# Yrðingar

Yrðingar eru setningar sem eru annaðhvort sannar eða ósannar.

**Dæmi um yrðingar:**

Himininn er blár

$$2 + 2 = 4$$

**Dæmi um ekki yrðingar:**

Er himinninn blár?

$$x + 2 = 4$$

# Helstu tákn

$\neg$ : ekki, t.d. $\neg p$ ekki $p$ þar sem $p$ er einhver yrðing.

$\wedge$ : og, t.d. $p \wedge q$ : er satt ef $p$ og $q$ eru sönn.

$\vee$ : eða, t.d. $p \vee q$ : er satt er p$ eða $q$ eða bæði eru sönn.

$\oplus$ : annaðhvort (en ekki bæði), t.d. $p \oplus q$ : er satt ef $p$ er satt eða ef $q$ er satt.

$\rightarrow$ : leiðir til, t.d. $p \rightarrow q$ : ef $p$ er satt, þá hlýtur $q$ að vera satt. (Ef $p$ er ósatt getur $q$ verið satt eða ósatt). Sagt ef $p$ þá $q$.

$\leftrightarrow$ : Tvíleiðing, t.d. $p \leftrightarrow q$ : jafngildir $p \rightarrow q \wedge q \rightarrow p$.

$\equiv$ : jafngildir, t.d. $p \equiv p$ : $p$ jafngildir $p$. ### Quantifiers $\forall$, fyrir öll.

$\exists$, er til.

$\therefore$ "therefore", "ergo". (því, svo, þannig).

## Sanntöflur

| $p$ | $q$ | $\neg p$ | $p \wedge q$ | $p \vee q$ | $p \oplus q$ |
|---|---|---|---|---|---|
| F | F | T | F | F | F |
| F | T | T | F | T | T |
| T | F | F | F | T | T |
| T | T | F | T | T | F |

| $p$ | $q$ | $p \to q$ | $p \leftrightarrow q$ | $\neg(p \oplus q)$ |
|---|---|---|---|---|
| F | F | T | T | T |
| F | T | T | F | F |
| T | F | F | F | F |
| T | T | T | T | T |

# Rökfræði reglur

### Identity laws

$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

### Domination laws

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

### Indempotent laws

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

### Double neagtion law

$$\neg(\neg p) \equiv p$$

### Commutative laws

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

### Associative laws

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

### Distributive laws

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

### DeMorgans laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

### Absorption laws

$$p \lor (p \land q) \equiv p$$
$$p \land (p \lor q) \equiv p$$

### Negation laws

$$p \lor \neg p \equiv T$$
$$p \land \neg p \equiv F$$

## Logical equivalences

$$p \to q \equiv \neg p \lor q \equiv \neg q \to \neg p$$

$$p \lor q \equiv \neg p \to q$$
$$p \land q \equiv \neg(p \to \neg q)$$

$$\neg(p \to q) \equiv p \lor \neg q$$

$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$
$$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$$

$$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$$
$$(p \to r) \lor (r \to r) \equiv (p \lor q) \to r$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

## Rules of inference

### Modus ponens

$$(p \land (p \to q)) \to q$$

### Modus tollens

$$(\neg q \land (p \to q)) \to \neg p$$

**Hypothetical syllogism**

$$((p \to q) \land (q \to r)) \to (p \to r)$$

**Disjunctive syllogism**

$$((p \lor q) \land \neg p) \to q$$

**Addition**

$$p \to p \lor q$$

**Simplification**

$$(p \land q) \to p$$

**Conjuction**

$$((p) \land (q)) \to p \land q$$

**Resolution**

$$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$$