Seoul National University
Dept. of Electrical and Computer Engineering

430.523: Random Signal Theory

Spring Semester, 2018
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Quiz1 (March 29, 75 minutes)

You can use ONLY class notes. Other than that no other things (electronic device, computer, book) is allowed. The purpose of this test is to check your basic knowledge on probability.

Following quantities might be helpful in your problem solving: $e^1 = 2.7$, $e^2 = 7.4$, $e^3 = 20$, $e^{-1} = 0.4$, $e^{0.5} = 1.6$, $e^{-0.5} = 0.6$

Problem 1)[20pt] Let Y follows $B(n, p)$. Show that $E\left(\frac{1}{Y+1}\right) = \frac{1-(1-p)^{n+1}}{(n+1)p}$.

Problem 2)[20pt] A discrete random variable X and Y are expressed as follows.

	Y=-1	Y=0	Y=1	Y=2
X = 0	0	0.15	0	0.35
X = 2	0.2	0	0.3	0

(a) Are X and Y independent?

(b) Find $E[7X - 3Y]$.

Problem 3)[20pt] In each week, you buy a lottery ticket from the store. It is known that the chance of winning the lottery is only 0.0015. In every week, you buy a ticket for 5 consecutive years (let's assume that there are 50 weeks in one year). You want to find out the probability that you win at least one time during this period.

(a) Find out the exact probability (you don't need to calculate the exact value. Just get the simplest expression).

(b) Use a proper approximation of the random variable and get the probability.

Problem 4)[20pt] Let X_i, $i = 1, 2, \cdots, 20$ be the Gaussian random variable with mean $\mu = 0$ and variance $\sigma^2 = 9$. Suppose $Z_1 = \min_i X_i$ and $Z_2 = \max_i X_i$.

(a) Find the $P(Z_1 \leq z)$.

(b) Find the $P(Z_2 \leq z)$.

Problem 5)[20pt] Each computer in a lab has a 20% chance to be infected with a virus. If a computer is infected, an antivirus software finds the virus with probability 0.9. If a computer is not infected, the software will still generate a false alarm and report a virus with probability 0.1. If the antivirus software reports a virus, what is the probability that indeed, the computer is infected?

Problem 6)[20pt] Find out the variance of Binomial random variable X with parameter n and p.