# Problem Set 2.

## Question 1

a)
$$P = P_0 e^{-z/H}$$

$$\frac{dP}{dz} = \frac{-1}{H} P_0 e^{-z/H}$$

$$\left|\frac{dP}{dz}\right| = \frac{P}{H}$$

b) For a plane

$$\frac{1}{\rho}\frac{dp}{dz} = -g$$

Simillary

For a spherical surface of mass m and Radius R

$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM}{r^2}$$

$$\frac{dp}{dr} = \frac{-P}{c_s^2}\frac{GM}{r^2} \qquad \because \rho = \frac{P}{c_s^2}$$

$$\int_{p_0}^{p}\frac{dp}{p} = -\gamma\int_{R}^{r}\frac{dr}{r^2} \qquad \text{where } \gamma = \frac{GM}{c_s^2}$$

$$\ln\left(\frac{P}{P_0}\right) = \gamma\left[\frac{1}{r} - \frac{1}{R}\right]$$

but $H_0 = \dfrac{c_s^2 R^2}{GM}$ $\therefore \gamma = \dfrac{R^2}{H_0}$

$$\therefore \ln\left(\frac{P}{P_0}\right) = \frac{R^2}{H_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{R}{r} - 1\right)$$

$$\therefore P = P_0 e^{R/H_0\left(\frac{R}{r} - 1\right)}$$

1 c) from $\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{R}{r} - 1\right)$

$$z \equiv r - R \ll R.$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{R}{R+z} - 1\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{-z}{R+z}\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{-z/R}{1 + z/R}\right)$$

Since $z \ll R$

Consider

$$f(x) = \frac{-x}{1+x}$$

By Taylor Expansion about $x = 0$.

$$f(x) = f(0) + f'(0)x$$

$$= 0 + (-1)x = -x$$

$$\therefore \ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{-z}{R}\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{-z}{H_0}$$

$$P = P_0 e^{-z/H_0}$$

d) from

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}\left(\frac{R}{r} - 1\right)$$

at $r \to \infty$

$$\ln\left(\frac{P_\infty}{P_0}\right) = \lim_{r \to \infty} R/H_0\left(\frac{R}{r} - 1\right)$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{R}{H_0}(-1)$$

$$\therefore P = P_0 e^{-R/H_0}$$

Qn 2, 3 and 4 Solutions in Notebook