AY 260 Winter 2025
Homework 1, Signal to Noise

There are seven problems in total, and five questions that ask you to comment on your results.

Part 1 due Thursday, Jan 16 by class time, 3:20 pm.
Parts 2-4 due before class on Thursday, Jan 23.

Part 1

These parameters will be constant for all the calculations in Part 1, unless specified otherwise:

- exposure time: 600 seconds
- pixel size: 15 microns (squares, 15 microns on a side)
- dark current rate: 3 e-/pixel/hour
- readnoise: 3 e-
- sky count rate: 150 photons/second/arcsecond^2
- count rate from the source: 10 photons/second

Your object of interest is an unresolved point source, so in a normal image it has the brightness distribution of the point spread function (PSF). For ground-based telescopes, the PSF is usually dominated by the seeing, the blur from turbulence in the atmosphere. That PSF looks something like a Gaussian distribution, with peak brightness at the location of the source. To make it possible to do quick calculations here, make the simplifying assumption that the brightness distribution of the PSF is a uniform circular disk, with a diameter that is the FWHM of the seeing. So if the PSF FWHM is given as 1.4 arcseconds, the area of the source is $\pi 1.4^2/4$ and the flux from the source is uniformly distributed over that area.

Another simplifying assumption: don't worry about fractional pixels. You don't need to account for the fact that the pixels at at the edge of our circular PSF aren't completely within the area of the PSF, so will get fewer photons than pixels closer to the center of the PSF. In other words, you can just compute the the area of the PSF in pixels and assume the flux from the source is evenly distributed over those pixels, even fractional pixels.

Question 1: If you did project your circular PSF onto the pixel array more correctly, would you expect the signal to noise of the pixels at the edge to be lower or higher than the ones in the center? Why?

Problem 1) With those preliminaries out of the way, compute the signal to noise for Scenarios 1-4 below, which are a few variations on a set of observation and instrument parameters.

Scenario 1:

- The Sloan Digital Sky Survey (SDSS) telescope is 2.5 m in diameter, and has a focal plane scale (the mapping of the sky onto the detector) of 16.5 arcseconds/mm.
- Use a point source size specified by PSF full width at half maximum (FWHM) of 1.4 arcseconds.

Scenario 2:

- Use the SDSS focal plane scale of 16.5 arcseconds/mm again
- Point source size PSF FWHM: 0.7 arcseconds

Question 2: Compare the signal to noise in Scenarios 1 and 2 to the PSF sizes. How does signal to noise scale with the size (diameter) of the PSF?

Scenario 3:

- The Keck telescope is 10 m in diameter and has a focal plane scale of 1.375 arcseconds/mm. (There are two Keck telescopes, both alike, you are using just one.)
- Point source size PSF FWHM: 1.4 arcseconds

Scenario 4: Now a more complete version of Scenario 3. The detector parameters do not change. But this time, the sky and source counts are scaled correctly by the increase in telescope collecting area from the SDSS 2.5 m to the Keck 10 m.

To be clear: these homework problems specify the counts/second actually collected by the detector. Those numbers take into account the size of the telescope, the efficiency of the instrument and the detector: for example, how reflective are the mirrors? Is dust on the optics absorbing light? etc. When I scale the detected counts/second to different telescope sizes, the luminosity of the source and the surface brightness of the sky in magnitudes/arcsec^2 are not changing. What changes is the number of photons that are emitted by the sky and the source that are recorded as e- in the detector. Take a look at the Observables section from the 1st set of lecture slides if you want to refer back to definitions. We will do the whole process of going from source magnitude and sky brightness to detected counts/sec later in the quarter.

- Focal plane scale 1.375 arcsec/mm
- Point source size PSF FWHM: 1.4 arcseconds
- sky count rate: 2400 photons/second/arcsecond^2
- count rate from the source: 160 photons/second

Question 3: The APF (Automated Planet Finder) telescope on Mt. Hamilton is 2.4 m in diameter and has a focal plane scale of 5.73 arcsec/mm. Since it is the same diameter as the SDSS telescope (at least for the purposes of Astro 260 homework), the source and background count rates are the same. If you used the APF in scenarios 1 and 2 instead of the SDSS telescope, how would you expect the signal to noise to change? You don't have to compute the signal to noise for this case, but you must justify your answer.

Part 2: Fractional contribution of each noise source to the signal to noise.

- pixel size 15 microns (squares, 15 microns on a side)
- dark current rate 3 e-/pixel/hour
- readnoise 3 e-
- sky count rate: 150 photons/second/arcsecond^2
- PSF FWHM 1.4
- focal plane scale 16.5 arcsec/mm

Problem 2) Use a count rate from the source of 10 photons/second. Vary the exposure time from 0 to 60 seconds, and plot the fractional contribution of each term in the noise (source, background, dark current and readnoise) as a function of exposure time.

Problem 3) Keep the exposure time fixed at 10 seconds. Vary the photon rate from the source from 0 to 500 photons/second. Plot the fractional contribution from each term in the noise.

Part 3: The readnoise limit and why you want to avoid it.

Observation parameters:

- pixel size 15 microns (squares, 15 microns on a side)
- dark current rate 3 e-/pixel/hour
- readnoise 3 e-
- sky count rate: 150 photons/second/arcsecond^2.
- Point source size PSF FWHM: 1.4 arcseconds
- focal plane scale 16.5 arcsec/mm
- sky count rate: 0 photons/second/arcsecond^2

Problem 4) Vary the exposure time from 10 to 250 seconds using steps no bigger than 1 second. For each exposure time, compute the signal to noise. Plot the fractional contribution of each term in the noise. Plot the signal to noise vs. exposure time. Then compute the signal to noise you would get by coadding two exposures at each value of the exposure time. Plot the results.

Problem 5) Now double the exposure times at each step, recompute the signal to noise and plot. Make a plot that compares signal to noise vs. exposure time for the doubled exposure time and the coadds of two exposures at the original (factor of 2 shorter) exposure time. Is doubling the exposure time equivalent to coadding two exposures? The comparison is easiest if you plot the ratio of signal to noise for the coadds to the signal to noise of the doubled exposure time.

Problem 6) Make the same comparison, this time with the original sky count rate of 150 photons/second/arcsec^2.

Question 4: What does all this tell you about the problems with small exposure times and low backgrounds, and how to avoid those problems?

Part 4: Estimating the sky background.

Observation parameters:

- exposure time 600 seconds
- pixel size 15 microns (squares, 15 microns on a side)
- dark current rate 3 e-/pixel/hour
- readnoise 3 e-
- sky count rate: 150 photons/second/arcsecond^2
- count rate from the source: 10 photons/second
- Point source size PSF FWHM: 1.4 arcseconds
- focal plane scale 16.5 arcsec/mm
- Your object is a point source, and you can make the same simplifying assumption that the PSF is a circular disk with the diameter of the FWHM of the image.

Now add a noise term to account for the uncertainty in how well you estimate the background count rate. Remember this is different from the Poisson uncertainty in the background counts in your source aperture if you know the background count rate perfectly. Assume you use the same exposure time to measure the background; perhaps you are using the same image.

Problem 7) Compute that noise term as a function of the area you use to estimate the background rate. Use a minimum aperture about the size of a single pixel, and the largest aperture should be 10 times the area of your source. Plot the fractional contribution of all the noise terms as a function of the background aperture size. Then plot signal to noise as a function of background aperture size.

Question 5: What would you suggest as a good rule of thumb for measuring the sky background?