430.523: Random Signal Theory

Electrical and Computer Engineering, Seoul National Univ.
Spring Semester, 2018
Homework #2, Due: In class @ April 11

Note: No late homework will be accepted.

Problem 1) X, Y, and Z are independent and uniformly distributed RVs defined over $(0, 10)$. Compute $P(X \geq YZ)$.

Problem 2) Prove a rectified linear unit (a.k.a. ReLu) has the transfer function $y = xu(x)$ where $u(x) = 1$ for $x \geq 0$. Suppose X is uniform over $(0, 2)$. Plot the CDF $F_Y(y)$.

Problem 3) Let X_1, X_2, and X_3 be i.i.d. continuous random variables with the CDF $F(x)$. Also, let $Y_i = F(X_i)$ be the random variables where $F(X_i)$ is the CDF of X_i. Find the joint distribution of Y_1, Y_2, and Y_3.

Problem 4) Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables. Also, let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the order statistic of X_i. Suppose that $Y_i = g(X_{(i)})$ where g is some monotonically increasing and invertible function.

(a) Find the distribution of Y_1

(b) Find the distribution of Y_n

(c) Find the joint distribution of Y_1 and Y_n

(d) Are Y_1 and Y_n independent?

Problem 5) Show that if X and Y are independent gamma RVs with parameter (α, λ) and (β, λ), respectively, then the $X + Y$ is also gamma RV with parameters $(\alpha + \beta, \lambda)$.

Problem 6) X and Y are independent gamma RVs with parameter (α, λ) and (β, λ), respectively.

Compute the joint density of $U = X + Y$ and $V = \frac{X}{X+Y}$.

Hint: U will be gamma distributed with parameters $(\alpha + \beta, \lambda)$ (see also previous problem) and V will be beta distributed with parameters (α, β) and they will be independent each other.

Problem 7) The joint pdf of RVs X and Y is defined as

$$f_{X,Y}(x, y) = \alpha e^{-5y}$$

where $0 < x < 2$ and $y > 0$.

(a) Find α

(b) Find the marginal pdfs of X and Y.

(c) What is the covariance of X and Y?

Problem 8) Show that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$