Qn 1.

from Kinematics

$$\underbrace{s}_{\text{distance}} = \frac{1}{2} \underbrace{a}_{\text{acceleration.}} t^2$$

$$\therefore \tau_{dynamic} = \sqrt{\frac{2s}{a}}$$

when $a = \frac{GM}{R^2}$, $s = R$

$$\tau_{dynamic} = \sqrt{\frac{2R}{\frac{GM}{R^2}}}$$

$$\tau_{dyn} \sim \left(\frac{R^3}{GM}\right)^{\frac{1}{2}}$$

$$\boxed{\tau_{dyn} \sim \left(\frac{1}{G\rho}\right)^{\frac{1}{2}}} \quad \because \rho = \frac{M}{\frac{4}{3}\pi R^3}$$

Sound Crossing time, τ_{sc}

$$\tau_{sc} \sim \frac{\text{distance}}{\text{Speed}} \sim \frac{R}{c_s}$$

→ or hydrostatic Equilibrium.

$$\frac{1}{\rho}\frac{dp}{dr} \simeq \frac{GM}{r^2} \Rightarrow \frac{1}{\rho}\frac{P}{R} \simeq \frac{GM}{R^2}$$

$$\frac{P}{\rho} = \frac{GM}{R}$$

$$c_s^2 = \frac{GM}{R}$$

$$\therefore \tau_{sc} \sim \frac{R}{\left(\frac{GM}{R}\right)^{\frac{1}{2}}} \sim R\left(\frac{R}{GM}\right)^{\frac{1}{2}}$$

$$\boxed{\tau_{sc} \sim \left(\frac{R^3}{GM}\right)^{\frac{1}{2}} \sim \left(\frac{1}{G\rho}\right)^{\frac{1}{2}}}$$

$$\therefore \underline{\tau_{sc} \sim \tau_{dyn}}$$

If $\tau_{dyn} < \tau_{sc}$

$$\left(\frac{r^3}{GM}\right)^{\frac{1}{2}} < \frac{r}{c_s}$$

$$\left(\frac{r^3}{GM}\right) < \left(\frac{r}{c_s}\right)^2 \Rightarrow \frac{r^3}{GM} < \frac{r^2}{c_s^2}$$

$$\Rightarrow \frac{r}{GM} < \frac{1}{c_s^2}$$

$$\therefore \frac{GM}{r} > c_s^2 \Rightarrow \frac{GM}{R} > \frac{P}{\rho}$$

$$\therefore \underbrace{\frac{GM}{R^2}}_{\text{Gravitational force}} > \underbrace{\frac{1}{\rho}\left(\frac{P}{R}\right)}_{\text{pressure force.}}$$

Gravitational force will outweigh pressure force and the Cloud will collapse.

If $c_s > V_{esc}$

$$c_s > \left(\frac{GM}{R}\right)^{\frac{1}{2}} \Rightarrow c_s^2 > \frac{GM}{R}$$

$$\frac{P}{\rho} > \frac{GM}{R}$$

$$\underbrace{\frac{1}{\rho}\left(\frac{P}{R}\right)}_{\text{pressure force}} > \underbrace{\frac{GM}{R^2}}_{\text{Gravitational force}}$$

When the pressure force is greater, clouds will fly off

Qn 2.

for Steady flow

$$\dot{M} \cong \rho v r^2 = \text{constant}$$

let $\rho v r^2 = A$ - - - - $*$

from Bernoulli's Eqn.

$$B = \frac{1}{2} v^2 + h + \phi$$

$$h = \int_s \frac{dP}{\rho}$$

from ideal Gas Eqn.

$$P = \frac{\rho k T}{\mu} = \rho v^2$$

$$dp = d\rho\, c_s^2$$

$$\therefore h = \int_s \frac{d\rho}{\rho} c_s^2 = c_s^2 \ln\rho + A_0.$$

Also $\phi = -\frac{GM}{r} = \frac{2c_s^2 r_s}{r}$

Since $r_s = \frac{GM}{(2c_s^2)}$

$$\therefore B = \frac{1}{2} v^2 + c_s^2 \ln\rho + A_0 - \frac{2c_s^2 r_s}{r}$$

Simplifying

$$B_0 = \frac{1}{2} v^2 + c_s^2 \ln\rho - \frac{2c_s^2 r_s}{r}$$

Substituting Eqn $*$

$$B_0 = \frac{1}{2} v^2 + c_s^2 \ln\frac{A}{vr^2} - \frac{2c_s^2 r_s}{r}$$

$$B_0 - \ln A^{c_s^2} = \frac{1}{2} v^2 - c_s^2 \ln v - c_s^2 \ln r^2 - \frac{2 r_s c_s^2}{r}$$

$$\therefore \ln v - \frac{1}{2}\frac{v^2}{c_s^2} = -2\ln r - \frac{2r_s}{r} + C$$

where $\left(\frac{B_0 - \ln A^{c_s^2}}{c_s^2}\right)$ some constant

Since at $r = r_s$
$v = c_s$

$$\therefore \ln c_s - \frac{1}{2}\frac{c_s^2}{c_s^2} = -2\ln r_s - \frac{2r_s}{r_s} + C$$

$$\ln c_s \cdot r_s^2 + \frac{3}{2} = C.$$

$$\therefore \ln v - \frac{1}{2}\frac{v^2}{c_s^2} = -2\ln r - \frac{2r_s}{r} + \ln c_s \cdot r_s^2 + \frac{3}{2}$$

$$\ln v + 2\ln r - \ln c_s \cdot r_s^2 = \frac{1}{2}\frac{v^2}{c_s^2} - \frac{2r_s}{r} + \frac{3}{2}$$

$$\ln \frac{vr^2}{c_s r_s^2} = \frac{1}{2}\frac{v^2}{c_s^2} - \frac{2r_s}{r} + \frac{3}{2}$$

$$\frac{vr^2}{c_s r_s^2} = e^{\frac{3}{2} - \frac{2r_s}{r}} \cdot e^{\frac{1}{2}\frac{v^2}{c_s^2}}$$

$$\therefore \underline{\underline{v(r) \cdot e^{-v^2/2c_s^2} = c_s\left(\frac{r_s}{r}\right)^2 e^{3/2 - \frac{2r_s}{r}}}}$$

Sunday, October 29, 2023 5:02 AM

Qn 2 b>

Recall

① mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \vec{v}) = 0$$

for Steady State

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla\cdot(\rho\vec{v}) = 0$$

Spherical Symmetry

$$\frac{\partial}{\partial r}(\rho r^2 v) = 0.$$

$$r^2 v\frac{\partial \rho}{\partial r} + 2r\rho v + \rho r^2\frac{\partial v}{\partial r} = 0$$

Divide by $\rho r^2 v$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial r} + \frac{2}{r} + \frac{1}{v}\frac{\partial v}{\partial r} = 0$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial r} = -\frac{2}{r} - \frac{1}{v}\frac{\partial v}{\partial r} \quad \text{--- } *$$

② momentum Eqn.

$$\rho v\frac{\partial v}{\partial r} = -c_s^2\frac{\partial \rho}{\partial r} - \rho\frac{GM}{r^2}$$

Substituting Eqn *

$$v\frac{\partial v}{\partial r} = -c_s^2\left(-\frac{2}{r} - \frac{1}{v}\frac{\partial v}{\partial r}\right) - \frac{GM}{r^2}$$

$$v\frac{\partial v}{\partial r} = \frac{2c_s^2}{r} + \frac{c_s^2}{v}\frac{\partial v}{\partial r} - \frac{GM}{r^2}$$

$$v\frac{\partial v}{\partial r} - \frac{c_s^2}{v}\frac{\partial v}{\partial r} = \frac{2c_s^2}{r} - \frac{GM}{r^2}$$

$$\frac{\partial v}{\partial r}\left(v - \frac{c_s^2}{v}\right) = \frac{2c_s^2}{r} - \frac{GM}{r^2}$$

$$\therefore \frac{1}{v}\frac{dv}{dr}(v^2 - c_s^2) = \frac{2c_s^2}{r} - \frac{GM}{r^2}$$

Qn 2 c> On Notebook

Qn 2 d>

$$\dot{M} = 4\pi r^2 \rho v$$

$$\rho = \frac{\dot{M}}{4\pi r^2 \rho}\,\frac{1}{v}$$

$$\rho = \frac{4\pi r_s^2 c_s\cdot\rho_s}{4\pi r^2 v}$$

$$\rho = \rho_s\left(\frac{r_s}{r}\right)^2\cdot\frac{c_s}{v}$$

Simillary

$$\rho = \rho_0\left(\frac{r_0}{r}\right)\frac{v_0}{v}$$

From Bernoulli's Eqn.

$$\frac{1}{2}\left(\frac{\dot{M}}{4\pi r^2\rho}\right)^2 + c_s^2\ln\rho - \frac{GM}{r} = \text{Const}$$

$$\frac{1}{2}\left(\frac{r_0^2}{r^2}\right)^2\left(\frac{\rho_0}{\rho}\right)^2\left(\frac{v_0}{c_s}\right)^2 + \ln\rho - \frac{2r_s}{r} = C$$

where at $r = r_0$ $\rho = \rho_0$ and $v = v_0$

$$\therefore \rho e^{\frac{1}{2}\left(\frac{r_0^2\rho_0 v_0}{r^2\rho v}\right)^2} = Ce^{\frac{2r_s}{r}}$$

$$\therefore \rho_0 e^{\frac{1}{2}\left(\frac{v_0}{c_s}\right)^2} = Ce^{\frac{2r_s}{r_0}}$$

$$C = \rho_0 e^{-\frac{2r_s}{r_0} + \frac{1}{2}\left(\frac{v_0}{c_s}\right)^2}$$

$$\therefore \rho e^{\frac{1}{2}\left(\frac{r_0^2\rho_0 v_0}{r^2\rho v}\right)} = \rho_0 e^{-\frac{2r_s}{r_0} + \frac{1}{2}\left(\frac{v_0}{c_s}\right)^2}\cdot e^{\frac{2r_s}{r}}$$

$$\frac{\rho}{\rho_0}e^{\frac{1}{2}\left(\frac{r_0^2\rho_0 v_0}{r^2\rho c_s}\right)^2} = e^{-\frac{2r_s}{r_0}\left(1-\frac{r_0}{r}\right) + \frac{1}{2}\left(\frac{v_0}{c_s}\right)^2}$$

Qn 2 e>

$\tau_s \rightarrow$ Time to get to a parker wind.

from

$$a = \frac{\Delta v}{t}$$

$$t = \Delta v\cdot\frac{1}{a}$$

Consider out-flow

acceleration due to pressure force will be greater

To accelerate to c_s

$$\Delta v = c_s - 0 = c_s$$

$$a \sim \frac{1}{\rho}\frac{P}{H} \quad \text{where } H = \underbrace{\frac{r^2c_s^2}{GM_p}}_{\text{from Hydrostatic Eqn}}$$

$$H = \frac{r^2}{2r_s} = \frac{r_s}{2} \text{ at } r = r_s$$

$$\therefore \tau_s \sim \frac{c_s}{\frac{1}{\rho}\cdot P\cdot\frac{2}{r_s}} \sim \frac{c_s}{\frac{1}{\rho}\cdot\rho c_s^2\cdot\frac{2}{r_s}}$$

$$\therefore \tau_s \sim \frac{r_s}{c_s} \quad (\text{O.O.M})$$

$$\tau_s \sim \frac{r_s}{c_s} = \frac{GM_p}{2c_s^3}$$

$$c_s = \sqrt{\frac{KT}{\mu}} =$$

$$M_p = 6.4\times10^8\,g$$

Actuall calculation in Notebook

from Notebook

$$\tau_s = 2.34 \text{ hrs}$$

τ_s is with in a factor of 2 of the simulation.