Metallicity and star formation rate impact study with *cosmoRate*

Lorenzo Buriola, Filippo Conforto, Lorenzo Domenichetti
Università degli studi di Padova

5 August 2021

ABSTRACT
CosmoRate [Santoliquido et al. (2020)] is a *python* script that allows to simulate merging events between binary systems - namely, BBH, BNS, and BHNS. In this paper, some of the main simulation results related to run's parameters will be discussed. In particular, the mass distribution and merger rate density of binary objects will be evaluated and compared for different theoretical models. Moreover, an analysis on how these quantities are affected by the metallicity models and star formation rate will be carried out. Finally, an in-depth look at the performance of CosmoRate will be shown.

Key words: Merger Rate Density – Binary systems – Metallicity effects

1 INTRODUCTION

The final stage in the evolution of a massive star is the formation of a *compact object* - either a black hole or a neutron star. One of the most important topics nowadays in astrophysics is the study of gravitational waves produced from the collision of two compact objects - binary black holes (BBH), one black hole and one neutron star (BHNS), and binary neutron stars (BNS). Each class of binary objects has its unique properties, as the underlying compact objects are not equal. These differences arise the necessity to split the analysis of such classes.

Since the Big Bang, the Universe is expanding, and its expansion speed is growing ever since. In astrophysics, a particularly relevant quantity is the *redshift* (z). This quantity is associated with the speed at which a determined object is moving away from us. In particular, next-generation interferometers will be able to look for merging binaries up to redshift ≈ 10. Providing accurate predictions on the merger rate of such systems at never-observed redshifts is paramount to interpret the incoming data properly. This is the fundamental aim of many simulations run nowadays.

Dealing with redshift may sound puzzling: a more intuitive quantity is instead the *look-back time* (t_L). It describes the elapsed time from the emission of light signals to the collection throughout instruments - up to now, ground-based interferometers. From the study of the evolution of the universe, it is possible to relate this two quantities.

$$t_L = t_H \int_0^z \frac{dz'}{(1+z')E(z')}$$

where t_H is related to the Hubble constant $H_0 \approx 67.8\,km/(Mpc\ s)$, z is the redshift, and $E(z')$ is a function of the redshift containing parameters linked to the cosmological constant Λ and the matter density in the Universe. Many libraries allow the translation of one variable to the other - the one exploited for this analysis was *astropy*.

One of the main parameters affecting the merger rate of binaries per given volume is the *Star Formation Rate* (SFR). This quantity returns the number of stars formed at a given redshift in a given volume. SFR is directly linked to the number of compact objects born at the end of a star's life. A useful parametrization of the SFR is the one given by Madau & Dickinson (2014), obtained by fitting experimental data.

$$\psi = 0.01 \frac{(1+z)^{2.6}}{1+[(1+z)/3.2]^{6.2}} \, [M_\circ yr^{-1} Mpc^{-3}] \qquad (1)$$

Another quantity that has a major importance on the merger rate is the *metallicity (Z)* - which in astronomy is defined as the abundance of elements present in an object that are heavier than hydrogen and helium. Elements in a star are converted through reactions mediated by different forces from hydrogen to heavier elements, up to when the core collapses under gravitational force due to *"lack of fuel"*. The metallicity may be then defined as :

$$Z = 1 - X - Y$$

where X and Y are the percentages of the mass of hydrogen and helium, respectively.

Every star keeps converting lower mass nuclei to higher ones, and when at the end of a star's cycle a supernova explodes, heavier elements pollute the surrounding environment. Therefore, if a star forms in a polluted environment, it may already have a non-null metallicity. It is possible to think of metallicity as a measure of the age of the universe, as the heavier element formation process is not reversible.

Nonetheless, linking metallicity to the age of the universe, and then to redshift, is a quite complicated task. Linear models were proposed by De Cia et al. (2018) and Gallazzi et al. (2007), but these still have to deal with high uncertainties in measurement.

Metallicity directly affects the properties of merging binaries, as the Z of the progenitor massive star has a large impact on the mass of the black hole formed. On the other hand, the percentage of heavy elements in the progenitor star does not affect the properties of neutron stars, so BNS will not be much affected by this variable.

The study of the merger rate density (MRD) of compact binaries has to take into account many quantities to give a reliable estimate. *CosmoRate* provides a set of tools and algorithms for MRD estimation of binary systems of neutron stars, black holes, or a mix of the two. Using Monte Carlo techniques, it is possible to get a consistent estimation of the number of merging binary events during the universe evolution. Moreover, *CosmoRate* allows the creation of catalogues of merging objects per redshift bin, where the bin width can be chosen by the user itself. The approach followed by *CosmoRate* is the combination of compact binary catalogues, star formation rate (SFR), and metallicity evolution models.

2 METHODS

The analysis was carried out using a set of tools including the *Python* language, *jupyter notebook* environment and the *CosmoRate* script itself.

The main script can be easily interfaced through a user-friendly Python file (*cosmo_input*), where almost all parameters can be set. All functions needed during the simulation are included in a third Python file, *cosmo_functions*.

In *cosmo_input* are many options to be set. The most relevant ones, as far as the presented analysis is concerned, are:

- Astrophysical model (In the analyzed case A1/A5/A10);
- Binary type;
- Redshift range and binning;
- Metallicity model and uncertainty (logical);
- Number of loops for uncertainty calculation;
- Logical set for catalogue production.

Having chosen a model, the list of metallicites used in the simulation, a redshift range, its binning, and the kind of binary system to simulate, the script picks the right catalogues and produces the required output.

The simulation parameters have a large impact on the resulting output. Apart from the catalogues themselves, a crucial ingredient for *CosmoRate* is the astrophysical model considered. For the sake of this analysis, three of them have been tested, namely $\alpha 1$, $\alpha 5$, and $\alpha 10$. Each model is associated to a unique value of α (1,5, or 10 for this analysis), defining the envelope behaviour during the binary evolution: smaller values of α imply that the binary's envelope tightens more, while for larger α the envelope may be lost during the evolution. In the simulation, the chosen model properties are applied to data coming from catalogues given the binary type and metallicity.

Accordingly to the information given in the input file, the main script begins its simulation combining the three main variables affecting the final merger rate density:

- **SFR**: Star Formation Rate;
- **Astrophysical Model**;
- **Metallicity**.

Metallicity estimation as a function of the redshift is not easy and, throughout the analysis, has been obtained using different methods. In one case, linear models were exploited, in another one a dataset interpolation ('galaxy' option). In the former case, the linear model intercept and slope are drawn from a Gaussian distribution, and then the metallicity value at a fixed z is again drawn from a Gaussian PDF with a given variance centered in the corresponding point of the line. The resulting metallicity values will be then spread - this will be a source of uncertainty on the final MRD.

Star Formation Rate concerns instead the number of stars produced in the universe in a given unit of volume and time. SFR can be defined using the equation 1 described in the introduction.

CosmoRate combines the information coming from these main elements to generate a value of merger rate density for each redshift bin. It is also possible to get an uncertainty evaluation for the MRD, by looping the calculations and producing a certain number of MRD estimations, whose spread can be seen as a probability distribution. The uncertainty will mainly be due to the metallicity estimation, which is characterized by a large variance as seen before, while the SFR uncertainty has a lighter effect on the final result.

The script's output may consist of two kinds of files: catalogues of binaries and MRD estimation per redshift bin. In the output catalogues are collected all produced binaries and their main features (mass of the original objects and formation time), categorized by their merger redshift z_{merg}. The file containing MRD estimation collects the MRD estimates. If more than one iteration is set, the file contains also the result for each iteration, again categorized by the merger time.

Once collected the output files (either containing MRD per redshift or catalogues of CB per redshift bin), the analysis shifted to *jupyter notebooks*. The main libraries used are *pandas* for data handling, *matplotlib* for plotting and *astropy* to simplify the management of astronomical quantities.

The following analysis will then focus on *CosmoRate*'s outputs to highlight the main differences between different configuration options. Indeed, a broader objective is the understanding of the main features of the distribution of compact binaries.

3 RESULTS

3.1 Mass and delay time distributions

One useful analysis to start with is the distribution of the masses of the simulated binary system for different starting models. For each class of binary objects, simulations are run using the three binary star evolution models mentioned before, named $\alpha 1$, $\alpha 5$ and $\alpha 10$ (A1, A5, and A10). The distributions obtained from the output catalogues are shown in Fig. 1, 2 and 3. Single objects' and total masses are reported in the histograms. From the plot, it is straightforward to see that the black hole masses are distributed between 3 and 50 $M_\odot$ while the neutron stars are always by default less than 3 $M_\odot$. (This is a cutoff value of *CosmoRate* that classifies every object more massive than 3$M_\odot$ to be a black hole). Further comments on the mass distribution will be presented in the following paragraph, analysing figure 5. Concerning BHNS, a simple result that one can check is how many times the most massive star becomes the black hole. It is sufficient to count how many times the first mass is the biggest between the two. This fact seems to be very dependent on the model as it passes from a fraction of 0.97 for A1, to 0.35 for A5 and 0.17 for A10.

Moreover, in the figures, the distribution of the delay time - the time difference between the formation and the merging of the binaries - is shown. It is clear that the vast majority of binary systems have a delay time between 10^7 and 10^8 years.

3.2 Merger Rate Density

The core of the analysis is the study of the Merger Rate Density for different binary objects as a function of the lookback time. The main results are reported in the figure 4. *CosmoRate* has been run many times, varying the cosmological model and the class of compact objects. Different numbers of iterations for uncertainty calculation were chosen (ranging between 100 and 10000) turning on both uncertainties related to metallicity and SFR. In the plot, the median value of these iterations is shown. The shaded areas lay between the .25 and .75 percentiles. In the plot, the Star Formation Rate as a function of lookback time has been also reported to allow a visual

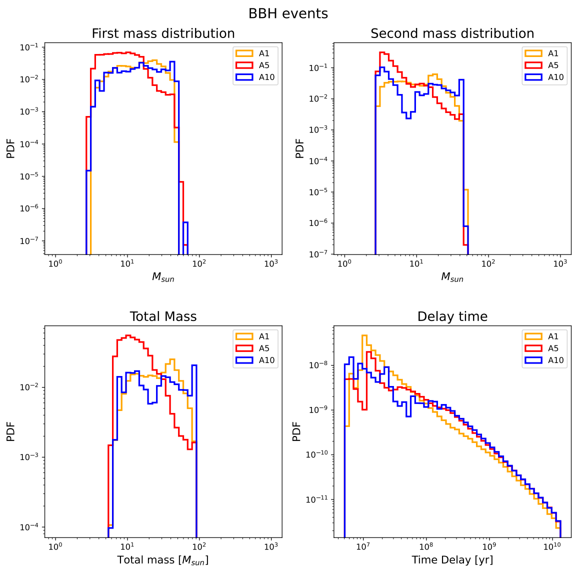

Figure 1. Mass and time delay distribution for BBH binaries

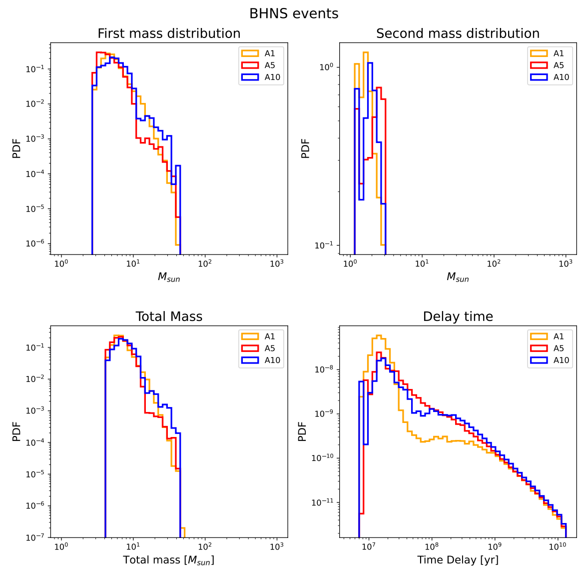

Figure 3. Mass and time delay distribution for BHNS binaries

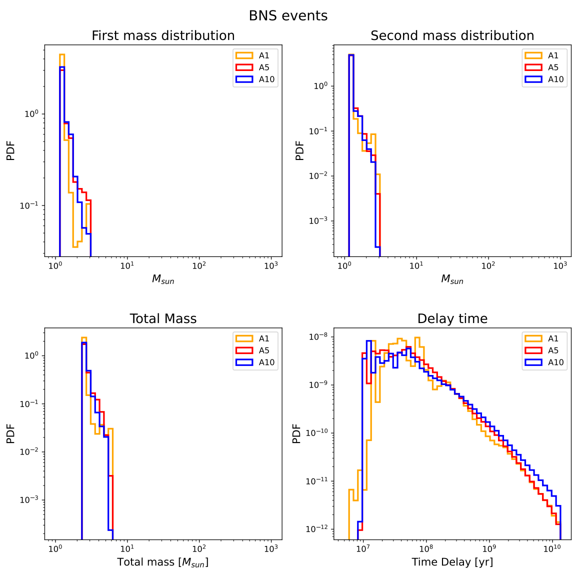

Figure 2. Mass and time delay distribution for BNS binaries

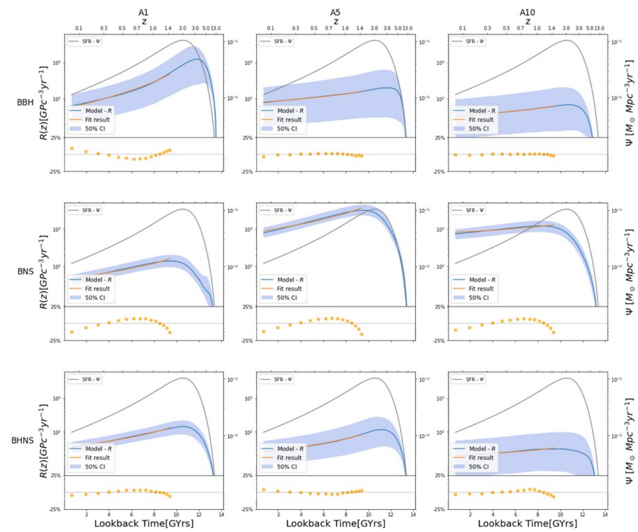

Figure 4. Merger rate density and its fit for different models and binaries

comparison and the curve follows the expected behaviour [Santoliquido et al. (2020)]. For BBH and BHNS, the MRD depends on the cosmological model used for simulation. It is useful to remind that the parameter α that differs among the three models is the **common envelope efficiency**. This parameter is highly connected to the initial position of the two objects after the envelope removal. An higher α means that statistically the two compact objects are more distant one to each other and the merging process could not happen. This explains why the MRD is higher for A1 and becomes lower for A10. As far as BNS are concerned, the dependence on the cosmological model is not so evident. Nonetheless, with α = 1 the MRD seems to be lower than the other cases. The initial distance between the compact objects is probably too low and many objects may merge before becoming neutron stars.

For each model-binary couple, a fit has been performed for redshift values smaller than to 1.5 (figure 4). In that range, the MRD should follow a distribution $\propto (1+z)^k$. Looking at the residuals, it is noticeable that the simulated MRD follows the expected trend up to a certain redshift value. Then, more complicated effects appear, and the expected behaviour is no more followed - as the last high-redshift residues show. Data has been fitted with and without considering the errors (68% confidence level around the mean), obtaining very similar results.

CosmoRate simulations produce the same number of catalogues for each redshift bin. Once obtained the MRD as a function of the redshift, one can use it to weigh the 2-dimensional output mass vs z_{merg} distributions. The 2D histogram is reported in fig. 5. Analyzing the plots, some important features come out. For what concerns BNS, the distribution is peaked at a total mass of 2.5 $M_{\odot}$ - as the majority of neutron stars have a mass of 1.25, and a redshift between 2 and 2.5 corresponding to the maximum of the SFR translated to allow

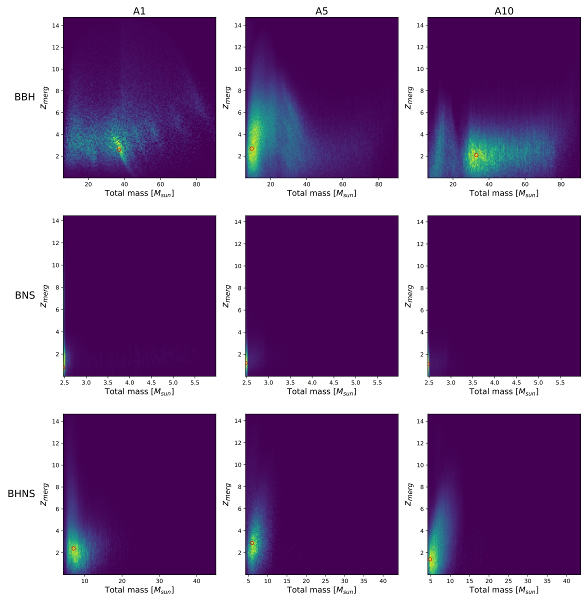

Figure 5. 2D histogram for total masses and redshifts

the binary to merge. This is due to the fact that Binary Neutron Stars are mainly influenced by SFR and not by metallicity. On the other hand, BBHs follow a different distribution, which depends both on the metallicity and the cosmological model. One can notice that, considering the cosmological model A1, large masses (a total of 40 $M_{\odot}$) are favoured. It has been mentioned before that in these conditions many binary star systems merge before becoming black holes because of the short initial distance between objects. Moreover, comparing A1 with the other models, it can be observed that binaries with larger masses tend to merge more. This comes from the fact that large binaries are often born with two objects further between each other. In the other two models, such distances may lead to an envelope loss.

A long tail in the mass distribution appears in the A10-case - this is due to the fact that the two black holes start the merging process further than the other cases. Binaries have a coalescence time that scales as $1/M^3$ and large masses are once more favoured. The initial distance is the cause of the low values of the merging redshifts. A5 is an intermediate case between the other two cosmological models, but here lower masses are favoured. From the 2D plot, it is noticeable that the weighted BBH distribution assumes high values for z_{merg} between 2 and 6. For BBH, the distributions tend to extend up to larger redshift values. This is because the merging of black holes is highly influenced by metallicity: a binary system that arrives at coalescence usually comes from a population of low metallicity stars. Indeed, this characteristic is associated with higher redshifts. Finally, BHNS case is positioned in between the other two classes of binary objects. Both low mass and redshifts are preferred - nonetheless there is a tail of binaries that merges at higher z.

3.3 Metallicity models

Analyses in the previous chapter have been performed using the default option of metallicity built in *CosmoRate*. The model chosen was linear, whose intercept has been set to 1. One can study how the metallicity model affects the MRD trend. In fig. 6 the MRD curves obtained using different intercept values for a linear metallicity

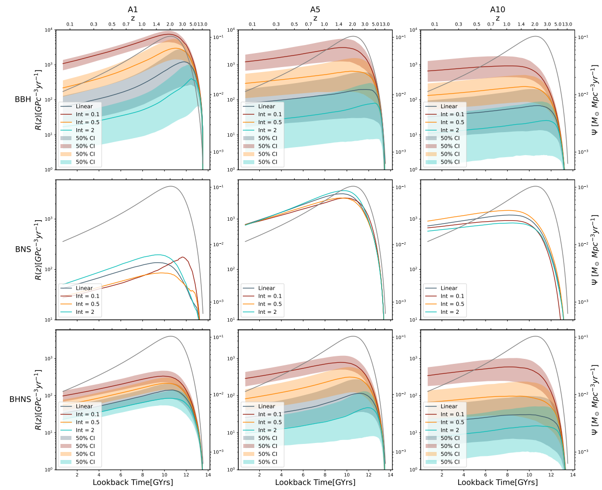

Figure 6. Merger rate density for different values of metallicity linear model

model are shown. Concerning BBH and BHNS plots, it is possible to see that the MRD assumes higher values for lower intercepts. This is an expected result, as the black hole formation is highly favoured by lower values of metallicity. On the other hand, as far as BNSs are concerned, the change in the intercept does not seem to be relevant. Once more, this proofs that neutron stars are not influenced by metallicity.

As explained in the previous section, other models of metallicity have been implemented (the 'galaxy' option). The two models used exploit the estimation of the metallicity of the galaxy population. One is the *Mass Metallicity Relation (MZR)* and the other one is the *Fundamental Metallicity Relation (FMR)*. These two models have not an analytical expression and have been extracted through interpolation of two given datasets. The results of the simulation employing such models are shown in fig. 7. Note that in this plot the error bars reported for the two nonlinear models are computed only using the uncertainty retrieved from *Star Formation Rate*. As expected, a change in the metallicity model affects both BBH and BHNS cases. The MRD computed from FMR is quite different from the linear case - indeed the two models show large differences. On the other hand, the MRD obtained using the MZR estimate is lower than the default curve for low redshifts and tends to reach it for $z > 5$. Such behaviour is justified comparing the metallicity models themselves. Furthermore, it is possible to see that the metallicity model does not affect much the MRD for BNS.

3.4 A brief discussion about uncertainties

The uncertainties computed for the MRD are obtained from the analysis of multiple iterations of *CosmoRate*, once set the errors for metallicity and SFR. The whole simulation procedure may take much time, depending on the number of input catalogues and used computational resources. To avoid a too long execution time, one can try to reduce the number of iterations. The risk of such a procedure is to lose precision on the uncertainty estimate, as statistical effects may be more evident.

Nevertheless, it is important to study how the number of iterations set affects the uncertainty. In fig. 8 runs with different number of iterations are reported. It is clear that there are some differences between the median values of the MRD - anyway the shaded areas are large enough to contain these differences. Moreover, most of the

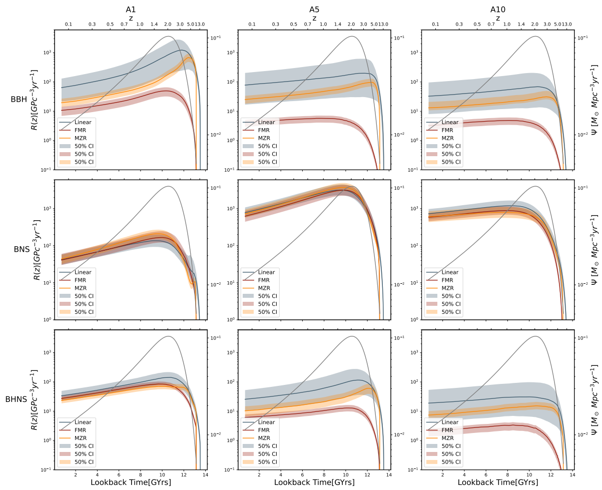

Figure 7. Merger rate density for "Galaxy" models

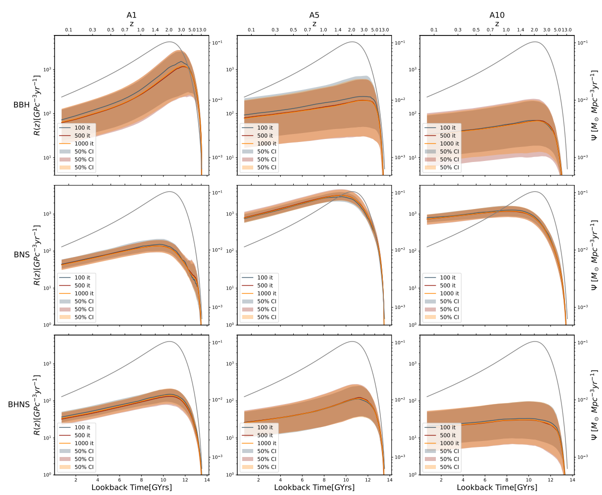

Figure 8. Merger rate density distribution comparison with multiple iterations

time uncertainties are comparable among each other. From such an analysis, it seems that it is not necessary to run many iterations to get a reasonable approximation.

4 CONCLUSION

CosmoRate script allowed to understand how simulated binary systems properties depend on *Cosmo_input* options - including metallicity, star formation rate, and redshift interval. The main task was to study how the results change considering different astrophysical models and binary types. The graphs show how the expected features, such as the total mass distribution in Fig. 5, are reproduced in the simulations.

The other main topic of the work was the study of the influence of the metallicity models on the final binary population. Changing the intercept of the default linear model, different MRD values were compared. As expected from the theory, higher intercepts lead to lower merger rate values for BBH and BHNS systems. As an alternative, *galaxy* models were produced by fitting experimental data describing the metallicity trends in galaxies and compared to the usual linear model.

Finally, one last test was done by comparing the simulations obtained with different iteration values, to understand how large was their impact on the resulting merger rate density distribution. The uncertainty values associated with these different simulations show how even with a lower number of iterations it is possible to have a good estimation of the error.

In conclusion, *CosmoRate* has proved to be a very powerful, scalable, and complete algorithm for merger rate density estimation and binary system simulation.

5 CONTRIBUTIONS

The work presented has been carried out by a team with a strong collaboration sense. Even if it is a tough task, in this paragraph are presented the main contributions given by each team member.

Simulations were equally split for each model - A1 LD, A5 FC, A10 LB; the fit of the MRD, with and without considering errors, and the residual subplot comes mainly from FC. The galaxy model implementation, the study of the uncertainty per iteration, and the respective simulations were carried out by LD. No specific contributions were found in the other topics, which may be assigned equally to all three members.

As far as the final report is concerned, the introduction section was primarily written by LD, the results one by LB, and the methods and conclusion by FC. However, the intersection between each other's work is by far different from zero - all authors have given a significant contribution to each part of the document.

REFERENCES

De Cia A., Ledoux C., Petitjean P., Savaglio S., 2018, A&A, 611, A76

Gallazzi A., Brinchmann J., Charlot S., White S. D. M., 2007, Monthly Notices of the Royal Astronomical Society, 383, 1439–1458

Madau P., Dickinson M., 2014, Annual Review of Astronomy and Astrophysics, 52, 415

Santoliquido F., Mapelli M., Bouffanais Y., Giacobbo N., Di Carlo U. N., Rastello S., Artale M. C., Ballone A., 2020, The Astrophysical Journal, 898, 152