430.306: Signals and Systems

Electrical and Computer Engineering, Seoul National Univ.
Spring Semester, 2018
Homework #1

Problem 1) We can write $x[n]$ as $x[n] = r[n] + js[n]$, where $r[n]$ and $s[n]$ are real-valued signals. Then, by definition, the corresponding output signal $y[n]$ is $y[n] = r[n]$. Now suppose that $jx[n]$ is an input signal to the system. Then, since $jx[n] = -s[n] + jr[n]$, the output signal $\tilde{y}[n]$ should be $\tilde{y}[n] = -s[n]$ which is not $jy[n] = jr[n]$. Therefore, this system is not linear.
(The scaling factor could be a complex number.)

Problem 2) (a) $x_1(t)$ is not periodic because it is zero for $t < 0$.
(b) $x_2[n] = 1$ for all n except $n = 0$ ($x_2[0] = 2$). Therefore, $x_2[n]$ is not periodic.
(c) $x_3[n]$ is as shown in the Figure S1.6.

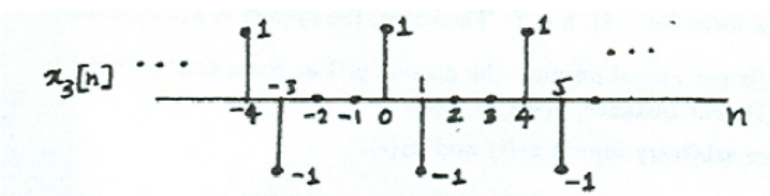


Figure S1.6

Therefore, it is periodic with a fundamental period of 4.

Problem 3) (a) The system is not memoryless because $y[n]$ depends on past values of $x[n]$.
(b) The output of the system will be $y[n] = A^2\delta[n]\delta[n-2] = 0$.
(c) From the result of part (b), we may conclude that the system output is always zero for inputs of the form $A\delta[n]$, $A \in \mathbb{C}$. Therefore, the system is not invertible.

Problem 4) (a) The system is not causal because the output $y(t)$ at some time may depend on future values of $x(t)$. For instance, $y(-\pi) = x(0)$.

(b) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$$x_1(t) \to y_1(t) = x_1(\sin(t))$$

$$x_2(t) \to y_2(t) = x_2(\sin(t))$$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t)$ is

$$\begin{aligned} y_3(t) &= x_3(\sin(t)) \\ &= ax_1(\sin(t)) + bx_2(\sin(t)) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, the system is linear.

Problem 5) (a) Consider two arbitrary inputs $x_1[n]$ and $x_2[n]$.

$$x_1[n] \rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \rightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is,

$$x_3[n] = ax_1[n] + bx_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output $y_3[n]$ is

$$\begin{aligned} y_3[n] &= \sum_{k=n-n_0}^{n+n_0} x_3[k] \\ &= \sum_{k=n-n_0}^{n+n_0} (ax_1[k] + bx_2[k]) \\ &= a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Therefore, the system is linear.

(b) Consider an arbitrary input $x_1[n]$. Let

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

be the corresponding output. Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n - n_1]$$

The output corresponding to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k - n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that

$$y_1[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k].$$

Therefore,

$$y_2[n] = y_1[n-n_1]$$

This implies that the system is time-invariant.

(c) If $|x[n]| < B$, then

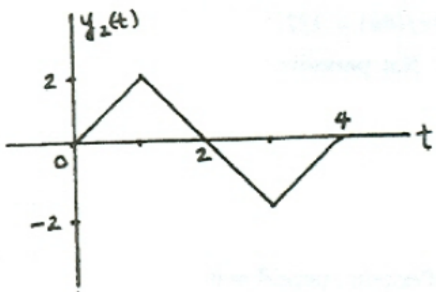
$$|y[n]| = |\sum_{k=n-n_0}^{n-n_0} x[k]| \leq \sum_{k=n-n_0}^{n-n_0} |x[k]| < (2n_0+1)B.$$

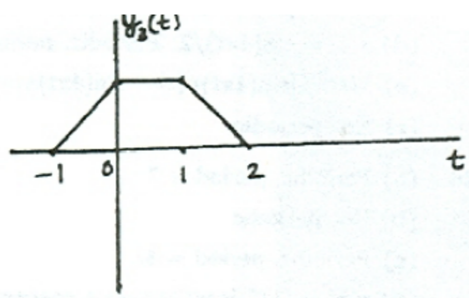
Therefore, $C = (2n_0+1)B$

Problem 6) (a) Periodic.
Note that $x[n+7] = x[n]$. You can check that its fundamental period is 7.
(b) Not periodic.
Suppose that $x[n+N] = x[n]$ for some $N \in \mathbb{N}$. Then $\cos(\frac{n+N}{8}) = \cos(\frac{n}{8})$ for all $n \in \mathbb{Z}$. Put $n = 0$. Then $\cos(\frac{N}{8}) = 1$, which means that $N = 16\pi k$ for some $k \in \mathbb{N}$. This is a contradiction.
(d) Periodic.
Note that $x[n+8] = \cos(4\pi + \frac{n\pi}{2})\cos(2\pi + \frac{n\pi}{4}) = x[n]$. You can easily check that its fundamental period is 8.

Problem 7) (a) Invertible. Inverse system: $y(t) = x(t+4)$.
(b) Non invertible. The signals $x(t)$ and $x_1(t) = x(t) + 2\pi$ give the same output.
(c) Non invertible. $\delta[n]$ and $2\delta[n]$ give the same output.
(d) Invertible. Inverse system: $y(t) = \frac{dx(t)}{dt}$.
(e) Invertible. Inverse system: $y[n] = x[n+1]$ for $n \geq 0$ and $y[n] = x[n]$ for $n < 0$.
(f) Non invertible. $x[n]$ and $-x[n]$ give the same result.
(g) Invertible. Inverse system: $y[n] = x[1-n]$.
(h) Invertible. Inverse system: $y(t) = x(t) + \frac{dx(t)}{dt}$.
(i) Invertible. Inverse system: $y[n] = x[n] - \frac{1}{2}x[n-1]$.
(j) Non invertible. If $x(t)$ is any constant, then $y(t) = 0$.
(k) Non invertible. $\delta[n]$ and $2\delta[n]$ result in $y[n] = 0$.
(l) Invertible. Inverse system: $y(t) = x(\frac{t}{2})$.
(m) Non invertible. $x_1[n] = \delta[n] + \delta[n-1]$ and $x_2[n] = \delta[n]$ give $y[n] = \delta[n]$.
(n) Invertible. Inverse system: $y[n] = x[2n]$.

Problem 8) (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Then since the system is LTI, we can acquire $y_2(t) = y_1(t) - y_1(t-2)$.
(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Then since the system is LTI, we can get $y_3(t) = y_1(t) + y_1(t+1)$.


$y_2(t)$
2
-2
0
2
4
t


$y_3(t)$
-1
0
1
2
t