430.523: Random Signal Theory

Electrical and Computer Engineering, Seoul National Univ.
Spring Semester, 2018
Homework #1, Due: In class @ March 29

Note: No late homework will be accepted.

Problem 1) Show that $P(\cup_i E_i) \leq \sum_i P(E_i)$

Problem 2) Show that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

Problem 3) A random variable X is called to have gamma distribution with parameters (α, λ), $\alpha > 0, \lambda > 0$, if its density function is given by

$$f_X(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}, \quad x \geq 0 \tag{1}$$

where $\Gamma(\alpha) = \int_0^\infty e^{-y} y^{\alpha-1} dy$.

Show that $Var[X] = \frac{\alpha}{\lambda^2}$

Problem 4) Show that the pdf of Gaussian RV X is valid pdf. You need to show that integration of $f_X(x)$ for all real line (i.e., $x \in (-\infty, \infty)$) should be 1.

Problem 5) Show that the variance of the Binomial random variable Z with parameter n, p (i.e., $B(n,p)$) is $Var(Z) = np(1-p)$

Problem 6) Let Y follows $B(n,p)$. Show that $E\left(\frac{1}{Y+1}\right) = \frac{1-(1-p)^{n+1}}{(n+1)p}$

Problem 7) Let T be the random variable that takes on all positive real t. Show that if $P(t_0 \leq T \leq t_0 + t_1 | T \geq t_0) = P(T \leq t_1)$ for all t_0 and t_1, then $P(T \leq t_1) = 1 - e^{-ct_1}$.

Problem 8) Suppose a jar contains $2N$ cards, two of them marked 1, two marked 2, and so on. Draw out m cards at random. What is the expected number of pairs that still remain in the jar?
Hint: this problem is posed and solved by D. Bernoulli, the great mathematician in 18th century. You may define a Bernoulli random variable X_i that takes on value 1 when i-th pair remains in the jar and 0 otherwise.

Problem 9) Find out the expected value of the Rayleigh random variable R whose density function is given by

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

Problem 10) Show that

$$\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$