Signal To Noise

Often abbreviated as S/N or SN

Assume:
- photon counting detector
- not the same as integrating energy of arriving photons!

- sources of noise are independent, so they add in quadrature:

$$N_{total} = \sqrt{N_1^2 + N_2^2 + N_3^2 + ...}$$

In the optical and shorter wavelengths, photon arrival obeys Poisson statistics

Probability of observing n photons in time t: $\mathrm{Prob(n)} = \dfrac{(\lambda t)^n}{n!} e^{-\lambda t}$

For photon arrival rate λ, which is related to
the flux from the object and the sensitivity of the system you are observing with.

Probability of observing n photons in time t: Prob(n) = $\frac{(\lambda t)^n}{n!} e^{-\lambda t}$

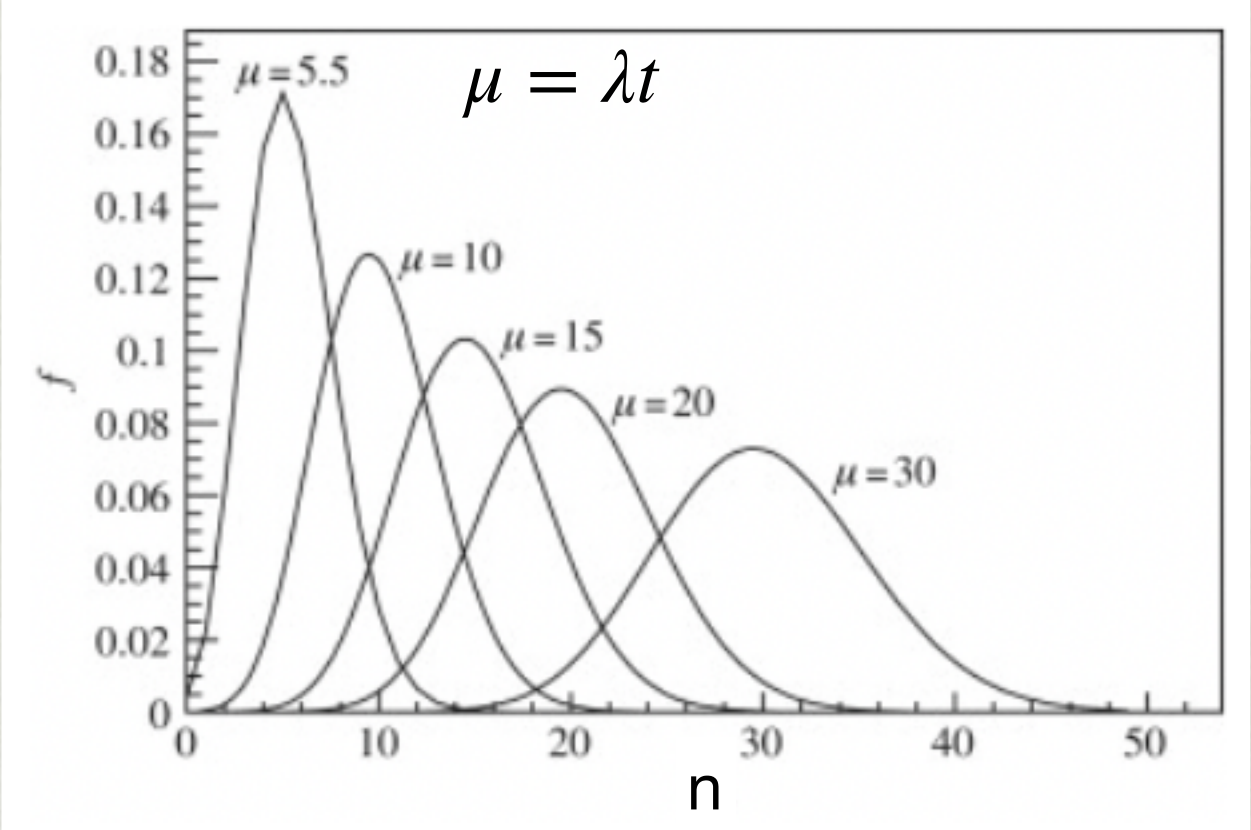
For photon arrival rate λ, which is related to the luminosity of the object and the sensitivity of the system you are observing with.

Even if you know λ perfectly, the distribution of the number of photons that arrive in time t will follow the Poisson distribution

How to reduce this uncertainty?

Measure N photons collected in many pixels to measure a uniform background, like the sky

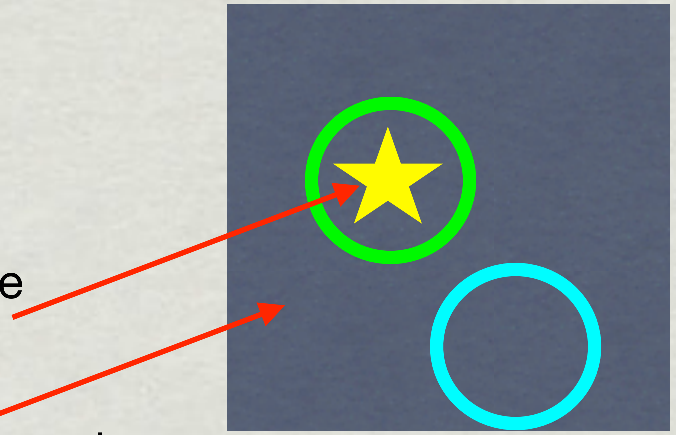
Measure N photons in a pixel in many exposures of your source, or use a longer exposure time so N is larger



Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN^2_{e-} N_{pix}}}$$



$S_\gamma t$ = your signal, the number of photons from your source

Written as the product of the rate of photon arrival from the source S_γ in counts/second and the exposure time **t** you spent collecting the photons

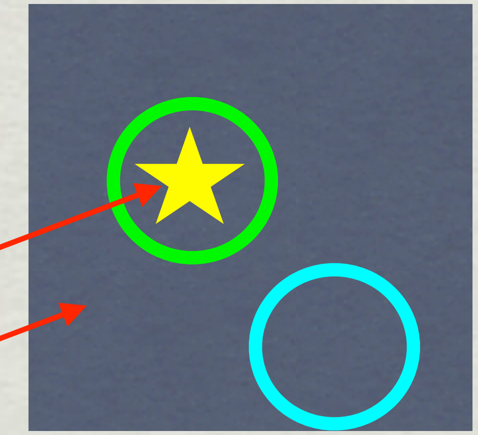
Denominator is the “noise”, the uncertainty in your measurement

- quadrature sum of the random noise processes that matters most for optical and NIR astronomy data. Adjust for radio, submm, other wavelengths
- does not include systematics, which can easily dominate any measurement error or uncertainty
    - examples: unknown gain, unstable bias or background,…

Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN_{e-}^2 N_{pix}}}$$



$S_\gamma t$ = your signal, N, the number of photons from your source

$B_\gamma t A_\Omega$ = number of photons from the background ***in the source region***

$B_\gamma t$ = number of photons from the sky detected per square arcsecond in that region. Rate of photon arrival from the sky per square arcsecond B_γ times exposure time ***t.***

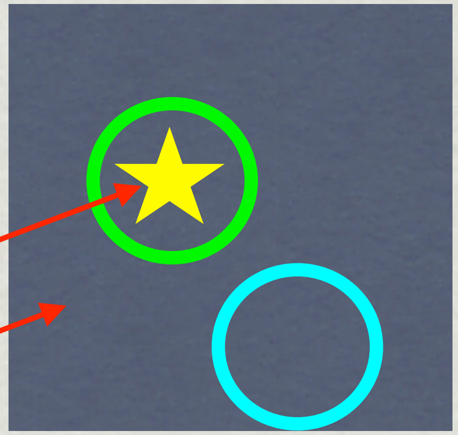
This expression for signal to noise assumes you know B_γ perfectly

A_Ω = area of source region, in square arcseconds (same sky units as B_γ).

Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN^2_{e-} N_{pix}}}$$



Source

Region of image dominated by background

N total photons collected in the green circle is source plus sky plus dark current

$S_\gamma t$ = your signal, the number of photons from your source, N_S

Written as the product of the rate of photon arrival from the source S_γ and the exposure time **t** you spent collecting the photons

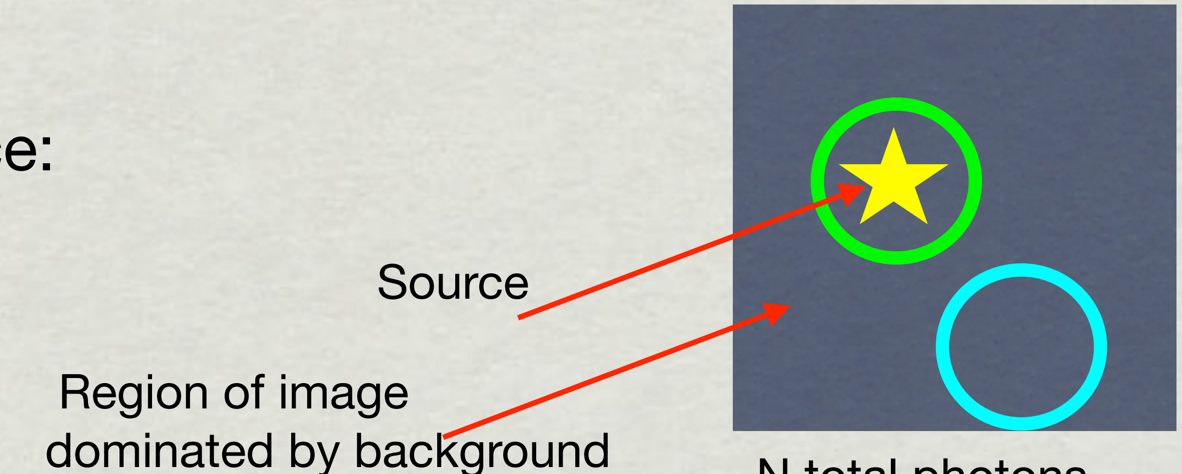
You measure some number of counts in your source region N_{total} by adding up the counts in the green circle in the cartoon

Subtract an estimate of the background contribution by adding up the counts in a background region, the cyan circle in this cartoon.
Background estimate is $N_B = B_\gamma t A_\Omega$

Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN_{e-}^2 N_{pix}}}$$



N total photons collected in the green circle is source plus sky plus dark current

Propagation of the uncertainties: assume you only need to include the Poisson uncertainty in the number of photons you collect in your source region

$S_\gamma t + B_\gamma t A_\Omega =$ N_{total} the total number of photons collected in the source region

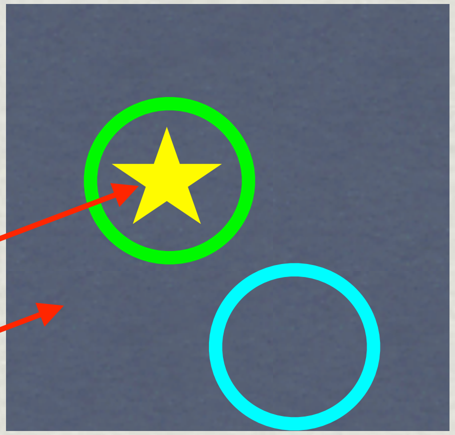
$\sqrt{S_\gamma t + B_\gamma t A_\Omega}$ is the Poisson uncertainty on N_{total}

That assumes you measure B_γ over a large enough area there is no additional uncertainty in your background estimate to include in the uncertainty on $S_\gamma t$. To make this assumption valid, you should use as large a region as you can to estimate the background. We will explore that in a homework and discuss.

Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN_{e-}^2 N_{pix}}}$$



N total photons collected in the green circle is source plus sky plus dark current

Some detectors generate their own background.

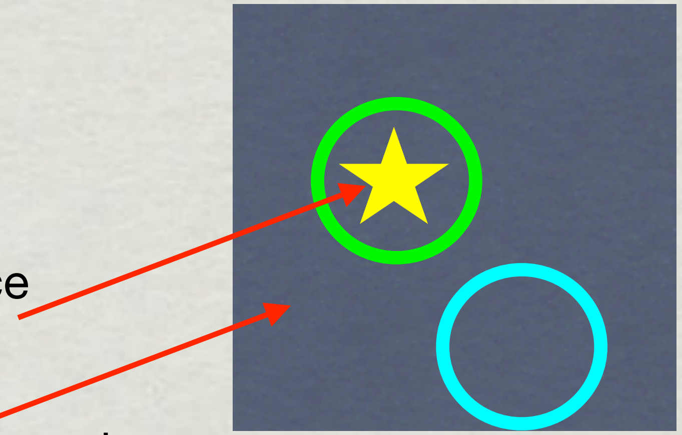
“dark current” is electrons that get a boost from thermal energy and are collected just like electrons that interact with astronomical photons

Same kind of measurement considerations apply: try to measure detector background rate well enough that you can neglect that source of uncertainty - impossible if that detector backgroud is not stable — beware!

Signal To Noise

Signal to Noise ratio for an unresolved source:

$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN^2_{e-} N_{pix}}}$$



N total photons collected in the green circle is source plus sky plus dark current

Readnoise RN: uncertainty in the measurement of the signal in any pixel

- in photon counting detectors, units are electrons
- quantifies detector's (like a CCD or an IR detector) imperfect ability to measure the signal in any pixel
- NOT the same thing as uncertainty due to Poisson arrival statistics

- RN is per pixel. It contributes the quadrature sum of the contribution per pixel in your source region, assuming RN is the same for every pixel:

$$\sqrt{RN_1^2 + RN_2^2 + RN_3^2 + \ldots + RN_N^2} = \sqrt{N_{pix} \times RN^2}$$

Signal To Noise

Signal to Noise ratio for an unresolved source:
$$\frac{S_\gamma t}{\sqrt{S_\gamma t + B_\gamma t A_\Omega + RN^2_{e-} N_{pix}}}$$

Ways to improve SN:

Lower readnoise
- want source to occupy minimum number of pixels: fewer reads, more flux/pixel
- “minimum number of pixels” can only go so far
- this is what “binning” is for on detectors that can do it. Fewer reads per binned pixel
- read slower

Lower the dark current
- why you always hear about “filling dewars” and cryo-coolers

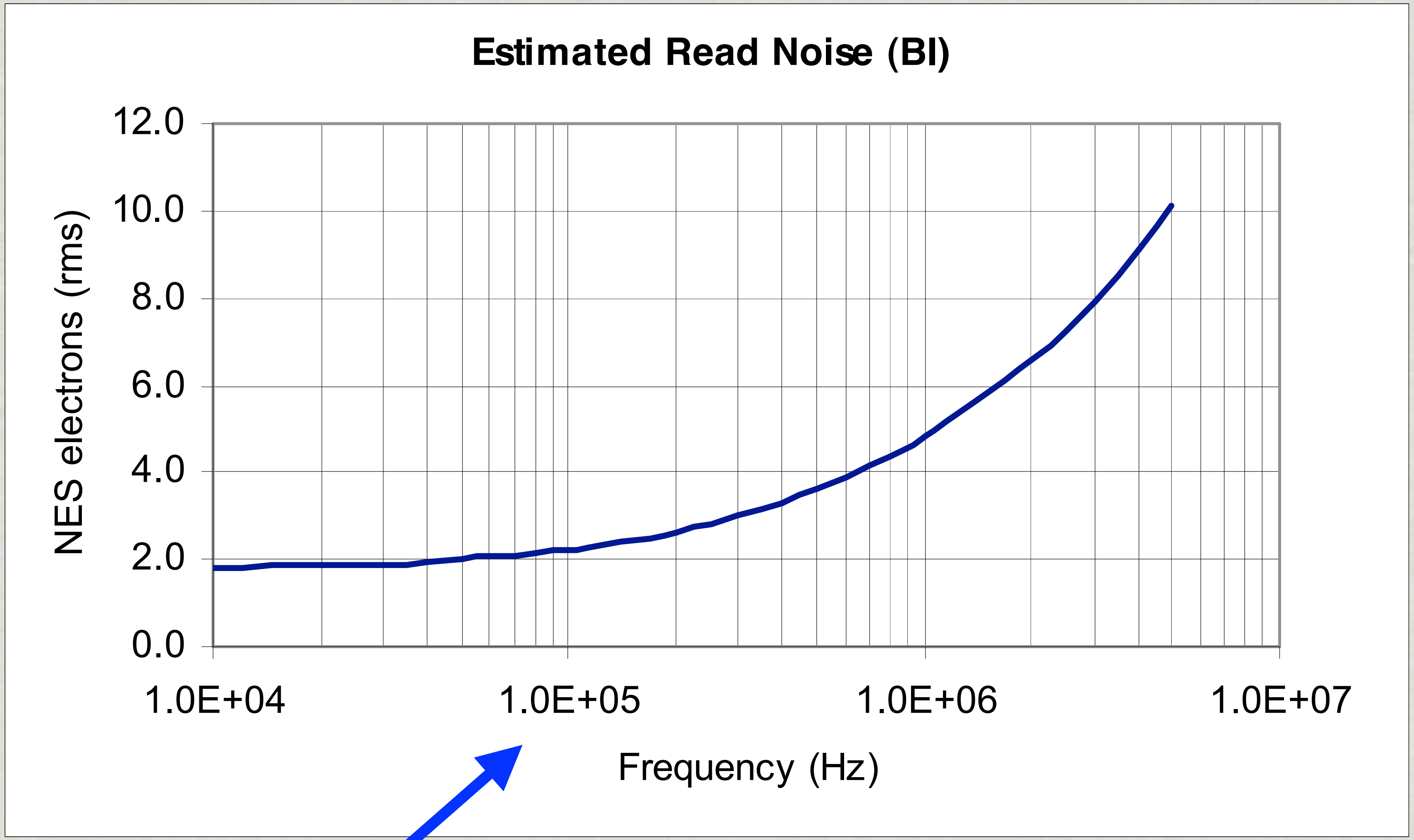
Read the detector slower (see next slide)

Reduce the sky background
- smaller A_Ω using adaptive optics
- cool and baffle your instrument, go to space

Improve efficiency: collect more of the photons you get from the telescope
- anti-reflection coatings, better materials

Increase exposure time, take more exposures!

CCD Readout, Double Correlated Sampling



readout time increases
(lower pixel rate)

100 kHz = 1e5 Hz: time to read out 2048 x 2048 pixels = 42 seconds
(or 1 4k x 4k CCD with 4 amplifiers)

One way to lower readnoise: average over current fluctuations in the output amplifier (and any other random noise)
Penalty: increased time to read each pixel to have time to average

Dark Current

\# of electrons in conduction band set by thermal distribution, ~ $\exp \frac{-E_{gap}}{kT}$

Density of states: at T=0 all electrons in states below the Fermi energy
no electrons at higher energy

At larger temperatures, e- start to occupy nearby higher energy states
At high enough temperatures, can get into conduction band

Density of e- in conduction band of Si at 300K: 1.38×10^{13} cm^{-3}

at 77K (conveniently, the temperature of liquid nitrogen): 1.8×10^{-18} cm^{-3}

This is not quite true for real Si, with impurities, but you get the idea. Cool your detector!
IR detectors with smaller bandgaps have even bigger sensitivity to thermal e
-> go colder!