Seoul National University
School of Electrical and Computer Engineering

430.523: Random Signal Theory

Spring Semester, 2018
Instructor : Prof. Byonghyo Shim

Midterm Exam 1
April 17, 2018
75 minutes

This is closed book test. However, one A4 page cheating sheet is allowed.
Make sure to clearly show your work and full justification to get the full credit for the problem.
You have 75 minutes to finish the exam.

Please do not turn this page until requested to do so

Problem 1)[20pt] Let X_i ($i = 1, 2, \cdots, 10$) be the i.i.d. exponential random variables with the parameters λ_i ($i = 1, 2, \cdots, 10$), respectively. Also, let $Y = \min_i X_i$.

(a) What is the PDF of Y? Can you tell me what kind of random variable is Y?

(b) Find the probability that $Y = X_1$.

Problem 2)[20pt] Let X and Y be two jointly continuous random variables with the joint PDF as

$$f_{X,Y}(x, y) = \begin{cases} e^{-x} + \lambda y^2 & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{else} \end{cases}$$

(a) Find λ.

(b) What is $P(X \le \frac{Y}{2})$?

(c) Are X and Y independent? Justify your answer in details.

Problem 3)[20pt] Let X be a random variable. Suppose that the moment generating function $M_X(t)$ of X is $M_X(t) = \exp(e^t - 1)$. Show that

$$P(X \ge \alpha) \le \exp(-\alpha \ln(\alpha) + \alpha - 1), \ \alpha \ge 1.$$

Problem 4)[20pt] Let X and Y be discrete random variables. Show that

$$E[YE[X \mid Y]] = E[E[XY \mid Y]].$$

Problem 5)[20pt] Let $f_{X,Y}(x,y)$ be given by

	Y=0	Y=1
X = 0	$\frac{1}{4}$	$\frac{1}{4}$
X = 1	0	$\frac{1}{2}$

Find:

(a) $H(X), H(Y)$.

(b) $H(X,Y)$.

(c) $H(X \mid Y), H(Y \mid X)$.

(d) $I(X;Y)$.

Problem 6)[20pt] Let a_i and b_i $(i = 1, 2, \cdots, n)$ be positive numbers. Show that

$$\sum_{i=1}^{n} a_i \ln(\frac{a_i}{b_i}) \geq (\sum_{i=1}^{n} a_i) \ln \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i},$$