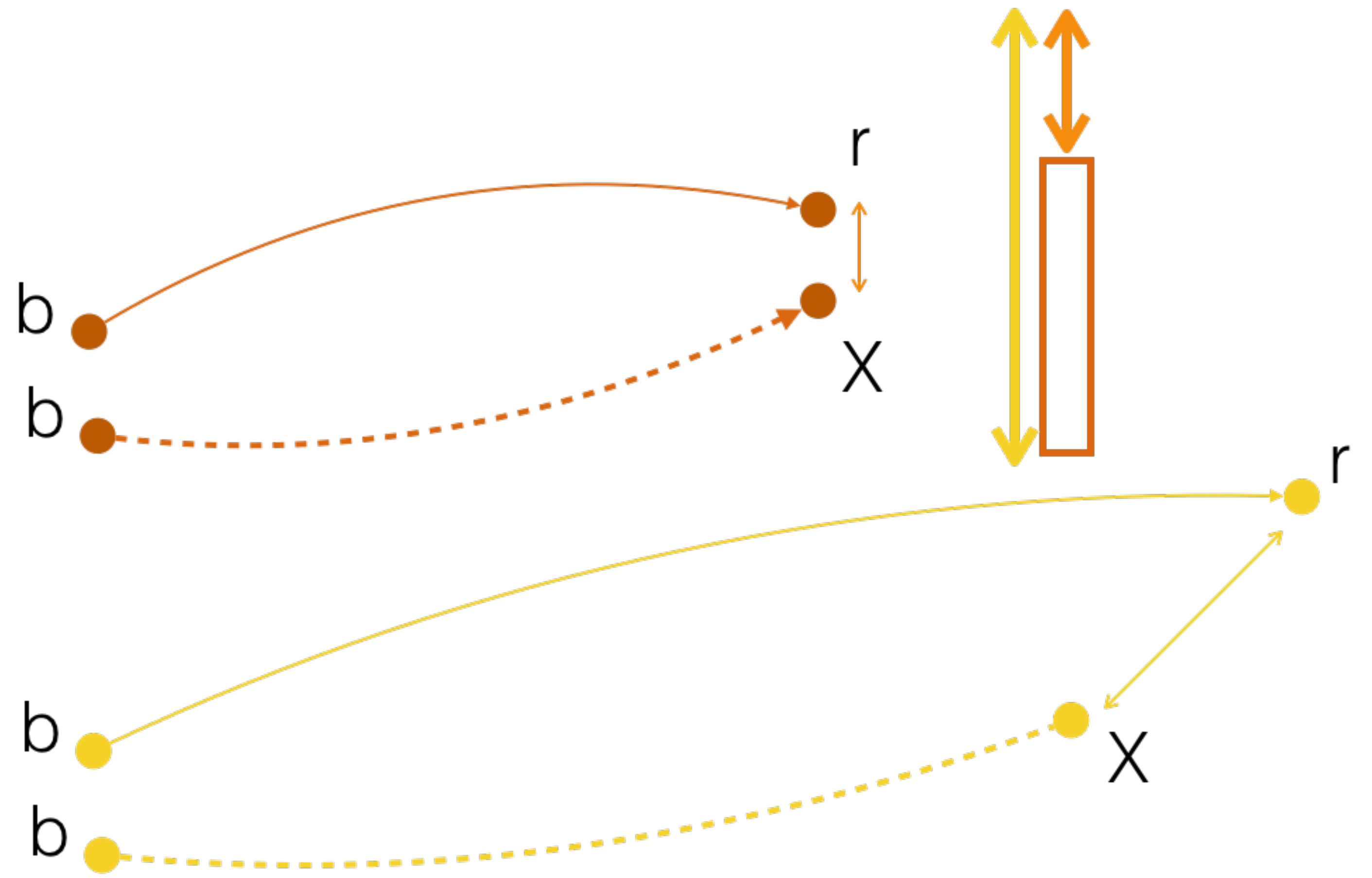
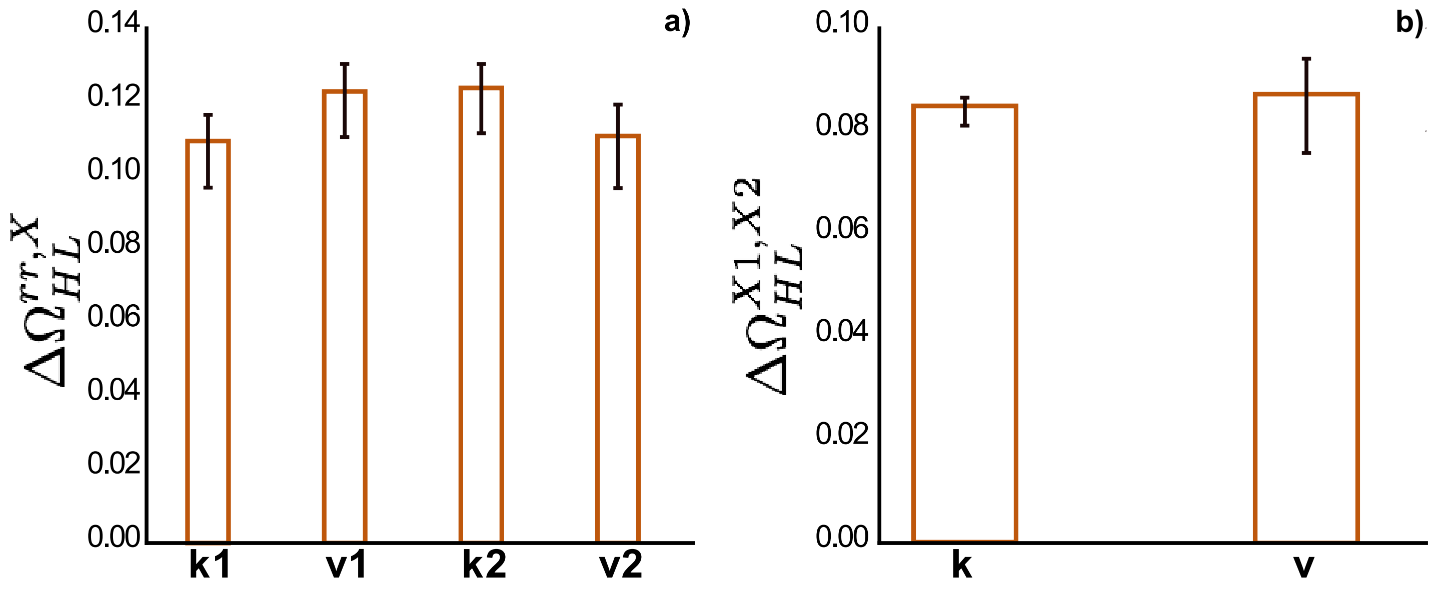


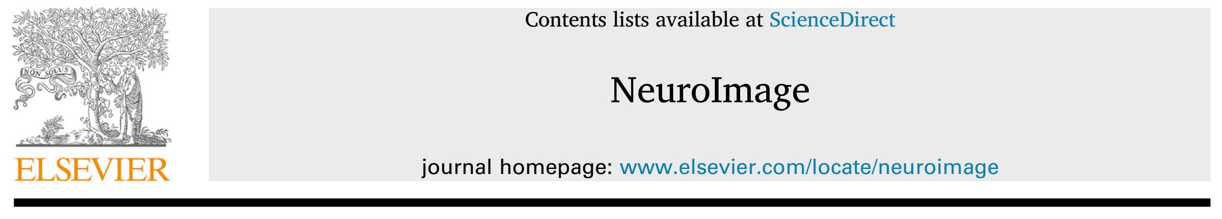


(ISI+IMT+S.Lucia)

Funct. Equival.







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ELSEVIER

Topology highlights mesoscopic functional equivalence between imagery and perception: The case of hypnotizability

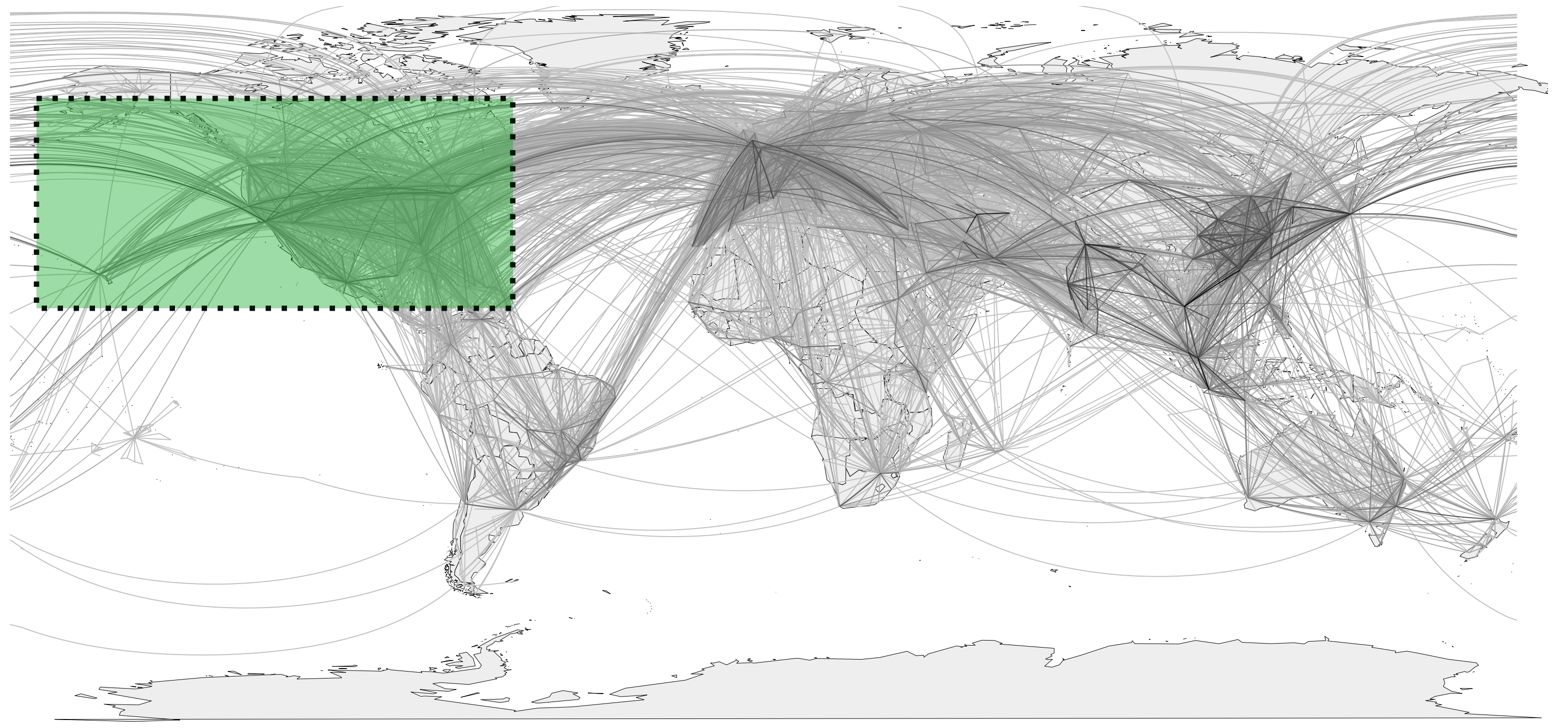
Esther Ibáñez-Marcelo[a], Lisa Campioni[b], Angkoon Phinyomark[c], Giovanni Petri[a,d,*], Enrica L. Santarcangelo[b]

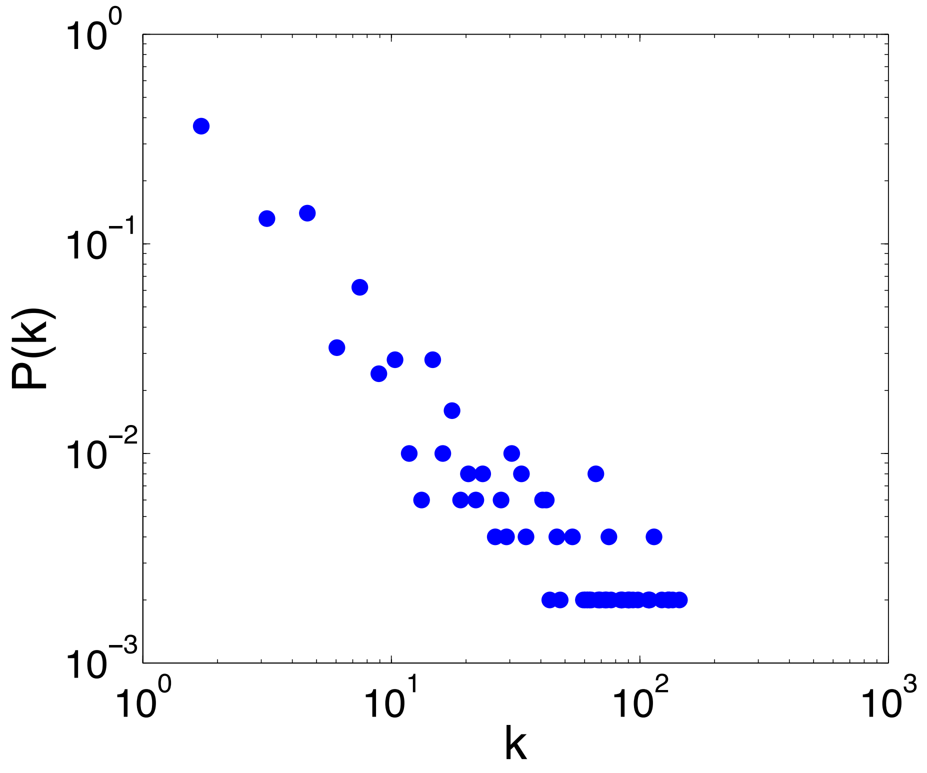
What about networks?

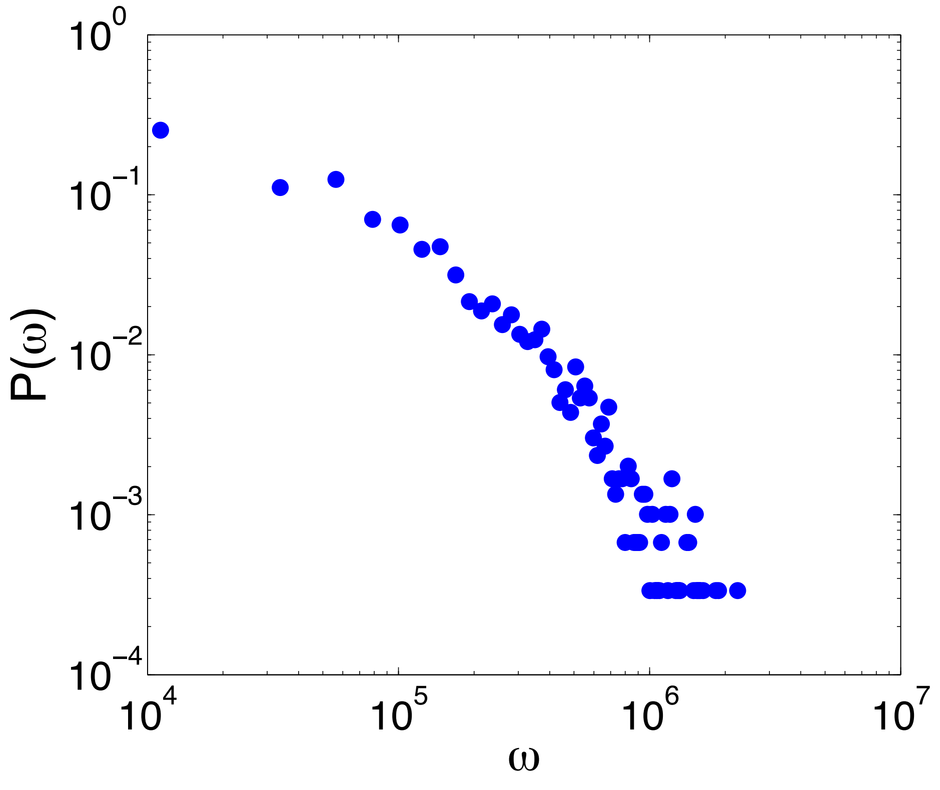
Metrical persistent homology

No metric space, but one can define a few: shortest path, commute time, other Kernel distances..

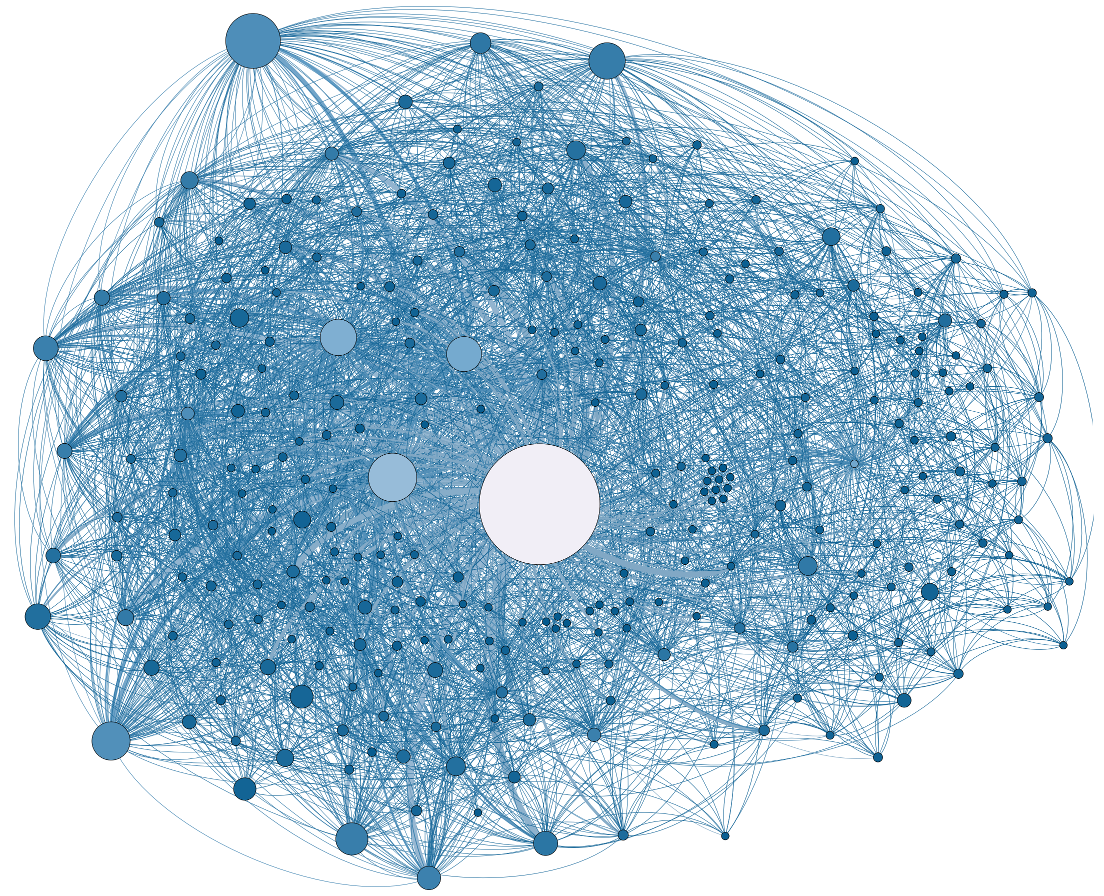
US Top-500 airports network

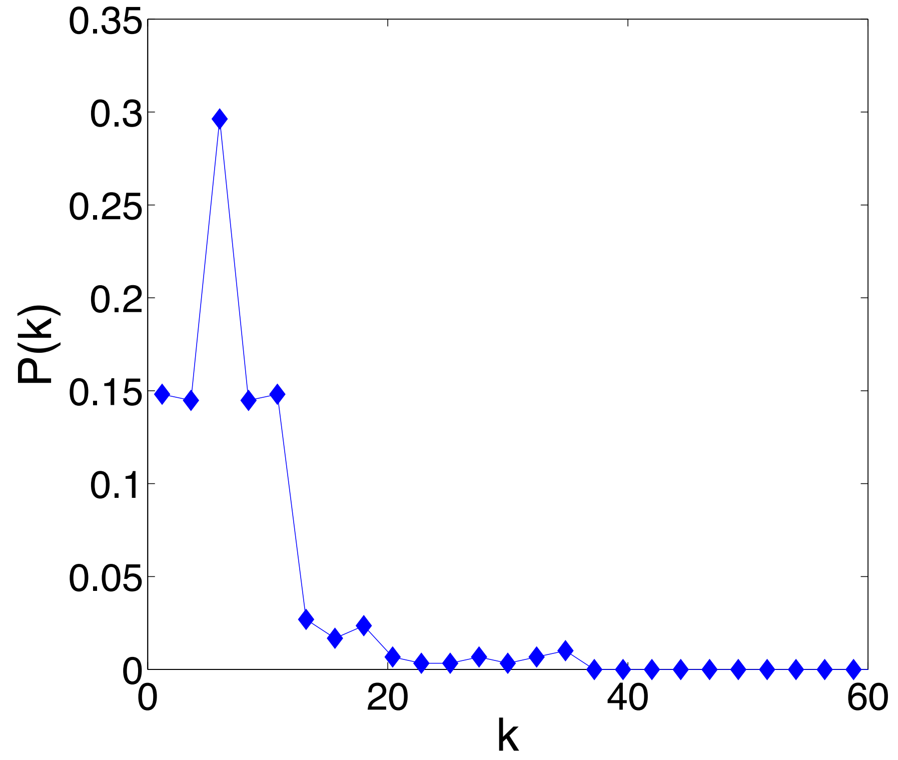


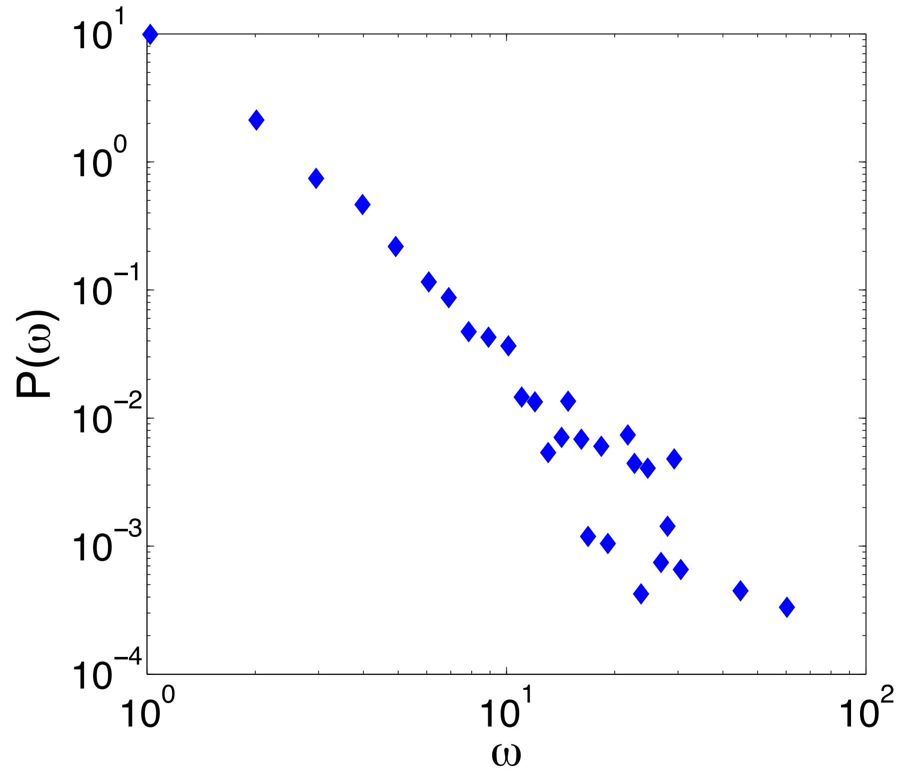




C. Elegans neural network

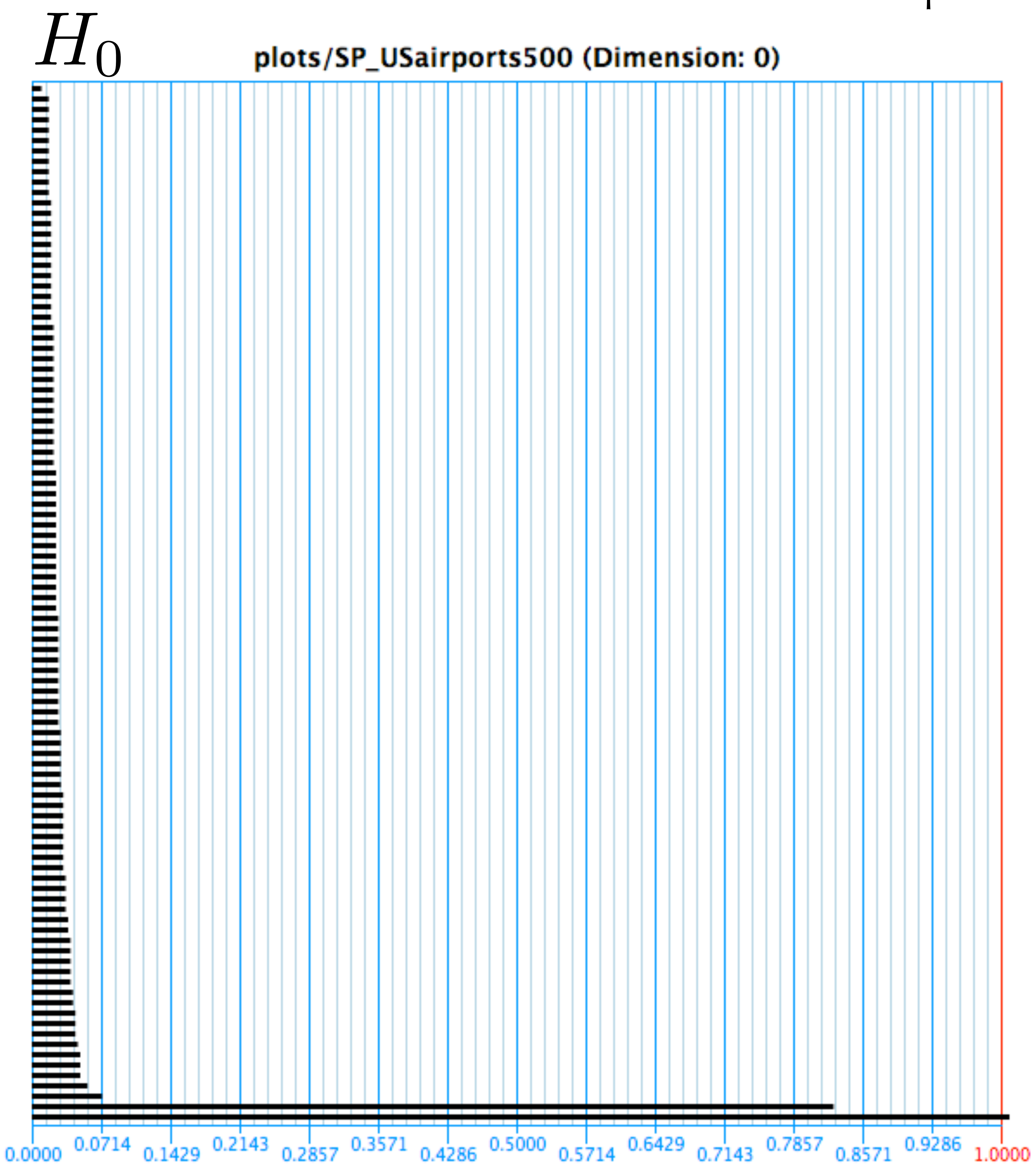




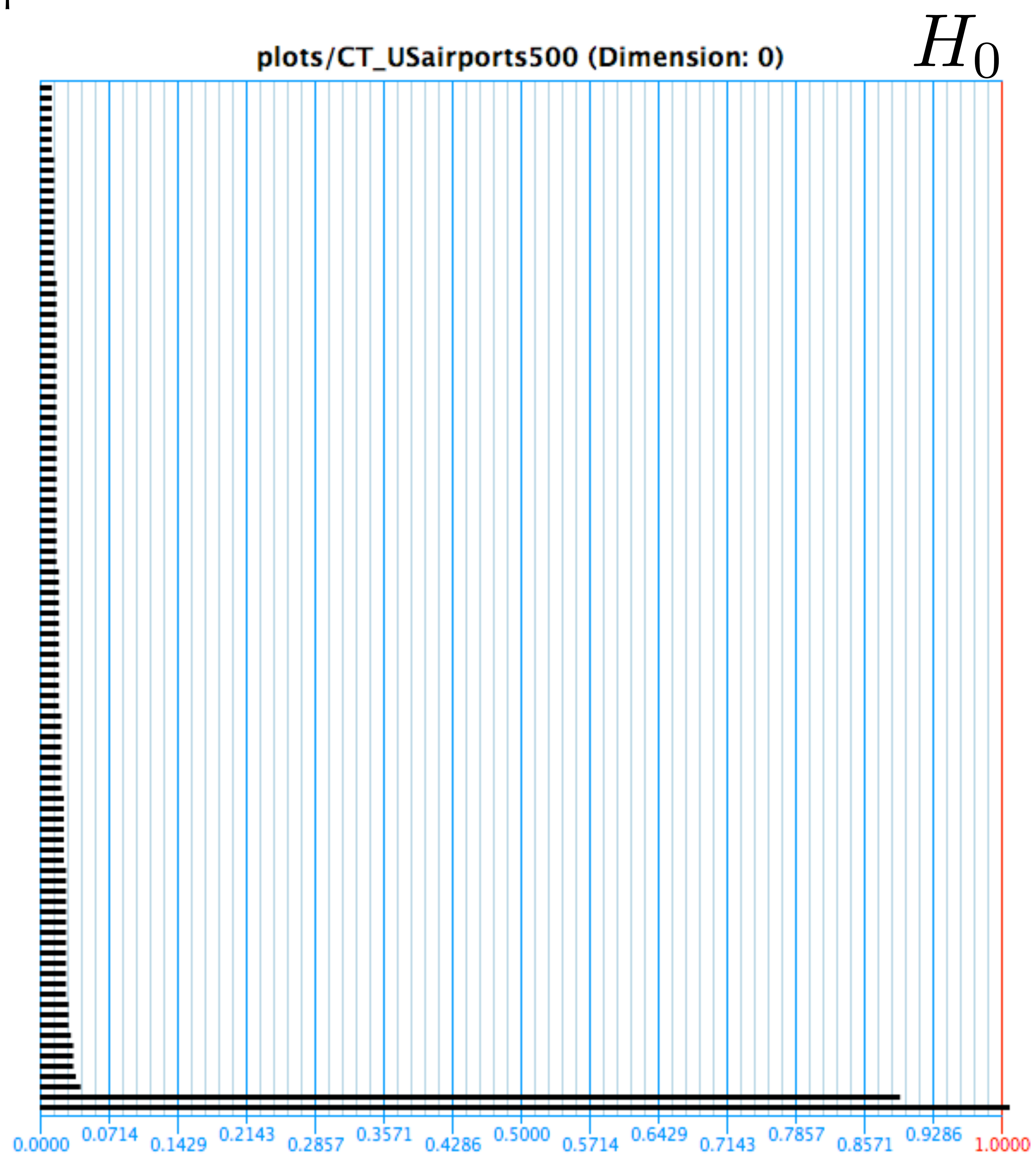


Metrical persistent homology II

US Top500 airports network

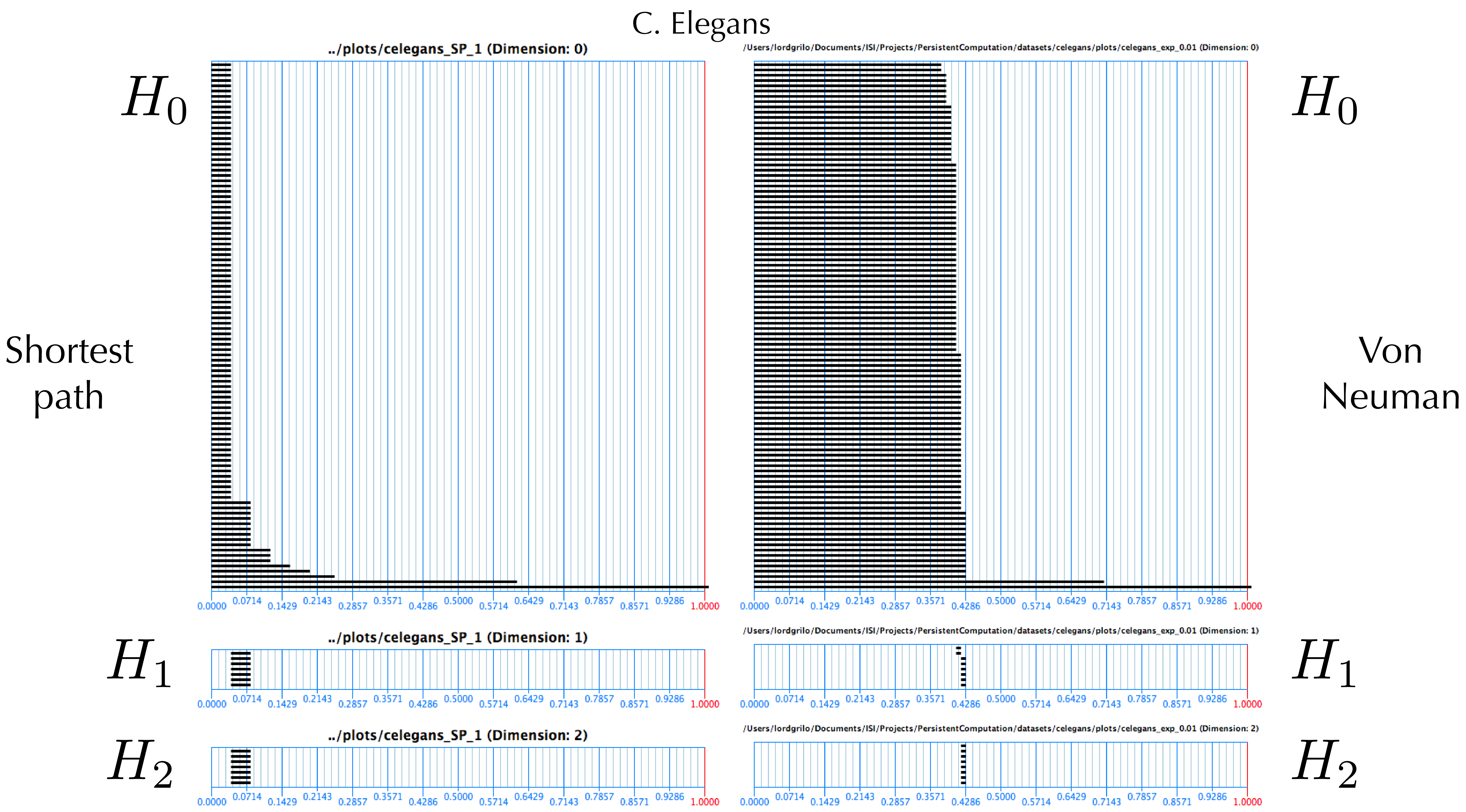


Shortest path



Commute time

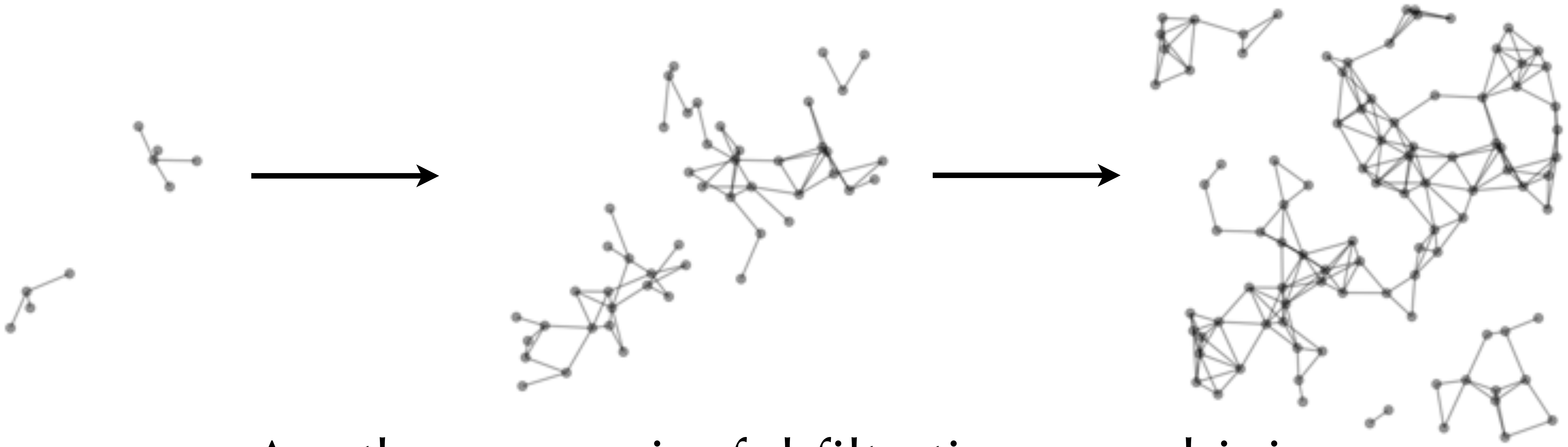
Metrical persistent homology II_{bis}



1. Different metrics ~ same information
2. Cycles appear at emergence of GCC
3. No need of homology to do this!!

Metrical persistent homology III

Metrics do not convey much information.
What now? The important ingredient is the **filtration**.



Are there meaningful filtrations combining:

Network "linking patterns"

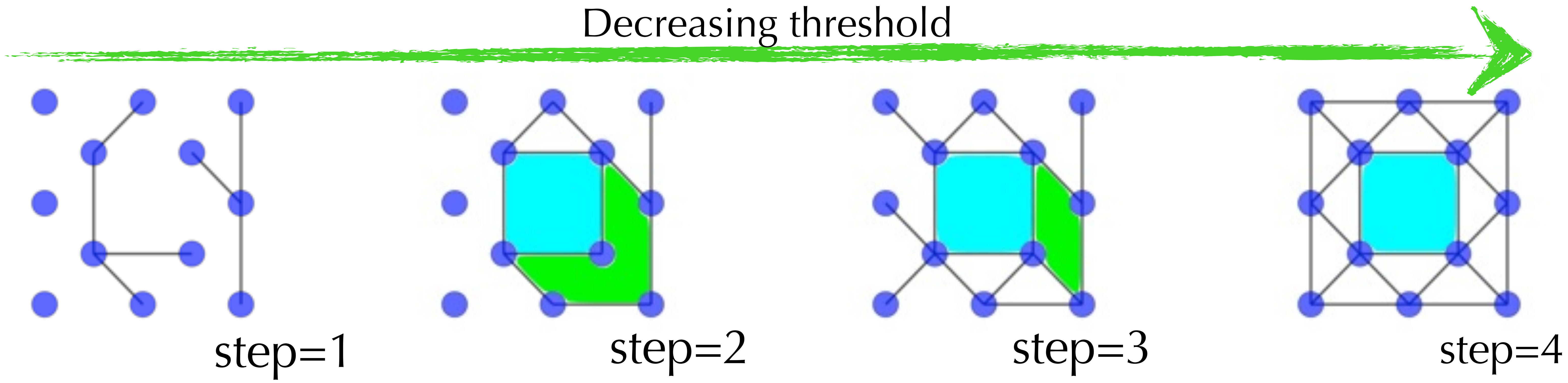
&

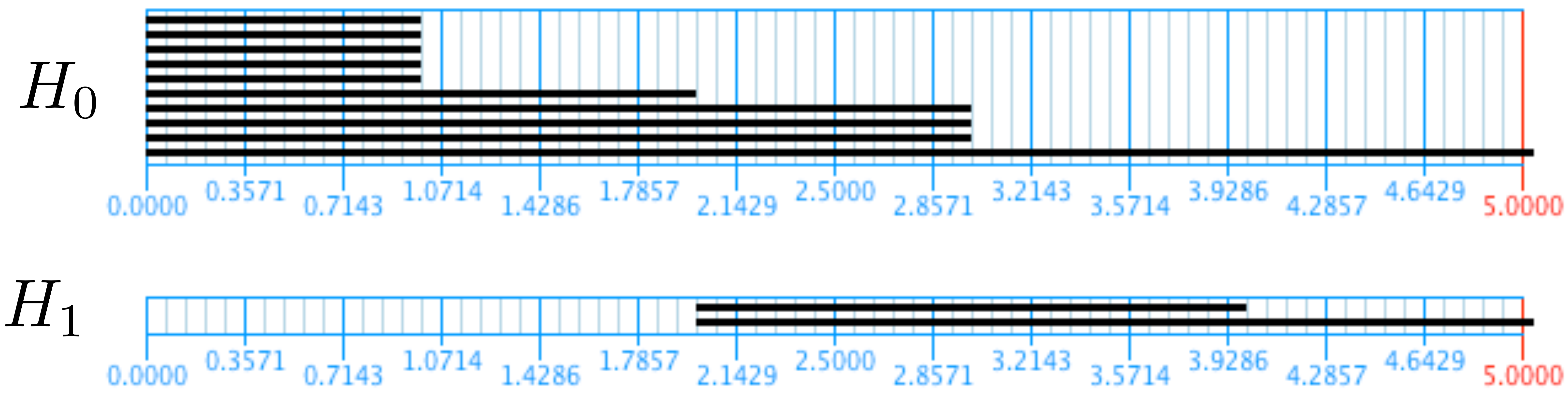
Weight structure

?

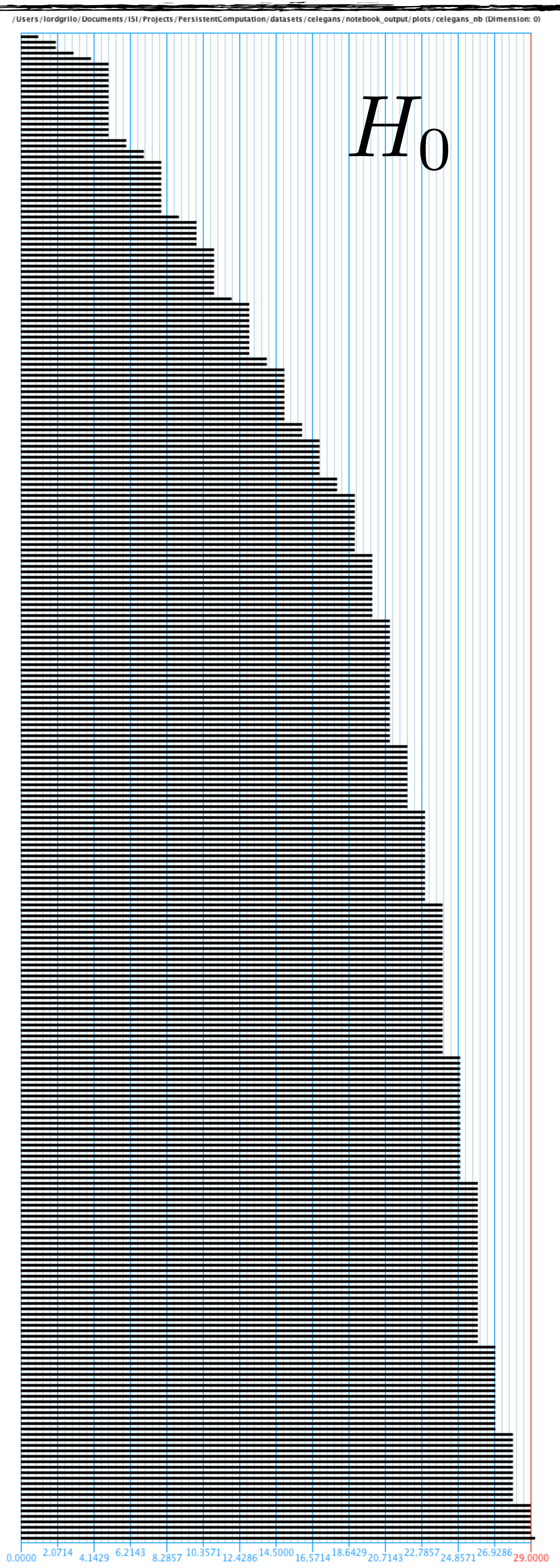
Weighted clique filtration I

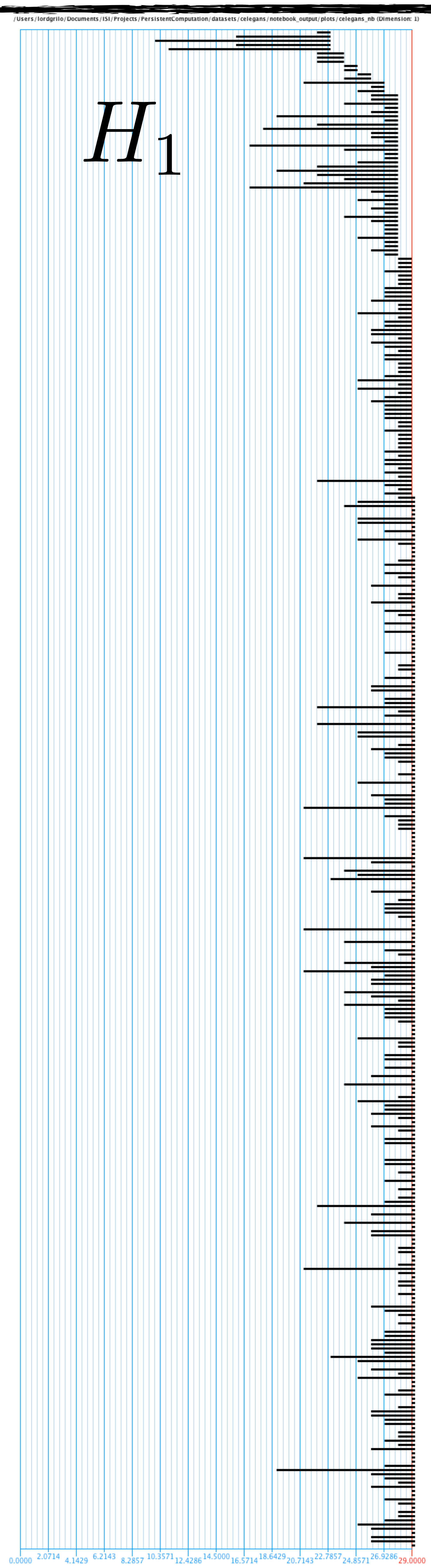
1) Given a real network G=(V,E), consider another one with the same number of node N=|V| and no edges.
2) Start adding edges from G, in decreasing order of weight.
3) At each step, calculate the associated clique complex.

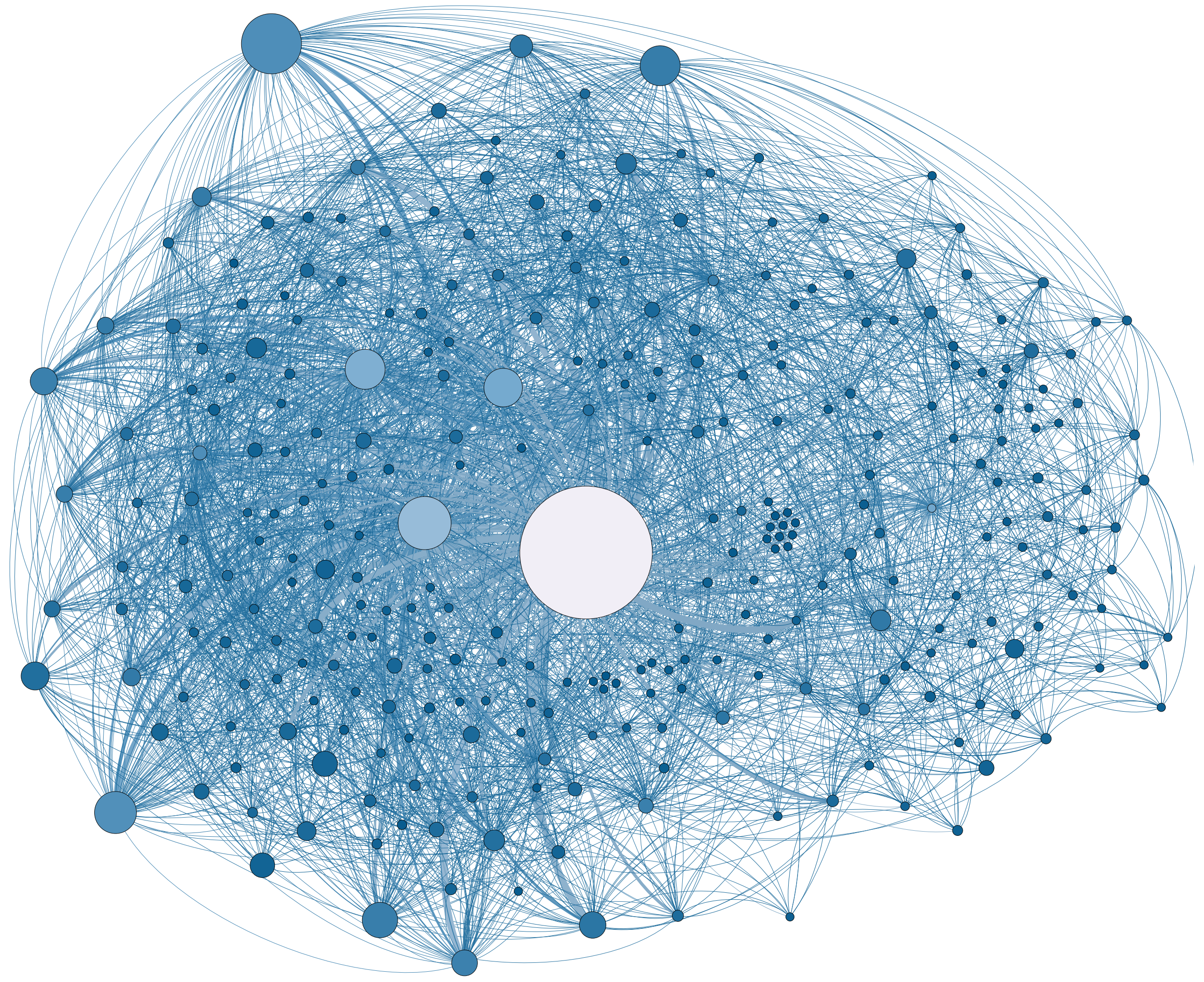




Weighted clique filtration II$_{\text{bis}}$

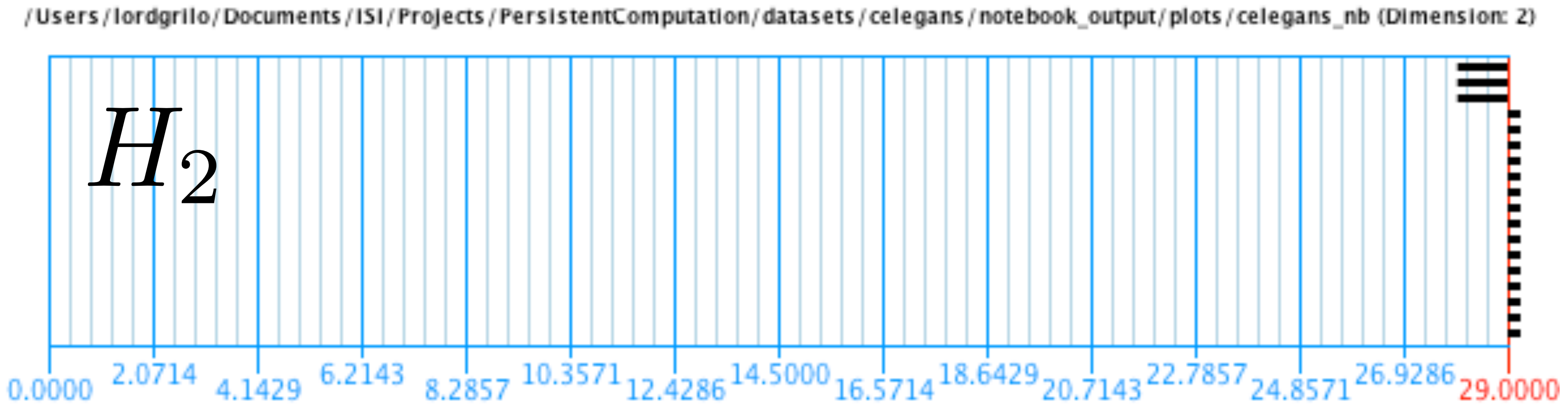




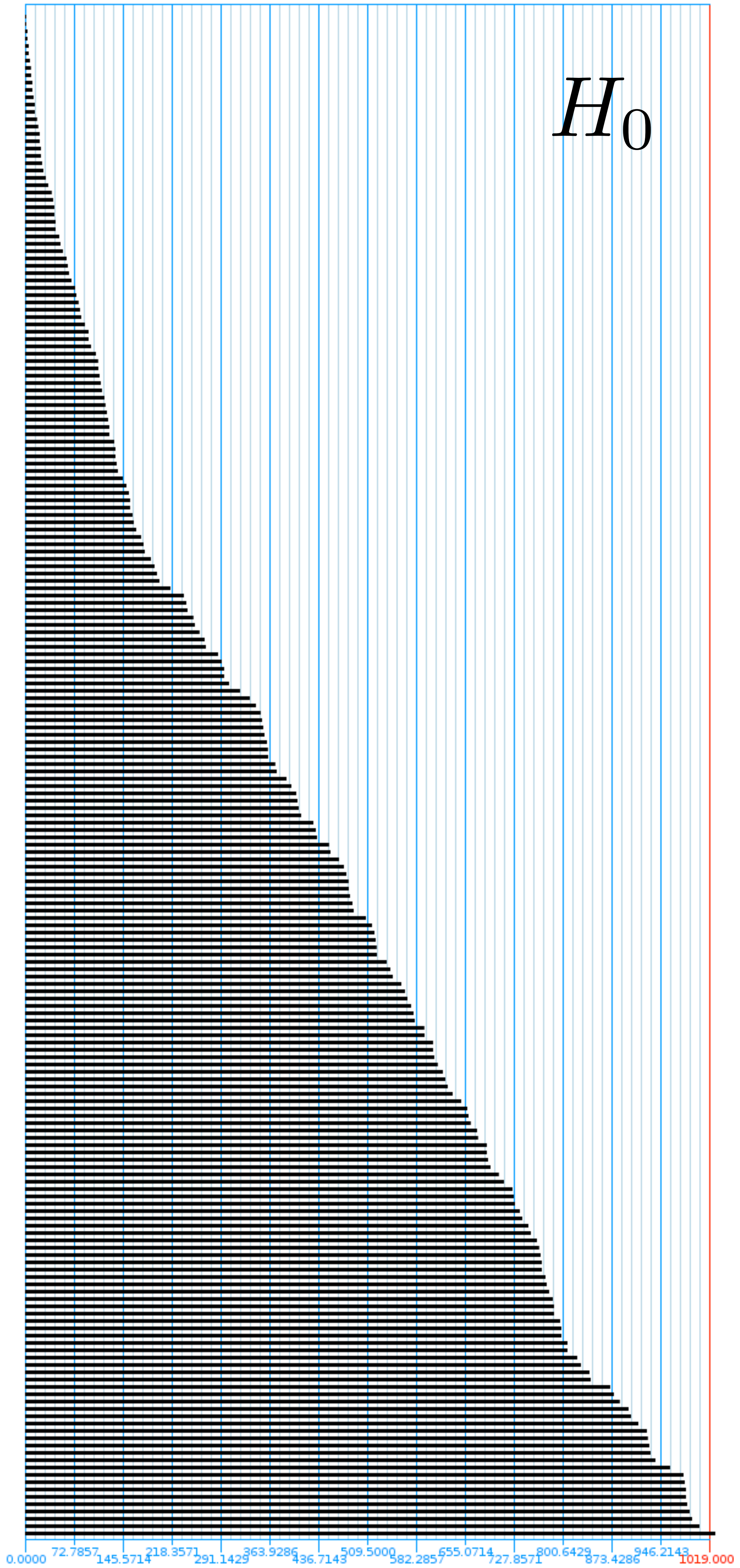


Much more structure appears!

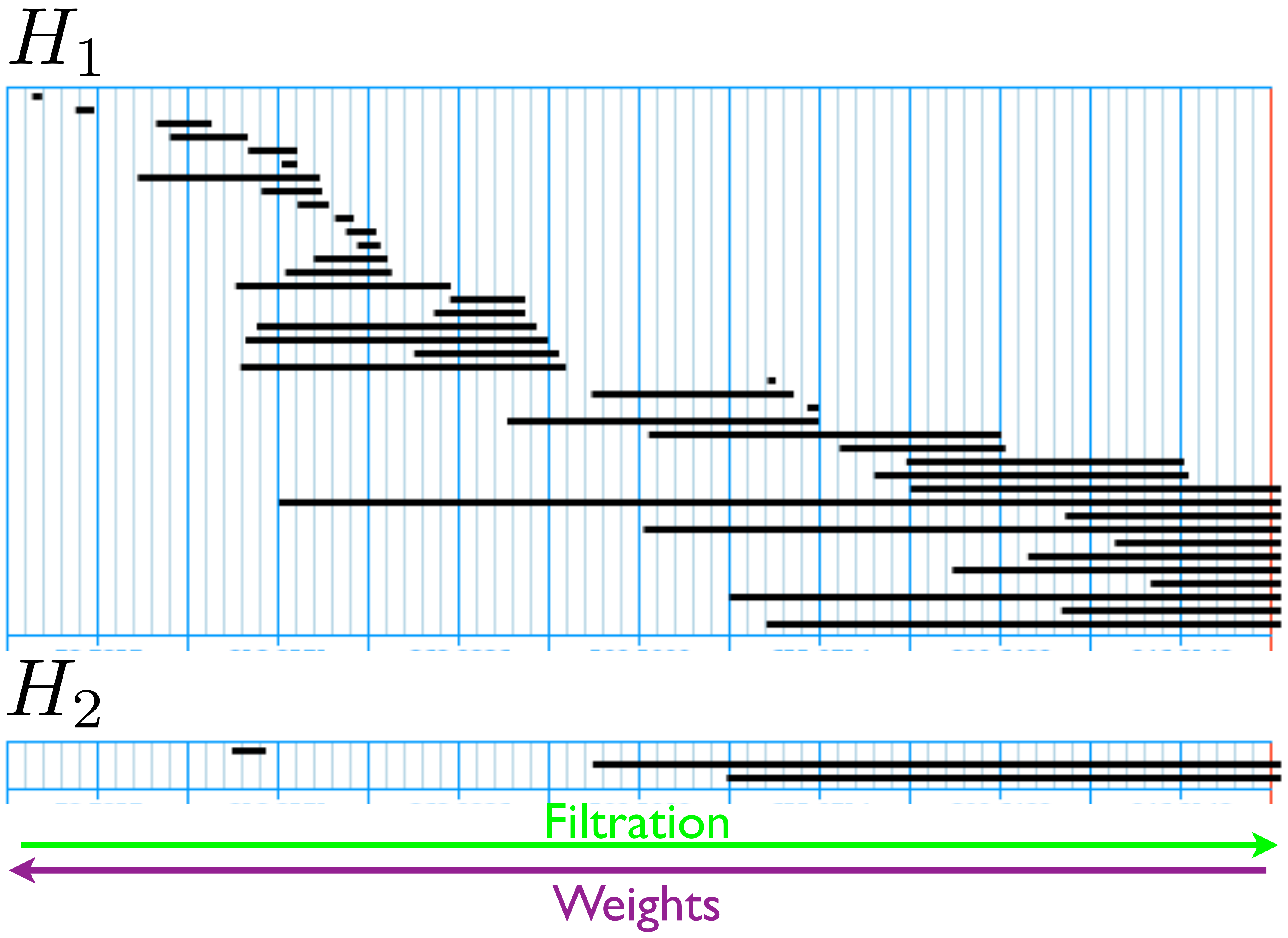
- Conn. comps. smoothly coalesce.
- A large number of loops.
- Even some “3D”-holes for low weights.



Weighted clique filtration II

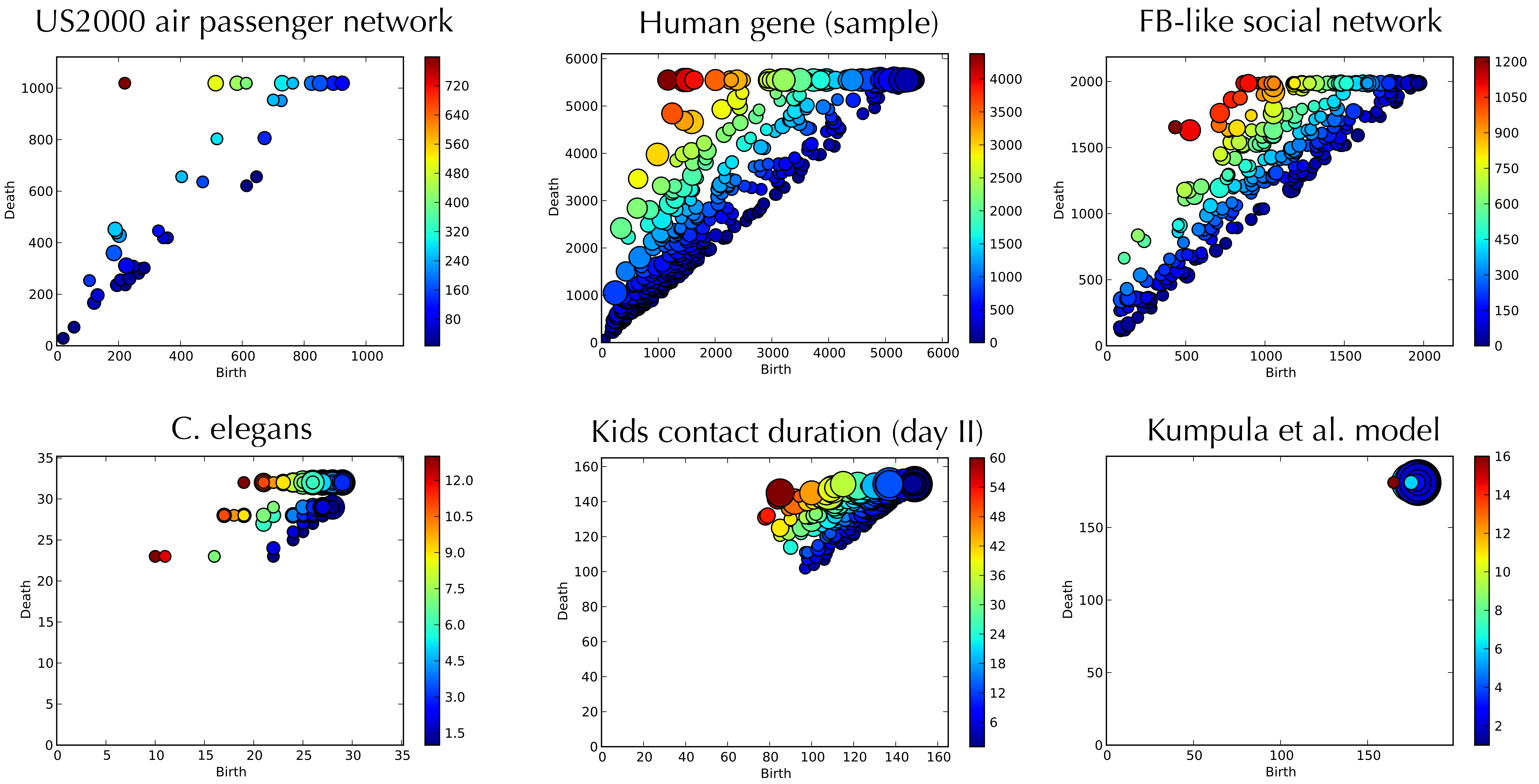


H_0	Connected components
H_1	1D Cycles (2-simplexes)
H_2	3D holes (3-simplexes)



Notebook 08-09

Are there homological features in graphs?

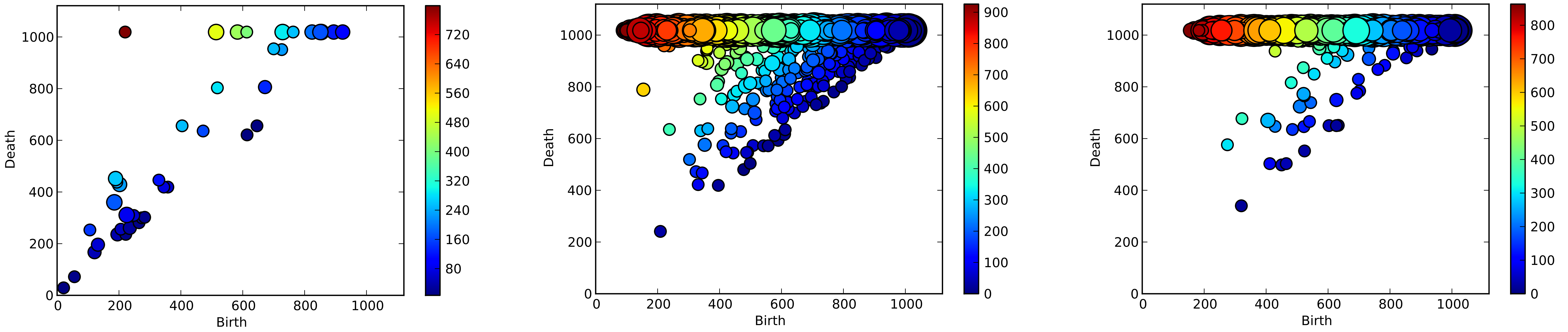


$$H_1$$

Are there homological features in graphs?

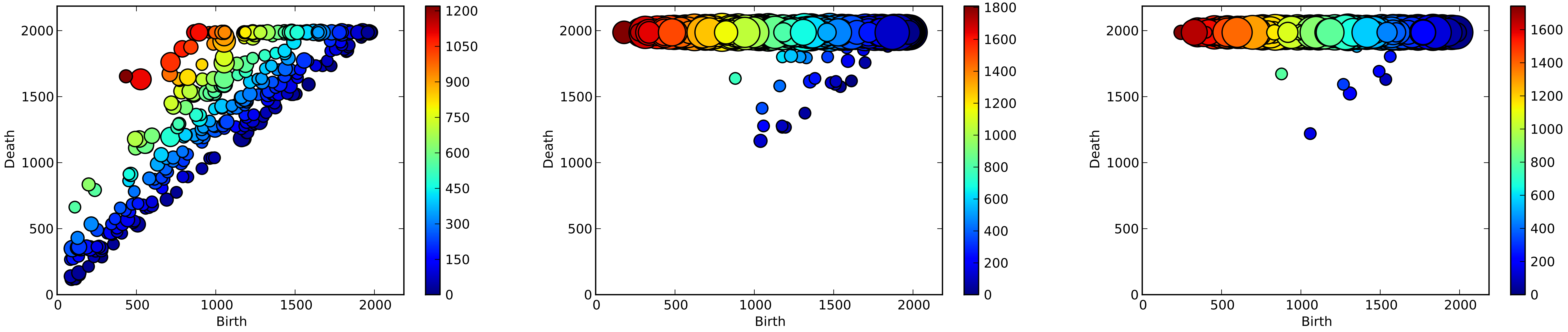
US 2000 air passenger network

Data | Weight shuffled | Degree-preserving



FB-like social network

Data | Weight shuffled | Degree-preserving

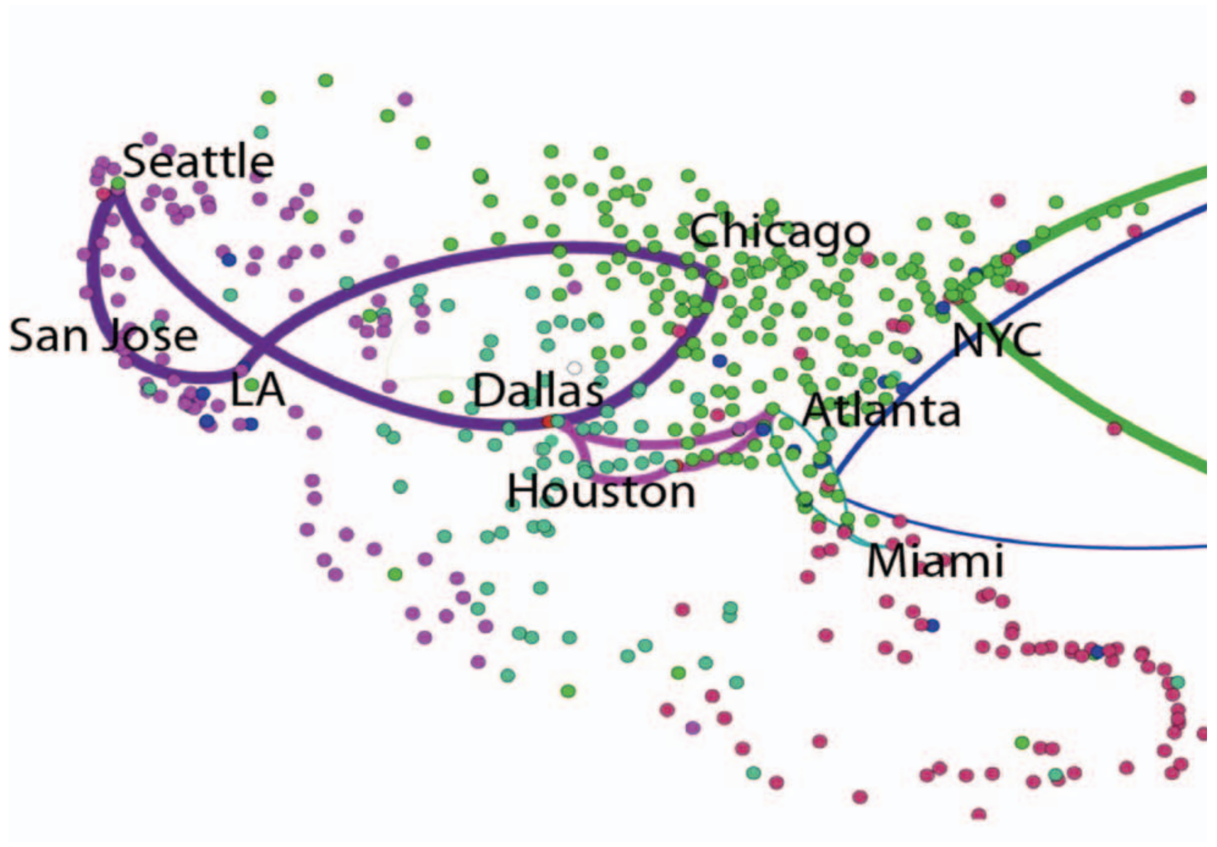


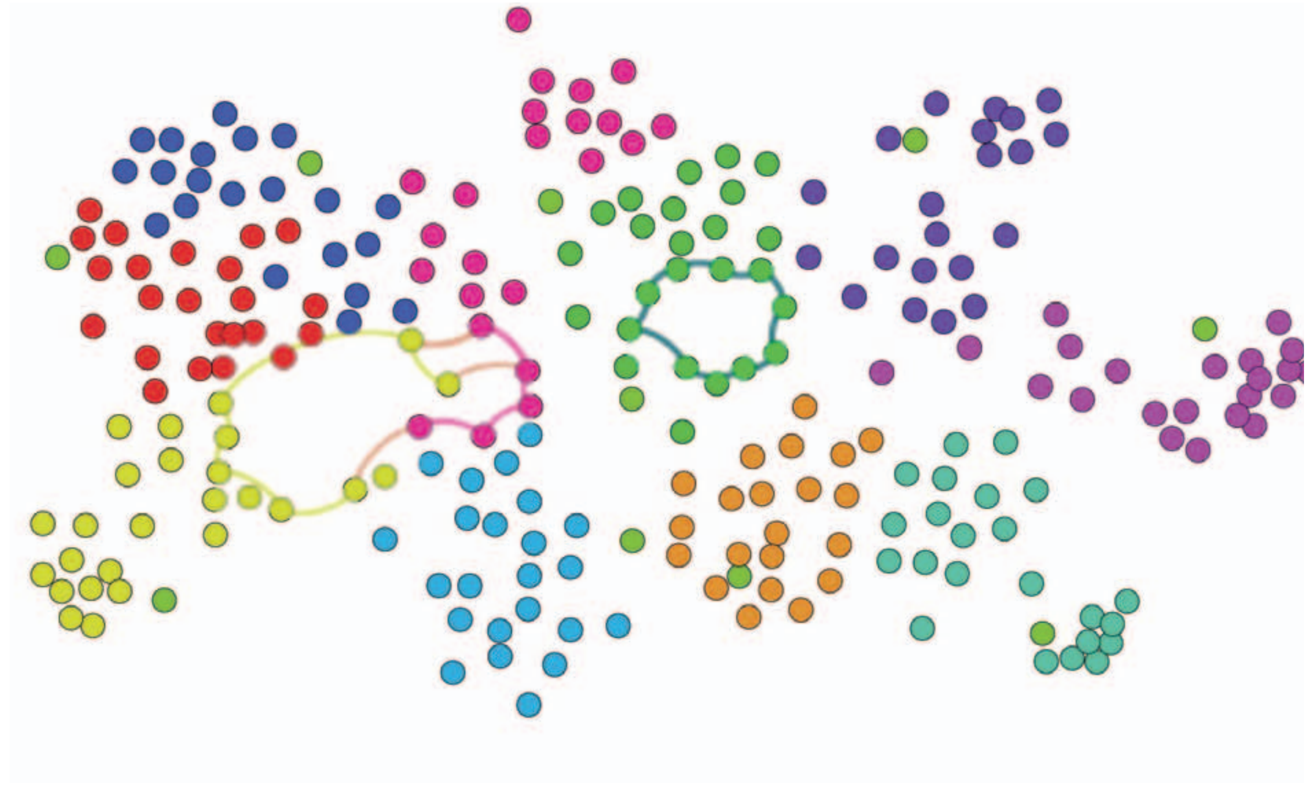
Topological cycles are not simply reproduced by statistical network properties

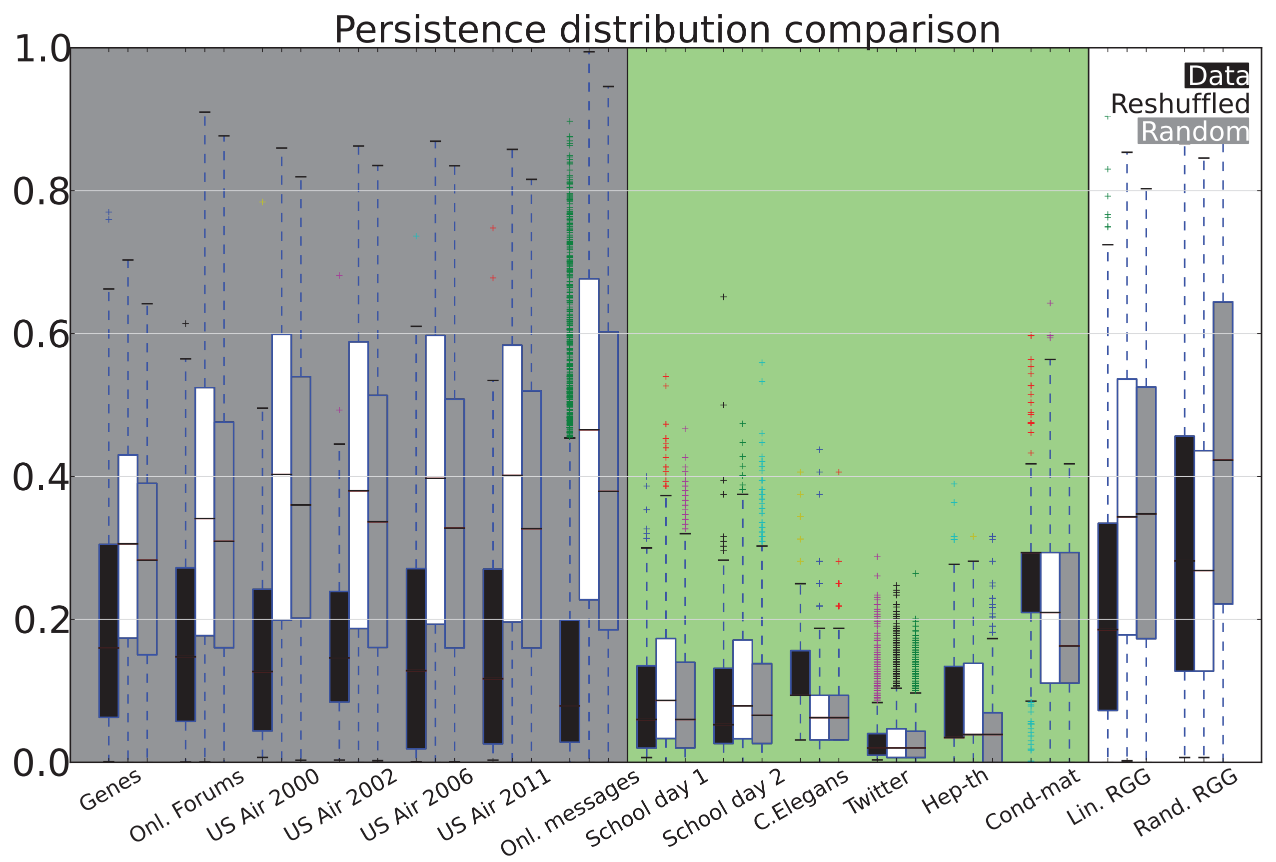
Are there homological features in graphs?

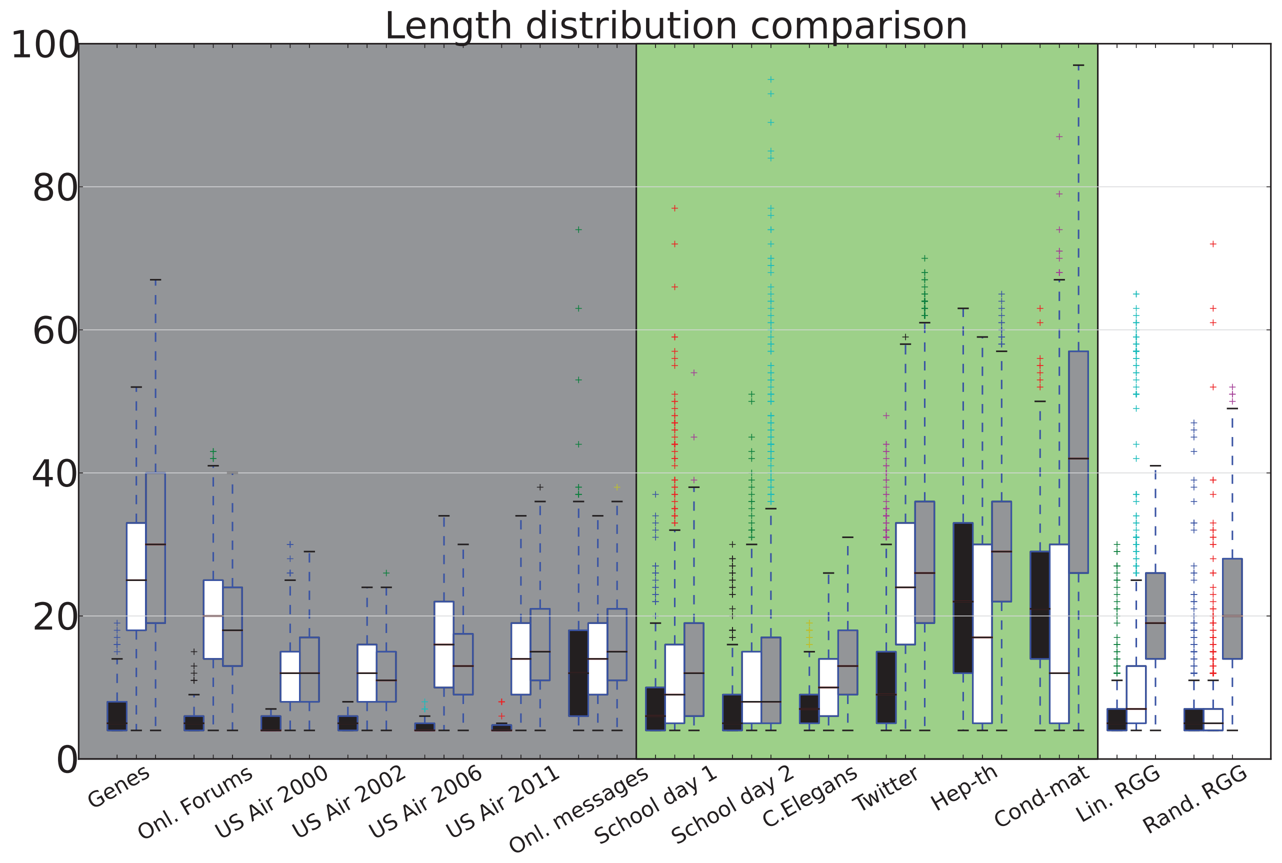
Are there homological features in graphs?

LANGUAGE







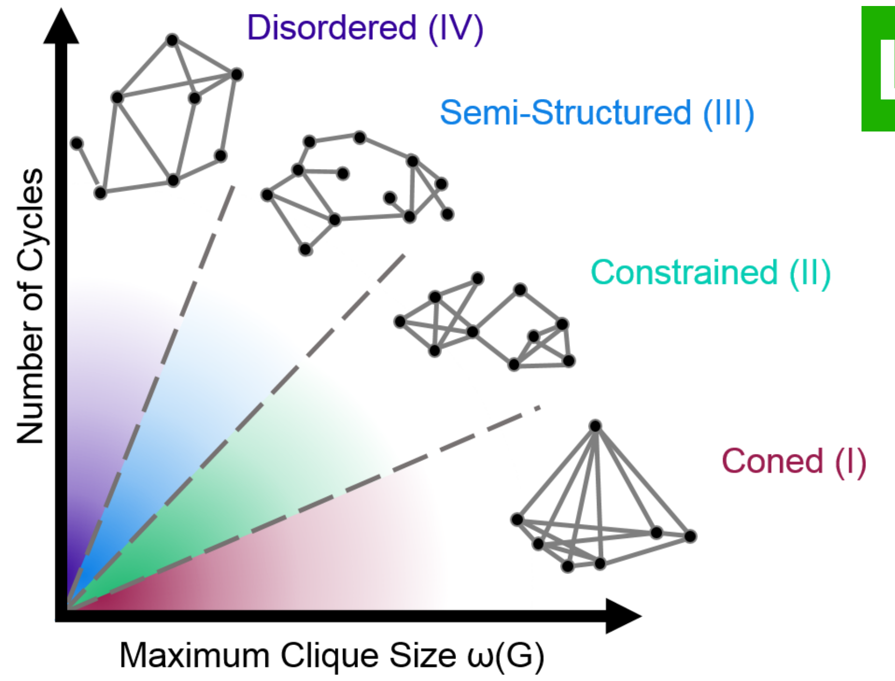


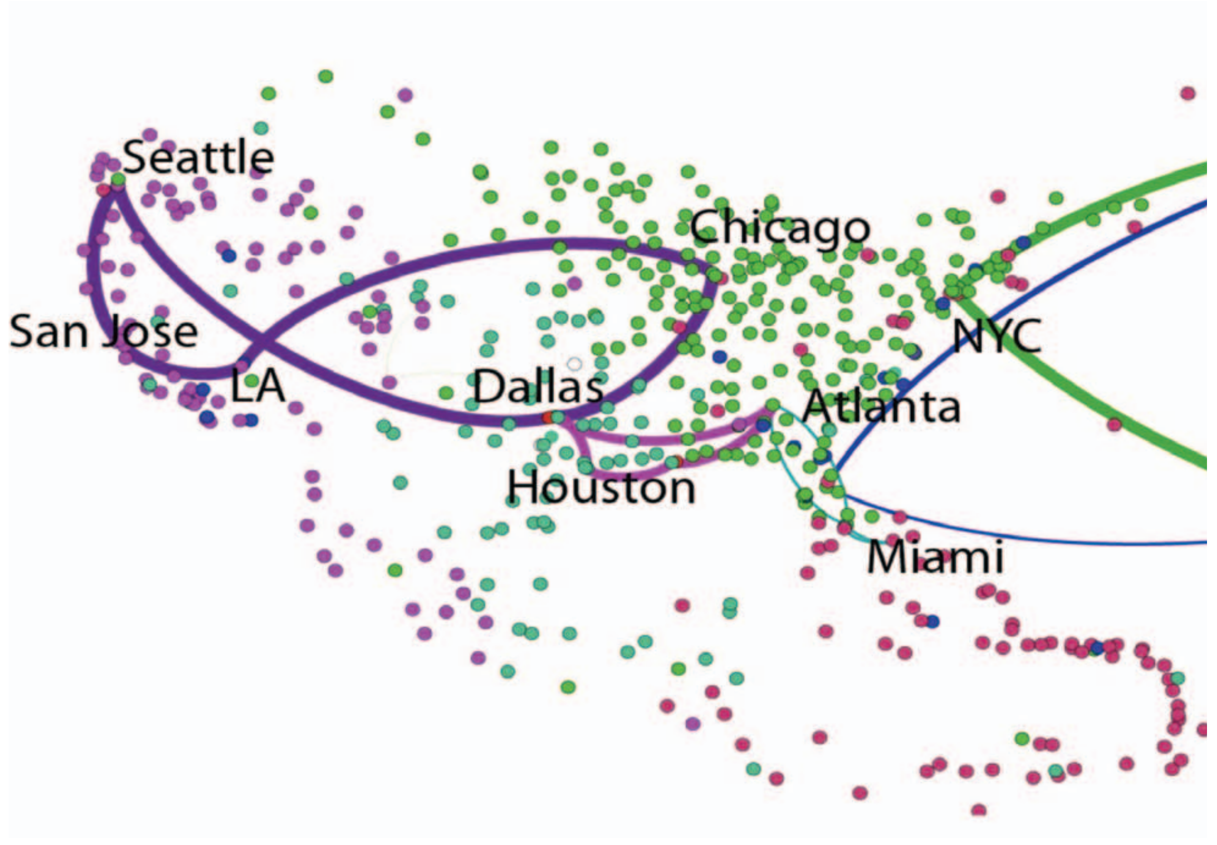
Petri, Giovanni, et al. "Topological strata of weighted complex networks."
PloS one 8.6 (2013): e66506.

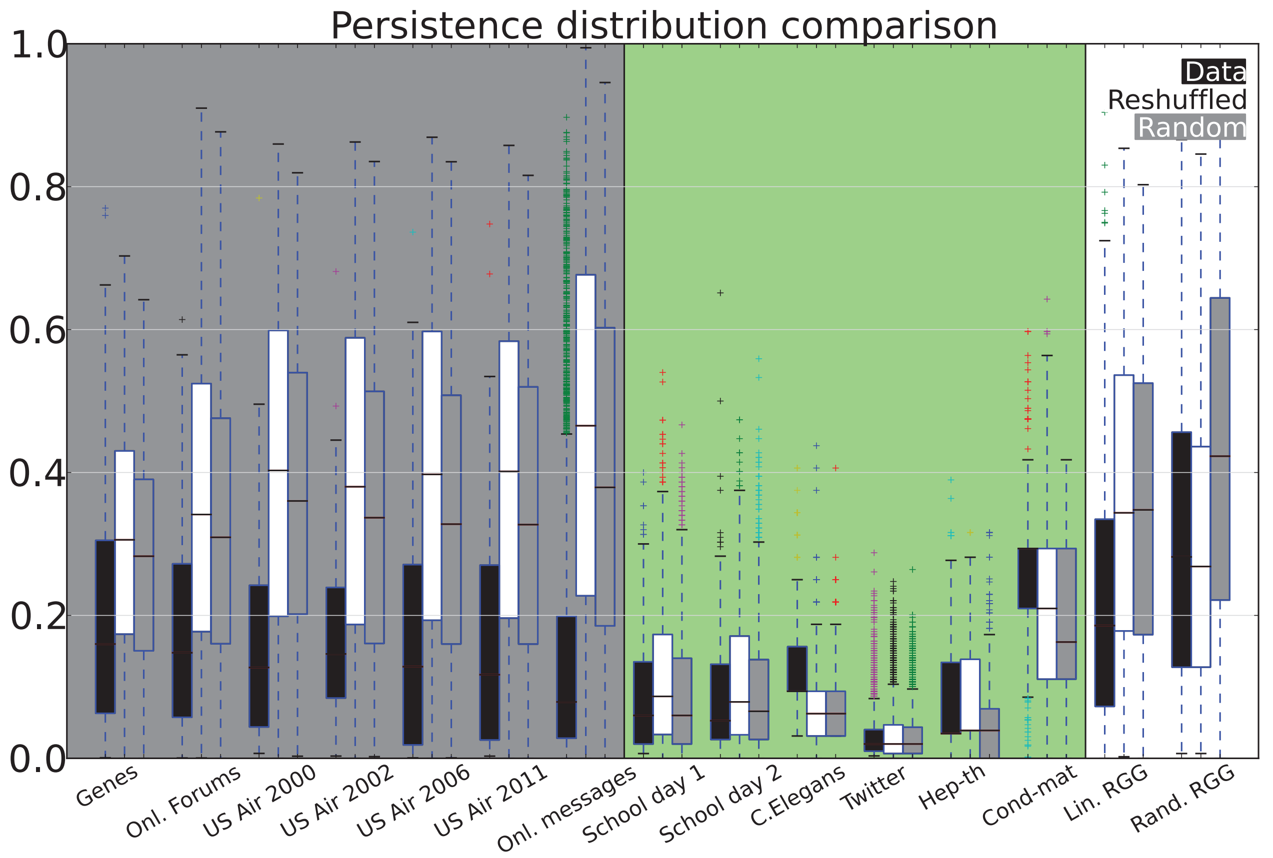
PHOM

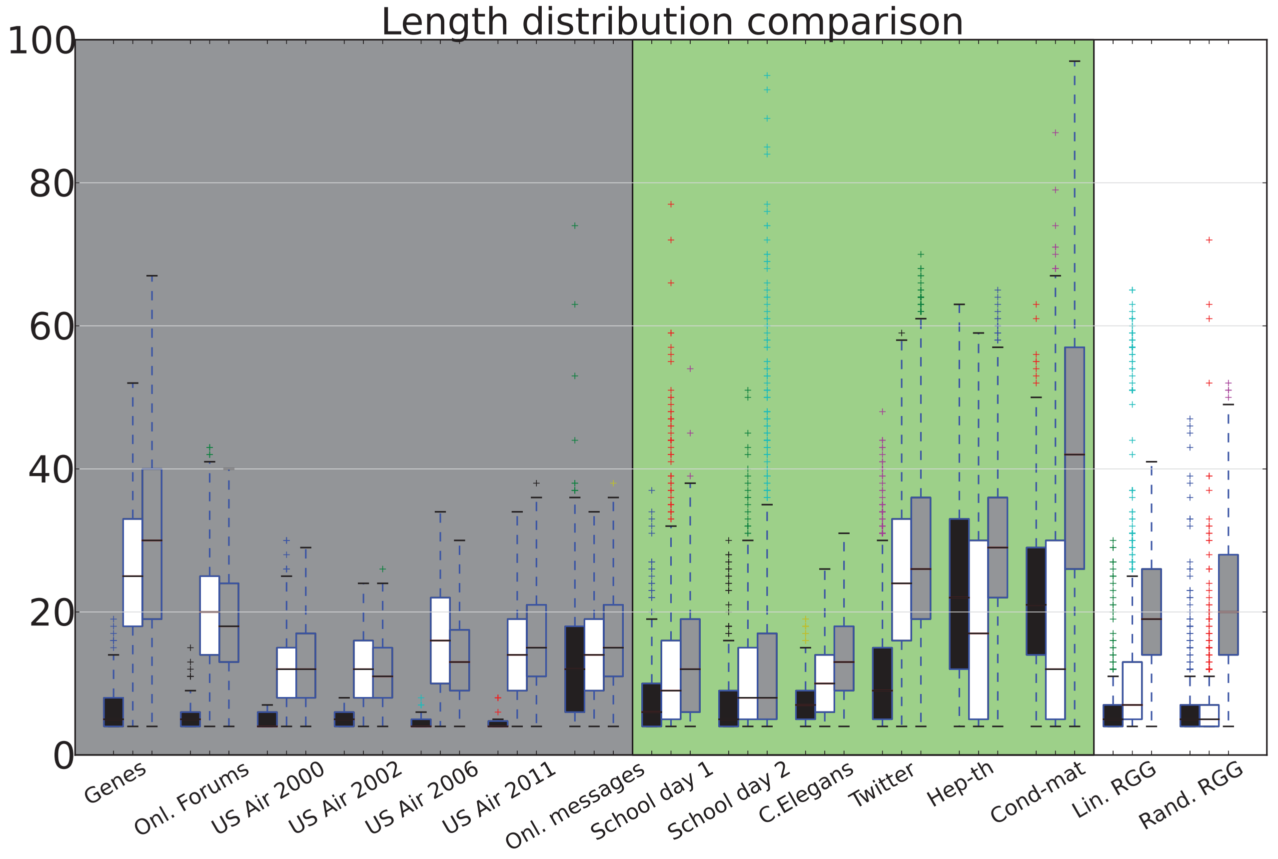
Are there homological features in graphs?

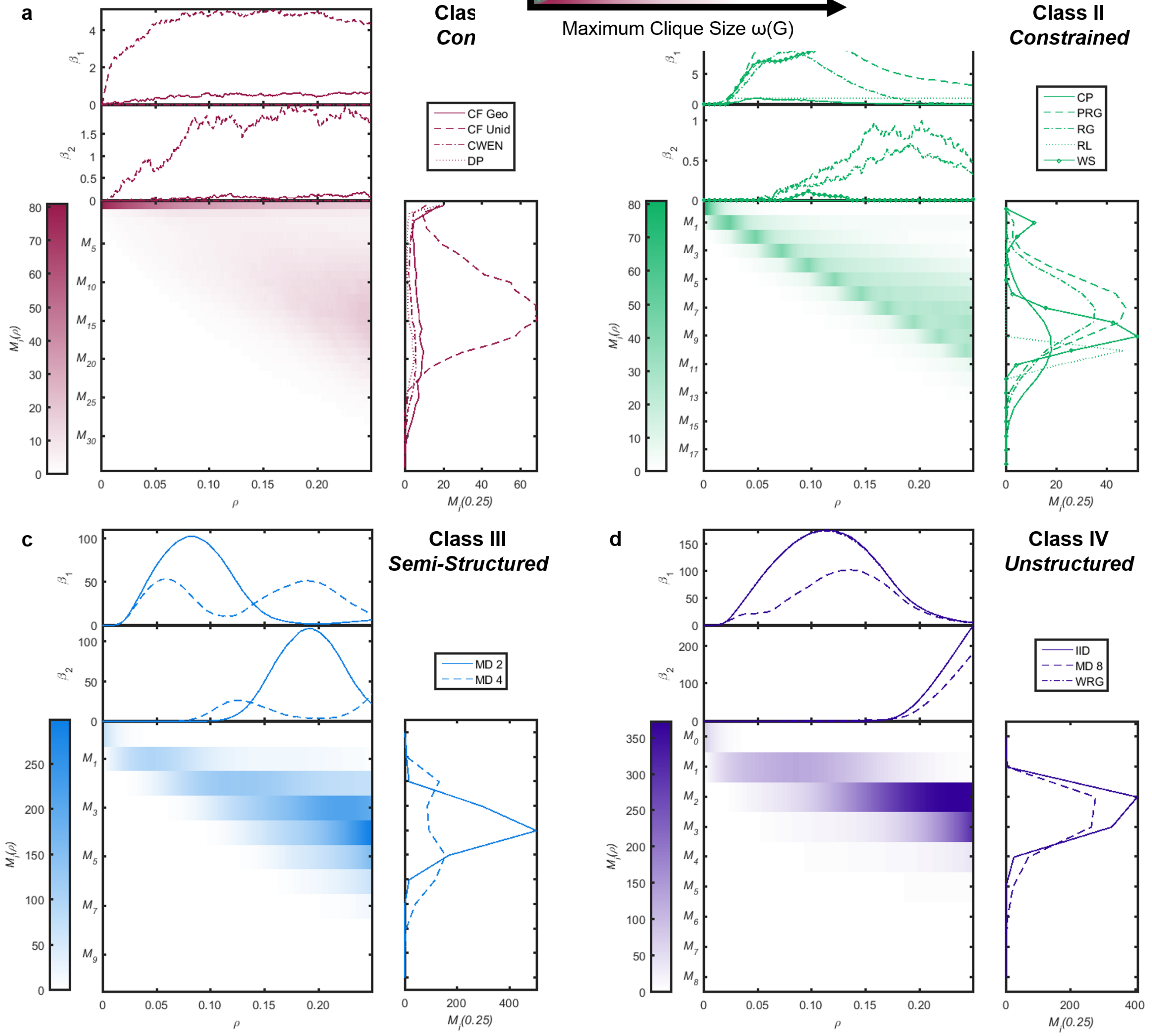
LANGUAGE











Petri, Giovanni, et al. "Topological strata of weighted complex networks." *PloS one* 8.6 (2013): e66506.

Sizemore, Ann, Chad Giusti, and Danielle S. Bassett. "Classification of weighted networks through mesoscale homological features." *Journal of Complex Networks* 5.2 (2016): 245-273.

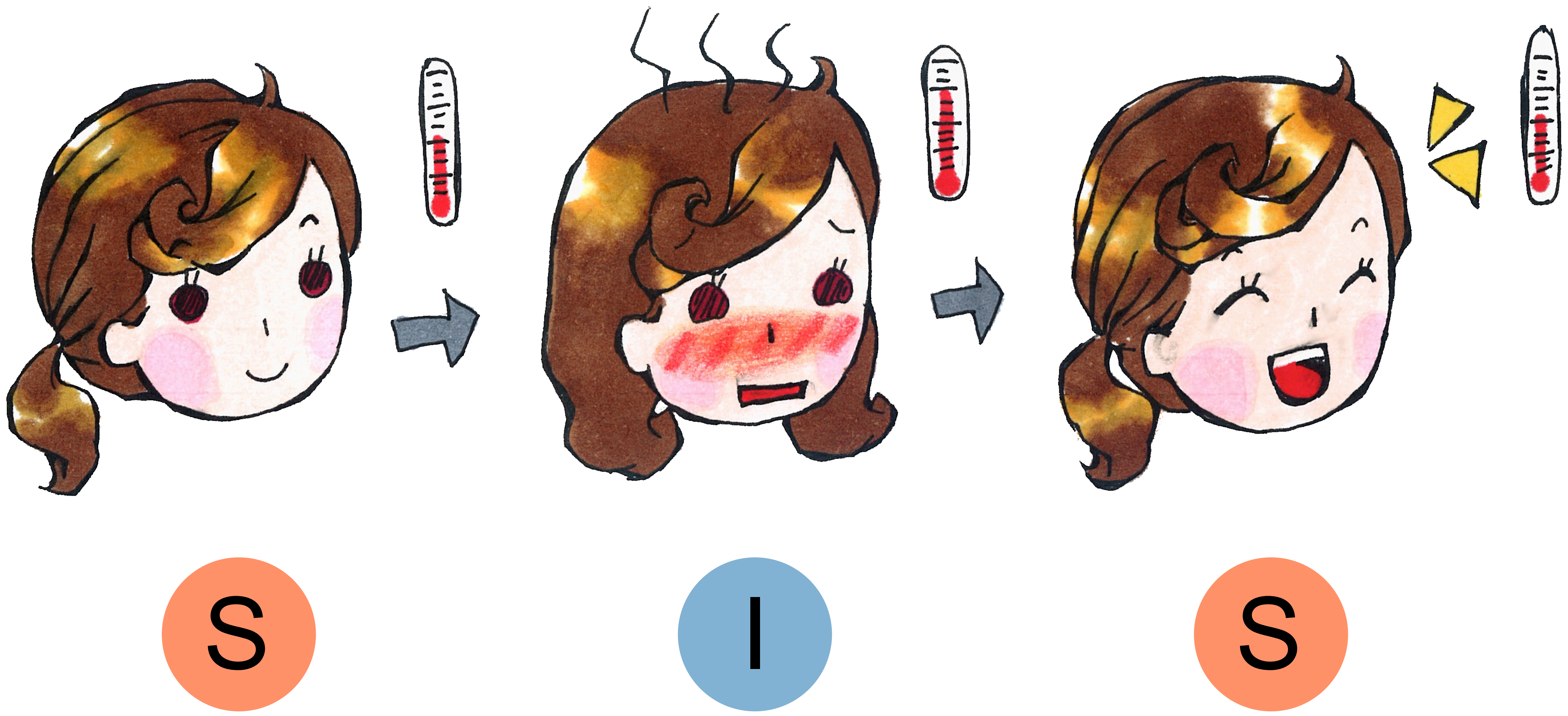
PHOM

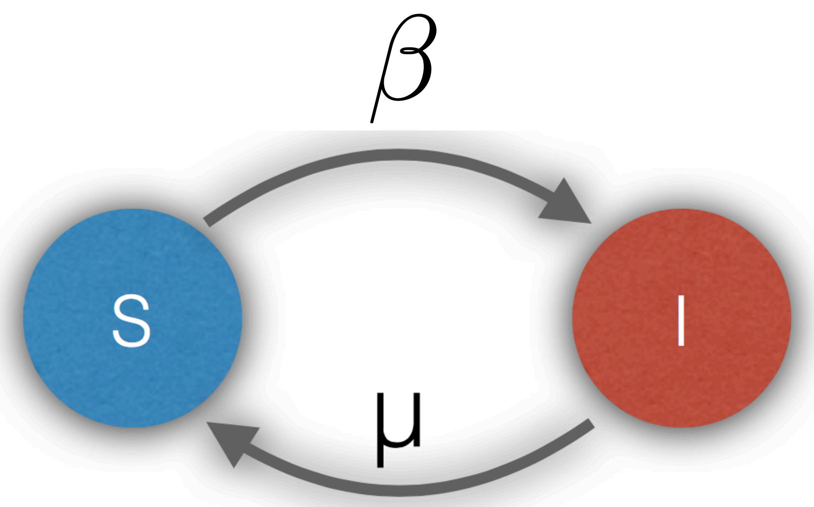
Break!!

Higher-order mechanisms

Simple Contagion

Spreading of infectious diseases

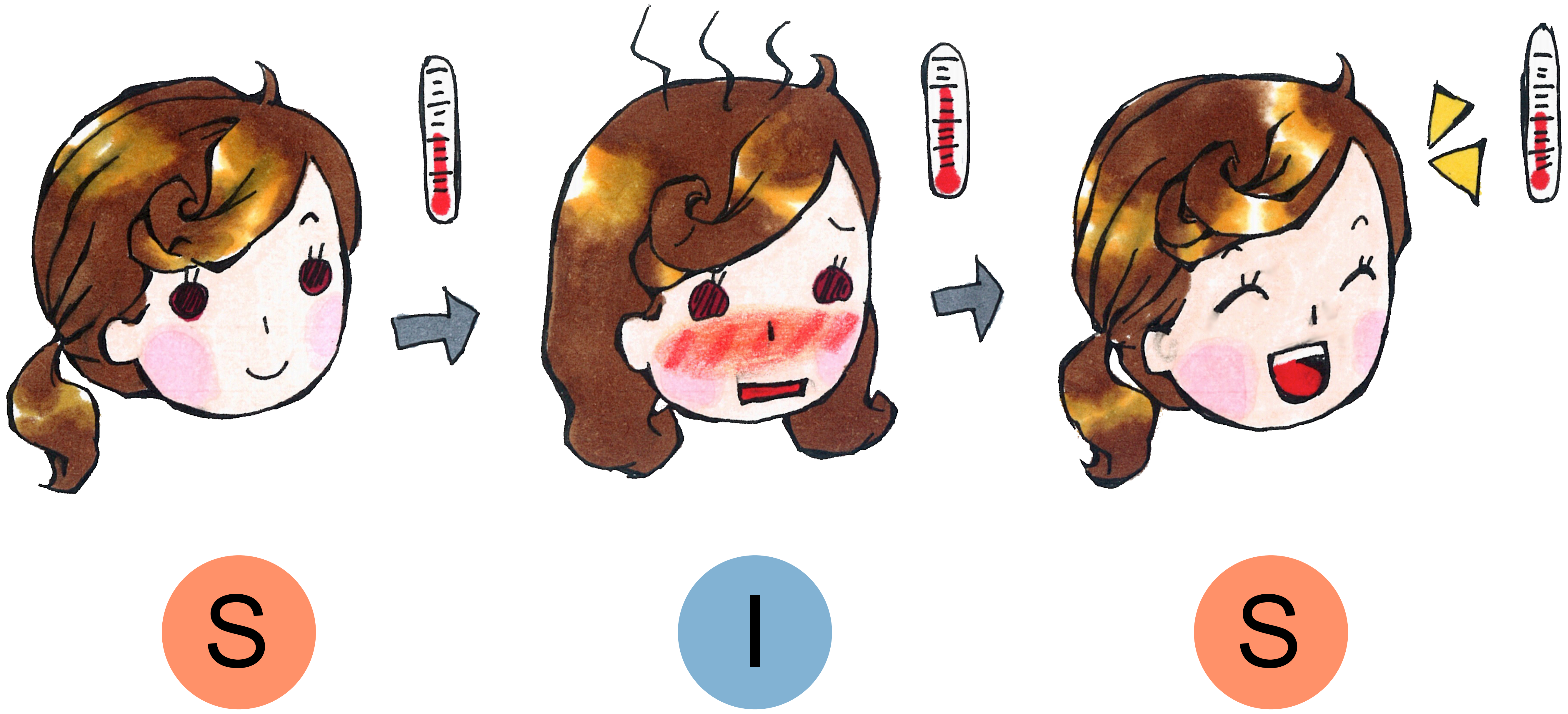


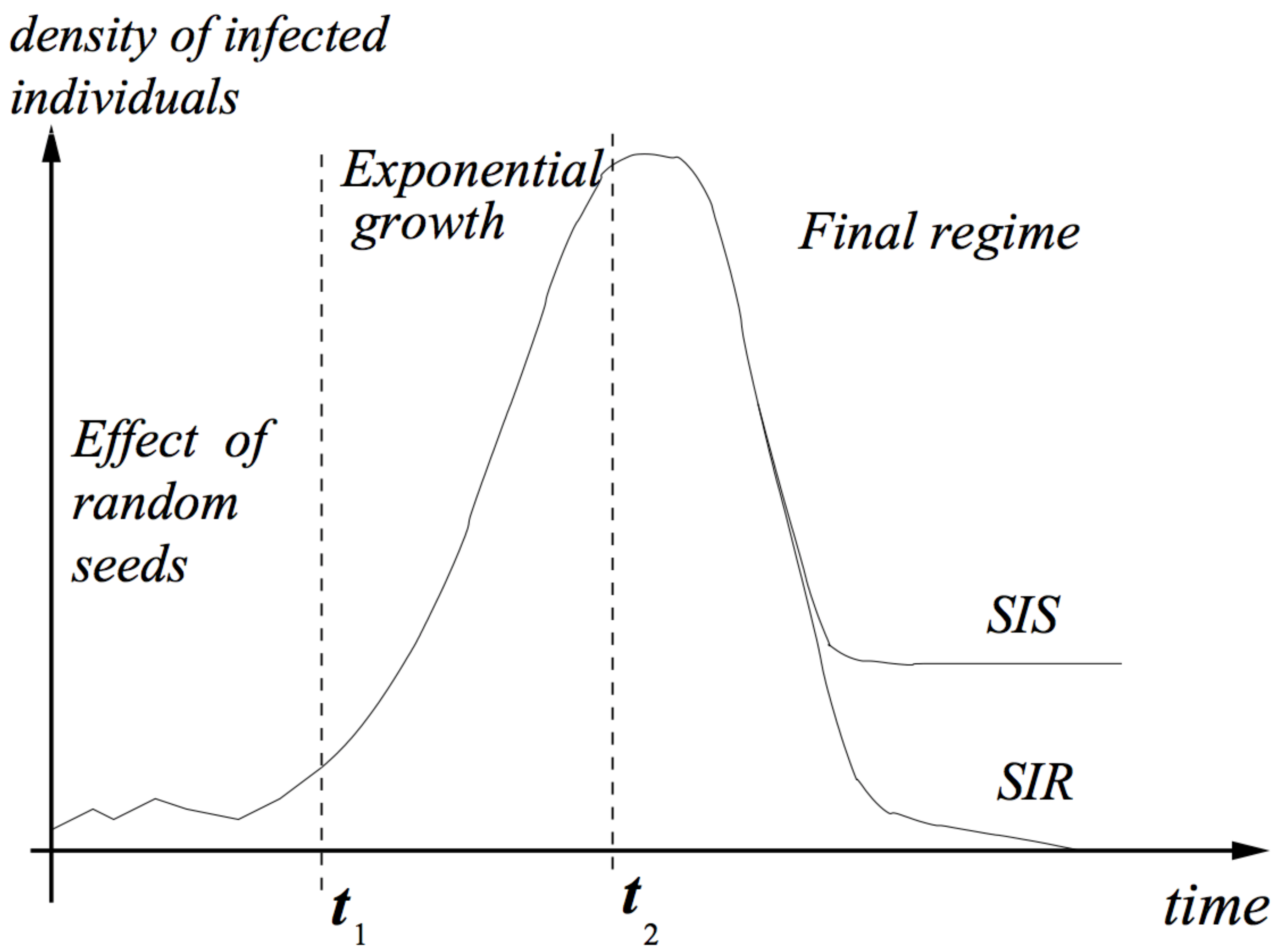


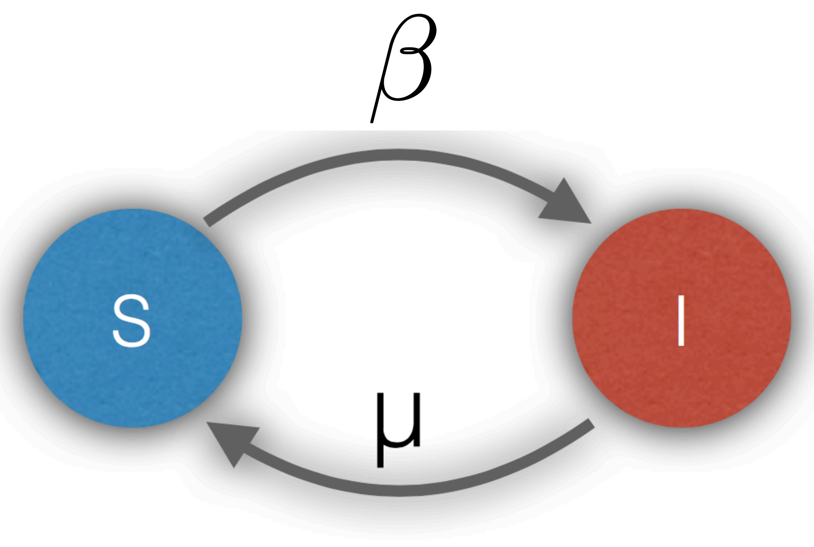
Artwork by Mi Jin Lee

Simple Contagion

Spreading of infectious diseases



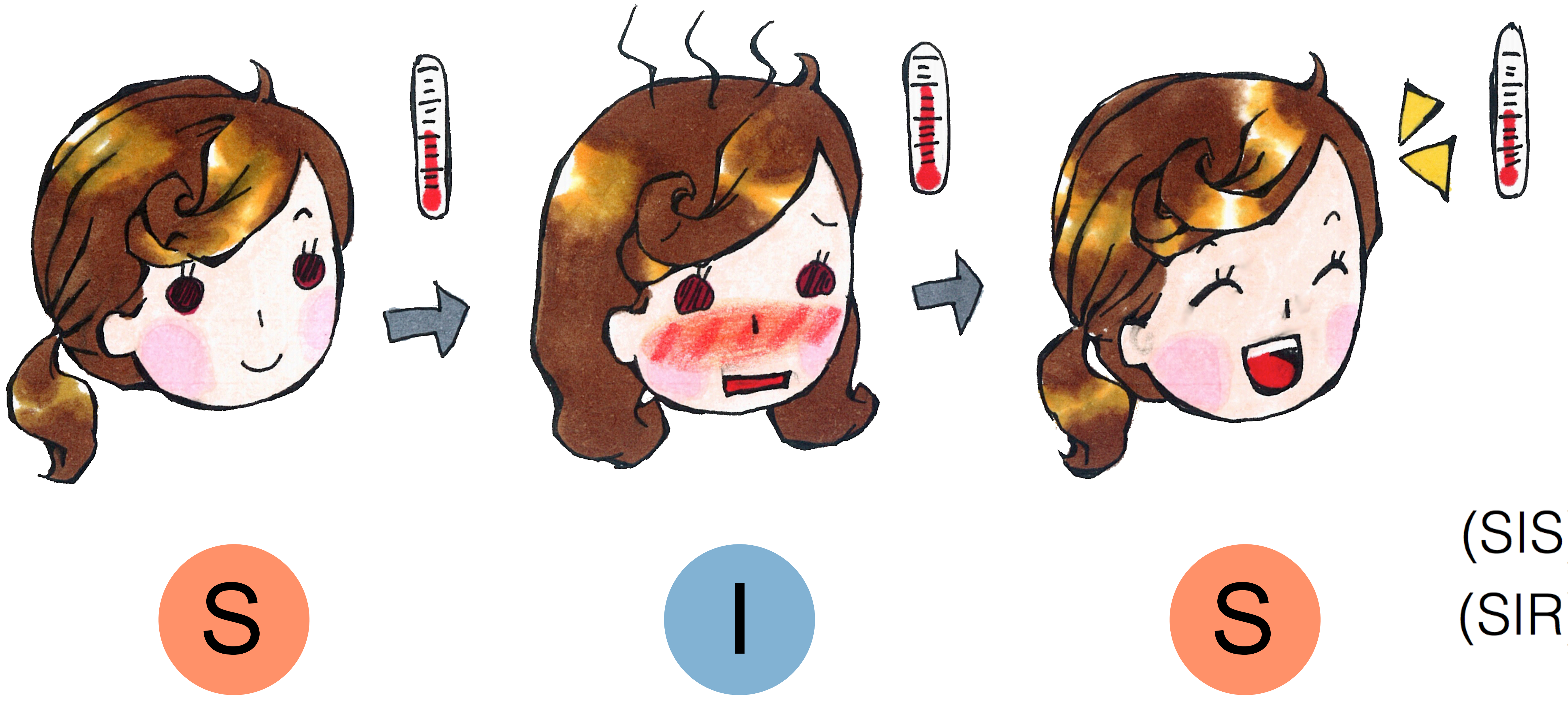


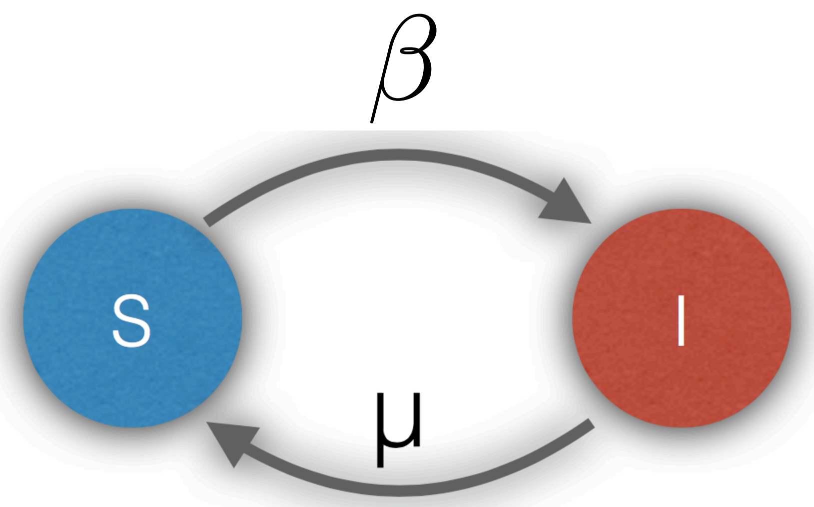


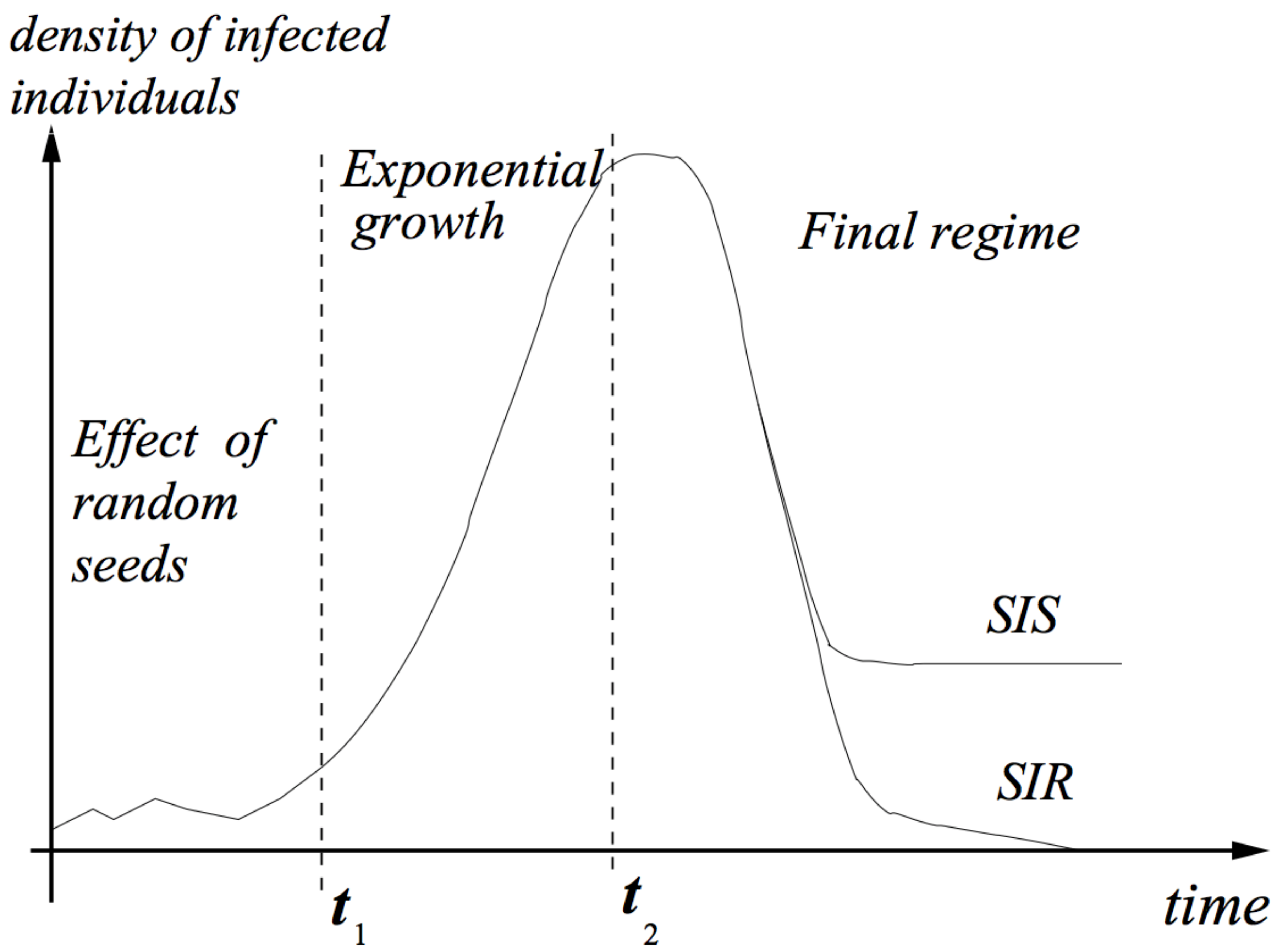
Artwork by Mi Jin Lee

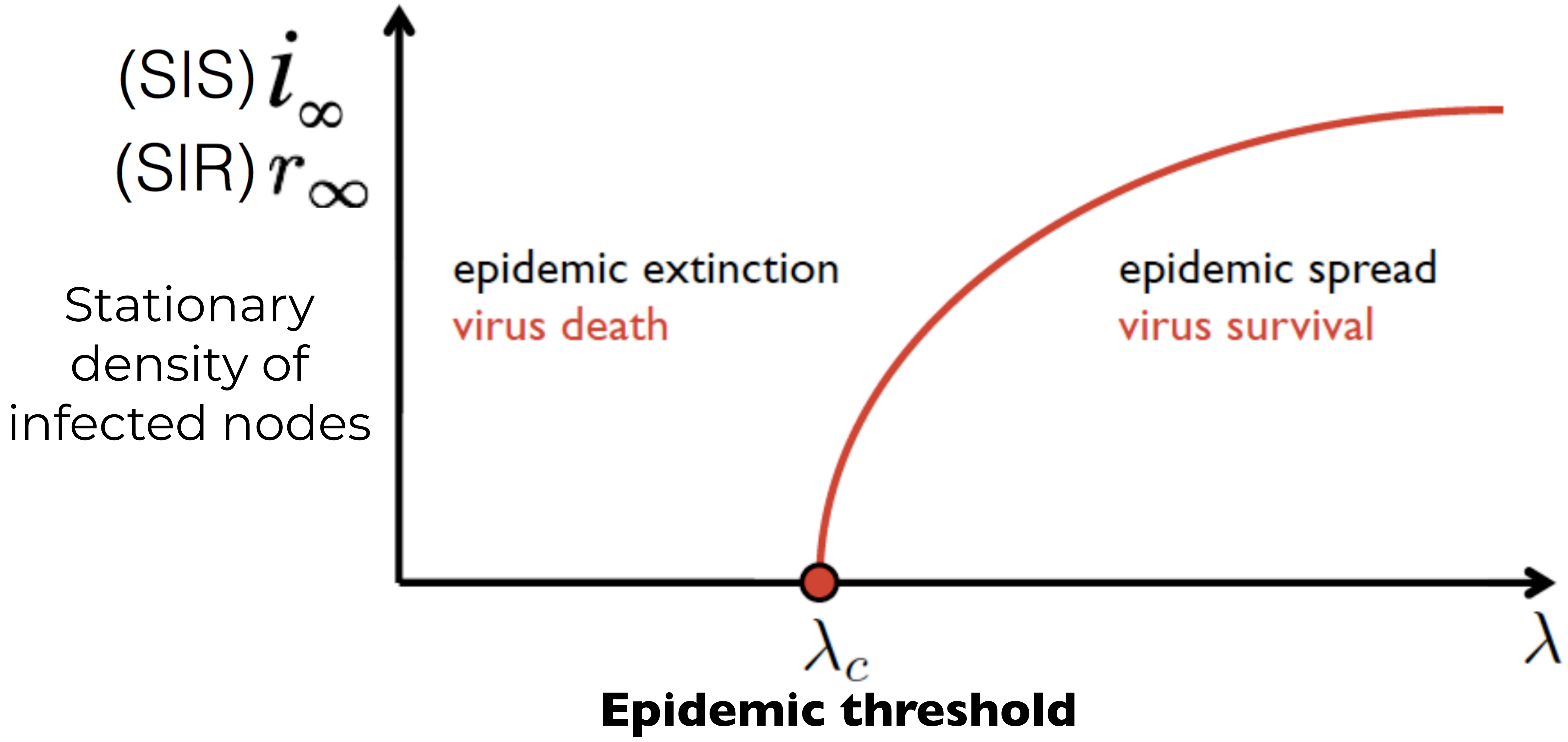
Simple Contagion

Spreading of infectious diseases









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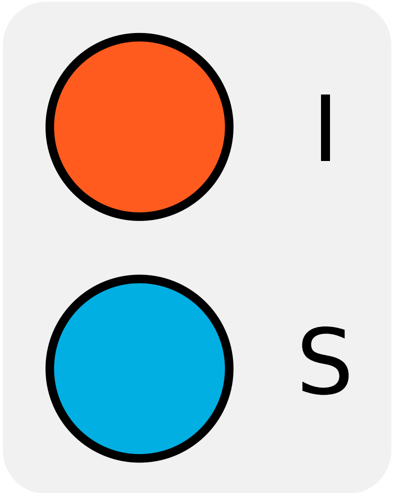
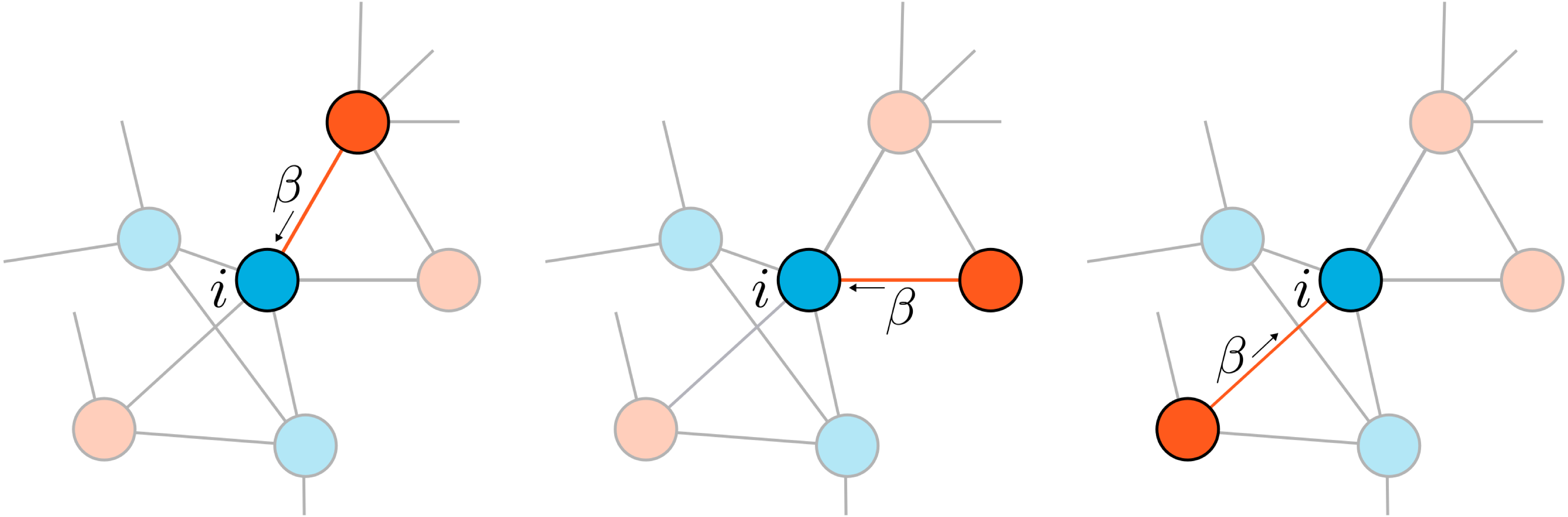
Simple Contagion

Spreading of infectious diseases





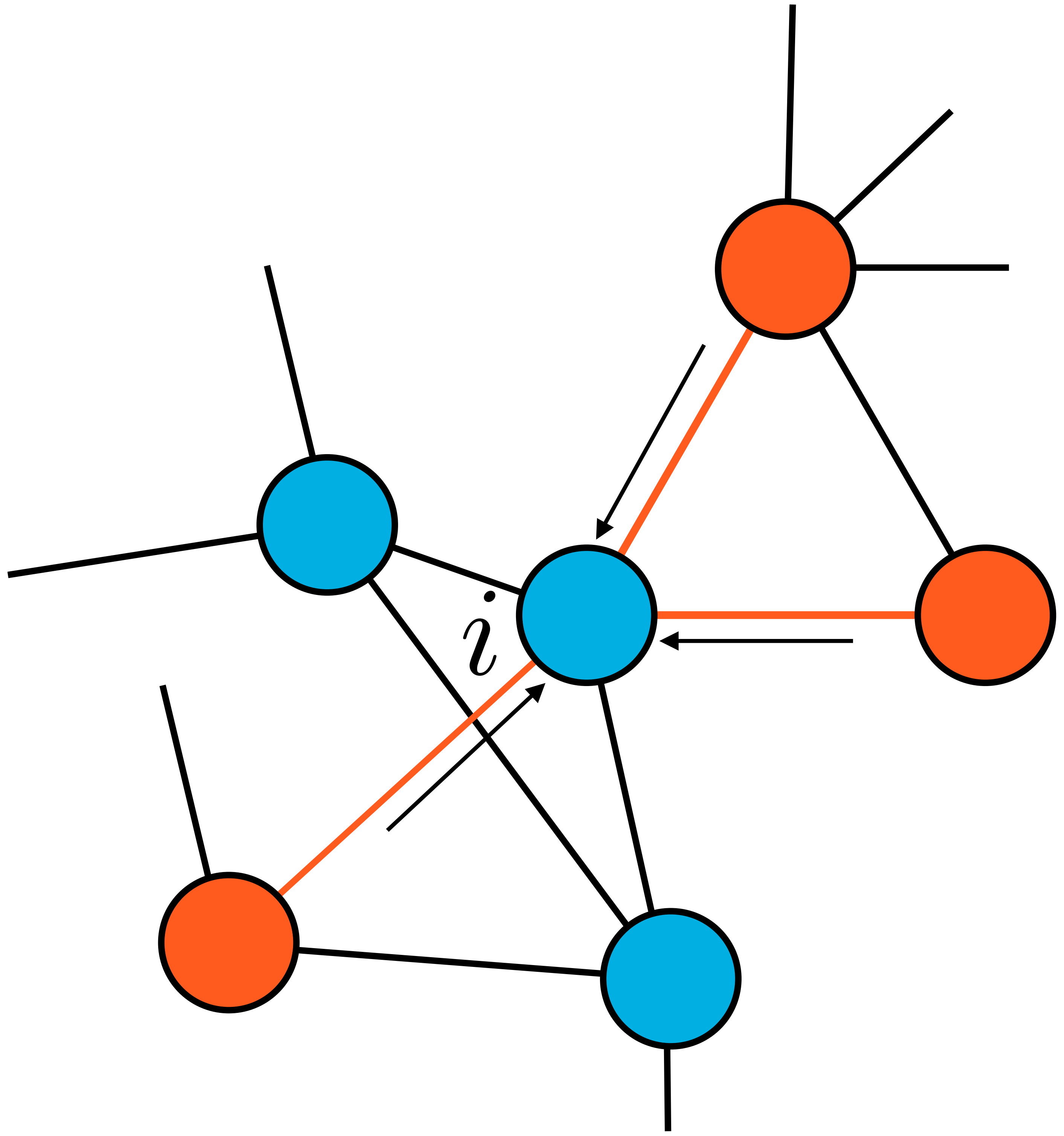
Independent
sources of infection

I+S $\longrightarrow$ 2I

β: probability of infection

Complex Contagion

Social contagion



Multiple sources of activation are required for a transmission

REPORT

The Spread of Behavior in an Online Social Network Experiment

Damon Centola

+ See all authors and affiliations

Science 03 Sep 2010:
Vol. 329, Issue 5996, pp. 1194-1197
DOI: 10.1126/science.1185231

JOURNAL OF THE ROYAL SOCIETY Interface

Complex contagion process in spreading of online innovation

Márton Karsai[1,2,3,4], Gerardo Iñiguez[2], Kimmo Kaski[2,5] and János Kertész[2,6,7]

Structural diversity in social contagion

Johan Ugander, Lars Backstrom, Cameron Marlow, and Jon Kleinberg

PNAS April 17, 2012 109 (16) 5962-5966; https://doi.org/10.1073/pnas.1116502109

Edited by Ronald L. Graham, University of California at San Diego, La Jolla, CA, and approved February 21, 2012

PLOS | ONE

RESEARCH ARTICLE

Evidence of complex contagion of information in social media: An experiment using Twitter bots

Bjarke Mønsted[1☯], Piotr Sapieżyński[1☯], Emilio Ferrara[2,3☯], Sune Lehmann[1☯*]

PRL 115, 218702 (2015) PHYSICAL REVIEW LETTERS week ending 20 NOVEMBER 2015

Kinetics of Social Contagion

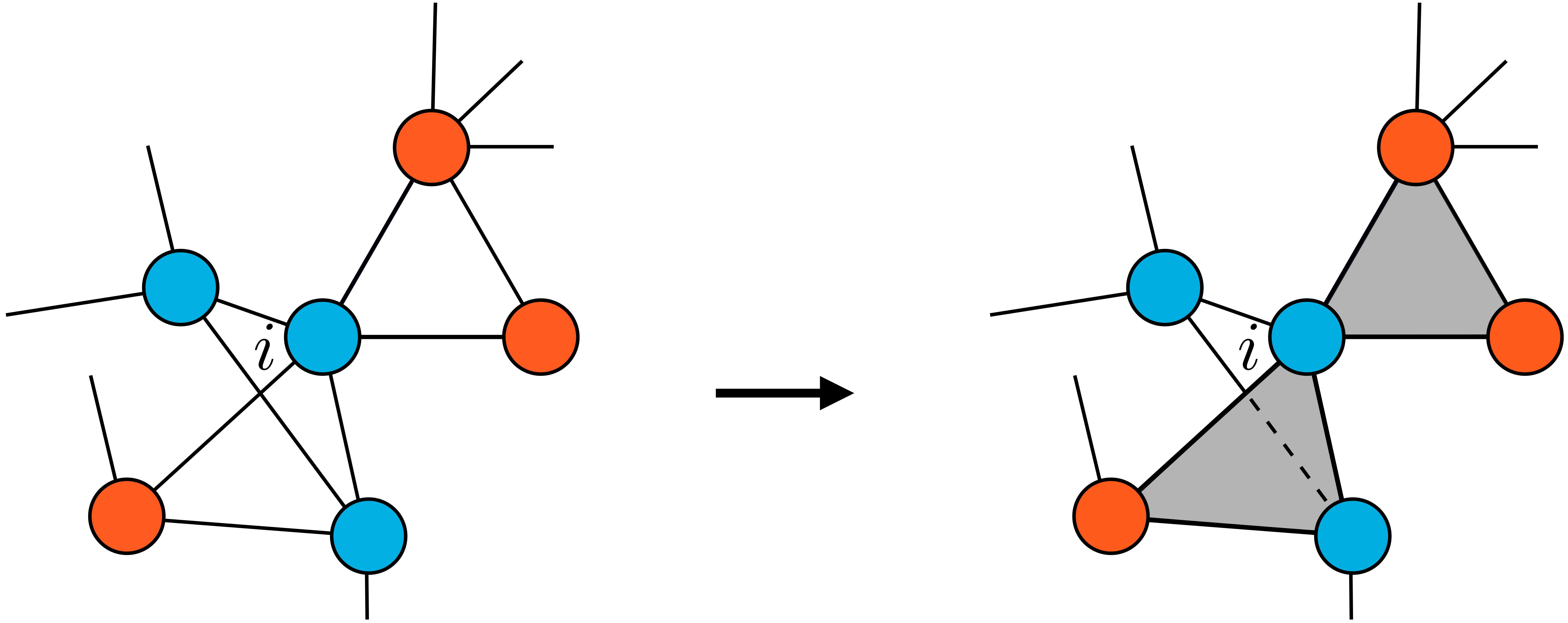
Zhongyuan Ruan,[1,2] Gerardo Iñiguez,[3,4] Márton Karsai,[5] and János Kertész[1,2,4,*]

[1]Center for Network Science, Central European University, H-1051 Budapest, Hungary
[2]Institute of Physics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary
[3]Centro de Investigación y Docencia Económicas, Consejo Nacional de Ciencia y Tecnología, 01210 México D.F., Mexico
[4]Department of Computer Science, Aalto University School of Science, FI-00076 AALTO, Finland
[5]Laboratoire de l'Informatique du Parallélisme, INRIA-UMR 5668, IXXI, ENS de Lyon, 69364 Lyon, France

D. Centola & M. Macy, Americ. J. Sociol. (2007); N. O. Todas & K. Lerman, Sci. Rep. (2014); D. Guilbeault et al., Springer (2018)

What about group interactions?

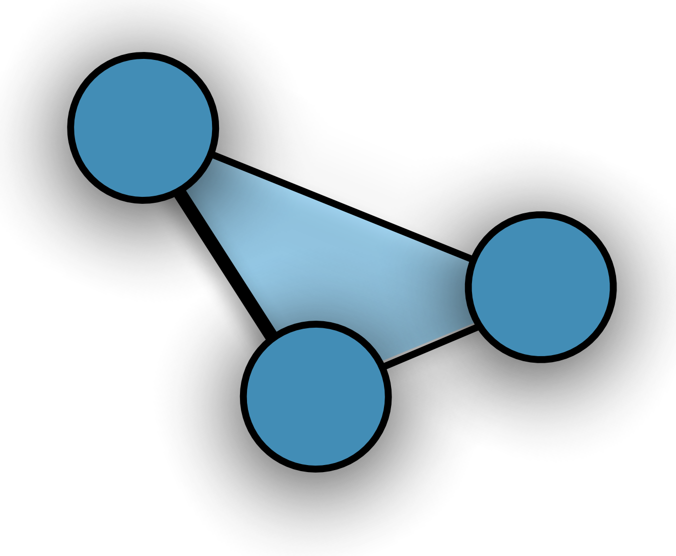
SIMPLicial ContAGION



Network representation of the social structure

Simplicial complex

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019). Simplicial models of social contagion. Nature communications, 10(1), 2485.

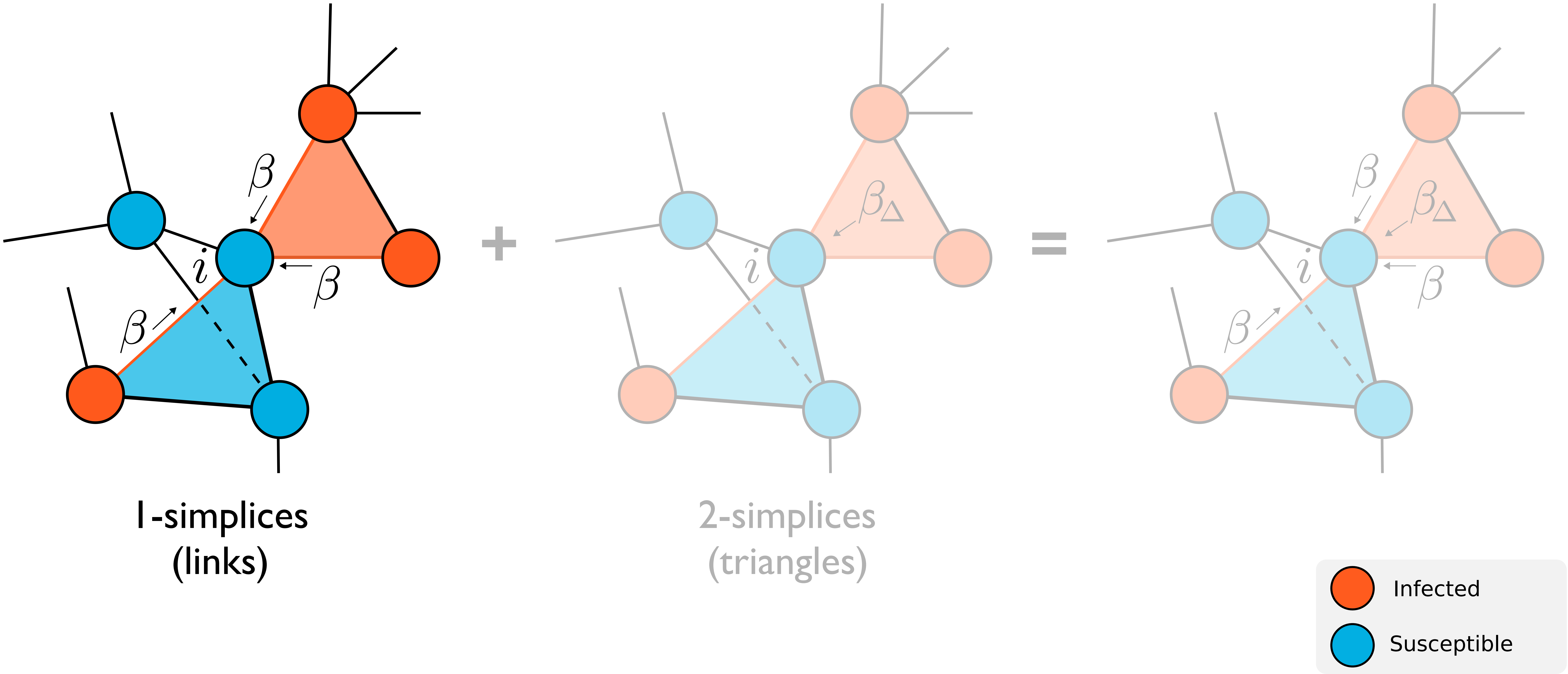


The Simplicial Contagion Model

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019).
Simplicial models of social contagion. Nature communications, 10(1), 2485.

SIMPLicial ContAGION

The Model (D=2)

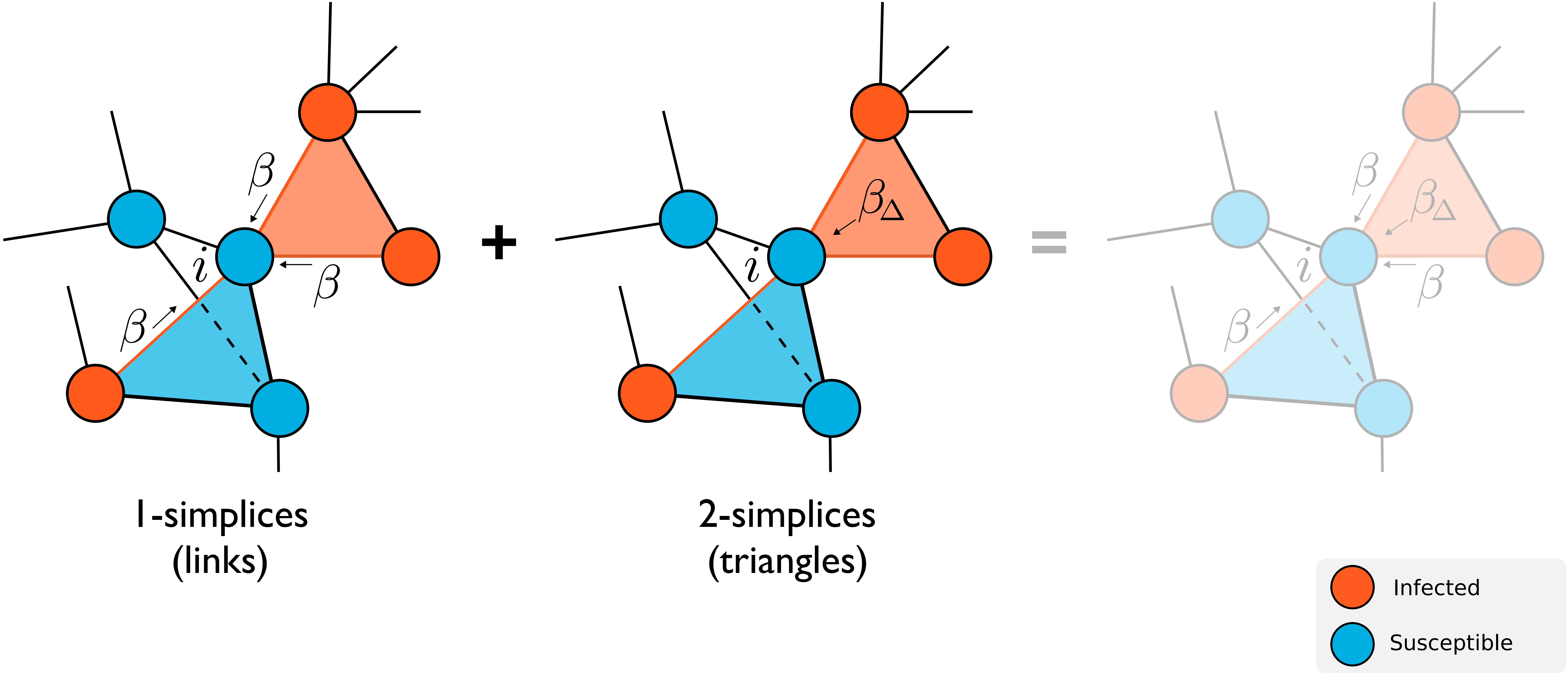


1-simplices
(links)

2-simplices
(triangles)

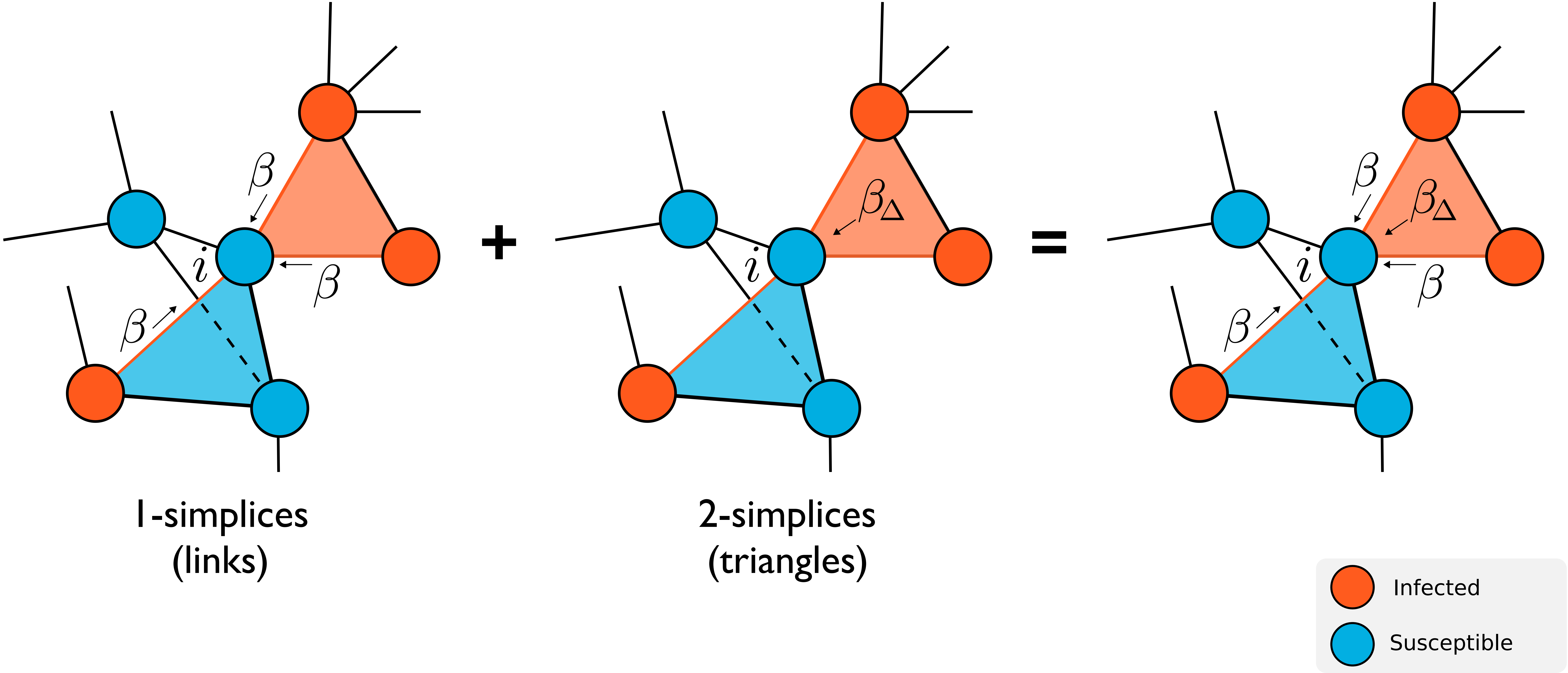
SIMPLicial ContAGION

The Model (D=2)



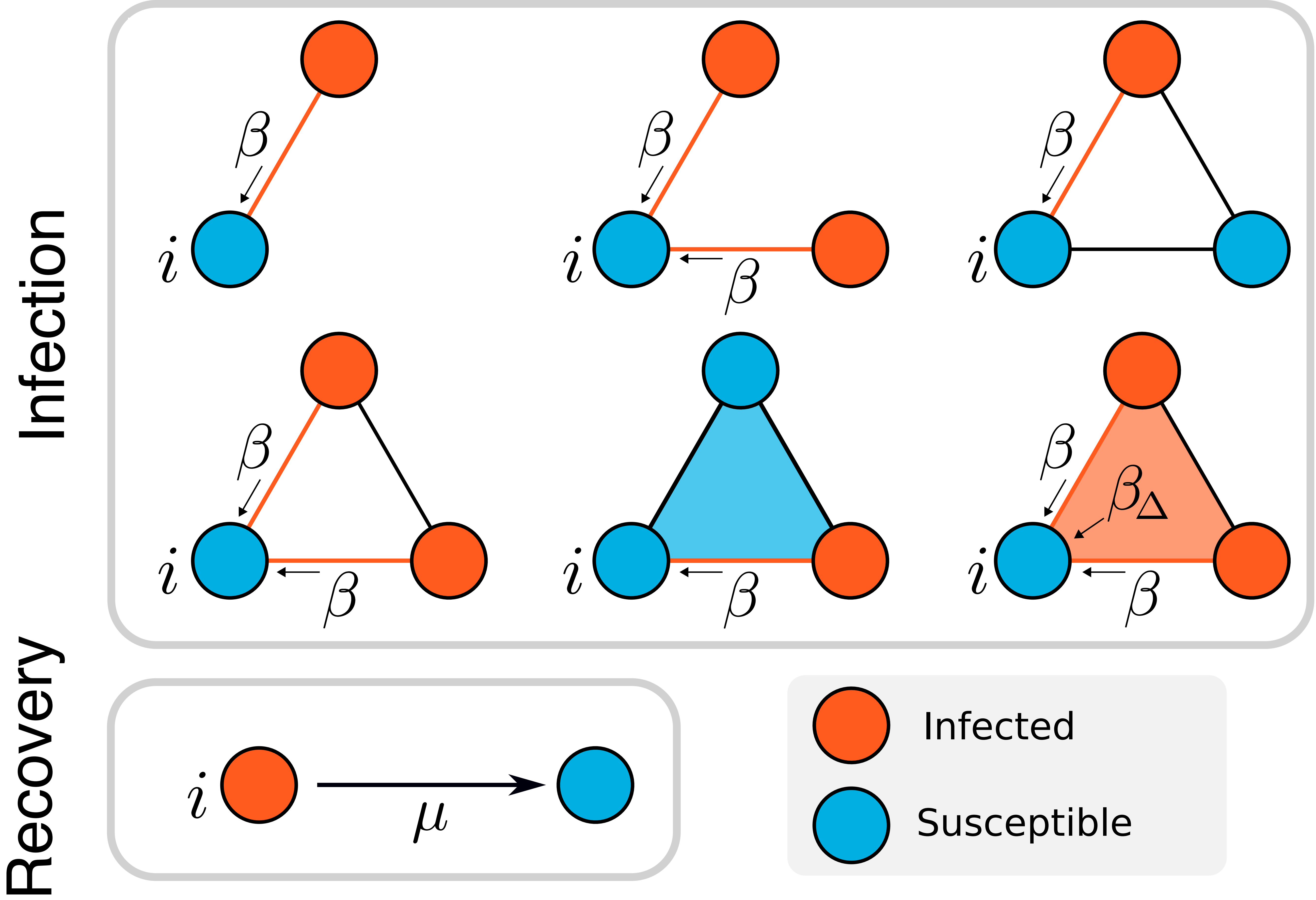
SIMPLicial ContAGION

The Model (D=2)



SIMPLicial contAGION

The Model (D=2)



dynamical state variable

$$x_i(t) \in \{0,1\}$$

control parameters

$$\boxed{\begin{aligned}\lambda &= \beta\langle k\rangle/\mu \\ \lambda_\Delta &= \beta_\Delta\langle k_\Delta\rangle/\mu\end{aligned}}$$

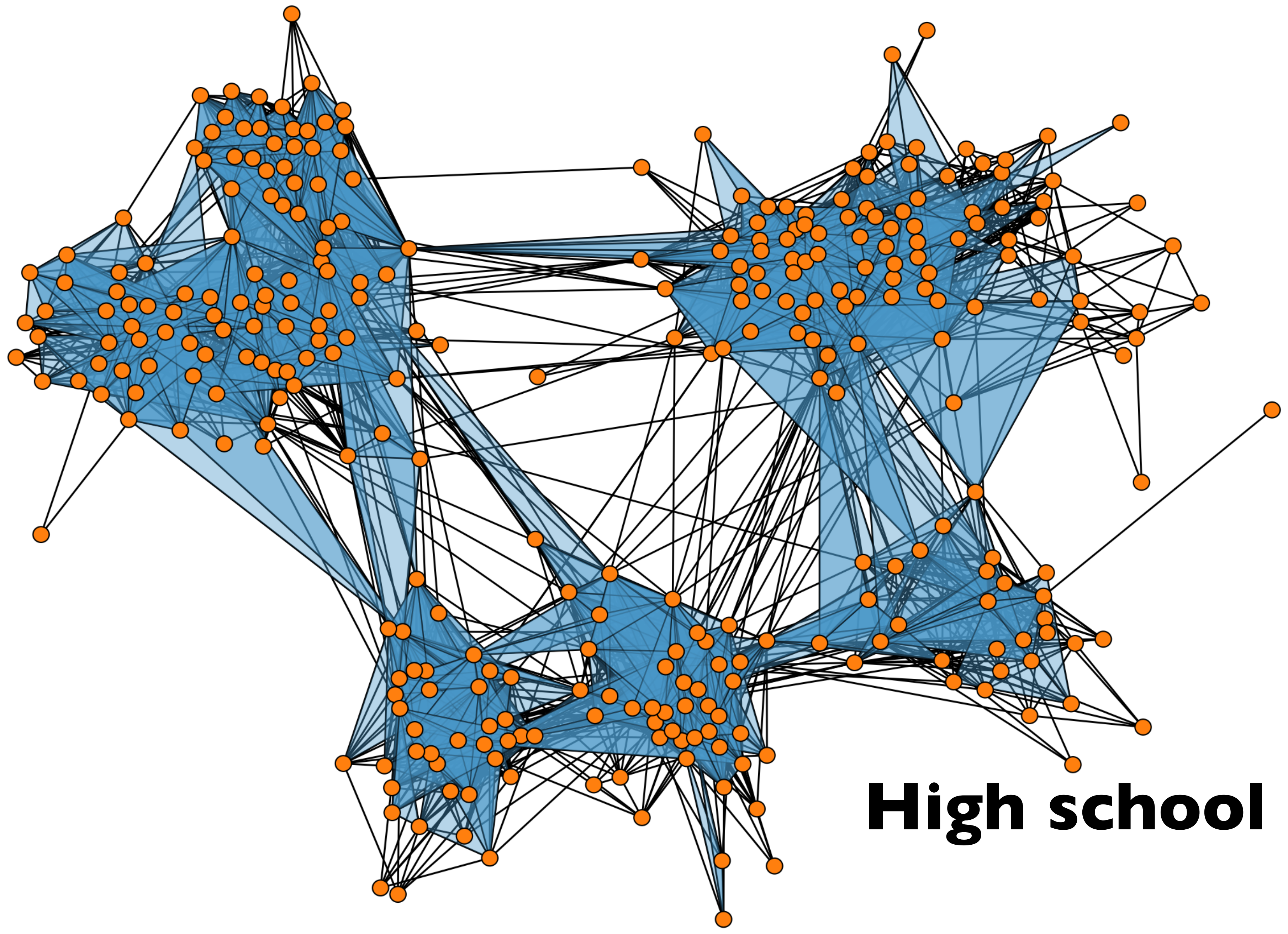
macroscopic order parameter

$$\rho(t) = \frac{1}{N}\sum_i x_i(t)$$

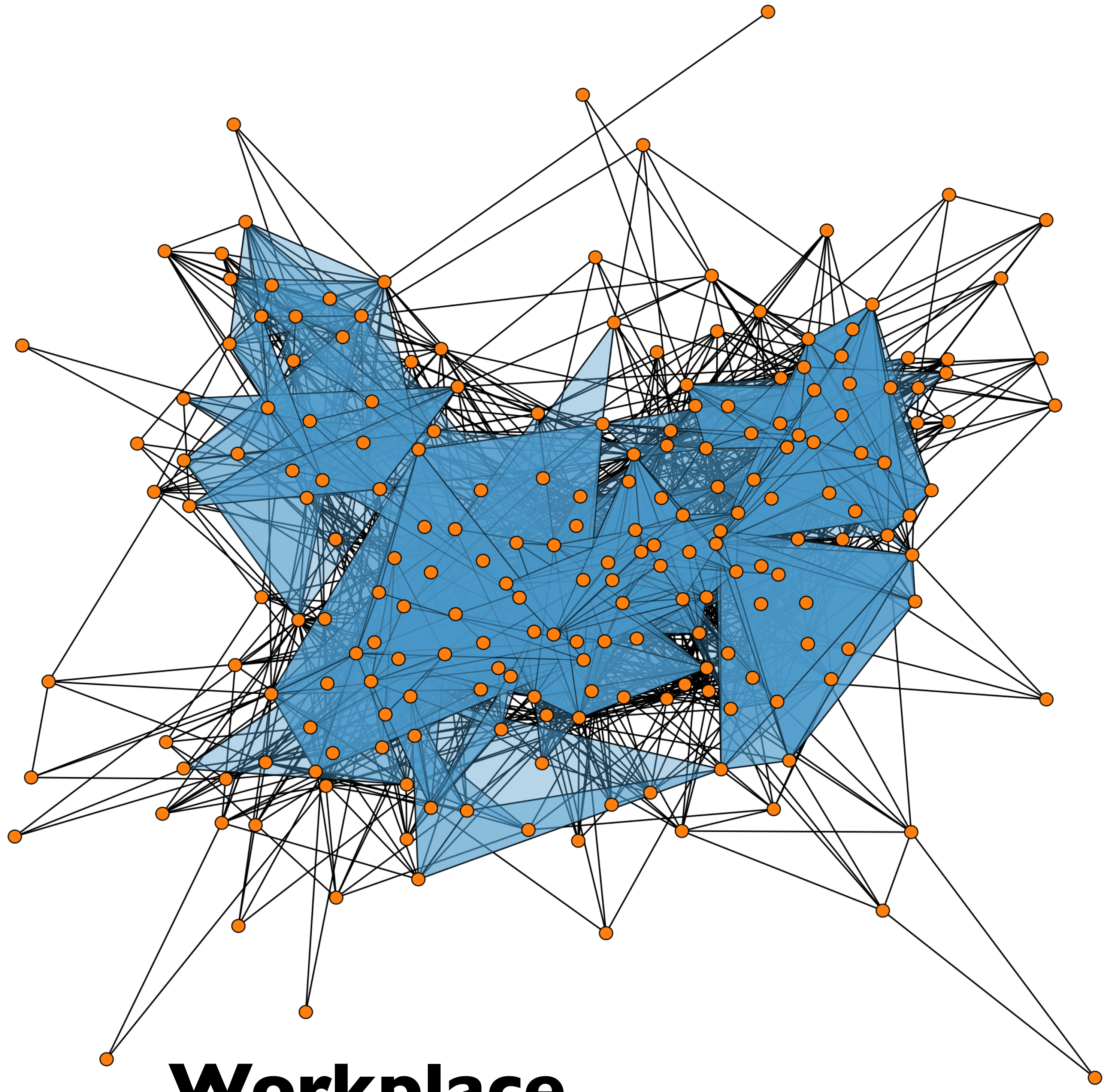
Empirical Social Structures

Real world simplicial complexes

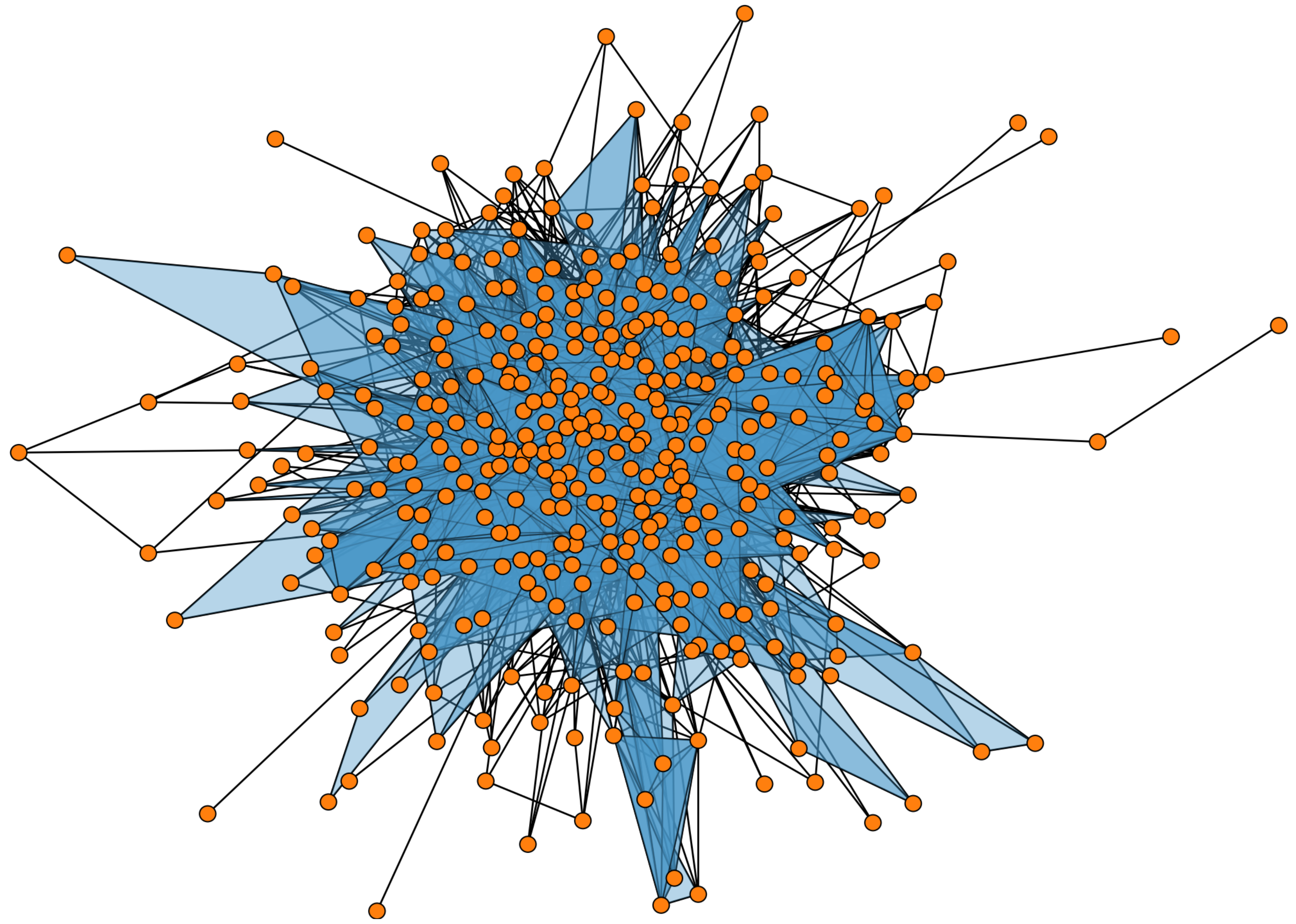
High-resolution proximity data



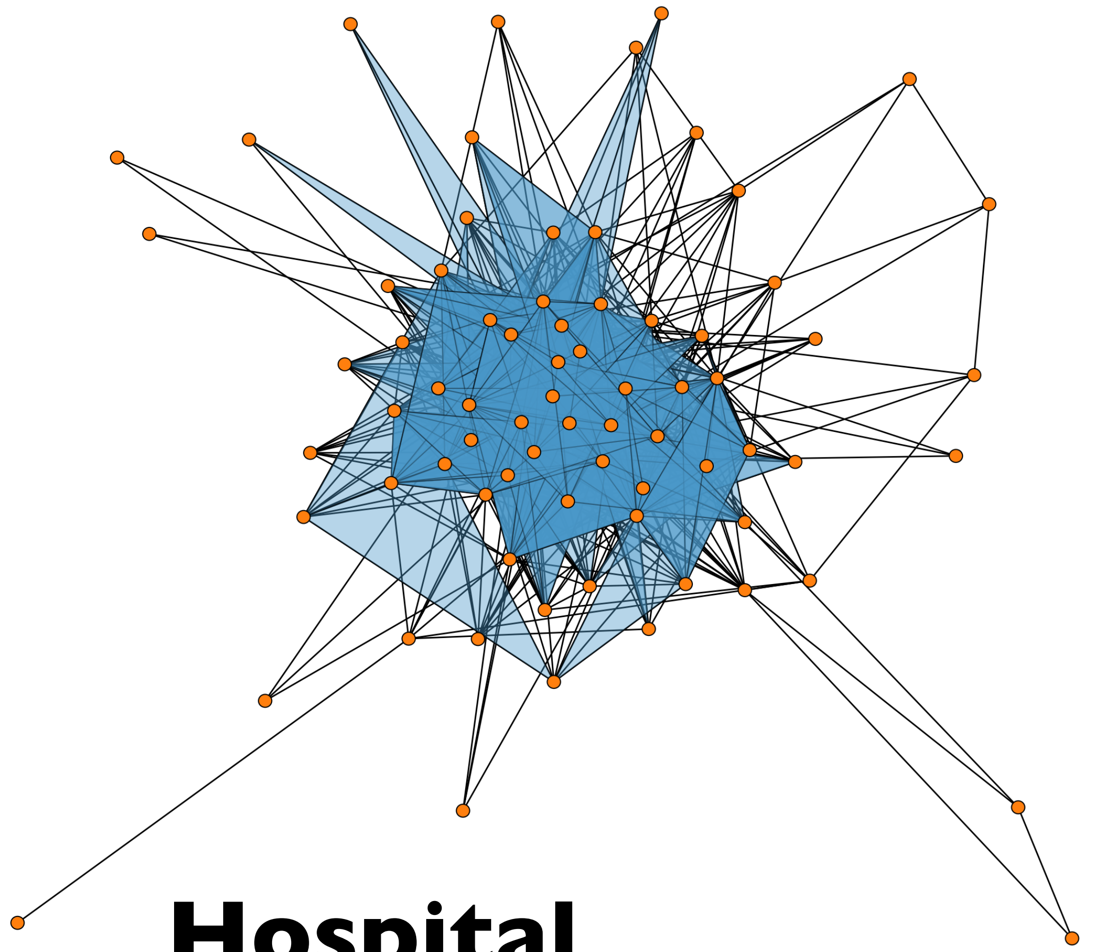
High school



Workplace



Conference



Hospital

SocioPatterns M. Génois and A. Barrat, EPJ Data Science (2018)

Results

High-resolution proximity data

Results

High-resolution proximity data

Results

High-resolution proximity data

Mean Field Approach

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$

D=2

$$d_t \rho(t)$$

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$

D=2

loss of infectiousness

$$d_t\rho(t) = -\mu\rho(t)$$

Mean Field approach

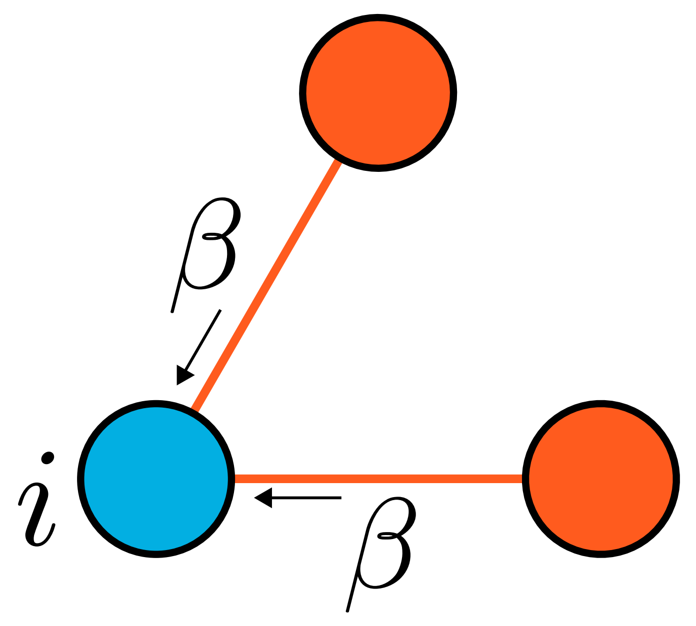
Temporal evolution of the density of infected nodes $\rho(t)$

D=2

loss of infectiousness

$$d_t\rho(t) = \underbrace{-\mu\rho(t)}_{} + \underbrace{\beta\langle k\rangle\rho(t)\left[1-\rho(t)\right]}_{}$$

new infections
from 1-simplices



Mean Field approach

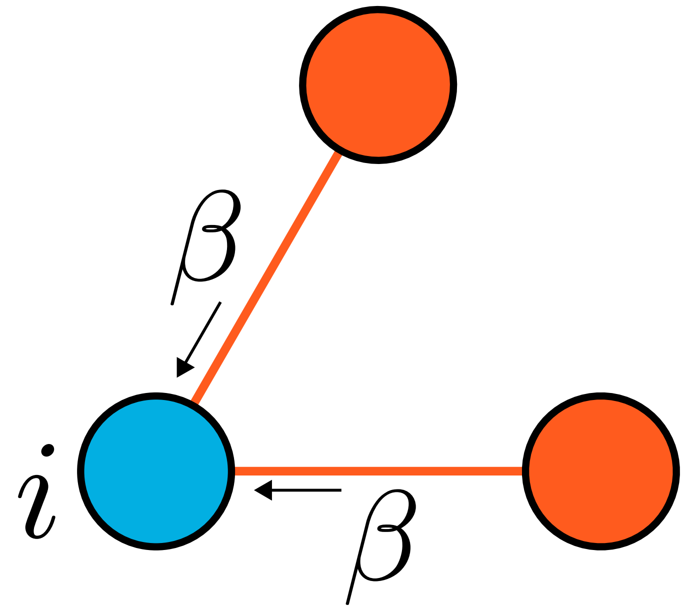
Temporal evolution of the density of infected nodes $\rho(t)$

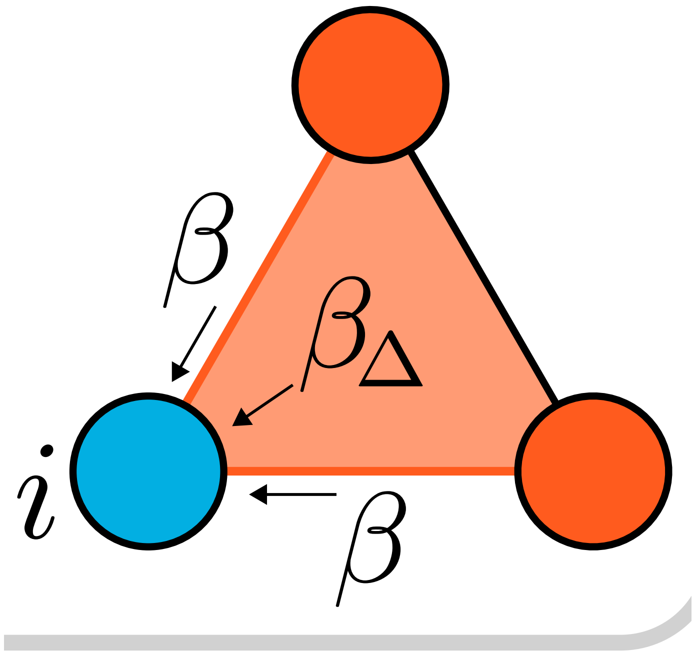
D=2 loss of infectiousness

$$d_t\rho(t) = -\mu\rho(t) + \beta\langle k\rangle\rho(t)\left[1-\rho(t)\right] + \beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]$$

new infections from 1-simplices

new infections from 2-simplices





Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$

D=2

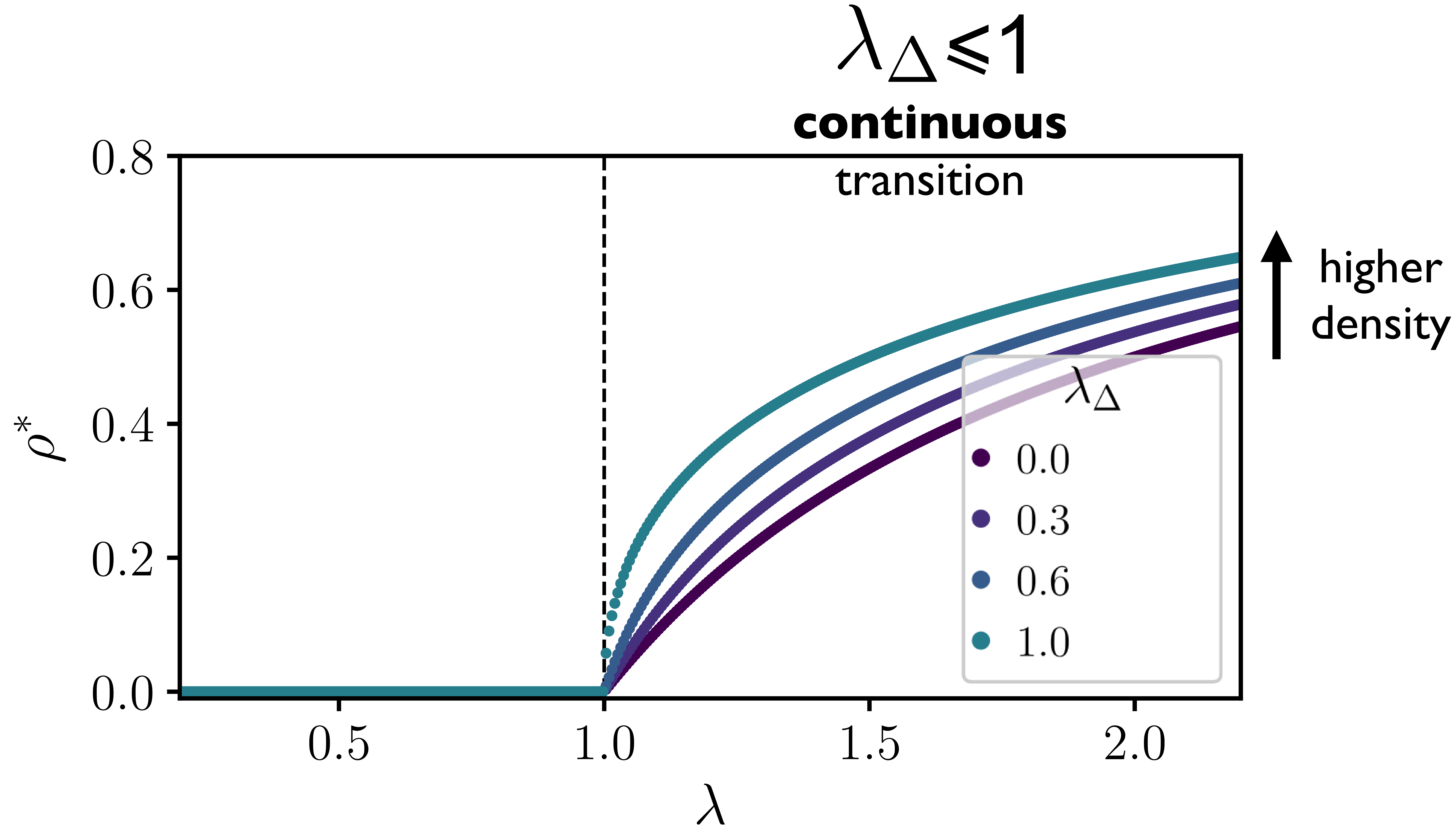
$$\underbrace{d_t\rho(t) = -\mu\rho(t)}_{\text{loss of infectiousness}} + \underbrace{\beta\langle k\rangle\rho(t)\left[1-\rho(t)\right]}_{\text{new infections from 1-simplices}} + \underbrace{\beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]}_{\text{new infections from 2-simplices}}$$

Set of infection probabilities $B \equiv \{\beta_\omega, \omega = 1, \cdots, D\}$

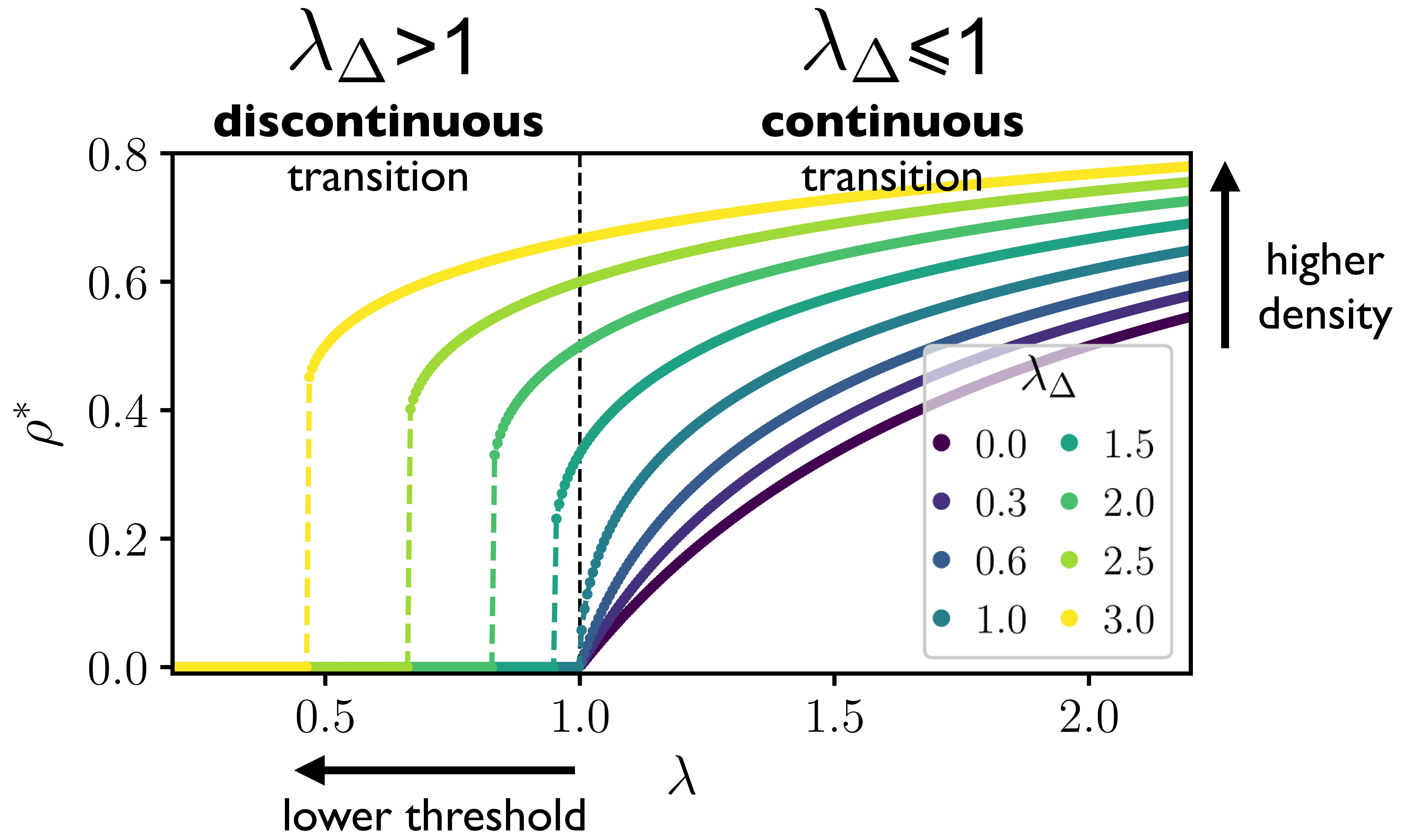
D

$$d_t\rho(t) = \underbrace{-\mu\rho(t)}_{\text{loss of infectiousness}} + \sum_{\omega=1}^{D} \underbrace{\beta_\omega\langle k_\omega\rangle\rho^\omega(t)\left[1-\rho(t)\right]}_{\text{new infections}}$$

Mean Field approach



Mean Field approach



Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$

D=2

loss of infectiousness

$$d_t\rho(t) = -\mu\rho(t) + \beta\langle k\rangle\rho(t)\left[1-\rho(t)\right] + \beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]$$

new infections from 1-simplices

new infections from 2-simplices

$$d_t\rho(t) = -\rho(t)(\rho(t)-\rho^*_{2+})(\rho(t)-\rho^*_{2-})$$

The **steady state** equation $d_t\rho(t) = 0$ has up to three solution in the acceptable range $\rho \in [0,1]$

$$\rho^*_1 = 0$$

absorbing epidemic-free state

$$\rho^*_{2\pm} = \frac{\lambda_\Delta - \lambda \pm \sqrt{(\lambda-\lambda_\Delta)^2 - 4\lambda_\Delta(1-\lambda)}}{2\lambda_\Delta}$$

Mean Field approach

Temporal evolution of the density of infected nodes $\rho(t)$

D=2

loss of infectiousness

$$d_t\rho(t) = \underbrace{-\mu\rho(t)}_{\text{loss of infectiousness}} + \underbrace{\beta\langle k\rangle\rho(t)\left[1-\rho(t)\right]}_{\text{new infections from 1-simplices}} + \underbrace{\beta_\Delta\langle k_\Delta\rangle\rho^2(t)\left[1-\rho(t)\right]}_{\text{new infections from 2-simplices}}$$

new infections from 1-simplices

new infections from 2-simplices

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The **steady state** equation $d_t\rho(t) = 0$ has up to three solution in the acceptable range $\rho \in [0,1]$

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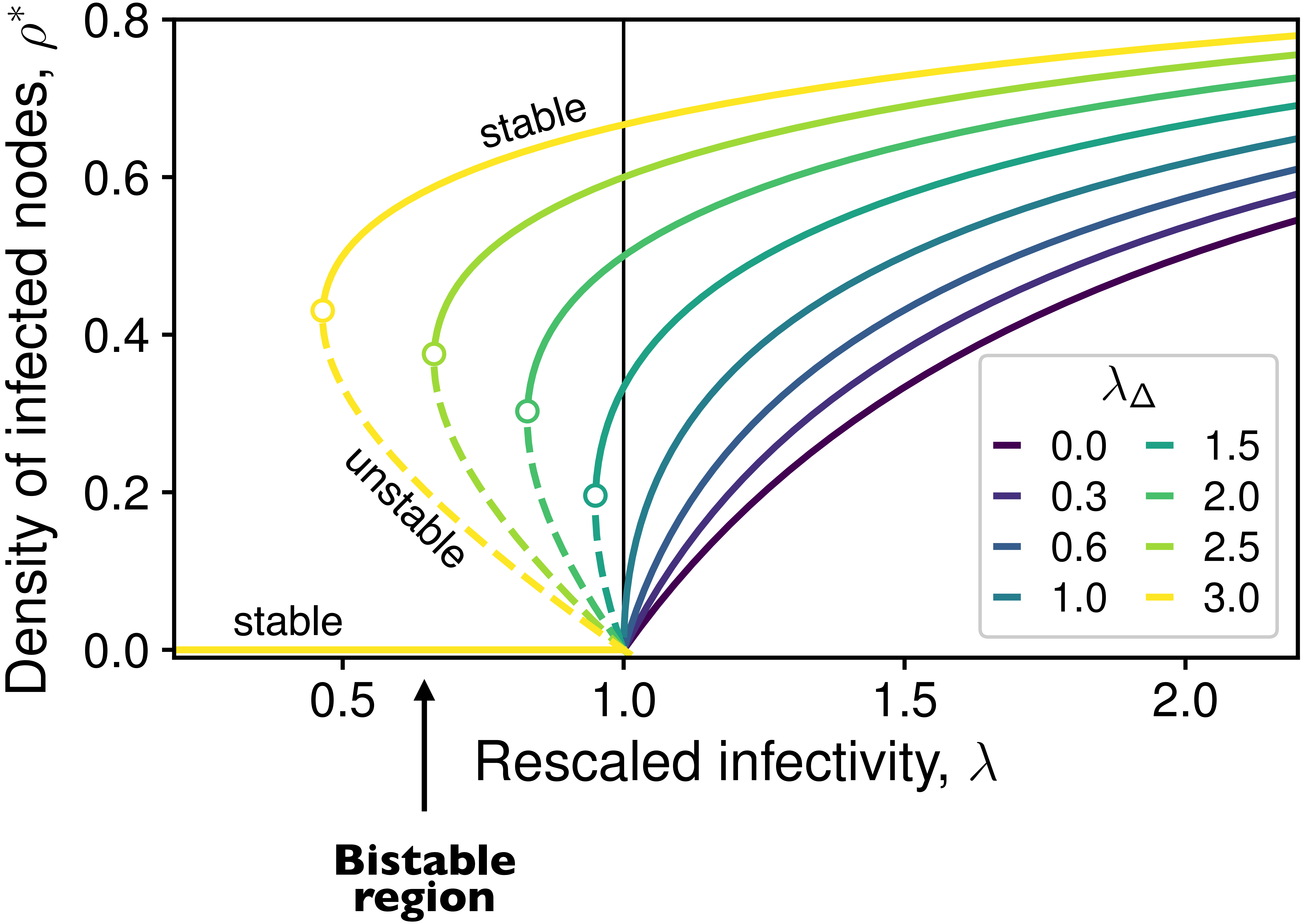
$$\boxed{\rho^*_{2\pm} = \frac{\lambda_\Delta - \lambda \pm \sqrt{(\lambda-\lambda_\Delta)^2 - 4\lambda_\Delta(1-\lambda)}}{2\lambda_\Delta}}$$

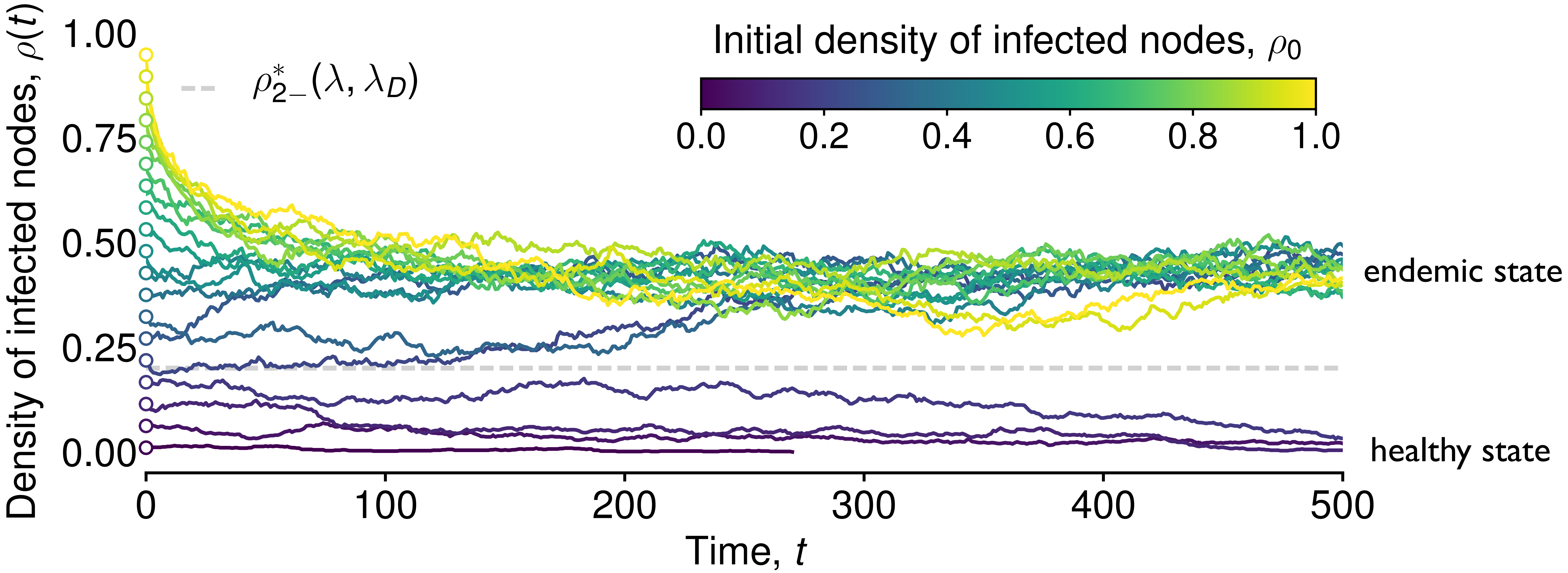
Mean Field approach



Notebook Simplagion 1

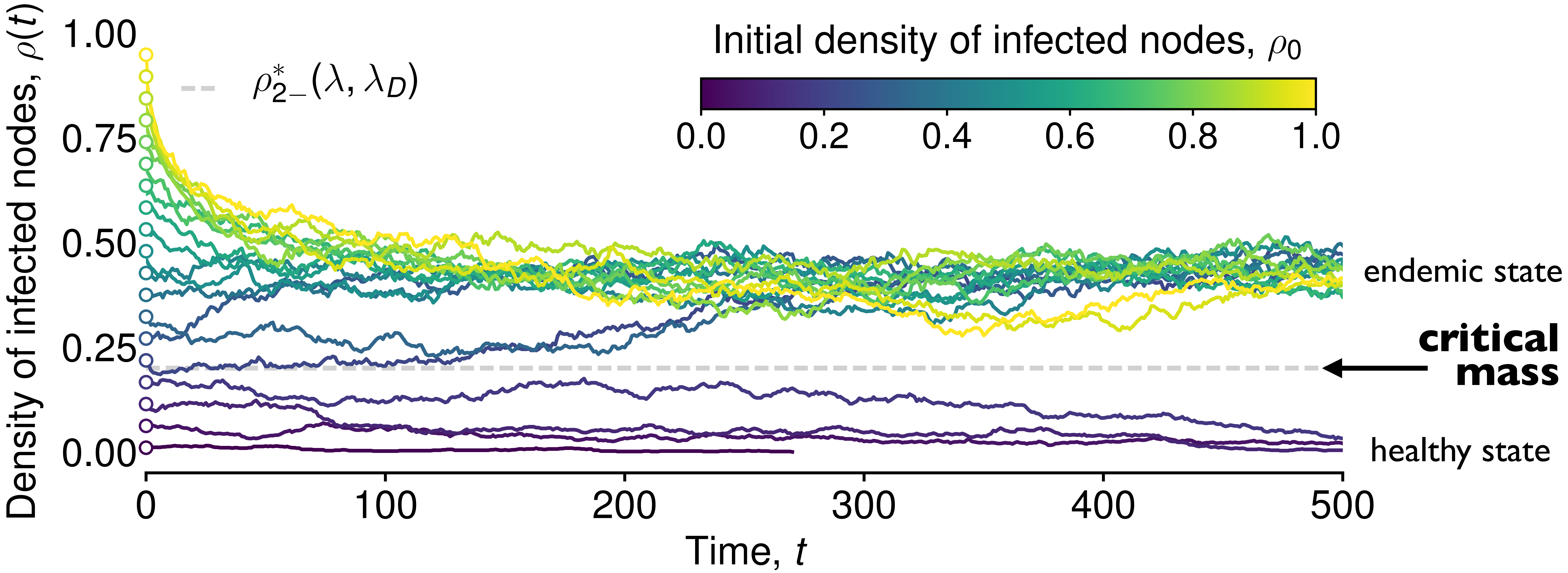
Mean Field approach

Dependency on initial conditions



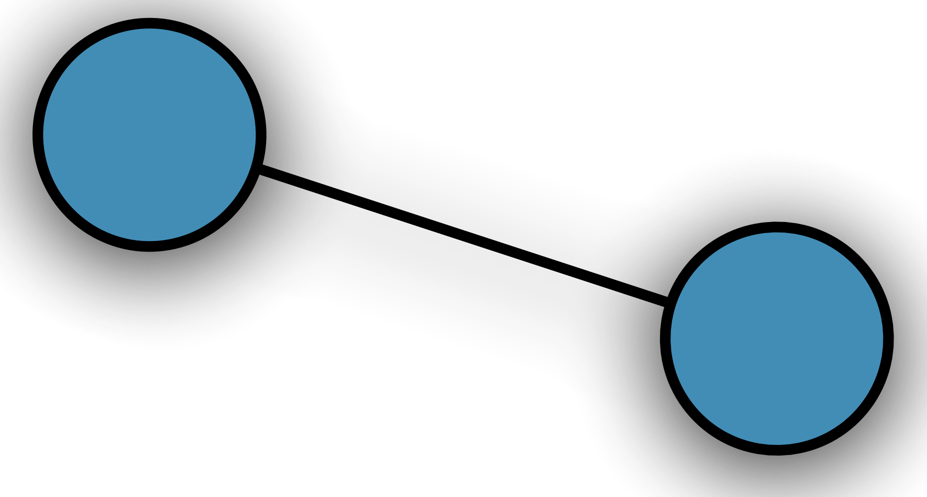
Mean Field approach

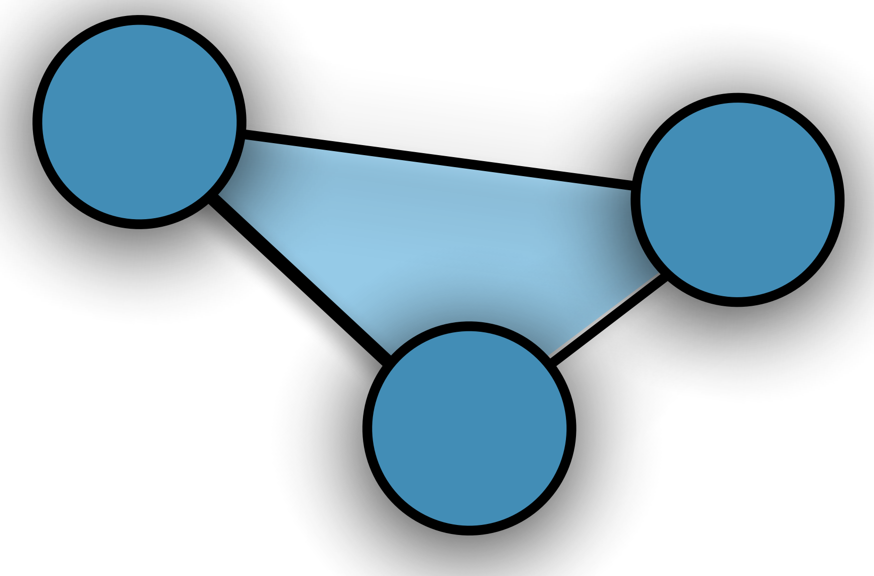
Dependency on initial conditions

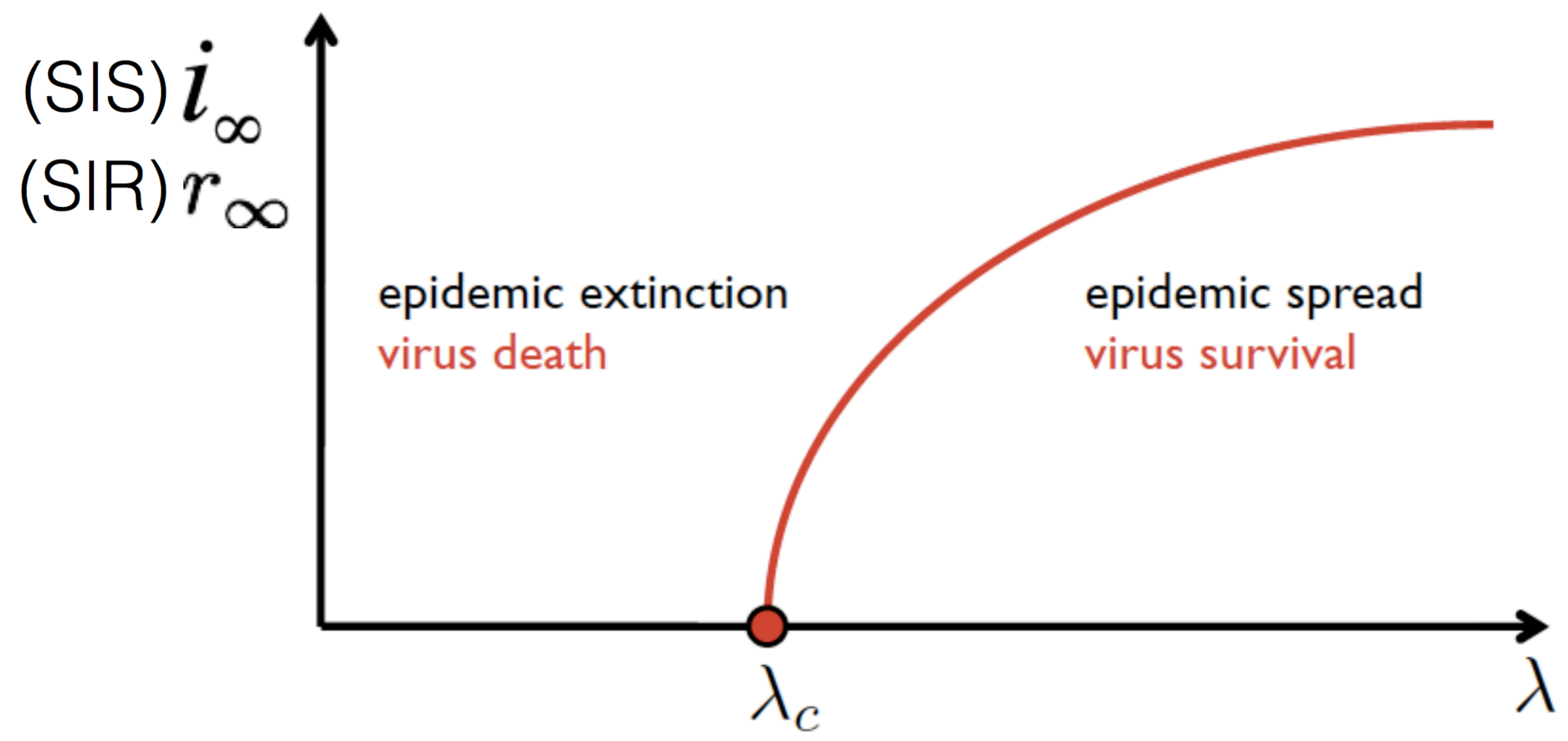


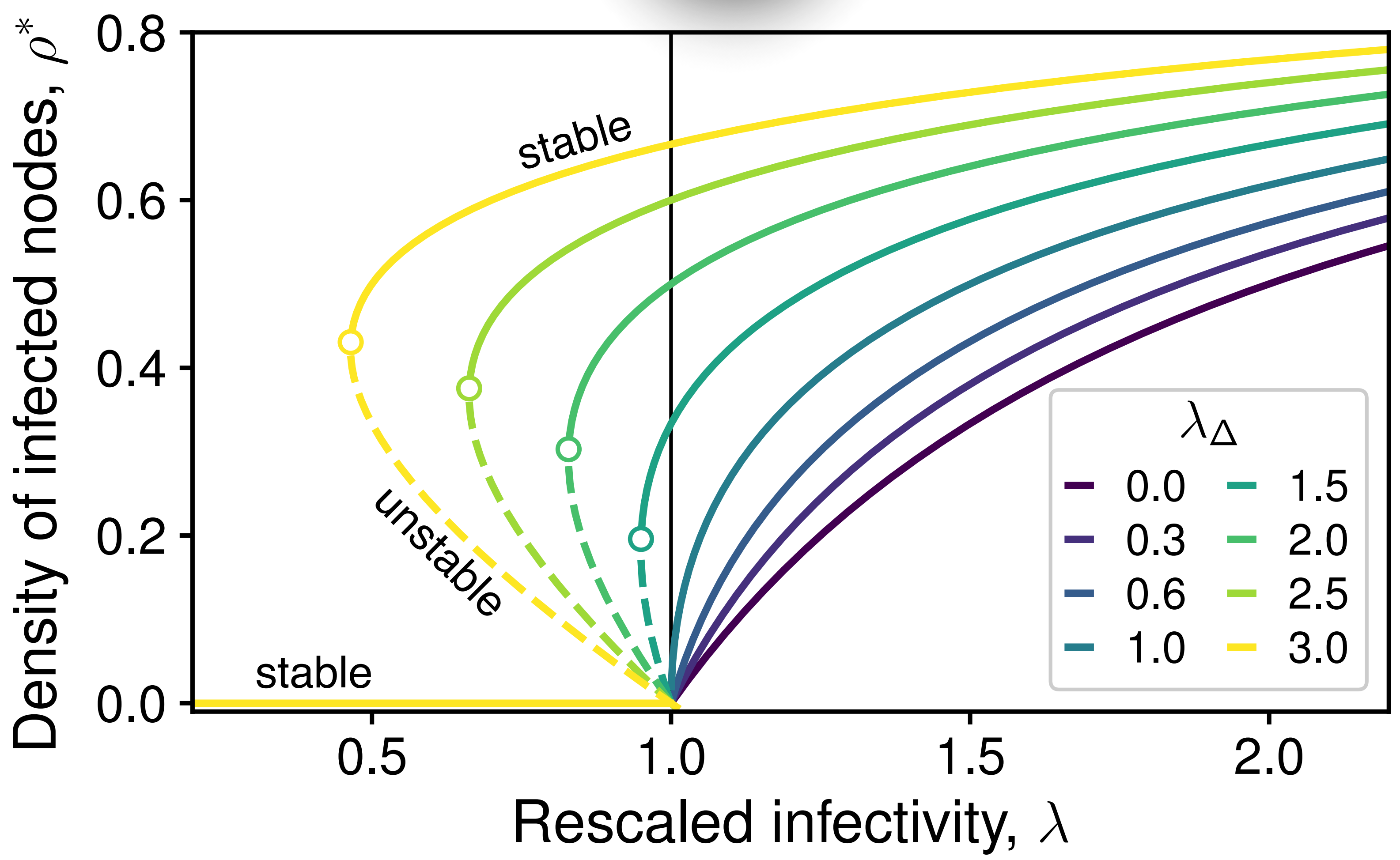
Notebook
Simplagion 2

Summing up - “SIMPLAGION”







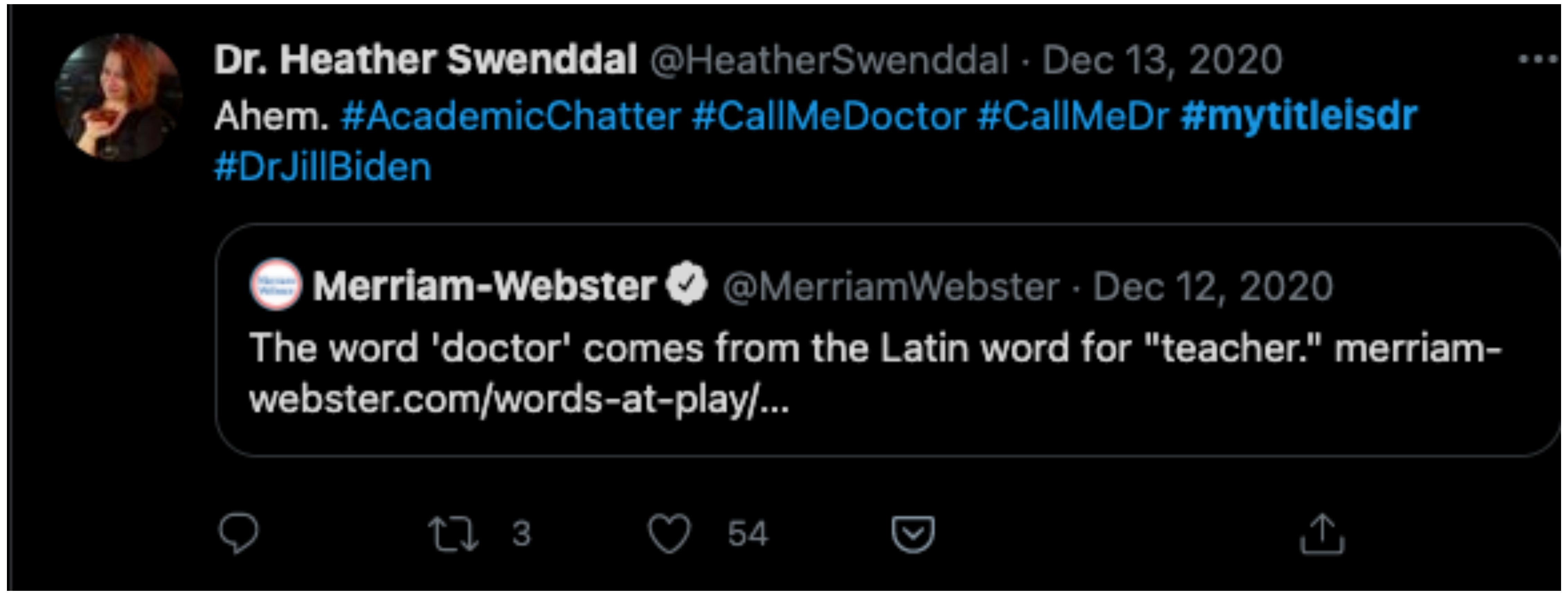


- **Model:**
  - Social structure modelled as a **simplicial complex/hypergraph**
  - Contagion occurs in **group interactions** (with different transmission rates)
- **New phenomenology**
  - Discontinuous transition
  - **Dependence on the size of the seed <u>(critical mass)</u>**
  - Confirmed in hypergraphs (De Arruda et al 2020)

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019). Simplicial models of social contagion. Nature communications, 10(1), 2485.

Are there other interesting cases in which critical masses appear?

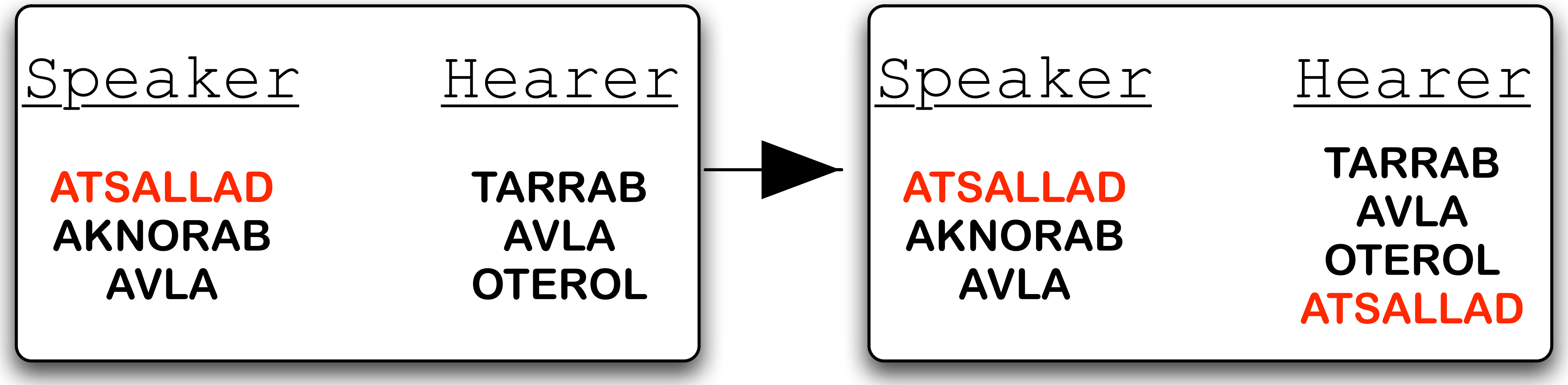
Tipping points in social conventions



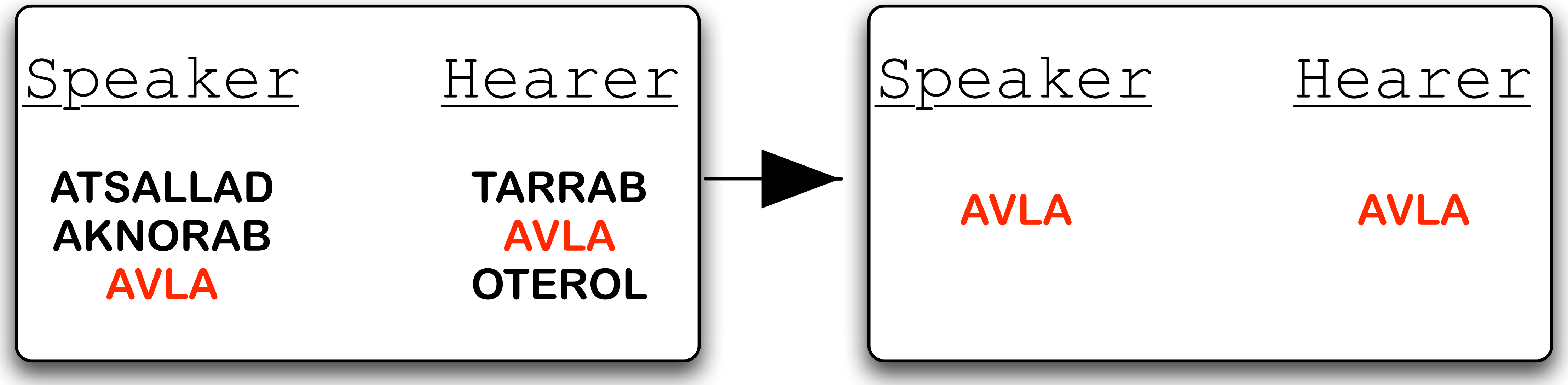
Naming Game

Naming Game

Failure

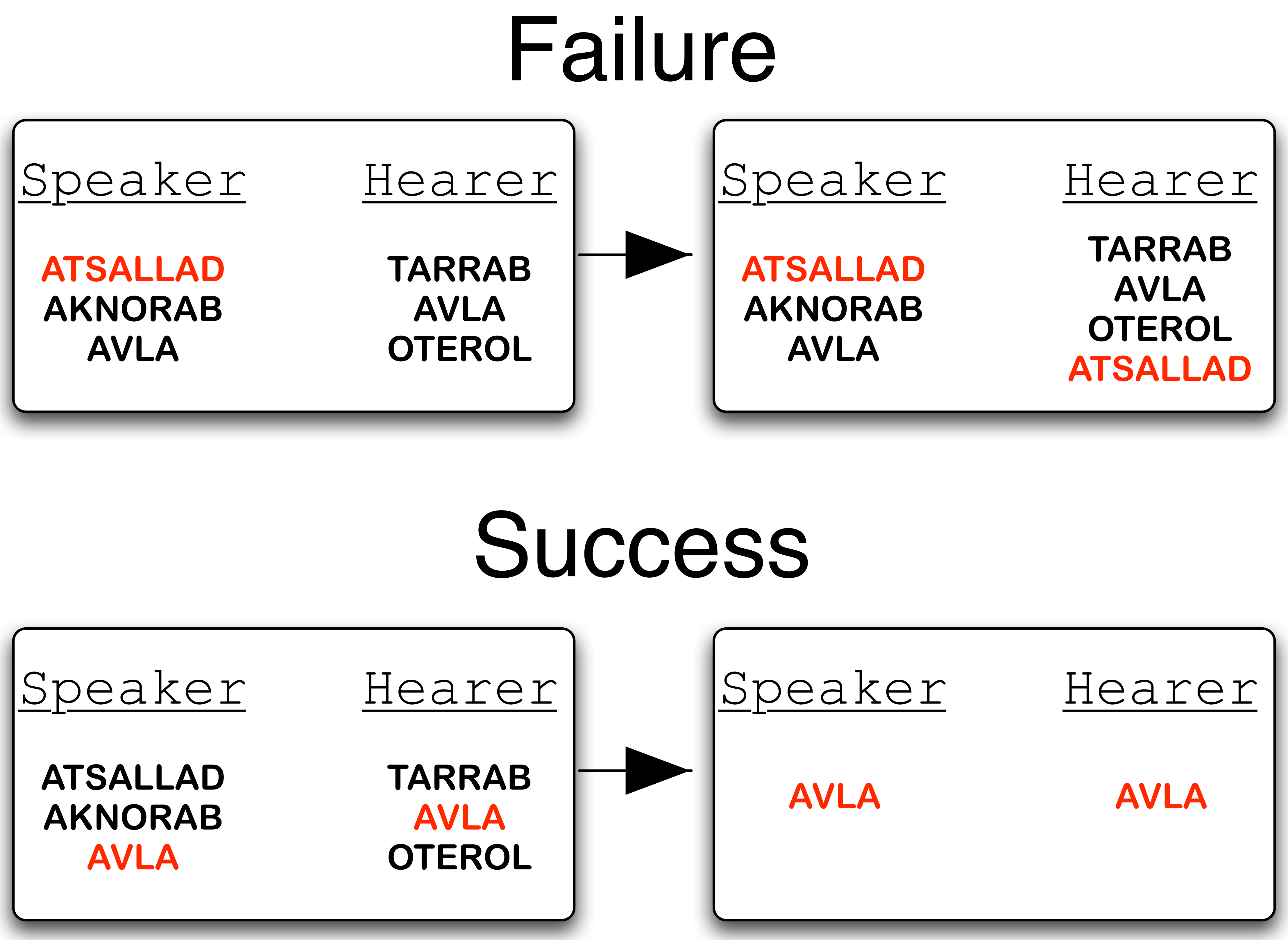


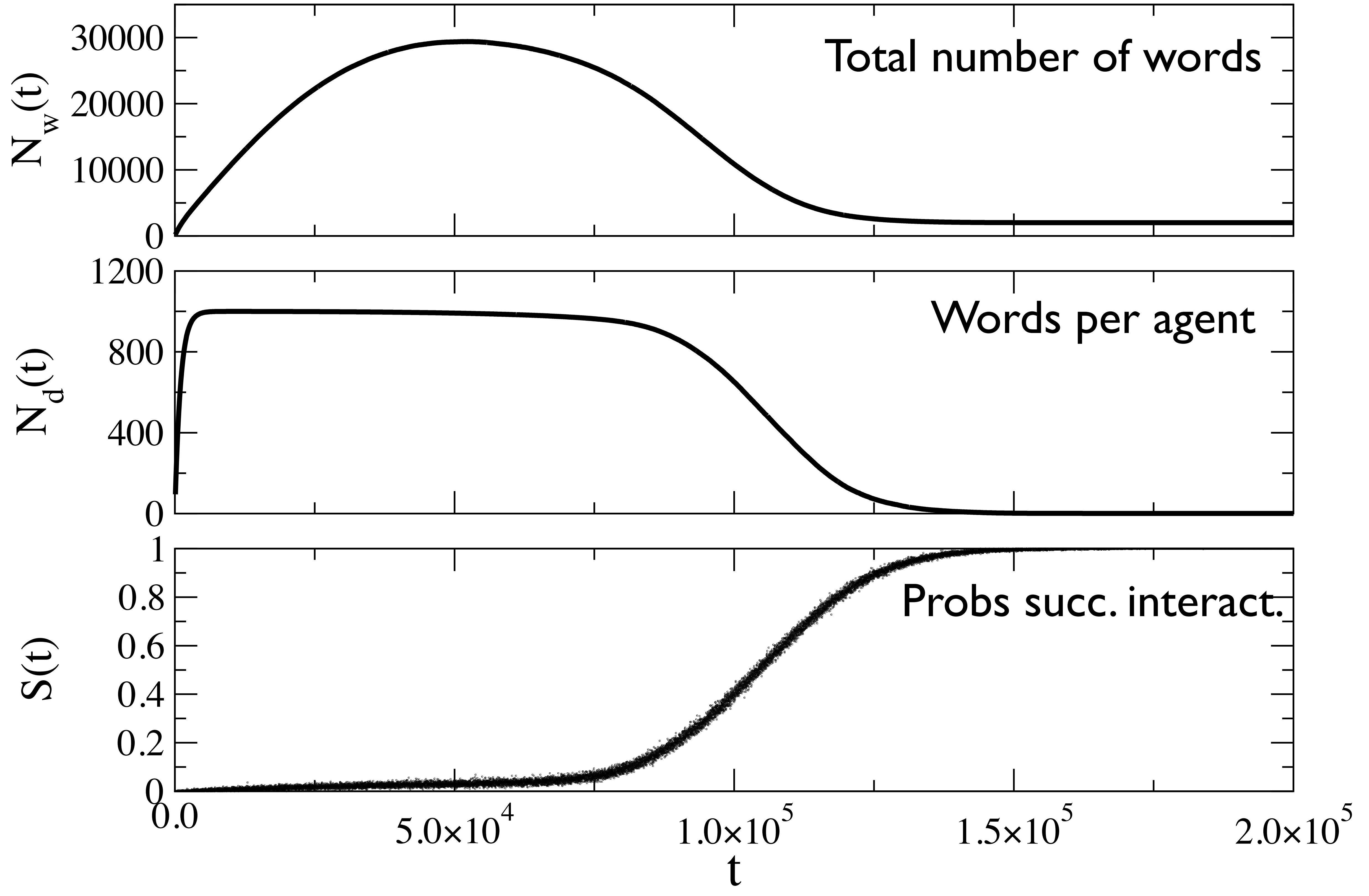
Success



Baronchelli, Andrea. "A gentle introduction to the minimal Naming Game." *Belgian Journal of Linguistics* 30.1 (2016): 171-192.

Naming Game





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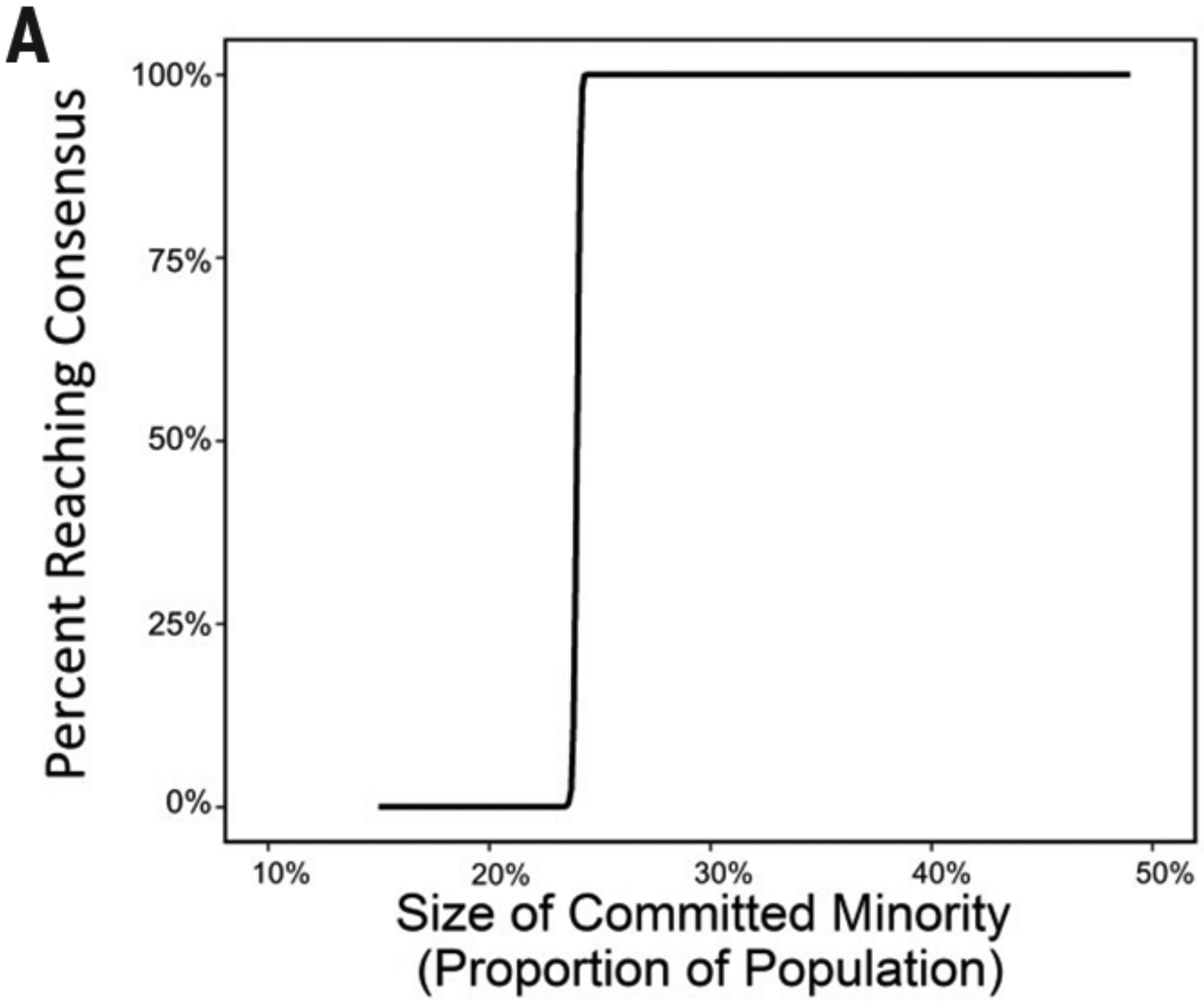
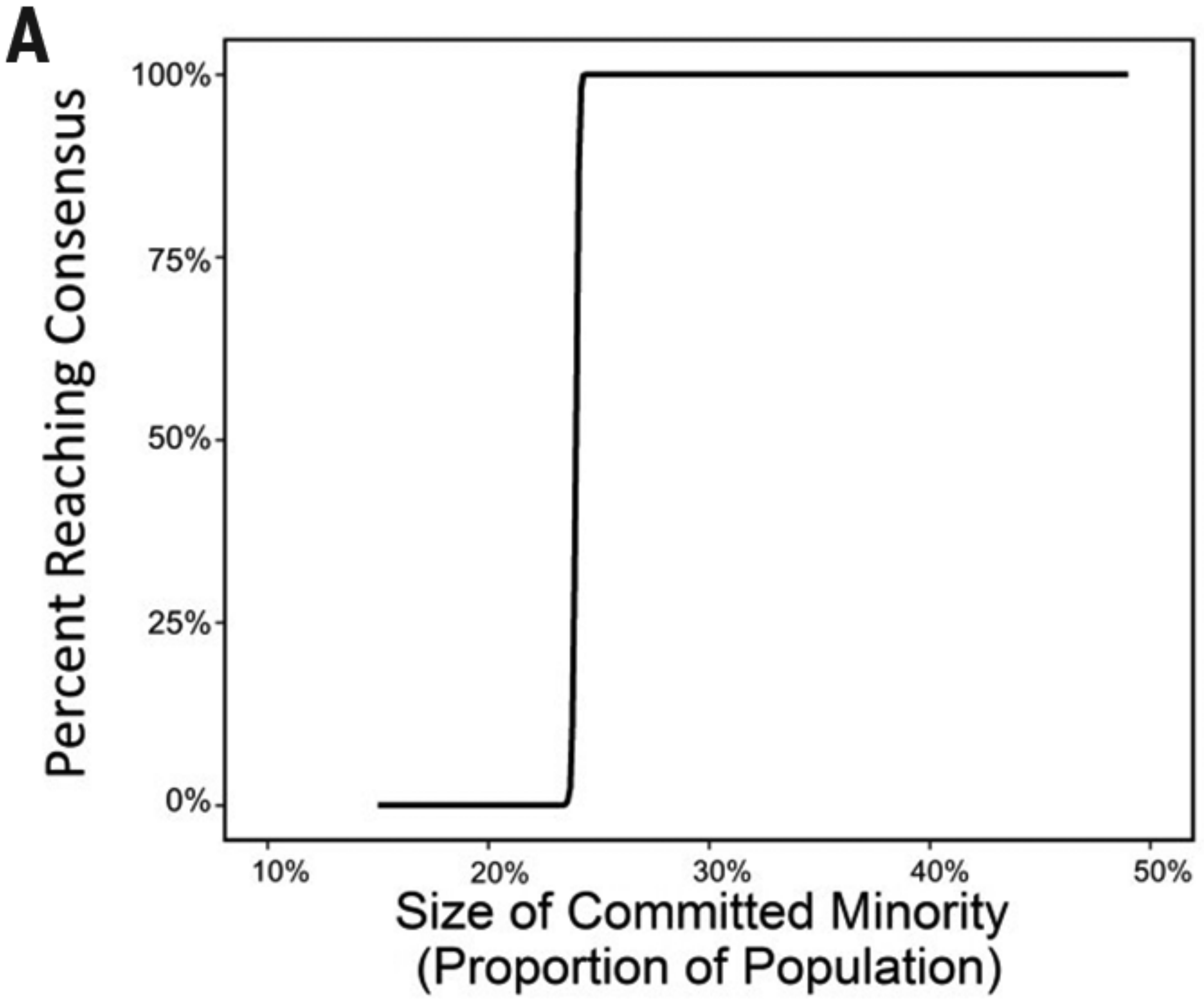
Naming Game with committed minorities

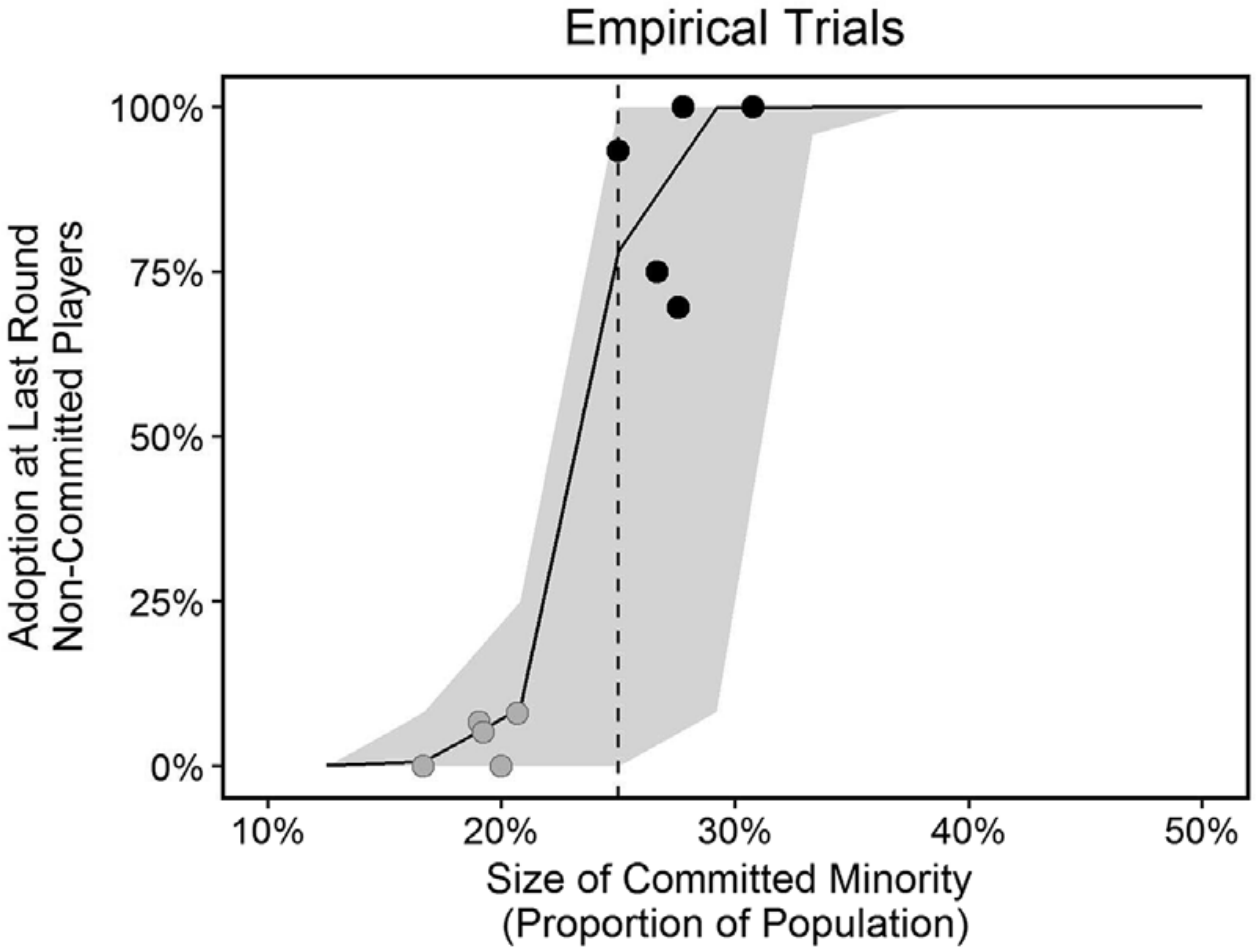
Naming Game with committed minorities



Centola, Damon, et al. "Experimental evidence for tipping points in social convention." *Science* 360.6393 (2018): 1116-1119.

Naming Game with committed minorities





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25% is a big minority.
How do we get to it?

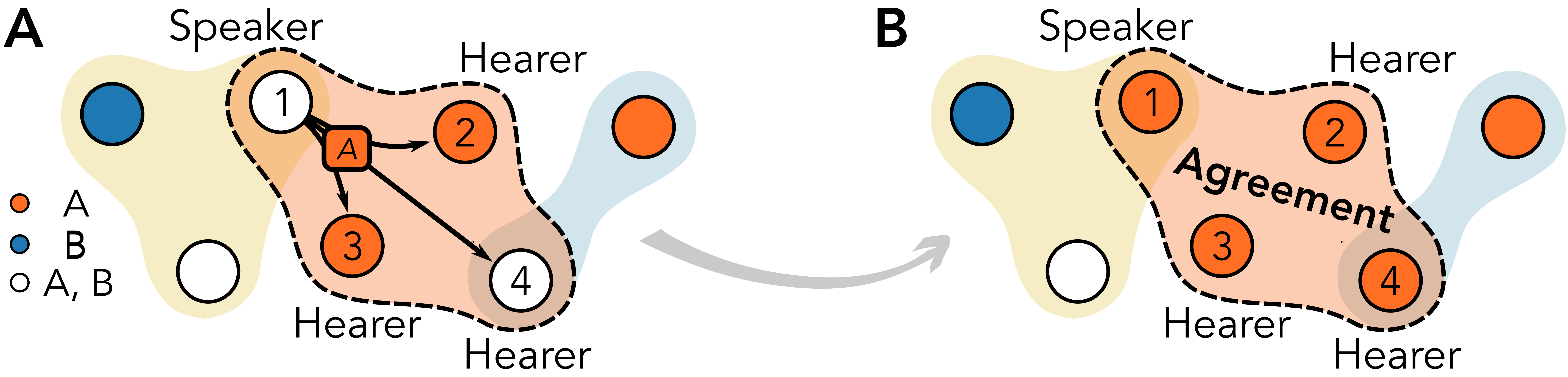
25% is a big minority.
How do we get to it?

And, do groups play a role?

Naming Game with groups

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

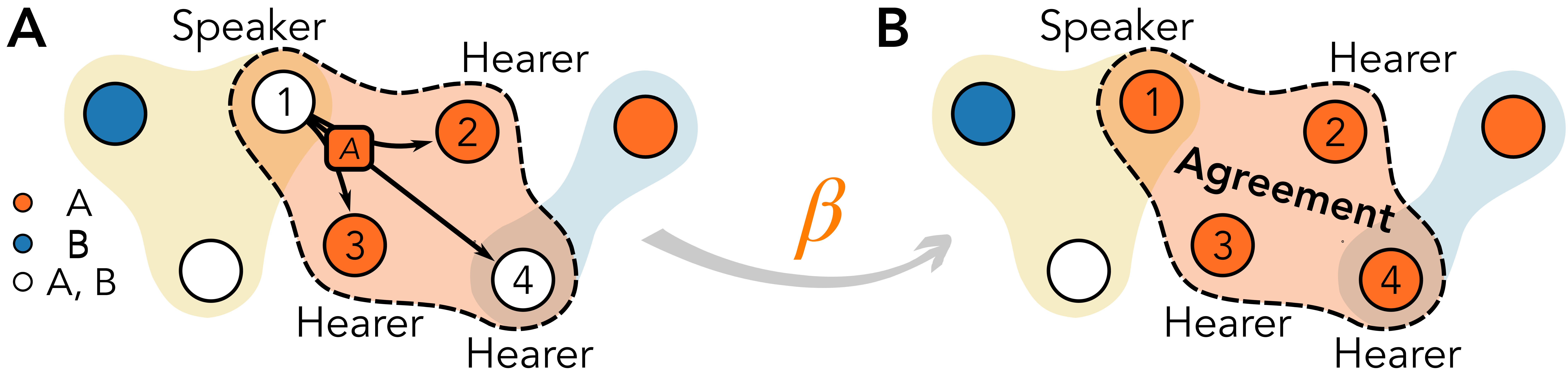
Naming Game with groups



Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with groups

We add another twist: agreement happens with probability β



Iacopini, Petri, Baronchelli, Barrat (2021), In prep

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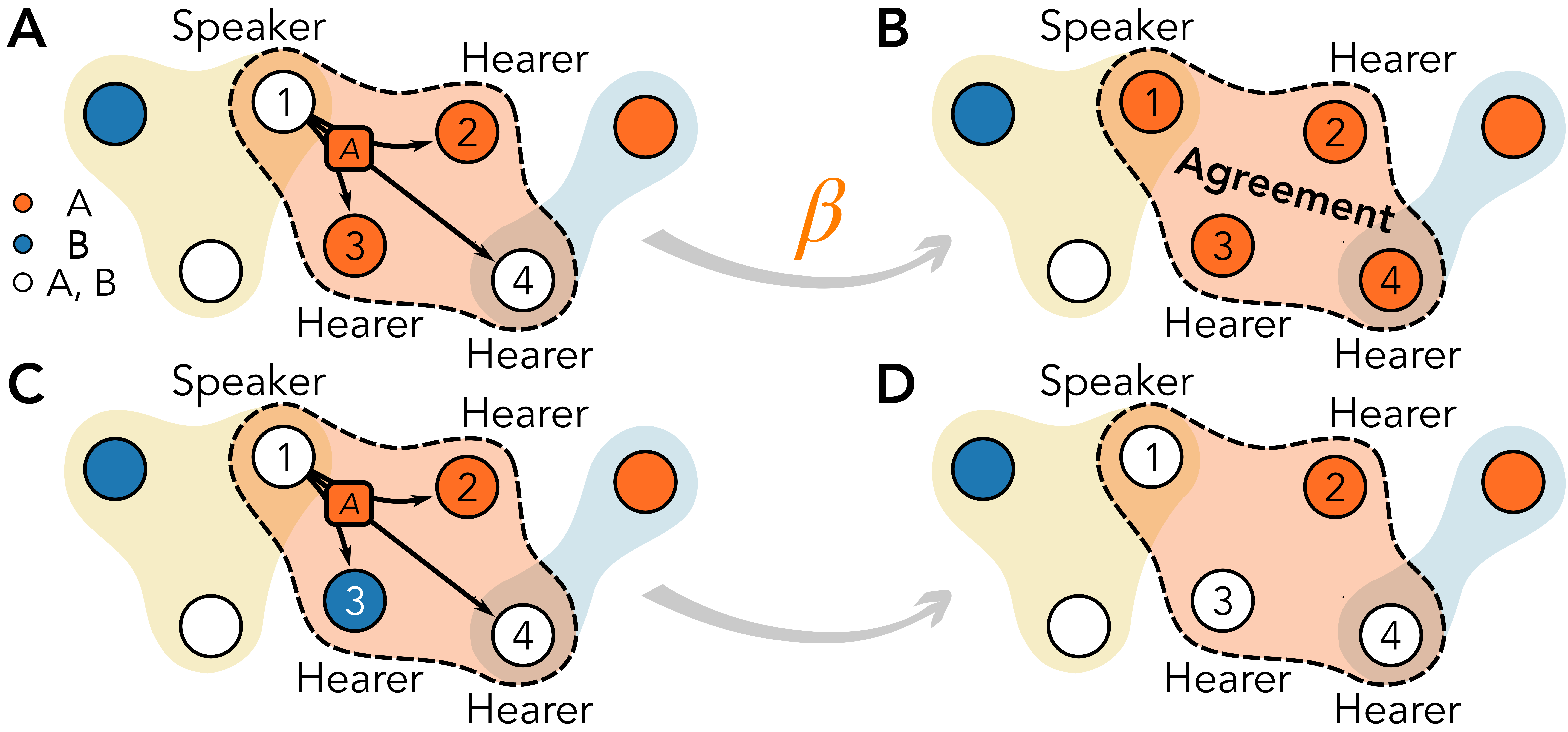
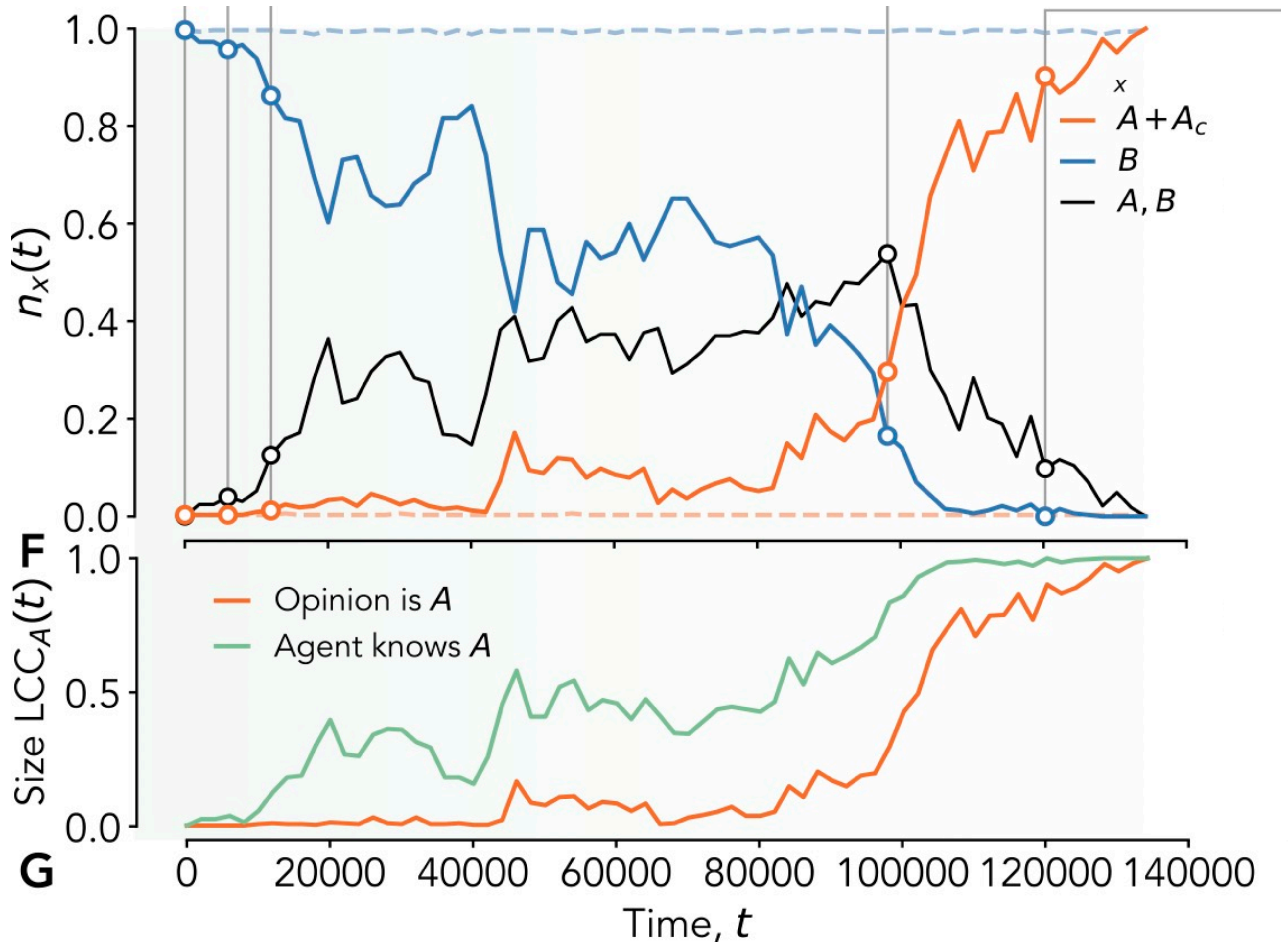


Iacopini, Petri, Baronchelli, Barrat (2021), In prep

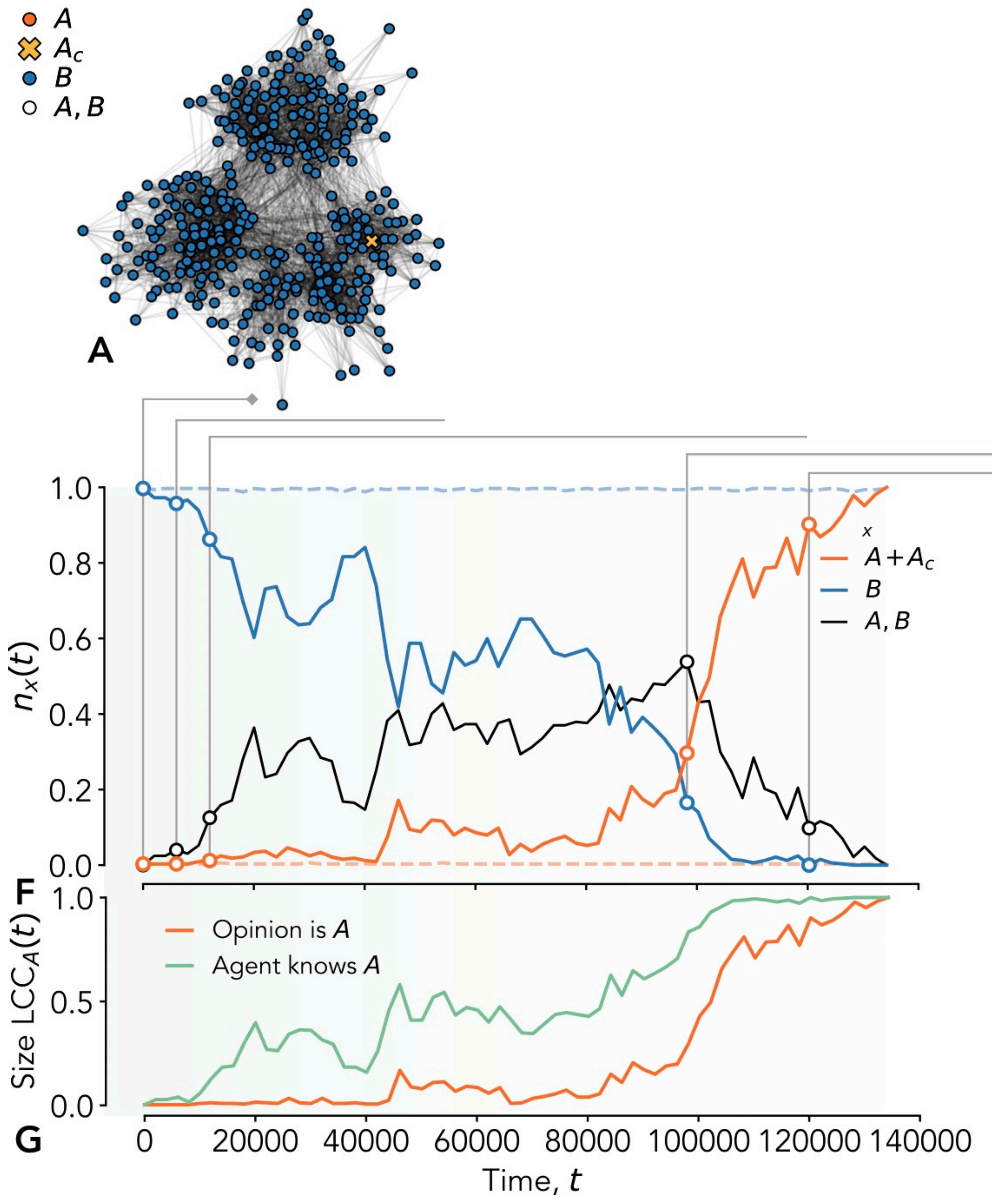
Empirical Social Structures

Naming Game with comm. minority + groups



Iacopini, Petri, Baronchelli, Barrat (2021), In prep

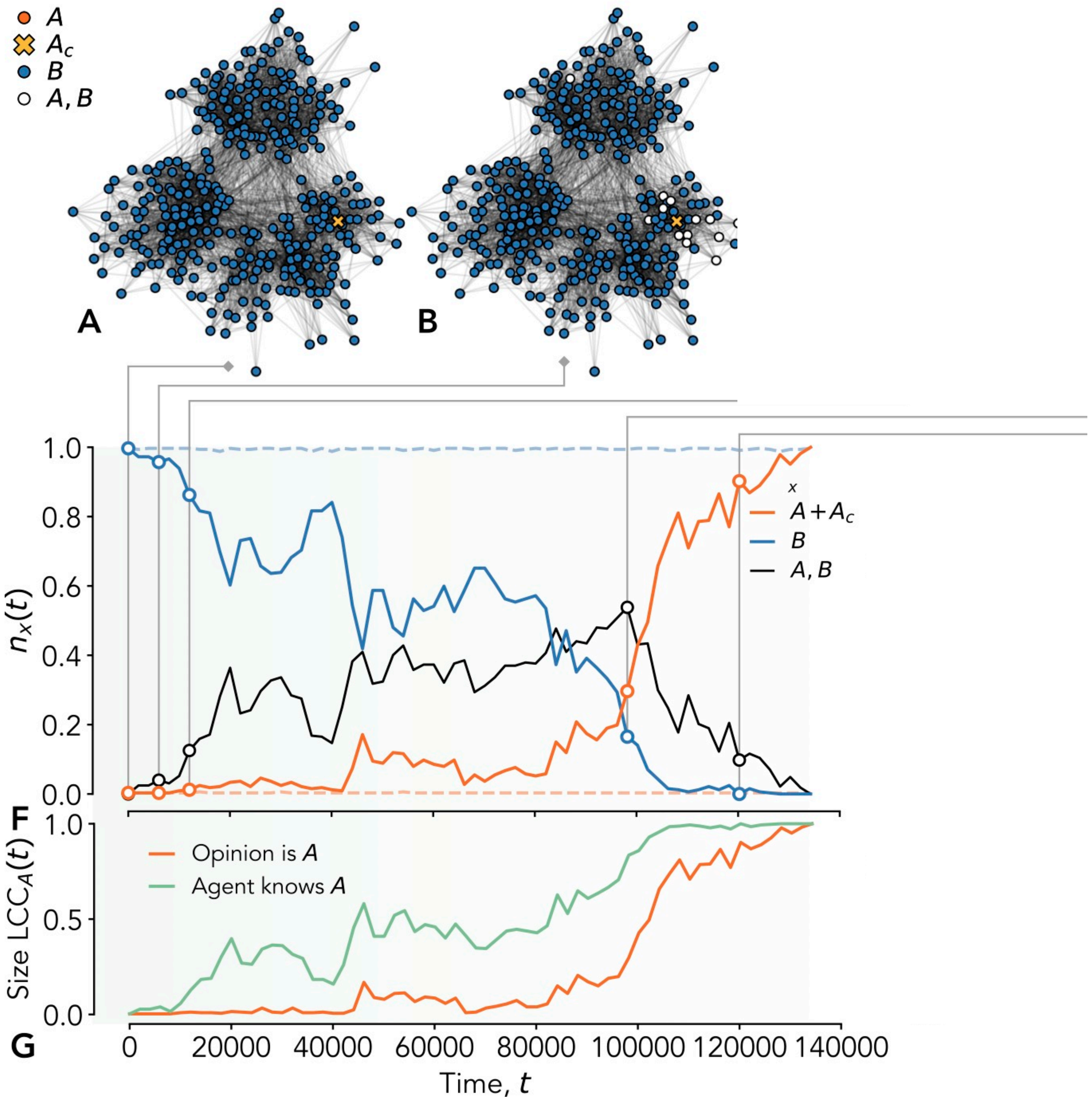
Naming Game **with comm. minority + groups**



- we start with **one** committed individual

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups



- we start with **one** committed individual
- The committed agent starts seeding (A,B) agents

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

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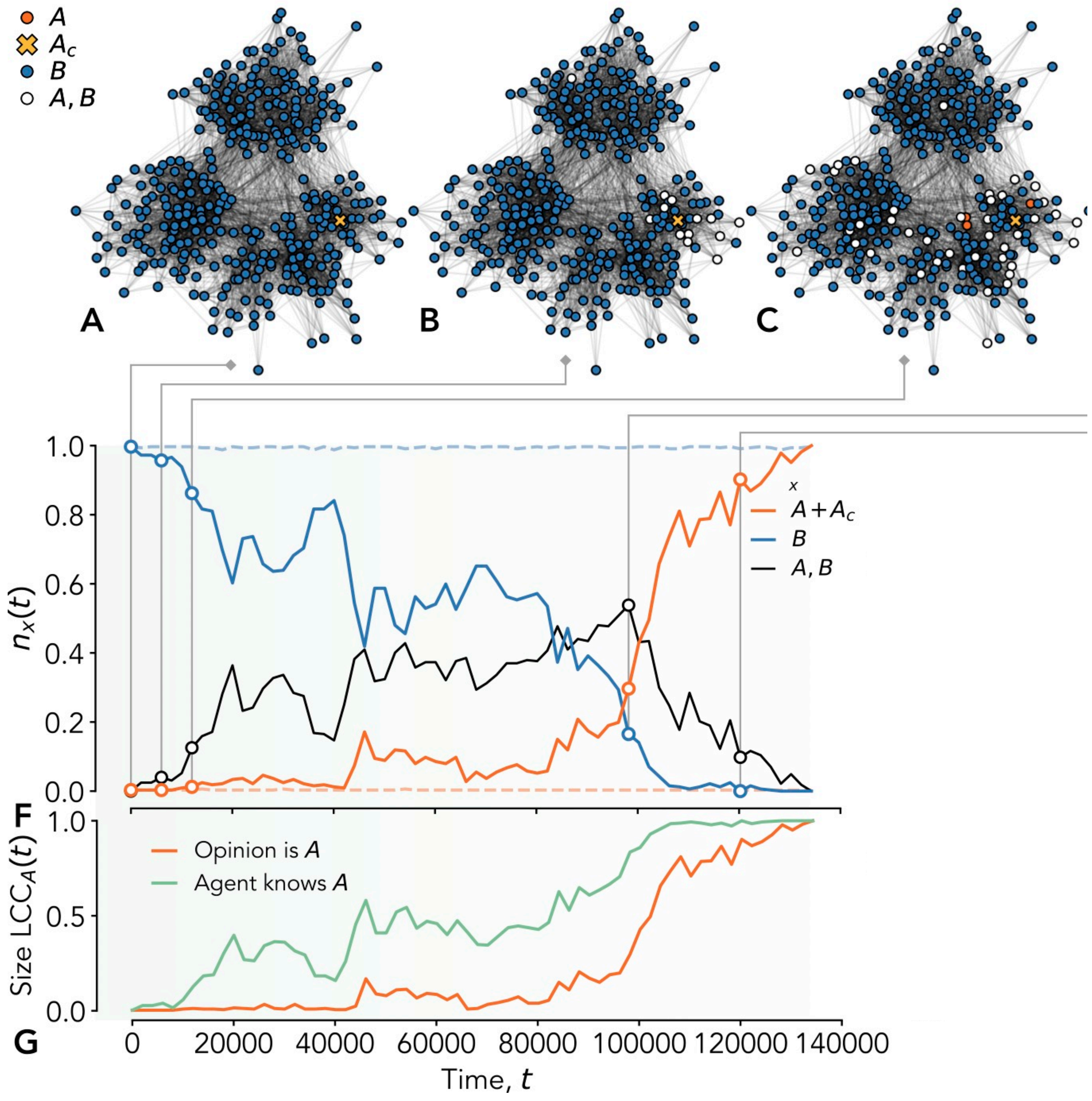

- we start with **one** committed individual
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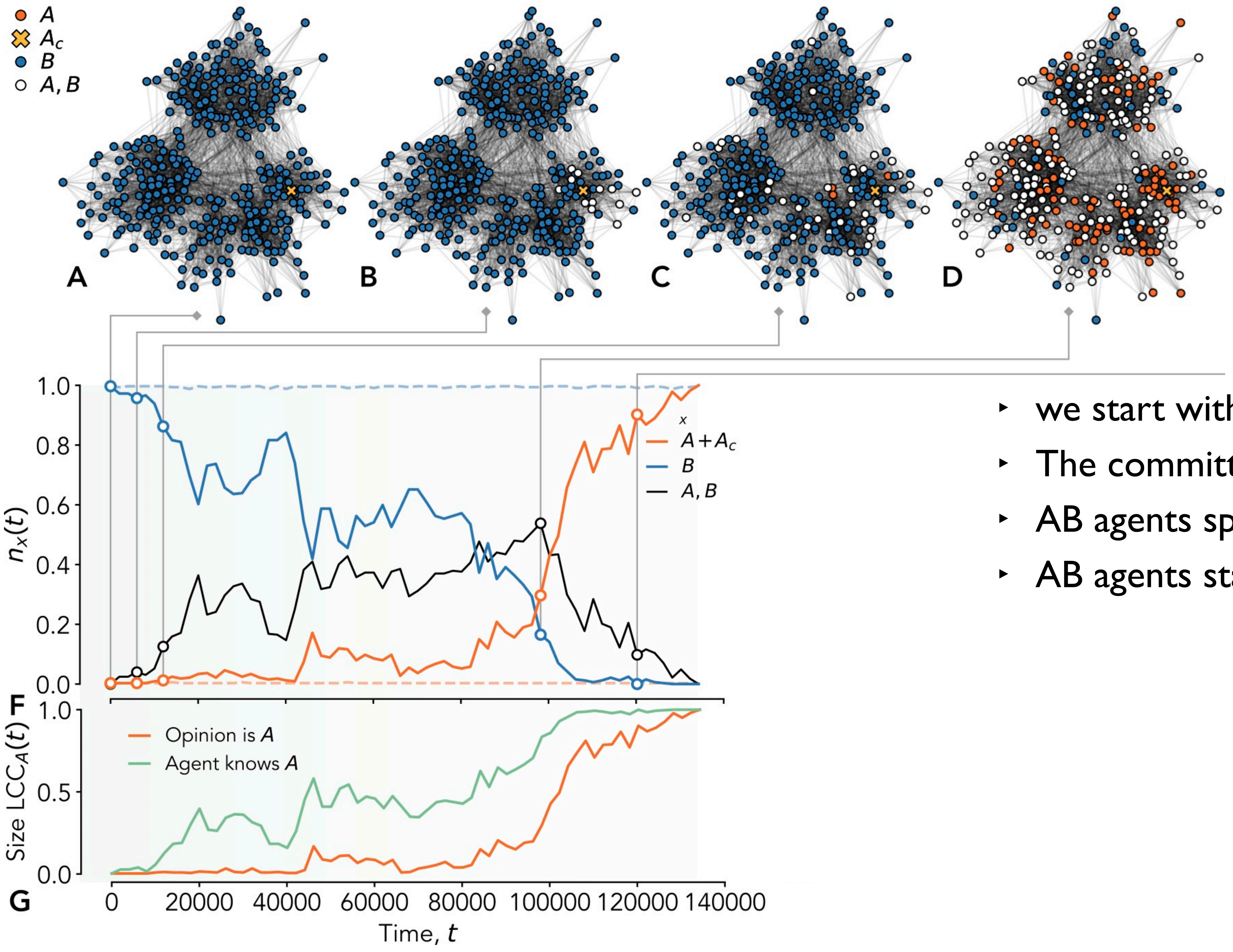
- AB agents spread widely

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups



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Iacopini, Petri, Baronchelli, Barrat (2021), In prep

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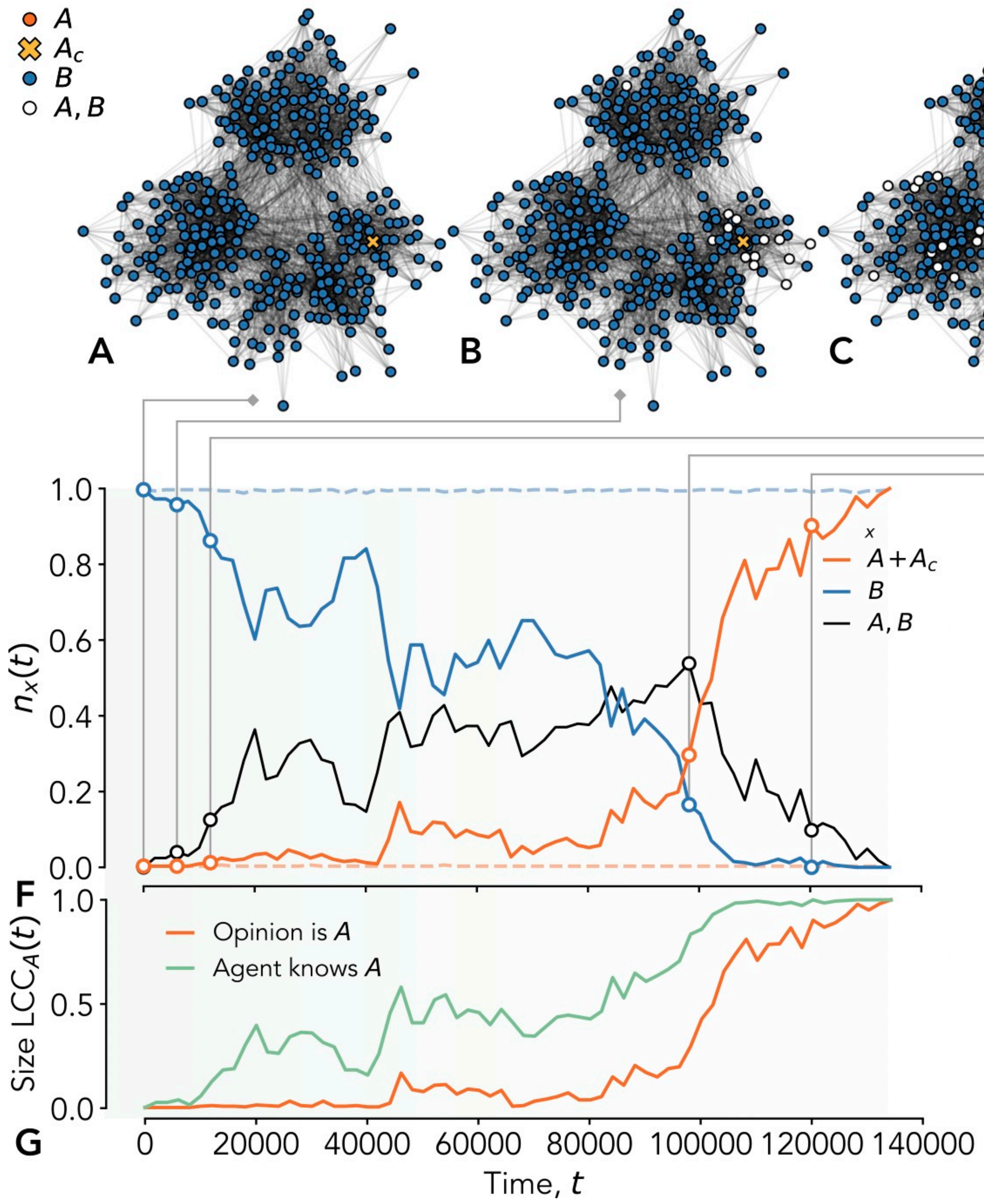

- we start with **one** committed individual
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Iacopini, Petri, Baronchelli, Barrat (2021), In prep

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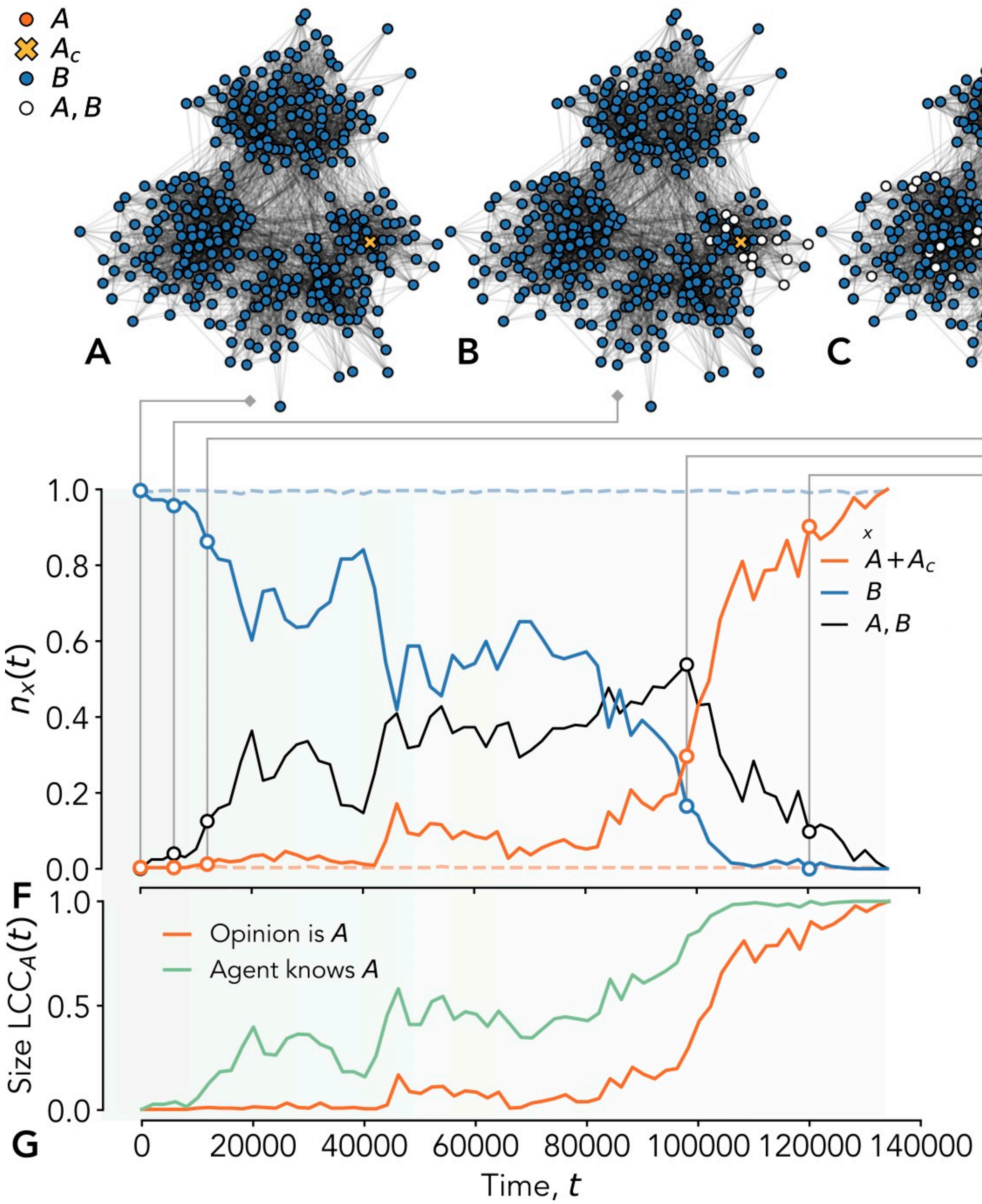

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- The committed agent starts seeding (A,B) agents
- AB agents spread widely
- AB agents start converting to A
- eventually the system converges to A (95% of the time)

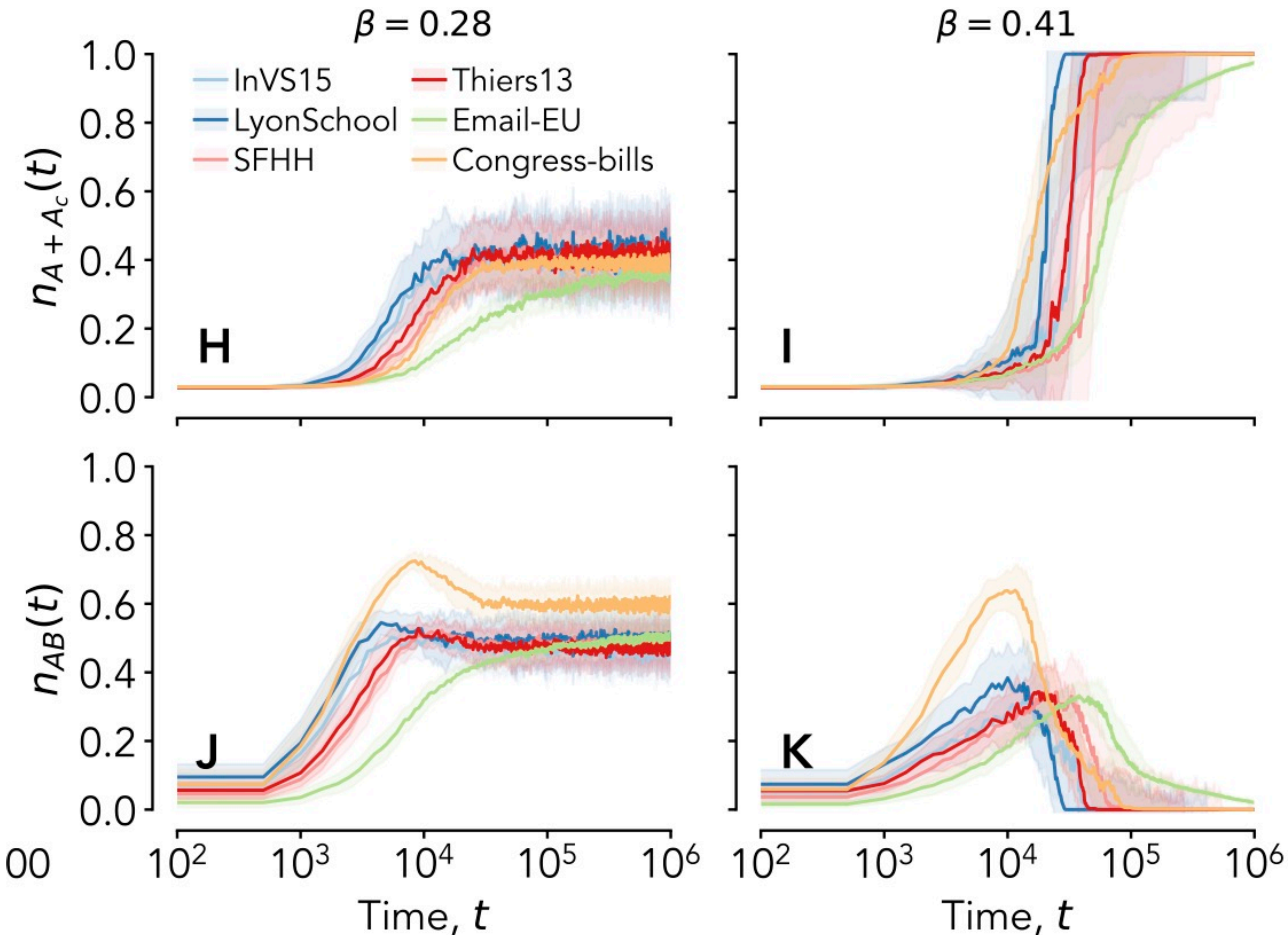
Main result:
vanishing critical mass of committed agents
(with well chosen β)

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

Temporal evolution with committed fraction p = 0.03(!)

Coexistence



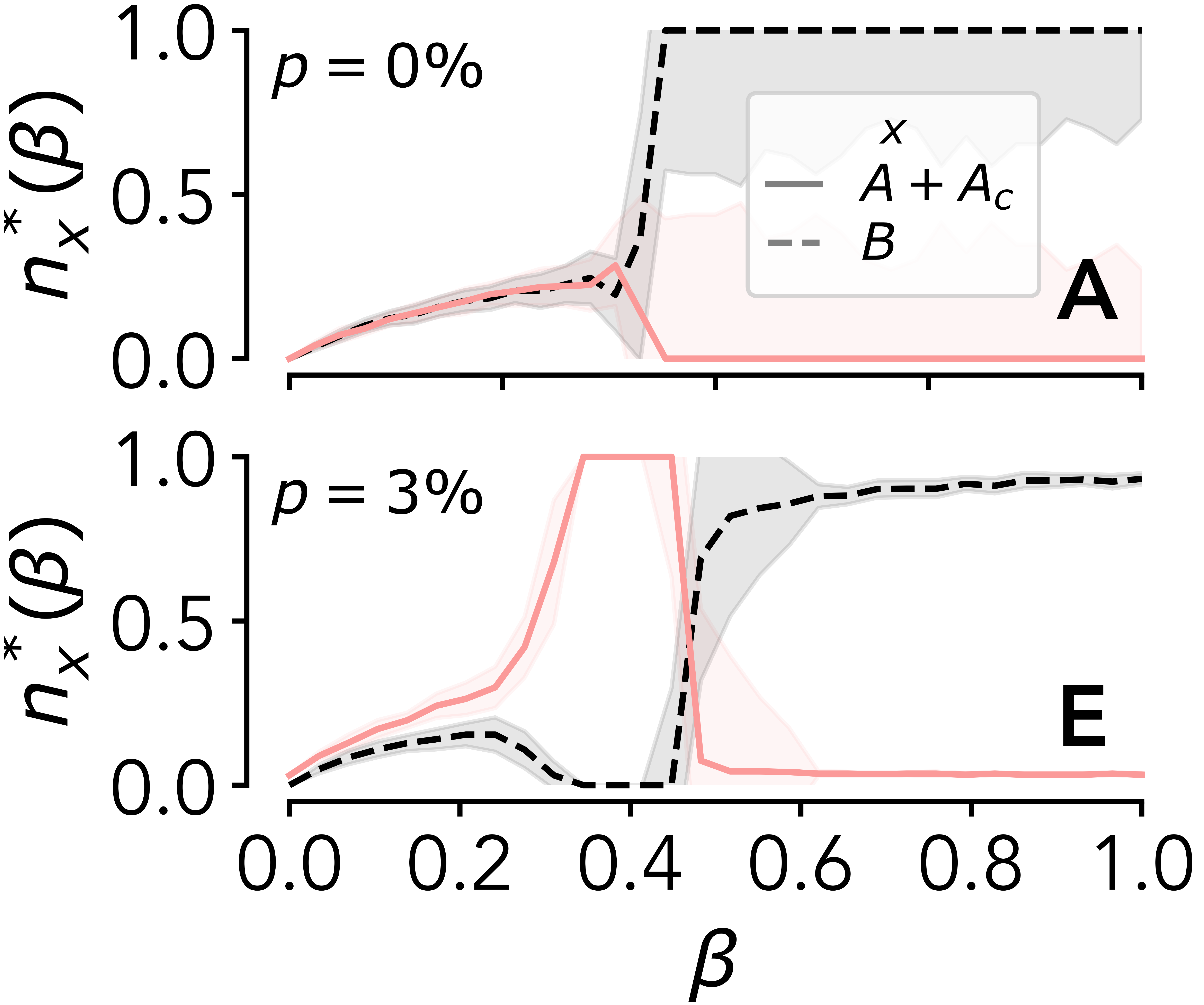
Minority takeover

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

The role of β

SFHH dataset



Naming Game with comm. minority + groups

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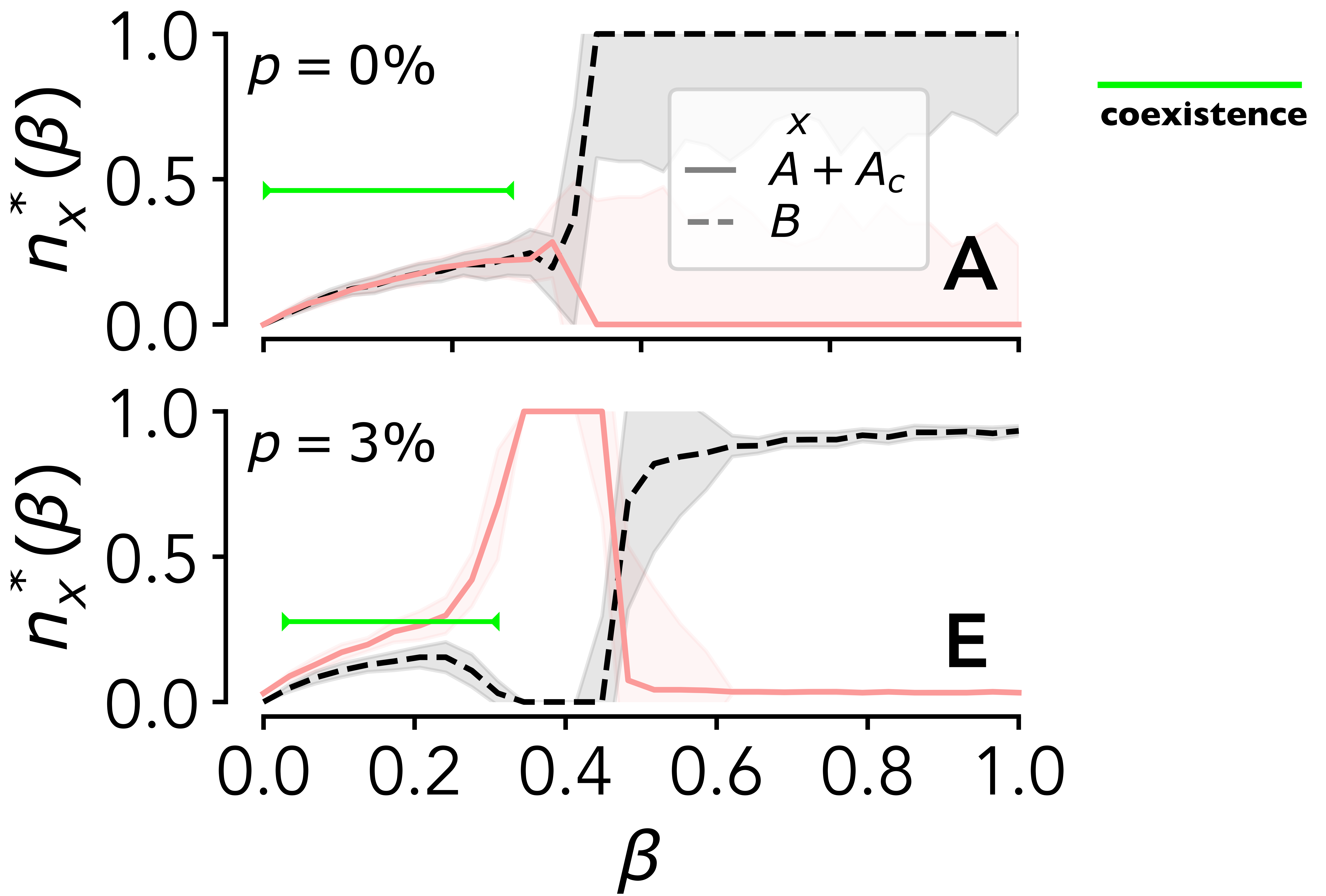

Naming Game with comm. minority + groups

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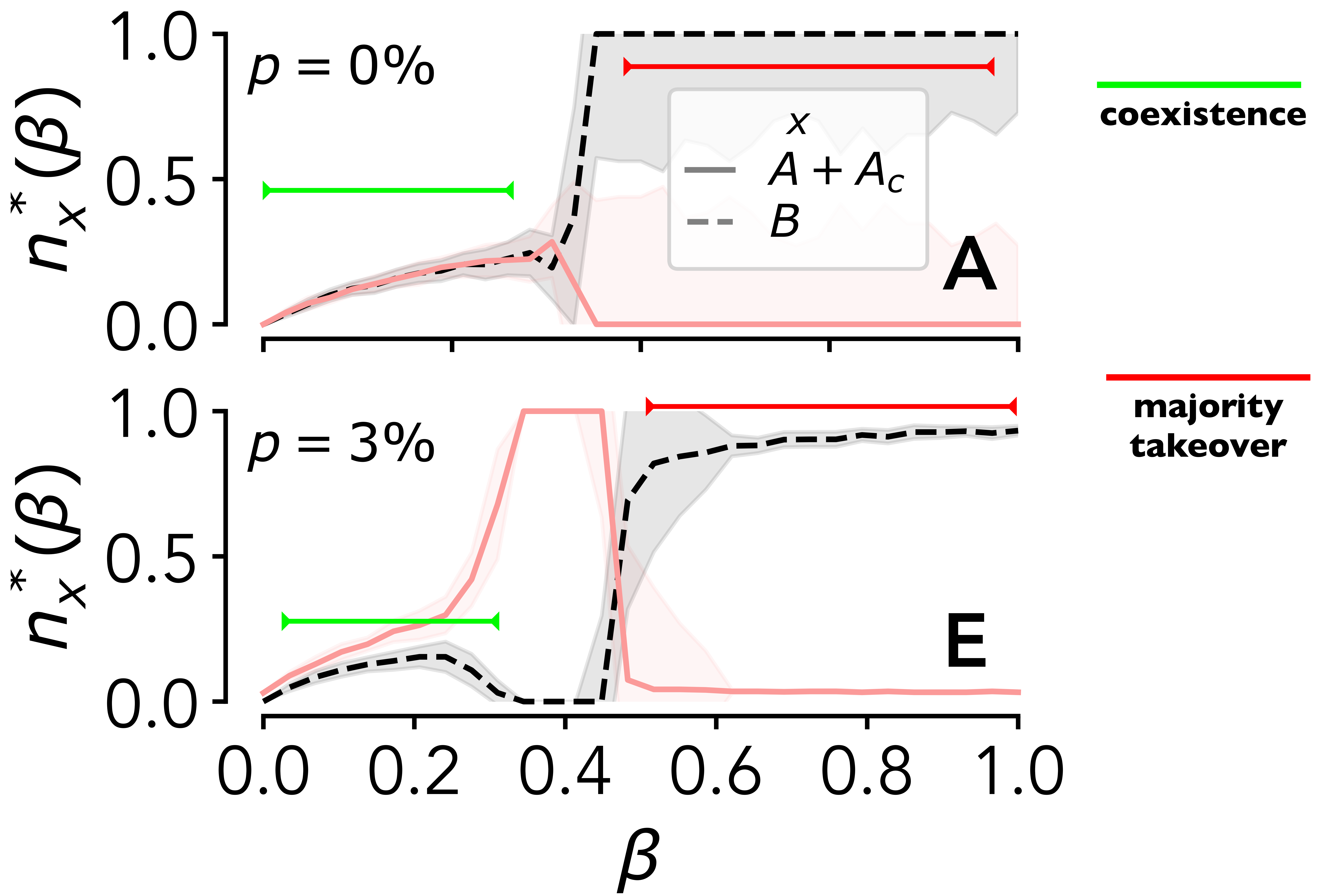

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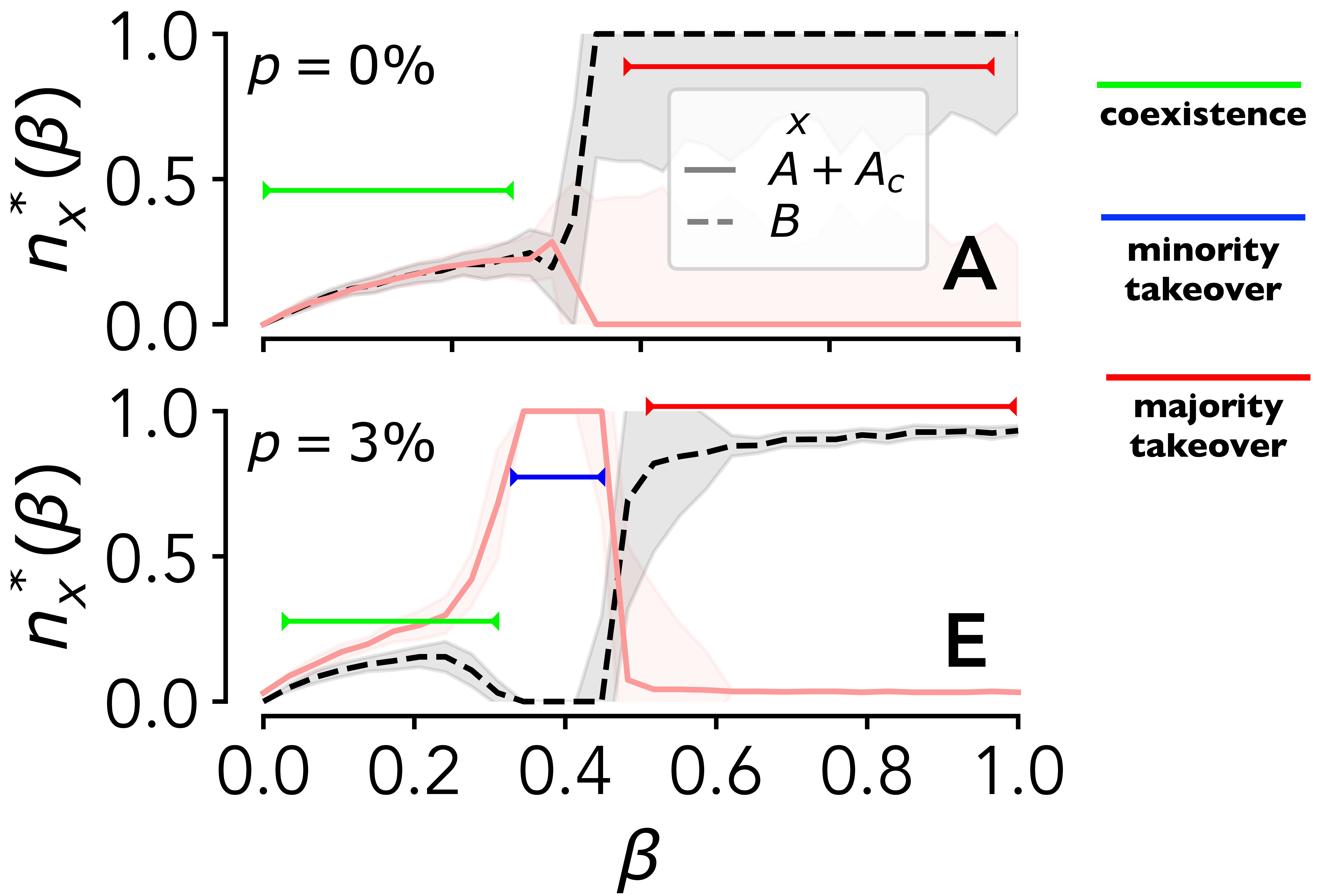

Naming Game with comm. minority + groups

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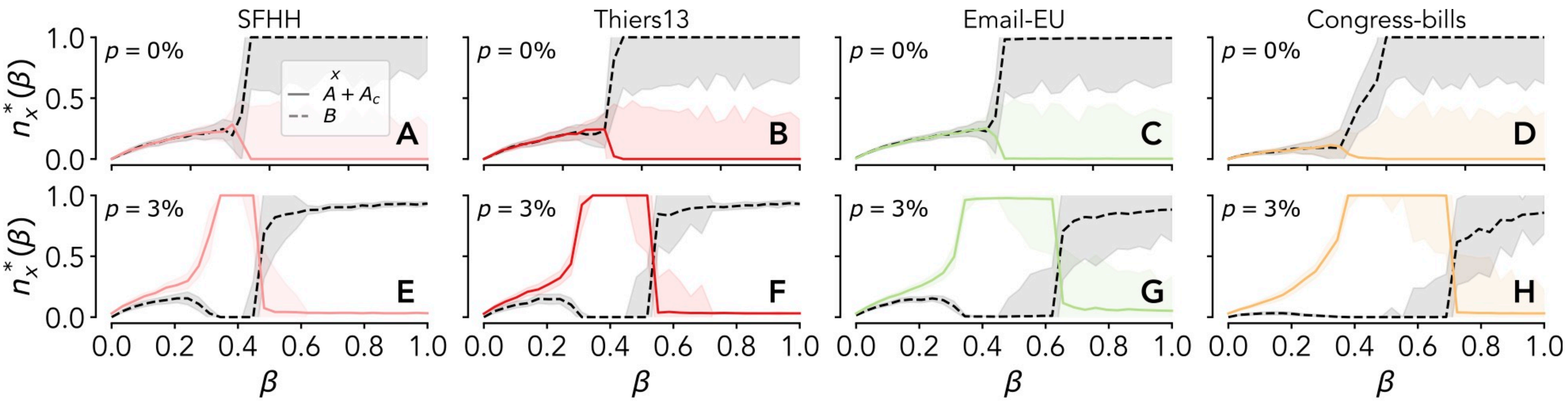

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

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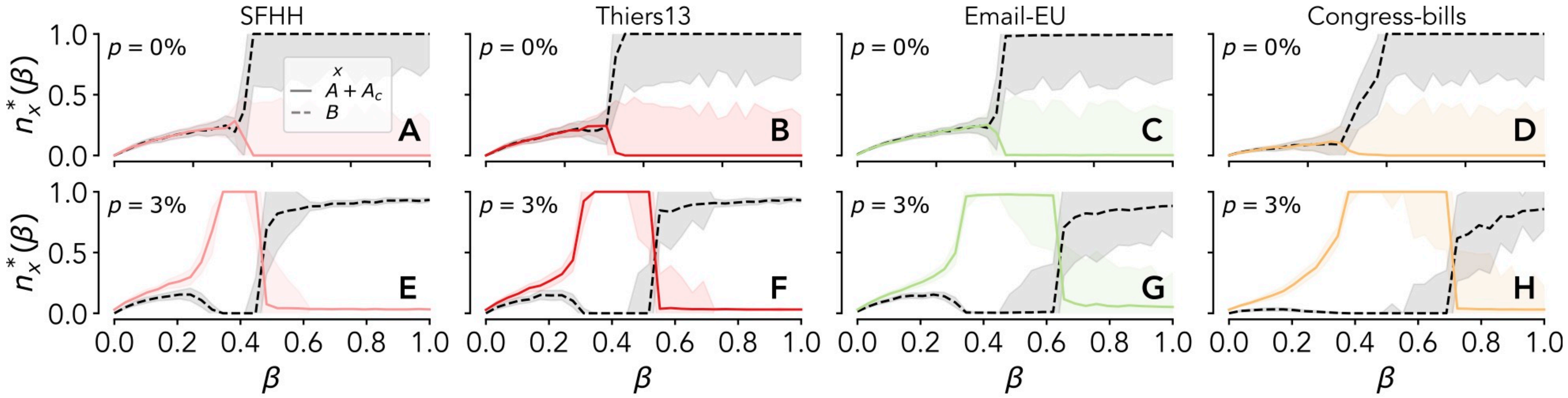


What is the interplay between β and p?

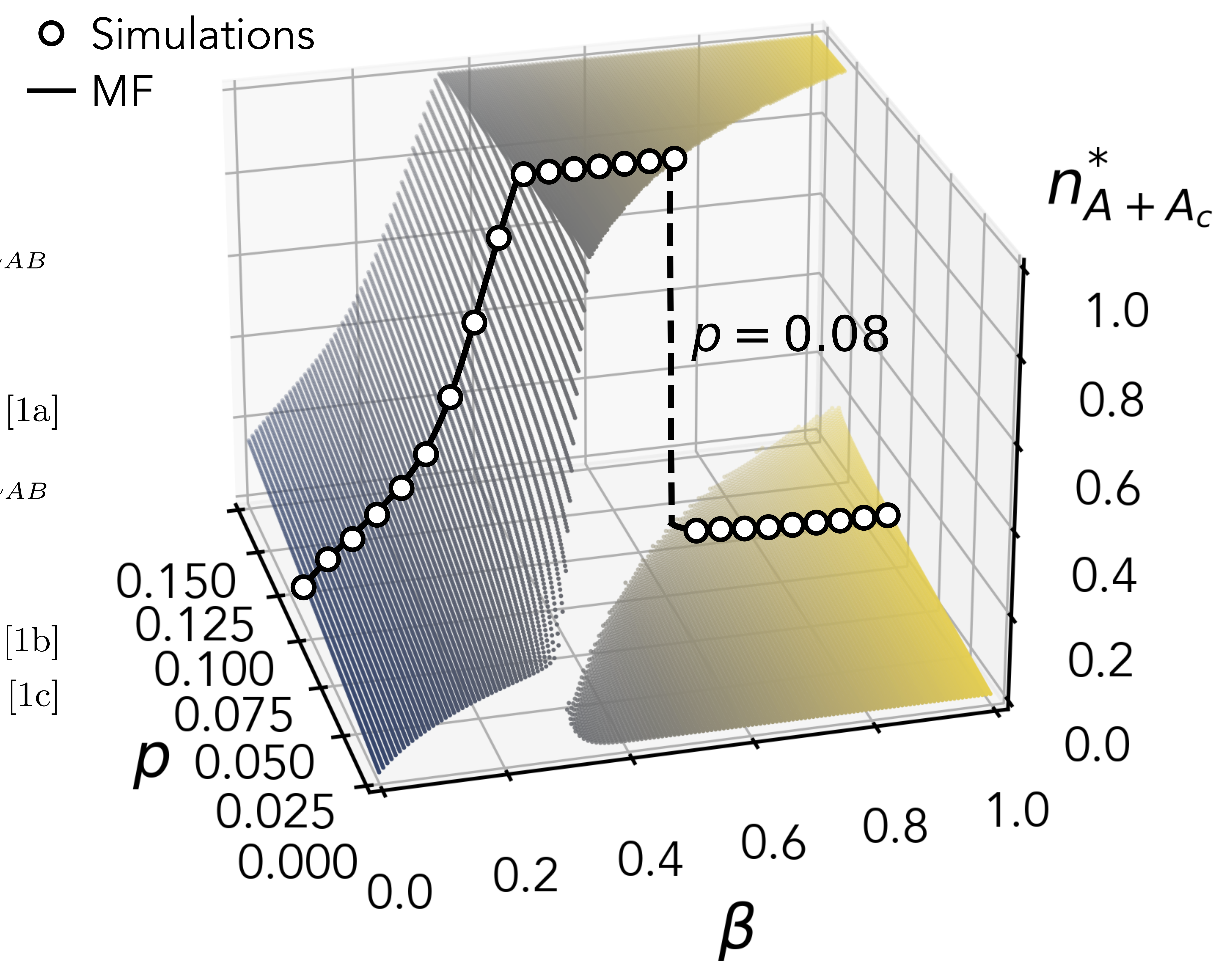
Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

The (analytical) interplay of β and p

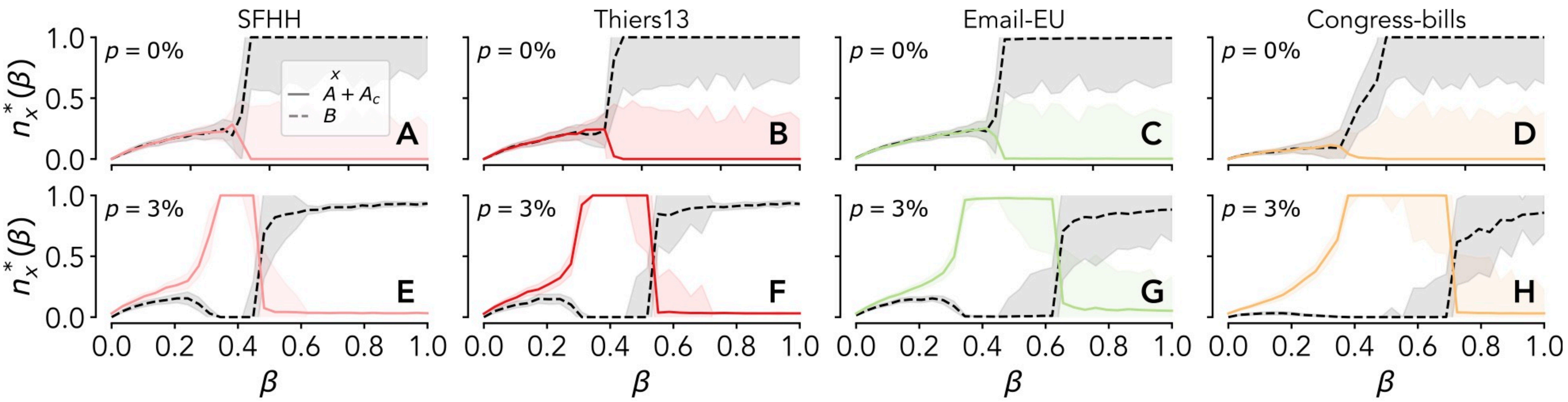
Structure: 3-regular hypergraph

$$d_t n_A = -2n_A^2 n_B + (\frac{5}{2}\beta - 1)n_A^2 n_{AB} - 2n_A n_B^2 - 3n_A n_B n_{AB}$$
$$+ (4\beta - 1)n_A n_{AB}^2 + \frac{3}{2}\beta n_{AB}^3 + \frac{5}{2}\beta p^2 n_{AB}$$
$$+ p[-2n_A n_B + (5\beta - 1)n_A n_{AB} + 4\beta n_{AB}^2] \quad [1a]$$
$$d_t n_B = -2n_B^2 n_A + (\frac{5}{2}\beta - 1)n_B^2 n_{AB} - 2n_B n_A^2 - 3n_B n_A n_{AB}$$
$$+ (4\beta - 1)n_B n_{AB}^2 + \frac{3}{2}\beta n_{AB}^3 - 2p^2 n_B$$
$$- p[4n_A n_B + 2n_B^2 + 3n_b n_{AB}] \quad [1b]$$
$$n_{AB} = 1 - n_A - n_B - p \quad [1c]$$



Naming Game with comm. minority + groups

Fine tuning of β ?



Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

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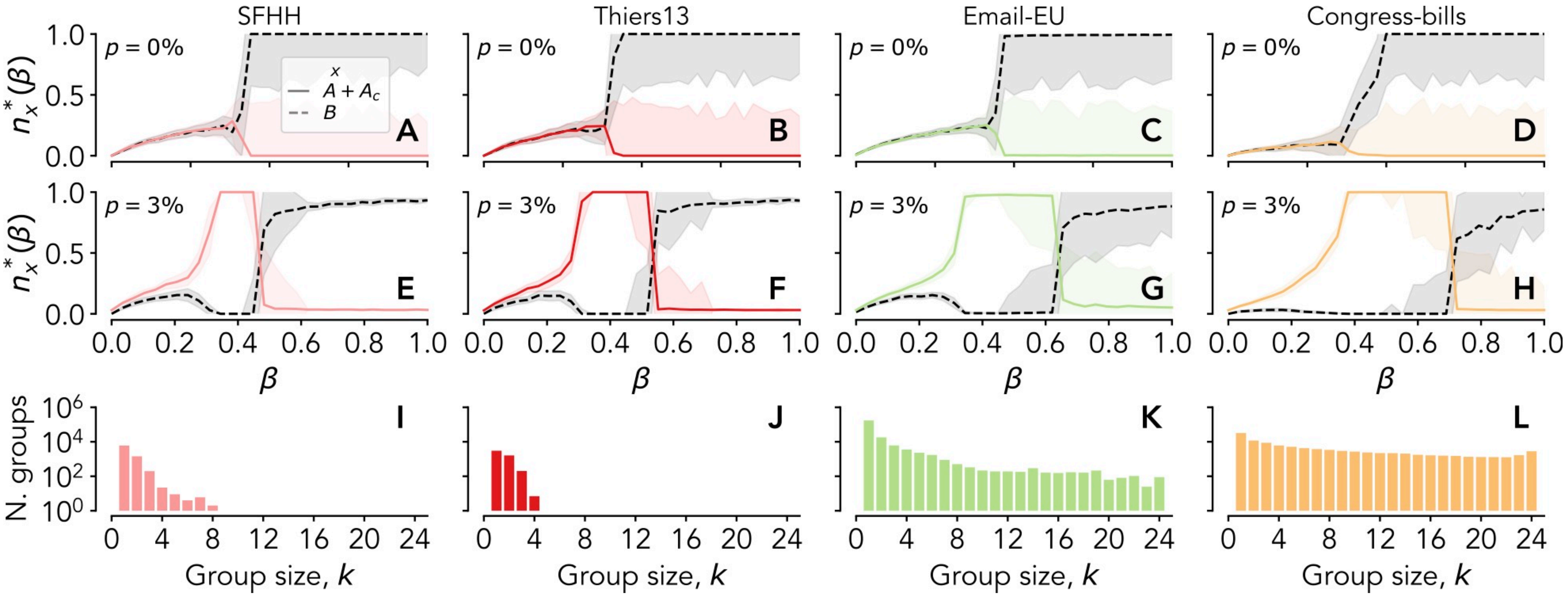

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

The interplay of β and group sizes

Structures: k-regular hypergraphs

Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Naming Game with comm. minority + groups

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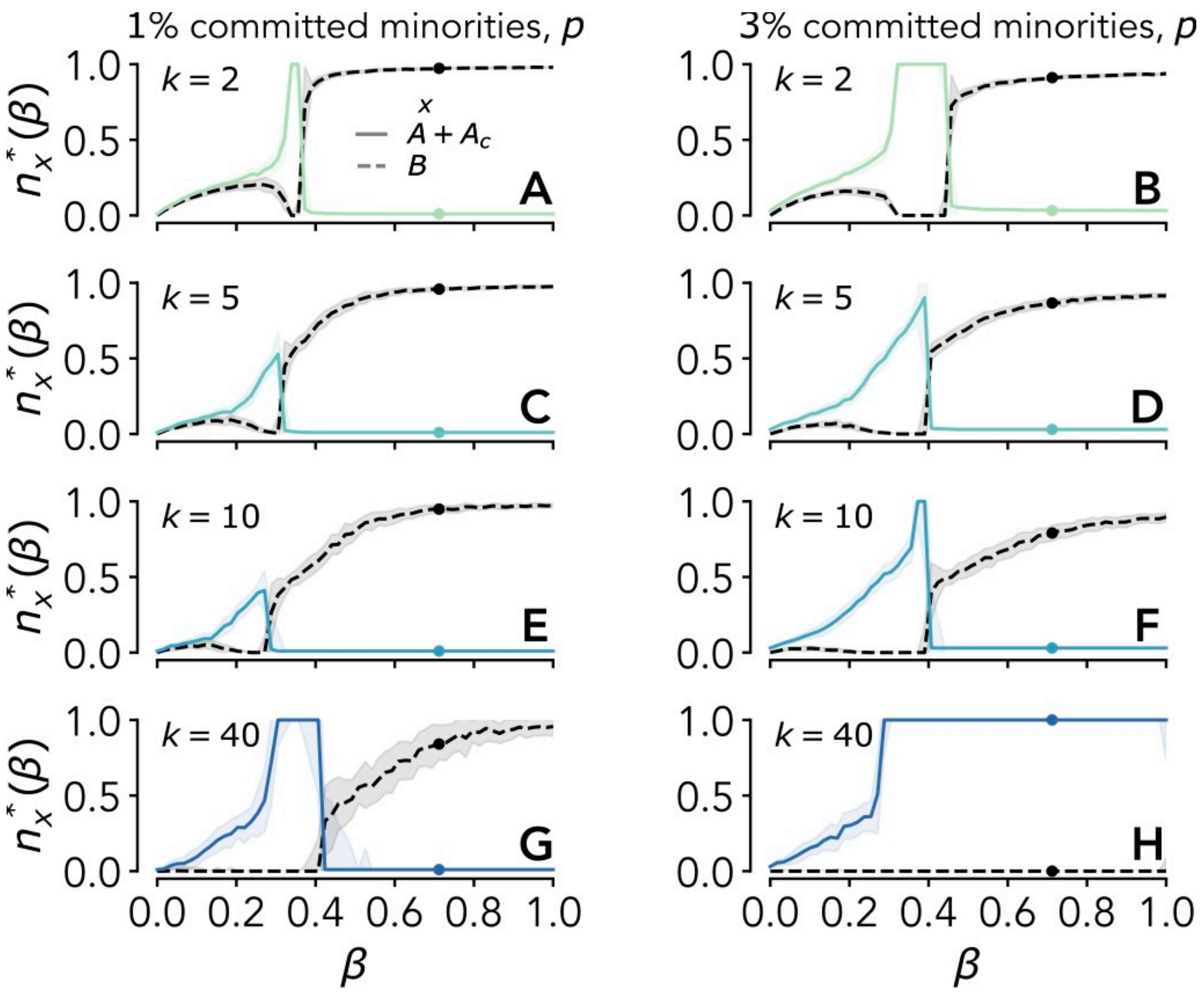

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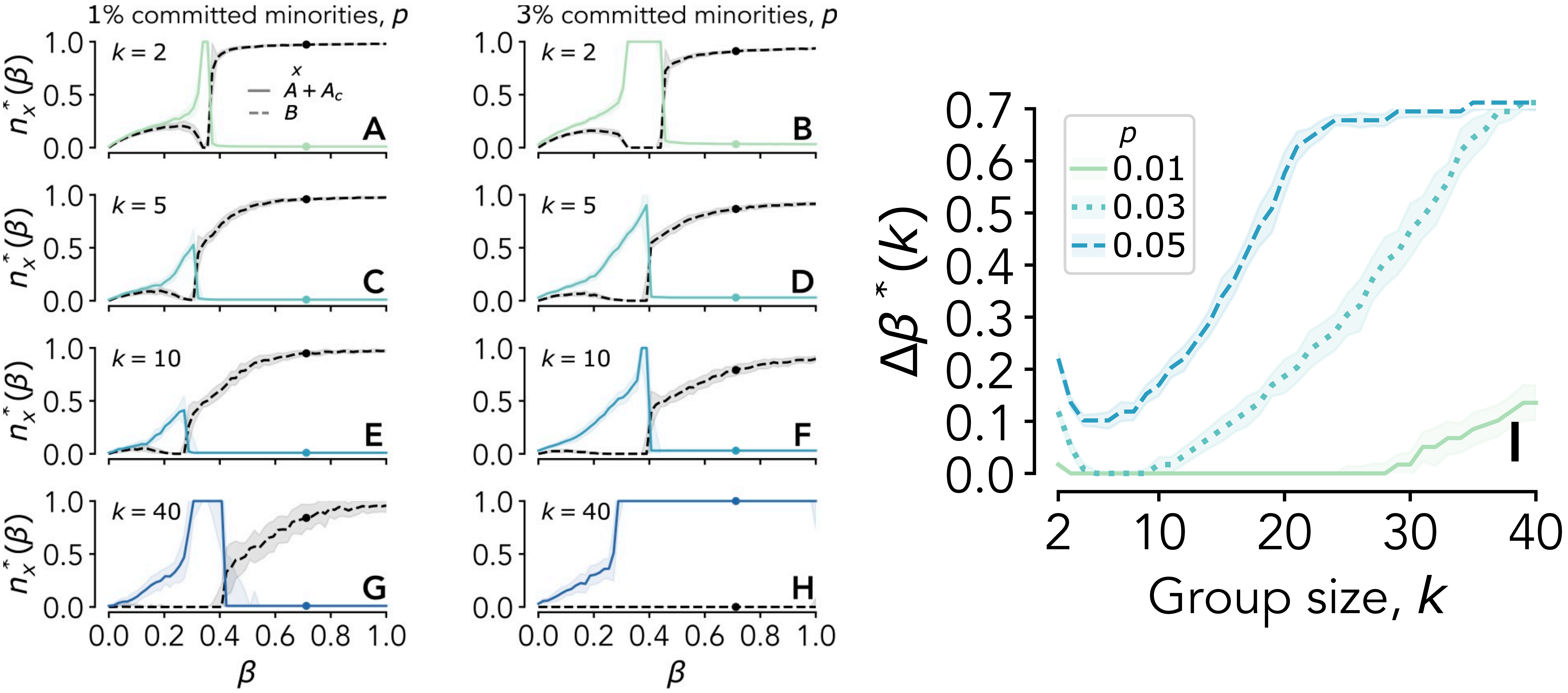
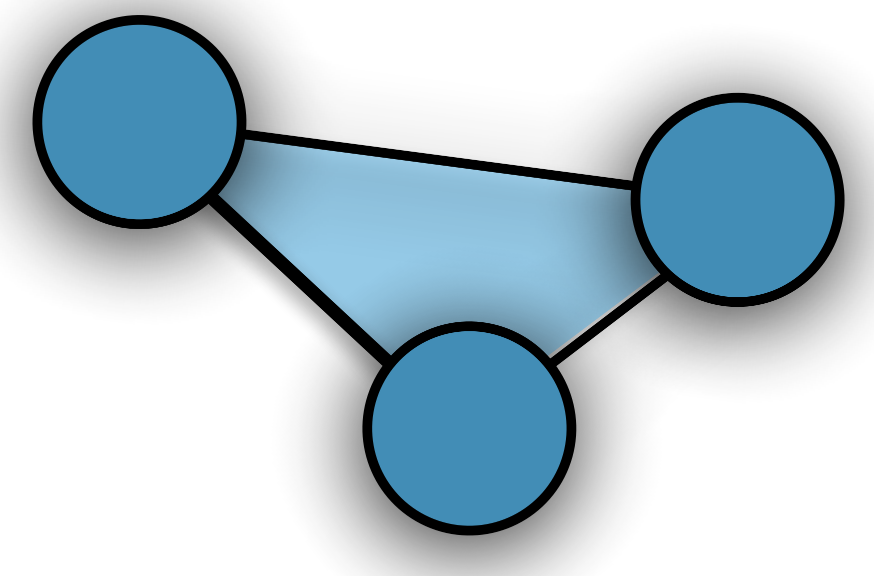


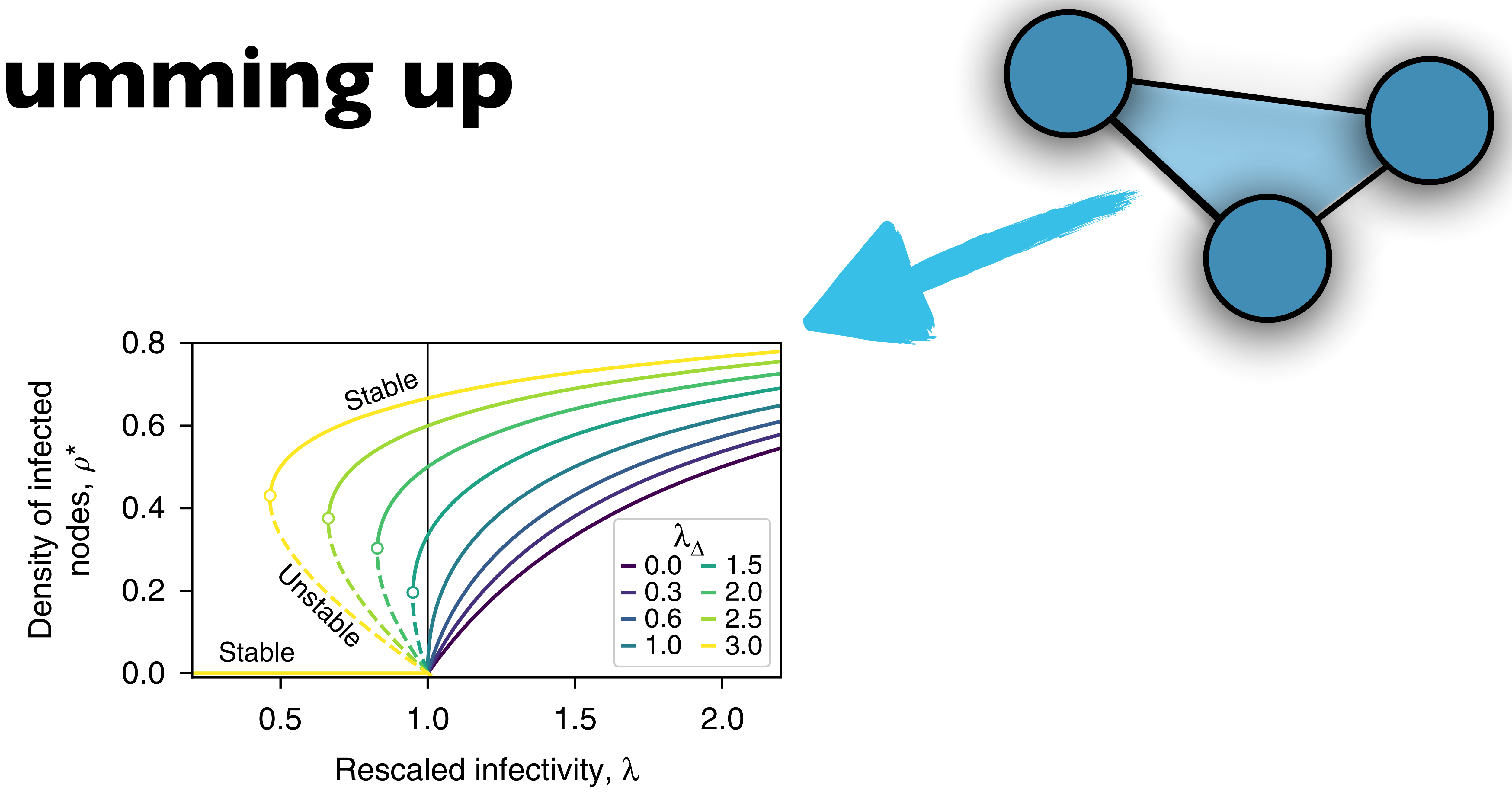
Iacopini, Petri, Baronchelli, Barrat (2021), In prep

Notebooks HONG

Summing up



Summing up



- **Model:**
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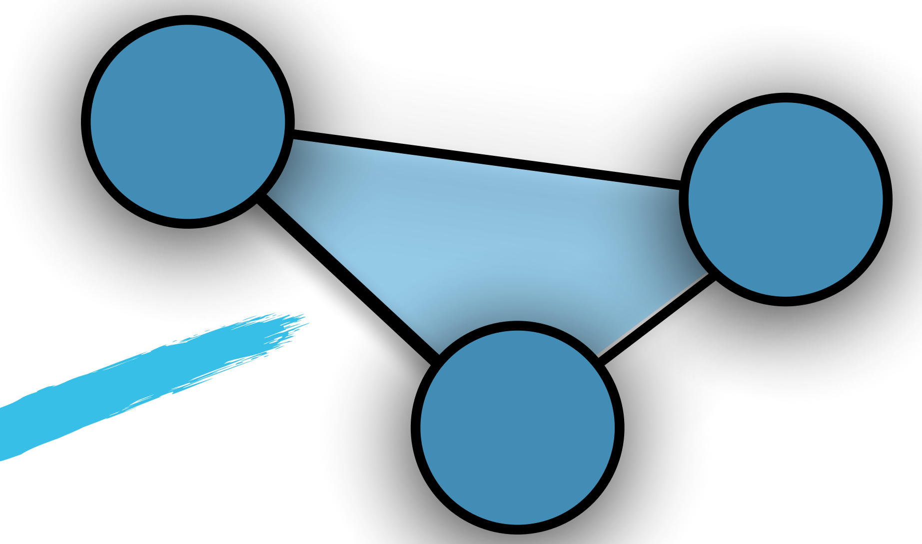
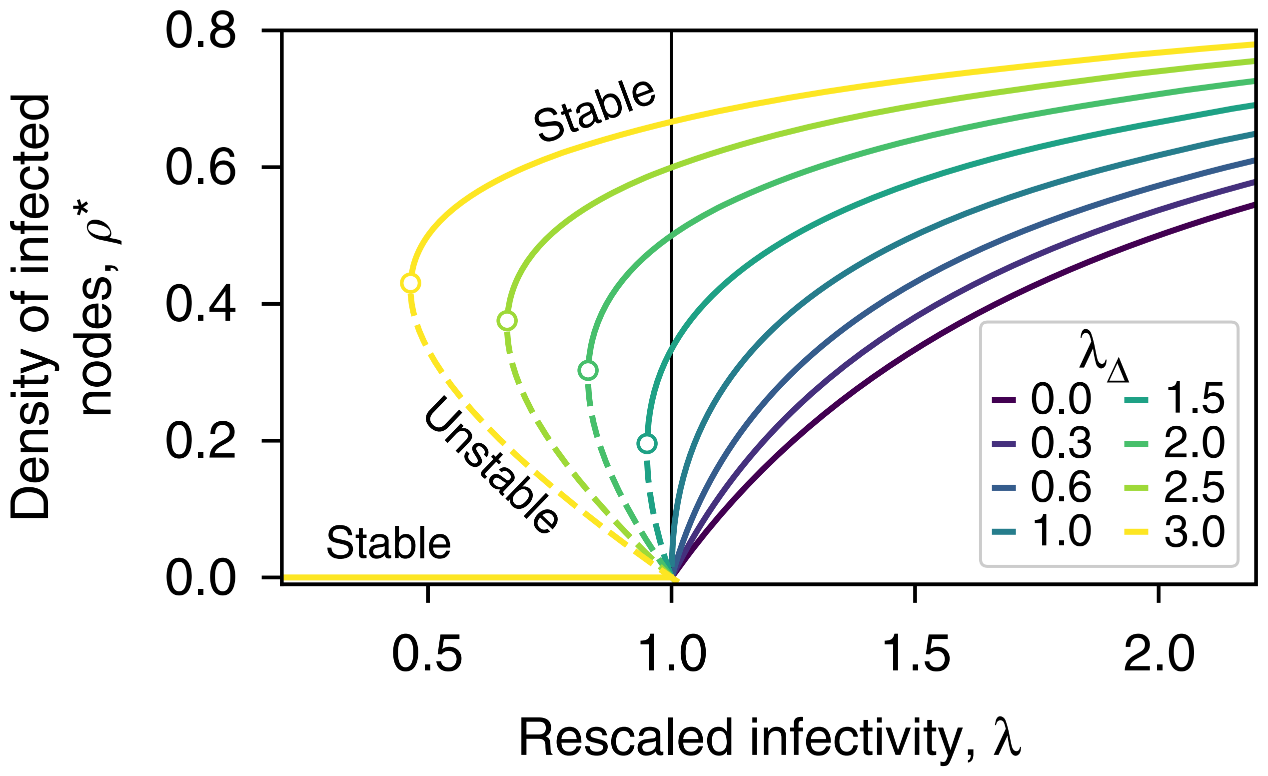
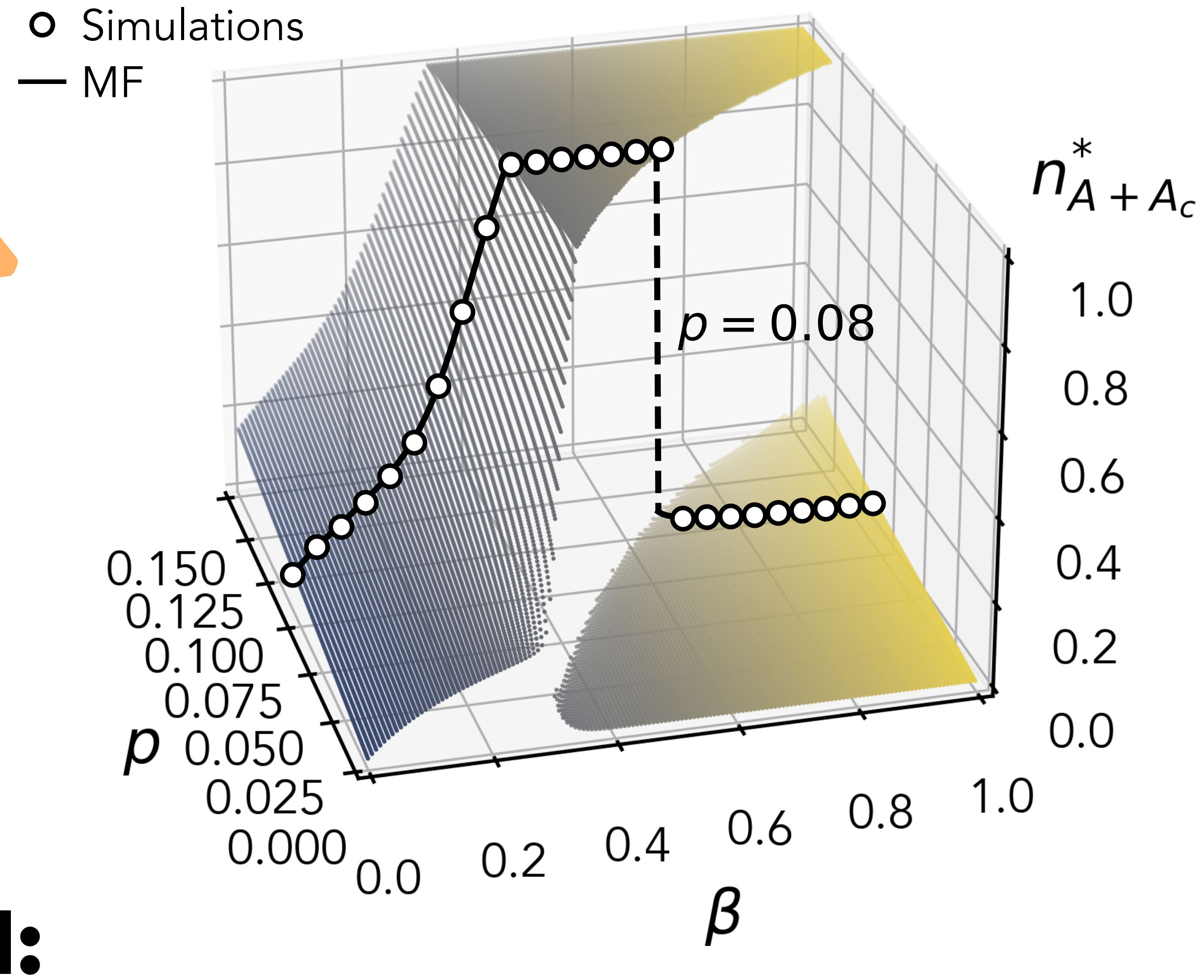
Summing up







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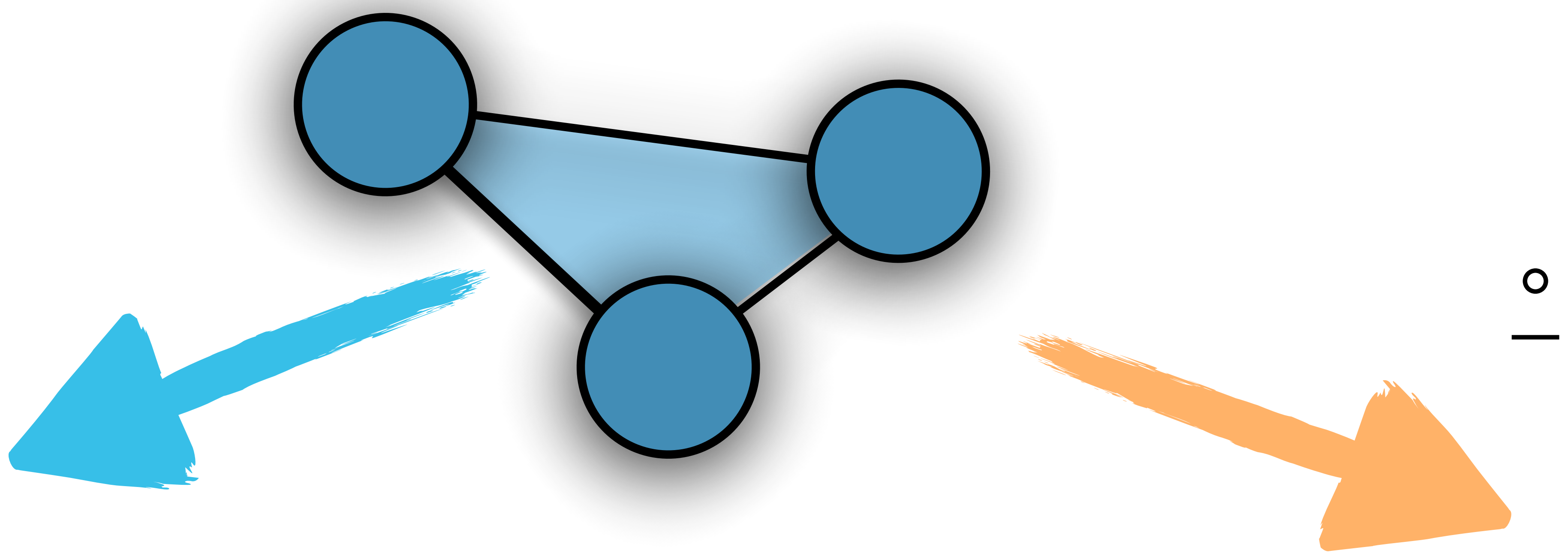
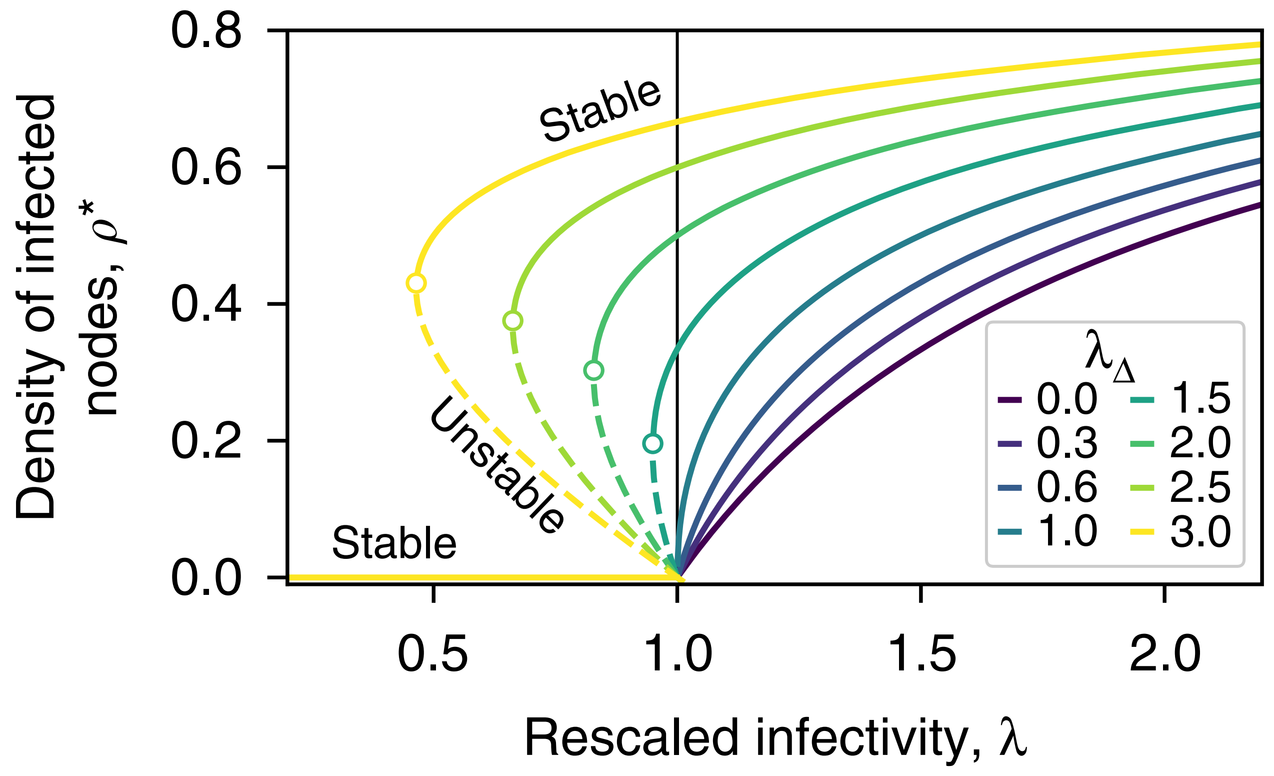
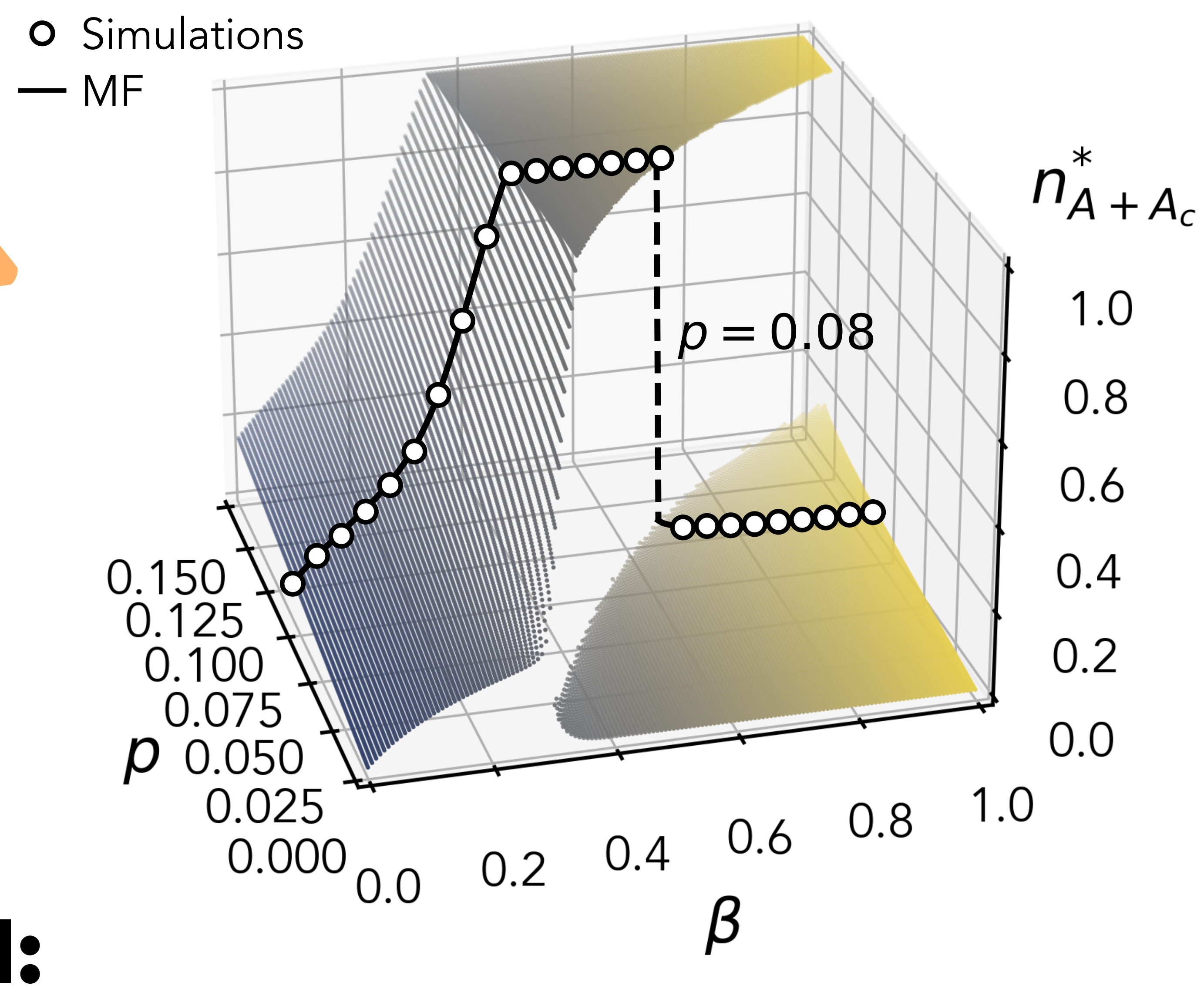
  - **Vanishing critical mass for comm. min.**
  - **Group sizes modulate interplay**

Summing up







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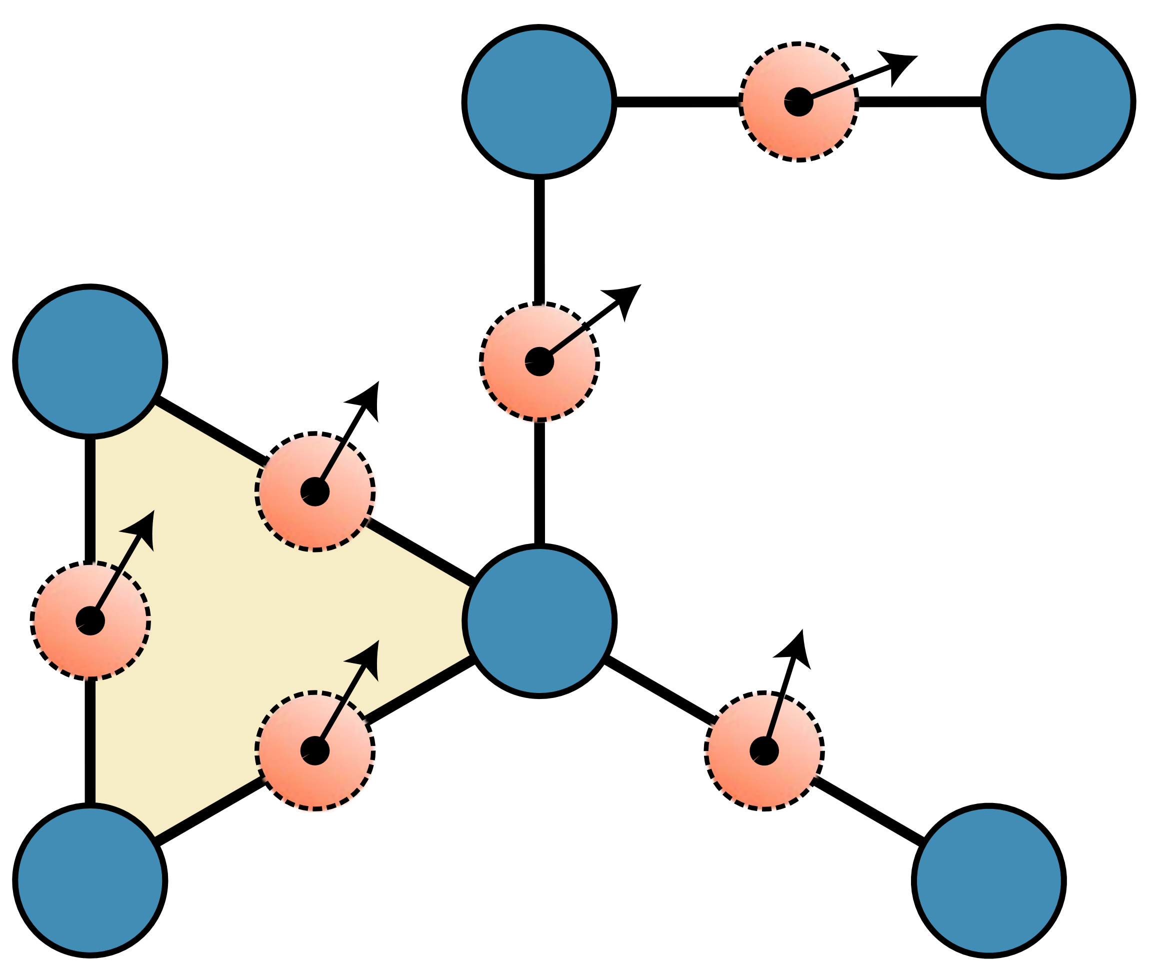
Next steps: experiments! Data!

Topological dynamical processes

Higher-order systems are fully dynamical.

a



Millán, A. P., Torres, J. J., & Bianconi, G. (2020). Explosive higher-order Kuramoto dynamics on simplicial complexes. *Physical Review Letters*, *124*(21), 218301.

Topological dynamical processes

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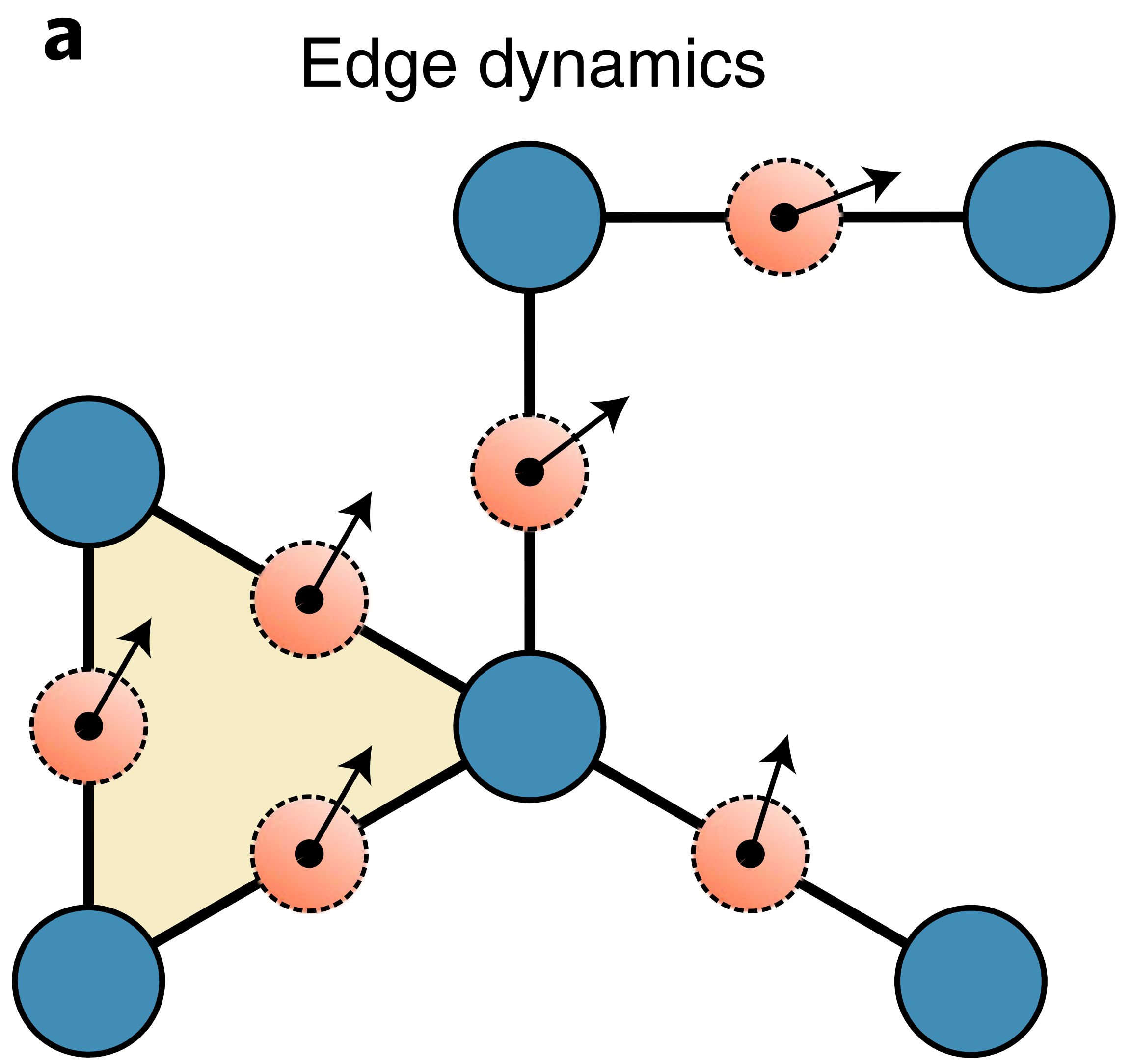

Node Kuramoto

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega} - \sigma \mathbf{B}_{[1]} \sin \mathbf{B}_{[1]}^{\top} \boldsymbol{\theta},$$

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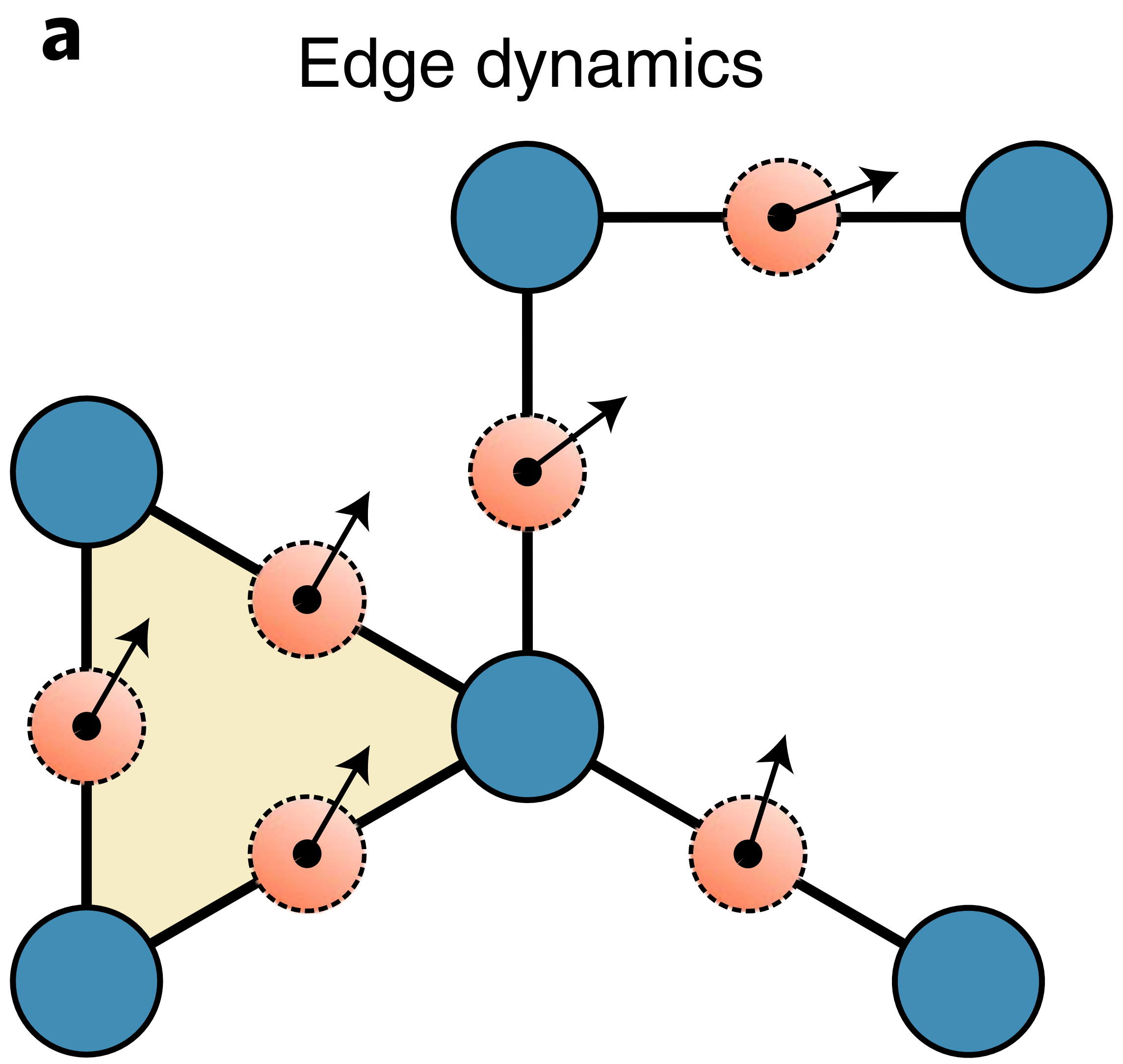

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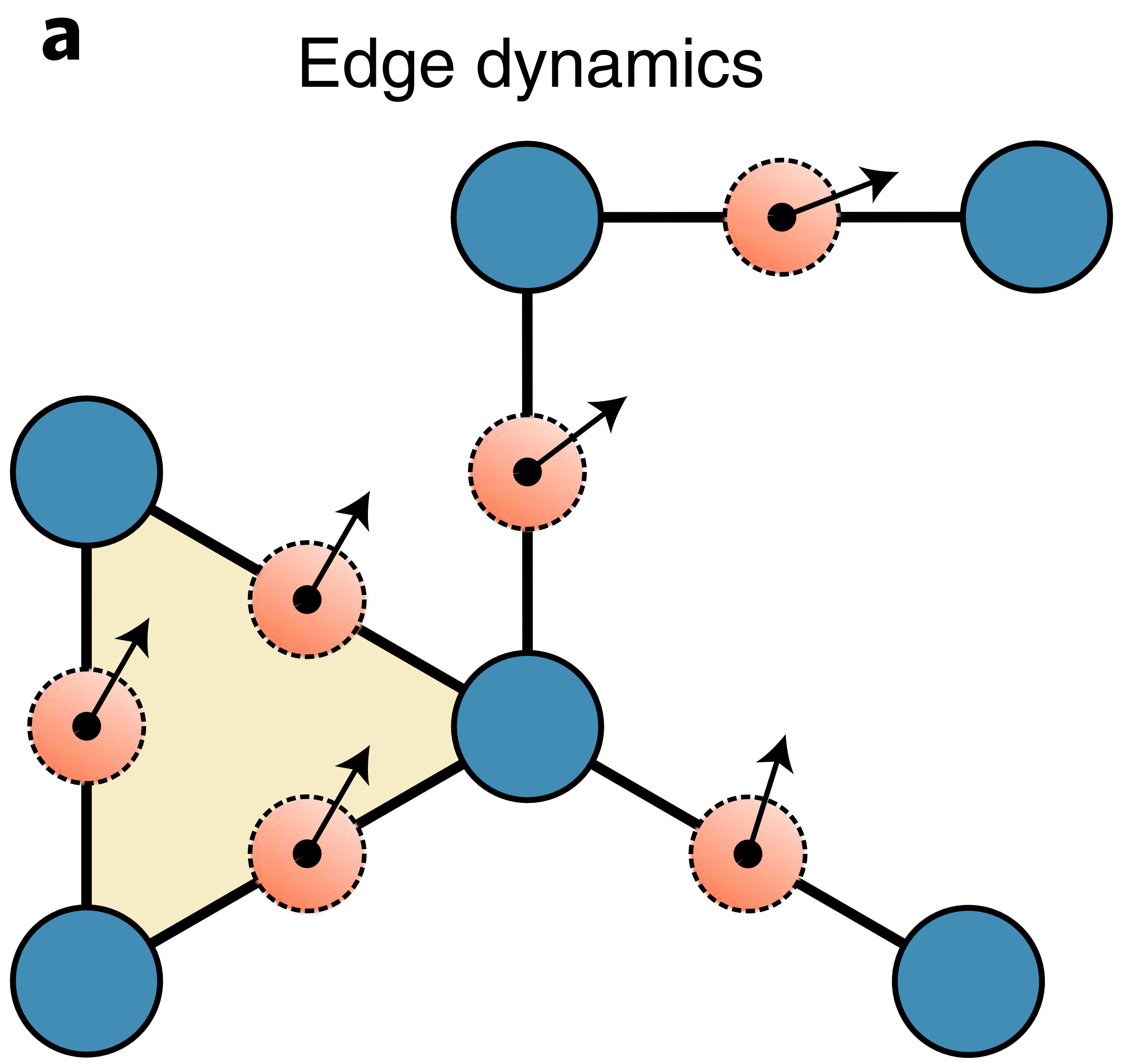

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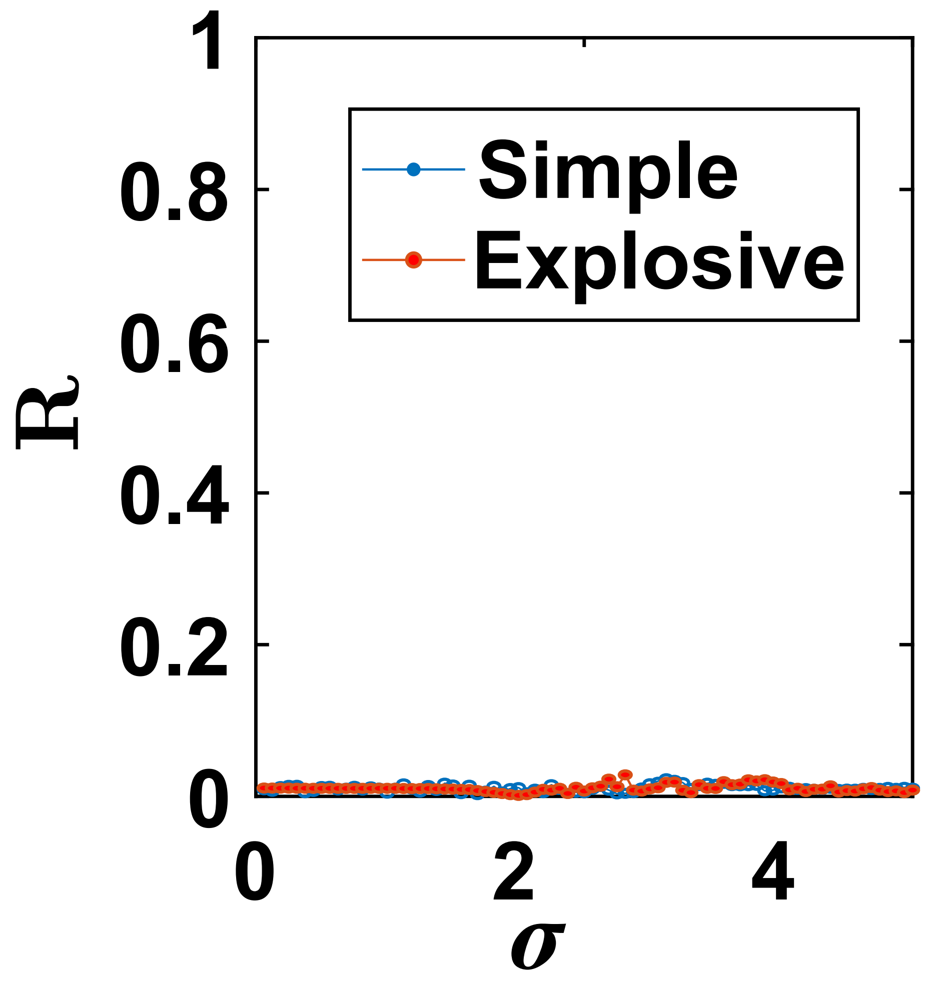

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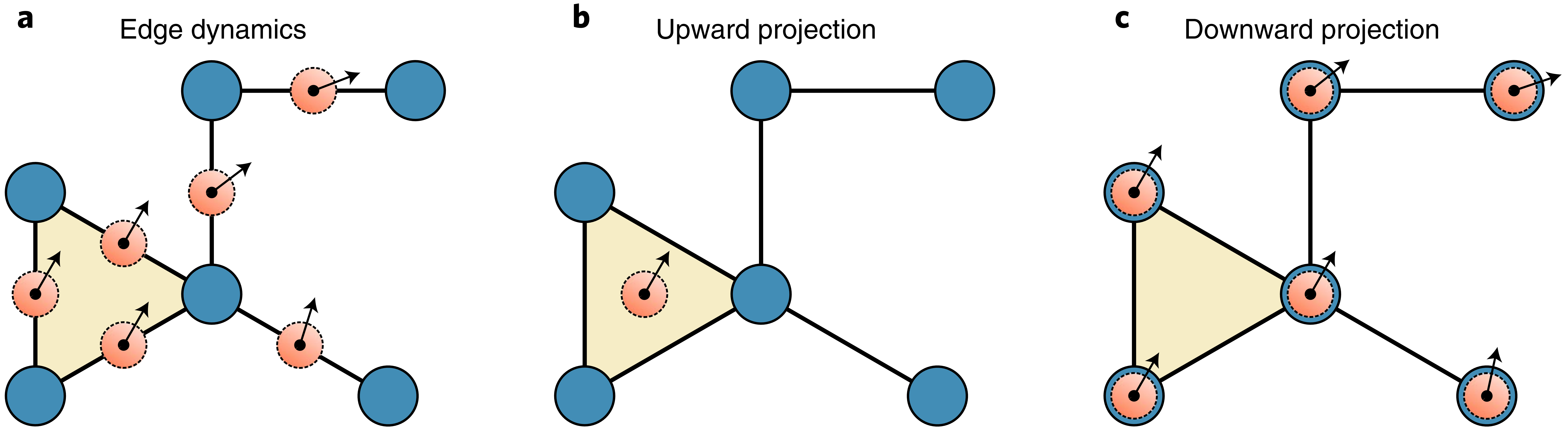
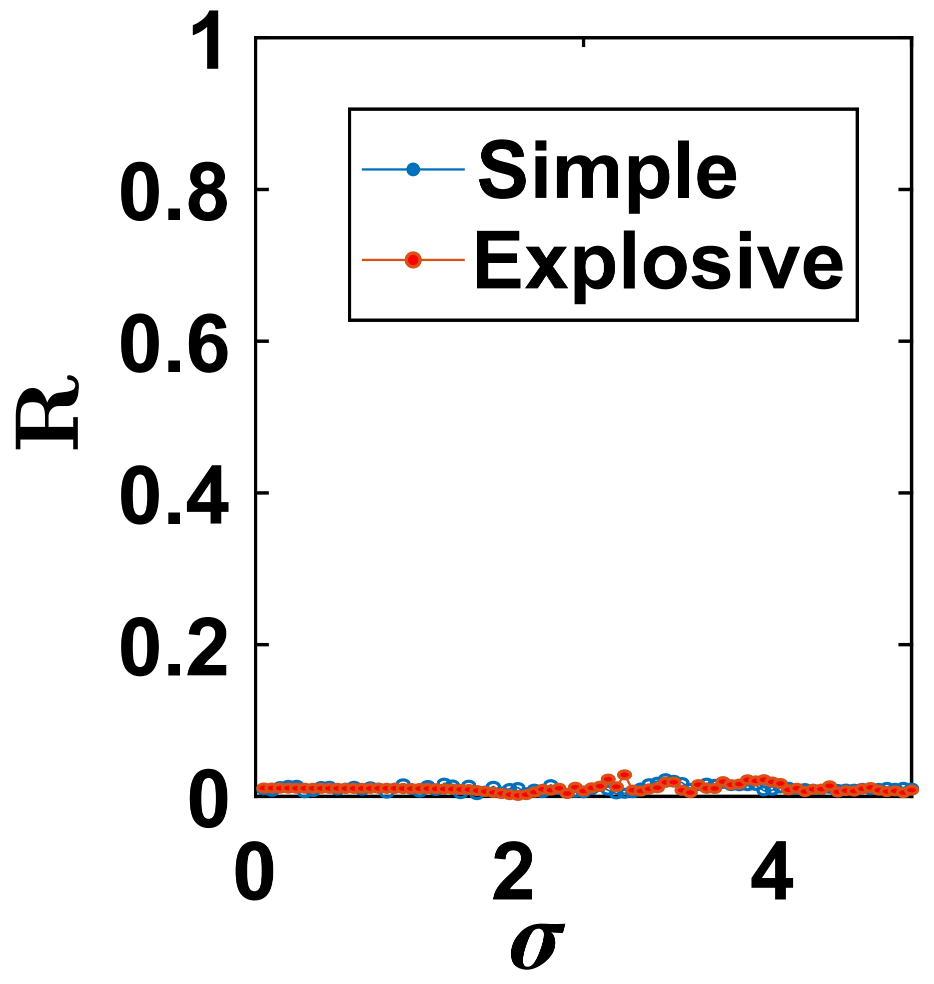


Node Kuramoto

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega} - \sigma \mathbf{B}_{[1]} \sin \mathbf{B}_{[1]}^{\top} \boldsymbol{\theta},$$

Edge Kuramoto

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega} - \sigma \mathbf{B}_{[n+1]} \sin \mathbf{B}_{[n+1]}^{\top} \boldsymbol{\theta} - \sigma \mathbf{B}_{[n]}^{\top} \sin \mathbf{B}_{[n]} \boldsymbol{\theta},$$



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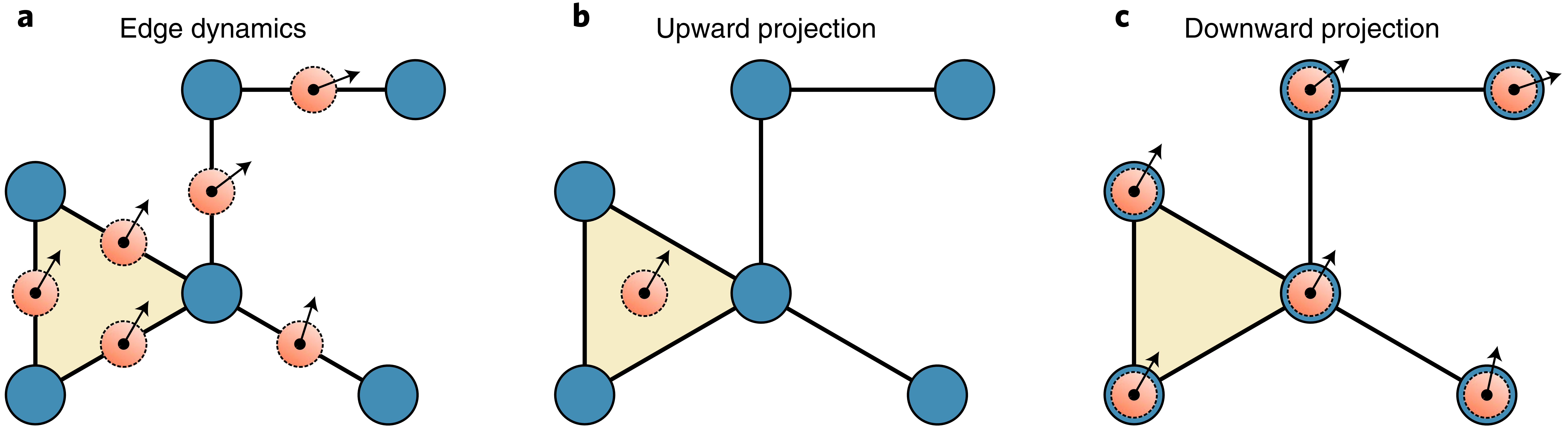

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Coupled Edge Kuramoto

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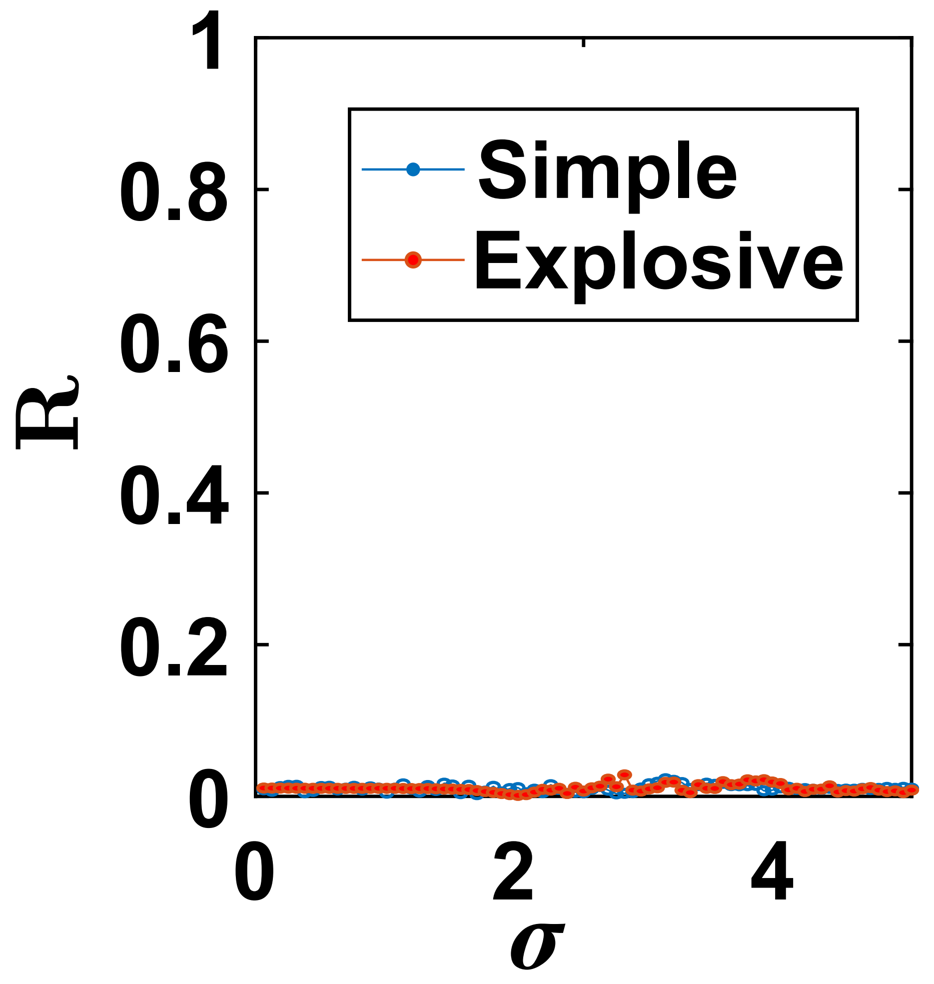

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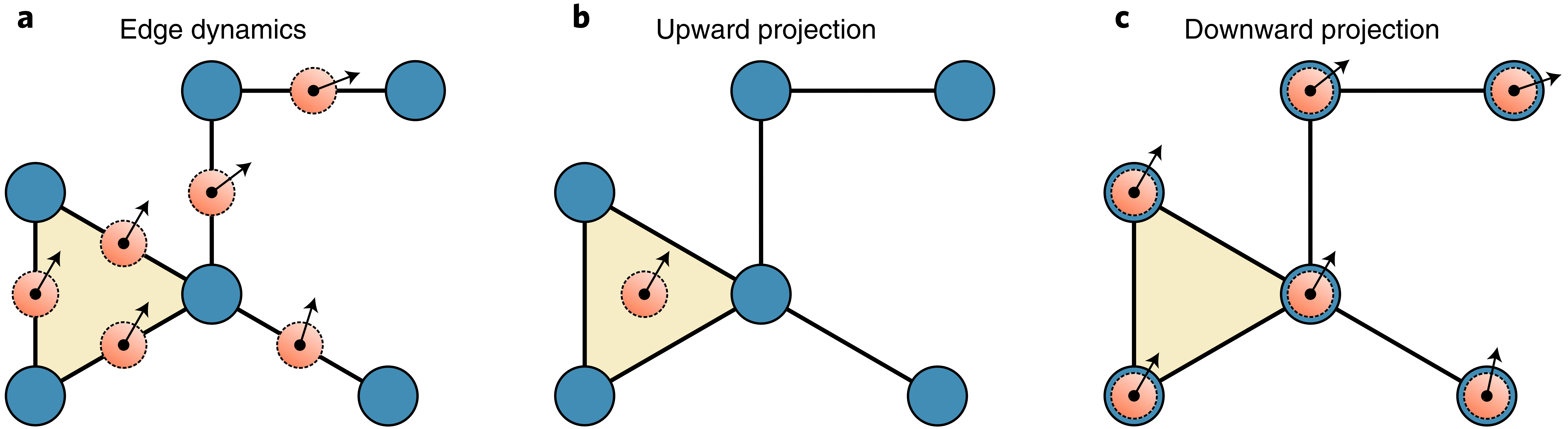

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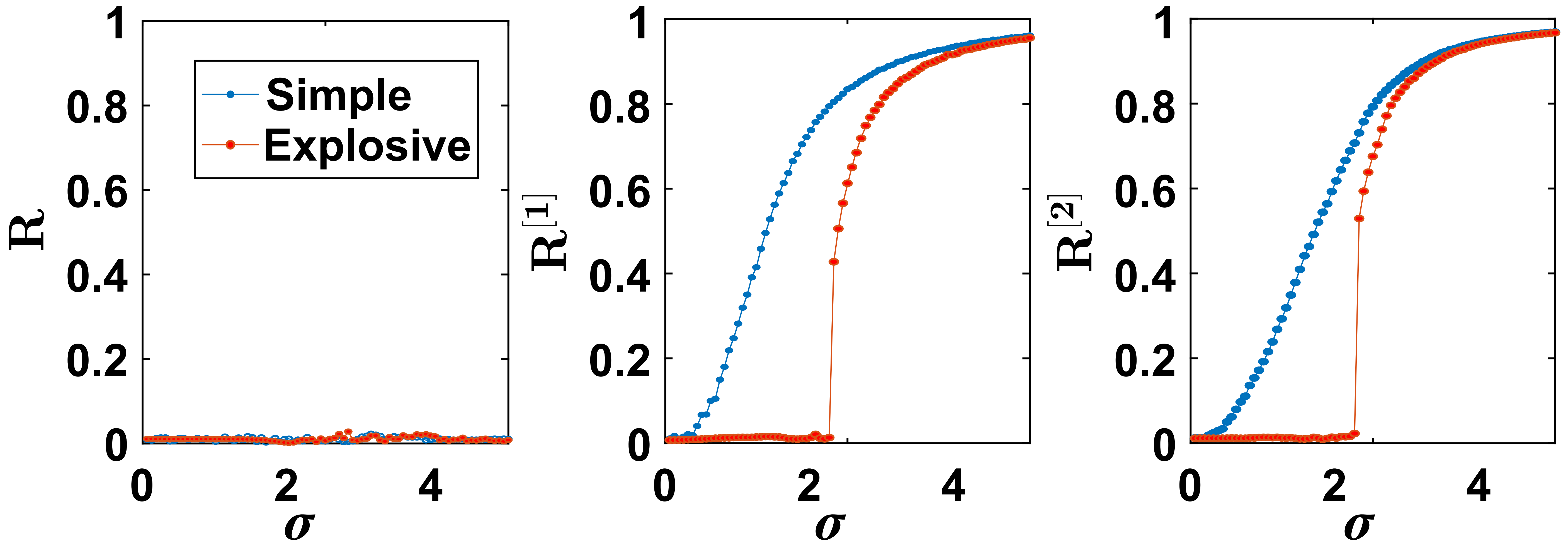

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Topological dynamical processes

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Expert, Arnaudon, Peach, Petri, (2021) https://arxiv.org/abs/2111.11073

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Black simplicial magic (Lifts in 2N space, frustration operators, axe parane' etc…)

$$N_k = B_k \,, \qquad N_k^* = W_k B_k^T W_{k+1}^{-1}$$

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Frustrated Simplicial Kuramoto on nodes

$$\dot{\theta}^{(0)} = -\alpha_0 - (N_0^* V_1)^- \sin\left(V_1 N_0 \theta^{(0)} + \alpha_1\right)$$

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Proper frequencies are just higher order frustrations

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Topological dynamical processes

Frustrated simplicial Kuramoto is general and rich.

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Hodge decomposition!

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Harmonic

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Harmonic **Irrotational**

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$$\mathbf{x}_{[n]} = \underbrace{\mathbf{x}^H_{[n]}}_{\text{Harmonic}} + \underbrace{\mathbf{B}^\top_{[n]}\mathbf{z}_{[n-1]}}_{\text{Irrotational}} + \underbrace{\mathbf{B}_{[n+1]}\mathbf{z}_{[n+1]}}_{\text{Solenoidal}},$$

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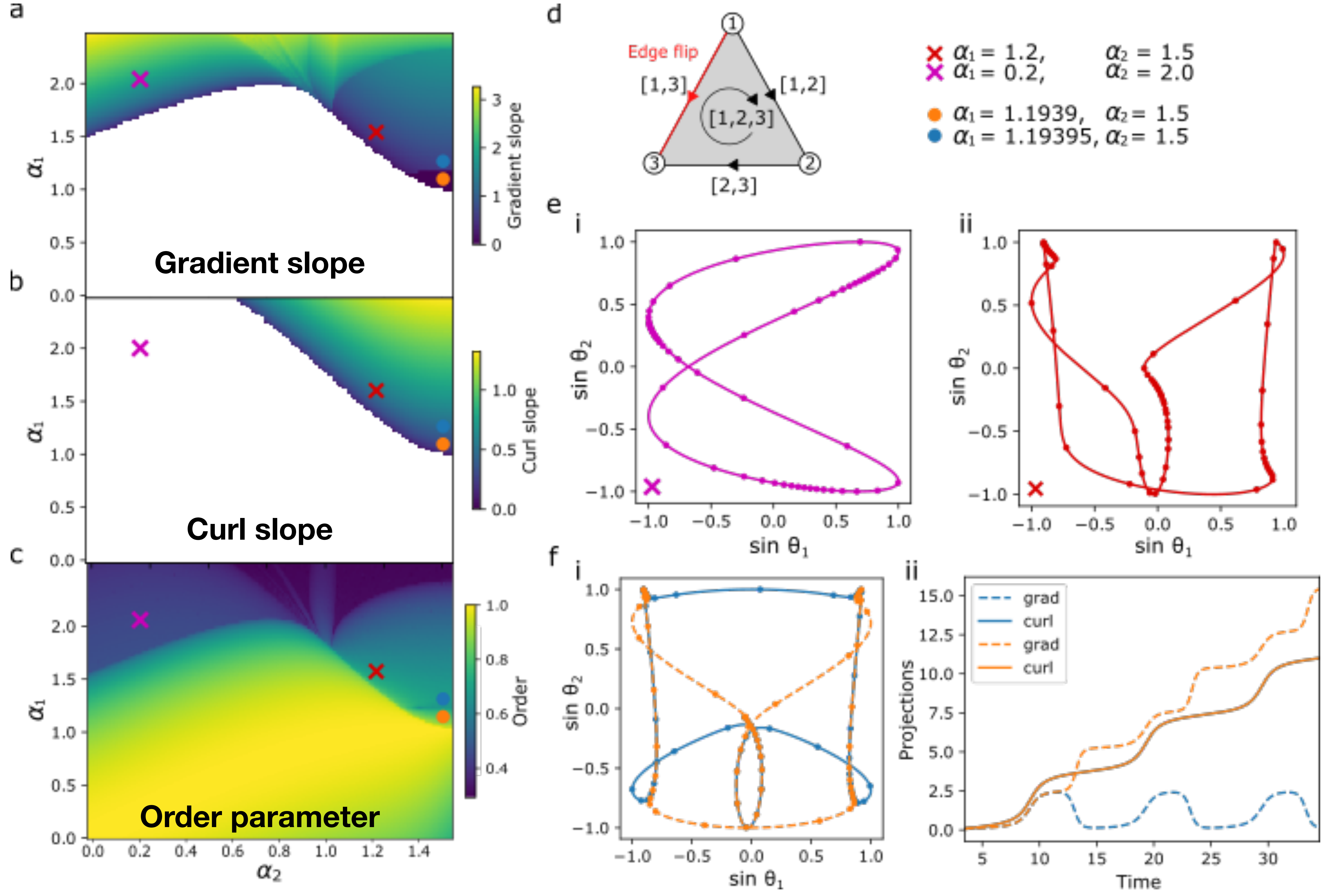
Harmonic **Irrotational** **Solenoidal**



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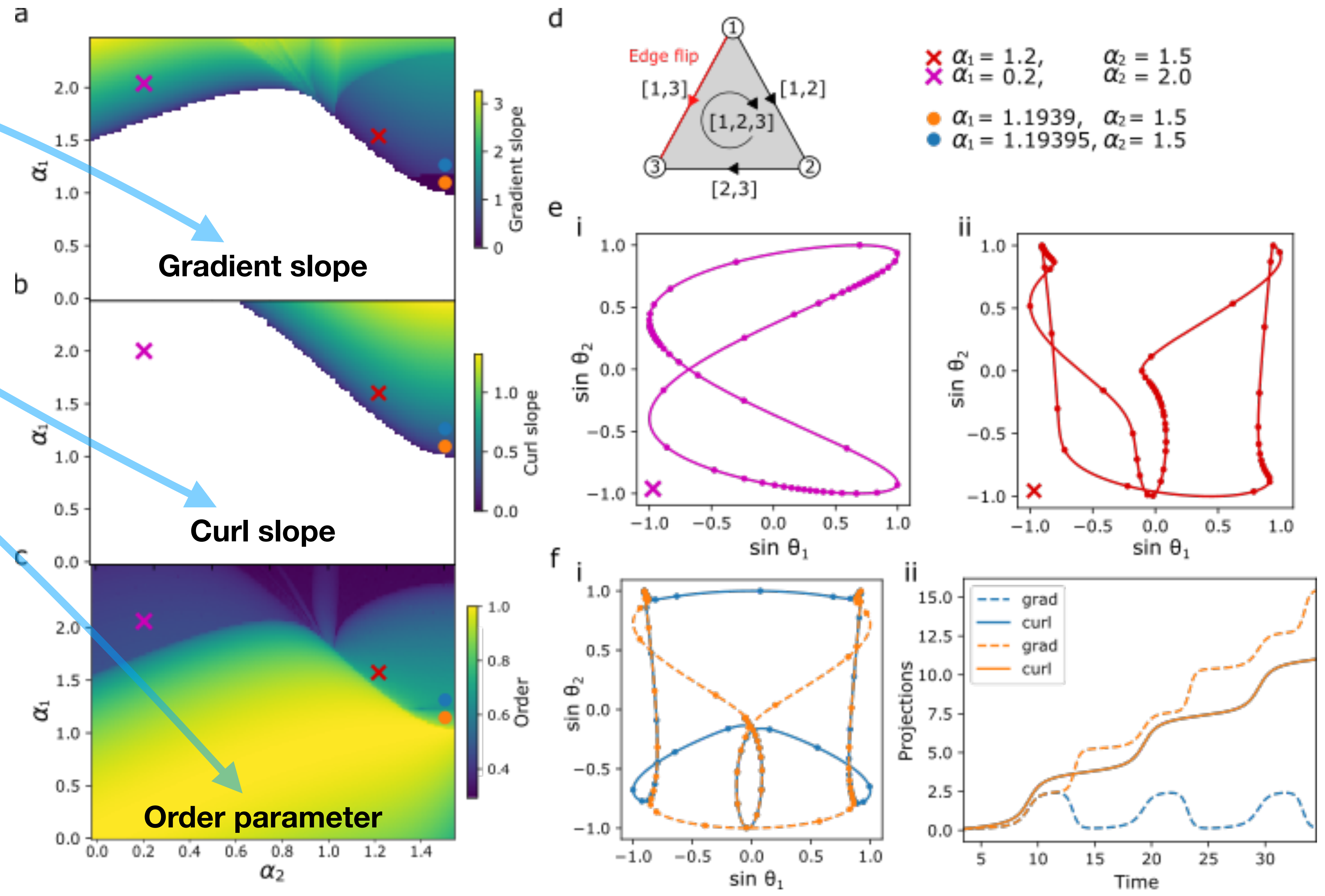

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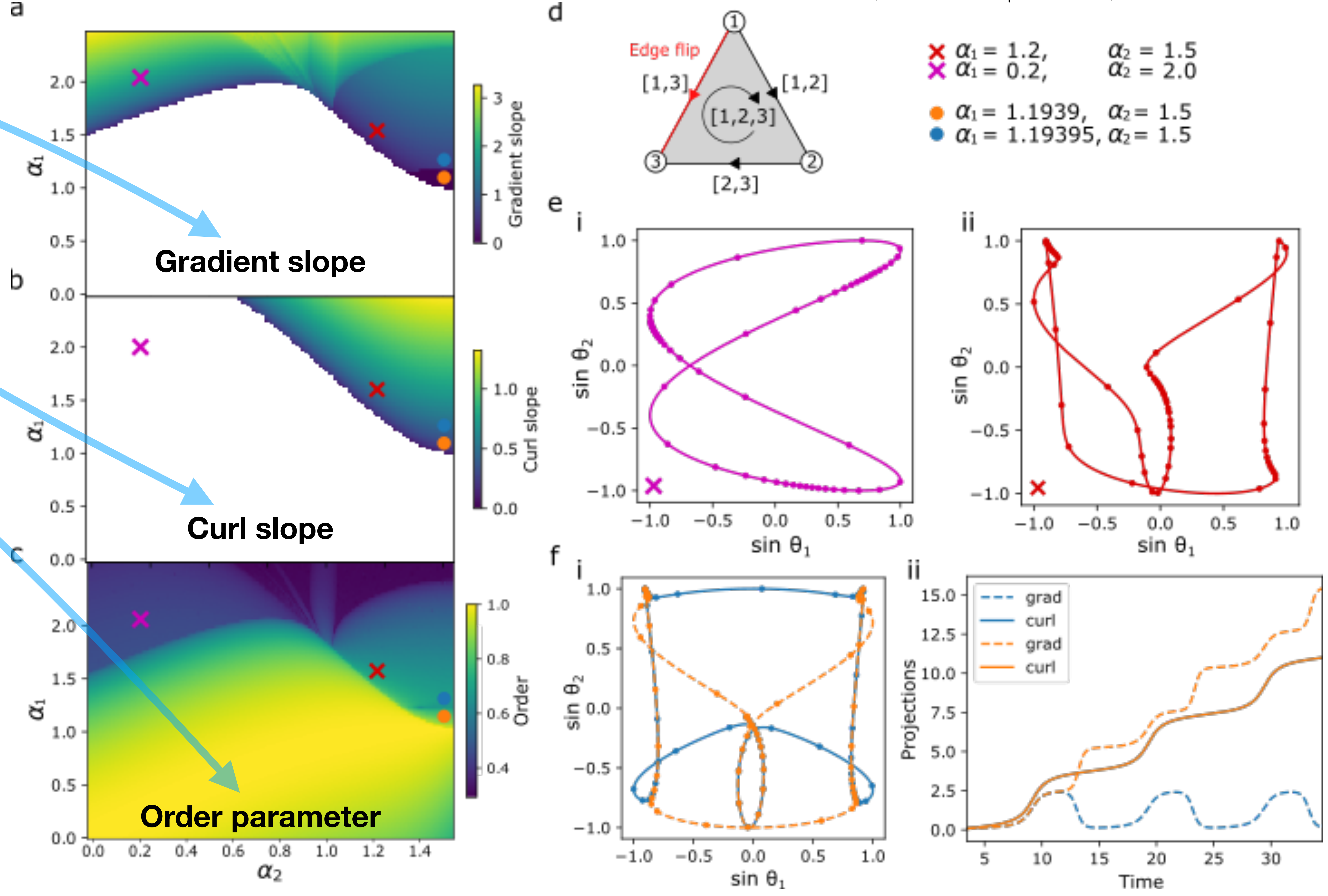
Harmonic Irrotational Solenoidal

Harmonic Order Parameter

$$\mathcal{R}_k^2(\theta) = \frac{1_{n_{k-1}} \cdot W_{k-1}^{-1}\cos(N_k^*\theta) + 1_{n_{k+1}} \cdot W_{k+1}^{-1}\cos(N_{k+1}\theta)}{1_{n_{k-1}} \cdot W_{k-1}^{-1} 1_{n_{k-1}} + 1_{n_{k+1}} \cdot W_{k+1}^{-1} 1_{n_{k+1}}}$$



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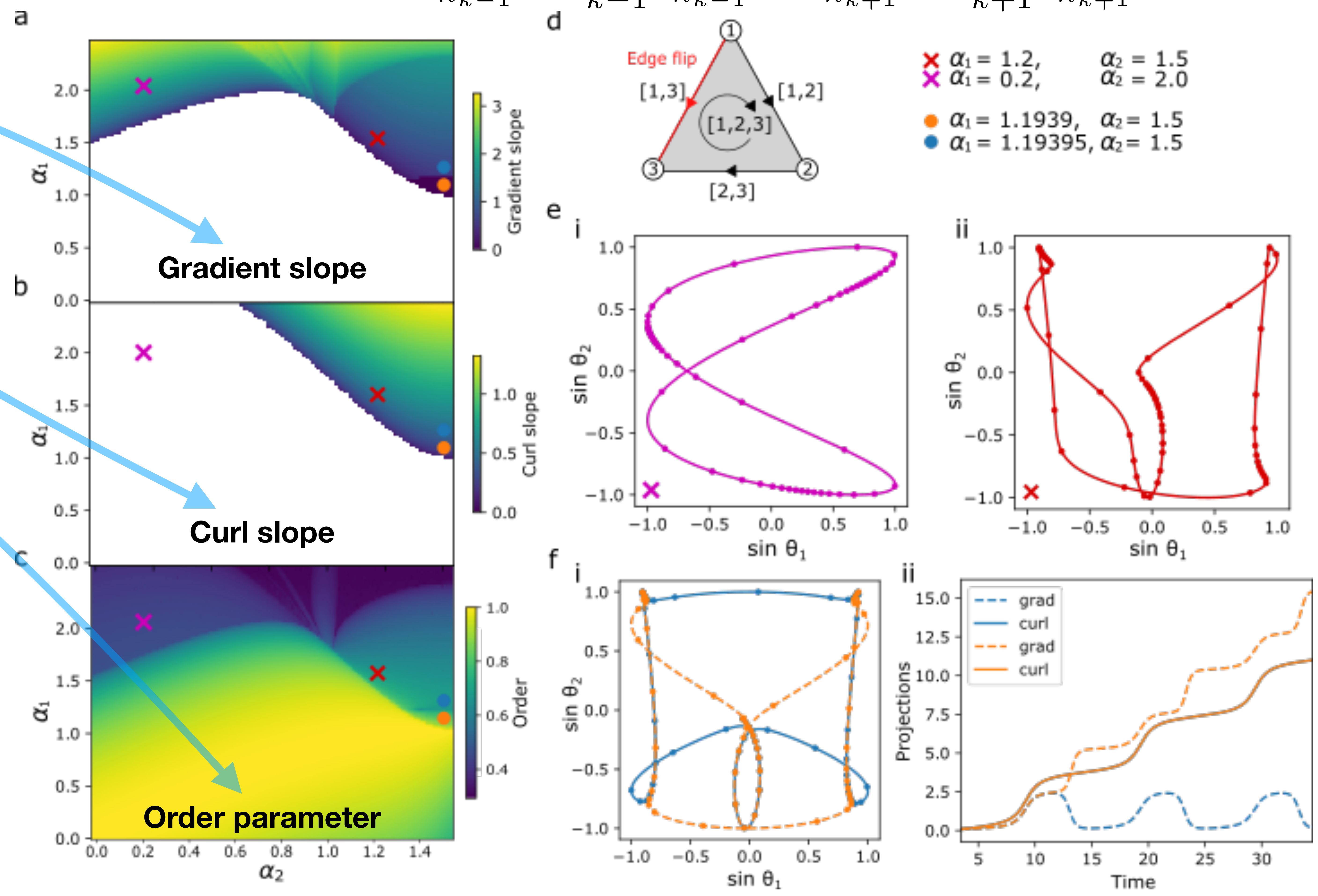
Potential-like Kuramoto formalism

$$\dot{\theta} = -W_k \nabla_\theta R^2_k$$



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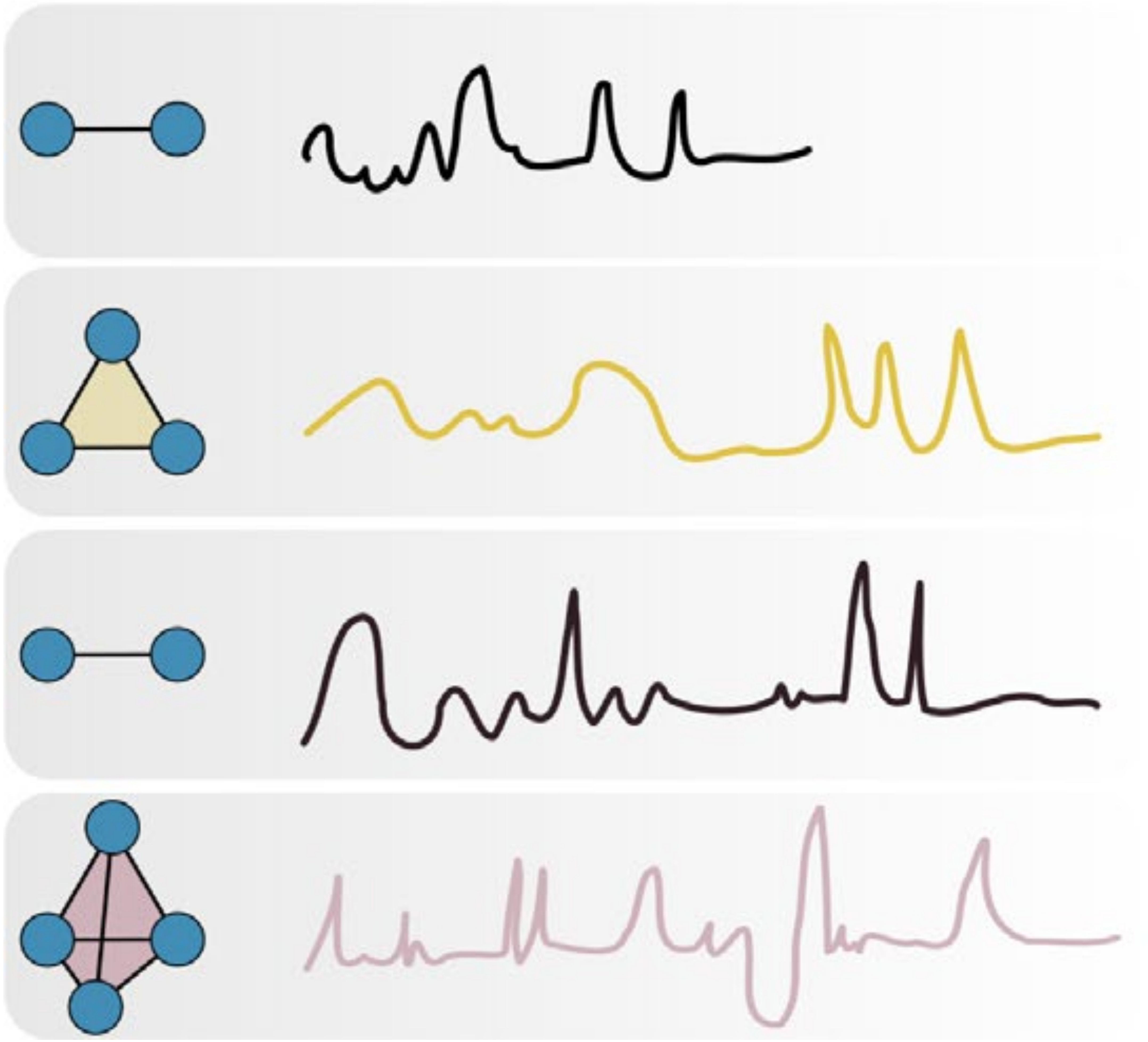
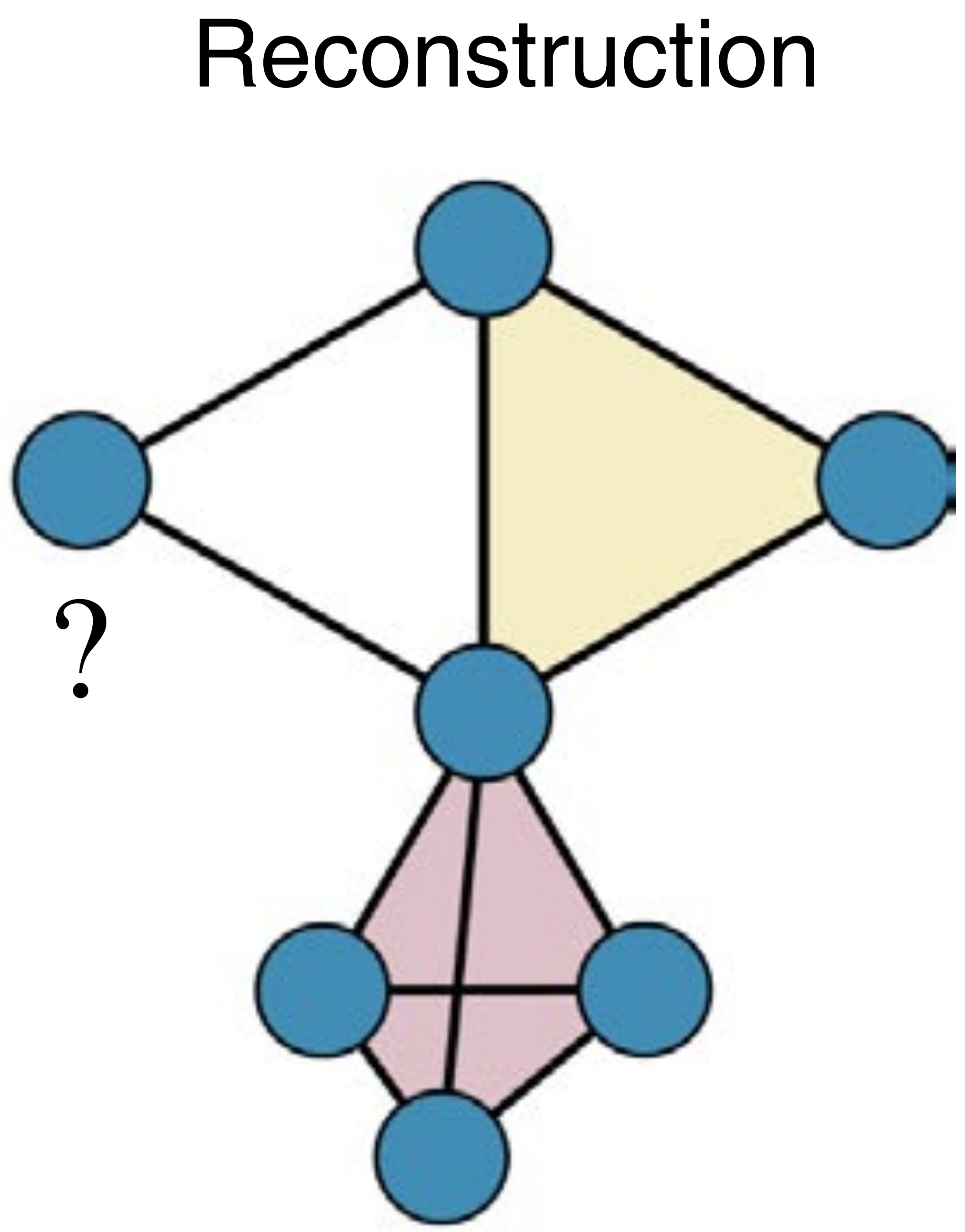
Notebook HoSaKu

Inferring higher-order interactions from data





Inferring higher-order interactions from data

Inferring higher-order interactions from data

From higher-order phenomena
to mechanisms

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Mechanisms/dynamical models

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Phenomena/observables

High-order observables:
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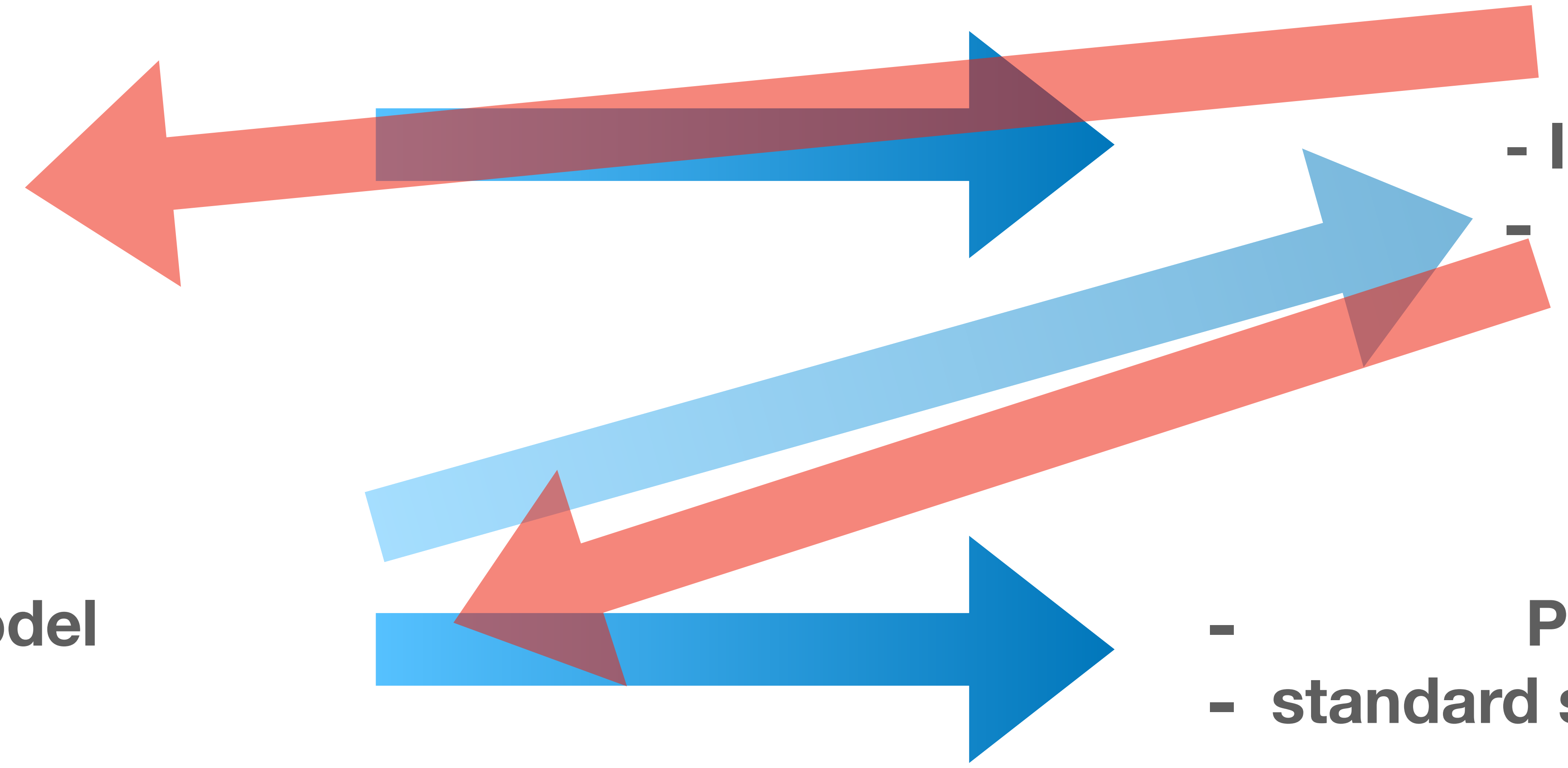

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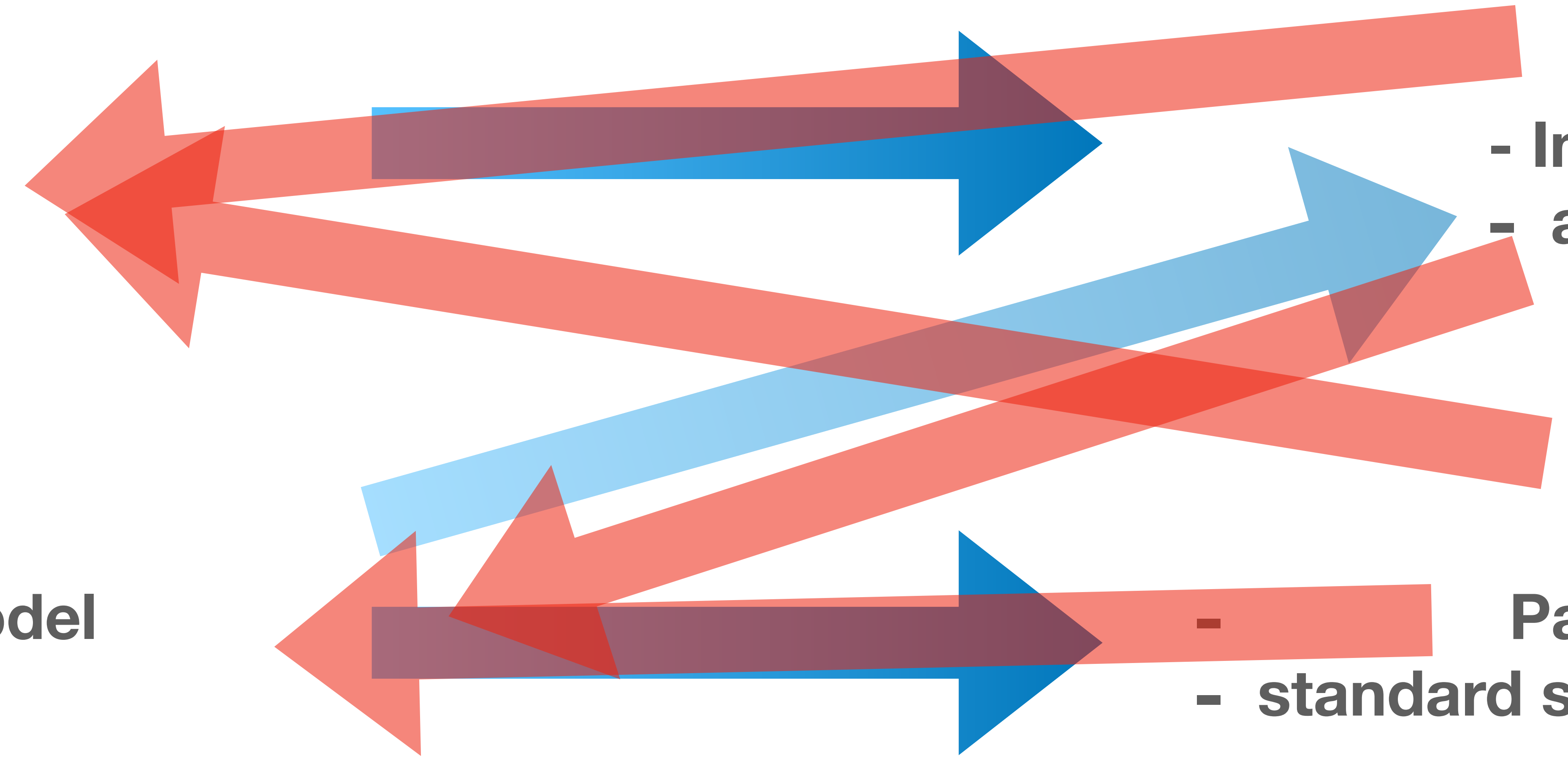

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Low-order (pairwise) mechanistic model

“Low-order” observables:
- Pairwise correlations, mutual info, etc..
- standard scalar observables (prevalence, order params)

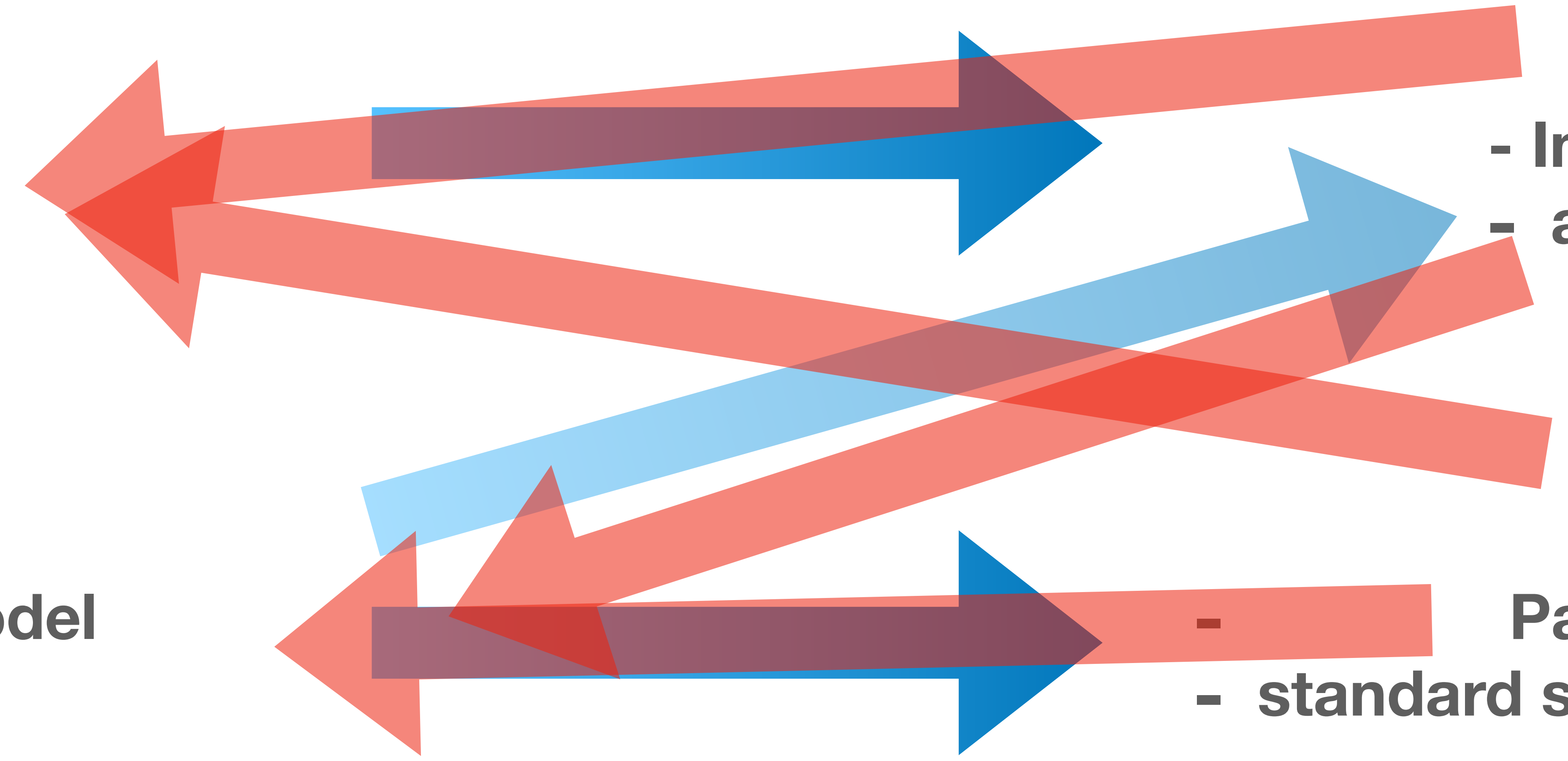
Inferring higher-order interactions from data

From higher-order phenomena
to mechanisms

Mechanisms/dynamical models

High-order mechanistic model

Low-order (pairwise) mechanistic model



Phenomena/observables

High-order observables:
- Information-based (PID, O-synergy, etc)
- algebraic-topological (homological info)

“Low-order” observables:
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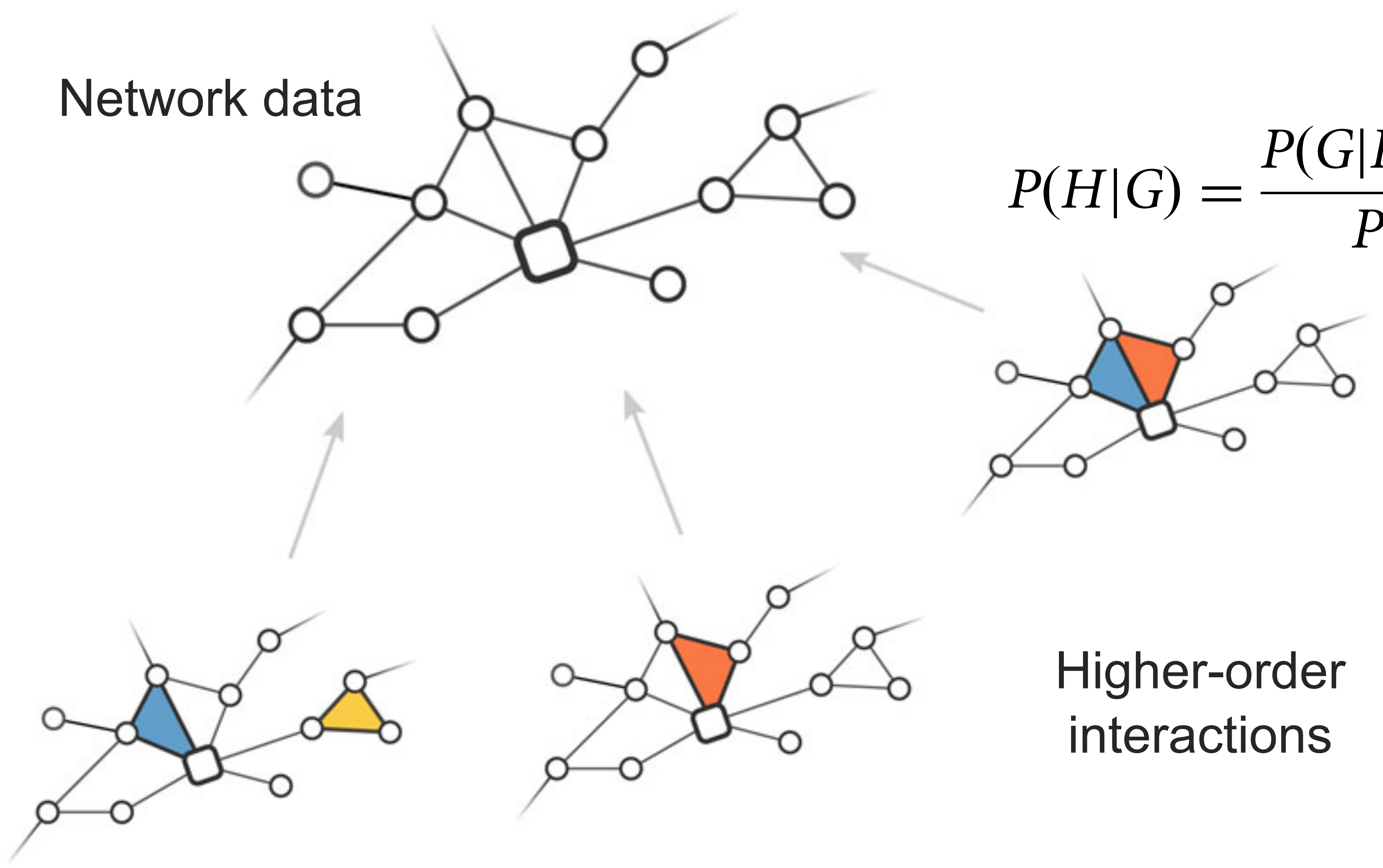
So what can we do?

Strategy #1: Infer higher-order structural relations from pairwise ones



$$P(H|G) = \frac{P(G|H)P(H)}{P(G)}$$

Hypergraph reconstruction from network data

Jean-Gabriel Young [1,2,3], Giovanni Petri [4] & Tiago P. Peixoto [4,5,6]

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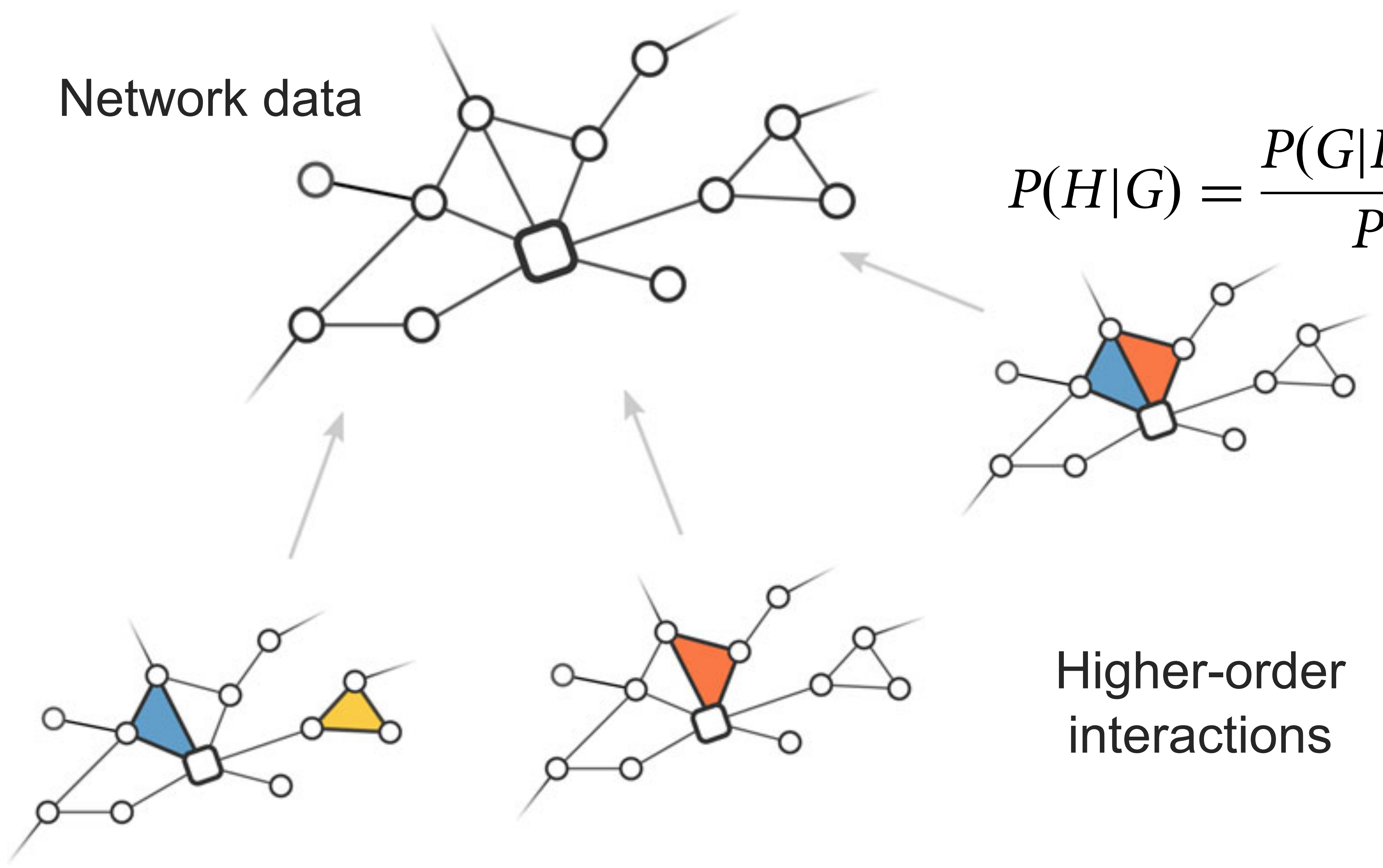

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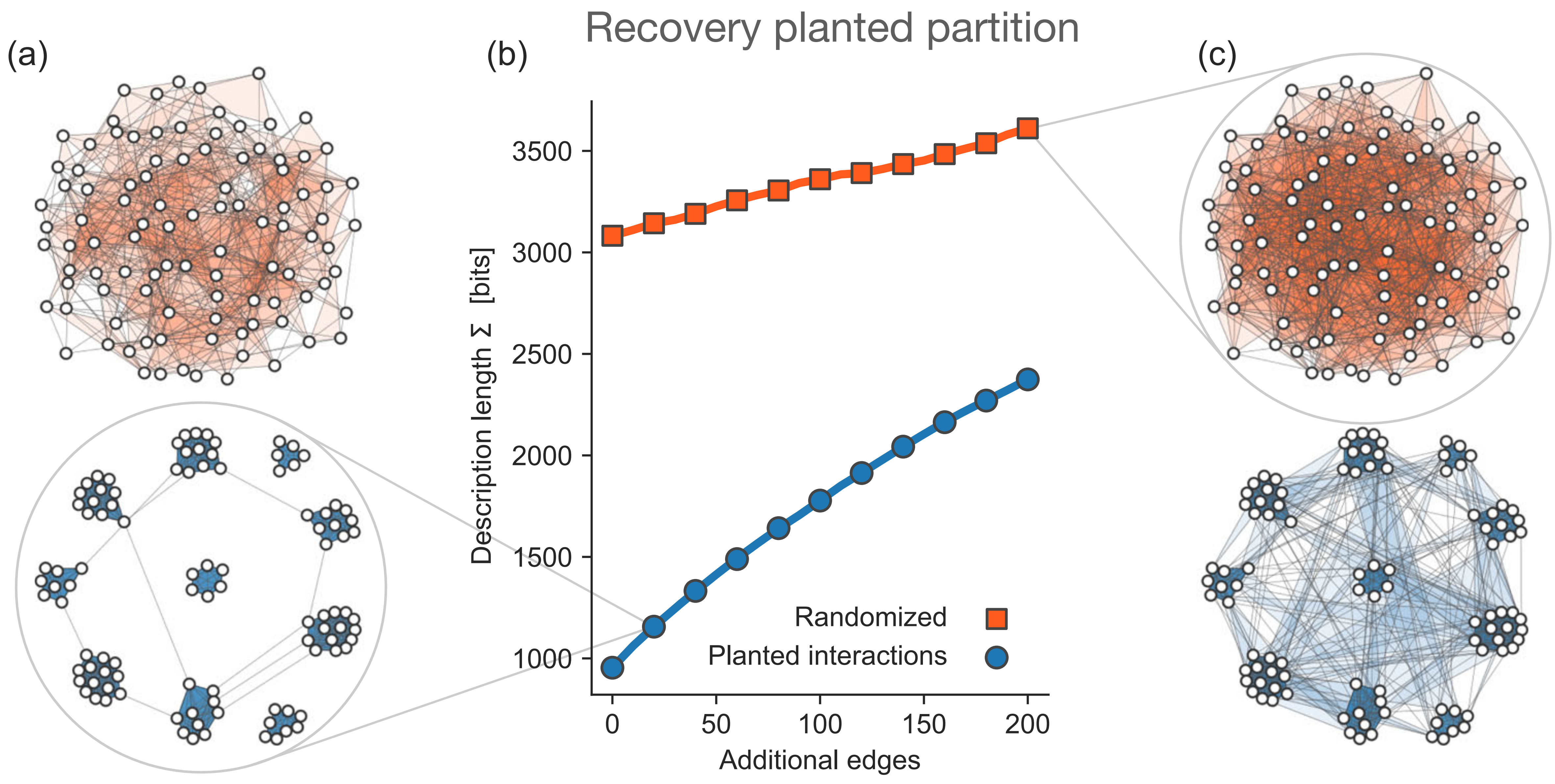

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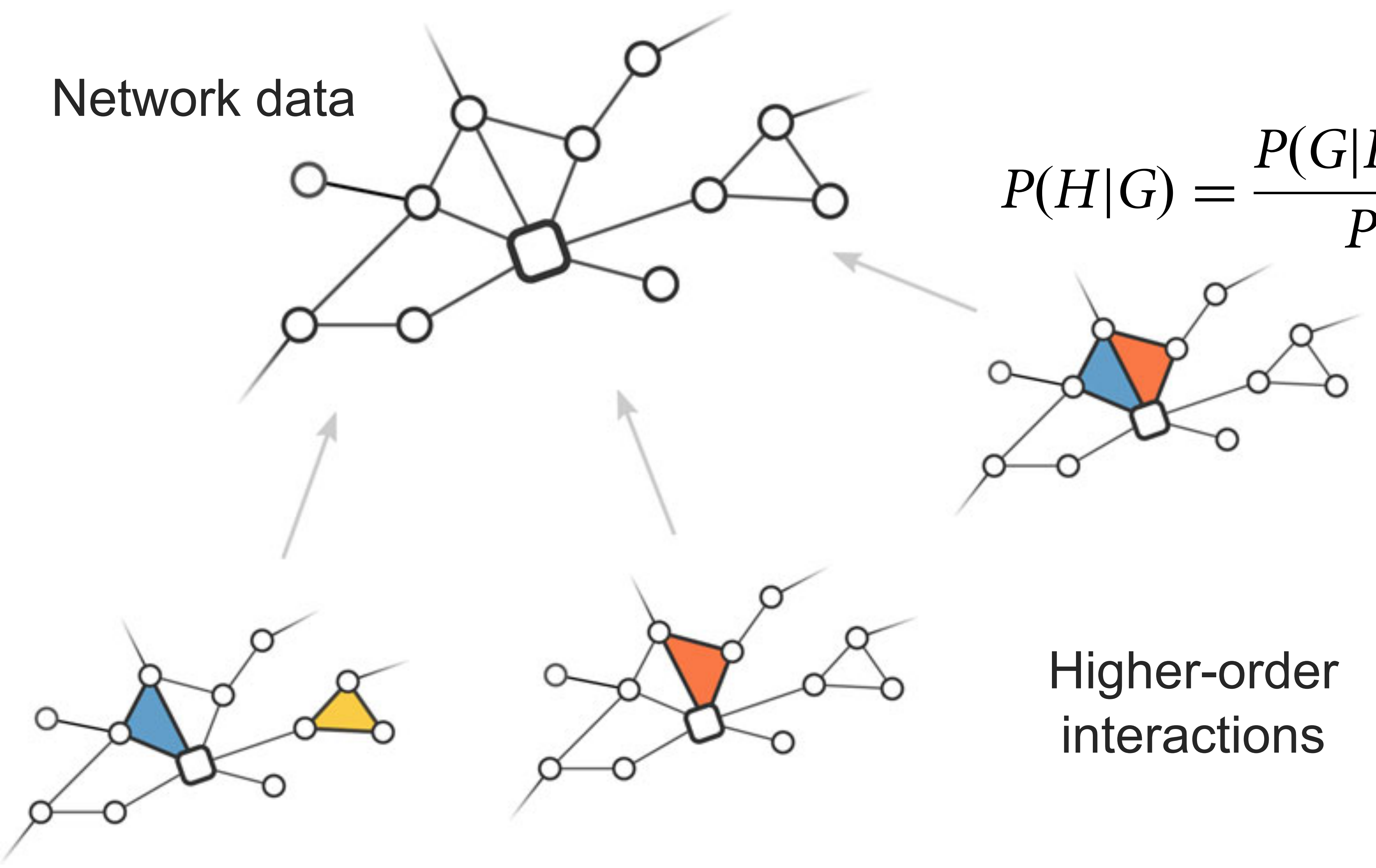

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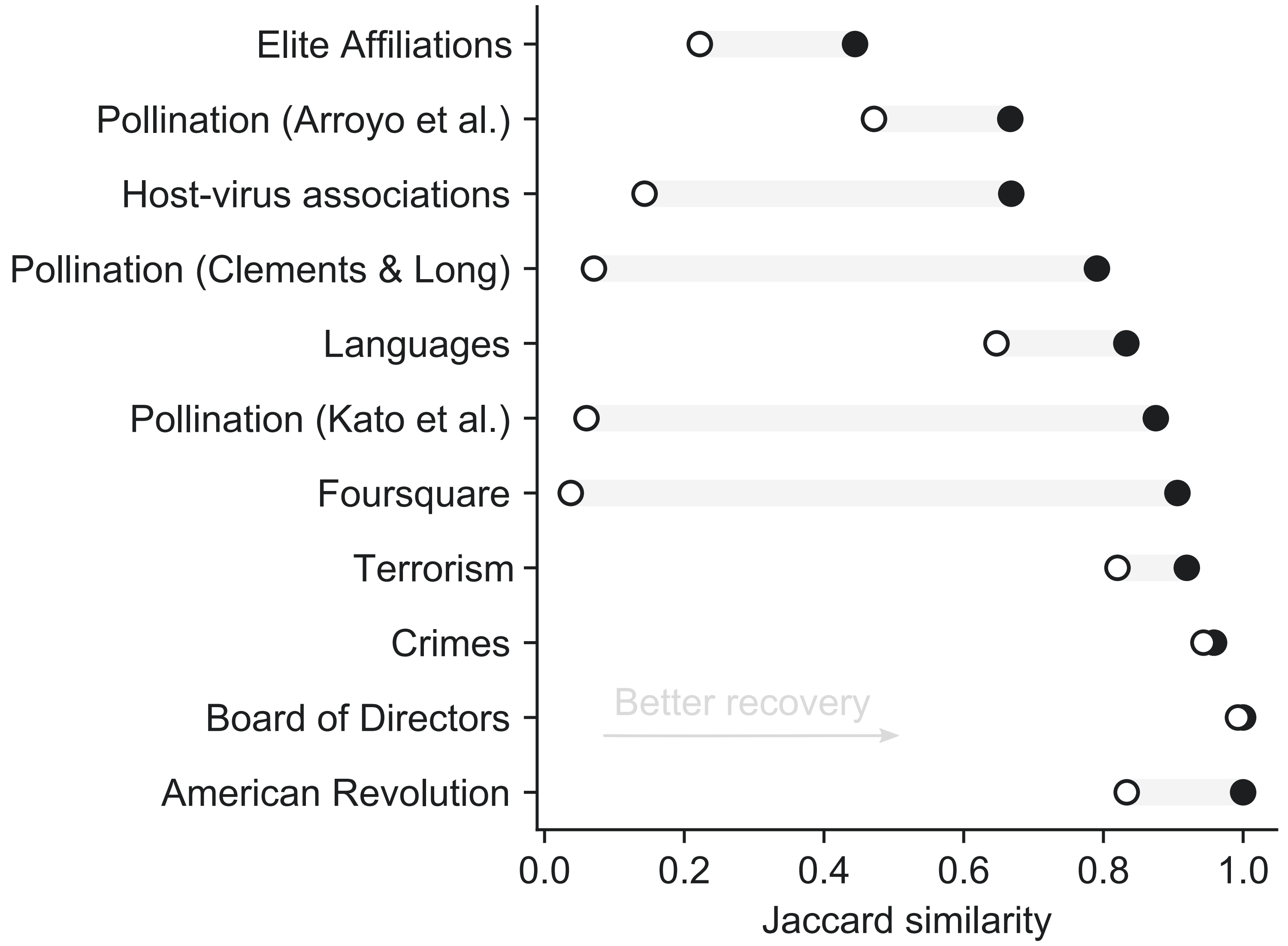
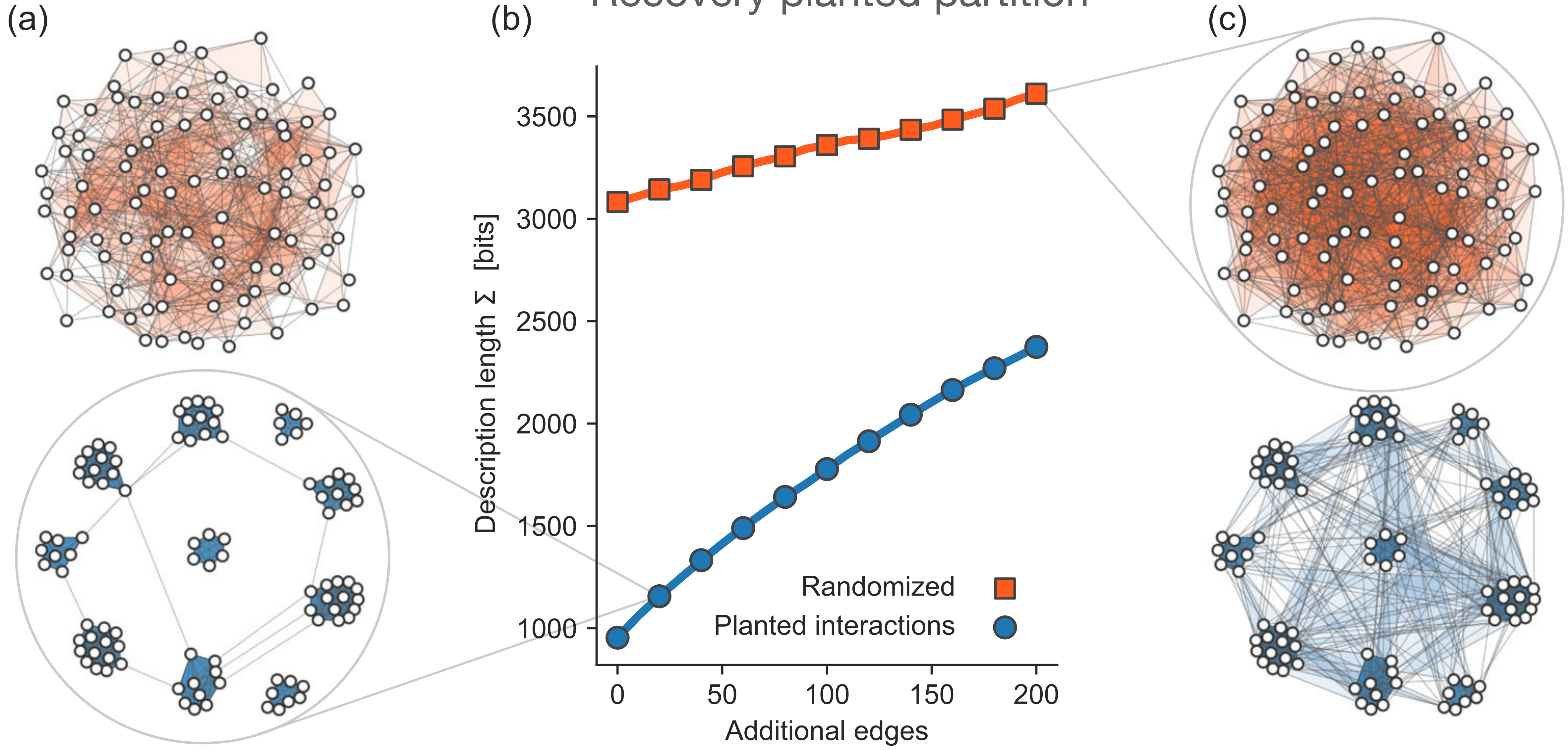
Reconstruction projected bipartite graphs



Recovery planted partition



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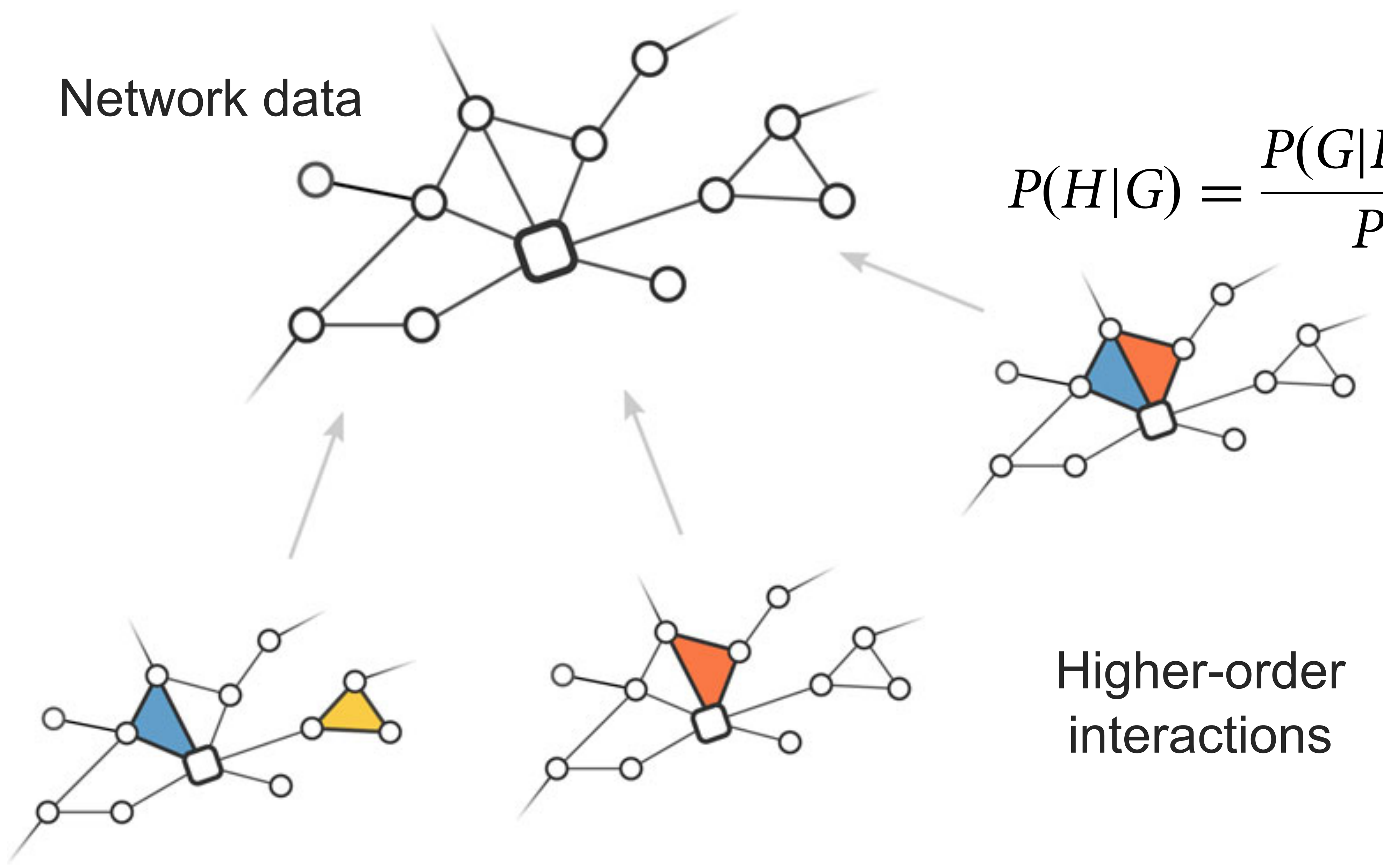

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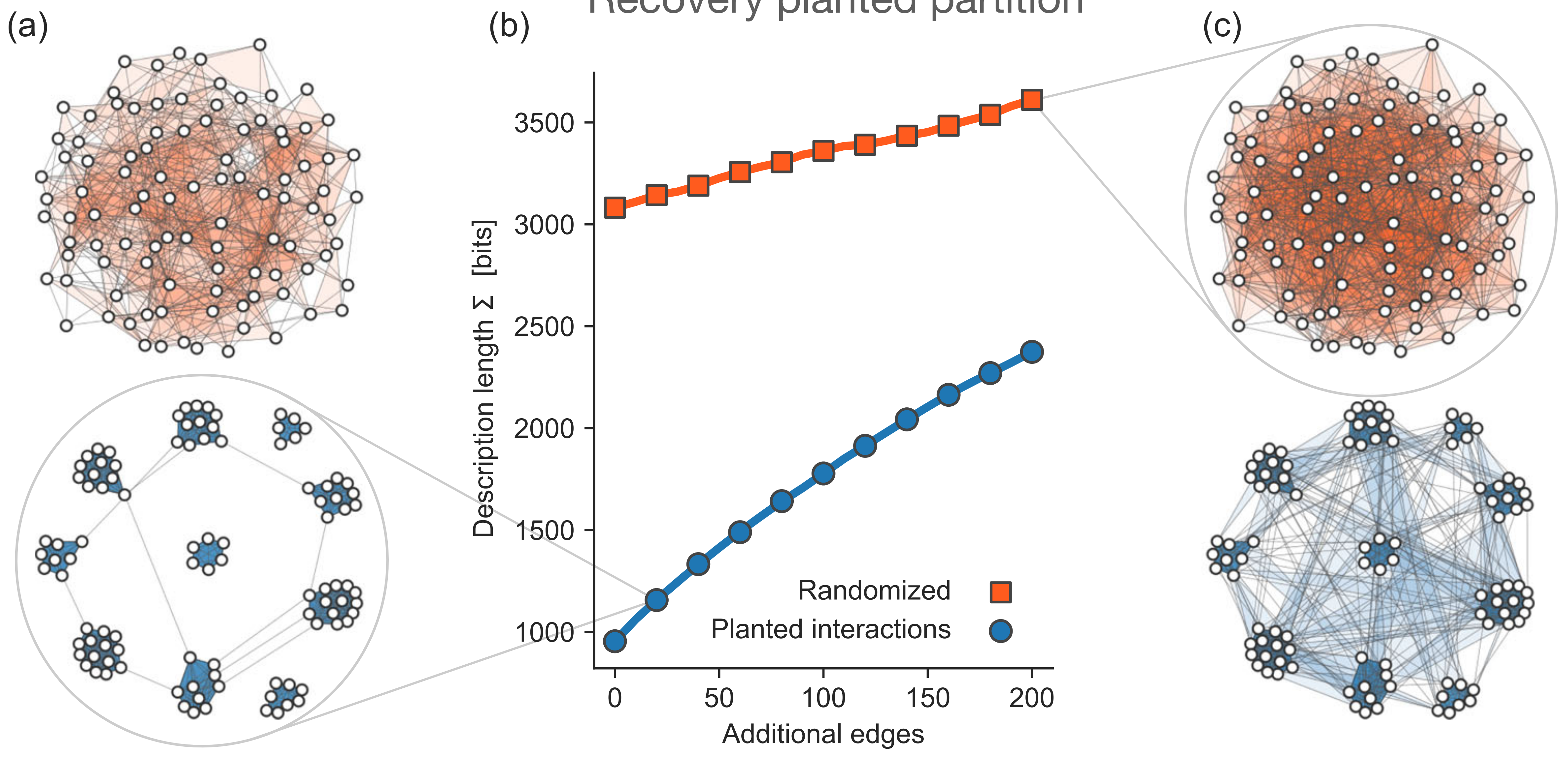

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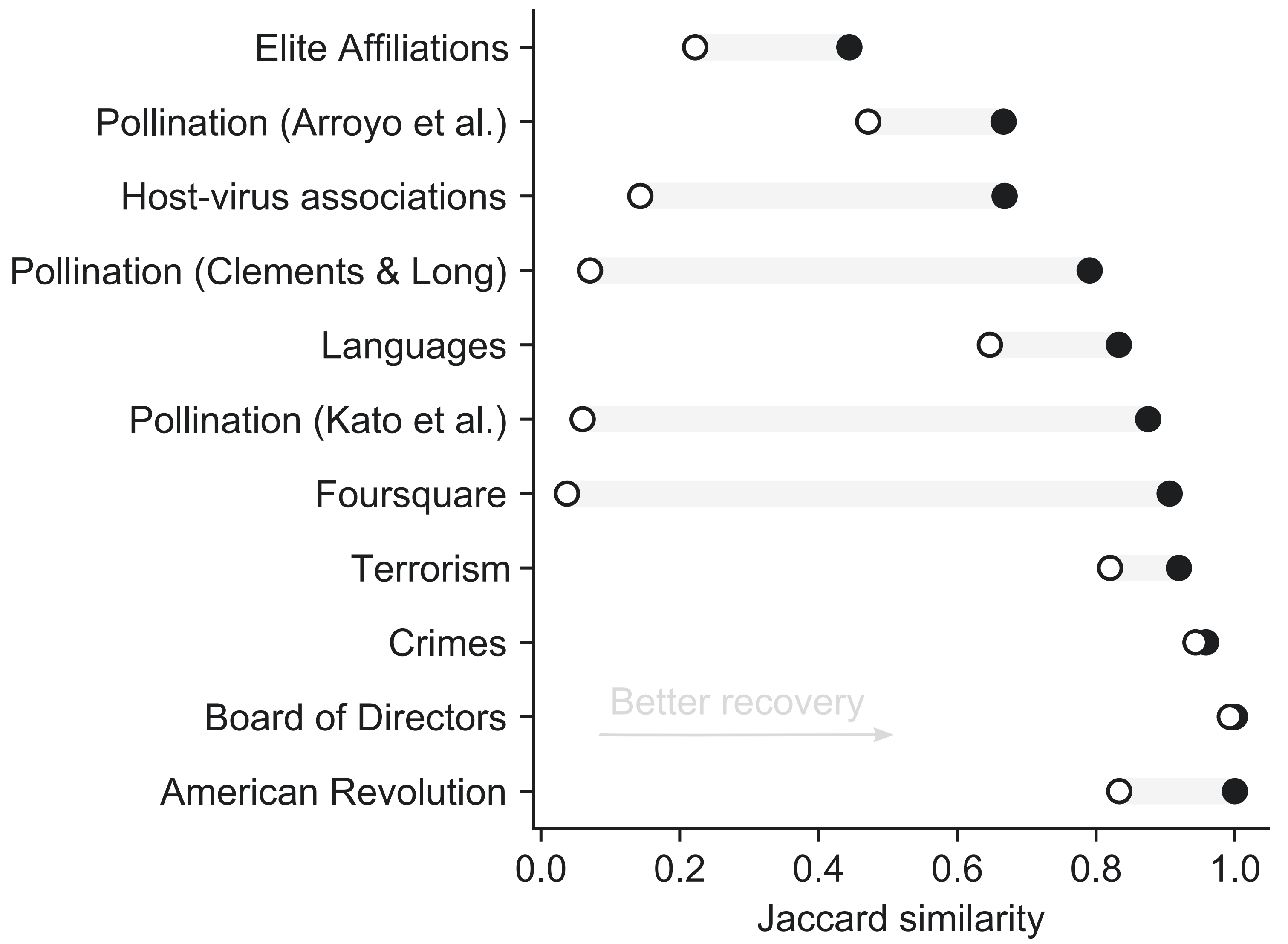
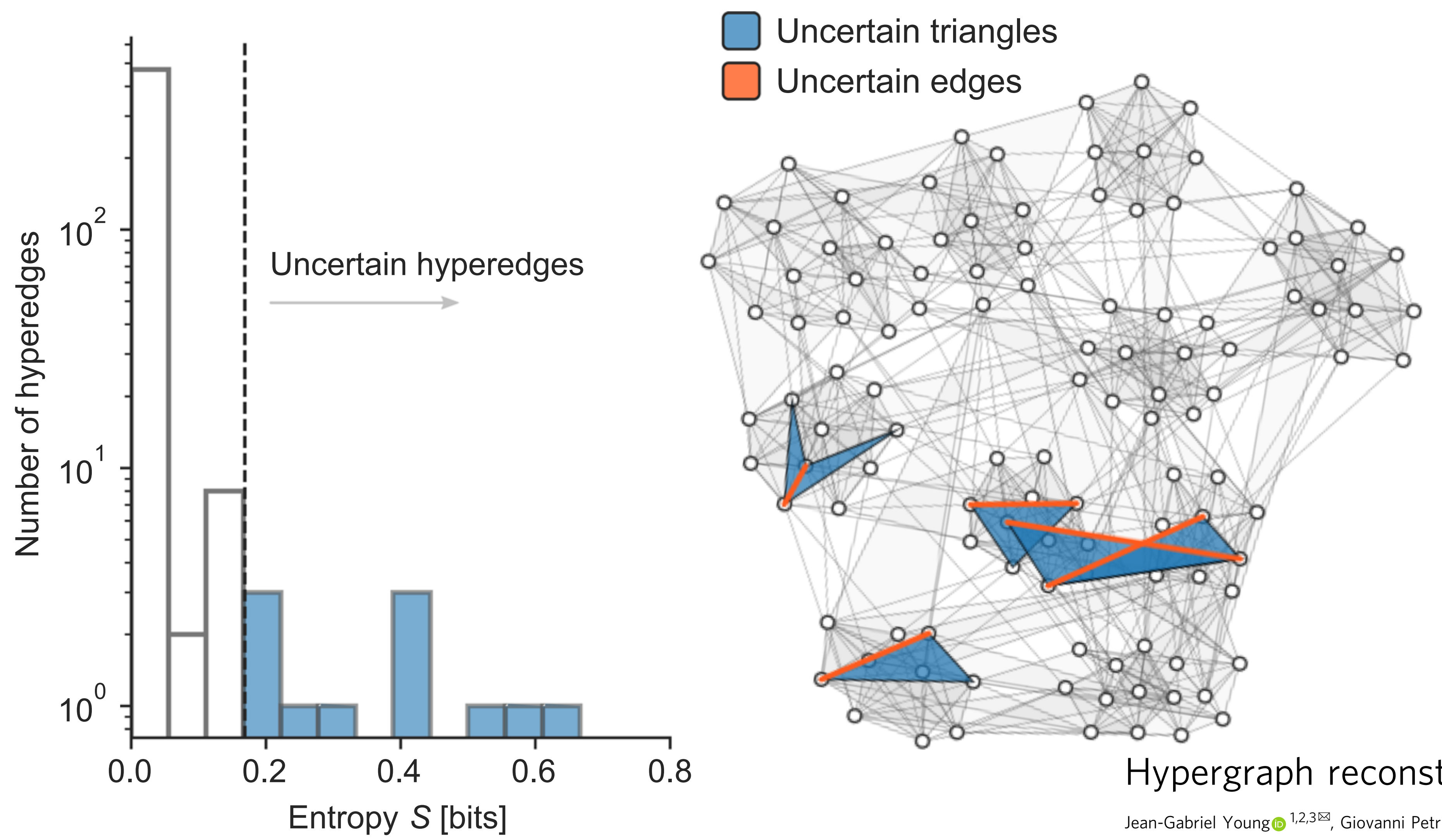


Qualify edge confidence



Hypergraph reconstruction from network data

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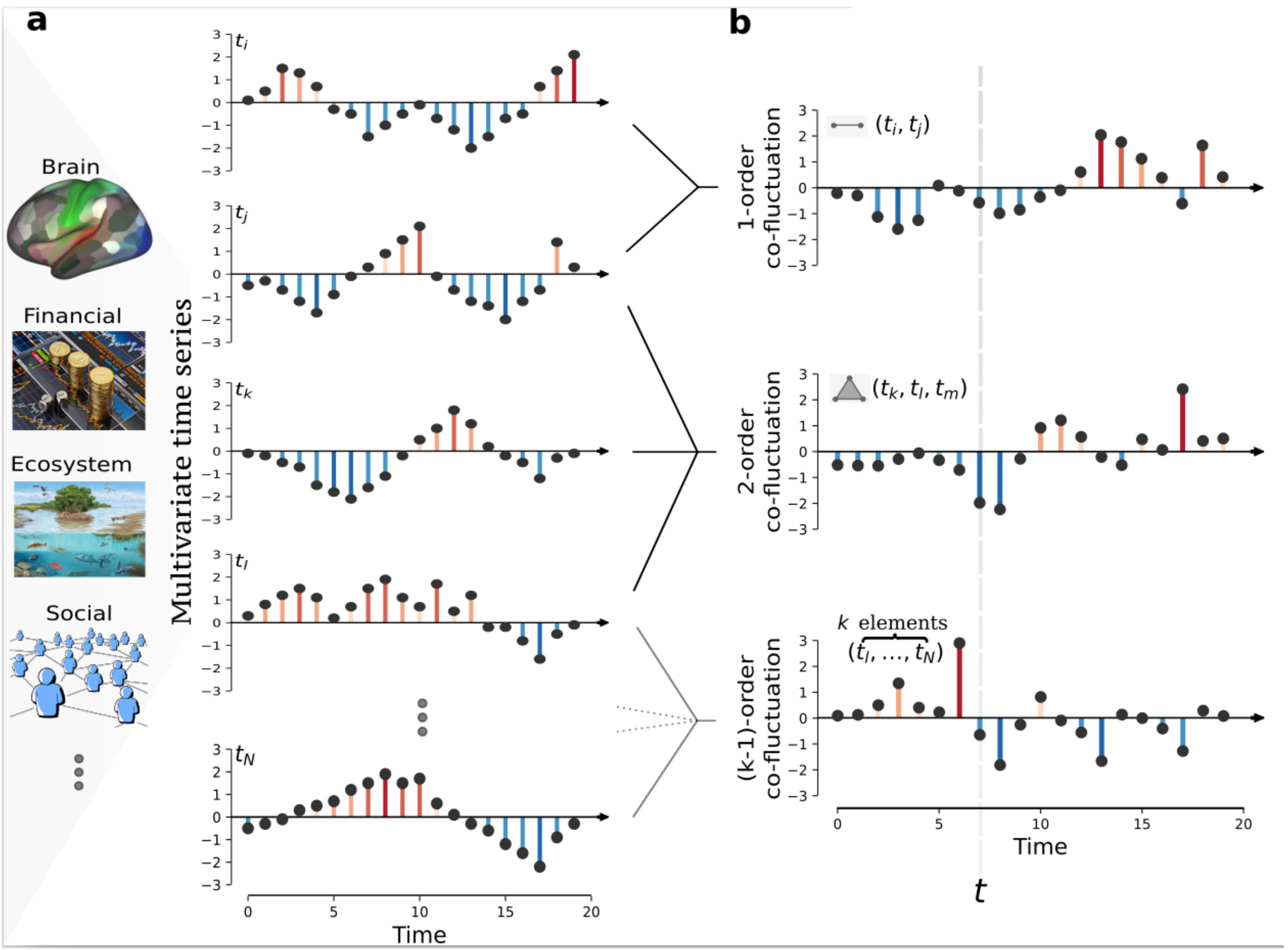
Notebooks Reconstruction

Strategy #2: *Construct* higher-order signals from low order signals



Santoro, Petri, Battiston, Amico (2021), in prep

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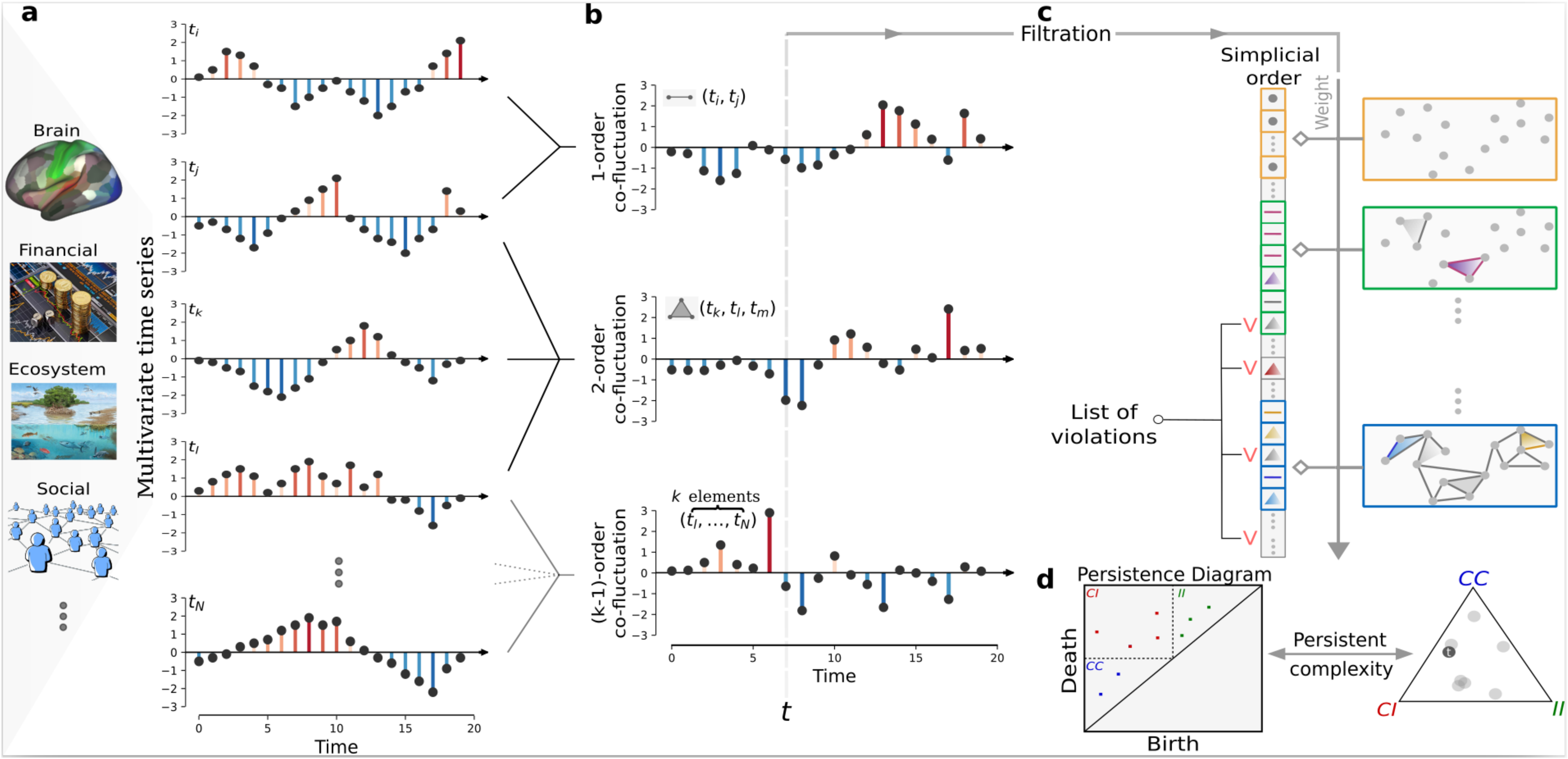

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Check out Andrea Santoro’s talk

(Tomorrow,Session 03B)

Strategy #3: *Constrain* higher-order mechanisms using high order signals/observables

PHYSICAL REVIEW LETTERS **123,** 128301 (2019)

Network Reconstruction and Community Detection from Dynamics

Tiago P. Peixoto[1,2,3,*]
[1]*Department of Network and Data Science, Central European University, H-1051 Budapest, Hungary*
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DOI: 10.1103/PhysRevLett.123.128301

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Example: SI on networks $\sigma \in \{0, 1\} \sim \text{Susc, Inf}$

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$\sigma \in \{0,1\} \sim \text{Susc, Inf}$

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Connectivity

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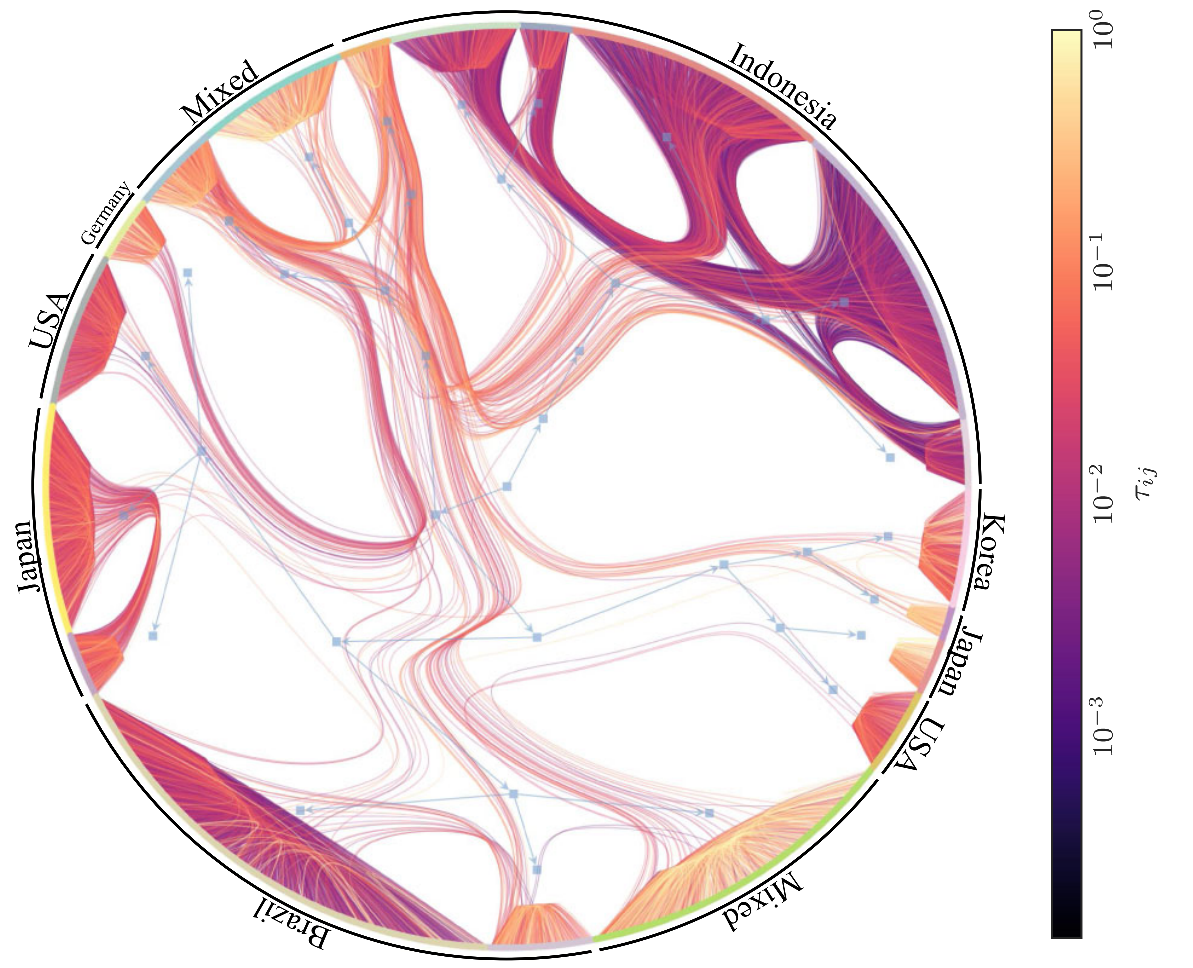

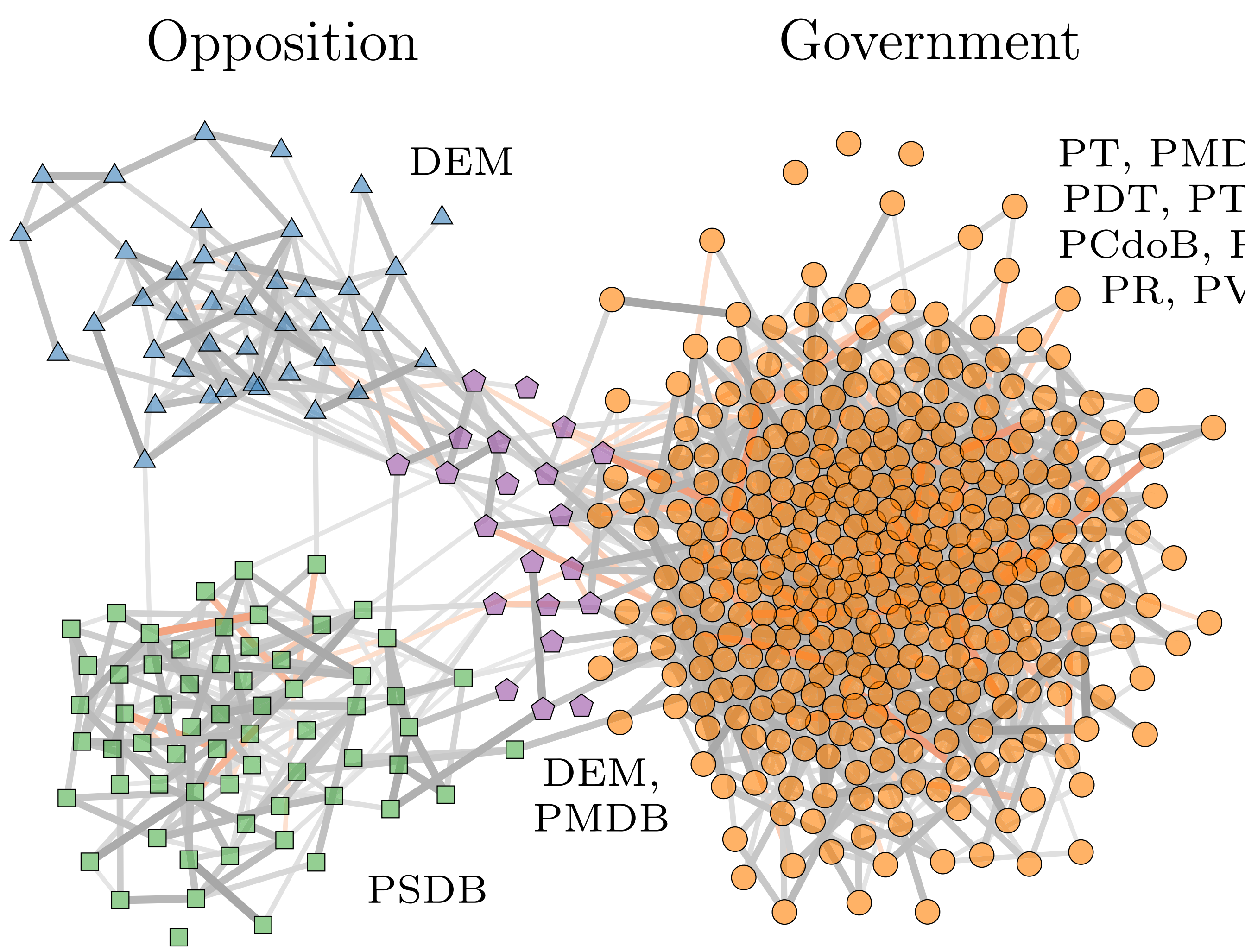

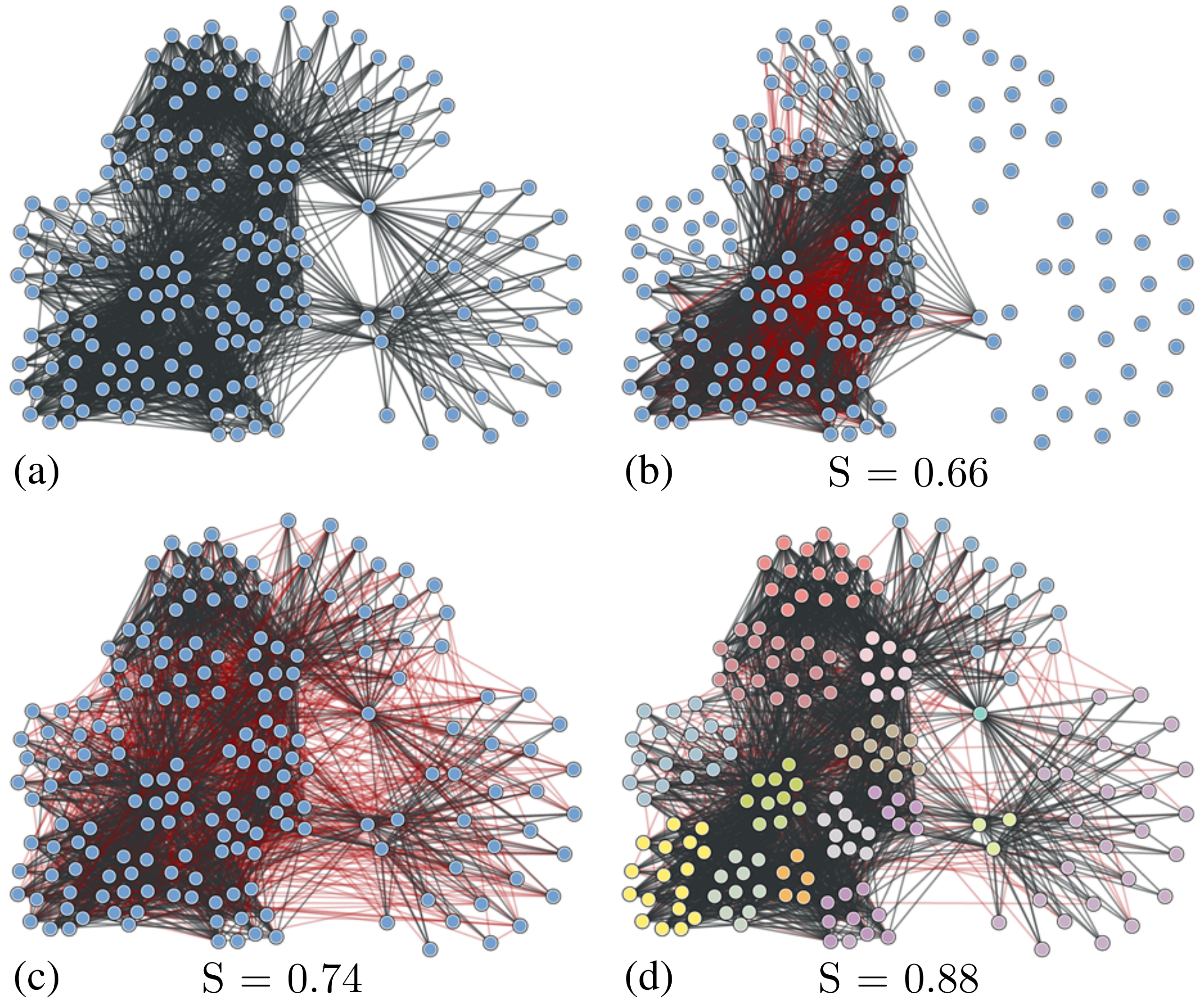

Summing up
(general)

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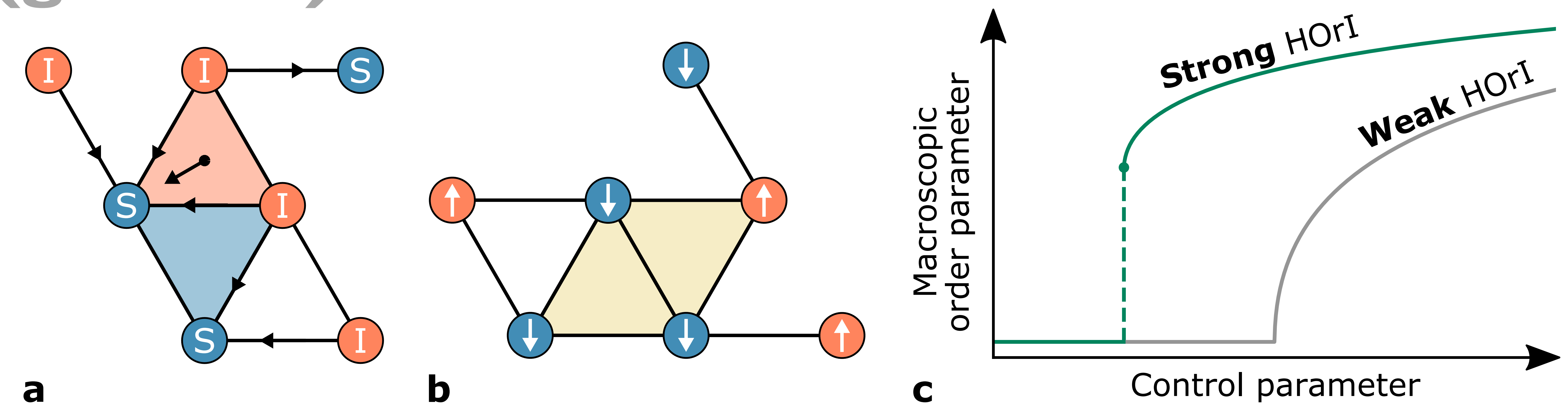

New phenomenologies!

- abrupt transitions
- bi-stability/coexistences
- Critical masses/tipping points

Summing up
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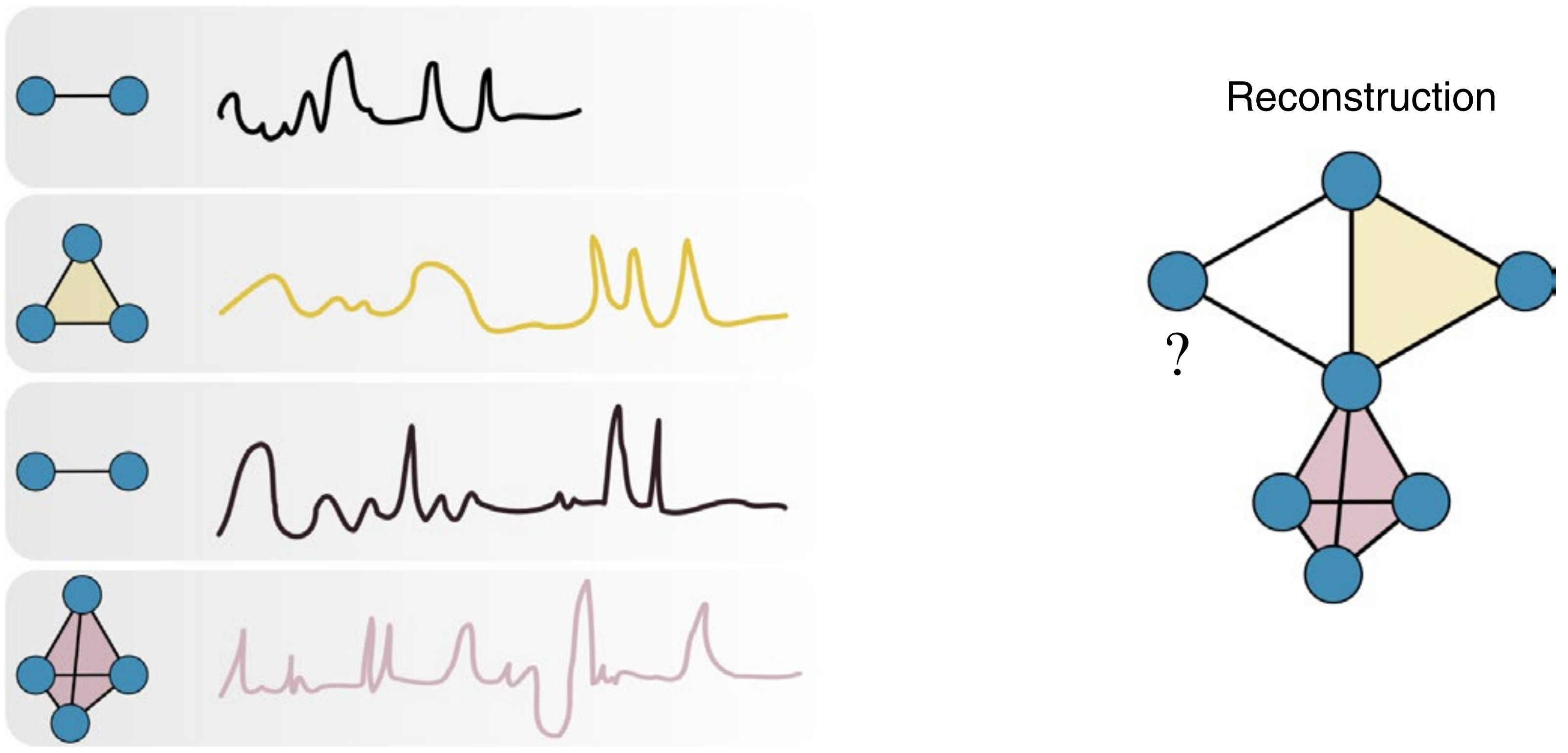

Exciting times for reconstruction!

- Promising steps from various directions
- Early steps on higher-order model inference
- A lot more attention needed!

Summing up
(general)

New phenomenologies!
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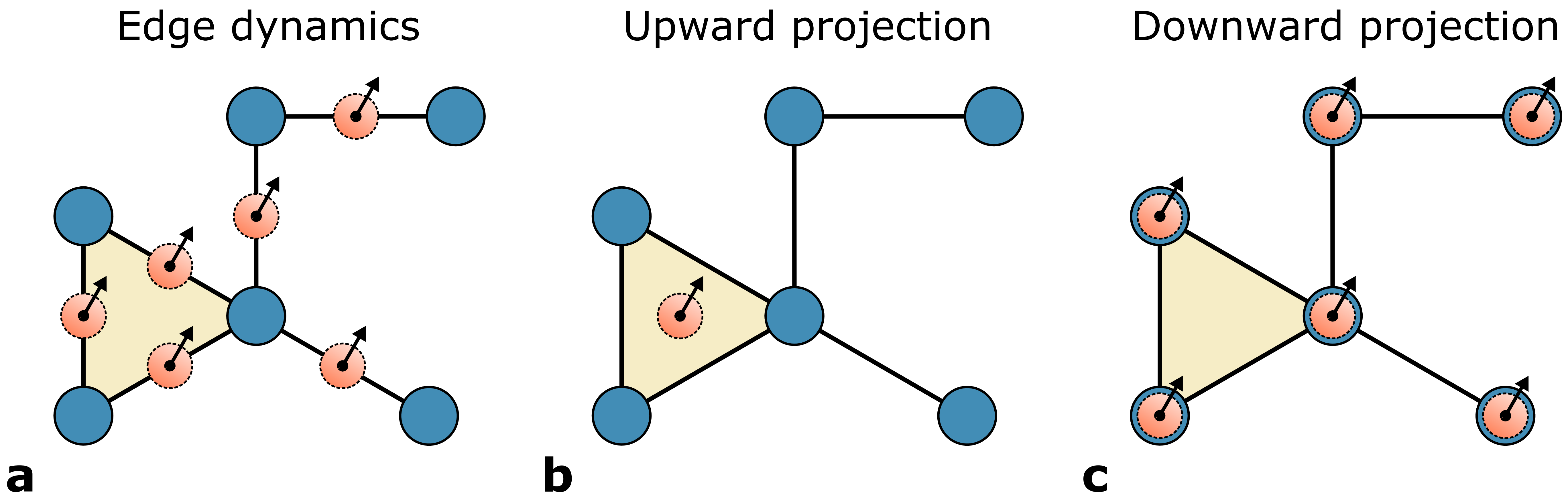
Exciting times for reconstruction!
- Promising steps from various directions
- Early steps on higher-order model inference
- A lot more attention needed!

New objects!
- Systems that are fully *alive/dynamical*
- “Hidden” transitions, novel chimeras
- Interesting developments for signals?



Millán, A. P., Torres, J. J., & Bianconi, G. PRL (2020)
Expert, Arnaudon, Peach, Petri, (2021) https://arxiv.org/abs/2111.11073

Thank you.