430.306: Signal and Systems

Electrical and Computer Engineering, Seoul National Univ.
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Quiz #1

Problem 1)[20pt] Answer the following questions.
(a)[10pt] Compute the convolution $y[n] = x[n] * h[n]$, where

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1] \quad \text{and} \quad h[n] = u[n-1].$$

(b)[10pt] Compute the convolution $y(t) = x(t) * h(t)$, where

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and} \quad h(t) = \delta(t+2) + 2\delta(t+1).$$

Solution. (a) $y[n] = \begin{cases} \frac{1}{2} & \text{if} \quad n \ge 0 \\ \frac{3^n}{2} & \text{if} \quad n < 0 \end{cases}.$

(b) Note that $y(t) = x(t+2) + 2x(t+1)$. Therefore

$$y(t) = \begin{cases} t+3 & \text{if} \quad -2 \le t < -1 \\ 4 & \text{if} \quad t = -1 \\ t+4 & \text{if} \quad -1 < t \le 0 \\ 2-2t & \text{if} \quad 0 < t \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

Problem 2)[20pt] Consider the causal LTI system described by the difference equation

$$y[n] - \frac{1}{5}y[n-1] = x[n].$$

(a)[10pt] Find out the impulse response $h[n]$ for this system.

(b)[10pt] Determine whether this system satisfies BIBO stability or not. You should justify your answer.

Solution. (a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$.

(b)Note that $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{5}{4}$. Since the impulse response function $h[n]$ is absolutely summable, this system satisfies the BIBO stability.

Problem 3)[20pt] Check whether each of the following statements is true or false. In order to get the full credit, you should justify your answer.

(a)[5pt] The system whose input-output relationship is given by $y(t) = \cos[x(t)]$ is invertible.

(b)[5pt] The system whose input-output relationship is given by $y[n] = \mathrm{Im}(x[n])$ is linear ($\mathrm{Im}(x[n])$ denotes the imaginary part of $x[n]$).

(c)[10pt] $y(t) = 4e^{j3t}$ could be the output signal for some LTI system corresponding to the input signal $x(t) = e^{j5t}$.

Solution. (a)False

$x_1(t) = 0$ and $x_2(t) = 2\pi$ yield the same output.

(b)False

Let $x[n] = j$ be an input signal. Then the corresponding output signal $y[n]$ is $y[n] = 1$. If this system is linear, the output for $jx[n] = -1$ should be $jy[n] = j$. However this is a contradiction. Therefore this system is not linear.

(c)False

Let $h(t)$ be an impulse response function for this system. If the system is LTI, the corresponding output $y(t)$ to the input signal $x(t) = e^{j5t}$ is given by

$$\begin{aligned} y(t) &= x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) e^{j5(t-\tau)} d\tau \\ &= \left[\int_{-\infty}^{\infty} h(\tau) e^{-j5\tau} d\tau \right] e^{j5t}. \end{aligned}$$

This means that if the system is LTI, the output should be the complex exponential of the same frequency. Therefore this system is not LTI.