ASTR 204 PS1.

Refer to fig at End.

Qn 1 a)

Change in momentum = impulse.

For a test particle with mass m

$$m\,\Delta v = F \cdot \Delta t$$

Assuming $\Delta t \sim \frac{2b}{v_{rel}}$.

$$m\,\Delta v = \frac{GMm}{b^2} \cdot \frac{2b}{v_{rel}}$$

$$\nabla v = \frac{2GM}{b} \cdot \frac{1}{v_{rel}}.$$

for $\boxed{\nabla v = v_{rel}}$

$$\cancel{b = \left[\frac{v_{esc}}{\ }\right]}$$

$$v_{rel} = \frac{2GM}{b}\,\frac{1}{v_{rel}}$$

$$b = \frac{2GM}{v_{rel}^2}$$

$$b = \left[\frac{v_{esc}}{v_{rel}}\right]^2 \cdot R \quad \text{where } v_{esc} = \sqrt{\frac{2GM}{R}}.$$

1 (b) From energy Conservation Principle

$$KE_1 + U_1 = K.E_2 + U_2 \quad \text{---(i)}$$

Taking point 1 as a reference point

let $U_1 = 0$

$$K.E_1 = \frac{1}{2} m v_{rel}^2 \quad \text{---(ii)}$$

If it grazes mass m at Origin

$$U_2 = -\frac{GMm}{R} \quad \text{---(iii)}$$

$$K.E_2 = \frac{1}{2} m v_2^2 \quad \text{---(iv)}$$

but $L = mbv_{rel} = mRv_2$ --- Conservation of Angular momentum

$$\therefore v_2 = \frac{b}{R} v_{rel} \quad \text{---(v)}$$

pluging (ii), (iii), (iv) and (v) into (i) yields.

$$\frac{1}{2} m v_{rel}^2 + 0 = \frac{1}{2} m \frac{b^2}{R^2} v_{rel}^2 - \frac{GMm}{R}$$

$$v_{rel}^2 = \frac{b^2}{R^2} v_{rel}^2 - \frac{2GM}{R}$$

$$v_{rel}^2 = \frac{b^2}{R^2} v_{rel}^2 - v_{esc}^2$$

$$\therefore b = R\sqrt{1 + \left(\frac{v_{esc}}{v_{rel}}\right)^2}$$

Fig