Analysis of different satellite constellations

GNSS, WS 2025/26

Orbits

Overview

- Data
  - Satellite coordinates ECEF
  - Satellite coordinates ECSF
- Wanted
  - Visualization of satellite coordinates (ECEF and ECSF) → all epochs
  - Groundtrack plot from at least 2 satellites → all epochs

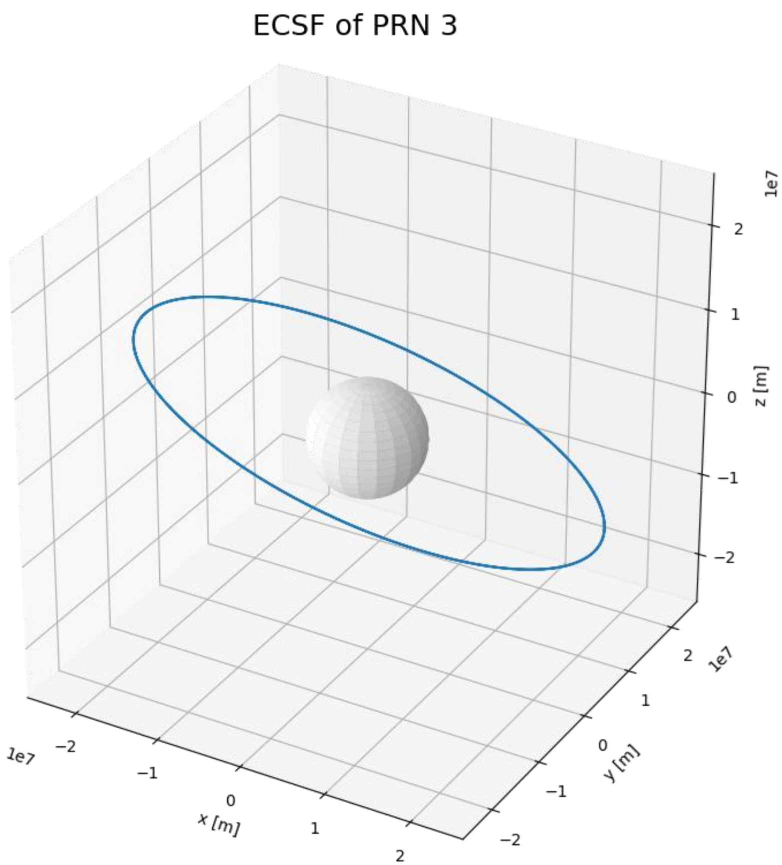
Input data

- Data structure (pos_ECSF.txt and pos_ECEF.txt)

```
# UEBUNG GNSS
#
# Positionsdaten von Satelliten
#
# Satellit   Zeit     Position X Y Z
GPS 30 2023-06-29 00:00:00 5.953525788999999641e+05 2.597461778000000119e+07 4.894085983000000007e+06
GPS 30 2023-06-29 00:01:00 5.688540505000000121e+05 2.600795346000000089e+07 4.710838166000000201e+06
GPS 30 2023-06-29 00:02:00 5.426082839000000386e+05 2.603999803999999911e+07 4.527224515999999829e+06
GPS 30 2023-06-29 00:03:00 5.166054929000000120e+05 2.607074721999999881e+07 4.343259250000000000e+06
GPS 30 2023-06-29 00:04:00 4.908358411000000197e+05 2.610019687999999896e+07 4.158956615000000224e+06
GPS 30 2023-06-29 00:05:00 4.652894432999999844e+05 2.612834305000000075e+07 3.974330887999999803e+06
GPS 30 2023-06-29 00:06:00 4.399563679000000120e+05 2.615518192999999970e+07 3.789396373999999836e+06
GPS 30 2023-06-29 00:07:00 4.148266386000000057e+05 2.618070987999999896e+07 3.604167407999999821e+06
GPS 30 2023-06-29 00:08:00 3.898902368000000133e+05 2.620492342000000179e+07 3.418658350000000093e+06
GPS 30 2023-06-29 00:09:00 3.651371034000000218e+05 2.622781926999999955e+07 3.232883584999999963e+06
GPS 30 2023-06-29 00:10:00 3.405571406999999890e+05 2.624939426999999955e+07 3.046857524000000209e+06
GPS 30 2023-06-29 00:11:00 3.161402147000000114e+05 2.626964546000000089e+07 2.860594600000000093e+06
GPS 30 2023-06-29 00:12:00 2.918761573000000208e+05 2.628857005000000075e+07 2.674109268000000156e+06
GPS 30 2023-06-29 00:13:00 2.677547677000000258e+05 2.630616539000000060e+07 2.487416004999999888e+06
GPS 30 2023-06-29 00:14:00 2.437658152999999875e+05 2.632242901999999955e+07 2.300529305000000168e+06
GPS 30 2023-06-29 00:15:00 2.198990412000000069e+05 2.633735867000000179e+07 2.113463683000000194e+06
GPS 30 2023-06-29 00:16:00 1.961441602999999886e+05 2.635095219000000134e+07 1.926233671000000089e+06
GPS 30 2023-06-29 00:17:00 1.724908638999999966e+05 2.636320764000000060e+07 1.738853816000000108e+06
```

Plot satellite orbits

- read data
- Python *matplotlib.pyplot*
  - Plot earth
  - *plot_surface(x,y,z)*
  - Sphere/ellipsoid

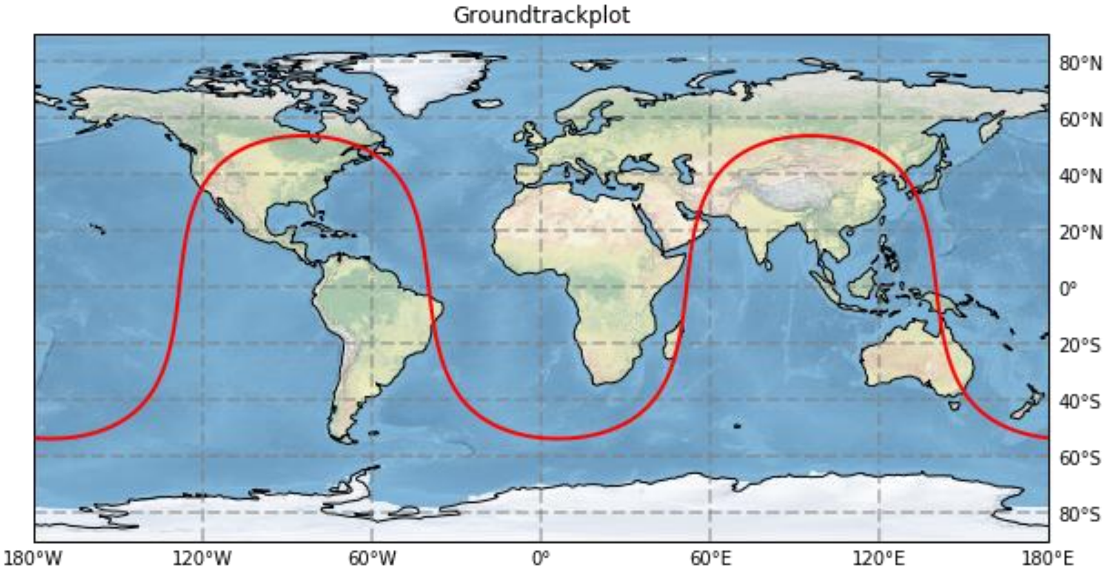


Plot coastlines and countries

- *import cartopy.crs*
- *import matplotlib.pyplot*

- Coordinate transformation

- Plot data PRN
- Plot φ, λ



DOP

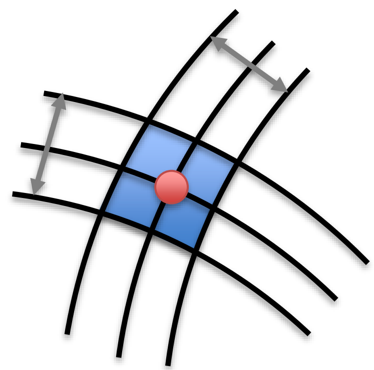
Overview

- Data
  - Satellite coordinates ECEF (t=900 to 1200 min)
  - Satellite coordinates ECSF (t=900 to 1200 min)

- Wanted
  - DOP (P-, H-, VDOP)values within t=900 to t=1200min
    - Compare number of visible satellites
    - Vary mask angle and visible satellites
    - For your location and a place in Norway

Dilution of precision
Concept

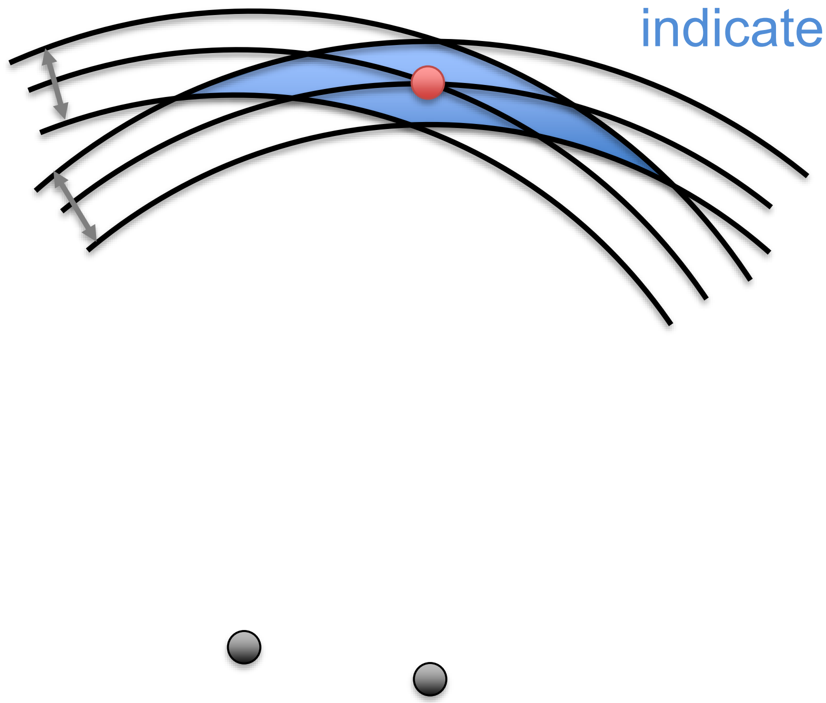
Variation in range ring due to range errors



Blue region: locations using data from within indicated error bounds

Dilution of precision
Concept

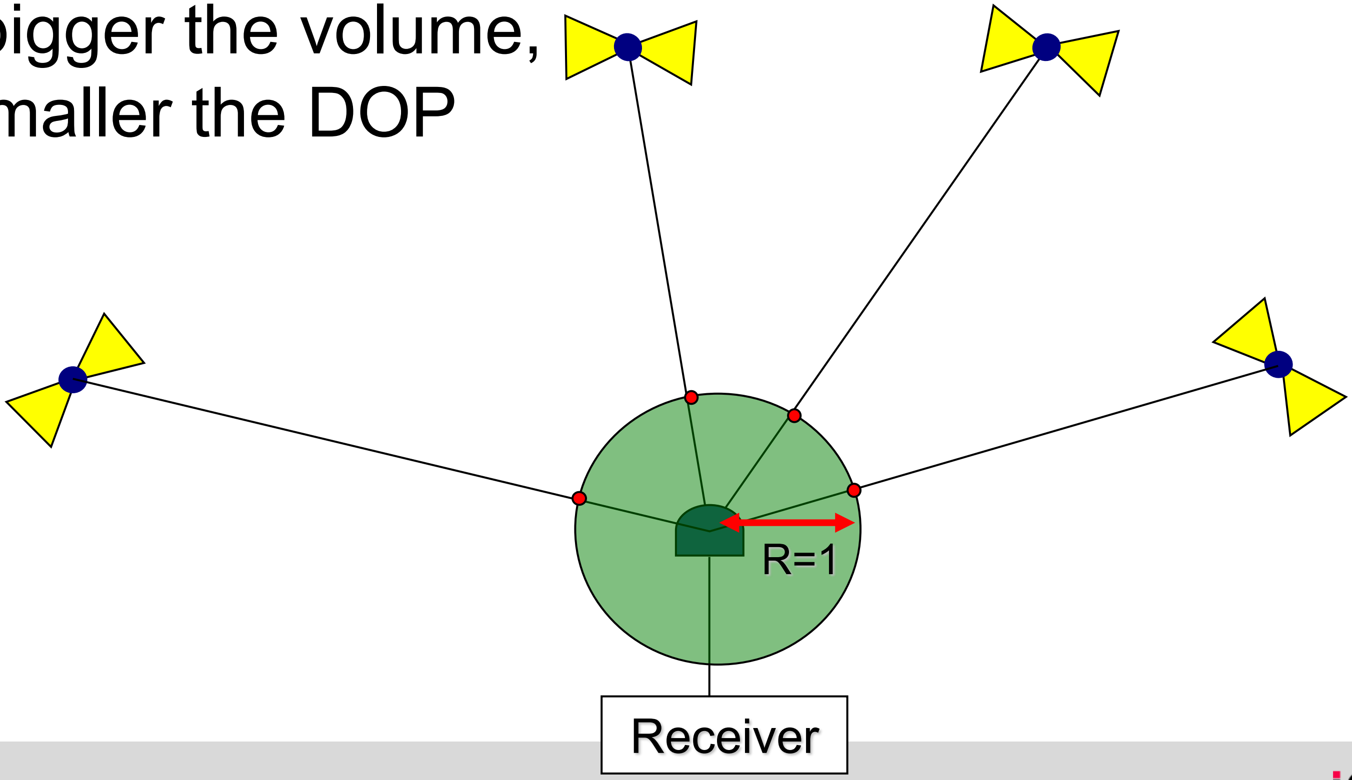
Variation in range ring due to range errors



Blue region: locations using data from within indicated error bounds

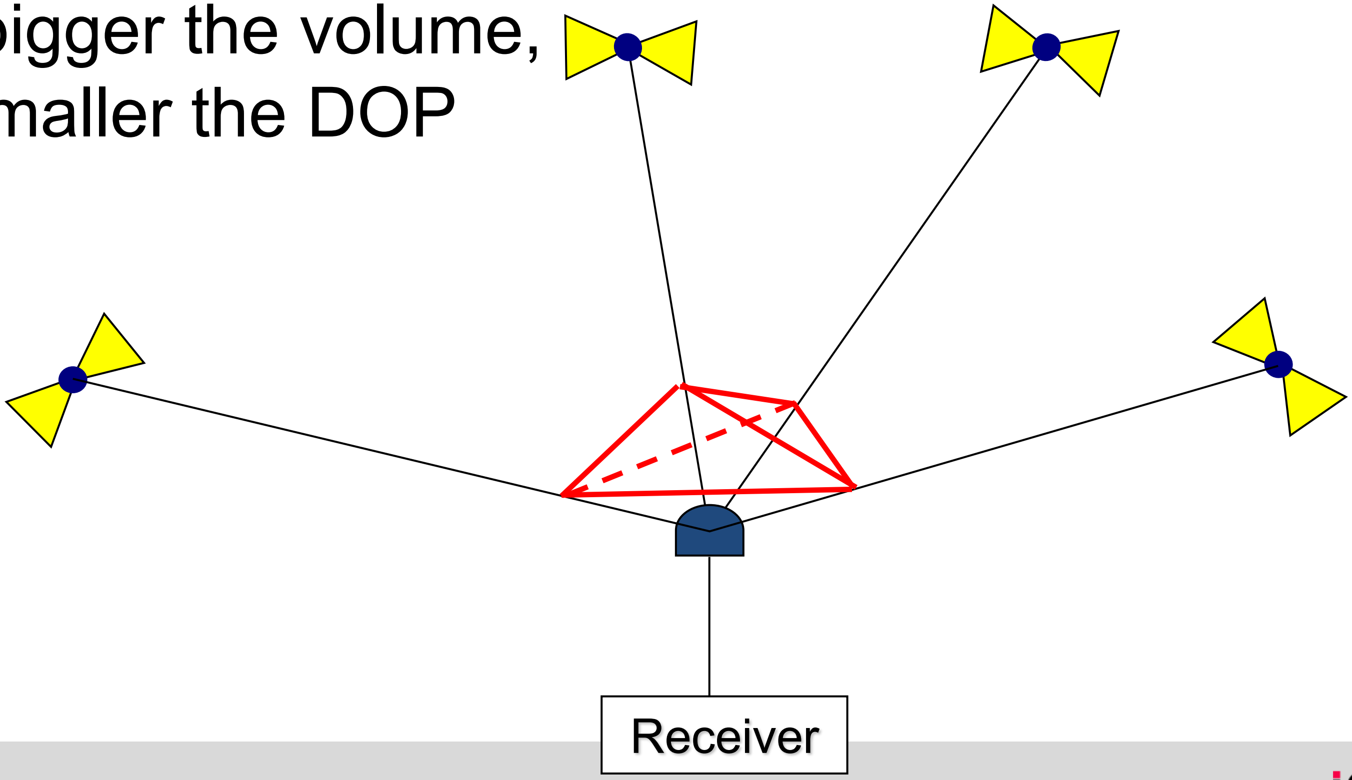
Dilution of precision

- Measure of geometry of visible satellites
- Inverse volume of triangular pyramid
  - The bigger the volume, the smaller the DOP



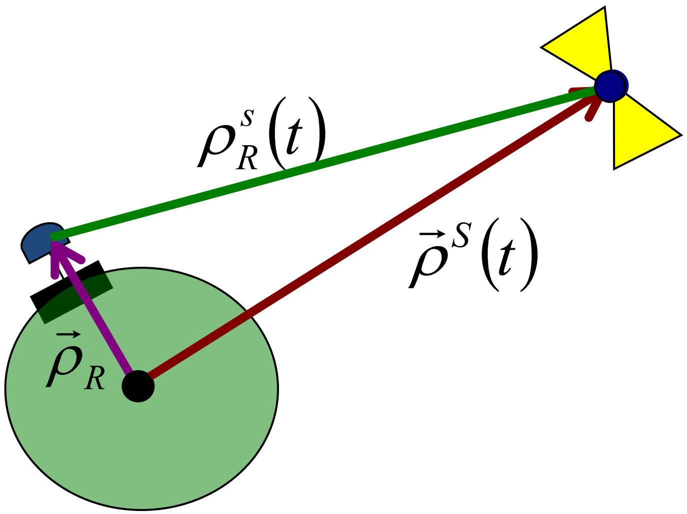
Dilution of precision

- Measure of geometry of visible satellites
- Inverse volume of triangular pyramid
  - The bigger the volume, the smaller the DOP



Mathematical principle

- ECEF coordinates
  - $\vec{\boldsymbol{\rho}}^S(t) = [X^S(t), Y^S(t), Z^S(t)]$ … satellite
  - $\vec{\boldsymbol{\rho}}_R = [X_R, Y_R, Z_R]$ … user
- $\rho_R^S(t) = \sqrt{(X^S(t) - X_R)^2 + (Y^S(t) - Y_R)^2 + (Z^S(t) - Z_R)^2}$



Non-linear equation

Mathematical principle

- Measured: pseudorange
  - $R_R^S = \rho_R^S + c \cdot \delta_R$
- Linearization
  - $dR_R^S = -\frac{X^S(t) - X_R}{\rho_R^S(t)} dX_R - \frac{Y^S(t) - Y_R}{\rho_R^S(t)} dY_R - \frac{Z^S(t) - Z_R}{\rho_R^S(t)} dZ_R + c \cdot d\delta_R$

Design matrix for one epoch t

$$A = \begin{bmatrix} -\dfrac{X^1(t)-X_R}{\rho_R^1(t)} & -\dfrac{Y^1(t)-Y_R}{\rho_R^1(t)} & -\dfrac{Z^1(t)-Z_R}{\rho_R^1(t)} & c \\ -\dfrac{X^2(t)-X_R}{\rho_R^2(t)} & -\dfrac{Y^2(t)-Y_R}{\rho_R^2(t)} & -\dfrac{Z^2(t)-Z_R}{\rho_R^2(t)} & c \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{array}{l} \textbf{SV 1} \\ \\ \textbf{SV 2} \\ \\ \textbf{Additional visible satellites} \end{array}$$

- Set 4th column to 1 instead of c for numerical reasons

Dilution of precision
PDOP

- $\mathbf{Q_x} = \mathbf{N}^{-1} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}$
- $\mathbf{Q_x} = \begin{bmatrix} q_{xx} & q_{yx} & q_{zx} & q_{xt} \\ q_{xy} & q_{yy} & q_{zy} & q_{yt} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{bmatrix}$
- $\mathbf{Q_x}$ … in ECEF
- PDOP
  - $PDOP = \sqrt{q_{xx} + q_{yy} + q_{zz}}$

Dilution of precision HDOP, VDOP

- $\mathbf{Q_x} = \mathbf{N}^{-1} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}$

- $\mathbf{Q_x} = \begin{bmatrix} q_{xx} & q_{yx} & q_{zx} & q_{xt} \\ q_{xy} & q_{yy} & q_{zy} & \mathbf{Q_{xyz}} \\ q_{xz} & q_{yz} & q_{zz} & q_{zt} \\ q_{xt} & q_{yt} & q_{zt} & q_{tt} \end{bmatrix}$

- HDOP, VDOP: Rotation to local level frame (N - E - D)
  - $\mathbf{Q}_{x,ll} = \mathbf{R}^T(\varphi,\lambda)\mathbf{Q}_{xyz}\mathbf{R}(\varphi,\lambda)$
  - $R(\varphi,\lambda) = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & -\cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & -\cos\varphi\sin\lambda \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$

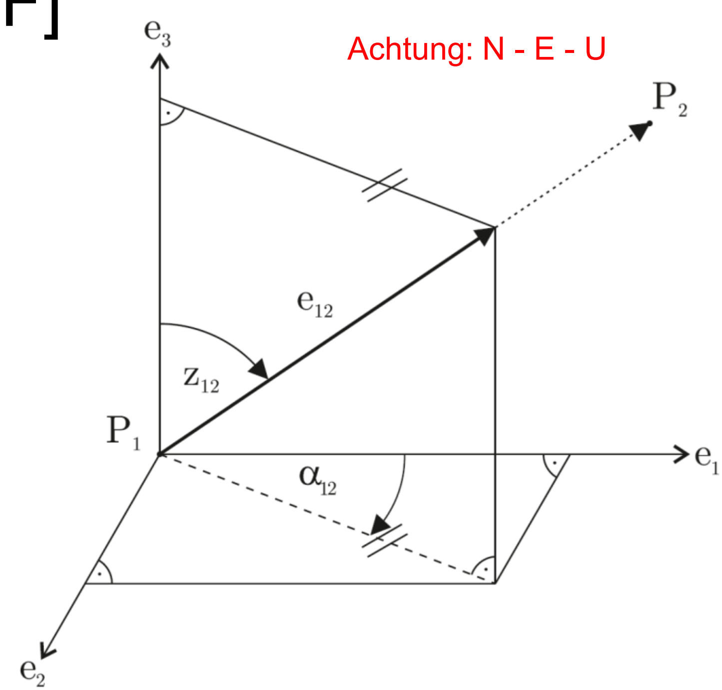
Dilution of precision HDOP, VDOP

- $$\mathbf{Q}_{x,ll} = \begin{bmatrix} q_{nn} & q_{en} & q_{dn} \\ q_{ne} & q_{ee} & q_{de} \\ q_{nd} & q_{ed} & q_{dd} \end{bmatrix}$$

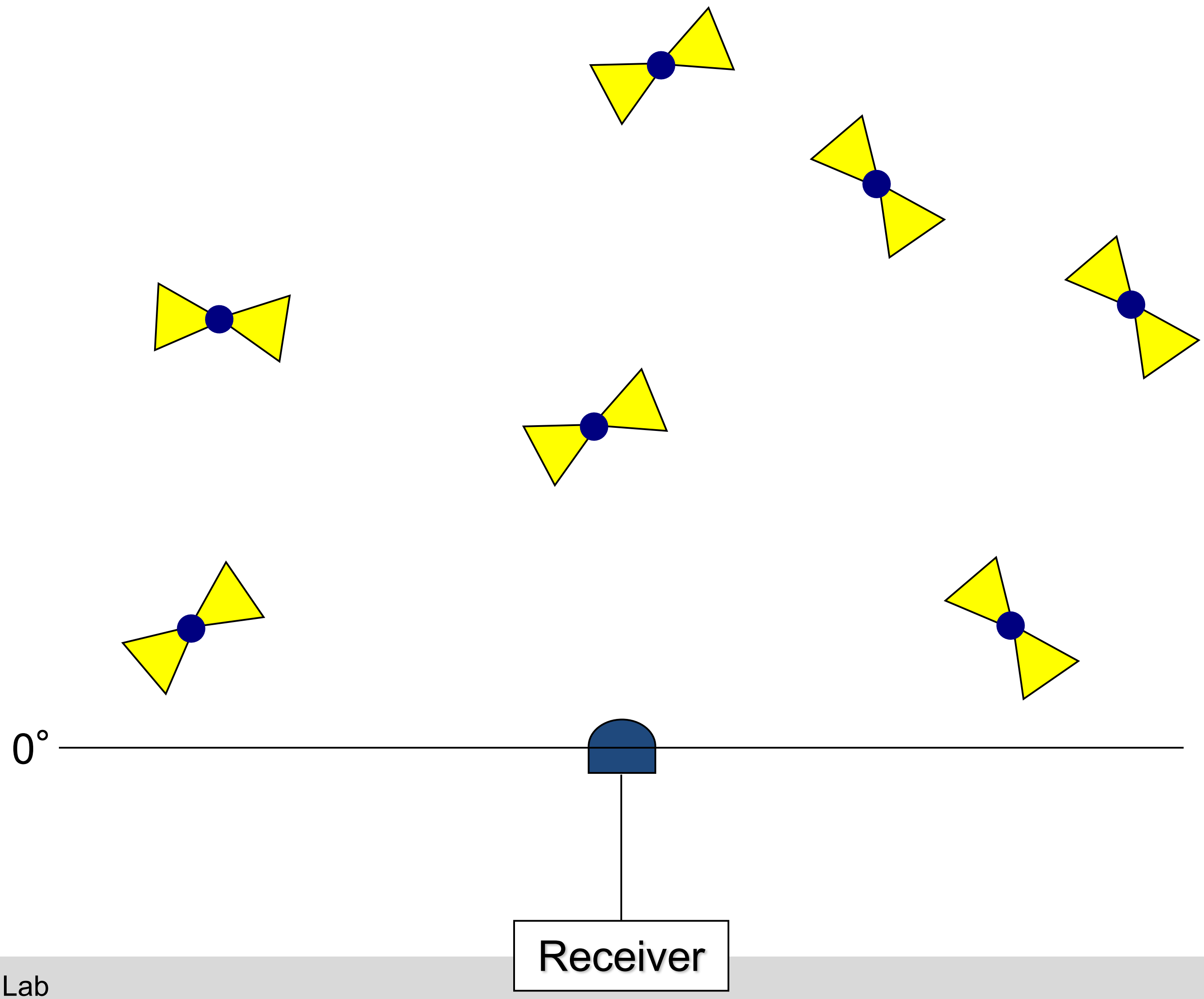
- HDOP
  - $HDOP = \sqrt{q_{nn} + q_{ee}}$
- VDOP
  - $VDOP = \sqrt{q_{dd}}$

Visible satellites

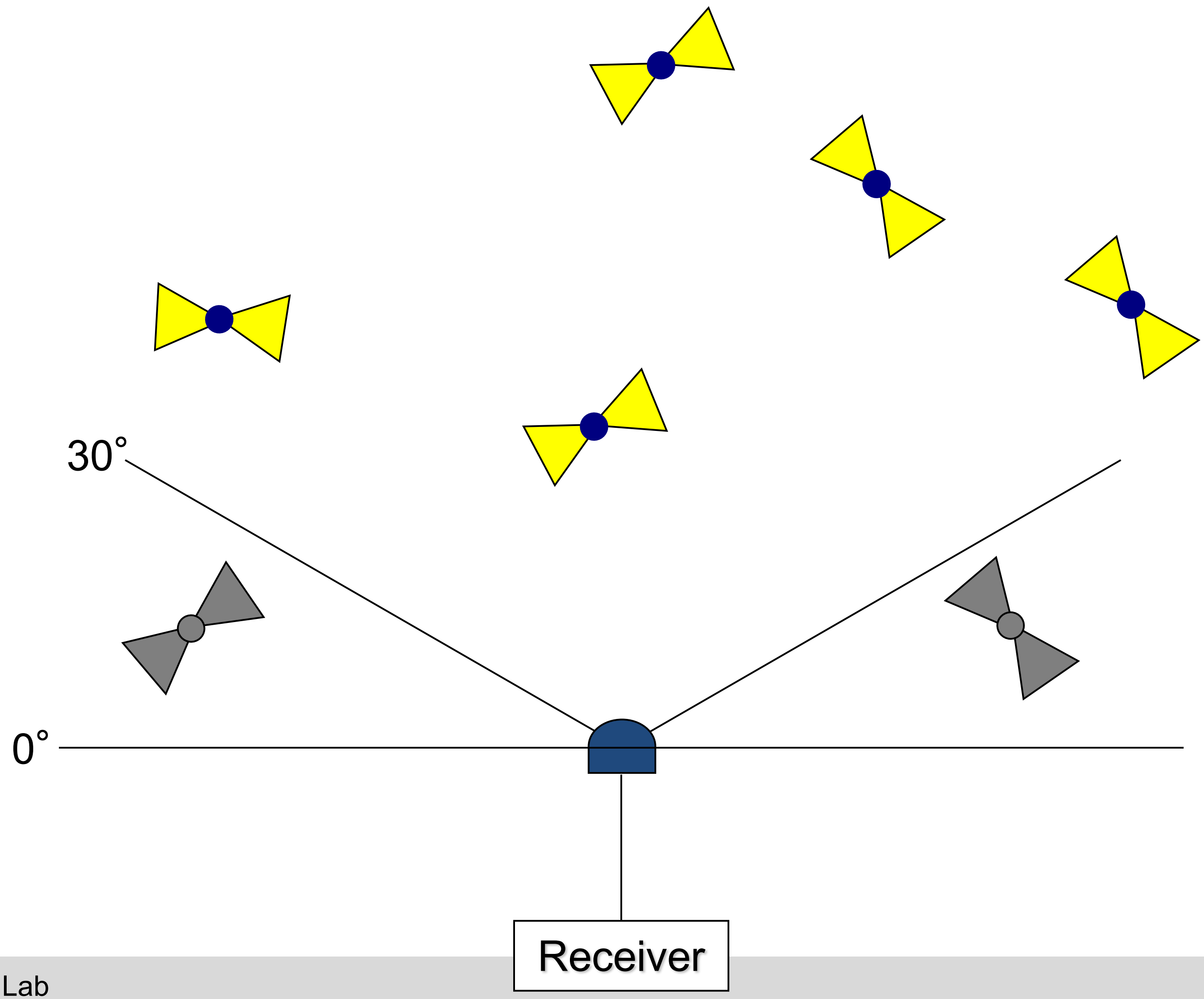
- Use only visible satellites for computing DOP values!
- Compute difference vector from user to satellite
  - $\mathbf{\Delta X_{user}^{sat}} = \mathbf{X_{sat}} - \mathbf{X_{user}}$ [ECEF]
- Rotate to local level frame
  - $\mathbf{\Delta X_{ll}} = \mathbf{R^T} \cdot \mathbf{\Delta X_{user}^{sat}}$
- Calculate zenith angle
  - $z = \arccos\left(\frac{-\mathbf{\Delta X_{ll}(3)}}{||\mathbf{\Delta X_{ll}}||}\right)$
  - 90 ° z → 0° elevation



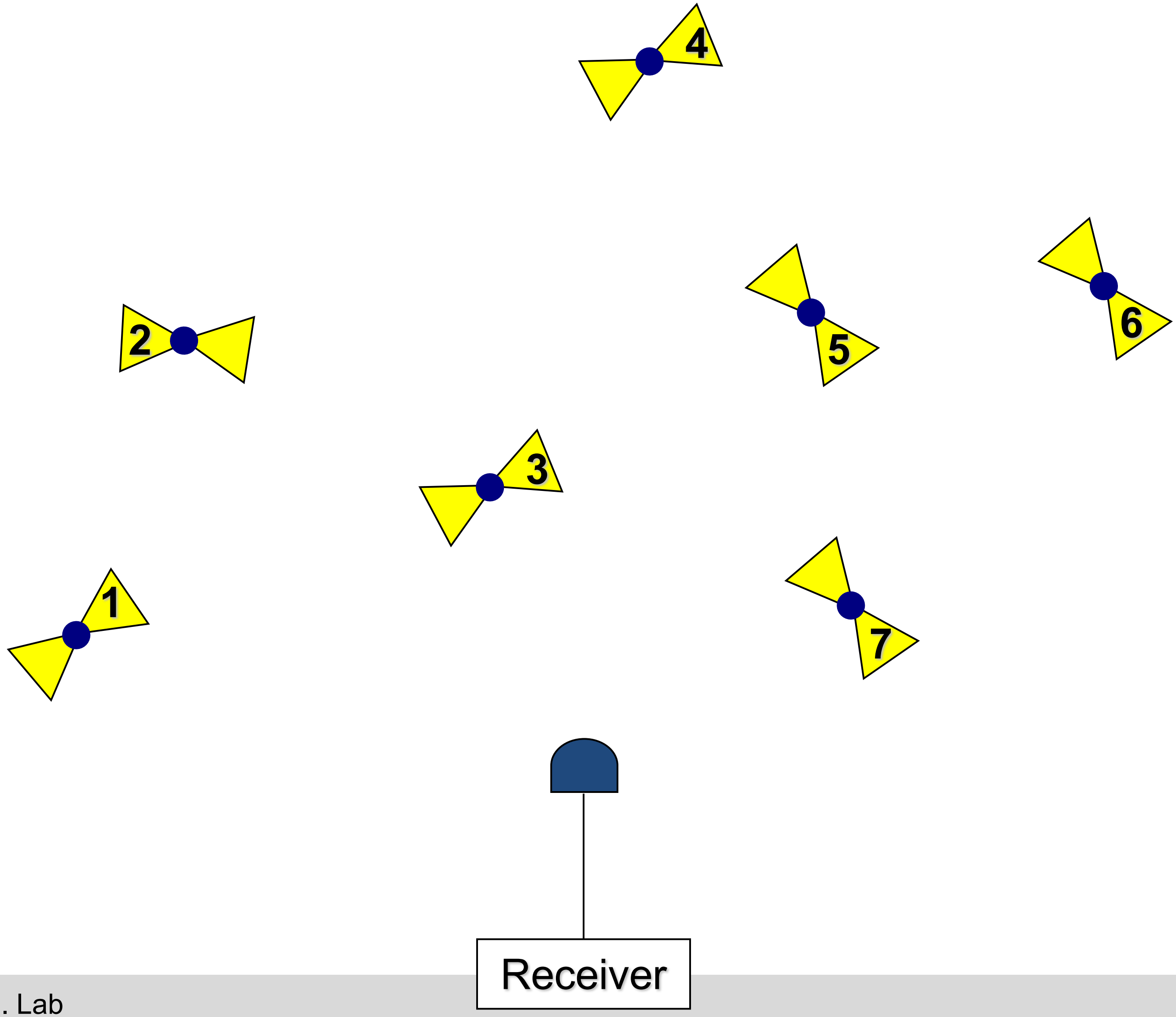
Variation of mask angle



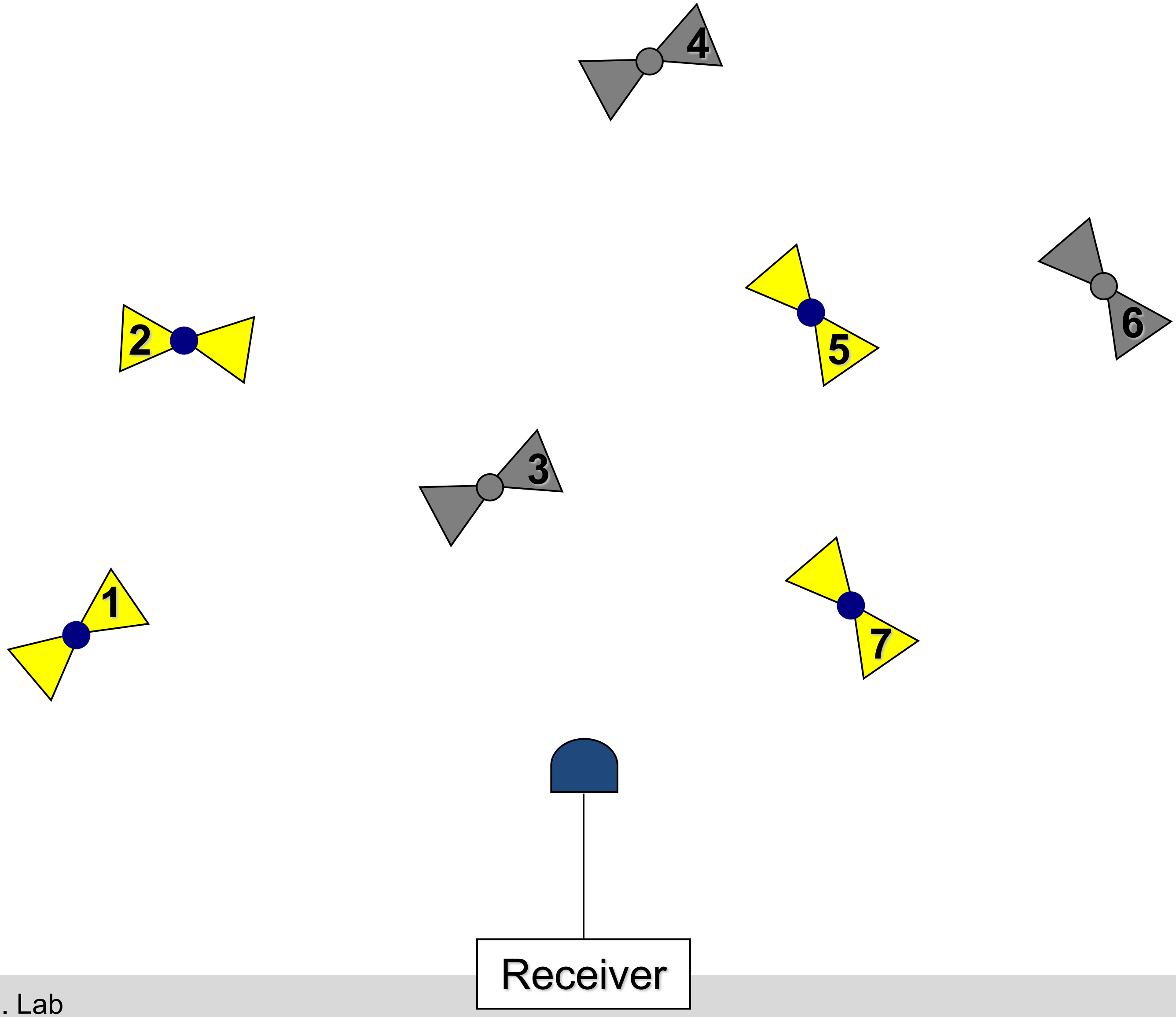
Variation of mask angle



Variation of visible satellite vehicles



Variation of visible satellite vehicles



Lab 2

- Group work (2 people per group) – New groups!!
- Hand in source code and technical report (max. 20 pdf pages):
  - 20.01.2026, 12:00 p.m.
- Interview:
  - 29.01.2026 (Timeslots → TC)
- Questions:
  - salloker@tugraz.at
  - s.laller@tugraz.at
- Remember: The use of AI is prohibited throughout the entire course

Abgabe Source-Code & Technischer Bericht

- Der abgegebene Code muss exakt die Ergebnisse im technischen Bericht liefern!
- Keine absoluten Pfade im Code!
- Source Code muss in üblichen Entwicklungsumgebungen (Pycharm, Spider, …) ausführbar sein.
  - Jupyter Notebook Codes müssen gegebenenfalls selbst in „.py“ konvertiert werden.

- Sowohl der technische Bericht als auch der Source Code müssen folgende Struktur besitzen:
  - → GNSSWS2526_Nachname_Matrikelnummer_TechnischerBericht.pdf
  - → GNSSWS2526 _Nachname_Matrikelnummer_main.py
  - → GNSSWS2526 _Nachname_Matrikelnummer_function.py
  - → GNSSWS2526 _Nachname_Matrikelnummer_Abgabe2UEP.zip
- Abgegeben wird nur eine .zip Datei, welche alle Dateien beinhaltet (auch gegebene Daten)