Designing Bus Bridging Services for Regular Egress

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Abstract. We are concerned with the regular egress problem after a major event at a known location. Without properly design complementary transport services, such sudden crowd build-ups will overwhelm the existing infrastructure. In this paper, we introduce a novel flow-rate based model to model the dynamic movement of passengers over the transportation flow network. Based on this basic model, an integer linear programming model is proposed to solve the bus bridging problem permanently. We validate our model against a real scenario in Singapore, where a newly constructed mega-stadium hosts various large events regularly. The results show that the proposed approach effectively enables regular egress, and achieves almost 24.1% travel time reduction with an addition of 40 buses serving 18.7% of the passengers.

Keywords: egress design, bus bridging service,crowd control

1 Introduction

In architectural and urban design community, there is a growing trend to design and build increasingly larger facilities that integrate diverse functions [15]. Examples of such facilities include stadiums, convention centers and airports. Operating such facilities with high volumes of human traffic is very challenging and needs to be carefully planned. Issues related to the operation of such facilities include, but not limited to, wayfinding inside the facility, regular egress, and emergency egress. In particular, to serve the transportation needs of crowds moving into and out of such facilities, an important consideration is to integrate mass transit to the facilities.

In this paper we focus on designing a bus bridging service to complement mass transit during regular egress in order to minimze total journey time of crowds. While regular egress is predictable in both crowd volume and timing (the planner should know exactly how many people will be leaving the facility, and at what time), and all utilities can be assumed to be in perfect working condition (which contrasts the case of emergency egress, where the timing is uncertain, and some utilities could be faulty), the planning problem is still challenging. The major challenge in regular egress is to avoid bottlenecks and crowd buildups, which is hard to avoid since mass transit is designed to satisfy regular transport demands

and not demand surges. A popular solution adopted by many planners is to complement mass transit with bus bridging services, yet despite the long history of using such services, optimizing its delivery has not received much attention; as a result, the design of bus bridging services for regular egress is usually ad hoc and static.

In the area of disruption management, however, there are rich literature on how to optimally utilize bus bridging services to make up for the lost link or capacity due to disruptions (for instance, in [4], [6], a two-step framework for bus bridging service planning is proposed). Despite the similarity between disruption management and regular egress, they are fundamentally different, in the following aspects. For disruption management, the priority is on restoring as much connectivity as possible, and as a result, the modeling effort has been mostly on maximizing the *amount of flow* that can pass through the point of disconnection. For regular egress, on the other hand, the focus is on experience management, which aims at minimizing total journey time including both travel and waiting time. To accurately account for the journey time, we have to modify the classical flow network so that both travel and waiting times can be quantified and thus minimized.

The objective of this paper is to formulate and study the design of bus bridging services for regular egress at crowded facilities. In doing so, we make the following three major contributions:

1. We formulate regular egress as a normalized flow network in which the total journey time can be easily calculated.
2. We model the introduction of bus bridging services as an increase to the link capacities in the above normalized flow network, and we create an integer programming model to derive the *optimal design* of the bus bridging service that would minimize the total journey time for all flows to reach destinations.
3. We demonstrate the practical usage of our model by solving instances inspired by a real-world scenario. The key parameters of this scenario are derived from a real-world public transport dataset in Singapore.

2 Literature Review

There are extensive works in the literatures discussing about the strategic planning for public transportation services. The general process consists of three major problems: network design, line planning and timetabling. On solving these three major problems, various approaches and techniques were proposed. Network design problems focus on altering the configuration of the transportation network for achieving a specific target. This problem was modelled in different aspects, such as elastic demand [7], directness of both transfer and routes [16]. Due to the high combinatorial complexity of the problem, approximation methods were proposed in two major streams: heuristics (e.g. [2],[17]) and meta-heuristics (e.g. [9],[11]). Line planning problems mainly discuss the design of routes and their relative frequencies over the public transportation network. In [14], Anita et al. proposed an integer programming approach and apply the

Dantzig-Wolfe decomposition method to get the solution. Year after, Ralf et al. presented the branch and price method to solve the problem in [1]. Later in 2012, A comprehensive survey of line planning problem was summarized in [13]. Timetabling problems refer to the generation of schedules for a set of vehicles or trains that under specific operational constraints. Various works in the literature solve this problem with different objective functions. In [8], Liebchen et al. proposed a periodic event-scheduling method to minimize passengers' waiting time at transfers by optimizing the Berlin subway timetable. In [5], with the objective of minimizing both user inconvenience and operational costs, Kaspi et al. solve this problem using a cross-entropy metaheuristic method.

Unlike the above works, whose focus were on the strategic planning under normal situations and improving the service quality during a long term period, our work put the emphasis on minimizing the negative effect caused by the large events through establishing the temporary bus links. Reasonable bus planning strategy in context of the normal cases might not be applicable under the large event cases as the passengers demand change dramatically during special hours. Moreover, regular bus service that is suitable for the long term period is unnecessary with respect to the special cases, since the impact results from the event only last for a few hours. Therefore, we seek strategic planning for the transportation services under special situations.

One of the special cases is the metro infrastructure disruption management problem. Contingency plans were investigated in case of disruption, which can be found in [3] and [10]. A survey did by Pender et al. on the various practices to manage the disruption in [12] , which indicated that bus bridging service is the most common way to minimize the negative impact of the disruption.

In [6], Konstantinos et al. proposed a methodological framework for planning the bus services. There were two key steps: bus routes planning on the network and shuttle bus assignment over the selected routes. The optimal bus routes were generated by using a shortest path algorithm and improved by a heuristic approach. Following this framework, Jin et al. in [4] formulated the problem by applying a different approach for generating candidate bus routes compared to [6]. Though the the two-step framework makes the problem tractable to some extent, separation of the two processes, namely candidate routes selection and resources assignment, may cause some inconsistencies. Whereas we optimize the planning problem in an integrated manner, which covers both processes in one optimization model and guarantees optimality for the two simultaneously.

## 3 Normalized Flow Model for Regular Egress at Facilities with Ultra High Demand

3.1 Background

Defined formally, our problem can be represented by a graph incorporating existing public transport service lines, where stations are denoted as nodes and connectivities denoted as directed links. An example can be seen in Figure 1a,

where there are three lines, each represented by a different line style. Stations along all lines are represented as hollow nodes in Figure 1a, the node with ultra-high demand is shaded as node s. Note that node s is not necessarily connected to existing stations, and visitors at node s might need to find their ways to the closest station. This might be feasible during normal circumstances, yet when the demand is beyond planned capacity, this sudden inflow of demands might overwhelm the service provided at the nearby stations.
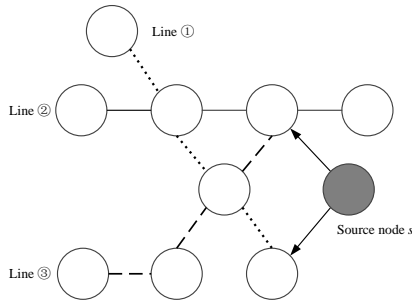
(a) An example of public transportation network with a node emitting ultra high demands.
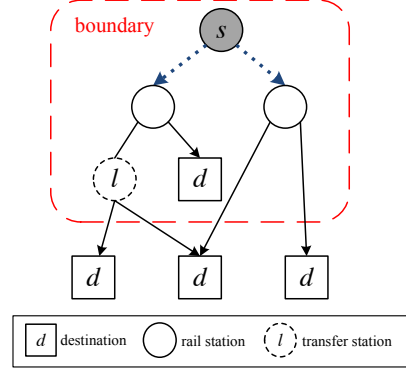
(b) An example of individual's trip over the transportation networks

Fig. 1: Problem description

3.2 Overview

The problem is defined on a graph $\hat{G} = (\hat{N}, \hat{E})$, where the set $\hat{N}$ represents all stations, and the set $\hat{E}$ represents directed links connecting stations. Every link is defined with a link-specific flow capacity, which will be defined next. In this context, the bus bridging service between two stations is essentially a way to *add capacity* to the graph: if the selected two stations are not already connected, a link with corresponding flow capacity will be created; if the selected two stations are already connected, its flow capacity will be increased accordingly. The planning horizon is discretized into T time units with equal intervals, where T is large enough for all travelers to reach their destinations even without any bridging service. The planned bus bridging service can be dynamic, which means that it can change over time. The total number of buses that can be deployed is bounded by B.

Let $s \in \hat{N}$ be the source node where surge demands originate. To focus only on the part of graph where the bridging service can reach within *reasonable amount of time*, we define $N \subseteq \hat{N}$ to be the set containing only nodes that can be reached from node s within X minutes (X is empirically set to be large

enough to contain all nodes we will ever consider). Similarly, we define $E \subseteq \hat{E}$ to contain all edges between nodes in N. The reduced graph $G = (N, E)$ will be our focus for the rest of the paper. To accurately estimate total journey time, for a passenger who travels to a destination node d not in N, a transfer node $l \in N$ that's closest to node d will be chosen as the transfer node, and the remaining travel time will be accounted for from l to d. In other words, the total journey time should contain two components: (1) from node s to transfer node l, and (2) from transfer node l to destination d. For travelers whose destination nodes are already in N, the transfer time will be set to 0. Figure 1b illustrates an example of individual's trip over the transportation networks.

3.3   Normalized Flow Network Model

In classical flow network models (such as the one introduced in [6]), the primary focus is on flows, and journey time cannot be derived from the model directly. To enable the quantification of journey time from flow networks, we introduce *time periods* to the model. To ensure that the model is still tractable after we introduce the time dimension, we make following assumptions. (a). The time period has equal length; (b). Train arrives with equal frequencies; (c). Travel time on each link is equivalent to the train frequency. With these assumptions, we can then simply recover journey time by summing up flows waiting at all nodes across all time periods. However, having uniform time periods implies that the travel time between any pair of nodes has to be set to the same (single time period) as well. To enable such normalization, for each edge we will calculate the *normalized flow rate* to replace capacity, which intuitively refers to the amount of flow that can pass through the edge within a single time period. (To understand how this works, assume that it takes 5 minutes for a train with the capacity of 100 to travel from a to b, the normalized flow rate for edge (a, b) is then 20 per minute.)

Formally speaking, we define $n_{u,t}^{l,d}$ to be the amount of flow waiting at node u in time t, with destination node being d and transfer node being l. Similarly, we define $x_{u,v,t}^{l,d}$ to be the flow going through the edge (u, v) in time t, with destination node being d and transfer node being l. The normalized capacity of the edge (u, v) in time t is defined as $c_{u,v,t}$. In other words, for time period t, at most $c_{u,v,t}$ units of flow can pass from u to v. The normalization procedure will be described in detail in Section 3.4.

3.4   Deriving Normalized Flow Capacity

As highlighted earlier, total journey time is composed of two major components: the time required to move from s to the transfer node l, which is denoted as $\delta_{s,l}$, and the time required to move from l to the final destination d, which is denoted as $\phi_{l,d}$. If $d \in N$, $\phi_{l,d}$ will be set to 0, otherwise it will be pre-computed. $\delta_{s,l}$, on the other hand, will be computed from the normalized flow network as follows:

$$\delta_{s,l} = \sum_{t,u,d,l,u \neq l} n_{u,t}^{l,d}. \tag{1}$$

The journey time can be computed as above since flow waiting at any nodes other than the transfer node will require one time period to move forward. Next we will explain how we can compute the normalized capacity.

t=0 $n_s = \epsilon$ $n_d = 0$
t=1 $n_s = \epsilon - \epsilon/\Delta_t$ $n_d = \epsilon/\Delta_t$
t=2 $n_s = \epsilon - 2\epsilon/\Delta_t$ $n_d = 2\epsilon/\Delta_t$

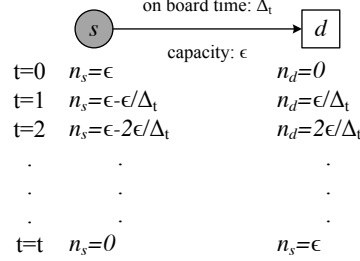t=t $n_s = 0$ $n_s = \epsilon$

Fig. 2: An example to illustrate the normalization procedure.

Figure 2 shows an example with two nodes explaining the rationality of the normalization procedure, where ϵ is the capacity between the start point s and destination d and Δ_t is the travel time from s to d before normalization.

The purpose of the normalization procedure is to normalize the capacity of the link to be the amount of flow that can pass through in one time unit. The normalized capacity is therefore ϵ/Δ_t. After normalization, at the time step $t = 1$, the total journey time, which consists of wait time and travel time, for the first ϵ/Δ_t unit of flow who successfully pass through the link is $(1 + 0) \times \epsilon/\Delta_t = \epsilon/\Delta_t$. Similarly, the total journey time for the second ϵ/Δ_t unit of flow is $(1 + 1) \times \epsilon/t = 2\epsilon/t$. Generally, the total journey time for the $i^{th}\epsilon/\Delta_t$ unit of flow is $(i - 1 + 1) \times \epsilon/t = i\epsilon/\Delta_t$. Thus the total journey time for all ϵ units of flow is $\sum_{i=1}^{t} i\epsilon/t = (1 + t)\epsilon/2$, which indicates the journey time over the link is $(1+\Delta_t)/2$. To correct the bias, we can adding a constant $\Delta_t - (1+\Delta_t)/2 = (\Delta_t - 1)/2$ to the final average journey time via normalization. As in an optimization model, adding constant to the objective function does not change the solution, therefore, we maintain the calculation of $\delta_{s,l}$ as formula 1.

Total travel time out of the boundary can be measured according to the number of passengers at transfer node l at the last time step $n_{l,T}^{l,d}$ and the estimated shortest travel time from l to d: $\phi_{l,d}$. Therefore, the total journey time that we seek to minimize will be:

$$\sum_{s,l} \delta_{s,l} + \sum_{l,d} n_{l,T}^{l,d} \cdot \phi_{l,d}. \tag{2}$$

Finally, the bus bridging service in our context can be thought of as either creating a new edge with capacity $\alpha_{u,v}$ or increasing the existing capacity $c_{u,v,t}$ by $\alpha_{u,v}$. $\alpha_{u,v}$ is the normalized capacity that corresponds to a particular bus service connecting u and v. The bridging can be time dependent, yet every assigned bus need to complete its current service before being re-assigned to

serve another route. Our goal is to come up with a bus bridging service that would minimize the above total journey time.

4 The Integer Linear Programming (ILP) Model for Dynamic Bus Bridging Service During Regular Egress

As highlighted in the previous section, the objective of introducing bus bridging service in our paper focuses on reducing total journey time, not making up for the lost capacity (as in the cases of disruption management). The major innovations we introduce in our mathematical model are: 1) normalization of link capacity to reflect uniform time period length, and 2) the separation of node delay and link delay. With these two modeling innovations, we can now formally introduce the integer linear programming model for optimizing dynamic bus bridging service during regular egress. In our model, the objective is to minimize the total journey time experienced by all travelers.

$$\mathbf{min} \sum_{s,l} \delta_{s,l} + \sum_{l,d} n_{l,T}^{l,d} \cdot \phi_{l,d}.$$

Let $q_{s,t}^d$ be the demand size that comes out of node s with destination d in time t. With such dynamic demand, (3) is to ensure the flow conservation for demand node s, which states that the flow at s in time t is constrained by the flow in time $(t-1)$ plus the difference between new demands and outgoing flow. Flow conservation for other nodes is described by (4). The next two constraints, (5) and (6), state that outgoing flow of a node u should not exceed the flow at node u as well as the capacity of the edge taken.

$$\sum_l n_{s,t}^{l,d} = \sum_l n_{s,t-1}^{l,d} + q_{s,t}^d - \sum_{l,u} x_{s,u,t-1}^{l,d} \quad \forall s,t,d, \tag{3}$$

$$n_{u,t+1}^{l,d} = n_{u,t}^{l,d} + \sum_w x_{w,u,t}^{l,d} - \sum_v x_{u,v,t}^{l,d} \quad \forall u,l,d,t, \tag{4}$$

$$\sum_v x_{u,v,t}^{l,d} \le n_{u,t}^{l,d} \quad \forall u,l,d,t, \tag{5}$$

$$\sum_v x_{u,v,t}^{l,d} \le c_{u,v,t} \quad \forall u,l,d,t. \tag{6}$$

The decision variable $a_{s,u,r}^{k,t}$ is set to 1 if bus k is assigned to link (s,u) in time t. This decision will add additional capacity of $\alpha_{s,u}$ units to edge (s,u), and is expressed in constraint (7).

$$c_{s,u,t} = c_{s,u,0} + \sum_{k,r} a_{s,u,r}^{k,t} \cdot \alpha_{s,u}, \quad \forall s,u,t. \tag{7}$$

In our formulation, each bus k is allowed to make one intermediate stop before reaching its destination. In other words, a bus route should contain leg 1 and leg

2, and is denoted as index r. The dependency between two legs of the same bus route is specified in constraint (8).

$$a_{s,u,1}^{k,t} \leq b_{s,u,1}^{k,t-1} + \sum_w a_{w,s,0}^{k,t-1} \quad \forall s, u, k, t. \tag{8}$$

Although our model allows the same bus to be assigned to different routes over time, it cannot be re-assigned unless it has completed the current assignment. This temporal relationship is ensured by both (9) and (10). $b_{s,u,r}^{k,t}$ is a derived decision variable that is set to 1 when bus k starts its current trip, and its value would increase monotonically by 1 at a time, until it ends its current service. After the service terminates, the value of $b_{s,u,r}^{k,t}$ will be reset to 0, and bus k can be utilized in other service route, as noted by (11).

$$b_{s,u,r}^{k,t} \leq \tau_{s,u} \cdot a_{s,u,r}^{k,t} \quad \forall s, u, k, t, r, \tag{9}$$

$$b_{s,u,r}^{k,t} = \begin{cases} 0 & \text{if } b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t} = \tau_{s,u}; \\ b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t} & \text{otherwise}; \end{cases} \quad \forall s, u, k, t, r, \tag{10}$$

$$\sum_w a_{u,w,0}^{k,t} = 1 \text{ if } a_{s,u,0}^{k,t-1} + b_{s,u,0}^{k,t-1} = 1 \quad \forall s, u, k, t. \tag{11}$$

Both (10) and (11) are nonlinear and have to be linearized. To linearize (10), we introduce two additional variables $y_{s,u,r}^{k,t}$ and $\lambda_{s,u,r}^{k,t}$. Let L and U be the lower and upper bounds of $b_{s,u,r}^{k,t}$, which equal 0 and $\tau_{s,u}-1$ respectively. The nonlinear constraint (10) can be re-written as:

$$\begin{cases} b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t} \geq L \cdot y_{s,u,r}^{k,t} + \tau_{s,u} \lambda_{s,u,r}^{k,t} \\ b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t} \leq \tau_{s,u} \lambda_{s,u,r}^{k,t} + U y_{s,u,r}^{k,t} \\ y_{s,u,r}^{k,t} + \lambda_{s,u,r}^{k,t} = 1 \\ U(1 - \lambda_{s,u,r}^{k,t}) \geq b_{s,u,r}^{k,t} \\ (1 - U)\lambda_{s,u,r}^{k,t} \leq b_{s,u,r}^{k,t} - (b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t}) \\ U(1 - y_{s,u,r}^{k,t}) \geq b_{s,u,r}^{k,t} - (b_{s,u,r}^{k,t-1} + a_{s,u,r}^{k,t}) \\ y_{s,u,r}^{k,t}, \lambda_{s,u,r}^{k,t} \in \{0, 1\} \end{cases} \tag{12}$$

(11) can be linearized similarly, and in the interest of space, we will skip it. Budget constraint (14) is to reflect the limited number of buses that are available. The amount of demand with destination d is represented by β_d and (13) is to make sure that in the last time period, all travelers must reach their respective final destinations (either their true destinations, or the transfer nodes leading to their real destinations that are outside of the boundary). Finally, the domains

of decision variables are listed as the last two constraints.

$$\sum_l n_{l,T}^{l,d} = \beta_d \quad \forall d, \tag{13}$$

$$\sum_{k,s,u,r} a_{s,u,r}^{k,t} \leq B \quad \forall t, \tag{14}$$

$$a_{s,u,r}^{k,t} \in \{0,1\} \quad \forall s, u, k, t, r, \tag{15}$$

$$b_{s,u,r}^{k,t} \in \{0, \tau_{s,u} - 1\} \quad \forall s, u, k, t, r. \tag{16}$$

5 Experiment

The effectiveness of our model is demonstrated by a real-world inspired scenario in Singapore, where a newly constructed multi-purpose national stadium is designed to host large events. In this section, we first estimate passengers' travel demands based on a real-world public transport dataset. We then perform computational experiments to measure the effectiveness of our ILP model. We solve the ILP model using CPLEX 12.5.

5.1 Dataset Description

The public transport dataset we obtained from our industry partners is called EZlink[1] dataset, which contains each passenger's boarding and alighting information (the boarding/alighting stations and times). It contains over one million card users' tap records from 1 *Nov 2011* to 31 *Jan 2012*. We use only records from work days for consistency.

Inferring Destinations For each card holder h, we maintain a list of candidate destinations and append the station s' to the list if: (1) s' is the first station that h registered as boarding in the morning (before $12 : 00$) of a day; or (2) s' is the last station that h has registered as alighting during a day after $16 : 00$. The intuition behind this filtering process is based on the assumption that majority of public transport users would depart from homes in the morning, and leave their workplaces in the afternoon. By aggregating records collected over three months, we can obtain the frequency of visited stations from the list.

Figure 3 plots an example of the candidate list extracted from 4 card holders. Card holders maintain a set candidate destination stations. It is observed that most of the card holders (1,2,3) in the figure have one dominant station s' whose frequency is much higher than the rest. This pattern is common when we process the dataset. We consider a station s' to be the home location for card holder h if its appearance frequency is over 80%.

If no dominant station can be detected in the list, we will try to cluster stations based on their distance. If the combined frequency of all stations belonging to the same cluster is high enough, we will consider the home station to

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[1] http://www.transitlink.com.sg/PSdetail.aspx?ty=catart&Id=1

(a) Card holder 1   (b) Card holder 2

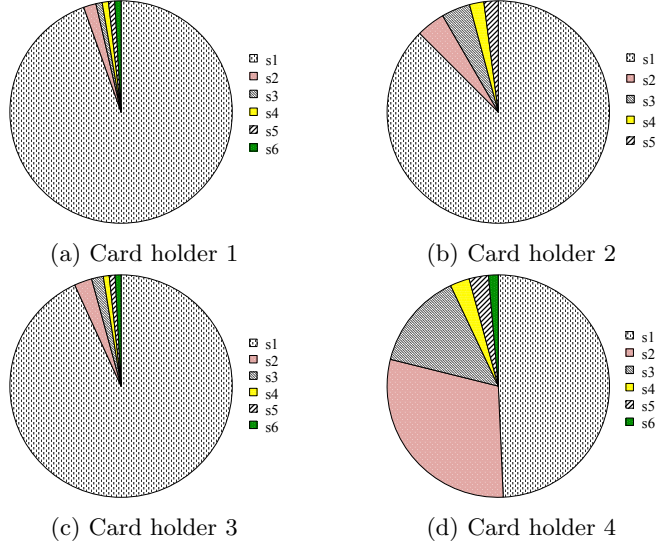(c) Card holder 3   (d) Card holder 4

Fig. 3: Candidate destination list

be within the cluster. For example, card holder 4 in Figure 3d maintains a list of 6 stations. Although no dominant station can be found, we can identify the cluster of $(s1, s2, s3)$ as they are less than 800 meters from each other. And the combined frequency for these 3 stations is 90.3%, well above the threshold. In this case, we conclude that all of them are close enough to card holder 4's real home and thus we use station $s1$ as the representative home station. If dominant home station cannot be found for a card hold after the above two checks, we will remove this card from consideration.

In total, we have extracted $|\mathcal{D}| = 22$ important destinations. The distribution of each station s and the travel times from the national stadium s_0 to $s \in \mathcal{D}$ are shown in table 1. We treat the set of card users that we identified as representative of the whole city.

Inferring Edge Capacity We obtain the aggregate number of passengers on rail way link (u, v) at time t from the EZLink dataset. The actual flow rate $c_{u,v,t}$ on each link is extracted according to train frequencies. The edge capacity $c_{u,v,t}$ in the model is represented by the additional flow rate, which is defined as the spare space available for passengers and can be inferred from the designed flow rate and actual flow rate .

$$c_{u,v,t} = dc_{u,v} - ac_{u,v,t}. \tag{17}$$

We assume that the capacity of each bus is 140. In our experiment, the capacity of bus is small compared to the capacity on the links. To reduce the solution space (and make the numerical experiments tractable), we assume that we will assign 4 buses at a time.

Table 1: Destination distributions.

station s	percentage	$\Delta t_{s_0,s}$	station s	percentage	$\Delta t_{s_0,s}$
Yishun	7.6%	34	Aljunied	4.0%	10
Sembawang	7.4%	38	Simei	3.9%	21
Admiralty	6.9%	41	Hougang	3.8%	20
Yew Tee	6.0%	44	Boon Lay	3.8%	40
Ang Mo Kio	5.4%	24	Tiong Bahru	3.8%	16
Khatib	4.9%	32	Sengkang	3.7%	24
Tampines	4.5%	23	Bukit Batok	3.7%	36
Lakeside	4.5%	37	Bukit Gombak	3.7%	38
Pioneer	4.3%	42	Woodlands	3.6%	44
Choa Chu Kang	4.0%	42	Clementi	3.4%	29
Serangoon	4.0%	16	Toa Payoh	3.3%	22

5.2 The Scenario

Let the stadium be the demand originator (s_0) with up to $30,000$ people. Time horizon is assumed to be $T = 12$ and is enough for all passengers to reach their respective destinations. Each time period refers to 6 minutes. In total, there are 19 nodes (stations) and 38 links within the boundary.

5.3 Effectiveness

We first discuss the effectiveness of our approach by comparing our ILP model to a rule-of-thumb assignment policy. After consulting industry experts on regular egress from a sports complex, they recommend a rule-of-thumb assignment policy to create a recurrent bus service line running between the sports facility and a major nearby station (called Cityhall station). Finally, we also assume that the egress for all 30,000 visitors would occur at the same time. Travel time reduction for both ILP model and rule-of-thumb approach is shown in Figure 4a. Travel time decreases along the y-axis for both approaches as the number of available buses increases along the x-axis. When the number of buses is set to be 0, it shows the very baseline indicating the situation of assigning no bus. On one hand, in terms of the average waiting time reduction, our ILP model improve the total journey by 15.7 minutes, i.e., a 24.1% reduction, compared to the no bus situation. On the other hand, with a fixed budget, the journey time reduction of our ILP model is larger than the ad hoc method, which indicates that the ILP model is more effective in planning the bus bridging services. For example, with 40 buses, the ILP model saves almost 4 minutes (8% reduction) for each passenger compared to the rule-of-thumb assignment. This result is not surprising as the rule-of-thumb assignment only considers the approximate demand and does not change its strategy over time as need.

Figure 4b depicts the number of passengers that arrives at their target transfer nodes over time horizon under 3 different situations. Arriving at the transfer

(a) Average journey time.
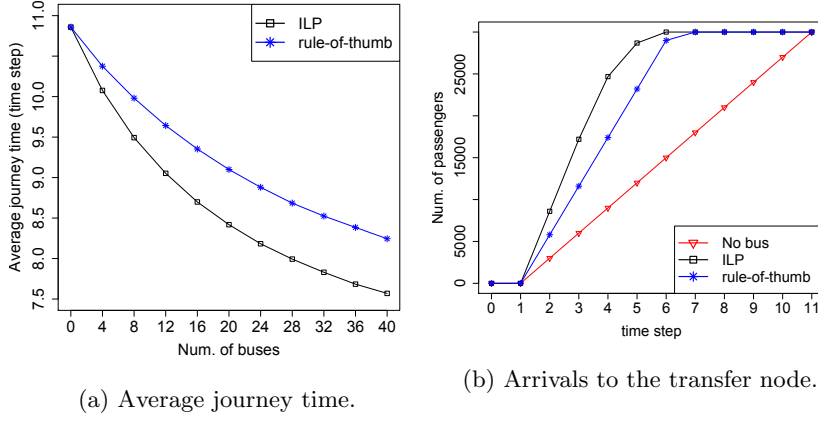

(b) Arrivals to the transfer node.

Fig. 4: Effectiveness.

node is the first step of the whole trip. When there is no bus services incorporated, passengers gradually approach the target transfer node with a much slower pace. Before $t = 7$, when all things being equal, the naive rule-of-thumb approach serves a increase of around 20% passengers when the planning horizon is going up. This is a stark contrast to our ILP model, in which almost 82% of passengers were sent to the transfer node in time $t = 4$.

One phenomenon observed from Figure 4b is that the ILP approach is not only effective in sending people to the final destinations but also efficient in sending passengers to the transfer node. However, the rate of sending passengers towards the transfer node after $t = 4$ decreases significantly. On the other hand, such rate for the rule-of-thumb approach keeps the same even after $t = 4$. This is because the rule-of-thumb strategy requests all of the buses serving on the edge from s_0 to the preselected station, Cityhall, hence passengers quickly diffuse to the transfer node. When $t = 6$, most of the passengers are sent to the transfer node and serving on the route $(s_0, \text{Cityhall})$ does not help any more. Rest of passengers who did not reach the transfer node would have to rely on regular train services and this accounts for the slower movement pace during this $t = 6$ and $t = 7$.

In our problem, besides the trip within the boundary, another significant factor that affect the total journey time is the trip from transfer nodes to destinations. In addition to the fact that the ILP model can disperse the crowd to the transfer nodes quickly, it also assign passengers to the optimal nodes l for making transfers. While for the rule-of-thumb approach, it fails to assign passengers to the optimal transfers which lead to the situation that people may take longer time to their final destinations.

5.4 Effect of Stop

In this section, we discuss the effect of having alighting stops in our bus bridging service. For simplicity, we assume that buses can only start their services from the demand node s_0 and operate in 2 different ways: 1) creating a *direct* link connecting two stations, and 2) creating a route with one alighting stop. We assume that all $30,000$ passengers exit the source node simultaneously at time $t = 0$.

We show the map illustrating the optimal bus routes taken by the above two services in Figure 5. In Figure 5a, buses start from the national stadium (label A with red circle) to a set of stations with label B. In figure 5b, buses start from the national stadium and have one alighting stop labeled as B and then at the same station, start their service towards the end of the service stop.

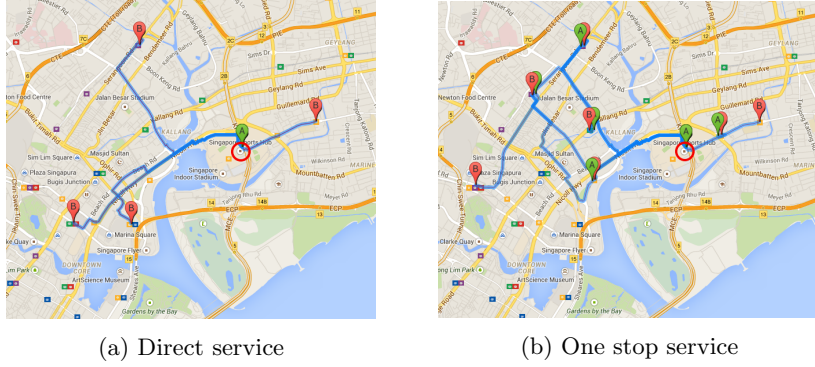

(a) Direct service   (b) One stop service
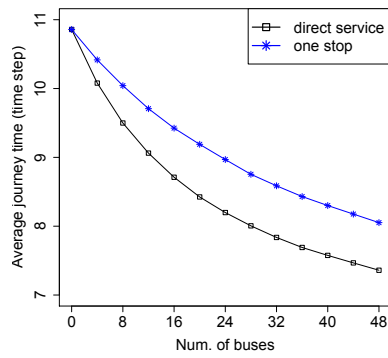
Fig. 5: Bus routes

Fig. 6: Direct service vs one stop service

Figure 6 plots the average journey time experienced by passengers when the number of employed buses varies along the x-axis. Intuitively, we observe that for both services, the average journey time reduces as the number of bus increases. Another observation from Figure 6 is that direct bus service is more effective compared to setting an alighting stop under our scenario.

In our ILP model, buses serve for two major roles: (1) facilitate the movement of crowds out of congested area near the demand node s_0 (which reduce the travel time within the boundary); (2) adjust passengers' trip and accommodate them to the proper transfer node l at lower-density area, which reduce the travel time beyond the boundary. Easing the congestion near the demand node is the key factor that affect passengers' total journey time. Adjusting the transfer node further improve passengers' travel experience. As the number of buses is too small to handle the demand (48 buses provides services for over 2% passengers), plenty of passengers are clogged near station s_0.

6 Conclusion

In this work, we presented a novel normalized flow network approach to model the regular egress of a large-scale facility. With such movement model, we propose an ILP-based approach to generate the optimal bus bridging services. The results from a real-world scenario show that our ILP formulation obtains 24.1% journey time reduction with only 40 buses providing services for 18.7% of the passengers. Even compared to the rule-of-thumb strategy, where authorities set a bus route based on experience, it is able to save 8% of the journey time for each passenger. Furthermore, we learn that setting an alighting node along the bus route is not a good choice when the number of available buses is not enough. The future research is to seek more efficient approaches to handle problem instances on a larger scale. In addition, we also look at extending the existing model to provide online planning strategies.

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