Sparsity-Aware Ordered Successive Interference Cancellation for Massive Machine-Type Communications

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Abstract—In massive machine-type communication (mMTC) systems, by exploiting sporadic device activities, compressed sensing based multi-user detection (CS-MUD) is used to recover sparse multi-user vectors. In CS-MUD, multi-user vectors are detected based on a sparsity-aware maximum a posteriori probability (S-MAP) criterion. To reduce the computational complexity of S-MAP detection, a sparsity-aware successive interference cancellation (SA-SIC) technique can be used. However, SA-SIC does not perform well without proper layer sorting due to error propagation. In this paper, we propose a novel sparsity-aware ordered SIC scheme that finds the optimal detection order based on the activity probabilities and channel gains of devices. Simulation results verify that the proposed scheme greatly improves the performance of SA-SIC.

Index Terms—Massive machine-type communication, sporadic communication, compressed sensing, successive interference cancellation, multi-user detection.

I. INTRODUCTION

In recent years, massive machine-type communication (mMTC) has received much attention due to its wide range of applications such as smart metering, factory automation, autonomous driving, surveillance, and health monitoring [1]. As the term speaks for itself, mMTC concerns massive access by a large number of machine-type communication (MTC) devices. mMTC has many distinctive features over human-centric communications from a traffic point of view. For instance, MTC traffic is *uplink-dominated*, and devices are *sporadically active* only for a short period of time to transmit *short packets* with *low data rates* [2].

By exploiting the low activity of MTC devices, the multiuser detection (MUD) problem can be formulated as a compressed sensing based multi-user detection (CS-MUD) problem [3], [4]. Since inactive devices do not transmit information, a symbol vector consisting of information of both active and inactive devices can be readily modeled as a sparse vector.

Recently, several approaches to cast the CS-MUD problem into a sparsity-aware maximum *a posteriori* probability (S-MAP) detection problem have been suggested. In a nutshell, the main goal of S-MAP detection is to perform a MAP detection of sparse multi-user symbol vectors from a zero-augmented finite alphabet. In [5], linear relaxed S-MAP detectors have been suggested. In [6], sparsity-aware sphere decoding (SA-SD) has been proposed. In [7], as a variant of SA-SD, K-Best detector has been proposed.

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With an aim to reduce the computational complexity, sparsity-aware successive interference cancellation (SA-SIC) has been studied in [8]. SA-SIC is a simple low-complexity scheme that recovers transmit symbols in a sequential manner. The difference between SA-SIC and the conventional successive interference cancellation (SIC) is that SA-SIC incorporates a sparsity constraint into the detection process. Although SA-SIC can find a sparse solution with very low computational effort, error propagation during the successive detection severely degrades the performance of SA-SIC. In order to minimize such error propagation and thus enhance the performance, the detection order should be sorted properly prior to the SA-SIC operation. To this end, sorted QR decomposition (SQRD) and its variants ensuring that devices with a high channel gain are detected in the early layers have been proposed. It has been shown that SORD determines the optimal detection order with high probability in the recovery of non-sparse multi-user vectors [9].

However, when multi-user vectors are sparse and each device is active with a distinct probability, the detection order determined solely by channel gains might not be optimal. In practice, owing to their heterogeneous traffic demands, each MTC device is active with a distinct probability [10]. In this situation, the detection ordering should incorporate the (heterogeneous) activity probabilities of devices.

An aim of this paper is to propose a novel sparsity-aware ordered SIC scheme for the recovery of sparse multiuser vectors in mMTC systems. Specifically, we present an activity-aware sorted QR decomposition (A-SQRD) algorithm that finds the optimal detection order to reduce the error propagation and enhance the performance of SA-SIC. The proposed A-SQRD algorithm is distinct from existing methods in that the detection order is determined not only by the channel gains but also by the activity probabilities of devices. We show from numerical simulations that the proposed technique achieves a significant enhancement in the detection performance over conventional schemes without significantly increasing computational complexity.

II. SYSTEM MODEL

We consider the uplink of mMTC systems where N MTC devices access a single base station, as illustrated in Fig. 1. The symbol of each device is spread with a user-specific sequence with a length of M. Here, we assume that devices are synchronized in time, meaning that all devices switch activities and draw symbols in the same time slot. In this setup, the received signal at the base station can be described as

$$y = Hx + w, \tag{1}$$

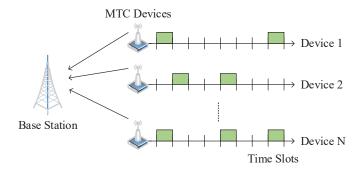


Fig. 1. Massive machine-type communication (mMTC) uplink scenario with N devices sporadically transmitting data symbols to a base station.

where **H** is a $M \times N$ complex-valued matrix capturing the spreading sequences and channel impulse responses between devices and the base station, **x** is the symbol vector of all (active and inactive) devices, and **w** is the complex Gaussian noise vector with the noise variance σ_w^2 . The symbol x_n is drawn from an equi-probable finite modulation alphabet \mathcal{A} when the n-th device is active, and zero otherwise. When M < N, the system is said to be under-determined. While it is in general not possible to recover the symbol vector in this scenario, theory of compressed sensing (CS) guarantees that **x** can be recovered accurately if **x** is a sparse vector.

III. Sparsity-Aware Successive Interference Cancellation (SA-SIC)

A. Derivation of S-MAP Detection

The output of the S-MAP detector maximizing the *a posteriori* probability $Pr(\mathbf{x}|\mathbf{y})$ is given by Bayes' rule as

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathcal{A}_0^N} \Pr(\mathbf{x}|\mathbf{y})
= \arg \min_{\mathbf{x} \in \mathcal{A}_0^N} - \ln \Pr(\mathbf{y}|\mathbf{x}) - \ln \Pr(\mathbf{x}),$$
(2)

where $A_0 = A \cup \{0\}$ is the augmented modulation alphabet.

When the n-th device is active with the activity probability p_n and further p_n is very small for all n, \mathbf{x} can be modeled as a sparse vector containing many zeros. Considering that the activity of each device is independent of each other, the prior distribution of \mathbf{x} can then be described as

$$\Pr(\mathbf{x}) = \prod_{n=1}^{N} \Pr(x_n) = \prod_{n=1}^{N} (1 - p_n)^{1 - |x_n|_0} (p_n/|\mathcal{A}|)^{|x_n|_0},$$
(3)

where $|x_n|_0$ is the element-wise l_0 -norm that is equal to 1 if x_n is a non-zero value, otherwise it is zero.

From (2) and (3), we have

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|_0, \qquad (4)$$

where $\lambda_n = \ln[(1 - p_n)/(p_n/|\mathcal{A}|)]$ is an element-wise regularization parameter.

The goal of the S-MAP detection is to find a vector in A_0^N that maximizes the cost function in (4). The optimization problem in (4) is in essence a regularized least squares

minimization problem. The regularization term accounts for the heterogeneous activity probabilities of N MTC devices. Since λ_n is inversely proportional to p_n , the regularization term promotes the sparsity of \mathbf{x} .

B. Sparsity-Aware SIC (SA-SIC) Detection

In this section, we discuss the SA-SIC technique to solve the problem (4). SA-SIC uses the QR decomposition of matrix $\mathbf{H} = \mathbf{Q}\mathbf{R}$, where \mathbf{Q} is an $M \times N$ unitary matrix and \mathbf{R} is a $N \times N$ upper triangular matrix. Using QR decomposition, we have

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|_0$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y} - (\mathbf{Q}\mathbf{R})\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|_0$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{Q}^H \mathbf{y} - \mathbf{Q}^H (\mathbf{Q}\mathbf{R})\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|_0$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\tilde{\mathbf{y}} - \mathbf{R}\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|_0$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \sum_{n=1}^N \left[\left| \tilde{y}_n - \sum_{l=n}^N R_{nl} x_l \right|^2 + \sigma_w^2 \lambda_n |x_n|_0 \right], \quad (5)$$

where $\tilde{\mathbf{y}} = \mathbf{Q}^H \mathbf{y}$. Once the estimates of previous symbols $\{\hat{x}_l\}_{l=n+1}^N$ are available, the optimal \hat{x}_n can be obtained by minimizing the n-th per-symbol cost function $d_n(x_n)$ over one scalar variable $x_n \in \mathcal{A}_0$ as

$$\hat{x}_n = \arg\min_{x_n \in \mathcal{A}_0} \left[\left| \tilde{y}_n - \sum_{l=n+1}^N R_{nl} \hat{x}_l - R_{nn} x_n \right|^2 + \sigma_w^2 \lambda_n |x_n|_0 \right]$$

$$\triangleq d_n(x_n)$$
(6)

The detection process begins with the highest layer (n = N) and goes down to the lowest layer sequentially, $n = N - 1, \dots, 1$. In doing so, SA-SIC achieves an acceptable detection performance with much lower complexity compared to other optimal but complex S-MAP detectors. However, the main drawback of SA-SIC is that it is sensitive to the error propagation from the early layers. Hence, the selection of an appropriate detection order is crucial to mitigate the error propagation.

IV. PROPOSED ACTIVITY-AWARE SORTED-QRD (A-SQRD) ALGORITHM

In this section, we describe the proposed activity-aware sorted QR decomposition (A-SQRD) algorithm to enhance the detection performance of SA-SIC. The conventional SQRD algorithm sorts the columns of channel matrix \mathbf{H} and find its sorted QR decomposition based on channel gains. Since the cost function of the S-MAP detection contains an additional sparsity-promoting regularization term, the regularization parameters λ_n as well as the noise variance σ_w^2 and channel matrix \mathbf{H} needs to be considered in the detection ordering.

To this end, we construct an augmented system and then sort the columns of the augmented system matrix such that devices with high post-detection signal-to-noise-ratio (SNR) are detected first.

In this work, we restrict ourselves to the constant modulus alphabet (i.e., $\|\mathbf{x}\|_0 = \|\mathbf{x}\|_2^2 = \|\mathbf{x}\|_p^p$, $p \ge 1$). Noting that the data rate of MTC devices is in general low (typically in orders of tens of kilobits per seconds), it is reasonable to assume that data symbols are modulated with constant modulus alphabets (e.g., phase shift keying constellations). Replacing the l_0 -norm with the l_2 -norm in (4), we have

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \sigma_w^2 \sum_{n=1}^N \lambda_n |x_n|^2$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \|\sigma_w \operatorname{diag}(\sqrt{\lambda})\mathbf{x}\|_2^2$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\begin{bmatrix} \mathbf{y} \\ \mathbf{0}_N \end{bmatrix} - \begin{bmatrix} \mathbf{H} \\ \sigma_w \operatorname{diag}(\sqrt{\lambda}) \end{bmatrix} \mathbf{x}\|_2^2$$

$$= \arg\min_{\mathbf{x} \in \mathcal{A}_0^N} \|\mathbf{y}_0 - \mathbf{H}'\mathbf{x}\|_2^2, \tag{7}$$

where $\mathbf{y}_0 \in \mathbb{C}^{M+N}$ is the zero-augmented observations, $\mathbf{H}' \in \mathbb{C}^{(M+N)\times N}$ is an augmented system matrix and $\boldsymbol{\lambda}$ is the regularization parameters of N devices.

The objective of the proposed A-SQRD algorithm is to find the optimal permutation of columns of \mathbf{H}' and the corresponding QR decomposition that maximizes the detection SNRs in the early layers. Here, we find the QR decomposition of the augmented channel matrix \mathbf{H}' as $\mathbf{H}'\mathbf{P} = \mathbf{Q}\mathbf{R}$, where \mathbf{P} is the binary permutation matrix. To find the optimal permutation, we employ the modified Gram-Schmidt algorithm [11] and extend it to reorder the columns of \mathbf{H}' before each orthogonalization step.

Detailed steps of the proposed algorithm are as follows: First, we set $\mathbf{Q} = \mathbf{H}'$ and $\mathbf{R} = \mathbf{0}_{N \times N}$, respectively. In the next step, for each n-th iteration, the column index with the smallest l_2 -norm of \mathbf{q}_j $(j=n+1,\cdots,N)$ is determined. Then the n-th column of each of \mathbf{Q} , \mathbf{R} and \mathbf{P} is exchanged with the one corresponding to the smallest norm. After the columns are exchanged, R_{nn} is set to $\|\mathbf{q}_n\|$ and \mathbf{q}_n is normalized. In this way, the diagonal elements of \mathbf{R} are ordered such that $R_{ii} < R_{nn}$ for i < n, and thus the post-detection SNR is higher in the early layers.

After the columns are sorted, we perform the orthogonalization of the columns such that $\mathbf{q}_{n+1}, \dots, \mathbf{q}_N$ are orthogonalized with respect to \mathbf{q}_n (i.e., $\mathbf{q}_j \perp \mathbf{q}_n$, $j=n+1, \dots, N$). R_{nj} is computed as $R_{nj} = \mathbf{q}_n^H \mathbf{q}_j$ and \mathbf{q}_j is calculated as

$$\mathbf{q}_i = \mathbf{q}_i - R_{ni}\mathbf{q}_n. \tag{8}$$

Using the sorted QR decomposition $\mathbf{H}'\mathbf{P} = \mathbf{Q}\mathbf{R}$ found by the A-SQRD algorithm, (7) can be rewritten as

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathcal{A}_0^N} \left\| \mathbf{y}_0 - \mathbf{Q} \mathbf{R} \mathbf{P}^H \mathbf{x} \right\|_2^2$$

$$= \arg \min_{\mathbf{x} \in \mathcal{A}_0^N} \left\| \tilde{\mathbf{y}}_0 - \mathbf{R} \mathbf{P}^H \mathbf{x} \right\|_2^2, \tag{9}$$

where $\tilde{\mathbf{y}}_0 = \mathbf{Q}^H \mathbf{y}_0$. Similarly to (5), we can decompose (9) into the sum of N per-symbol cost functions. Then, for n = 0

Algorithm 1 SA-SIC with Activity-Aware SQRD

Input:
$$\mathbf{y}, \mathbf{H}, \mathcal{A}_0, \sigma_w^2, \{p_n\}_{n=1}^N$$

Output: $\hat{\mathbf{x}}$
 $%$ Initialization

1: $\lambda(n) \leftarrow \ln[(1-p_n)/(p_n/|\mathcal{A}|)]$

2: $\mathbf{y}_0 \leftarrow [\mathbf{y}; \operatorname{zeros}(N,1)], \mathbf{Q} \leftarrow [\mathbf{H}; \sigma_w \operatorname{diag}(\sqrt{\lambda})], \mathbf{R} \leftarrow \mathbf{0}_{N \times N}, \mathbf{P} \leftarrow \mathbf{I}_N$

3: for $n=1, \cdots, N$ do

 $%$ Column Sorting

4: $n_{\min} \leftarrow \arg \min_{j=n,\cdots,N} \|\mathbf{q}_j\|^2$

5: exchange columns n and n_{\min} in \mathbf{Q}, \mathbf{R} , and \mathbf{P}

6: $R_{nn} \leftarrow \|\mathbf{q}_n\|$

7: $\mathbf{q}_n \leftarrow \mathbf{q}_n/R_{nn}$
 $%$ Column Orthogonalization

8: for $j=n+1,\cdots,N$ do

9: $R_{nj} \leftarrow \mathbf{q}_n^H \mathbf{q}_j$

10: $\mathbf{q}_j \leftarrow \mathbf{q}_j - R_{nj} \mathbf{q}_n$

11: end for

12: end for

13: $\tilde{\mathbf{y}}_0 \leftarrow \mathbf{Q}^H \mathbf{y}_0$

14: for $n=N,\cdots,1$ do

15: $x_n^{\mathrm{ls}} \leftarrow (\tilde{y}_{0,n} - \sum_{l=n+1}^N R_{nl} \hat{x}_l)/R_{nn}$

16: $\hat{x}_n \leftarrow Q_{\mathcal{A}_0}(x_n^{\mathrm{ls}})$

17: end for

 $N, \dots, 1$, the optimal solution of \hat{x}_n can be found in closed form as

18: $\hat{\mathbf{x}} \leftarrow \hat{\mathbf{x}} \mathbf{P}^H$

$$\hat{x}_n = Q_{\mathcal{A}_0}(x_n^{\text{ls}}),\tag{10}$$

where $x_n^{\mathrm{ls}} = (\tilde{y}_{0,n} - \sum_{l=n+1}^N R_{nl} \hat{x}_l)/R_{nn}$ is the least squares solution to the n-th per-symbol cost, and $Q_{\mathcal{A}_0}(\cdot)$ is the quantization operator that maps the input to the closest point in \mathcal{A}_0 . Finally, by right-multiplying \mathbf{P}^H by $\hat{\mathbf{x}}$ (de-sorting), the final solution is obtained. The proposed algorithm is summarized in Algorithm 1.

V. COMPLEXITY ANALYSIS

The computational complexity of the A-SQRD and SQRD algorithm is analyzed below by counting each required numerical operation as one complex floating point operation.

$$C_{\text{A-SQRD}} = 2N^3 + (2M+2)N^2 + (M-1)N, \quad (11)$$

$$C_{\text{SQRD}} = (2M+1)N^2 + (M-1)N.$$
 (12)

Since the A-SQRD algorithm performs the QR decomposition of the augmented channel matrix, it causes a slight computational order increase compared to the conventional SQRD algorithm.

However, the computational burden of A-SQRD algorithm is less than that of K-Best detector and SA-SD. The K-Best detector performs a breadth-first search to find the best K paths minimizing the sum of per-symbol cost functions. The computational complexity of K-Best detector is

$$C_{K-Best} = K|\mathcal{A}_0|(\frac{N^3}{3} + 2N^2 + \frac{5}{3}N + \log^2(K|\mathcal{A}_0|)),$$
 (13)

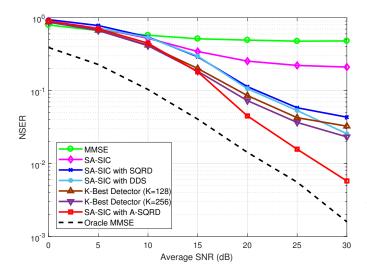


Fig. 2. NSER performance as a function of average SNR.

which scales with K. Increasing K will improve the performance of K-Best detector so that it performs close to the SA-SD. However, it causes an infeasible complexity compared to the A-SQRD algorithm.

VI. NUMERICAL RESULTS

In this section, we describe the numerical experiment that demonstrates the effectiveness of the proposed algorithm. We compare the proposed A-SQRD algorithm with the linear minimum mean squared error (LMMSE) estimator¹, unsorted SA-SIC, conventional SQRD algorithm, data-dependent sorting and regularization (DDS) algorithm, and K-Best detector. DDS algorithm is a heuristic approach that rescales λ_n based on the correlation between \mathbf{y} and \mathbf{H} to enhance the SQRD algorithm [8]. As a lower bound of the detection algorithms, we also test the Oracle LMMSE detector. Note that the Oracle detector has the support² (index set of nonzero entries) information of \mathbf{x} and thus solves an over-determined system.

A. Simulation Setup

We simulate an under-determined mMTC system with N=128 MTC devices and unit-norm random sequences with a length of M=64 for spreading. We consider non-dispersive independent Rayleigh fading channels between devices and a base station from a complex Gaussian distribution of $\mathcal{CN}(0,1)$. Thus, the average SNR is set to $1/\sigma_w^2$. We assume that the base station has perfect knowledge of the matrix H. Data symbols of active devices are modulated with quadrature phase shift keying (QPSK). We consider the net symbol error rate (NSER) as a detection performance measure. NSER refers to the symbol error rate of active devices.

B. Simulation Results

Fig. 2 shows the NSER performance for each algorithm as a function of the average SNR. The activity probabilities

TABLE I COMPARISON OF COMPUTATIONAL COMPLEXITY $(M=64,N=128\ {\rm AND}\ {\rm QPSK}\ {\rm MODULATION})$

SQRD	DDS	A-SQRD	K-Best ($K = 128$)
2.12×10^{6}	2.14×10^{6}	6.33×10^{6}	4.68×10^{8}

 $\{p_n\}_{n=1}^N$ are drawn uniformly at random in [0.1, 0.3]. We observe that SA-SIC with the proposed A-SQRD algorithm outperforms the conventional algorithms. LMMSE exhibits poor performance since the system is under-determined. Due to error propagation, the unsorted SA-SIC does not perform well. Since the SQRD and DDS algorithm do not consider the heterogeneous activity probabilities, the performance gains over the unsorted SA-SIC are marginal. In contrast, the A-SQRD algorithm is effective since it considers both the activity probabilities and channel gains to find the best detection order. The K-Best detector performs worse than the proposed algorithm with K=128 and 256. In Table I, we provide the number of required numerical operations of detection algorithms. When compared to the A-SQRD algorithm, the K-Best detector requires higher computational complexity to achieve the performance comparable to the A-SQRD algorithm.

VII. CONCLUSION

In this letter, we have proposed a novel sparsity-aware ordered SIC scheme for mMTC systems. Based on activity probabilities and channel gains of devices, the proposed scheme constructs an augmented system and finds the optimal detection order. We have demonstrated that our proposed algorithm greatly improves the performance of SA-SIC.

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¹We use the quantized version of the output of the LMMSE estimator. ²For example, if $\mathbf{x} = [-1\ 0\ 1\ 0\ 0\ 0]^T$, then the support T is $T = \{1, 3\}$.