430.523: Random Signal Theory

Electrical and Computer Engineering, Seoul National Univ.
Spring Semester, 2018
Homework #3, Due: In class @ May 10

Note: No late homework will be accepted.

Problem 1) Find out the moment generating function (MGF) of the RV X when:

(a) X is the Gaussian RV $\mathcal{N}(\mu, \sigma^2)$.

(b) X is the uniform RV distributed in (0,1).

(c) X is the exponential RV with the parameter λ.

Problem 2) Show that the expected value of negative Binomial RV X with parameters l and p (l is the number of success and p is the success probability in a trial) is $\frac{l}{p}$.

(a) Show the answer by direct way.

(b) Show by using MGF.

Problem 3) Let X_1, X_2, and X_3 be the i.i.d. exponential RVs with parameter $\lambda = 2$. Suppose $Y = \max_i X_i$ and $Z = median\ X_i$ where *median* is the middle value (e.g., if $X_1 = 3$, $X_2 = 7$, and $X_3 = 1$, then *median* is 3).

(a) Find the CDF of Y by direct calculation.

(b) Find the CDF of Y and Z using order statistics approaches.

Problem 4) Fair dice is rolled. Let X and Y be the number of rolls to obtain 3 and 5. What is $E[X]$? Also, what is $E[X|Y = 1]$?

Problem 5) Let X and Y be i.i.d. geometric RVs with the parameter p. Also, let $Z = e^X + Y$. Find $E[Z|X]$.

Problem 6) Show the following equality

$$E[E[Z|X,Y]] = E[Z].$$

Assume X, Y, and Z are discrete RVs. The first expectation is w.r.t X, Y and the second expectation is w.r.t Z.

Problem 7) Suppose X and Y be the i.i.d. geometric RVs with parameter $p = 0.2$. Find $P(X < Y)$.

Problem 8) Let X_1, X_2, and X_3 be the i.i.d. geometric RVs with parameter p. Suppose $Y_k = \sum_{j=1}^{k} X_j$.

(a) Find the joint probability mass function of Y_1, Y_2, Y_3.

(b) Find the probability mass functions of Y_1, Y_2, and Y_3.

Problem 9) Suppose X and Y are normal random variables, both with mean 1 and variance 10. Suppose $\rho(X,Y) = 0.4$. Find the variance of $3X + 5Y$

Problem 10) Let $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$ be jointly normal distributed RVs. Suppose that X and Y are uncorrelated. Show that X and Y are independent.