Synchronization — transition, structure, time delay, music

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Outline

- Metronomes
- Fireflies
- Brain (epilepsy and music)
- Kuramoto
- Pulses and time delay

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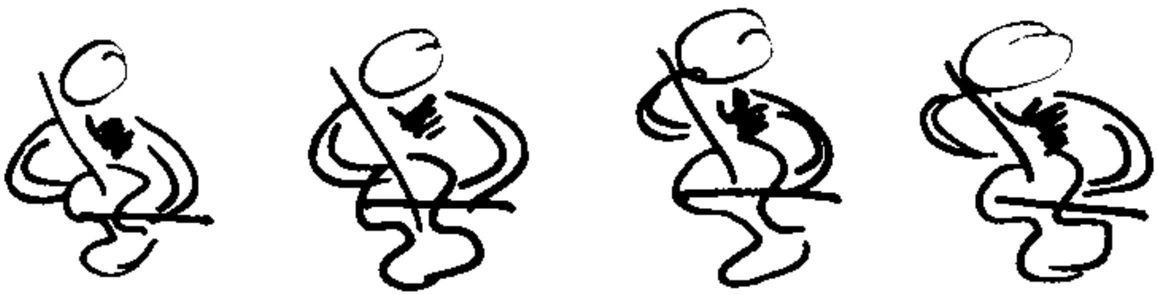
Interaction and the emergence of collective modes



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Synchronization

No interaction -> no synchronization

$$\left.\begin{aligned} \dot{\theta}_1(t) &= \omega_1 \\ &\vdots \\ \dot{\theta}_N(t) &= \omega_N \end{aligned}\right\}$$

Individual "confused" incoherent motion

Turn on interaction -> synchronization

$$\left.\begin{aligned} \dot{\theta}_1(t) &= \omega_1 + F(\theta_1(t), ..., \theta_N(t)) \\ &\vdots \\ \dot{\theta}_N(t) &= \omega_N + F(\theta_1(t), ..., \theta_N(t)) \end{aligned}\right\}$$

Collective coherent motion.
New mode of change is generated

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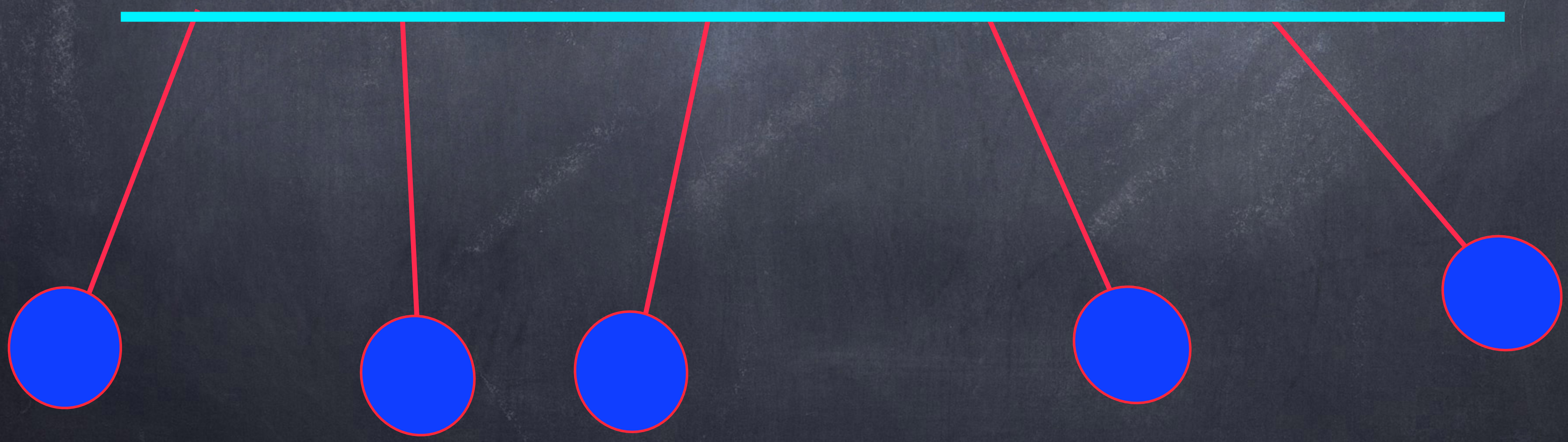
From micro randomness to macro collective coordination

Synchronization through interaction

Synchronisation of 5 coupled metronomes done in Lancaster University, Physics Dep (http://www.youtube.com/user/lancaster...), Nonlinear dynamics and medical physics group.

From micro to macro time

Onset of synchronization



NATURE REVIEWS | NEUROSCIENCE VOLUME 2 | APRIL 2001 | 229

THE BRAINWEB: PHASE SYNCHRONIZATION AND LARGE-SCALE INTEGRATION

Francisco Varela, Jean-Philippe Lachaux*, Eugenio Rodriguez‡ and Jacques Martinerie**

The emergence of a unified cognitive moment relies on the coordination of scattered mosaics of functionally specialized brain regions. Here we review the mechanisms of large-scale integration that counterbalance the distributed anatomical and functional organization of brain activity to enable the emergence of coherent behaviour and cognition. Although the mechanisms involved in large-scale integration are still largely unknown, we argue that the most plausible candidate is the formation of dynamic links mediated by synchrony over multiple frequency bands.

So understanding the dynamics of the brain will involve:

- > topology and synchronisation
    - > on static networks
    - > on evolving networks

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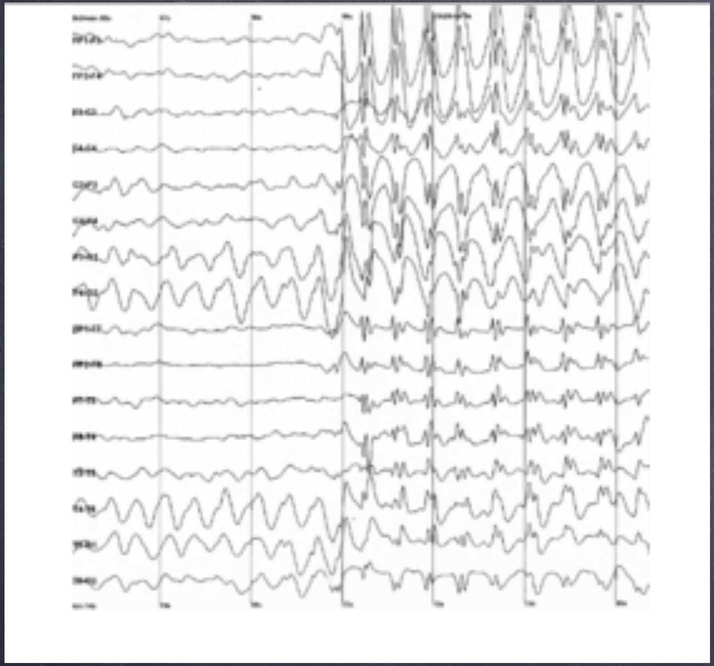
Epileptic seizure - too much synchronization



http://en.wikipedia.org/wiki/Epileptic_seizure

http://www.medicalhomeportal.org/diagnoses-and-conditions/seizures-epilepsy

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Kuramoto model

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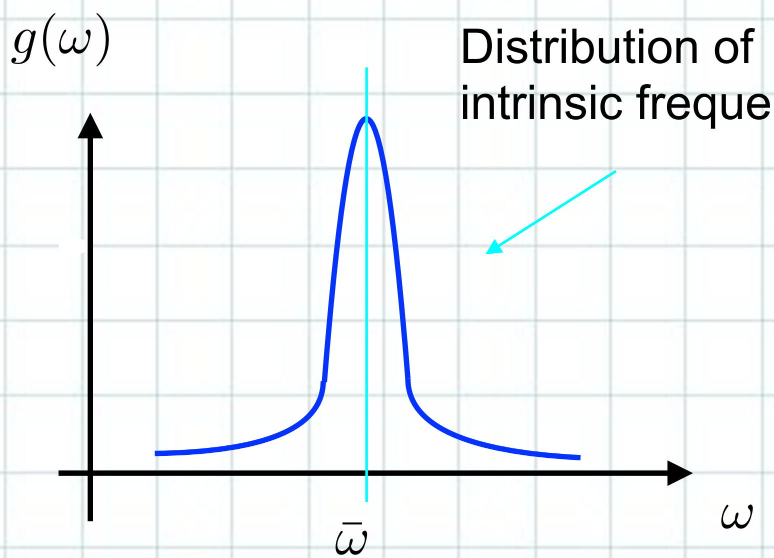
Coupling strength

$$\frac{d\phi_k}{dt} = \omega_k + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$

phase variables

Distribution of intrinsic frequencies



Note:
Neurones are not like the Kuramoto oscillators.
They exchange pulses

Reference:
Synchronization: A Universal Concept in Nonlinear Sciences
Cambridge University Press 2001
by Arkady Pikovsky, Michael Rosenblum, Jürgen Kurths

Henrik Jeldtoft Jensen

Kuramoto model

Analysis of $$\frac{d\phi_k}{dt} = \omega_k + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$

Note
$$\begin{cases} \phi_j < \phi_k \rightarrow \phi_k & \text{speed up to catch up with } \phi_j \\ \phi_j < \phi_k \rightarrow \phi_k & \text{slow down towards } \phi_j \end{cases}$$

$\epsilon \rightarrow 0$ $$\frac{d\phi_k}{dt} = \omega_k \rightarrow \phi_k = \omega_k t$$ all rotors move at individual frequency

$\epsilon \gg 1$ $$\frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$ will dominante

effective equation $$\frac{d\phi_k}{dt} = \bar{\omega} + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$

Kuramoto model

$\epsilon \gg 1$ $\qquad \dfrac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$ will dominante

effective equation

$$\frac{d\phi_k}{dt} = \bar{\omega} + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$

the 2nd term will vanish

$$\frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k) \to 0$$

which leads to synchronisation

$$\frac{d\phi_k}{dt} = \bar{\omega} \to \phi_k = \bar{\omega}t$$

Hence we expect two regimes

$\phi_k = \omega_k t$ $\qquad\qquad$ $\phi_k = \bar{\omega}t$

ϵ/σ_ω

1

no synchronization $\qquad\qquad$ synchronization

Kuramoto model

Hence we expect two regimes

$\phi_k = \omega_k t$ | $\phi_k = \bar{\omega} t$

ϵ / σ_ω

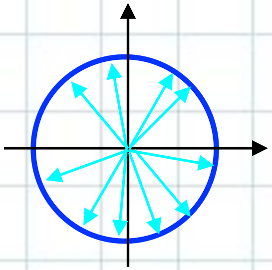
1

no synchronization | synchronization

Want a parameter that can distinguish between the two regimes.
Consider

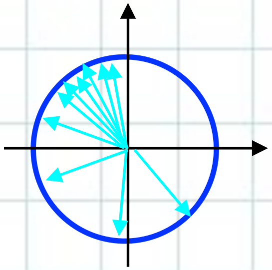
$$K e^{i\theta} = \frac{1}{N} \sum_{k=1}^{N} e^{i\phi_k}$$

Random



$$\sum_{k=1}^{N} e^{i\phi_k} = 0 \to K = 0$$

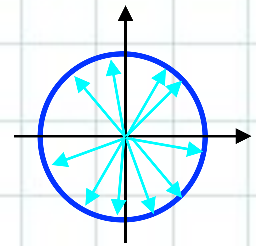
Directed



$$\sum_{k=1}^{N} e^{i\phi_k} \neq 0 \to K > 0$$

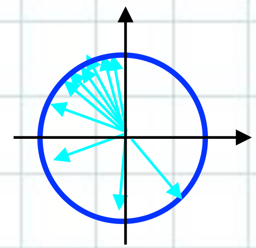
$$Ke^{i\theta} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k}$$

Random



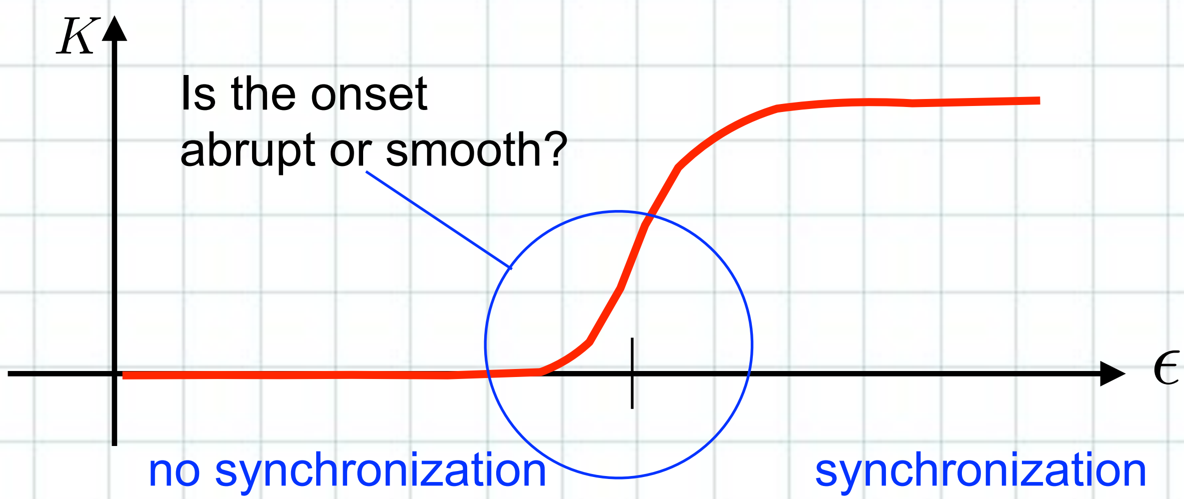
$$\sum_{k=1}^{N} e^{i\phi_k} = 0 \rightarrow K = 0$$

Directed



$$\sum_{k=1}^{N} e^{i\phi_k} \neq 0 \rightarrow K > 0$$

Two regimes



Introduce the order parameter into the Eq. of Motion $Ke^{i\theta} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k}$

$$\frac{d\phi_k}{dt} = \omega_k + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$$
$$= \omega_k + \frac{\epsilon}{N}\mathrm{Im}\sum_{j=1}^{N} e^{i(\phi_j - \phi_k)}$$
$$= \omega_k + \frac{\epsilon}{N}\mathrm{Im}\{(\sum_{j=1}^{N} e^{i\phi_j})e^{-i\phi_k}\}$$
$$= \omega_k + \epsilon\mathrm{Im}\{Ke^{i\theta}e^{-i\phi_k}\}$$
$$= \omega_k + \epsilon K\sin(\theta - \phi_k)$$

Closer look at $\frac{d\phi_k}{dt} = \omega_k + \epsilon K\sin(\theta - \phi_k)$

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Closer look at $\frac{d\phi_k}{dt} = \omega_k + \epsilon K \sin(\theta - \phi_k)$

$$K e^{i\theta} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k}$$

Consider deviation from average $\phi_k(t) = \bar{\omega}t + \psi_k(t)$

Make the natural assumption that $\theta(t) = \bar{\omega}t$

Substitute
$$\begin{aligned}\theta - \phi_k &= \theta - (\bar{\omega}t + \psi_k(t)) \\ &= -\psi_k(t)\end{aligned}$$

into eq. of motion to obtain

$$\frac{d\psi_k}{dt} = \omega_k - \bar{\omega} - \epsilon K \sin(\psi_k)$$

Synchronization and the behaviour of ψ_k

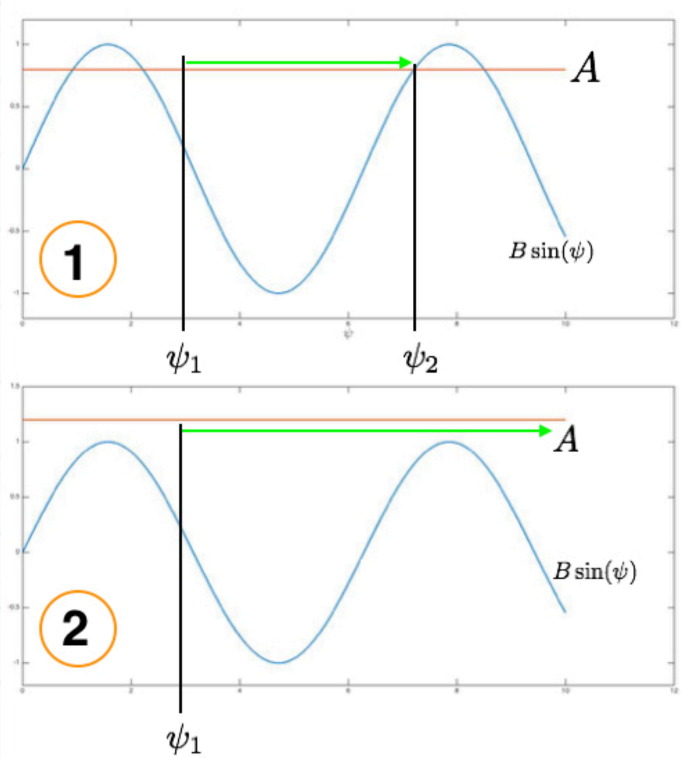
(A) $\forall \psi_k(t) \rightarrow$ constant — synchronization

(B) $\forall \psi_k(t)$ monotonically increase — no synchronization

Closer look at $\dfrac{d\psi_k}{dt} = \omega_k - \bar{\omega} - \epsilon K \sin(\psi_k)$

Equation of the form

$$\frac{d\psi}{dt} = A - B\sin(\psi)$$



We have

1 $A < B \Rightarrow \lim_{t\to\infty} \dfrac{d\psi}{dt} = 0$

so ψ stops

2 $A > B \Rightarrow \lim_{t\to\infty} \dfrac{d\psi}{dt} > 0$

so ψ keeps monotonically increasing

Hence two classes of rotors:

- $|\omega - \bar{\omega}| < \epsilon K$ class 1 - rotors lock in -> synch
- $|\omega - \bar{\omega}| > \epsilon K$ class 2 - no synch

We want to compute $Ke^{i\theta} = \frac{1}{N}\sum_{k=1}^{N} e^{i\phi_k}$

by dividing sum into the two cases of rotors

First substitute $\phi_k(t) = \bar{\omega}t + \psi_k(t)$ into expression for $Ke^{i\theta}$

$$Ke^{i\bar{\omega}t} = \frac{1}{N}\sum_{k=1}^{N} e^{i(\bar{\omega}t+\psi_k)}$$

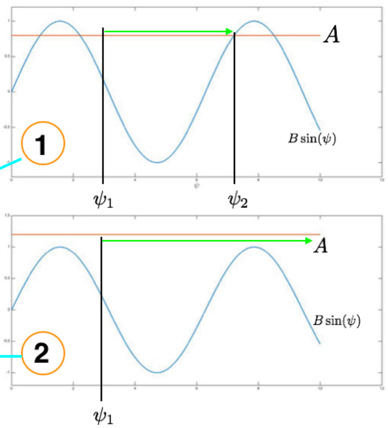
$$\Downarrow$$

$$K = \sum_{k=1}^{N} e^{i\psi_k}.$$

Next assume there is so many rotors that we can replace the sum by an integral

$$K = \int_{-\infty}^{\infty} d\psi n(\psi) e^{i\psi}$$

Where $n(\psi)d\psi$ denotes the number of rotors
with their phase in the interval $[\psi, \psi + d\psi]$



We divide $n(\psi)$ according to the two classes:

$$n(\psi) = n_s(\psi) + n_{as}(\psi)$$

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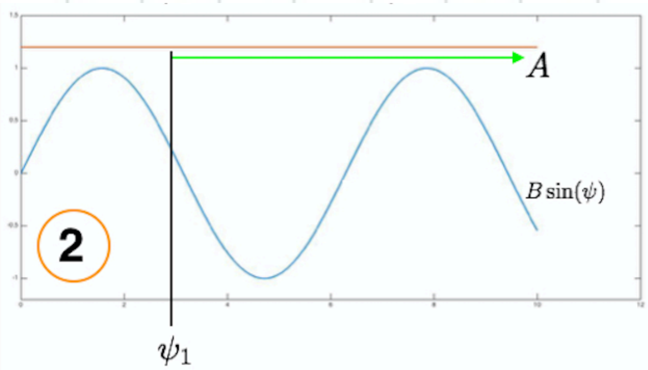
We have to determine the contribution of each class $K = \int_{-\infty}^{\infty} d\psi [n_s(\psi) + n_{as}(\psi)] e^{i\psi}$

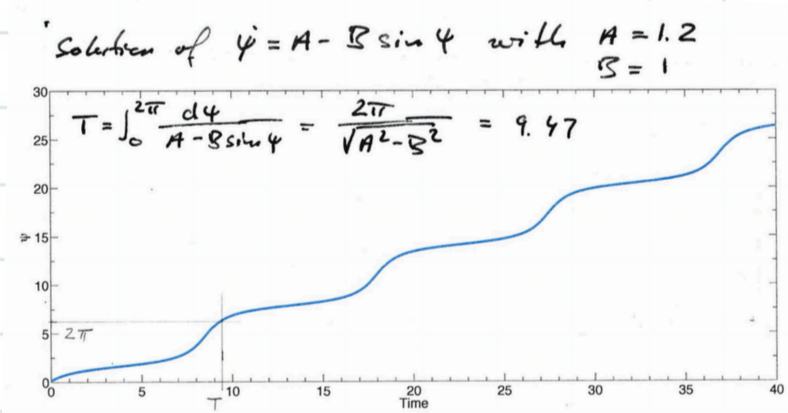
Start with non-synch, so rotors moving according to

$$\frac{d\psi}{dt} = A - B\sin(\psi)$$ with $A > B$



Integrate to get

$$t = \int \frac{d\psi}{A - B\sin\psi}$$
$$= \frac{2}{\sqrt{A^2 - B^2}} \arctan \frac{A\tan\frac{\psi}{2} - B}{\sqrt{A^2 - B^2}}$$



The periodicity of the solution ensures that

$$n_{as}(\psi + \pi) = n_{as}(\psi)$$

and therefore $K = \int_{-\infty}^{\infty} d\psi n_{as}(\psi) e^{i\psi} = 0$

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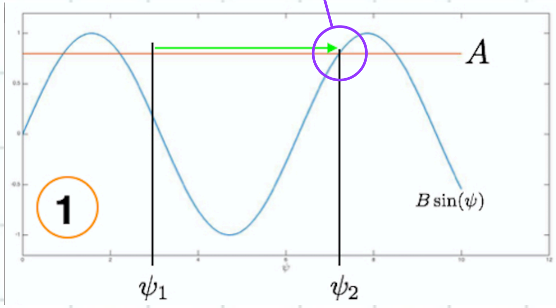
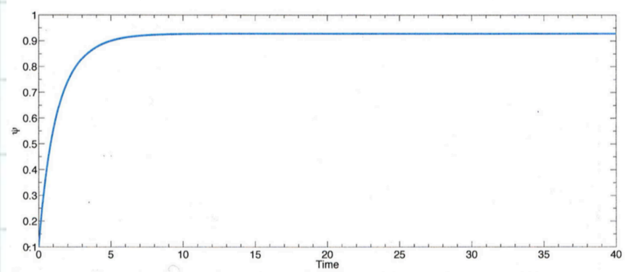
Contribution to K from the synchronizing rotors

$$\frac{d\psi_k}{dt} = \omega_k - \bar{\omega} - \epsilon K \sin(\psi_k)$$

$$\frac{d\psi_k}{dt} = 0$$



solution for $A = 0.8$, $B = 1.$



The distribution of the ψ_k will be given by the asymptotic stationary solution

$$\omega - \bar{\omega} - \epsilon K \sin(\psi) = 0$$

$$\Downarrow$$

$$\omega = \bar{\omega} + \epsilon K \sin(\psi)$$

use

So $n_s(\psi) = g(\omega)|\frac{d\omega}{d\psi}|$

$$\Downarrow$$

$$n_s(\psi) = g(\bar{\omega} + \epsilon K \sin(\psi))\epsilon K \cos(\psi)$$

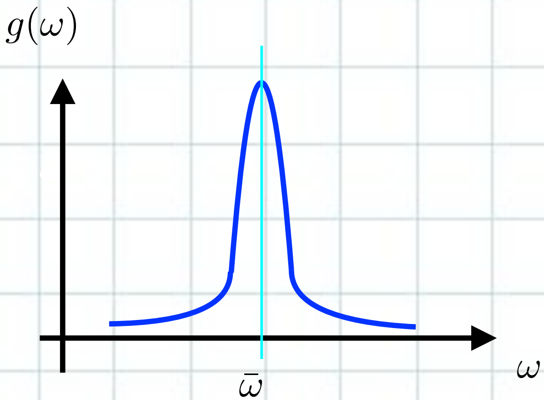
Next substitute this expression into $K = \int_{-\infty}^{\infty} d\psi n_s(\psi) e^{i\psi}$

to obtain $K = \int d\psi n_s(\psi) e^{i\psi} = \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi)\epsilon K \cos\psi e^{i\psi}$

The periodicity allows us to restrict the domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Analysis of $K = \int d\psi n_s(\psi) e^{i\psi} = \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \epsilon K \cos\psi e^{i\psi}$

To proceed assume $g(\omega)$ symmetric about a peak



$$g(\bar{\omega} + \omega) = g(\bar{\omega} - \omega),\ \omega \in \mathbb{R}$$

We the get for
$$K = \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \epsilon K \cos\psi e^{i\psi}$$
$$= \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \epsilon K \cos\psi [\cos(\psi) + i \sin(\psi)]$$

with

$$\int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \epsilon K \cos^2\psi > 0$$ integral of even function

$$\int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \epsilon K \cos\psi \sin(\psi) = 0$$ integral of odd function

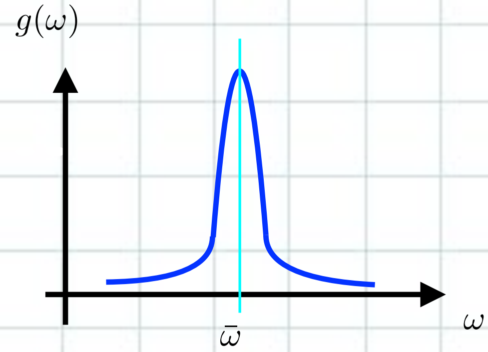
and therefore
if $K > 0$

$$\boxed{1 = \epsilon \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \cos\psi \cos^2(\psi)}$$

Solve eq. for K
given ϵ

Solve eq. for K given ϵ

$$1 = \epsilon \int_{-\pi/2}^{\pi/2} d\psi g(\bar{\omega} + \epsilon K \sin\psi) \cos\psi \cos^2(\psi)$$



To obtain a general expression expand about peak of $g(\omega)$ at $\bar{\omega}$ so $g'(\bar{\omega}) = 0$

$$g(\bar{\omega} + \epsilon K \sin\psi) \simeq g(\bar{\omega}) + \frac{g''(\bar{\omega})}{2}(\epsilon K \sin\psi)^2$$

Use $\int_{-\pi/2}^{\pi/2} d\psi \cos^2\psi = \frac{\pi}{2}$ and $\int_{-\pi/2}^{\pi/2} d\psi \sin^2\psi \cos^2\psi = \frac{\pi}{8}$

This leads to the equation $\epsilon g(\bar{\omega})\frac{\pi}{2} - \frac{\epsilon^3 |g''(\bar{\omega})|}{2} K^2 \frac{\pi}{8} = 1$ Recall $g''(\bar{\omega}) < 0$

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So $\epsilon g(\bar{\omega})\frac{\pi}{2} - \frac{\epsilon^3|g''(\bar{\omega})|}{2}K^2\frac{\pi}{8} = 1$ or $K^2 = \frac{\epsilon g(\bar{\omega})\frac{\pi}{2} - 1}{\frac{\pi}{8}\frac{\epsilon^3 g''(\bar{\omega})}{2}}$

A solution exist if $\epsilon > \epsilon_c = \frac{2}{\pi g(\bar{\omega})}$

Remember our equation is $\frac{d\phi_k}{dt} = \omega_k + \frac{\epsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_k)$

The larger the number of rotors that have a frequency near the average, i.e $g(\bar{\omega})$ large, the smaller the interaction strength $\epsilon_c = 2/\pi g(\bar{\omega})$ needed to induce synchronisation

Conclusion

$$\boxed{K = \begin{cases} 0 & \text{for } \epsilon < \epsilon_c \\ \sqrt{\frac{8g(\bar{\omega})}{|g''(\bar{\omega})|\epsilon^3}}(\epsilon - \epsilon_c)^{\frac{1}{2}} & \text{for } \epsilon > \epsilon_c \end{cases}}$$

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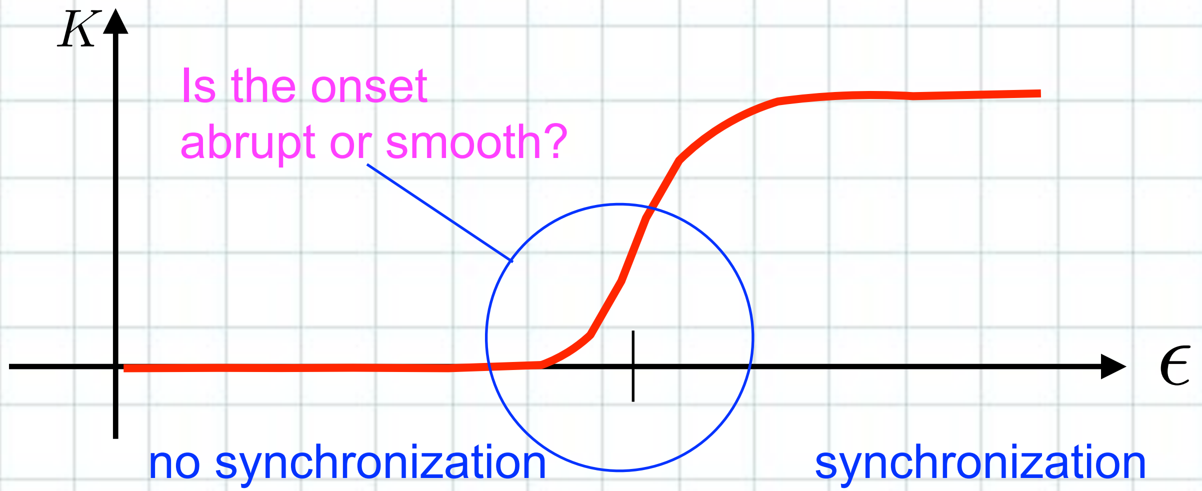
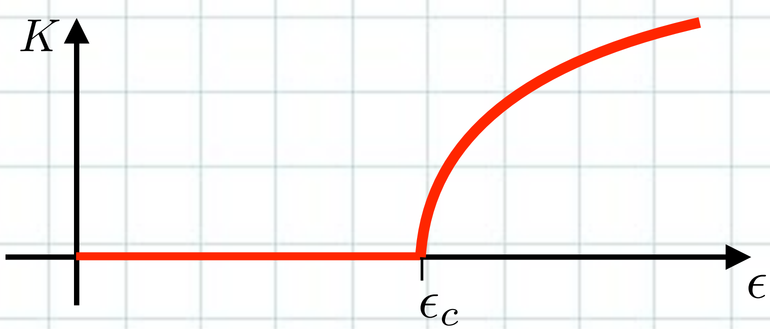
Conclusion $$K = \begin{cases} 0 & \text{for } \epsilon < \epsilon_c \\ \sqrt{\frac{8g(\bar{\omega})}{|g''(\bar{\omega})|\epsilon^3}}(\epsilon-\epsilon_c)^{\frac{1}{2}} & \text{for } \epsilon > \epsilon_c \end{cases}$$

We can now answer the question



The onset is sharp and $K \propto (\epsilon - \epsilon_c)^{\frac{1}{2}}$
in the vicinity above ϵ_c



Example $g(\omega) = \dfrac{\gamma}{\pi[(\omega - \bar{\omega})^2 + \gamma^2]}$ $\rightarrow$

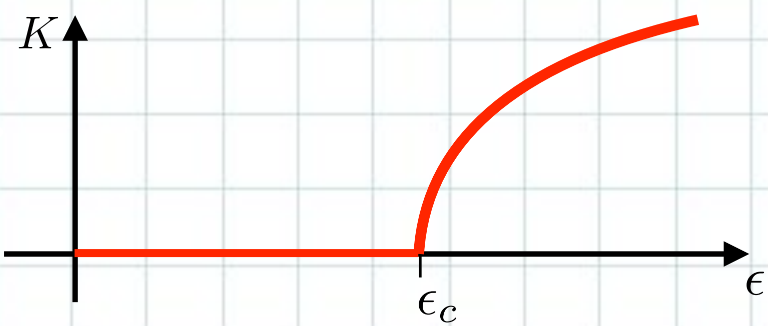
$$\begin{cases} g(\bar{\omega}) = \dfrac{1}{\pi\gamma} \\ g(\bar{\omega}+\gamma) = \dfrac{\gamma}{\pi[\gamma^2+\gamma^2]} = \dfrac{1}{2}\dfrac{1}{\pi\gamma} = \dfrac{1}{2} g(\bar{\omega}) \end{cases}$$

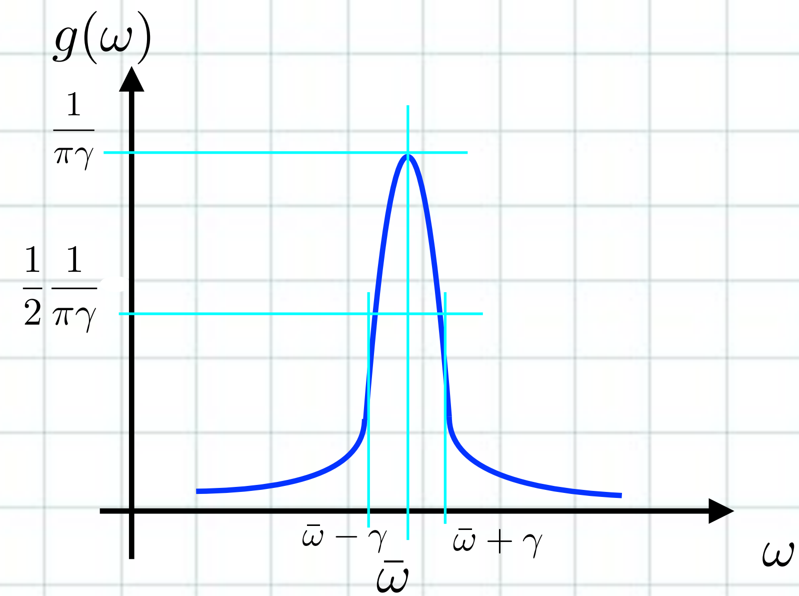
Assume

Then

$$\epsilon_c = 2\gamma$$

$$K = \sqrt{1 - 2\gamma/\epsilon} \quad \text{for} \quad \epsilon > \epsilon_c$$

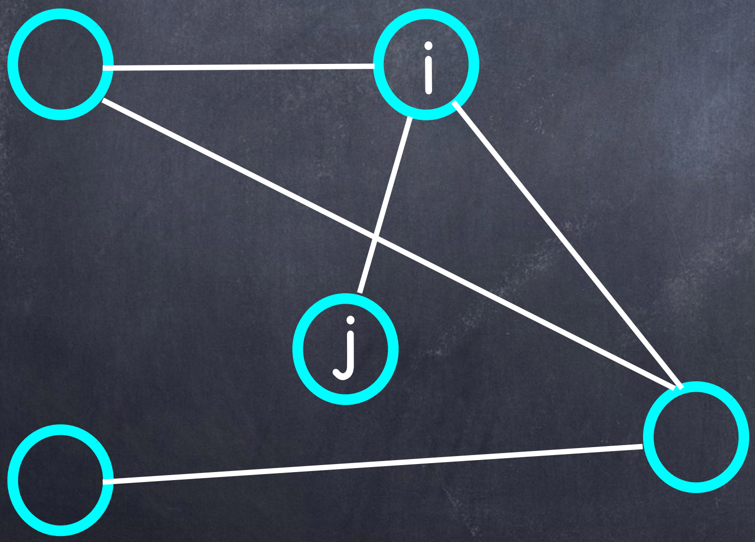




Synchronization and the topology of networks

Kuramoto model: transient reveals structure

$$\frac{d\theta_i}{dt} = \omega_i + \sum_j K_{ij} \sin(\theta_j - \theta_i), \quad i = 1, ..., N$$



PRL 96, 114102 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Synchronization Reveals Topological Scales in Complex Networks

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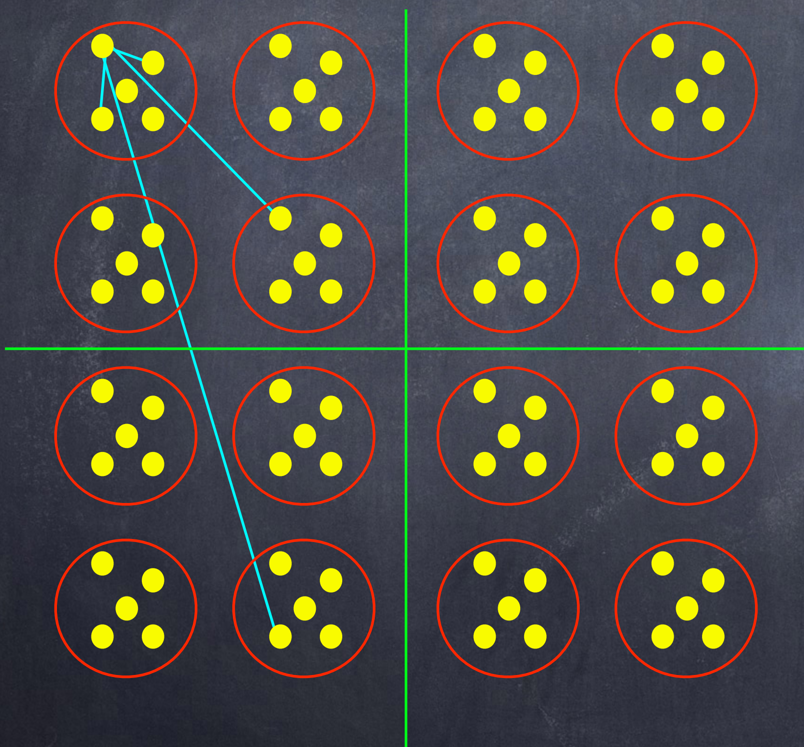
(Received 30 November 2005; published 22 March 2006)

Synchronization and the topology of networks

$$\frac{d\theta_i}{dt} = \omega_i + \sum_j K_{ij} \sin(\theta_j - \theta_i), \quad i = 1, ..., N$$



$$z_1 + z_2 + z_3 = Z_{tot}$$

Rank the eigenvalues of the Laplacian Matrix

$$0 \leq \lambda_1 \leq \lambda_2 ... \leq \lambda_N$$

Linearised Kuramoto

$$\frac{d\theta_i}{dt} = -k \sum_j L_{ij} \theta_j i = 1, \ldots, N, \qquad (4)$$

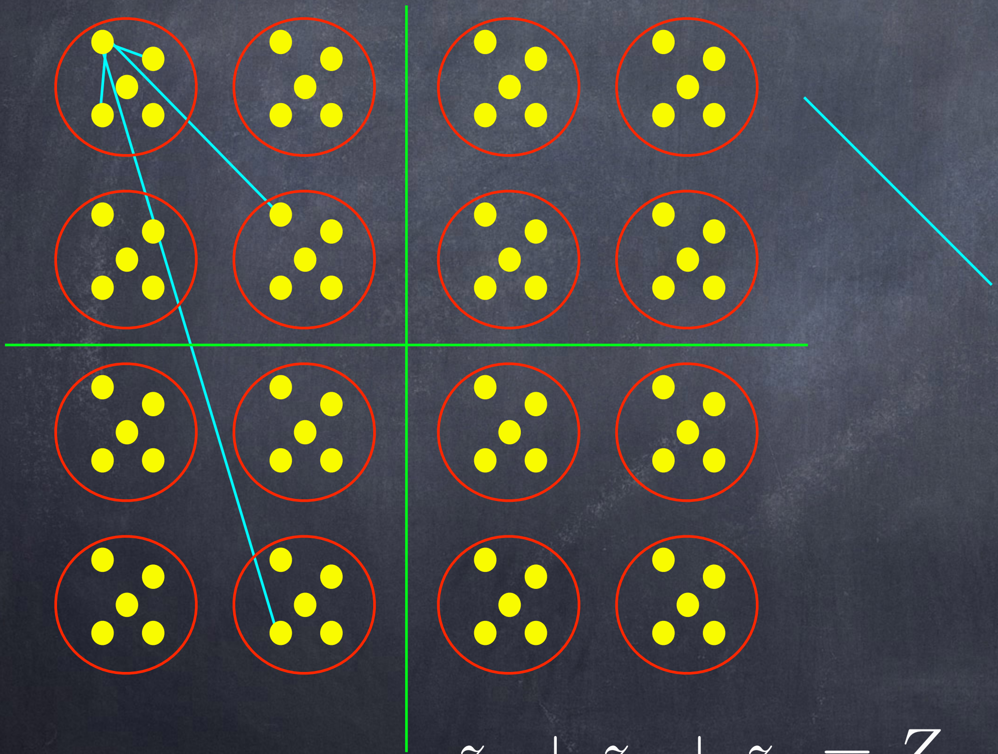
whose solution in terms of the normal modes $\varphi_i(t)$ reads

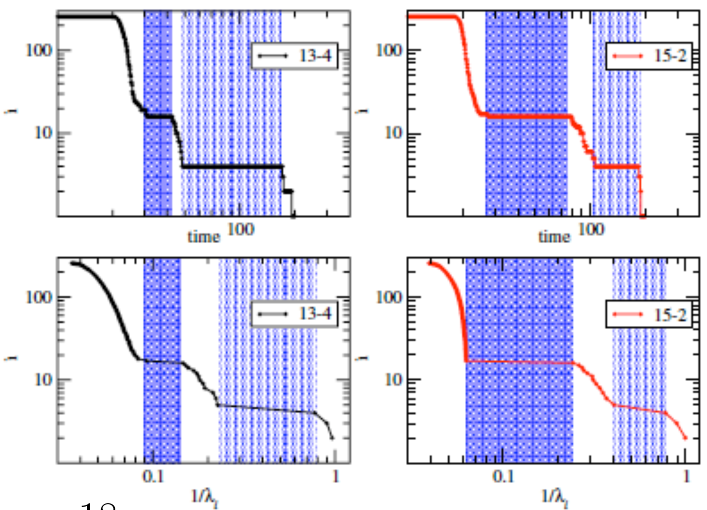
$$\varphi_i(t) = \sum_j B_{ij} \theta_j = \varphi_i(0) e^{-\lambda_i t} i = 1, \ldots, N, \qquad (5)$$

where λ_i are the eigenvalues of the Laplacian matrix, and B is the eigenvectors matrix.

Synchronization and the topology of networks

$$\frac{d\theta_i}{dt} = \omega + \sum_j K_{ij} \sin(\theta_i - \theta_j) \; i = 1, 2, ..., N$$





$Z_{tot} = 18$

FIG. 2 (color online). Top: Number of disconnected synchronized components (equivalent to number of null eigenvalues of $S[\mathcal{D}_T(t)]$) as a function of time for the two networks of Fig. 1 at $T = 0.99$. Bottom: Rank index i (see text) versus the inverse of the corresponding eigenvalues of the Laplacian matrix $\mathcal{L}$. The shadow regions indicate the stability plateaus for 16 (dark) and 4 (light) communities. The same representation is used for the plateaus in the eigenvalue spectrum corresponding to indices 16 and 4.

$$z_1 + z_2 + z_3 = Z_{tot}$$

Arenas, Diaz-Guilera and Perez-Vicente Phys. Rev. Lett. **96**, 114102 (2006)

Recent very useful references:

PHYSICAL REVIEW E **82**, 036201 (2010)

Hub synchronization in scale-free networks

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(Received 14 January 2010; revised manuscript received 24 June 2010; published 1 September 2010)

Heterogeneity in the degree distribution is known to suppress global synchronization in complex networks of symmetrically coupled oscillators. Scale-free networks display a great deal of heterogeneity, containing a few nodes, termed hubs, that are highly connected, while most nodes receive only a few connections. Here, we show that a group of synchronized nodes may appear in scale-free networks: hubs undergo a transition to synchronization while the other nodes remain unsynchronized. This general phenomenon can occur even in the absence of global synchronization. Our results suggest that scale-free networks may have evolved to complement various levels of synchronization.

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Stability of Synchronized Motion in Complex Networks

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Lecture notes, arXiv:1112.2297

Connectivity driven Coherence in Complex Networks

Tiago Pereira[1], Deniz Eroglu[2], G. Baris Bagci[2], Ugur Tirnakli[2], and Henrik Jeldtoft Jensen[1]
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We study the emergence of coherence in complex networks of mutually coupled non-identical elements. We uncover the precise dependence of the dynamical coherence on the network connectivity, on the isolated dynamics of the elements and the coupling function. These findings predict that in random graphs, the enhancement of coherence is inversely proportional to the mean degree. In locally connected networks, coherence is no longer controlled by the mean degree, but rather on how the mean degree scales with the network size. In these networks, even when the coherence is absent, adding a fraction s of random connections leads to an enhancement of coherence proportional to $1/s$. Our results provide a way to control the emergent properties by the manipulation of the dynamics of the elements and the network connectivity.

Phys. Rev. Lett. **110**, 234103 (2013)

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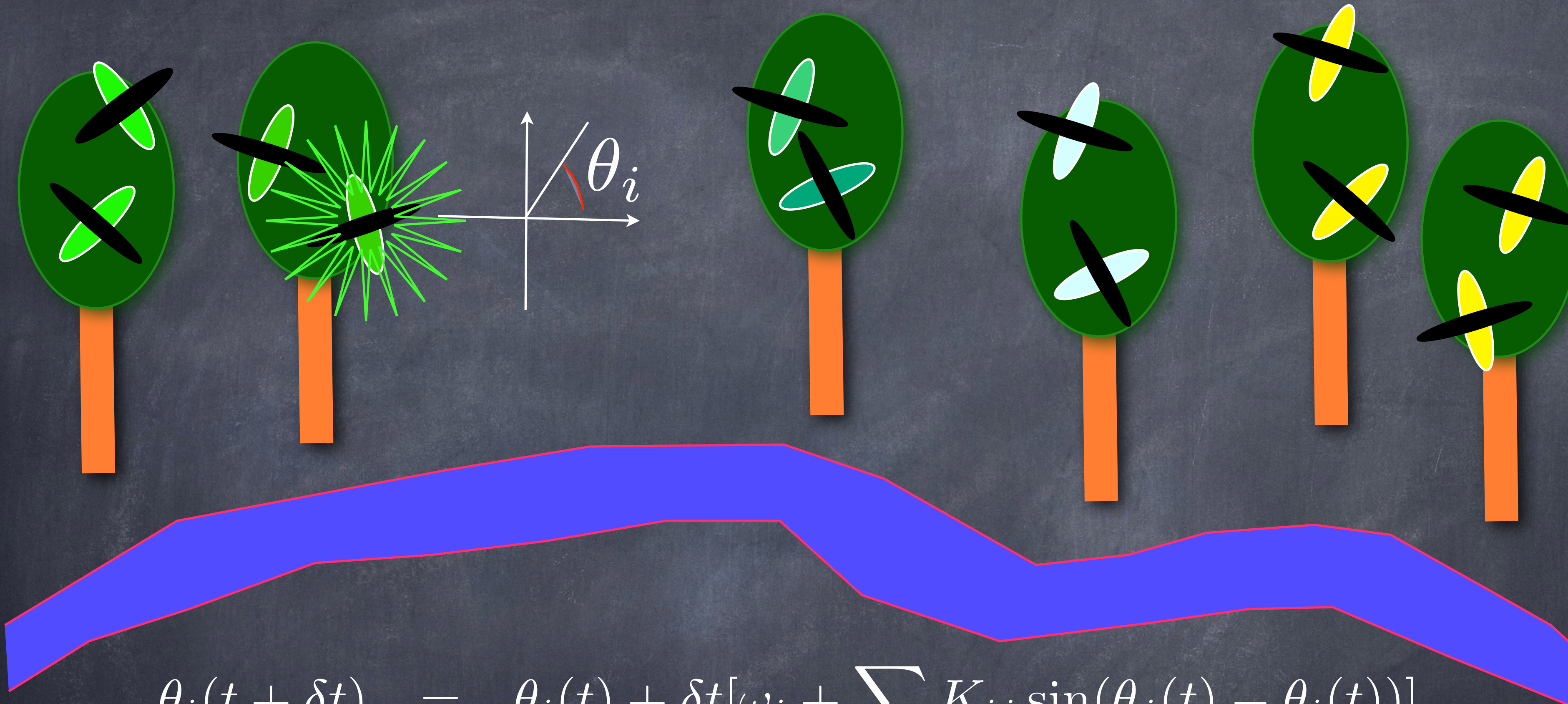
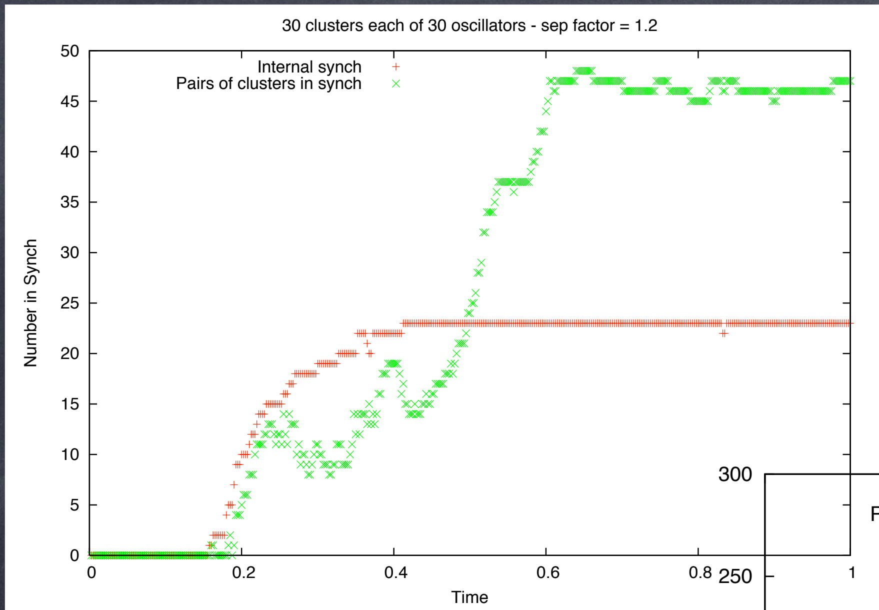
Fireflies – The Movie

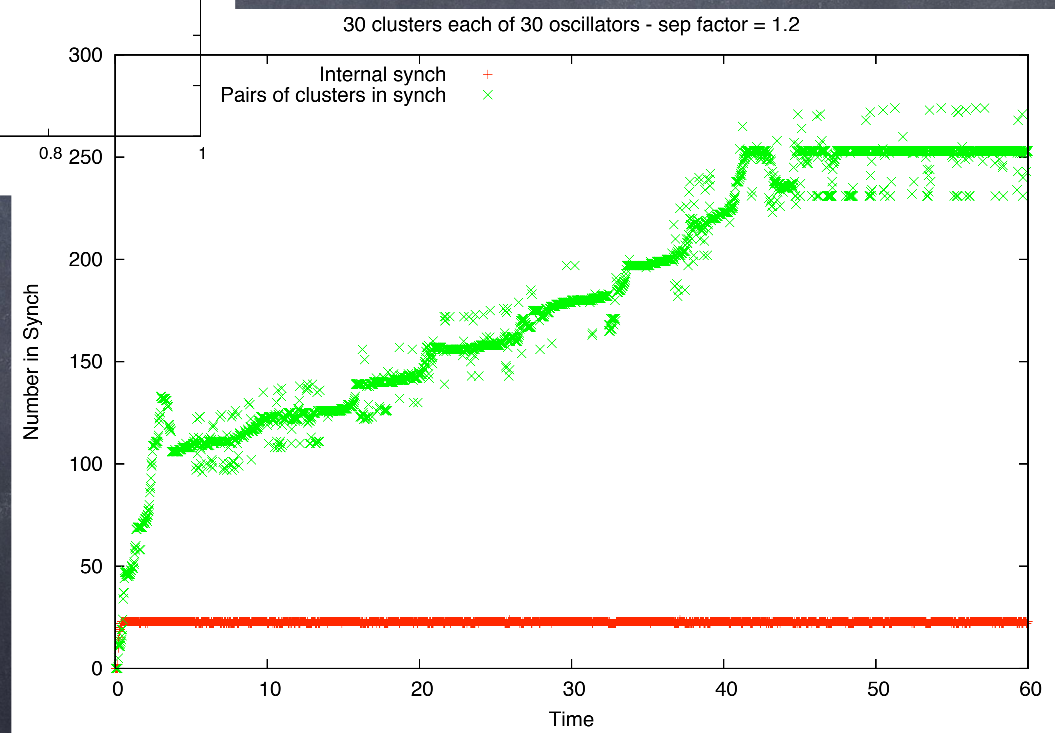
BBC – Trails of Life

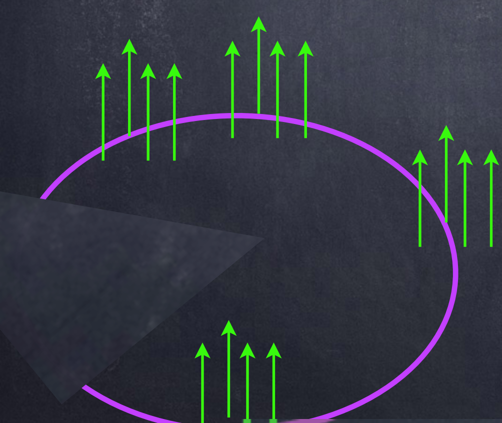


$$
\begin{aligned}
\theta_i(t+\delta t) &= \theta_i(t) + \delta t\left[\omega_i + \sum_j K_{ij}\sin(\theta_j(t) - \theta_i(t))\right] \\
K_{ij} &= \frac{L(j)}{r_{ij}^2} \\
L(j) &= \text{increasing with synch of cluster } j
\end{aligned}
$$

"Exponential" separation







$$\mathbf{r}_i = \xi + \gamma^{k-1}, \quad \xi \in [-0.2, 0.2], \quad \gamma = 1.2, \quad k = \text{cluster } \#$$

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Not all synchronisation involve the Kuramoto rotors

- Consider pulse oscillators
- General coupling functions and focus on network structure

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Pulse synchronisation

$$\dot{\theta}_1(t) = \omega + J \sum_{n \in \mathbb{Z}} \delta(\theta_2(t - \delta t) - n))$$

$$\dot{\theta}_2(t) = \omega + J \sum_{n \in \mathbb{Z}} \delta(\theta_1(t - \delta t) - n))$$

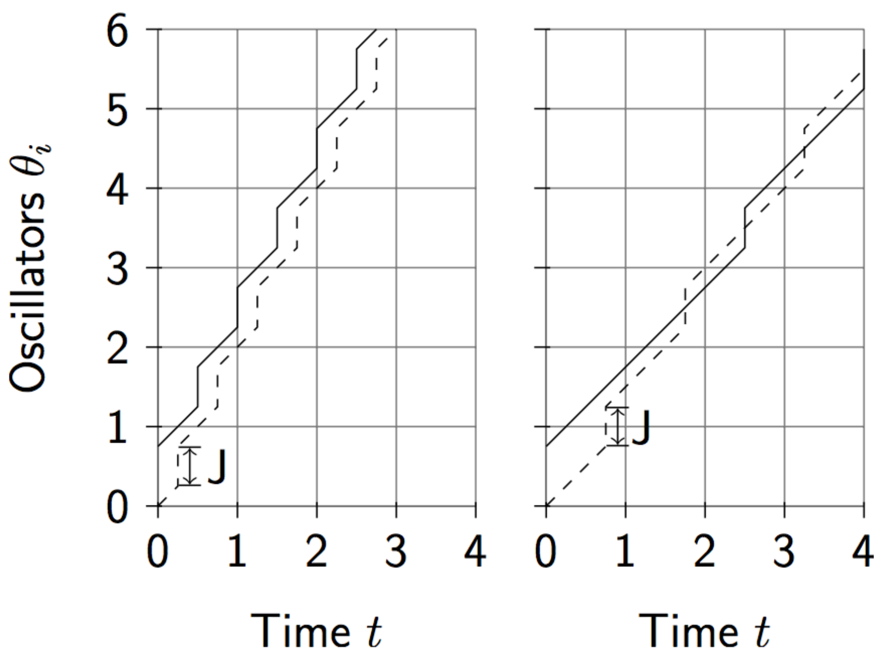

Figure 1. The time evolution of two oscillators (solid and dashed lines) exchanging pulses according to Eq. 1. Left panel: $\delta t = 0$, right panel: $\delta t = 0.5$.

G Pruessner, S Cheang and H.J. Jensen, *Synchronisation by small time delays*, Physica A, **420**, 8-13 (2015)

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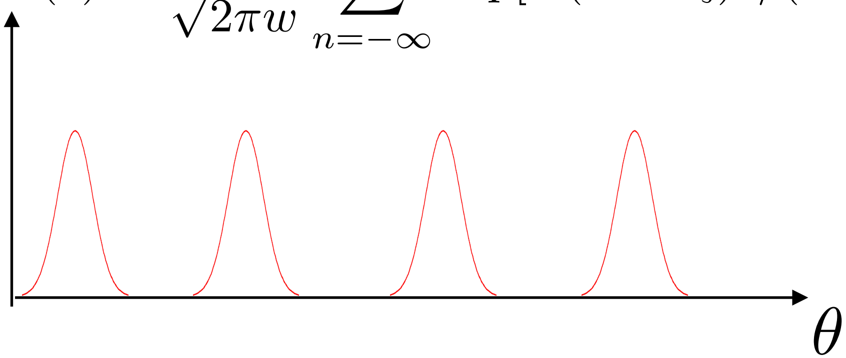
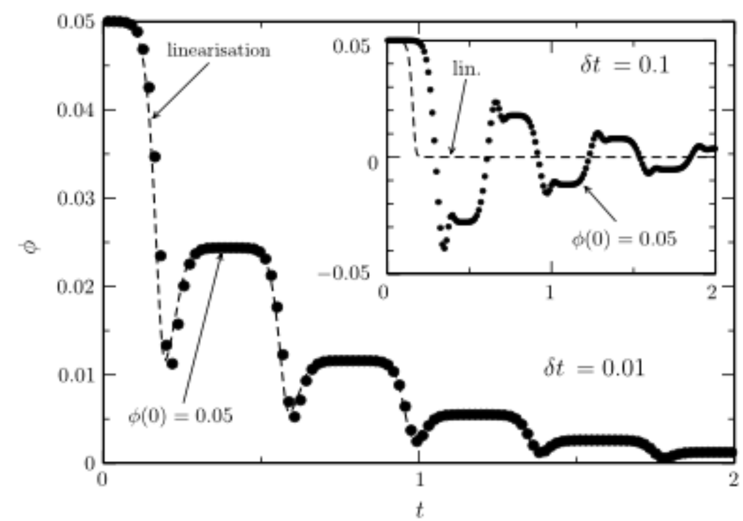
Smooth pulses

$$\dot{\theta}_i(t) = \omega + \sum_j J_{ij}\sigma(\theta_j(t-\delta t))$$

$$\sigma(\theta) = \frac{1}{\sqrt{2\pi}w}\sum_{n=-\infty}^{\infty} \exp[-(\theta + n\xi)^2/(2w^2)]$$



$$\phi = \theta_1 - \theta_2$$



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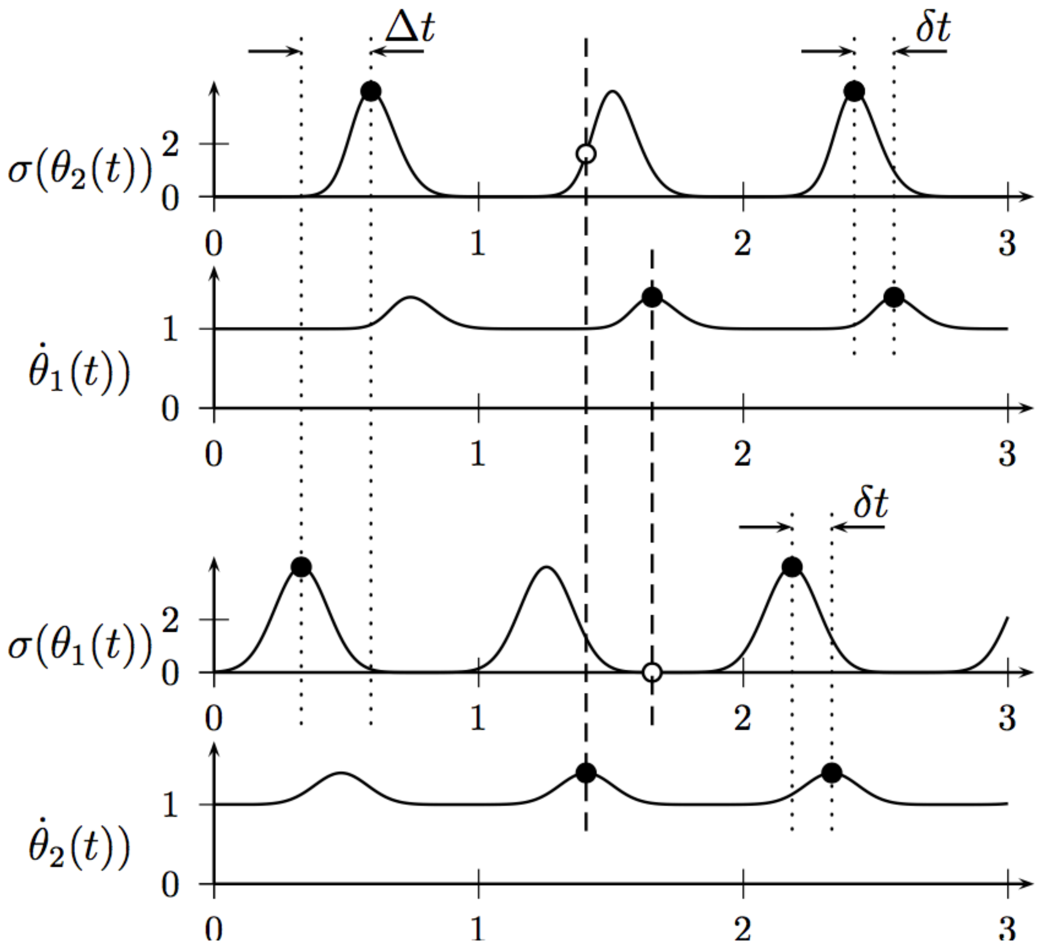

Figure 4. We see that θ_1 is ahead of θ_2, with the maximum of $\sigma(\theta_2(t))$ displaced by Δt (as indicated) to the right relative to that of $\sigma(\theta_1(t))$, as initialised. At a given time, θ_2 still has to pass through the maximum of $\sigma(\theta_2(t))$ when θ_1 already has. The phase speed $\dot{\theta}_{1,2}(t)$ is essentially $\sigma(\theta_{2,1}(t))$ shifted by δt to the right, as indicated by the dotted lines. As a result, the maximum of $\dot{\theta}_2$ nearly aligns with the maximum of $\sigma(\theta_2(t))$, *i.e.* θ_2 is fast when $\sigma(\theta_2(t))$ goes through the maximum (dashed line), thereby narrowing it. In turn, θ_1 passes very quickly through a low value of $\sigma(\theta_1(t))$, relatively broadening in turn the maximum of $\sigma(\theta_1(t))$.

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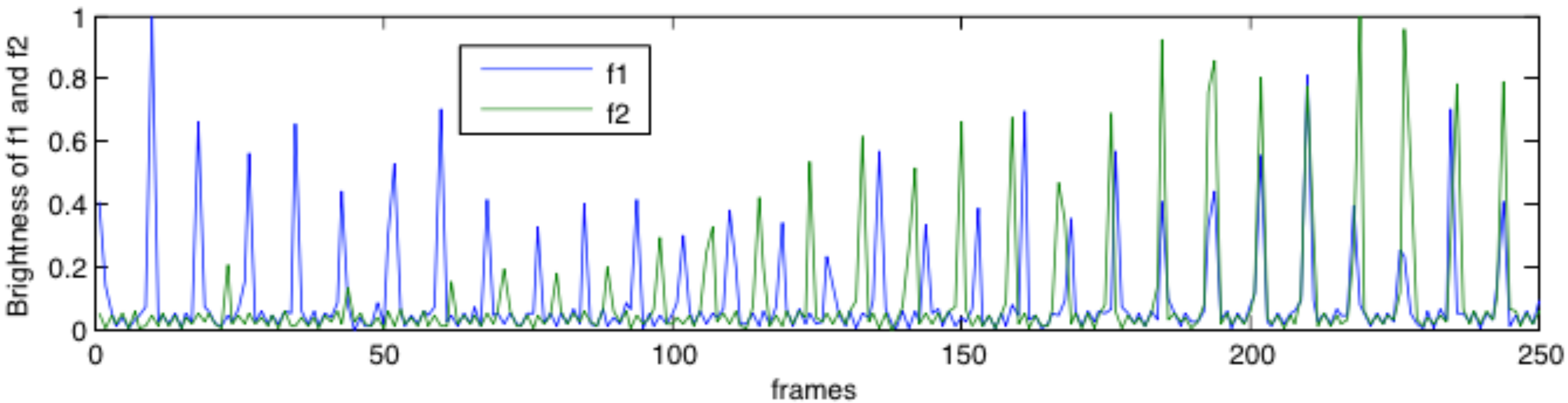
But real fireflies are not Kuramoto oscillators



Time series from two fireflies

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How the top level can influence the lower level — in humans

From mind to molecules

Henrik Jeldtoft Jensen

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Music as pacemaker

The flow of somebody else's mind may change the rate of your neurones

From joint research with
Guildhall School of Music and Drama

Work together with David Dolan, John Sloboda, and Björn Crütz
Trio Anima: Drew Balch, Matthew Featherstone and Anne Hodnett

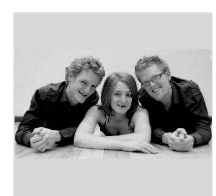
Music played in different modes:

- Composed, improvised
- “Strict” or “let-go”

David Dolan

John Sloboda

Björn Crütz



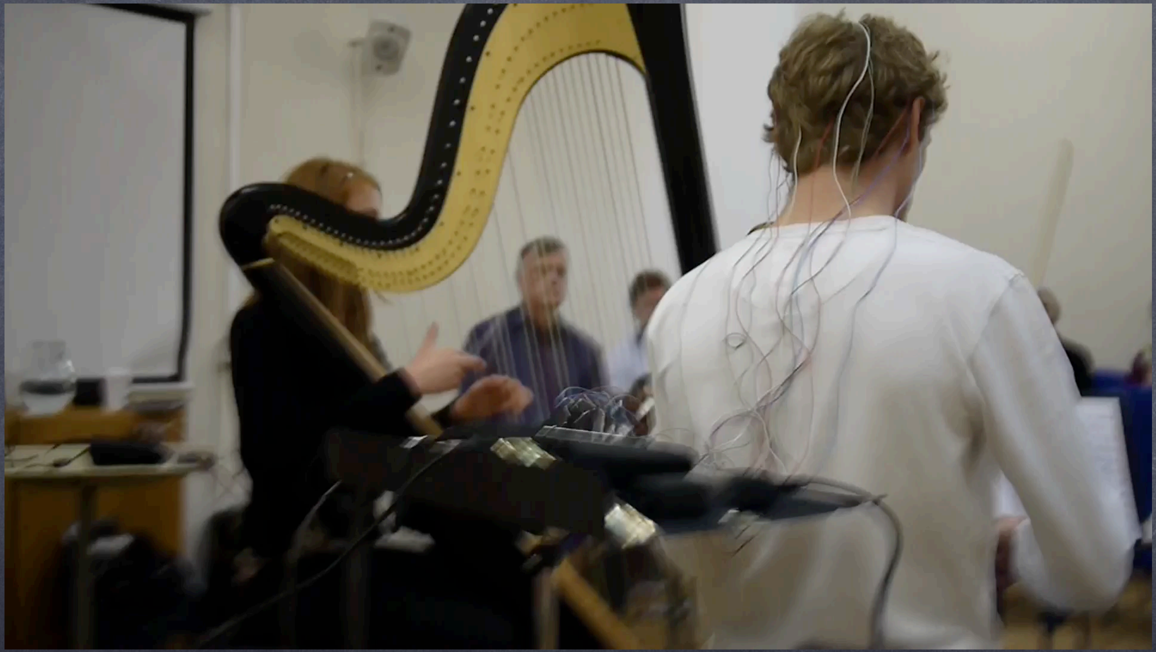
Trio Anima:
Drew Balch, Anneke Hodnett, Matthew Featherstone

Henrik Jeldtoft Jensen

Example: Telemann

measurement03

Example: Telemann



measurement03

Example: Telemann

measurement04

Example: Telemann

measurement04

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Special Moments and the coherence between musicians and listeners

Example: Ibert

strict

Work together with David Dolan, John Sloboda, and Björn Crütz
Trio Anima: Drew Balch, Matthew Featherstone and Anne Hodnett

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Special moments - when tingle sets in

Ibert

let-go

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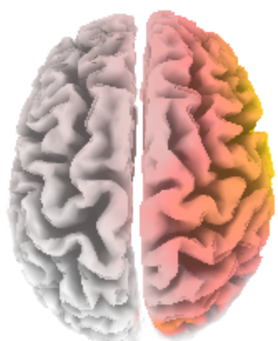
Music can set the pace of events

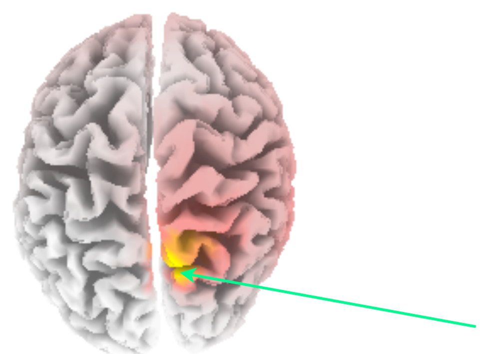
Special moment versus over all state

Averaged map of difference between musician's and listener's “brain state”.

Over all



During special moments



Musicians and listener become more similar during the special moments

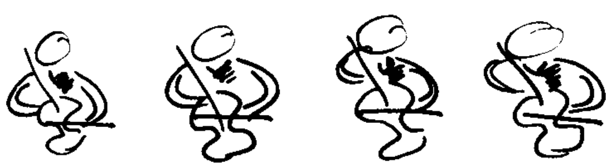
Synchronization occurs - the musicians set the rate of firing in the brains of the listeners

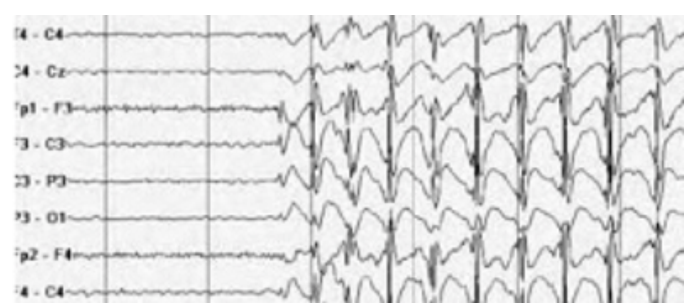
Henrik Jeldtoft Jensen

Summary

Synchronisation

- occurs frequently
- = macroscopic coherent time evolution
- many different forms: attractive phases, pulses
- various mathematical approaches
- can be good or can be bad





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Thank you